Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/141-4.7.7-Trig-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [950]. This is test number [141].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.37 (944)	0.63 (6)
Mathematica	98.74 (938)	1.26 (12)
Fricas	95.79 (910)	4.21 (40)
Maple	95.58 (908)	4.42 (42)
Giac	76.32 (725)	23.68 (225)
Mupad	73.68 (700)	26.32 (250)
Maxima	68.95 (655)	31.05 (295)
Sympy	44.53 (423)	55.47 (527)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

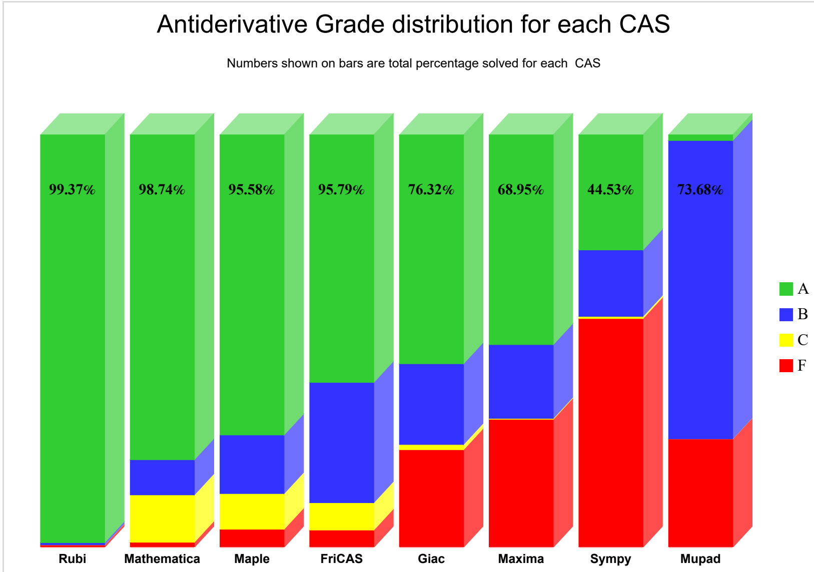
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

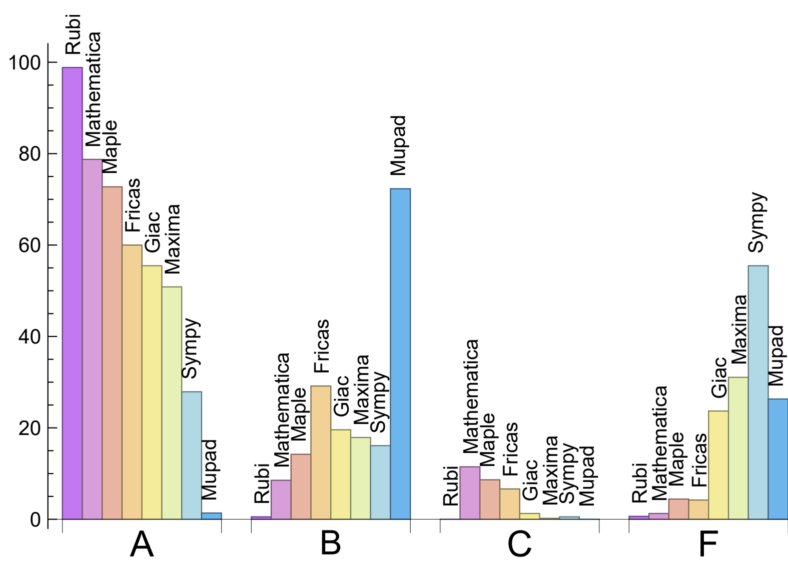
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.84	0.53	0.00	0.63
Mathematica	78.74	8.53	11.47	1.26
Maple	72.74	14.21	8.63	4.42
Fricas	60.00	29.16	6.63	4.21
Giac	55.47	19.58	1.26	23.68
Maxima	50.84	17.89	0.21	31.05
Sympy	27.89	16.11	0.53	55.47
Mupad	N/A	72.32	0.00	26.32

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00 %	0.00 %	0.00 %
Mathematica	12	66.67 %	33.33 %	0.00 %
Maple	42	100.00 %	0.00 %	0.00 %
Fricas	40	45.00 %	0.00 %	55.00 %
Giac	225	78.67 %	16.00 %	5.33 %
Maxima	295	65.42 %	1.36 %	33.22 %
Sympy	527	65.84 %	28.08 %	6.07 %
Mupad	250	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

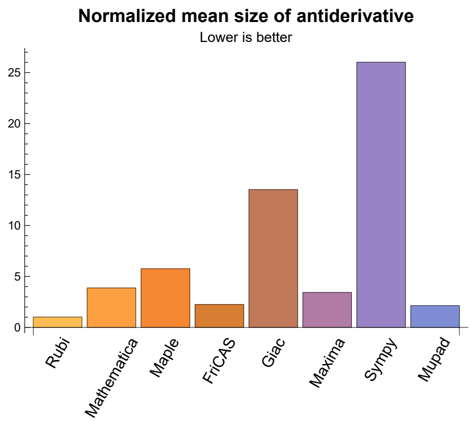
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

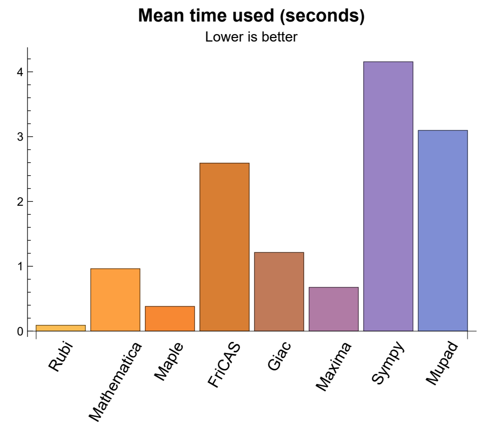
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	76.23	1.01	43.50	1.00
Mathematica	0.96	366.68	3.87	42.00	1.00
Maple	0.38	1148.63	5.76	40.50	1.13
Maxima	0.67	203.85	3.43	29.00	1.00
Fricas	2.59	251.81	2.25	50.50	1.40
Sympy	4.16	476.99	26.02	36.00	1.37
Giac	1.21	835.26	13.52	39.00	1.11
Mupad	3.10	146.31	2.13	29.00	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
25
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {37, 38, 87, 89, 107, 109, 160, 163, 240, 242, 244, 245, 246, 403, 407, 408, 409, 410, 411, 412, 414, 415, 416, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 556, 557, 558, 559, 560, 561, 588, 597, 630, 631, 859}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

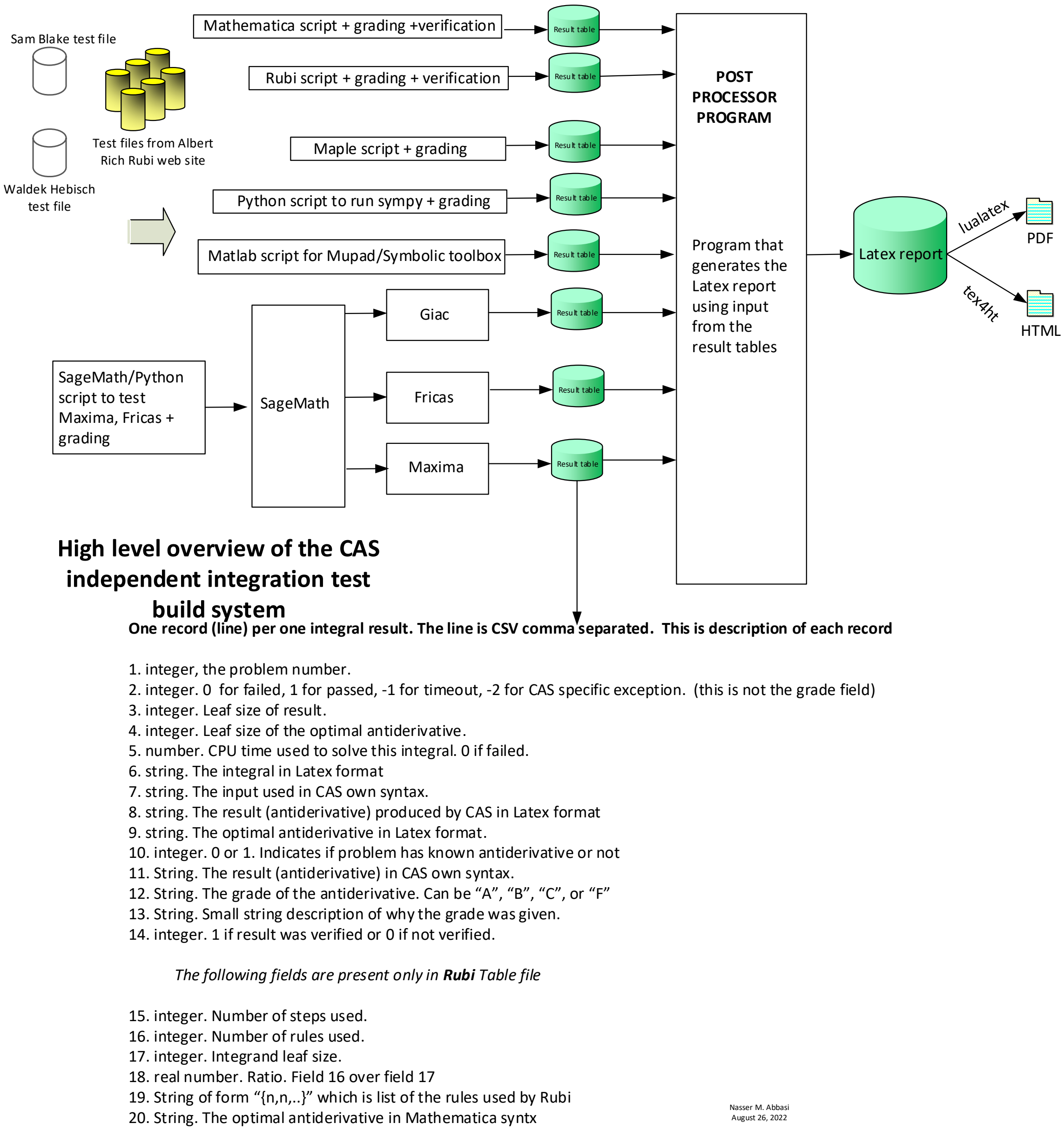
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 555, 759, 858, 860, 912 }

C grade: { }

F grade: { 796, 859, 914, 915, 931, 933 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 115, 116, 117, 119, 121, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 268, 270, 272, 273, 275, 276, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 357, 358, 359, 360, 363, 364, 365, 367, 368, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 385, 386, 388, 389, 390, 392, 393, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 423, 424, 425, 426, 443, 444, 445, 446, 447, 458, 459, 460, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 515, 516, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 642, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 782, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 804, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 833, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 946, 947, 948, 949, 950 }

B grade: { 35, 84, 90, 94, 106, 108, 110, 114, 123, 127, 160, 163, 185, 189, 263, 267, 269, 271, 274, 277, 278, 280, 281, 287, 297, 306, 310, 312, 325, 329, 331, 341, 343, 361, 362, 366, 369, 370, 378, 380, 384, 387, 391, 394, 402, 461, 475, 497, 548, 549, 581, 638, 640, 650, 654, 673, 677, 691, 705, 709, 710, 711, 712, 713, 728, 759, 760, 781, 784, 802, 803, 805, 806, 807, 826, 834, 861, 885, 904, 927, 945 }

C grade: { 31, 34, 36, 37, 38, 46, 51, 52, 62, 64, 65, 77, 78, 85, 87, 89, 92, 105, 107, 109, 112, 118, 120, 125, 174, 175, 176, 182, 228, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 351, 354, 355, 356, 379, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 427, 428, 429, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 455, 462, 463, 464, 465, 466, 469, 503, 510, 511, 512, 513, 514, 517, 556, 557, 558, 559, 560, 561, 588, 597, 657, 910, 912 }

F grade: { 435, 436, 441, 442, 453, 454, 456, 457, 467, 468, 470, 471 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 150, 151, 152, 159, 162, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 387, 388, 389, 390, 392, 395, 396, 397, 398, 399, 400, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 436, 437, 438, 442, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 490, 491, 492, 496, 499, 500, 503, 504, 505, 506, 509, 510, 511, 512, 513, 514, 516, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 538, 539, 540, 541, 543, 544, 545, 546, 548, 549, 550, 551, 553, 554, 562, 563, 567, 568, 569, 570, 571, 572, 576, 582, 583, 584, 590, 593, 599, 602, 603, 604, 605, 606, 611, 612, 613, 614, 637, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 861, 862, 863, 866, 867, 868, 869, 870, 873, 874, 875, 876, 877, 879, 880, 881, 882, 883, 884, 886, 887, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 913, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 944, 945, 946, 947, 948, 949 }

B grade: { 76, 78, 145, 146, 147, 148, 149, 160, 161, 163, 164, 193, 194, 195, 196, 197, 198, 205, 219, 230, 249, 250, 251, 255, 271, 273, 274, 275, 293, 294, 295, 333, 334, 335, 336, 337, 338, 339, 340, 355, 361, 362, 384, 386, 391, 393, 394, 401, 402, 410, 411, 412, 413, 414, 415, 416, 433, 434, 435, 439, 440, 441, 497, 498, 501, 502, 507, 508, 517, 525, 532, 537, 542, 547, 552, 555, 556, 557, 558, 559, 560, 561, 564, 565, 573, 574, 575, 577, 578, 579, 580, 581, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 670, 722, 729, 759, 793, 795, 834, 836, 864, 871, 872, 885, 888, 891, 904, 926, 943, 950 }

C grade: { 34, 139, 140, 141, 142, 153, 154, 155, 156, 157, 158, 199, 200, 319, 320, 321, 403, 404, 405, 406, 407, 408, 409, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 487, 488, 489, 493, 494, 495, 515, 588, 589, 591, 597, 598, 600, 633, 634, 635, 636, 709, 710, 711, 712, 713, 741, 742, 743, 744, 745, 746, 747, 748, 796, 809, 859, 860, 878, 912, 914, 927, 933 }

F grade: { 39, 40, 41, 53, 54, 55, 60, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 318, 566, 585, 586, 587, 592, 594, 595, 596, 601, 657, 865, 918 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 121, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 159, 162, 180, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 274, 276, 277, 279, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 322, 323, 324, 325, 327, 329, 331, 333, 341, 342, 343, 344, 345, 346, 347, 348, 350, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 504, 505, 510, 511, 512, 514, 515, 516, 517, 518, 519, 522, 523, 528, 567, 568, 569, 582, 583, 584, 590, 599, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 745, 746, 749, 751, 753, 754, 755, 756, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 798, 799, 800, 801, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 843, 844, 845, 852, 854, 857, 862, 863, 864, 867, 868, 869, 870, 871, 872, 873, 880, 881, 882, 883, 884, 885, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 908, 909, 910, 911, 913, 914, 916, 917, 920, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 943, 945, 946, 948, 950 }

B grade: { 61, 64, 65, 74, 80, 81, 82, 85, 86, 90, 91, 92, 105, 110, 111, 112, 116, 117, 122, 123, 124, 125, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 189, 190, 197, 198, 219, 228, 230, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 268, 270, 272, 273, 275, 278, 280, 281, 282, 288, 290, 292, 293, 294, 295, 307, 309, 311, 313, 314, 315, 316, 317, 326, 328, 330, 332, 334, 335, 336, 349, 351, 352, 353, 354, 377, 379, 384, 385, 386, 387, 391, 392, 393, 394, 444, 513, 529, 530, 531, 532, 533, 534, 589, 591, 593, 598, 600, 602, 607, 608, 609, 610, 615, 616, 654, 657, 670, 715, 722, 728, 750, 752, 757, 761, 770, 771, 796, 797, 804, 805, 810, 827, 829, 836, 841, 842, 850, 851, 853, 855, 856, 858, 859, 860, 866, 874, 876, 877, 878, 879, 886, 887, 888, 904, 915, 918, 919, 927, 928, 930, 933, 944, 947, 949 }

C grade: { 907, 921 }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 63, 75, 76, 77, 78, 79, 83, 84, 87, 88, 89, 93, 94, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 126, 127, 153, 154, 155, 156, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 199, 200, 205, 206, 210, 211, 215, 216, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 269, 271, 289, 291, 318, 319, 320, 321, 337, 338, 339, 340, 360, 361, 362, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 501, 502, 503, 506, 507, 508, 509, 520, 521, 524, 525, 526, 527, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 592, 594, 595, 596, 597, 601, 603, 604, 605, 606, 611, 612, 613, 614, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 743, 744, 747, 748, 762, 846, 847, 848, 849, 861, 865, 875, 906, 912, 922, 931, 938, 939, 942 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 42, 43, 44, 45, 48, 50, 51, 52, 56, 57, 58, 59, 61, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 86, 88, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 109, 111, 113, 117, 124, 126, 129, 130, 132, 134, 136, 138, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 227, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 273, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 313, 314, 315, 316, 317, 321, 322, 323, 324, 326, 333, 334, 335, 336, 337, 341, 342, 344, 345, 346, 348, 355, 356, 357, 358, 359, 360, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 417, 418, 419, 423, 424, 425, 426, 430, 431, 432, 433, 437, 438, 439, 440, 444, 447, 461, 472, 473, 474, 476, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 505, 506, 507, 508, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 555, 562, 563, 567, 568, 569, 570, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 641, 642, 644, 645, 646, 647, 648, 649, 654, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 696, 700, 705, 706, 708, 715, 717, 723, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 794, 796, 799, 800, 802, 803, 804, 806, 807, 808, 809, 811, 812, 813, 814, 815, 817, 819, 820, 821, 822, 824, 827, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 866, 867, 874, 880, 881, 882, 883, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 898, 900, 901, 903, 905, 907, 908, 909, 910, 911, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 930, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 49, 62, 64, 65, 74, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 93, 94, 105, 106, 108, 110, 112, 114, 116, 118, 119, 120, 121, 122, 123, 125, 127, 128, 131, 133, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 154, 156, 159, 160, 161, 162, 163, 164, 195, 196, 197, 198, 214, 215, 216, 219, 226, 228, 229, 230, 231, 267, 269, 271, 272, 274, 281, 283, 287, 289, 291, 304, 306, 308, 312, 318, 319, 320, 325, 327, 328, 329, 330, 331, 332, 338, 339, 340, 343, 347, 349, 350, 351, 352, 353, 354, 361, 362, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 399, 400, 401, 402, 420, 421, 422, 427, 428, 429, 434, 435, 436, 441, 442, 443, 445, 446, 458, 459, 460, 475, 477, 478, 479, 480, 496, 497, 498, 502, 503, 509, 513, 521, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 564, 565, 571, 572, 579, 580, 581, 592, 593, 601, 602, 608, 616, 633, 634, 638, 639, 640, 643, 650, 651, 652, 653, 655, 657, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 690, 691, 694, 695, 697, 698, 699, 701, 702, 703, 704, 707, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 722, 728, 732, 733, 738, 752, 759, 775, 791, 793, 795, 797, 798, 801, 805, 810, 816, 818, 823, 825, 826, 829, 836, 846, 847, 859, 862, 863, 864, 868, 869, 870, 871, 872, 873, 875, 876, 877, 878, 879, 885, 888, 897, 899, 902, 904, 906, 912, 913, 924 }

C grade: { 31, 32, 46, 47, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 576, 938, 939 }

F grade: { 34, 39, 40, 41, 53, 54, 55, 60, 63, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 217, 218, 566, 573, 574, 575, 577, 578, 865, 931, 932 }

2.1.6 Sympy

A grade: { 1, 8, 15, 17, 18, 19, 22, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 62, 66, 67, 68, 70, 71, 72, 73, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 124, 136, 180, 185, 186, 187, 188, 203, 204, 225, 248, 249, 250, 251, 252, 253, 254, 255, 256, 264, 265, 266, 267, 274, 276, 277, 284, 285, 286, 287, 294, 296, 303, 304, 305, 322, 323, 324, 341, 342, 343, 344, 357, 358, 363, 365, 374, 376, 381, 383, 388, 390, 396, 397, 398, 444, 481, 510, 511, 526, 527, 528, 538, 539, 543, 544, 548, 549, 553, 554, 567, 569, 582, 589, 590, 598, 599, 644, 645, 646, 647, 648, 651, 652, 654, 655, 656, 665, 674, 675, 676, 677, 678, 679, 680, 681, 701, 702, 703, 704, 705, 706, 707, 714, 715, 718, 720, 721, 722, 723, 724, 725, 726, 728, 730, 731, 732, 735, 736, 737, 754, 755, 756, 758, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 799, 800, 801, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 823, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 852, 855, 856, 884, 887, 889, 890, 891, 892, 893, 894, 895, 900, 901, 902, 905, 907, 908, 911, 913, 916, 920, 921, 923, 924, 925, 926, 929, 932, 933, 937, 943, 944 }

B grade: { 3, 4, 5, 10, 11, 12, 24, 25, 26, 69, 80, 90, 92, 104, 110, 121, 122, 123, 125, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 146, 201, 202, 205, 206, 207, 208, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 263, 268, 270, 272, 273, 275, 278, 280, 282, 283, 293, 295, 297, 306, 325, 355, 356, 364, 366, 370, 371, 372, 373, 375, 377, 382, 384, 389, 391, 395, 399, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 499, 500, 562, 568, 637, 638, 639, 640, 641, 642, 643, 649, 653, 659, 660, 661, 666, 672, 684, 685, 686, 694, 695, 719, 727, 738, 751, 753, 759, 760, 775, 793, 798, 802, 803, 804, 805, 821, 822, 828, 836, 841, 848, 849, 854, 862, 880, 881, 882, 883, 885, 888, 909, 934, 935, 936, 940, 941, 942, 945, 946 }

C grade: { 349, 352, 512, 529, 532 }

F grade: { 2, 6, 7, 9, 13, 14, 16, 20, 21, 23, 27, 28, 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 61, 63, 64, 65, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 209, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 257, 258, 259, 260, 261, 262, 269, 271, 279, 281, 288, 289, 290, 291, 292, 298, 299, 300, 301, 302, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 350, 351, 353, 354, 359, 360, 361, 362, 367, 368, 369, 378, 379, 380, 385, 386, 387, 392, 393, 394, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 501, 502, 503, 504, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 583, 584, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 650, 657, 658, 662, 663, 664, 667, 668, 669, 670, 671, 673, 682, 683, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 708, 709, 710, 711, 712, 713, 716, 717, 729, 733, 734, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 752, 757, 761, 762, 774, 796, 797, 810, 829, 850, 851, 853, 857, 858, 859, 860, 861, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 886, 896, 897, 898, 899, 903, 904, 906, 910, 912, 914, 915, 917, 918, 919, 922, 927, 928, 930, 931, 938, 939, 947, 948, 949, 950 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 42, 43, 44, 45, 48, 49, 50, 56, 57, 58, 59, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 117, 119, 120, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 150, 151, 152, 159, 165, 166, 167, 174, 180, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 229, 231, 248, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 268, 269, 270, 272, 274, 276, 279, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 308, 310, 312, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 337, 338, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 400, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 506, 507, 512, 516, 517, 520, 522, 523, 525, 526, 527, 529, 530, 531, 532, 533, 535, 536, 540, 541, 545, 546, 550, 551, 562, 563, 567, 568, 569, 570, 571, 572, 582, 583, 584, 589, 591, 598, 600, 640, 641, 643, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 668, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 684, 685, 686, 687, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 708, 710, 711, 713, 714, 715, 716, 724, 725, 726, 727, 729, 730, 731, 735, 736, 737, 738, 739, 749, 751, 753, 754, 755, 756, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 798, 799, 800, 801, 802, 803, 804, 806, 807, 808, 810, 811, 812, 814, 817, 818, 819, 820, 821, 822, 823, 824, 828, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 851, 852, 853, 854, 855, 856, 857, 860, 866, 867, 880, 881, 882, 883, 884, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 916, 919, 920, 921, 922, 923, 924, 925, 926, 929, 932, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade: { 36, 37, 38, 51, 52, 64, 65, 75, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 94, 110, 112, 114, 116, 118, 121, 122, 123, 125, 127, 139, 140, 141, 142, 143, 144, 145, 146, 157, 158, 162, 171, 172, 173, 175, 176, 182, 183, 185, 196, 197, 199, 200, 219, 221, 228, 230, 249, 250, 251, 261, 262, 267, 271, 273, 275, 277, 278, 280, 282, 284, 304, 307, 309, 311, 313, 325, 336, 339, 340, 341, 342, 343, 347, 354, 362, 384, 385, 386, 387, 391, 392, 393, 394, 401, 402, 475, 478, 496, 501, 502, 508, 509, 510, 511, 513, 514, 515, 518, 519, 521, 524, 528, 534, 537, 538, 539, 542, 543, 544, 547, 548, 549, 552, 553, 554, 555, 564, 565, 590, 599, 605, 606, 637, 638, 642, 650, 667, 670, 676, 683, 707, 717, 718, 719, 720, 721, 722, 728, 732, 743, 744, 747, 748, 752, 757, 759, 775, 793, 796, 797, 805, 813, 815, 816, 825, 826, 827, 829, 842, 848, 849, 850, 861, 862, 863, 864, 885, 886, 888, 904, 910, 915, 927, 928, 930, 933 }

C grade: { 168, 169, 170, 585, 586, 587, 588, 594, 595, 596, 597, 712 }

F grade: { 31, 32, 34, 39, 40, 41, 46, 47, 53, 54, 55, 60, 63, 74, 76, 77, 78, 79, 105, 106, 107, 108, 109, 115, 147, 148, 149, 153, 154, 155, 156, 160, 161, 163, 164, 177, 178, 179, 181, 184, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 314, 315, 316, 317, 318, 319, 320, 321, 333, 334, 335, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 503, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 592, 593, 601, 602, 603, 604, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 639, 662, 663, 664, 669, 688, 689, 693, 709, 723, 733, 734, 740, 741, 742, 745, 746, 750, 809, 858, 859, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 912, 914, 917, 918, 931, 938, 939 }

2.1.8 Mupad

A grade: { 42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 155, 159, 162, 165, 166, 167, 171, 172, 173, 183, 185, 186, 187, 188, 189, 190, 191, 192, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 419, 426, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 589, 590, 598, 599, 603, 604, 605, 606, 611, 612, 613, 614, 633, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 855, 856, 857, 858, 861, 862, 863, 864, 865, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 923, 924, 925, 926, 927, 928, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

C grade: { }

F grade: { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 63, 79, 115, 148, 149, 151, 152, 153, 154, 156, 157, 158, 160, 161, 163, 164, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 184, 193, 194, 195, 196, 197, 198, 199, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 260, 261, 262, 314, 315, 316, 318, 319, 320, 321, 333, 334, 335, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 504, 505, 506, 507, 514, 515, 516, 517, 522, 523, 524, 525, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 657, 658, 662, 663, 664, 679, 687, 688, 689, 709, 710, 711, 713, 850, 851, 853, 859, 860, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 921, 922 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	32	57	35
N.S.	1	1.00	0.50	0.39	0.59	0.75	0.73	1.30	0.80
time (sec)	N/A	0.027	0.030	0.128	0.477	1.562	0.117	0.407	2.582

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	22	17	26	33	0	57	16
N.S.	1	1.00	0.50	0.39	0.59	0.75	0.00	1.30	0.36
time (sec)	N/A	0.028	0.023	0.381	0.479	1.545	0.000	0.448	2.689

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
N.S.	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.015	0.034	0.210	0.472	1.667	3.356	0.419	2.486

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
N.S.	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.015	0.045	0.131	0.469	1.796	3.364	0.414	2.380

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	246	57	35
N.S.	1	1.00	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.019	0.021	0.264	0.482	1.628	3.336	0.403	2.376

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	16	16
N.S.	1	1.00	0.46	0.35	0.33	0.90	0.00	0.33	0.33
time (sec)	N/A	0.029	0.017	0.345	0.473	1.777	0.000	0.954	2.402

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	17	16	43	0	57	16
N.S.	1	1.00	0.46	0.35	0.33	0.90	0.00	1.19	0.33
time (sec)	N/A	0.026	0.017	0.286	0.484	1.699	0.000	0.511	2.398

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	42	39	16
N.S.	1	1.00	0.37	0.28	0.90	1.23	0.70	0.65	0.27
time (sec)	N/A	0.017	0.029	0.095	0.471	2.085	0.138	0.421	2.566

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	54	74	0	39	16
N.S.	1	1.00	0.37	0.28	0.90	1.23	0.00	0.65	0.27
time (sec)	N/A	0.033	0.030	0.359	0.475	1.269	0.000	0.468	2.624

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
N.S.	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.014	0.043	0.174	0.490	1.603	5.953	0.396	2.453

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
N.S.	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.013	0.051	0.115	0.476	1.094	5.768	0.431	2.440

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	1644	39	16
N.S.	1	1.00	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.021	0.022	0.253	0.482	1.378	6.045	0.432	2.404

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
N.S.	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.031	0.019	0.314	0.481	1.672	0.000	0.950	2.402

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	34	86	0	39	16
N.S.	1	1.00	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.031	0.026	0.264	0.477	1.412	0.000	0.525	2.410

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	34	57	36
N.S.	1	1.00	0.52	0.43	0.64	0.74	0.81	1.36	0.86
time (sec)	N/A	0.026	0.024	0.079	0.482	1.604	0.124	0.392	2.536

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	18	27	31	0	57	17
N.S.	1	1.00	0.52	0.43	0.64	0.74	0.00	1.36	0.40
time (sec)	N/A	0.026	0.018	0.372	0.476	1.669	0.000	0.462	2.689

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
N.S.	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.013	0.018	0.160	0.474	1.904	0.286	0.407	2.438

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
N.S.	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.012	0.031	0.113	0.480	2.769	0.257	0.403	2.367

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	76	57	36
N.S.	1	1.00	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.018	0.017	0.246	0.479	1.445	0.305	0.428	2.367

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	17	17
N.S.	1	1.00	0.46	0.38	0.35	0.90	0.00	0.35	0.35
time (sec)	N/A	0.027	0.016	0.309	0.469	1.523	0.000	0.847	2.371

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	18	17	43	0	57	17
N.S.	1	1.00	0.46	0.38	0.35	0.90	0.00	1.19	0.35
time (sec)	N/A	0.029	0.016	0.303	0.485	2.592	0.000	0.532	2.380

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	39	39	17
N.S.	1	1.00	0.36	0.30	0.87	1.21	0.64	0.64	0.28
time (sec)	N/A	0.021	0.021	0.082	0.490	2.290	0.133	0.404	2.501

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	53	74	0	39	17
N.S.	1	1.00	0.36	0.30	0.87	1.21	0.00	0.64	0.28
time (sec)	N/A	0.031	0.026	0.381	0.492	2.529	0.000	0.452	2.713

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
N.S.	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.014	0.046	0.172	0.485	2.234	6.000	0.412	2.515

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
N.S.	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.014	0.040	0.121	0.489	2.315	5.905	0.436	2.443

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	1481	39	17
N.S.	1	1.00	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.021	0.021	0.264	0.482	2.539	5.974	0.449	2.418

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	37	17
N.S.	1	1.00	0.36	0.30	0.52	1.39	0.00	0.61	0.28
time (sec)	N/A	0.028	0.017	0.320	0.493	2.109	0.000	0.838	2.400

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	22	18	32	85	0	39	17
N.S.	1	1.00	0.36	0.30	0.52	1.39	0.00	0.64	0.28
time (sec)	N/A	0.031	0.027	0.302	0.474	1.426	0.000	0.548	2.334

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.024	0.034	0.076	0.267	2.995	0.067	0.409	0.057

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	57	48	46	85	46	46
N.S.	1	1.00	0.86	1.02	0.86	0.82	1.52	0.82	0.82
time (sec)	N/A	0.044	0.048	0.105	0.273	3.395	0.094	0.403	2.308

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	233	0	187	0	0	-1
N.S.	1	1.00	0.81	1.09	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.200	0.236	0.000	3.344	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	238	328	0	447	0	0	-1
N.S.	1	1.00	0.88	1.21	0.00	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.524	0.236	0.356	0.000	3.443	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.013	0.020	0.090	0.274	3.580	0.096	0.391	2.419

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	72	0	0	0	0	-1
N.S.	1	1.00	1.64	2.57	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.315	0.460	0.113	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	26	12	17	22	10	11	11
N.S.	1	1.00	2.17	1.00	1.42	1.83	0.83	0.92	0.92
time (sec)	N/A	0.017	0.016	0.109	0.269	2.982	0.664	0.401	2.399

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	272	142	0	139	0	630	-1
N.S.	1	1.00	2.72	1.42	0.00	1.39	0.00	6.30	-0.01
time (sec)	N/A	0.111	3.724	0.257	0.000	2.771	0.000	3.351	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	330	195	0	149	0	681	-1
N.S.	1	1.00	3.08	1.82	0.00	1.39	0.00	6.36	-0.01
time (sec)	N/A	0.127	5.015	0.129	0.000	1.887	0.000	22.575	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	417	295	0	277	0	1239	-1
N.S.	1	1.00	2.15	1.52	0.00	1.43	0.00	6.39	-0.01
time (sec)	N/A	0.215	6.419	0.151	0.000	2.500	0.000	61.518	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.066	0.487	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.051	0.207	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	0.029	0.039	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	2.739	0.149	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	9.852	0.213	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	24	23	41	24	24
N.S.	1	1.00	0.87	0.83	0.80	0.77	1.37	0.80	0.80
time (sec)	N/A	0.023	0.044	0.092	0.290	2.027	0.056	0.415	0.049

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	57	46	44	85	46	48
N.S.	1	1.00	0.91	1.02	0.82	0.79	1.52	0.82	0.86
time (sec)	N/A	0.044	0.056	0.069	0.265	1.790	0.094	0.405	0.076

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	172	231	0	189	0	0	-1
N.S.	1	1.00	0.81	1.08	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.191	0.110	0.000	1.977	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	236	326	0	449	0	0	-1
N.S.	1	1.00	0.87	1.20	0.00	1.66	0.00	0.00	-0.00
time (sec)	N/A	0.410	0.219	0.207	0.000	1.937	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.091	0.035	0.084	0.273	1.908	0.132	0.395	2.260

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	18	17	37	20	17	15
N.S.	1	1.00	1.00	0.82	0.77	1.68	0.91	0.77	0.68
time (sec)	N/A	0.123	0.042	0.147	0.479	2.026	0.192	0.439	2.316

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	16	16	15	15	19	16
N.S.	1	1.00	0.83	0.67	0.67	0.62	0.62	0.79	0.67
time (sec)	N/A	0.330	0.085	0.579	0.265	1.919	3.126	0.391	2.349

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	260	142	0	138	0	633	-1
N.S.	1	1.00	2.57	1.41	0.00	1.37	0.00	6.27	-0.01
time (sec)	N/A	0.094	3.815	0.115	0.000	2.157	0.000	4.785	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	329	195	0	144	0	683	-1
N.S.	1	1.00	3.07	1.82	0.00	1.35	0.00	6.38	-0.01
time (sec)	N/A	0.112	4.606	0.157	0.000	2.161	0.000	22.260	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.016	0.647	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.025	0.305	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.006	0.070	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	1.623	0.086	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	7.056	0.230	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	13	10	8	19
N.S.	1	1.00	1.00	0.89	0.78	1.44	1.11	0.89	2.11
time (sec)	N/A	0.006	0.009	0.023	0.268	2.046	0.064	0.406	3.179

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	12	14	12	20
N.S.	1	1.00	1.12	0.94	0.75	0.75	0.88	0.75	1.25
time (sec)	N/A	0.011	0.024	0.052	0.484	1.784	0.069	0.404	2.584

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	80	0	0	0	-1
N.S.	1	1.00	1.00	0.00	1.14	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.015	0.088	0.489	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	83	23	0	18	21
N.S.	1	1.00	0.95	0.95	4.37	1.21	0.00	0.95	1.11
time (sec)	N/A	0.015	0.411	0.223	0.309	3.556	0.000	3.560	2.511

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	16	14	28	15	22	20
N.S.	1	1.00	1.62	1.00	0.88	1.75	0.94	1.38	1.25
time (sec)	N/A	0.014	0.030	0.026	0.493	3.412	0.075	0.407	2.549

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	85	150	0	0	0	0	-1
N.S.	1	1.00	0.92	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.218	2.786	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	331	43	6257	72	0	66	29
N.S.	1	1.00	9.46	1.23	178.77	2.06	0.00	1.89	0.83
time (sec)	N/A	0.023	0.455	0.319	88.540	2.817	0.000	0.431	0.089

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	331	43	6257	72	0	66	29
N.S.	1	1.00	9.46	1.23	178.77	2.06	0.00	1.89	0.83
time (sec)	N/A	0.022	0.085	0.000	91.142	2.774	0.000	0.425	0.002

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	10	20	6	6
N.S.	1	1.00	1.00	0.47	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.006	0.007	0.067	0.294	1.831	0.121	0.411	0.027

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.005	0.010	0.125	0.273	1.682	0.126	0.418	0.029

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.006	0.011	0.117	0.319	2.191	0.117	0.401	0.029

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	26	78	29	64
N.S.	1	1.00	0.71	0.80	0.80	0.74	2.23	0.83	1.83
time (sec)	N/A	0.022	0.034	0.161	0.283	3.102	0.194	0.406	2.323

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	9	20	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.60	1.33	0.73	0.60
time (sec)	N/A	0.006	0.007	0.089	0.276	2.284	0.119	0.397	0.023

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.006	0.007	0.119	0.264	1.639	0.129	0.404	0.025

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	17	20	13	17
N.S.	1	1.00	1.00	0.82	0.76	1.00	1.18	0.76	1.00
time (sec)	N/A	0.006	0.009	0.119	0.263	2.296	0.121	0.408	0.028

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	28	24	37	29	55
N.S.	1	1.00	0.69	0.80	0.80	0.69	1.06	0.83	1.57
time (sec)	N/A	0.018	0.030	0.082	0.280	2.017	0.196	0.399	0.100

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	141	38	0	0	17
N.S.	1	1.00	1.00	0.90	7.05	1.90	0.00	0.00	0.85
time (sec)	N/A	0.015	0.011	0.211	0.502	2.744	0.000	0.000	2.394

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	21	38	0	39	0	364	26
N.S.	1	1.00	0.45	0.81	0.00	0.83	0.00	7.74	0.55
time (sec)	N/A	0.034	0.036	0.331	0.000	1.203	0.000	0.460	2.339

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	115	0	101	0	0	103
N.S.	1	1.00	0.97	1.62	0.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.070	0.068	0.475	0.000	2.472	0.000	0.000	2.562

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	248	84	0	136	0	0	107
N.S.	1	1.00	2.21	0.75	0.00	1.21	0.00	0.00	0.96
time (sec)	N/A	0.107	0.124	0.489	0.000	1.940	0.000	0.000	2.885

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	366	256	0	134	0	0	131
N.S.	1	1.00	4.11	2.88	0.00	1.51	0.00	0.00	1.47
time (sec)	N/A	0.177	0.338	0.720	0.000	2.745	0.000	0.000	3.113

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	200	0	0	0	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.114	0.050	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	37	19	19	19	10
N.S.	1	1.00	1.00	1.20	3.70	1.90	1.90	1.90	1.00
time (sec)	N/A	0.015	0.007	0.092	0.471	1.249	0.462	0.408	2.319

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	127	36	0	34	16
N.S.	1	1.00	1.00	0.85	6.35	1.80	0.00	1.70	0.80
time (sec)	N/A	0.017	0.013	0.158	0.491	2.339	0.000	0.417	2.366

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	173	52	0	50	29
N.S.	1	1.00	1.00	1.07	6.18	1.86	0.00	1.79	1.04
time (sec)	N/A	0.033	0.024	0.176	0.500	1.001	0.000	0.419	2.378

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	76	70	0	127	0	111	119
N.S.	1	1.00	0.93	0.85	0.00	1.55	0.00	1.35	1.45
time (sec)	N/A	0.131	0.144	0.306	0.000	2.116	0.000	0.479	2.609

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	99	49	0	70	0	70	37
N.S.	1	1.00	2.61	1.29	0.00	1.84	0.00	1.84	0.97
time (sec)	N/A	0.054	0.069	0.253	0.000	1.042	0.000	0.405	2.500

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	129	33	0	49	12
N.S.	1	1.00	11.60	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.011	0.211	0.177	0.495	1.959	0.000	0.424	2.289

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	81	19	0	24	15
N.S.	1	1.00	0.81	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.017	0.007	0.210	0.482	1.195	0.000	0.409	2.274

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	4852	54	0	121	0	133	112
N.S.	1	1.00	68.34	0.76	0.00	1.70	0.00	1.87	1.58
time (sec)	N/A	0.045	50.074	0.282	0.000	2.415	0.000	0.411	0.088

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	43	0	72	0	67	47
N.S.	1	1.00	0.92	0.69	0.00	1.16	0.00	1.08	0.76
time (sec)	N/A	0.050	0.064	0.311	0.000	1.095	0.000	0.407	0.535

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	627	80	0	153	0	182	118
N.S.	1	1.00	7.38	0.94	0.00	1.80	0.00	2.14	1.39
time (sec)	N/A	0.043	6.264	0.352	0.000	2.087	0.000	0.446	2.283

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	9	35	17	15	17	5
N.S.	1	1.00	5.29	1.29	5.00	2.43	2.14	2.43	0.71
time (sec)	N/A	0.008	0.006	0.092	0.484	1.624	0.616	0.414	0.109

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	15	14	125	58	76	31	17
N.S.	1	1.00	0.33	0.31	2.78	1.29	1.69	0.69	0.38
time (sec)	N/A	0.026	0.014	0.215	0.506	1.370	0.942	0.407	2.793

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	218	28	171	50	294	48	27
N.S.	1	1.00	8.38	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.017	0.280	0.254	0.498	2.603	3.614	0.413	2.433

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	84	66	0	231	0	105	217
N.S.	1	1.00	0.51	0.40	0.00	1.40	0.00	0.64	1.32
time (sec)	N/A	0.096	0.073	0.337	0.000	2.110	0.000	0.459	2.592

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	95	47	0	68	0	68	35
N.S.	1	1.00	2.64	1.31	0.00	1.89	0.00	1.89	0.97
time (sec)	N/A	0.032	0.045	0.346	0.000	2.055	0.000	0.408	2.460

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	6	9	6	8	5	6	6
N.S.	1	1.00	0.75	1.12	0.75	1.00	0.62	0.75	0.75
time (sec)	N/A	0.022	0.009	0.109	0.294	1.959	1.081	0.413	2.252

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	6	6	7	6	6
N.S.	1	1.00	1.00	1.12	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.010	0.004	0.069	0.273	1.596	1.838	0.438	0.027

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	11	6	22	6	6
N.S.	1	1.00	1.00	0.47	0.73	0.40	1.47	0.40	0.40
time (sec)	N/A	0.005	0.007	0.068	0.272	1.132	0.222	0.411	0.018

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	13	13	13	22	13	13
N.S.	1	1.00	1.00	0.76	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.006	0.006	0.139	0.274	2.062	0.116	0.402	0.026

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	14
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.82
time (sec)	N/A	0.010	0.008	0.104	0.292	1.565	0.115	0.378	0.024

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	27	25	44	29	57
N.S.	1	1.00	0.74	0.80	0.77	0.71	1.26	0.83	1.63
time (sec)	N/A	0.020	0.033	0.079	0.272	2.133	0.190	0.412	2.285

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.006	0.007	0.082	0.280	1.732	0.215	0.419	0.025

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	7	20	13	7
N.S.	1	1.00	1.00	0.82	0.76	0.41	1.18	0.76	0.41
time (sec)	N/A	0.006	0.008	0.092	0.274	1.844	0.116	0.398	0.022

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.006	0.008	0.111	0.270	2.043	0.204	0.397	0.025

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	58
N.S.	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	1.66
time (sec)	N/A	0.020	0.028	0.077	0.267	1.839	0.190	0.407	0.124

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	183	18	133	38	0	0	42
N.S.	1	1.00	9.15	0.90	6.65	1.90	0.00	0.00	2.10
time (sec)	N/A	0.017	0.117	0.075	0.503	2.387	0.000	0.000	2.358

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	38	0	0	42
N.S.	1	1.00	2.29	0.90	0.00	1.81	0.00	0.00	2.00
time (sec)	N/A	0.016	0.036	0.098	0.000	1.329	0.000	0.000	2.312

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	6196	68	0	101	0	0	295
N.S.	1	1.00	87.27	0.96	0.00	1.42	0.00	0.00	4.15
time (sec)	N/A	0.059	53.026	0.129	0.000	1.678	0.000	0.000	2.446

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	215	72	0	129	0	0	407
N.S.	1	1.00	2.56	0.86	0.00	1.54	0.00	0.00	4.85
time (sec)	N/A	0.072	0.439	0.180	0.000	2.165	0.000	0.000	2.497

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	628	104	0	134	0	0	787
N.S.	1	1.00	7.06	1.17	0.00	1.51	0.00	0.00	8.84
time (sec)	N/A	0.165	6.885	0.168	0.000	1.837	0.000	0.000	4.074

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	25	14	37	21	19	19	20
N.S.	1	1.00	2.50	1.40	3.70	2.10	1.90	1.90	2.00
time (sec)	N/A	0.014	0.013	0.108	0.273	1.857	0.510	0.415	2.336

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	36	131	39	0	39	39
N.S.	1	1.00	1.04	0.80	2.91	0.87	0.00	0.87	0.87
time (sec)	N/A	0.033	0.016	0.348	0.478	1.748	0.000	0.438	2.369

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	73	30	165	53	0	50	67
N.S.	1	1.00	2.61	1.07	5.89	1.89	0.00	1.79	2.39
time (sec)	N/A	0.033	0.082	0.460	0.495	1.588	0.000	0.408	2.349

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	133	82	0	137	0	117	611
N.S.	1	1.00	1.21	0.75	0.00	1.25	0.00	1.06	5.55
time (sec)	N/A	0.099	0.092	0.567	0.000	1.774	0.000	0.416	3.034

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	49	0	71	0	70	86
N.S.	1	1.00	2.29	1.29	0.00	1.87	0.00	1.84	2.26
time (sec)	N/A	0.049	0.067	0.649	0.000	1.836	0.000	0.417	2.435

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	179	0	0	0	0	0	-1
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.130	0.079	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	137	33	0	31	12
N.S.	1	1.00	1.00	0.87	9.13	2.20	0.00	2.07	0.80
time (sec)	N/A	0.010	0.006	0.159	0.492	1.547	0.000	0.417	0.114

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	15	13	76	53	0	31	16
N.S.	1	1.00	0.34	0.30	1.73	1.20	0.00	0.70	0.36
time (sec)	N/A	0.026	0.012	0.204	0.485	3.153	0.000	0.426	2.664

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	91	54	0	121	0	99	95
N.S.	1	1.00	1.28	0.76	0.00	1.70	0.00	1.39	1.34
time (sec)	N/A	0.038	0.027	0.291	0.000	2.397	0.000	0.447	2.272

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	84	68	0	231	0	105	217
N.S.	1	1.00	0.52	0.42	0.00	1.42	0.00	0.64	1.33
time (sec)	N/A	0.096	0.072	0.310	0.000	2.479	0.000	0.473	2.667

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	356	80	0	154	0	132	118
N.S.	1	1.00	4.19	0.94	0.00	1.81	0.00	1.55	1.39
time (sec)	N/A	0.044	0.288	0.371	0.000	2.224	0.000	0.460	2.292

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	19	21	20	21	10
N.S.	1	1.00	1.00	1.40	1.90	2.10	2.00	2.10	1.00
time (sec)	N/A	0.011	0.007	0.154	0.269	1.691	0.617	0.410	2.247

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	129	25	427	25	12
N.S.	1	1.00	1.00	1.29	9.21	1.79	30.50	1.79	0.86
time (sec)	N/A	0.012	0.007	0.154	0.491	2.182	2.090	0.394	0.025

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	21	11	35	19	15	17	5
N.S.	1	1.00	3.00	1.57	5.00	2.71	2.14	2.43	0.71
time (sec)	N/A	0.008	0.004	0.102	0.279	2.318	0.507	0.415	0.027

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	129	19	17	24	17
N.S.	1	1.00	1.00	1.62	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.017	0.007	0.234	0.499	2.396	0.676	0.411	0.103

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	163	52	248	48	55
N.S.	1	1.00	2.54	1.08	6.27	2.00	9.54	1.85	2.12
time (sec)	N/A	0.017	0.041	0.279	0.485	2.814	3.043	0.416	2.307

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	80	0	72	0	67	51
N.S.	1	1.00	0.92	1.29	0.00	1.16	0.00	1.08	0.82
time (sec)	N/A	0.049	0.046	0.334	0.000	2.838	0.000	0.430	2.679

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	83	47	0	70	0	68	74
N.S.	1	1.00	2.31	1.31	0.00	1.94	0.00	1.89	2.06
time (sec)	N/A	0.028	0.051	0.359	0.000	2.525	0.000	0.402	2.368

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	57	63	25	57
N.S.	1	1.00	1.00	0.79	0.76	1.73	1.91	0.76	1.73
time (sec)	N/A	0.021	0.021	0.256	0.266	2.933	0.800	0.401	0.080

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	39	71	25	78
N.S.	1	1.00	1.00	0.79	0.76	1.18	2.15	0.76	2.36
time (sec)	N/A	0.021	0.021	0.147	0.269	2.259	0.797	0.407	2.469

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	48	19	25
N.S.	1	1.00	1.00	0.80	0.76	1.28	1.92	0.76	1.00
time (sec)	N/A	0.018	0.018	0.159	0.280	2.308	0.306	0.401	2.289

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	42	42	17	17
N.S.	1	1.00	1.00	0.78	0.74	1.83	1.83	0.74	0.74
time (sec)	N/A	0.017	0.014	0.164	0.265	4.783	0.304	0.394	2.487

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	49	65	49	150
N.S.	1	1.00	1.00	0.79	0.76	1.48	1.97	1.48	4.55
time (sec)	N/A	0.021	0.018	0.148	0.267	2.260	0.809	0.406	2.698

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	67	70	23	198
N.S.	1	1.00	1.00	0.77	0.74	2.16	2.26	0.74	6.39
time (sec)	N/A	0.020	0.019	0.440	0.262	2.412	0.769	0.416	3.179

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	25	226	31	25
N.S.	1	1.00	1.00	0.78	0.76	0.61	5.51	0.76	0.61
time (sec)	N/A	0.030	0.025	0.187	0.269	1.167	2.635	0.402	2.271

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.017	0.035	0.132	0.260	1.637	0.274	0.417	2.476

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	61	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.26	0.85	1.33
time (sec)	N/A	0.017	0.025	0.118	0.267	2.999	0.310	0.415	2.461

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.012	0.019	0.105	0.268	1.557	0.199	0.394	2.275

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.15	0.85	1.33
time (sec)	N/A	0.013	0.018	0.111	0.271	1.615	0.244	0.401	2.264

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	173	371	145	7672	242	207
N.S.	1	1.00	0.79	4.44	9.51	3.72	196.72	6.21	5.31
time (sec)	N/A	0.046	0.371	0.129	0.296	1.566	6.592	0.425	4.989

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	145	290	145	7679	245	196
N.S.	1	1.00	0.82	4.26	8.53	4.26	225.85	7.21	5.76
time (sec)	N/A	0.049	0.390	0.122	0.288	1.667	5.469	0.458	4.998

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	177	549	118	7404	348	207
N.S.	1	1.00	0.79	4.54	14.08	3.03	189.85	8.92	5.31
time (sec)	N/A	0.024	0.376	0.161	0.323	2.610	15.707	0.458	4.815

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	149	432	118	7417	345	200
N.S.	1	1.00	0.88	4.38	12.71	3.47	218.15	10.15	5.88
time (sec)	N/A	0.021	0.369	0.156	0.294	4.553	16.570	0.444	5.055

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	54	349	107	0	171	249
N.S.	1	1.00	0.78	1.50	9.69	2.97	0.00	4.75	6.92
time (sec)	N/A	0.013	0.166	0.606	0.282	1.630	0.000	0.448	7.837

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	322	93	0	169	249
N.S.	1	1.00	0.79	1.61	9.76	2.82	0.00	5.12	7.55
time (sec)	N/A	0.013	0.175	0.552	0.288	1.444	0.000	0.451	7.728

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	79	564	110	0	396	249
N.S.	1	1.00	0.78	2.19	15.67	3.06	0.00	11.00	6.92
time (sec)	N/A	0.012	0.184	0.560	0.302	1.396	0.000	0.447	7.774

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	80	536	96	1824	397	249
N.S.	1	1.00	0.88	2.42	16.24	2.91	55.27	12.03	7.55
time (sec)	N/A	0.013	0.166	0.542	0.298	2.778	53.633	0.438	7.873

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	177	57	22	0	0	20
N.S.	1	1.00	1.00	13.62	4.38	1.69	0.00	0.00	1.54
time (sec)	N/A	0.021	0.029	0.624	0.506	2.748	0.000	0.000	2.562

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	587	57	26	0	0	-1
N.S.	1	1.00	0.74	18.94	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.038	0.028	0.319	0.485	2.411	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	324	82	38	0	0	-1
N.S.	1	1.00	0.58	6.48	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.086	0.378	0.495	1.512	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	12	18
N.S.	1	1.00	1.00	1.54	14.46	1.46	0.00	0.92	1.38
time (sec)	N/A	0.025	0.049	0.435	0.541	1.300	0.000	0.387	2.653

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	19	-1
N.S.	1	1.00	0.68	0.84	10.13	0.74	0.00	0.61	-0.03
time (sec)	N/A	0.045	0.028	0.296	0.558	1.637	0.000	0.404	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	27	-1
N.S.	1	1.00	0.58	0.68	8.54	0.70	0.00	0.54	-0.02
time (sec)	N/A	0.063	0.075	0.314	0.563	1.479	0.000	0.404	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	159	0	236	0	0	-1
N.S.	1	1.00	0.97	2.74	0.00	4.07	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.101	0.485	0.000	1.173	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	257	0	459	0	0	-1
N.S.	1	1.00	0.99	3.02	0.00	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.193	0.751	0.000	1.330	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	58	154	0	227	0	0	132
N.S.	1	1.00	0.98	2.61	0.00	3.85	0.00	0.00	2.24
time (sec)	N/A	0.046	0.075	0.253	0.000	1.162	0.000	0.000	3.292

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	250	0	417	0	0	-1
N.S.	1	1.00	0.97	2.84	0.00	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.234	0.417	0.000	1.410	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	86	250	80	0	322	-1
N.S.	1	1.00	0.96	1.72	5.00	1.60	0.00	6.44	-0.02
time (sec)	N/A	0.057	0.135	0.309	0.274	3.878	0.000	0.495	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	87	250	81	0	322	-1
N.S.	1	1.00	0.96	1.74	5.00	1.62	0.00	6.44	-0.02
time (sec)	N/A	0.055	0.137	0.292	0.284	2.231	0.000	0.483	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
N.S.	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.040	0.069	0.443	0.473	2.293	0.000	0.601	2.560

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	512	1003	0	3284	0	0	-1
N.S.	1	1.00	2.43	4.75	0.00	15.56	0.00	0.00	-0.00
time (sec)	N/A	0.367	3.937	0.464	0.000	4.990	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	294	1251	0	4568	0	0	-1
N.S.	1	1.00	0.87	3.71	0.00	13.55	0.00	0.00	-0.00
time (sec)	N/A	0.632	0.718	0.394	0.000	5.230	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	43	300	0	76	45
N.S.	1	1.00	1.00	0.85	1.08	7.50	0.00	1.90	1.12
time (sec)	N/A	0.452	0.172	0.467	0.478	2.833	0.000	0.781	2.597

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	751	1670	0	4100	0	0	-1
N.S.	1	1.00	2.81	6.25	0.00	15.36	0.00	0.00	-0.00
time (sec)	N/A	0.474	2.827	0.606	0.000	5.364	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	499	2061	0	5696	0	0	-1
N.S.	1	1.00	1.23	5.06	0.00	14.00	0.00	0.00	-0.00
time (sec)	N/A	0.706	1.389	0.483	0.000	4.801	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	61	0	0	0	0	156	111
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	1.01	0.72
time (sec)	N/A	0.128	0.286	0.180	0.000	0.000	0.000	0.433	3.053

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	54	0	0	0	0	119	86
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	1.01	0.73
time (sec)	N/A	0.105	0.204	0.118	0.000	0.000	0.000	0.424	2.808

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	0	0	0	0	82	61
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	1.11	0.82
time (sec)	N/A	0.064	0.139	0.109	0.000	0.000	0.000	0.460	2.692

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	0	274	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	3.19	-0.01
time (sec)	N/A	0.114	0.117	0.114	0.000	0.000	0.000	0.462	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	0	0	886	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	7.20	-0.01
time (sec)	N/A	0.131	0.163	0.112	0.000	0.000	0.000	0.511	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	87	0	0	0	0	1022	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	5.81	-0.01
time (sec)	N/A	0.155	0.177	0.111	0.000	0.000	0.000	0.549	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	113	0	0	0	0	1487	216
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	3.78	0.55
time (sec)	N/A	0.232	0.750	0.110	0.000	0.000	0.000	0.631	4.147

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	95	0	0	0	0	739	159
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	2.79	0.60
time (sec)	N/A	0.183	0.511	0.105	0.000	0.000	0.000	0.543	3.765

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	73	0	0	0	0	311	123
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	1.85	0.73
time (sec)	N/A	0.089	0.414	0.102	0.000	0.000	0.000	0.469	1.206

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	150	0	0	0	0	160	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.86	-0.01
time (sec)	N/A	0.448	0.754	0.102	0.000	0.000	0.000	0.446	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	231	0	0	0	0	608	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	2.23	-0.00
time (sec)	N/A	0.470	0.907	0.109	0.000	0.000	0.000	0.505	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	317	0	0	0	0	1502	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	3.90	-0.00
time (sec)	N/A	0.500	1.090	0.106	0.000	0.000	0.000	0.600	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	767	247	0	0	0	0	0	-1
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.897	1.530	0.138	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	194	0	0	0	0	0	-1
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.614	1.089	0.121	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	154	0	0	0	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	0.796	0.297	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.476	2.517	0.108	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	193	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.582	1.406	0.069	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	154	0	0	0	0	1577	-1
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	5.63	-0.00
time (sec)	N/A	1.636	1.028	0.075	0.000	0.000	0.000	0.703	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	150	0	0	78	0	446	88
N.S.	1	1.00	0.88	0.00	0.00	0.46	0.00	2.61	0.51
time (sec)	N/A	0.772	0.348	0.069	0.000	1.364	0.000	0.504	3.316

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	85	154	56	75	0	0	-1
N.S.	1	1.00	0.28	0.51	0.19	0.25	0.00	0.00	-0.00
time (sec)	N/A	0.293	0.063	0.145	0.490	1.377	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	51	24	23	32	27	51	54
N.S.	1	1.00	2.83	1.33	1.28	1.78	1.50	2.83	3.00
time (sec)	N/A	0.071	0.015	0.125	0.262	1.966	1.251	0.412	2.476

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	67	56	54	60	61	100	403
N.S.	1	1.00	1.18	0.98	0.95	1.05	1.07	1.75	7.07
time (sec)	N/A	0.137	0.060	0.131	0.270	2.821	2.113	0.397	2.464

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	80	87	84	77	92	125	431
N.S.	1	1.00	1.07	1.16	1.12	1.03	1.23	1.67	5.75
time (sec)	N/A	0.205	0.077	0.149	0.274	2.260	4.649	0.415	2.477

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	97	124	118	89	116	149	460
N.S.	1	1.00	0.93	1.19	1.13	0.86	1.12	1.43	4.42
time (sec)	N/A	0.282	0.092	0.166	0.274	2.224	9.820	0.422	2.523

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	71	38	63	47	0	46	25
N.S.	1	1.00	2.84	1.52	2.52	1.88	0.00	1.84	1.00
time (sec)	N/A	0.065	0.065	0.116	0.268	4.808	0.000	0.435	2.364

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	58	93	85	0	77	50
N.S.	1	1.00	1.58	1.21	1.94	1.77	0.00	1.60	1.04
time (sec)	N/A	0.128	0.153	0.123	0.282	3.617	0.000	0.413	2.349

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	76	119	122	0	102	92
N.S.	1	1.00	1.17	1.01	1.59	1.63	0.00	1.36	1.23
time (sec)	N/A	0.198	0.260	0.145	0.282	3.004	0.000	0.418	2.366

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	104	93	143	158	0	126	140
N.S.	1	1.00	1.08	0.97	1.49	1.65	0.00	1.31	1.46
time (sec)	N/A	0.283	0.556	0.158	0.279	5.249	0.000	0.426	2.340

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	230	43	123	0	134	-1
N.S.	1	1.00	0.80	2.35	0.44	1.26	0.00	1.37	-0.01
time (sec)	N/A	0.282	0.114	0.517	0.524	2.506	0.000	0.428	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	201	26	99	0	92	-1
N.S.	1	1.00	0.86	2.79	0.36	1.38	0.00	1.28	-0.01
time (sec)	N/A	0.190	0.073	0.437	0.554	4.824	0.000	0.407	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	154	13	81	0	61	-1
N.S.	1	1.00	1.07	3.50	0.30	1.84	0.00	1.39	-0.02
time (sec)	N/A	0.110	0.027	0.419	0.531	2.965	0.000	0.422	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	194	58	116	0	120	-1
N.S.	1	1.00	0.76	2.85	0.85	1.71	0.00	1.76	-0.01
time (sec)	N/A	0.129	0.036	0.461	0.538	3.676	0.000	0.437	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	270	9153	187	0	158	-1
N.S.	1	1.00	0.79	2.93	99.49	2.03	0.00	1.72	-0.01
time (sec)	N/A	0.216	0.274	0.504	1.086	2.931	0.000	0.432	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	95	322	50224	234	0	183	-1
N.S.	1	1.00	0.79	2.68	418.53	1.95	0.00	1.52	-0.01
time (sec)	N/A	0.315	0.367	0.602	23.505	3.461	0.000	0.438	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	73	0	35	0	283	-1
N.S.	1	1.00	1.00	2.92	0.00	1.40	0.00	11.32	-0.04
time (sec)	N/A	0.035	0.074	0.614	0.000	3.482	0.000	0.484	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	67	0	36	0	397	68
N.S.	1	1.00	1.00	2.79	0.00	1.50	0.00	16.54	2.83
time (sec)	N/A	0.038	0.071	0.312	0.000	2.689	0.000	0.513	2.745

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	8	15	41	8	8
N.S.	1	1.00	1.00	1.38	1.00	1.88	5.12	1.00	1.00
time (sec)	N/A	0.031	0.010	0.102	0.496	2.454	0.393	0.403	2.307

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	24	18	15	35	248	49	26
N.S.	1	1.00	0.67	0.50	0.42	0.97	6.89	1.36	0.72
time (sec)	N/A	0.029	0.020	0.118	0.485	2.370	23.496	0.402	2.324

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	13	10	15	12	16	8
N.S.	1	1.00	1.00	1.62	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.029	0.007	0.087	0.480	2.222	0.315	0.406	2.287

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	19	16	35	61	49	27
N.S.	1	1.00	0.62	0.51	0.43	0.95	1.65	1.32	0.73
time (sec)	N/A	0.027	0.024	0.108	0.479	2.606	0.821	0.399	2.286

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	31	0	13	61	26	13
N.S.	1	1.00	1.00	2.21	0.00	0.93	4.36	1.86	0.93
time (sec)	N/A	0.091	0.009	0.133	0.000	2.284	34.398	0.384	2.474

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	88	0	254	2608	110	2429
N.S.	1	1.00	0.70	0.84	0.00	2.42	24.84	1.05	23.13
time (sec)	N/A	0.189	0.117	0.232	0.000	2.463	68.560	0.414	4.043

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	30	29	45	143	62	242
N.S.	1	1.00	0.60	0.53	0.51	0.79	2.51	1.09	4.25
time (sec)	N/A	0.094	0.088	0.228	0.480	3.835	4.420	0.396	2.427

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	17	19	24	29	13
N.S.	1	1.00	1.00	1.20	1.13	1.27	1.60	1.93	0.87
time (sec)	N/A	0.061	0.010	0.128	0.488	2.643	0.457	0.382	2.309

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	44	40	228	0	58	1987
N.S.	1	1.00	0.96	0.90	0.82	4.65	0.00	1.18	40.55
time (sec)	N/A	0.109	0.121	0.297	0.478	4.041	0.000	0.410	2.848

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	26	0	12	56	22	13
N.S.	1	1.00	1.00	2.00	0.00	0.92	4.31	1.69	1.00
time (sec)	N/A	0.084	0.009	0.155	0.000	1.805	60.730	0.383	2.461

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	72	86	0	262	2035	93	1646
N.S.	1	1.00	0.72	0.86	0.00	2.62	20.35	0.93	16.46
time (sec)	N/A	0.170	0.133	0.192	0.000	2.025	156.125	0.401	5.150

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	29	28	45	520	62	249
N.S.	1	1.00	0.55	0.52	0.50	0.80	9.29	1.11	4.45
time (sec)	N/A	0.137	0.061	0.199	0.498	1.131	25.924	0.417	2.387

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	51	23	12
N.S.	1	1.00	1.00	1.07	1.00	1.29	3.64	1.64	0.86
time (sec)	N/A	0.038	0.011	0.134	0.515	2.315	0.484	0.401	2.315

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	46	42	255	0	70	1774
N.S.	1	1.00	0.96	0.94	0.86	5.20	0.00	1.43	36.20
time (sec)	N/A	0.120	0.120	0.288	0.502	1.024	0.000	0.411	2.946

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	98	106	0	318	0	125	1302
N.S.	1	1.00	1.32	1.43	0.00	4.30	0.00	1.69	17.59
time (sec)	N/A	0.170	0.336	0.323	0.000	2.760	0.000	0.418	3.514

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	102	90	0	332	0	110	463
N.S.	1	1.00	1.42	1.25	0.00	4.61	0.00	1.53	6.43
time (sec)	N/A	0.168	0.398	0.335	0.000	1.927	0.000	0.396	2.832

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	94	0	0	0	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.180	0.309	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.124	0.332	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	246	321	257	257	461	316	422
N.S.	1	1.00	1.94	2.53	2.02	2.02	3.63	2.49	3.32
time (sec)	N/A	0.055	0.723	0.592	0.263	1.315	0.692	0.636	6.158

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	192	285	238	219	770	235	519
N.S.	1	1.00	1.19	1.77	1.48	1.36	4.78	1.46	3.22
time (sec)	N/A	0.051	0.533	0.507	0.280	1.426	0.502	0.509	4.117

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	172	155	267	187	248
N.S.	1	1.00	1.66	1.86	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.032	0.339	0.435	0.263	2.203	0.316	0.477	2.720

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	136	121	381	122	320
N.S.	1	1.00	0.99	1.42	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.029	0.280	0.387	0.268	1.097	0.233	0.441	3.461

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	84	77	117	91	104
N.S.	1	1.00	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.018	0.246	0.000	0.270	3.357	0.130	0.420	2.480

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	68	52	128	50	63
N.S.	1	1.00	0.95	1.27	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.013	0.077	0.341	0.265	2.562	0.090	0.402	2.418

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	24	23	31	24	38
N.S.	1	1.00	1.92	1.04	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.009	0.012	0.158	0.268	3.300	0.056	0.399	2.324

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	0	74	39
N.S.	1	1.00	0.96	0.91	1.70	2.79	0.00	1.57	0.83
time (sec)	N/A	0.019	0.048	0.000	0.486	2.185	0.000	0.439	2.801

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
N.S.	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.012	0.032	0.593	0.278	2.405	0.000	0.414	2.342

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
N.S.	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.042	0.214	0.648	0.484	1.581	0.000	0.453	4.555

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
N.S.	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.031	0.220	0.733	0.288	1.962	0.000	0.431	3.116

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	157	514	822	544	0	588	719
N.S.	1	1.00	1.01	3.29	5.27	3.49	0.00	3.77	4.61
time (sec)	N/A	0.066	0.858	1.050	0.525	2.272	0.000	0.465	6.093

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	182	125	174	441	0	118	470
N.S.	1	1.00	1.21	0.83	1.15	2.92	0.00	0.78	3.11
time (sec)	N/A	0.049	0.370	1.046	0.309	4.184	0.000	0.448	5.151

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	205	185	0	261	0	0	-1
N.S.	1	1.00	1.10	0.99	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.310	0.636	0.000	0.624	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	256	250	0	247	0	0	-1
N.S.	1	1.00	1.95	1.91	0.00	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.043	1.208	0.477	0.000	0.733	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	143	165	0	159	0	0	-1
N.S.	1	1.00	1.09	1.26	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.946	0.424	0.000	0.908	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	268	163	0	163	0	0	-1
N.S.	1	1.00	3.57	2.17	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.797	1.519	0.000	0.521	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	92	124	0	128	0	0	-1
N.S.	1	1.00	1.23	1.65	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.137	0.433	0.000	0.594	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	219	232	0	278	0	0	-1
N.S.	1	1.00	1.59	1.68	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.044	2.236	0.452	0.000	0.756	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	145	182	0	406	0	0	-1
N.S.	1	1.00	1.02	1.28	0.00	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.040	1.252	0.457	0.000	1.043	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	277	297	0	552	0	0	-1
N.S.	1	1.00	1.41	1.51	0.00	2.80	0.00	0.00	-0.01
time (sec)	N/A	0.062	1.756	0.466	0.000	0.740	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	153	128	0	118	0	0	-1
N.S.	1	1.00	1.28	1.07	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.364	0.598	0.000	1.522	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	199	174	0	111	0	0	-1
N.S.	1	1.00	2.65	2.32	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.640	0.504	0.000	0.731	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	133	108	0	96	0	0	-1
N.S.	1	1.00	1.77	1.44	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.243	0.414	0.000	0.736	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	184	112	0	63	0	0	-1
N.S.	1	1.00	6.81	4.15	0.00	2.33	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.625	0.740	0.000	0.602	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	88	85	0	58	0	0	-1
N.S.	1	1.00	3.26	3.15	0.00	2.15	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.082	0.379	0.000	0.491	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	190	162	0	158	0	0	-1
N.S.	1	1.00	2.60	2.22	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.798	0.430	0.000	0.313	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	157	118	0	189	0	0	-1
N.S.	1	1.00	2.09	1.57	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.537	0.415	0.000	0.323	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	224	205	0	264	0	0	-1
N.S.	1	1.00	1.87	1.71	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.497	0.500	0.000	0.870	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	59	23	42	23	-1
N.S.	1	1.00	0.97	0.97	1.84	0.72	1.31	0.72	-0.03
time (sec)	N/A	0.012	0.062	0.516	0.481	1.797	0.120	0.491	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	151	132	17	36	52	84
N.S.	1	1.00	1.00	4.87	4.26	0.55	1.16	1.68	2.71
time (sec)	N/A	0.011	0.108	0.491	0.261	3.313	0.066	0.441	2.555

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	76	83	17	36	52	66
N.S.	1	1.00	1.00	2.45	2.68	0.55	1.16	1.68	2.13
time (sec)	N/A	0.013	0.062	0.445	0.282	2.265	0.067	0.406	2.465

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	73	69	17	36	52	44
N.S.	1	1.00	1.00	2.35	2.23	0.55	1.16	1.68	1.42
time (sec)	N/A	0.011	0.043	0.398	0.278	3.629	0.061	0.405	2.425

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	51	26	24	15	26	24	20
N.S.	1	1.00	1.96	1.00	0.92	0.58	1.00	0.92	0.77
time (sec)	N/A	0.010	0.012	0.159	0.269	3.075	0.052	0.412	2.389

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	29	17	31	21	25
N.S.	1	1.00	1.00	0.79	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.011	0.027	0.558	0.274	3.265	0.065	0.426	2.389

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	22	17	44	30	31
N.S.	1	1.00	1.00	0.74	0.71	0.55	1.42	0.97	1.00
time (sec)	N/A	0.012	0.030	0.001	0.274	2.541	0.071	0.399	2.412

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	29	17	44	36	68
N.S.	1	1.00	1.00	1.84	0.94	0.55	1.42	1.16	2.19
time (sec)	N/A	0.013	0.038	0.617	0.282	3.501	0.073	0.417	2.465

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	36	29	17	44	44	91
N.S.	1	1.00	1.00	1.16	0.94	0.55	1.42	1.42	2.94
time (sec)	N/A	0.012	0.033	0.681	0.276	3.265	0.072	0.426	2.558

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	35
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.06
time (sec)	N/A	0.012	0.025	0.413	0.501	2.334	0.000	0.578	0.432

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	25	33
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	0.76	1.00
time (sec)	N/A	0.012	0.026	0.378	0.487	2.665	0.000	0.425	2.378

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	25	-1
N.S.	1	1.00	0.97	0.90	1.65	0.55	0.00	0.81	-0.03
time (sec)	N/A	0.012	0.020	0.373	0.495	1.350	0.000	0.416	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	51	17	0	25	-1
N.S.	1	1.00	0.97	0.90	1.65	0.55	0.00	0.81	-0.03
time (sec)	N/A	0.013	0.029	0.386	0.545	0.963	0.000	0.413	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	65	-1
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	1.97	-0.03
time (sec)	N/A	0.013	0.028	0.385	0.491	1.199	0.000	0.892	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	67	-1
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	2.03	-0.03
time (sec)	N/A	0.012	0.031	0.447	0.506	1.192	0.000	0.745	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	303	164	204	166	308	178	272
N.S.	1	1.00	2.03	1.10	1.37	1.11	2.07	1.19	1.83
time (sec)	N/A	0.163	0.888	0.332	0.276	2.828	4.228	0.405	2.883

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	96	72	80	97	131	115
N.S.	1	1.00	0.96	0.96	0.72	0.80	0.97	1.31	1.15
time (sec)	N/A	0.135	0.217	0.171	0.477	2.056	2.418	0.420	2.532

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	123	74	95	85	122	86	126
N.S.	1	1.00	1.64	0.99	1.27	1.13	1.63	1.15	1.68
time (sec)	N/A	0.097	0.438	0.165	0.277	3.057	2.912	0.411	2.503

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	26	29	22	40	40
N.S.	1	1.00	0.93	0.96	0.96	1.07	0.81	1.48	1.48
time (sec)	N/A	0.038	0.034	0.106	0.496	2.402	0.711	0.408	2.380

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	42	16	14	25	24	34	37
N.S.	1	1.00	3.50	1.33	1.17	2.08	2.00	2.83	3.08
time (sec)	N/A	0.005	0.007	0.082	0.267	3.045	0.032	0.415	2.493

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	50	11	65	12	55
N.S.	1	1.00	1.00	1.09	4.55	1.00	5.91	1.09	5.00
time (sec)	N/A	0.023	0.007	0.162	0.486	2.601	0.228	0.419	3.792

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	344	92	0	308	0	94	604
N.S.	1	1.00	5.21	1.39	0.00	4.67	0.00	1.42	9.15
time (sec)	N/A	0.080	1.492	0.346	0.000	3.259	0.000	0.404	2.806

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	67	50	201	83	554	43	106
N.S.	1	1.00	1.31	0.98	3.94	1.63	10.86	0.84	2.08
time (sec)	N/A	0.055	0.143	0.330	0.504	1.988	1.256	0.441	2.698

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	2661	358	0	931	0	369	2782
N.S.	1	1.00	17.06	2.29	0.00	5.97	0.00	2.37	17.83
time (sec)	N/A	0.221	6.245	0.652	0.000	2.365	0.000	0.419	6.373

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	138	102	483	217	1787	91	541
N.S.	1	1.00	1.37	1.01	4.78	2.15	17.69	0.90	5.36
time (sec)	N/A	0.082	0.270	0.817	0.518	3.149	7.663	0.400	3.837

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	54	106	141	38	68	62	59
N.S.	1	1.00	1.80	3.53	4.70	1.27	2.27	2.07	1.97
time (sec)	N/A	0.035	0.071	0.158	0.306	2.027	4.162	0.409	2.438

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	64	71	28	61	44	20	20
N.S.	1	1.00	2.13	2.37	0.93	2.03	1.47	0.67	0.67
time (sec)	N/A	0.066	0.105	0.131	0.497	2.705	2.361	0.393	2.373

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	31	45	52	21	44	48	43
N.S.	1	1.00	1.72	2.50	2.89	1.17	2.44	2.67	2.39
time (sec)	N/A	0.030	0.023	0.121	0.308	3.175	2.871	0.390	2.395

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	28	10	14	14
N.S.	1	1.00	0.88	0.94	0.88	1.75	0.62	0.88	0.88
time (sec)	N/A	0.048	0.009	0.082	0.507	1.551	0.659	0.409	2.357

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	9	38	13	10	9	20	31	19
N.S.	1	0.69	2.92	1.00	0.77	0.69	1.54	2.38	1.46
time (sec)	N/A	0.004	0.006	0.075	0.288	2.405	0.033	0.387	2.399

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
N.S.	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.015	0.011	0.127	0.273	1.571	0.061	0.399	2.776

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	27	19	28	25	0	14	14
N.S.	1	1.00	1.93	1.36	2.00	1.79	0.00	1.00	1.00
time (sec)	N/A	0.025	0.019	0.171	0.487	1.724	0.000	0.406	2.342

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	34	17	64	20	301	45	41
N.S.	1	1.00	2.12	1.06	4.00	1.25	18.81	2.81	2.56
time (sec)	N/A	0.029	0.016	0.186	0.479	1.444	0.413	0.410	2.359

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	62	29	64	63	0	20	19
N.S.	1	1.00	2.38	1.12	2.46	2.42	0.00	0.77	0.73
time (sec)	N/A	0.042	0.055	0.188	0.479	1.251	0.000	0.410	2.340

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	39	23	92	35	1059	64	61
N.S.	1	1.00	1.77	1.05	4.18	1.59	48.14	2.91	2.77
time (sec)	N/A	0.031	0.037	0.223	0.550	2.846	1.268	0.407	2.379

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	143	167	188	292	308	169	174
N.S.	1	1.00	0.94	1.10	1.24	1.92	2.03	1.11	1.14
time (sec)	N/A	0.151	0.508	0.260	0.331	2.318	61.677	0.429	2.580

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	95	93	80	95	97	215	127
N.S.	1	1.00	0.94	0.92	0.79	0.94	0.96	2.13	1.26
time (sec)	N/A	0.144	0.191	0.108	0.489	1.994	19.591	0.391	2.530

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	80	87	128	124	86	82
N.S.	1	1.00	1.03	1.04	1.13	1.66	1.61	1.12	1.06
time (sec)	N/A	0.096	0.209	0.141	0.291	3.050	8.309	0.393	2.448

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	29	29	28	31	52	30
N.S.	1	1.00	0.83	1.00	1.00	0.97	1.07	1.79	1.03
time (sec)	N/A	0.039	0.100	0.086	0.479	4.995	1.517	0.411	2.417

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	16	15	27	24	15	27
N.S.	1	1.00	2.08	1.33	1.25	2.25	2.00	1.25	2.25
time (sec)	N/A	0.005	0.010	0.080	0.270	3.299	0.032	0.413	2.408

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	45	12	0	13	36
N.S.	1	1.00	1.00	1.08	3.75	1.00	0.00	1.08	3.00
time (sec)	N/A	0.024	0.014	0.152	0.475	1.906	0.000	0.394	3.283

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	71	86	0	307	0	107	440
N.S.	1	1.00	1.06	1.28	0.00	4.58	0.00	1.60	6.57
time (sec)	N/A	0.088	0.202	0.252	0.000	2.698	0.000	0.394	3.044

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	77	49	177	70	0	45	311
N.S.	1	1.00	1.54	0.98	3.54	1.40	0.00	0.90	6.22
time (sec)	N/A	0.056	0.088	0.204	0.489	2.286	0.000	0.389	2.770

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	150	197	0	878	0	282	3068
N.S.	1	1.00	0.94	1.24	0.00	5.52	0.00	1.77	19.30
time (sec)	N/A	0.245	0.353	0.339	0.000	3.051	0.000	0.414	8.246

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	138	107	497	166	0	93	538
N.S.	1	1.00	1.38	1.07	4.97	1.66	0.00	0.93	5.38
time (sec)	N/A	0.087	0.241	0.320	0.541	3.124	0.000	0.406	3.680

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	32	105	125	37	68	22	34
N.S.	1	1.00	1.14	3.75	4.46	1.32	2.43	0.79	1.21
time (sec)	N/A	0.034	0.052	0.128	0.296	3.078	82.111	0.388	2.417

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	68	56	36	44	20	16
N.S.	1	1.00	1.00	2.27	1.87	1.20	1.47	0.67	0.53
time (sec)	N/A	0.068	0.033	0.111	0.494	1.640	25.048	0.398	2.413

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	46	22	46	18	25
N.S.	1	1.00	1.00	2.45	2.30	1.10	2.30	0.90	1.25
time (sec)	N/A	0.032	0.030	0.105	0.272	2.058	9.290	0.387	2.385

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	15	16	16	17	12	10
N.S.	1	1.00	0.75	0.94	1.00	1.00	1.06	0.75	0.62
time (sec)	N/A	0.046	0.017	0.067	0.488	2.319	1.448	0.401	2.404

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	13	12	7	20	11	19
N.S.	1	1.00	2.22	1.44	1.33	0.78	2.22	1.22	2.11
time (sec)	N/A	0.004	0.005	0.075	0.274	2.790	0.031	0.405	2.409

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	14	9	0	7	9
N.S.	1	1.00	1.29	1.14	2.00	1.29	0.00	1.00	1.29
time (sec)	N/A	0.017	0.011	0.141	0.489	3.071	0.000	0.405	2.857

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	15	23	18	0	10	10
N.S.	1	1.00	0.86	1.07	1.64	1.29	0.00	0.71	0.71
time (sec)	N/A	0.027	0.010	0.148	0.491	2.429	0.000	0.376	2.407

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	28	21	0	14	18
N.S.	1	1.00	1.29	1.07	2.00	1.50	0.00	1.00	1.29
time (sec)	N/A	0.032	0.011	0.175	0.482	2.927	0.000	0.388	2.345

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	35	40	0	16	16
N.S.	1	1.00	1.15	0.88	1.35	1.54	0.00	0.62	0.62
time (sec)	N/A	0.048	0.012	0.174	0.487	2.541	0.000	0.389	2.371

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	25	39	38	0	22	26
N.S.	1	1.00	1.33	1.04	1.62	1.58	0.00	0.92	1.08
time (sec)	N/A	0.035	0.012	0.198	0.503	5.471	0.000	0.400	2.408

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	39	36	51	44	39	59
N.S.	1	1.00	0.86	0.89	0.82	1.16	1.00	0.89	1.34
time (sec)	N/A	0.024	0.025	0.118	0.276	2.773	4.958	0.394	2.494

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	61	32	37	57	42	99	75
N.S.	1	1.00	1.79	0.94	1.09	1.68	1.24	2.91	2.21
time (sec)	N/A	0.033	0.016	0.119	0.270	2.071	2.119	0.405	2.488

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	15	16	20	15	23	21
N.S.	1	1.00	0.82	0.68	0.73	0.91	0.68	1.05	0.95
time (sec)	N/A	0.018	0.012	0.085	0.270	3.890	0.912	0.400	2.404

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	11	21	19	9	8
N.S.	1	1.00	2.38	1.50	1.38	2.62	2.38	1.12	1.00
time (sec)	N/A	0.004	0.004	0.052	0.290	3.422	0.033	0.394	0.021

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	17	4	0	17	12
N.S.	1	1.00	1.00	2.50	8.50	2.00	0.00	8.50	6.00
time (sec)	N/A	0.013	0.004	0.131	0.266	2.498	0.000	0.397	2.462

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	0	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	0.00	0.75	0.75
time (sec)	N/A	0.011	0.002	0.158	0.267	2.638	0.000	0.391	2.406

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	103	14	0	59	13
N.S.	1	1.00	1.00	0.82	6.06	0.82	0.00	3.47	0.76
time (sec)	N/A	0.025	0.016	0.197	0.279	2.429	0.000	0.410	2.547

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	13	26	0	13	23
N.S.	1	1.00	2.18	0.82	0.76	1.53	0.00	0.76	1.35
time (sec)	N/A	0.015	0.013	0.254	0.264	2.600	0.000	0.392	2.566

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	187	20	0	101	19
N.S.	1	1.00	1.00	0.80	7.48	0.80	0.00	4.04	0.76
time (sec)	N/A	0.030	0.015	0.214	0.290	2.242	0.000	0.406	2.944

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	19	40	0	19	33
N.S.	1	1.00	2.28	0.80	0.76	1.60	0.00	0.76	1.32
time (sec)	N/A	0.016	0.015	0.240	0.319	2.944	0.000	0.398	2.915

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	271	26	0	143	25
N.S.	1	1.00	1.00	0.79	8.21	0.79	0.00	4.33	0.76
time (sec)	N/A	0.030	0.014	0.263	0.286	4.183	0.000	0.408	3.538

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	40	578	44	0	0	-1
N.S.	1	1.00	0.51	0.55	7.92	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.058	0.401	0.586	3.277	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	34	427	35	0	0	-1
N.S.	1	1.00	0.58	0.68	8.54	0.70	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.062	0.336	0.577	2.927	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	26	314	23	0	0	-1
N.S.	1	1.00	0.68	0.84	10.13	0.74	0.00	0.00	-0.03
time (sec)	N/A	0.056	0.029	0.295	0.579	3.346	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	188	19	0	0	15
N.S.	1	1.00	1.00	1.54	14.46	1.46	0.00	0.00	1.15
time (sec)	N/A	0.034	0.021	0.298	0.548	3.275	0.000	0.000	2.483

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	0	0	124	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	2.07	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.199	0.220	0.000	3.519	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	465	0	152	0	0	-1
N.S.	1	1.00	0.75	5.81	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.183	0.582	0.000	1.838	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	382	0	165	0	0	-1
N.S.	1	1.00	0.70	3.86	0.00	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.388	0.473	0.000	4.372	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	74	487	0	167	0	0	-1
N.S.	1	1.00	0.63	4.13	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.190	0.477	0.000	1.814	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	40	26	37	44	35	80
N.S.	1	1.00	0.86	0.91	0.59	0.84	1.00	0.80	1.82
time (sec)	N/A	0.022	0.021	0.131	0.287	1.248	6.093	0.386	2.573

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	30	37	49	42	39	68
N.S.	1	1.00	1.12	0.88	1.09	1.44	1.24	1.15	2.00
time (sec)	N/A	0.029	0.008	0.128	0.267	1.276	2.256	0.394	2.464

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	16	13	12	22	14	18	49
N.S.	1	1.00	0.73	0.59	0.55	1.00	0.64	0.82	2.23
time (sec)	N/A	0.016	0.013	0.091	0.301	2.039	0.949	0.415	2.410

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	37	12	11	21	19	29	14
N.S.	1	1.00	4.62	1.50	1.38	2.62	2.38	3.62	1.75
time (sec)	N/A	0.004	0.004	0.056	0.303	1.574	0.030	0.372	2.300

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	7	21	6	0	6	6
N.S.	1	1.00	1.00	1.75	5.25	1.50	0.00	1.50	1.50
time (sec)	N/A	0.014	0.003	0.122	0.277	2.114	0.000	0.413	2.368

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	18	0	6	6
N.S.	1	1.00	1.00	0.88	0.75	2.25	0.00	0.75	0.75
time (sec)	N/A	0.011	0.002	0.129	0.272	2.352	0.000	0.413	2.450

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	73	28	0	14	14
N.S.	1	1.00	1.00	0.82	4.29	1.65	0.00	0.82	0.82
time (sec)	N/A	0.029	0.008	0.149	0.294	1.844	0.000	0.388	2.382

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	14	39	0	14	16
N.S.	1	1.00	2.18	0.82	0.82	2.29	0.00	0.82	0.94
time (sec)	N/A	0.013	0.016	0.152	0.282	1.242	0.000	0.411	2.431

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	121	46	0	20	20
N.S.	1	1.00	1.00	0.80	4.84	1.84	0.00	0.80	0.80
time (sec)	N/A	0.029	0.010	0.174	0.278	1.352	0.000	0.400	2.421

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	20	57	0	20	46
N.S.	1	1.00	2.28	0.80	0.80	2.28	0.00	0.80	1.84
time (sec)	N/A	0.017	0.015	0.175	0.269	2.493	0.000	0.393	2.446

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	169	64	0	26	109
N.S.	1	1.00	1.00	0.79	5.12	1.94	0.00	0.79	3.30
time (sec)	N/A	0.031	0.011	0.211	0.293	4.981	0.000	0.401	2.506

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	603	82	44	0	0	-1
N.S.	1	1.00	0.51	8.26	1.12	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.152	0.499	0.512	3.144	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	29	321	82	38	0	0	-1
N.S.	1	1.00	0.58	6.42	1.64	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.057	0.395	0.498	3.024	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	584	57	26	0	0	-1
N.S.	1	1.00	0.74	18.84	1.84	0.84	0.00	0.00	-0.03
time (sec)	N/A	0.044	0.028	0.208	0.487	2.755	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	174	57	22	0	46	20
N.S.	1	1.00	1.00	13.38	4.38	1.69	0.00	3.54	1.54
time (sec)	N/A	0.028	0.020	0.228	0.503	1.730	0.000	0.429	2.424

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	105	0	72	0	35	-1
N.S.	1	1.00	0.83	2.02	0.00	1.38	0.00	0.67	-0.02
time (sec)	N/A	0.052	0.054	0.210	0.000	2.920	0.000	0.446	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	265	0	119	0	95	-1
N.S.	1	1.00	0.78	3.68	0.00	1.65	0.00	1.32	-0.01
time (sec)	N/A	0.064	0.100	0.226	0.000	3.375	0.000	0.468	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	454	0	147	0	151	-1
N.S.	1	1.00	0.80	4.99	0.00	1.62	0.00	1.66	-0.01
time (sec)	N/A	0.085	0.497	0.220	0.000	3.054	0.000	0.493	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	494	0	171	0	210	-1
N.S.	1	1.00	0.67	4.49	0.00	1.55	0.00	1.91	-0.01
time (sec)	N/A	0.099	0.269	0.238	0.000	3.047	0.000	0.545	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	129	66	68	78	90	1375	88
N.S.	1	1.00	2.35	1.20	1.24	1.42	1.64	25.00	1.60
time (sec)	N/A	0.078	0.150	0.070	0.481	2.637	2.908	2.688	2.539

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	42	47	46	173	65
N.S.	1	1.00	1.05	1.03	1.11	1.24	1.21	4.55	1.71
time (sec)	N/A	0.032	0.030	0.070	0.262	1.967	3.598	0.564	2.450

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	60	25	28	44	31	177	61
N.S.	1	1.00	2.40	1.00	1.12	1.76	1.24	7.08	2.44
time (sec)	N/A	0.043	0.071	0.057	0.473	2.059	0.957	0.441	2.423

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	12	8	11	22
N.S.	1	1.00	1.00	1.10	0.80	1.20	0.80	1.10	2.20
time (sec)	N/A	0.005	0.004	0.033	0.264	1.628	0.010	0.412	2.399

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
N.S.	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.038	0.012	0.099	0.263	2.407	0.000	0.407	2.467

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	57	32	45	34	0	31	40
N.S.	1	1.00	1.73	0.97	1.36	1.03	0.00	0.94	1.21
time (sec)	N/A	0.080	0.013	0.116	0.279	1.127	0.000	0.401	2.393

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	83	56	73	130	0	95	48
N.S.	1	1.00	1.38	0.93	1.22	2.17	0.00	1.58	0.80
time (sec)	N/A	0.048	0.014	0.129	0.272	1.089	0.000	0.409	2.386

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	129	64	97	78	0	65	87
N.S.	1	1.00	1.98	0.98	1.49	1.20	0.00	1.00	1.34
time (sec)	N/A	0.138	0.015	0.142	0.262	1.409	0.000	0.414	2.453

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	117	153	144	634	131	695
N.S.	1	1.00	0.92	1.58	2.07	1.95	8.57	1.77	9.39
time (sec)	N/A	0.042	0.150	0.234	0.476	1.282	20.490	0.442	7.017

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	82	108	146	200	0	130	105
N.S.	1	1.00	1.09	1.44	1.95	2.67	0.00	1.73	1.40
time (sec)	N/A	0.044	0.236	0.226	0.480	1.460	0.000	0.436	2.583

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	177	338	279	0	199	227
N.S.	1	1.00	1.14	1.53	2.91	2.41	0.00	1.72	1.96
time (sec)	N/A	0.081	0.287	0.294	0.501	1.277	0.000	0.473	2.856

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	117	153	143	673	131	692
N.S.	1	1.00	0.92	1.60	2.10	1.96	9.22	1.79	9.48
time (sec)	N/A	0.036	0.106	0.244	0.493	1.505	20.699	0.456	6.525

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	109	156	201	0	132	126
N.S.	1	1.00	1.08	1.43	2.05	2.64	0.00	1.74	1.66
time (sec)	N/A	0.039	0.175	0.227	0.485	1.116	0.000	0.450	2.623

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	204	366	279	0	245	251
N.S.	1	1.00	1.02	1.76	3.16	2.41	0.00	2.11	2.16
time (sec)	N/A	0.075	0.250	0.280	0.491	0.877	0.000	0.438	2.879

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	238	514	363	230	857	287	522
N.S.	1	1.00	0.97	2.09	1.48	0.93	3.48	1.17	2.12
time (sec)	N/A	0.112	1.045	0.546	0.271	0.986	0.651	0.510	7.319

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	250	211	152	415	199	261
N.S.	1	1.00	0.92	1.40	1.19	0.85	2.33	1.12	1.47
time (sec)	N/A	0.071	0.452	0.395	0.264	0.804	0.387	0.438	7.324

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	124	115	86	192	92	100
N.S.	1	1.00	0.96	1.07	0.99	0.74	1.66	0.79	0.86
time (sec)	N/A	0.037	0.158	0.289	0.262	0.972	0.151	0.413	3.075

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	36	35	36	42	35	48
N.S.	1	1.00	0.97	0.97	0.95	0.97	1.14	0.95	1.30
time (sec)	N/A	0.011	0.027	0.137	0.254	0.879	0.057	0.405	2.672

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	41	81	0	43	38
N.S.	1	1.00	1.00	1.02	0.84	1.65	0.00	0.88	0.78
time (sec)	N/A	0.024	0.078	0.333	0.259	0.764	0.000	0.418	2.824

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	98	233	0	204	0	158	274
N.S.	1	1.00	0.76	1.81	0.00	1.58	0.00	1.22	2.12
time (sec)	N/A	0.060	0.186	0.536	0.000	0.665	0.000	0.419	3.673

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	420	496	0	511	0	342	592
N.S.	1	1.00	2.20	2.60	0.00	2.68	0.00	1.79	3.10
time (sec)	N/A	0.090	1.871	0.897	0.000	1.361	0.000	0.493	8.120

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	533	823	0	766	0	593	1004
N.S.	1	1.00	2.06	3.18	0.00	2.96	0.00	2.29	3.88
time (sec)	N/A	0.134	1.359	1.566	0.000	2.020	0.000	0.727	12.306

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	195	140	291	151	239
N.S.	1	1.00	0.86	1.13	1.24	0.89	1.85	0.96	1.52
time (sec)	N/A	0.099	0.302	0.263	0.276	1.484	0.172	0.435	2.575

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	101	75	170	78	96
N.S.	1	1.00	1.14	1.25	1.25	0.93	2.10	0.96	1.19
time (sec)	N/A	0.034	0.114	0.213	0.270	1.566	0.106	0.415	3.213

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	29	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	1.00	1.34	1.00	1.00
time (sec)	N/A	0.011	0.015	0.092	0.282	1.429	0.057	0.379	2.428

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	57	23	30	62	63	23	22
N.S.	1	1.00	2.28	0.92	1.20	2.48	2.52	0.92	0.88
time (sec)	N/A	0.018	0.041	0.309	0.269	1.715	0.642	0.399	2.820

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	115	68	95	168	0	83	79
N.S.	1	1.00	1.53	0.91	1.27	2.24	0.00	1.11	1.05
time (sec)	N/A	0.036	0.412	0.391	0.272	1.632	0.000	0.428	2.479

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	186	131	201	457	0	164	162
N.S.	1	1.00	1.39	0.98	1.50	3.41	0.00	1.22	1.21
time (sec)	N/A	0.079	2.143	0.499	0.290	1.619	0.000	0.426	2.481

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	492	221	324	825	0	291	260
N.S.	1	1.00	2.38	1.07	1.57	3.99	0.00	1.41	1.26
time (sec)	N/A	0.169	1.175	0.681	0.303	1.648	0.000	0.433	2.526

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	21	29	32	36	21	20
N.S.	1	1.00	2.17	0.91	1.26	1.39	1.57	0.91	0.87
time (sec)	N/A	0.014	0.024	0.306	0.283	1.274	0.315	0.406	2.488

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	93	48	85	113	168	65	59
N.S.	1	1.00	1.24	0.64	1.13	1.51	2.24	0.87	0.79
time (sec)	N/A	0.032	0.140	0.299	0.276	1.726	0.911	0.399	2.454

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	135	78	157	161	423	102	90
N.S.	1	1.00	1.10	0.63	1.28	1.31	3.44	0.83	0.73
time (sec)	N/A	0.074	0.410	0.381	0.291	1.314	3.140	0.428	2.446

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	247	106	225	265	792	132	161
N.S.	1	1.00	1.47	0.63	1.34	1.58	4.71	0.79	0.96
time (sec)	N/A	0.124	0.711	0.418	0.290	1.810	11.625	0.426	2.441

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	178	192	140	291	151	258
N.S.	1	1.00	0.87	1.13	1.22	0.89	1.85	0.96	1.64
time (sec)	N/A	0.095	0.330	0.263	0.256	1.677	0.162	0.428	3.214

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	100	75	170	78	84
N.S.	1	1.00	1.14	1.23	1.23	0.93	2.10	0.96	1.04
time (sec)	N/A	0.031	0.115	0.213	0.273	1.641	0.116	0.406	2.504

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	30	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	1.03	1.34	1.00	1.00
time (sec)	N/A	0.010	0.013	0.096	0.266	1.718	0.056	0.392	2.429

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	50	40	57	62	95	41	26
N.S.	1	1.00	2.00	1.60	2.28	2.48	3.80	1.64	1.04
time (sec)	N/A	0.014	0.112	0.316	0.294	1.661	0.673	0.418	2.617

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	229	88	146	176	0	110	91
N.S.	1	1.00	3.05	1.17	1.95	2.35	0.00	1.47	1.21
time (sec)	N/A	0.034	0.311	0.411	0.302	1.900	0.000	0.412	2.546

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	350	176	280	462	0	229	186
N.S.	1	1.00	2.61	1.31	2.09	3.45	0.00	1.71	1.39
time (sec)	N/A	0.079	0.455	0.552	0.284	1.461	0.000	0.439	4.468

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	494	253	403	830	0	347	301
N.S.	1	1.00	2.39	1.22	1.95	4.01	0.00	1.68	1.45
time (sec)	N/A	0.165	0.934	0.701	0.297	1.894	0.000	0.457	6.051

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	135	177	195	133	291	151	292
N.S.	1	1.00	0.86	1.13	1.24	0.85	1.85	0.96	1.86
time (sec)	N/A	0.098	0.342	0.257	0.271	1.263	0.165	0.418	3.603

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	101	101	76	170	79	127
N.S.	1	1.00	1.14	1.25	1.25	0.94	2.10	0.98	1.57
time (sec)	N/A	0.034	0.115	0.236	0.282	1.278	0.099	0.410	3.718

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	29	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	1.00	1.34	1.00	1.00
time (sec)	N/A	0.011	0.015	0.099	0.256	1.265	0.054	0.405	2.440

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	93	66	69	56	107	79	33
N.S.	1	1.00	2.82	2.00	2.09	1.70	3.24	2.39	1.00
time (sec)	N/A	0.016	0.055	0.325	0.272	1.222	0.959	0.440	2.830

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	162	122	194	162	0	187	126
N.S.	1	1.00	1.95	1.47	2.34	1.95	0.00	2.25	1.52
time (sec)	N/A	0.036	0.429	0.470	0.279	2.943	0.000	0.428	2.721

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	255	264	510	444	0	458	360
N.S.	1	1.00	1.80	1.86	3.59	3.13	0.00	3.23	2.54
time (sec)	N/A	0.080	1.762	0.660	0.304	3.253	0.000	0.441	6.445

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	632	398	988	763	0	957	730
N.S.	1	1.00	2.94	1.85	4.60	3.55	0.00	4.45	3.40
time (sec)	N/A	0.172	1.344	1.148	0.336	3.200	0.000	0.458	7.217

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	136	176	192	132	291	151	292
N.S.	1	1.00	0.87	1.12	1.22	0.84	1.85	0.96	1.86
time (sec)	N/A	0.092	0.324	0.263	0.300	3.069	0.184	0.423	3.439

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	100	100	74	170	79	128
N.S.	1	1.00	1.14	1.23	1.23	0.91	2.10	0.98	1.58
time (sec)	N/A	0.033	0.111	0.231	0.279	3.057	0.104	0.415	3.743

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	28	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	0.97	1.34	1.00	1.00
time (sec)	N/A	0.010	0.013	0.095	0.273	2.642	0.060	0.386	2.445

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	96	59	65	59	109	79	32
N.S.	1	1.00	2.91	1.79	1.97	1.79	3.30	2.39	0.97
time (sec)	N/A	0.015	0.070	0.327	0.275	2.761	1.008	0.417	2.739

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	166	130	191	168	0	189	126
N.S.	1	1.00	2.00	1.57	2.30	2.02	0.00	2.28	1.52
time (sec)	N/A	0.035	0.529	0.449	0.280	2.332	0.000	0.431	2.738

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	261	275	508	447	0	458	361
N.S.	1	1.00	1.84	1.94	3.58	3.15	0.00	3.23	2.54
time (sec)	N/A	0.077	1.973	0.640	0.296	2.747	0.000	0.435	6.482

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	636	410	984	769	0	957	731
N.S.	1	1.00	2.96	1.91	4.58	3.58	0.00	4.45	3.40
time (sec)	N/A	0.164	1.363	1.050	0.342	2.142	0.000	0.471	7.088

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	237	335	339	263	682	286	376
N.S.	1	1.00	0.91	1.29	1.30	1.01	2.62	1.10	1.45
time (sec)	N/A	0.275	0.790	0.296	0.279	2.594	0.318	0.452	3.374

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	144	177	193	153	294	167	333
N.S.	1	1.00	0.85	1.04	1.14	0.90	1.73	0.98	1.96
time (sec)	N/A	0.127	0.323	0.249	0.267	2.698	0.175	0.441	3.700

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	77	99	102	77	162	81	125
N.S.	1	1.00	0.85	1.09	1.12	0.85	1.78	0.89	1.37
time (sec)	N/A	0.032	0.130	0.214	0.268	2.909	0.099	0.412	3.784

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	49	28	27	28	34	27	40
N.S.	1	1.00	1.81	1.04	1.00	1.04	1.26	1.00	1.48
time (sec)	N/A	0.011	0.013	0.086	0.275	2.638	0.055	0.385	2.506

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	61	0	448	3179	89	75
N.S.	1	1.00	0.93	1.00	0.00	7.34	52.11	1.46	1.23
time (sec)	N/A	0.063	0.093	0.327	0.000	2.327	91.740	0.413	4.011

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	116	203	0	853	0	214	195
N.S.	1	1.00	0.96	1.68	0.00	7.05	0.00	1.77	1.61
time (sec)	N/A	0.086	0.266	0.506	0.000	3.136	0.000	0.409	3.057

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	200	611	0	1997	0	856	700
N.S.	1	1.00	1.02	3.10	0.00	10.14	0.00	4.35	3.55
time (sec)	N/A	0.154	0.671	0.849	0.000	3.163	0.000	0.451	6.055

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	606	1656	0	4135	0	2558	1946
N.S.	1	1.00	2.08	5.67	0.00	14.16	0.00	8.76	6.66
time (sec)	N/A	0.275	1.420	1.723	0.000	3.595	0.000	0.508	4.806

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	399	821	0	193	0	0	-1
N.S.	1	1.00	2.16	4.44	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.190	3.999	0.646	0.000	0.943	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	349	806	0	170	0	0	-1
N.S.	1	1.00	2.51	5.80	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.095	2.461	0.430	0.000	0.724	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	326	461	0	127	0	0	-1
N.S.	1	1.00	7.24	10.24	0.00	2.82	0.00	0.00	-0.02
time (sec)	N/A	0.023	1.776	0.739	0.000	0.696	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	128	152	0	63	0	0	-1
N.S.	1	1.00	2.84	3.38	0.00	1.40	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.206	0.277	0.000	0.622	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	390	425	0	303	0	0	-1
N.S.	1	1.00	4.15	4.52	0.00	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.038	4.325	0.424	0.000	0.467	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	430	586	0	470	0	0	-1
N.S.	1	1.00	2.30	3.13	0.00	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.141	2.321	0.647	0.000	1.010	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	436	631	0	630	0	0	-1
N.S.	1	1.00	1.87	2.71	0.00	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.183	2.748	0.679	0.000	0.910	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	3767	2270	0	1608	0	0	-1
N.S.	1	1.00	10.86	6.54	0.00	4.63	0.00	0.00	-0.00
time (sec)	N/A	0.371	6.456	1.218	0.000	0.784	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	2190	1494	0	1494	0	0	-1
N.S.	1	1.00	7.74	5.28	0.00	5.28	0.00	0.00	-0.00
time (sec)	N/A	0.190	6.219	0.629	0.000	2.047	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	1408	691	0	1378	0	0	-1
N.S.	1	1.00	13.04	6.40	0.00	12.76	0.00	0.00	-0.01
time (sec)	N/A	0.047	6.174	1.280	0.000	0.883	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	285	295	0	509	0	0	-1
N.S.	1	1.00	2.64	2.73	0.00	4.71	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.435	0.345	0.000	0.660	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	1540	2645	0	1762	0	0	-1
N.S.	1	1.00	8.28	14.22	0.00	9.47	0.00	0.00	-0.01
time (sec)	N/A	0.069	6.326	1.016	0.000	1.563	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	2408	3642	0	2844	0	0	-1
N.S.	1	1.00	6.30	9.53	0.00	7.45	0.00	0.00	-0.00
time (sec)	N/A	0.260	6.474	4.914	0.000	2.200	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	4116	5028	0	5008	0	0	-1
N.S.	1	1.00	8.40	10.26	0.00	10.22	0.00	0.00	-0.00
time (sec)	N/A	0.419	6.685	20.096	0.000	2.003	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	130	74	0	114	0	0	-1
N.S.	1	1.00	0.94	0.53	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.679	0.335	0.000	1.327	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	104	60	0	92	0	0	-1
N.S.	1	1.00	1.12	0.65	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.375	0.268	0.000	1.716	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	70	0	0	39
N.S.	1	1.00	1.70	1.14	0.00	1.59	0.00	0.00	0.89
time (sec)	N/A	0.012	0.045	0.256	0.000	2.005	0.000	0.000	0.310

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	101	77	0	158	0	0	-1
N.S.	1	1.00	2.10	1.60	0.00	3.29	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.123	0.640	0.000	2.034	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	154	117	0	297	0	0	-1
N.S.	1	1.00	1.60	1.22	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.336	0.322	0.000	1.305	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	180	190	0	378	0	0	-1
N.S.	1	1.00	1.27	1.34	0.00	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.455	0.348	0.000	1.711	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	151	86	0	136	0	0	-1
N.S.	1	1.00	0.82	0.46	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.092	0.337	0.000	1.988	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	127	74	0	114	0	0	-1
N.S.	1	1.00	0.91	0.53	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.549	0.372	0.000	1.352	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	103	60	0	91	0	0	-1
N.S.	1	1.00	1.11	0.65	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.250	0.260	0.000	3.964	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	68	0	0	39
N.S.	1	1.00	1.70	1.14	0.00	1.55	0.00	0.00	0.89
time (sec)	N/A	0.013	0.039	0.283	0.000	2.703	0.000	0.000	0.423

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	99	77	0	93	0	0	-1
N.S.	1	1.00	2.02	1.57	0.00	1.90	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.107	0.838	0.000	2.764	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	118	0	233	0	0	-1
N.S.	1	1.00	1.58	1.23	0.00	2.43	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.335	0.324	0.000	2.759	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	178	190	0	311	0	0	-1
N.S.	1	1.00	1.25	1.34	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.445	0.361	0.000	2.348	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	11888	306	0	283	0	0	-1
N.S.	1	1.00	46.08	1.19	0.00	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.128	47.140	0.459	0.000	2.202	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	11771	200	0	202	0	0	-1
N.S.	1	1.00	61.95	1.05	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.085	38.672	0.342	0.000	2.068	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	11679	126	0	136	0	0	-1
N.S.	1	1.00	92.69	1.00	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.053	21.848	0.367	0.000	2.600	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	11586	113	0	88	0	0	-1
N.S.	1	1.00	210.65	2.05	0.00	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.025	32.016	0.362	0.000	2.305	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	63264	172	0	366	0	0	-1
N.S.	1	1.00	718.91	1.95	0.00	4.16	0.00	0.00	-0.01
time (sec)	N/A	0.079	51.136	1.234	0.000	3.640	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	0	350	0	683	0	0	-1
N.S.	1	1.00	0.00	2.19	0.00	4.27	0.00	0.00	-0.01
time (sec)	N/A	0.090	180.002	0.418	0.000	3.098	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	350	0	942	0	0	-1
N.S.	1	1.00	0.00	1.55	0.00	4.17	0.00	0.00	-0.00
time (sec)	N/A	0.123	180.054	0.429	0.000	3.503	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	11602	204	0	205	0	0	-1
N.S.	1	1.00	59.19	1.04	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.091	50.972	0.409	0.000	2.798	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	11512	130	0	138	0	0	-1
N.S.	1	1.00	88.55	1.00	0.00	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.056	27.522	0.375	0.000	2.409	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	5053	117	0	88	0	0	-1
N.S.	1	1.00	88.65	2.05	0.00	1.54	0.00	0.00	-0.02
time (sec)	N/A	0.025	17.446	0.358	0.000	1.937	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	61904	175	0	112	0	0	-1
N.S.	1	1.00	680.26	1.92	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.069	55.272	0.971	0.000	2.612	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	B	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	0	363	0	467	0	0	-1
N.S.	1	1.00	0.00	2.21	0.00	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.084	180.001	0.441	0.000	2.800	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	A	F(-2)	B	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	0	363	0	690	0	0	-1
N.S.	1	1.00	0.00	1.56	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.121	180.028	0.438	0.000	3.013	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	176	0	579	0	160	950
N.S.	1	1.00	0.79	1.74	0.00	5.73	0.00	1.58	9.41
time (sec)	N/A	0.079	0.161	0.336	0.000	2.777	0.000	0.474	11.444

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	22	27	41	11	22	25	34
N.S.	1	1.36	1.00	1.23	1.86	0.50	1.00	1.14	1.55
time (sec)	N/A	0.021	0.030	0.102	0.476	2.204	0.111	0.423	2.789

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	79	171	0	553	0	158	988
N.S.	1	1.00	0.81	1.76	0.00	5.70	0.00	1.63	10.19
time (sec)	N/A	0.087	0.141	0.283	0.000	3.013	0.000	0.453	13.029

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
N.S.	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.049	0.034	0.189	0.000	3.027	0.000	0.503	2.777

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	120	180	0	663	0	161	977
N.S.	1	1.00	0.85	1.27	0.00	4.67	0.00	1.13	6.88
time (sec)	N/A	0.353	0.207	0.401	0.000	6.057	0.000	0.425	11.437

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	70673	0	1518	0	0	-1
N.S.	1	1.00	6.71	190.49	0.00	4.09	0.00	0.00	-0.00
time (sec)	N/A	0.302	6.316	8.422	0.000	2.078	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	45041	0	1374	0	0	-1
N.S.	1	1.00	13.39	381.70	0.00	11.64	0.00	0.00	-0.01
time (sec)	N/A	0.099	6.188	2.329	0.000	2.513	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	1133	0	507	0	0	-1
N.S.	1	1.00	2.87	9.60	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.718	1.295	0.000	1.123	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	44834	0	1752	0	0	-1
N.S.	1	1.00	7.22	186.81	0.00	7.30	0.00	0.00	-0.00
time (sec)	N/A	0.145	6.300	0.990	0.000	2.594	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	171436	0	2812	0	0	-1
N.S.	1	1.00	5.50	348.45	0.00	5.72	0.00	0.00	-0.00
time (sec)	N/A	0.353	6.337	2.398	0.000	2.353	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	70663	0	1513	0	0	-1
N.S.	1	1.00	0.00	190.47	0.00	4.08	0.00	0.00	-0.00
time (sec)	N/A	0.257	173.161	1.645	0.000	1.999	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	44690	0	1378	0	0	-1
N.S.	1	1.00	0.00	378.73	0.00	11.68	0.00	0.00	-0.01
time (sec)	N/A	0.096	13.707	1.105	0.000	2.340	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	506	1125	0	509	0	0	-1
N.S.	1	1.00	4.29	9.53	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.096	2.079	1.184	0.000	0.857	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	44824	0	1779	0	0	-1
N.S.	1	1.00	0.00	186.77	0.00	7.41	0.00	0.00	-0.00
time (sec)	N/A	0.142	18.662	0.870	0.000	2.368	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	169879	0	2857	0	0	-1
N.S.	1	1.00	0.00	345.28	0.00	5.81	0.00	0.00	-0.00
time (sec)	N/A	0.343	32.560	1.750	0.000	3.105	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	176	0	555	0	158	965
N.S.	1	1.00	0.82	1.80	0.00	5.66	0.00	1.61	9.85
time (sec)	N/A	0.076	0.161	0.291	0.000	3.553	0.000	0.407	13.762

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	53	0	349	0	73	47
N.S.	1	1.00	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.048	0.035	0.164	0.000	3.868	0.000	0.426	2.787

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	104	123	0	669	0	142	531
N.S.	1	1.00	0.87	1.02	0.00	5.58	0.00	1.18	4.42
time (sec)	N/A	0.366	0.205	0.363	0.000	7.684	0.000	0.451	8.698

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	51	10	14	24	0	22	9
N.S.	1	1.00	2.43	0.48	0.67	1.14	0.00	1.05	0.43
time (sec)	N/A	0.032	0.018	0.112	0.480	2.784	0.000	0.425	3.141

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	2490	20867	0	1525	0	0	-1
N.S.	1	1.00	6.71	56.25	0.00	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.292	6.312	5.533	0.000	2.657	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	12367	0	1368	0	0	-1
N.S.	1	1.00	13.39	104.81	0.00	11.59	0.00	0.00	-0.01
time (sec)	N/A	0.094	6.148	1.773	0.000	2.774	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	339	701	0	511	0	0	-1
N.S.	1	1.00	2.87	5.94	0.00	4.33	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.524	2.027	0.000	0.664	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	1732	12477	0	1739	0	0	-1
N.S.	1	1.00	7.22	51.99	0.00	7.25	0.00	0.00	-0.00
time (sec)	N/A	0.139	6.294	0.885	0.000	1.959	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	2708	64203	0	2820	0	0	-1
N.S.	1	1.00	5.50	130.49	0.00	5.73	0.00	0.00	-0.00
time (sec)	N/A	0.345	6.320	1.628	0.000	3.387	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	0	20858	0	1513	0	0	-1
N.S.	1	1.00	0.00	56.22	0.00	4.08	0.00	0.00	-0.00
time (sec)	N/A	0.259	91.440	1.116	0.000	3.026	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	0	12365	0	1378	0	0	-1
N.S.	1	1.00	0.00	104.79	0.00	11.68	0.00	0.00	-0.01
time (sec)	N/A	0.098	7.891	0.848	0.000	2.378	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	519	695	0	509	0	0	-1
N.S.	1	1.00	4.40	5.89	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.100	2.072	0.855	0.000	0.676	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	0	12464	0	1773	0	0	-1
N.S.	1	1.00	0.00	51.93	0.00	7.39	0.00	0.00	-0.00
time (sec)	N/A	0.140	16.759	0.772	0.000	1.514	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	0	64185	0	2839	0	0	-1
N.S.	1	1.00	0.00	130.46	0.00	5.77	0.00	0.00	-0.00
time (sec)	N/A	0.348	31.065	1.307	0.000	2.097	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.007	0.000	0.065	0.468	2.061	0.163	0.435	2.645

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.007	0.000	0.059	0.473	2.334	0.313	0.436	2.630

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.006	0.000	0.068	0.471	2.841	0.595	0.423	2.594

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	36	33	3
N.S.	1	1.00	2.09	0.36	1.36	2.09	3.27	3.00	0.27
time (sec)	N/A	0.010	0.008	0.084	0.258	2.202	0.153	0.432	2.905

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	8	18	12	15	48	6	6
N.S.	1	1.00	0.62	1.38	0.92	1.15	3.69	0.46	0.46
time (sec)	N/A	0.018	0.004	0.092	0.269	2.838	0.608	0.429	2.633

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	48	38	74	765	37	32
N.S.	1	1.00	0.69	1.50	1.19	2.31	23.91	1.16	1.00
time (sec)	N/A	0.020	0.006	0.122	0.264	3.066	1.557	0.449	2.651

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	35	12007	20	9
N.S.	1	1.00	1.00	1.11	1.00	3.89	1334.11	2.22	1.00
time (sec)	N/A	0.013	0.024	0.116	0.478	2.941	10.295	0.445	2.834

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	31	30189	22	11
N.S.	1	1.00	1.00	1.09	1.00	2.82	2744.45	2.00	1.00
time (sec)	N/A	0.013	0.024	0.125	0.469	2.215	12.758	0.428	2.832

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.017	0.034	0.128	0.480	2.438	18.696	0.452	2.845

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	25	18	17	43	87	61	36
N.S.	1	1.00	0.47	0.34	0.32	0.81	1.64	1.15	0.68
time (sec)	N/A	0.024	0.033	0.210	0.476	2.885	0.303	0.425	2.759

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	38	35	182	201	48	51
N.S.	1	1.00	0.84	0.88	0.81	4.23	4.67	1.12	1.19
time (sec)	N/A	0.103	0.070	0.167	0.465	2.510	0.605	0.413	2.730

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	38	35	181	226	48	48
N.S.	1	1.00	0.84	0.88	0.81	4.21	5.26	1.12	1.12
time (sec)	N/A	0.075	0.041	0.151	0.484	2.598	0.631	0.439	2.672

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	19	18	15	35	0	15	15
N.S.	1	1.00	0.53	0.50	0.42	0.97	0.00	0.42	0.42
time (sec)	N/A	0.019	0.034	0.125	0.474	2.540	0.000	0.405	2.652

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	29	27	68	0	27	27
N.S.	1	1.00	0.86	0.59	0.55	1.39	0.00	0.55	0.55
time (sec)	N/A	0.030	0.100	0.124	0.478	3.002	0.000	0.405	2.664

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	42	45	100	0	39	40
N.S.	1	1.00	1.07	0.57	0.61	1.35	0.00	0.53	0.54
time (sec)	N/A	0.036	0.131	0.142	0.474	2.464	0.000	0.419	2.698

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
N.S.	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.008	0.000	0.085	0.468	1.895	0.000	0.451	2.746

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
N.S.	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.008	0.000	0.064	0.487	2.863	0.000	0.424	2.581

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
N.S.	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.008	0.000	0.065	0.469	2.382	0.000	0.430	2.571

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	19	19	16	35	0	49	16
N.S.	1	1.00	0.51	0.51	0.43	0.95	0.00	1.32	0.43
time (sec)	N/A	0.019	0.031	0.153	0.486	3.058	0.000	0.422	2.713

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	30	27	66	0	60	27
N.S.	1	1.00	1.36	0.64	0.57	1.40	0.00	1.28	0.57
time (sec)	N/A	0.025	0.077	0.161	0.462	2.663	0.000	0.420	2.688

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	41	42	98	0	69	43
N.S.	1	1.00	0.92	0.57	0.58	1.36	0.00	0.96	0.60
time (sec)	N/A	0.046	0.116	0.191	0.476	2.329	0.000	0.423	2.683

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.081	0.486	2.456	0.000	0.406	2.729

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	4	1	1	0	1	1
N.S.	1	1.00	1.00	4.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.008	0.000	0.065	0.462	2.395	0.000	0.425	2.650

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	0	3	3
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.009	0.000	0.073	0.470	2.518	0.000	0.425	2.615

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	259	0	61	43
N.S.	1	1.00	1.00	0.82	0.79	7.85	0.00	1.85	1.30
time (sec)	N/A	0.032	0.047	0.146	0.483	3.518	0.000	0.424	2.855

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	507	820	0	2869	0	0	-1
N.S.	1	1.00	2.12	3.43	0.00	12.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	2.129	0.210	0.000	5.285	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	258	1161	0	4289	0	0	-1
N.S.	1	1.00	0.71	3.18	0.00	11.75	0.00	0.00	-0.00
time (sec)	N/A	0.481	2.728	0.185	0.000	5.583	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	149	255	265	156	566	158	456
N.S.	1	1.00	0.76	1.31	1.36	0.80	2.90	0.81	2.34
time (sec)	N/A	0.259	0.645	0.189	0.266	2.635	0.388	0.441	4.490

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	115	120	80	204	79	88
N.S.	1	1.00	0.71	1.06	1.10	0.73	1.87	0.72	0.81
time (sec)	N/A	0.065	0.208	0.119	0.287	2.794	0.142	0.405	2.898

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	54	0	27	0	50	39
N.S.	1	1.00	1.00	2.35	0.00	1.17	0.00	2.17	1.70
time (sec)	N/A	0.059	0.045	0.323	0.000	2.293	0.000	0.427	2.841

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	140	398	0	832	0	435	497
N.S.	1	1.00	0.89	2.54	0.00	5.30	0.00	2.77	3.17
time (sec)	N/A	0.251	0.674	0.683	0.000	2.769	0.000	0.453	6.063

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	286	263	0	6647	0	0	2500
N.S.	1	1.00	1.18	1.09	0.00	27.47	0.00	0.00	10.33
time (sec)	N/A	0.609	0.470	4.987	0.000	20.504	0.000	0.000	17.109

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	140	269	595	117	0	229	-1
N.S.	1	1.00	0.42	0.81	1.80	0.35	0.00	0.69	-0.00
time (sec)	N/A	0.217	0.595	1.055	0.516	2.243	0.000	0.494	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	107	202	46	0	94	-1
N.S.	1	1.00	0.38	0.58	1.09	0.25	0.00	0.51	-0.01
time (sec)	N/A	0.070	0.131	0.416	0.500	2.221	0.000	0.447	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	85	170	0	213	0	110	-1
N.S.	1	1.00	0.62	1.24	0.00	1.55	0.00	0.80	-0.01
time (sec)	N/A	0.120	0.127	0.344	0.000	2.013	0.000	0.440	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	144	726	0	556	0	292	-1
N.S.	1	1.00	0.60	3.04	0.00	2.33	0.00	1.22	-0.00
time (sec)	N/A	0.168	0.251	0.337	0.000	2.449	0.000	0.468	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	33	0	11	0	32	24
N.S.	1	1.00	1.00	3.00	0.00	1.00	0.00	2.91	2.18
time (sec)	N/A	0.053	0.039	0.107	0.000	2.128	0.000	0.447	2.937

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	241	254	0	6649	0	5301	2500
N.S.	1	1.00	0.98	1.03	0.00	27.03	0.00	21.55	10.16
time (sec)	N/A	0.556	0.376	4.927	0.000	23.185	0.000	1.990	16.769

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	153	173	155	154	248	2157	205
N.S.	1	1.00	1.06	1.20	1.08	1.07	1.72	14.98	1.42
time (sec)	N/A	0.182	1.463	0.052	0.488	3.298	0.172	2.351	2.854

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	88	84	77	77	122	652	105
N.S.	1	1.00	1.22	1.17	1.07	1.07	1.69	9.06	1.46
time (sec)	N/A	0.056	0.237	0.030	0.480	3.641	0.091	0.776	2.770

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	187	125	164	201	1358	199	152
N.S.	1	1.00	1.85	1.24	1.62	1.99	13.45	1.97	1.50
time (sec)	N/A	0.172	1.676	0.194	0.474	2.722	0.759	0.699	3.089

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	308	218	426	600	0	437	388
N.S.	1	1.00	1.56	1.11	2.16	3.05	0.00	2.22	1.97
time (sec)	N/A	0.360	3.770	0.250	0.486	2.571	0.000	0.953	3.261

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	147	158	174	106	0	1615	-1
N.S.	1	1.00	0.52	0.56	0.61	0.37	0.00	5.69	-0.00
time (sec)	N/A	0.154	1.372	0.382	0.483	2.717	0.000	1.355	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	58	75	69	39	0	363	-1
N.S.	1	1.00	0.48	0.61	0.57	0.32	0.00	2.98	-0.01
time (sec)	N/A	0.067	0.203	0.234	0.473	2.390	0.000	0.548	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	88	114	142	74	0	161	-1
N.S.	1	1.00	0.64	0.83	1.03	0.54	0.00	1.17	-0.01
time (sec)	N/A	0.127	0.473	0.195	0.478	2.535	0.000	0.535	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	268	620	509	370	0	481	-1
N.S.	1	1.00	0.85	1.96	1.61	1.17	0.00	1.52	-0.00
time (sec)	N/A	0.272	2.251	0.175	0.496	2.176	0.000	0.732	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	130	256	323	208	0	447	323
N.S.	1	1.00	0.71	1.39	1.76	1.13	0.00	2.43	1.76
time (sec)	N/A	0.282	0.549	0.159	0.282	3.030	0.000	0.528	3.304

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	112	136	133	0	182	160
N.S.	1	1.00	0.84	1.47	1.79	1.75	0.00	2.39	2.11
time (sec)	N/A	0.051	0.192	0.128	0.267	2.896	0.000	0.461	2.908

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	120	0	303	0	139	444
N.S.	1	1.00	1.05	1.30	0.00	3.29	0.00	1.51	4.83
time (sec)	N/A	0.207	0.266	0.302	0.000	4.017	0.000	0.473	3.023

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	276	307	0	1383	0	469	2500
N.S.	1	1.00	1.20	1.33	0.00	6.01	0.00	2.04	10.87
time (sec)	N/A	0.568	1.034	0.572	0.000	3.726	0.000	0.535	11.473

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	128	390	469	171	0	605	-1
N.S.	1	1.00	0.36	1.09	1.31	0.48	0.00	1.69	-0.00
time (sec)	N/A	0.192	0.567	1.289	0.495	3.834	0.000	0.570	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	67	210	177	92	0	208	-1
N.S.	1	1.00	0.39	1.21	1.02	0.53	0.00	1.20	-0.01
time (sec)	N/A	0.081	0.191	0.418	0.496	3.756	0.000	0.475	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	92	157	0	192	0	383	-1
N.S.	1	1.00	0.65	1.11	0.00	1.35	0.00	2.70	-0.01
time (sec)	N/A	0.141	0.259	0.478	0.000	3.659	0.000	0.831	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	216	756	0	834	0	405	-1
N.S.	1	1.00	0.65	2.29	0.00	2.53	0.00	1.23	-0.00
time (sec)	N/A	0.375	0.726	0.497	0.000	3.431	0.000	0.541	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	8	14	16
N.S.	1	1.00	1.12	0.47	0.00	0.35	0.47	0.82	0.94
time (sec)	N/A	0.026	0.009	0.194	0.000	2.752	0.026	0.424	2.760

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	10	14	13
N.S.	1	1.00	1.12	0.47	0.00	0.35	0.59	0.82	0.76
time (sec)	N/A	0.024	0.009	0.181	0.000	2.862	0.024	0.416	2.745

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	11	7	16	32
N.S.	1	1.00	1.00	1.17	1.00	1.83	1.17	2.67	5.33
time (sec)	N/A	0.015	0.020	0.070	0.258	3.113	0.048	0.447	2.918

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	66	181	59	360	77	1976
N.S.	1	1.00	0.83	1.40	3.85	1.26	7.66	1.64	42.04
time (sec)	N/A	0.027	0.102	0.151	0.477	2.917	0.423	0.462	12.819

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	113	271	194	0	132	129
N.S.	1	1.00	1.01	1.53	3.66	2.62	0.00	1.78	1.74
time (sec)	N/A	0.048	0.177	0.261	0.476	3.027	0.000	0.455	3.007

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	37	199	152	0	26	95
N.S.	1	1.00	0.97	0.56	3.02	2.30	0.00	0.39	1.44
time (sec)	N/A	0.040	0.139	0.247	0.285	2.696	0.000	0.447	2.823

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	167	243	155	1030	148	1099
N.S.	1	1.00	0.93	1.99	2.89	1.85	12.26	1.76	13.08
time (sec)	N/A	0.043	0.180	0.302	0.482	2.668	21.549	0.478	9.406

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	92	124	286	226	0	150	141
N.S.	1	1.00	1.08	1.46	3.36	2.66	0.00	1.76	1.66
time (sec)	N/A	0.040	0.198	0.237	0.514	1.995	0.000	0.450	3.051

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	218	451	311	0	270	264
N.S.	1	1.00	0.95	1.69	3.50	2.41	0.00	2.09	2.05
time (sec)	N/A	0.091	0.458	0.300	0.509	2.780	0.000	0.496	3.439

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	185	0	625	0	178	1709
N.S.	1	1.00	0.83	1.61	0.00	5.43	0.00	1.55	14.86
time (sec)	N/A	0.097	0.201	0.435	0.000	2.897	0.000	0.435	25.561

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	206	0	1277	0	209	205
N.S.	1	1.00	1.04	1.82	0.00	11.30	0.00	1.85	1.81
time (sec)	N/A	0.085	0.244	0.342	0.000	2.818	0.000	0.410	3.253

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	326	853	0	3402	0	1162	946
N.S.	1	1.00	1.63	4.26	0.00	17.01	0.00	5.81	4.73
time (sec)	N/A	0.188	0.624	0.792	0.000	4.438	0.000	0.470	6.845

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	104	0	72	94	168	99
N.S.	1	1.00	1.75	1.24	0.00	0.86	1.12	2.00	1.18
time (sec)	N/A	0.030	0.155	0.300	0.000	3.000	0.315	0.414	4.333

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	147	116	0	56	73	168	584
N.S.	1	1.00	1.75	1.38	0.00	0.67	0.87	2.00	6.95
time (sec)	N/A	0.029	0.134	0.299	0.000	2.677	0.318	0.412	8.386

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	187	0	625	0	177	1741
N.S.	1	1.00	0.83	1.61	0.00	5.39	0.00	1.53	15.01
time (sec)	N/A	0.081	0.180	0.444	0.000	4.479	0.000	0.408	24.863

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	123	207	0	1301	0	206	204
N.S.	1	1.00	1.08	1.82	0.00	11.41	0.00	1.81	1.79
time (sec)	N/A	0.080	0.277	0.339	0.000	4.022	0.000	0.427	3.191

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	361	832	0	3513	0	1054	912
N.S.	1	1.00	1.80	4.16	0.00	17.56	0.00	5.27	4.56
time (sec)	N/A	0.175	0.651	0.794	0.000	4.341	0.000	0.480	6.734

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	107	0	71	100	169	96
N.S.	1	1.00	1.79	1.26	0.00	0.84	1.18	1.99	1.13
time (sec)	N/A	0.030	0.202	0.273	0.000	2.993	0.418	0.427	4.354

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	152	119	0	57	78	169	96
N.S.	1	1.00	1.79	1.40	0.00	0.67	0.92	1.99	1.13
time (sec)	N/A	0.032	0.183	0.287	0.000	2.784	0.403	0.425	4.343

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	98	217	0	687	0	187	1864
N.S.	1	1.00	0.82	1.82	0.00	5.77	0.00	1.57	15.66
time (sec)	N/A	0.088	0.247	0.510	0.000	3.394	0.000	0.426	28.555

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	116	206	0	1316	0	205	202
N.S.	1	1.00	1.05	1.87	0.00	11.96	0.00	1.86	1.84
time (sec)	N/A	0.077	0.311	0.358	0.000	2.885	0.000	0.414	3.238

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	311	795	0	3264	0	1034	923
N.S.	1	1.00	1.58	4.04	0.00	16.57	0.00	5.25	4.69
time (sec)	N/A	0.155	0.643	0.726	0.000	3.306	0.000	0.505	6.362

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	87	195	123	0	78	110	178	118
N.S.	1	0.95	2.12	1.34	0.00	0.85	1.20	1.93	1.28
time (sec)	N/A	0.051	0.226	0.302	0.000	2.392	0.463	0.420	5.319

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	195	133	0	68	99	178	118
N.S.	1	0.94	2.17	1.48	0.00	0.76	1.10	1.98	1.31
time (sec)	N/A	0.052	0.216	0.289	0.000	2.265	0.430	0.436	4.506

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	227	0	711	0	199	2711
N.S.	1	1.00	0.84	1.73	0.00	5.43	0.00	1.52	20.69
time (sec)	N/A	0.098	0.262	0.806	0.000	2.900	0.000	0.446	55.105

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	137	231	0	1556	0	241	227
N.S.	1	1.00	1.08	1.82	0.00	12.25	0.00	1.90	1.79
time (sec)	N/A	0.096	0.359	0.381	0.000	2.657	0.000	0.426	3.487

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	452	1080	0	4240	0	1506	1160
N.S.	1	1.00	1.91	4.56	0.00	17.89	0.00	6.35	4.89
time (sec)	N/A	0.213	0.871	0.917	0.000	2.881	0.000	0.500	8.215

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	165	136	0	89	129	203	132
N.S.	1	1.00	1.57	1.30	0.00	0.85	1.23	1.93	1.26
time (sec)	N/A	0.051	0.298	0.297	0.000	2.828	0.738	0.421	6.927

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	148	0	73	105	203	133
N.S.	1	1.00	1.62	1.44	0.00	0.71	1.02	1.97	1.29
time (sec)	N/A	0.052	0.323	0.295	0.000	3.229	0.691	0.433	6.858

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	68	32	70	0	24	0	68	62
N.S.	1	2.83	1.33	2.92	0.00	1.00	0.00	2.83	2.58
time (sec)	N/A	0.047	0.068	0.233	0.000	4.309	0.000	0.434	3.031

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	7823	3452	0	2027	0	0	-1
N.S.	1	1.00	20.06	8.85	0.00	5.20	0.00	0.00	-0.00
time (sec)	N/A	0.611	6.675	1.816	0.000	4.386	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	5218	2205	0	1703	0	0	-1
N.S.	1	1.00	17.75	7.50	0.00	5.79	0.00	0.00	-0.00
time (sec)	N/A	0.369	6.397	1.006	0.000	1.742	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	3006	1438	0	1506	0	0	-1
N.S.	1	1.00	13.13	6.28	0.00	6.58	0.00	0.00	-0.00
time (sec)	N/A	0.219	6.249	0.678	0.000	1.999	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	570	766	0	1358	0	0	-1
N.S.	1	1.00	3.17	4.26	0.00	7.54	0.00	0.00	-0.01
time (sec)	N/A	0.129	5.557	1.095	0.000	1.833	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	3176	2912	0	2106	0	0	-1
N.S.	1	1.00	12.70	11.65	0.00	8.42	0.00	0.00	-0.00
time (sec)	N/A	0.225	6.395	5.618	0.000	2.054	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	5554	4020	0	4092	0	0	-1
N.S.	1	1.00	14.69	10.63	0.00	10.83	0.00	0.00	-0.00
time (sec)	N/A	0.386	6.640	9.223	0.000	3.630	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	80	128	0	359	1110	136	1143
N.S.	1	1.00	0.95	1.52	0.00	4.27	13.21	1.62	13.61
time (sec)	N/A	0.113	0.353	0.394	0.000	4.076	27.027	0.428	9.625

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	155	0	477	0	180	227
N.S.	1	1.00	0.97	1.31	0.00	4.04	0.00	1.53	1.92
time (sec)	N/A	0.110	0.352	0.461	0.000	2.778	0.000	0.421	3.327

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	174	419	0	909	0	571	557
N.S.	1	1.00	0.94	2.26	0.00	4.91	0.00	3.09	3.01
time (sec)	N/A	0.171	0.620	0.645	0.000	4.573	0.000	0.446	5.272

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	244	860	0	1448	0	1281	1085
N.S.	1	1.00	0.95	3.33	0.00	5.61	0.00	4.97	4.21
time (sec)	N/A	0.276	1.720	0.908	0.000	3.438	0.000	0.462	6.218

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	145	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.427	0.231	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	75	106	80	97	190	75	125
N.S.	1	1.00	0.70	0.99	0.75	0.91	1.78	0.70	1.17
time (sec)	N/A	0.057	0.197	0.313	0.297	2.869	0.440	0.427	3.437

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	69	48	63	129	46	78
N.S.	1	1.00	0.79	1.13	0.79	1.03	2.11	0.75	1.28
time (sec)	N/A	0.023	0.117	0.241	0.263	3.008	0.186	0.424	3.034

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	22	26	18	22
N.S.	1	1.00	1.90	0.95	0.90	1.10	1.30	0.90	1.10
time (sec)	N/A	0.010	0.007	0.106	0.268	3.384	0.063	0.411	2.942

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	0	290	0	61	44
N.S.	1	1.00	1.00	0.94	0.00	6.04	0.00	1.27	0.92
time (sec)	N/A	0.046	0.052	0.290	0.000	2.482	0.000	0.401	3.114

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	94	114	0	493	0	116	181
N.S.	1	1.00	0.99	1.20	0.00	5.19	0.00	1.22	1.91
time (sec)	N/A	0.068	0.290	0.465	0.000	2.840	0.000	0.428	3.050

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	120	271	0	969	0	252	396
N.S.	1	1.00	0.81	1.82	0.00	6.50	0.00	1.69	2.66
time (sec)	N/A	0.116	0.648	0.660	0.000	2.706	0.000	0.436	3.874

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	202	1138	0	0	0	0	-1
N.S.	1	1.00	0.76	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	1.283	0.550	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	167	844	0	0	0	0	-1
N.S.	1	1.00	0.79	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	1.051	0.418	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	312	0	0	0	0	-1
N.S.	1	1.00	0.99	4.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.077	0.340	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	165	0	302	0	0	-1
N.S.	1	1.00	0.92	2.17	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.099	0.283	0.000	0.487	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	101	570	0	0	0	0	-1
N.S.	1	1.00	0.71	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.306	0.385	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	201	1553	0	0	0	0	-1
N.S.	1	1.00	0.68	5.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	1.058	0.559	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	340	2282	0	3308	0	0	-1
N.S.	1	1.00	0.74	4.95	0.00	7.18	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.534	0.206	0.000	3.721	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	256	1782	0	2492	0	0	-1
N.S.	1	1.00	0.75	5.24	0.00	7.33	0.00	0.00	-0.00
time (sec)	N/A	0.361	0.438	0.151	0.000	4.835	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	788	1284	0	1676	0	0	-1
N.S.	1	1.00	3.50	5.71	0.00	7.45	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.986	0.151	0.000	4.746	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.054	0.918	0.116	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	3.400	0.847	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	3.146	0.812	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	198	0	0	186	0	7347	-1
N.S.	1	1.00	1.13	0.00	0.00	1.06	0.00	41.98	-0.01
time (sec)	N/A	0.198	0.692	180.000	0.000	2.723	0.000	0.818	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	142	0	0	142	0	4175	-1
N.S.	1	1.00	1.08	0.00	0.00	1.08	0.00	31.87	-0.01
time (sec)	N/A	0.162	0.495	180.000	0.000	3.203	0.000	0.667	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	0	77	0	1033	-1
N.S.	1	1.00	0.96	0.00	0.00	0.96	0.00	12.91	-0.01
time (sec)	N/A	0.089	0.400	180.000	0.000	2.389	0.000	0.580	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	242	108	0	45	0	496	-1
N.S.	1	1.00	4.32	1.93	0.00	0.80	0.00	8.86	-0.02
time (sec)	N/A	0.071	4.745	2.981	0.000	3.026	0.000	0.477	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	54	114	24	20	39	24
N.S.	1	1.00	0.69	1.54	3.26	0.69	0.57	1.11	0.69
time (sec)	N/A	0.017	0.221	1.158	0.259	2.592	1.812	0.440	3.028

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	21	19	42	23
N.S.	1	1.00	1.00	1.05	1.00	1.05	0.95	2.10	1.15
time (sec)	N/A	0.027	0.022	0.253	0.270	2.279	1.779	0.423	0.118

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	100	34	0	53	-1
N.S.	1	1.00	0.91	1.11	2.86	0.97	0.00	1.51	-0.03
time (sec)	N/A	0.027	0.167	0.875	0.266	3.655	0.000	0.451	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	157	0	0	295	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.500	0.851	0.000	2.756	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	172	594	410	0	0	-1
N.S.	1	1.00	0.80	1.35	4.68	3.23	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.615	0.766	0.307	2.966	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	194	0	0	162	0	7279	-1
N.S.	1	1.00	1.10	0.00	0.00	0.92	0.00	41.36	-0.01
time (sec)	N/A	0.210	0.562	180.000	0.000	3.350	0.000	0.806	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	136	0	0	185	0	3130	-1
N.S.	1	1.00	1.03	0.00	0.00	1.40	0.00	23.71	-0.01
time (sec)	N/A	0.171	0.410	180.000	0.000	2.747	0.000	0.634	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	73	0	997	-1
N.S.	1	1.00	0.89	0.00	0.00	0.91	0.00	12.46	-0.01
time (sec)	N/A	0.091	0.329	180.000	0.000	2.795	0.000	0.552	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	237	106	0	62	0	366	-1
N.S.	1	1.00	4.23	1.89	0.00	1.11	0.00	6.54	-0.02
time (sec)	N/A	0.070	4.689	3.092	0.000	2.154	0.000	0.503	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	22	55	114	23	20	32	22
N.S.	1	1.00	0.65	1.62	3.35	0.68	0.59	0.94	0.65
time (sec)	N/A	0.017	0.156	1.167	0.282	2.056	1.692	0.439	0.163

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	20	40	22
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.05	2.11	1.16
time (sec)	N/A	0.041	0.016	0.257	0.272	2.600	1.712	0.450	0.093

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	40	100	36	0	52	-1
N.S.	1	1.00	0.94	1.21	3.03	1.09	0.00	1.58	-0.03
time (sec)	N/A	0.026	0.128	0.842	0.276	2.365	0.000	0.408	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	0	0	290	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.511	0.910	0.000	2.523	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	130	141	372	382	0	0	-1
N.S.	1	1.00	1.05	1.14	3.00	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.476	0.868	0.506	3.178	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	64	98	0	106	0	0	463
N.S.	1	1.00	0.41	0.62	0.00	0.68	0.00	0.00	2.95
time (sec)	N/A	0.319	0.168	2.680	0.000	2.788	0.000	0.000	8.884

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	62	88	0	84	0	0	148
N.S.	1	1.00	0.56	0.80	0.00	0.76	0.00	0.00	1.35
time (sec)	N/A	0.202	0.127	0.783	0.000	2.711	0.000	0.000	11.027

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	44	78	0	64	0	413066	129
N.S.	1	1.00	0.61	1.08	0.00	0.89	0.00	5737.03	1.79
time (sec)	N/A	0.139	0.127	0.945	0.000	3.457	0.000	120.437	7.343

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	52	0	40	0	80097	87
N.S.	1	1.00	0.91	1.58	0.00	1.21	0.00	2427.18	2.64
time (sec)	N/A	0.044	0.061	0.806	0.000	3.370	0.000	30.725	3.677

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	73	136	430	201	0	0	-1
N.S.	1	1.00	1.62	3.02	9.56	4.47	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.090	0.894	0.516	3.646	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	391	1049	351	0	0	-1
N.S.	1	1.00	1.10	4.65	12.49	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.170	1.108	0.593	3.209	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	105	657	1421	419	0	0	-1
N.S.	1	1.00	0.81	5.09	11.02	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.178	0.816	0.616	3.039	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	116	933	2333	481	0	0	-1
N.S.	1	1.00	0.66	5.30	13.26	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.202	0.794	0.891	2.901	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	85	105	0	132	0	0	594
N.S.	1	1.00	0.41	0.50	0.00	0.63	0.00	0.00	2.86
time (sec)	N/A	0.343	0.237	1.160	0.000	2.437	0.000	0.000	10.183

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	73	95	0	111	0	0	479
N.S.	1	1.00	0.49	0.64	0.00	0.75	0.00	0.00	3.24
time (sec)	N/A	0.237	0.153	0.654	0.000	2.494	0.000	0.000	9.262

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	59	85	0	88	0	0	149
N.S.	1	1.00	0.54	0.77	0.00	0.80	0.00	0.00	1.35
time (sec)	N/A	0.176	0.152	0.598	0.000	2.520	0.000	0.000	11.037

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	51	61	0	67	0	0	158
N.S.	1	1.00	0.68	0.81	0.00	0.89	0.00	0.00	2.11
time (sec)	N/A	0.072	0.112	0.551	0.000	2.702	0.000	0.000	7.866

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	253	1317	296	0	0	-1
N.S.	1	1.00	1.08	3.16	16.46	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.110	3.352	0.613	3.002	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	93	518	1058	369	0	0	-1
N.S.	1	1.00	1.08	6.02	12.30	4.29	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.168	1.051	0.597	2.732	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	105	792	0	437	0	0	-1
N.S.	1	1.00	0.79	5.95	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.197	0.699	0.000	2.517	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	117	1078	0	503	0	0	-1
N.S.	1	1.00	0.64	5.92	0.00	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.170	0.742	0.000	2.786	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	112	984	0	380	0	0	-1
N.S.	1	1.00	0.64	5.62	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.417	0.494	1.719	0.000	2.812	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	89	677	0	294	0	0	-1
N.S.	1	1.00	0.69	5.25	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.280	0.813	0.000	2.503	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	478	0	245	0	0	-1
N.S.	1	1.00	0.76	5.43	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.172	1.010	0.000	1.910	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	236	0	146	0	0	-1
N.S.	1	1.00	1.16	4.29	0.00	2.65	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.119	0.707	0.000	2.050	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	298	0	309	0	0	-1
N.S.	1	1.00	0.96	2.98	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.407	0.697	0.000	1.713	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	163	1023	0	481	0	0	-1
N.S.	1	1.00	1.18	7.41	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.797	1.139	0.000	2.346	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	186	1828	0	569	0	0	-1
N.S.	1	1.00	1.02	10.04	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.251	2.653	0.754	0.000	2.310	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	100	1211	0	350	0	0	-1
N.S.	1	1.00	0.56	6.73	0.00	1.94	0.00	0.00	-0.01
time (sec)	N/A	0.342	0.907	1.965	0.000	1.821	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	94	930	0	276	0	0	-1
N.S.	1	1.00	0.73	7.27	0.00	2.16	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.424	0.752	0.000	1.742	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	433	0	272	0	0	-1
N.S.	1	1.00	0.90	4.66	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.393	0.713	0.000	2.663	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	599	0	269	0	0	-1
N.S.	1	1.00	0.89	6.44	0.00	2.89	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.424	0.705	0.000	3.369	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	198	561	0	438	0	0	-1
N.S.	1	1.00	1.43	4.07	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.584	0.652	0.000	2.739	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	217	1157	0	528	0	0	-1
N.S.	1	1.00	1.22	6.50	0.00	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.226	4.967	1.082	0.000	2.643	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	357	1787	0	616	0	0	-1
N.S.	1	1.00	1.53	7.64	0.00	2.63	0.00	0.00	-0.00
time (sec)	N/A	0.336	6.163	0.701	0.000	2.598	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	119	0	29	0	0	14
N.S.	1	1.00	1.00	7.44	0.00	1.81	0.00	0.00	0.88
time (sec)	N/A	0.062	0.028	0.205	0.000	2.546	0.000	0.000	3.099

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	396	0	120	0	0	-1
N.S.	1	1.00	1.16	5.74	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.255	0.818	0.224	0.000	2.808	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	122	396	0	120	0	0	-1
N.S.	1	1.00	1.54	5.01	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.400	0.246	0.217	0.000	2.997	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	761	0	136	0	0	-1
N.S.	1	1.00	0.97	8.01	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.412	3.260	0.333	0.000	3.011	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	51	31	30	59	138	85	63
N.S.	1	1.00	1.70	1.03	1.00	1.97	4.60	2.83	2.10
time (sec)	N/A	0.044	0.859	0.594	0.293	2.659	19.492	1.111	5.540

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	938	187	24	122	129	142	185
N.S.	1	1.00	36.08	7.19	0.92	4.69	4.96	5.46	7.12
time (sec)	N/A	0.029	6.386	0.564	0.272	2.801	3.526	0.916	3.566

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	152	24	92	100	0	100
N.S.	1	1.00	1.19	5.85	0.92	3.54	3.85	0.00	3.85
time (sec)	N/A	0.029	0.836	0.504	0.286	2.416	1.408	0.000	3.223

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	67	57	24	61	73	45	61
N.S.	1	1.00	2.58	2.19	0.92	2.35	2.81	1.73	2.35
time (sec)	N/A	0.018	0.030	0.457	0.280	2.163	0.555	0.566	3.177

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	29	23	22	33	61	42	133
N.S.	1	1.00	1.32	1.05	1.00	1.50	2.77	1.91	6.05
time (sec)	N/A	0.034	0.332	0.611	0.274	2.338	3.764	111.898	4.859

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	27	25	24	29	49	108	47
N.S.	1	1.00	1.12	1.04	1.00	1.21	2.04	4.50	1.96
time (sec)	N/A	0.031	0.214	0.655	0.264	2.658	11.292	0.697	3.236

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	25	24	63	80	37	291
N.S.	1	1.00	1.12	0.96	0.92	2.42	3.08	1.42	11.19
time (sec)	N/A	0.031	0.507	0.735	0.272	2.273	21.729	121.278	6.313

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.031	0.017	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.009	0.025	0.013	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.059	0.024	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.057	0.033	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.017	0.014	0.049	0.265	3.633	0.154	0.422	0.057

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	23	63	20	20
N.S.	1	1.00	0.95	1.05	1.00	1.15	3.15	1.00	1.00
time (sec)	N/A	0.019	0.033	1.046	0.283	4.293	0.505	0.408	3.150

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	5	36	0	14	5
N.S.	1	1.00	3.20	1.20	1.00	7.20	0.00	2.80	1.00
time (sec)	N/A	0.017	0.024	0.063	0.482	3.798	0.000	0.419	3.078

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.006	1.717	0.030	0.276	3.183	0.129	0.395	0.088

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	21	23	17	73	34	17	22
N.S.	1	1.00	0.75	0.82	0.61	2.61	1.21	0.61	0.79
time (sec)	N/A	0.018	0.948	0.063	0.280	3.275	0.813	0.408	3.046

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	168	54	20	20
N.S.	1	1.00	1.00	0.81	0.77	6.46	2.08	0.77	0.77
time (sec)	N/A	0.033	3.034	0.145	0.278	2.932	3.990	0.416	3.134

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	137	40	103	39	46	39	39
N.S.	1	1.00	3.81	1.11	2.86	1.08	1.28	1.08	1.08
time (sec)	N/A	0.060	0.232	0.085	0.275	2.500	1.355	0.417	0.090

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	22	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.44	0.78	0.78	0.78
time (sec)	N/A	0.009	1.763	0.027	0.268	2.960	0.130	0.448	3.010

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	24	26	17	17	26	17	17
N.S.	1	1.00	0.77	0.84	0.55	0.55	0.84	0.55	0.55
time (sec)	N/A	0.016	0.085	0.116	0.268	2.269	0.158	0.419	0.116

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	162	0	18	29	0	7	-1
N.S.	1	1.00	18.00	0.00	2.00	3.22	0.00	0.78	-0.11
time (sec)	N/A	0.049	1.460	0.652	0.487	1.710	0.000	0.443	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	49	51	34	0	75	-1
N.S.	1	1.00	0.83	0.69	0.72	0.48	0.00	1.06	-0.01
time (sec)	N/A	0.045	0.109	0.250	0.264	2.300	0.000	0.400	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	17	39	17	17
N.S.	1	1.00	1.00	1.00	0.94	0.94	2.17	0.94	0.94
time (sec)	N/A	0.010	0.038	0.039	0.273	2.951	0.121	0.416	0.101

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	53	23	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	2.30	1.00	1.00
time (sec)	N/A	0.011	0.104	0.089	0.270	1.775	5.553	0.443	3.160

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	24	23	23	51	23	23
N.S.	1	1.00	0.96	1.00	0.96	0.96	2.12	0.96	0.96
time (sec)	N/A	0.011	0.027	0.051	0.270	1.633	0.311	0.400	3.023

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	14	14	0	0	-1
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.00	0.00	-0.07
time (sec)	N/A	0.017	0.028	0.444	0.320	2.312	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	20	20	0	0	-1
N.S.	1	1.00	1.00	1.16	1.05	1.05	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.045	0.572	0.311	2.672	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	22	20	20	0	0	-1
N.S.	1	1.00	0.95	1.10	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.043	0.046	0.332	3.177	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	14	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.27	1.09	1.00
time (sec)	N/A	0.016	0.006	0.046	0.272	2.243	0.152	0.409	0.034

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	22	56	19	19
N.S.	1	1.00	0.95	1.05	1.00	1.16	2.95	1.00	1.00
time (sec)	N/A	0.015	0.016	0.127	0.282	2.795	0.534	0.418	3.135

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	39	0	16	9
N.S.	1	1.00	1.00	1.33	1.00	13.00	0.00	5.33	3.00
time (sec)	N/A	0.016	0.006	0.074	0.489	2.553	0.000	0.400	0.021

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	53	0	5	5
N.S.	1	1.00	1.00	0.86	0.71	7.57	0.00	0.71	0.71
time (sec)	N/A	0.017	0.007	0.078	0.487	2.459	0.000	0.428	2.979

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	71	0	0	9
N.S.	1	1.00	1.00	0.77	0.69	5.46	0.00	0.00	0.69
time (sec)	N/A	0.018	0.020	0.095	0.493	3.445	0.000	0.000	2.984

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	48	38	79	0	38	47
N.S.	1	1.00	1.00	2.29	1.81	3.76	0.00	1.81	2.24
time (sec)	N/A	0.018	0.012	0.122	0.266	2.615	0.000	0.413	3.095

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	61	0	22	20
N.S.	1	1.00	1.00	0.82	0.79	2.18	0.00	0.79	0.71
time (sec)	N/A	0.019	0.013	0.055	0.490	2.575	0.000	0.407	2.965

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	12	27	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.86	1.93	0.71	0.71
time (sec)	N/A	0.025	0.006	0.017	0.276	4.028	0.162	0.392	0.100

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	40	17	20	35	0	20	14
N.S.	1	1.00	2.11	0.89	1.05	1.84	0.00	1.05	0.74
time (sec)	N/A	0.022	0.014	0.188	0.272	5.247	0.000	0.424	3.170

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	17	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	5.67	1.00	1.00	1.00
time (sec)	N/A	0.006	0.920	0.027	0.308	2.961	0.126	0.430	2.953

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4
N.S.	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00
time (sec)	N/A	0.015	6.533	0.040	0.306	2.501	1.080	0.403	3.002

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	8	47	10	29	21
N.S.	1	1.00	1.00	2.25	2.00	11.75	2.50	7.25	5.25
time (sec)	N/A	0.004	0.005	0.033	0.328	2.776	0.742	0.418	3.240

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	40	39	103	44	39	73
N.S.	1	1.00	3.56	1.11	1.08	2.86	1.22	1.08	2.03
time (sec)	N/A	0.057	0.292	0.060	0.335	2.841	1.322	0.411	0.072

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	9	13	8	8	12	8	8
N.S.	1	1.00	0.64	0.93	0.57	0.57	0.86	0.57	0.57
time (sec)	N/A	0.011	0.012	0.043	0.314	2.996	0.154	0.414	2.915

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	14	19	28	36	13	-1
N.S.	1	1.00	0.80	0.56	0.76	1.12	1.44	0.52	-0.04
time (sec)	N/A	0.033	0.015	0.231	0.316	2.640	0.535	0.435	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.018	0.008	0.027	0.308	3.462	0.136	0.422	3.002

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	7	5	6
N.S.	1	1.00	1.00	1.00	0.83	0.83	1.17	0.83	1.00
time (sec)	N/A	0.007	0.008	0.028	0.303	2.323	0.127	0.416	2.908

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	7	6	0	12	7
N.S.	1	1.00	0.70	0.70	0.70	0.60	0.00	1.20	0.70
time (sec)	N/A	0.009	0.017	0.085	0.890	4.158	0.000	0.422	2.993

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	7	12	0	18	7
N.S.	1	1.00	1.00	1.30	0.70	1.20	0.00	1.80	0.70
time (sec)	N/A	0.008	0.007	0.089	0.328	3.762	0.000	0.408	2.952

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	16	36	16	16
N.S.	1	1.00	1.00	1.00	0.94	0.94	2.12	0.94	0.94
time (sec)	N/A	0.009	0.012	0.036	0.309	4.583	0.124	0.431	0.107

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	22	22	49	22	22
N.S.	1	1.00	1.00	1.05	1.00	1.00	2.23	1.00	1.00
time (sec)	N/A	0.010	0.094	0.074	0.313	2.535	5.256	0.418	3.080

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	22	22	48	22	22
N.S.	1	1.00	0.96	1.00	0.96	0.96	2.09	0.96	0.96
time (sec)	N/A	0.012	0.031	0.053	0.304	4.140	0.294	0.438	2.985

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	13	13	0	13	-1
N.S.	1	1.00	1.00	1.31	1.00	1.00	0.00	1.00	-0.08
time (sec)	N/A	0.015	0.029	0.440	0.362	3.551	0.000	0.408	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	23	19	19	0	0	-1
N.S.	1	1.00	1.00	1.28	1.06	1.06	0.00	0.00	-0.06
time (sec)	N/A	0.015	0.045	0.532	0.360	4.444	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	23	19	19	0	0	-1
N.S.	1	1.00	0.95	1.21	1.00	1.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.044	0.053	0.358	1.950	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	12	11	40	0	12	11
N.S.	1	1.00	1.82	1.09	1.00	3.64	0.00	1.09	1.00
time (sec)	N/A	0.025	0.042	0.110	0.304	3.508	0.000	0.402	3.026

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	15	23	0	17	3
N.S.	1	1.00	2.09	0.36	1.36	2.09	0.00	1.55	0.27
time (sec)	N/A	0.024	0.007	0.098	0.295	3.444	0.000	0.413	3.082

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	7	21	0	7	7
N.S.	1	1.00	0.33	0.30	0.26	0.78	0.00	0.26	0.26
time (sec)	N/A	0.023	0.020	0.094	0.536	3.457	0.000	0.405	2.882

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	20	19	37	0	0	37
N.S.	1	1.00	0.95	1.05	1.00	1.95	0.00	0.00	1.95
time (sec)	N/A	0.027	0.121	0.151	0.275	3.939	0.000	0.000	3.563

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
N.S.	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.031	0.005	0.092	0.539	4.084	0.286	0.422	2.937

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	27	4	4
N.S.	1	1.00	1.00	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.047	0.003	0.105	0.529	2.442	0.294	0.433	2.997

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	6	5	35	0	5	5
N.S.	1	1.00	0.94	0.18	0.15	1.06	0.00	0.15	0.15
time (sec)	N/A	0.034	0.029	0.142	0.516	2.803	0.000	0.410	3.122

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	18	17	36	0	19	16
N.S.	1	1.50	1.60	1.80	1.70	3.60	0.00	1.90	1.60
time (sec)	N/A	0.034	0.025	0.110	0.364	1.801	0.000	0.434	3.098

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	15	16	16	15	36	0	17	14
N.S.	1	1.50	1.60	1.60	1.50	3.60	0.00	1.70	1.40
time (sec)	N/A	0.035	0.027	0.106	0.286	2.572	0.000	0.412	2.984

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	74	80	89	441	0	61	75
N.S.	1	1.00	0.42	0.45	0.51	2.51	0.00	0.35	0.43
time (sec)	N/A	0.099	0.084	0.117	0.502	4.168	0.000	0.399	3.308

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	22	18	17	48	0	17	17
N.S.	1	1.00	0.42	0.34	0.32	0.91	0.00	0.32	0.32
time (sec)	N/A	0.053	0.042	0.161	0.540	4.227	0.000	0.390	3.115

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	54	28	28	71	29	29	27
N.S.	1	1.00	1.93	1.00	1.00	2.54	1.04	1.04	0.96
time (sec)	N/A	0.059	0.290	0.158	0.315	2.980	2.267	0.415	3.093

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	60	63	122	56	64	65
N.S.	1	1.00	1.17	1.13	1.19	2.30	1.06	1.21	1.23
time (sec)	N/A	0.088	0.436	0.186	0.321	2.774	3.119	0.433	2.984

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	133	116	118	201	95	123	122
N.S.	1	1.00	1.71	1.49	1.51	2.58	1.22	1.58	1.56
time (sec)	N/A	0.093	0.680	0.164	0.314	2.441	4.614	0.424	2.955

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	36	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	3.00	0.83	0.83	1.00
time (sec)	N/A	0.050	0.028	0.118	0.308	2.462	170.871	0.421	2.930

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	67	42	25	46	27	25	25
N.S.	1	1.00	2.03	1.27	0.76	1.39	0.82	0.76	0.76
time (sec)	N/A	0.061	0.019	0.072	0.294	2.776	13.768	0.431	2.922

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	24	23	52	41	24	30
N.S.	1	1.00	0.70	0.52	0.50	1.13	0.89	0.52	0.65
time (sec)	N/A	0.059	0.164	0.167	0.510	3.556	4.554	0.415	2.965

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	3	15	4
N.S.	1	1.00	1.00	1.25	1.00	3.00	0.75	3.75	1.00
time (sec)	N/A	0.013	0.003	0.059	0.509	4.406	3.908	0.416	2.882

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	13	29	0	15	9
N.S.	1	1.00	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.077	0.026	0.136	0.290	3.312	0.000	0.422	3.508

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	43	103	8	25	0	0	-1
N.S.	1	1.00	4.78	11.44	0.89	2.78	0.00	0.00	-0.11
time (sec)	N/A	0.030	0.030	0.320	0.504	3.045	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	47	171	7	45	0	7	-1
N.S.	1	1.00	5.22	19.00	0.78	5.00	0.00	0.78	-0.11
time (sec)	N/A	0.031	0.041	0.622	0.513	2.961	0.000	0.433	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	46	171	16	67	0	17	-1
N.S.	1	1.00	3.29	12.21	1.14	4.79	0.00	1.21	-0.07
time (sec)	N/A	0.029	0.040	0.717	0.288	2.840	0.000	0.417	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	220	30	78	0	142	19
N.S.	1	1.00	2.74	11.58	1.58	4.11	0.00	7.47	1.00
time (sec)	N/A	0.032	0.341	0.580	0.512	2.472	0.000	0.436	3.061

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	492	20	72	0	20	-1
N.S.	1	1.00	2.42	18.92	0.77	2.77	0.00	0.77	-0.04
time (sec)	N/A	0.030	0.081	0.315	0.500	2.491	0.000	0.405	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	8	3	3	3
N.S.	1	1.00	1.00	1.00	0.75	2.00	0.75	0.75	0.75
time (sec)	N/A	0.008	0.045	0.040	0.289	2.758	0.550	0.425	3.102

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	25	20	42	20	19	20	14
N.S.	1	1.00	1.47	1.18	2.47	1.18	1.12	1.18	0.82
time (sec)	N/A	0.043	0.014	0.060	0.292	2.618	1.078	0.426	2.923

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	45	0	22	16
N.S.	1	1.00	1.67	1.08	1.00	3.75	0.00	1.83	1.33
time (sec)	N/A	0.027	0.043	0.090	0.304	2.497	0.000	0.431	3.012

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	38	0	41	43
N.S.	1	1.00	0.95	1.05	1.00	1.90	0.00	2.05	2.15
time (sec)	N/A	0.030	0.136	0.125	0.277	2.119	0.000	0.414	3.186

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	3	16	6
N.S.	1	1.00	1.00	1.17	1.33	2.33	0.50	2.67	1.00
time (sec)	N/A	0.012	0.003	0.053	0.511	2.821	2.559	0.415	2.935

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	27	16	6
N.S.	1	1.00	1.00	1.17	1.33	2.33	4.50	2.67	1.00
time (sec)	N/A	0.034	0.005	0.071	0.512	3.442	0.251	0.428	2.942

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	56	47	46	76	31	68	35
N.S.	1	1.00	2.00	1.68	1.64	2.71	1.11	2.43	1.25
time (sec)	N/A	0.056	0.270	0.132	0.295	2.509	3.547	0.424	3.069

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	94	92	182	58	139	92
N.S.	1	1.00	1.17	1.77	1.74	3.43	1.09	2.62	1.74
time (sec)	N/A	0.088	0.388	0.129	0.315	3.429	7.722	0.440	3.072

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	135	158	161	320	97	232	141
N.S.	1	1.00	1.73	2.03	2.06	4.10	1.24	2.97	1.81
time (sec)	N/A	0.090	0.915	0.164	0.302	3.154	13.599	0.451	3.080

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	9	5	0	7
N.S.	1	1.00	1.00	1.00	0.83	1.50	0.83	0.00	1.17
time (sec)	N/A	0.010	0.062	0.053	0.303	3.503	14.036	0.000	2.938

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	11	19	14	22	48
N.S.	1	1.82	1.82	1.09	1.00	1.73	1.27	2.00	4.36
time (sec)	N/A	0.031	0.013	0.069	0.289	3.696	0.195	0.427	3.246

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	5	5	3	5	7
N.S.	1	1.00	1.00	0.80	1.00	1.00	0.60	1.00	1.40
time (sec)	N/A	0.024	0.017	0.062	0.503	3.667	0.081	0.423	2.950

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	13
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	1.18
time (sec)	N/A	0.024	0.022	0.061	0.505	3.648	0.094	0.417	3.044

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	12	7	9	15	7	9
N.S.	1	1.00	1.29	1.71	1.00	1.29	2.14	1.00	1.29
time (sec)	N/A	0.021	0.005	0.072	0.292	3.314	0.085	0.453	3.072

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	38	6	33	27	5	36	7
N.S.	1	1.00	7.60	1.20	6.60	5.40	1.00	7.20	1.40
time (sec)	N/A	0.030	0.021	0.076	0.296	3.697	0.398	0.409	3.102

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	13	16	0	21	13
N.S.	1	1.00	1.00	1.92	1.00	1.23	0.00	1.62	1.00
time (sec)	N/A	0.052	0.012	0.106	0.502	4.489	0.000	0.425	0.106

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	5	3	5	5
N.S.	1	1.00	1.00	1.00	0.75	1.25	0.75	1.25	1.25
time (sec)	N/A	0.014	0.007	0.030	0.310	3.792	0.183	0.419	3.093

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	11	11
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.22	1.22
time (sec)	N/A	0.015	0.006	0.085	0.298	3.195	0.181	0.420	3.123

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	11	10	29	12	31	18
N.S.	1	1.00	1.67	0.92	0.83	2.42	1.00	2.58	1.50
time (sec)	N/A	0.035	0.052	0.089	0.290	3.568	0.394	0.426	3.114

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	65	35	122	0	0	36
N.S.	1	1.00	1.00	1.51	0.81	2.84	0.00	0.00	0.84
time (sec)	N/A	0.063	0.038	0.109	0.509	3.576	0.000	0.000	3.257

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	20	20	0	0	18
N.S.	1	1.00	1.00	1.55	0.91	0.91	0.00	0.00	0.82
time (sec)	N/A	0.062	0.019	0.083	0.505	2.413	0.000	0.000	3.026

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	20	20	13	12	20	17	22	31
N.S.	1	1.67	1.67	1.08	1.00	1.67	1.42	1.83	2.58
time (sec)	N/A	0.028	0.011	0.072	0.299	2.640	0.195	0.405	3.180

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	14	12	14	14
N.S.	1	1.00	1.00	0.93	0.86	1.00	0.86	1.00	1.00
time (sec)	N/A	0.016	0.014	0.065	0.290	3.324	0.177	0.461	2.968

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	5	3	26
N.S.	1	1.00	1.00	2.00	1.00	1.00	1.67	1.00	8.67
time (sec)	N/A	0.023	0.010	0.053	0.508	3.645	0.086	0.413	3.215

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	49	73	151	48	57
N.S.	1	1.00	0.95	0.81	1.14	1.70	3.51	1.12	1.33
time (sec)	N/A	0.038	0.471	0.086	0.527	4.741	0.513	0.465	3.148

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	0	21	34
N.S.	1	1.00	1.00	1.07	1.00	1.50	0.00	1.50	2.43
time (sec)	N/A	0.052	0.014	0.216	0.509	3.202	0.000	0.429	3.114

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	38	37	57	0	0	28
N.S.	1	1.00	0.65	0.88	0.86	1.33	0.00	0.00	0.65
time (sec)	N/A	0.072	0.040	0.280	0.501	3.253	0.000	0.000	3.144

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
N.S.	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.024	0.044	0.079	0.307	3.699	0.000	0.000	3.178

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	104	37	27	0	0	27
N.S.	1	1.00	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.024	0.023	0.000	0.316	4.433	0.000	0.000	0.002

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
N.S.	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.026	0.050	0.092	0.000	3.508	0.000	0.463	3.177

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	122	0	33	0	138	33
N.S.	1	1.00	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.025	0.023	0.000	0.000	3.164	0.000	0.472	0.002

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
N.S.	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.023	0.075	0.065	0.318	3.386	0.000	0.000	3.214

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	28	105	37	27	0	0	27
N.S.	1	1.00	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.023	0.023	0.001	0.310	2.080	0.000	0.000	0.002

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
N.S.	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.025	0.084	0.079	0.000	2.051	0.000	0.461	3.174

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	123	0	33	0	195	33
N.S.	1	1.00	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.025	0.022	0.000	0.000	2.681	0.000	0.473	0.002

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
N.S.	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.016	0.005	0.195	0.290	3.029	0.000	0.414	5.268

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	265	7	0	0	27
N.S.	1	1.00	1.00	0.89	29.44	0.78	0.00	0.00	3.00
time (sec)	N/A	0.014	0.008	0.120	0.513	4.705	0.000	0.000	3.527

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	5	4	4	14	4	4
N.S.	1	1.00	1.00	1.67	1.33	1.33	4.67	1.33	1.33
time (sec)	N/A	0.006	0.001	0.054	0.501	4.615	0.045	0.413	0.026

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	0	103	8
N.S.	1	1.00	1.00	1.12	9.25	2.25	0.00	12.88	1.00
time (sec)	N/A	0.012	0.004	0.027	0.512	3.003	0.000	0.425	0.024

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	26	22	27	76	22	22
N.S.	1	1.00	0.82	0.76	0.65	0.79	2.24	0.65	0.65
time (sec)	N/A	0.014	0.014	0.070	0.283	4.034	0.188	0.413	2.973

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	10	6	6
N.S.	1	1.00	1.00	0.70	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.009	0.002	0.038	0.285	4.538	0.101	0.406	0.067

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	14	15	9	12
N.S.	1	1.00	1.00	0.82	0.73	1.27	1.36	0.82	1.09
time (sec)	N/A	0.007	0.007	0.043	0.294	3.336	0.164	0.404	3.536

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	7	9	8	7	7
N.S.	1	1.00	1.29	1.14	1.00	1.29	1.14	1.00	1.00
time (sec)	N/A	0.033	0.005	0.073	0.302	3.387	1.234	0.399	2.924

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	15
N.S.	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	0.79
time (sec)	N/A	0.013	0.008	0.052	0.493	2.908	0.000	0.445	2.897

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	31	26	29	32	23	25
N.S.	1	1.00	0.70	0.84	0.70	0.78	0.86	0.62	0.68
time (sec)	N/A	0.009	0.022	0.083	0.287	4.323	0.174	0.409	2.901

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	129	49	176	8	39	236	37	49
N.S.	1	10.75	4.08	14.67	0.67	3.25	19.67	3.08	4.08
time (sec)	N/A	0.222	0.032	0.151	0.288	4.133	0.023	0.419	2.986

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	19	8	7	13	19	8	19
N.S.	1	1.00	2.11	0.89	0.78	1.44	2.11	0.89	2.11
time (sec)	N/A	0.006	0.007	0.017	0.286	4.092	0.059	0.403	0.117

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	12	0	6	14
N.S.	1	1.00	1.00	0.88	4.38	1.50	0.00	0.75	1.75
time (sec)	N/A	0.009	0.012	0.034	0.282	3.155	0.000	0.392	0.099

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	13	0	11	13
N.S.	1	1.00	1.00	0.80	0.00	0.87	0.00	0.73	0.87
time (sec)	N/A	0.020	0.013	0.135	0.000	3.561	0.000	0.408	2.965

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	16	15	15	19	15	15
N.S.	1	1.00	0.87	0.70	0.65	0.65	0.83	0.65	0.65
time (sec)	N/A	0.016	0.009	0.052	0.500	3.426	0.083	0.419	0.074

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	29	29	42	29	33
N.S.	1	1.00	1.00	0.81	0.78	0.78	1.14	0.78	0.89
time (sec)	N/A	0.025	0.022	0.198	0.288	2.637	0.147	0.396	2.973

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	10	8	10	8
N.S.	1	1.00	1.20	0.90	0.80	1.00	0.80	1.00	0.80
time (sec)	N/A	0.024	0.015	0.040	0.286	3.652	0.057	0.407	2.918

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.005	0.002	0.033	0.281	3.649	0.057	0.410	0.056

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.007	0.003	0.036	0.294	3.827	0.074	0.403	0.059

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.006	0.003	0.038	0.301	3.778	0.118	0.416	0.067

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.005	0.011	0.013	0.292	3.750	0.052	0.400	2.924

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	56	8	5	8	8
N.S.	1	1.00	1.00	0.88	7.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.044	0.006	0.027	0.275	3.270	0.092	0.424	0.075

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	13	67	13	7	10	10
N.S.	1	1.00	1.20	1.30	6.70	1.30	0.70	1.00	1.00
time (sec)	N/A	0.012	0.018	0.032	0.293	3.915	0.077	0.398	2.916

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	12	10	13
N.S.	1	1.00	1.00	0.91	0.82	1.36	1.09	0.91	1.18
time (sec)	N/A	0.007	0.014	0.019	0.285	2.518	0.043	0.450	0.283

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	10	13
N.S.	1	1.00	1.00	0.92	0.83	1.00	1.00	0.83	1.08
time (sec)	N/A	0.003	0.004	0.044	0.283	3.408	0.456	0.402	2.923

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	20	18	29	0	18	16
N.S.	1	1.00	1.19	0.95	0.86	1.38	0.00	0.86	0.76
time (sec)	N/A	0.040	0.014	0.130	0.299	2.897	0.000	0.409	3.001

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	40	32	39	15
N.S.	1	1.00	1.00	0.84	0.79	2.11	1.68	2.05	0.79
time (sec)	N/A	0.025	0.008	0.169	0.290	3.379	0.780	0.429	3.027

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	33	32	32	48	32	43
N.S.	1	1.00	0.74	0.70	0.68	0.68	1.02	0.68	0.91
time (sec)	N/A	0.040	0.025	0.017	0.300	3.185	4.884	0.449	3.004

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	36	25	29	32	25	25
N.S.	1	1.00	0.80	1.03	0.71	0.83	0.91	0.71	0.71
time (sec)	N/A	0.111	0.034	0.034	0.297	3.064	0.430	0.421	2.966

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.005	0.002	0.023	0.281	3.577	0.050	0.427	2.924

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.008	0.003	0.043	0.284	3.479	0.079	0.404	0.057

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.003	0.006	0.083	0.499	3.564	0.026	0.439	0.055

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
N.S.	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.006	0.011	0.021	0.283	3.865	0.053	0.423	0.042

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.006	0.004	0.039	0.292	3.694	0.053	0.410	2.981

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.006	0.003	0.043	0.292	2.819	0.081	0.429	0.046

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
N.S.	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.008	0.011	0.025	0.294	2.798	0.080	0.426	0.047

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.006	0.002	0.036	0.283	2.669	0.074	0.428	0.051

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.007	0.026	0.040	0.304	3.754	0.049	0.407	2.945

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.012	0.008	0.053	0.286	3.509	0.272	0.405	2.965

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	15	17	20	15	15
N.S.	1	1.00	0.70	0.78	0.65	0.74	0.87	0.65	0.65
time (sec)	N/A	0.006	0.010	0.036	0.300	3.088	0.169	0.422	0.021

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.012	0.003	0.026	0.287	3.354	0.123	0.430	2.959

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.012	0.007	0.038	0.301	2.974	0.118	0.405	0.050

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	19	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.11	0.78	0.78	0.78
time (sec)	N/A	0.007	0.948	0.069	0.285	2.611	0.129	0.392	0.072

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	11	10	9	17
N.S.	1	1.00	1.00	0.91	0.82	1.00	0.91	0.82	1.55
time (sec)	N/A	0.021	0.024	0.039	0.279	3.672	0.056	0.435	3.008

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	65	8	45	36	61	8
N.S.	1	1.00	1.00	6.50	0.80	4.50	3.60	6.10	0.80
time (sec)	N/A	0.017	0.014	0.093	0.279	3.789	0.131	0.410	3.153

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	12	13	10	8	8	6
N.S.	1	1.00	1.00	1.33	1.44	1.11	0.89	0.89	0.67
time (sec)	N/A	0.015	0.005	0.046	0.292	5.086	0.971	0.398	2.926

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	7
N.S.	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	1.40
time (sec)	N/A	0.009	0.009	0.033	0.286	4.506	0.018	0.426	3.001

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	A	F(-2)	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	13	30	88	14	0	794	14
N.S.	1	0.00	1.00	2.31	6.77	1.08	0.00	61.08	1.08
time (sec)	N/A	0.441	0.214	0.256	0.737	3.432	0.000	0.423	3.121

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	0	52	9
N.S.	1	1.00	1.00	1.11	11.56	2.22	0.00	5.78	1.00
time (sec)	N/A	0.012	0.013	0.023	0.288	3.733	0.000	0.432	0.028

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
N.S.	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.012	0.010	0.125	0.290	3.383	0.138	0.414	0.028

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	15	15	22	15	14
N.S.	1	1.00	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.010	0.010	0.027	0.285	2.361	0.123	0.398	2.952

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
N.S.	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.010	0.005	0.043	0.284	2.625	0.021	0.407	2.950

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	20	37	20	28	32
N.S.	1	1.00	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.021	0.018	0.052	0.284	3.394	0.028	0.409	2.971

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	5	5	12	5	5
N.S.	1	1.00	7.20	1.20	1.00	1.00	2.40	1.00	1.00
time (sec)	N/A	0.010	0.008	0.044	0.284	3.276	1.130	0.404	2.938

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	5	7	12	7	5
N.S.	1	1.00	2.86	0.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.010	0.006	0.046	0.295	2.175	1.034	0.452	2.907

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	11	5	14	9	6
N.S.	1	1.00	1.60	1.20	2.20	1.00	2.80	1.80	1.20
time (sec)	N/A	0.015	0.004	0.043	0.281	1.863	1.850	0.388	2.888

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	69	35	32	29	24
N.S.	1	1.00	4.07	1.33	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.033	0.010	0.208	0.505	2.789	1.116	0.415	3.110

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	11	15	12	15	11
N.S.	1	1.00	2.36	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.031	0.066	0.064	0.279	3.737	0.076	0.398	0.085

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
N.S.	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.036	0.121	0.050	0.489	3.543	0.135	0.425	2.905

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.015	0.007	0.072	0.286	2.685	0.061	0.410	2.979

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	674	34	27	36	0	29
N.S.	1	1.00	1.00	25.92	1.31	1.04	1.38	0.00	1.12
time (sec)	N/A	0.032	0.033	0.379	0.509	3.061	0.337	0.000	3.075

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
N.S.	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.020	0.025	0.151	0.291	2.299	0.000	0.420	3.073

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	38	26	39	26	34
N.S.	1	1.00	0.72	0.88	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.045	0.039	0.052	0.295	3.333	0.083	0.401	2.900

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	19	21	26	19	19
N.S.	1	1.00	0.81	0.81	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.008	0.023	0.030	0.295	2.566	0.174	0.426	0.031

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	42	12
N.S.	1	1.00	1.35	0.78	0.74	0.52	1.13	1.83	0.52
time (sec)	N/A	0.026	0.019	0.085	0.288	2.943	0.094	0.401	0.102

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	41	24	36	27	24	24
N.S.	1	1.00	1.53	1.37	0.80	1.20	0.90	0.80	0.80
time (sec)	N/A	0.021	0.022	0.076	0.294	1.917	0.013	0.418	2.998

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	14	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	2.33	1.00
time (sec)	N/A	0.008	0.002	0.070	0.289	2.169	0.658	0.432	2.945

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	15	8	30	12
N.S.	1	1.00	1.00	1.14	1.00	2.14	1.14	4.29	1.71
time (sec)	N/A	0.028	0.004	0.102	0.283	2.837	3.502	0.417	2.967

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	10	10	13	10	14	13	6
N.S.	1	1.00	0.77	0.77	1.00	0.77	1.08	1.00	0.46
time (sec)	N/A	0.009	0.004	0.051	0.288	3.975	0.015	0.415	2.930

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	22	12	11	14
N.S.	1	1.00	1.00	0.91	1.00	2.00	1.09	1.00	1.27
time (sec)	N/A	0.006	0.008	0.080	0.292	2.892	0.012	0.423	2.918

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	10	18	10	11	24
N.S.	1	1.00	1.00	1.70	1.00	1.80	1.00	1.10	2.40
time (sec)	N/A	0.005	0.013	0.066	0.284	2.729	0.012	0.405	3.083

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.005	0.010	0.011	0.287	2.986	0.060	0.416	0.050

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	14	32	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.78	1.78	0.67	0.67
time (sec)	N/A	0.028	0.026	0.075	0.284	3.200	0.167	0.390	2.993

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	31	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	3.10	0.80	0.80
time (sec)	N/A	0.009	0.005	0.020	0.294	4.683	0.546	0.426	2.919

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	11	10	21	12	10	14
N.S.	1	1.00	1.56	0.69	0.62	1.31	0.75	0.62	0.88
time (sec)	N/A	0.012	0.021	0.060	0.285	2.706	0.128	0.461	0.079

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	13	8	9	9
N.S.	1	1.00	1.00	1.14	1.00	1.86	1.14	1.29	1.29
time (sec)	N/A	0.005	0.006	0.019	0.289	3.900	0.042	0.419	0.074

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	16	25	15	41	15
N.S.	1	1.00	1.00	1.31	1.23	1.92	1.15	3.15	1.15
time (sec)	N/A	0.008	0.013	0.014	0.286	2.141	0.572	0.397	3.526

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	21	11	10	23	10	43	31
N.S.	1	1.00	4.20	2.20	2.00	4.60	2.00	8.60	6.20
time (sec)	N/A	0.007	0.012	0.023	0.294	3.020	0.695	0.413	3.681

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	9	7	8	15	7	8	16	26
N.S.	1	1.29	1.00	1.14	2.14	1.00	1.14	2.29	3.71
time (sec)	N/A	0.032	0.007	0.132	0.286	1.847	1.330	0.421	3.254

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	20	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.012	0.002	0.213	0.279	3.253	0.141	0.421	3.074

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	21	14	74	27	0	81	13
N.S.	1	1.00	1.62	1.08	5.69	2.08	0.00	6.23	1.00
time (sec)	N/A	0.013	0.006	0.041	0.495	3.575	0.000	0.422	2.994

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	28	20	22	20	20	18
N.S.	1	1.00	1.12	1.75	1.25	1.38	1.25	1.25	1.12
time (sec)	N/A	0.020	0.018	0.051	0.496	4.305	0.010	0.410	2.961

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	34	35	28	36	31	26
N.S.	1	1.00	0.81	1.06	1.09	0.88	1.12	0.97	0.81
time (sec)	N/A	0.023	0.019	0.035	0.499	4.037	0.011	0.447	2.995

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	19	10	19	12	10	18
N.S.	1	1.00	0.89	1.06	0.56	1.06	0.67	0.56	1.00
time (sec)	N/A	0.020	0.008	0.018	0.288	3.570	0.013	0.442	0.047

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	29	18	25	32	22	24
N.S.	1	1.00	0.94	0.85	0.53	0.74	0.94	0.65	0.71
time (sec)	N/A	0.033	0.014	0.026	0.285	4.145	0.010	0.404	0.050

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	33	29	13	21	12	13	13
N.S.	1	1.00	2.54	2.23	1.00	1.62	0.92	1.00	1.00
time (sec)	N/A	0.016	0.014	0.027	0.291	3.293	0.010	0.426	0.039

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	24	36	16	31	31	16	33
N.S.	1	1.00	0.52	0.78	0.35	0.67	0.67	0.35	0.72
time (sec)	N/A	0.037	0.010	0.029	0.284	2.469	0.013	0.413	0.041

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	21	20	19	20	18	13
N.S.	1	1.00	1.00	2.33	2.22	2.11	2.22	2.00	1.44
time (sec)	N/A	0.022	0.004	0.062	0.293	3.209	0.098	0.443	3.008

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	17	13	13	12	12	13	11
N.S.	1	1.00	1.21	0.93	0.93	0.86	0.86	0.93	0.79
time (sec)	N/A	0.012	0.005	0.054	0.287	2.754	0.011	0.396	2.970

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.012	0.000	0.052	0.293	1.940	0.012	0.434	2.934

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.008	0.004	0.046	0.299	2.759	0.012	0.401	2.936

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.006	0.006	0.055	0.288	2.499	0.097	0.436	0.072

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	6	22	6	6
N.S.	1	1.00	1.00	0.88	4.38	0.75	2.75	0.75	0.75
time (sec)	N/A	0.011	0.003	0.035	0.314	1.868	0.048	0.425	2.945

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	9	51	8	5	26	8
N.S.	1	1.00	1.00	1.50	8.50	1.33	0.83	4.33	1.33
time (sec)	N/A	0.132	0.020	0.027	0.589	1.942	0.140	0.407	0.085

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	8	8	8
N.S.	1	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.152	0.014	0.066	0.327	1.617	0.091	0.416	3.069

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	8	7	8	8
N.S.	1	1.00	1.50	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.008	0.013	0.042	0.333	1.598	0.090	0.405	3.051

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	8
N.S.	1	1.00	1.00	0.88	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.135	0.012	0.050	0.326	1.915	0.095	0.431	3.005

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	55	80	0	320	162	77	1108
N.S.	1	1.27	1.00	1.45	0.00	5.82	2.95	1.40	20.15
time (sec)	N/A	0.108	0.063	0.243	0.000	1.806	4.661	0.405	4.178

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	70	54	72	0	322	162	77	1374
N.S.	1	1.27	0.98	1.31	0.00	5.85	2.95	1.40	24.98
time (sec)	N/A	0.088	0.047	0.230	0.000	2.510	4.680	0.424	3.438

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	49	0	225	432	141	108
N.S.	1	1.00	0.92	0.94	0.00	4.33	8.31	2.71	2.08
time (sec)	N/A	0.095	0.064	0.164	0.000	3.654	17.701	0.417	3.434

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	51	0	224	432	159	684
N.S.	1	1.00	0.96	0.98	0.00	4.31	8.31	3.06	13.15
time (sec)	N/A	0.068	0.040	0.135	0.000	3.835	17.688	0.408	3.319

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	30	76	91	0	56	-1
N.S.	1	1.00	1.03	1.00	2.53	3.03	0.00	1.87	-0.03
time (sec)	N/A	0.022	0.027	0.129	0.536	2.184	0.000	0.530	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	31	51	84	0	27	-1
N.S.	1	1.00	1.58	1.00	1.65	2.71	0.00	0.87	-0.03
time (sec)	N/A	0.023	0.041	0.131	0.330	2.164	0.000	0.416	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.002	0.054	0.323	2.165	0.088	0.397	3.185

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	25	41	21	0	14	-1
N.S.	1	1.00	0.95	1.32	2.16	1.11	0.00	0.74	-0.05
time (sec)	N/A	0.022	0.010	0.249	0.541	2.214	0.000	0.422	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	16	24	12	23	476	12	12
N.S.	1	1.00	0.55	0.83	0.41	0.79	16.41	0.41	0.41
time (sec)	N/A	0.039	0.015	0.053	0.325	2.382	6.980	0.410	3.201

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	36	103	26	32	34	45
N.S.	1	1.28	1.00	1.24	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.092	0.071	0.161	0.546	2.053	0.118	0.428	3.309

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	29	36	103	26	32	34	45
N.S.	1	1.28	1.00	1.24	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.064	0.056	0.142	0.565	2.295	0.109	0.411	3.192

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	31	30	46	0	34	32
N.S.	1	1.00	0.86	0.70	0.68	1.05	0.00	0.77	0.73
time (sec)	N/A	0.039	0.054	0.168	0.301	2.272	0.000	0.410	0.066

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	88	19	16	145	15	0	0	15
N.S.	1	4.63	1.00	0.84	7.63	0.79	0.00	0.00	0.79
time (sec)	N/A	0.840	0.045	0.480	0.570	1.770	0.000	0.000	3.786

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	0	68	1911	518	96	0	0	-1
N.S.	1	0.00	1.55	43.43	11.77	2.18	0.00	0.00	-0.02
time (sec)	N/A	1.601	0.683	0.740	0.744	2.206	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	72	17	12372	329	3	0	11	-1
N.S.	1	3.79	0.89	651.16	17.32	0.16	0.00	0.58	-0.05
time (sec)	N/A	1.173	0.009	0.744	0.560	2.650	0.000	0.429	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	37	10	0	21	0	55	14
N.S.	1	1.00	2.85	0.77	0.00	1.62	0.00	4.23	1.08
time (sec)	N/A	0.105	0.031	0.177	0.000	1.799	0.000	0.534	0.292

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	25	29	68	29
N.S.	1	1.00	1.00	0.79	0.71	1.79	2.07	4.86	2.07
time (sec)	N/A	0.029	0.037	0.050	0.319	1.683	0.178	0.402	3.231

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	34	21	35	0	128	24
N.S.	1	1.00	1.20	1.36	0.84	1.40	0.00	5.12	0.96
time (sec)	N/A	0.056	0.137	0.117	0.326	3.436	0.000	0.448	3.334

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	18	38	21	44	0	128	24
N.S.	1	1.00	0.72	1.52	0.84	1.76	0.00	5.12	0.96
time (sec)	N/A	0.058	0.027	0.164	0.301	3.443	0.000	0.411	3.418

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	77
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	3.85
time (sec)	N/A	0.108	0.305	0.263	0.000	0.000	0.000	0.000	3.460

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	40	54	136	34	0	97	-1
N.S.	1	1.11	0.53	0.71	1.79	0.45	0.00	1.28	-0.01
time (sec)	N/A	0.108	0.063	0.277	0.509	2.412	0.000	0.440	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	65	86	41	0	44	-1
N.S.	1	1.00	0.68	0.80	1.06	0.51	0.00	0.54	-0.01
time (sec)	N/A	0.110	0.046	0.236	0.299	2.810	0.000	0.426	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	69	98	79	124	0	0	-1
N.S.	1	1.00	0.91	1.29	1.04	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.378	0.039	0.132	0.519	2.718	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	99	132	107	227	0	0	-1
N.S.	1	1.00	0.77	1.03	0.84	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.416	0.045	0.118	0.528	2.379	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	147	172	131	327	0	0	-1
N.S.	1	1.00	0.79	0.92	0.70	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.419	0.064	0.113	0.559	3.143	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	50	147	83	138	0	0	-1
N.S.	1	1.00	0.62	1.81	1.02	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.026	0.128	0.531	2.175	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	75	183	113	248	0	0	-1
N.S.	1	1.00	0.69	1.68	1.04	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.413	0.042	0.115	0.520	2.714	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	87	221	137	356	0	0	-1
N.S.	1	1.00	0.61	1.55	0.96	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.428	0.045	0.121	0.522	2.868	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	108	86	294	140	0	0	-1
N.S.	1	1.00	1.03	0.82	2.80	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.054	0.157	0.514	2.248	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	174	200	0	337	0	0	-1
N.S.	1	1.00	0.77	0.89	0.00	1.50	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.095	0.191	0.000	2.828	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	290	250	567	539	0	0	-1
N.S.	1	1.00	0.85	0.73	1.66	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.275	0.276	0.571	3.007	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	85	165	423	270	0	0	-1
N.S.	1	1.00	0.60	1.16	2.98	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.164	0.118	0.581	2.996	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	138	254	639	550	0	0	-1
N.S.	1	1.00	0.63	1.15	2.90	2.50	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.433	0.142	0.575	3.610	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	191	324	852	740	0	0	-1
N.S.	1	1.00	0.54	0.91	2.39	2.08	0.00	0.00	-0.00
time (sec)	N/A	0.461	0.699	0.133	0.616	2.387	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	116	17	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.64	0.68	0.56
time (sec)	N/A	0.020	0.014	0.098	0.278	2.034	1.071	0.422	2.936

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73
time (sec)	N/A	0.023	0.015	0.114	0.304	2.554	1.077	0.404	3.037

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	0.73
time (sec)	N/A	0.020	0.013	0.071	0.288	2.199	1.056	0.421	3.008

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	114	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	4.56	0.76	0.76
time (sec)	N/A	0.020	0.012	0.064	0.281	2.369	1.041	0.394	3.178

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.005	0.006	0.012	0.300	2.156	0.049	0.395	0.046

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	97	9	34	46	19	20
N.S.	1	1.00	2.27	8.82	0.82	3.09	4.18	1.73	1.82
time (sec)	N/A	0.014	0.053	0.108	0.291	2.454	0.302	0.417	3.190

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	18	12	133	15	0	52	16
N.S.	1	1.00	1.64	1.09	12.09	1.36	0.00	4.73	1.45
time (sec)	N/A	0.015	0.009	0.053	0.291	1.590	0.000	0.433	3.089

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	18	13	7	8	8
N.S.	1	1.00	1.00	0.75	1.50	1.08	0.58	0.67	0.67
time (sec)	N/A	0.033	0.014	0.096	0.297	1.839	0.792	0.420	2.928

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	17	17	27	26	33	29	24	19
N.S.	1	1.42	1.42	2.25	2.17	2.75	2.42	2.00	1.58
time (sec)	N/A	0.027	0.011	0.072	0.284	1.745	0.355	0.401	0.087

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	20	20	19	20	19
N.S.	1	1.00	1.00	0.95	1.05	1.05	1.00	1.05	1.00
time (sec)	N/A	0.029	0.017	0.086	0.281	1.453	2.597	0.447	3.672

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	52	50	49	65	51	51	69
N.S.	1	1.00	1.18	1.14	1.11	1.48	1.16	1.16	1.57
time (sec)	N/A	0.026	0.031	0.101	0.305	2.070	0.040	0.452	3.112

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	60	33	32	34	33	30
N.S.	1	1.00	0.95	1.62	0.89	0.86	0.92	0.89	0.81
time (sec)	N/A	0.022	0.023	0.088	0.284	1.880	0.028	0.655	3.099

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	58	57	73	61	59	85
N.S.	1	1.00	1.26	1.07	1.06	1.35	1.13	1.09	1.57
time (sec)	N/A	0.028	0.068	0.102	0.285	2.768	0.043	0.538	7.214

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	31	34	42	33	33
N.S.	1	1.00	1.00	0.86	0.84	0.92	1.14	0.89	0.89
time (sec)	N/A	0.028	0.022	0.102	0.296	2.890	108.924	0.427	2.944

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	42	46	46	49	49	42	69
N.S.	1	1.00	1.24	1.35	1.35	1.44	1.44	1.24	2.03
time (sec)	N/A	0.017	0.007	0.062	0.285	3.039	0.053	0.413	3.038

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	68	41	52	42	41	42
N.S.	1	1.00	1.00	1.58	0.95	1.21	0.98	0.95	0.98
time (sec)	N/A	0.026	0.022	0.088	0.287	3.124	0.035	0.408	3.056

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	72	73	92	0	73	74
N.S.	1	1.00	1.00	0.83	0.84	1.06	0.00	0.84	0.85
time (sec)	N/A	0.096	0.043	0.141	0.305	2.472	0.000	0.507	2.970

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	44	79	0	52	71
N.S.	1	1.00	1.00	0.88	1.05	1.88	0.00	1.24	1.69
time (sec)	N/A	0.085	0.031	0.118	0.291	2.587	0.000	0.419	3.161

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	57	84	0	57	57
N.S.	1	1.00	1.00	0.89	0.90	1.33	0.00	0.90	0.90
time (sec)	N/A	0.087	0.025	0.125	0.285	2.205	0.000	0.413	2.967

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	52	103	0	60	84
N.S.	1	1.00	0.87	0.82	0.87	1.72	0.00	1.00	1.40
time (sec)	N/A	0.085	0.086	0.129	0.296	3.166	0.000	0.465	4.674

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	38	41	42	44	41	42
N.S.	1	1.00	1.26	0.81	0.87	0.89	0.94	0.87	0.89
time (sec)	N/A	0.069	0.036	0.109	0.301	2.103	3.320	60.801	5.378

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	38	37	41	44	39	37
N.S.	1	1.00	1.65	0.88	0.86	0.95	1.02	0.91	0.86
time (sec)	N/A	0.078	0.021	0.090	0.303	2.660	0.513	0.387	0.077

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	69	44	79	41	52	71
N.S.	1	1.00	1.00	1.64	1.05	1.88	0.98	1.24	1.69
time (sec)	N/A	0.029	0.024	0.086	0.281	2.844	0.036	0.415	3.051

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	55	91	82	0	72	41
N.S.	1	1.00	0.85	0.74	1.23	1.11	0.00	0.97	0.55
time (sec)	N/A	0.168	0.056	0.092	0.502	2.768	0.000	0.445	3.137

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	47	32	176	39	0	1179	34
N.S.	1	1.00	3.36	2.29	12.57	2.79	0.00	84.21	2.43
time (sec)	N/A	0.007	0.030	0.050	0.518	2.419	0.000	0.588	3.173

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	10	12	12	10
N.S.	1	1.00	1.00	0.93	0.86	0.71	0.86	0.86	0.71
time (sec)	N/A	0.027	0.012	0.096	0.288	2.524	0.737	0.434	0.166

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	24	0	9	18
N.S.	1	1.00	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.037	0.012	0.124	0.000	3.024	0.000	0.399	3.142

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	35	19	32	32	34	33
N.S.	1	1.00	0.94	1.03	0.56	0.94	0.94	1.00	0.97
time (sec)	N/A	0.034	0.020	0.074	0.328	2.237	0.677	0.408	2.998

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	24	17	17	22	17	18
N.S.	1	1.00	1.19	1.14	0.81	0.81	1.05	0.81	0.86
time (sec)	N/A	0.019	0.011	0.114	0.308	2.602	0.307	0.414	0.069

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	55	97	51	66	189	58	69
N.S.	1	1.00	0.56	0.98	0.52	0.67	1.91	0.59	0.70
time (sec)	N/A	0.040	0.115	0.115	0.309	2.215	0.229	0.409	3.063

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	55	59	51	0	70	64
N.S.	1	1.00	1.08	1.49	1.59	1.38	0.00	1.89	1.73
time (sec)	N/A	0.063	0.042	0.111	0.524	2.396	0.000	0.436	3.128

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	41	73	90	41	45
N.S.	1	1.00	0.86	0.81	0.72	1.28	1.58	0.72	0.79
time (sec)	N/A	0.052	0.066	0.114	0.292	2.444	0.838	0.425	3.048

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	243	68	137	0	85	0	0	51
N.S.	1	4.26	1.19	2.40	0.00	1.49	0.00	0.00	0.89
time (sec)	N/A	0.144	0.040	0.231	0.000	3.108	0.000	0.000	4.619

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	17	7	10	10
N.S.	1	1.00	1.00	1.60	1.40	3.40	1.40	2.00	2.00
time (sec)	N/A	0.015	0.007	0.049	0.295	2.826	1.244	0.424	2.966

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	A	A	F	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	30	11	11	0	0	11
N.S.	1	0.00	1.00	2.73	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.192	0.201	0.140	0.387	2.325	0.000	0.000	3.158

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	A	F	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	24	22	74	27	0	93	38
N.S.	1	0.00	0.89	0.81	2.74	1.00	0.00	3.44	1.41
time (sec)	N/A	0.044	0.102	0.264	0.517	1.361	0.000	0.417	0.482

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	62	66	50	100	51	65
N.S.	1	1.00	0.77	0.95	1.02	0.77	1.54	0.78	1.00
time (sec)	N/A	0.045	0.047	0.092	0.294	3.432	0.208	0.384	3.029

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	10	14	0	0	10
N.S.	1	1.00	1.00	1.38	1.25	1.75	0.00	0.00	1.25
time (sec)	N/A	0.015	0.011	0.742	0.296	1.895	0.000	0.000	0.145

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	125	41	0	0	45
N.S.	1	1.00	1.00	0.00	3.91	1.28	0.00	0.00	1.41
time (sec)	N/A	0.027	0.057	0.191	0.517	2.286	0.000	0.000	3.430

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	45	3	0	3	26
N.S.	1	1.00	1.00	1.33	15.00	1.00	0.00	1.00	8.67
time (sec)	N/A	0.021	0.013	0.131	0.293	2.421	0.000	0.397	3.190

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	35	27	39	27	34
N.S.	1	1.00	0.91	1.03	1.00	0.77	1.11	0.77	0.97
time (sec)	N/A	0.043	0.033	0.036	0.298	3.832	0.095	0.405	0.061

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	6	5	6	-1
N.S.	1	1.00	1.00	0.88	2.12	0.75	0.62	0.75	-0.12
time (sec)	N/A	0.005	0.003	0.049	0.318	2.479	0.314	0.403	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	29	30	0	43	0	29	-1
N.S.	1	1.00	0.78	0.81	0.00	1.16	0.00	0.78	-0.03
time (sec)	N/A	0.126	0.036	0.067	0.000	3.511	0.000	0.414	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	41
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	2.93
time (sec)	N/A	0.009	0.004	0.029	0.282	3.256	6.318	0.430	3.143

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	8	20	7	8	19
N.S.	1	1.00	1.00	1.50	0.80	2.00	0.70	0.80	1.90
time (sec)	N/A	0.025	0.002	0.075	0.294	2.665	0.582	0.393	3.094

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	27	15	56
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.59	0.88	3.29
time (sec)	N/A	0.016	0.014	0.131	0.280	2.661	1.779	0.397	3.261

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	120	403	126	85	241	155	147
N.S.	1	1.00	0.93	3.12	0.98	0.66	1.87	1.20	1.14
time (sec)	N/A	0.083	0.354	0.684	0.307	2.464	4.778	0.422	3.455

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	352	160	3830	115	0	1363	730
N.S.	1	1.00	3.20	1.45	34.82	1.05	0.00	12.39	6.64
time (sec)	N/A	0.076	0.594	0.191	0.696	2.663	0.000	0.887	13.421

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	36	12	0	20	14
N.S.	1	1.00	1.00	1.17	6.00	2.00	0.00	3.33	2.33
time (sec)	N/A	0.013	0.013	0.042	0.299	2.734	0.000	0.407	3.028

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.006	0.003	0.031	0.283	2.610	0.080	0.427	2.952

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	24	98	39	0	943	23
N.S.	1	1.00	1.11	0.89	3.63	1.44	0.00	34.93	0.85
time (sec)	N/A	0.014	0.013	0.050	0.504	2.219	0.000	0.521	0.095

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	F	F(-2)	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	26	28	0	0	0	0	22
N.S.	1	0.00	1.00	1.08	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.603	0.222	0.635	0.000	0.000	0.000	0.000	3.475

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.078	5.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	C	B	A	A	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	9	24	82	10	10	83	10
N.S.	1	0.00	1.00	2.67	9.11	1.11	1.11	9.22	1.11
time (sec)	N/A	0.276	0.057	0.148	0.426	3.246	0.138	0.396	2.955

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	60	57	60	150	62	111
N.S.	1	1.00	0.99	0.78	0.74	0.78	1.95	0.81	1.44
time (sec)	N/A	0.100	0.118	0.101	0.295	2.781	0.206	0.417	6.559

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	134	125	131	123	326	143	210
N.S.	1	1.00	0.83	0.78	0.81	0.76	2.02	0.89	1.30
time (sec)	N/A	0.204	0.148	0.165	0.272	2.476	0.724	0.427	6.744

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	105	84	79	72	201	70	73
N.S.	1	1.00	1.18	0.94	0.89	0.81	2.26	0.79	0.82
time (sec)	N/A	0.078	0.114	0.126	0.292	2.239	0.212	0.432	3.183

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	167	213	218	162	541	186	456
N.S.	1	1.00	0.58	0.74	0.76	0.56	1.88	0.65	1.58
time (sec)	N/A	0.308	0.314	0.198	0.303	1.866	0.770	0.418	4.517

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	136	0	108	0	0	51
N.S.	1	1.00	0.90	2.23	0.00	1.77	0.00	0.00	0.84
time (sec)	N/A	0.205	0.132	0.291	0.000	0.494	0.000	0.000	3.247

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	266	0	210	0	0	129
N.S.	1	1.00	0.93	1.80	0.00	1.42	0.00	0.00	0.87
time (sec)	N/A	0.169	0.191	0.375	0.000	0.487	0.000	0.000	6.684

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	32	50	30	107	31	35
N.S.	1	1.00	1.26	0.94	1.47	0.88	3.15	0.91	1.03
time (sec)	N/A	0.073	0.069	0.307	0.299	2.273	1.417	0.569	3.456

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	34	62	30	105	31	35
N.S.	1	1.00	1.19	0.94	1.72	0.83	2.92	0.86	0.97
time (sec)	N/A	0.066	0.056	0.324	0.303	2.437	1.408	0.607	3.351

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	73	111	0	150	379	128	226
N.S.	1	1.00	0.61	0.92	0.00	1.25	3.16	1.07	1.88
time (sec)	N/A	0.500	0.387	0.711	0.000	2.698	94.351	0.598	4.216

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	168	58	74	122	48	23
N.S.	1	1.00	0.35	2.33	0.81	1.03	1.69	0.67	0.32
time (sec)	N/A	0.111	0.023	0.359	0.515	3.391	1.869	0.471	3.166

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	51	154	42	76	52	105
N.S.	1	1.00	0.76	0.93	2.80	0.76	1.38	0.95	1.91
time (sec)	N/A	0.295	0.150	0.599	0.513	2.918	0.422	0.496	3.305

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	17	22	16	32	14	14
N.S.	1	1.00	2.06	1.06	1.38	1.00	2.00	0.88	0.88
time (sec)	N/A	0.037	0.010	0.204	0.291	2.969	0.094	0.423	3.034

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	31	29	50
N.S.	1	1.00	1.00	1.06	1.00	1.22	1.72	1.61	2.78
time (sec)	N/A	0.018	0.032	0.270	0.305	2.518	0.188	0.409	3.136

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	30	70	22	0	29	50
N.S.	1	1.00	1.00	1.58	3.68	1.16	0.00	1.53	2.63
time (sec)	N/A	0.235	0.043	0.506	0.509	3.150	0.000	0.463	3.371

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	18	23	17	0	14	14
N.S.	1	1.00	1.94	1.06	1.35	1.00	0.00	0.82	0.82
time (sec)	N/A	0.124	0.011	0.293	0.299	3.204	0.000	0.472	3.054

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	51	154	42	0	52	106
N.S.	1	1.00	0.78	0.94	2.85	0.78	0.00	0.96	1.96
time (sec)	N/A	0.386	0.168	0.793	0.503	2.299	0.000	0.526	3.233

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	26	168	58	74	0	48	23
N.S.	1	1.00	0.36	2.33	0.81	1.03	0.00	0.67	0.32
time (sec)	N/A	0.915	0.018	0.399	0.534	3.342	0.000	0.529	3.089

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [637] had the largest ratio of [43]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	14	0.143
2	A	3	3	1.00	27	0.111
3	A	2	2	1.00	12	0.167
4	A	2	2	1.00	14	0.143
5	A	2	1	1.00	21	0.048
6	A	2	2	1.00	23	0.087
7	A	2	2	1.00	21	0.095
8	A	3	3	1.00	14	0.214
9	A	4	4	1.00	25	0.160
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	14	0.143
12	A	2	1	1.00	23	0.043
13	A	2	2	1.00	23	0.087
14	A	2	2	1.00	23	0.087
15	A	2	2	1.00	12	0.167
16	A	3	3	1.00	25	0.120
17	A	2	2	1.00	14	0.143
18	A	2	2	1.00	12	0.167
19	A	2	1	1.00	21	0.048
20	A	2	2	1.00	21	0.095
21	A	2	2	1.00	23	0.087
22	A	3	3	1.00	14	0.214
23	A	4	4	1.00	25	0.160
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	1	1.00	21	0.048
27	A	2	2	1.00	21	0.095
28	A	2	2	1.00	23	0.087
29	A	6	5	1.00	6	0.833
30	A	9	6	1.00	6	1.000
31	A	8	4	1.00	16	0.250
32	A	8	4	1.00	19	0.210
33	A	3	3	1.00	16	0.188
34	A	3	3	1.00	27	0.111
35	A	2	1	1.00	11	0.091
36	A	5	5	1.00	14	0.357
37	A	6	6	1.00	16	0.375
38	A	9	5	1.00	16	0.312
39	A	5	3	1.00	36	0.083
40	A	4	3	1.00	36	0.083
41	A	2	2	1.00	34	0.059
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	6	5	1.00	6	0.833
45	A	9	6	1.00	6	1.000
46	A	8	4	1.00	16	0.250
47	A	8	4	1.00	19	0.210
48	A	4	4	1.00	21	0.190
49	A	4	4	1.00	27	0.148
50	A	5	4	1.00	37	0.108
51	A	5	5	1.00	14	0.357
52	A	6	6	1.00	16	0.375
53	A	5	3	1.00	36	0.083
54	A	4	3	1.00	36	0.083
55	A	2	2	1.00	34	0.059
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	2	2	1.00	12	0.167
59	A	3	3	1.00	14	0.214
60	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	1	1.00	33	0.030
62	A	3	3	1.00	14	0.214
63	A	2	2	1.00	25	0.080
64	A	4	3	1.00	16	0.188
65	A	4	3	1.00	15	0.200
66	A	1	1	1.00	7	0.143
67	A	1	1	1.00	7	0.143
68	A	1	1	1.00	7	0.143
69	A	4	2	1.00	7	0.286
70	A	1	1	1.00	7	0.143
71	A	1	1	1.00	7	0.143
72	A	1	1	1.00	7	0.143
73	A	4	2	1.00	7	0.286
74	A	4	3	1.00	7	0.429
75	A	9	4	1.00	7	0.571
76	A	5	3	1.00	7	0.429
77	A	10	4	1.00	7	0.571
78	A	10	5	1.00	7	0.714
79	A	6	3	1.00	7	0.429
80	A	3	2	1.00	7	0.286
81	A	3	2	1.00	7	0.286
82	A	6	3	1.00	7	0.429
83	A	6	3	1.00	7	0.429
84	A	7	3	1.00	7	0.429
85	A	2	2	1.00	7	0.286
86	A	5	5	1.00	7	0.714
87	A	4	3	1.00	7	0.429
88	A	7	6	1.00	7	0.857
89	A	7	4	1.00	7	0.571
90	A	2	2	1.00	7	0.286
91	A	2	1	1.00	7	0.143
92	A	4	2	1.00	7	0.286
93	A	4	2	1.00	7	0.286
94	A	7	3	1.00	7	0.429
95	A	3	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	9	0.222
97	A	1	1	1.00	7	0.143
98	A	1	1	1.00	7	0.143
99	A	1	1	1.00	7	0.143
100	A	4	2	1.00	7	0.286
101	A	1	1	1.00	7	0.143
102	A	1	1	1.00	7	0.143
103	A	1	1	1.00	7	0.143
104	A	4	2	1.00	7	0.286
105	A	4	3	1.00	7	0.429
106	A	3	2	1.00	7	0.286
107	A	6	4	1.00	7	0.571
108	A	6	3	1.00	7	0.429
109	A	10	5	1.00	7	0.714
110	A	4	3	1.00	7	0.429
111	A	9	4	1.00	7	0.571
112	A	6	3	1.00	7	0.429
113	A	10	4	1.00	7	0.571
114	A	7	3	1.00	7	0.429
115	A	6	3	1.00	7	0.429
116	A	2	2	1.00	7	0.286
117	A	2	1	1.00	7	0.143
118	A	4	3	1.00	7	0.429
119	A	4	2	1.00	7	0.286
120	A	7	4	1.00	7	0.571
121	A	3	3	1.00	7	0.429
122	A	3	3	1.00	9	0.333
123	A	2	2	1.00	7	0.286
124	A	5	5	1.00	7	0.714
125	A	4	2	1.00	7	0.286
126	A	7	6	1.00	7	0.857
127	A	7	3	1.00	7	0.429
128	A	6	2	1.00	9	0.222
129	A	6	2	1.00	11	0.182
130	A	5	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	5	2	1.00	9	0.222
132	A	6	2	1.00	9	0.222
133	A	6	2	1.00	11	0.182
134	A	7	2	1.00	13	0.154
135	A	3	2	1.00	13	0.154
136	A	3	2	1.00	14	0.143
137	A	3	2	1.00	13	0.154
138	A	3	2	1.00	14	0.143
139	A	4	3	1.00	13	0.231
140	A	4	3	1.00	14	0.214
141	A	4	3	1.00	13	0.231
142	A	4	3	1.00	14	0.214
143	A	3	2	1.00	13	0.154
144	A	3	2	1.00	14	0.143
145	A	3	2	1.00	13	0.154
146	A	3	2	1.00	14	0.143
147	A	2	2	1.00	9	0.222
148	A	3	3	1.00	9	0.333
149	A	4	4	1.00	9	0.444
150	A	2	2	1.00	9	0.222
151	A	3	3	1.00	9	0.333
152	A	4	4	1.00	9	0.444
153	A	4	4	1.00	12	0.333
154	A	6	6	1.00	12	0.500
155	A	3	3	1.00	12	0.250
156	A	5	5	1.00	12	0.417
157	A	3	3	1.00	14	0.214
158	A	3	3	1.00	14	0.214
159	A	2	2	1.00	23	0.087
160	A	9	6	1.00	24	0.250
161	A	11	7	1.00	26	0.269
162	A	2	1	1.00	33	0.030
163	A	9	6	1.00	34	0.176
164	A	11	7	1.00	36	0.194
165	A	5	3	1.00	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	33	0.091
167	A	3	3	1.00	31	0.097
168	A	4	4	1.00	33	0.121
169	A	5	5	1.00	33	0.152
170	A	6	5	1.00	33	0.152
171	A	11	9	1.00	33	0.273
172	A	8	7	1.00	33	0.212
173	A	3	3	1.00	31	0.097
174	A	11	7	1.00	33	0.212
175	A	13	8	1.00	33	0.242
176	A	15	8	1.00	33	0.242
177	A	20	11	1.00	37	0.297
178	A	17	10	1.00	37	0.270
179	A	14	9	1.00	35	0.257
180	A	0	0	0.00	0	0.000
181	A	51	17	1.00	33	0.515
182	A	34	14	1.00	33	0.424
183	A	26	12	1.00	31	0.387
184	A	15	8	1.00	22	0.364
185	A	5	5	1.00	13	0.385
186	A	6	6	1.00	15	0.400
187	A	7	6	1.00	15	0.400
188	A	8	6	1.00	15	0.400
189	A	4	4	1.00	15	0.267
190	A	5	4	1.00	15	0.267
191	A	6	4	1.00	15	0.267
192	A	7	4	1.00	15	0.267
193	A	6	5	1.00	17	0.294
194	A	5	5	1.00	17	0.294
195	A	4	4	1.00	17	0.235
196	A	6	5	1.00	17	0.294
197	A	7	6	1.00	17	0.353
198	A	8	6	1.00	17	0.353
199	A	3	3	1.00	16	0.188
200	A	3	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.00	17	0.235
202	A	3	3	1.00	17	0.176
203	A	4	4	1.00	17	0.235
204	A	3	3	1.00	17	0.176
205	A	4	3	1.00	22	0.136
206	A	8	5	1.00	17	0.294
207	A	5	3	1.00	19	0.158
208	A	3	2	1.00	20	0.100
209	A	4	3	1.00	19	0.158
210	A	4	3	1.00	22	0.136
211	A	10	6	1.00	17	0.353
212	A	4	2	1.00	19	0.105
213	A	3	3	1.00	20	0.150
214	A	4	3	1.00	19	0.158
215	A	6	6	1.00	17	0.353
216	A	7	7	1.00	17	0.412
217	A	2	2	1.00	19	0.105
218	A	2	2	1.00	19	0.105
219	A	3	2	1.00	19	0.105
220	A	4	2	1.00	19	0.105
221	A	3	2	1.00	19	0.105
222	A	3	2	1.00	19	0.105
223	A	2	1	1.00	19	0.053
224	A	2	2	1.00	19	0.105
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	19	0.105
227	A	1	1	1.00	19	0.053
228	A	3	3	1.00	19	0.158
229	A	2	2	1.00	19	0.105
230	A	4	3	1.00	19	0.158
231	A	3	2	1.00	19	0.105
232	A	4	3	1.00	21	0.143
233	A	3	3	1.00	21	0.143
234	A	3	3	1.00	21	0.143
235	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	21	0.095
237	A	3	3	1.00	21	0.143
238	A	3	3	1.00	21	0.143
239	A	4	3	1.00	21	0.143
240	A	4	3	1.00	21	0.143
241	A	3	3	1.00	21	0.143
242	A	3	3	1.00	21	0.143
243	A	2	2	1.00	21	0.095
244	A	2	2	1.00	21	0.095
245	A	3	3	1.00	21	0.143
246	A	3	3	1.00	21	0.143
247	A	4	3	1.00	21	0.143
248	A	1	1	1.00	22	0.045
249	A	1	1	1.00	22	0.045
250	A	1	1	1.00	22	0.045
251	A	1	1	1.00	22	0.045
252	A	3	2	1.00	20	0.100
253	A	1	1	1.00	22	0.045
254	A	1	1	1.00	22	0.045
255	A	1	1	1.00	22	0.045
256	A	1	1	1.00	22	0.045
257	A	1	1	1.00	24	0.042
258	A	1	1	1.00	24	0.042
259	A	1	1	1.00	24	0.042
260	A	1	1	1.00	24	0.042
261	A	1	1	1.00	24	0.042
262	A	1	1	1.00	24	0.042
263	A	8	7	1.00	11	0.636
264	A	4	4	1.00	11	0.364
265	A	7	6	1.00	11	0.546
266	A	4	3	1.00	11	0.273
267	A	3	2	1.00	9	0.222
268	A	3	3	1.00	11	0.273
269	A	6	6	1.00	11	0.546
270	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	8	8	1.00	11	0.727
272	A	4	3	1.00	11	0.273
273	A	4	3	1.00	7	0.429
274	A	5	4	1.00	7	0.571
275	A	4	3	1.00	7	0.429
276	A	4	4	1.00	7	0.571
277	A	3	2	0.69	5	0.400
278	A	3	3	1.00	7	0.429
279	A	3	3	1.00	7	0.429
280	A	4	3	1.00	7	0.429
281	A	4	3	1.00	7	0.429
282	A	4	3	1.00	7	0.429
283	A	8	7	1.00	11	0.636
284	A	4	4	1.00	11	0.364
285	A	7	6	1.00	11	0.546
286	A	4	3	1.00	11	0.273
287	A	3	2	1.00	9	0.222
288	A	3	3	1.00	11	0.273
289	A	5	5	1.00	11	0.454
290	A	4	3	1.00	11	0.273
291	A	7	7	1.00	11	0.636
292	A	4	3	1.00	11	0.273
293	A	4	3	1.00	7	0.429
294	A	5	4	1.00	7	0.571
295	A	4	3	1.00	7	0.429
296	A	4	4	1.00	7	0.571
297	A	3	2	1.00	5	0.400
298	A	3	3	1.00	7	0.429
299	A	3	3	1.00	7	0.429
300	A	4	3	1.00	7	0.429
301	A	4	3	1.00	7	0.429
302	A	4	3	1.00	7	0.429
303	A	6	3	1.00	9	0.333
304	A	6	5	1.00	9	0.556
305	A	4	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	2	1.00	7	0.286
307	A	3	3	1.00	9	0.333
308	A	2	1	1.00	9	0.111
309	A	4	3	1.00	9	0.333
310	A	2	0	1.00	9	0.000
311	A	4	3	1.00	9	0.333
312	A	3	1	1.00	9	0.111
313	A	4	3	1.00	9	0.333
314	A	6	5	1.00	11	0.454
315	A	5	5	1.00	11	0.454
316	A	4	4	1.00	11	0.364
317	A	3	3	1.00	11	0.273
318	A	8	8	1.00	11	0.727
319	A	9	9	1.00	11	0.818
320	A	10	10	1.00	11	0.909
321	A	11	10	1.00	11	0.909
322	A	6	3	1.00	9	0.333
323	A	6	5	1.00	9	0.556
324	A	4	3	1.00	9	0.333
325	A	3	2	1.00	7	0.286
326	A	3	3	1.00	9	0.333
327	A	2	1	1.00	9	0.111
328	A	4	3	1.00	9	0.333
329	A	2	0	1.00	9	0.000
330	A	4	3	1.00	9	0.333
331	A	3	1	1.00	9	0.111
332	A	4	3	1.00	9	0.333
333	A	6	5	1.00	11	0.454
334	A	5	5	1.00	11	0.454
335	A	4	4	1.00	11	0.364
336	A	3	3	1.00	11	0.273
337	A	8	8	1.00	11	0.727
338	A	9	9	1.00	11	0.818
339	A	10	10	1.00	11	0.909
340	A	11	10	1.00	11	0.909

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	18	9	1.00	7	1.286
342	A	4	3	1.00	7	0.429
343	A	9	7	1.00	7	1.000
344	A	3	2	1.00	5	0.400
345	A	6	6	1.00	7	0.857
346	A	11	6	1.00	7	0.857
347	A	5	4	1.00	7	0.571
348	A	18	6	1.00	7	0.857
349	A	3	3	1.00	18	0.167
350	A	3	3	1.00	18	0.167
351	A	4	4	1.00	18	0.222
352	A	3	3	1.00	18	0.167
353	A	3	3	1.00	18	0.167
354	A	4	4	1.00	18	0.222
355	A	6	3	1.00	30	0.100
356	A	5	3	1.00	30	0.100
357	A	4	3	1.00	30	0.100
358	A	3	2	1.00	28	0.071
359	A	1	1	1.00	30	0.033
360	A	2	2	1.00	30	0.067
361	A	3	2	1.00	30	0.067
362	A	4	2	1.00	30	0.067
363	A	5	4	1.00	24	0.167
364	A	4	3	1.00	24	0.125
365	A	3	2	1.00	22	0.091
366	A	2	2	1.00	24	0.083
367	A	4	4	1.00	24	0.167
368	A	4	4	1.00	24	0.167
369	A	5	5	1.00	24	0.208
370	A	2	2	1.00	24	0.083
371	A	4	4	1.00	24	0.167
372	A	4	4	1.00	24	0.167
373	A	5	5	1.00	24	0.208
374	A	5	4	1.00	24	0.167
375	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	3	2	1.00	22	0.091
377	A	2	2	1.00	24	0.083
378	A	4	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	5	5	1.00	24	0.208
381	A	5	4	1.00	24	0.167
382	A	4	3	1.00	24	0.125
383	A	3	2	1.00	22	0.091
384	A	2	2	1.00	24	0.083
385	A	4	4	1.00	24	0.167
386	A	4	4	1.00	24	0.167
387	A	5	5	1.00	24	0.208
388	A	5	4	1.00	24	0.167
389	A	4	3	1.00	24	0.125
390	A	3	2	1.00	22	0.091
391	A	2	2	1.00	24	0.083
392	A	4	4	1.00	24	0.167
393	A	4	4	1.00	24	0.167
394	A	5	5	1.00	24	0.208
395	A	6	4	1.00	20	0.200
396	A	5	4	1.00	20	0.200
397	A	4	3	1.00	20	0.150
398	A	3	2	1.00	18	0.111
399	A	3	3	1.00	20	0.150
400	A	5	5	1.00	20	0.250
401	A	5	5	1.00	20	0.250
402	A	6	6	1.00	20	0.300
403	A	7	7	1.00	22	0.318
404	A	6	6	1.00	22	0.273
405	A	2	2	1.00	22	0.091
406	A	2	2	1.00	22	0.091
407	A	3	3	1.00	22	0.136
408	A	7	7	1.00	22	0.318
409	A	8	7	1.00	22	0.318
410	A	7	7	1.00	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	6	6	1.00	22	0.273
412	A	2	2	1.00	22	0.091
413	A	2	2	1.00	22	0.091
414	A	3	3	1.00	22	0.136
415	A	7	7	1.00	22	0.318
416	A	8	7	1.00	22	0.318
417	A	3	2	1.00	22	0.091
418	A	2	2	1.00	22	0.091
419	A	1	1	1.00	22	0.045
420	A	3	3	1.00	22	0.136
421	A	4	4	1.00	22	0.182
422	A	5	4	1.00	22	0.182
423	A	4	2	1.00	22	0.091
424	A	3	2	1.00	22	0.091
425	A	2	2	1.00	22	0.091
426	A	1	1	1.00	22	0.045
427	A	3	3	1.00	22	0.136
428	A	4	4	1.00	22	0.182
429	A	5	4	1.00	22	0.182
430	A	4	2	1.00	32	0.062
431	A	3	2	1.00	32	0.062
432	A	2	2	1.00	32	0.062
433	A	1	1	1.00	32	0.031
434	A	3	3	1.00	32	0.094
435	A	4	4	1.00	32	0.125
436	A	5	4	1.00	32	0.125
437	A	3	2	1.00	34	0.059
438	A	2	2	1.00	34	0.059
439	A	1	1	1.00	34	0.029
440	A	3	3	1.00	34	0.088
441	A	4	4	1.00	34	0.118
442	A	5	4	1.00	34	0.118
443	A	4	4	1.00	15	0.267
444	A	3	3	1.36	11	0.273
445	A	5	5	1.00	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
446	A	4	4	1.00	15	0.267
447	A	10	8	1.00	17	0.471
448	A	7	7	1.00	33	0.212
449	A	3	3	1.00	33	0.091
450	A	3	3	1.00	33	0.091
451	A	4	4	1.00	33	0.121
452	A	8	8	1.00	33	0.242
453	A	7	7	1.00	33	0.212
454	A	3	3	1.00	33	0.091
455	A	3	3	1.00	33	0.091
456	A	4	4	1.00	33	0.121
457	A	8	8	1.00	33	0.242
458	A	5	5	1.00	12	0.417
459	A	4	4	1.00	15	0.267
460	A	9	7	1.00	17	0.412
461	A	4	4	1.00	15	0.267
462	A	7	7	1.00	33	0.212
463	A	3	3	1.00	33	0.091
464	A	3	3	1.00	33	0.091
465	A	4	4	1.00	33	0.121
466	A	8	8	1.00	33	0.242
467	A	7	7	1.00	33	0.212
468	A	3	3	1.00	33	0.091
469	A	3	3	1.00	33	0.091
470	A	4	4	1.00	33	0.121
471	A	8	8	1.00	33	0.242
472	A	2	2	1.00	11	0.182
473	A	2	2	1.00	11	0.182
474	A	2	2	1.00	11	0.182
475	A	2	1	1.00	13	0.077
476	A	2	1	1.00	13	0.077
477	A	4	3	1.00	13	0.231
478	A	2	1	1.00	15	0.067
479	A	2	1	1.00	15	0.067
480	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	2	1	1.00	23	0.043
482	A	4	3	1.00	20	0.150
483	A	4	3	1.00	20	0.150
484	A	4	2	1.00	11	0.182
485	A	6	4	1.00	11	0.364
486	A	6	4	1.00	11	0.364
487	A	2	2	1.00	13	0.154
488	A	2	2	1.00	13	0.154
489	A	2	2	1.00	13	0.154
490	A	4	2	1.00	11	0.182
491	A	6	4	1.00	11	0.364
492	A	6	4	1.00	11	0.364
493	A	2	2	1.00	13	0.154
494	A	2	2	1.00	13	0.154
495	A	2	2	1.00	13	0.154
496	A	2	1	1.00	16	0.062
497	A	9	6	1.00	18	0.333
498	A	11	7	1.00	20	0.350
499	A	5	3	1.00	39	0.077
500	A	2	2	1.00	37	0.054
501	A	3	3	1.00	39	0.077
502	A	9	6	1.00	39	0.154
503	A	7	4	1.00	21	0.190
504	A	4	3	1.00	41	0.073
505	A	2	2	1.00	41	0.049
506	A	5	5	1.00	41	0.122
507	A	8	6	1.00	41	0.146
508	A	3	3	1.00	27	0.111
509	A	5	3	1.00	21	0.143
510	A	7	5	1.00	39	0.128
511	A	3	3	1.00	37	0.081
512	A	4	4	1.00	39	0.103
513	A	6	4	1.00	39	0.103
514	A	6	5	1.00	41	0.122
515	A	3	3	1.00	41	0.073

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	3	1.00	41	0.073
517	A	5	4	1.00	41	0.098
518	A	8	7	1.00	39	0.180
519	A	5	4	1.00	37	0.108
520	A	6	6	1.00	39	0.154
521	A	8	7	1.00	39	0.180
522	A	7	6	1.00	41	0.146
523	A	5	5	1.00	41	0.122
524	A	5	5	1.00	41	0.122
525	A	7	7	1.00	41	0.171
526	A	1	1	1.00	21	0.048
527	A	1	1	1.00	21	0.048
528	A	1	1	1.00	15	0.067
529	A	1	1	1.00	21	0.048
530	A	3	3	1.00	21	0.143
531	A	3	3	1.00	21	0.143
532	A	3	3	1.00	22	0.136
533	A	3	3	1.00	22	0.136
534	A	4	4	1.00	22	0.182
535	A	4	4	1.00	19	0.210
536	A	4	4	1.00	19	0.210
537	A	5	5	1.00	19	0.263
538	A	1	1	1.00	22	0.045
539	A	1	1	1.00	22	0.045
540	A	4	4	1.00	19	0.210
541	A	4	4	1.00	19	0.210
542	A	5	5	1.00	19	0.263
543	A	1	1	1.00	22	0.045
544	A	1	1	1.00	22	0.045
545	A	4	4	1.00	22	0.182
546	A	4	4	1.00	22	0.182
547	A	5	5	1.00	22	0.227
548	A	1	1	0.95	25	0.040
549	A	1	1	0.94	25	0.040
550	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	4	4	1.00	23	0.174
552	A	5	5	1.00	23	0.217
553	A	1	1	1.00	26	0.038
554	A	1	1	1.00	26	0.038
555	B	1	1	2.83	30	0.033
556	A	8	6	1.00	27	0.222
557	A	7	6	1.00	27	0.222
558	A	6	6	1.00	27	0.222
559	A	5	5	1.00	27	0.185
560	A	6	6	1.00	27	0.222
561	A	7	6	1.00	27	0.222
562	A	7	7	1.00	31	0.226
563	A	8	8	1.00	31	0.258
564	A	9	8	1.00	31	0.258
565	A	10	8	1.00	31	0.258
566	A	4	4	1.00	18	0.222
567	A	3	3	1.00	18	0.167
568	A	2	2	1.00	18	0.111
569	A	3	2	1.00	16	0.125
570	A	4	4	1.00	18	0.222
571	A	6	6	1.00	18	0.333
572	A	7	7	1.00	18	0.389
573	A	8	8	1.00	20	0.400
574	A	7	7	1.00	20	0.350
575	A	3	3	1.00	20	0.150
576	A	3	3	1.00	20	0.150
577	A	5	5	1.00	20	0.250
578	A	8	8	1.00	20	0.400
579	A	13	8	1.00	14	0.571
580	A	11	7	1.00	14	0.500
581	A	9	6	1.00	12	0.500
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	0	0	0.00	0	0.000
585	A	15	6	1.00	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	11	5	1.00	26	0.192
587	A	6	6	1.00	26	0.231
588	A	4	3	1.00	26	0.115
589	A	1	1	1.00	23	0.043
590	A	1	1	1.00	22	0.045
591	A	3	3	1.00	20	0.150
592	A	7	5	1.00	24	0.208
593	A	9	9	1.00	26	0.346
594	A	15	6	1.00	24	0.250
595	A	11	5	1.00	24	0.208
596	A	6	6	1.00	24	0.250
597	A	4	3	1.00	24	0.125
598	A	1	1	1.00	21	0.048
599	A	1	1	1.00	20	0.050
600	A	3	3	1.00	18	0.167
601	A	7	5	1.00	22	0.227
602	A	9	9	1.00	24	0.375
603	A	5	5	1.00	31	0.161
604	A	4	4	1.00	31	0.129
605	A	3	3	1.00	31	0.097
606	A	2	2	1.00	29	0.069
607	A	3	3	1.00	20	0.150
608	A	4	4	1.00	29	0.138
609	A	5	4	1.00	31	0.129
610	A	6	4	1.00	31	0.129
611	A	7	7	1.00	31	0.226
612	A	5	5	1.00	31	0.161
613	A	4	4	1.00	31	0.129
614	A	3	3	1.00	29	0.103
615	A	5	5	1.00	20	0.250
616	A	6	6	1.00	29	0.207
617	A	6	6	1.00	31	0.194
618	A	7	6	1.00	31	0.194
619	A	6	6	1.00	31	0.194
620	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	4	4	1.00	31	0.129
622	A	3	3	1.00	29	0.103
623	A	6	5	1.00	20	0.250
624	A	7	6	1.00	29	0.207
625	A	8	7	1.00	31	0.226
626	A	6	6	1.00	31	0.194
627	A	5	5	1.00	31	0.161
628	A	4	4	1.00	31	0.129
629	A	4	4	1.00	29	0.138
630	A	7	6	1.00	20	0.300
631	A	8	7	1.00	29	0.241
632	A	9	7	1.00	31	0.226
633	A	3	2	1.00	13	0.154
634	A	6	4	1.00	21	0.190
635	A	6	4	1.00	28	0.143
636	A	6	4	1.00	30	0.133
637	A	1	1	1.00	43	0.023
638	A	1	1	1.00	43	0.023
639	A	1	1	1.00	43	0.023
640	A	1	1	1.00	41	0.024
641	A	1	1	1.00	43	0.023
642	A	1	1	1.00	43	0.023
643	A	1	1	1.00	43	0.023
644	A	0	0	0.00	0	0.000
645	A	0	0	0.00	0	0.000
646	A	0	0	0.00	0	0.000
647	A	0	0	0.00	0	0.000
648	A	2	2	1.00	11	0.182
649	A	2	2	1.00	11	0.182
650	A	2	2	1.00	13	0.154
651	A	2	2	1.00	6	0.333
652	A	4	4	1.00	11	0.364
653	A	6	3	1.00	13	0.231
654	A	4	3	1.00	17	0.176
655	A	2	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	3	3	1.00	21	0.143
657	A	3	3	1.00	19	0.158
658	A	3	2	1.00	15	0.133
659	A	2	2	1.00	17	0.118
660	A	2	2	1.00	22	0.091
661	A	2	2	1.00	22	0.091
662	A	2	2	1.00	17	0.118
663	A	2	2	1.00	22	0.091
664	A	2	2	1.00	22	0.091
665	A	2	2	1.00	11	0.182
666	A	2	2	1.00	11	0.182
667	A	2	2	1.00	13	0.154
668	A	2	2	1.00	15	0.133
669	A	2	2	1.00	19	0.105
670	A	4	4	1.00	11	0.364
671	A	3	3	1.00	15	0.200
672	A	2	2	1.00	15	0.133
673	A	3	3	1.00	16	0.188
674	A	2	2	1.00	6	0.333
675	A	3	2	1.00	10	0.200
676	A	2	2	1.00	6	0.333
677	A	4	3	1.00	17	0.176
678	A	3	3	1.00	9	0.333
679	A	4	3	1.00	13	0.231
680	A	2	2	1.00	17	0.118
681	A	2	2	1.00	9	0.222
682	A	2	2	1.00	12	0.167
683	A	2	2	1.00	18	0.111
684	A	2	2	1.00	17	0.118
685	A	2	2	1.00	22	0.091
686	A	2	2	1.00	22	0.091
687	A	2	2	1.00	17	0.118
688	A	2	2	1.00	22	0.091
689	A	2	2	1.00	22	0.091
690	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	2	2	1.00	15	0.133
692	A	2	2	1.00	13	0.154
693	A	2	2	1.00	13	0.154
694	A	3	1	1.00	15	0.067
695	A	4	3	1.00	19	0.158
696	A	3	3	1.00	17	0.176
697	A	3	2	1.50	16	0.125
698	A	3	2	1.50	18	0.111
699	A	7	7	1.00	15	0.467
700	A	3	3	1.00	19	0.158
701	A	3	2	1.00	19	0.105
702	A	3	2	1.00	21	0.095
703	A	3	2	1.00	21	0.095
704	A	2	2	1.00	17	0.118
705	A	4	3	1.00	17	0.176
706	A	5	5	1.00	19	0.263
707	A	2	2	1.00	11	0.182
708	A	4	2	1.00	17	0.118
709	A	2	2	1.00	17	0.118
710	A	2	2	1.00	17	0.118
711	A	3	3	1.00	15	0.200
712	A	3	3	1.00	17	0.176
713	A	3	3	1.00	17	0.176
714	A	2	2	1.00	9	0.222
715	A	4	3	1.00	15	0.200
716	A	2	2	1.00	13	0.154
717	A	2	2	1.00	13	0.154
718	A	2	2	1.00	11	0.182
719	A	4	2	1.00	15	0.133
720	A	3	2	1.00	19	0.105
721	A	3	2	1.00	21	0.095
722	A	3	2	1.00	21	0.095
723	A	2	2	1.00	11	0.182
724	A	4	4	1.82	13	0.308
725	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	2	2	1.00	15	0.133
727	A	2	2	1.00	14	0.143
728	A	3	3	1.00	15	0.200
729	A	2	1	1.00	15	0.067
730	A	2	2	1.00	9	0.222
731	A	2	2	1.00	9	0.222
732	A	2	2	1.00	19	0.105
733	A	3	2	1.00	23	0.087
734	A	2	1	1.00	23	0.043
735	A	4	4	1.67	13	0.308
736	A	2	2	1.00	15	0.133
737	A	2	2	1.00	13	0.154
738	A	3	3	1.00	21	0.143
739	A	2	1	1.00	15	0.067
740	A	3	2	1.00	23	0.087
741	A	4	3	1.00	20	0.150
742	A	4	3	1.00	19	0.158
743	A	4	3	1.00	24	0.125
744	A	4	3	1.00	21	0.143
745	A	4	3	1.00	20	0.150
746	A	4	3	1.00	19	0.158
747	A	4	3	1.00	24	0.125
748	A	4	3	1.00	21	0.143
749	A	1	3	1.00	8	0.375
750	A	1	2	1.00	8	0.250
751	A	3	2	1.00	11	0.182
752	A	2	2	1.00	6	0.333
753	A	4	3	1.00	8	0.375
754	A	2	2	1.00	9	0.222
755	A	2	2	1.00	12	0.167
756	A	3	2	1.00	13	0.154
757	A	2	2	1.00	8	0.250
758	A	1	1	1.00	12	0.083
759	B	25	3	10.75	20	0.150
760	A	2	2	1.00	6	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	3	1.00	8	0.375
762	A	4	3	1.00	15	0.200
763	A	2	2	1.00	15	0.133
764	A	8	4	1.00	17	0.235
765	A	3	2	1.00	13	0.154
766	A	2	2	1.00	6	0.333
767	A	2	2	1.00	10	0.200
768	A	2	2	1.00	8	0.250
769	A	2	2	1.00	10	0.200
770	A	3	3	1.00	10	0.300
771	A	3	3	1.00	10	0.300
772	A	2	2	1.00	8	0.250
773	A	1	1	1.00	8	0.125
774	A	3	1	1.00	18	0.056
775	A	6	3	1.00	16	0.188
776	A	6	3	1.00	8	0.375
777	A	2	2	1.00	17	0.118
778	A	3	3	1.00	7	0.429
779	A	3	3	1.00	11	0.273
780	A	3	2	1.00	10	0.200
781	A	2	2	1.00	8	0.250
782	A	2	2	1.00	8	0.250
783	A	3	2	1.00	10	0.200
784	A	2	2	1.00	10	0.200
785	A	3	3	1.00	9	0.333
786	A	2	2	1.00	8	0.250
787	A	3	3	1.00	8	0.375
788	A	1	1	1.00	8	0.125
789	A	3	3	1.00	8	0.375
790	A	3	3	1.00	8	0.375
791	A	2	2	1.00	8	0.250
792	A	2	2	1.00	13	0.154
793	A	1	1	1.00	11	0.091
794	A	3	3	1.00	9	0.333
795	A	3	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
796	F	0	0	N/A	0.	N/A
797	A	2	2	1.00	6	0.333
798	A	3	2	1.00	11	0.182
799	A	3	2	1.00	8	0.250
800	A	3	2	1.00	7	0.286
801	A	4	3	1.00	9	0.333
802	A	2	2	1.00	9	0.222
803	A	2	2	1.00	7	0.286
804	A	2	2	1.00	13	0.154
805	A	6	4	1.00	10	0.400
806	A	2	2	1.00	21	0.095
807	A	3	3	1.00	15	0.200
808	A	2	2	1.00	12	0.167
809	A	5	5	1.00	16	0.312
810	A	4	2	1.00	11	0.182
811	A	8	4	1.00	13	0.308
812	A	1	1	1.00	10	0.100
813	A	3	2	1.00	13	0.154
814	A	4	2	1.00	12	0.167
815	A	3	2	1.00	8	0.250
816	A	5	4	1.00	10	0.400
817	A	6	4	1.00	15	0.267
818	A	4	3	1.00	11	0.273
819	A	3	2	1.00	13	0.154
820	A	2	2	1.00	8	0.250
821	A	2	2	1.00	28	0.071
822	A	1	1	1.00	12	0.083
823	A	2	2	1.00	23	0.087
824	A	3	3	1.00	7	0.429
825	A	2	2	1.00	10	0.200
826	A	2	2	1.00	8	0.250
827	A	4	2	1.29	13	0.154
828	A	3	1	1.00	20	0.050
829	A	3	3	1.00	9	0.333
830	A	5	5	1.00	10	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	5	4	1.00	9	0.444
832	A	4	4	1.00	10	0.400
833	A	5	4	1.00	10	0.400
834	A	4	3	1.00	10	0.300
835	A	6	4	1.00	10	0.400
836	A	5	5	1.00	14	0.357
837	A	5	4	1.00	13	0.308
838	A	5	2	1.00	9	0.222
839	A	5	2	1.00	11	0.182
840	A	2	2	1.00	7	0.286
841	A	6	2	1.00	9	0.222
842	A	13	5	1.00	10	0.500
843	A	3	3	1.00	18	0.167
844	A	1	1	1.00	18	0.056
845	A	3	3	1.00	18	0.167
846	A	9	7	1.27	15	0.467
847	A	8	7	1.27	15	0.467
848	A	4	2	1.00	15	0.133
849	A	4	2	1.00	15	0.133
850	A	3	3	1.00	21	0.143
851	A	3	3	1.00	21	0.143
852	A	1	1	1.00	14	0.071
853	A	4	4	1.00	15	0.267
854	A	4	3	1.00	14	0.214
855	A	7	5	1.28	16	0.312
856	A	7	5	1.28	16	0.312
857	A	4	2	1.00	15	0.133
858	B	5	4	4.63	18	0.222
859	F	0	0	N/A	0.	N/A
860	B	17	9	3.79	16	0.562
861	A	4	4	1.00	12	0.333
862	A	2	2	1.00	15	0.133
863	A	6	5	1.00	15	0.333
864	A	6	5	1.00	15	0.333
865	A	8	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	10	8	1.11	23	0.348
867	A	7	6	1.00	23	0.261
868	A	6	4	1.00	16	0.250
869	A	8	5	1.00	18	0.278
870	A	10	6	1.00	18	0.333
871	A	5	5	1.00	16	0.312
872	A	6	6	1.00	18	0.333
873	A	7	7	1.00	18	0.389
874	A	10	10	1.00	16	0.625
875	A	17	14	1.00	18	0.778
876	A	21	13	1.00	18	0.722
877	A	12	11	1.00	16	0.688
878	A	16	13	1.00	18	0.722
879	A	21	17	1.00	18	0.944
880	A	5	2	1.00	11	0.182
881	A	5	2	1.00	11	0.182
882	A	5	2	1.00	11	0.182
883	A	5	2	1.00	11	0.182
884	A	2	2	1.00	6	0.333
885	A	1	1	1.00	15	0.067
886	A	4	4	1.00	9	0.444
887	A	3	2	1.00	15	0.133
888	A	4	3	1.42	16	0.188
889	A	3	2	1.00	15	0.133
890	A	5	4	1.00	13	0.308
891	A	3	2	1.00	13	0.154
892	A	5	4	1.00	13	0.308
893	A	4	3	1.00	15	0.200
894	A	5	4	1.00	7	0.571
895	A	3	2	1.00	13	0.154
896	A	5	4	1.00	27	0.148
897	A	5	4	1.00	27	0.148
898	A	5	4	1.00	27	0.148
899	A	5	4	1.00	27	0.148
900	A	5	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
901	A	7	5	1.00	23	0.217
902	A	4	3	1.00	13	0.231
903	A	7	7	1.00	15	0.467
904	A	2	2	1.00	10	0.200
905	A	3	2	1.00	17	0.118
906	A	5	4	1.00	17	0.235
907	A	5	3	1.00	8	0.375
908	A	3	2	1.00	13	0.154
909	A	4	3	1.00	16	0.188
910	A	6	5	1.00	13	0.385
911	A	8	3	1.00	12	0.250
912	B	22	9	4.26	18	0.500
913	A	3	3	1.00	9	0.333
914	F	0	0	N/A	0.	N/A
915	F	0	0	N/A	0.	N/A
916	A	6	4	1.00	12	0.333
917	A	2	2	1.00	9	0.222
918	A	1	1	1.00	21	0.048
919	A	3	3	1.00	10	0.300
920	A	8	4	1.00	13	0.308
921	A	1	1	1.00	8	0.125
922	A	7	5	1.00	12	0.417
923	A	1	1	1.00	18	0.056
924	A	1	1	1.00	14	0.071
925	A	1	1	1.00	22	0.045
926	A	7	4	1.00	22	0.182
927	A	6	4	1.00	22	0.182
928	A	3	3	1.00	10	0.300
929	A	3	3	1.00	9	0.333
930	A	2	2	1.00	14	0.143
931	F	0	0	N/A	0.	N/A
932	A	0	0	0.00	0	0.000
933	F	0	0	N/A	0.	N/A
934	A	7	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
935	A	9	6	1.00	30	0.200
936	A	7	6	1.00	36	0.167
937	A	16	5	1.00	38	0.132
938	A	7	6	1.00	29	0.207
939	A	11	8	1.00	31	0.258
940	A	4	4	1.00	27	0.148
941	A	4	4	1.00	27	0.148
942	A	7	4	1.00	39	0.103
943	A	4	2	1.00	39	0.051
944	A	5	3	1.00	39	0.077
945	A	6	3	1.00	39	0.077
946	A	1	1	1.00	31	0.032
947	A	4	2	1.00	31	0.065
948	A	2	1	1.00	39	0.026
949	A	5	3	1.00	39	0.077
950	A	4	2	1.00	39	0.051

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{2}{3-\cos(4+6x)} \, dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\operatorname{ArcTan}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 2736}

$$\frac{\operatorname{ArcTan}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[2/(3 - Cos[4 + 6*x]),x]

[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{2}{3-\cos(4+6x)} \, dx &= 2\int \frac{1}{3-\cos(4+6x)} \, dx\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.50

$$\frac{\text{ArcTan}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 - Cos[4 + 6*x]),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.13, size = 17, normalized size = 0.39

method	result	size
derivativedivides	$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3-cos(4+6*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A]
time = 1.56, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

Sympy [A]
time = 0.12, size = 32, normalized size = 0.73

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(3x+2\right)\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x)
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-2\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-2\cos\left(6\,x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3-cos(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.58, size = 35, normalized size = 0.80

$$\frac{\sqrt{2}\,\left(3\,x-\mathrm{atan}\left(\tan\left(3\,x+2\right)\right)\right)}{6}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(cos(6*x + 4) - 3),x)
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

3.2 $\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=44

$$\frac{x}{\sqrt{2}}+\frac{\mathrm{ArcTan}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 3245, 2736}

$$\frac{\mathrm{ArcTan}\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = 2\int \frac{\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$$
$$= 2\int \frac{1}{3-\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.50

$$\frac{\text{ArcTan}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.38, size = 17, normalized size = 0.39

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.59

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6x+4\right)}{\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A]
time = 1.55, size = 33, normalized size = 0.75

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(6x+4\right)-\sqrt{2}}{4\sin\left(6x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\csc\left(6x+4\right)}{\cot\left(6x+4\right)-3\csc\left(6x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
[Out] -2*Integral(csc(6*x + 4)/(cot(6*x + 4) - 3*csc(6*x + 4)), x)
```

Giac [A]
time = 0.45, size = 57, normalized size = 1.30

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.69, size = 16, normalized size = 0.36

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(cot(6*x + 4) - 3/sin(6*x + 4))),x)
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.3 $\int \frac{1}{1+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.21, size = 17, normalized size = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.67, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3\,x+2\right)^{2}-2\sqrt{2}}{4\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(44) = 88$.
time = 3.36, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\,\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{8119\sqrt{2}\,\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

Giac [A]
time = 0.42, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.49, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\,\left(3x-\operatorname{atan}(\tan(3x+2))\right)}{6}+\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 + 1),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

3.4 $\int \frac{1}{2-\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(2 - Cos[2 + 3*x]^2)^(-1),x]

[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^2)]/(3*Sqrt[2])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\cot(2+3x)\right)\right)\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\,\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Cos[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.13, size = 17, normalized size = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-cos(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 16, normalized size = 0.33

$$\frac{1}{6}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.80, size = 43, normalized size = 0.90

$$-\frac{1}{12}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(3\,x+2\right)^{2}-2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(44) = 88$.
time = 3.36, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\,(3x-\operatorname{atan}(\tan(3x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(3*x + 2)^2 - 2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

3.5 $\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {209}

$$\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\,\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.26, size = 17, normalized size = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.63, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(3\,x+2\right)^{2}-2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(44) = 88$.
time = 3.34, size = 246, normalized size = 5.12

$$\frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}+\frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] 47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt(3
 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/2 -
 pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2) + 3)
*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x/2 + 1
)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) +
117606)
```

Giac [A]
time = 0.40, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.38, size = 35, normalized size = 0.73

$$\frac{\sqrt{2}\left(3x-\operatorname{atan}(\tan(3x+2))\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(3*x + 2)^2 + cos(3*x + 2)^2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)
))/6
```

3.6 $\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.34, size = 17, normalized size = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.78, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-2\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{2\tan^{2}\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(1+2*tan(2+3*x)**2),x)
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 + 1), x)
```

Giac [A]
time = 0.95, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Mupad [B]
time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 + 1)),x)
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.7 $\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3756, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\sin^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Sin[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

Maple [A]
time = 0.29, size = 17, normalized size = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2+cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.33

$$\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.70, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-2\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
2)*sin(3*x + 2)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(3x+2\right)}{\cot^{2}\left(3x+2\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2+cot(2+3*x)**2),x)
[Out] Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 + 2), x)
```

Giac [A]
time = 0.51, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-2\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-2\cos\left(6x+4\right)+2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2)
*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

Mupad [B]
time = 2.40, size = 16, normalized size = 0.33

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 + 2)),x)
[Out] (2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.8 $\int \frac{2}{1-3\cos(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\,\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\,\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 2738, 213}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\,\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = 2\int \frac{1}{1-3\cos(4+6x)}\,dx$$
$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(1 - 3*Cos[4 + 6*x]),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.10, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(1-3*cos(4+6*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 54, normalized size = 0.90

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{2\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A]
time = 2.09, size = 74, normalized size = 1.23

$$\frac{1}{24}\,\sqrt{2}\,\log\left(-\frac{7\,\cos\left(6\,x+4\right)^{2}-4\,\left(\sqrt{2}\,\cos\left(6\,x+4\right)-3\,\sqrt{2}\right)\sin\left(6\,x+4\right)+6\,\cos\left(6\,x+4\right)-17}{9\,\cos\left(6\,x+4\right)^{2}-6\,\cos\left(6\,x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

Sympy [A]
time = 0.14, size = 42, normalized size = 0.70

$$\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\frac{\sqrt{2}}{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2)/2)/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(
2)/2)/12
```

Giac [A]
time = 0.42, size = 39, normalized size = 0.65

$$\frac{1}{12}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(1-3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.57, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3\,x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(3*cos(6*x + 4) - 1),x)
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.9 $\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {12, 3245, 2738, 213}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
```

```
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx &= 2\int \frac{\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx\\ &= 2\int \frac{1}{1-3\cos(4+6x)}\,dx\\ &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-2+4x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2\right.}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.36, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 54, normalized size = 0.90

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{2\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A]
time = 1.27, size = 74, normalized size = 1.23

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2-4\left(\sqrt{2}\cos(6x+4)-3\sqrt{2}\right)\sin(6x+4)+6\cos(6x+4)-17}{9\cos(6x+4)^2-6\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))*
sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) + 1)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) - csc(6*x + 4)), x)
```

Giac [A]
time = 0.47, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan(3x+2)\right|}{\left|2\sqrt{2}+4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.62, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\tan(3\,x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) - 1/sin(6*x + 4))),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.10 $\int \frac{1}{-1+3\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 213}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+3\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\,\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.17, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+3*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-2\,\tan\left(3\,x+2\right)}{\sqrt{2}+2\,\tan\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A]
time = 1.60, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\,\log\left(-\frac{7\,\cos\left(3\,x+2\right)^{4}-4\,\cos\left(3\,x+2\right)^{2}-4\left(\sqrt{2}\,\cos\left(3\,x+2\right)^{3}-2\,\sqrt{2}\,\cos\left(3\,x+2\right)\right)\sin\left(3\,x+2\right)-4}{9\,\cos\left(3\,x+2\right)^{4}-12\,\cos\left(3\,x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.
time = 5.95, size = 1644, normalized size = 27.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
```

```
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

Giac [A]
time = 0.40, size = 39, normalized size = 0.65

$$\frac{1}{12}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.45, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3\,x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(3*x + 2)^2 - 1),x)
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.11 $\int \frac{1}{2-3\cos^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3260, 212}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-3\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\cot(2+3x)\right)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.12, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A]
time = 1.09, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.
time = 5.77, size = 1644, normalized size = 27.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
```

```
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

Giac [A]
time = 0.43, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="giac")
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.44, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*cos(3*x + 2)^2 - 2),x)
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.12 $\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {213}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.25, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A]
time = 1.38, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.

time = 6.04, size = 1644, normalized size = 27.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

```
[Out] -1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(
6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6))*l
og(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 19104891739654
8*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 137649929
5618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467
972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 -
 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqr
t(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x
/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6)
+ 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473
741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 1247944371758796*sq
rt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-4679723635326
75 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6)
)*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6
) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) +
sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597
568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 334737410739183
39*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 47005690897992*sqrt(6)*sqrt(5
 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 -
 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sq
rt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
5)) + 115139957707068*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(5 - 2*s
qrt(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sq
rt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 -
2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12353375735168316*sqrt(5 - 2*sqrt(6))*log
(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*
sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)
 + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 50432445253
40232*sqrt(6)*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))
/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6
))*sqrt(2*sqrt(6) + 5)) + 4748539075824*sqrt(6)*sqrt(2*sqrt(6) + 5)*log(tan
(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt
```

```
(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 3
3473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 11631497759436*
sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(2*sqrt(6) + 5))/(-46797236353
2675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5)) - 140186421619524*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(2
*sqrt(6) + 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857
156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqr
t(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 57230866972417*sqrt(6)*sqrt(2*sqrt(
6) + 5)*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 19104
8917396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*s
qrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) +
13625938289227872*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) +
 5))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6
)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 5562766012543373*sqrt(6)*sqrt(5 - 2*sqrt(6))*
log(tan(3*x/2 + 1) + sqrt(2*sqrt(6) + 5))/(-467972363532675 - 1910489173965
48*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) +
 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5))
```

Giac [A]
time = 0.43, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\,\log\left(\frac{\left|-2\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.40, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3\,x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(3*x + 2)^2 - cos(3*x + 2)^2),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.13 $\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3756, 213}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.31, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan\left(3x+2\right)}{\sqrt{2}+2\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A]
time = 1.67, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-4\cos\left(3x+2\right)^{2}-4\left(\sqrt{2}\cos\left(3x+2\right)^{3}-2\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-4}{9\cos\left(3x+2\right)^{4}-12\cos\left(3x+2\right)^{2}+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{2\tan^{2}\left(3x+2\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-1+2*tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 - 1), x)
```

Giac [A]
time = 0.95, size = 39, normalized size = 0.65

$$-\frac{1}{12}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\tan\left(3x+2\right)\right|\right)+\frac{1}{12}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\tan\left(3x+2\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(1/2*sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(-
1/2*sqrt(2) + tan(3*x + 2)))
```

Mupad [B]
time = 2.40, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 - 1)),x)
```

```
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.14 $\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx$

Optimal. Leaf size=60

$$\frac{\log\left(\cos(2+3x)-\sqrt{2}\,\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\cos(2+3x)+\sqrt{2}\,\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*x
)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 212}

$$\frac{\log\left(\cos(3x+2)-\sqrt{2}\,\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\sin(3x+2)+\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Cos[2 + 3*x] - Sqrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Cos[2 + 3*x] + S
qrt[2]*Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.37

$$-\frac{\tanh^{-1}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

Maple [A]
time = 0.26, size = 17, normalized size = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(2-cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.57

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

Fricas [A]
time = 1.41, size = 86, normalized size = 1.43

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-4}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="fricas")
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*x
 + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 12*
cos(3*x + 2)^2 + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^2(3x+2)}{\cot^2(3x+2)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(2-cot(2+3*x)**2),x)
[Out] -Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 - 2), x)
```

Giac [A]
time = 0.53, size = 39, normalized size = 0.65

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan(3x+2)\right|}{\left|2\sqrt{2}+4\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="giac")
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.41, size = 16, normalized size = 0.27

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\tan(3x+2)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 - 2)),x)
[Out] -(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.15 $\int \frac{2}{3+\cos(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 2736}

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[2/(3 + Cos[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{2}{3+\cos(4+6x)}\,dx &= 2\int \frac{1}{3+\cos(4+6x)}\,dx\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.52

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(3 + Cos[4 + 6*x]),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.43

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(3+cos(4+6*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.64

$$\frac{1}{6}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sin\left(6\,x+4\right)}{2\,\left(\cos\left(6\,x+4\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A]
time = 1.60, size = 31, normalized size = 0.74

$$-\frac{1}{12}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(6\,x+4\right)+\sqrt{2}}{4\,\sin\left(6\,x+4\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

Sympy [A]
time = 0.12, size = 34, normalized size = 0.81

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan\left(3x+2\right)}{2}\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)/2) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

Giac [A]
time = 0.39, size = 57, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(3+cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.54, size = 36, normalized size = 0.86

$$\frac{\sqrt{2}\left(3x-\operatorname{atan}(\tan(3x+2))\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(cos(6*x + 4) + 3),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

3.16 $\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$

Optimal. Leaf size=42

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {12, 3245, 2736}

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

```
[Out] x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(3*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = 2\int \frac{\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$$
$$= 2\int \frac{1}{3+\cos(4+6x)}\,dx$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.52

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.37, size = 18, normalized size = 0.43

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.64

$$\frac{1}{6}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sin\left(6\,x+4\right)}{2\left(\cos\left(6\,x+4\right)+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

Fricas [A]
time = 1.67, size = 31, normalized size = 0.74

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)+\sqrt{2}}{4\sin(6x+4)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\int\frac{\csc(6x+4)}{\cot(6x+4)+3\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
[Out] 2*Integral(csc(6*x + 4)/(cot(6*x + 4) + 3*csc(6*x + 4)), x)
```

Giac [A]
time = 0.46, size = 57, normalized size = 1.36

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.69, size = 17, normalized size = 0.40

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(sin(6*x + 4)*(cot(6*x + 4) + 3/sin(6*x + 4))),x)
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.17 $\int \frac{1}{2-\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3260, 209}

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

<u>**Mathematica**</u> [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - Sin[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

<u>**Maple**</u> [A]
time = 0.16, size = 18, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-sin(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

<u>**Maxima**</u> [A]
time = 0.47, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

<u>**Fricas**</u> [A]
time = 1.90, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A]
time = 0.29, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A]
time = 0.41, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.44, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\,(3x-\operatorname{atan}(\tan(3x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2 - 2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

3.18 $\int \frac{1}{1+\cos^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3260, 209}

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\cot(2+3x)\right)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[2 + 3*x]^2)^(-1),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.11, size = 18, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 2.77, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A]
time = 0.26, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A]
time = 0.40, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\left(3x-\operatorname{atan}(\tan(3x+2))\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2 + 1),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

3.19 $\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {209}

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.25, size = 18, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.44, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\,\arctan\left(\frac{3\sqrt{2}\,\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [A]
time = 0.31, size = 76, normalized size = 1.58

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}+\frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi)
)/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2 +
1)/pi))/6
```

Giac [A]
time = 0.43, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.37, size = 36, normalized size = 0.75

$$\frac{\sqrt{2}\,(3x-\operatorname{atan}(\tan(3x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 + 2*cos(3*x + 2)^2),x)
```

```
[Out] (2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x + 2
))/2))/6
```

3.20 $\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3756, 209}

$$\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.31, size = 18, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12} - \frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(2+tan(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="maxima")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 1.52, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="fricas")
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(3x+2)}{\tan^2(3x+2)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(2+tan(2+3*x)**2),x)
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 + 2), x)
```

Giac [A]
time = 0.85, size = 17, normalized size = 0.35

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="giac")
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Mupad [B]
time = 2.37, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 + 2)),x)
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.21 $\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

```
[Out] 1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2^
(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 209}

$$\frac{x}{\sqrt{2}}-\frac{\operatorname{ArcTan}\left(\frac{\sin(3x+2)\cos(3x+2)}{\cos^2(3x+2)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] x/Sqrt[2] - ArcTan[(Cos[2 + 3*x]*Sin[2 + 3*x])/(1 + Sqrt[2] + Cos[2 + 3*x]^
2)]/(3*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

```
[Out] ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

Maple [A]
time = 0.30, size = 18, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12} - \frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.35

$$\frac{1}{6}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(3\,x+2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

Fricas [A]
time = 2.59, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^{2}-\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2)
*sin(3*x + 2)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{2}\left(3x+2\right)}{2\cot^{2}\left(3x+2\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1+2*cot(2+3*x)**2),x)
```

```
[Out] Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 + 1), x)
```

Giac [A]
time = 0.53, size = 57, normalized size = 1.19

$$\frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*c
os(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

Mupad [B]
time = 2.38, size = 17, normalized size = 0.35

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan\left(3x+2\right)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 + 1)),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.22 $\int -\frac{2}{1+3\cos(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\,\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {12, 2738, 212}

$$\frac{\log\left(\sqrt{2}\,\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\,\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[-2/(1 + 3*Cos[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/(1 + 3*Cos[4 + 6*x]),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(1+3*cos(4+6*x)),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A]
time = 2.29, size = 74, normalized size = 1.21

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6x+4\right)^{2}+4\left(\sqrt{2}\cos\left(6x+4\right)+3\sqrt{2}\right)\sin\left(6x+4\right)-6\cos\left(6x+4\right)-17}{9\cos\left(6x+4\right)^{2}+6\cos\left(6x+4\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 0.64

$$\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\sqrt{2}\right)}{12}-\frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\sqrt{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x)
```

```
[Out] sqrt(2)*log(tan(3*x + 2) - sqrt(2))/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(2)
)/12
```

Giac [A]
time = 0.40, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/(1+3*cos(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.50, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\ \mathrm{atanh}\left(\frac{\sqrt{2}\ \tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(3*cos(6*x + 4) + 1),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.23 $\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {12, 3245, 2738, 212}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
```

```
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx &= -\left(2\int \frac{\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx\right)\\
&= -\left(2\int \frac{1}{1+3\cos(4+6x)}\,dx\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4-2x^2}\,dx, x, \tan\left(\frac{1}{2}(4+6x)\right)\right)\right)\\
&= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(\right.}{6\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.38, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 53, normalized size = 0.87

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + si
n(6*x + 4)/(cos(6*x + 4) + 1)))
```

Fricas [A]
time = 2.53, size = 74, normalized size = 1.21

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2+4\left(\sqrt{2}\cos(6x+4)+3\sqrt{2}\right)\sin(6x+4)-6\cos(6x+4)-17}{9\cos(6x+4)^2+6\cos(6x+4)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))*
sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) + 1)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-2\int\frac{\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
```

```
[Out] -2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) + csc(6*x + 4)), x)
```

Giac [A]
time = 0.45, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.71, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3\,x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) + 1/sin(6*x + 4))),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.24 $\int \frac{1}{-2+3\sin^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 213}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 + 3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2+3\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.17, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+3*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A]
time = 2.23, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-1}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*cos(3*x + 2)^2 + 1))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.
time = 6.00, size = 1481, normalized size = 24.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-2+3*sin(2+3*x)**2),x)

[Out] -2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 769310*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1277630*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 737640*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2

```
 - sqrt(3))*sqrt(sqrt(3) + 2)) + 1332484*sqrt(sqrt(3) + 2)*log(tan(3*x/2 +
1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 84
6881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
)) - 564917*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2
))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 146684
1 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 215985*sqrt(sqrt
(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
sqrt(3))*sqrt(sqrt(3) + 2)) + 124699*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/
2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
- 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
 + 2)) + 978465*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(
-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 +
9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
```

Giac [A]
time = 0.41, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.51, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(3*x + 2)^2 - 2),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.25 $\int \frac{1}{1-3\cos^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\,\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3260, 212}

$$\frac{\log\left(\sqrt{2}\,\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\,\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-3\cos^2(2+3x)}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)\\ &= \frac{\log\left(\sqrt{2}\,\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.12, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3*cos(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan\left(3x+2\right)}{\sqrt{2}+\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A]
time = 2.31, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-10\cos\left(3x+2\right)^{2}+4\left(\sqrt{2}\cos\left(3x+2\right)^{3}+\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-1}{9\cos\left(3x+2\right)^{4}-6\cos\left(3x+2\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="fricas")

[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*cos(3*x + 2)^2 + 1))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.
time = 5.90, size = 1481, normalized size = 24.28

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-3*cos(2+3*x)**2),x)

[Out] -2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 769310*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 1277630*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 737640*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2

```
 - sqrt(3))*sqrt(sqrt(3) + 2)) + 1332484*sqrt(sqrt(3) + 2)*log(tan(3*x/2 +
1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 84
6881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
)) - 564917*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2
))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 146684
1 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 215985*sqrt(sqrt
(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
sqrt(3))*sqrt(sqrt(3) + 2)) + 124699*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/
2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
- 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
 + 2)) + 978465*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(
-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 +
9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
```

Giac [**A**]
time = 0.44, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="giac")
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [**B**]
time = 2.44, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(3*cos(3*x + 2)^2 - 1),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.26 $\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {213}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)\\ &= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(\right.}{6\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

```
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.26, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 32, normalized size = 0.52

$$\frac{1}{12}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\tan\left(3\,x+2\right)}{\sqrt{2}+\tan\left(3\,x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A]
time = 2.54, size = 85, normalized size = 1.39

$$\frac{1}{24}\,\sqrt{2}\,\log\left(-\frac{7\,\cos\left(3\,x+2\right)^{4}-10\,\cos\left(3\,x+2\right)^{2}+4\,\left(\sqrt{2}\,\cos\left(3\,x+2\right)^{3}+\sqrt{2}\,\cos\left(3\,x+2\right)\right)\sin\left(3\,x+2\right)-1}{9\,\cos\left(3\,x+2\right)^{4}-6\,\cos\left(3\,x+2\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.

time = 5.97, size = 1481, normalized size = 24.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

```
[Out] -2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/
(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 +
 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - sqrt
(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*s
qrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt
(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1
) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846
881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-165
57465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559
457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(sqrt(3) + 2
)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt
(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3)
)*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1)
+ sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 84688
1*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
+ 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-165574
65*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457
*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*sqrt(3)*sqrt(sqrt(3
) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))
*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sq
rt(3))*sqrt(sqrt(3) + 2)) - 769310*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2
+ 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) -
846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) +
 2)) - 1277630*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-
16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9
559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 737640*sqrt(3)*sqrt(2
 - sqrt(3))*log(tan(3*x/2 + 1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqr
t(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2
 - sqrt(3))*sqrt(sqrt(3) + 2)) + 1332484*sqrt(sqrt(3) + 2)*log(tan(3*x/2 +
1) - sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 84
6881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
)) - 564917*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2
))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 146684
1 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 215985*sqrt(sqrt
(3) + 2)*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3
```

```
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
sqrt(3))*sqrt(sqrt(3) + 2)) + 124699*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/
2 + 1) + sqrt(sqrt(3) + 2))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)
- 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3)
 + 2)) + 978465*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(sqrt(3) + 2))/(
-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 +
9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2))
```

Giac [A]
time = 0.45, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.42, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(3*x + 2)^2 - 2*cos(3*x + 2)^2),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.27 $\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\,\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\,\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3756, 213}

$$\frac{\log\left(\sqrt{2}\,\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\,\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \tan(2+3x)\right)$$
$$= \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.32, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan\left(3x+2\right)}{\sqrt{2}+\tan\left(3x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A]
time = 2.11, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-10\cos\left(3x+2\right)^{2}+4\left(\sqrt{2}\cos\left(3x+2\right)^{3}+\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-1}{9\cos\left(3x+2\right)^{4}-6\cos\left(3x+2\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(3x+2\right)}{\tan^{2}\left(3x+2\right)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)**2/(-2+tan(2+3*x)**2),x)
```

```
[Out] Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 - 2), x)
```

Giac [A]
time = 0.84, size = 37, normalized size = 0.61

$$-\frac{1}{12}\sqrt{2}\log\left(\left|\sqrt{2}+\tan\left(3x+2\right)\right|\right)+\frac{1}{12}\sqrt{2}\log\left(\left|-\sqrt{2}+\tan\left(3x+2\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(-sqrt
(2) + tan(3*x + 2)))
```

Mupad [B]
time = 2.40, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 - 2)),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.28 $\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx$

Optimal. Leaf size=61

$$\frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}}-\frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

```
[Out] 1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3*
x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 212}

$$\frac{\log\left(\sqrt{2}\cos(3x+2)-\sin(3x+2)\right)}{6\sqrt{2}}-\frac{\log\left(\sin(3x+2)+\sqrt{2}\cos(3x+2)\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

```
[Out] Log[Sqrt[2]*Cos[2 + 3*x] - Sin[2 + 3*x]]/(6*Sqrt[2]) - Log[Sqrt[2]*Cos[2 +
3*x] + Sin[2 + 3*x]]/(6*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \cot(2+3x)\right)\right)$$
$$= \frac{\log\left(\sqrt{2}\,\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\,\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
[Out] -1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.30, size = 18, normalized size = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x,method=_RETURNVERBOSE)
[Out] -1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 32, normalized size = 0.52

$$\frac{1}{12}\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

Fricas [A]
time = 1.43, size = 85, normalized size = 1.39

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^{4}-10\cos\left(3x+2\right)^{2}+4\left(\sqrt{2}\cos\left(3x+2\right)^{3}+\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)-1}{9\cos\left(3x+2\right)^{4}-6\cos\left(3x+2\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3*
x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*co
s(3*x + 2)^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^{2}\left(3x+2\right)}{2\cot^{2}\left(3x+2\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)**2/(1-2*cot(2+3*x)**2),x)
```

```
[Out] -Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 - 1), x)
```

Giac [A]
time = 0.55, size = 39, normalized size = 0.64

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*x
 + 2)))
```

Mupad [B]
time = 2.33, size = 17, normalized size = 0.28

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\tan(3x+2)}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 - 1)),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.29 $\int (x+\sin(x))^2\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6874, 3377, 2717, 2715, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\sin(x))^2\,dx &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\sin(x)\,dx+\int \sin^2(x)\,dx\\ &= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}+2\int \cos(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{1}{6}x\left(3+2x^2\right)-2x\cos(x)+2\sin(x)-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^2,x]
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
default	$\frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{\cos(x)\sin(x)}{2}$	25
risch	$\frac{x^3}{3}+\frac{x}{2}-2x\cos(x)+2\sin(x)-\frac{\sin(2x)}{4}$	25
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{3x}{2}+\frac{x^3}{3}+5\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{5x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{2x^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3-2x\cos(x)+\frac{1}{2}x-\frac{1}{4}\sin(2x)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Fricas [A]
time = 3.00, size = 22, normalized size = 0.73

$$\frac{1}{3}\,x^3 - 2\,x\cos\left(x\right) - \frac{1}{2}\left(\cos\left(x\right) - 4\right)\sin\left(x\right) + \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

Sympy [A]
time = 0.07, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2\left(x\right)}{2} + \frac{x\cos^2\left(x\right)}{2} - 2x\cos\left(x\right) - \frac{\sin\left(x\right)\cos\left(x\right)}{2} + 2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**2,x)
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

Giac [A]
time = 0.41, size = 24, normalized size = 0.80

$$\frac{1}{3}\,x^3 - 2\,x\cos\left(x\right) + \frac{1}{2}\,x - \frac{1}{4}\,\sin\left(2\,x\right) + 2\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Mupad [B]
time = 0.06, size = 24, normalized size = 0.80

$$\frac{x}{2} + 2\,\sin\left(x\right) - \frac{\cos\left(x\right)\,\sin\left(x\right)}{2} - 2\,x\,\cos\left(x\right) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))^2,x)
[Out] x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3
```

3.30 $\int (x+\sin(x))^3\,dx$

Optimal. Leaf size=56

$$\frac{3x^2}{4}+\frac{x^4}{4}+5\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}$$

```
[Out] 3/4*x^2+1/4*x^4+5*cos(x)-3*x^2*cos(x)+1/3*cos(x)^3+6*x*sin(x)-3/2*x*cos(x)*
sin(x)+3/4*sin(x)^2
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6874, 3377, 2718, 3391, 30, 2713}

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 5*Cos[x] - 3*x^2*Cos[x] + Cos[x]^3/3 + 6*x*Sin[x] - (3*
x*Cos[x]*Sin[x])/2 + (3*Sin[x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\sin(x))^3\,dx &= \int \left(x^3+3x^2\sin(x)+3x\sin^2(x)+\sin^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\sin(x)\,dx+3\int x\sin^2(x)\,dx+\int \sin^3(x)\,dx\\ &= \frac{x^4}{4}-3x^2\cos(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}+\frac{3\int x\,dx}{2}+6\int x\cos(x)\,dx-\text{Subst}\Big(\\ &= \frac{3x^2}{4}+\frac{x^4}{4}-\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}-\\ &= \frac{3x^2}{4}+\frac{x^4}{4}+5\cos(x)-3x^2\cos(x)+\frac{\cos^3(x)}{3}+6x\sin(x)-\frac{3}{2}x\cos(x)\sin(x)+\frac{3\sin^2(x)}{4}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.86

$$\frac{1}{24}\left(-18\left(-7+4x^2\right)\cos(x)-9\cos(2x)+2\cos(3x)+6x\left(3x+x^3+24\sin(x)-3\sin(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^3,x]
```

```
[Out] (-18*(-7 + 4*x^2)*Cos[x] - 9*Cos[2*x] + 2*Cos[3*x] + 6*x*(3*x + x^3 + 24*Si
n[x] - 3*Sin[2*x]))/24
```

Maple [A]
time = 0.10, size = 57, normalized size = 1.02

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+\left(\frac{21}{4}-3x^2\right)\cos\left(x\right)+6x\sin\left(x\right)+\frac{\cos(3x)}{12}-\frac{3\cos(2x)}{8}-\frac{3x\sin(2x)}{4}$

default	$-\frac{(2+\sin^2(x))\cos(x)}{3}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}+\frac{3(\sin^2(x))}{4}-3x^2\cos(x)+6\cos(x)+6x\sin(x)$ -
norman	$\frac{-5(\tan^6(\frac{x}{2}))+8(\tan^2(\frac{x}{2}))-\frac{9x^2}{4}+\frac{x^4}{4}+9x\tan(\frac{x}{2})+24x(\tan^3(\frac{x}{2}))+15x(\tan^5(\frac{x}{2}))-\frac{3x^2(\tan^2(\frac{x}{2}))}{4}+\frac{21x^2(\tan^4(\frac{x}{2}))}{4}+\frac{15x^2(\tan^6(\frac{x}{2}}{4}}{(1+\tan^2(\frac{x}{2}))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)+3*x*(-1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2+3/4*sin(x)^
2-3*x^2*cos(x)+6*cos(x)+6*x*sin(x)+1/4*x^4
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.86

$$\frac{1}{4}x^4+\frac{1}{3}\cos(x)^3+\frac{3}{4}x^2-3(x^2-2)\cos(x)-\frac{3}{4}x\sin(2x)+6x\sin(x)-\frac{3}{8}\cos(2x)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - 3*(x^2 - 2)*cos(x) - 3/4*x*sin(2*x) + 6*
x*sin(x) - 3/8*cos(2*x) - cos(x)
```

Fricas [A]
time = 3.39, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4+\frac{1}{3}\cos(x)^3+\frac{3}{4}x^2-(3x^2-5)\cos(x)-\frac{3}{4}\cos(x)^2-\frac{3}{2}(x\cos(x)-4x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - (3*x^2 - 5)*cos(x) - 3/4*cos(x)^2 - 3/2*
(x*cos(x) - 4*x)*sin(x)
```

Sympy [A]
time = 0.09, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sin^2(x)}{4}+\frac{3x^2\cos^2(x)}{4}-3x^2\cos(x)-\frac{3x\sin(x)\cos(x)}{2}+6x\sin(x)-\sin^2(x)\cos(x)-\frac{2\cos^3(x)}{3}-\frac{3\cos^2(x)}{4}+6\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*cos(x)**2/4 - 3*x**2*cos(x) - 3*x*sin(
x)*cos(x)/2 + 6*x*sin(x) - sin(x)**2*cos(x) - 2*cos(x)**3/3 - 3*cos(x)**2/4
 + 6*cos(x)
```

Giac [A]
time = 0.40, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{3}{4}\left(4x^2 - 7\right)\cos\left(x\right) - \frac{3}{4}x\sin\left(2x\right) + 6x\sin\left(x\right) + \frac{1}{12}\cos\left(3x\right) - \frac{3}{8}\cos\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 3/4*(4*x^2 - 7)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) +
1/12*cos(3*x) - 3/8*cos(2*x)
```

Mupad [B]
time = 2.31, size = 46, normalized size = 0.82

$$5\cos\left(x\right) - 3x^2\cos\left(x\right) - \frac{3\cos\left(x\right)^2}{4} + \frac{\cos\left(x\right)^3}{3} + 6x\sin\left(x\right) + \frac{3x^2}{4} + \frac{x^4}{4} - \frac{3x\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + sin(x))^3,x)
```

```
[Out] 5*cos(x) - 3*x^2*cos(x) - (3*cos(x)^2)/4 + cos(x)^3/3 + 6*x*sin(x) + (3*x^2
)/4 + x^4/4 - (3*x*cos(x)*sin(x))/2
```

3.31 $\int \frac{\sin(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$-\frac{\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a+\right.}{}$$

```
[Out] 1/2*cos(a+b*(-c)^(1/2)/d^(1/2))*Si(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d^(
1/2)-1/2*cos(a-b*(-c)^(1/2)/d^(1/2))*Si(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2
)/d^(1/2)-1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))/(-c)
^(1/2)/d^(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^(1/2)
)/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3414, 3384, 3380, 3383}

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/2*(CosIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sin[a - (b*Sqrt[-c])/Sqrt[d]]
)/(Sqrt[-c]*Sqrt[d]) + (CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sin[a + (b*
Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*S
inIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*
Sqrt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqr
t[d])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{c+dx^2}\,dx &= \int \left(\frac{\sqrt{-c}\,\sin(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\sin(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx \\
&= -\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\sin(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}} \\
&= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sin\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\cdots \\
&= -\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\text{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}-\cdots
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 172, normalized size = 0.81

$$\frac{i\left(\text{CosIntegral}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)-\text{CosIntegral}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)\sin\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)+\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)+\cos\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\text{Si}\left(\frac{ib\sqrt{c}}{\sqrt{d}}-bx\right)\right)}{2\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((I/2)*(CosIntegral[b*((I*Sqrt[c])/Sqrt[d] + x)]*Sin[a - (I*b*Sqrt[c])/Sqrt
[d]] - CosIntegral[b*(((-I)*Sqrt[c])/Sqrt[d] + x)]*Sin[a + (I*b*Sqrt[c])/Sq
rt[d]] + Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[d]
+ x)] + Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d] -
b*x]))/(Sqrt[c]*Sqrt[d])
```

Maple [A]
time = 0.24, size = 233, normalized size = 1.09

method	result
derivativedivides	$b\left(-\frac{-\text{sinIntegral}\left(-bx-a+\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\text{cosineIntegral}\left(bx+a-\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b}{}\right.}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}\right.$
default	$b\left(-\frac{-\text{sinIntegral}\left(-bx-a+\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\text{cosineIntegral}\left(bx+a-\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b}{}\right.}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}\right.$
risch	$-\frac{\sqrt{cd}\,\mathrm{e}^{\frac{iad+b\sqrt{cd}}{d}}\,\text{expIntegral}\left(1,\frac{iad+b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}+\frac{\sqrt{cd}\,\mathrm{e}^{\frac{iad-b\sqrt{cd}}{d}}\,\text{expIntegral}\left(1,\frac{iad-b\sqrt{cd}-}{d}\right.}{4cd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/2/d/(-(b*(-c*d)^(1/2)+a*d)/d+a)*(-Si(-b*x-a+(b*(-c*d)^(1/2)+a*d)/d)*c
os((b*(-c*d)^(1/2)+a*d)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*sin((b*(-c*d)^(
1/2)+a*d)/d))-1/2/d/((b*(-c*d)^(1/2)-a*d)/d+a)*(-Si(-b*x-a-(b*(-c*d)^(1/2)-
a*d)/d)*cos((b*(-c*d)^(1/2)-a*d)/d)-Ci(b*x+a+(b*(-c*d)^(1/2)-a*d)/d)*sin((b
*(-c*d)^(1/2)-a*d)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.34, size = 187, normalized size = 0.88

$$\frac{\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia-\sqrt{\frac{b^2c}{d}}\right)}+\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-ia+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(-ia-\sqrt{\frac{b^2c}{d}}\right)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) - sqrt
(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + sqrt(b^2*c/d)
*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(-I*
b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{(a+bx)}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x**2+c),x)
[Out] Integral(sin(a + b*x)/(c + d*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*x^2 + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x^2),x)
[Out] int(sin(a + b*x)/(c + d*x^2), x)
```

3.32 $\int \frac{\sin(a+bx)}{c+dx+ex^2} \, dx$

Optimal. Leaf size=271

$$\frac{\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}-\frac{\text{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

```
[Out] cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e
)/(-4*c*e+d^2)^(1/2)-cos(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(d+
(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2
))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^
2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6860, 3384, 3380, 3383}

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d - Sqrt[
d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CosIntegral[(b*(d + Sqrt[d^2 -
4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2
- 4*c*e] + (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d -
Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt
[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]
)/Sqrt[d^2 - 4*c*e]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\,\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\,\left(d+\sqrt{d^2-4ce}+2ex\right)}\right) d$$

$$= \frac{(2e)\int \frac{\sin(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sin(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sin\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cos\left(a-\frac{b(}{}\right.\right.}{}$$

$$= \frac{\mathrm{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\mathrm{Ci}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+b\right.}{\sqrt{}}$$

Mathematica [A]
time = 0.24, size = 238, normalized size = 0.88

$$\frac{\mathrm{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)\sin\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)-\mathrm{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)-\cos\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Si}\left(\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}-bx\right)-\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]*Sin[a + (b*(-d + Sq
rt[d^2 - 4*c*e]))/(2*e)] - CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/
(2*e)]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)] - Cos[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
```

```
- Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

Maple [A]
time = 0.36, size = 328, normalized size = 1.21

method	result
derivativedivides	$b\left(\frac{-\operatorname{sinIntegral}\left(-bx-a+\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\operatorname{cosineIntegral}\left(\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
default	$b\left(\frac{-\operatorname{sinIntegral}\left(-bx-a+\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\operatorname{cosineIntegral}\left(\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
risch	$-\frac{b\,e^{\frac{2iae-ibd+\sqrt{4b^2ce-b^2d^2}}{2e}}\operatorname{expIntegral}\left(1,\frac{2iae-ibd-2e(ibx+ia)+\sqrt{4b^2ce-b^2d^2}}{2e}\right)}{2\sqrt{4b^2ce-b^2d^2}}+\frac{b\,e^{\frac{2iae-ibd-\sqrt{4b^2ce-b^2}}{2e}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-b*x-a+1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^
2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1
/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c
*e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-b*x-a-1/2*(-2*a*e+d
*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^
(1/2))/e)-Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(
-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(x^2*e + d*x + c), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.44, size = 447, normalized size = 1.65

$$-\frac{\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,\operatorname{Ei}\left(\frac{1}{2}\left(-2ibxe-ibd-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(ibd-2iae+\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,\operatorname{Ei}\left(\frac{1}{2}\left(-2ibxe-ibd+\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(ibd-2iae-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}+\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,\operatorname{Ei}\left(\frac{1}{2}\left(2ibxe+ibd-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(-ibd+2iae+\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,\operatorname{Ei}\left(\frac{1}{2}\left(2ibxe+ibd+\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(-ibd+2iae-\sqrt{-(b^2d^2-4b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}}{2\left(bd^2-4bce\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*Ei(1/2*(-2*I*b*x*e - I*b*d - sqrt
(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1))*e^(1/2*(I*b*d - 2*I*a*e + sqrt(-
(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1) + 1) - sqrt(-(b^2*d^2 - 4*b^2*c*e)*
e^(-2))*Ei(1/2*(-2*I*b*x*e - I*b*d + sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)
*e^(-1))*e^(1/2*(I*b*d - 2*I*a*e - sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e
^(-1) + 1) + sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*Ei(1/2*(2*I*b*x*e + I*b*d
- sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1))*e^(1/2*(-I*b*d + 2*I*a*e +
 sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1) + 1) - sqrt(-(b^2*d^2 - 4*b^
2*c*e)*e^(-2))*Ei(1/2*(2*I*b*x*e + I*b*d + sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-
2))*e)*e^(-1))*e^(1/2*(-I*b*d + 2*I*a*e - sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2
))*e)*e^(-1) + 1))/(b*d^2 - 4*b*c*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(sin(a + b*x)/(c + d*x + e*x^2), x)
```

3.33 $\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{-7+x}\right)$$

[Out] -2*cos((-7+x)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3512, 15, 2718}

$$-2\cos\left(\sqrt{x-7}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]

[Out] -2*Cos[Sqrt[-7 + x]]

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3512

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = 2\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{-7+x}\right)$$
$$= 2\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{-7+x}\right)$$
$$= -2\cos\left(\sqrt{-7+x}\right)$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{-7+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
[Out] -2*Cos[Sqrt[-7 + x]]
```

Maple [A]
time = 0.09, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-2\cos\left(\sqrt{-7+x}\right)$	9
default	$-2\cos\left(\sqrt{-7+x}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-7+x)^(1/2))/(-7+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos((-7+x)^(1/2))
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="maxima")
[Out] -2*cos(sqrt(x - 7))
```

Fricas [A]
time = 3.58, size = 8, normalized size = 0.80

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="fricas")
[Out] -2*cos(sqrt(x - 7))
```

Sympy [A]
time = 0.10, size = 10, normalized size = 1.00

$$-2\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)**(1/2))/(-7+x)**(1/2),x)
[Out] -2*cos(sqrt(x - 7))
```

Giac [A]
time = 0.39, size = 8, normalized size = 0.80

$$-2\,\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="giac")
[Out] -2*cos(sqrt(x - 7))
```

Mupad [B]
time = 2.42, size = 8, normalized size = 0.80

$$-2\,\cos\left(\sqrt{x-7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((x - 7)^(1/2))/(x - 7)^(1/2),x)
[Out] -2*cos((x - 7)^(1/2))
```

3.34 $\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x\text{Si}(x)}{\sqrt{a-bx^2}}$$

[Out] x*Si(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A]
time = 0.31, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6853, 23, 3380}

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}\sin(x)}{x\sqrt{a-bx^2}}\,dx}{\sqrt{1-\frac{bx^2}{a}}}$$

$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sin(x)}{x}\,dx}{\sqrt{a-bx^2}}$$

$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x\mathrm{Si}(x)}{\sqrt{a-bx^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 46, normalized size = 1.64

$$\frac{i\sqrt{b-\frac{a}{x^2}}\,x(\mathrm{Ei}(-ix)-\mathrm{Ei}(ix))}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
```

```
[Out] ((I/2)*Sqrt[b - a/x^2]*x*(ExpIntegralEi[(-I)*x] - ExpIntegralEi[I*x]))/Sqrt
[a - b*x^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 72, normalized size = 2.57

method	result	size
risch	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)x\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}\left(-i\,\mathrm{sinIntegral}(x)+\frac{i\pi\,\mathrm{csgn}(x)}{2}\right)}{(-b\,x^2+a)^{\frac{3}{2}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)*(b*x^2-a)/(-b*x^2+a)^(3/2)*x*(1/(b*x^2-a)*(-b*x^2+
a))^(1/2)*(-I*Si(x)+1/2*I*Pi*csgn(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-b*x^2 + a)*sqrt((b*x^2 - a)/x^2)*sin(x)/(b*x^2 - a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}\,\sin(x)}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-a/x**2 + b)*sin(x)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sin(x)\,\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)
```

3.35 $\int \frac{1}{x(1+\sin(\log(x)))} dx$

Optimal. Leaf size=12

$$-\frac{\cos(\log(x))}{1+\sin(\log(x))}$$

```
[Out] -cos(ln(x))/(1+sin(ln(x)))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2727}

$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Sin[Log[x]])),x]
```

```
[Out] -(Cos[Log[x]]/(1 + Sin[Log[x]]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+\sin(\log(x)))} dx &= \text{Subst}\left(\int \frac{1}{1+\sin(x)} dx, x, \log(x)\right) \\ &= -\frac{\cos(\log(x))}{1+\sin(\log(x))} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.02, size = 26, normalized size = 2.17

$$\frac{2\sin\left(\frac{\log(x)}{2}\right)}{\cos\left(\frac{\log(x)}{2}\right)+\sin\left(\frac{\log(x)}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Sin[Log[x]])),x]
[Out] (2*Sin[Log[x]/2])/(Cos[Log[x]/2] + Sin[Log[x]/2])
```

Maple [A]
time = 0.11, size = 12, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	12
default	$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	12
risch	$-\frac{2}{x^i+i}$	12
norman	$\frac{2\tan\left(\frac{\ln(x)}{2}\right)}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+sin(ln(x))),x,method=_RETURNVERBOSE)
[Out] -2/(tan(1/2*ln(x))+1)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.42

$$-\frac{2}{\frac{\sin(\log(x))}{\cos(\log(x))+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="maxima")
[Out] -2/(sin(log(x))/(cos(log(x)) + 1) + 1)
```

Fricas [A]
time = 2.98, size = 22, normalized size = 1.83

$$-\frac{\cos(\log(x)) - \sin(\log(x)) + 1}{\cos(\log(x)) + \sin(\log(x)) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="fricas")
[Out] -(cos(log(x)) - sin(log(x)) + 1)/(cos(log(x)) + sin(log(x)) + 1)
```

Sympy [A]
time = 0.66, size = 10, normalized size = 0.83

$$-\frac{2}{\tan\left(\frac{\log(x)}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(ln(x))),x)
```

```
[Out] -2/(tan(log(x)/2) + 1)
```

Giac [A]
time = 0.40, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{1}{2}\log\left(x\right)\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+sin(log(x))),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*log(x)) + 1)
```

Mupad [B]
time = 2.40, size = 11, normalized size = 0.92

$$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(sin(log(x)) + 1)),x)
```

```
[Out] -2/(tan(log(x)/2) + 1)
```

3.36 $\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=100

$$\frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

```
[Out] (-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2+(-a*d+b*c)*Si((-a*d+b*c)/d
/(d*x+c))*sin(b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4659, 3378, 3384, 3380, 3383}

$$\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c + d
*x)*Sin[(a + b*x)/(c + d*x)])/d + ((b*c - a*d)*Sin[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4659

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Dist[-d^(-1), Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin\left(\frac{a+bx}{c+dx}\right) dx &= -\frac{\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2} dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{((bc-
\end{aligned}$$

$$= \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.72, size = 272, normalized size = 2.72

$$\frac{2(bc-ad)\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{-bc+ad}{d(c+dx)}\right)+de^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}}\left(ic\left(e^{\frac{2ibc}{d(c+dx)}}-e^{2i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+d\left(e^{\frac{ib(3c+dx)}{d(c+dx)}}+e^{i\left(\frac{b}{d}+\frac{2a}{c+dx}\right)}\right)x\sin\left(\frac{b}{d}\right)+2de^{\frac{i(2bc+ad+bdx)}{d(c+dx)}}x\cos\left(\frac{b}{d}\right)\sin\left(\frac{-bc+ad}{d(c+dx)}\right)\right)-2(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{-bc+ad}{d(c+dx)}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)],x]
```

```
[Out] (2*(b*c - a*d)*Cos[b/d]*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + (d*(I*c
*(E^(((2*I)*b*c)/(d*(c + d*x))) - E^((2*I)*(b/d + a/(c + d*x)))) + d*(E^((I
*b*(3*c + d*x))/(d*(c + d*x))) + E^(I*(b/d + (2*a)/(c + d*x))))*x*Sin[b/d]
+ 2*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Cos[b/d]*Sin[(-(b*c) +
a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 2*(b*c
- a*d)*Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(2*d^2)
```

Maple [A]
time = 0.26, size = 142, normalized size = 1.42

method	result
derivativedivides	$-(ad-cb)\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\text{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)$
default	$-(ad-cb)\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\text{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)$
risch	$\frac{\text{expIntegral}\left(1,-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}a}{2d}-\frac{\text{expIntegral}\left(1,-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}cb}{2d^2}-\frac{ie^{-\frac{ib}{d}}\pi\,\text{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}+\frac{ie^{-\frac{ib}{d}}\pi\,\text{csgn}\left(\frac{ad-}{d(dx+}\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*d-b*c)*(-sin(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d
+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin((b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 2.77, size = 139, normalized size = 1.39

$$-\frac{2\,(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-\left((bc-ad)\,\text{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\,\text{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cos\left(\frac{b}{d}\right)-2\left(d^2x+cd\right)\sin\left(\frac{bx+a}{dx+c}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - ((b
*c - a*d)*cos_integral((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integra
l(-(b*c - a*d)/(d^2*x + c*d)))*cos(b/d) - 2*(d^2*x + c*d)*sin((b*x + a)/(d*
x + c)))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x)
[Out] Integral(sin((a + b*x)/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(100) = 200.
time = 3.35, size = 630, normalized size = 6.30

$$\frac{\left(b^{3}c^{2}\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-2\,ab^{2}cd\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-\frac{(bx+a)b^{2}c^{2}d\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+a^{2}bd^{2}\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)+\frac{2\,(bx+a)abcd^{2}\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}-\frac{(bx+a)a^{2}d^{3}\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+b^{3}c^{2}\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-2\,ab^{2}cd\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-\frac{(bx+a)b^{2}c^{2}d\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+a^{2}bd^{2}\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)+\frac{2\,(bx+a)abcd^{2}\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}-\frac{(bx+a)a^{2}d^{3}\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+b^{2}c^{2}d\sin\left(\frac{bx+a}{dx+c}\right)-2\,abcd^{2}\sin\left(\frac{bx+a}{dx+c}\right)+a^{2}d^{3}\sin\left(\frac{bx+a}{dx+c}\right)\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)}{bd^{2}-\frac{(bx+a)d^{3}}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c)),x, algorithm="giac")
[Out] (b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 2*a*b^2*c*
d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2*c^2
*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2*b*
d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a*b
*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (b
*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) + b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*a*b
^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2
*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2
*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*a
*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - (
b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) + b^2*c^2*d*sin((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*sin((b*x + a)/(d*x
+ c)) + a^2*d^3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a
*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x)),x)
[Out] int(sin((a + b*x)/(c + d*x)), x)
```

3.37 $\int \sin^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=107

$$\frac{(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

```
[Out] -(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Ci(2*(-a*d
+b*c)/d/(d*x+c))*sin(2*b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4659, 3394, 12, 3384, 3380, 3383}

$$\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}-\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CosIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 +
 ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4659

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Dist[-d^(-1), Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{\text{Subst}\left(\int \frac{\sin^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{((l}{}$$

$$= \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-a}{d(c+dx}\right.}{d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.02, size = 330, normalized size = 3.08

$$\frac{8(bc-ad)\text{CosIntegral}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right) - de^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}}\left(2c\left(e^{\frac{4ibc}{d(c+dx)}}+e^{4i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+d\left(e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}+e^{\frac{4ib(2c+dx)}{d(c+dx)}}-4e^{\frac{2i(2bc+ad+bdx)}{d(c+dx)}}+e^{4i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)x-4de^{\frac{2i(2bc+ad+bdx)}{d(c+dx)}}x\sin\left(\frac{2b}{d}\right)\sin\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\right)+8(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)}{8d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (8*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d] -
 (d*(2*c*(E^(((4*I)*b*c)/(d*(c + d*x))) + E^((4*I)*(b/d + a/(c + d*x)))) +
d*(E^(((4*I)*a)/(c + d*x)) + E^(((4*I)*b*c)/(d*(c + d*x))) + E^(((4*I)*b*(2
*c + d*x))/(d*(c + d*x))) - 4*E^(((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x)
)) + E^((4*I)*(b/d + a/(c + d*x))))*x - 4*d*E^(((2*I)*(2*b*c + a*d + b*d*x)
)/(d*(c + d*x)))*x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))]))/E^(
((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) + 8*(b*c - a*d)*Cos[(2*b)/d]*S
inIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(8*d^2)
```

Maple [A]
time = 0.13, size = 195, normalized size = 1.82

method	result
derivativedivides	$-\frac{(ad-cb)\left(-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)}-\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(\frac{2a}{d(}\right.}{d}\right.}{d}\right)}{4}\right)}{d^2}$
default	$-\frac{(ad-cb)\left(-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)}-\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(\frac{2a}{d(}\right.}{d}\right.}{d}\right)}{4}\right)}{d^2}$
risch	$\frac{e^{-\frac{2ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}-\frac{e^{-\frac{2ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{2d^2}-\frac{e^{-\frac{2ib}{d}}\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)a}{d}+\frac{e^{-\frac{2ib}{d}}\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)-1/4*d^2*(-2*cos(2*
(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a*d
-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 1.89, size = 149, normalized size = 1.39

$$\frac{2\,d^2x-2\,(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)^2+2\,(bc-ad)\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)+\left((bc-ad)\operatorname{Ci}\left(\frac{2\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\operatorname{Ci}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\right)\sin\left(\frac{2\,b}{d}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*x - 2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + 2*(b*c - a*d)*c
os(2*b/d)*sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) + ((b*c - a*d)*cos_int
egral(2*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d
)/(d^2*x + c*d)))*sin(2*b/d))/d^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(107) = 214.
time = 22.58, size = 681, normalized size = 6.36

$$\frac{\left(2\,b^3c^2\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-4\,ab^2cd\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-\frac{2\,(bx+a)b^2c^2d\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)}{dx+c}+2\,a^2bd^2\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)+\frac{4\,(bx+a)abcd^2\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)}{dx+c}-\frac{2\,(bx+a)a^2d^3\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)}{dx+c}-2\,b^3c^2\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)+4\,ab^2cd\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)+\frac{2\,(bx+a)b^2c^2d\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}-2\,a^2bd^2\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)-\frac{4\,(bx+a)abcd^2\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}+\frac{2\,(bx+a)a^2d^3\cos\left(\frac{2\,b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}-b^2c^2d\cos\left(\frac{2\,(bx+a)}{dx+c}\right)+2\,abcd^2\cos\left(\frac{2\,(bx+a)}{dx+c}\right)-a^2d^3\cos\left(\frac{2\,(bx+a)}{dx+c}\right)+b^2c^2d-2\,abcd^2+a^2d^3\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)}{2\left(bd^2-\frac{(bx+a)d^3}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) - 2*(
b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d
)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*si
n(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c)
)/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b - (b*x +
 a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin_integra
l(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_integral(2*
```

```
(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)*sin_integ
ral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos(2*b/d)*sin
_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d^2*cos(2*b/
d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*(b*x + a)*a^
2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) -
b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*x + a)/(d*x + c
)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) + b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x))^2,x)
```

```
[Out] int(sin((a + b*x)/(c + d*x))^2, x)
```

3.38 $\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=194

$$\frac{3(bc-ad)\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3(bc-ad)\cos\left(\frac{3b}{d}\right)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d}+$$

```
[Out] 3/4*(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2-3/4*(-a*d+b*c)*Ci(3*(-
a*d+b*c)/d/(d*x+c))*cos(3*b/d)/d^2+3/4*(-a*d+b*c)*Si((-a*d+b*c)/d/(d*x+c))*
sin(b/d)/d^2-3/4*(-a*d+b*c)*Si(3*(-a*d+b*c)/d/(d*x+c))*sin(3*b/d)/d^2+(d*x+
c)*sin((b*x+a)/(d*x+c))^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4659, 3394, 3384, 3380, 3383}

$$\frac{3\cos\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\cos\left(\frac{3b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{3\sin\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2}-\frac{3\sin\left(\frac{3b}{d}\right)(bc-ad)\text{Si}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (3*(b*c - a*d)*Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (
3*(b*c - a*d)*Cos[(3*b)/d]*CosIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d
^2) + ((c + d*x)*Sin[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sin[b/d]*Si
nIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sin[(3*b)/d]
*SinIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4659

```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Dist[-d^(-1), Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sin^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\operatorname{Subst}\left(\int\left(-\frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right) d\right.}{d^2}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\operatorname{Subst}\left(\int \frac{\cos\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{(3(bc-}{}$$

$$= \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left(3(bc-ad)\cos\left(\frac{b}{d}\right)\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(b}{}$$

$$= \frac{3(bc-ad)\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\cos\left(\frac{3b}{d}\right)\operatorname{Ci}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sin^3\left(\frac{a+}{c+}\right.}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.42, size = 417, normalized size = 2.15

$$\frac{3icde^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}}\left(e^{\frac{2ibc}{d(c+dx)}}-e^{2i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+icde^{-\frac{3i(2bc+ad+bdx)}{d(c+dx)}}\left(-e^{\frac{6ibc}{d(c+dx)}}+e^{6i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+6d^2x\cos\left(\frac{-bc+ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)-2d^2x\cos\left(\frac{3(-bc+ad)}{d(c+dx)}\right)\sin\left(\frac{3b}{d}\right)+6d^2x\cos\left(\frac{b}{d}\right)\sin\left(\frac{-bc+ad}{d(c+dx)}\right)-2d^2x\cos\left(\frac{3b}{d}\right)\sin\left(\frac{3(-bc+ad)}{d(c+dx)}\right)+6(bc-ad)\left(\cos\left(\frac{b}{d}\right)\operatorname{CosIntegral}\left(\frac{-bc+ad}{d(c+dx)}\right)-\cos\left(\frac{3b}{d}\right)\operatorname{CosIntegral}\left(\frac{3(-bc+ad)}{d(c+dx)}\right)-\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{-bc+ad}{d(c+dx)}\right)+\sin\left(\frac{3b}{d}\right)\operatorname{Si}\left(\frac{3(-bc+ad)}{d(c+dx)}\right)\right)}{8d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (((3*I)*c*d*(E^(((2*I)*b*c)/(d*(c + d*x))) - E^((2*I)*(b/d + a/(c + d*x))))
)/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) + (I*c*d*(-E^(((6*I)*b*c)/(d*
```

```
(c + d*x))) + E^((6*I)*(b/d + a/(c + d*x)))))/E^(((3*I)*(2*b*c + a*d + b*d*
x))/(d*(c + d*x))) + 6*d^2*x*Cos[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] - 2
*d^2*x*Cos[(3*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(3*b)/d] + 6*d^2*x*Cos[b/d
]*Sin[(-(b*c) + a*d)/(d*(c + d*x))] - 2*d^2*x*Cos[(3*b)/d]*Sin[(3*(-(b*c) +
 a*d))/(d*(c + d*x))] + 6*(b*c - a*d)*(Cos[b/d]*CosIntegral[(-(b*c) + a*d)/
(d*(c + d*x))] - Cos[(3*b)/d]*CosIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]
 - Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Sin[(3*b)/d]*SinInt
egral[(3*(-(b*c) + a*d))/(d*(c + d*x))]))/(8*d^2)
```

Maple [A]
time = 0.15, size = 295, normalized size = 1.52

method	result
derivativedivides	$-\frac{(ad-cb)\left(-\frac{d^2\left(-\frac{3\sin\left(\frac{3ad-3cb}{d(dx+c)}+\frac{3b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(\frac{3ad-3cb}{d(dx+c)}\right)\sin\left(\frac{3b}{d}\right)}{d}+\frac{9\,\mathrm{cosineIntegral}\left(\frac{3ad-3cb}{d(dx+c)}\right)\cos\left(\frac{3b}{d}\right)}{d}}{d}\right)}{12}+\frac{3d^2\cdots}{\cdots}\right)}{d^2}$
default	$-\frac{(ad-cb)\left(-\frac{d^2\left(-\frac{3\sin\left(\frac{3ad-3cb}{d(dx+c)}+\frac{3b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{9\,\mathrm{sinIntegral}\left(\frac{3ad-3cb}{d(dx+c)}\right)\sin\left(\frac{3b}{d}\right)}{d}+\frac{9\,\mathrm{cosineIntegral}\left(\frac{3ad-3cb}{d(dx+c)}\right)\cos\left(\frac{3b}{d}\right)}{d}}{d}\right)}{12}+\frac{3d^2\cdots}{\cdots}\right)}{d^2}$
risch	$-\frac{3ie^{-\frac{3ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{8d^2}-\frac{3ie^{-\frac{ib}{d}}\,\mathrm{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{4d^2}+\frac{3ie^{-\frac{ib}{d}}\,\mathrm{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)a}{4d}+\frac{3ie^{-\frac{3ib}{d}}\,\mathrm{sinIn}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(a*d-b*c)*(-1/12*d^2*(-3*sin(3*(a*d-b*c)/d/(d*x+c)+3*b/d)/((b/d+(a*d
-b*c)/d/(d*x+c))*d-b)/d+3*(-3*Si(3*(a*d-b*c)/d/(d*x+c))*sin(3*b/d)/d+3*Ci(3
*(a*d-b*c)/d/(d*x+c))*cos(3*b/d)/d)/d)+3/4*d^2*(-sin(b/d+(a*d-b*c)/d/(d*x+c
))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+C
i((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

[Out] integrate(sin((b*x + a)/(d*x + c))^3, x)

Fricas [A]
time = 2.50, size = 277, normalized size = 1.43

$$-\frac{6\,(bc-ad)\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-6\,(bc-ad)\sin\left(\frac{3\,b}{d}\right)\operatorname{Si}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)+3\left((bc-ad)\operatorname{Ci}\left(\frac{3\,(bc-ad)}{d^2x+cd}\right)+(bc-ad)\operatorname{Ci}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\right)\cos\left(\frac{3\,b}{d}\right)-3\left((bc-ad)\operatorname{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\operatorname{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cos\left(\frac{b}{d}\right)-8\left(d^2x-(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)^2+cd\right)\sin\left(\frac{bx+a}{dx+c}\right)}{8\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="fricas")

[Out] -1/8*(6*(b*c - a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 6*(
b*c - a*d)*sin(3*b/d)*sin_integral(-3*(b*c - a*d)/(d^2*x + c*d)) + 3*((b*c
- a*d)*cos_integral(3*(b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral
(-3*(b*c - a*d)/(d^2*x + c*d)))*cos(3*b/d) - 3*((b*c - a*d)*cos_integral((b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c
*d)))*cos(b/d) - 8*(d^2*x - (d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + c*d)
*sin((b*x + a)/(d*x + c)))/d^2

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))**3,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(186) = 372$.
time = 61.52, size = 1239, normalized size = 6.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="giac")

[Out] 1/4*(3*b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 6*a*
b^2*c*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 3*(b*x + a)
*b^2*c^2*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c)
+ 3*a^2*b*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 6*(b*
x + a)*a*b*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) - 3*(b*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x +
c))/d)/(d*x + c) - 3*b^3*c^2*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*d/(d
*x + c))/d) + 6*a*b^2*c*d*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*d/(d*x
+ c))/d) + 3*(b*x + a)*b^2*c^2*d*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)*

```
d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*cos(3*b/d)*cos_integral(-3*(b - (b*
x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*c*d^2*cos(3*b/d)*cos_integral(-3*(
b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*cos(3*b/d)*co
s_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*b^3*c^2*sin(3*b/
d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 6*a*b^2*c*d*sin(3*b/d)*s
in_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 3*(b*x + a)*b^2*c^2*d*sin(3*
b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*
sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*
c*d^2*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) +
3*(b*x + a)*a^2*d^3*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d
)/(d*x + c) + 3*b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d
) - 6*a*b^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 3*(b
*x + a)*b^2*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) + 3*a^2*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) +
6*(b*x + a)*a*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/
(d*x + c) - 3*(b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x
 + c))/d)/(d*x + c) - b^2*c^2*d*sin(3*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*si
n(3*(b*x + a)/(d*x + c)) - a^2*d^3*sin(3*(b*x + a)/(d*x + c)) + 3*b^2*c^2*d
*sin((b*x + a)/(d*x + c)) - 6*a*b*c*d^2*sin((b*x + a)/(d*x + c)) + 3*a^2*d^
3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2
- (b*x + a)*d^3/(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((a + b*x)/(c + d*x))^3,x)
```

```
[Out] int(sin((a + b*x)/(c + d*x))^3, x)
```

3.39 $\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}+\frac{\mathrm{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

```
[Out] -3/4*Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/4*Si(3*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3380}

$$\frac{\mathrm{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{3\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) + SinIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4x} - \frac{\sin(3x)}{4x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$
$$= -\frac{3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 0.91

$$\frac{-3\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + SinIntegral[(3*Sqrt[1 - a*x]
)/Sqrt[1 + a*x]])/(4*a)
```

Maple [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(
a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.40 $\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] 1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/2
))/a
```

Rubi [A]
time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3383}

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]/(2*a)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cos(2x)}{2x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}\\ &= \frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 57, normalized size = 0.98

$$\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log(1-ax)}{4a}+\frac{\log(1+ax)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) + L
og[1 + a*x]/(4*a)
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x
) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*co
s(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/sq
rt(a*x + 1))^2), x) + log(a*x + 1) - log(a*x - 1))/a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.41 $\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=26

$$-\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6813, 3380}

$$-\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$-\frac{\mathrm{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^{2}x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

```
[Out] -integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.42 $\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Csc[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\csc(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 2.74, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)-\sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sin(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.43 $\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\operatorname{Int}\left(\frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csc[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\csc^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 9.85, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sin \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] ((a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*s
in(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)/s
qrt(a*x + 1)) - a)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*cos(sqr
t(-a*x + 1)/sqrt(a*x + 1))^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*sin(sqrt(-a*x
+ 1)/sqrt(a*x + 1))^2 - a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*cos(sqrt(-a*x
 + 1)/sqrt(a*x + 1)) + 1), x) - (a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/s
qrt(a*x + 1))^2 + (a^2*x - a)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 2*(a^
2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - a)*integrate(sqrt(a*x + 1)*s
qrt(-a*x + 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^2 + (a^3*x
^3 - a^2*x^2 - a*x + 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^3*x^3 - a^
2*x^2 - a*x + 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - a*x - 2*(a^3*x^3 - a
^2*x^2 - a*x + 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1), x) - 2*sqrt(a*x +
 1)*sqrt(-a*x + 1)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1)))/(a^2*x + (a^2*x - a
)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*sin(2*sqrt(-a*x + 1)/
sqrt(a*x + 1))^2 - 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - a)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/(a^2*x^2 - (a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 -
1), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

```
[Out] -Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sin(sqrt(-a*x
 + 1)/sqrt(a*x + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.44 $\int (x+\cos(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)$$

```
[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6874, 3377, 2718, 2715, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^2,x]
```

```
[Out] x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (x+\cos(x))^2\,dx &= \int \left(x^2+2x\cos(x)+\cos^2(x)\right)\,dx \\
&= \frac{x^3}{3}+2\int x\cos(x)\,dx+\int \cos^2(x)\,dx \\
&= \frac{x^3}{3}+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}-2\int \sin(x)\,dx \\
&= \frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{1}{2}\cos(x)\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 0.87

$$\frac{1}{6}\left(3\cos(x)(4+\sin(x))+x\left(3+2x^2+12\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^2,x]
[Out] (3*Cos[x]*(4 + Sin[x]) + x*(3 + 2*x^2 + 12*Sin[x]))/6
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result
default	$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{\cos(x)\sin(x)}{2}$
risch	$\frac{x^3}{3}+\frac{x}{2}+2\cos(x)+2x\sin(x)+\frac{\sin(2x)}{4}$
norman	$\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)-2\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{x}{2}+\frac{x^3}{3}-\left(\tan^3\left(\frac{x}{2}\right)\right)+4x\tan\left(\frac{x}{2}\right)+4x\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{2x^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{x^3\left(\tan^4\left(\frac{x}{2}\right)\right)}{3}+2+\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cos(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3+2x\sin(x)+\frac{1}{2}x+2\cos(x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

Fricas [A]
time = 2.03, size = 23, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}(4x + \cos(x))\sin(x) + \frac{1}{2}x + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*(4*x + cos(x))*sin(x) + 1/2*x + 2*cos(x)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2(x)}{2} + 2x\sin(x) + \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + 2*x*sin(x) + x*cos(x)**2/2 + sin(x)*cos(x)/2 + 2*c
os(x)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + 2x\sin(x) + \frac{1}{2}x + 2\cos(x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

Mupad [B]
time = 0.05, size = 24, normalized size = 0.80

$$\frac{x}{2} + 2\cos(x) + \frac{\cos(x)\sin(x)}{2} + 2x\sin(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cos(x))^2,x)
```

```
[Out] x/2 + 2*cos(x) + (cos(x)*sin(x))/2 + 2*x*sin(x) + x^3/3
```

3.45 $\int (x+\cos(x))^3 \, dx$

Optimal. Leaf size=56

$$\frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}-5\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] 3/4*x^2+1/4*x^4+6*x*cos(x)+3/4*cos(x)^2-5*sin(x)+3*x^2*sin(x)+3/2*x*cos(x)*
sin(x)-1/3*sin(x)^3
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6874, 3377, 2717, 3391, 30, 2713}

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)-\frac{\sin^3(x)}{3}-5\sin(x)+\frac{3\cos^2(x)}{4}+6x\cos(x)+\frac{3}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cos[x])^3,x]
```

```
[Out] (3*x^2)/4 + x^4/4 + 6*x*Cos[x] + (3*Cos[x]^2)/4 - 5*Sin[x] + 3*x^2*Sin[x] +
 (3*x*Cos[x]*Sin[x])/2 - Sin[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int (x+\cos(x))^3\,dx &= \int \left(x^3+3x^2\cos(x)+3x\cos^2(x)+\cos^3(x)\right)\,dx\\ &= \frac{x^4}{4}+3\int x^2\cos(x)\,dx+3\int x\cos^2(x)\,dx+\int \cos^3(x)\,dx\\ &= \frac{x^4}{4}+\frac{3\cos^2(x)}{4}+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)+\frac{3\int x\,dx}{2}-6\int x\sin(x)\,dx-\text{Subst}\\ &= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}+\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}\\ &= \frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}-5\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x}{3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.91

$$6x\cos(x)+\frac{3}{8}\cos(2x)+\frac{1}{12}\left(9x^2+3x^4+9\left(-7+4x^2\right)\sin(x)+9x\sin(2x)+\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cos[x])^3,x]
```

```
[Out] 6*x*Cos[x] + (3*Cos[2*x])/8 + (9*x^2 + 3*x^4 + 9*(-7 + 4*x^2)*Sin[x] + 9*x*
Sin[2*x] + Sin[3*x])/12
```

Maple [A]
time = 0.07, size = 57, normalized size = 1.02

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+6x\cos(x)+\frac{3(4x^2-7)\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{3\cos(2x)}{8}+\frac{3x\sin(2x)}{4}$

default	$\frac{(2+\cos^2(x))\sin(x)}{3}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}-\frac{3(\sin^2(x))}{4}+3x^2\sin(x)-6\sin(x)+6x\cos(x)+\frac{x^4}{4}$
norman	$\frac{\tan^6(\frac{x}{2})+6x+\frac{3x^2}{4}+\frac{x^4}{4}-\frac{68(\tan^3(\frac{x}{2}))}{3}-10(\tan^5(\frac{x}{2}))+3x\tan(\frac{x}{2})+6x(\tan^2(\frac{x}{2}))-6x(\tan^4(\frac{x}{2}))-3x(\tan^5(\frac{x}{2}))-6x(\tan^6(\frac{x}{2}))+6x^2\text{ta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cos(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)+3*x*(1/2*cos(x)*sin(x)+1/2*x)-3/4*x^2-3/4*sin(x)^2+
3*x^2*sin(x)-6*sin(x)+6*x*cos(x)+1/4*x^4
```

Maxima [A]
time = 0.26, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4-\frac{1}{3}\sin(x)^3+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}x\sin(2x)+3(x^2-2)\sin(x)+\frac{3}{8}\cos(2x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 1/3*sin(x)^3 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3*(x^2 - 2
)*sin(x) + 3/8*cos(2*x) + sin(x)
```

Fricas [A]
time = 1.79, size = 44, normalized size = 0.79

$$\frac{1}{4}x^4+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}\cos(x)^2+\frac{1}{6}(18x^2+9x\cos(x)+2\cos(x)^2-32)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*cos(x)^2 + 1/6*(18*x^2 + 9*x*cos(x) +
2*cos(x)^2 - 32)*sin(x)
```

Sympy [A]
time = 0.09, size = 85, normalized size = 1.52

$$\frac{x^4}{4}+\frac{3x^2\sin^2(x)}{4}+3x^2\sin(x)+\frac{3x^2\cos^2(x)}{4}+\frac{3x\sin(x)\cos(x)}{2}+6x\cos(x)+\frac{2\sin^3(x)}{3}+\sin(x)\cos^2(x)-6\sin(x)+\frac{3\cos^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))**3,x)
```

```
[Out] x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*sin(x) + 3*x**2*cos(x)**2/4 + 3*x*sin(
x)*cos(x)/2 + 6*x*cos(x) + 2*sin(x)**3/3 + sin(x)*cos(x)**2 - 6*sin(x) + 3*
cos(x)**2/4
```

Giac [A]
time = 0.41, size = 46, normalized size = 0.82

$$\frac{1}{4}x^4+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}x\sin(2x)+\frac{3}{4}\left(4x^2-7\right)\sin(x)+\frac{3}{8}\cos(2x)+\frac{1}{12}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cos(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3/4*(4*x^2 - 7)*sin(x) +
3/8*cos(2*x) + 1/12*sin(3*x)
```

Mupad [B]
time = 0.08, size = 48, normalized size = 0.86

$$3x^2\sin(x)-\frac{16\sin(x)}{3}+\frac{3\cos(x)^2}{4}+\frac{\cos(x)^2\sin(x)}{3}+6x\cos(x)+\frac{3x^2}{4}+\frac{x^4}{4}+\frac{3x\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + cos(x))^3,x)
```

```
[Out] 3*x^2*sin(x) - (16*sin(x))/3 + (3*cos(x)^2)/4 + (cos(x)^2*sin(x))/3 + 6*x*c
os(x) + (3*x^2)/4 + x^4/4 + (3*x*cos(x)*sin(x))/2
```

3.46 $\int \frac{\cos(a+bx)}{c+dx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{}}{}\right.}{}$$

```
[Out] -1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*cos(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d^
(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*cos(a+b*(-c)^(1/2)/d^(1/2))/(-c)^(1
/2)/d^(1/2)+1/2*Si(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))/(-
c)^(1/2)/d^(1/2)-1/2*Si(b*x-b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^(1/2
))/(-c)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3415, 3384, 3380, 3383}

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(xb+\frac{\sqrt{-c}\,b}{\sqrt{d}}\right)}{2\sqrt{-c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] (Cos[a + (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*
Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])
/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a + (b*Sqrt[-c])/Sqrt[d]]*SinI
ntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a - (b*Sqr
t[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d
])
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{c+dx^2}\,dx &= \int \left(\frac{\sqrt{-c}\,\cos(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}\,x\right)}+\frac{\sqrt{-c}\,\cos(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}\,x\right)}\right)dx\\
&= -\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\cos(a+bx)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}\\
&= -\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}\,x}\,dx}{2\sqrt{-c}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cos\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}\,x}\,dx}{2\sqrt{-c}}+\\
&= \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\,\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Ci}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\,\sqrt{d}}+\frac{\sin\left(a\right.}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.19, size = 172, normalized size = 0.81

$$-\frac{i\left(\cos\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-\cos\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{CosIntegral}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)+\sin\left(a-\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)+\sin\left(a+\frac{ib\sqrt{c}}{\sqrt{d}}\right)\mathrm{Si}\left(\frac{ib\sqrt{c}}{\sqrt{d}}-bx\right)\right)}{2\sqrt{c}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/2*I)*(Cos[a + (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*(((-I)*Sqrt[c])/Sqr
t[d] + x)] - Cos[a - (I*b*Sqrt[c])/Sqrt[d]]*CosIntegral[b*((I*Sqrt[c])/Sqrt
[d] + x)] + Sin[a - (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[b*((I*Sqrt[c])/Sqrt[
d] + x)] + Sin[a + (I*b*Sqrt[c])/Sqrt[d]]*SinIntegral[(I*b*Sqrt[c])/Sqrt[d]
 - b*x]))/(Sqrt[c]*Sqrt[d])
```

Maple [A]
time = 0.11, size = 231, normalized size = 1.08

method	result
derivativedivides	$b\left(-\frac{\mathrm{sinIntegral}\left(-bx-a+\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\mathrm{cosineIntegral}\left(bx+a-\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-}}{}\right.}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}\right.$
default	$b\left(-\frac{\mathrm{sinIntegral}\left(-bx-a+\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\mathrm{cosineIntegral}\left(bx+a-\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-}}{}\right.}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}\right.$
risch	$\frac{i\sqrt{cd}\,\mathrm{e}^{-\frac{iad-b\sqrt{cd}}{d}}\mathrm{expIntegral}\left(1,-\frac{iad-b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}-\frac{i\sqrt{cd}\,\mathrm{e}^{-\frac{iad+b\sqrt{cd}}{d}}\mathrm{expIntegral}\left(1,-\frac{iad+b\sqrt{c}}{}\right.}{4cd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/2/d/(-(b*(-c*d)^(1/2)+a*d)/d+a)*(Si(-b*x-a+(b*(-c*d)^(1/2)+a*d)/d)*si
n((b*(-c*d)^(1/2)+a*d)/d)+Ci(b*x+a-(b*(-c*d)^(1/2)+a*d)/d)*cos((b*(-c*d)^(1
/2)+a*d)/d))-1/2/d/((b*(-c*d)^(1/2)-a*d)/d+a)*(-Si(-b*x-a-(b*(-c*d)^(1/2)-a
*d)/d)*sin((b*(-c*d)^(1/2)-a*d)/d)+Ci(b*x+a+(b*(-c*d)^(1/2)-a*d)/d)*cos((b*
(-c*d)^(1/2)-a*d)/d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.98, size = 189, normalized size = 0.89

$$-\frac{-i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(ibx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia+\sqrt{\frac{b^2c}{d}}\right)}+i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(ibx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(ia-\sqrt{\frac{b^2c}{d}}\right)}+i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-ibx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-ia+\sqrt{\frac{b^2c}{d}}\right)}-i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-ibx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(-ia-\sqrt{\frac{b^2c}{d}}\right)}}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*(-I*sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) +
I*sqrt(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + I*sqrt(
b^2*c/d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - I*sqrt(b^2*c
/d)*Ei(-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x**2+c),x)
[Out] Integral(cos(a + b*x)/(c + d*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)/(d*x^2 + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)}{d\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x^2),x)
[Out] int(cos(a + b*x)/(c + d*x^2), x)
```

3.47 $\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$

Optimal. Leaf size=271

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegr}}{\sqrt{d^2-4ce}}$$

[Out] Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Si(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)

Rubi [A]
time = 0.41, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6860, 3384, 3380, 3383}

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]/(c + d*x + e*x^2),x]

[Out] (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e]

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx = \int \left(\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)$$

$$= \frac{(2e)\int \frac{\cos(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\cos(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cos\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{\left(2e\cos\left(a-\cdots\right.\right.}{\cdots}$$

$$= \frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{Ci}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4c\cdots}\right.}{2e}\right.}{\cdots}$$

Mathematica [A]
time = 0.22, size = 236, normalized size = 0.87

$$\frac{\cos\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)-\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+\sin\left(a+\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(-d+\sqrt{d^2-4ce}\right)}{2e}-bx\right)+\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}+2ex\right)}{2e}\right)}{\sqrt{d^2-4ce}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (Cos[a + (b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)] - Cos[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIn
tegral[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + Sin[a + (b*(-d + Sqrt[d
^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(-d + Sqrt[d^2 - 4*c*e]))/(2*e) - b*x]
```

```
+ Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 -
 4*c*e] + 2*e*x))/(2*e)])/Sqrt[d^2 - 4*c*e]
```

Maple [A]
time = 0.21, size = 326, normalized size = 1.20

method	result
derivativedivides	$b\left(\frac{\mathrm{sinIntegral}\left(-bx-a+\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\mathrm{cosineIntegral}\left(bx+\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
default	$b\left(\frac{\mathrm{sinIntegral}\left(-bx-a+\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-db+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\mathrm{cosineIntegral}\left(bx+\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
risch	$-\frac{ib\,\mathrm{e}^{-\frac{2iae-ibd+\sqrt{4b^2ce-b^2d^2}}{2e}}\,\mathrm{expIntegral}\left(1,-\frac{2iae-ibd-2e(ibx+ia)+\sqrt{4b^2ce-b^2d^2}}{2e}\right)}{2\sqrt{4b^2ce-b^2d^2}}+\frac{ib\,\mathrm{e}^{-\frac{2iae-ibd-\sqrt{4b^2c}}{2e}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(-b*x-a+1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2
*d^2)^(1/2)))*sin(1/2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(b*x+a-1/
2/e*(2*a*e-d*b+(-4*b^2*c*e+b^2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-d*b+(-4*b^2*c*
e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-b*x-a-1/2*(-2*a*e+d*
b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(
1/2))/e)+Ci(b*x+a+1/2*(-2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-
2*a*e+d*b+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(x^2*e + d*x + c), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.94, size = 449, normalized size = 1.66

$$-\frac{-i\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,\mathrm{Ei}\left(\frac{1}{2}\left(-2i\,bxe-i\,bd-\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(i\,bd-2i\,ae+\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}+i\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,\mathrm{Ei}\left(\frac{1}{2}\left(-2i\,bxe-i\,bd+\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(i\,bd-2i\,ae-\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}+i\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,\mathrm{Ei}\left(\frac{1}{2}\left(2i\,bxe+i\,bd-\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(-i\,bd+2i\,ae+\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}-i\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,\mathrm{Ei}\left(\frac{1}{2}\left(2i\,bxe+i\,bd+\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}\right)e^{\left(\frac{1}{2}\left(-i\,bd+2i\,ae-\sqrt{-(b^2d^2-4\,b^2ce)e^{(-2)}}\,e\right)e^{(-1)}+1\right)}}{2\,(bd^2-4\,bce)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(-I*sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*Ei(1/2*(-2*I*b*x*e - I*b*d - s
qrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1))*e^(1/2*(I*b*d - 2*I*a*e + sqr
t(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1) + 1) + I*sqrt(-(b^2*d^2 - 4*b^2*
c*e)*e^(-2))*Ei(1/2*(-2*I*b*x*e - I*b*d + sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2
))*e)*e^(-1))*e^(1/2*(I*b*d - 2*I*a*e - sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))
*e)*e^(-1) + 1) + I*sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*Ei(1/2*(2*I*b*x*e +
 I*b*d - sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1))*e^(1/2*(-I*b*d + 2*
I*a*e + sqrt(-(b^2*d^2 - 4*b^2*c*e)*e^(-2))*e)*e^(-1) + 1) - I*sqrt(-(b^2*d
^2 - 4*b^2*c*e)*e^(-2))*Ei(1/2*(2*I*b*x*e + I*b*d + sqrt(-(b^2*d^2 - 4*b^2*
c*e)*e^(-2))*e)*e^(-1))*e^(1/2*(-I*b*d + 2*I*a*e - sqrt(-(b^2*d^2 - 4*b^2*c
*e)*e^(-2))*e)*e^(-1) + 1))/(b*d^2 - 4*b*c*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)/(c + d*x + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(cos(a + b*x)/(c + d*x + e*x^2), x)
```

3.48 $\int \frac{x \cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=10

$$\sin\left(\sqrt{1+x^2}\right)$$

```
[Out] sin((x^2+1)^(1/2))
```

Rubi [A]
time = 0.09, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6847, 3513, 15, 2717}

$$\sin\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sin[Sqrt[1 + x^2]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\cos\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx, x, x^2\right)\\ &= \text{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x^2}\right)\\ &= 1\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{1+x^2}\right)\\ &= \sin\left(\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 10, normalized size = 1.00

$$\sin\left(\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
[Out] Sin[Sqrt[1 + x^2]]
```

Maple [A]
time = 0.08, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\sin\left(\sqrt{x^2+1}\right)$	9
default	$\sin\left(\sqrt{x^2+1}\right)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] sin((x^2+1)^(1/2))
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sin(sqrt(x^2 + 1))
```

Fricas [A]
time = 1.91, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] sin(sqrt(x^2 + 1))
```

Sympy [A]
time = 0.13, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
[Out] sin(sqrt(x**2 + 1))
```

Giac [A]
time = 0.39, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
[Out] sin(sqrt(x^2 + 1))
```

Mupad [B]
time = 2.26, size = 8, normalized size = 0.80

$$\sin\left(\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
[Out] sin((x^2 + 1)^(1/2))
```

3.49 $\int \frac{x \cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sin\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{3}}$$

[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6847, 3513, 15, 2717}

$$\frac{\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]

[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
```

```
fQ[x^(m + 1), u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos\left(\sqrt{3}\,\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\cos\left(\sqrt{3}\,\sqrt{2+x}\right)}{\sqrt{2+x}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{x\cos\left(\sqrt{3}\,x\right)}{\sqrt{x^2}}\,dx, x, \sqrt{2+x^2}\right)\\
&= 1\text{Subst}\left(\int \cos\left(\sqrt{3}\,x\right)\,dx, x, \sqrt{2+x^2}\right)\\
&= \frac{\sin\left(\sqrt{3}\,\sqrt{2+x^2}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 22, normalized size = 1.00

$$\frac{\sin\left(\sqrt{3}\,\sqrt{2+x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
[Out] Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

Maple [**A**]
time = 0.15, size = 18, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\sin\left(\sqrt{3}\,\sqrt{x^2+2}\right)\sqrt{3}}{3}$	18
default	$\frac{\sin\left(\sqrt{3}\,\sqrt{x^2+2}\right)\sqrt{3}}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 2.03, size = 37, normalized size = 1.68

$$\frac{2\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)}{3\left(\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*tan(1/2*sqrt(3)*sqrt(x^2 + 2))/(tan(1/2*sqrt(3)*sqrt(x^2 + 2))^
2 + 1)
```

Sympy [A]
time = 0.19, size = 20, normalized size = 0.91

$$\frac{\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3**(1/2)*(x**2+2)**(1/2))/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(3)*sin(sqrt(3)*sqrt(x**2 + 2))/3
```

Giac [A]
time = 0.44, size = 17, normalized size = 0.77

$$\frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

Mupad [B]
time = 2.32, size = 15, normalized size = 0.68

$$\frac{\sqrt{3}\,\sin\left(\sqrt{3\,x^2+6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(3^(1/2)*(x^2 + 2)^(1/2)))/(x^2 + 2)^(1/2),x)
[Out] (3^(1/2)*sin((3*x^2 + 6)^(1/2)))/3
```

3.50 $\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx$

Optimal. Leaf size=24

$$\frac{1}{6}\sin\left(\sqrt{3}\,\sqrt{2+(-1+2x)^2}\right)$$

```
[Out] 1/6*sin(3^(1/2)*(2+(-1+2*x)^2)^(1/2))
```

Rubi [A]

time = 0.33, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6847, 3513, 15, 2717}

$$\frac{1}{6}\sin\left(\sqrt{3}\,\sqrt{(2x-1)^2+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^2],x]
```

```
[Out] Sin[Sqrt[3]*Sqrt[2 + (-1 + 2*x)^2]]/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3513

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\cos\left(\sqrt{6+3x^2}\right)}{\sqrt{6+3x^2}}\,dx, x, -1+2x\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{\cos\left(\sqrt{6+3x}\right)}{\sqrt{6+3x}}\,dx, x, (-1+2x)^2\right)$$
$$= \frac{1}{6}\text{Subst}\left(\int \frac{x\cos(x)}{\sqrt{x^2}}\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)$$
$$= \frac{1}{6}\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{3}\sqrt{2+(-1+2x)^2}\right)$$
$$= \frac{1}{6}\sin\left(\sqrt{3}\sqrt{2+(-1+2x)^2}\right)$$

Mathematica [A]
time = 0.08, size = 20, normalized size = 0.83

$$\frac{1}{6}\sin\left(\sqrt{6+3(1-2x)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^
2],x]
```

```
[Out] Sin[Sqrt[6 + 3*(1 - 2*x)^2]]/6
```

Maple [A]
time = 0.58, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\sin\left(\sqrt{12x^2-12x+9}\right)}{6}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/6*sin((12*x^2-12*x+9)^(1/2))
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.67

$$\frac{1}{6}\sin\left(\sqrt{3\,(2\,x-1)^2+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="maxima")
```

```
[Out] 1/6*sin(sqrt(3*(2*x - 1)^2 + 6))
```

Fricas [A]
time = 1.92, size = 15, normalized size = 0.62

$$\frac{1}{6}\sin\left(\sqrt{12\,x^2-12\,x+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/6*sin(sqrt(12*x^2 - 12*x + 9))
```

Sympy [A]
time = 3.13, size = 15, normalized size = 0.62

$$\frac{\sin\left(\sqrt{3\left(2x-1\right)^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)**2)**(1/2))/(6+3*(-1+2*x)**2)**(1/2),x
)
```

```
[Out] sin(sqrt(3*(2*x - 1)**2 + 6))/6
```

Giac [A]
time = 0.39, size = 19, normalized size = 0.79

$$\frac{1}{6}\sin\left(\sqrt{3}\,\sqrt{4\,x^2-4\,x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/6*sin(sqrt(3)*sqrt(4*x^2 - 4*x + 3))
```

Mupad [B]
time = 2.35, size = 16, normalized size = 0.67

$$\frac{\sin\left(\sqrt{3\,{\left(2\,x-1\right)}^2+6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos((3*(2*x - 1)^2 + 6)^(1/2))*(2*x - 1))/(3*(2*x - 1)^2 + 6)^(1/2),x)
[Out] sin((3*(2*x - 1)^2 + 6)^(1/2))/6
```

3.51 $\int \cos\left(\frac{a+bx}{c+dx}\right) dx$

Optimal. Leaf size=101

$$\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)}{d^2}+\frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

```
[Out] (d*x+c)*cos((b*x+a)/(d*x+c))/d+(-a*d+b*c)*cos(b/d)*Si((-a*d+b*c)/d/(d*x+c))
/d^2-(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*sin(b/d)/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4660, 3378, 3384, 3380, 3383}

$$-\frac{\sin\left(\frac{b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{b}{d}\right)(bc-ad)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*CosIntegral[(b*c - a*
d)/(d*(c + d*x))]*Sin[b/d])/d^2 + ((b*c - a*d)*Cos[b/d]*SinIntegral[(b*c -
a*d)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4660

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Dist[-d^(-1), Subst[Int[Cos[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \cos\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{\text{Subst}\left(\int \frac{\cos\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sin\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{((bc-}{}$$

$$= \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{bc-ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.81, size = 260, normalized size = 2.57

$$\frac{-4(bc-ad)\text{CosIntegral}\left(\frac{-bc+ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)+de^{-\frac{i(2bc+ad+bdx)}{d(c+dx)}}\left(2c\left(e^{\frac{2ibc}{d(c+dx)}}+e^{2i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+d\left(1+e^{\frac{2ib}{d}}\right)\left(e^{\frac{2ia}{c+dx}}+e^{\frac{2ibc}{d(c+dx)}}\right)x-4de^{\frac{i(2bc+ad+bdx)}{d(c+dx)}}x\sin\left(\frac{b}{d}\right)\sin\left(\frac{-bc+ad}{d(c+dx)}\right)\right)-4(bc-ad)\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{-bc+ad}{d(c+dx)}\right)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)],x]
```

```
[Out] (-4*(b*c - a*d)*CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] + (d*(2*
c*(E^(((2*I)*b*c)/(d*(c + d*x))) + E^((2*I)*(b/d + a/(c + d*x)))) + d*(1 +
E^(((2*I)*b)/d))*(E^(((2*I)*a)/(c + d*x)) + E^(((2*I)*b*c)/(d*(c + d*x))))*
x - 4*d*E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x)))*x*Sin[b/d]*Sin[(-(b*c)
+ a*d)/(d*(c + d*x))]))/E^((I*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 4*(b*
c - a*d)*Cos[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))])/(4*d^2)
```

Maple [A]
time = 0.12, size = 142, normalized size = 1.41

method	result
derivativedivides	$-(ad-cb)\left(-\frac{\cos\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{\frac{\text{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d}\right)$
default	$-(ad-cb)\left(-\frac{\cos\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{\frac{\text{sinIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}+\frac{\text{cosineIntegral}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d}\right)$
risch	$\frac{i\,\text{expIntegral}\left(1,-\frac{i(ad-cb)}{d(dx+c)}\right)\mathrm{e}^{\frac{ib}{d}}a}{2d}-\frac{i\,\text{expIntegral}\left(1,-\frac{i(ad-cb)}{d(dx+c)}\right)\mathrm{e}^{\frac{ib}{d}}cb}{2d^2}-\frac{\mathrm{e}^{-\frac{ib}{d}}\pi\,\text{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}+\frac{\mathrm{e}^{-\frac{ib}{d}}\pi\,\text{csgn}\left(\frac{a}{d(}\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*d-b*c)*(-cos(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d
-(Si((a*d-b*c)/d/(d*x+c))*cos(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*sin(b/d)/d)/d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos((b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 2.16, size = 138, normalized size = 1.37

$$-\frac{2\,(bc-ad)\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-2\,(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)+\left((bc-ad)\,\mathrm{Ci}\left(\frac{bc-ad}{d^2x+cd}\right)+(bc-ad)\,\mathrm{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\sin\left(\frac{b}{d}\right)}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(b*c - a*d)*cos(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - 2*(
d^2*x + c*d)*cos((b*x + a)/(d*x + c)) + ((b*c - a*d)*cos_integral((b*c - a*
d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)))*s
in(b/d))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(\frac{a+bx}{c+dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x)
```

```
[Out] Integral(cos((a + b*x)/(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(101) = 202.

time = 4.78, size = 633, normalized size = 6.27

$$-\frac{\left(b^{3}c^{2}\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)-2\,ab^{2}cd\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)-\frac{(bx+a)b^{2}c^{2}d\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)}{dx+c}+a^{2}bd^{2}\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)+\frac{2\,(bx+a)abcd^{2}\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)}{dx+c}-\frac{(bx+a)a^{2}d^{3}\,\mathrm{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)\sin\left(\frac{b}{d}\right)}{dx+c}-b^{3}c^{2}\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)+2\,ab^{2}cd\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)+\frac{(bx+a)b^{2}c^{2}d\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}-a^{2}bd^{2}\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)-\frac{2\,(bx+a)abcd^{2}\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}+\frac{(bx+a)a^{2}d^{3}\cos\left(\frac{b}{d}\right)\mathrm{Si}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c}-b^{2}c^{2}d\cos\left(\frac{bx+a}{dx+c}\right)+2\,abcd^{2}\cos\left(\frac{bx+a}{dx+c}\right)-a^{2}d^{3}\cos\left(\frac{bx+a}{dx+c}\right)\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)}{bd^{2}-\frac{(bx+a)d^{3}}{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^3*c^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - 2*a*b^2*c
*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - (b*x + a)*b^2*c^
2*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) + a^2*b
*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) + 2*(b*x + a)*a*
b*c*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) - (
b*x + a)*a^2*d^3*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x
 + c) - b^3*c^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*a*
b^2*c*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + (b*x + a)*b^
2*c^2*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - a^
2*b*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*(b*x + a)*
a*b*c*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) +
(b*x + a)*a^2*d^3*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
 + c) - b^2*c^2*d*cos((b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos((b*x + a)/(d*x
 + c)) - a^2*d^3*cos((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c -
a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)/(c + d*x)),x)
```

```
[Out] int(cos((a + b*x)/(c + d*x)), x)
```

3.52 $\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal. Leaf size=107

$$\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2}+\frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

```
[Out] (d*x+c)*cos((b*x+a)/(d*x+c))^2/d+(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(d
*x+c))/d^2-(-a*d+b*c)*Ci(2*(-a*d+b*c)/d/(d*x+c))*sin(2*b/d)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4660, 3394, 12, 3384, 3380, 3383}

$$-\frac{\sin\left(\frac{2b}{d}\right)(bc-ad)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{\cos\left(\frac{2b}{d}\right)(bc-ad)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Cos[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*CosIntegral[(2*(b*c
 - a*d))/(d*(c + d*x))]*Sin[(2*b)/d])/d^2 + ((b*c - a*d)*Cos[(2*b)/d]*SinIn
tegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4660

```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Dist[-d^(-1), Subst[Int[Cos[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x], x
, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0]
```

Rubi steps

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(2(bc-ad))\text{Subst}\left(\int -\frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sin\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cos\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{((b\ldots}{\ldots}$$

$$= \frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\text{Ci}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad\ldots}{d(c+dx)}\right.}{d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.61, size = 329, normalized size = 3.07

$$\frac{-8(bc-ad)\text{CosIntegral}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)+de^{-\frac{2i(2bc+ad+bdx)}{d(c+dx)}}\left(2c\left(e^{\frac{4ibc}{d(c+dx)}}+e^{4i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)+d\left(e^{\frac{4ia}{c+dx}}+e^{\frac{4ibc}{d(c+dx)}}+e^{\frac{4ib(2c+dx)}{d(c+dx)}}+4e^{\frac{2i(2bc+ad+bdx)}{d(c+dx)}}+e^{4i\left(\frac{b}{d}+\frac{a}{c+dx}\right)}\right)x-4de^{\frac{2i(2bc+ad+bdx)}{d(c+dx)}}x\sin\left(\frac{2b}{d}\right)\sin\left(\frac{2(-bc+ad)}{d(c+dx)}\right)\right)-8(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] (-8*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(2*b)/d]
+ (d*(2*c*(E^(((4*I)*b*c)/(d*(c + d*x))) + E^((4*I)*(b/d + a/(c + d*x)))) +
 d*(E^(((4*I)*a)/(c + d*x)) + E^(((4*I)*b*c)/(d*(c + d*x))) + E^(((4*I)*b*(
2*c + d*x))/(d*(c + d*x))) + 4*E^(((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x
))) + E^((4*I)*(b/d + a/(c + d*x))))*x - 4*d*E^(((2*I)*(2*b*c + a*d + b*d*x
))/(d*(c + d*x)))*x*Sin[(2*b)/d]*Sin[(2*(-(b*c) + a*d))/(d*(c + d*x))]))/E^
(((2*I)*(2*b*c + a*d + b*d*x))/(d*(c + d*x))) - 8*(b*c - a*d)*Cos[(2*b)/d]*
SinIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))])/(8*d^2)
```

Maple [A]

time = 0.16, size = 195, normalized size = 1.82

method	result
derivativedivides	$-\frac{(ad-cb)\left(\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{\ }{2(\ }\right.}{d^2}$
default	$-\frac{(ad-cb)\left(\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{cosineIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{\ }{2(\ }\right.}{d^2}$
risch	$-\frac{e^{-\frac{2ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}+\frac{e^{-\frac{2ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{2d^2}+\frac{e^{-\frac{2ib}{d}}\,\mathrm{sinIntegral}\left(\frac{2ad-2cb}{d(dx+c)}\right)a}{d}-\frac{e^{-\frac{2ib}{d}}\,\mathrm{sinIntegral}\left(\frac{2ad}{d(dx}\right.}{d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(a*d-b*c)*(1/4*d^2*(-2*cos(2*(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b
*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a*d-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a
*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d)-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

Fricas [A]
time = 2.16, size = 144, normalized size = 1.35

$$\frac{2\left(d^{2}x+cd\right)\cos\left(\frac{bx+a}{dx+c}\right)^{2}-2\left(bc-ad\right)\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(-\frac{2\left(bc-ad\right)}{d^{2}x+cd}\right)-\left(\left(bc-ad\right)\operatorname{Ci}\left(\frac{2\left(bc-ad\right)}{d^{2}x+cd}\right)+\left(bc-ad\right)\operatorname{Ci}\left(-\frac{2\left(bc-ad\right)}{d^{2}x+cd}\right)\right)\sin\left(\frac{2b}{d}\right)}{2\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 - 2*(b*c - a*d)*cos(2*b/d)*
sin_integral(-2*(b*c - a*d)/(d^2*x + c*d)) - ((b*c - a*d)*cos_integral(2*(b
*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*cos_integral(-2*(b*c - a*d)/(d^2*x +
 c*d)))*sin(2*b/d))/d^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(107) = 214.
time = 22.26, size = 683, normalized size = 6.38

$$-\frac{\left(2\,b^{3}c^{2}\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)-4\,ab^{2}cd\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)-\frac{2\,(bx+a)b^{2}c^{2}d\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)}{dx+c}+2\,a^{2}bd^{2}\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)+\frac{4\,(bx+a)abcd^{2}\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)}{dx+c}-\frac{2\,(bx+a)a^{2}d^{3}\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2b}{d}\right)}{dx+c}-2\,b^{3}c^{2}\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)+4\,ab^{2}cd\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)+\frac{2\,(bx+a)b^{2}c^{2}d\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}-2\,a^{2}bd^{2}\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)-\frac{4\,(bx+a)abcd^{2}\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}+\frac{2\,(bx+a)a^{2}d^{3}\cos\left(\frac{2b}{d}\right)\operatorname{Si}\left(\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}-b^{2}c^{2}d\cos\left(\frac{2(bx+a)}{dx+c}\right)+2\,abcd^{2}\cos\left(\frac{2(bx+a)}{dx+c}\right)-a^{2}d^{3}\cos\left(\frac{2(bx+a)}{dx+c}\right)-b^{2}c^{2}d+2\,abcd^{2}-a^{2}d^{3}\right)\left(\frac{bc}{(bc-ad)^{2}}-\frac{ad}{(bc-ad)^{2}}\right)}{2\left(bd^{2}-\frac{(bx+a)d^{3}}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
 4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) - 2*
(b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/
d)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*s
in(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c
))/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b - (b*x
+ a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin_integr
al(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_integral(2
```

```
*(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)*sin_inte
gral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos(2*b/d)*si
n_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d^2*cos(2*b
/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*(b*x + a)*a
^2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) -
 b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*x + a)/(d*x +
c)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) - b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)/(c + d*x))^2,x)
```

```
[Out] int(cos((a + b*x)/(c + d*x))^2, x)
```

3.53 $\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

```
[Out] -3/4*Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Ci(3*(-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6813, 3393, 3383}

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (-3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - CosIntegral[(3*Sqrt[1
 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4x}+\frac{\cos(3x)}{4x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{3\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} \\
&= -\frac{3\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Ci}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.91

$$-\frac{3\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
[Out] -1/4*(3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + CosIntegral[(3*Sqrt[1 -
a*x])/Sqrt[1 + a*x]])/a
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos^{3}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.54 $\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6813, 3393, 3383}

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/(2*a)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
```

```
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cos(2x)}{2x}\right)\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$
$$= -\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx,x,\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$
$$= -\frac{\text{Ci}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.88

$$-\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
[Out] -1/2*(CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]])/a
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1),
x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*c
os(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)/s
qrt(a*x + 1))^2), x) - log(a*x + 1) + log(a*x - 1))/a
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="fr
icas")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="gi
ac")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.55 $\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6813, 3383}

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a} = -\frac{\text{Ci}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
[Out] int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
[Out] -integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

```
[Out] integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] -Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

```
[Out] integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] -int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.56 $\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=40

$$\text{Int}\left(\frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Sec[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cos \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

```
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="ma
xima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="fr
icas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^{2} x^{2} \cos \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)-\cos \left(\frac{\sqrt{-a x+1}}{\sqrt{a x+1}}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cos(sqrt(-a*x +
1)/sqrt(a*x + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="gi
ac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

```
[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.57 $\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=42

$$\text{Int}\left(\frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Sec[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a
)
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\sec^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A]
time = 7.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cos \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
maxima")
```

```
[Out] 2*((a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)
*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)
/sqrt(a*x + 1)) - a)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(2*sqrt(-a*x
 + 1)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*cos
(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*sin(2*sq
rt(-a*x + 1)/sqrt(a*x + 1))^2 - a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*cos(2
*sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1), x) + sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(
2*sqrt(-a*x + 1)/sqrt(a*x + 1)))/(a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/
sqrt(a*x + 1))^2 + (a^2*x - a)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + 2*(a
^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - a)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
fricas")
```

```
[Out] integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{a^2x^2\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
[Out] -Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cos(sqrt(-a*x
 + 1)/sqrt(a*x + 1))**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm="
giac")
[Out] integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
[Out] -int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.58 $\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=9

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

```
[Out] -2*ln(cos(x^(1/2)))
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3832, 3556}

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Log[Cos[Sqrt[x]]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \tan(x)\,dx, x, \sqrt{x}\right)\\ &= -2\log\left(\cos\left(\sqrt{x}\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$-2\log\left(\cos\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Log[Cos[Sqrt[x]]]
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$-2\ln\left(\cos\left(\sqrt{x}\,\right)\right)$	8
default	$-2\ln\left(\cos\left(\sqrt{x}\,\right)\right)$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*ln(cos(x^(1/2)))
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.78

$$2\,\log\left(\sec\left(\sqrt{x}\,\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*log(sec(sqrt(x)))
```

Fricas [A]
time = 2.05, size = 13, normalized size = 1.44

$$-\log\left(\frac{1}{\tan\left(\sqrt{x}\,\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -log(1/(tan(sqrt(x))^2 + 1))
```

Sympy [A]
time = 0.06, size = 10, normalized size = 1.11

$$\log\left(\tan^{2}\left(\sqrt{x}\,\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))/x**(1/2),x)
```

```
[Out] log(tan(sqrt(x))**2 + 1)
```

Giac [A]
time = 0.41, size = 8, normalized size = 0.89

$$-2\,\log\left(\left|\cos\left(\sqrt{x}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(cos(sqrt(x))))
```

Mupad [B]
time = 3.18, size = 19, normalized size = 2.11

$$-2\,\ln\left(\mathrm{e}^{\sqrt{x}\,2\mathrm{i}}+1\right)+\sqrt{x}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/x^(1/2),x)
```

```
[Out] x^(1/2)*2i - 2*log(exp(x^(1/2)*2i) + 1)
```

3.59 $\int \frac{\tan^2(\sqrt{x})}{\sqrt{x}}\, dx$

Optimal. Leaf size=16

$$-2\sqrt{x}+2\tan\left(\sqrt{x}\right)$$

```
[Out] -2*x^(1/2)+2*tan(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3832, 3554, 8}

$$2\tan\left(\sqrt{x}\right)-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + 2*Tan[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\, dx &= 2\text{Subst}\left(\int \tan^2(x)\, dx, x, \sqrt{x}\right)\\
&= 2\tan\left(\sqrt{x}\right)-2\text{Subst}\left(\int 1\, dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}+2\tan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.12

$$-2\text{ArcTan}\left(\tan\left(\sqrt{x}\right)\right)+2\tan\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[Sqrt[x]]^2/Sqrt[x],x]
[Out] -2*ArcTan[Tan[Sqrt[x]]] + 2*Tan[Sqrt[x]]
```

Maple [A]
time = 0.05, size = 15, normalized size = 0.94

method	result	size
derivativedivides	$2\tan\left(\sqrt{x}\right)-2\arctan\left(\tan\left(\sqrt{x}\right)\right)$	15
default	$2\tan\left(\sqrt{x}\right)-2\arctan\left(\tan\left(\sqrt{x}\right)\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*tan(x^(1/2))-2*arctan(tan(x^(1/2)))
```

Maxima [A]
time = 0.48, size = 12, normalized size = 0.75

$$-2\sqrt{x}+2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Fricas [A]
time = 1.78, size = 12, normalized size = 0.75

$$-2\sqrt{x}+2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Sympy [A]
time = 0.07, size = 14, normalized size = 0.88

$$-2\sqrt{x}+2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x**(1/2))**2/x**(1/2),x)
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Giac [A]
time = 0.40, size = 12, normalized size = 0.75

$$-2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(x) + 2*tan(sqrt(x))
```

Mupad [B]
time = 2.58, size = 20, normalized size = 1.25

$$-2\sqrt{x} + \frac{4\mathrm{i}}{\mathrm{e}^{\sqrt{x}\,2\mathrm{i}} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))^2/x^(1/2),x)
```

```
[Out] 4i/(exp(x^(1/2)*2i) + 1) - 2*x^(1/2)
```

3.60 $\int \sqrt{x} \tan\left(\sqrt{x}\right) dx$

Optimal. Leaf size=70

$$\frac{2}{3}ix^{3/2} - 2x\log\left(1 + e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\mathrm{PolyLog}\left(2, -e^{2i\sqrt{x}}\right) - \mathrm{PolyLog}\left(3, -e^{2i\sqrt{x}}\right)$$

```
[Out] 2/3*I*x^(3/2)-2*x*ln(1+exp(2*I*x^(1/2)))-polylog(3,-exp(2*I*x^(1/2)))+2*I*p
olylog(2,-exp(2*I*x^(1/2)))*x^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3832, 3800, 2221, 2611, 2320, 6724}

$$2i\sqrt{x}\,\mathrm{Li}_2\left(-e^{2i\sqrt{x}}\right) - \mathrm{Li}_3\left(-e^{2i\sqrt{x}}\right) + \frac{2}{3}ix^{3/2} - 2x\log\left(1 + e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x} \tan\left(\sqrt{x}\right) dx &= 2\text{Subst}\left(\int x^2 \tan(x)\,dx, x, \sqrt{x}\right) \\
&= \frac{2}{3} i x^{3/2} - 4i\text{Subst}\left(\int \frac{e^{2ix} x^2}{1+e^{2ix}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2}{3} i x^{3/2} - 2x \log\left(1+e^{2i\sqrt{x}}\right) + 4\text{Subst}\left(\int x \log\left(1+e^{2ix}\right) dx, x, \sqrt{x}\right) \\
&= \frac{2}{3} i x^{3/2} - 2x \log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - 2i\text{Subst}\left(\int \text{Li}_2\left(-e^{2ix}\right) dx, x, \right. \\
&= \frac{2}{3} i x^{3/2} - 2x \log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - \text{Subst}\left(\int \frac{\text{Li}_2(-x)}{x}\,dx, x, e^{2i\sqrt{x}}\right. \\
&= \frac{2}{3} i x^{3/2} - 2x \log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{Li}_2\left(-e^{2i\sqrt{x}}\right) - \text{Li}_3\left(-e^{2i\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 1.00

$$\frac{2}{3} i x^{3/2} - 2x \log\left(1+e^{2i\sqrt{x}}\right) + 2i\sqrt{x}\,\text{PolyLog}\left(2, -e^{2i\sqrt{x}}\right) - \text{PolyLog}\left(3, -e^{2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Tan[Sqrt[x]],x]
```

```
[Out] ((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog[
2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{x}\, \tan\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*tan(x^(1/2)),x)
```

```
[Out] int(x^(1/2)*tan(x^(1/2)),x)
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.14

$$-2i\,x\arctan\left(\sin\left(2\sqrt{x}\right),\cos\left(2\sqrt{x}\right)+1\right)-x\log\left(\cos\left(2\sqrt{x}\right)^{2}+\sin\left(2\sqrt{x}\right)^{2}+2\cos\left(2\sqrt{x}\right)+1\right)+\frac{2}{3}i\,x^{\frac{3}{2}}+2i\sqrt{x}\,\mathrm{Li}_2\left(-e^{\left(2i\sqrt{x}\right)}\right)-\mathrm{Li}_3\left(-e^{\left(2i\sqrt{x}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*I*x*arctan2(sin(2*sqrt(x)), cos(2*sqrt(x)) + 1) - x*log(cos(2*sqrt(x))^2
 + sin(2*sqrt(x))^2 + 2*cos(2*sqrt(x)) + 1) + 2/3*I*x^(3/2) + 2*I*sqrt(x)*d
ilog(-e^(2*I*sqrt(x))) - polylog(3, -e^(2*I*sqrt(x)))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)*tan(sqrt(x)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\tan\left(\sqrt{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*tan(x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)*tan(sqrt(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="giac")
[Out] integrate(sqrt(x)*tan(sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\tan\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*tan(x^(1/2)),x)
[Out] int(x^(1/2)*tan(x^(1/2)), x)
```

3.61 $\int \left(\frac{b\tan(a+bx+cx^2)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx$

Optimal. Leaf size=19

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

```
[Out] -1/2*ln(cos(c*x^2+b*x+a))/c
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {3848}

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

```
[Out] -1/2*Log[Cos[a + b*x + c*x^2]]/c
```

Rule 3848

```
Int[((d_.) + (e_.)*(x_))*Tan[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> Simp[(-e)*(Log[Cos[a + b*x + c*x^2]]/(2*c)), x] + Dist[(2*c*d - b*e)/(2*
c), Int[Tan[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*
c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx &= \frac{b\int \tan\left(a+bx+cx^2\right)\,dx}{2c}+\int x\tan\left(a+bx+cx^2\right)\\ &= -\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 18, normalized size = 0.95

$$-\frac{\log(\cos(a+x(b+cx)))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

```
[Out] -1/2*Log[Cos[a + x*(b + c*x)]]/c
```

Maple [A]
time = 0.22, size = 18, normalized size = 0.95

method	result	size
default	$-\frac{\ln\left(\cos\left(c\,x^{2}+b x+a\right)\right)}{2 c}$	18
norman	$\frac{\ln\left(1+\tan^{2}\left(c\,x^{2}+b x+a\right)\right)}{4 c}$	22
risch	$\frac{i x^{2}}{2}+\frac{i b x}{2 c}+\frac{i a}{c}-\frac{\ln\left({\mathrm e}^{2 i \left(c\,x^{2}+b x+a\right)}+1\right)}{2 c}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(cos(c*x^2+b*x+a))/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(17) = 34.
time = 0.31, size = 83, normalized size = 4.37

$$-\frac{\log\left(\cos\left(2\,cx^{2}\right)^{2}+2\,\cos\left(2\,cx^{2}\right)\cos\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)^{2}+\sin\left(2\,cx^{2}\right)^{2}-2\,\sin\left(2\,cx^{2}\right)\sin\left(2\,bx+2\,a\right)+\sin\left(2\,bx+2\,a\right)^{2}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="maxima"
)
```

```
[Out] -1/4*log(cos(2*c*x^2)^2 + 2*cos(2*c*x^2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a
)^2 + sin(2*c*x^2)^2 - 2*sin(2*c*x^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2
)/c
```

Fricas [A]
time = 3.56, size = 23, normalized size = 1.21

$$-\frac{\log\left(\frac{1}{\tan\left(cx^{2}+bx+a\right)^{2}+1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="fricas"
)
```

```
[Out] -1/4*log(1/(tan(c*x^2 + b*x + a)^2 + 1))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int b\tan\left(a+bx+cx^{2}\right)dx+\int 2cx\tan\left(a+bx+cx^{2}\right)dx}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x**2+b*x+a)/c+x*tan(c*x**2+b*x+a),x)
```

```
[Out] (Integral(b*tan(a + b*x + c*x**2), x) + Integral(2*c*x*tan(a + b*x + c*x**2
), x))/(2*c)
```

Giac [A]
time = 3.56, size = 18, normalized size = 0.95

$$-\frac{\log\left(\left|\cos\left(cx^2+bx+a\right)\right|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(cos(c*x^2 + b*x + a)))/c
```

Mupad [B]
time = 2.51, size = 21, normalized size = 1.11

$$\frac{\ln\left({\mathrm{tan}\left(c\,x^2+b\,x+a\right)}^2+1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(a + b*x + c*x^2) + (b*tan(a + b*x + c*x^2))/(2*c),x)
```

```
[Out] log(tan(a + b*x + c*x^2)^2 + 1)/(4*c)
```

3.62 $\int \frac{\cot^2(\sqrt{x})}{\sqrt{x}} dx$

Optimal. Leaf size=16

$$-2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

```
[Out] -2*cot(x^(1/2))-2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3833, 3554, 8}

$$-2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - 2*Cot[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3833

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}} dx &= 2\text{Subst}\left(\int \cot^2(x)\, dx, x, \sqrt{x}\right)\\
&= -2\cot\left(\sqrt{x}\right) - 2\text{Subst}\left(\int 1\, dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x} - 2\cot\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 26, normalized size = 1.62

$$-2\cot\left(\sqrt{x}\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[Sqrt[x]]^2/Sqrt[x],x]
[Out] -2*Cot[Sqrt[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[Sqrt[x]]^2]
```

Maple [A]
time = 0.03, size = 16, normalized size = 1.00

method	result	size
derivativedivides	$-2\cot\left(\sqrt{x}\right)+\pi-2\,\mathrm{arccot}\left(\cot\left(\sqrt{x}\right)\right)$	16
default	$-2\cot\left(\sqrt{x}\right)+\pi-2\,\mathrm{arccot}\left(\cot\left(\sqrt{x}\right)\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cot(x^(1/2))+Pi-2*arccot(cot(x^(1/2)))
```

Maxima [A]
time = 0.49, size = 14, normalized size = 0.88

$$-2\sqrt{x}-\frac{2}{\tan\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(x) - 2/tan(sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 3.41, size = 28, normalized size = 1.75

$$-\frac{2\left(\sqrt{x}\sin\left(2\sqrt{x}\right)+\cos\left(2\sqrt{x}\right)+1\right)}{\sin\left(2\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(x)*sin(2*sqrt(x)) + cos(2*sqrt(x)) + 1)/sin(2*sqrt(x))
```

Sympy [A]
time = 0.08, size = 15, normalized size = 0.94

$$-2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))**2/x**(1/2),x)
[Out] -2*sqrt(x) - 2*cot(sqrt(x))
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.38

$$-2\sqrt{x} - \frac{1}{\tan\left(\frac{1}{2}\sqrt{x}\right)} + \tan\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] -2*sqrt(x) - 1/tan(1/2*sqrt(x)) + tan(1/2*sqrt(x))
```

Mupad [B]
time = 2.55, size = 20, normalized size = 1.25

$$-2\sqrt{x} - \frac{4\mathrm{i}}{\mathrm{e}^{\sqrt{x}\,2\mathrm{i}} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))^2/x^(1/2),x)
[Out] - 4i/(exp(x^(1/2)*2i) - 1) - 2*x^(1/2)
```

3.63 $\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{E\left(\text{ArcSin}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}$$

```
[Out] EllipticE(tan(d*x+c)/(1+sec(d*x+c)),((a-b)/(a+b))^(1/2))*(1/(1+sec(d*x+c)))
^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((a+b*sec(d*x+c))/(a+b)/(1+sec(d*x+c)))^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2908, 4053}

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}\,E\left(\text{ArcSin}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(a + b*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))])
```

Rule 2908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_)*((a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(m_.), x_Symbol] :> Int[(b + a*Csc[e + f*x])^m*((c + d*Csc[e +
f*x])^n/Csc[e + f*x]^m), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !Integer
Q[n] && IntegerQ[m]
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
```

```
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{1+\sec(c+dx)}\,dx$$
$$= \frac{E\left(\sin^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}$$

Mathematica [A]
time = 1.22, size = 85, normalized size = 0.92

$$\frac{E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x
])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))])
```

Maple [A]
time = 2.79, size = 150, normalized size = 1.63

method	result
default	$-\frac{\text{EllipticE}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c)-1)\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}(1+\cos(dx+c))}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*EllipticE((cos(d*x+c)-1)/sin(d*x+c),((a-b)/(a+b))^(1/2))*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(cos(d*x
```

```
+c)-1)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c)
)/sin(d*x+c)^2*(-a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec{\left(c + dx \right)}}}{\cos{\left(c + dx \right)} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/(1+cos(d*x+c)),x)
[Out] Integral(sqrt(a + b*sec(c + d*x))/(cos(c + d*x) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\cos\left(c+d\,x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1),x)
[Out] int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1), x)
```

3.64 $\int \sec(a+bx)\sec(2a+2bx)\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}(\sin(a+bx))}{b}+\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

[Out] -arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4449, 1107, 213}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]*Sec[2*a + 2*b*x],x]

[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 331, normalized size = 9.46

$$\frac{\frac{(2+2i)\left((-1-i)+\sqrt{2}\right)\text{ArcTan}\left(\frac{\cos\left(\frac{1}{2}(a+bx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{2}(a+bx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)}\right)}{(-1+i)+\sqrt{2}} - 2i\sqrt{2}\,\text{ArcTan}\left(\frac{\cos\left(\frac{1}{2}(a+bx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{2}(a+bx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)}\right) + 4\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right) - 4\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right) + 2\sqrt{2}\log\left(\sqrt{2}+2\sin(a+bx)\right) - \sqrt{2}\log\left(2-\sqrt{2}\cos(a+bx)-\sqrt{2}\sin(a+bx)\right) + \frac{(1-i)\left((-1-i)+\sqrt{2}\right)\log\left(2+\sqrt{2}\cos(a+bx)-\sqrt{2}\sin(a+bx)\right)}{(-1+i)+\sqrt{2}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```

```
[Out] (((2 + 2*I)*((-1 - I) + Sqrt[2])*ArcTan[(Cos[(a + b*x)/2] - (-1 + Sqrt[2])*
Sin[(a + b*x)/2])/((1 + Sqrt[2])*Cos[(a + b*x)/2] - Sin[(a + b*x)/2])])/((-
1 + I) + Sqrt[2]) - (2*I)*Sqrt[2]*ArcTan[(Cos[(a + b*x)/2] - (1 + Sqrt[2])*
Sin[(a + b*x)/2])/((-1 + Sqrt[2])*Cos[(a + b*x)/2] - Sin[(a + b*x)/2])] + 4
*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - 4*Log[Cos[(a + b*x)/2] + Sin[(a
 + b*x)/2]] + 2*Sqrt[2]*Log[Sqrt[2] + 2*Sin[a + b*x]] - Sqrt[2]*Log[2 - Sqr
t[2]*Cos[a + b*x] - Sqrt[2]*Sin[a + b*x]] + ((1 - I)*((-1 - I) + Sqrt[2])*L
og[2 + Sqrt[2]*Cos[a + b*x] - Sqrt[2]*Sin[a + b*x]])/((-1 + I) + Sqrt[2]))/
(4*b)
```

Maple [A]
time = 0.32, size = 43, normalized size = 1.23

method	result	s
default	$\frac{-\frac{\ln(\sin(bx+a)+1)}{2}+\frac{\ln(\sin(bx+a)-1)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\sin(bx+a)\sqrt{2}\right)}{b}$	4
risch	$-\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}+\frac{\sqrt{2}\,\ln\left(e^{2i(bx+a)}+i\sqrt{2}\,e^{i(bx+a)}-1\right)}{2b}-\frac{\sqrt{2}\,\ln\left(e^{2i(bx+a)}-i\sqrt{2}\,e^{i(bx+a)}-1\right)}{2b}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*ln(sin(b*x+a)+1)+1/2*ln(sin(b*x+a)-1)+2^(1/2)*arctanh(sin(b*x+a)*
2^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6257 vs. $2(31) = 62$.
time = 88.54, size = 6257, normalized size = 178.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*co
s(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))
))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt(2)*
cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), c
os(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(
4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*(sqrt(2)*
cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(4*a), cos(4
*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*arctan2(-sqrt(
2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos(b*x)*sin(1/4
*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(2*
b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqrt(2)*cos(b*x)*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(
4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(4*a))) - sin(2*
b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt(2)*cos(3*a)*cos(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a)
, cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))
) + (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*
a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a))))*arctan2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a)
, cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/
2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))
))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a
)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*sin(b*x) - cos(
1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a
))))/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a),
 cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(
sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi +
```

```
1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arct
an2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), c
os(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a),
cos(4*a)))^2), (2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2
(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)
))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + cos(1/4*a
rctan2(sin(4*a), cos(4*a)))^2 - sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)
))^2 - 2*(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arc
tan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^
2)/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(si
n(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a
), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1
/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt
(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a)
)))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.
time = 2.82, size = 72, normalized size = 2.06

$$\frac{\sqrt{2}\log\left(-\frac{2\cos\left(bx+a\right)^{2}-2\sqrt{2}\sin\left(bx+a\right)-3}{2\cos\left(bx+a\right)^{2}-1}\right)-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(a+bx\right)\sec\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.
time = 0.43, size = 66, normalized size = 1.89

$$-\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(bx+a)\right|}{\left|2\sqrt{2}+4\sin(bx+a)\right|}\right)+\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(b*x + a))/abs(2*sqrt(2) + 4*sin(b*
x + a))) + log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.83

$$-\frac{\mathrm{atanh}(\sin\left(a+b\,x\right))-\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
```

```
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

3.65 $\int \sec(a+bx)\sec(2(a+bx))\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}(\sin(a+bx))}{b}+\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

[Out] -arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4449, 1107, 213}

$$\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]*Sec[2*(a + b*x)],x]

[Out] -(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-3x^2+2x^4}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sin(a+bx)\right)}{b} - \frac{2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(a+bx\right.}{b}$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{b} + \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 331, normalized size = 9.46

$$\frac{\frac{(2+2i)\left((-1-i)+\sqrt{2}\right)\text{ArcTan}\left(\frac{\cos\left(\frac{1}{2}(a+bx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{2}(a+bx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)}\right)}{(-1+i)+\sqrt{2}} - 2i\sqrt{2}\,\text{ArcTan}\left(\frac{\cos\left(\frac{1}{2}(a+bx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{2}(a+bx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)}\right) + 4\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right) - 4\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right) + 2\sqrt{2}\log\left(\sqrt{2}+2\sin(a+bx)\right) - \sqrt{2}\log\left(2-\sqrt{2}\cos(a+bx)-\sqrt{2}\sin(a+bx)\right) + \frac{(1-i)\left((-1-i)+\sqrt{2}\right)\log\left(2+\sqrt{2}\cos(a+bx)-\sqrt{2}\sin(a+bx)\right)}{(-1+i)+\sqrt{2}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

```
[Out] (((2 + 2*I)*((-1 - I) + Sqrt[2])*ArcTan[(Cos[(a + b*x)/2] - (-1 + Sqrt[2])*
Sin[(a + b*x)/2])/((1 + Sqrt[2])*Cos[(a + b*x)/2] - Sin[(a + b*x)/2])])/((-
1 + I) + Sqrt[2]) - (2*I)*Sqrt[2]*ArcTan[(Cos[(a + b*x)/2] - (1 + Sqrt[2])*
Sin[(a + b*x)/2])/((-1 + Sqrt[2])*Cos[(a + b*x)/2] - Sin[(a + b*x)/2])] + 4
*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - 4*Log[Cos[(a + b*x)/2] + Sin[(a
 + b*x)/2]] + 2*Sqrt[2]*Log[Sqrt[2] + 2*Sin[a + b*x]] - Sqrt[2]*Log[2 - Sqr
t[2]*Cos[a + b*x] - Sqrt[2]*Sin[a + b*x]] + ((1 - I)*((-1 - I) + Sqrt[2])*L
og[2 + Sqrt[2]*Cos[a + b*x] - Sqrt[2]*Sin[a + b*x]])/((-1 + I) + Sqrt[2]))/
(4*b)
```

Maple [A]
time = 0.00, size = 43, normalized size = 1.23

method	result
default	$\frac{-\frac{\ln(\sin(bx+a)+1)}{2}+\frac{\ln(\sin(bx+a)-1)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\sin(bx+a)\sqrt{2}\right)}{b}$
risch	$-\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}+\frac{\sqrt{2}\,\ln\left(e^{2i(bx+a)}+i\sqrt{2}\,e^{i(bx+a)}-1\right)}{2b}-\frac{\sqrt{2}\,\ln\left(e^{2i(bx+a)}-i\sqrt{2}\,e^{i(bx+a)}-1\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*ln(sin(b*x+a)+1)+1/2*ln(sin(b*x+a)-1)+2^(1/2)*arctanh(sin(b*x+a)*
2^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6257 vs. $2(31) = 62$.
time = 91.14, size = 6257, normalized size = 178.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*co
s(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan
2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))
))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt(2)*
cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), c
os(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x)*sin(1
/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(
4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*(sqrt(2)*
cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(4*a), cos(4
*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*arctan2(-sqrt(
2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos(b*x)*sin(1/4
*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(2*
b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqrt(2)*cos(b*x)*c
os(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(1/4*arctan2(sin(
4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), cos(4*a))) - sin(2*
b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt(2)*cos(3*a)*cos(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1/2*arctan2(sin(4*a)
, cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))
) + (sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(3*a) - sqrt(2)*cos(3*
a)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(a))*sin(1/2*pi + 1/4*
arctan2(sin(4*a), cos(4*a))))*arctan2(-2*((sqrt(2)*cos(1/2*arctan2(sin(4*a)
, cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(1/
2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))
))*cos(b*x) + cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a))) - (sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a
)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1/4*arctan2
(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))))*sin(b*x) - cos(
1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a
))))/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a),
 cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin
(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(
sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(
4*a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4
*a), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi +
```

```
1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sq
rt(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*ar
ctan2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin
(4*a), cos(4*a)))*sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(
4*a))) - sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4
*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arct
an2(sin(4*a), cos(4*a))))*sin(b*x) + 2*sin(1/2*pi + 1/4*arctan2(sin(4*a), c
os(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))) + sin(1/4*arctan2(sin(4*a),
cos(4*a)))^2), (2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2
(sin(4*a), cos(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)
))^2 + sin(1/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 - cos(1/2*pi + 1/
4*arctan2(sin(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*arctan2(sin(4*a), cos
(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*arctan2(sin(
4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a))))*cos(b*x) + cos(1/4*a
rctan2(sin(4*a), cos(4*a)))^2 - sin(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)
))^2 - 2*(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*cos(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arc
tan2(sin(4*a), cos(4*a))))*sin(b*x) + sin(1/4*arctan2(sin(4*a), cos(4*a)))^
2)/(2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1/2*arctan2(sin(4*a), c
os(4*a)))^2)*cos(b*x)^2 + 2*(cos(1/2*arctan2(sin(4*a), cos(4*a)))^2 + sin(1
/2*arctan2(sin(4*a), cos(4*a)))^2)*sin(b*x)^2 + cos(1/2*pi + 1/4*arctan2(si
n(4*a), cos(4*a)))^2 + 2*(sqrt(2)*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*
a)))*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(1/2*arctan2(sin(4*a
), cos(4*a)))*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(1/2*pi + 1
/4*arctan2(sin(4*a), cos(4*a)))*sin(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt
(2)*sin(1/2*arctan2(sin(4*a), cos(4*a)))*sin(1/4*arctan2(sin(4*a), cos(4*a)
)))*cos(b*x) + 2*cos(1/2*pi + 1/4*arctan2(sin(4*a), cos(4*a)))*cos(1/4*arct
an2(sin(4*a), cos(4*a))) + cos(1/4*arctan2(sin(...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.
time = 2.77, size = 72, normalized size = 2.06

$$\frac{\sqrt{2}\,\log\left(-\frac{2\,\cos\left(bx+a\right)^{2}-2\,\sqrt{2}\,\sin\left(bx+a\right)-3}{2\,\cos\left(bx+a\right)^{2}-1}\right)-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b*
x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(a+bx)\sec(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.42, size = 66, normalized size = 1.89

$$-\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(bx+a)\right|}{\left|2\sqrt{2}+4\sin(bx+a)\right|}\right)+\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(b*x + a))/abs(2*sqrt(2) + 4*sin(b*
x + a))) + log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.00, size = 29, normalized size = 0.83

$$-\frac{\mathrm{atanh}(\sin\left(a+b\,x\right))-\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
```

```
[Out] -(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

3.66 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

[Out] 1/2*sin(x)-1/6*sin(3*x)

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4367}

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[2*x],x]

[Out] Sin[x]/2 - Sin[3*x]/6

Maple [A]
time = 0.07, size = 7, normalized size = 0.47

method	result	size
derivativedivides	$\frac{2(\sin^3(x))}{3}$	7
default	$\frac{2(\sin^3(x))}{3}$	7
risch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{2\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{4\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{2\tan(x)}{3}-\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2(x)+1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*sin(x)^3
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.73

$$-\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A]
time = 1.83, size = 10, normalized size = 0.67

$$-\frac{2}{3}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.40

$$\frac{2}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] 2/3*sin(x)^3
```

Mupad [B]
time = 0.03, size = 6, normalized size = 0.40

$$\frac{2\sin(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(x),x)
```

```
[Out] (2*sin(x)^3)/3
```

3.67 $\int \sin(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

[Out] 1/4*sin(2*x)-1/8*sin(4*x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4367}

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Sin[3*x],x]

[Out] Sin[2*x]/4 - Sin[4*x]/8

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(3x)\,dx=\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]*Sin[3*x],x]

[Out] Sin[2*x]/4 - Sin[4*x]/8

Maple [A]
time = 0.12, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
risch	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{4}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{3x}{2}\right)}{4}-\frac{3\tan\left(\frac{x}{2}\right)}{4}+\frac{\tan\left(\frac{3x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(2*x)-1/8*sin(4*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$-\frac{1}{8}\sin\left(4\,x\right)+\frac{1}{4}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A]
time = 1.68, size = 13, normalized size = 0.76

$$-\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -(cos(x)^3 - cos(x))*sin(x)
```

Sympy [A]
time = 0.13, size = 20, normalized size = 1.18

$$-\frac{3\sin\left(x\right)\cos\left(3x\right)}{8}+\frac{\sin\left(3x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x)
```

```
[Out] -3*sin(x)*cos(3*x)/8 + sin(3*x)*cos(x)/8
```

Giac [A]
time = 0.42, size = 13, normalized size = 0.76

$$-\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*sin(4*x) + 1/4*sin(2*x)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin(2x)}{4} - \frac{\sin(4x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(x),x)
```

```
[Out] sin(2*x)/4 - sin(4*x)/8
```

3.68 $\int \sin(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

```
[Out] 1/6*sin(3*x)-1/10*sin(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4367}

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[4*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(4x)\,dx=\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[4*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[5*x]/10
```

Maple [A]
time = 0.12, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{2\tan(2x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{8\left(\tan^2(2x)\right)\tan\left(\frac{x}{2}\right)}{15}+\frac{2\tan(2x)}{15}-\frac{8\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sin(3*x)-1/10*sin(5*x)
```

Maxima [A]
time = 0.32, size = 13, normalized size = 0.76

$$-\frac{1}{10}\sin\left(5x\right)+\frac{1}{6}\sin\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A]
time = 2.19, size = 18, normalized size = 1.06

$$-\frac{4}{15}\left(6\cos\left(x\right)^{4}-7\cos\left(x\right)^{2}+1\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.18

$$-\frac{4\sin\left(x\right)\cos\left(4x\right)}{15}+\frac{\sin\left(4x\right)\cos\left(x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x)
```

```
[Out] -4*sin(x)*cos(4*x)/15 + sin(4*x)*cos(x)/15
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.76

$$-\frac{8}{5}\sin\left(x\right)^{5}+\frac{4}{3}\sin\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(4*x),x, algorithm="giac")
[Out] -8/5*sin(x)^5 + 4/3*sin(x)^3
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$\frac{\sin\left(3\,x\right)}{6}-\frac{\sin\left(5\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)*sin(x),x)
[Out] sin(3*x)/6 - sin(5*x)/10
```

3.69 $\int \sin(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((1+m)x)}{2(1+m)}$$

```
[Out] 1/2*sin((1-m)*x)/(1-m)-1/2*sin((1+m)*x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4665, 2717}

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[m*x],x]
[Out] Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(mx)\,dx&=\int \left(\frac{1}{2}\cos((1-m)x)-\frac{1}{2}\cos((1+m)x)\right)\,dx\\&=\frac{1}{2}\int \cos((1-m)x)\,dx-\frac{1}{2}\int \cos((1+m)x)\,dx\\&=\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.71

$$\frac{-m\cos(mx)\sin(x)+\cos(x)\sin(mx)}{-1+m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[m*x],x]
[Out] (-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A]
time = 0.16, size = 28, normalized size = 0.80

method	result	size
default	$\frac{\sin((-1+m)x)}{-2+2m}-\frac{\sin((1+m)x)}{2(1+m)}$	28
risch	$\frac{\sin((-1+m)x)}{-2+2m}+\frac{\sin((1+m)x)}{2(-1+m)(1+m)}-\frac{\sin((1+m)x)m}{2(-1+m)(1+m)}$	52
norman	$\frac{\frac{2\tan\left(\frac{mx}{2}\right)}{m^2-1}-\frac{2m\tan\left(\frac{x}{2}\right)}{m^2-1}-\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}+\frac{2m\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{mx}{2}\right)\right)}{m^2-1}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{mx}{2}\right)\right)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(m*x),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+m)*sin((-1+m)*x)-1/2*sin((1+m)*x)/(1+m)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.80

$$-\frac{\sin\left((m+1)x\right)}{2\,(m+1)}-\frac{\sin\left(-(m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="maxima")
[Out] -1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Fricas [A]
time = 3.10, size = 26, normalized size = 0.74

$$-\frac{m\cos\left(mx\right)\sin\left(x\right)-\cos\left(x\right)\sin\left(mx\right)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="fricas")
[Out] -(m*cos(m*x)*sin(x) - cos(x)*sin(m*x))/(m^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

time = 0.19, size = 78, normalized size = 2.23

$$\begin{cases} -\frac{x\sin^2(x)}{2} - \frac{x\cos^2(x)}{2} + \frac{\sin(x)\cos(x)}{2} & \text{for } m = -1 \\ \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - \frac{\sin(x)\cos(x)}{2} & \text{for } m = 1 \\ -\frac{m\sin(x)\cos(mx)}{m^2-1} + \frac{\sin(mx)\cos(x)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x)
```

```
[Out] Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (x
*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos(m
*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.83

$$-\frac{\sin(mx+x)}{2(m+1)} + \frac{\sin(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(m*x),x, algorithm="giac")
```

```
[Out] -1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

Mupad [B]
time = 2.32, size = 64, normalized size = 1.83

$$\begin{cases} \frac{x}{2} - \frac{\sin(2x)}{4} & \text{if } m = 1 \\ \frac{\sin(2x)}{4} - \frac{x}{2} & \text{if } m = -1 \\ \frac{\sin(x(m-1))}{2m-2} - \frac{\sin(x(m+1))}{2m+2} & \text{if } m \neq -1 \land m \neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(m*x)*sin(x),x)
```

```
[Out] piecewise(m == 1, x/2 - sin(2*x)/4, m == -1, - x/2 + sin(2*x)/4, m ~= -1 &
m ~= 1, sin(x*(m - 1))/(2*m - 2) - sin(x*(m + 1))/(2*m + 2))
```

3.70 $\int \cos(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] 1/2*cos(x)-1/6*cos(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4369}

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[x],x]
```

```
[Out] Cos[x]/2 - Cos[3*x]/6
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(2x)\sin(x)\,dx=\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[x],x]
```

```
[Out] Cos[x]/2 - Cos[3*x]/6
```

Maple [A]
time = 0.09, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
norman	$\frac{\frac{2\left(\tan^2(x)\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{8\tan(x)\tan\left(\frac{x}{2}\right)}{3}+\frac{2}{3}}{(\tan^2(x)+1)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(x)-1/6*cos(3*x)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos(3x)+\frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

Fricas [A]
time = 2.28, size = 9, normalized size = 0.60

$$-\frac{2}{3}\cos(x)^3+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^3 + cos(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.33

$$\frac{2\sin(x)\sin(2x)}{3}+\frac{\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x)
```

```
[Out] 2*sin(x)*sin(2*x)/3 + cos(x)*cos(2*x)/3
```

Giac [A]
time = 0.40, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*cos(3*x) + 1/2*cos(x)
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.60

$$\cos(x) - \frac{2\cos(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(x),x)
```

```
[Out] cos(x) - (2*cos(x)^3)/3
```

3.71 $\int \cos(3x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

```
[Out] 1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[x],x]
[Out] Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\cos^2(x)}{2} - \frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[x],x]
[Out] Cos[x]^2/2 - Cos[4*x]/8
```

Maple [A]
time = 0.12, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
norman	$\frac{-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}-\frac{\left(\tan^2\left(\frac{3x}{2}\right)\right)}{4}+\frac{3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*cos(2*x)-1/8*cos(4*x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$-\frac{1}{8}\cos(4x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) + 1/4*cos(2*x)
```

Fricas [A]
time = 1.64, size = 13, normalized size = 0.76

$$-\cos(x)^4+\frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 3/2*cos(x)^2
```

Sympy [A]
time = 0.13, size = 20, normalized size = 1.18

$$\frac{3\sin(x)\sin(3x)}{8}+\frac{\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x)
```

```
[Out] 3*sin(x)*sin(3*x)/8 + cos(x)*cos(3*x)/8
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.76

$$-\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -sin(x)^4 + 1/2*sin(x)^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{3\cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(x),x)
```

```
[Out] (3*cos(x)^2)/2 - cos(x)^4
```

3.72 $\int \cos(4x)\sin(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

[Out] 1/6*cos(3*x)-1/10*cos(5*x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4369\}$

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

[In] Int[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(4x)\sin(x)\,dx = \frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[4*x]*Sin[x],x]

[Out] Cos[3*x]/6 - Cos[5*x]/10

Maple [A]
time = 0.12, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
risch	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
norman	$\frac{\frac{2\left(\tan^2(2x)\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{16\tan(2x)\tan\left(\frac{x}{2}\right)}{15}+\frac{2}{15}}{(1+\tan^2(2x))\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*cos(3*x)-1/10*cos(5*x)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos\left(5\,x\right)+\frac{1}{6}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

Fricas [A]
time = 2.30, size = 17, normalized size = 1.00

$$-\frac{8}{5}\cos\left(x\right)^{5}+\frac{8}{3}\cos\left(x\right)^{3}-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.18

$$\frac{4\sin\left(x\right)\sin\left(4x\right)}{15}+\frac{\cos\left(x\right)\cos\left(4x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x)
```

```
[Out] 4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/6*cos(3*x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 1.00

$$-\frac{8\cos(x)^5}{5} + \frac{8\cos(x)^3}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sin(x),x)
```

```
[Out] (8*cos(x)^3)/3 - cos(x) - (8*cos(x)^5)/5
```

3.73 $\int \cos(mx)\sin(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}$$

```
[Out] -1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4670, 2718}

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[m*x]*Sin[x],x]
```

```
[Out] -1/2*Cos[(1 - m)*x]/(1 - m) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \cos(mx)\sin(x)\,dx &= \int \left(\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)\,dx\\
&= \frac{1}{2}\int \sin((1-m)x)\,dx+\frac{1}{2}\int \sin((1+m)x)\,dx\\
&= -\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.69

$$\frac{\cos(x)\cos(mx)+m\sin(x)\sin(mx)}{-1+m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[m*x]*Sin[x],x]
[Out] (Cos[x]*Cos[m*x] + m*Sin[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.80

method	result	size
default	$\frac{\cos((-1+m)x)}{-2+2m}-\frac{\cos((1+m)x)}{2(1+m)}$	28
risch	$\frac{\cos((-1+m)x)}{-2+2m}+\frac{\cos((1+m)x)}{2(1+m)(-1+m)}-\frac{\cos((1+m)x)m}{2(1+m)(-1+m)}$	52
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{m^2-1}-\frac{2\left(\tan^2\left(\frac{mx}{2}\right)\right)}{m^2-1}+\frac{4m\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan^2\left(\frac{mx}{2}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*cos((-1+m)*x)/(-1+m)-1/2*cos((1+m)*x)/(1+m)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.80

$$-\frac{\cos\left((m+1)x\right)}{2\,(m+1)}+\frac{\cos\left(-(m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)
```

Fricas [A]
time = 2.02, size = 24, normalized size = 0.69

$$\frac{m\sin\left(mx\right)\sin\left(x\right)+\cos\left(mx\right)\cos\left(x\right)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="fricas")
[Out] (m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)
```

Sympy [A]
time = 0.20, size = 37, normalized size = 1.06

$$\begin{cases}\frac{\sin^2\left(x\right)}{2} & \text{for } m=-1\vee m=1\\ \frac{m\sin\left(x\right)\sin\left(mx\right)}{m^2-1}+\frac{\cos\left(x\right)\cos\left(mx\right)}{m^2-1} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x)
```

```
[Out] Piecewise((sin(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 - 1
) + cos(x)*cos(m*x)/(m**2 - 1), True))
```

Giac [A]
time = 0.40, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)}+\frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(m*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)
```

Mupad [B]
time = 0.10, size = 55, normalized size = 1.57

$$\begin{cases} \frac{\sin(x)^2}{2} & \text{if } m=-1\vee m=1 \\ \frac{\cos(x\,(m-1))}{2\,m-2}-\frac{\cos(x\,(m+1))}{2\,m+2} & \text{if } m\neq -1\wedge m\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*sin(x),x)
```

```
[Out] piecewise(m == -1 | m == 1, sin(x)^2/2, m ~= -1 & m ~= 1, cos(x*(m - 1))/(2
*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

3.74 $\int \sin(x)\tan(2x)\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\sin(x)$$

```
[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {12, 327, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sin(x)\tan(2x)\,dx = \text{Subst}\left(\int \frac{2x^2}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{x^2}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= -\sin(x) + \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

Maple [A]
time = 0.21, size = 18, normalized size = 0.90

method	result	size
default	$-\sin(x)+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$	18
risch	$\frac{i\mathrm{e}^{ix}}{2}-\frac{i\mathrm{e}^{-ix}}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(17) = 34.
time = 0.50, size = 141, normalized size = 7.05

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)+\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 2.74, size = 38, normalized size = 1.90

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - si
n(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x)
```

```
[Out] Integral(sin(x)*tan(2*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(2*x), x)
```

Mupad [B]
time = 2.39, size = 17, normalized size = 0.85

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{2} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)*sin(x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - sin(x)
```

3.75 $\int \sin(x) \tan(3x) \, dx$

Optimal. Leaf size=47

$$-\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(1+\sin(x))+\frac{1}{6}\log(1+2\sin(x))-\sin(x)$$

```
[Out] -1/6*ln(1-2*sin(x))-1/6*ln(1-sin(x))+1/6*ln(1+sin(x))+1/6*ln(1+2*sin(x))-si
n(x)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1293, 1175, 630, 31}

$$-\sin(x)-\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(\sin(x)+1)+\frac{1}{6}\log(2\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[3*x],x]
```

```
[Out] -1/6*Log[1 - 2*Sin[x]] - Log[1 - Sin[x]]/6 + Log[1 + Sin[x]]/6 + Log[1 + 2*
Sin[x]]/6 - Sin[x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(3x)\,dx &= \text{Subst}\left(\int \frac{x^2(3-4x^2)}{1-5x^2+4x^4}\,dx,x,\sin(x)\right)\\ &= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx,x,\sin(x)\right)\\ &= -\sin(x)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx,x,\sin(x)\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx,x,\right.\\ &= -\sin(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\sin(x)\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx,x,\sin(x)\right)+\\ &= -\frac{1}{6}\log(1-2\sin(x))-\frac{1}{6}\log(1-\sin(x))+\frac{1}{6}\log(1+\sin(x))+\frac{1}{6}\log(1+2\sin(x))-\end{aligned}$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 0.45

$$\frac{1}{3}\tanh^{-1}(\sin(x))+\frac{1}{3}\tanh^{-1}(2\sin(x))-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[3*x],x]
[Out] ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]
```

Maple [A]
time = 0.33, size = 38, normalized size = 0.81

method	result	size
default	$-\frac{\ln(2\sin(x)-1)}{6}+\frac{\ln(1+2\sin(x))}{6}-\frac{\ln(\sin(x)-1)}{6}+\frac{\ln(1+\sin(x))}{6}-\sin(x)$	38
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\ln(e^{ix}-i)}{3}+\frac{\ln(e^{ix}+i)}{3}+\frac{\ln(i e^{ix}+e^{2ix}-1)}{6}-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{6}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(2*sin(x)-1)+1/6*ln(1+2*sin(x))-1/6*ln(sin(x)-1)+1/6*ln(1+sin(x))-si
n(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - co
s(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)
*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6*log
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2*sin(
x) + 1) - sin(x)
```

Fricas [A]
time = 1.20, size = 39, normalized size = 0.83

$$\frac{1}{6}\,\log\left(2\,\sin\left(x\right)+1\right)+\frac{1}{6}\,\log\left(\sin\left(x\right)+1\right)-\frac{1}{6}\,\log\left(-\sin\left(x\right)+1\right)-\frac{1}{6}\,\log\left(-2\,\sin\left(x\right)+1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*lo
g(-2*sin(x) + 1) - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(x\right)\tan\left(3x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x)
```

```
[Out] Integral(sin(x)*tan(3*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(39) = 78.
time = 0.46, size = 364, normalized size = 7.74

$$\frac{\log\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{4}+8\,\tan\left(\frac{1}{2}\,x\right)^{3}+18\,\tan\left(\frac{1}{2}\,x\right)^{2}+8\,\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}-\log\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{4}-8\,\tan\left(\frac{1}{2}\,x\right)^{3}+18\,\tan\left(\frac{1}{2}\,x\right)^{2}-8\,\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+2\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+\log\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{4}+8\,\tan\left(\frac{1}{2}\,x\right)^{3}+18\,\tan\left(\frac{1}{2}\,x\right)^{2}+8\,\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-\log\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{4}-8\,\tan\left(\frac{1}{2}\,x\right)^{3}+18\,\tan\left(\frac{1}{2}\,x\right)^{2}-8\,\tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)+2\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-2\,\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-24\,\tan\left(\frac{1}{2}\,x\right)}{12\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(3*x),x, algorithm="giac")
```

```
[Out] 1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) +
 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4 -
 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan
(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(t
an(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/
(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*
tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) - log
((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(
1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1)) - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1)) - 24*tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 2.34, size = 26, normalized size = 0.55

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}+\frac{\mathrm{atanh}(2\,\sin\left(x\right))}{3}-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)*sin(x),x)
```

```
[Out] (2*atanh(sin(x/2)/cos(x/2)))/3 + atanh(2*sin(x))/3 - sin(x)
```

3.76 $\int \sin(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-\sin(x)$$

```
[Out] -sin(x)+1/4*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arcta
nh(2*sin(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1293, 1180, 213}

$$-\sin(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4 - Sin[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
```

```
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan(4x)\,dx &= \text{Subst}\left(\int \frac{x^2(4-8x^2)}{1-8x^2+8x^4}\,dx,x,\sin(x)\right)\\ &= -\sin(x)-\frac{1}{8}\text{Subst}\left(\int \frac{-8+32x^2}{1-8x^2+8x^4}\,dx,x,\sin(x)\right)\\ &= -\sin(x)-\left(2-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx,x,\sin(x)\right)-\left(2+\sqrt{2}\right)\text{Subst}\\ &= \frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-\sin(x\end{aligned}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.97

$$\frac{1}{4}\left(\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)-4\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[4*x],x]
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2]
]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(51) = 102.
time = 0.48, size = 115, normalized size = 1.62

method	result
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\left(\sum_{\_R=\text{RootOf}\left(128\_Z^4-32\_Z^2+1\right)}\_R\ln\left(e^{2ix}-4i\_R\,e^{ix}-1\right)\right)}{2}$
default	$\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2+\sqrt{2}}\,\sqrt{2}\,\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}-\sin(x)+\frac{\sqrt{2}\,\text{arctan}}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2^(1/2)-2)*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2
))+1/4*(2+2^(1/2))^(1/2)*2^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-sin(x)
+1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + co
s(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)
```

Fricas [A]
time = 2.47, size = 101, normalized size = 1.42

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\sin\left(x\right)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\sin\left(x\right)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\sin\left(x\right)\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\sin\left(x\right)\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
qrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
*sin(x)) - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(x\right)\tan\left(4x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x)
```

```
[Out] Integral(sin(x)*tan(4*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(4*x), x)
```

Mupad [B]
time = 2.56, size = 103, normalized size = 1.45

$$\frac{\operatorname{atanh}\left(\frac{34\sin(x)\sqrt{\sqrt{2}+2}+24\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{41\sqrt{2}+58}\right)\sqrt{\sqrt{2}+2}}{4}-\sin(x)-\frac{\operatorname{atanh}\left(\frac{34\sin(x)\sqrt{2-\sqrt{2}}-24\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{41\sqrt{2}-58}\right)\sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)*sin(x),x)
```

```
[Out] (atanh((34*sin(x)*(2^(1/2) + 2)^(1/2) + 24*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/
2))/(41*2^(1/2) + 58))*(2^(1/2) + 2)^(1/2))/4 - sin(x) - (atanh((34*sin(x)*
(2 - 2^(1/2))^(1/2) - 24*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(41*2^(1/2) -
58))*(2 - 2^(1/2))^(1/2))/4
```

3.77 $\int \sin(x)\tan(5x)\,dx$

Optimal. Leaf size=112

$$\frac{1}{5}\tanh^{-1}(\sin(x))-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)+\frac{1}{20}\left(1\right.$$

```
[Out] 1/5*arctanh(sin(x))-sin(x)-1/20*ln(1-4*sin(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln
(1+4*sin(x)-5^(1/2))*(-5^(1/2)+1)-1/20*ln(1-4*sin(x)+5^(1/2))*(5^(1/2)+1)+1
/20*ln(1+4*sin(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2100, 213, 646, 31}

$$-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\sin(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\sin(x)+\sqrt{5}+1\right)+\frac{1}{5}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[5*x],x]
```

```
[Out] ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5] +
 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
```

```
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \sin(x)\tan(5x)\,dx = \text{Subst}\left(\int \frac{x^2(5-20x^2+16x^4)}{1-13x^2+28x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-1-\frac{1}{5\,(-1+x^2)}-\frac{2(1+x)}{5\,(-1-2x+4x^2)}+\frac{2(-1+x)}{5\,(-1+2x+4x^2)}\right)\,dx, x, \sin($$
$$= -\sin(x)-\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)-\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \sin$$
$$= \frac{1}{5}\tanh^{-1}(\sin(x))-\sin(x)+\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \sin(x)\right)-\frac{1}{5}$$
$$= \frac{1}{5}\tanh^{-1}(\sin(x))-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 248, normalized size = 2.21

$$\frac{1}{20}\left(\text{RootSum}\left[1-\#1^2+\#1^4-\#1^6+\#1^8\&,\frac{6\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-3i\log\left(1-2\cos(x)\#1+\#1^2\right)-2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2+i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2-2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4+i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4+6\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-3i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{-\#1+2\#1^3-3\#1^5+4\#1^7}\&\right]-4\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+5\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[5*x],x]
```

```
[Out] (RootSum[1 - #1^2 + #1^4 - #1^6 + #1^8 & , (6*ArcTan[Sin[x]/(Cos[x] - #1)]
- (3*I)*Log[1 - 2*Cos[x]*#1 + #1^2] - 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 +
 I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 - 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 +
 I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 6*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 -
 (3*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-#1 + 2*#1^3 - 3*#1^5 + 4*#1^7) &
 ] - 4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + 5*Sin[x]))/20
```

Maple [A]
time = 0.49, size = 84, normalized size = 0.75

method	result
default	$-\frac{\ln(\sin(x)-1)}{10}+\frac{\ln(4(\sin^2(x))+2\sin(x)-1)}{20}+\frac{\sqrt{5}\ \text{arctanh}\left(\frac{(8\sin(x)+2)\sqrt{5}}{10}\right)}{10}-\frac{\ln(4(\sin^2(x))-2\sin(x)-1)}{20}+\frac{\sqrt{5}\ \text{arct}}{}$
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\ln(e^{ix}-i)}{5}+\frac{\ln(e^{ix}+i)}{5}+\frac{\ln\left(e^{2ix}-\frac{i\left(\sqrt{5}-1\right)e^{ix}}{2}-1\right)}{20}-\frac{\ln\left(e^{2ix}-\frac{i\left(\sqrt{5}-1\right)e^{ix}}{2}-1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{2ix}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*ln(sin(x)-1)+1/20*ln(4*sin(x)^2+2*sin(x)-1)+1/10*5^(1/2)*arctanh(1/10
*(8*sin(x)+2)*5^(1/2))-1/20*ln(4*sin(x)^2-2*sin(x)-1)+1/10*5^(1/2)*arctanh(
1/10*(8*sin(x)-2)*5^(1/2))+1/10*ln(1+sin(x))-sin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3*
(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*co
s(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(x))
*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) - sin
(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2*x))*
sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - sin(2*x)
)*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*sin(2*x
)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos
(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) -
 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))
*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/10*lo
g(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + sin(x)^2 - 2*si
n(x) + 1) - sin(x)
```

Fricas [A]
time = 1.94, size = 136, normalized size = 1.21

$$\frac{1}{20}\sqrt{5}\log\left(\frac{8\cos(x)^2-4\left(\sqrt{5}-1\right)\sin(x)+\sqrt{5}-11}{4\cos(x)^2+2\sin(x)-3}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{8\cos(x)^2-4\left(\sqrt{5}+1\right)\sin(x)-\sqrt{5}-11}{4\cos(x)^2-2\sin(x)-3}\right)-\frac{1}{20}\log\left(4\cos(x)^2+2\sin(x)-3\right)+\frac{1}{20}\log\left(4\cos(x)^2-2\sin(x)-3\right)+\frac{1}{10}\log\left(\sin(x)+1\right)-\frac{1}{10}\log\left(-\sin(x)+1\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*co
s(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)*s
in(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2 +
2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) + 1)
- 1/10*log(-sin(x) + 1) - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x)
```

```
[Out] Integral(sin(x)*tan(5*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(5*x), x)
```

Mupad [B]
time = 2.89, size = 107, normalized size = 0.96

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{5}+\frac{\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10}-\frac{\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10}-\sin(x)-\frac{\sqrt{5}\,\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(5*x)*sin(x),x)
```

```
[Out] (atan((sin(x)*1042i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10 - at
anh(sin(x) - 5^(1/2)*sin(x))/10 + (2*atanh(sin(x/2)/cos(x/2)))/5 - sin(x) -
 (5^(1/2)*atanh(sin(x) - 5^(1/2)*sin(x)))/10 - (5^(1/2)*atan((sin(x)*1042i
- 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10
```

3.78 $\int \sin(x) \tan(6x)\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)-\sin(x$$

```
[Out] -sin(x)+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]

time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {12, 6874, 2098, 213, 1180}

$$-\sin(x)+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqr
t[3]]])/6 - Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \sin(x)\tan(6x)\,dx &= \text{Subst}\left(\int \frac{2x^2(3-16x^2+16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int \frac{x^2(3-16x^2+16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{1}{2}+\frac{1-12x^2+16x^4}{2\,(1-18x^2+48x^4-32x^6)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= -\sin(x)+\text{Subst}\left(\int \left(-\frac{1}{3\,(-1+2x^2)}-\frac{2(-1+8x^2)}{3\,(1-16x^2+16x^4)}\right)dx, x, \sin(x)\right)\\
&= -\sin(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right)-\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx, x,\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{3\sqrt{2}}-\sin(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx, x\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\,\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\,\tanh^{-1}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.34, size = 366, normalized size = 4.11

$$\frac{1}{24}\left(\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{4\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-2i\log\left(1-2\cos(x)\#1+\#1^2\right)-2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2+i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2-2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4+i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4+4\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-2i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{-\#1^3+2\#1^7}\&\right]-\sqrt{2}\left(2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)+2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)-2\log\left(\sqrt{2}+2\sin(x)\right)+\log\left(2-\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)+\log\left(2+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)+12\sqrt{2}\sin(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[6*x],x]
```

```
[Out] (RootSum[1 - #1^4 + #1^8 & , (4*ArcTan[Sin[x]/(Cos[x] - #1)] - (2*I)*Log[1
- 2*Cos[x]*#1 + #1^2] - 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + I*Log[1 - 2*C
os[x]*#1 + #1^2]*#1^2 - 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 + I*Log[1 - 2*C
os[x]*#1 + #1^2]*#1^4 + 4*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - (2*I)*Log[1 -
 2*Cos[x]*#1 + #1^2]*#1^6)/(-#1^3 + 2*#1^7) & ] - Sqrt[2]*((2*I)*ArcTan[(Co
s[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] + (2
*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt[2])*Cos[x/2] - S
in[x/2])] - 2*Log[Sqrt[2] + 2*Sin[x]] + Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Si
n[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + 12*Sqrt[2]*Sin[x]))/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(79) = 158$.
time = 0.72, size = 256, normalized size = 2.88

method	result
risch	$\frac{ie^{ix}}{2} - \frac{ie^{-ix}}{2} - \frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{12} + \frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{12} - \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1296\_Z^4-144\_Z^2+1\right)} \_R1\right.}{2}$
default	$\frac{\sqrt{3}\,\left(3+2\sqrt{3}\right)\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{6\sqrt{6}+6\sqrt{2}} + \frac{\left(-3+2\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{6\sqrt{6}-6\sqrt{2}} + \frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*(3+2*3^(1/2))/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)
+2*2^(1/2)))+1/3*(-3+2*3^(1/2))*3^(1/2)/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin
(x)/(2*6^(1/2)-2*2^(1/2)))+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)-4/3/(2*6^(1/
2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))-4/3/(2*6^(1/2)-2*2^(1
/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2)))-sin(x)+1/9*(-3+2*3^(1/2))*3^(1
/2)/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)+2*2^(1/2)))+1/9*3^(1/
2)*(3+2*3^(1/2))/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2*6^(1/2)-2*2^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
```

```
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((2*cos(7*x) - c
os(5*x) - cos(3*x) + 2*cos(x))*cos(8*x) - 2*(cos(4*x) - 1)*cos(7*x) + (cos(
4*x) - 1)*cos(5*x) + (cos(3*x) - 2*cos(x))*cos(4*x) + (2*sin(7*x) - sin(5*x
) - sin(3*x) + 2*sin(x))*sin(8*x) + (sin(3*x) - 2*sin(x))*sin(4*x) - 2*sin(
7*x)*sin(4*x) + sin(5*x)*sin(4*x) - cos(3*x) + 2*cos(x))/(2*(cos(4*x) - 1)*
cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin
(4*x)^2 + 2*cos(4*x) - 1), x) - sin(x)
```

Fricas [A]
time = 2.75, size = 134, normalized size = 1.51

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\sin\left(x\right)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\sin\left(x\right)\right)-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}-2\sqrt{2}\sin\left(x\right)-3}{2\cos\left(x\right)^{2}-1}\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos
(x)^2 - 1)) - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(x\right)\tan\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x)
```

```
[Out] Integral(sin(x)*tan(6*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(6*x), x)
```

Mupad [B]
time = 3.11, size = 131, normalized size = 1.47

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{6}-\sin\left(x\right)-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)-\sqrt{6}\,\sin\left(x\right)\right)}{12}-\frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)-\sqrt{6}\,\sin\left(x\right)\right)}{12}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i}-\sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6}-140452}\right)\,1\mathrm{i}}{12}-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i}-\sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6}-140452}\right)\,1\mathrm{i}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(6*x)*sin(x),x)

[Out] (2^(1/2)*atan((2^(1/2)*sin(x)*102818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/2)*6^(1/2) - 140452))*1i)/12 - sin(x) - (6^(1/2)*atan((2^(1/2)*sin(x)*102818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/2)*6^(1/2) - 140452))*1i)/12 + (2^(1/2)*atanh(2^(1/2)*sin(x)))/6 - (2^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin(x)))/12 - (6^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin(x)))/12

3.79 $\int \sin(x)\tan(nx)\,dx$

Optimal. Leaf size=105

$$\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)$$

```
[Out] 1/2*I/exp(I*x)+1/2*I*exp(I*x)-I*hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*I*n*
x))/exp(I*x)-I*exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*I*n*x))
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4653, 2225, 2283}

$$-ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)-ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[n*x],x]
```

```
[Out] (I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*
n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 +
 n^(-1))/2, -E^((2*I)*n*x)]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4653

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin(x)\tan(nx)\,dx &= \int \left(\frac{e^{-ix}}{2} - \frac{e^{ix}}{2} - \frac{e^{-ix}}{1+e^{2inx}} + \frac{e^{ix}}{1+e^{2inx}}\right) dx \\
&= \frac{1}{2}\int e^{-ix}\,dx - \frac{1}{2}\int e^{ix}\,dx - \int \frac{e^{-ix}}{1+e^{2inx}}\,dx + \int \frac{e^{ix}}{1+e^{2inx}}\,dx \\
&= \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - ie^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right) - ie^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 200, normalized size = 1.90

$$
-\frac{ie^{-2ix}\left(e^{i(x+2nx)}(1+2n)\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};-e^{2inx}\right)+(-1+2n)\left(-e^{i(3+2n)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};-e^{2inx}\right)+e^{ix}(1+2n)\left({}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};-e^{2inx}\right)+e^{2ix}\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};-e^{2inx}\right)\right)\right)\right)}{2\left(-1+4n^2\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[n*x],x]
```

```
[Out] ((-1/2*I)*(E^(I*(x + 2*n*x))*(1 + 2*n)*Hypergeometric2F1[1, 1 - 1/(2*n), 2
- 1/(2*n), -E^((2*I)*n*x)] + (-1 + 2*n)*(-(E^(I*(3 + 2*n)*x)*Hypergeometric
2F1[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^((2*I)*n*x)]) + E^(I*x)*(1 + 2*n)*(Hype
rgeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^((2*I)*n*x)] + E^((2*I)*x)*Hyper
geometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)*x)*(-1
+ 4*n^2))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$
\int \sin(x)\tan(nx)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(n*x),x)
```

```
[Out] int(sin(x)*tan(n*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)*tan(n*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="fricas")
```

```
[Out] integral(sin(x)*tan(n*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\tan(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x)
```

```
[Out] Integral(sin(x)*tan(n*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(n*x),x, algorithm="giac")
```

```
[Out] integrate(sin(x)*tan(n*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(n\,x)\,\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(n*x)*sin(x),x)
```

```
[Out] int(tan(n*x)*sin(x), x)
```

3.80 $\int \cot(2x)\sin(x)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\tanh^{-1}(\sin(x))+\sin(x)$$

```
[Out] -1/2*arctanh(sin(x))+sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {396, 212}

$$\sin(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*Sin[x],x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(2x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-2x^2}{2-2x^2}\,dx,x,\sin(x)\right)\\ &= \sin(x)-\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\sin(x))+\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}(\sin(x))+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*Sin[x],x]
```

```
[Out] -1/2*ArcTanh[Sin[x]] + Sin[x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 1.20

method	result	size
default	$\sin(x)-\frac{\ln(\sec(x)+\tan(x))}{2}$	12
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\ln(e^{ix}-i)}{2}-\frac{\ln(e^{ix}+i)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(8) = 16$.
time = 0.47, size = 37, normalized size = 3.70

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1) + sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 1.25, size = 19, normalized size = 1.90

$$-\frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(-\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.46, size = 19, normalized size = 1.90

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 + sin(x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.41, size = 19, normalized size = 1.90

$$-\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

Mupad [**B**]
time = 2.32, size = 10, normalized size = 1.00

$$\sin\left(x\right)-\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*sin(x),x)
```

```
[Out] sin(x) - atanh(tan(x/2))
```

3.81 $\int \cot(3x)\sin(x)\,dx$

Optimal. Leaf size=20

$$-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\sin(x)$$

```
[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {396, 212}

$$\sin(x)-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*Sin[x],x]
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(3x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx, x, \sin(x)\right)\\ &= \sin(x)-2\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\sin(x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*Sin[x],x]
[Out] -(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

Maple [**A**]
time = 0.16, size = 17, normalized size = 0.85

method	result	size
default	$\sin\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+i\sqrt{3}\,e^{ix}-1\right)}{6}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(16) = 32$.
time = 0.49, size = 127, normalized size = 6.35

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="maxima")
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 2.34, size = 36, normalized size = 1.80

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + si
n(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(3*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.42, size = 34, normalized size = 1.70

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + sin
(x)
```

Mupad [B]
time = 2.37, size = 16, normalized size = 0.80

$$\sin(x)-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,\sin(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*sin(x),x)
```

```
[Out] sin(x) - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/3
```

3.82 $\int \cot(4x)\sin(x)\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4}\tanh^{-1}(\sin(x))-\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{2\sqrt{2}}+\sin(x)$$

```
[Out] -1/4*arctanh(sin(x))+sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1690, 1180, 213}

$$\sin(x)-\frac{1}{4}\tanh^{-1}(\sin(x))-\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[4*x]*Sin[x],x]
[Out] -1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\int \cot(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(1-\frac{3-4x^2}{4-12x^2+8x^4}\right)\,dx, x, \sin(x)\right)\\
&= \sin(x) - \text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\
&= \sin(x) + 2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right) + 2\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4}\tanh^{-1}(\sin(x)) - \frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[4*x]*Sin[x],x]
[Out] -1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

Maple [A]
time = 0.18, size = 30, normalized size = 1.07

method	result	size
default	$\sin(x)-\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(1+\sin(x))}{8}$	30
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}-\frac{\ln(e^{ix}+i)}{4}+\frac{\ln(e^{ix}-i)}{4}+\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{8}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(sin(x)-1)-1/8*ln(1+sin(x)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(20) = 40.
time = 0.50, size = 173, normalized size = 6.18

$$-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 +
2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.
time = 1.00, size = 52, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin(x)\cot(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(4*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.
time = 0.42, size = 50, normalized size = 1.79

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/8
*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

Mupad [B]
time = 2.38, size = 29, normalized size = 1.04

$$\sin(x) - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2} - \frac{\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*sin(x),x)
[Out] sin(x) - atanh(sin(x/2)/cos(x/2))/2 - (2^(1/2)*atanh(2^(1/2)*sin(x)))/4
```

3.83 $\int \cot(5x)\sin(x)\,dx$

Optimal. Leaf size=82

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)-$$

```
[Out] sin(x)-1/10*arctanh(1/5*sin(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-
1/10*arctanh(2*sin(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1690, 1180, 213}

$$\sin(x)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[5*x]*Sin[x],x]
```

```
[Out] -1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]]) - (Sqr
t[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cot(5x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1-\frac{4(1-2x^2)}{5-20x^2+16x^4}\right)\,dx, x, \sin(x)\right)$$
$$= \sin(x) - 4\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \sin(x)\right)$$
$$= \sin(x) + \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \sin(x)\right) + \frac{1}{5}\left(4\left(5+\sqrt{\right.\right.$$
$$= -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\right.$$

Mathematica [A]
time = 0.14, size = 76, normalized size = 0.93

$$\frac{1}{10}\left(-\sqrt{10-2\sqrt{5}}\tanh^{-1}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sin(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) + 10\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[5*x]*Sin[x],x]
```

```
[Out] (-(Sqrt[10 - 2*Sqrt[5]]*ArcTanh[Sqrt[2 + 2/Sqrt[5]]*Sin[x]]) - Sqrt[2*(5 +
Sqrt[5])]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]] + 10*Sin[x])/10
```

Maple [A]
time = 0.31, size = 70, normalized size = 0.85

method	result	size
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(125\_Z^4-25\_Z^2+1\right)}\_R\ln\left(e^{2ix}-5i\_R e^{ix}-1\right)\right)}{2}$	55
default	$\sin(x)-\frac{\left(\sqrt{5}-1\right)\sqrt{5}\,\text{arctanh}\left(\frac{4\sin(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\text{arctanh}\left(\frac{4\sin(x)}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)-1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*sin(x)/(10-2*
5^(1/2))^(1/2))-1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*sin(
x)/(10+2*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*c
os(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + c
os(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x))*s
in(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(x))/
(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) +
 cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 +
 cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 + 2*(sin
(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + si
n(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x) + cos(x))
*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (2*cos(x) -
 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + sin(x))*sin(4
*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)
*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x) - 1)*cos(4*x)
 - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2 + 2*(cos(x)
 - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x) + sin(x))*s
in(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*
x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(54) = 108$.
time = 2.12, size = 127, normalized size = 1.55

$$-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\sin(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\sin(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}+4\sin(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}-4\sin(x)\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*sin(x)) +
 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*sin(x)) -
 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*sin(x))
 + 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*sin(x
)) + sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\cot(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(5*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(54) = 108.
time = 0.48, size = 111, normalized size = 1.35

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + sin(x)
)) + 1/20*sqrt(2*sqrt(5) + 10)*log(abs(-1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + s
in(x))) - 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(-1/8*sqrt(5) + 5/8) + sin
(x))) + 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(-1/8*sqrt(5) + 5/8) + sin(
x))) + sin(x)
```

Mupad [B]
time = 2.61, size = 119, normalized size = 1.45

$$\sin(x)-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\,\sin(x)\,\sqrt{\sqrt{5}+5}}{2}+\frac{11\sqrt{2}\,\sqrt{5}\,\sin(x)\,\sqrt{\sqrt{5}+5}}{2}}{20\sqrt{5}+45}\right)\sqrt{\sqrt{5}+5}}{10}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\,\sin(x)\,\sqrt{5-\sqrt{5}}}{2}-\frac{11\sqrt{2}\,\sqrt{5}\,\sin(x)\,\sqrt{5-\sqrt{5}}}{2}}{20\sqrt{5}-45}\right)\sqrt{5-\sqrt{5}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*sin(x),x)
```

```
[Out] sin(x) - (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2 + (11*2^
(1/2)*5^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2)/(20*5^(1/2) + 45))*(5^(1/2) +
5)^(1/2))/10 + (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2 -
(11*2^(1/2)*5^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2)/(20*5^(1/2) - 45))*(5 -
5^(1/2))^(1/2))/10
```

3.84 $\int \cot(6x)\sin(x)\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sin(x)$$

```
[Out] -1/6*arctanh(sin(x))-1/6*arctanh(2*sin(x))+sin(x)-1/6*arctanh(2/3*sin(x)*3^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2098, 213}

$$\sin(x)-\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[6*x]*Sin[x],x]
```

```
[Out] -1/6*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2
*Sqrt[3]) + Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cot(6x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{2(3-19x^2+32x^4-16x^6)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1-18x^2+48x^4-32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(2+\frac{1}{3(-1+x^2)}+\frac{2}{-3+4x^2}+\frac{2}{3(-1+4x^2)}\right) dx, x, \sin(x)\right)$$
$$= \sin(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+$$
$$= -\frac{1}{6}\tanh^{-1}(\sin(x))-\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\sin(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(38) = 76.
time = 0.07, size = 99, normalized size = 2.61

$$\frac{1}{12}\left(-2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\sqrt{3}\tanh^{-1}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\log(1-2\sin(x))-\log(1+2\sin(x))+12\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[6*x]*Sin[x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Tan[x/2]/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x/2]]
 + 2*Log[Cos[x/2] - Sin[x/2]] - 2*Log[Cos[x/2] + Sin[x/2]] + Log[1 - 2*Sin[
x]] - Log[1 + 2*Sin[x]] + 12*Sin[x])/12
```

Maple [A]
time = 0.25, size = 49, normalized size = 1.29

method	result
default	$\sin(x)+\frac{\ln(2\sin(x)-1)}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(1+2\sin(x))}{12}+\frac{\ln(\sin(x)-1)}{12}-\frac{\ln(1+\sin(x))}{12}$
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}-\frac{\ln(e^{ix}+i)}{6}+\frac{\ln(e^{ix}-i)}{6}-\frac{\ln(i e^{ix}+e^{2ix}-1)}{12}+\frac{\ln(-i e^{ix}+e^{2ix}-1)}{12}+\frac{\sqrt{3}\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{12}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)+1/12*ln(2*sin(x)-1)-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)-1/12*ln(
1+2*sin(x))+1/12*ln(sin(x)-1)-1/12*ln(1+sin(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) - integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) + 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.
time = 1.04, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)-\frac{1}{12}\log(2\sin(x)+1)-\frac{1}{12}\log(\sin(x)+1)+\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{12}\log(-2\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) - 1
/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/12
*log(-2*sin(x) + 1) + sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin(x)\cot(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*cot(6*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.
time = 0.40, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)-\frac{1}{12}\log(\sin(x)+1)+\frac{1}{12}\log(-\sin(x)+1)-\frac{1}{12}\log(|2\sin(x)+1|)+\frac{1}{12}\log(|2\sin(x)-1|)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1/
12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1)) +
1/12*log(abs(2*sin(x) - 1)) + sin(x)
```

Mupad [B]
time = 2.50, size = 37, normalized size = 0.97

$$\sin(x) - \frac{\operatorname{atanh}(2\sin(x))}{6} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*sin(x),x)
```

```
[Out] sin(x) - atanh(2*sin(x))/6 - atanh(sin(x/2)/cos(x/2))/3 - (3^(1/2)*atanh((2
*3^(1/2)*sin(x))/3))/6
```

3.85 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4442, 213}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(2x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 174, normalized size = 11.60

$$\frac{2i\text{ArcTan}\left(\frac{\cos(\frac{x}{2})-\left(-1+\sqrt{2}\right)\sin(\frac{x}{2})}{\left(1+\sqrt{2}\right)\cos(\frac{x}{2})-\sin(\frac{x}{2})}\right)-2i\text{ArcTan}\left(\frac{\cos(\frac{x}{2})-\left(1+\sqrt{2}\right)\sin(\frac{x}{2})}{\left(-1+\sqrt{2}\right)\cos(\frac{x}{2})-\sin(\frac{x}{2})}\right)+4\tanh^{-1}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)-\log\left(2-\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)+\log\left(2+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2
]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*S
qrt[2])
```

Maple [A]
time = 0.18, size = 13, normalized size = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(12) = 24.
time = 0.49, size = 129, normalized size = 8.60

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+1\right)-\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x)-2\left(\sqrt{2}\cos(x)-1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 1.96, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x)
[Out] Integral(sin(x)*sec(2*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.
time = 0.42, size = 49, normalized size = 3.27

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}{\left|4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*sin(x),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

Mupad [B]
time = 2.29, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\cos(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

3.86 $\int \sec(3x) \sin(x) \, dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3 - 4\cos^2(x)\right)$$

```
[Out] 1/3*ln(cos(x))-1/6*ln(3-4*cos(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {4442, 272, 36, 29, 31}

$$\frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3 - 4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[3*x]*Sin[x],x]
```

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
```

```
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(3x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{x\left(-3+4x^2\right)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-3+4x)}\,dx, x, \cos^2(x)\right)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-3+4x}\,dx, x, \cos^2(x)\right)\\ &= \frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3-4\cos^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$-\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(-5+8\sin^2(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[3*x]*Sin[x],x]
[Out] -1/3*ArcTanh[(-5 + 8*Sin[x]^2)/3]
```

Maple [A]
time = 0.21, size = 18, normalized size = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln\left(4\left(\cos^2(x)\right)-3\right)}{6}$	18
risch	$\frac{\ln\left(e^{2ix}+1\right)}{3}-\frac{\ln\left(e^{4ix}-e^{2ix}+1\right)}{6}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(cos(x))-1/6*ln(4*cos(x)^2-3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(17) = 34$.
time = 0.48, size = 81, normalized size = 3.86

$$-\frac{1}{12}\log\left(-2\left(\cos\left(2x\right)-1\right)\cos\left(4x\right)+\cos\left(4x\right)^2+\cos\left(2x\right)^2+\sin\left(4x\right)^2-2\sin\left(4x\right)\sin\left(2x\right)+\sin\left(2x\right)^2-2\cos\left(2x\right)+1\right)+\frac{1}{6}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\cos\left(2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 1.20, size = 19, normalized size = 0.90

$$-\frac{1}{6}\log\left(4\cos\left(x\right)^{2}-3\right)+\frac{1}{3}\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(x\right)\sec\left(3x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(3*x), x)
```

Giac [A]
time = 0.41, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\sin\left(x\right)^{2}+1\right)-\frac{1}{6}\log\left(\left|4\sin\left(x\right)^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/6*log(-sin(x)^2 + 1) - 1/6*log(abs(4*sin(x)^2 - 1))
```

Mupad [B]
time = 2.27, size = 15, normalized size = 0.71

$$\frac{\ln\left(\cos\left(x\right)\right)}{3}-\frac{\ln\left(\cos\left(x\right)^{2}-\frac{3}{4}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(3*x),x)
```

```
[Out] log(cos(x))/3 - log(cos(x)^2 - 3/4)/6
```

3.87 $\int \sec(4x)\sin(x)\,dx$

Optimal. Leaf size=71

$$-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] -1/2*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctanh(2*
cos(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4442, 1107, 213}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[4*x]*Sin[x],x]
```

```
[Out] -1/2*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/Sqrt[2*(2 - Sqrt[2])] + ArcTanh[
(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
```

```
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \sec(4x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \cos(x)\right) \\
&= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)\right) + \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8\ldots}\right. \\
&= -\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 50.07, size = 4852, normalized size = 68.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[4*x]*Sin[x],x]
```

```
[Out] ((-2*(-1)^(3/8)*(1 + Sqrt[2])*x - (2*(-1)^(1/4)*(-2 - (1 - I)*(-1)^(5/8) +
(-1)^(5/8)*Sqrt[2])*ArcTan[(-Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(3/8) +
 Cos[x] - Sqrt[2]*Cos[x] + Sin[x])])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) -
(2*(1 - I)^(3/2)*2^(1/4)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 +
2*I)*(-1)^(3/8)*Sqrt[2] + 2*(-1)^(5/8)*Sqrt[2])*ArcTan[((1 + I) + I*Sqrt[2]
 + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])/(Sqrt[1 - I]*2^(3/4))])/((
-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) + 2*(-1)^(3/8)*Log[Sec[x/2]^2] + ((-1)^(3
/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*Log[-(Sec[x/2]^4*(-2 + (
1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Cos[2*x] - 2*
(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x]))])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]
))*((-1/2 - I/2)/(((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1
 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos
[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - Sin[x]/(Sqrt[-1 -
 I]*(1 - I)^(1/4)*(1 + I)^(1/4)*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1
 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*S
qrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - ((I
/2)*Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*Sin[x])/(((-1 + I) + Sqrt[1 -
I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Co
```

s[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt
[1 + I]*Sin[x]))))/(-2*(-1)^(3/8)*(1 + Sqrt[2]) - (2*(-1)^(1/4)*(-2 - (1 -
I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*(((1 + Sqrt[2])*Cos[x] + Sin[x])/(2*(-1
)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x]) - ((Cos[x] - Sin[x] + Sqrt[2]*S
in[x])*(-Cos[x] + (1 + Sqrt[2])*Sin[x]))/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*C
os[x] + Sin[x])^2))/(((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*(1 + (-Cos[x] + (1
 + Sqrt[2])*Sin[x])^2/(2*(-1)^(3/8) + Cos[x] - Sqrt[2]*Cos[x] + Sin[x])^2))
 + 2*(-1)^(3/8)*Tan[x/2] - ((-1)^(3/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8
)*Sqrt[2])*Cos[x/2]^4*(-(Sec[x/2]^4*(-2*(-1)^(3/8)*Cos[x] + 2*Sqrt[2]*Cos[2
x] - 2(-1)^(3/8)*(-1 + Sqrt[2])*Sin[x] - 2*Sqrt[2]*Sin[2*x])) - 2*Sec[x/2
]^4*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Co
s[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])*Tan[x/2]))/(((-1 + I) + 2*
(-1)^(3/8) + Sqrt[2])*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*C
os[x] + Sqrt[2]*Cos[2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x])) - ((1 -
 I)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (2 + 2*I)*(-1)^(3/8)*Sqrt[
2] + 2*(-1)^(5/8)*Sqrt[2])*Sec[x/2]^2)/(Sqrt[2]*(1 + ((1/4 + I/4)*((1 + I)
+ I*Sqrt[2] + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])^2)/Sqrt[2]))) +
 ((-4*Sqrt[-1 - I]*(-1 + Sqrt[2])*ArcTanh[((-I)*((1 + I) + Sqrt[2]) + ((1 +
 I) + 2*(-1)^(5/8) - Sqrt[2])*Tan[x/2])/(Sqrt[-1 - I]*2^(3/4))] + (-1)^(1/8
)*2^(1/4)*(2*ArcTan[(Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(5/8) + (-1 + S
qrt[2])*Cos[x] + Sin[x])] - I*(2*(1 + Sqrt[2])*x + 2*Log[Sec[x/2]^2] - Log[
Sec[x/2]^4*(2 - (1 + I)*Sqrt[2] + 2*(-1)^(5/8)*(-1 + Sqrt[2])*Cos[x] - Sqrt
[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x] + Sqrt[2]*Sin[2*x])])))*((-1 - I) + Sqrt
[2] - (2*2^(1/4)*Sin[x])/Sqrt[-1 + I]))/(2^(1/4)*((-1 - I) + Sqrt[2])*(2*Sq
rt[-1 + I]*2^(1/4)*((-1 - I) + Sqrt[2]) - 4*(-1 + Sqrt[2])*Cos[x] - 4*Sin[x
])*((2*(-1)^(1/8)*(-2 - (1 + I)*Sqrt[2] + (-1)^(1/8)*((1 + I) + I*Sqrt[2])*
Cos[x] + (2*I)*(1 + Sqrt[2])*Cos[2*x] + (-1)^(1/8)*Sin[x] - (-1)^(5/8)*Sin[
x] + 3*(-1)^(1/8)*Sqrt[2]*Sin[x] - (2*I)*Sin[2*x]))/(2 - (1 + I)*Sqrt[2] +
2*(-1)^(5/8)*(-1 + Sqrt[2])*Cos[x] - Sqrt[2]*Cos[2*x] + 2*(-1)^(5/8)*Sin[x]
 + Sqrt[2]*Sin[2*x]) - (((1 + I) + 2*(-1)^(5/8) - Sqrt[2])*(-1 + Sqrt[2])*S
ec[x/2]^2)/(1 + ((1/4 - I/4)*(I*((1 + I) + Sqrt[2]) + ((-1 - I) - 2*(-1)^(5
/8) + Sqrt[2])*Tan[x/2])^2)/Sqrt[2]))) + ((-2*(-1)^(3/8)*Sqrt[2]*(1 + (-1)^
(1/4))*x + (2*(-2*I + 2*(-1)^(3/4) + 2*(-1)^(1/8)*Sqrt[2] - (-1)^(3/8)*Sqrt
[2] + (-1)^(7/8)*Sqrt[2])*ArcTan[Cos[x]/(-((-1)^(1/8)*Sqrt[2]) + (-1)^(3/4)
*Cos[x] + (1 + (-1)^(1/4))*Sin[x])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2])
 - ((4 + 4*I)*(-1)^(5/8)*((3 - 3*I) - (2 - 2*I)*Sqrt[2] + (-1)^(1/8)*Sqrt[2
] - (-1)^(3/8)*Sqrt[2] + (1 - I)*(-1)^(5/8)*Sqrt[2] + (1 + I)*(-1)^(7/8)*Sq
rt[2])*ArcTanh[(1/2 + I/2)*(-1)^(5/8)*(-1 - (-1)^(1/4) + (-I + (-1)^(3/4) +
 (-1)^(1/8)*Sqrt[2])*Tan[x/2])])/(-I + (-1)^(3/4) + (-1)^(1/8)*Sqrt[2]) - 2
*(-1)^(7/8)*Sqrt[2]*(-1 + (-1)^(1/4))*Log[Sec[x/2]^2] - ((-1 + (-1)^(1/4))*
(2 - (-1)^(3/8)*Sqrt[2] + (-1)^(5/8)*Sqrt[2])*Log[(1/4 + I/4)*Sec[x/2]^4*((
2 - 2*I) + 6*Sqrt[2] - (4 - 4*I)*(-1)^(7/8)*Sqrt[2]*Cos[x] - 2*((1 + I) + S
qrt[2])*Cos[2*x] - (4 - 4*I)*(-1)^(1/8)*Sqrt[2]*Sin[x] - (4 - 4*I)*(-1)^(3/
8)*Sqrt[2]*Sin[x] - (2 - 2*I)*Sin[2*x] + (2*I)*Sqrt[2]*Sin[2*x])])/(-I + (-
1)^(3/4) + (-1)^(1/8)*Sqrt[2]))*(I/(Sqrt[1 - I]*((-1 + I) + Sqrt[1 - I]*Sqr

```
t[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/4)*(1 + I)^(1/4) + Sqrt[1 - I]*Cos[x]
- Sqrt[1 + I]*Cos[x] + I*Sqrt[1 - I]*Sin[x] + I*Sqrt[1 + I]*Sin[x])) + 1/(S
qrt[1 + I]*((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])^2*(Sqrt[-1 - I]*(1 - I)^(3/
4)*(1 + I)^(1/4) + Sqrt[1 - I]*Cos[x] - Sqrt[1 ...
```

Maple [A]
time = 0.28, size = 54, normalized size = 0.76

method	result	size
risch	$-i\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^4+128\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-512i\_R^3-24i\_R\right)\mathrm{e}^{ix}+1\right)\right)$	47
default	$-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))+1/4*2^(1
/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(sec(4*x)*sin(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(49) = 98$.
time = 2.42, size = 121, normalized size = 1.70

$$-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos(x)\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\cos(x)\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*cos(x)) + 1/
8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*cos(x)) + 1/8*s
qrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(4*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(49) = 98.
time = 0.41, size = 133, normalized size = 1.87

$$-\frac{2.16139547686000\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0395661298966000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+28.1312524456150}-\frac{4.18450863968000\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.446462692172000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+44.3876588494000}-\frac{20.9929814212000\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-2.23982880884000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+404.466590643000}-\frac{1380.66111446200\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-25.2741423691000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}-10892.9855019000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -2.16139547686000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.0395661298966000)/(140
*(cos(x) - 1)/(cos(x) + 1) + 28.1312524456150) - 4.18450863968000*log(-(cos
(x) - 1)/(cos(x) + 1) - 0.446462692172000)/(140*(cos(x) - 1)/(cos(x) + 1) +
 44.3876588494000) - 20.9929814212000*log(-(cos(x) - 1)/(cos(x) + 1) - 2.23
982880884000)/(140*(cos(x) - 1)/(cos(x) + 1) + 404.466590643000) - 1380.661
11446200*log(-(cos(x) - 1)/(cos(x) + 1) - 25.2741423691000)/(140*(cos(x) -
1)/(cos(x) + 1) - 10892.9855019000)
```

Mupad [B]
time = 0.09, size = 112, normalized size = 1.58

$$\frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}-\frac{\sqrt{2}\,\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}\right)\sqrt{2-\sqrt{2}}}{4}-\frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}+\frac{\sqrt{2}\,\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}\right)\sqrt{\sqrt{2}+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(4*x),x)
```

```
[Out] (atanh((cos(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)) - (2^(1/2)*co
s(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)))*(2 - 2^(1/2))^(1/2))/4
 - (atanh((cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)) + (2^(1/2)
*cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)))*(2^(1/2) + 2)^(1/2)
)/4
```

3.88 $\int \sec(5x)\sin(x)\,dx$

Optimal. Leaf size=62

$$-\frac{1}{5}\log(\cos(x))+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cos^2(x)\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cos^2(x)\right)$$

```
[Out] -1/5*ln(cos(x))+1/20*ln(5-8*cos(x)^2+5^(1/2))*(-5^(1/2)+1)+1/20*ln(5-8*cos(
x)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4442, 1128, 719, 29, 646, 31}

$$\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\cos^2(x)-\sqrt{5}+5\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\cos^2(x)+\sqrt{5}+5\right)-\frac{1}{5}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[5*x]*Sin[x],x]
```

```
[Out] -1/5*Log[Cos[x]] + ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Cos[x]^2])/20 + ((1 -
 Sqrt[5])*Log[5 + Sqrt[5] - 8*Cos[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(5x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{x\,(5-20x^2+16x^4)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\,(5-20x+16x^2)}\,dx, x, \cos^2(x)\right)\right)\\ &= -\left(\frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos^2(x)\right)\right) - \frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \cos^2(x)\right)\\ &= -\frac{1}{5}\log(\cos(x)) + \frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \cos^2(x)\right) + \frac{1}{5}\\ &= -\frac{1}{5}\log(\cos(x)) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cos^2(x)\right) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(-4\log(\cos(x)) + \left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(2x)\right) - \left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(2x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[5*x]*Sin[x],x]
```

```
[Out] (-4*Log[Cos[x]] + (1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[2*x]] - (-1 + Sqrt[
5])*Log[1 + Sqrt[5] - 4*Cos[2*x]])/20
```

Maple [A]
time = 0.31, size = 43, normalized size = 0.69

method	result
default	$-\frac{\ln(\cos(x))}{5}+\frac{\ln(16(\cos^4(x))-20(\cos^2(x))+5)}{20}+\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{\left(32\left(\cos^2(x)\right)-20\right)\sqrt{5}}{20}\right)}{10}$
risch	$-\frac{\ln(e^{2ix}+1)}{5}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)}{20}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{4ix}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(cos(x))+1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/2
0*(32*cos(x)^2-20)*5^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(sin
(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x)
- cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arctan2(
sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^2 +
cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x), cos(4
*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*sin(4*x)
 + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x), cos(4*x)
)))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*x), cos(4*
x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x))) + sin
(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5*integrate(-
(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2
*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(cos(6*x) - cos(
4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*co
s(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 +
 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - s
in(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x
)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arctan2(sin(6*x), cos(6
*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(1/3*ar
ctan2(sin(6*x), cos(6*x)))*sin(6*x) - cos(6*x)*sin(4/3*arctan2(sin(6*x), co
s(6*x))) - cos(6*x)*sin(2/3*arctan2(sin(6*x), cos(6*x))) + cos(6*x)*sin(1/3
```

```
*arctan2(sin(6*x), cos(6*x))) + sin(6*x))/(cos(6*x)^2 - 2*(cos(6*x) - cos(2
/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)
*cos(4/3*arctan2(sin(6*x), cos(6*x))) + cos(4/3*arctan2(sin(6*x), cos(6*x))
)^2 - 2*(cos(6*x) + cos(1/3*arctan2(sin(6*x), cos(6*x))) - 1)*cos(2/3*arcta
n2(sin(6*x), cos(6*x))) + cos(2/3*arctan2(sin(6*x), cos(6*x)))^2 + 2*(cos(6
*x) - 1)*cos(1/3*arctan2(sin(6*x), cos(6*x))) + cos(1/3*arctan2(sin(6*x), c
os(6*x)))^2 + sin(6*x)^2 - 2*(sin(6*x) - sin(2/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x))))*sin(4/3*arctan2(sin(6*x), cos(6*
x))) + sin(4/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*(sin(6*x) + sin(1/3*arcta
n2(sin(6*x), cos(6*x))))*sin(2/3*arctan2(sin(6*x), cos(6*x))) + sin(2/3*arc
tan2(sin(6*x), cos(6*x)))^2 + 2*sin(6*x)*sin(1/3*arctan2(sin(6*x), cos(6*x)
)) + sin(1/3*arctan2(sin(6*x), cos(6*x)))^2 - 2*cos(6*x) + 1), x) - 2/5*int
egrate(-(sin(8*x) - sin(6*x) + sin(4*x) - sin(2*x))/(2*(cos(6*x) - cos(4*x)
 + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*
x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(
sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2
*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2
+ 2*cos(2*x) - 1), x) - 1/10*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 1.09, size = 72, normalized size = 1.16

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{32\,\cos\left(x\right)^{4}+8\,\left(\sqrt{5}-5\right)\cos\left(x\right)^{2}-5\,\sqrt{5}+15}{16\,\cos\left(x\right)^{4}-20\,\cos\left(x\right)^{2}+5}\right)+\frac{1}{20}\,\log\left(16\,\cos\left(x\right)^{4}-20\,\cos\left(x\right)^{2}+5\right)-\frac{1}{5}\,\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="fricas")
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)/
(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5)
- 1/5*log(-cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin\left(x\right)\sec\left(5x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x)
[Out] Integral(sin(x)*sec(5*x), x)
```

Giac [A]
time = 0.41, size = 67, normalized size = 1.08

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{\left|32\,\sin\left(x\right)^{2}-4\,\sqrt{5}-12\right|}{\left|32\,\sin\left(x\right)^{2}+4\,\sqrt{5}-12\right|}\right)-\frac{1}{10}\,\log\left(-\sin\left(x\right)^{2}+1\right)+\frac{1}{20}\,\log\left(\left|16\,\sin\left(x\right)^{4}-12\,\sin\left(x\right)^{2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(32*sin(x)^2 - 4*sqrt(5) - 12)/abs(32*sin(x)^2 + 4*sqrt
(5) - 12)) - 1/10*log(-sin(x)^2 + 1) + 1/20*log(abs(16*sin(x)^4 - 12*sin(x)
^2 + 1))
```

Mupad [B]
time = 0.54, size = 47, normalized size = 0.76

$$\ln\left(\cos(x)^2+\frac{\sqrt{5}}{8}-\frac{5}{8}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)-\ln\left(\cos(x)^2-\frac{\sqrt{5}}{8}-\frac{5}{8}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\frac{\ln(\cos(x))}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(5*x),x)
```

```
[Out] log(cos(x)^2 + 5^(1/2)/8 - 5/8)*(5^(1/2)/20 + 1/20) - log(cos(x)^2 - 5^(1/2
)/8 - 5/8)*(5^(1/2)/20 - 1/20) - log(cos(x))/5
```

3.89 $\int \sec(6x)\sin(x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4442, 2082, 213, 1180}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[6*x]*Sin[x],x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \sec(6x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+18x^2-48x^4+32x^6}\,dx, x, \cos(x)\right) \\
&= -\text{Subst}\left(\int \left(-\frac{1}{3\left(-1+2x^2\right)}+\frac{4\left(-1+2x^2\right)}{3\left(1-16x^2+16x^4\right)}\right)\,dx, x, \cos(x)\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \cos(x)\right) \\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \cos(x)\right)-\frac{4}{3}\text{Subst}\Bigg(\int \\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.26, size = 627, normalized size = 7.38

$$\frac{1}{24}\left((-4-4i)(-1)^{3/4}\tanh^{-1}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-(4-4i)\sqrt[4]{-1}\tanh^{-1}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2\left(1+\sqrt{2}\right)\left(x+2\sqrt{3}\tanh^{-1}\left(\frac{2+\left(2+\sqrt{2}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{6}}\right)-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{2}-2\cos(x)+2\sin(x)\right)\right)\right)}{2+\sqrt{2}}-\sqrt{2}\left(x-2\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{2}+\left(-1+\sqrt{2}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(\sec^2\left(\frac{x}{2}\right)\left(1+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)\right)\right)+\frac{2\left(2\left(\sqrt{2}+\sqrt{3}\right)\tanh^{-1}\left(\frac{2+\left(2+\sqrt{6}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\left(3+\sqrt{6}\right)\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{6}-2\cos(x)+2\sin(x)\right)\right)\right)\right)\left(1+\sqrt{6}\sin(x)\right)\left(3+\sqrt{6}-\left(2+\sqrt{6}\right)\cos(x)+\left(2+\sqrt{6}\right)\sin(x)\right)}{\left(12+5\sqrt{6}\right)\cos(2x)+2\cos(x)\left(5+2\sqrt{6}+5\sqrt{6}\sin(x)\right)-2\left(12+5\sqrt{6}+4\left(5+2\sqrt{6}\right)\sin(x)-6\sin(2x)\right)}+\frac{\left(-2\left(-2+\sqrt{6}\right)\tanh^{-1}\left(\sqrt{2}+\left(\sqrt{2}-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)+\left(3\sqrt{2}-2\sqrt{3}\right)\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{3}+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)\right)\right)\right)\left(\sqrt{2}-2\sqrt{3}\sin(x)\right)\left(-3+\sqrt{6}-\left(-2+\sqrt{6}\right)\cos(x)+\left(-2+\sqrt{6}\right)\sin(x)\right)}{\left(-12+5\sqrt{6}\right)\cos(2x)+2\cos(x)\left(-5+2\sqrt{6}+5\sqrt{6}\sin(x)\right)-2\left(-12+5\sqrt{6}+4\left(-5+2\sqrt{6}\right)\sin(x)+6\sin(2x)\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[6*x]*Sin[x],x]

[Out] ((-4 - 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (4 - 4*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + (2*(1 + Sqrt[2])*(x + 2*Sqrt[3]*ArcTa
nh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x/2]
^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) - Sqrt[2]*(x - 2*Sqrt[
3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]^2] +
 Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) + (2*(2*(Sqrt[2] +
```

```
Sqrt[3])*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x -
 Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(1
+ Sqrt[6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x
]))/((12 + 5*Sqrt[6])*Cos[2*x] + 2*Cos[x]*(5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]
) - 2*(12 + 5*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Sin[x] - 6*Sin[2*x])) + ((-2*(-2
+ Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2] - 2
*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Cos[x]
 - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - 2*Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 + S
qrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/((-12 + 5*Sqrt[6])*Cos[2*x] + 2*Co
s[x]*(-5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) - 2*(-12 + 5*Sqrt[6] + 4*(-5 + 2*S
qrt[6])*Sin[x] + 6*Sin[2*x])))/24
```

Maple [A]
time = 0.35, size = 80, normalized size = 0.94

method	result
default	$-\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{6}+\frac{2\operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\operatorname{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}$
risch	$-i\left(\sum_{\_R=\operatorname{RootOf}\left(20736\_Z^4+576\_Z^2+1\right)}\_R\ln\left(e^{2ix}+\left(-1728i\_R^3-48i\_R\right)e^{ix}+1\right)\right)+\frac{\sqrt{2}\ \ln\left(e^{2ix}-\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*co
s(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2*6
^(1/2)+2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) - integrate(1/3*((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*
x) - (sin(3*x) - sin(x))*cos(4*x) - (cos(7*x) - cos(5*x) + cos(3*x) - cos(x
))*sin(8*x) - (cos(4*x) - 1)*sin(7*x) + (cos(4*x) - 1)*sin(5*x) + (cos(3*x)
```

```
 - cos(x))*sin(4*x) + cos(7*x)*sin(4*x) - cos(5*x)*sin(4*x) + sin(3*x) - si
n(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2
*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(67) = 134$.
time = 2.09, size = 153, normalized size = 1.80

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\cos\left(x\right)\right)-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{2}\log\left(\frac{2\cos\left(x\right)^{2}-2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*cos(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*cos(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*cos(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*cos(x)) + 1/1
2*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(x\right)\sec\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*sec(6*x), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(67) = 134$.
time = 0.45, size = 182, normalized size = 2.14

$$-\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)-\frac{2.39014968180000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0173323801210000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+60.0540532247402}+\frac{5.82951931426000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.588790706481000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+121.584934401100}+\frac{16.8155413244667\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-1.69839637242000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+559.622604171000}-\frac{7956.25491093333\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-57.6954805410000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}-168981.261592000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*s
qrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 2.39014968180000*log(-(cos(x)
- 1)/(cos(x) + 1) - 0.0173323801210000)/(268*(cos(x) - 1)/(cos(x) + 1) + 60
.0540532247402) + 5.82951931426000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.58879
0706481000)/(268*(cos(x) - 1)/(cos(x) + 1) + 121.584934401100) + 16.8155413
244667*log(-(cos(x) - 1)/(cos(x) + 1) - 1.69839637242000)/(268*(cos(x) - 1)
/(cos(x) + 1) + 559.622604171000) - 7956.25491093333*log(-(cos(x) - 1)/(cos
```

```
(x) + 1) - 57.6954805410000)/(268*(cos(x) - 1)/(cos(x) + 1) - 168981.261592
000)
```

Mupad [B]
time = 2.28, size = 118, normalized size = 1.39

$$\operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}+\frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right)-\operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}-\frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right)-\frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\cos(x)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(6*x),x)
```

```
[Out] atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
+ (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(2^
(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)) - (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4
194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/2)*co
s(x)))/6
```

3.90 $\int \csc(2x)\sin(x)\,dx$

Optimal. Leaf size=7

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] 1/2*arctanh(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4373, 3855}

$$\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sin[x]]/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \csc(2x)\sin(x)\,dx &= \frac{1}{2}\int \sec(x)\,dx \\ &= \frac{1}{2}\tanh^{-1}(\sin(x)) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(7) = 14.
time = 0.01, size = 37, normalized size = 5.29

$$\frac{1}{2}\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Sin[x],x]
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/2
```

Maple [A]
time = 0.09, size = 9, normalized size = 1.29

method	result	size
default	$\frac{\ln(\sec(x)+\tan(x))}{2}$	9
risch	$-\frac{\ln(e^{ix}-i)}{2}+\frac{\ln(e^{ix}+i)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sec(x)+tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(5) = 10.
time = 0.48, size = 35, normalized size = 5.00

$$\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="maxima")
[Out] 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2 -
 2*sin(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.
time = 1.62, size = 17, normalized size = 2.43

$$\frac{1}{4}\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="fricas")
[Out] 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.
time = 0.62, size = 15, normalized size = 2.14

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{4}+\frac{\log\left(\sin\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.
time = 0.41, size = 17, normalized size = 2.43

$$\frac{1}{4}\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Mupad [B]
time = 0.11, size = 5, normalized size = 0.71

$$\frac{\mathrm{atanh}(\sin\left(x\right))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(2*x),x)
```

```
[Out] atanh(sin(x))/2
```

3.91 $\int \csc(3x)\sin(x)\,dx$

Optimal. Leaf size=45

$$-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(-sin(x)+cos(x)*3^(1/2))*3^(1/2)+1/6*ln(sin(x)+cos(x)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {212}

$$\frac{\log\left(\sin(x)+\sqrt{3}\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[3*x]*Sin[x],x]
```

```
[Out] -1/2*Log[Sqrt[3]*Cos[x] - Sin[x]]/Sqrt[3] + Log[Sqrt[3]*Cos[x] + Sin[x]]/(2
*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(3x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.33

$$\frac{\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[x],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

Maple [A]
time = 0.22, size = 14, normalized size = 0.31

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{3}$	14
risch	$\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(33) = 66.
time = 0.51, size = 125, normalized size = 2.78

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3)
```

Fricas [A]
time = 1.37, size = 58, normalized size = 1.29

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4-16\cos(x)^2-4\left(2\sqrt{3}\cos(x)^3+\sqrt{3}\cos(x)\right)\sin(x)-1}{16\cos(x)^4-8\cos(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(
3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))
```

Sympy [A]
time = 0.94, size = 76, normalized size = 1.69

$$\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x)
```

```
[Out] sqrt(3)*log(tan(x/2) - sqrt(3))/6 - sqrt(3)*log(tan(x/2) - sqrt(3)/3)/6 + s
qrt(3)*log(tan(x/2) + sqrt(3)/3)/6 - sqrt(3)*log(tan(x/2) + sqrt(3))/6
```

Giac [A]
time = 0.41, size = 31, normalized size = 0.69

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(x\right)\right|}{\left|2\sqrt{3}+2\tan\left(x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x)))
```

Mupad [B]
time = 2.79, size = 17, normalized size = 0.38

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}\sin(x)}{3\cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(3*x),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*sin(x))/(3*cos(x))))/3
```

3.92 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1107, 213}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx,x,\sin(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sin(x)\right)-2\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 218, normalized size = 8.38

$$\frac{-2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)-2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)+2\sqrt{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\sqrt{2}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+2\log\left(\sqrt{2}+2\sin(x)\right)-\log\left(2-\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)-\log\left(2+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x],x]
```

```
[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
 - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqr
t[2])*Cos[x/2] - Sin[x/2])] + 2*Sqrt[2]*Log[Cos[x/2] - Sin[x/2]] - 2*Sqrt[2
]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Co
s[x] - Sqrt[2]*Sin[x]] - Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(8*Sqrt[
2])
```

Maple [A]
time = 0.25, size = 28, normalized size = 1.08

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}+\frac{\ln(\sin(x)-1)}{8}-\frac{\ln(1+\sin(x))}{8}$	28
risch	$-\frac{\ln(e^{ix}+i)}{4}+\frac{\ln(e^{ix}-i)}{4}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{8}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/8*ln(sin(x)-1)-1/8*ln(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(18) = 36.
time = 0.50, size = 171, normalized size = 6.58

$$\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.
time = 2.60, size = 50, normalized size = 1.92

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.
time = 3.61, size = 294, normalized size = 11.31

$$\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} - \frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x)
```

```
[Out] 27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt(2
) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808) + 2
7720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*
log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1 +
 sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2)
- 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/(110880*
sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(
2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.41, size = 48, normalized size = 1.85

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Mupad [B]
time = 2.43, size = 27, normalized size = 1.04

$$\frac{\sqrt{2}\ \operatorname{atanh}\left(\sqrt{2}\ \sin\left(x\right)\right)}{4}-\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(4*x),x)
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

3.93 $\int \csc(5x)\sin(x)\,dx$

Optimal. Leaf size=165

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\text{si}\right.$$

```
[Out] -1/20*ln(-sin(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(s
in(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(-sin(x)+cos(
x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)-1/20*ln(sin(x)+cos(x)*(5+2*5^(
1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1180, 213}

$$-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sin(x)+\sqrt{5-2\sqrt{5}}\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sin(x)+\sqrt{5+2\sqrt{5}}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[5*x]*Sin[x],x]
```

```
[Out] -1/10*(Sqrt[(5 - Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] - Sin[x]]) + (S
qrt[(5 + Sqrt[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] - Sin[x]])/10 + (Sqrt[(
5 - Sqrt[5])/2]*Log[Sqrt[5 - 2*Sqrt[5]]*Cos[x] + Sin[x]])/10 - (Sqrt[(5 + S
qrt[5])/2]*Log[Sqrt[5 + 2*Sqrt[5]]*Cos[x] + Sin[x]])/10
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \csc(5x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{5-10x^2+x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{10}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+x^2}\,dx, x, \tan(x)\right) + \frac{1}{10}\left(5+3\sqrt{5}\right)\text{Subst}\Bigg($$
$$= -\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right) + \frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\Bigg($$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.51

$$\frac{\sqrt{5+\sqrt{5}}\tanh^{-1}\left(\frac{\left(-3+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tanh^{-1}\left(\frac{\left(3+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[5*x]*Sin[x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [A]
time = 0.34, size = 66, normalized size = 0.40

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(2000\_Z^4-100\_Z^2+1\right)}\_R\ln\left(\mathrm{e}^{2ix}-500i\_R^3+50\_R^2+15i\_R-1\right)$	42
default	$-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(5^(1/2)-3)*5^(1/2)/(5-2*5^(1/2))^(1/2)*arctanh(tan(x)/(5-2*5^(1/2))^
(1/2))-1/10*(3+5^(1/2))*5^(1/2)/(5+2*5^(1/2))^(1/2)*arctanh(tan(x)/(5+2*5^(
1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="maxima")
```

```
[Out] integrate(csc(5*x)*sin(x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(113) = 226.
time = 2.11, size = 231, normalized size = 1.40

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)-\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)+\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) - 3) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) - 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(5*x), x)
```

Giac [A]
time = 0.46, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right)-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(5*x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(2*sqrt(5) + 10)*log(abs(sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sq
rt(2*sqrt(5) + 10)*log(abs(-sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sqrt(-2*s
qrt(5) + 10)*log(abs(sqrt(-2*sqrt(5) + 5) + tan(x))) - 1/20*sqrt(-2*sqrt(5)
 + 10)*log(abs(-sqrt(-2*sqrt(5) + 5) + tan(x)))
```

Mupad [B]
time = 2.59, size = 217, normalized size = 1.32

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)\,\sqrt{\sqrt{5}+5}}{1953125\,\left(\frac{90194313216\,\sqrt{5}}{1953125}-\frac{90194313216\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}-\frac{201863462912\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}+\frac{201863462912}{1953125}\right)}-\frac{77309411328\,\sqrt{2}\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\,\sqrt{\sqrt{5}+5}}{9765625\,\left(\frac{90194313216\,\sqrt{5}}{1953125}-\frac{90194313216\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}-\frac{201863462912\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}+\frac{201863462912}{1953125}\right)}\right)\,\sqrt{\sqrt{5}+5}}{10}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\,\sqrt{2}\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\,\sqrt{5-\sqrt{5}}}{9765625\,\left(\frac{90194313216\,\sqrt{5}}{1953125}-\frac{90194313216\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}+\frac{201863462912\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}-\frac{201863462912}{1953125}\right)}-\frac{34359738368\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)\,\sqrt{5-\sqrt{5}}}{1953125\,\left(\frac{90194313216\,\sqrt{5}}{1953125}-\frac{90194313216\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}+\frac{201863462912\,{\tan\left(\frac{x}{2}\right)}^2}{1953125}-\frac{201863462912}{1953125}\right)}\right)\,\sqrt{5-\sqrt{5}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(5*x),x)
```

```
[Out] (2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(195312
5*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125
 - (201863462912*tan(x/2)^2)/1953125 + 201863462912/1953125)) - (7730941132
8*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(9765625*((90194313216*5^(1
/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 - (201863462912*tan
(x/2)^2)/1953125 + 201863462912/1953125)))*(5^(1/2) + 5)^(1/2))/10 - (2^(1/
2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(976562
5*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125
 + (201863462912*tan(x/2)^2)/1953125 - 201863462912/1953125)) - (3435973836
8*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(1953125*((90194313216*5^(1/2))/195
3125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 + (201863462912*tan(x/2)^2)
/1953125 - 201863462912/1953125)))*(5 - 5^(1/2))^(1/2))/10
```

3.94 $\int \csc(6x)\sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctanh(sin(x))+1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*3
^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2082, 213}

$$\frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[6*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sin[x]]/6 + ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \csc(6x)\sin(x)\,dx = \text{Subst}\left(\int \frac{1}{2(3-19x^2+32x^4-16x^6)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3(-1+x^2)}+\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right)dx, x, \sin(x)\right)$$
$$= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \sin(x)\right)+\text{Su}$$
$$= \frac{1}{6}\tanh^{-1}(\sin(x))+\frac{1}{6}\tanh^{-1}(2\sin(x))-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.
time = 0.05, size = 95, normalized size = 2.64

$$\frac{1}{12}\left(-2\sqrt{3}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\sqrt{3}\tanh^{-1}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\log(1-2\sin(x))+\log(1+2\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[6*x]*Sin[x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[Tan[x/2]/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x/2]]
 - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - Log[1 - 2*Sin[
x]] + Log[1 + 2*Sin[x]])/12
```

Maple [A]
time = 0.35, size = 47, normalized size = 1.31

method	result
default	$-\frac{\ln(2\sin(x)-1)}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(1+2\sin(x))}{12}-\frac{\ln(\sin(x)-1)}{12}+\frac{\ln(1+\sin(x))}{12}$
risch	$\frac{\ln(e^{ix}+i)}{6}-\frac{\ln(e^{ix}-i)}{6}-\frac{\ln(-ie^{ix}+e^{2ix}-1)}{12}+\frac{\ln(ie^{ix}+e^{2ix}-1)}{12}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{12}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+i\sqrt{3}\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(2*sin(x)-1)-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)+1/12*ln(1+2*si
n(x))-1/12*ln(sin(x)-1)+1/12*ln(1+sin(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*co
s(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*si
n(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 - 4
/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3
*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((cos(3
*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3
*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(
cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 2.06, size = 68, normalized size = 1.89

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right)+\frac{1}{12}\log(2\sin(x)+1)+\frac{1}{12}\log(\sin(x)+1)-\frac{1}{12}\log(-\sin(x)+1)-\frac{1}{12}\log(-2\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + 1
/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/12
*log(-2*sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sin(x)\csc(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x)
```

```
[Out] Integral(sin(x)*csc(6*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 0.41, size = 68, normalized size = 1.89

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right)+\frac{1}{12}\log(\sin(x)+1)-\frac{1}{12}\log(-\sin(x)+1)+\frac{1}{12}\log(|2\sin(x)+1|)-\frac{1}{12}\log(|2\sin(x)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(6*x)*sin(x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1/
12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1)) -
1/12*log(abs(2*sin(x) - 1))
```

Mupad [B]
time = 2.46, size = 35, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3}+\frac{\operatorname{atanh}(2\,\sin\left(x\right))}{6}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\,\sqrt{3}\,\sin(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(6*x),x)
```

```
[Out] atanh(sin(x/2)/cos(x/2))/3 + atanh(2*sin(x))/6 - (3^(1/2)*atanh((2*3^(1/2)*
sin(x))/3))/6
```

3.95 $\int \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=8

$$x + 2\cos(x)\sin(x)$$

```
[Out] x+2*cos(x)*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {393, 209}

$$x + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sin[3*x],x]
```

```
[Out] x + 2*Cos[x]*Sin[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\int \csc(x)\sin(3x)\,dx &= \text{Subst}\left(\int \frac{3-x^2}{(1+x^2)^2}\,dx, x, \tan(x)\right)\\
&= 2\cos(x)\sin(x) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= x + 2\cos(x)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sin[3*x],x]
```

```
[Out] x + Sin[2*x]
```

Maple [A]
time = 0.11, size = 9, normalized size = 1.12

method	result	size
risch	$x+\sin(2x)$	7
default	$x+2\cos(x)\sin(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*cos(x)*sin(x)
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.75

$$x+\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] x + sin(2*x)
```

Fricas [A]
time = 1.96, size = 8, normalized size = 1.00

$$2\cos(x)\sin(x)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 2*cos(x)*sin(x) + x
```

Sympy [A]
time = 1.08, size = 5, normalized size = 0.62

$$x+\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x)
```

```
[Out] x + sin(2*x)
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] x + sin(2*x)
```

Mupad [B]
time = 2.25, size = 6, normalized size = 0.75

$$x + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/sin(x),x)
```

```
[Out] x + sin(2*x)
```

3.96 $\int \csc(3x)\sin(6x)\,dx$

Optimal. Leaf size=8

$$\frac{2}{3}\sin(3x)$$

```
[Out] 2/3*sin(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4373, 2717}

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[3*x]*Sin[6*x],x]
```

```
[Out] (2*Sin[3*x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \csc(3x)\sin(6x)\,dx &= 2\int \cos(3x)\,dx \\ &= \frac{2}{3}\sin(3x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{2}{3}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[3*x]*Sin[6*x],x]
```

```
[Out] (2*Sin[3*x])/3
```

Maple [A]
time = 0.07, size = 9, normalized size = 1.12

method	result	size
risch	$\frac{2\sin(3x)}{3}$	7
derivativedivides	$\frac{2}{3\csc(3x)}$	9
default	$\frac{2}{3\csc(3x)}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(3*x)*sin(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/csc(3*x)
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$\frac{2}{3}\sin\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")
```

```
[Out] 2/3*sin(3*x)
```

Fricas [A]
time = 1.60, size = 6, normalized size = 0.75

$$\frac{2}{3}\sin\left(3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")
```

```
[Out] 2/3*sin(3*x)
```

Sympy [A]
time = 1.84, size = 7, normalized size = 0.88

$$\frac{2\sin\left(3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x)
```

```
[Out] 2*sin(3*x)/3
```

Giac [A]
time = 0.44, size = 6, normalized size = 0.75

$$\frac{2}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(3*x)*sin(6*x),x, algorithm="giac")
```

```
[Out] 2/3*sin(3*x)
```

Mupad [B]
time = 0.03, size = 6, normalized size = 0.75

$$\frac{2\sin(3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)/sin(3*x),x)
```

```
[Out] (2*sin(3*x))/3
```

3.97 $\int \cos(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

```
[Out] -1/2*cos(x)-1/6*cos(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4369}

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x],x]
```

```
[Out] -1/2*Cos[x] - Cos[3*x]/6
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(2x)\,dx=-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x],x]
```

```
[Out] -1/2*Cos[x] - Cos[3*x]/6
```

Maple [A]
time = 0.07, size = 7, normalized size = 0.47

method	result	size
derivativedivides	$-\frac{2(\cos^3(x))}{3}$	7
default	$-\frac{2(\cos^3(x))}{3}$	7
risch	$-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
norman	$\frac{\frac{4(\tan^2(x))}{3}+\frac{4(\tan^2(\frac{x}{2}))}{3}-\frac{4\tan(x)\tan(\frac{x}{2})}{3}}{(1+\tan^2(\frac{x}{2}))(\tan^2(x)+1)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*cos(x)^3
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$-\frac{1}{6}\cos(3x)-\frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*x) - 1/2*cos(x)
```

Fricas [A]
time = 1.13, size = 6, normalized size = 0.40

$$-\frac{2}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*cos(x)^3
```

Sympy [A]
time = 0.22, size = 22, normalized size = 1.47

$$-\frac{\sin(x)\sin(2x)}{3}-\frac{2\cos(x)\cos(2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x)
```

```
[Out] -sin(x)*sin(2*x)/3 - 2*cos(x)*cos(2*x)/3
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.40

$$-\frac{2}{3}\cos\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -2/3*cos(x)^3
```

Mupad [B]
time = 0.02, size = 6, normalized size = 0.40

$$-\frac{2\cos\left(x\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*cos(x),x)
```

```
[Out] -(2*cos(x)^3)/3
```

3.98 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -1/4*cos(2*x)-1/8*cos(4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4369}

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -1/4*Cos[2*x] - Cos[4*x]/8
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(3x)\,dx=-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[3*x],x]
```

```
[Out] -1/2*Cos[x]^2 - Cos[4*x]/8
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.76

method	result	size
derivativedivides	$-\frac{\left(4\left(\cos^2(x)\right)-1\right)^2}{16}$	13
default	$-\frac{\left(4\left(\cos^2(x)\right)-1\right)^2}{16}$	13
risch	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
norman	$\frac{-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{4}-\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}-\frac{3}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(4*cos(x)^2-1)^2
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$-\frac{1}{8}\cos\left(4\,x\right)-\frac{1}{4}\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

Fricas [A]
time = 2.06, size = 13, normalized size = 0.76

$$-\cos\left(x\right)^4+\frac{1}{2}\cos\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Sympy [A]
time = 0.12, size = 22, normalized size = 1.29

$$-\frac{\sin\left(x\right)\sin\left(3x\right)}{8}-\frac{3\cos\left(x\right)\cos\left(3x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.76

$$-\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$\frac{\cos(x)^2}{2} - \cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*cos(x),x)
```

```
[Out] cos(x)^2/2 - cos(x)^4
```

3.99 $\int \cos(x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

```
[Out] -1/6*cos(3*x)-1/10*cos(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{4369\}$

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[4*x],x]
```

```
[Out] -1/6*Cos[3*x] - Cos[5*x]/10
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(4x)\,dx=-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[4*x],x]
```

```
[Out] -1/6*Cos[3*x] - Cos[5*x]/10
```

Maple [A]
time = 0.10, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{8(\cos^5(x))}{5}+\frac{4(\cos^3(x))}{3}$	14
default	$-\frac{8(\cos^5(x))}{5}+\frac{4(\cos^3(x))}{3}$	14
risch	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
norman	$\frac{\frac{8\left(\tan^2(2x)\right)}{15}+\frac{8\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}-\frac{4\tan(2x)\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] -8/5*cos(x)^5+4/3*cos(x)^3
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$-\frac{1}{10}\cos(5x)-\frac{1}{6}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) - 1/6*cos(3*x)
```

Fricas [A]
time = 1.56, size = 13, normalized size = 0.76

$$-\frac{8}{5}\cos(x)^5+\frac{4}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3
```

Sympy [A]
time = 0.12, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(4x)}{15}-\frac{4\cos(x)\cos(4x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x)
```

```
[Out] -sin(x)*sin(4*x)/15 - 4*cos(x)*cos(4*x)/15
```

Giac [A]
time = 0.38, size = 13, normalized size = 0.76

$$-\frac{8}{5}\cos(x)^5 + \frac{4}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(4*x),x, algorithm="giac")
```

```
[Out] -8/5*cos(x)^5 + 4/3*cos(x)^3
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.82

$$-\frac{4\cos(x)^3\left(6\cos(x)^2-5\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)*cos(x),x)
```

```
[Out] -(4*cos(x)^3*(6*cos(x)^2 - 5))/15
```

3.100 $\int \cos(x)\sin(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}$$

```
[Out] 1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4670, 2718}

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[m*x],x]
[Out] Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(mx)\,dx &= \int \left(-\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((1+m)x)\right)\,dx\\ &= -\left(\frac{1}{2}\int \sin((1-m)x)\,dx\right)+\frac{1}{2}\int \sin((1+m)x)\,dx\\ &= \frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.74

$$\frac{m\cos(x)\cos(mx)+\sin(x)\sin(mx)}{1-m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[m*x],x]
[Out] (m*Cos[x]*Cos[m*x] + Sin[x]*Sin[m*x])/(1 - m^2)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{\cos((-1+m)x)}{2(-1+m)}-\frac{\cos((1+m)x)}{2(1+m)}$	28
risch	$-\frac{\cos((-1+m)x)}{2(-1+m)}-\frac{\cos((1+m)x)}{2(1+m)}$	28
norman	$\frac{\frac{2m\left(\tan^2\left(\frac{x}{2}\right)\right)}{m^2-1}+\frac{2m\left(\tan^2\left(\frac{mx}{2}\right)\right)}{m^2-1}-\frac{4\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{mx}{2}\right)\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(m*x),x,method=_RETURNVERBOSE)
[Out] -1/2*cos((-1+m)*x)/(-1+m)-1/2*cos((1+m)*x)/(1+m)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.77

$$-\frac{\cos\left((m+1)x\right)}{2\,(m+1)}-\frac{\cos\left((m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="maxima")
[Out] -1/2*cos((m + 1)*x)/(m + 1) - 1/2*cos((m - 1)*x)/(m - 1)
```

Fricas [A]
time = 2.13, size = 25, normalized size = 0.71

$$-\frac{m\cos\left(mx\right)\cos\left(x\right)+\sin\left(mx\right)\sin\left(x\right)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="fricas")
[Out] -(m*cos(m*x)*cos(x) + sin(m*x)*sin(x))/(m^2 - 1)
```

Sympy [A]
time = 0.19, size = 44, normalized size = 1.26

$$\begin{cases} -\frac{\sin^2\left(x\right)}{2} & \text{for}\ m=-1 \\ \frac{\sin^2\left(x\right)}{2} & \text{for}\ m=1 \\ -\frac{m\cos\left(x\right)\cos\left(mx\right)}{m^2-1}-\frac{\sin\left(x\right)\sin\left(mx\right)}{m^2-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x)
```

```
[Out] Piecewise((-sin(x)**2/2, Eq(m, -1)), (sin(x)**2/2, Eq(m, 1)), (-m*cos(x)*co
s(m*x)/(m**2 - 1) - sin(x)*sin(m*x)/(m**2 - 1), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.83

$$-\frac{\cos(mx+x)}{2(m+1)}-\frac{\cos(mx-x)}{2(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(m*x),x, algorithm="giac")
```

```
[Out] -1/2*cos(m*x + x)/(m + 1) - 1/2*cos(m*x - x)/(m - 1)
```

Mupad [B]
time = 2.29, size = 57, normalized size = 1.63

$$\begin{cases} \frac{\sin(x)^2}{2} & \text{if } m=1 \\ \frac{\cos(x)^2}{2} & \text{if } m=-1 \\ -\frac{\cos(x\,(m-1))}{2\,m-2}-\frac{\cos(x\,(m+1))}{2\,m+2} & \text{if } m\neq -1\wedge m\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(m*x)*cos(x),x)
```

```
[Out] piecewise(m == 1, sin(x)^2/2, m == -1, cos(x)^2/2, m ~= -1 & m ~= 1, - cos(
x*(m - 1))/(2*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

3.101 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

[Out] 1/2*sin(x)+1/6*sin(3*x)

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4368}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[2*x],x]

[Out] Sin[x]/2 + Sin[3*x]/6

Maple [A]
time = 0.08, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{4\tan(x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{3}+\frac{2\left(\tan^2(x)\right)\tan\left(\frac{x}{2}\right)}{3}+\frac{4\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(\tan^2(x)+1\right)}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A]
time = 1.73, size = 12, normalized size = 0.80

$$\frac{1}{3}\left(2\cos(x)^2+1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A]
time = 0.21, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3}+\frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

Giac [A]
time = 0.42, size = 11, normalized size = 0.73

$$\frac{1}{6}\sin\left(3x\right)+\frac{1}{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Mupad [B]
time = 0.02, size = 9, normalized size = 0.60

$$\sin\left(x\right)-\frac{2\sin\left(x\right)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(x),x)
```

```
[Out] sin(x) - (2*sin(x)^3)/3
```

3.102 $\int \cos(x)\cos(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

[Out] 1/4*sin(2*x)+1/8*sin(4*x)

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4368}

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[3*x],x]

[Out] Sin[2*x]/4 + Sin[4*x]/8

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(3x)\,dx=\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Cos[3*x],x]

[Out] Sin[2*x]/4 + Sin[4*x]/8

Maple [A]
time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$	14
risch	$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$	14
norman	$\frac{\frac{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{3x}{2}\right)\right)}{4}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{3x}{2}\right)}{4}-\frac{\tan\left(\frac{x}{2}\right)}{4}+\frac{3\tan\left(\frac{3x}{2}\right)}{4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(2*x)+1/8*sin(4*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{8}\sin\left(4\,x\right)+\frac{1}{4}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A]
time = 1.84, size = 7, normalized size = 0.41

$$\cos\left(x\right)^3\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] cos(x)^3*sin(x)
```

Sympy [A]
time = 0.12, size = 20, normalized size = 1.18

$$-\frac{\sin\left(x\right)\cos\left(3x\right)}{8}+\frac{3\sin\left(3x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x)
```

```
[Out] -sin(x)*cos(3*x)/8 + 3*sin(3*x)*cos(x)/8
```

<u>**Giac**</u> [**A**]
time = 0.40, size = 13, normalized size = 0.76

$$\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/8*sin(4*x) + 1/4*sin(2*x)
```

<u>**Mupad**</u> [**B**]
time = 0.02, size = 7, normalized size = 0.41

$$\cos(x)^3 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(x),x)
```

```
[Out] cos(x)^3*sin(x)
```

3.103 $\int \cos(x)\cos(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4368}

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4368

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(4x)\,dx=\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[4*x],x]
```

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Maple [A]
time = 0.11, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8\tan(2x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{15}+\frac{2\left(\tan^2(2x)\right)\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(2x)\right)}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin(5x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A]
time = 2.04, size = 18, normalized size = 1.06

$$\frac{1}{15}\left(24\cos(x)^4-8\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A]
time = 0.20, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(4x)}{15}+\frac{4\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x)
```

```
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.76

$$\frac{1}{10}\sin\left(5\,x\right) + \frac{1}{6}\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{\sin\left(3\,x\right)}{6} + \frac{\sin\left(5\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*cos(x),x)
```

```
[Out] sin(3*x)/6 + sin(5*x)/10
```

3.104 $\int \cos(x)\cos(mx)\,dx$

Optimal. Leaf size=35

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((1+m)x)}{2(1+m)}$$

```
[Out] 1/2*sin((1-m)*x)/(1-m)+1/2*sin((1+m)*x)/(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4666, 2717}

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[m*x],x]
```

```
[Out] Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(mx)\,dx &= \int \left(\frac{1}{2}\cos((1-m)x)+\frac{1}{2}\cos((1+m)x)\right)dx\\ &= \frac{1}{2}\int \cos((1-m)x)\,dx+\frac{1}{2}\int \cos((1+m)x)\,dx\\ &= \frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((1+m)x)}{2(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.71

$$\frac{-\cos(mx)\sin(x)+m\cos(x)\sin(mx)}{-1+m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[m*x],x]
[Out] (-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.80

method	result	size
default	$\frac{\sin((-1+m)x)}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$	28
risch	$\frac{\sin((-1+m)x)}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$	28
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{m^2-1}+\frac{2m\tan\left(\frac{mx}{2}\right)}{m^2-1}+\frac{2\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{mx}{2}\right)\right)}{m^2-1}-\frac{2m\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2\left(\frac{mx}{2}\right)\right)}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(m*x),x,method=_RETURNVERBOSE)
[Out] 1/2/(-1+m)*sin((-1+m)*x)+1/2*sin((1+m)*x)/(1+m)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.80

$$\frac{\sin\left((m+1)x\right)}{2\,(m+1)}-\frac{\sin\left(-(m-1)x\right)}{2\,(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="maxima")
[Out] 1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

Fricas [A]
time = 1.84, size = 25, normalized size = 0.71

$$\frac{m\cos(x)\sin(mx)-\cos(mx)\sin(x)}{m^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="fricas")
[Out] (m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(22) = 44$.
time = 0.19, size = 56, normalized size = 1.60

$$\begin{cases}\frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2} & \text{for } m=-1\vee m=1\\ \frac{m\sin(mx)\cos(x)}{m^2-1}-\frac{\sin(x)\cos(mx)}{m^2-1} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq(
m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.83

$$\frac{\sin\left(mx+x\right)}{2\left(m+1\right)}+\frac{\sin\left(mx-x\right)}{2\left(m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(m*x),x, algorithm="giac")
```

```
[Out] 1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

Mupad [B]
time = 0.12, size = 58, normalized size = 1.66

$$\begin{cases} \frac{x}{2}+\frac{\sin(2\,x)}{4} & \text{if} \ \ m=-1\vee m=1 \\ \frac{\sin(x\,(m-1))}{2\,m-2}+\frac{\sin(x\,(m+1))}{2\,m+2} & \text{if} \ \ m\neq -1\wedge m\neq 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(m*x)*cos(x),x)
```

```
[Out] piecewise(m == -1 | m == 1, x/2 + sin(2*x)/4, m ~= -1 & m ~= 1, sin(x*(m -
1))/(2*m - 2) + sin(x*(m + 1))/(2*m + 2))
```

3.105 $\int \cos(x)\tan(2x)\,dx$

Optimal. Leaf size=20

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

```
[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 327, 213}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\cos(x)\right)}{\sqrt{2}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\tan(2x)\,dx &= -\text{Subst}\left(\int \frac{2x^2}{-1+2x^2}\,dx, x, \cos(x)\right) \\
&= -\left(2\text{Subst}\left(\int \frac{x^2}{-1+2x^2}\,dx, x, \cos(x)\right)\right) \\
&= -\cos(x) - \text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right) \\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}} - \cos(x)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 183, normalized size = 9.15

$$
\frac{2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right) - 2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right) + 4\tanh^{-1}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right) - 4\sqrt{2}\cos(x) - \log\left(2-\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right) + \log\left(2+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[2*x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - 4*Sqrt[2]*Cos[
x] - Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sq
rt[2]*Sin[x]])/(4*Sqrt[2])
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.90

method	result	size
derivativedivides	$-\cos(x)+\frac{\text{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	18
default	$-\cos(x)+\frac{\text{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	18
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(17) = 34$.
time = 0.50, size = 133, normalized size = 6.65

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+1\right)-\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x)-2\left(\sqrt{2}\cos(x)-1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+1\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1) - cos(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 2.39, size = 38, normalized size = 1.90

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - co
s(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\tan(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x)
```

```
[Out] Integral(cos(x)*tan(2*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(2*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(2*x), x)
```

Mupad [B]
time = 2.36, size = 42, normalized size = 2.10

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{8\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)^2}{12\tan\left(\frac{x}{2}\right)^2-4}\right)}{2}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)*cos(x),x)
```

```
[Out] - (2^(1/2)*atanh((8*2^(1/2)*tan(x/2)^2)/(12*tan(x/2)^2 - 4)))/2 - 2/(tan(x/
2)^2 + 1)
```

3.106 $\int \cos(x)\tan(3x)\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

```
[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {396, 212}

$$\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[3*x],x]
```

```
[Out] ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\tan(3x)\,dx &= -\text{Subst}\left(\int \frac{1-4x^2}{3-4x^2}\,dx,x,\cos(x)\right)\\ &= -\cos(x)+2\text{Subst}\left(\int \frac{1}{3-4x^2}\,dx,x,\cos(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.
time = 0.04, size = 48, normalized size = 2.29

$$-\frac{\tanh^{-1}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[3*x],x]
```

```
[Out] -(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[3
]]/Sqrt[3] - Cos[x]
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.90

method	result	size
derivativedivides	$-\cos(x)+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
default	$-\cos(x)+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{6}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin
(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos
(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2
 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.
time = 1.33, size = 38, normalized size = 1.81

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - co
s(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\tan(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x)
[Out] Integral(cos(x)*tan(3*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(3*x),x, algorithm="giac")
[Out] integrate(cos(x)*tan(3*x), x)
```

Mupad [B]
time = 2.31, size = 42, normalized size = 2.00

$$-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{32\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{3\left(\frac{56\tan\left(\frac{x}{2}\right)^2}{3}-\frac{8}{3}\right)}\right)}{3}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)*cos(x),x)
[Out] - (3^(1/2)*atanh((32*3^(1/2)*tan(x/2)^2)/(3*((56*tan(x/2)^2)/3 - 8/3))))/3
- 2/(tan(x/2)^2 + 1)
```

3.107 $\int \cos(x)\tan(4x)\,dx$

Optimal. Leaf size=71

$$\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)-\cos(x)$$

```
[Out] -cos(x)+1/4*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arcta
nh(2*cos(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {12, 1293, 1180, 213}

$$-\cos(x)+\frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[4*x],x]
```

```
[Out] (Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2 + Sqr
t[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\tan(4x)\,dx &= -\text{Subst}\left(\int \frac{4x^2(-1+2x^2)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\ &= -\left(4\text{Subst}\left(\int \frac{x^2(-1+2x^2)}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\right)\\ &= -\cos(x)+\frac{1}{2}\text{Subst}\left(\int \frac{2-8x^2}{1-8x^2+8x^4}\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,dx, x, \cos(x)\right)-\left(2+\sqrt{2}\right)\text{Su}\\ &= \frac{1}{4}\sqrt{2-\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)-\cos(\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 53.03, size = 6196, normalized size = 87.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[4*x],x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.96

method	result	si
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\text{RootOf}(2048\_Z^4+128\_Z^2+1)}\_R\ln\left(e^{2ix}-8i\_R\,e^{ix}+1\right)\right)$	5

derivativedivides	$-\cos(x)+\frac{\sqrt{2}\,(\sqrt{2}-1)\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{(1+\sqrt{2})\,\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$
default	$-\cos(x)+\frac{\sqrt{2}\,(\sqrt{2}-1)\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}+\frac{(1+\sqrt{2})\,\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+1/4*2^(1/2)*(2^(1/2)-1)/(2-2^(1/2))^(1/2)*arctanh(2*cos(x)/(2-2^(1/
2))^(1/2))+1/4*(1+2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*cos(x)/(2+2^
(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(-((sin(7*x) - sin(x))*cos(8*x) - (cos(7*x) - cos(x))*si
n(8*x) + sin(7*x) - sin(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x)
```

Fricas [A]
time = 1.68, size = 101, normalized size = 1.42

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\cos(x)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\cos(x)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\cos(x)\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2)
+ 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-s
qrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2) - 2
*cos(x)) - cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(4x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x)
```

```
[Out] Integral(cos(x)*tan(4*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(4*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(4*x), x)
```

Mupad [B]
time = 2.45, size = 295, normalized size = 4.15

$$-\frac{\operatorname{atanh}\left(\frac{219747975168\tan\left(\frac{x}{2}\right)^2\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}-\frac{15971909632\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}-\frac{130056978432\sqrt{2}\tan\left(\frac{x}{2}\right)^2\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}\right)\sqrt{2-\sqrt{2}}}{4}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}-\frac{\operatorname{atanh}\left(\frac{15971909632\sqrt{\sqrt{2}+2}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2-386664497152\tan\left(\frac{x}{2}\right)^2+20887633920}-\frac{219747975168\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{2}+2}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2-386664497152\tan\left(\frac{x}{2}\right)^2+20887633920}-\frac{130056978432\sqrt{2}\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{2}+2}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2-386664497152\tan\left(\frac{x}{2}\right)^2+20887633920}\right)\sqrt{\sqrt{2}+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)*cos(x),x)
```

```
[Out] - (atanh((219747975168*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2)
- 254015438848*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920)
- (15971909632*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2^(1
/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920) - (130056978432*2^(
1/2)*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2^(
1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920))*(2 - 2^(1/2))^(1/
2))/4 - 2/(tan(x/2)^2 + 1) - (atanh((15971909632*(2^(1/2) + 2)^(1/2))/(6098
518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2
+ 20887633920) - (219747975168*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(6098518016*
2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2 + 20887
633920) - (130056978432*2^(1/2)*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(6098518016
*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)^2 + 2088
7633920))*(2^(1/2) + 2)^(1/2))/4
```

3.108 $\int \cos(x)\tan(5x)\,dx$

Optimal. Leaf size=84

$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)-c$$

```
[Out] -cos(x)+1/10*arctanh(1/5*cos(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)
+1/10*arctanh(2*cos(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1690, 1180, 213}

$$-\cos(x)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[5*x],x]
```

```
[Out] (Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[(
5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cos(x)\tan(5x)\,dx = -\text{Subst}\left(\int \frac{1-12x^2+16x^4}{5-20x^2+16x^4}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(1-\frac{4(1-2x^2)}{5-20x^2+16x^4}\right)\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + 4\text{Subst}\left(\int \frac{1-2x^2}{5-20x^2+16x^4}\,dx, x, \cos(x)\right)$$
$$= -\cos(x) - \frac{1}{5}\left(4\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10+2\sqrt{5}+16x^2}\,dx, x, \cos(x)\right) - \frac{1}{5}\left(4\left(5-\right.\right.$$
$$= \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tanh^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{5}}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(84) = 168.
time = 0.44, size = 215, normalized size = 2.56

$$\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4-\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4+\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\left(5+\sqrt{5}\right)}}+\frac{\left(-1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4-\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}}+\frac{\left(-1+\sqrt{5}\right)\tanh^{-1}\left(\frac{4+\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Tan[5*x],x]
```

```
[Out] ((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]]
)/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan[x
/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*ArcTa
nh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]])/Sqrt[50 - 10*Sqrt[5]
] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5
]]])/Sqrt[50 - 10*Sqrt[5]] - Cos[x]
```

Maple [A]
time = 0.18, size = 72, normalized size = 0.86

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\text{RootOf}\left(2000\_Z^4+100\_Z^2+1\right)}\_R\ln\left(e^{2ix}-10i\_R\,e^{ix}+1\right)\right)$
derivativedivides	$-\cos(x)+\frac{\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}$

default	$-\cos(x)+\frac{(\sqrt{5}-1)\sqrt{5}\,\operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{(\sqrt{5}+1)\sqrt{5}\,\operatorname{arctanh}\left(\frac{4\cos(x)}{\sqrt{10+2\sqrt{5}}}\right.}{5\sqrt{10+2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+1/5*(5^(1/2)-1)*5^(1/2)/(10-2*5^(1/2))^(1/2)*arctanh(4*cos(x)/(10-2
*5^(1/2))^(1/2))+1/5*(5^(1/2)+1)*5^(1/2)/(10+2*5^(1/2))^(1/2)*arctanh(4*cos
(x)/(10+2*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="maxima")
```

```
[Out] -cos(x) - integrate(((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*x) + (
sin(6*x) - sin(4*x) + sin(2*x))*cos(7*x) + (sin(5*x) - sin(3*x) + sin(x))*c
os(6*x) + (sin(4*x) - sin(2*x))*cos(5*x) + (sin(3*x) - sin(x))*cos(4*x) - (
cos(7*x) - cos(5*x) + cos(3*x) - cos(x))*sin(8*x) - (cos(6*x) - cos(4*x) +
cos(2*x) - 1)*sin(7*x) - (cos(5*x) - cos(3*x) + cos(x))*sin(6*x) - (cos(4*x
) - cos(2*x) + 1)*sin(5*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x) - 1)*
sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - sin(x))/
(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x)
 - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*
x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^
2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)
*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.
time = 2.16, size = 129, normalized size = 1.54

$$\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\cos(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\cos(x)\right)+\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}+4\cos(x)\right)-\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}-4\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*cos(x)) -
1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*cos(x)) +
1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*cos(x))
```

```
- 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*cos(x)
) - cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\tan(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x)
```

```
[Out] Integral(cos(x)*tan(5*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(5*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(5*x), x)
```

Mupad [B]
time = 2.50, size = 407, normalized size = 4.85

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{18032420192256\sqrt{2}\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{5}+5}}{\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816}-\frac{867583393792\sqrt{2}\sqrt{5}\sqrt{\sqrt{5}+5}}{25\left(\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816\right)}-\frac{3805341024256\sqrt{2}\sqrt{\sqrt{5}+5}}{5\left(\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816\right)}+\frac{6886980059136\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{5}+5}}{\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816}\right)\sqrt{\sqrt{5}+5}}{10}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{867583393792\sqrt{2}\sqrt{5}\sqrt{5-\sqrt{5}}}{25\left(\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}-2398739234816\right)}-\frac{3805341024256\sqrt{2}\sqrt{5-\sqrt{5}}}{5\left(\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}-2398739234816\right)}+\frac{18032420192256\sqrt{2}\tan\left(\frac{x}{2}\right)^2\sqrt{5-\sqrt{5}}}{\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}-2398739234816}-\frac{6886980059136\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)^2\sqrt{5-\sqrt{5}}}{\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}-2398739234816}\right)\sqrt{5-\sqrt{5}}}{10}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(5*x)*cos(x),x)
```

```
[Out] (2^(1/2)*atanh((18032420192256*2^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/2))/((88
51927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433
343574016*tan(x/2)^2)/5 + 2398739234816) - (867583393792*2^(1/2)*5^(1/2)*(5
^(1/2) + 5)^(1/2))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/
2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816)) - (380
5341024256*2^(1/2)*(5^(1/2) + 5)^(1/2))/(5*((8851927597056*5^(1/2))/25 - (6
76375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 23
98739234816)) + (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/
2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 -
 (333433343574016*tan(x/2)^2)/5 + 2398739234816))*(5^(1/2) + 5)^(1/2))/10 -
 (2^(1/2)*atanh((867583393792*2^(1/2)*5^(1/2)*(5 - 5^(1/2))^(1/2))/(25*((88
51927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433
343574016*tan(x/2)^2)/5 - 2398739234816)) - (3805341024256*2^(1/2)*(5 - 5^(
1/2))^(1/2))/(5*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(
```

```
x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)) + (1803242019
2256*2^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1/2))/((8851927597056*5^(1/2))/25 -
(676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 -
2398739234816) - (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1
/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25
+ (333433343574016*tan(x/2)^2)/5 - 2398739234816))*(5 - 5^(1/2))^(1/2))/10
- 2/(tan(x/2)^2 + 1)
```

3.109 $\int \cos(x)\tan(6x)\,dx$

Optimal. Leaf size=89

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)-\cos(x)$$

```
[Out] -cos(x)+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/
2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {12, 6874, 2098, 213, 1180}

$$-\cos(x)+\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Tan[6*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/(3*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos[x])
/Sqrt[2 - Sqrt[3]]])/6 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqr
t[3]]])/6 - Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\tan(6x)\,dx &= -\text{Subst}\left(\int \frac{2x^2(-3+16x^2-16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{x^2(-3+16x^2-16x^4)}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\right)\\
&= -\left(2\text{Subst}\left(\int \left(\frac{1}{2}-\frac{1-12x^2+16x^4}{2\,(1-18x^2+48x^4-32x^6)}\right)dx, x, \cos(x)\right)\right)\\
&= -\cos(x)+\text{Subst}\left(\int \frac{1-12x^2+16x^4}{1-18x^2+48x^4-32x^6}\,dx, x, \cos(x)\right)\\
&= -\cos(x)+\text{Subst}\left(\int \left(-\frac{1}{3\,(-1+2x^2)}-\frac{2(-1+8x^2)}{3\,(1-16x^2+16x^4)}\right)dx, x, \cos(x)\right)\\
&= -\cos(x)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)-\frac{2}{3}\text{Subst}\left(\int \frac{-1+8x^2}{1-16x^2+16x^4}\,dx,\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}-\cos(x)-\frac{1}{3}\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{-8+4\sqrt{3}+16x^2}\,dx\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{1}{6}\sqrt{2-\sqrt{3}}\,\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{6}\sqrt{2+\sqrt{3}}\,\tanh\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.89, size = 628, normalized size = 7.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[x]*Tan[6*x],x]
```

```
[Out] ((4 + 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + (4 - 4*I)*(-1)^(1/
4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] - 24*Cos[x] - (2*(1 + Sqrt[2])*(x - 2*Sq
```

```
rt[3]*ArcTanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log
[-(Sec[x/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]]))/(2 + Sqrt[2]) + Sqrt[2]*(
x + 2*Sqrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Se
c[x/2]^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) - (2*(2*
(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*Sqrt[2]
 - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sqrt[2]*Co
s[x] - Sqrt[2]*Sin[x]))]]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6] - (-2 +
 Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(-36 + 15*Sqrt[6] + (20 - 8*Sqrt
[6])*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]*Sin[x] + 1
2*Sin[2*x] - 5*Sqrt[6]*Sin[2*x]) + (2*(-2*(Sqrt[2] + Sqrt[3])*ArcTanh[(2 +
(2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[x/2]^2] + Log
[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]]))*(2 + Sqrt[6]*Sin[x])*(3 +
 Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(-36 - 15*Sqrt[6]
+ 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] - 20*Sqr
t[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))/24
```

Maple [**A**]
time = 0.17, size = 104, normalized size = 1.17

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\mathrm{RootOf}\left(20736\_Z^{4}+576\_Z^{2}+1\right)}\_R\ln\left(e^{2ix}-12i\_R\,e^{ix}+1\right)\right)+\frac{\sqrt{2}\,\ln\left(e^{2i\cdots\right.}{\cdots}$
derivativedivides	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{6}+\frac{2\left(-3+2\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{8\cos\left(x\right)}{2\sqrt{6}-2\sqrt{2}}\right)}{9\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\sqrt{3}\,\left(3+2\sqrt{\cdots}\right.}{9\left(\cdots\right.}$
default	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{6}+\frac{2\left(-3+2\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{8\cos\left(x\right)}{2\sqrt{6}-2\sqrt{2}}\right)}{9\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\sqrt{3}\,\left(3+2\sqrt{\cdots}\right.}{9\left(\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+2/9*(-3+2*3^(1/2))*3^(1/2)/(2*6
^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)-2*2^(1/2)))+2/9*3^(1/2)*(3+2*
3^(1/2))/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) - 2*si
n(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5*x) -
cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x) - 1)*
sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + cos(5*x)*
sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - c
os(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1)
, x)
```

Fricas [A]
time = 1.84, size = 134, normalized size = 1.51

$$\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\cos\left(x\right)\right)-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\cos\left(x\right)\right)-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\cos\left(x\right)\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^{2}-1}\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(3
) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sqrt
(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) + 2)
 - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos
(x)^2 - 1)) - cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(x\right)\tan\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x)
```

```
[Out] Integral(cos(x)*tan(6*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*tan(6*x),x, algorithm="giac")
```

```
[Out] integrate(cos(x)*tan(6*x), x)
```

Mupad [B]
time = 4.07, size = 787, normalized size = 8.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(6*x)*cos(x),x)
```

```
[Out] (6^(1/2)*(atan((2^(1/2)*321030945816576i)/(2132548963043330400*tan(x/2)^4
 - 1292758292627954380800*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*88840
5273481134080i)/(2132548963043330400*tan(x/2)^4 - 1292758292627954380800*t
an(x/2)^2 + 2176593611144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/
(2132548963043330400*tan(x/2)^4 - 1292758292627954380800*tan(x/2)^2 + 2176
593611144037376) + (2^(1/2)*tan(x/2)^4*10905601889064960i)/(21325489630433
3030400*tan(x/2)^4 - 1292758292627954380800*tan(x/2)^2 + 2176593611144037376)
 - (6^(1/2)*tan(x/2)^2*527658334623522877441i)/(2132548963043330400*tan(x/
2)^4 - 1292758292627954380800*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*t
an(x/2)^4*870546504971060121601i)/(2132548963043330400*tan(x/2)^4 - 129275
829262795438080*tan(x/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255
045898017881904843325441i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) +
 1191297171699098884409493395865600*tan(x/2)^2 - 3436727172698795994646686620
39040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572925441) - (6^(1
/2)*852047139771204346616741888000i)/(589232404262260650654553866240*2^(1/2
)*6^(1/2) + 1191297171699098884409493395865600*tan(x/2)^2 - 343672717269879 5
994646686620390400*2^(1/2)*6^(1/2)*tan(x/2)^2 - 208709030945079899783455729 25
44) - (2^(1/2)*tan(x/2)^2*84182283571305304543568582410240i)/(5892324042622
60650654553866240*2^(1/2)*6^(1/2) + 1191297171699098884409493395865600*tan(x
/2)^2 - 343672717269879599464668620390400*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870
903094507989978345572925441) + (6^(1/2)*tan(x/2)^2*486345010752364865048734 2
4060416i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717169909
888440949339586560*tan(x/2)^2 - 343672717269879599464668620390400*2^(1/2)*6^
(1/2)*tan(x/2)^2 - 208709030945079899783455729254 4)))*1i)/12 - 2/(tan(x/2)^
2 + 1) - (2^(1/2)*(2*atan((2^(1/2)*227680384600318033434103303372 8i)/(18766
876017666378997952094928896*tan(x/2)^2 - 321988687788455255352932093132 8) -
 (2^(1/2)*tan(x/2)^2*132701852937786461100817409638 40i)/(187668760176663789
97952094928896*tan(x/2)^2 - 321988687788455255352932093132 8)) - atan((2^(1/
2)*321030945816576i)/(2132548963043330400*tan(x/2)^4 - 1292758292627954 38
080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*888405273481134080i)/(2132
548963043330400*tan(x/2)^4 - 1292758292627954380800*tan(x/2)^2 + 217659361
1144037376) - (2^(1/2)*tan(x/2)^2*18711054724802560i)/(2132548963043330 40
0*tan(x/2)^4 - 1292758292627954380800*tan(x/2)^2 + 2176593611144037376) + (2
^(1/2)*tan(x/2)^4*10905601889064960i)/(2132548963043330400*tan(x/2)^4 - 1
292758292627954380800*tan(x/2)^2 + 2176593611144037376) - (6^(1/2)*tan(x/2)^
2*527658334623522877441i)/(2132548963043330400*tan(x/2)^4 - 12927582926279
5438080*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*tan(x/2)^4*87054650497
```

106012160i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x
/2)^2 + 2176593611144037376)) + atan((2^(1/2)*14433255045898017881904843325
44i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 11912971716990988844
0949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)
*tan(x/2)^2 - 2087090309450798997834557292544) - (6^(1/2)*85204713977120434
6616741888000i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 119129717
169909888440949339586560*tan(x/2)^2 - 34367271726987959946466862039040*2^(1
/2)*6^(1/2)*tan(x/2)^2 - 2087090309450798997834557292544) - (2^(1/2)*tan(x/
2)^2*84182283571305304543568582410240i)/(589232404262260650654553866240*2^(
1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 - 3436727172698
7959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572
92544) + (6^(1/2)*tan(x/2)^2*48634501075236486504873424060416i)/(5892324042
62260650654553866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*ta
n(x/2)^2 - 34367271726987959946466862039040*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20
87090309450798997834557292544))*1i)/12

3.110 $\int \cos(x)\cot(2x)\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\tanh^{-1}(\cos(x))+\cos(x)$$

```
[Out] -1/2*arctanh(cos(x))+cos(x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 396, 212}

$$\cos(x)-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[2*x],x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] + Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(2x)\,dx &= -\text{Subst}\left(\int \frac{-1+2x^2}{2(1-x^2)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x^2}{1-x^2}\,dx, x, \cos(x)\right)\right)\\ &= \cos(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \cos(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.
time = 0.01, size = 25, normalized size = 2.50

$$\cos(x) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[2*x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

Maple [A]
time = 0.11, size = 14, normalized size = 1.40

method	result	size
default	$\cos(x) + \frac{\ln(-\cot(x)+\csc(x))}{2}$	14
risch	$\frac{e^{ix}}{2} + \frac{e^{-ix}}{2} + \frac{\ln(e^{ix}-1)}{2} - \frac{\ln(e^{ix}+1)}{2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)+1/2*ln(-cot(x)+csc(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(8) = 16.
time = 0.27, size = 37, normalized size = 3.70

$$\cos(x) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(8) = 16$.
time = 1.86, size = 21, normalized size = 2.10

$$\cos\left(x\right) - \frac{1}{4}\,\log\left(\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right) + \frac{1}{4}\,\log\left(-\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.51, size = 19, normalized size = 1.90

$$\frac{\log\left(\cos\left(x\right) - 1\right)}{4} - \frac{\log\left(\cos\left(x\right) + 1\right)}{4} + \cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.41, size = 19, normalized size = 1.90

$$\cos\left(x\right) - \frac{1}{4}\,\log\left(\cos\left(x\right) + 1\right) + \frac{1}{4}\,\log\left(-\cos\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(2*x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

Mupad [B]
time = 2.34, size = 20, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{2}{{\tan\left(\frac{x}{2}\right)}^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*cos(x),x)
```

```
[Out] log(tan(x/2))/2 + 2/(tan(x/2)^2 + 1)
```

3.111 $\int \cos(x)\cot(3x)\,dx$

Optimal. Leaf size=45

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(1+\cos(x))-\frac{1}{6}\log(1+2\cos(x))$$

```
[Out] cos(x)+1/6*ln(1-2*cos(x))+1/6*ln(1-cos(x))-1/6*ln(cos(x)+1)-1/6*ln(1+2*cos(
x))
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1293, 1175, 630, 31}

$$\cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(\cos(x)+1)-\frac{1}{6}\log(2\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] + Log[1 - 2*Cos[x]]/6 + Log[1 - Cos[x]]/6 - Log[1 + Cos[x]]/6 - Log[
1 + 2*Cos[x]]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(3x)\,dx &= -\text{Subst}\left(\int \frac{x^2(3-4x^2)}{1-5x^2+4x^4}\,dx, x, \cos(x)\right)\\ &= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{-4+8x^2}{1-5x^2+4x^4}\,dx, x, \cos(x)\right)\\ &= \cos(x)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx, x, \cos(x)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx, x, \text{co}\right.\\ &= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \cos(x)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+x}\,dx, x, \cos(x)\right)-\frac{1}{6}\\ &= \cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(1+\cos(x))-\frac{1}{6}\log(1+2\,\text{co}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.04

$$\cos(x)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cos(x))-\frac{1}{6}\log(1+2\cos(x))+\frac{1}{3}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[3*x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log[
Sin[x/2]]/3
```

Maple [A]
time = 0.35, size = 36, normalized size = 0.80

method	result	size
default	$\frac{\ln(2\cos(x)-1)}{6}-\frac{\ln(1+2\cos(x))}{6}+\frac{\ln(\cos(x)-1)}{6}-\frac{\ln(1+\cos(x))}{6}+\cos(x)$	36
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln(e^{ix}+1)}{3}+\frac{\ln(e^{ix}-1)}{3}+\frac{\ln(e^{2ix}-e^{ix}+1)}{6}-\frac{\ln(e^{2ix}+e^{ix}+1)}{6}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(2*cos(x)-1)-1/6*ln(1+2*cos(x))+1/6*ln(cos(x)-1)-1/6*ln(1+cos(x))+cos
(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(37) = 74$.
time = 0.48, size = 131, normalized size = 2.91

$$\cos(x) - \frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{12}\log\left(-2(\cos(x)-1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 - 2\sin(2x)\sin(x) + \sin(x)^2 - 2\cos(x) + 1\right) - \frac{1}{6}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{6}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x
)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(-2*(cos(x) -
1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(
x)^2 - 2*cos(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A]
time = 1.75, size = 39, normalized size = 0.87

$$\cos(x) - \frac{1}{6}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{6}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{6}\log(-2\cos(x) + 1) - \frac{1}{6}\log(-2\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(-
2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x)
```

```
[Out] Integral(cos(x)*cot(3*x), x)
```

Giac [A]
time = 0.44, size = 39, normalized size = 0.87

$$\cos(x) - \frac{1}{6}\log(\cos(x) + 1) + \frac{1}{6}\log(-\cos(x) + 1) - \frac{1}{6}\log(|2\cos(x) + 1|) + \frac{1}{6}\log(|2\cos(x) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(3*x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x)
+ 1)) + 1/6*log(abs(2*cos(x) - 1))
```

Mupad [B]
time = 2.37, size = 39, normalized size = 0.87

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{3} + \frac{\operatorname{atanh}\left(\frac{8}{183\left(\frac{488\tan\left(\frac{x}{2}\right)^2}{243} - \frac{56}{81}\right)} + \frac{121}{122}\right)}{3} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*cos(x),x)
```

```
[Out] log(tan(x/2))/3 + atanh(8/(183*((488*tan(x/2)^2)/243 - 56/81)) + 121/122)/3
 + 2/(tan(x/2)^2 + 1)
```

3.112 $\int \cos(x)\cot(4x)\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4}\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}+\cos(x)$$

```
[Out] -1/4*arctanh(cos(x))+cos(x)-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {1690, 1180, 213}

$$\cos(x)-\frac{1}{4}\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[4*x],x]
```

```
[Out] -1/4*ArcTanh[Cos[x]] - ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) + Cos[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \cos(x)\cot(4x)\,dx = -\text{Subst}\left(\int \frac{-1+8x^2-8x^4}{4-12x^2+8x^4}\,dx,x,\cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-1+\frac{3-4x^2}{4-12x^2+8x^4}\right)\,dx,x,\cos(x)\right)$$
$$= \cos(x)-\text{Subst}\left(\int \frac{3-4x^2}{4-12x^2+8x^4}\,dx,x,\cos(x)\right)$$
$$= \cos(x)+2\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\cos(x)\right)+2\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\cos(x)\right)$$
$$= -\frac{1}{4}\tanh^{-1}(\cos(x))-\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}+\cos(x)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 73, normalized size = 2.61

$$\frac{1}{4}\left((-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-(1-i)\sqrt[4]{-1}\tanh^{-1}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+4\cos(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[4*x],x]
```

```
[Out] ((-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^(1/4)*
ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]])
/4
```

Maple [A]
time = 0.46, size = 30, normalized size = 1.07

method	result	size
default	$\frac{\ln(\cos(x)-1)}{8}-\frac{\ln(1+\cos(x))}{8}-\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}+\cos(x)$	30
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln(e^{ix}-1)}{4}-\frac{\ln(e^{ix}+1)}{4}-\frac{\sqrt{2}\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{8}+\frac{\sqrt{2}\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{8}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(cos(x)-1)-1/8*ln(1+cos(x))-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)+cos(x
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(20) = 40$.
time = 0.49, size = 165, normalized size = 5.89

$$-\frac{1}{16}\,\sqrt{2}\,\log\left(2\,\sqrt{2}\,\sin\left(2\,x\right)\sin\left(x\right)+2\,\left(\sqrt{2}\,\cos\left(x\right)+1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+2\,\cos\left(x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\sin\left(x\right)^{2}+2\,\sqrt{2}\,\cos\left(x\right)+1\right)+\frac{1}{16}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\sin\left(2\,x\right)\sin\left(x\right)-2\,\left(\sqrt{2}\,\cos\left(x\right)-1\right)\cos\left(2\,x\right)+\cos\left(2\,x\right)^{2}+2\,\cos\left(x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\sin\left(x\right)^{2}-2\,\sqrt{2}\,\cos\left(x\right)+1\right)+\cos\left(x\right)-\frac{1}{8}\,\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)+\frac{1}{8}\,\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) + 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*co
s(x) + 1) + cos(x) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.
time = 1.59, size = 53, normalized size = 1.89

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{2\,\cos\left(x\right)^{2}-2\,\sqrt{2}\,\cos\left(x\right)+1}{2\,\cos\left(x\right)^{2}-1}\right)+\cos\left(x\right)-\frac{1}{8}\,\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{8}\,\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + cos
(x) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(x\right)\cot\left(4x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x)
```

```
[Out] Integral(cos(x)*cot(4*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.
time = 0.41, size = 50, normalized size = 1.79

$$\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\cos\left(x\right)\right|}{\left|2\,\sqrt{2}+4\,\cos\left(x\right)\right|}\right)+\cos\left(x\right)-\frac{1}{8}\,\log\left(\cos\left(x\right)+1\right)+\frac{1}{8}\,\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(4*x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) + cos
(x) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

Mupad [B]
time = 2.35, size = 67, normalized size = 2.39

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{7\sqrt{2}}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}-\frac{41\sqrt{2}\,\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}\right)}{4}+\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(4*x)*cos(x),x)
```

```
[Out] log(tan(x/2))/4 - (2^(1/2)*atanh((7*2^(1/2))/(8*((29*tan(x/2)^2)/4 - 5/4))
- (41*2^(1/2)*tan(x/2)^2)/(8*((29*tan(x/2)^2)/4 - 5/4))))/4 + 2/(tan(x/2)^2
 + 1)
```

3.113 $\int \cos(x)\cot(5x)\,dx$

Optimal. Leaf size=110

$$-\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(x)\right)$$

```
[Out] -1/5*arctanh(cos(x))+cos(x)+1/20*ln(1-4*cos(x)-5^(1/2))*(-5^(1/2)+1)-1/20*l
n(1+4*cos(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cos(x)+5^(1/2))*(5^(1/2)+1)-
1/20*ln(1+4*cos(x)+5^(1/2))*(5^(1/2)+1)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2100, 213, 646, 31}

$$\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\cos(x)+\sqrt{5}+1\right)-\frac{1}{5}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[5*x],x]
```

```
[Out] -1/5*ArcTanh[Cos[x]] + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]])
/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log[1
 - Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2100

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
```

```
x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(5x)\,dx &= -\text{Subst}\left(\int \frac{x^2(5-20x^2+16x^4)}{1-13x^2+28x^4-16x^6}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1-\frac{1}{5\left(-1+x^2\right)}-\frac{2(1+x)}{5\left(-1-2x+4x^2\right)}+\frac{2(-1+x)}{5\left(-1+2x+4x^2\right)}\right)\,dx, x, \text{co}\right.\\ &= \cos(x)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{2}{5}\text{Subst}\left(\int \frac{1+x}{-1-2x+4x^2}\,dx, x, \cos(\right.\\ &= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)-\frac{1}{5}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}+4x}\,dx, x, \cos(x)\right)+\\ &= -\frac{1}{5}\tanh^{-1}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)+\frac{1}{20}\left(1+\sqrt{5}\right.\end{aligned}$$

Mathematica [A]
time = 0.09, size = 133, normalized size = 1.21

$$\frac{1}{100}\left(100\cos(x)-20\log\left(\cos\left(\frac{x}{2}\right)\right)+\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)+\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(x)\right)-\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(x)\right)-\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(x)\right)+20\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[5*x],x]
```

```
[Out] (100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] - 4
*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-5
+ Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt
[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100
```

Maple [A]
time = 0.57, size = 82, normalized size = 0.75

method	result
default	$\frac{\ln(\cos(x)-1)}{10}-\frac{\ln(4(\cos^2(x))+2\cos(x)-1)}{20}-\frac{\sqrt{5}\,\text{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10}+\frac{\ln(4(\cos^2(x))-2\cos(x)-1)}{20}-\frac{\sqrt{5}\,\text{arcta}}{}$
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln(e^{ix}+1)}{5}+\frac{\ln(e^{ix}-1)}{5}+\frac{\ln\left(e^{2ix}+\frac{\left(\sqrt{5}-1\right)e^{ix}}{2}+1\right)}{20}-\frac{\ln\left(e^{2ix}+\frac{\left(\sqrt{5}-1\right)e^{ix}}{2}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{2ix}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*ln(cos(x)-1)-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*
(8*cos(x)+2)*5^(1/2))+1/20*ln(4*cos(x)^2-2*cos(x)-1)-1/10*5^(1/2)*arctanh(1
/10*(8*cos(x)-2)*5^(1/2))-1/10*ln(1+cos(x))+cos(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="maxima")
```

```
[Out] cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*a
rctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))
*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/
2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos
(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), c
os(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2
*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), c
os(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x
)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x)
, cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*
x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*
x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*
integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x)
, cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos
(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2
*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2
*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*
cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))
^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos
(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + si
n(2*x)^2 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*s
in(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^
2 + 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*
arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(
x)*sin(4*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x
)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*
cos(4*x) + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2
*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + s
in(x))*sin(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2
+ sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) - 1/10*inte
grate(-(cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(
x) + cos(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) +
cos(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - co
```

```
s(3*x)^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) -
sin(2*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) -
 sin(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x)
 + 3/10*integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3
*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)
))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(
2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3
*x))) + sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos
(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x
), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(
1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2
(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2
 + 2*(sin(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(s
in(3*x), cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))
*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))
)^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin
(3*x), cos(3*x)))^2 + 2*cos(3*x) + 1), x) + 3/10*integrate(-(cos(4/3*arctan
2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(
3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arc
tan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) +
 cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*
(cos(3*x) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x)
, cos(3*x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(si
n(3*x), cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) -
 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*
x)))^2 + 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*ar
ctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2
(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x
) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x
))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(s
in(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x),...
```

<u>Fricas</u> [A]
time = 1.77, size = 137, normalized size = 1.25

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}-1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}+\sqrt{5}-3}{4\cos\left(x\right)^{2}+2\cos\left(x\right)-1}\right)+\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}+1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}-\sqrt{5}-3}{4\cos\left(x\right)^{2}-2\cos\left(x\right)-1}\right)+\cos\left(x\right)-\frac{1}{20}\log\left(4\cos\left(x\right)^{2}+2\cos\left(x\right)-1\right)+\frac{1}{20}\log\left(4\cos\left(x\right)^{2}-2\cos\left(x\right)-1\right)-\frac{1}{10}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{10}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*co
s(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*cos
```

```
(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*cos
(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(1/2*
cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x)
```

```
[Out] Integral(cos(x)*cot(5*x), x)
```

Giac [**A**]
time = 0.42, size = 117, normalized size = 1.06

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)+2\right|}{\left|2\sqrt{5}+8\cos(x)+2\right|}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)-2\right|}{\left|2\sqrt{5}+8\cos(x)-2\right|}\right)+\cos(x)-\frac{1}{10}\log(\cos(x)+1)+\frac{1}{10}\log(-\cos(x)+1)-\frac{1}{20}\log\left(\left|4\cos(x)^2+2\cos(x)-1\right|\right)+\frac{1}{20}\log\left(\left|4\cos(x)^2-2\cos(x)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(5*x),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) +
2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*cos
(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20*lo
g(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x) - 1)
)
```

Mupad [**B**]
time = 3.03, size = 611, normalized size = 5.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*cos(x),x)
```

```
[Out] (atan((tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))
/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (495922908
5483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) - 954873235374
08i/(244140625*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^
(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 1
10872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*((21348
5644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220
703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/2441406
25)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((213485644414976
```

*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (
4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625))*1i)/
10 + (atan(95487323537408i/(244140625*((213485644414976*5^(1/2))/1220703125
 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan
(x/2)^2)/1220703125 - 110872433262592/244140625)) - (5^(1/2)*24788779558502
4i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5
^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 -
110872433262592/244140625)) - (tan(x/2)^2*4813499234516992i)/(1220703125*((
213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)
/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/24
4140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((2134856444
14976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/122070312
5 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)))
1i)/10 + log(tan(x/2))/5 + 2/(tan(x/2)^2 + 1) + (5^(1/2)(atan((tan(x/2)^2
4813499234516992i)/(1220703125((213485644414976*5^(1/2))/1220703125 - (21
52646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^
2)/1220703125 + 110872433262592/244140625)) - 95487323537408i/(244140625*((
213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)
/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/24
4140625)) - (5^(1/2)*247887795585024i)/(1220703125*((213485644414976*5^(1/2
))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229
085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) + (5^(1/2)*t
an(x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^(1/2))/1220703
125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*
tan(x/2)^2)/1220703125 + 110872433262592/244140625))) - atan(95487323537408
i/(244140625*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1
/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110
872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*((2134856
44414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/122070
3125 + (4959229085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625
)) - (tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2))/
1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085
483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)) + (5^(1/2)*tan(
x/2)^2*2217818569310208i)/(1220703125*((213485644414976*5^(1/2))/1220703125
 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan
(x/2)^2)/1220703125 - 110872433262592/244140625))))*1i)/10

3.114 $\int \cos(x)\cot(6x)\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cos(x)$$

```
[Out] -1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+cos(x)-1/6*arctanh(2/3*cos(x)*3^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 2098, 213}

$$\cos(x)-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[6*x],x]
```

```
[Out] -1/6*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/6 - ArcTanh[(2*Cos[x])/Sqrt[3]]/(2
*Sqrt[3]) + Cos[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2098

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \cos(x)\cot(6x)\,dx = -\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{2(3-19x^2+32x^4-16x^6)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+18x^2-48x^4+32x^6}{3-19x^2+32x^4-16x^6}\,dx, x, \cos(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-2-\frac{1}{3(-1+x^2)}-\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right)dx, x, \cos(x)\right)\right)$$
$$= \cos(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cos(x)\right)+$$
$$= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))-\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}+\cos(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(38) = 76.
time = 0.07, size = 87, normalized size = 2.29

$$\frac{1}{12}\left(2\sqrt{3}\tanh^{-1}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\sqrt{3}\tanh^{-1}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+12\cos(x)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(1+2\cos(x))+2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[6*x],x]
```

```
[Out] (2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x/
2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*
Cos[x]] + 2*Log[Sin[x/2]])/12
```

Maple [A]
time = 0.65, size = 49, normalized size = 1.29

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(2\cos(x)-1)}{12}-\frac{\ln(1+2\cos(x))}{12}+\frac{\ln(\cos(x)-1)}{12}-\frac{\ln(1+\cos(x))}{12}+\cos(x)$
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln(e^{ix}-1)}{6}-\frac{\ln(e^{ix}+1)}{6}-\frac{\ln(e^{2ix}+e^{ix}+1)}{12}+\frac{\ln(e^{2ix}-e^{ix}+1)}{12}-\frac{\sqrt{3}\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\sqrt{3}\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)+1/12*ln(2*cos(x)-1)-1/12*ln(1+2*co
s(x))+1/12*ln(cos(x)-1)-1/12*ln(1+cos(x))+cos(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="maxima")
```

```
[Out] cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*
sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) +
cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) -
 1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.
time = 1.84, size = 71, normalized size = 1.87

$$\frac{1}{12}\sqrt{3}\log\left(\frac{4\cos(x)^2 - 4\sqrt{3}\cos(x) + 3}{4\cos(x)^2 - 3}\right) + \cos(x) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log(-2\cos(x) + 1) - \frac{1}{12}\log(-2\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + co
s(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(
-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(6x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x)
```

```
[Out] Integral(cos(x)*cot(6*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.
time = 0.42, size = 70, normalized size = 1.84

$$\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3} + 8\cos(x)\right|}{\left|4\sqrt{3} + 8\cos(x)\right|}\right) + \cos(x) - \frac{1}{12}\log(\cos(x) + 1) + \frac{1}{12}\log(-\cos(x) + 1) - \frac{1}{12}\log(|2\cos(x) + 1|) + \frac{1}{12}\log(|2\cos(x) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(6*x),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + co
s(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x)
 + 1)) + 1/12*log(abs(2*cos(x) - 1))
```

Mupad [B]
time = 2.44, size = 86, normalized size = 2.26

$$\frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{303181204553728\sqrt{3}}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}-\frac{4222769432625152\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}\right)}{6}+\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*cos(x),x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 - (3^(1/2)*atanh((30
3181204553728*3^(1/2))/(4782969*((7314051205955584*tan(x/2)^2)/4782969 - 52
5125250187264/4782969)) - (4222769432625152*3^(1/2)*tan(x/2)^2)/(4782969*((
7314051205955584*tan(x/2)^2)/4782969 - 525125250187264/4782969))))/6 + 2/(t
an(x/2)^2 + 1)
```

3.115 $\int \cos(x)\cot(nx)\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}e^{-ix}+\frac{e^{ix}}{2}+e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)$$

```
[Out] -1/2/exp(I*x)+1/2*exp(I*x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*I*n*x))/ex
p(I*x)-exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*I*n*x))
```

Rubi [A]

time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4654, 2225, 2283}

$$e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[n*x],x]
```

```
[Out] -1/2*1/E^(I*x) + E^(I*x)/2 + Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E^
((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2
, E^((2*I)*n*x)]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4654

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cot(nx)\,dx &= \int \left(\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}-\frac{ie^{-ix}}{1-e^{2inx}}-\frac{ie^{ix}}{1-e^{2inx}}\right)dx\\ &= \frac{1}{2}i\int e^{-ix}\,dx+\frac{1}{2}i\int e^{ix}\,dx-i\int \frac{e^{-ix}}{1-e^{2inx}}\,dx-i\int \frac{e^{ix}}{1-e^{2inx}}\,dx\\ &= -\frac{1}{2}e^{-ix}+\frac{e^{ix}}{2}+e^{-ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{ix}\,{}_2F_1\left(1,\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);e^{2inx}\right)\end{aligned}$$

Mathematica [A]
time = 0.13, size = 179, normalized size = 1.95

$$\frac{1}{2}e^{-2ix}\left(-\frac{e^{i(x+2nx)}\,{}_2F_1\left(1,1-\frac{1}{2n};2-\frac{1}{2n};e^{2inx}\right)}{-1+2n}-\frac{e^{i(3+2n)x}\,{}_2F_1\left(1,1+\frac{1}{2n};2+\frac{1}{2n};e^{2inx}\right)}{1+2n}+e^{ix}\,{}_2F_1\left(1,-\frac{1}{2n};1-\frac{1}{2n};e^{2inx}\right)-e^{3ix}\,{}_2F_1\left(1,\frac{1}{2n};1+\frac{1}{2n};e^{2inx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[n*x],x]
```

```
[Out] (-((E^(I*(x + 2*n*x))*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^((2*
I)*n*x)])/(-1 + 2*n)) - (E^(I*(3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*n)
, 2 + 1/(2*n), E^((2*I)*n*x)])/(1 + 2*n) + E^(I*x)*Hypergeometric2F1[1, -1/
2*1/n, 1 - 1/(2*n), E^((2*I)*n*x)] - E^((3*I)*x)*Hypergeometric2F1[1, 1/(2*
n), 1 + 1/(2*n), E^((2*I)*n*x)])/(2*E^((2*I)*x))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cot(n*x),x)
```

```
[Out] int(cos(x)*cot(n*x),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*cot(n*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="fricas")
[Out] integral(cos(x)*cot(n*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\cot(nx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x)
[Out] Integral(cos(x)*cot(n*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cot(n*x),x, algorithm="giac")
[Out] integrate(cos(x)*cot(n*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(n\,x)\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(n*x)*cos(x),x)
[Out] int(cot(n*x)*cos(x), x)
```

3.116 $\int \cos(x)\sec(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4441, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(2x)\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\sin(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[2*x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

Maple [A]
time = 0.16, size = 13, normalized size = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(12) = 24$.
time = 0.49, size = 137, normalized size = 9.13

$$\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)+\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(
x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sq
rt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*s
qrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 1.55, size = 33, normalized size = 2.20

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x)
```

```
[Out] Integral(cos(x)*sec(2*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.42, size = 31, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt(
2) + sin(x)))
```

Mupad [B]
time = 0.11, size = 12, normalized size = 0.80

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(2*x),x)
```

```
[Out] (2^(1/2)*atanh(2^(1/2)*sin(x)))/2
```

3.117 $\int \cos(x)\sec(3x)\,dx$

Optimal. Leaf size=44

$$-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)+1/6*ln(cos(x)+sin(x)*3^(1/2))*3^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {212}

$$\frac{\log\left(\sqrt{3}\sin(x)+\cos(x)\right)}{2\sqrt{3}}-\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[3*x],x]
```

```
[Out] -1/2*Log[Cos[x] - Sqrt[3]*Sin[x]]/Sqrt[3] + Log[Cos[x] + Sqrt[3]*Sin[x]]/(2
*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(3x)\,dx &= \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \tan(x)\right)\\ &= -\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.34

$$\frac{\tanh^{-1}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[3*x],x]
```

```
[Out] ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

Maple [A]
time = 0.20, size = 13, normalized size = 0.30

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\tan(x)\sqrt{3}\right)}{3}$	13
risch	$\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*arctanh(tan(x)*3^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.
time = 0.48, size = 76, normalized size = 1.73

$$\frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2+\frac{4}{3}\sin(2x)^2+\frac{4}{3}\sqrt{3}\sin(2x)-\frac{4}{3}\cos(2x)+\frac{4}{3}\right)-\log\left(\frac{4}{3}\cos(2x)^2+\frac{4}{3}\sin(2x)^2-\frac{4}{3}\sqrt{3}\sin(2x)-\frac{4}{3}\cos(2x)+\frac{4}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 + 4/3*sqrt(3)*sin(2*x) -
4/3*cos(2*x) + 4/3) - log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 - 4/3*sqrt(3)*sin
(2*x) - 4/3*cos(2*x) + 4/3))
```

Fricas [A]
time = 3.15, size = 53, normalized size = 1.20

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4+4\left(2\sqrt{3}\cos(x)^3-3\sqrt{3}\cos(x)\right)\sin(x)-9}{16\cos(x)^4-24\cos(x)^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(8*cos(x)^4 + 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*s
in(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x) \sec(3x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x)
[Out] Integral(cos(x)*sec(3*x), x)
```

Giac [**A**]
time = 0.43, size = 31, normalized size = 0.70

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(3*x),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x)))
```

Mupad [**B**]
time = 2.66, size = 16, normalized size = 0.36

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sin(x)}{\cos(x)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(3*x),x)
[Out] (3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x)))/3
```

3.118 $\int \cos(x)\sec(4x)\,dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

```
[Out] 1/2*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*s
in(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4441, 1107, 213}

$$\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[4*x],x]
```

```
[Out] ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[(
2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
```

```
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(4x)\,dx &= \text{Subst}\left(\int \frac{1}{1-8x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4-2\sqrt{2}+8x^2}\,dx, x, \sin(x)\right) - \sqrt{2}\,\text{Subst}\left(\int \frac{1}{-4+2\sqrt{2}+8x^2}\,d\right.\\ &= \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 91, normalized size = 1.28

$$\frac{1}{16}\text{RootSum}\left[1+\#1^8\&, \frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right) - i\log\left(1-2\cos(x)\#1+\#1^2\right) + 2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2 - i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2}{\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[4*x],x]
```

```
[Out] RootSum[1 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#
1 + #1^2] + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - I*Log[1 - 2*Cos[x]*#1 + #
1^2]*#1^2)/#1^5 & ]/16
```

Maple [A]
time = 0.29, size = 54, normalized size = 0.76

method	result	size
risch	$2\left(\sum_{\_R=\text{RootOf}(32768\_Z^4-512\_Z^2+1)} \_R\ln\left(e^{2ix}+\left(4096i\_R^3-48i\_R\right)e^{ix}-1\right)\right)$	46
default	$\frac{\sqrt{2}\,\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}} - \frac{\sqrt{2}\,\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-1/4*2^(1/
2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(4*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.
time = 2.40, size = 121, normalized size = 1.70

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\sin\left(x\right)\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\sin\left(x\right)\right)-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\sin\left(x\right)\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/8
*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*sq
rt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*sqr
t(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(x\right)\sec\left(4x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x)
```

```
[Out] Integral(cos(x)*sec(4*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.
time = 0.45, size = 99, normalized size = 1.39

$$-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right)-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt
(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(2)
 + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2)*l
og(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))
```

Mupad [B]
time = 2.27, size = 95, normalized size = 1.34

$$\frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{\sqrt{2}+2}+2\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{\sqrt{2}+2}\right)\sqrt{\sqrt{2}+2}}{4}-\frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{2-\sqrt{2}}-2\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{\sqrt{2}-2}\right)\sqrt{2-\sqrt{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(4*x),x)
```

```
[Out] (atanh((2*sin(x)*(2^(1/2) + 2)^(1/2) + 2*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/2)
)/(2^(1/2) + 2))*(2^(1/2) + 2)^(1/2))/4 - (atanh((2*sin(x)*(2 - 2^(1/2))^(1
/2) - 2*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(2^(1/2) - 2))*(2 - 2^(1/2))^(1
/2))/4
```

3.119 $\int \cos(x)\sec(5x)\,dx$

Optimal. Leaf size=163

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\,\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\cos(x)+\sqrt{5-2\sqrt{5}}\,\sin(\right.$$

[Out] 1/20*ln(cos(x)-sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)+sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)-sin(x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)+1/20*ln(cos(x)+sin(x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1180, 213}

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\,\sin(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\sqrt{5-2\sqrt{5}}\,\sin(x)+\cos(x)\right)-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\log\left(\cos(x)-\sqrt{5+2\sqrt{5}}\,\sin(x)\right)+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\log\left(\sqrt{5+2\sqrt{5}}\,\sin(x)+\cos(x)\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sec[5*x],x]

[Out] (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 - Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 - 2*Sqrt[5]]*Sin[x]])/10 - (Sqrt[(5 + Sqrt[5])/2]*Log[Cos[x] - Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10 + (Sqrt[(5 + Sqrt[5])/2]*Log[Cos[x] + Sqrt[5 + 2*Sqrt[5]]*Sin[x]])/10

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \cos(x)\sec(5x)\,dx = \text{Subst}\left(\int \frac{1+x^2}{1-10x^2+5x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-5+2\sqrt{5}+5x^2}\,dx, x, \tan(x)\right) + \frac{1}{2}\left(1+\sqrt{5}\right)\text{Subst}\left(\int -\right.$$
$$= \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right) - \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\mathrm{c}\right.$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.52

$$\frac{\sqrt{5+\sqrt{5}}\tanh^{-1}\left(\frac{\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\tanh^{-1}\left(\frac{\left(-5+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[5*x],x]
```

```
[Out] (Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] + S
qrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/(5
*Sqrt[2])
```

Maple [A]
time = 0.31, size = 68, normalized size = 0.42

method	result	size
risch	$2\left(\sum_{\_R=\text{RootOf}\left(32000\_Z^4-400\_Z^2+1\right)}\_R\ln\left(e^{2ix}+4000i\_R^3-200\_R^2-30i\_R+1\right)\right)$	44
default	$-\frac{\sqrt{5}\left(\sqrt{5}-5\right)\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)}{10\sqrt{25-10\sqrt{5}}}-\frac{\left(5+\sqrt{5}\right)\sqrt{5}\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)}{10\sqrt{25+10\sqrt{5}}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*5^(1/2)*(5^(1/2)-5)/(25-10*5^(1/2))^(1/2)*arctanh(5*tan(x)/(25-10*5^(
1/2))^(1/2))-1/10*(5+5^(1/2))*5^(1/2)/(25+10*5^(1/2))^(1/2)*arctanh(5*tan(x
)/(25+10*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)*sec(5*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(111) = 222$.
time = 2.48, size = 231, normalized size = 1.42

$$-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)-\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}+5\right)+\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="fricas")
```

```
[Out] -1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5
) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) + 1/40*sqrt(
2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*cos
(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) - 1/40*sqrt(2)*sqrt(-s
qrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x
) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) + 5) + 1/40*sqrt(2)*sqrt(-sqrt(5) +
5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x) + 2*(s
qrt(5) - 1)*cos(x)^2 - sqrt(5) + 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(x)\sec(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x)
```

```
[Out] Integral(cos(x)*sec(5*x), x)
```

Giac [A]
time = 0.47, size = 105, normalized size = 0.64

$$-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(5*x),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20
*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20*sqr
t(2*sqrt(5) + 10)*log(abs(sqrt(-2/5*sqrt(5) + 1) + tan(x))) - 1/20*sqrt(2*s
qrt(5) + 10)*log(abs(-sqrt(-2/5*sqrt(5) + 1) + tan(x)))
```

Mupad [B]
time = 2.67, size = 217, normalized size = 1.33

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)\,\sqrt{5-\sqrt{5}}}{5\,\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{25}-\frac{55834574848\,{\tan\left(\frac{x}{2}\right)}^2}{5}+\frac{55834574848}{5}\right)}-\frac{77309411328\,\sqrt{2}\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\,\sqrt{5-\sqrt{5}}}{25\,\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{25}-\frac{55834574848\,{\tan\left(\frac{x}{2}\right)}^2}{5}+\frac{55834574848}{5}\right)}\right)\,\sqrt{5-\sqrt{5}}}{10}-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\,\sqrt{2}\,\sqrt{5}\,\tan\left(\frac{x}{2}\right)\,\sqrt{\sqrt{5}+5}}{25\,\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{25}+\frac{55834574848\,{\tan\left(\frac{x}{2}\right)}^2}{5}-\frac{55834574848}{5}\right)}-\frac{34359738368\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)\,\sqrt{\sqrt{5}+5}}{5\,\left(\frac{124554051584\,\sqrt{5}}{25}-\frac{124554051584\,\sqrt{5}\,{\tan\left(\frac{x}{2}\right)}^2}{25}+\frac{55834574848\,{\tan\left(\frac{x}{2}\right)}^2}{5}-\frac{55834574848}{5}\right)}\right)\,\sqrt{\sqrt{5}+5}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(5*x),x)
```

```
[Out] (2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(5*((12
4554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2)^2)/25 - (5583457484
8*tan(x/2)^2)/5 + 55834574848/5)) - (77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(
5 - 5^(1/2))^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*
tan(x/2)^2)/25 - (55834574848*tan(x/2)^2)/5 + 55834574848/5)))*(5 - 5^(1/2)
)^(1/2))/10 - (2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2)
 + 5)^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2
)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)) - (34359738368*2^(1/
2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(5*((124554051584*5^(1/2))/25 - (124554051
584*5^(1/2)*tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)))*
(5^(1/2) + 5)^(1/2))/10
```

3.120 $\int \cos(x)\sec(6x)\,dx$

Optimal. Leaf size=85

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

```
[Out] -1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)-1/2*
2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)+1/2*2
^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4441, 2082, 213, 1180}

$$-\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[6*x],x]
```

```
[Out] -1/3*ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]]
]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2 +
 Sqrt[3]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\sec(6x)\,dx &= \text{Subst}\left(\int \frac{1}{1-18x^2+48x^4-32x^6}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{3(-1+2x^2)} - \frac{4(-1+2x^2)}{3(1-16x^2+16x^4)}\right)\,dx, x, \sin(x)\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sin(x)\right) - \frac{4}{3}\text{Subst}\left(\int \frac{-1+2x^2}{1-16x^2+16x^4}\,dx, x, \sin(x)\right)\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}} - \frac{4}{3}\text{Subst}\left(\int \frac{1}{-8-4\sqrt{3}+16x^2}\,dx, x, \sin(x)\right) - \frac{4}{3}\text{Subst}\Bigg(\\
&= -\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\tanh^{-1}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.29, size = 356, normalized size = 4.19

$$\frac{1}{24}\left(\sqrt{2}\left(2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)+2i\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{x}{2}\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}\right)-2\log\left(\sqrt{2}+2\sin(x)\right)+\log\left(2-\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)+\log\left(2+\sqrt{2}\cos(x)-\sqrt{2}\sin(x)\right)\right)+\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)+2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^2+2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^4-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^4+2\text{ArcTan}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^6-i\log\left(1-2\cos(x)\#1+\#1^2\right)\#1^6}{-\#1^3+2\#1^7}\&\right]\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[6*x],x]
```

```
[Out] (Sqrt[2]*((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*
Cos[x/2] - Sin[x/2])] + (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((
-1 + Sqrt[2])*Cos[x/2] - Sin[x/2])] - 2*Log[Sqrt[2] + 2*Sin[x]] + Log[2 - S
qrt[2]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])
 + RootSum[1 - #1^4 + #1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 -
```

```
2*Cos[x]*#1 + #1^2] + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - I*Log[1 - 2*Cos
[x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - I*Log[1 - 2*Cos
[x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - I*Log[1 - 2*Cos
[x]*#1 + #1^2]*#1^6)/(-#1^3 + 2*#1^7) & ])/24
```

Maple [A]
time = 0.37, size = 80, normalized size = 0.94

method	result
default	$-\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}+\frac{2\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}+\frac{2\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}$
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{12}+2\left(\sum_{\_R=\operatorname{RootOf}(331776\_Z^4-2304\_Z^2+1)}\_R\ln\left(e^{2ix}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*si
n(x)/(2*6^(1/2)-2*2^(1/2)))+2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*sin(x)/(2*6
^(1/2)+2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x) + co
s(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos(4*x)
- 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x) + sin(3
*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*sin(4*x) -
 sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*
x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4
*x) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

time = 2.22, size = 154, normalized size = 1.81

$$-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\sin\left(x\right)\right)-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\sin\left(x\right)\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^2+2\sqrt{2}\sin\left(x\right)-3}{2\cos\left(x\right)^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) + 1
/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*sin(x)) - 1/1
2*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*sin(x)) + 1/1
2*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(x\right)\sec\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x)
```

```
[Out] Integral(cos(x)*sec(6*x), x)
```

Giac [A]
time = 0.46, size = 132, normalized size = 1.55

$$\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)+\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(6*x),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/2
4*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24*(
sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*(sq
rt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*sqrt(
2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))
```

Mupad [B]
time = 2.29, size = 118, normalized size = 1.39

$$\mathrm{atanh}\left(\frac{5\sqrt{2}\sin\left(x\right)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}+\frac{3\sqrt{6}\sin\left(x\right)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right)-\mathrm{atanh}\left(\frac{5\sqrt{2}\sin\left(x\right)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}-\frac{3\sqrt{6}\sin\left(x\right)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right)-\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\sin\left(x\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(6*x),x)
```

```
[Out] atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
+ (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(2^
(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)) - (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4
194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/2)*si
n(x)))/6
```

3.121 $\int \cos(2x)\sec(x)\,dx$

Optimal. Leaf size=10

$$-\tanh^{-1}(\sin(x))+2\sin(x)$$

```
[Out] -arctanh(sin(x))+2*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4449, 396, 212}

$$2\sin(x)-\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sec[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \cos(2x)\sec(x)\,dx = \text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(x)\right)$$
$$= 2\sin(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)$$
$$= -\tanh^{-1}(\sin(x)) + 2\sin(x)$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\tanh^{-1}(\sin(x)) + 2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sec[x],x]
[Out] -ArcTanh[Sin[x]] + 2*Sin[x]
```

Maple [A]
time = 0.15, size = 14, normalized size = 1.40

method	result	size
default	$-\ln(\sec(x)+\tan(x))+2\sin(x)$	14
risch	$-i e^{ix}+i e^{-ix}-\ln(e^{ix}+i)+\ln(e^{ix}-i)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
[Out] -ln(sec(x)+tan(x))+2*sin(x)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.90

$$-\frac{1}{2}\log(\sin(x)+1) + \frac{1}{2}\log(\sin(x)-1) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="maxima")
[Out] -1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 1.69, size = 21, normalized size = 2.10

$$-\frac{1}{2}\log(\sin(x)+1) + \frac{1}{2}\log(-\sin(x)+1) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.
time = 0.62, size = 20, normalized size = 2.00

$$\frac{\log\left(\sin\left(x\right)-1\right)}{2}-\frac{\log\left(\sin\left(x\right)+1\right)}{2}+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.41, size = 21, normalized size = 2.10

$$-\frac{1}{2}\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)+2\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sec(x),x, algorithm="giac")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

Mupad [B]
time = 2.25, size = 10, normalized size = 1.00

$$2\sin\left(x\right)-\mathrm{atanh}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/cos(x),x)
```

```
[Out] 2*sin(x) - atanh(sin(x))
```

3.122 $\int \cos(4x)\sec(2x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}(\sin(2x))+\sin(2x)$$

```
[Out] -1/2*arctanh(sin(2*x))+sin(2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4449, 396, 212}

$$\sin(2x)-\frac{1}{2}\tanh^{-1}(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[2*x],x]
```

```
[Out] -1/2*ArcTanh[Sin[2*x]] + Sin[2*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 4449

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\int \cos(4x)\sec(2x)\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1-2x^2}{1-x^2}\,dx, x, \sin(2x)\right) \\
&= \sin(2x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(2x)\right) \\
&= -\frac{1}{2}\tanh^{-1}(\sin(2x)) + \sin(2x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}(\sin(2x)) + \sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[2*x],x]
```

```
[Out] -1/2*ArcTanh[Sin[2*x]] + Sin[2*x]
```

Maple [A]
time = 0.15, size = 18, normalized size = 1.29

method	result	size
default	$-\frac{\ln(\sec(2x)+\tan(2x))}{2} + \sin(2x)$	18
risch	$-\frac{i e^{2ix}}{2} + \frac{i e^{-2ix}}{2} - \frac{\ln(e^{2ix}+i)}{2} + \frac{\ln(e^{2ix}-i)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*sec(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(sec(2*x)+tan(2*x))+sin(2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.
time = 0.49, size = 129, normalized size = 9.21

$$\frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2)
- 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2
```

```
) - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) +
 2) + 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x)
 + 2) + sin(2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.
time = 2.18, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2x\right)+1\right)+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(12) = 24$.
time = 2.09, size = 427, normalized size = 30.50

$$-4x+\frac{32x\tan^{4}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}+\frac{64x\tan^{2}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}+\frac{32x}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}-\frac{3\log\left(\tan^{2}\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}+\frac{3\log\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}+\frac{8\log\left(\tan^{2}\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)\tan^{4}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}+\frac{16\log\left(\tan^{2}\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)\tan^{2}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}+\frac{8\log\left(\tan^{2}\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}-\frac{8\log\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)\tan^{4}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}-\frac{16\log\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)\tan^{2}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}-\frac{8\log\left(\tan^{2}\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}-\frac{32\tan^{3}\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}+\frac{32\tan\left(\frac{x}{2}\right)}{8\tan^{4}\left(\frac{x}{2}\right)+16\tan^{2}\left(\frac{x}{2}\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x)
```

```
[Out] -4*x + 32*x*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 64*x*tan(x/2
)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*x/(8*tan(x/2)**4 + 16*tan(x/
2)**2 + 8) - 3*log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 + 3*log(tan(x/2)**2 + 2*
tan(x/2) - 1)/2 + 8*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(8*tan(x/
2)**4 + 16*tan(x/2)**2 + 8) + 16*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)
**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 8*log(tan(x/2)**2 - 2*tan(x/2) -
 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 16*log(tan(x/2)**2 +
 2*tan(x/2) - 1)*tan(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(t
an(x/2)**2 + 2*tan(x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 32*tan(
x/2)**3/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*tan(x/2)/(8*tan(x/2)**4 +
 16*tan(x/2)**2 + 8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.
time = 0.39, size = 25, normalized size = 1.79

$$-\frac{1}{4}\log\left(\sin\left(2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2x\right)+1\right)+\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)*sec(2*x),x, algorithm="giac")
```

```
[Out] -1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\sin(2x) - \frac{\operatorname{atanh}(\sin(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(2*x),x)
```

```
[Out] sin(2*x) - atanh(sin(2*x))/2
```

3.123 $\int \cos(x)\csc(2x)\,dx$

Optimal. Leaf size=7

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -1/2*arctanh(cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4372, 3855}

$$-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[2*x],x]
```

```
[Out] -1/2*ArcTanh[Cos[x]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(2x)\,dx &= \frac{1}{2}\int \csc(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.
time = 0.00, size = 21, normalized size = 3.00

$$\frac{1}{2}\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[2*x],x]
```

```
[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]])/2
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.57

method	result	size
default	$\frac{\ln(-\cot(x)+\csc(x))}{2}$	11
risch	$\frac{\ln(e^{ix}-1)}{2}-\frac{\ln(e^{ix}+1)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-cot(x)+csc(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.
time = 0.28, size = 35, normalized size = 5.00

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.
time = 2.32, size = 19, normalized size = 2.71

$$-\frac{1}{4}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.51, size = 15, normalized size = 2.14

$$\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.
time = 0.42, size = 17, normalized size = 2.43

$$-\frac{1}{4}\log\left(\cos\left(x\right)+1\right)+\frac{1}{4}\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(2*x),x, algorithm="giac")
```

```
[Out] -1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.71

$$-\frac{\mathrm{atanh}(\cos\left(x\right))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x),x)
```

```
[Out] -atanh(cos(x))/2
```

3.124 $\int \cos(x)\csc(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

```
[Out] 1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.714$, Rules used = {4441, 272, 36, 31, 29}

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
```

```
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\csc(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3-4x^2\right)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{1}{3}\log(\sin(x))-\frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x],x]
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Maple [A]
time = 0.23, size = 34, normalized size = 1.62

method	result	size
risch	$\frac{\ln\left(e^{2 i x}-1\right)}{3}-\frac{\ln\left(e^{4 i x}+e^{2 i x}+1\right)}{6}$	27
default	$\frac{\ln(1+\cos(x))}{6}-\frac{\ln(2\cos(x)-1)}{6}-\frac{\ln(1+2\cos(x))}{6}+\frac{\ln(\cos(x)-1)}{6}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(3*x),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(1+cos(x))-1/6*ln(2*cos(x)-1)-1/6*ln(1+2*cos(x))+1/6*ln(cos(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(17) = 34.
time = 0.50, size = 129, normalized size = 6.14

$$-\frac{1}{12}\log\left(2\left(\cos(x)+1\right)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2+2\sin(2x)\sin(x)+\sin(x)^2+2\cos(x)+1\right)-\frac{1}{12}\log\left(-2\left(\cos(x)-1\right)\cos(2x)+\cos(2x)^2+\cos(x)^2+\sin(2x)^2-2\sin(2x)\sin(x)+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{6}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{6}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A]
time = 2.40, size = 19, normalized size = 0.90

$$-\frac{1}{6}\log\left(4\cos\left(x\right)^{2}-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A]
time = 0.68, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^{2}\left(x\right)-3\right)}{6}+\frac{\log\left(\sin\left(x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

Giac [A]
time = 0.41, size = 24, normalized size = 1.14

$$\frac{1}{6}\log\left(-\cos\left(x\right)^{2}+1\right)-\frac{1}{6}\log\left(\left|4\cos\left(x\right)^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(3*x),x, algorithm="giac")
```

```
[Out] 1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

Mupad [B]
time = 0.10, size = 17, normalized size = 0.81

$$\frac{\ln\left(\sin\left(x\right)\right)}{3}-\frac{\ln\left(\frac{1}{4}-{\cos\left(x\right)}^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x)
```

```
[Out] log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

3.125 $\int \cos(x)\csc(4x)\,dx$

Optimal. Leaf size=26

$$-\frac{1}{4}\tanh^{-1}(\cos(x))+\frac{\tanh^{-1}\left(\sqrt{2}\,\cos(x)\right)}{2\sqrt{2}}$$

```
[Out] -1/4*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1107, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[4*x],x]
```

```
[Out] -1/4*ArcTanh[Cos[x]] + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\csc(4x)\,dx &= -\text{Subst}\left(\int \frac{1}{-4+12x^2-8x^4}\,dx,x,\cos(x)\right)\\
&= 2\text{Subst}\left(\int \frac{1}{4-8x^2}\,dx,x,\cos(x)\right)-2\text{Subst}\left(\int \frac{1}{8-8x^2}\,dx,x,\cos(x)\right)\\
&= -\frac{1}{4}\tanh^{-1}(\cos(x))+\frac{\tanh^{-1}\left(\sqrt{2}\,\cos(x)\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 66, normalized size = 2.54

$$\frac{1}{4}\left((1+i)(-1)^{3/4}\tanh^{-1}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\sqrt{2}\tanh^{-1}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[4*x],x]
[Out] ((1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + Sqrt[2]*ArcTanh[(1 +
 Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]])/4
```

Maple [A]
time = 0.28, size = 28, normalized size = 1.08

method	result	size
default	$\frac{\ln(\cos(x)-1)}{8}-\frac{\ln(1+\cos(x))}{8}+\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}$	28
risch	$\frac{\ln(e^{ix}-1)}{4}-\frac{\ln(e^{ix}+1)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{8}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(4*x),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(cos(x)-1)-1/8*ln(1+cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(18) = 36.
time = 0.49, size = 163, normalized size = 6.27

$$\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x)+2\left(\sqrt{2}\cos(x)+1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+1\right)-\frac{1}{16}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x)-2\left(\sqrt{2}\cos(x)-1\right)\cos(2x)+\cos(2x)^2+2\cos(x)^2+\sin(2x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+1\right)-\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="maxima")
[Out] 1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*c
os(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos
(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.
time = 2.81, size = 52, normalized size = 2.00

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\frac{1}{8}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{8}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1/
8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(22) = 44$.
time = 3.04, size = 248, normalized size = 9.54

$$-\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808}-\frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808}+\frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808}+\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808}+\frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808}+\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808}-\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}-\frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}+\frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2}+156808}+\frac{39202\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2}+156808}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x)
```

```
[Out] -19601*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 2772
0*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2
) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) + 1
 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) - sqrt(2) - 1)/(
110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/(11088
0*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110880*sqr
t(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) + 156808
) + 27720*sqrt(2)*log(tan(x/2))/(110880*sqrt(2) + 156808) + 39202*log(tan(x
/2))/(110880*sqrt(2) + 156808)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.
time = 0.42, size = 48, normalized size = 1.85

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos\left(x\right)\right|}{\left|2\sqrt{2}+4\cos\left(x\right)\right|}\right)-\frac{1}{8}\log\left(\cos\left(x\right)+1\right)+\frac{1}{8}\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) - 1/
8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

Mupad [B]
time = 2.31, size = 55, normalized size = 2.12

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4}+\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{41\sqrt{2}}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}-\frac{239\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(4*x),x)
```

```
[Out] log(tan(x/2))/4 + (2^(1/2)*atanh((41*2^(1/2))/(8*((169*tan(x/2)^2)/4 - 29/4
)) - (239*2^(1/2)*tan(x/2)^2)/(8*((169*tan(x/2)^2)/4 - 29/4))))/4
```

3.126 $\int \cos(x)\csc(5x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{5}\log(\sin(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\sin^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\sin^2(x)\right)$$

```
[Out] 1/5*ln(sin(x))-1/20*ln(5-8*sin(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*sin(x
)^2-5^(1/2))*(5^(1/2)+1)
```

Rubi [A]
time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$,
Rules used = {4441, 1128, 719, 29, 646, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-8\sin^2(x)-\sqrt{5}+5\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-8\sin^2(x)+\sqrt{5}+5\right)+\frac{1}{5}\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[5*x],x]
```

```
[Out] Log[Sin[x]]/5 - ((1 + Sqrt[5])*Log[5 - Sqrt[5] - 8*Sin[x]^2])/20 - ((1 - Sq
rt[5])*Log[5 + Sqrt[5] - 8*Sin[x]^2])/20
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\csc(5x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(5-20x^2+16x^4\right)}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(5-20x+16x^2\right)}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{10}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{20-16x}{5-20x+16x^2}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{5}\log(\sin(x))-\frac{1}{5}\left(4\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-10-2\sqrt{5}+16x}\,dx, x, \sin^2(x)\right)-\frac{1}{5}\Big(\\
&= \frac{1}{5}\log(\sin(x))-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\sin^2(x)\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{}\right.
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 0.92

$$\frac{1}{20}\left(-\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(2x)\right)\right)+\left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(2x)\right)+4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[5*x],x]
```

```
[Out] (-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + Sq
rt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20
```

Maple [A]
time = 0.33, size = 80, normalized size = 1.29

method	result
default	$\frac{\ln(1+\cos(x))}{10}-\frac{\ln(4(\cos^2(x))+2\cos(x)-1)}{20}-\frac{\sqrt{5}\ \operatorname{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10}-\frac{\ln(4(\cos^2(x))-2\cos(x)-1)}{20}+\frac{\sqrt{5}\ \operatorname{arcta}}{}$
risch	$\frac{\ln(e^{2ix}-1)}{5}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(5*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*ln(1+cos(x))-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*
(8*cos(x)+2)*5^(1/2))-1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctanh(1
/10*(8*cos(x)-2)*5^(1/2))+1/10*ln(cos(x)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

```
[Out] -1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(s
in(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x
) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arctan
2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2
+ cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x))
) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos
(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))
) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*
x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x), cos(2*
x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(
2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + s
in(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1/10*integrat
e((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin(2*x), cos(2*
x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(2*x)*si
n(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan2(sin(2*x), cos
(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 +
2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)*cos(3/2*
arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(
cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2*x))) + cos(1/2*arc
tan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2
 + 2*(sin(4*x) + sin(2*x) + sin(1/2*arctan2(sin(2*x), cos(2*x))))*sin(3/2*a
```

```
rctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*(s
in(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))) + sin(1/2*arctan2(
sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) - 1/10*integrate((cos(x)*sin(4
*x) + cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) - cos(3*x)*sin(x)
 - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x)
 + cos(4*x)^2 + 2*(cos(2*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x)
 + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*s
in(4*x) + sin(4*x)^2 + 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*
x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1), x) + 1/10*integrate(-(
cos(x)*sin(4*x) - cos(x)*sin(3*x) + cos(x)*sin(2*x) - cos(4*x)*sin(x) + cos
(3*x)*sin(x) - cos(2*x)*sin(x) - sin(x))/(2*(cos(3*x) - cos(2*x) + cos(x) -
 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)^2
 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2*x)
 + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x
)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) + 3/10*
integrate(-(cos(4/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2
(sin(3*x), cos(3*x)))*sin(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3
*x) - cos(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arct
an2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) +
sin(3*x))/(cos(3*x)^2 + 2*(cos(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))
+ cos(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(4/3*arctan2(sin(3*x), cos(3
*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + cos(1/3*arct
an2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/
3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) + 1)*cos(1/3*arctan2(sin(3*x
), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x)))^2 + sin(3*x)^2 + 2*(si
n(3*x) + sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) + sin(4/3*arctan2(sin(3*x),
 cos(3*x)))^2 + 2*(sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3
*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*
sin(3*x)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arctan2(sin(3*x), c
os(3*x)))^2 + 2*cos(3*x) + 1), x) - 3/10*integrate(-(cos(4/3*arctan2(sin(3*
x), cos(3*x)))*sin(3*x) + cos(2/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - c
os(1/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(3*x)*sin(4/3*arctan2(sin
(3*x), cos(3*x))) - cos(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x
)*sin(1/3*arctan2(sin(3*x), cos(3*x))) + sin(3*x))/(cos(3*x)^2 - 2*(cos(3*x
) - cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*
x))) - 1)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x),
cos(3*x)))^2 - 2*(cos(3*x) + cos(1/3*arctan2(sin(3*x), cos(3*x))) - 1)*cos(
2/3*arctan2(sin(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 +
 2*(cos(3*x) - 1)*cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(si
n(3*x), cos(3*x)))^2 + sin(3*x)^2 - 2*(sin(3*x) - sin(2/3*arctan2(sin(3*x),
 cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x
), cos(3*x))) + sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(sin(3*x) + sin(
1/3*arctan2(sin(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + si
n(2/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*sin(3*x)*sin(1/3*arctan2(sin(3*x),
```

```
cos(3*x))) + sin(1/3*arctan2(sin(3*x), cos(3*x...
```

Fricas [A]
time = 2.84, size = 72, normalized size = 1.16

$$\frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4+8\left(\sqrt{5}-3\right)\cos(x)^2-3\sqrt{5}+7}{16\cos(x)^4-12\cos(x)^2+1}\right)-\frac{1}{20}\log\left(16\cos(x)^4-12\cos(x)^2+1\right)+\frac{1}{5}\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="fricas")
[Out] 1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/(
16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1) +
 1/5*log(1/2*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos(x)\csc(5x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x)
[Out] Integral(cos(x)*csc(5*x), x)
```

Giac [A]
time = 0.43, size = 67, normalized size = 1.08

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\cos(x)^2-4\sqrt{5}-12\right|}{\left|32\cos(x)^2+4\sqrt{5}-12\right|}\right)+\frac{1}{10}\log\left(-\cos(x)^2+1\right)-\frac{1}{20}\log\left(\left|16\cos(x)^4-12\cos(x)^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(5*x),x, algorithm="giac")
[Out] -1/20*sqrt(5)*log(abs(32*cos(x)^2 - 4*sqrt(5) - 12)/abs(32*cos(x)^2 + 4*sqr
t(5) - 12)) + 1/10*log(-cos(x)^2 + 1) - 1/20*log(abs(16*cos(x)^4 - 12*cos(x
)^2 + 1))
```

Mupad [B]
time = 2.68, size = 51, normalized size = 0.82

$$\frac{\ln(\sin(x))}{5}+\ln\left(-\cos(x)^2-\frac{\sqrt{5}}{8}+\frac{3}{8}\right)\left(\frac{\sqrt{5}}{20}-\frac{1}{20}\right)-\ln\left(-\cos(x)^2+\frac{\sqrt{5}}{8}+\frac{3}{8}\right)\left(\frac{\sqrt{5}}{20}+\frac{1}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(5*x),x)
[Out] log(sin(x))/5 + log(3/8 - 5^(1/2)/8 - cos(x)^2)*(5^(1/2)/20 - 1/20) - log(5
^(1/2)/8 - cos(x)^2 + 3/8)*(5^(1/2)/20 + 1/20)
```

3.127 $\int \cos(x)\csc(6x)\,dx$

Optimal. Leaf size=36

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+1/6*arctanh(2/3*cos(x)*3^(1/2))*
3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {12, 2082, 213}

$$-\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[6*x],x]
```

```
[Out] -1/6*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/6 + ArcTanh[(2*Cos[x])/Sqrt[3]]/(2
*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2082

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(6x)\,dx &= -\text{Subst}\left(\int \frac{1}{2(3-19x^2+32x^4-16x^6)}\,dx, x, \cos(x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx, x, \cos(x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3(-1+x^2)}+\frac{2}{-3+4x^2}-\frac{2}{3(-1+4x^2)}\right)dx, x, \cos(x)\right)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+4x^2}\,dx, x, \cos(x)\right)-\text{Subst}\Big(\\ &= -\frac{1}{6}\tanh^{-1}(\cos(x))-\frac{1}{6}\tanh^{-1}(2\cos(x))+\frac{\tanh^{-1}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.
time = 0.05, size = 83, normalized size = 2.31

$$\frac{1}{12}\left(-2\sqrt{3}\tanh^{-1}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\sqrt{3}\tanh^{-1}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(1+2\cos(x))+2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[6*x],x]
```

```
[Out] (-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[x
/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] + 2
*Log[Sin[x/2]])/12
```

Maple [A]
time = 0.36, size = 47, normalized size = 1.31

method	result
default	$-\frac{\ln(1+\cos(x))}{12}+\frac{\ln(2\cos(x)-1)}{12}-\frac{\ln(1+2\cos(x))}{12}+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(\cos(x)-1)}{12}$
risch	$-\frac{\ln(e^{ix}+1)}{6}+\frac{\ln(e^{ix}-1)}{6}-\frac{\ln(e^{2ix}+e^{ix}+1)}{12}-\frac{\sqrt{3}\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\sqrt{3}\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\ln(e^{2ix}-e^{ix}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*ln(1+cos(x))+1/12*ln(2*cos(x)-1)-1/12*ln(1+2*cos(x))+1/6*arctanh(2/3*
cos(x)*3^(1/2))*3^(1/2)+1/12*ln(cos(x)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x)
 - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)
*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - si
n(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/24*lo
g(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin(2*x)
*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x) + co
s(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*cos(x)
+ 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.
time = 2.52, size = 70, normalized size = 1.94

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos\left(x\right)^{2}+4\sqrt{3}\cos\left(x\right)+3}{4\cos\left(x\right)^{2}-3}\right)-\frac{1}{12}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-2\cos\left(x\right)+1\right)-\frac{1}{12}\log\left(-2\cos\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - 1
/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos(x
) + 1) - 1/12*log(-2*cos(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(x\right)\csc\left(6x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x)
```

```
[Out] Integral(cos(x)*csc(6*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.
time = 0.40, size = 68, normalized size = 1.89

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos\left(x\right)\right|}{\left|4\sqrt{3}+8\cos\left(x\right)\right|}\right)-\frac{1}{12}\log\left(\cos\left(x\right)+1\right)+\frac{1}{12}\log\left(-\cos\left(x\right)+1\right)-\frac{1}{12}\log\left(\left|2\cos\left(x\right)+1\right|\right)+\frac{1}{12}\log\left(\left|2\cos\left(x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(6*x),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) - 1
/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1)) +
 1/12*log(abs(2*cos(x) - 1))
```

Mupad [B]
time = 2.37, size = 74, normalized size = 2.06

$$\frac{\mathrm{atanh}\left(\frac{1073741824}{10761687\,\left(\frac{427973089951744\,\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6}+\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{4222769432625152\,\sqrt{3}}{4782969\,\left(\frac{101871591633190912\,\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)}-\frac{19605196950732800\,\sqrt{3}\,\tan\left(\frac{x}{2}\right)^2}{1594323\,\left(\frac{101871591633190912\,\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(6*x),x)
```

```
[Out] atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 4755280
4159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 + (3^(1/2)*atanh((42
22769432625152*3^(1/2))/(4782969*((101871591633190912*tan(x/2)^2)/4782969 -
 7314051205955584/4782969)) - (19605196950732800*3^(1/2)*tan(x/2)^2)/(15943
23*((101871591633190912*tan(x/2)^2)/4782969 - 7314051205955584/4782969))))/
6
```

3.128 $\int \cos^3(6x)\sin(x)\,dx$

Optimal. Leaf size=33

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

```
[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4439, 2718}

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[x],x]
```

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(6x)\sin(x)\,dx &= \int \left(-\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)+\frac{1}{8}\sin(19x)\right)\,dx\\ &= -\left(\frac{1}{8}\int \sin(17x)\,dx\right)+\frac{1}{8}\int \sin(19x)\,dx-\frac{3}{8}\int \sin(5x)\,dx+\frac{3}{8}\int \sin(7x)\,dx\\ &= \frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[x],x]
```

```
[Out] (3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

Maple [A]
time = 0.26, size = 26, normalized size = 0.79

method	result	size
default	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26
risch	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] 3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.76

$$-\frac{1}{152}\cos\left(19\,x\right)+\frac{1}{136}\cos\left(17\,x\right)-\frac{3}{56}\cos\left(7\,x\right)+\frac{3}{40}\cos\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")
```

```
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.
time = 2.93, size = 57, normalized size = 1.73

$$-\frac{32768}{19}\cos\left(x\right)^{19}+\frac{147456}{17}\cos\left(x\right)^{17}-18432\cos\left(x\right)^{15}+21504\cos\left(x\right)^{13}-14976\cos\left(x\right)^{11}+6336\cos\left(x\right)^{9}-\frac{11112}{7}\cos\left(x\right)^{7}+\frac{1116}{5}\cos\left(x\right)^{5}-18\cos\left(x\right)^{3}+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -32768/19*cos(x)^19 + 147456/17*cos(x)^17 - 18432*cos(x)^15 + 21504*cos(x)^
13 - 14976*cos(x)^11 + 6336*cos(x)^9 - 11112/7*cos(x)^7 + 1116/5*cos(x)^5 -
 18*cos(x)^3 + cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.
time = 0.80, size = 63, normalized size = 1.91

$$\frac{1296\sin\left(x\right)\sin^{3}\left(6x\right)}{11305}+\frac{1926\sin\left(x\right)\sin\left(6x\right)\cos^{2}\left(6x\right)}{11305}+\frac{216\sin^{2}\left(6x\right)\cos\left(x\right)\cos\left(6x\right)}{11305}+\frac{251\cos\left(x\right)\cos^{3}\left(6x\right)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(x),x)
```

```
[Out] 1296*sin(x)*sin(6*x)**3/11305 + 1926*sin(x)*sin(6*x)*cos(6*x)**2/11305 + 21
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 + 251*cos(x)*cos(6*x)**3/11305
```

Giac [A]
time = 0.40, size = 25, normalized size = 0.76

$$-\frac{1}{152}\cos\left(19\,x\right)+\frac{1}{136}\cos\left(17\,x\right)-\frac{3}{56}\cos\left(7\,x\right)+\frac{3}{40}\cos\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

Mupad [B]
time = 0.08, size = 57, normalized size = 1.73

$$-\frac{32768\cos\left(x\right)^{19}}{19}+\frac{147456\cos\left(x\right)^{17}}{17}-18432\cos\left(x\right)^{15}+21504\cos\left(x\right)^{13}-14976\cos\left(x\right)^{11}+6336\cos\left(x\right)^{9}-\frac{11112\cos\left(x\right)^{7}}{7}+\frac{1116\cos\left(x\right)^{5}}{5}-18\cos\left(x\right)^{3}+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(x),x)
```

```
[Out] cos(x) - 18*cos(x)^3 + (1116*cos(x)^5)/5 - (11112*cos(x)^7)/7 + 6336*cos(x)
^9 - 14976*cos(x)^11 + 21504*cos(x)^13 - 18432*cos(x)^15 + (147456*cos(x)^1
7)/17 - (32768*cos(x)^19)/19
```

3.129 $\int \cos^3(6x)\sin(9x)\,dx$

Optimal. Leaf size=33

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

```
[Out] -1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4439, 2718}

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[6*x]^3*Sin[9*x],x]
[Out] -1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(6x)\sin(9x)\,dx &= \int \left(\frac{3}{8}\sin(3x)-\frac{1}{8}\sin(9x)+\frac{3}{8}\sin(15x)+\frac{1}{8}\sin(27x)\right)dx\\
&= -\left(\frac{1}{8}\int \sin(9x)\,dx\right)+\frac{1}{8}\int \sin(27x)\,dx+\frac{3}{8}\int \sin(3x)\,dx+\frac{3}{8}\int \sin(15x)\,dx\\
&= -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[6*x]^3*Sin[9*x],x]
```

```
[Out] -1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

Maple [A]
time = 0.15, size = 26, normalized size = 0.79

method	result	size
default	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26
risch	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(9*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.76

$$-\frac{1}{216}\cos(27x)-\frac{1}{40}\cos(15x)+\frac{1}{72}\cos(9x)-\frac{1}{8}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")
```

```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

Fricas [A]
time = 2.26, size = 39, normalized size = 1.18

$$-\frac{32}{27}\cos(3x)^9+\frac{8}{3}\cos(3x)^7-\frac{12}{5}\cos(3x)^5+\frac{10}{9}\cos(3x)^3-\frac{1}{3}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")
```

```
[Out] -32/27*cos(3*x)^9 + 8/3*cos(3*x)^7 - 12/5*cos(3*x)^5 + 10/9*cos(3*x)^3 - 1/
3*cos(3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(26)=52$.
time = 0.80, size = 71, normalized size = 2.15

$$-\frac{16\sin^3(6x)\sin(9x)}{135}-\frac{8\sin^2(6x)\cos(6x)\cos(9x)}{45}-\frac{2\sin(6x)\sin(9x)\cos^2(6x)}{45}-\frac{19\cos^3(6x)\cos(9x)}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)**3*sin(9*x),x)
```

```
[Out] -16*sin(6*x)**3*sin(9*x)/135 - 8*sin(6*x)**2*cos(6*x)*cos(9*x)/45 - 2*sin(6
*x)*sin(9*x)*cos(6*x)**2/45 - 19*cos(6*x)**3*cos(9*x)/135
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.76

$$-\frac{1}{216}\cos\left(27\,x\right)-\frac{1}{40}\cos\left(15\,x\right)+\frac{1}{72}\cos\left(9\,x\right)-\frac{1}{8}\cos\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")
```

```
[Out] -1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

Mupad [B]
time = 2.47, size = 78, normalized size = 2.36

$$-\frac{2\left(135\tan\left(\frac{3x}{2}\right)^{16}-900\tan\left(\frac{3x}{2}\right)^{14}+5640\tan\left(\frac{3x}{2}\right)^{12}-13140\tan\left(\frac{3x}{2}\right)^{10}+15534\tan\left(\frac{3x}{2}\right)^{8}-4044\tan\left(\frac{3x}{2}\right)^{6}+1584\tan\left(\frac{3x}{2}\right)^{4}+36\tan\left(\frac{3x}{2}\right)^{2}+19\right)}{135\left(\tan\left(\frac{3x}{2}\right)^{2}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(6*x)^3*sin(9*x),x)
```

```
[Out] -(2*(36*tan((3*x)/2)^2 + 1584*tan((3*x)/2)^4 - 4044*tan((3*x)/2)^6 + 15534*
tan((3*x)/2)^8 - 13140*tan((3*x)/2)^10 + 5640*tan((3*x)/2)^12 - 900*tan((3*
x)/2)^14 + 135*tan((3*x)/2)^16 + 19))/(135*(tan((3*x)/2)^2 + 1)^9)
```

3.130 $\int \cos(2x)\sin^2(6x)\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

```
[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4439, 2717}

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sin[6*x]^2,x]
```

```
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sin^2(6x)\,dx &= \int \left(\frac{1}{2}\cos(2x)-\frac{1}{4}\cos(10x)-\frac{1}{4}\cos(14x)\right)\,dx\\ &= -\left(\frac{1}{4}\int \cos(10x)\,dx\right)-\frac{1}{4}\int \cos(14x)\,dx+\frac{1}{2}\int \cos(2x)\,dx\\ &= \frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sin[6*x]^2,x]
[Out] Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

Maple [A]
time = 0.16, size = 20, normalized size = 0.80

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
risch	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
norman	$\frac{\frac{6\left(\tan^3(3x)\right)}{35}+\frac{32\tan(x)\left(\tan^2(3x)\right)}{35}+\frac{18\tan(x)\left(\tan^4(3x)\right)}{35}+\frac{6\left(\tan^2(x)\right)\tan(3x)}{35}-\frac{6\left(\tan^2(x)\right)\left(\tan^3(3x)\right)}{35}+\frac{18\tan(x)}{35}-\frac{6\tan(3x)}{35}}{(\tan^2(x)+1)(1+\tan^2(3x))^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$-\frac{1}{56}\sin(14x)-\frac{1}{40}\sin(10x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

Fricas [A]
time = 2.31, size = 32, normalized size = 1.28

$$-\frac{1}{70}\left(80\cos(2x)^6-72\cos(2x)^4+9\cos(2x)^2-17\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")
[Out] -1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.
time = 0.31, size = 48, normalized size = 1.92

$$\frac{17\sin(2x)\sin^2(6x)}{70} + \frac{9\sin(2x)\cos^2(6x)}{35} - \frac{3\sin(6x)\cos(2x)\cos(6x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)**2,x)
```

```
[Out] 17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*x
)*cos(6*x)/35
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.76

$$-\frac{1}{56}\sin(14x) - \frac{1}{40}\sin(10x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")
```

```
[Out] -1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

Mupad [B]
time = 2.29, size = 25, normalized size = 1.00

$$\frac{8\sin(2x)^7}{7} - \frac{12\sin(2x)^5}{5} + \frac{3\sin(2x)^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(6*x)^2,x)
```

```
[Out] (3*sin(2*x)^3)/2 - (12*sin(2*x)^5)/5 + (8*sin(2*x)^7)/7
```

3.131 $\int \cos(x)\sin^2(6x)\,dx$

Optimal. Leaf size=23

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

```
[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4439, 2717}

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^2,x]
```

```
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^2(6x)\,dx &= \int \left(\frac{\cos(x)}{2}-\frac{1}{4}\cos(11x)-\frac{1}{4}\cos(13x)\right)dx\\ &= -\left(\frac{1}{4}\int \cos(11x)\,dx\right)-\frac{1}{4}\int \cos(13x)\,dx+\frac{1}{2}\int \cos(x)\,dx\\ &= \frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^2,x]
[Out] Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

Maple [A]
time = 0.16, size = 18, normalized size = 0.78

method	result
default	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$
risch	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$
norman	$\frac{\frac{24\left(\tan^3(3x)\right)}{143}+\frac{24\tan(3x)\left(\tan^2\left(\frac{x}{2}\right)\right)}{143}+\frac{280\left(\tan^2(3x)\right)\tan\left(\frac{x}{2}\right)}{143}-\frac{24\left(\tan^3(3x)\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{143}+\frac{144\left(\tan^4(3x)\right)\tan\left(\frac{x}{2}\right)}{143}-\frac{24\tan(3x)}{143}+\frac{144\tan\left(\frac{x}{2}\right)}{143}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(1+\tan^2(3x)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.74

$$-\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 4.78, size = 42, normalized size = 1.83

$$-\frac{4}{143}\left(2816\cos(x)^{12} - 6912\cos(x)^{10} + 6048\cos(x)^{8} - 2240\cos(x)^{6} + 315\cos(x)^{4} - 9\cos(x)^{2} - 18\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")
```

```
[Out] -4/143*(2816*cos(x)^12 - 6912*cos(x)^10 + 6048*cos(x)^8 - 2240*cos(x)^6 + 3
15*cos(x)^4 - 9*cos(x)^2 - 18)*sin(x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 0.30, size = 42, normalized size = 1.83

$$\frac{71\sin(x)\sin^2(6x)}{143} + \frac{72\sin(x)\cos^2(6x)}{143} - \frac{12\sin(6x)\cos(x)\cos(6x)}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**2,x)
```

```
[Out] 71*sin(x)*sin(6*x)**2/143 + 72*sin(x)*cos(6*x)**2/143 - 12*sin(6*x)*cos(x)*
cos(6*x)/143
```

Giac [**A**]
time = 0.39, size = 17, normalized size = 0.74

$$-\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")
```

```
[Out] -1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

Mupad [**B**]
time = 2.49, size = 17, normalized size = 0.74

$$\frac{\sin(x)}{2} - \frac{\sin(13x)}{52} - \frac{\sin(11x)}{44}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^2*cos(x),x)
```

```
[Out] sin(x)/2 - sin(13*x)/52 - sin(11*x)/44
```

3.132 $\int \cos(x)\sin^3(6x)\,dx$

Optimal. Leaf size=33

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

```
[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4439, 2718}

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[6*x]^3,x]
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin^3(6x)\,dx &= \int \left(\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)-\frac{1}{8}\sin(19x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(17x)\,dx\right)-\frac{1}{8}\int \sin(19x)\,dx+\frac{3}{8}\int \sin(5x)\,dx+\frac{3}{8}\int \sin(7x)\,dx\\ &= -\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[6*x]^3,x]
[Out] (-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

Maple [A]
time = 0.15, size = 26, normalized size = 0.79

method	result	size
default	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26
risch	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
[Out] -3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.76

$$\frac{1}{152}\cos(19x)+\frac{1}{136}\cos(17x)-\frac{3}{56}\cos(7x)-\frac{3}{40}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")
[Out] 1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

Fricas [A]
time = 2.26, size = 49, normalized size = 1.48

$$\frac{32768}{19}\cos(x)^{19}-\frac{131072}{17}\cos(x)^{17}+14336\cos(x)^{15}-14336\cos(x)^{13}+8320\cos(x)^{11}-2816\cos(x)^{9}+\frac{3672}{7}\cos(x)^{7}-\frac{216}{5}\cos(x)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")
[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^1
3 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.81, size = 65, normalized size = 1.97

$$-\frac{251\sin(x)\sin^3(6x)}{11305}-\frac{216\sin(x)\sin(6x)\cos^2(6x)}{11305}-\frac{1926\sin^2(6x)\cos(x)\cos(6x)}{11305}-\frac{1296\cos(x)\cos^3(6x)}{11305}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)**3,x)
```

```
[Out] -251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 192
6*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305
```

Giac [A]
time = 0.41, size = 49, normalized size = 1.48

$$\frac{32768}{19}\cos\left(x\right)^{19}-\frac{131072}{17}\cos\left(x\right)^{17}+14336\cos\left(x\right)^{15}-14336\cos\left(x\right)^{13}+8320\cos\left(x\right)^{11}-2816\cos\left(x\right)^{9}+\frac{3672}{7}\cos\left(x\right)^{7}-\frac{216}{5}\cos\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")
```

```
[Out] 32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^1
3 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

Mupad [B]
time = 2.70, size = 150, normalized size = 4.55

$$-\frac{32\left(305235\tan\left(\frac{x}{2}\right)^{34}-9665775\tan\left(\frac{x}{2}\right)^{32}+153838440\tan\left(\frac{x}{2}\right)^{30}-1348695544\tan\left(\frac{x}{2}\right)^{28}+7083812484\tan\left(\frac{x}{2}\right)^{26}-23578828164\tan\left(\frac{x}{2}\right)^{24}+51613490424\tan\left(\frac{x}{2}\right)^{22}-75928491144\tan\left(\frac{x}{2}\right)^{20}+75935973762\tan\left(\frac{x}{2}\right)^{18}-51607368282\tan\left(\frac{x}{2}\right)^{16}+23582909592\tan\left(\frac{x}{2}\right)^{14}-7081614792\tan\left(\frac{x}{2}\right)^{12}+1349637412\tan\left(\frac{x}{2}\right)^{10}-153524484\tan\left(\frac{x}{2}\right)^{8}+9744264\tan\left(\frac{x}{2}\right)^{6}-291384\tan\left(\frac{x}{2}\right)^{4}+1539\tan\left(\frac{x}{2}\right)^{2}+81\right)}{11305\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^3*cos(x),x)
```

```
[Out] -(32*(1539*tan(x/2)^2 - 291384*tan(x/2)^4 + 9744264*tan(x/2)^6 - 153524484*
tan(x/2)^8 + 1349637412*tan(x/2)^10 - 7081614792*tan(x/2)^12 + 23582909592*
tan(x/2)^14 - 51607368282*tan(x/2)^16 + 75935973762*tan(x/2)^18 - 759284911
44*tan(x/2)^20 + 51613490424*tan(x/2)^22 - 23578828164*tan(x/2)^24 + 708381
2484*tan(x/2)^26 - 1348695544*tan(x/2)^28 + 153838440*tan(x/2)^30 - 9665775
*tan(x/2)^32 + 305235*tan(x/2)^34 + 81))/(11305*(tan(x/2)^2 + 1)^19)
```

3.133 $\int \cos(7x)\sin^3(6x)\,dx$

Optimal. Leaf size=31

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

```
[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4439, 2718}

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[7*x]*Sin[6*x]^3,x]
```

```
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(7x)\sin^3(6x)\,dx &= \int \left(-\frac{3\sin(x)}{8}-\frac{1}{8}\sin(11x)+\frac{3}{8}\sin(13x)-\frac{1}{8}\sin(25x)\right)dx\\ &= -\left(\frac{1}{8}\int \sin(11x)\,dx\right)-\frac{1}{8}\int \sin(25x)\,dx-\frac{3}{8}\int \sin(x)\,dx+\frac{3}{8}\int \sin(13x)\,dx\\ &= \frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[7*x]*Sin[6*x]^3,x]
[Out] (3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

Maple [A]
time = 0.44, size = 24, normalized size = 0.77

method	result	size
default	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$	24
risch	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(7*x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
[Out] 3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.74

$$\frac{1}{200}\cos\left(25\,x\right)-\frac{3}{104}\cos\left(13\,x\right)+\frac{1}{88}\cos\left(11\,x\right)+\frac{3}{8}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.
time = 2.41, size = 67, normalized size = 2.16

$$\frac{2097152}{25}\cos\left(x\right)^{25}-524288\cos\left(x\right)^{23}+1441792\cos\left(x\right)^{21}-2293760\cos\left(x\right)^{19}+2334720\cos\left(x\right)^{17}-\frac{7938048}{5}\cos\left(x\right)^{15}+\frac{9503232}{13}\cos\left(x\right)^{13}-\frac{2484992}{11}\cos\left(x\right)^{11}+45248\cos\left(x\right)^{9}-5400\cos\left(x\right)^{7}+\frac{1512}{5}\cos\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")
[Out] 2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(x
)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 248
4992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.
time = 0.77, size = 70, normalized size = 2.26

$$\frac{1421\sin^3(6x)\sin(7x)}{3575} + \frac{1062\sin^2(6x)\cos(6x)\cos(7x)}{3575} + \frac{1512\sin(6x)\sin(7x)\cos^2(6x)}{3575} + \frac{1296\cos^3(6x)\cos(7x)}{3575}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)**3,x)
```

```
[Out] 1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +
1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

Giac [A]
time = 0.42, size = 23, normalized size = 0.74

$$\frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")
```

```
[Out] 1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

Mupad [B]
time = 3.18, size = 198, normalized size = 6.39

$$\frac{32\left(-96525\tan\left(\frac{x}{2}\right)^{46}+8655075\tan\left(\frac{x}{2}\right)^{44}-300482325\tan\left(\frac{x}{2}\right)^{42}+5743927475\tan\left(\frac{x}{2}\right)^{40}-67792485475\tan\left(\frac{x}{2}\right)^{38}+523868412925\tan\left(\frac{x}{2}\right)^{36}-2750448633075\tan\left(\frac{x}{2}\right)^{34}+10084506042325\tan\left(\frac{x}{2}\right)^{32}-26325778958050\tan\left(\frac{x}{2}\right)^{30}+49575817586750\tan\left(\frac{x}{2}\right)^{28}-67895787973650\tan\left(\frac{x}{2}\right)^{26}+67896209197950\tan\left(\frac{x}{2}\right)^{24}-49575456537350\tan\left(\frac{x}{2}\right)^{22}+26326043727610\tan\left(\frac{x}{2}\right)^{20}-10084340561350\tan\left(\frac{x}{2}\right)^{18}+2750536240650\tan\left(\frac{x}{2}\right)^{16}-523829476225\tan\left(\frac{x}{2}\right)^{14}+67806830575\tan\left(\frac{x}{2}\right)^{12}-5739623945\tan\left(\frac{x}{2}\right)^{10}+301506975\tan\left(\frac{x}{2}\right)^{8}-8468775\tan\left(\frac{x}{2}\right)^{6}+120825\tan\left(\frac{x}{2}\right)^{4}+2025\tan\left(\frac{x}{2}\right)^{2}+81\right)}{3575\left(\tan\left(\frac{x}{2}\right)^2+1\right)^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(7*x)*sin(6*x)^3,x)
```

```
[Out] (32*(2025*tan(x/2)^2 + 120825*tan(x/2)^4 - 8468775*tan(x/2)^6 + 301506975*t
an(x/2)^8 - 5739623945*tan(x/2)^10 + 67806830575*tan(x/2)^12 - 523829476225
*tan(x/2)^14 + 2750536240650*tan(x/2)^16 - 10084340561350*tan(x/2)^18 + 263
26043727610*tan(x/2)^20 - 49575456537350*tan(x/2)^22 + 67896209197950*tan(x
/2)^24 - 67895787973650*tan(x/2)^26 + 49575817586750*tan(x/2)^28 - 26325778
958050*tan(x/2)^30 + 10084506042325*tan(x/2)^32 - 2750448633075*tan(x/2)^34
 + 523868412925*tan(x/2)^36 - 67792485475*tan(x/2)^38 + 5743927475*tan(x/2)
^40 - 300482325*tan(x/2)^42 + 8655075*tan(x/2)^44 - 96525*tan(x/2)^46 + 81)
)/(3575*(tan(x/2)^2 + 1)^25)
```

3.134 $\int \cos^2(3x)\sin^3(2x)\,dx$

Optimal. Leaf size=41

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

```
[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4439, 2718}

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]^2*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(3x)\sin^3(2x)\,dx &= \int \left(\frac{3}{8}\sin(2x)-\frac{3}{16}\sin(4x)-\frac{1}{8}\sin(6x)+\frac{3}{16}\sin(8x)-\frac{1}{16}\sin(12x)\right)dx\\
&= -\left(\frac{1}{16}\int \sin(12x)\,dx\right)-\frac{1}{8}\int \sin(6x)\,dx-\frac{3}{16}\int \sin(4x)\,dx+\frac{3}{16}\int \sin(8x)\,dx+\\
&= -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 1.00

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]^2*Sin[2*x]^3,x]
```

```
[Out] (-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[1
2*x]/192
```

Maple [A]
time = 0.19, size = 32, normalized size = 0.78

method	result	size
default	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$	32
risch	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)^2*sin(2*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.76

$$\frac{1}{192}\cos(12x)-\frac{3}{128}\cos(8x)+\frac{1}{48}\cos(6x)+\frac{3}{64}\cos(4x)-\frac{3}{16}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

Fricas [A]
time = 1.17, size = 25, normalized size = 0.61

$$\frac{32}{3}\cos(x)^{12}-32\cos(x)^{10}+33\cos(x)^{8}-12\cos(x)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")
```

```
[Out] 32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(37) = 74$.
time = 2.63, size = 226, normalized size = 5.51

$$-\frac{x\sin^{3}(2x)\sin^{2}(3x)}{16}+\frac{x\sin^{3}(2x)\cos^{2}(3x)}{16}-\frac{3x\sin^{2}(2x)\sin(3x)\cos(2x)\cos(3x)}{8}+\frac{3x\sin(2x)\sin^{2}(3x)\cos^{2}(2x)}{16}-\frac{3x\sin(2x)\cos^{2}(2x)\cos^{2}(3x)}{16}+\frac{x\sin(3x)\cos^{3}(2x)\cos(3x)}{8}-\frac{\sin^{3}(2x)\sin(3x)\cos(3x)}{48}-\frac{\sin^{2}(2x)\cos(2x)\cos^{2}(3x)}{2}+\frac{5\sin(2x)\sin(3x)\cos^{2}(2x)\cos(3x)}{8}-\frac{9\sin^{2}(3x)\cos^{3}(2x)}{32}-\frac{5\cos^{3}(2x)\cos^{2}(3x)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)**2*sin(2*x)**3,x)
```

```
[Out] -x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x)
**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/16
- 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3*x)
/8 - sin(2*x)**3*sin(3*x)*cos(3*x)/48 - sin(2*x)**2*cos(2*x)*cos(3*x)**2/2
+ 5*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/8 - 9*sin(3*x)**2*cos(2*x)**3/32
 - 5*cos(2*x)**3*cos(3*x)**2/96
```

Giac [**A**]
time = 0.40, size = 31, normalized size = 0.76

$$\frac{1}{192}\cos\left(12\,x\right)-\frac{3}{128}\cos\left(8\,x\right)+\frac{1}{48}\cos\left(6\,x\right)+\frac{3}{64}\cos\left(4\,x\right)-\frac{3}{16}\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*cos
(2*x)
```

Mupad [**B**]
time = 2.27, size = 25, normalized size = 0.61

$$\frac{32\cos\left(x\right)^{12}}{3}-32\cos\left(x\right)^{10}+33\cos\left(x\right)^{8}-12\cos\left(x\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)^2*sin(2*x)^3,x)
```

```
[Out] 33*cos(x)^8 - 12*cos(x)^6 - 32*cos(x)^10 + (32*cos(x)^12)/3
```

3.135 $\int \sin(a+bx)\sin(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+c+2bx)}{4b}$$

```
[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4665, 2717}

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+c+2bx)\right)\,dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)-\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.96

$$-\frac{-2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

```
[Out] -1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b
```

Maple [A]
time = 0.13, size = 24, normalized size = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}-\frac{\sin(2bx+a+c)}{4b}$
norman	$\frac{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}{2}+2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)(}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(-a+c\right)-\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.
time = 1.64, size = 50, normalized size = 1.85

$$\frac{bx\cos\left(-a+c\right)-\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)+\cos\left(bx+c\right)^{2}\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c)
^2*sin(-a + c))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.
time = 0.27, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} - \frac{\sin(a+bx)\cos(bx+c)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
sin(a + b*x)*cos(b*x + c)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

Giac [A]
time = 0.42, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(a-c\right) - \frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

Mupad [B]
time = 2.48, size = 36, normalized size = 1.33

$$\begin{cases} x\,\sin\left(a\right)\,\sin\left(c\right) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2} - \frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + b*x),x)
```

```
[Out] piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, (x*cos(a - c))/2 - sin(a + c + 2
*b*x)/(4*b))
```

3.136 $\int \sin(c-bx)\sin(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}$$

```
[Out] -1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4665, 2717}

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c - b*x]*Sin[a + b*x],x]
```

```
[Out] -1/2*(x*Cos[a + c]) + Sin[a - c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c-bx)\sin(a+bx)\,dx &= \int \left(-\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)\,dx\\ &= -\frac{1}{2}x\cos(a+c)+\frac{1}{2}\int \cos(a-c+2bx)\,dx\\ &= -\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 0.96

$$\frac{-2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c - b*x]*Sin[a + b*x],x]
[Out] (-2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Maple [A]
time = 0.12, size = 24, normalized size = 0.89

method	result
default	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
risch	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{x\left(\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)}{2}-2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{bx}{2}-\frac{c}{2}\right)-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2 \right.}{b}}{\left(1+\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(b*x-c)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x\cos\left(a+c\right)+\frac{\sin\left(2\,bx+a-c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Fricas [A]
time = 3.00, size = 44, normalized size = 1.63

$$-\frac{bx\cos\left(a+c\right)-\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)^2\sin\left(a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(b*x*cos(a + c) - cos(b*x + a)*cos(a + c)*sin(b*x + a) + cos(b*x + a)^
2*sin(a + c))/b
```

Sympy [A]
time = 0.31, size = 61, normalized size = 2.26

$$-\begin{cases}\frac{x\sin\left(a+bx\right)\sin\left(bx-c\right)}{2}+\frac{x\cos\left(a+bx\right)\cos\left(bx-c\right)}{2}-\frac{\sin\left(a+bx\right)\cos\left(bx-c\right)}{2b} & \text{for}\: b\neq 0\\-x\sin\left(a\right)\sin\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x)
```

```
[Out] -Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 -
 sin(a + b*x)*cos(b*x - c)/(2*b), Ne(b, 0)), (-x*sin(a)*sin(c), True))
```

Giac [A]
time = 0.41, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x\cos\left(a+c\right)+\frac{\sin\left(2\,bx+a-c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Mupad [B]
time = 2.46, size = 36, normalized size = 1.33

$$\begin{cases} x\,\sin\left(a\right)\,\sin\left(c\right) & \text{if}\ \ b=0\\ \frac{\sin(a-c+2\,b\,x)}{4\,b}-\frac{x\,\cos(a+c)}{2} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c - b*x),x)
```

```
[Out] piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) - (x*co
s(a + c))/2)
```

3.137 $\int \cos(a+bx)\cos(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cos(a-c)+\frac{\sin(a+c+2bx)}{4b}$$

```
[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{4666, 2717\}$

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos(c+bx)\,dx &= \int \left(\frac{1}{2}\cos(a-c)+\frac{1}{2}\cos(a+c+2bx)\right)\,dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{1}{2}\int \cos(a+c+2bx)\,dx\\ &= \frac{1}{2}x\cos(a-c)+\frac{\sin(a+c+2bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.96

$$\frac{2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

```
[Out] (2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

Maple [A]
time = 0.10, size = 24, normalized size = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}+\frac{\sin(2bx+a+c)}{4b}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}{2}+2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{bx}{2}+\frac{c}{2}\right)+\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\text{ta}\right.}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(-a+c\right)+\frac{\sin\left(2\,bx+a+c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.
time = 1.56, size = 50, normalized size = 1.85

$$\frac{bx\cos\left(-a+c\right)+\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)-\cos\left(bx+c\right)^{2}\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c)
^2*sin(-a + c))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.
time = 0.20, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} + \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for}\ b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Giac [A]
time = 0.39, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos(a-c) + \frac{\sin(2\,bx + a + c)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

Mupad [B]
time = 2.27, size = 36, normalized size = 1.33

$$\begin{cases} x\cos(a)\cos(c) & \text{if}\ b = 0 \\ \frac{x\cos(a-c)}{2} + \frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if}\ b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + b*x),x)
```

```
[Out] piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, (x*cos(a - c))/2 + sin(a + c + 2
*b*x)/(4*b))
```

3.138 $\int \cos(c-bx)\cos(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}$$

[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4666, 2717}

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

Antiderivative was successfully verified.

[In] Int[Cos[c - b*x]*Cos[a + b*x],x]

[Out] (x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c-bx)\cos(a+bx)\,dx&=\int\left(\frac{1}{2}\cos(a+c)+\frac{1}{2}\cos(a-c+2bx)\right)dx\\&=\frac{1}{2}x\cos(a+c)+\frac{1}{2}\int\cos(a-c+2bx)\,dx\\&=\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.96

$$\frac{2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c - b*x]*Cos[a + b*x],x]
```

```
[Out] (2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

Maple [A]
time = 0.11, size = 24, normalized size = 0.89

method	result
default	$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
risch	$\frac{x\cos(a+c)}{2}+\frac{\sin(2bx+a-c)}{4b}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}-\frac{x\left(\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)}{2}+2x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{bx}{2}-\frac{c}{2}\right)+\frac{x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\right.}{b}}{\left(1+\tan^2\left(\frac{bx}{2}-\frac{c}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x-c)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.85

$$\frac{1}{2}\,x\cos\left(a+c\right)+\frac{\sin\left(2\,bx+a-c\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Fricas [A]
time = 1.62, size = 44, normalized size = 1.63

$$\frac{bx\cos\left(a+c\right)+\cos\left(bx+a\right)\cos\left(a+c\right)\sin\left(bx+a\right)-\cos\left(bx+a\right)^{2}\sin\left(a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^2
*sin(a + c))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

time = 0.24, size = 58, normalized size = 2.15

$$\begin{cases} \frac{x\sin{(a+bx)}\sin{(bx-c)}}{2} + \frac{x\cos{(a+bx)}\cos{(bx-c)}}{2} + \frac{\sin{(a+bx)}\cos{(bx-c)}}{2b} & \text{for } b \neq 0 \\ x\cos{(a)}\cos{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 +
sin(a + b*x)*cos(b*x - c)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

Giac [A]
time = 0.40, size = 23, normalized size = 0.85

$$\frac{1}{2}x\cos{(a+c)} + \frac{\sin{(2bx+a-c)}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

Mupad [B]
time = 2.26, size = 36, normalized size = 1.33

$$\begin{cases} x\cos{(a)}\cos{(c)} & \text{if } b = 0 \\ \frac{\sin(a-c+2bx)}{4b} + \frac{x\cos(a+c)}{2} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c - b*x),x)
```

```
[Out] piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) + (x*co
s(a + c))/2)
```

3.139 $\int \tan(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=39

$$-x-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(c+bx))}{b}$$

```
[Out] -x-cot(a-c)*ln(cos(b*x+a))/b+cot(a-c)*ln(cos(b*x+c))/b
```

Rubi [A]
time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4708, 4706, 3556}

$$-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]*Tan[c + b*x],x]
[Out] -x - (Cot[a - c]*Log[Cos[a + b*x]])/b + (Cot[a - c]*Log[Cos[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4706

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4708

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Dist[(b/d)*Cos[(b*c - a*d)/d], Int[Sec[a + b*x]*Sec[c + d*x], x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \tan(a+bx)\tan(c+bx)\,dx &= -x+\cos(a-c)\int \sec(a+bx)\sec(c+bx)\,dx\\ &= -x+\cot(a-c)\int \tan(a+bx)\,dx-\cot(a-c)\int \tan(c+bx)\,dx\\ &= -x-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(c+bx))}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.37, size = 31, normalized size = 0.79

$$-x+\frac{\cot(a-c)(-\log(\cos(a+bx))+\log(\cos(c+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Cos[a + b*x]] + Log[Cos[c + b*x]]))/b
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.13, size = 173, normalized size = 4.44

method	result	size
risch	$-x+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}$	173

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -x+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*
a)+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I*
c)-I/b/(exp(2*I*a)-exp(2*I*c))*ln(1+exp(2*I*(b*x+a)))*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(1+exp(2*I*(b*x+a)))*exp(2*I*c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(39) = 78.
time = 0.30, size = 371, normalized size = 9.51

$$-\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^{2}+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^{2}-\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}\right)b\right)x+\left(\cos\left(2\,a\right)^{2}-\cos\left(2\,c\right)^{2}+\sin\left(2\,a\right)^{2}-\sin\left(2\,c\right)^{2}\right)\arctan\left(\sin\left(2\,bx\right)-\sin\left(2\,a\right),\cos\left(2\,bx\right)+\cos\left(2\,a\right)\right)-\left(\cos\left(2\,a\right)^{2}-\cos\left(2\,c\right)^{2}+\sin\left(2\,a\right)^{2}-\sin\left(2\,c\right)^{2}\right)\arctan\left(\sin\left(2\,bx\right)-\sin\left(2\,c\right),\cos\left(2\,bx\right)+\cos\left(2\,c\right)\right)-\left(\cos\left(2\,c\right)\sin\left(2\,a\right)-\cos\left(2\,a\right)\sin\left(2\,c\right)\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right)^{2}\right)+\left(\cos\left(2\,c\right)\sin\left(2\,a\right)-\cos\left(2\,a\right)\sin\left(2\,c\right)\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)+\sin\left(2\,c\right)^{2}\right)}{2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^{2}+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^{2}-\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - (co
s(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
os(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b
*x)*sin(2*a) + sin(2*a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(co
s(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*
x)*sin(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(
2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(39) = 78$.
time = 1.57, size = 145, normalized size = 3.72

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos\left(-2\,a+2\,c\right)-1\right)\tan\left(bx+c\right)^{2}-2\,\sin\left(-2\,a+2\,c\right)\tan\left(bx+c\right)-\cos\left(-2\,a+2\,c\right)-1}{\left(\cos\left(-2\,a+2\,c\right)+1\right)\tan\left(bx+c\right)^{2}+\cos\left(-2\,a+2\,c\right)+1}\right)+\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(\frac{1}{\tan\left(bx+c\right)^{2}+1}\right)}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-((cos(-2*a + 2*c)
- 1)*tan(b*x + c)^2 - 2*sin(-2*a + 2*c)*tan(b*x + c) - cos(-2*a + 2*c) - 1)
/((cos(-2*a + 2*c) + 1)*tan(b*x + c)^2 + cos(-2*a + 2*c) + 1)) + (cos(-2*a
+ 2*c) + 1)*log(1/(tan(b*x + c)^2 + 1)))/(b*sin(-2*a + 2*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. $2(31) = 62$.
time = 6.59, size = 7672, normalized size = 196.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (b*x*tan(c)**4*tan(b*x)/(b*t
an(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 +
b*tan(c)**2*tan(b*x) - b*tan(c)) - b*x*tan(c)**3/(b*tan(c)**6*tan(b*x) - b*
tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) -
 b*tan(c)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) +
b*x*tan(c)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2
*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + 2*log(tan(b*x) - 1/tan(c)
)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - 2*log(tan(b*x)
- 1/tan(c))*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*t
an(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - log(tan(b*x)**
2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)*
*4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) + log(tan(b*
x)**2 + 1)*tan(c)**2/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)) - tan(c)**2/(b*ta
n(c)**6*tan(b*x) - b*tan(c)**5 + 2*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b
*tan(c)**2*tan(b*x) - b*tan(c)) - 1/(b*tan(c)**6*tan(b*x) - b*tan(c)**5 + 2
*b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + b*tan(c)**2*tan(b*x) - b*tan(c)), E
q(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (0, Eq(b, 0)), (-2*b*x*tan(c
)/(2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**3
+ 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**3 + 2*b*tan(c))
```

```
, Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**3 + 2*b*tan(a)) - 2*log(tan(b*x) -
 1/tan(a))/(2*b*tan(a)**3 + 2*b*tan(a)) - log(tan(b*x)**2 + 1)*tan(a)**2/(2
*b*tan(a)**3 + 2*b*tan(a)), Eq(c, 0)), (2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)*
*3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(
c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*
x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(
a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)*
*2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(c)/(2*b*tan(a)**3
*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c)
 + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(
tan(b*x) - 1/tan(a))*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2
*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*
tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2
*log(tan(b*x) - 1/tan(c))*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**
3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(c))/(2*b
*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)
**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)
) - log(tan(b*x)**2 + 1)*tan(a)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(c)**2/(
2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan
(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan
(c)), True)) + Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-4*b*x*tan(c
)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b
*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 4*b*x*tan(c)/(2*b*tan(c)
**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(2*
b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)*
*2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(2*b
*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**
2 + 2*b*tan(c)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)*tan(b*x)
/(2*b*tan(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan
(c)**2 + 2*b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c
)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan
(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 +
2*b*tan(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*
tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan
(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*t
an(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(39) = 78$.

time = 0.43, size = 242, normalized size = 6.21

$$-\frac{2\,bx+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{3}+4\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)\right)\log\left(\left|2\,\tan\left(bx\right)\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,a\right)^{2}-1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)}-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,c\right)\right)\log\left(\left|2\,\tan\left(bx\right)\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,c\right)^{2}-1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^{2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + (tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^3 + 4*tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(2*tan(b*x)*tan(1/2
*a) + tan(1/2*a)^2 - 1))/(tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^2*tan(1/2*c)
^2 + tan(1/2*a)^2 - tan(1/2*a)*tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^3 - t
an(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2
*c))*log(abs(2*tan(b*x)*tan(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^2*tan(1
/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2))/
b
```

Mupad [B]
time = 4.99, size = 207, normalized size = 5.31

$$-\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^{2}-\frac{1}{2}\right)}{\sin\left(a-c\right)^{2}}-\frac{\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^{2}\,2\mathrm{i}-\sin(a+b\,x)^{2}\,2\mathrm{i}+\sin(3\,a-2\,c+b\,x)^{2}\,2\mathrm{i}+\sin(4\,a-4\,c)+\sin(6\,a-4\,c+2\,b\,x)-\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^{2}\,2\mathrm{i}-\sin(c+b\,x)^{2}\,2\mathrm{i}+\sin(2\,a-c+b\,x)^{2}\,2\mathrm{i}+\sin(4\,a-4\,c)+\sin(4\,a-2\,c+2\,b\,x)-\sin(2\,c+2\,b\,x)\right)}{2}}{b\sin\left(a-c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)*tan(c + b*x),x)
```

```
[Out] - (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4*
a - 4*c) + sin(6*a - 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2i
- sin(a + b*x)^2*2i + sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*log(s
in(4*a - 4*c) + sin(4*a - 2*c + 2*b*x) - sin(2*c + 2*b*x) + sin(2*a - 2*c)^
2*2i - sin(c + b*x)^2*2i + sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^2)
```

3.140 $\int \tan(c-bx)\tan(a+bx)\,dx$

Optimal. Leaf size=34

$$x-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}$$

```
[Out] x-cot(a+c)*ln(cos(b*x-c))/b+cot(a+c)*ln(cos(b*x+a))/b
```

Rubi [A]
time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4708, 4706, 3556}

$$-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c - b*x]*Tan[a + b*x],x]
```

```
[Out] x - (Cot[a + c]*Log[Cos[c - b*x]])/b + (Cot[a + c]*Log[Cos[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4706

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 4708

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Dist[(b/d)*Cos[(b*c - a*d)/d], Int[Sec[a + b*x]*Sec[c + d*x], x
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \tan(c-bx)\tan(a+bx)\,dx &= x-\cos(a+c)\int \sec(c-bx)\sec(a+bx)\,dx\\ &= x-\cot(a+c)\int \tan(c-bx)\,dx-\cot(a+c)\int \tan(a+bx)\,dx\\ &= x-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 28, normalized size = 0.82

$$x+\frac{\cot(a+c)(-\log(\cos(c-bx))+\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c - b*x]*Tan[a + b*x],x]
```

```
[Out] x + (Cot[a + c]*(-Log[Cos[c - b*x]] + Log[Cos[a + b*x]]))/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 145, normalized size = 4.26

method	result	size
risch	$x+\frac{i\ln\left(1+{\mathrm e}^{2 i (b x+a)}\right){\mathrm e}^{2 i (a+c)}}{b\left({\mathrm e}^{2 i (a+c)}-1\right)}+\frac{i\ln\left(1+{\mathrm e}^{2 i (b x+a)}\right)}{b\left({\mathrm e}^{2 i (a+c)}-1\right)}-\frac{i\ln\left({\mathrm e}^{2 i (a+c)}+{\mathrm e}^{2 i (b x+a)}\right){\mathrm e}^{2 i (a+c)}}{b\left({\mathrm e}^{2 i (a+c)}-1\right)}-\frac{i\ln\left({\mathrm e}^{2 i (a+c)}+{\mathrm e}^{2 i (b x+a)}\right)}{b\left({\mathrm e}^{2 i (a+c)}-1\right)}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(b*x-c)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))*exp(2*I*(a+c))+I/b/(exp(2*I
*(a+c))-1)*ln(1+exp(2*I*(b*x+a)))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+
exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c))+e
xp(2*I*(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(35) = 70.
time = 0.29, size = 290, normalized size = 8.53

$$\frac{(b\cos(2a+2c)^2+b\sin(2a+2c)^2-2b\cos(2a+2c)+b)x-(\cos(2a+2c)^2+\sin(2a+2c)^2-1)\arctan(\sin(2bx)-\sin(2a),\cos(2bx)+\cos(2a))+(\cos(2a+2c)^2+\sin(2a+2c)^2-1)\arctan(\sin(2bx)+\sin(2c),\cos(2bx)+\cos(2c))+\log(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin(2a)+\sin(2a)^2)\sin(2a+2c)-\log(\cos(2bx)^2+2\cos(2bx)\cos(2c)+\cos(2c)^2+\sin(2bx)^2+2\sin(2bx)\sin(2c)+\sin(2c)^2)\sin(2a+2c)}{b\cos(2a+2c)^2+b\sin(2a+2c)^2-2b\cos(2a+2c)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos(2
*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2
*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b*x)*
cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)*
sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + si
n(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(2*a + 2*c))/(b*cos(2*a
 + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(35) = 70.

time = 1.67, size = 145, normalized size = 4.26

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos\left(2\,a+2\,c\right)-1\right)\tan\left(bx+a\right)^{2}-2\,\sin\left(2\,a+2\,c\right)\tan\left(bx+a\right)-\cos\left(2\,a+2\,c\right)-1}{\left(\cos\left(2\,a+2\,c\right)+1\right)\tan\left(bx+a\right)^{2}+\cos\left(2\,a+2\,c\right)+1}\right)+\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(\frac{1}{\tan\left(bx+a\right)^{2}+1}\right)}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1)
*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((cos
(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c) +
1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. $2(31) = 62$.
time = 5.47, size = 7679, normalized size = 225.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)**
2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x)*
*2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*t
an(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2
+ 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*t
an(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2
*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x)
 + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(
b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(b*x)
+ 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b
*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(
b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**5*tan(b*x) + 2*b*
tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) +
2*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*
b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
+ 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c
)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b)
- log(tan(b*x)**2 + 1)*tan(c)*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)
**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) -
 log(tan(b*x)**2 + 1)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)*
*3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*tan(c)**2/(2*b
*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**
2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4
```

```
+ 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b), Eq(a
, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (0, Eq(b, 0)), (2*b*x*tan(a)/(
2*b*tan(a)**2 + 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - l
og(tan(b*x)**2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (2*b*x*tan(a)**2/(2*b
*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)
**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)
) - 2*b*x*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**
2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*
b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)*tan(c)**2/(2*
b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a
)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c
)) + 2*log(tan(b*x) - 1/tan(a))*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a
)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**
2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*t
an(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*ta
n(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*ta
n(c)**3 + 2*b*tan(c)) + 2*log(tan(b*x) + 1/tan(c))*tan(c)/(2*b*tan(a)**3*ta
n(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) +
2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b
*x)**2 + 1)*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b
*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*ta
n(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)*tan(c)**2/
(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*ta
n(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*ta
n(c)) - log(tan(b*x)**2 + 1)*tan(a)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**
3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(c)/(2*
b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a
)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c
)), True))*tan(a) - Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*
tan(c)/(2*b*tan(c)**2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 +
2*b) - log(tan(b*x)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-4*b*x*tan(c
)**2*tan(b*x)/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b
*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)
**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*
b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)*
*2 + 2*b*tan(c)*tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b
*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**
2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)
/(2*b*tan(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan
(c)**2 + 2*b*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c
)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*t...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(35) =

70.
time = 0.46, size = 245, normalized size = 7.21

$$\frac{2\,bx-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{3}-4\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)\right)\log\left(\left|2\,\tan\left(bx\right)\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,a\right)^{2}-1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{2}-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)}+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,c\right)\right)\log\left(\left|2\,\tan\left(bx\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^{2}+1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^{2}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x - (tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^3 - 4*tan(1/2*a)^2*tan
(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(2*tan(b*x)*tan(1/2*
a) + tan(1/2*a)^2 - 1))/(tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^
2 - tan(1/2*a)^2 - tan(1/2*a)*tan(1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^3 - ta
n(1/2*a)^2*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*
c))*log(abs(2*tan(b*x)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2))/b
```

Mupad [B]
time = 5.00, size = 196, normalized size = 5.76

$$\frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^{2}-\frac{1}{2}\right)}{\sin\left(a+c\right)^{2}}+\frac{\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+2\,c)^{2}\,2\mathrm{i}-\sin(a+b\,x)^{2}\,2\mathrm{i}+\sin(3\,a+2\,c+b\,x)^{2}\,2\mathrm{i}+\sin(4\,a+4\,c)+\sin(6\,a+4\,c+2\,b\,x)-\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+c+b\,x)^{2}\,2\mathrm{i}+\sin(2\,a+2\,c)^{2}\,2\mathrm{i}-\sin(c-b\,x)^{2}\,2\mathrm{i}+\sin(4\,a+4\,c)+\sin(4\,a+2\,c+2\,b\,x)+\sin(2\,c-2\,b\,x)\right)}{2}}{b\sin\left(a+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)*tan(c - b*x),x)
```

```
[Out] (x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
+ 4*c) + sin(6*a + 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i -
sin(a + b*x)^2*2i + sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(sin
(4*a + 4*c) + sin(4*a + 2*c + 2*b*x) + sin(2*c - 2*b*x) + sin(2*a + c + b*x
)^2*2i + sin(2*a + 2*c)^2*2i - sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

3.141 $\int \cot(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=39

$$-x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}$$

```
[Out] -x-cot(a-c)*ln(sin(b*x+a))/b+cot(a-c)*ln(sin(b*x+c))/b
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4709, 4707, 3556}

$$-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(bx+c))}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x - (Cot[a - c]*Log[Sin[a + b*x]])/b + (Cot[a - c]*Log[Sin[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4707

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 4709

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \cot(a+bx)\cot(c+bx)\,dx &= -x+\cos(a-c)\int \csc(a+bx)\csc(c+bx)\,dx\\
&= -x-\cot(a-c)\int \cot(a+bx)\,dx+\cot(a-c)\int \cot(c+bx)\,dx\\
&= -x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.38, size = 31, normalized size = 0.79

$$-x+\frac{\cot(a-c)(-\log(\sin(a+bx))+\log(\sin(c+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -x + (Cot[a - c]*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]]))/b
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.16, size = 177, normalized size = 4.54

method	result	size
risch	$-x-\frac{i\ln\left(e^{2i(bx+a)}-1\right)e^{2ia}}{b(e^{2ia}-e^{2ic})}-\frac{i\ln\left(e^{2i(bx+a)}-1\right)e^{2ic}}{b(e^{2ia}-e^{2ic})}+\frac{i\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{2ia}}{b(e^{2ia}-e^{2ic})}+\frac{i\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{2ic}}{b(e^{2ia}-e^{2ic})}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(2
*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(2*I
*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(39) = 78.
time = 0.32, size = 549, normalized size = 14.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c
)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a)^2
 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(a), cos(b*x
) + cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(s
in(b*x) + sin(c), cos(b*x) - cos(c)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^
2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) - (cos(2*c)*s
in(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a) - cos(2*a
)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(a) + sin(a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(c
```

```
os(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) +
 sin(c)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2))/(2*b*
cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (
cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(39) = 78$.
time = 2.61, size = 118, normalized size = 3.03

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos\left(2\,bx+2\,c\right)\cos\left(-2\,a+2\,c\right)+\sin\left(2\,bx+2\,c\right)\sin\left(-2\,a+2\,c\right)-1}{\cos\left(-2\,a+2\,c\right)+1}\right)+\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,c\right)+\frac{1}{2}\right)}{2\,b\sin\left(-2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)*
cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c) +
1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*a +
 2*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(31) = 62$.
time = 15.71, size = 7404, normalized size = 189.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x)
```

```
[Out] Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq
(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (-b*x*tan(c)**5/(b*tan(c)**5
+ b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c)
+ b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x)
+ 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan
(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*
tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(b*tan(c)**5 + b
*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b
*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*
tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x))
- 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*ta
n(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) +
log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*ta
n(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + log(tan(b*x)**2
 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**
3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**6/(b*tan(c)**
```

5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c
) + b*tan(b*x)) - tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c
)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)), Eq(a, atan(tan(c))
+ pi*floor((c - pi/2)/pi))), (x/(cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) +
zoo), Eq(b, 0)), (-2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b
*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*ta
n(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + 2*b*x*tan(a)**2*tan(c)
3/(2*b*tan(a)3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*
b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*
b*tan(c)) + 2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3
 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 +
2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(
a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*t
an(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2
*log(tan(a) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*
tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan
(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) - 2*log(tan(a) + tan(b*x)
)*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2
*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b
*tan(c)**3 - 2*b*tan(c)) + 2*log(tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*
b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a
)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c
)) + 2*log(tan(c) + tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2
*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*
tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)) + log(tan(b*x)**2 + 1)
*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*
tan(c)**3 - 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*
tan(c)**3 - 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)
3*tan(c)2 + 2*b*tan(a)**3 - 2*b*tan(a)**2*tan(c)**3 - 2*b*tan(a)**2*tan
(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) - 2*b*tan(c)**3 - 2*b*tan(c)), True
)) + Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b
, 0) & Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (b*x*tan(c)**5/(b*ta
n(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b
*tan(c) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*t
an(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) -
 b*x*tan(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*ta
n(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*tan(b*x)/(b*tan(c
)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*ta
n(c) + b*tan(b*x)) + 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*ta
n(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*ta
n(b*x)) + 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(
c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(
b*x)) - log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(b*x)
+ 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - log(tan
(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*

```
tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*
tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) +
 b*tan(c) + b*tan(b*x)) - tan(c)**2/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2
*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)), Eq(a, atan(
tan(c)) + pi*floor((c - pi/2)/pi))), (zoo*x/(cot(a)*cot(c) + zoo*cot(a) + z
oo*cot(c) + zoo), Eq(b, 0)), (2*b*x*tan(a)**3*t...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(39) = 78$.
time = 0.46, size = 348, normalized size = 8.92

$$-\frac{2\,bx+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^4\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^4+4\,\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+2\,\tan\left(\frac{1}{2}\,a\right)^2-4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,c\right)^2-1\right)\log\left(\left|\tan\left(bx\right)\tan\left(\frac{1}{2}\,a\right)^2-\tan\left(bx\right)-2\,\tan\left(\frac{1}{2}\,a\right)\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^4\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^3-2\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,c\right)}-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^4-2\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,c\right)^4+\tan\left(\frac{1}{2}\,a\right)^2-4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+2\,\tan\left(\frac{1}{2}\,c\right)^2-1\right)\log\left(\left|\tan\left(bx\right)\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(bx\right)-2\,\tan\left(\frac{1}{2}\,c\right)\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^4-\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)+2\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,c\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*ta
n(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 - 4*tan(1/2*a)*tan(
1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*tan
(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)
^3 - 2*tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) + tan
(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan
(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 - 4*tan(1/2*a)*tan(1/2*c
) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) - 2*tan(1/
2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^2*
tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a) + tan(1/
2*c)))/b
```

Mupad [B]
time = 4.82, size = 207, normalized size = 5.31

$$-\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^2-\frac{1}{2}\right)}{\sin\left(a-c\right)^2}-\frac{\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^2\,2\mathrm{i}+\sin(a+b\,x)^2\,2\mathrm{i}-\sin(3\,a-2\,c+b\,x)^2\,2\mathrm{i}+\sin(4\,a-4\,c)-\sin(6\,a-4\,c+2\,b\,x)+\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a-2\,c)\,\ln\left(\sin(2\,a-2\,c)^2\,2\mathrm{i}+\sin(c+b\,x)^2\,2\mathrm{i}-\sin(2\,a-c+b\,x)^2\,2\mathrm{i}+\sin(4\,a-4\,c)-\sin(4\,a-2\,c+2\,b\,x)+\sin(2\,c+2\,b\,x)\right)}{2}}{b\sin\left(a-c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*cot(c + b*x),x)
```

```
[Out] - (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4*
a - 4*c) - sin(6*a - 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2i
+ sin(a + b*x)^2*2i - sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*log(s
in(4*a - 4*c) - sin(4*a - 2*c + 2*b*x) + sin(2*c + 2*b*x) + sin(2*a - 2*c)^
2*2i + sin(c + b*x)^2*2i - sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^2)
```

3.142 $\int \cot(c-bx)\cot(a+bx)\,dx$

Optimal. Leaf size=34

$$x-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}$$

```
[Out] x-cot(a+c)*ln(-sin(b*x-c))/b+cot(a+c)*ln(sin(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4709, 4707, 3556}

$$-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x - (Cot[a + c]*Log[Sin[c - b*x]])/b + (Cot[a + c]*Log[Sin[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4707

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rule 4709

```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Dist[Cos[(b*c - a*d)/d], Int[Csc[a + b*x]*Csc[c + d*x], x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c-bx)\cot(a+bx)\,dx &= x+\cos(a+c)\int \csc(c-bx)\csc(a+bx)\,dx\\ &= x+\cot(a+c)\int \cot(c-bx)\,dx+\cot(a+c)\int \cot(a+bx)\,dx\\ &= x-\frac{\cot(a+c)\log(\sin(c-bx))}{b}+\frac{\cot(a+c)\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 30, normalized size = 0.88

$$x+\frac{\cot(a+c)(-\log(\sin(c-bx))+\log(-\sin(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c - b*x]*Cot[a + b*x],x]
```

```
[Out] x + (Cot[a + c]*(-Log[Sin[c - b*x]] + Log[-Sin[a + b*x]]))/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 149, normalized size = 4.38

method	result	size
risch	$x+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}+\frac{i\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}-\frac{i\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}-\frac{i\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(b*x-c)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I
*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))
+exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))
+exp(2*I*(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(37) = 74.
time = 0.29, size = 432, normalized size = 12.71

$$\frac{\left(b\cos(2a+2c)^2+b\sin(2a+2c)^2-2b\cos(2a+2c)+b\right)x-\left(\cos(2a+2c)^2+\sin(2a+2c)^2-1\right)\arctan\left(\sin(bx)+\sin(a),\cos(bx)-\cos(a)\right)-\left(\cos(2a+2c)^2+\sin(2a+2c)^2-1\right)\arctan\left(\sin(bx)-\sin(a),\cos(bx)+\cos(a)\right)+\left(\cos(2a+2c)^2+\sin(2a+2c)^2-1\right)\arctan\left(\sin(bx)+\sin(c),\cos(bx)+\cos(c)\right)+\left(\cos(2a+2c)^2+\sin(2a+2c)^2-1\right)\arctan\left(\sin(bx)-\sin(c),\cos(bx)-\cos(c)\right)+\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)\sin(2a+2c)+\log\left(\cos(bx)^2-2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2+2\sin(bx)\sin(a)+\sin(a)^2\right)\sin(2a+2c)-\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2+2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(2a+2c)-\log\left(\cos(bx)^2-2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(2a+2c)}{b\cos(2a+2c)^2+b\sin(2a+2c)^2-2b\cos(2a+2c)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] ((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (co
s(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x)
- cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) - si
n(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan
2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c
)^2 - 1)*arctan2(sin(b*x) - sin(c), cos(b*x) - cos(c)) + log(cos(b*x)^2 + 2
*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)*si
n(2*a + 2*c) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 +
 2*sin(b*x)*sin(a) + sin(a)^2)*sin(2*a + 2*c) - log(cos(b*x)^2 + 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(2*a + 2
*c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*
```

```
x)*sin(c) + sin(c)^2)*sin(2*a + 2*c))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c
)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.
time = 4.55, size = 118, normalized size = 3.47

$$\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos\left(2\,bx+2\,a\right)\cos\left(2\,a+2\,c\right)+\sin\left(2\,bx+2\,a\right)\sin\left(2\,a+2\,c\right)-1}{\cos\left(2\,a+2\,c\right)+1}\right)+\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)}{2\,b\sin\left(2\,a+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*cos
(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1)) +
(cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1722 vs. $2(32) = 64$.
time = 16.57, size = 7417, normalized size = 218.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x)
```

```
[Out] -Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & E
q(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (b*x*tan(c)**5/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c
) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*
x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - b*x*
tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)
**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c
) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**4/(-b*tan(c)**5 + b*tan
(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan
(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan
(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan
(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x
) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - log(t
an(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2
*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**6/
(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*
x) - b*tan(c) + b*tan(b*x)) - tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x
) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)), Eq(a,
-atan(tan(c)) - pi*floor((c - pi/2)/pi))), (x/(-cot(a)*cot(c) + zoo*cot(a)
```

+ zoo*cot(c) + zoo), Eq(b, 0)), (-2*b*x*tan(a)**3*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)**2*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)**2*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*b*x*tan(a)*tan(c)**2/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + 2*log(tan(a) + tan(b*x))*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)**3*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - 2*log(-tan(c) + tan(b*x))*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) - log(tan(b*x)**2 + 1)*tan(a)**3*tan(c)/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)) + log(tan(b*x)**2 + 1)*tan(a)*tan(c)**3/(2*b*tan(a)**3*tan(c)**2 + 2*b*tan(a)**3 + 2*b*tan(a)**2*tan(c)**3 + 2*b*tan(a)**2*tan(c) + 2*b*tan(a)*tan(c)**2 + 2*b*tan(a) + 2*b*tan(c)**3 + 2*b*tan(c)), True)) + Piecewise((zoo*x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (-b*x*tan(c)**5/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - b*x*tan(c)**2*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**4/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**2/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*ta

```
n(c) + b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (zoo*x
/(-cot(a)*cot(c) + zoo*cot(a) + zoo*cot(c) + zo...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(37) = 74.
time = 0.44, size = 345, normalized size = 10.15

$$\frac{2\,bx-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{4}-4\,\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+2\,\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,c\right)^{2}-1\right)\log\left(\left|\tan\left(bx\right)\tan\left(\frac{1}{2}\,a\right)^{2}-\tan\left(bx\right)-2\,\tan\left(\frac{1}{2}\,a\right)\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,a\right)^{3}-2\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,c\right)}+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{4}-2\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,c\right)^{4}+\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+2\,\tan\left(\frac{1}{2}\,c\right)^{2}-1\right)\log\left(\left|\tan\left(bx\right)\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(bx\right)+2\,\tan\left(\frac{1}{2}\,c\right)\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{4}-\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,c\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x - (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 - 4*tan(1/2*a)^3*tan
(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1
/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*tan(
1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^
3 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) + tan(
1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 - 4*tan(
1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c)
 + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) + 2*tan(1/2
*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*a)^2*t
an(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a) + tan(1/2
*c)))/b
```

Mupad [B]
time = 5.05, size = 200, normalized size = 5.88

$$\frac{\frac{x}{2}+x\left(\sin\left(a+c\right)^{2}-\frac{1}{2}\right)}{\sin\left(a+c\right)^{2}}+\frac{\frac{\sin(2\,a+2\,c)\,\ln\left(\sin(2\,a+2\,c)^{2}\,2\mathrm{i}+\sin(a+b\,x)^{2}\,2\mathrm{i}-\sin(3\,a+2\,c+b\,x)^{2}\,2\mathrm{i}+\sin(4\,a+4\,c)-\sin(6\,a+4\,c+2\,b\,x)+\sin(2\,a+2\,b\,x)\right)}{2}-\frac{\sin(2\,a+2\,c)\,\ln\left(-\sin(2\,a+c+b\,x)^{2}\,2\mathrm{i}+\sin(2\,a+2\,c)^{2}\,2\mathrm{i}+\sin(c-b\,x)^{2}\,2\mathrm{i}+\sin(4\,a+4\,c)-\sin(4\,a+2\,c+2\,b\,x)-\sin(2\,c-2\,b\,x)\right)}{2}}{b\sin\left(a+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*cot(c - b*x),x)
```

```
[Out] (x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
+ 4*c) - sin(6*a + 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i +
sin(a + b*x)^2*2i - sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(sin
(4*a + 4*c) - sin(4*a + 2*c + 2*b*x) - sin(2*c - 2*b*x) - sin(2*a + c + b*x
)^2*2i + sin(2*a + 2*c)^2*2i + sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

3.143 $\int \sec(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}$$

```
[Out] -csc(a-c)*ln(cos(b*x+a))/b+csc(a-c)*ln(cos(b*x+c))/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4706, 3556}

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4706

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\sec(c+bx)\,dx &= \csc(a-c)\int \tan(a+bx)\,dx-\csc(a-c)\int \tan(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\cos(a+bx))}{b}+\frac{\csc(a-c)\log(\cos(c+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\cos(a+bx))-\log(\cos(c+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

Maple [A]
time = 0.61, size = 54, normalized size = 1.50

method	result	size
default	$\frac{\ln(\tan(bx+a)\sin(a)\cos(c)-\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{b(\sin(a)\cos(c)-\cos(a)\sin(c))}$	54
risch	$\frac{2i\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{i(a+c)}}{(\mathrm{e}^{2ia}-\mathrm{e}^{2ic})b}-\frac{2i\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)\mathrm{e}^{i(a+c)}}{(\mathrm{e}^{2ia}-\mathrm{e}^{2ic})b}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*co
s(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(36) = 72.
time = 0.28, size = 349, normalized size = 9.69

$$-\frac{2\left(\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\cos\left(a+c\right)+\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\sin\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2a\right),\cos\left(2bx\right)+\cos\left(2a\right)\right)-2\left(\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\cos\left(a+c\right)+\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\sin\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2c\right),\cos\left(2bx\right)+\cos\left(2c\right)\right)-\left(\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\cos\left(a+c\right)-\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\sin\left(a+c\right)\right)\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2a\right)+\cos\left(2a\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2a\right)+\sin\left(2a\right)^2\right)+\left(\left(\sin\left(2a\right)-\sin\left(2c\right)\right)\cos\left(a+c\right)-\left(\cos\left(2a\right)-\cos\left(2c\right)\right)\sin\left(a+c\right)\right)\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2c\right)+\cos\left(2c\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2c\right)+\sin\left(2c\right)^2\right)}{2b\cos\left(2a\right)\cos\left(2c\right)-b\cos\left(2c\right)^2+2b\sin\left(2a\right)\sin\left(2c\right)-b\sin\left(2c\right)^2-\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos(2
*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) - si
n(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2
*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(
2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2*a) -
sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 +
 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c)
+ sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c
) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.
time = 1.63, size = 107, normalized size = 2.97

$$-\frac{\log\left(\cos\left(bx+c\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)}{\cos(-a+c)^2+2\cos(-a+c)+1}\right)}{2b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)*
sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(co
s(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec{(a+bx)}\sec{(bx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x)
```

```
[Out] Integral(sec(a + b*x)*sec(b*x + c), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(36) = 72.
time = 0.45, size = 171, normalized size = 4.75

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}a\right)^2-2\tan\left(bx+a\right)\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right|\right)}{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(2
*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c
)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2
- 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/
((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*
c))*b)
```

Mupad [B]
time = 7.84, size = 249, normalized size = 6.92

$$\frac{2\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}-c\,2{}\mathrm{i}}}\,\left(\ln\left(-\frac{2\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}}\,\left(4\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\right)}{b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}-1\right)}+{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,1{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\,4{}\mathrm{i}\right)-\ln\left(-\frac{2\sqrt{-{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}}\,\left(4\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\right)}{b-b\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,2{}\mathrm{i}}}+{\mathrm{e}}^{a\,1{}\mathrm{i}}\,{\mathrm{e}}^{a\,2{}\mathrm{i}}\,{\mathrm{e}}^{-c\,1{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2{}\mathrm{i}}\,4{}\mathrm{i}\right)\right)}{b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}-c\,2{}\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

```
[Out] (2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i
)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp
(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*e
xp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i - (2*(
-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)*exp(
b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(-c*2i
)))))/(b*(exp(a*2i - c*2i) - 1))
```

3.144 $\int \sec(c-bx)\sec(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

```
[Out] csc(a+c)*ln(cos(b*x-c))/b-csc(a+c)*ln(cos(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4706, 3556}

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4706

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[-Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c-bx)\sec(a+bx)\,dx&=\csc(a+c)\int \tan(c-bx)\,dx+\csc(a+c)\int \tan(a+bx)\,dx\\&=\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 26, normalized size = 0.79

$$\frac{\csc(a+c)(\log(\cos(c-bx))-\log(\cos(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c - b*x]*Sec[a + b*x],x]
```

```
[Out] (Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b
```

Maple [A]
time = 0.55, size = 53, normalized size = 1.61

method	result	size
default	$\frac{\ln(\tan(bx+a)\sin(a)\cos(c)+\tan(bx+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}$	53
risch	$-\frac{2i\ln\left(1+e^{2i(bx+a)}\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}+\frac{2i\ln\left(e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x-c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)+tan(b*x+a)*co
s(a)*sin(c)+cos(a)*cos(c)-sin(a)*sin(c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(34) = 68.
time = 0.29, size = 322, normalized size = 9.76

$$\frac{2\left(\cos\left(2a+2c\right)\cos\left(a+c\right)+\sin\left(2a+2c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)-\sin\left(2a\right),\cos\left(2bx\right)+\cos\left(2a\right)\right)-2\left(\cos\left(2a+2c\right)\cos\left(a+c\right)+\sin\left(2a+2c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(2bx\right)+\sin\left(2c\right),\cos\left(2bx\right)+\cos\left(2c\right)\right)-\left(\cos\left(a+c\right)\sin\left(2a+2c\right)-\cos\left(2a+2c\right)\sin\left(a+c\right)+\sin\left(a+c\right)\right)\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2a\right)+\cos\left(2a\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2a\right)+\sin\left(2a\right)^2\right)+\left(\cos\left(a+c\right)\sin\left(2a+2c\right)-\cos\left(2a+2c\right)\sin\left(a+c\right)+\sin\left(a+c\right)\right)\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2c\right)+\cos\left(2c\right)^2+\sin\left(2bx\right)^2+2\sin\left(2bx\right)\sin\left(2c\right)+\sin\left(2c\right)^2\right)}{b\cos\left(2a+2c\right)^2+b\sin\left(2a+2c\right)^2-2b\cos\left(2a+2c\right)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arc
tan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*(cos(2*a + 2*c)*cos(
a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(2*b*x) + sin(2
*c), cos(2*b*x) + cos(2*c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*s
in(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)
^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(a + c)*sin(2
*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*co
s(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin
(2*c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(34) = 68.
time = 1.44, size = 93, normalized size = 2.82

$$-\frac{\log\left(\cos\left(bx+a\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2+1\right)}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(cos(b*x + a)^2) - log(4*(2*cos(b*x + a)*cos(a + c)*sin(b*x + a)*s
in(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2 + 1)/(cos(a
+ c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec(a+bx)\sec(bx-c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)*sec(b*x - c), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(34) = 68.
time = 0.45, size = 169, normalized size = 5.12

$$-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)+\tan\left(\frac{1}{2}a\right)^2-2\tan\left(bx+a\right)\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^2-1\right|\right)}{2\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*
c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a)^2
 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1))
/((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2
*c))*b)
```

Mupad [**B**]
time = 7.73, size = 249, normalized size = 7.55

$$\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}}\left(\ln\left(-\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}\,\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}-1\right)}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(-\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}\,\left(4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b-b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*cos(c - b*x)),x)
```

```
[Out] (2*(-exp(a*2i + c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)
*4i - (2*(-exp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*
2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*exp(c
*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i - (2*(-exp(a
*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*2i)*exp(b*x*2i)
+ 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(c*2i)))))/(b*(
exp(a*2i + c*2i) - 1))
```

3.145 $\int \csc(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\csc(a-c)\log(\sin(a+bx))}{b}+\frac{\csc(a-c)\log(\sin(c+bx))}{b}$$

```
[Out] -csc(a-c)*ln(sin(b*x+a))/b+csc(a-c)*ln(sin(b*x+c))/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4707, 3556}

$$\frac{\csc(a-c)\log(\sin(bx+c))}{b}-\frac{\csc(a-c)\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*Log[Sin[a + b*x]])/b) + (Csc[a - c]*Log[Sin[c + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4707

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\csc(c+bx)\,dx &= -\left(\csc(a-c)\int \cot(a+bx)\,dx\right)+\csc(a-c)\int \cot(c+bx)\,dx\\ &= -\frac{\csc(a-c)\log(\sin(a+bx))}{b}+\frac{\csc(a-c)\log(\sin(c+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 28, normalized size = 0.78

$$-\frac{\csc(a-c)(\log(\sin(a+bx))-\log(\sin(c+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

```
[Out] -((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

time = 0.56, size = 79, normalized size = 2.19

method	result	size
default	$\frac{-\frac{\ln(\tan(bx+a))}{\sin(a)\cos(c)-\cos(a)\sin(c)}+\frac{\ln(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{\sin(a)\cos(c)-\cos(a)\sin(c)}}{b}$	79
risch	$\frac{2i\ln\left(e^{2i(bx+a)}-e^{2i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}-\frac{2i\ln\left(e^{2i(bx+a)}-1\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)-cos(a
)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(c
)+cos(a)*sin(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(36) = 72$.

time = 0.30, size = 564, normalized size = 15.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

```
[Out] -(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*a
rctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*cos
(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), cos(
b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c)
)*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(2*a)
- cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x)
 - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*
a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c))*cos(a +
 c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(2*a) - sin(
2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos
(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + ((si
n(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b
```

```
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin
(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*s
in(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(36) = 72$.
time = 1.40, size = 110, normalized size = 3.06

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^{2}+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^{2}-1\right)\cos(bx+c)^{2}-\cos(-a+c)^{2}}{\cos(-a+c)^{2}+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*sin
(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^
2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(a+bx\right)\csc\left(bx+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*csc(b*x + c), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(36) = 72$.
time = 0.45, size = 396, normalized size = 11.00

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log\left(\left|\tan(bx+a)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^{2}+4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan(bx+a)\tan\left(\frac{1}{2}c\right)^{2}+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan(bx+a)-2\tan\left(\frac{1}{2}a\right)+2\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{2}-6\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}a\right)^{3}+6\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-6\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\left|\tan(bx+a)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 +
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1/2
```

```
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B]
time = 7.77, size = 249, normalized size = 6.92

$$\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}\,\left(\ln\left(\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}}\,\left(-4\,b\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,4\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}\right)}{b\,\left({\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}-1\right)}-{\mathrm{e}}^{a\,1\mathrm{i}}\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,1\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(\frac{2\,\sqrt{-{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}}\,\left(-4\,b\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}+2\,b\,{\mathrm{e}}^{a\,4\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}\right)}{b-b\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,2\mathrm{i}}}-{\mathrm{e}}^{a\,1\mathrm{i}}\,{\mathrm{e}}^{a\,2\mathrm{i}}\,{\mathrm{e}}^{-c\,1\mathrm{i}}\,{\mathrm{e}}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)\right)}{b\,\left({\mathrm{e}}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(c + b*x)),x)
```

```
[Out] (2*(-exp(a*2i - c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp
(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*ex
p(b*x*2i)))/(b*(exp(a*2i)*exp(-c*2i) - 1)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)
*exp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp(a*2i)*exp(
b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))
/(b - b*exp(a*2i)*exp(-c*2i)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*
4i)))/(b*(exp(a*2i - c*2i) - 1))
```

3.146 $\int \csc(c-bx)\csc(a+bx)\,dx$

Optimal. Leaf size=33

$$-\frac{\csc(a+c)\log(\sin(c-bx))}{b}+\frac{\csc(a+c)\log(\sin(a+bx))}{b}$$

```
[Out] -csc(a+c)*ln(-sin(b*x-c))/b+csc(a+c)*ln(sin(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4707, 3556}

$$\frac{\csc(a+c)\log(\sin(a+bx))}{b}-\frac{\csc(a+c)\log(\sin(c-bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c - b*x]*Csc[a + b*x],x]
[Out] -((Csc[a + c]*Log[Sin[c - b*x]])/b) + (Csc[a + c]*Log[Sin[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4707

```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Dist[Csc[(
b*c - a*d)/b], Int[Cot[a + b*x], x], x] - Dist[Csc[(b*c - a*d)/d], Int[Cot[
c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
- a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c-bx)\csc(a+bx)\,dx&=\csc(a+c)\int \cot(c-bx)\,dx+\csc(a+c)\int \cot(a+bx)\,dx\\&=-\frac{\csc(a+c)\log(\sin(c-bx))}{b}+\frac{\csc(a+c)\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 29, normalized size = 0.88

$$-\frac{\csc(a+c)(\log(\sin(c-bx))-\log(-\sin(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c - b*x]*Csc[a + b*x],x]
```

```
[Out] -((Csc[a + c]*(Log[Sin[c - b*x]] - Log[-Sin[a + b*x]]))/b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.
time = 0.54, size = 80, normalized size = 2.42

method	result	size
default	$-\frac{-\frac{\ln(\tan(bx+a))}{\sin(a)\cos(c)+\cos(a)\sin(c)}+\frac{\ln(\tan(bx+a)\cos(a)\cos(c)-\tan(bx+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))}{\sin(a)\cos(c)+\cos(a)\sin(c)}}{b}$	80
risch	$-\frac{2i\ln\left(-e^{2i(a+c)}+e^{2i(bx+a)}\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}+\frac{2i\ln\left(e^{2i(bx+a)}-1\right)e^{i(a+c)}}{\left(e^{2i(a+c)}-1\right)b}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(b*x-c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-1/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)+cos(
a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos(
c)-cos(a)*sin(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(36) = 72$.
time = 0.30, size = 536, normalized size = 16.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*ar
ctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*(cos(2*a + 2*c)*cos(a + c)
+ sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - sin(a), cos(b*
x) + cos(a)) - 2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - c
os(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) - 2*(cos(2*a + 2*c
)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - s
in(c), cos(b*x) - cos(c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*sin
(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(a + c)*sin(2*a + 2*c) - cos(2*
a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(a + c)*sin(2*a + 2
*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*c
os(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(a + c)
*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 -
```

```
2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2))/
(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.
time = 2.78, size = 96, normalized size = 2.91

$$\frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(-1/4*cos(b*x + a)^2 + 1/4) - log(-(2*cos(b*x + a)*cos(a + c)*sin(b
*x + a)*sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2)/(c
os(a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1824 vs. $2(31) = 62$.
time = 53.63, size = 1824, normalized size = 55.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x)
```

```
[Out] Piecewise((-tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c
/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(c
/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)
**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - tan(b*x/2)/(-2*b*tan(c/2
)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan
(c/2)*tan(b*x/2)), Eq(a, 2*atan(1/tan(c/2)))), (tan(c/2)**4*tan(b*x/2)/(-2*
b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2
+ 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan
(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*ta
n(b*x/2)) + tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b
*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, -2*atan(tan(c/
2)) - 2*pi*floor((c/2 - pi/2)/pi))), (x/(sin(a)*sin(c)), Eq(b, 0)), (-log(-
tan(c/2) + tan(b*x/2))*tan(c/2)/(2*b) - log(-tan(c/2) + tan(b*x/2))/(2*b*ta
n(c/2)) - log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(2*b) - log(tan(b*x/2) + 1/
tan(c/2))/(2*b*tan(c/2)) + log(tan(b*x/2))*tan(c/2)/(2*b) + log(tan(b*x/2))
/(2*b*tan(c/2)), Eq(a, 0)), (log(tan(a/2) + tan(b*x/2))*tan(a/2)/(2*b) + lo
g(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)) + log(tan(b*x/2) - 1/tan(a/2))*tan(
a/2)/(2*b) + log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a/2)) - log(tan(b*x/2))*
tan(a/2)/(2*b) - log(tan(b*x/2))/(2*b*tan(a/2)), Eq(c, 0)), (-log(tan(a/2)
```

```
+ tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a
/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))
*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan
(a/2) - 2*b*tan(c/2)) - log(tan(a/2) + tan(b*x/2))*tan(c/2)**2/(2*b*tan(a/2
)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - l
og(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)
**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(a/2)**
2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*ta
n(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x/2))*tan(a/2)**2/(2*b*tan(a
/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) +
 log(-tan(c/2) + tan(b*x/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*ta
n(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(-tan(c/2) + tan(b*x
/2))/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) -
2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**2*tan(c/2)**2/(2*b*t
an(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2
)) - log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2
*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1
/tan(a/2))*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2
 - 2*b*tan(a/2) - 2*b*tan(c/2)) - log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a/2
)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + l
og(tan(b*x/2) + 1/tan(c/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)**2*tan(c/
2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/
2) + 1/tan(c/2))*tan(a/2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c
/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))*tan(c/
2)**2/(2*b*tan(a/2)**2*tan(c/2) + 2*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) -
 2*b*tan(c/2)) + log(tan(b*x/2) + 1/tan(c/2))/(2*b*tan(a/2)**2*tan(c/2) + 2
*b*tan(a/2)*tan(c/2)**2 - 2*b*tan(a/2) - 2*b*tan(c/2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(36) = 72.

time = 0.44, size = 397, normalized size = 12.03

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^4-4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log\left(\left|\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2-4\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan(bx+a)\tan\left(\frac{1}{2}c\right)^2+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan(bx+a)-2\tan\left(\frac{1}{2}a\right)-2\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^4-\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)-6\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^2-6\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^4+\tan\left(\frac{1}{2}a\right)^3+6\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|\tan(bx+a)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^
4 - 4*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 -
4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2
 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2*tan
(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^
2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/2*c)^3
 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a)^3*tan
(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 + tan(1/2
```

```
*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^
3 - tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c) + tan(1/2
*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B]
time = 7.87, size = 249, normalized size = 7.55

$$\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}}\,\left(\ln\left(\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}\,\left(-4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}-1\right)}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)-\ln\left(\frac{2\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}\,\left(-4\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+2\,b\,\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\right)}{b-b\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,2\mathrm{i}}}+\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}\,4\mathrm{i}\right)\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+c\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(c - b*x)),x)
```

```
[Out] (2*(-exp(a*2i + c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(
a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b
*x*2i)))/(b*(exp(a*2i)*exp(c*2i) - 1)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(
b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(a*2i)*exp(b*x*2i
) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*
exp(a*2i)*exp(c*2i)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i)))/(b*(
exp(a*2i + c*2i) - 1))
```

3.147 $\int \sqrt{\sin(x)\tan(x)}\, dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

[Out] -2*cot(x)*(sin(x)*tan(x))^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4485, 2669}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[x]*Tan[x]],x]

[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sin(x)\tan(x)}\, dx &= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\, dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\ &= -2\cot(x)\sqrt{\sin(x)\tan(x)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 1.00

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sin[x]*Tan[x]],x]

[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(11) = 22$.

time = 0.62, size = 177, normalized size = 13.62

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{-\frac{\left(e^{2ix}-1\right)^{2}e^{-ix}}{e^{2ix}+1}}\,\left(e^{2ix}+1\right)}{e^{2ix}-1}$
default	$\frac{(\cos(x)-1)\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^{2}}}+\ln\left(-\frac{2\left(2\left(\cos^{2}(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^{2}}}-\left(\cos^{2}(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^{2}}}-1\right)}{\sin(x)^{2}}\right)-\ln}{4\sin(x)^{3}\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(x)*tan(x))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(cos(x)-1)*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+4*(-cos(x)/(1+cos(x))^2)^(1/2))*cos(x)*(-(cos(x)^2-1)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x))^2)^(1/2)*4^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.

time = 0.51, size = 57, normalized size = 4.38

$$\frac{2\left(\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\,\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\,\sqrt{\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="maxima")

[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))

Fricas [A]
time = 2.75, size = 22, normalized size = 1.69

$$-\frac{2 \sqrt{-\frac{\cos \left(x\right)^{2} - 1}{\cos \left(x\right)}} \cos \left(x\right)}{\sin \left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin \left(x\right) \tan \left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(1/2),x)
[Out] Integral(sqrt(sin(x)*tan(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sin(x)*tan(x)), x)
```

Mupad [B]
time = 2.56, size = 20, normalized size = 1.54

$$-\frac{2 \sin \left(x\right)}{\sqrt{\frac{1}{\cos \left(x\right)}} \sqrt{1 - \cos \left(x\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(1/2),x)
[Out] -(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

3.148 $\int (\sin(x)\tan(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4485, 2678, 2669}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.74

$$\frac{2}{3}\left(-1+4\csc^2(x)\right)\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(3/2),x]
```

```
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(23) = 46$.
time = 0.32, size = 587, normalized size = 18.94

method	result	size
default	Expression too large to display	587

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(cos(x)-1)^2*(3*cos(x)^3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2
)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1
+cos(x))^2)^(3/2)-3*cos(x)^3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-c
os(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+co
s(x))^2)^(3/2)+9*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*
cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(
3/2)*cos(x)^2-9*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos
(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2
)*cos(x)^2+9*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2
+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2
)^(3/2)-9*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*co
s(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/
```

```
2)+3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-c
os(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)-3*ln(-(
2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos
(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(x)^3+12*cos(x
))*(1+cos(x))^2*(-(cos(x)^2-1)/cos(x))^(3/2)/sin(x)^7*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.
time = 0.48, size = 57, normalized size = 1.84

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

Fricas [A]
time = 2.41, size = 26, normalized size = 0.84

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(\sin(x)\tan(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(3/2),x)
```

```
[Out] Integral((sin(x)*tan(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sin(x)*tan(x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (\sin(x)\ \tan(x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(3/2),x)
```

```
[Out] int((sin(x)*tan(x))^(3/2), x)
```

3.149 $\int (\sin(x)\tan(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*s
in(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4485, 2678, 2674, 2669}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]*Tan[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)\,dx}{5\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{\left(32\sqrt{\sin(x)\tan}\right.}{15\sqrt{}}$$
$$= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(}$$

Mathematica [A]
time = 0.09, size = 29, normalized size = 0.58

$$\frac{2}{15}\left(5+3\cos^2(x)+32\cot^2(x)\right)\tan(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]*Tan[x])^(5/2),x]
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(38) = 76$.
time = 0.38, size = 324, normalized size = 6.48

method	result
default	$-\frac{(\cos(x)-1)^2\left(6(\cos^4(x))-15(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\ \ln\left(-\frac{2(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-(\cos^2(x))+2\cos(x)-2\sqrt{-\frac{\mathrm{co}}{(1+\mathrm{c}}}}{\sin(x)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(cos(x)-1)^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(
-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2
*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-(2*
cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x
))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2*(2*co
s(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))
^2)^(1/2)-1)/sin(x)^2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(cos(x)^2-1)/c
os(x))^(5/2)/sin(x)^9*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.
time = 0.50, size = 82, normalized size = 1.64

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

Fricas [A]
time = 1.51, size = 38, normalized size = 0.76

$$-\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)*tan(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((sin(x)*tan(x))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (\sin(x)\ \tan(x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)*tan(x))^(5/2),x)
```

```
[Out] int((sin(x)*tan(x))^(5/2), x)
```

3.150 $\int \sqrt{\cos(x)\cot(x)}\, dx$

Optimal. Leaf size=13

$$2\sqrt{\cos(x)\cot(x)}\,\tan(x)$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4485, 2669}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]*Cot[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)\cot(x)}\, dx &= \frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}\, dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\ &= 2\sqrt{\cos(x)\cot(x)}\,\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 13, normalized size = 1.00

$$2\sqrt{\cos(x)\cot(x)}\,\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]*Cot[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A]
time = 0.44, size = 20, normalized size = 1.54

method	result	size
default	$\frac{2\sin(x)\sqrt{\frac{\cos^2(x)}{\sin(x)}}}{\cos(x)}$	20
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i(e^{2ix}+1)^2 e^{-ix}}{e^{2ix}-1}}\,(e^{2ix}-1)}{e^{2ix}+1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(11) = 22$.
time = 0.54, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)-\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)}{\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A]
time = 1.30, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(x)\cot(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x)*cot(x)), x)
```

Giac [A]
time = 0.39, size = 12, normalized size = 0.92

$$2\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sgn(cos(x))*sgn(sin(x))*sqrt(sin(x))
```

Mupad [B]
time = 2.65, size = 18, normalized size = 1.38

$$\frac{2\,|\cos(x)|\,\sin(x)^{3/2}}{|\sin(x)|\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(1/2),x)
```

```
[Out] (2*abs(cos(x))*sin(x)^(3/2))/(abs(sin(x))*cos(x))
```

3.151 $\int (\cos(x)\cot(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sqrt{\cos(x)\cot(x)}\,\sec(x)$$

```
[Out] 2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4485, 2678, 2669}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cot[x])^(3/2),x]
```

```
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\,\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\,\sqrt{\cot(x)}}$$
$$= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 0.68

$$\frac{2}{3}\left(-4+\cos^2(x)\right)\sqrt{\cos(x)\cot(x)}\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(3/2),x]
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Maple [A]
time = 0.30, size = 26, normalized size = 0.84

method	result	size
default	$\frac{2\left(\cos^2(x)-4\right)\left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{3}{2}}\sin(x)}{3\cos(x)^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(23) = 46.
time = 0.56, size = 314, normalized size = 10.13

$$\frac{\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^{\frac{1}{4}}\left(\left(\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{9}{2}x\right)+15\sin\left(\frac{5}{2}x\right)-\sin\left(\frac{3}{2}x\right)-15\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{9}{2}x\right)-15\sin\left(\frac{5}{2}x\right)+\sin\left(\frac{3}{2}x\right)+15\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)+\left(\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{9}{2}x\right)-15\sin\left(\frac{5}{2}x\right)+\sin\left(\frac{3}{2}x\right)+15\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{9}{2}x\right)+15\sin\left(\frac{5}{2}x\right)-\sin\left(\frac{3}{2}x\right)-15\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)\right)}{6\left(\cos(x)^4+\sin(x)^4+2\left(\cos(x)^2+1\right)\sin(x)^2-2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="maxima")
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
```

```
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

Fricas [A]
time = 1.64, size = 23, normalized size = 0.74

$$\frac{2\left(\cos\left(x\right)^{2}-4\right)\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}}{3\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\cos\left(x\right)\cot\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(3/2),x)
```

```
[Out] Integral((cos(x)*cot(x))**(3/2), x)
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.61

$$-\frac{2}{3}\left(\sin\left(x\right)^{\frac{3}{2}}+\frac{3}{\sqrt{\sin\left(x\right)}}\right)\operatorname{sgn}(\cos\left(x\right))\operatorname{sgn}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(sin(x)^(3/2) + 3/sqrt(sin(x)))*sgn(cos(x))*sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (\cos (x) \cot(x))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(3/2),x)
[Out] int((cos(x)*cot(x))^(3/2), x)
```

3.152 $\int (\cos(x)\cot(x))^{5/2}\, dx$

Optimal. Leaf size=50

$$-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] -16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1/
2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4485, 2678, 2674, 2669}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cot[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{\left(32\sqrt{\cos(x)\cot}\right.}{15\sqrt{}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\sqrt{\cos(x)\cot(}$$

Mathematica [A]
time = 0.08, size = 29, normalized size = 0.58

$$-\frac{2}{15}\sqrt{\cos(x)\cot(x)}\left(32+3\cos^2(x)+5\cot^2(x)\right)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cot[x])^(5/2),x]
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

Maple [A]
time = 0.31, size = 34, normalized size = 0.68

method	result	size
default	$\frac{2\left(3\left(\cos^4(x)\right)+24\left(\cos^2(x)\right)-32\right)\left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{5}{2}}\sin(x)}{15\cos(x)^5}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(38) = 76$.
time = 0.56, size = 427, normalized size = 8.54

$$-\frac{\left(\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)+3\sin\left(\frac{15}{2}x\right)+105\sin\left(\frac{11}{2}x\right)-410\sin\left(\frac{7}{2}x\right)+3\sin\left(\frac{5}{2}x\right)+410\sin\left(\frac{3}{2}x\right)+105\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{15}{2}x\right)-105\sin\left(\frac{11}{2}x\right)+410\sin\left(\frac{7}{2}x\right)-3\sin\left(\frac{5}{2}x\right)-410\sin\left(\frac{3}{2}x\right)-105\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)-\left(\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{15}{2}x\right)-105\sin\left(\frac{11}{2}x\right)+410\sin\left(\frac{7}{2}x\right)-3\sin\left(\frac{5}{2}x\right)-410\sin\left(\frac{3}{2}x\right)-105\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)+3\sin\left(\frac{15}{2}x\right)+105\sin\left(\frac{11}{2}x\right)-410\sin\left(\frac{7}{2}x\right)+3\sin\left(\frac{5}{2}x\right)+410\sin\left(\frac{3}{2}x\right)+105\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)}{60\left(\cos(x)^4+\sin(x)^4+2\left(\cos(x)^2+1\right)\sin(x)^2-2\cos(x)^2+1\right)\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

Fricas [A]
time = 1.48, size = 35, normalized size = 0.70

$$\frac{2\left(3\cos(x)^4+24\cos(x)^2-32\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.54

$$\frac{2}{15}\left(3\sin\left(x\right)^{\frac{5}{2}}-30\sqrt{\sin\left(x\right)}-\frac{5}{\sin\left(x\right)^{\frac{3}{2}}}\right)\operatorname{sgn}(\cos\left(x\right))\operatorname{sgn}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)*cot(x))^(5/2),x, algorithm="giac")
[Out] 2/15*(3*sin(x)^(5/2) - 30*sqrt(sin(x)) - 5/sin(x)^(3/2))*sgn(cos(x))*sgn(si
n(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\left(\cos\left(x\right)\,\cot(x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*cot(x))^(5/2),x)
[Out] int((cos(x)*cot(x))^(5/2), x)
```

3.153 $\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=58

$$\frac{2\text{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

[Out] -x/b/(a+b*sin(x))+2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4507, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[x])/(a + b*Sin[x])^2,x]

[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2]) - x/(b*(a + b*Sin[x]))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx &= -\frac{x}{b(a+b\sin(x))}+\frac{\int \frac{1}{a+b\sin(x)}\,dx}{b}\\ &= -\frac{x}{b(a+b\sin(x))}+\frac{2\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= -\frac{x}{b(a+b\sin(x))}-\frac{4\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.97

$$\frac{\frac{2\mathrm{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{x}{a+b\sin(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

```
[Out] ((2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - x/(a + b*Si
n[x]))/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.48, size = 159, normalized size = 2.74

method	result	size
risch	$-\frac{2ix\,e^{ix}}{b(b\,e^{2ix}-b+2ia\,e^{ix})}-\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}$	159

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*x*exp(I*x)/b/(b*exp(2*I*x)-b+2*I*a*exp(I*x))-1/(-a^2+b^2)^(1/2)/b*ln(e
xp(I*x)+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(1/
2)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.17, size = 236, normalized size = 4.07

$$\left[-\frac{\sqrt{-a^2+b^2}\,(b\sin(x)+a)\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+2\,(a^2-b^2)x}{2\,(a^3b-ab^3+(a^2b^2-b^4)\sin(x))},\ -\frac{\sqrt{a^2-b^2}\,(b\sin(x)+a)\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+(a^2-b^2)x}{a^3b-ab^3+(a^2b^2-b^4)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*sin(x) + a)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (
a^2*b^2 - b^4)*sin(x)), -(sqrt(a^2 - b^2)*(b*sin(x) + a)*arctan(-(a*sin(x)
+ b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 -
 b^4)*sin(x))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos\left(x\right)}{{\left(a+b\,\sin\left(x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x))/(a + b*sin(x))^2,x)
```

```
[Out] int((x*cos(x))/(a + b*sin(x))^2, x)
```

3.154 $\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$

Optimal. Leaf size=85

$$\frac{a\text{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}-\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}$$

```
[Out] a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)-1/2*x/b/(a+b*s
in(x))^2+1/2*cos(x)/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A]
time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4507, 2743, 12, 2739, 632, 210}

$$\frac{a\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{x}{2b(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) - x/(2*b
*(a + b*Sin[x])^2) + Cos[x]/(2*(a^2 - b^2)*(a + b*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx &= -\frac{x}{2b(a+b\sin(x))^2}+\frac{\int \frac{1}{(a+b\sin(x))^2}\,dx}{2b}\\
&= -\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}+\frac{\int \frac{a}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= -\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}+\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= -\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}+\frac{a\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}\\
&= -\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}-\frac{(2a)\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b-\right.}{b\left(a^2-b^2\right)}\\
&= \frac{a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}-\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\left(a^2-b^2\right)(a+b\sin(x))}
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 84, normalized size = 0.99

$$
\frac{a\text{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}}+\frac{-\frac{x}{b}+\frac{\cos(x)(a+b\sin(x))}{(a-b)(a+b)}}{2(a+b\sin(x))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

```
[Out] (a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) + (-(x/b
) + (Cos[x]*(a + b*Sin[x]))/((a - b)*(a + b)))/(2*(a + b*Sin[x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.75, size = 257, normalized size = 3.02

method	result
risch	$\frac{2ia^2 e^{2ix}+ib^2 e^{2ix}+2x\,a^2 e^{2ix}+ba\,e^{3ix}-2b^2 x\,e^{2ix}-ib^2-3ab\,e^{ix}}{\left(b\,e^{2ix}-b+2ia\,e^{ix}\right)^2 (a^2-b^2) b} - \frac{a \ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{2\sqrt{-a^2+b^2}\,(a+b)(a-b)b} + \frac{a \ln\left(e^{ix}+\frac{ia\sqrt{-a^2 \cdots}}{\sqrt{-a \cdots}}\right)}{2\sqrt{-a^2+b^2}\,\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (2*I*a^2*exp(2*I*x)+I*b^2*exp(2*I*x)+2*x*a^2*exp(2*I*x)+b*a*exp(3*I*x)-2*b^
2*x*exp(2*I*x)-I*b^2-3*a*b*exp(I*x))/(b*exp(2*I*x)-b+2*I*a*exp(I*x))^2/(a^2
-b^2)/b-1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1
/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(ex
p(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(75) = 150.
time = 1.33, size = 459, normalized size = 5.40

$$\left[\frac{2\,(a^2b^2 - b^4)\cos(x)\sin(x) - \left(ab^2\cos(x)^2 - 2\,a^2b\sin(x) - a^3 - ab^2\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2\,a^2-b^2)\cos(x)^2 - 2\,ab\sin(x) - a^2 - b^2 - 2\,(a\cos(x)\sin(x) + b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2 - 2\,ab\sin(x) - a^2 - b^2}\right) - 2\,(a^4 - 2\,a^2b^2 + b^4)x + 2\,(a^3b - ab^3)\cos(x)}{4\,\left(a^6b - a^4b^3 - a^2b^5 + b^7 - (a^4b^3 - 2\,a^2b^5 + b^7)\cos(x)^2 + 2\,(a^5b^2 - 2\,a^3b^4 + ab^6)\sin(x)\right)}, \frac{(a^2b^2 - b^4)\cos(x)\sin(x) + \left(ab^2\cos(x)^2 - 2\,a^2b\sin(x) - a^3 - ab^2\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right) - (a^4 - 2\,a^2b^2 + b^4)x + (a^3b - ab^3)\cos(x)}{2\,\left(a^6b - a^4b^3 - a^2b^5 + b^7 - (a^4b^3 - 2\,a^2b^5 + b^7)\cos(x)^2 + 2\,(a^5b^2 - 2\,a^3b^4 + ab^6)\sin(x)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b^2 - b^4)*cos(x)*sin(x) - (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) -
a^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) -
 a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2
 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 2*(a^3*b - a*
b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)
*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x)), 1/2*((a^2*b^2 - b^4)*c
os(x)*sin(x) + (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) - a^3 - a*b^2)*sqrt(a^2 - b
^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^4 - 2*a^2*b^2 + b
^4)*x + (a^3*b - a*b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3
 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)/(b*sin(x) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(x\right)}{{\left(a+b\,\sin\left(x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x))/(a + b*sin(x))^3,x)
```

```
[Out] int((x*cos(x))/(a + b*sin(x))^3, x)
```

3.155 $\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}+\frac{x}{b(a+b\cos(x))}$$

```
[Out] x/b/(a+b*cos(x))-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4508, 2738, 211}

$$\frac{x}{b(a+b\cos(x))}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

```
[Out] (-2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b])
 + x/(b*(a + b*Cos[x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 4508

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cos[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx &= \frac{x}{b(a+b\cos(x))}-\frac{\int \frac{1}{a+b\cos(x)}\,dx}{b}\\ &= \frac{x}{b(a+b\cos(x))}-\frac{2\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b}\\ &= -\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}}+\frac{x}{b(a+b\cos(x))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 0.98

$$\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}+\frac{x}{b(a+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

```
[Out] (2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) + x/(
b*(a + b*Cos[x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 154, normalized size = 2.61

method	result	size
risch	$\frac{2x\,e^{ix}}{b(b\,e^{2ix}+2a\,e^{ix}+b)}-\frac{i\ln\left(e^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}+\frac{i\ln\left(e^{ix}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}$	154

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*exp(I*x)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)-I/(a^2-b^2)^(1/2)/b*ln(exp(I*x
)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+I/(a^2-b^2)^(1/2)/b*ln(exp
(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 1.16, size = 227, normalized size = 3.85

$$\left[-\frac{\sqrt{-a^2+b^2}\,(b\cos(x)+a)\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)-2\,(a^2-b^2)x}{2\,(a^3b-ab^3+(a^2b^2-b^4)\cos(x))},\ -\frac{\sqrt{a^2-b^2}\,(b\cos(x)+a)\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)-(a^2-b^2)x}{a^3b-ab^3+(a^2b^2-b^4)\cos(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*(b*cos(x) + a)*log((2*a*b*cos(x) + (2*a^2 - b^2)*co
s(x)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x
)^2 + 2*a*b*cos(x) + a^2)) - 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b
^4)*cos(x)), -(sqrt(a^2 - b^2)*(b*cos(x) + a)*arctan(-(a*cos(x) + b)/(sqrt(
a^2 - b^2)*sin(x))) - (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x
))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)/(b*cos(x) + a)^2, x)
```

Mupad [B]
time = 3.29, size = 132, normalized size = 2.24

$$\frac{2\,x\,\mathrm{e}^{x\,1\mathrm{i}}}{b\,\left(2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+2\,b\,\mathrm{e}^{x\,1\mathrm{i}}\,\cos\left(x\right)\right)}+\frac{\ln\left(2\,\mathrm{e}^{x\,1\mathrm{i}}-\frac{\left(b+a\,\mathrm{e}^{x\,1\mathrm{i}}\right)\,2\mathrm{i}}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{\ln\left(2\,\mathrm{e}^{x\,1\mathrm{i}}+\frac{\left(b+a\,\mathrm{e}^{x\,1\mathrm{i}}\right)\,2\mathrm{i}}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x))/(a + b*cos(x))^2,x)
```

```
[Out] (2*x*exp(x*1i))/(b*(2*a*exp(x*1i) + 2*b*exp(x*1i)*cos(x))) + log(2*exp(x*1i
) - ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b - a)^(1/2)))/(b*(a + b)^(1/2)*
(b - a)^(1/2)) - log(2*exp(x*1i) + ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b
 - a)^(1/2)))/(b*(a + b)^(1/2)*(b - a)^(1/2))
```

3.156 $\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}+\frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}$$

```
[Out] -a*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)+1/2
*x/b/(a+b*cos(x))^2+1/2*sin(x)/(a^2-b^2)/(a+b*cos(x))
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4508, 2743, 12, 2738, 211}

$$\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}-\frac{a\text{ArcTan}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b(a-b)^{3/2}(a+b)^{3/2}}+\frac{x}{2b(a+b\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

```
[Out] -((a*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*b*(a + b)^(
3/2))) + x/(2*b*(a + b*Cos[x])^2) + Sin[x]/(2*(a^2 - b^2)*(a + b*Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4508

```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cos[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^
(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx &= \frac{x}{2b(a+b\cos(x))^2} - \frac{\int \frac{1}{(a+b\cos(x))^2}\,dx}{2b}\\
&= \frac{x}{2b(a+b\cos(x))^2} + \frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))} - \frac{\int \frac{a}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2} + \frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))} - \frac{a\int \frac{1}{a+b\cos(x)}\,dx}{2b\left(a^2-b^2\right)}\\
&= \frac{x}{2b(a+b\cos(x))^2} + \frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))} - \frac{a\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b\left(a^2-b^2\right)}\\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}} + \frac{x}{2b(a+b\cos(x))^2} + \frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 85, normalized size = 0.97

$$-\frac{a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\left(-a^2+b^2\right)^{3/2}} + \frac{\frac{x}{b}+\frac{(a+b\cos(x))\sin(x)}{(a-b)(a+b)}}{2(a+b\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[x])/(a + b*Cos[x])^3,x]
[Out] -((a*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*(-a^2 + b^2)^(3/2)))
+ (x/b + ((a + b*Cos[x])*Sin[x])/((a - b)*(a + b)))/(2*(a + b*Cos[x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 250, normalized size = 2.84

method	result
risch	$\frac{i\left(-2ia^2x\,\mathrm{e}^{2ix}+2ib^2x\,\mathrm{e}^{2ix}+ba\,\mathrm{e}^{3ix}+2a^2\mathrm{e}^{2ix}+b^2\mathrm{e}^{2ix}+3ab\,\mathrm{e}^{ix}+b^2\right)}{b\left(b\,\mathrm{e}^{2ix}+2a\,\mathrm{e}^{ix}+b\right)^2\left(a^2-b^2\right)}-\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}+\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-}}{\sqrt{a^2}}\right.}{2\sqrt{a^2-b^2}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/(a+b*cos(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] I*(-2*I*a^2*x*exp(2*I*x)+2*I*b^2*x*exp(2*I*x)+b*a*exp(3*I*x)+2*a^2*exp(2*I*
x)+b^2*exp(2*I*x)+3*a*b*exp(I*x)+b^2)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)^2/(a^
2-b^2)-1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1/2)
+a^2-b^2)/(a^2-b^2)^(1/2)/b)+1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I
*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(74) = 148$.
time = 1.41, size = 417, normalized size = 4.74

$$\left[\frac{\left(ab^2\cos\left(x\right)^2+2\,a^2b\cos\left(x\right)+a^3\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,\left(a\cos(x)+b\right)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)+2\left(a^4-2\,a^2b^2+b^4\right)x+2\left(a^3b-ab^3+\left(a^2b^2-b^4\right)\cos\left(x\right)\right)\sin\left(x\right)}{4\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos\left(x\right)^2+2\left(a^5b^2-2\,a^3b^4+ab^6\right)\cos\left(x\right)\right)},-\frac{\left(ab^2\cos\left(x\right)^2+2\,a^2b\cos\left(x\right)+a^3\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\,\sin(x)}\right)-\left(a^4-2\,a^2b^2+b^4\right)x-\left(a^3b-ab^3+\left(a^2b^2-b^4\right)\cos\left(x\right)\right)\sin\left(x\right)}{2\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos\left(x\right)^2+2\left(a^5b^2-2\,a^3b^4+ab^6\right)\cos\left(x\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(-a^2 + b^2)*log((2*a*b*c
os(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) -
 a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) + 2*(a^4 - 2*a^2*b^2 + b
^4)*x + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*b
^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*cos(x)), -1/2*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(a^2
- b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^4 - 2*a^2*b^2
+ b^4)*x - (a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a^4*
```

```
b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b
^4 + a*b^6)*cos(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="giac")
[Out] integrate(x*sin(x)/(b*cos(x) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin\left(x\right)}{{\left(a+b\,\cos\left(x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(x))/(a + b*cos(x))^3,x)
[Out] int((x*sin(x))/(a + b*cos(x))^3, x)
```

3.157 $\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx$

Optimal. Leaf size=50

$$\frac{ax}{b\,(a^2+b^2)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

```
[Out] a*x/b/(a^2+b^2)+ln(a*cos(x)+b*sin(x))/(a^2+b^2)-x/b/(a+b*tan(x))
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4509, 3565, 3611}

$$\frac{ax}{b\,(a^2+b^2)}+\frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{x}{b(a+b\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (a*x)/(b*(a^2 + b^2)) + Log[a*Cos[x] + b*Sin[x]]/(a^2 + b^2) - x/(b*(a + b*
Tan[x]))
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4509

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Tan[c + d
*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(
m - 1)*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
 x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx = -\frac{x}{b(a+b\tan(x))} + \frac{\int \frac{1}{a+b\tan(x)}\,dx}{b}$$
$$= \frac{ax}{b\,(a^2+b^2)} - \frac{x}{b(a+b\tan(x))} + \frac{\int \frac{b-a\tan(x)}{a+b\tan(x)}\,dx}{a^2+b^2}$$
$$= \frac{ax}{b\,(a^2+b^2)} + \frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2} - \frac{x}{b(a+b\tan(x))}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 0.96

$$\frac{-bx+a\log(a\cos(x)+b\sin(x))}{a^3+ab^2} + \frac{x\sin(x)}{a^2\cos(x)+ab\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

```
[Out] (-(b*x) + a*Log[a*Cos[x] + b*Sin[x]])/(a^3 + a*b^2) + (x*Sin[x])/(a^2*Cos[x
] + a*b*Sin[x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 86, normalized size = 1.72

method	result	size
risch	$-\frac{2ix}{a^2+b^2}+\frac{2ix}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)(-ib+a)}+\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^2+b^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2/(a+b*tan(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(a^2+b^2)*x+2*I*x/(-I*b*exp(2*I*x)+a*exp(2*I*x)+I*b+a)/(-I*b+a)+1/(a^2
+b^2)*ln(exp(2*I*x)-(I*b+a)/(I*b-a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(50) = 100$.
time = 0.27, size = 250, normalized size = 5.00

$$-\frac{8\,abx\cos(2\,x)-4\,(a^2-b^2)x\sin(2\,x)-\left((a^2+b^2)\cos(2\,x)^2+4\,ab\sin(2\,x)+(a^2+b^2)\sin(2\,x)^2+a^2+b^2+2\,(a^2-b^2)\cos(2\,x)\right)\log\left(\frac{(a^2+b^2)\cos(2\,x)^2+4\,ab\sin(2\,x)+(a^2+b^2)\sin(2\,x)^2+a^2+b^2+2\,(a^2-b^2)\cos(2\,x)}{a^2+b^2}\right)}{2\,\left(a^4+2\,a^2b^2+b^4+(a^4+2\,a^2b^2+b^4)\cos(2\,x)^2+(a^4+2\,a^2b^2+b^4)\sin(2\,x)^2+2\,(a^4-b^4)\cos(2\,x)+4\,(a^3b+ab^3)\sin(2\,x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) - 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
+ 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

Fricas [**A**]
time = 3.88, size = 80, normalized size = 1.60

$$-\frac{2\,bx\cos\left(x\right)-2\,ax\sin\left(x\right)-\left(a\cos\left(x\right)+b\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)}{2\left(\left(a^{3}+ab^{2}\right)\cos\left(x\right)+\left(a^{2}b+b^{3}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x*cos(x) - 2*a*x*sin(x) - (a*cos(x) + b*sin(x))*log(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)
*sin(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\sec^{2}\left(x\right)}{\left(a+b\tan\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2/(a+b*tan(x))**2,x)
```

```
[Out] Integral(x*sec(x)**2/(a + b*tan(x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(50) = 100.
time = 0.49, size = 322, normalized size = 6.44

$$-\frac{2\,bx\tan\left(\frac{1}{2}\,x\right)^{2}-a\log\left(\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}-2\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ax\tan\left(\frac{1}{2}\,x\right)+2\,b\log\left(\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}-2\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)-2\,bx+a\log\left(\frac{4\left(a^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}-2\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)}{2\left(a^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+ab^{2}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,a^{2}b\tan\left(\frac{1}{2}\,x\right)-2\,b^{3}\tan\left(\frac{1}{2}\,x\right)-a^{3}-ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*tan(1/2*x)^2 - a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 -
 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*a*x*tan(1/2*x) + 2*b*log(4*(a
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*b*x + a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2
```

```
*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^3*tan(1/2*x)^2 + a*b^2*tan(1/2*x)^2 - 2*a^2*b*tan(1/2*x)
 - 2*b^3*tan(1/2*x) - a^3 - a*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\cos(x)^2 (a + b\tan(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)^2*(a + b*tan(x))^2),x)
[Out] int(x/(cos(x)^2*(a + b*tan(x))^2), x)
```

3.158 $\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx$

Optimal. Leaf size=50

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{x}{b(a+b\cot(x))}+\frac{\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

```
[Out] -a*x/b/(a^2+b^2)+x/b/(a+b*cot(x))+ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4510, 3565, 3611}

$$-\frac{ax}{b\left(a^2+b^2\right)}+\frac{\log(a\sin(x)+b\cos(x))}{a^2+b^2}+\frac{x}{b(a+b\cot(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 + b^2))) + x/(b*(a + b*Cot[x])) + Log[b*Cos[x] + a*Sin[x]]/
(a^2 + b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4510

```
Int[Csc[(c_.) + (d_.)*(x_)]^2*(Cot[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*
((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cot[c
+ d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x
)^(m - 1)*(a + b*Cot[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = \frac{x}{b(a+b\cot(x))} - \frac{\int \frac{1}{a+b\cot(x)}\,dx}{b}$$
$$= -\frac{ax}{b\left(a^2+b^2\right)} + \frac{x}{b(a+b\cot(x))} + \frac{\int \frac{-b+a\cot(x)}{a+b\cot(x)}\,dx}{a^2+b^2}$$
$$= -\frac{ax}{b\left(a^2+b^2\right)} + \frac{x}{b(a+b\cot(x))} + \frac{\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 0.96

$$\frac{-ax+b\log(b\cos(x)+a\sin(x))}{a^2b+b^3} + \frac{x\sin(x)}{b^2\cos(x)+ab\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

```
[Out] (-(a*x) + b*Log[b*Cos[x] + a*Sin[x]])/(a^2*b + b^3) + (x*Sin[x])/(b^2*Cos[x
] + a*b*Sin[x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 87, normalized size = 1.74

method	result	size
risch	$-\frac{2ix}{a^2+b^2} - \frac{2ix}{(ib\,e^{2ix}+a\,e^{2ix}+ib-a)(ib+a)} + \frac{\ln\left(e^{2ix}+\frac{ib-a}{ib+a}\right)}{a^2+b^2}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2/(a+b*cot(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(a^2+b^2)*x-2*I*x/(I*b*exp(2*I*x)+a*exp(2*I*x)+I*b-a)/(I*b+a)+1/(a^2+b
^2)*ln(exp(2*I*x)+(I*b-a)/(I*b+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(50) = 100.
time = 0.28, size = 250, normalized size = 5.00

$$-\frac{8\,abx\cos(2\,x) + 4\,(a^2-b^2)x\sin(2\,x) - \left((a^2+b^2)\cos(2\,x)^2 + 4\,ab\sin(2\,x) + (a^2+b^2)\sin(2\,x)^2 + a^2 + b^2 - 2\,(a^2-b^2)\cos(2\,x)\right)\log\left(\frac{(a^2+b^2)\cos(2\,x)^2+4\,ab\sin(2\,x)+(a^2+b^2)\sin(2\,x)^2+a^2+b^2-2\,(a^2-b^2)\cos(2\,x)}{a^2+b^2}\right)}{2\left(a^4+2\,a^2b^2+b^4+(a^4+2\,a^2b^2+b^4)\cos(2\,x)^2+(a^4+2\,a^2b^2+b^4)\sin(2\,x)^2-2\,(a^4-b^4)\cos(2\,x)+4\,(a^3b+ab^3)\sin(2\,x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(8*a*b*x*cos(2*x) + 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^2
 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(
2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2
 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 + b^4
 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*x)^2
- 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

Fricas [A]
time = 2.23, size = 81, normalized size = 1.62

$$-\frac{2\,ax\cos\left(x\right)-2\,bx\sin\left(x\right)-\left(b\cos\left(x\right)+a\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+a^{2}\right)}{2\left(\left(a^{2}b+b^{3}\right)\cos\left(x\right)+\left(a^{3}+ab^{2}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cos(x) - 2*b*x*sin(x) - (b*cos(x) + a*sin(x))*log(2*a*b*cos(x)*
sin(x) - (a^2 - b^2)*cos(x)^2 + a^2))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)
*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\csc^{2}\left(x\right)}{\left(a+b\cot\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2/(a+b*cot(x))**2,x)
```

```
[Out] Integral(x*csc(x)**2/(a + b*cot(x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(50) = 100.
time = 0.48, size = 322, normalized size = 6.44

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,x\right)^{2}-b\log\left(\frac{4\left(b^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}+4\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+b^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+4\,bx\tan\left(\frac{1}{2}\,x\right)+2\,a\log\left(\frac{4\left(b^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}+4\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+b^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)-2\,ax+b\log\left(\frac{4\left(b^{2}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,ab\tan\left(\frac{1}{2}\,x\right)^{3}+4\,a^{2}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b^{2}\tan\left(\frac{1}{2}\,x\right)^{2}+4\,ab\tan\left(\frac{1}{2}\,x\right)+b^{2}\right)}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)}{2\left(a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}+b^{3}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,a^{3}\tan\left(\frac{1}{2}\,x\right)-2\,ab^{2}\tan\left(\frac{1}{2}\,x\right)-a^{2}b-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*x*tan(1/2*x)^2 - b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 +
 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2
*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*b*x*tan(1/2*x) + 2*a*log(4*(b
^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x
)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*
x) - 2*a*x + b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2
```

```
*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(
1/2*x)^2 + 1)))/(a^2*b*tan(1/2*x)^2 + b^3*tan(1/2*x)^2 - 2*a^3*tan(1/2*x) -
 2*a*b^2*tan(1/2*x) - a^2*b - b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sin(x)^2 (a + b\cot(x))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(sin(x)^2*(a + b*cot(x))^2),x)
```

```
[Out] int(x/(sin(x)^2*(a + b*cot(x))^2), x)
```

3.159 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

```
[Out] arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3756, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
[Out] ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A]
time = 0.44, size = 24, normalized size = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+a\sqrt{-ab}+b\sqrt{-ab}}{\sqrt{-ab}\,(a-b)}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{-2iab+a\sqrt{-ab}+b\sqrt{-ab}}{\sqrt{-ab}\,(a-b)}\right)}{2\sqrt{-ab}\,d}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
[Out] 1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

Maxima [A]
time = 0.47, size = 23, normalized size = 0.72

$$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
[Out] arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

time = 2.29, size = 205, normalized size = 6.41

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\,\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\,\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4\,abd},\ -\frac{\sqrt{ab}\,\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{2\,abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)*
cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*sin
(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos(d*
x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x + c)^2
 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [A]

time = 0.60, size = 40, normalized size = 1.25

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(s
qrt(a*b)*d)
```

Mupad [B]

time = 2.56, size = 24, normalized size = 0.75

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

```
[Out] atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

3.160 $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=211

$$-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}+\frac{\text{PolyLog}\left(2,-\right.}{4\sqrt{a}}$$

```
[Out] -1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1/2)
+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b^(1/2)
-1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^(1/2)/b^(
1/2)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d^2/a^(1/2)
/b^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4684, 3402, 2296, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}+\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\,\sqrt{b}\,d^2}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/2*I)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])/(
Sqrt[a]*Sqrt[b]*d) + ((I/2)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a
] + Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c +
d*x)))/(Sqrt[a] - Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2) + PolyLog[2, -(((a -
 b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)]/(4*Sqrt[a]*Sqrt[b]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4684

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx &= 2\int \frac{x}{a+b+(a-b)\cos(2c+2dx)}\,dx \\
&= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx \\
&= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{4\sqrt{a}\sqrt{b}+2(a+b)+2(\cdots}}{\sqrt{a}\sqrt{b}} \\
&= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{i\int\log\left(1+\frac{2(a\cdots}{-4\sqrt{\cdots}}\right.}{2\sqrt{a}\cdots} \\
&= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\cdots\right.}{\cdots}\right.}{\cdots} \\
&= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+\cdots}}{\left(\sqrt{a}-\sqrt{b}\cdots\right.}\right.}{4\sqrt{a}\sqrt{b}d^2\cdots}
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. $2(211) = 422$.
time = 3.94, size = 512, normalized size = 2.43

$$
\frac{x\left(4i\sqrt{-a}\,c\operatorname{ArcTan}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)-\sqrt{a}\log(1+i\tan(c+dx))\log\left(\frac{\sqrt{-a}-\sqrt{b}\tan(c+dx)}{\sqrt{-a}-i\sqrt{b}}\right)+\sqrt{a}\log(1-i\tan(c+dx))\log\left(\frac{\sqrt{-a}-\sqrt{b}\tan(c+dx)}{\sqrt{-a}+i\sqrt{b}}\right)-\sqrt{a}\log(1-i\tan(c+dx))\log\left(\frac{\sqrt{-a}+\sqrt{b}\tan(c+dx)}{\sqrt{-a}-i\sqrt{b}}\right)+\sqrt{a}\log(1+i\tan(c+dx))\log\left(\frac{\sqrt{-a}+\sqrt{b}\tan(c+dx)}{\sqrt{-a}+i\sqrt{b}}\right)-\sqrt{a}\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(1-i\tan(c+dx))}{i\sqrt{-a}+\sqrt{b}}\right)-\sqrt{a}\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(1+i\tan(c+dx))}{i\sqrt{-a}+\sqrt{b}}\right)+\sqrt{a}\operatorname{PolyLog}\left(2,-\frac{\sqrt{b}(-i+\tan(c+dx))}{\sqrt{-a}+i\sqrt{b}}\right)+\sqrt{a}\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(i+\tan(c+dx))}{\sqrt{-a}+i\sqrt{b}}\right)\right)}{2\sqrt{-a^2}\sqrt{b}\,d(2ic+\log(1-i\tan(c+dx))-\log(1+i\tan(c+dx)))}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*((4*I)*Sqrt[-a]*c*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Log[1
 + I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt
[b])] + Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x
])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a]
+ Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 + I*Tan[c +
 d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt
[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] - Sqr
t[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])] + Sq
rt[a]*PolyLog[2, -((Sqrt[b]*(-I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b]))] +
 Sqrt[a]*PolyLog[2, (Sqrt[b]*(I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqrt[b])]))/
```

```
(2*Sqrt[-a^2]*Sqrt[b]*d*((2*I)*c + Log[1 - I*Tan[c + d*x]] - Log[1 + I*Tan[
c + d*x]]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(161) = 322$.
time = 0.46, size = 1003, normalized size = 4.75

method	result
risch	$-\frac{i \ln\left(1-\frac{(a-b)\mathrm{e}^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right) ac}{2d^2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{i \ln\left(1-\frac{(a-b)\mathrm{e}^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right) bc}{2d^2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{i \ln\left(1-\frac{(a-b)\mathrm{e}^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right) ax}{2d\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{i \ln\left(1-\frac{(a-b)\mathrm{e}^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)}{2d\sqrt{ab}\left(-2\sqrt{ab}-a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d^2/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1
/2)-a-b))-1/4/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/
2)-a-b))-1/2/d^2/(a*b)^(1/2)*c^2-1/d^2/(-2*(a*b)^(1/2)-a-b)*c^2-1/2/d^2/(a*
b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^2-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b
)*b*c^2-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d
*x+c))/(-2*(a*b)^(1/2)-a-b))*a-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*pol
ylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b-I/d/(-2*(a*b)^(1/2)-a
-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2*I/d/(a*b)^(1/2)
*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x-I/d^2/(-2*(a*b)^(1/2)-a
-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c-1/2*I/d^2/(a*b)^(1/
2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*c-I/d^2*c/(a*b)^(1/2)*a
rctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^(1/2))-1/2*I/d^2/(a*b)^
(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b)
)*a*c-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c)
)/(-2*(a*b)^(1/2)-a-b))*b*c-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(
a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*a*x-1/2*I/d/(a*b)^(1/2)/(-2*(a*
b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b*x-1/2/(a*
b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x^2-2/d/(-2*(a*b)^(1/2)-a-b)*c*x-1/d/(a*b)^
(1/2)*c*x-1/2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*x^2-1/d/(a*b)^(1/2)/(-2*(a
*b)^(1/2)-a-b)*a*c*x-1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c*x-1/(-2*(a*b)
^(1/2)-a-b)*x^2-1/2/(a*b)^(1/2)*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(157) = 314$.
time = 4.99, size = 3284, normalized size = 15.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")

[Out] -1/4*(I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-(((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b) + 1) - (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog((((a + b)*cos(d*x + c) - (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - a + b)/(a - b) + 1) - (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-(((a + b)*cos(d*x + c) + (-I*a - I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b) + 1) - (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog((((a + b)*cos(d*x + c) - (-I*a - I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - a + b)/(a - b) + 1) + (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog(-(((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + a - b)/

```
(a - b) + 1) + (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog((((a + b)*cos(d*
x + c) - (I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*
sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2
- 2*a*b + b^2)) - a - b)/(a - b)) - a + b)/(a - b) + 1) + (a - b)*sqrt(a*b/
(a^2 - 2*a*b + b^2))*dilog(-(((a + b)*cos(d*x + c) + (-I*a - I*b)*sin(d*x +
 c) + 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2
*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b
)) + a - b)/(a - b) + 1) + (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*dilog((((a
 + b)*cos(d*x + c) - (-I*a - I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) +
(I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sq
rt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - a + b)/(a - b) + 1) - (I*(a
 - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log((((a + b)*cos(d*
x + c) + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*s
in(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2
- 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b)) - (-I*(a - b)*d*x - I*(
a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-(((a + b)*cos(d*x + c) - (I*a
+ I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*s
qrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)
) + a + b)/(a - b)) - a + b)/(a - b)) - (-I*(a - b)*d*x - I*(a - b)*c)*sqrt
(a*b/(a^2 - 2*a*b + b^2))*log((((a + b)*cos(d*x + c) + (-I*a - I*b)*sin(d*x
 + c) - 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2
- 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a
 - b)) + a - b)/(a - b)) - (I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*
a*b + b^2))*log(-(((a + b)*cos(d*x + c) - (-I*a - I*b)*sin(d*x + c) - 2*((a
 - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2
)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - a +
b)/(a - b)) - (-I*(a - b)*d*x - I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*
log((((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x
+ c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
 - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + a - b)/(a - b)) - (
I*(a - b)*d*x + I*(a - b)*c)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-(((a + b)*c
os(d*x + c) - (I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) + (-I*a +
I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
[Out] integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos\left(c+d\,x\right)^2\,\left(b\tan\left(c+d\,x\right)^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
[Out] int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

3.161 $\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}-\frac{x\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{x\text{PolyLog}}{2}$$

```
[Out] -1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1/
2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b^(
1/2)-1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^(1/
2)/b^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d^2
/a^(1/2)/b^(1/2)+1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a^(1/2)-b^(1/2))/(a^(1/
2)+b^(1/2)))/d^3/a^(1/2)/b^(1/2)-1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a^(1/2)
+b^(1/2))/(a^(1/2)-b^(1/2)))/d^3/a^(1/2)/b^(1/2)
```

Rubi [A]

time = 0.63, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4684, 3402, 2296, 2221, 2611, 2320, 6724}

$$\frac{i\text{Li}_3\left(-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d^3}-\frac{i\text{Li}_3\left(-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\,\sqrt{b}\,d^3}-\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}+\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}+\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2])
/(Sqrt[a]*Sqrt[b]*d) + ((I/2)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sq
rt[a] + Sqrt[b])^2])/(Sqrt[a]*Sqrt[b]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I
)*(c + d*x)))/(Sqrt[a] - Sqrt[b])^2)])/(2*Sqrt[a]*Sqrt[b]*d^2) + (x*PolyLog
[2, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)])/(2*Sqrt[a]*Sqr
t[b]*d^2) + ((I/4)*PolyLog[3, -(((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(
Sqrt[a] + Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^3) - ((I/4)*PolyLog[3, -(((Sqrt[a]
 + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))])/(Sqrt[a]*Sqrt[b]*d^
3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4684

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b + c + (b
- c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a}\sqrt{b}+2(a+b)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a}\sqrt{b}} - \frac{(2(a-b))\int \frac{e^{i(2c+2dx}}{4\sqrt{a}\sqrt{b}+2(a+b)+}}{\sqrt{a}\sqrt{b}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{i\int x\log\left(1+\right.}{}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\text{Li}_2\left(-\frac{(a-b)}{(\sqrt{a}}\right.}{2\sqrt{a}\sqrt{}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\text{Li}_2\left(-\frac{(a-b)}{(\sqrt{a}}\right.}{2\sqrt{a}\sqrt{}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} + \frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{x\text{Li}_2\left(-\frac{(a-b)}{(\sqrt{a}}\right.}{2\sqrt{a}\sqrt{}}$$

Mathematica [A]

time = 0.72, size = 294, normalized size = 0.87

$$\frac{i\left(2d^2x^2\log\left(1+\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)-2d^2x^2\log\left(1+\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)-2idx\text{PolyLog}\left(2,\frac{\left(-\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)+2idx\text{PolyLog}\left(2,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)+\text{PolyLog}\left(3,\frac{\left(-\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)-\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)\right)}{4\sqrt{a}\sqrt{b}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

```
[Out] ((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a
] + Sqrt[b])] - 2*d^2*x^2*Log[1 + ((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))
/(Sqrt[a] - Sqrt[b])] - (2*I)*d*x*PolyLog[2, ((-Sqrt[a] + Sqrt[b])*E^((2*I)
*(c + d*x)))/(Sqrt[a] + Sqrt[b])] + (2*I)*d*x*PolyLog[2, -(((Sqrt[a] + Sqrt
```

```
[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))] + PolyLog[3, ((-Sqrt[a] + S
qrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] - PolyLog[3, -(((Sqrt[a]
+ Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))]))/(Sqrt[a]*Sqrt[b]*d^
3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1250 vs. 2(253) = 506.
time = 0.39, size = 1251, normalized size = 3.71

method	result	size
risch	Expression too large to display	1251

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a*b)^(1/2)*x^3-2/3/(-2*(a*b)^(1/2)-a-b)*x^3-1/4*I/d^3/(a*b)^(1/2)/(-2
*(a*b)^(1/2)-a-b)*b*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))-
1/4*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*polylog(3,(a-b)*exp(2*I*(d*x+c
))/(-2*(a*b)^(1/2)-a-b))+4/3/d^3/(-2*(a*b)^(1/2)-a-b)*c^3+2/3/d^3/(a*b)^(1/
2)*c^3-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*ln(1-(a-b)*exp(2*I*(d*x+c
))/(-2*(a*b)^(1/2)-a-b))*x^2+1/2*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*l
n(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2+1/2*I/d^3/(a*b)^(1/2)/
(-2*(a*b)^(1/2)-a-b)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^
2-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-
2*(a*b)^(1/2)-a-b))*x^2+I/d^3*c^2/(a*b)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*
(d*x+c))+2*a+2*b)/(a*b)^(1/2))-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*
(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2+2/3/d^3*c^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-
a-b)*a+2/3/d^3*c^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b-1/2*I/d/(a*b)^(1/2)*l
n(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x^2+1/2*I/d^3*c^2/(a*b)^(1/
2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))+I/d^3/(-2*(a*b)^(1/2)-a
-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2+2/d^2/(-2*(a*b)^(
1/2)-a-b)*c^2*x+1/d^2/(a*b)^(1/2)*c^2*x-1/d^2/(-2*(a*b)^(1/2)-a-b)*polylog(
2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/2/d^2/(a*b)^(1/2)*polylo
g(2,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x-1/3/(a*b)^(1/2)/(-2*(a*b)
^(1/2)-a-b)*b*x^3-1/3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*x^3-1/2*I/d^3/(-2*
(a*b)^(1/2)-a-b)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))-1/4
*I/d^3/(a*b)^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))-1/
2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*polylog(2,(a-b)*exp(2*I*(d*x+c))/(
-2*(a*b)^(1/2)-a-b))*x+1/d^2*c^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*x+1/d^2
*c^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/
2)-a-b)*a*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4568 vs. 2(247) = 494.
time = 5.23, size = 4568, normalized size = 13.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-(((a + b)*cos(d*x +
 c) + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(
d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2
*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b) + 1) + 2*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2))*d*x*dilog((((a + b)*cos(d*x + c) - (I*a + I*b)*sin(d*x
+ c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 -
2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a -
 b)) - a + b)/(a - b) + 1) + 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*di
log(-(((a + b)*cos(d*x + c) + (-I*a - I*b)*sin(d*x + c) - 2*((a - b)*cos(d*
x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b) +
 1) + 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog((((a + b)*cos(d*x +
 c) - (-I*a - I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*si
n(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 -
 2*a*b + b^2)) + a + b)/(a - b)) - a + b)/(a - b) + 1) - 2*(a - b)*sqrt(a*b
/(a^2 - 2*a*b + b^2))*d*x*dilog(-(((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d
*x + c) + 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^
2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(
a - b)) + a - b)/(a - b) + 1) - 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x
*dilog((((a + b)*cos(d*x + c) - (I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d
*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - a + b)/(a - b) +
 1) - 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-(((a + b)*cos(d*x
+ c) + (-I*a - I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) - (I*a - I*b)*si
n(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2)) - a - b)/(a - b)) + a - b)/(a - b) + 1) - 2*(a - b)*sqrt(a*b/
(a^2 - 2*a*b + b^2))*d*x*dilog((((a + b)*cos(d*x + c) - (-I*a - I*b)*sin(d*
x + c) + 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2
- 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a
- b)) - a + b)/(a - b) + 1) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c^2*l
og(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*c
```

```
os(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c
^2*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) +
 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2
))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b
)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b +
 b^2))*c^2*log(2*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a
 - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a
*b + b^2))*c^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)
/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 -
2*a*b + b^2))*c^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a -
 b)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2
 - 2*a*b + b^2))*c^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) -
a - b)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(
a^2 - 2*a*b + b^2))*c^2*log(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))
 - a - b)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) + (I*(a - b)*d^2*x^
2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log((((a + b)*cos(d*x + c)
 + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x
 + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*
b + b^2)) + a + b)/(a - b)) + a - b)/(a - b)) + (-I*(a - b)*d^2*x^2 + I*(a
- b)*c^2)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log(-(((a + b)*cos(d*x + c) - (I*a
+ I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*s
qrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)
) + a + b)/(a - b)) - a + b)/(a - b)) + (-I*(a - b)*d^2*x^2 + I*(a - b)*c^2
)*sqrt(a*b/(a^2 - 2*a*b + b^2))*log((((a + b)*cos(d*x + c) + (-I*a - I*b)*s
in(d*x + c) - 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b
/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a +
 b)/(a - b)) + a - b)/(a - b)) + (I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a
*b/(a^2 - 2*a*b + b^2))*log(-(((a + b)*cos(d*x + c) - (-I*a - I*b)*sin(d*x
+ c) - 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 -
 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a
- b)) - a + b)/(a - b)) + (-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt(a*b/(a^
2 - 2*a*b + b^2))*log((((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d*x + c) + 2
*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b +
 b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + a
 - b)/(a - b)) + (I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt(a*b/(a^2 - 2*a*b
+ b^2))*log(-(((a + b)*cos(d*x + c) - (I*a + I*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec^2(c + dx)}{a + b\tan^2(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
[Out] integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos\left(c+d\,x\right)^2\,\left(b\,\tan\left(c+d\,x\right)^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
[Out] int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

3.162 $\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b+c}\,\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\,\sqrt{b+c}\,d}$$

```
[Out] arctan((b+c)^(1/2)*tan(d*x+c)/(a+c)^(1/2))/d/(a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b+c}\,\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\,\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+c+(b+c)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b+c}\,\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\,\sqrt{b+c}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 40, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b+c}\,\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\,\sqrt{b+c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

Maple [A]
time = 0.47, size = 34, normalized size = 0.85

method	result
derivativedivides	$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d\sqrt{(a+c)(b+c)}}$
default	$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d\sqrt{(a+c)(b+c)}}$
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iab+2iac+2ibc+2ic^2+a\sqrt{-ab-ac-cb-c^2}+b\sqrt{-ab-ac-cb-c^2}+2c\sqrt{-ab-\cdots}}{\sqrt{-ab-ac-cb-c^2}\,(a-b)}\right.}{2\sqrt{-ab-ac-cb-c^2}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/((a+c)*(b+c))^(1/2)*arctan((b+c)*tan(d*x+c)/((a+c)*(b+c))^(1/2))
```

Maxima [A]
time = 0.48, size = 43, normalized size = 1.08

$$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{ab+(a+b)c+c^2}}\right)}{\sqrt{ab+(a+b)c+c^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="maxi
ma")
```

```
[Out] arctan((b + c)*tan(d*x + c)/sqrt(a*b + (a + b)*c + c^2))/(sqrt(a*b + (a + b
)*c + c^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.
time = 2.83, size = 300, normalized size = 7.50

$$\left[-\frac{\sqrt{-ab-(a+b)c-c^2}\log\left(\frac{(a^2+6ab+b^2+8(a+b)c+8c^2)\cos(dx+c)^4-2(3ab+b^2+(3a+5b)c+4c^2)\cos(dx+c)^2+4\left((a+b+2c)\cos(dx+c)^3-(b+c)\cos(dx+c)\right)\sqrt{-ab-(a+b)c-c^2}\sin(dx+c)+b^2+2bc+c^2}{(a^2-2ab+b^2)\cos(dx+c)^4+2(ab-b^2+(a-b)c)\cos(dx+c)^2+b^2+2bc+c^2}\right)}{4(ab+(a+b)c+c^2)d},\ -\frac{\arctan\left(\frac{(a+b+2c)\cos(dx+c)^2-b-c}{2\sqrt{ab+(a+b)c+c^2}\cos(dx+c)\sin(dx+c)}\right)}{2\sqrt{ab+(a+b)c+c^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fric
as")
```

```
[Out] [-1/4*sqrt(-a*b - (a + b)*c - c^2)*log(((a^2 + 6*a*b + b^2 + 8*(a + b)*c +
8*c^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2 + (3*a + 5*b)*c + 4*c^2)*cos(d*x + c
)^2 + 4*((a + b + 2*c)*cos(d*x + c)^3 - (b + c)*cos(d*x + c))*sqrt(-a*b - (
a + b)*c - c^2)*sin(d*x + c) + b^2 + 2*b*c + c^2)/((a^2 - 2*a*b + b^2)*cos(
d*x + c)^4 + 2*(a*b - b^2 + (a - b)*c)*cos(d*x + c)^2 + b^2 + 2*b*c + c^2))
/((a*b + (a + b)*c + c^2)*d), -1/2*arctan(1/2*((a + b + 2*c)*cos(d*x + c)^2
 - b - c)/(sqrt(a*b + (a + b)*c + c^2)*cos(d*x + c)*sin(d*x + c)))/(sqrt(a*
b + (a + b)*c + c^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.
time = 0.78, size = 76, normalized size = 1.90

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2b+2c)+\arctan\left(\frac{b\tan(dx+c)+c\tan(dx+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{\sqrt{ab+ac+bc+c^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="giac
")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*b + 2*c) + arctan((b*tan(d*x + c) + c*t
an(d*x + c))/sqrt(a*b + a*c + b*c + c^2)))/(sqrt(a*b + a*c + b*c + c^2)*d)
```

Mupad [B]
time = 2.60, size = 45, normalized size = 1.12

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(b+c)}{\sqrt{a\,b+a\,c+b\,c+c^2}}\right)}{d\,\sqrt{a\,b+a\,c+b\,c+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

```
[Out] atan((tan(c + d*x)*(b + c))/(a*b + a*c + b*c + c^2)^(1/2))/(d*(a*b + a*c +
b*c + c^2)^(1/2))
```

3.163 $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

Optimal. Leaf size=267

$$-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\,\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d}-\frac{\text{PolyLog}\left(2,-\frac{(a}{a+b+2c-2}\right.}{4\sqrt{a+c}\,\sqrt{b}}$$

```
[Out] -1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d
/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+
c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/4*polylog(2,-(a-b)*exp(2
*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2
)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(1/2))
)/d^2/(a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4685, 3402, 2296, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{4d^2\sqrt{a+c}\,\sqrt{b+c}}+\frac{\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\,\sqrt{b+c}\right)}\right)}{4d^2\sqrt{a+c}\,\sqrt{b+c}}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\,\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\,\sqrt{b+c}}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\,\sqrt{b+c}+c\right)+a+b}\right)}{2d\sqrt{a+c}\,\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/2*I)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a +
 c]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x*Log[1 + ((a - b)*
E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[a +
c]*Sqrt[b + c]*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c
 - 2*Sqrt[a + c]*Sqrt[b + c]))]/(4*Sqrt[a + c]*Sqrt[b + c]*d^2) + PolyLog[2
, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))
)]/(4*Sqrt[a + c]*Sqrt[b + c]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4685

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rubi steps

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x}{-4\sqrt{a+c}\,\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\,\sqrt{b+c}} - \cdots$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{\cdots}{a+b+2(c\cdots}\right.}{2\sqrt{a+c}\cdots}$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{\cdots}{a+b+2(c\cdots}\right.}{2\sqrt{a+c}\cdots}$$

$$= -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d} + \frac{ix\log\left(1+\frac{\cdots}{a+b+2(c\cdots}\right.}{2\sqrt{a+c}\cdots}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 751 vs. $2(267) = 534$.
time = 2.83, size = 751, normalized size = 2.81

$$\frac{x\left(4\sqrt{-b-c}\,c\,\text{ArcTan}\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)-i\sqrt{b+c}\log(1+i\tan(c+dx))\log\left(\frac{i\left(\sqrt{a+c}-\sqrt{-b-c}\tan(c+dx)\right)}{\sqrt{-b-c}+i\sqrt{a+c}}\right)+i\sqrt{b+c}\log(1-i\tan(c+dx))\log\left(\frac{i\left(-\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)}{\sqrt{-b-c}-i\sqrt{a+c}}\right)+i\sqrt{b+c}\log(1+i\tan(c+dx))\log\left(-\frac{i\left(\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)}{\sqrt{-b-c}-i\sqrt{a+c}}\right)-i\sqrt{b+c}\log(1-i\tan(c+dx))\log\left(\frac{i\left(\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)}{\sqrt{-b-c}+i\sqrt{a+c}}\right)+i\sqrt{b+c}\,\text{PolyLog}\left(2,\frac{\sqrt{-b-c}(1-i\tan(c+dx))}{\sqrt{-b-c}-i\sqrt{a+c}}\right)-i\sqrt{b+c}\,\text{PolyLog}\left(2,\frac{\sqrt{-b-c}(1-i\tan(c+dx))}{\sqrt{-b-c}+i\sqrt{a+c}}\right)+i\sqrt{b+c}\,\text{PolyLog}\left(2,\frac{\sqrt{-b-c}(1+i\tan(c+dx))}{\sqrt{-b-c}-i\sqrt{a+c}}\right)-i\sqrt{b+c}\,\text{PolyLog}\left(2,\frac{\sqrt{-b-c}(1+i\tan(c+dx))}{\sqrt{-b-c}+i\sqrt{a+c}}\right)\right)\left(\sqrt{a+c}-\sqrt{-b-c}\tan(c+dx)\right)\left(\sqrt{a+c}+\sqrt{-b-c}\tan(c+dx)\right)}{2\sqrt{a+c}\,\sqrt{-(b+c)^2}\,d\left(2c-i\log(1-i\tan(c+dx))+i\log(1+i\tan(c+dx))\right)\left(a+c\sec^2(c+dx)+b\tan^2(c+dx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] (x*(4*Sqrt[-b - c]*c*ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]] - I*Sqr
t[b + c]*Log[1 + I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 - I*Tan[c + d
*x]]*Log[(I*(-Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*S
qrt[a + c])] + I*Sqrt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[((-I)*(Sqrt[a + c]
 + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b +
 c]*Log[1 - I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]
))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]
*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b + c]*Poly
Log[2, (Sqrt[-b - c]*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])]
+ I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c + d*x]))/(Sqrt[-b - c
```

```
] - I*Sqrt[a + c])] - I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c +
 d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])])*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c
+ d*x])*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(2*Sqrt[a + c]*Sqrt[-(b
+ c)^2]*d*(2*c - I*Log[1 - I*Tan[c + d*x]] + I*Log[1 + I*Tan[c + d*x]])*(a
+ c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(217) = 434$.
time = 0.61, size = 1670, normalized size = 6.25

method	result	size
risch	Expression too large to display	1670

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2/((a+c)*(b+c))^(1/2)*x^2-1/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x^2-2/d/((a
+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2*x-1/4/d^2/((a+c)*(b+c
))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/
(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a-1/4/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*
(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c))*b-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c
)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c-1/4/
d^2/((a+c)*(b+c))^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(
1/2)-a-b-2*c))-1/2/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp
(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/2/d^2/((a+c)*(b+c))^(1/2)
*c^2-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2-1/d/((a+c)*(b+c))^(1/2)/(-2
*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c*x-1/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c
))^(1/2)-a-b-2*c)*b*c*x-I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a
-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2-1
/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*c^2-1/2/d^2/(
(a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^2-1/2*I/d/((a+c)*(b
+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-I
/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*
(b+c))^(1/2)-a-b-2*c))*x-I/d^2*c/(a*b+a*c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-
b)*exp(2*I*(d*x+c))+2*a+2*b+4*c)/(a*b+a*c+b*c+c^2)^(1/2))-1/2*I/d^2/((a+c)*
(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c
-I/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c))*c-1/2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*
c))*a*c-1/2*I/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1
-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*b*c-1/2*I/d/((a+c
)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))
/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*x-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+
```

```
c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2
)-a-b-2*c))*b*x-I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln
(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c*x-1/((a+c)*(b
+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x^2-1/2/((a+c)*(b+c))^(1/2)/(
-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*x^2-1/2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b
+c))^(1/2)-a-b-2*c)*b*x^2-1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)
-a-b-2*c)*c^3-1/d/((a+c)*(b+c))^(1/2)*c*x-2/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c)*c*x
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="ma
xima")
```

```
[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4100 vs. 2(213) = 426.
time = 5.36, size = 4100, normalized size = 15.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fr
icas")
```

```
[Out] -1/4*(I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*s
qrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b +
 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b +
 (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x
+ c) - 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 -
2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) +
I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(
2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/
(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((a*b + (a
+ b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b
)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) +
2*I*sin(d*x + c)) + I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^
2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + I*(a - b
```

```
)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)
*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b))
- 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((a*b + (a + b)*c +
c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d
*x + c)) - (a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(
-(((a + b + 2*c)*cos(d*x + c) + (I*a + I*b + 2*I*c)*sin(d*x + c) - 2*((a -
b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) + a - b)/(a - b) + 1) - (a - b)*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog((((a + b + 2*c)*cos(d*x +
c) - (I*a + I*b + 2*I*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*
b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(
2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/
(a - b)) - a + b)/(a - b) + 1) - (a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
- 2*a*b + b^2))*dilog(-(((a + b + 2*c)*cos(d*x + c) + (-I*a - I*b - 2*I*c)*
sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a
 + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + a - b)/(a - b
) + 1) - (a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(((
(a + b + 2*c)*cos(d*x + c) - (-I*a - I*b - 2*I*c)*sin(d*x + c) - 2*((a - b)
*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^
2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a
*b + b^2)) + a + b + 2*c)/(a - b)) - a + b)/(a - b) + 1) + (a - b)*sqrt((a*
b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-(((a + b + 2*c)*cos(d*x +
c) + (I*a + I*b + 2*I*c)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) - (-I*a + I
*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((
2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/
(a - b)) + a - b)/(a - b) + 1) + (a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2
- 2*a*b + b^2))*dilog((((a + b + 2*c)*cos(d*x + c) - (I*a + I*b + 2*I*c)*si
n(d*x + c) + 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt((a*b
 + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - a + b)/(a - b) +
 1) + (a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-(((a
 + b + 2*c)*cos(d*x + c) + (-I*a - I*b - 2*I*c)*sin(d*x + c) + 2*((a - b)*c
os(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 -
 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)) - a - b - 2*c)/(a - b)) + a - b)/(a - b) + 1) + (a - b)*sqrt((a*b +
(a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog((((a + b + 2*c)*cos(d*x + c) -
(-I*a - I*b - 2*I*c)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) + (I*a - I*b)*s
in(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a
- b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a -
b)) - a + b)/(a - b) + 1) - (I*(a - b)*d*x + I*(a - b)*c)*sqrt((a*b + (a +
b)*c + c^2)/(a^2 - 2*a*b + b^2))*log((((a + b + 2*c)*cos(d*x + c) + (I*a +
I*b + 2*I*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x +
```

```
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + a
 - b)/(a - b)) - (-I*(a - b)*d*x - I*(a - b)*c)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2))*log(-(((a + b + 2*c)*cos(d*x + c) - (I*a + I*b + 2*I
*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="gi
ac")
```

```
[Out] integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\cos(c+d\,x)^2\,\left(a+\frac{c}{\cos(c+d\,x)^2}+b\tan(c+d\,x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

```
[Out] int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

3.164 $\int \frac{x^2 \sec^2(c+dx)}{a+c \sec^2(c+dx)+b \tan^2(c+dx)} \, dx$

Optimal. Leaf size=407

$$-\frac{ix^2 \log \left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d}+\frac{ix^2 \log \left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\,\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\,\sqrt{b+c}\,d}-\frac{x\text{PolyLog}\left(2,-\frac{\quad}{a+b+2c\cdots}\right.}{2\sqrt{a+c}\,\sqrt{\cdots}}$$

```
[Out] -1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))
/d/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2
*(a+c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/2*x*polylog(2,-(a-b)
*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c
)^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c
)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2)-1/4*I*polylog(3,-(a-b)*exp(2*I*(d*x+c
))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^3/(a+c)^(1/2)/(b+c)^(1/2)+1/4*I*p
olylog(3,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(1/2)))/d^3/(
a+c)^(1/2)/(b+c)^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4685, 3402, 2296, 2221, 2611, 2320, 6724}

$$-\frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{4d^3\sqrt{a+c}\,\sqrt{b+c}}+\frac{i\text{Li}_3\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\,\sqrt{b+c}\right)}\right)}{4d^3\sqrt{a+c}\,\sqrt{b+c}}-\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\,\sqrt{b+c}}\right)}{2d^2\sqrt{a+c}\,\sqrt{b+c}}+\frac{x\text{Li}_2\left(-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\,\sqrt{b+c}\right)}\right)}{2d^2\sqrt{a+c}\,\sqrt{b+c}}-\frac{ix^2 \log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\,\sqrt{b+c}+a+b+2c}\right)}{2d\sqrt{a+c}\,\sqrt{b+c}}+\frac{ix^2 \log\left(1+\frac{(a-b)e^{2i(c+dx)}}{2\left(\sqrt{a+c}\,\sqrt{b+c}+c\right)+a+b}\right)}{2d\sqrt{a+c}\,\sqrt{b+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a
 + c]*Sqrt[b + c])])/(Sqrt[a + c]*Sqrt[b + c]*d) + ((I/2)*x^2*Log[1 + ((a -
 b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[
a + c]*Sqrt[b + c]*d) - (x*PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a +
b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) + (
x*PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sq
rt[b + c])))])/(2*Sqrt[a + c]*Sqrt[b + c]*d^2) - ((I/4)*PolyLog[3, -(((a -
b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))])/(Sqrt[
a + c]*Sqrt[b + c]*d^3) + ((I/4)*PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))
/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))])/(Sqrt[a + c]*Sqrt[b + c]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4685

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> D
ist[2, Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = 2\int \frac{x^2}{a+b+2c+(a-b)\cos(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{i(2c+2dx)}x^2}{a-b+2(a+b+2c)e^{i(2c+2dx)}+(a-b)e^{2i(2c+2dx)}}\,dx$$

$$= \frac{(2(a-b))\int \frac{e^{i(2c+2dx)}x^2}{-4\sqrt{a+c}\sqrt{b+c}+2(a+b+2c)+2(a-b)e^{i(2c+2dx)}}\,dx}{\sqrt{a+c}\sqrt{b+c}} - \frac{(2(\ldots}{\ldots}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{\ldots}{a+b+2(\ldots}\right)}{2\sqrt{a+c\ldots}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{\ldots}{a+b+2(\ldots}\right)}{2\sqrt{a+c\ldots}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{\ldots}{a+b+2(\ldots}\right)}{2\sqrt{a+c\ldots}}$$

$$= -\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}\,d} + \frac{ix^2\log\left(1+\frac{\ldots}{a+b+2(\ldots}\right)}{2\sqrt{a+c\ldots}}$$

Mathematica [A]
time = 1.39, size = 499, normalized size = 1.23

$$-\frac{ie^{2ic}\left(2d^2x^2\log\left(1+\frac{(a-b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}-2\sqrt{(a+c)(b+c)e^{4ic}}}\right)-2d^2x^2\log\left(1+\frac{(a-b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}+2\sqrt{(a+c)(b+c)e^{4ic}}}\right)-2idx\,\mathrm{PolyLog}\left(2,\frac{(-a+b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}-2\sqrt{(a+c)(b+c)e^{4ic}}}\right)+2idx\,\mathrm{PolyLog}\left(2,\frac{(-a+b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}+2\sqrt{(a+c)(b+c)e^{4ic}}}\right)+\mathrm{PolyLog}\left(3,\frac{(-a+b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}-2\sqrt{(a+c)(b+c)e^{4ic}}}\right)-\mathrm{PolyLog}\left(3,\frac{(-a+b)e^{2i(2c+dx)}}{ae^{2ic}+be^{2ic}+2ce^{2ic}+2\sqrt{(a+c)(b+c)e^{4ic}}}\right)\right)}{4d^3\sqrt{(a+c)(b+c)e^{4ic}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

```
[Out] ((-1/4*I)*E^((2*I)*c)*(2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a
*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^(
(4*I)*c)])] - 2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)
```

```
*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)]
)] - (2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) +
 b*E^((2*I)*c) + 2*c*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] +
(2*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^
((2*I)*c) + 2*c*E^((2*I)*c) + 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] + PolyL
og[3, ((-a + b)*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c
*E^((2*I)*c) - 2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])] - PolyLog[3, ((-a + b)
*E^((2*I)*(2*c + d*x)))/(a*E^((2*I)*c) + b*E^((2*I)*c) + 2*c*E^((2*I)*c) +
2*Sqrt[(a + c)*(b + c)*E^((4*I)*c)])]))/(d^3*Sqrt[(a + c)*(b + c)*E^((4*I)*
c)])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2060 vs. $2(331) = 662$.
time = 0.48, size = 2061, normalized size = 5.06

method	result	size
risch	Expression too large to display	2061

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*polylog(2,(a-
b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/2/d^2/((a+c)*(b+c
))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*a*polylog(2,(a-b)*exp(2*I*(d*x+c)
)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/2/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c
)*(b+c))^(1/2)-a-b-2*c)*b*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c))*x+1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c)*a*c^2*x+1/d^2/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*c^
2*x+I/d^3*c^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-
b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/2*I/d^3/((a+c)*(b+c
))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*polylog(3,(a-b)*exp(2*I*(d*x+c)
)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))-1/4*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c
)*(b+c))^(1/2)-a-b-2*c)*a*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c))-1/4*I/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b
-2*c)*b*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))+
2/3/d^3/((a+c)*(b+c))^(1/2)*c^3+4/3/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^
3-I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*ln(1-(a-b)*exp
(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2+1/2*I/d^3*c^2/((a+c)*(b
+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2
*((a+c)*(b+c))^(1/2)-a-b-2*c))*b+1/2*I/d^3*c^2/((a+c)*(b+c))^(1/2)/(-2*((a+
c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2
)-a-b-2*c))*a-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*
I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/2/d^2/((a+c)*(b+c))^(1/2)*
polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^2/(
```

```
(a+c)*(b+c))^(1/2)*c^2*x+2/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2*x+4/3/d
^3*c^4/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)-1/4*I/d^3/((a+c
)*(b+c))^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-
2*c))-1/2*I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*polylog(3,(a-b)*exp(2*I*(d
*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))+2/d^2*c^3/((a+c)*(b+c))^(1/2)/(-2*
((a+c)*(b+c))^(1/2)-a-b-2*c)*x+2/3/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c)*a*c^3+2/3/d^3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-
a-b-2*c)*b*c^3-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x
+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*
((a+c)*(b+c))^(1/2)-a-b-2*c)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c)
)^(1/2)-a-b-2*c))*x^2-1/2*I/d/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a
-b-2*c)*b*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2
+I/d^3*c^2/(a*b+a*c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*
a+2*b+4*c)/(a*b+a*c+b*c+c^2)^(1/2))-1/2*I/d/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*
exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2+1/2*I/d^3*c^2/((a+c)*
(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))+I
/d^3*c^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(
(a+c)*(b+c))^(1/2)-a-b-2*c))-1/3/((a+c)*(b+c))^(1/2)*x^3-2/3/(-2*((a+c)*(b+
c))^(1/2)-a-b-2*c)*x^3-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-
2*c)*a*x^3-1/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*b*x^3-2
/3/((a+c)*(b+c))^(1/2)/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c*x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
maxima")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5696 vs. 2(325) = 650.
time = 4.80, size = 5696, normalized size = 14.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
fricas")
```

```
[Out] 1/4*(2*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(
-(((a + b + 2*c)*cos(d*x + c) + (I*a + I*b + 2*I*c)*sin(d*x + c) - 2*((a -
b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a
```

^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*
a*b + b^2)) + a + b + 2*c)/(a - b)) + a - b)/(a - b) + 1) + 2*(a - b)*d*x*s
qrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog((((a + b + 2*c)*cos(
d*x + c) - (I*a + I*b + 2*I*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*
a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*s
qrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b +
 2*c)/(a - b)) - a + b)/(a - b) + 1) + 2*(a - b)*d*x*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2))*dilog(-(((a + b + 2*c)*cos(d*x + c) + (-I*a - I
*b - 2*I*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x +
 c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sq
rt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + a
 - b)/(a - b) + 1) + 2*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*
b + b^2))*dilog((((a + b + 2*c)*cos(d*x + c) - (-I*a - I*b - 2*I*c)*sin(d*x
 + c) - 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a
 + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c
 + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - a + b)/(a - b) + 1)
 - 2*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog(-((
(a + b + 2*c)*cos(d*x + c) + (I*a + I*b + 2*I*c)*sin(d*x + c) + 2*((a - b)*
cos(d*x + c) - (-I*a + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2
 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2)) - a - b - 2*c)/(a - b)) + a - b)/(a - b) + 1) - 2*(a - b)*d*x*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*dilog((((a + b + 2*c)*cos(d*x
 + c) - (I*a + I*b + 2*I*c)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) + (-I*a
 + I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqr
t((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*
c)/(a - b)) - a + b)/(a - b) + 1) - 2*(a - b)*d*x*sqrt((a*b + (a + b)*c + c
^2)/(a^2 - 2*a*b + b^2))*dilog(-(((a + b + 2*c)*cos(d*x + c) + (-I*a - I*b
 - 2*I*c)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))
*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + a - b)
/(a - b) + 1) - 2*(a - b)*d*x*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b
^2))*dilog((((a + b + 2*c)*cos(d*x + c) - (-I*a - I*b - 2*I*c)*sin(d*x + c)
 + 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*sqrt((a*b + (a + b)*
c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)
/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - a + b)/(a - b) + 1) + I*(a
 - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(
a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a
 - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c^2*sqrt((a*b + (a +
 b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b
)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos(d*x + c) -
2*I*sin(d*x + c)) - I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
 + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b +
 b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) + I*(a
 - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(
a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a

```
- b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*c^2*sqrt((a*b + (a +
 b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)
*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) + 2
*I*sin(d*x + c)) + I*(a - b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b
+ b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b
^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x + c)) + I*(a -
b)*c^2*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a -
 b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b
)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c^2*sqrt((a*b + (a + b)
*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c
+ c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*
sin(d*x + c)) + (I*(a - b)*d^2*x^2 - I*(a - b)*c^2)*sqrt((a*b + (a + b)*c +
 c^2)/(a^2 - 2*a*b + b^2))*log((((a + b + 2*c)*cos(d*x + c) + (I*a + I*b +
2*I*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*sin(d*x + c))*s
qrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt((a*
b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + a - b)/
(a - b)) + (-I*(a - b)*d^2*x^2 + I*(a - b)*c^2)*sqrt((a*b + (a + b)*c + c^2
)/(a^2 - 2*a*b + b^2))*log(-(((a + b + 2*c)*cos(d*x + c) - (I*a + I*b + 2*I
*c)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (I...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

```
[Out] Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="
giac")
```

```
[Out] integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos\left(c+d\,x\right)^2\,\left(a+\frac{c}{\cos\left(c+d\,x\right)^2}+b\,\tan\left(c+d\,x\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
[Out] int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

3.165 $\int x^3 \sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=155

$$-\frac{6\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4}+\frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{6x\sqrt{a-a\sin(e$$

```
[Out] -6*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*x^2*(a-a*sin(f*x+e))
^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-6*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e
))^(1/2)*tan(f*x+e)/f^3+x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*t
an(f*x+e)/f
```

Rubi [A]
time = 0.13, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4700, 3377, 2718}

$$-\frac{6\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{6x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x^3\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (-6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*x^2*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (6*x*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^3*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
```

```
GeQ[n - m, 0]
```

Rubi steps

$$\int x^3 \sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx = \Big(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\Big)$$

$$= \frac{x^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx}{f}$$

$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x^3\sqrt{a}}{}$$

$$= \frac{3x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{6x\sqrt{a}}{}$$

$$= -\frac{6\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4} + \frac{3x^2\sqrt{a}}{}$$

Mathematica [**A**]
time = 0.29, size = 61, normalized size = 0.39

$$\frac{\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(-6+3f^2x^2+fx(-6+f^2x^2)\tan(e+fx))}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-6 + 3*f^2*x^2 + f*x*
(-6 + f^2*x^2)*Tan[e + f*x]))/f^4
```

Maple [**F**]
time = 0.18, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [A]
time = 0.43, size = 156, normalized size = 1.01

$$-\sqrt{a}\,\sqrt{c}\left(\frac{3\left(f^{2}x^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos\left(fx+e\right)}{f^{4}}+\frac{\left(f^{3}x^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-6\,fx\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin\left(fx+e\right)}{f^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] -sqrt(a)*sqrt(c)*(3*(f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)))*cos(f*x + e)/f^4 + (f^3*x^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*f*x*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f^4
)
```

Mupad [B]
time = 3.05, size = 111, normalized size = 0.72

$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(6\,\cos\left(2\,e+2\,f\,x\right)-3\,f^2\,x^2+6\,f\,x\,\sin\left(2\,e+2\,f\,x\right)-3\,f^2\,x^2\,\cos\left(2\,e+2\,f\,x\right)-f^3\,x^3\,\sin\left(2\,e+2\,f\,x\right)+6\right)}{f^4\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(6*cos(2*e + 2
*f*x) - 3*f^2*x^2 + 6*f*x*sin(2*e + 2*f*x) - 3*f^2*x^2*cos(2*e + 2*f*x) - f
^3*x^3*sin(2*e + 2*f*x) + 6))/(f^4*(cos(2*e + 2*f*x) + 1))
```

3.166 $\int x^2 \sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}\,dx$

Optimal. Leaf size=118

$$\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f^3}+\frac{x^2\sqrt{a}}{}$$

```
[Out] 2*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*(a-a*sin(f*x+e))^(1
/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3+x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4700, 3377, 2717}

$$-\frac{2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x^2\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (2*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f^3 + (x^2*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^2 \sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\right)$$

$$= \frac{x^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx}{f}$$

$$= \frac{2x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x^2\sqrt{a-}}{}$$

$$= \frac{2x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{2\sqrt{a-}}{}$$

Mathematica [A]
time = 0.20, size = 54, normalized size = 0.46

$$\frac{\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(2fx+(-2+f^2x^2)\tan(e+fx))}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(2*f*x + (-2 + f^2*x^2
)*Tan[e + f*x]))/f^3
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c(\sin(e+fx)+1)}\sqrt{-a(\sin(e+fx)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [A]
time = 0.42, size = 119, normalized size = 1.01

$$-\left(\frac{2\,x\cos(fx+e)\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f^2}+\frac{\left(f^2x^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-2\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sin(fx+e)}{f^3}\right)\sqrt{a}\,\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] -(2*x*cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))/f^2 + (f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f^3)*sqrt(a)*sqrt(c)
```

Mupad [B]
time = 2.81, size = 86, normalized size = 0.73

$$\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(2\,f\,x-2\,\sin\left(2\,e+2\,f\,x\right)+2\,f\,x\,\left(2\,{\cos\left(e+f\,x\right)}^2-1\right)+f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)\right)}{2\,f^3\,{\cos\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(2*f*x - 2*sin(
2*e + 2*f*x) + 2*f*x*(2*cos(e + f*x)^2 - 1) + f^2*x^2*sin(2*e + 2*f*x)))/(2
*f^3*cos(e + f*x)^2)
```

3.167 $\int x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}+\frac{x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx)}{f}$$

```
[Out] (a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+x*(a-a*sin(f*x+e))^(1/2)*
(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4700, 3377, 2718}

$$\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (x*Sqrt[a - a*Sin
[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int$$

$$= \frac{x\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\tan(e+fx)}{f} -$$

$$= \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x\sqrt{a-a\sin(}{}$$

Mathematica [**A**]
time = 0.14, size = 44, normalized size = 0.59

$$\frac{\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(1+fx\tan(e+fx))}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(1 + f*x*Tan[e + f*x])
)/f^2
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int x\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c(\sin(e+fx)+1)}\,\sqrt{-a(\sin(e+fx)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [A]
time = 0.46, size = 82, normalized size = 1.11

$$-\left(\frac{x\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin(fx+e)}{f}+\frac{\cos(fx+e)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f^2}\right)\sqrt{a}\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -(x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(f*x + e)/f + cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/f^2)*sqrt(a)*sqrt(c)
```

Mupad [B]
time = 2.69, size = 61, normalized size = 0.82

$$\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\cos\left(e+f\,x\right)^2+f\,x\,\sin\left(2\,e+2\,f\,x\right)\right)\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}}{2\,f^2\cos\left(e+f\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((-a*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x)^2 + f*x*sin(2*e + 2*f*x))*(c
*(sin(e + f*x) + 1))^(1/2))/(2*f^2*cos(e + f*x)^2)
```

3.168 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx$

Optimal. Leaf size=86

$\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}-\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+$

```
[Out] Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-sec
(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4700, 3384, 3380, 3383}

$$\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

```
[Out] Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Si
n[e + f*x]] - Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e
 + f*x]]*SinIntegral[f*x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
```

```
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int$$
$$= \left(\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+f$$
$$= \cos(e)\text{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin($$

Mathematica [**A**]
time = 0.12, size = 52, normalized size = 0.60

$$\sec(e+fx)\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(\cos(e)\text{CosIntegral}(fx)-\sin(e)\text{Si}(fx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

```
[Out] Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e]*Co
sIntegral[f*x] - Sin[e]*SinIntegral[f*x])
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 274, normalized size = 3.19

$$\frac{\left(\Re(\mathrm{Ci}\left(fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(\frac{1}{2}e\right)^{2}+\Re(\mathrm{Ci}\left(-fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(\frac{1}{2}e\right)^{2}+2\Im(\mathrm{Ci}\left(fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(\frac{1}{2}e\right)-2\Im(\mathrm{Ci}\left(-fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(\frac{1}{2}e\right)+4\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{Si}\left(fx\right)\tan\left(\frac{1}{2}e\right)-\Re(\mathrm{Ci}\left(fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\Re(\mathrm{Ci}\left(-fx\right))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{2\left(\tan\left(\frac{1}{2}e\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] 1/2*(real_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 + real_part(cos_integral(-f*x))
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*ta
n(1/2*e)^2 + 2*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) - 2*imag_part(cos_integ
ral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*tan(1/2*e) + 4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*e) - real_part(cos_integral(f
*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
```

```
) - real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/(tan(1/2*e)^2 + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x,x)
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x, x)
```

3.169 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}-f\text{CosIntegral}(fx)\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c$$

```
[Out] -(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-f*cos(e)*sec(f*x+e)*Si(f*x
)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-f*Ci(f*x)*sec(f*x+e)*sin(e)
*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4700, 3378, 3384, 3380, 3383}

$$-f\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-f\cos(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}-\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) - f*CosIntegral[f*
x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] -
f*Cos[e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin
Integral[f*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \Big(f\sec(e+$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \Big(f\cos(e)\,\mathrm{s}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - f\mathrm{Ci}(fx)\,\mathrm{se}$$

Mathematica [A]
time = 0.16, size = 65, normalized size = 0.53

$$-\frac{\sec(e+fx)\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(\cos(e+fx)+fx\mathrm{CosIntegral}(fx)\sin(e)+fx\cos(e)\mathrm{Si}(fx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

```
[Out] -((Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e
+ f*x] + f*x*CosIntegral[f*x]*Sin[e] + f*x*Cos[e]*SinIntegral[f*x]))/x)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c(\sin(e+fx)+1)}\,\sqrt{-a(\sin(e+fx)-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

Giac [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 886, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] -1/2*(f*x*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 - f*x*imag_
part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*f*x*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*ta
n(1/2*f*x)^2*tan(1/2*e)^2 - 2*f*x*real_part(cos_integral(f*x))*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*
tan(1/2*e) - 2*f*x*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e) - f
*x*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 + f*x*imag_part(cos_integral(-f
*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(1/2*f*x)^2 - 2*f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*f*x)^2 + f*x*imag_part(cos_
integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*tan(1/2*e)^2 - f*x*imag_part(cos_integral(-f*x))*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 + 2*
f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin_integral(f*x)*tan(1/2*e)^2 - 2*f*x*real_part(cos_integral(f*x))*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e
) - 2*f*x*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) - 2*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^
2 - f*x*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + f*x*imag_part(cos_integral(-f*x))*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*f*x*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin
_integral(f*x) + 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*tan(1/2*f*x)^2 + 8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)*tan(1/2*e) + 2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 - 2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqr
t(a)*sqrt(c)/(x*tan(1/2*f*x)^2*tan(1/2*e)^2 + x*tan(1/2*f*x)^2 + x*tan(1/2*
e)^2 + x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\sin\left(e+f\,x\right)}\,\sqrt{c+c\sin\left(e+f\,x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2, x)
```

3.170 $\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

Optimal. Leaf size=176

$$-\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2}-\frac{1}{2}f^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c$$

```
[Out] -1/2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*f^2*Ci(f*x)*cos(
e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f^2*sec(f*x
+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f*(a-a
*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/x
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4700, 3378, 3384, 3380, 3383}

$$-\frac{1}{2}f^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+\frac{1}{2}f^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x^2}+\frac{f\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] -1/2*(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2 - (f^2*Cos[e]*
CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]])/2 + (f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*SinIntegral[f*x])/2 + (f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[
e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}\Big(f\sec(e+$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin}}{}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} + \frac{f\sqrt{a-a\sin}}{}$$

$$= -\frac{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{1}{2}f^2\cos(e)\mathrm{C}$$

Mathematica [A]
time = 0.18, size = 87, normalized size = 0.49

$$\frac{\sec(e+fx)\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(-\cos(e+fx)-f^2x^2\cos(e)\mathrm{CosIntegral}(fx)+fx\sin(e+fx)+f^2x^2\sin(e)\mathrm{Si}(fx))}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

```
[Out] (Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-Cos[e +
 f*x] - f^2*x^2*Cos[e]*CosIntegral[f*x] + f*x*Sin[e + f*x] + f^2*x^2*Sin[e]
*SinIntegral[f*x]))/(2*x^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{c+c\sin(fx+e)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c(\sin(e+fx)+1)}\,\sqrt{-a(\sin(e+fx)-1)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 1022, normalized size = 5.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")
```

```
[Out] -1/4*(f^2*x^2*real_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + f^2*x
^2*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*f^2*x^2*imag_
part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e) - 2*f^2*x^2*imag_part(cos_in
tegral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e) + 4*f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*f*x
)^2*tan(1/2*e) - f^2*x^2*real_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 - f^2*x^2
*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 + f^2*x^2*real_part(cos_integral
(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*tan(1/2*e)^2 + f^2*x^2*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 + 2*f^2*
x^2*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) - 2*f^2*x^2*imag_part(cos_integral
(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))*tan(1/2*e) + 4*f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*e) - f^2*x^2*real_part(c
os_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) - f^2*x^2*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*f*x*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan
(1/2*e) - 4*f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))*tan(1/2*f*x)*tan(1/2*e)^2 - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 4*f*x
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*ta
n(1/2*f*x) + 4*f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*tan(1/2*e) + 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 + 8*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)*tan(1/2*e) + 2*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/
2*e)^2 - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
```

```
1/2*e)))*sqrt(a)*sqrt(c)/(x^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + x^2*tan(1/2*f*x
)^2 + x^2*tan(1/2*e)^2 + x^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a - a\sin(e + f\,x)}\,\sqrt{c + c\sin(e + f\,x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3,x)
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3, x)
```

3.171 $\int x^3 \sqrt{a - a\sin(e + fx)}\,(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=393

$$-\frac{6c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}+\frac{3cx\sec(e+f$$

```
[Out] 1/2*x^3*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f-6*c*(a
-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*c*x^2*(a-a*sin(f*x+e))^(1
/2)*(c+c*sin(f*x+e))^(1/2)/f^2+3/8*c*x*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c
+c*sin(f*x+e))^(1/2)/f^3-3/4*c*x^3*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)/f-3/8*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))
^(1/2)/f^4+3/4*c*x^2*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)/f^2-6*c*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3-3
/4*c*x*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/
f^3
```

Rubi [A]
time = 0.23, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4700, 4507, 3398, 3377, 2718, 3392, 30, 2715, 8}

$$-\frac{3c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{8f^4}-\frac{6c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^4}-\frac{3cx\sin(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^3}-\frac{6cx\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{8f^3}+\frac{3cx^2\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c\sin(e+fx)+c)^{5/2}}{2cf}-\frac{3cx^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-6*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^4 + (3*c*x^2*Sqr
t[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 + (3*c*x*Sec[e + f*x]*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^3) - (3*c*x^3*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) - (3*c*Sin[
e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(8*f^4) + (3*c*
x^2*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2)
 + (x^3*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(
2*c*f) - (6*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f
*x])/f^3 - (3*c*x*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]]*Tan[e + f*x])/(4*f^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
```

```
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^3\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\,\right)$$

$$= \frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/}}{2cf}$$

$$= \frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/}}{2cf}$$

$$= -\frac{cx^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+f}}{2f}$$

$$= \frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{cx^3\sec}{}$$

$$= \frac{3cx^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx^3\sec}{}$$

$$= -\frac{6c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^4} + \frac{3cx^2\sqrt{}}{}$$

Mathematica [A]
time = 0.75, size = 113, normalized size = 0.29

$$\frac{c\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(-fx(-3+2f^2x^2)\cos(2(e+fx))\sec(e+fx)+(-3+6f^2x^2)\sin(e+fx)+8(-6+3f^2x^2+fx(-6+f^2x^2)\tan(e+fx)))}{8f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-(f*x*(-3 + 2*f^2*x
^2)*Cos[2*(e + f*x)]*Sec[e + f*x]) + (-3 + 6*f^2*x^2)*Sin[e + f*x] + 8*(-6
+ 3*f^2*x^2 + f*x*(-6 + f^2*x^2)*Tan[e + f*x])))/(8*f^4)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int x^3(c+c\sin(fx+e))^{\frac{3}{2}}\,\sqrt{a-a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1487 vs. $2(345) = 690$.
time = 0.63, size = 1487, normalized size = 3.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")

[Out] 1/32*sqrt(a)*sqrt(c)*((pi^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 3*pi^2*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*pi*(pi - 2*f*x -
2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)) - (pi - 2*f*x - 2*e)^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi^2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*pi*(pi - 2*f*x - 2*e)*c*e*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*(pi -
 2*f*x - 2*e)^2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 12*pi*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 12*(pi - 2*f*x - 2*e)*c*e^2*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*c*e^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi*c*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*(pi
 - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 12*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*cos(2*f*x + 2*e)/f^3 - 24*(pi^2*c*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2
*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) + (pi - 2*f*x - 2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) - 4*pi*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c*e^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*c*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(f*x
 + e)/f^3 - 3*(pi^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*c*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*pi*c*
e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) +
 4*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) + 4*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(2*f*x + 2*e)/f^3 - 4*(pi^3*c*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*pi^2*(p
i - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 3*pi*(pi - 2*f*x - 2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)^3*c*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi^2*c
*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
+ 12*pi*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 6*(pi - 2*f*x - 2*e)^2*c*e*sgn(cos(-1/4*pi +

```
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*pi*c*e^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 12*(pi -
 2*f*x - 2*e)*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 8*c*e^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 24*pi*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*c*e*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f
^3)/f
```

Mupad [B]
time = 4.15, size = 216, normalized size = 0.55

$$-\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(3\,\sin\left(e+f\,x\right)+96\,\cos\left(2\,e+2\,f\,x\right)+3\,\sin\left(3\,e+3\,f\,x\right)-48\,f^2\,x^2-6\,f\,x\,\cos\left(3\,e+3\,f\,x\right)+96\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+4\,f^3\,x^3\,\cos\left(e+f\,x\right)-6\,f^2\,x^2\,\sin\left(e+f\,x\right)-6\,f\,x\,\cos\left(e+f\,x\right)-48\,f^2\,x^2\,\cos\left(2\,e+2\,f\,x\right)+4\,f^3\,x^3\,\cos\left(3\,e+3\,f\,x\right)-6\,f^2\,x^2\,\sin\left(3\,e+3\,f\,x\right)-16\,f^3\,x^3\,\sin\left(2\,e+2\,f\,x\right)+96\right)}{16\,f^4\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(3*sin(e + f
*x) + 96*cos(2*e + 2*f*x) + 3*sin(3*e + 3*f*x) - 48*f^2*x^2 - 6*f*x*cos(3*e
 + 3*f*x) + 96*f*x*sin(2*e + 2*f*x) + 4*f^3*x^3*cos(e + f*x) - 6*f^2*x^2*si
n(e + f*x) - 6*f*x*cos(e + f*x) - 48*f^2*x^2*cos(2*e + 2*f*x) + 4*f^3*x^3*c
os(3*e + 3*f*x) - 6*f^2*x^2*sin(3*e + 3*f*x) - 16*f^3*x^3*sin(2*e + 2*f*x)
+ 96))/(16*f^4*(cos(2*e + 2*f*x) + 1))
```

3.172 $\int x^2 \sqrt{a - a\sin(e + fx)}\, (c+c\sin(e+fx))^{3/2}\, dx$

Optimal. Leaf size=265

$$\frac{2cx\sqrt{a - a\sin(e + fx)}\,\sqrt{c + c\sin(e + fx)}}{f^2} - \frac{3cx^2\sec(e + fx)\sqrt{a - a\sin(e + fx)}\,\sqrt{c + c\sin(e + fx)}}{4f} + \frac{cx}{}$$

```
[Out] 1/2*x^2*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+2*c*x*
(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x^2*sec(f*x+e)*(a-a
*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/2*c*x*sin(f*x+e)*(a-a*sin(f*x
+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f
*x+e))^(1/2)*tan(f*x+e)/f^3-1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*si
n(f*x+e))^(1/2)*tan(f*x+e)/f^3
```

Rubi [A]

time = 0.18, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4700, 4507, 3398, 3377, 2717, 3391, 30}

$$-\frac{c\sin(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^3}-\frac{2c\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^3}+\frac{2cx\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2f^2}+\frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c\sin(e+fx)+c)^{5/2}}{2cf}-\frac{3cx^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*c*x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x^2*Se
c[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*x*
Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(2*f^2) + (
x^2*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*
f) - (2*c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/f
^3 - (c*Sin[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[
e + f*x])/(4*f^3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x^2 \sqrt{a - a\sin(e+fx)}\,(c + c\sin(e+fx))^{3/2}\,dx = \Big(\sec(e+fx)\sqrt{a - a\sin(e+fx)}\,\sqrt{c + c\sin(e+fx)}\,\Big)$$

$$= \frac{x^2\sec(e+fx)\sqrt{a - a\sin(e+fx)}\,(c + c\sin(e+fx))^{5/}}{2cf}$$

$$= \frac{x^2\sec(e+fx)\sqrt{a - a\sin(e+fx)}\,(c + c\sin(e+fx))^{5/}}{2cf}$$

$$= -\frac{cx^2\sec(e+fx)\sqrt{a - a\sin(e+fx)}\,\sqrt{c + c\sin(e+f}}{2f}$$

$$= \frac{2cx\sqrt{a - a\sin(e+fx)}\,\sqrt{c + c\sin(e+fx)}}{f^2} - \frac{cx^2\sec(}{}$$

$$= \frac{2cx\sqrt{a - a\sin(e+fx)}\,\sqrt{c + c\sin(e+fx)}}{f^2} - \frac{3cx^2\sec}{}$$

Mathematica [A]
time = 0.51, size = 95, normalized size = 0.36

$$\frac{c\sqrt{c(1+\sin(e+fx))}\,\sqrt{a - a\sin(e+fx)}\,(16fx - (-1+2f^2x^2)\cos(2(e+fx))\sec(e+fx) + 4fx\sin(e+fx) + 8(-2+f^2x^2)\tan(e+fx))}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(16*f*x - (-1 + 2*f^
2*x^2)*Cos[2*(e + f*x)]*Sec[e + f*x] + 4*f*x*Sin[e + f*x] + 8*(-2 + f^2*x^2
)*Tan[e + f*x]))/(8*f^3)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x^2(c + c\sin(fx+e))^{\frac{3}{2}}\sqrt{a - a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(233) = 466.
time = 0.54, size = 739, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/16*sqrt(a)*sqrt(c)*((pi^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*
c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 4*pi*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
```

```
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(2*f*x + 2*e)/f^2 - 16*(pi*c*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (p
i - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) - 2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*cos(f*x + e)/f^2 - 2*(pi*c*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*e*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(2*f*x + 2*e)/f^2 - 4*(pi^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*
c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 4*pi*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f^2)/f
```

Mupad [B]
time = 3.77, size = 159, normalized size = 0.60

$$\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-16\,\sin\left(2\,e+2\,f\,x\right)+16\,f\,x+16\,f\,x\,\cos\left(2\,e+2\,f\,x\right)+2\,f\,x\,\sin\left(3\,e+3\,f\,x\right)-2\,f^2\,x^2\,\cos\left(e+f\,x\right)+2\,f\,x\,\sin\left(e+f\,x\right)-2\,f^2\,x^2\,\cos\left(3\,e+3\,f\,x\right)+8\,f^2\,x^2\,\sin\left(2\,e+2\,f\,x\right)\right)}{8\,f^3\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(cos(e + f*x)
 + cos(3*e + 3*f*x) - 16*sin(2*e + 2*f*x) + 16*f*x + 16*f*x*cos(2*e + 2*f*x
) + 2*f*x*sin(3*e + 3*f*x) - 2*f^2*x^2*cos(e + f*x) + 2*f*x*sin(e + f*x) -
2*f^2*x^2*cos(3*e + 3*f*x) + 8*f^2*x^2*sin(2*e + 2*f*x)))/(8*f^3*(cos(2*e +
 2*f*x) + 1))
```

3.173 $\int x\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=168

$$\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{4f}+\frac{c\sin}{}$$

```
[Out] 1/2*x*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+c*(a-a*s
in(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x*sec(f*x+e)*(a-a*sin(f*x
+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)
*(c+c*sin(f*x+e))^(1/2)/f^2
```

Rubi [A]

time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4700, 4507, 2723}

$$\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f^2}+\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{f^2}+\frac{x\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c\sin(e+fx)+c)^{5/2}}{2cf}-\frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/f^2 - (3*c*x*Sec[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f) + (c*Sin[e +
f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/(4*f^2) + (x*Sec[e
+ f*x]*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(5/2))/(2*c*f)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4507

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x)^(m
- 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x
] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
```

```
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rubi steps

$$\int x\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)$$

$$= \frac{x\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{5/2}}{2cf}$$

$$= \frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{3cx\sec(e+}{}$$

Mathematica [A]
time = 0.41, size = 73, normalized size = 0.43

$$\frac{c\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}\,(4-fx\cos(2(e+fx))\sec(e+fx)+\sin(e+fx)+4fx\tan(e+fx))}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
[Out] (c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(4 - f*x*Cos[2*(e +
f*x)]*Sec[e + f*x] + Sin[e + f*x] + 4*f*x*Tan[e + f*x]))/(4*f^2)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int x(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
[Out] int(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(x*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(146) = 292.
time = 0.47, size = 311, normalized size = 1.85

$$-\frac{\left(\frac{8\,c\cos(fx+e)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}+\frac{c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin(2fx+2e)}{f}-\frac{\left(\pi c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,ce\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\cos(2fx+2e)}{f}+\frac{4\left(\pi c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-(\pi-2fx-2e)c\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\,ce\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sin(fx+e)}{f}\right)\sqrt{a}\sqrt{c}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/8*(8*c*cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/f + c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))*sin(2*f*x + 2*e)/f - (pi*c*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*e*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*co
s(2*f*x + 2*e)/f + 4*(pi*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*e*sgn(cos(-1/4*pi + 1/2*f
```

```
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f)*sqrt(a)*s
qrt(c)/f
```

Mupad [B]
time = 1.21, size = 123, normalized size = 0.73

$$-\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(-16\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+8\,f\,x\,\sin\left(2\,e+2\,f\,x\right)+2\,f\,x\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+2\,f\,x\,\left(2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-1\right)+16\right)}{8\,f^2\,\left(2\sin\left(e+f\,x\right)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*x
) + sin(3*e + 3*f*x) - 16*sin(e + f*x)^2 + 8*f*x*sin(2*e + 2*f*x) + 2*f*x*(
2*sin(e/2 + (f*x)/2)^2 - 1) + 2*f*x*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) + 16
))/(8*f^2*(2*sin(e + f*x)^2 - 2))
```

3.174 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x}\,dx$

Optimal. Leaf size=186

$$c\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}+\frac{1}{2}c\text{CosIntegral}(2fx)\sec(e+fx$$

```
[Out] c*Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1
/2*c*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^
(1/2)-c*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(
1/2)+1/2*c*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*
x+e))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4700, 6873, 12, 6874, 3384, 3380, 3383}

$$\frac{1}{2}c\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+c\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-c\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+\frac{1}{2}c\cos(2e)\text{Si}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] c*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*
Sin[e + f*x]] + (c*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[2*e]*Sqrt[a - a*Sin[
e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Si
n[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] + (c*Cos[2*e]*Sec[e +
 f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x])
/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x}\,dx &= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \\
&= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)\int \\
&= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \\
&= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \\
&= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right. \\
&= \left(c\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+\right. \\
&= c\cos(e)\mathrm{Ci}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.75, size = 150, normalized size = 0.81

$$\frac{ce^{-i(e-fx)}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(2e^{ie}\text{Ei}(-ifx)+2e^{3ie}\text{Ei}(ifx)+i\left(\text{Ei}(-2ifx)-e^{4ie}\text{Ei}(2ifx)\right)\right)\sqrt{a-a\sin(e+fx)}}{2\sqrt{2}\left(1+e^{2i(e+fx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(2*E^(I*e)*ExpInt
egralEi[(-I)*f*x] + 2*E^((3*I)*e)*ExpIntegralEi[I*f*x] + I*(ExpIntegralEi[(
-2*I)*f*x] - E^((4*I)*e)*ExpIntegralEi[(2*I)*f*x]))*Sqrt[a - a*Sin[e + f*x]
])/(2*Sqrt[2]*E^(I*(e - f*x))*(1 + E^((2*I)*(e + f*x))))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="fri
cas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}} \sqrt{-a(\sin(e+fx)-1)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x,x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

Giac [A]
time = 0.45, size = 160, normalized size = 0.86

$$-\frac{\left(2\,cf\cos(e)\,\mathrm{Ci}(fx)\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+cf\,\mathrm{Ci}(2fx)\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin(2e)+cf\cos(2e)\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{Si}(2fx)-2cf\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin(e)\,\mathrm{Si}(fx)\right)\sqrt{a}\,\sqrt{c}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="gia
c")
```

```
[Out] -1/2*(2*c*f*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + c*f*cos_integral(2*f*x)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) + c*f*cos
(2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))*sin_integral(2*f*x) - 2*c*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x))*sqrt(a)*sqrt(c)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin(e+f\,x)}\,\left(c+c\,\sin(e+f\,x)\right)^{3/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x, x)
```

3.175 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=273

$$-\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x}+cf\cos(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{}$$

```
[Out] -c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x+c*f*Ci(2*f*x)*cos(2*e)*s
ec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*cos(e)*sec(f*x+
e)*Si(f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*Ci(f*x)*sec(f*
x+e)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*sec(f*x+e)*Si
(2*f*x)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*sec(f*
x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x
```

Rubi [A]
time = 0.47, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4700, 6873, 12, 6874, 3378, 3384, 3380, 3383}

$$-cf\sin(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+cf\cos(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-cf\sin(2e)\text{Si}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-cf\cos(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{x}-\frac{c\sin(2e+2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] -((c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x) + c*f*Cos[2*e]*C
osIntegral[2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]] - c*f*CosIntegral[f*x]*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*S
qrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(2*x) - c*f*Cos[e]*Sec[e + f*x]*Sqrt[a -
 a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[f*x] - c*f*Sec[e + f*
x]*Sin[2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2
*f*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)$$

$$= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right)$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right.$$

$$= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right.$$

$$= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right.$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+}{}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} - \frac{c\sec(e+}{}$$

$$= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{x} + cf\cos(2e$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 231, normalized size = 0.85

$$\frac{ce^{-i(e+fx)}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(-i-2e^{i(e+fx)}-2e^{3i(e+fx)}+ie^{4i(e+fx)}-2ie^{i(e+2fx)}fx\text{Ei}(-ifx)+2ie^{3ie+2ifx}fx\text{Ei}(ifx)+2e^{2ifx}fx\text{Ei}(-2ifx)+2e^{2i(2e+fx)}fx\text{Ei}(2ifx)\right)\sqrt{a-a\sin(e+fx)}}{2\sqrt{2}\left(1+e^{2i(e+fx)}\right)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

```
[Out] (c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(-I - 2*E^(I*(e +
 f*x)) - 2*E^((3*I)*(e + f*x)) + I*E^((4*I)*(e + f*x)) - (2*I)*E^(I*(e + 2*
f*x))*f*x*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f*x*ExpIn
tegralEi[I*f*x] + 2*E^((2*I)*f*x)*f*x*ExpIntegralEi[(-2*I)*f*x] + 2*E^((2*I
)*(2*e + f*x))*f*x*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e + f*x]])/(2*S
qrt[2]*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)))*x)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-a\sin(fx+e)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**2,x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(247) = 494.
time = 0.50, size = 608, normalized size = 2.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="g
iac")
```

```
[Out] -1/2*(pi*c*f^2*cos(2*e)*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*f^2*cos(2*e
)*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - 2*c*e*f^2*cos(2*e)*cos_integral(2*f*x)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - pi*c*f^2*cos_int
egral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*sin(e) + (pi - 2*f*x - 2*e)*c*f^2*cos_integral(f*x)*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e) + 2*c*e*f^2
*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(e) - pi*c*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e)*sin_integral(2*f*x) + (pi - 2*f*x -
 2*e)*c*f^2sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(2*e)*sin_integral(2*f*x) + 2*c*e*f^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e)*sin_integral(2*f*x)
 - pi*c*f^2*cos(e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin_integral(f*x) + (pi - 2*f*x - 2*e)*c*f^2*cos(e)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integr
al(f*x) + 2*c*e*f^2*cos(e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x) - 2*c*f^2*cos(f*x + e)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - c*f^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*f*
x + 2*e))*sqrt(a)*sqrt(c)/(f^2*x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\left(c+c\,\sin\left(e+f\,x\right)\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2,x)
```

```
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2, x)
```

3.176 $\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^3}\,dx$

Optimal. Leaf size=385

$$-\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2}-\frac{cf\cos(2e+2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e}{2x}$$

```
[Out] -1/2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*c*f^2*Ci(f*x)*
cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*f*cos
(2*f*x+2*e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-c*f^
2*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)+1/2*c*f^2*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+
e))^(1/2)-c*f^2*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)-1/4*c*sec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+
c*sin(f*x+e))^(1/2)/x^2+1/2*c*f*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/
2)*tan(f*x+e)/x
```

Rubi [A]
time = 0.50, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4700, 6873, 12, 6874, 3378, 3384, 3380, 3383}

$$-cf^2\sin(2e)\text{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\frac{1}{2}cf^2\cos(e)\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}+\frac{1}{2}cf^2\sin(e)\text{Si}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-cf^2\cos(2e)\text{Si}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}-\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x^2}-\frac{c\sin(2e+2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{4x^2}+\frac{cf\tan(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x}-\frac{cf\cos(2e+2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2 - (c*f*Cos[2
*e + 2*f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
/(2*x) - (c*f^2*Cos[e]*CosIntegral[f*x]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]])/2 - c*f^2*CosIntegral[2*f*x]*Sec[e + f*x]*Sin[
2*e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]] - (c*Sec[e + f*x]*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*Sin[2*e + 2*f*x])/(4*x^2) +
 (c*f^2*Sec[e + f*x]*Sin[e]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x
]]*SinIntegral[f*x])/2 - c*f^2*Cos[2*e]*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x
]]*Sqrt[c + c*Sin[e + f*x]]*SinIntegral[2*f*x] + (c*f*Sqrt[a - a*Sin[e + f*
x]]*Sqrt[c + c*Sin[e + f*x]]*Tan[e + f*x])/(2*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{3/2}}{x^3}\,dx &= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \int \\
&= \left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \int \\
&= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \\
&= \left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right) \\
&= \frac{1}{2}\left(c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}\right. \\
&= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{c\sec(e+ \ldots}{} \\
&= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e \ldots}{} \\
&= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e \ldots}{} \\
&= -\frac{c\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}{2x^2} - \frac{cf\cos(2e \ldots}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.09, size = 317, normalized size = 0.82

$$
\frac{c^2 e^{-2i(e+fx)}\left(i+e^{i(e+fx)}\right)\left(-1+2ie^{i(e+fx)}+2ie^{3i(e+fx)}+e^{4i(e+fx)}+2ifx+2e^{i(e+fx)}fx-2e^{3i(e+fx)}fx+2ie^{4i(e+fx)}fx+2ie^{i(e+2fx)}f^2x^2\mathrm{Ei}(-ifx)+2ie^{3ie+2ifx}f^2x^2\mathrm{Ei}(ifx)-4e^{2ifx}f^2x^2\mathrm{Ei}(-2ifx)+4e^{2i(2e+fx)}f^2x^2\mathrm{Ei}(2ifx)\right)\sqrt{a-a\sin(e+fx)}}{4\sqrt{2}\left(-i+e^{i(e+fx)}\right)\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\,x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

```
[Out] (c^2*(I + E^(I*(e + f*x)))*(-1 + (2*I)*E^(I*(e + f*x)) + (2*I)*E^((3*I)*(e
+ f*x)) + E^((4*I)*(e + f*x)) + (2*I)*f*x + 2*E^(I*(e + f*x))*f*x - 2*E^((3
*I)*(e + f*x))*f*x + (2*I)*E^((4*I)*(e + f*x))*f*x + (2*I)*E^(I*(e + 2*f*x)
)*f^2*x^2*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f^2*x^2*E
xpIntegralEi[I*f*x] - 4*E^((2*I)*f*x)*f^2*x^2*ExpIntegralEi[(-2*I)*f*x] + 4
*E^((2*I)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e +
 f*x]])/(4*Sqrt[2]*E^((2*I)*(e + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*
(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*x^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$
\int \frac{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a-a\sin\left(fx+e\right)}}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

```
[Out] int((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**3,x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(341) = 682$.
time = 0.60, size = 1502, normalized size = 3.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="g
iac")

[Out] 1/2*(pi^2*c*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*f^3*cos(e
)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*c*f^3*cos(e)*cos_integral(f*x)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*pi*
c*e*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*f^3*cos(e)*cos_int
egral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) + 4*c*e^2*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*pi^2*c*f^3*cos_integral(2*f
*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(2*e) - 4*pi*(pi - 2*f*x - 2*e)*c*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) + 2*(pi
 - 2*f*x - 2*e)^2*c*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) - 8*pi*c*e*f^3*cos_integr
al(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(2*e) + 8*(pi - 2*f*x - 2*e)*c*e*f^3*cos_integral(2*f*x)*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) +
8*c*e^2*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) + 2*pi^2*c*f^3*cos(2*e)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(2*f
*x) - 4*pi*(pi - 2*f*x - 2*e)*c*f^3*cos(2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(2*f*x) + 2*(pi - 2*f
*x - 2*e)^2*c*f^3*cos(2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))*sin_integral(2*f*x) - 8*pi*c*e*f^3*cos(2*e)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integr
al(2*f*x) + 8*(pi - 2*f*x - 2*e)*c*e*f^3*cos(2*e)*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(2*f*x) + 8*c*e^
2*f^3*cos(2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin_integral(2*f*x) - pi^2*c*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x) + 2*pi*(
pi - 2*f*x - 2*e)*c*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x) - (pi - 2*f*x - 2*e)^2*c*f^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin
(e)*sin_integral(f*x) + 4*pi*c*e*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x) - 4*(pi - 2*f*x
- 2*e)*c*e*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(e)*sin_integral(f*x) - 4*c*e^2*f^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x) +
2*pi*c*f^3*cos(2*f*x + 2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) - 2*(pi - 2*f*x - 2*e)*c*f^3*cos(2*f*x + 2*e)*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*c*e
```

```
*f^3*cos(2*f*x + 2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - 2*pi*c*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(f*x + e) + 2*(pi - 2*f*x - 2*e)*c*f^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(f*x
 + e) + 4*c*e*f^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(f*x + e) + 4*c*f^3*cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*c*f^3*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*f*x + 2*e))*sq
rt(a)*sqrt(c)/((pi^2 - 2*pi*(pi - 2*f*x - 2*e) + (pi - 2*f*x - 2*e)^2 - 4*p
i*e + 4*(pi - 2*f*x - 2*e)*e + 4*e^2)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3,x)
[Out] int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3, x)
```

3.177 $\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=767

$$-\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}-\frac{2ia(g+hx)^3\mathrm{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}+\frac{a(g+hx)^3\cos(}{f\sqrt{a-a\sin(e+}$$

```
[Out] -1/4*I*a*(h*x+g)^4*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)^3*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^3*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3*I*a*h*(h*x+g)^2*cos(f*x+e)*p
olylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-3*I*a*h*(h*x+g)^2*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*
x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3/2*I*a*h*(h*x+g)^2*cos(f*x+e)*polylog(2
,-exp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-6*a*h
^2*(h*x+g)*cos(f*x+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+6*a*h^2*(h*x+g)*cos(f*x+e)*polylog(3,I*exp(I*(f*x
+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3/2*a*h^2*(h*x+g)*c
os(f*x+e)*polylog(3,-exp(2*I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(
f*x+e))^(1/2)-6*I*a*h^3*cos(f*x+e)*polylog(4,-I*exp(I*(f*x+e)))/f^4/(a-a*si
n(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+6*I*a*h^3*cos(f*x+e)*polylog(4,I*exp
(I*(f*x+e)))/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3/4*I*a*h^3*
cos(f*x+e)*polylog(4,-exp(2*I*(f*x+e)))/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin
(f*x+e))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 767, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {4700, 6873, 12, 6874, 4266, 2611, 6744, 2320, 6724, 3800, 2221}

$$-\frac{2ia(g+hx)^3\mathrm{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{6iah^3\mathrm{Li}_4\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{6iah^3\mathrm{Li}_4\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{3iah^3\mathrm{Li}_4\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{4f^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{6ah^2(g+hx)\mathrm{Li}_3\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{6ah^2(g+hx)\mathrm{Li}_3\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{3ah^2(g+hx)\mathrm{Li}_3\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{2f^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{3iah(g+hx)^2\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{3iah(g+hx)^2\mathrm{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{3iah(g+hx)^2\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{2f^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{a(g+hx)^3\log\left(1+e^{2i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-1/4*I)*a*(g + h*x)^4*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
c*Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^3*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x
])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^3*C
os[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[
c + c*Sin[e + f*x]]) + ((3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, (-I)*
E^(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) -
 ((3*I)*a*h*(g + h*x)^2*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sq
rt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (((3*I)/2)*a*h*(g + h*x)
```

```
^2*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f
*x]]*Sqrt[c + c*Sin[e + f*x]]) - (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3,
 (-I)*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]]) + (6*a*h^2*(g + h*x)*Cos[e + f*x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*
Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*h^2*(g + h*x)*Cos
[e + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3*Sqrt[a - a*Sin[e + f*x]]
*Sqrt[c + c*Sin[e + f*x]]) - ((6*I)*a*h^3*Cos[e + f*x]*PolyLog[4, (-I)*E^(I
*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + ((6
*I)*a*h^3*Cos[e + f*x]*PolyLog[4, I*E^(I*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e
 + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (((3*I)/4)*a*h^3*Cos[e + f*x]*PolyLog[
4, -E^((2*I)*(e + f*x))])/(f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e +
f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int(g+hx)^3\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{\cos(e+fx)\int a(g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx))\int(g+hx)^3\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx))\int\left((g+hx)^3\sec(e+fx)-(g+hx)^3\tan(e+fx)\right)\,c}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\
&= \frac{(a\cos(e+fx))\int(g+hx)^3\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int(g+}{\sqrt{a-a\sin(e+fx)}\,\cdot} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}} \\
&= -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\tan^{-}}{f\sqrt{a-a\sin(e+f}}
\end{aligned}
$$

Mathematica [A]
time = 1.53, size = 247, normalized size = 0.32

$$
\frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\left(\frac{(g+hx)^4}{h}-\frac{8i(g+hx)^3\log\left(1+ie^{-i(e+fx)}\right)}{f}+\frac{24h\left(f^2(g+hx)^2\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)-2h\left(if(g+hx)\text{PolyLog}\left(3,-ie^{-i(e+fx)}\right)+h\text{PolyLog}\left(4,-ie^{-i(e+fx)}\right)\right)\right)}{f^4}\right)\sqrt{a-a\sin(e+fx)}}{\sqrt{2}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]

[Out] ((1/4 + I/4)*(I + E^(I*(e + f*x)))*((g + h*x)^4/h - ((8*I)*(g + h*x)^3*Log[
1 + I/E^(I*(e + f*x))])/f + (24*h*(f^2*(g + h*x)^2*PolyLog[2, (-I)/E^(I*(e
```

```
+ f*x))] - 2*h*(I*f*(g + h*x)*PolyLog[3, (-I)/E^(I*(e + f*x))] + h*PolyLog[
4, (-I)/E^(I*(e + f*x))])))/f^4)*Sqrt[a - a*Sin[e + f*x]])/(Sqrt[2]*E^((I/2
)*(e + f*x))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)^3 \sqrt{a-a\sin(fx+e)}}{\sqrt{c+c\sin(fx+e)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e+fx)-1)}\,(g+hx)^3}{\sqrt{c(\sin(e+fx)+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**3/sqrt(c*(sin(e + f*x) + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{c+c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

3.178 $\int \frac{(g+hx)^2 \sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=555

$$-\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}-\frac{2ia(g+hx)^2\operatorname{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}+\frac{a(g+hx)^2\cos(e}{f\sqrt{a-a\sin(e+}$$

```
[Out] -1/3*I*a*(h*x+g)^3*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)^2*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^2*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*I*a*h*(h*x+g)*cos(f*x+e)*pol
ylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
-2*I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e)
)^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,-exp(2*I*
(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-2*a*h^2*cos(f*x
+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)
)^(1/2)+2*a*h^2*cos(f*x+e)*polylog(3,I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))
^(1/2)/(c+c*sin(f*x+e))^(1/2)+1/2*a*h^2*cos(f*x+e)*polylog(3,-exp(2*I*(f*x+
e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {4700, 6873, 12, 6874, 4266, 2611, 2320, 6724, 3800, 2221}

$$-\frac{2ia(g+hx)^2\operatorname{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2ah^2\operatorname{Li}_3\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2ah^2\operatorname{Li}_3\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{ah^2\operatorname{Li}_3\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{2f^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2iah(g+hx)\operatorname{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2iah(g+hx)\operatorname{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah(g+hx)\operatorname{Li}_2\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{a(g+hx)^2\log\left(1+e^{2i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-1/3*I)*a*(g + h*x)^3*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
c*Sin[e + f*x]]) - ((2*I)*a*(g + h*x)^2*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x
])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)^2*C
os[e + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[
c + c*Sin[e + f*x]]) + ((2*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, (-I)*E^
(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (
(2*I)*a*h*(g + h*x)*Cos[e + f*x]*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a
 - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*(g + h*x)*Cos[e + f*x
]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
c*Sin[e + f*x]]) - (2*a*h^2*Cos[e + f*x]*PolyLog[3, (-I)*E^(I*(e + f*x))])/
(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*h^2*Cos[e +
f*x]*PolyLog[3, I*E^(I*(e + f*x))])/(f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
```

```
c*Sin[e + f*x]]) + (a*h^2*Cos[e + f*x]*PolyLog[3, -E^((2*I)*(e + f*x))])/(2
*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int (g+hx)^2\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a(g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^2\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int \left((g+hx)^2\sec(e+fx)-(g+hx)^2\tan(e+fx)\right)\,d}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (g+hx)^2\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int (g+}{\sqrt{a-a\sin(e+fx)}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^-}{f\sqrt{a-a\sin(e+f}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^-}{f\sqrt{a-a\sin(e+f}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^-}{f\sqrt{a-a\sin(e+f}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^-}{f\sqrt{a-a\sin(e+f}}$$

$$= -\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^2\tan^-}{f\sqrt{a-a\sin(e+f}}$$

<u>Mathematica</u> [A]
time = 1.09, size = 194, normalized size = 0.35

$$\frac{\sqrt{2}\left(i+e^{i(e+fx)}\right)\left(f^2(g+hx)^2\left(f(g+hx)-6ih\log\left(1+ie^{-i(e+fx)}\right)\right)+12fh^2(g+hx)\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)-12ih^3\text{PolyLog}\left(3,-ie^{-i(e+fx)}\right)\right)\sqrt{a-a\sin(e+fx)}}{3\left(-i+e^{i(e+fx)}\right)\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\,f^3h}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x
]
```

```
[Out] (Sqrt[2]*(I + E^(I*(e + f*x)))*(f^2*(g + h*x)^2*(f*(g + h*x) - (6*I)*h*Log[
1 + I/E^(I*(e + f*x))]) + 12*f*h^2*(g + h*x)*PolyLog[2, (-I)/E^(I*(e + f*x)
)] - (12*I)*h^3*PolyLog[3, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*x]])
/(3*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e +
f*x))]*f^3*h)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)^2 \sqrt{a-a\sin(fx+e)}}{\sqrt{c+c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e+fx)-1)}\,(g+hx)^2}{\sqrt{c(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**2/sqrt(c*(sin(e + f*x) + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)^2\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

3.179 $\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=355

$$-\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}-\frac{2ia(g+hx)\text{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}+\frac{a(g+hx)\cos(e}{f\sqrt{a-a\sin(e+}}$$

```
[Out] -1/2*I*a*(h*x+g)^2*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-2*I*a*(h*x+g)*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/2)
/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(a-a*
sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+I*a*h*cos(f*x+e)*polylog(2,-I*exp(
I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*cos(f*x
+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))
^(1/2)-1/2*I*a*h*cos(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e
))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4700, 6873, 12, 6874, 4266, 2317, 2438, 3800, 2221}

$$-\frac{2ia(g+hx)\text{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{iah\text{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\text{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{iah\text{Li}_2\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{a(g+hx)\log\left(1+e^{2i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((-1/2*I)*a*(g + h*x)^2*Cos[e + f*x])/(h*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
c*Sin[e + f*x]]) - ((2*I)*a*(g + h*x)*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])
/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*(g + h*x)*Cos[e
 + f*x]*Log[1 + E^((2*I)*(e + f*x))])/(f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
c*Sin[e + f*x]]) + (I*a*h*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(f
^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (I*a*h*Cos[e + f*x]
*PolyLog[2, I*E^(I*(e + f*x))])/(f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Si
n[e + f*x]]) - ((I/2)*a*h*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(f
^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)\int(g+hx)\sec(e+fx)(a-a\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int a(g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int(g+hx)\sec(e+fx)(1-\sin(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int((g+hx)\sec(e+fx)-(g+hx)\tan(e+fx))\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int(g+hx)\sec(e+fx)\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{(a\cos(e+fx))\int(g+hx)}{\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}(}{f\sqrt{a-a\sin(e+fx)}}$$
$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}(}{f\sqrt{a-a\sin(e+fx)}}$$
$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}(}{f\sqrt{a-a\sin(e+fx)}}$$
$$= -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)\tan^{-1}(}{f\sqrt{a-a\sin(e+fx)}}$$

Mathematica [A]
time = 0.80, size = 154, normalized size = 0.43

$$\frac{\left(i+e^{i(e+fx)}\right)\left(f\left(fx(2g+hx)-4i(g+hx)\log\left(1+ie^{-i(e+fx)}\right)\right)+4h\text{PolyLog}\left(2,-ie^{-i(e+fx)}\right)\right)\sqrt{a-a\sin(e+fx)}}{\sqrt{2}\left(-i+e^{i(e+fx)}\right)\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

```
[Out] ((I + E^(I*(e + f*x)))*(f*(f*x*(2*g + h*x) - (4*I)*(g + h*x)*Log[1 + I/E^(I
*(e + f*x))]) + 4*h*PolyLog[2, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*
x]])/(Sqrt[2]*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/
E^(I*(e + f*x))]*f^2)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(hx+g)\sqrt{a-a\sin(fx+e)}}{\sqrt{c+c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a(\sin(e+fx)-1)}\,(g+hx)}{\sqrt{c(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)/sqrt(c*(sin(e + f*x) + 1)),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g+h\,x)\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

3.180 $\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$

Optimal. Leaf size=110

$$\frac{a\cos(e+fx)\text{Int}\left(\frac{\sec(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}-\frac{a\cos(e+fx)\text{Int}\left(\frac{\tan(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

```
[Out] a*cos(f*x+e)*Unintegrable(sec(f*x+e)/(h*x+g),x)/(a-a*sin(f*x+e))^(1/2)/(c+c
*sin(f*x+e))^(1/2)-a*cos(f*x+e)*Unintegrable(tan(f*x+e)/(h*x+g),x)/(a-a*sin
(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

```
[Out] (a*Cos[e + f*x]*Defer[Int][Sec[e + f*x]/(g + h*x), x])/(Sqrt[a - a*Sin[e +
f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Defer[Int][Tan[e + f*x]/(
g + h*x), x])/(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx &= \frac{\cos(e+fx)\int \frac{\sec(e+fx)(a-a\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\int \frac{a\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)(1-\sin(e+fx))}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{(a\cos(e+fx))\int \left(\frac{\sec(e+fx)}{g+hx}-\frac{\tan(e+fx)}{g+hx}\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&= \frac{(a\cos(e+fx))\int \frac{\sec(e+fx)}{g+hx}\,dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}-\frac{(a\cos(e+fx))\int \frac{\text{t}}{}}{\sqrt{a-a\sin(e+fx)}\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 2.52, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a\sin(e + fx)}}{(g + hx)\sqrt{c + c\sin(e + fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

```
[Out] Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - a\sin\left(fx + e\right)}}{\left(hx + g\right)\sqrt{c + c\sin\left(fx + e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\,\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**(1/2)/(h*x+g)/(c+c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1))/(sqrt(c*(sin(e + f*x) + 1))*(g + h*x))
, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)), x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\left(g+h\,x\right)\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)), x)
```

3.181 $\int \frac{x^3 \sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=536

$$-\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}-\frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}-\frac{12iax\mathrm{Ar}}{cf^3\sqrt{a-a\,\mathrm{si}}}$$

```
[Out] -3*a*x^2/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*I*a*x^2*cos(
f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-12*I*a*x*arctan(
exp(I*(f*x+e)))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1
/2)+6*a*x*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c
+c*sin(f*x+e))^(1/2)+6*I*a*cos(f*x+e)*polylog(2,-I*exp(I*(f*x+e)))/c/f^4/(a
-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-6*I*a*cos(f*x+e)*polylog(2,I*ex
p(I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*I*a*cos
(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(
f*x+e))^(1/2)-a*x^3*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^
(1/2)+3*a*x^2*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2
)+a*x^3*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 2.58, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {4700, 6873, 12, 6874, 4271, 4266, 2317, 2438, 2611, 6744, 2320, 6724, 3842, 4269, 3800, 2221, 4498}

$$-\frac{12iax\,\mathrm{ArcTan}\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{6ia\,\mathrm{Li}_2\left(-ie^{i(e+fx)}\right)\cos(e+fx)}{cf^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{6ia\,\mathrm{Li}_2\left(ie^{i(e+fx)}\right)\cos(e+fx)}{cf^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{3ia\,\mathrm{Li}_2\left(-e^{2i(e+fx)}\right)\cos(e+fx)}{cf^4\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{6ax\log\left(1+e^{2i(e+fx)}\right)\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{3ax^2\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{ax^3\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{ax^3\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a*x^2)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((3*
I)*a*x^2*Cos[e + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f
*x]]) - ((12*I)*a*x*ArcTan[E^(I*(e + f*x))]*Cos[e + f*x])/(c*f^3*Sqrt[a - a
*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (6*a*x*Cos[e + f*x]*Log[1 + E^((
2*I)*(e + f*x))])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
 + ((6*I)*a*Cos[e + f*x]*PolyLog[2, (-I)*E^(I*(e + f*x))])/(c*f^4*Sqrt[a -
a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - ((6*I)*a*Cos[e + f*x]*PolyLog[2
, I*E^(I*(e + f*x))])/(c*f^4*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*
x]]) - ((3*I)*a*Cos[e + f*x]*PolyLog[2, -E^((2*I)*(e + f*x))])/(c*f^4*Sqrt[
a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^3*Sec[e + f*x])/(c*f*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (3*a*x^2*Sin[e + f*x])/
(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^3*Tan[e +
f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
```

```
GeQ[n - m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^3\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\cos(e+fx)\int a^2x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int (x^3\sec^3(e+fx)-2x^3\sec^2(e+fx)\tan(e+fx)+x^3\sec(e}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{(a\cos(e+fx))\int x^3\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^3\sec(e+fx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+}}$$

$$= -\frac{3ax^2}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax^3\sec(e+}{cf\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{6iax\tan^{-1}\left(e^{i(e+fx)}\right)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iax^2\cos(e+}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

Mathematica [A]
time = 1.41, size = 193, normalized size = 0.36

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a-a\sin(e+fx)}\left(12i\text{PolyLog}\left(2,ie^{i(e+fx)}\right)(1+\sin(e+fx))+fx\left(3ifx+f^2x^2+3fx\cos(e+fx)-12\log\left(1-ie^{i(e+fx)}\right)+3i\left(fx+4i\log\left(1-ie^{i(e+fx)}\right)\right)\sin(e+fx)\right)\right)}{f^4\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((12*I)*P
olyLog[2, I*E^(I*(e + f*x))]*(1 + Sin[e + f*x]) + f*x*((3*I)*f*x + f^2*x^2
```

```
+ 3*f*x*Cos[e + f*x] - 12*Log[1 - I*E^(I*(e + f*x))] + (3*I)*(f*x + (4*I)*L
og[1 - I*E^(I*(e + f*x))])*Sin[e + f*x])))/(f^4*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{a-a \sin \left(f x+e\right)}}{\left(c+c \sin \left(f x+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3/(c^2*cos(f
*x + e)^2 - 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{- a \left(\sin \left(e+f x\right)-1\right)}}{\left(c \left(\sin \left(e+f x\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

3.182 $\int \frac{x^2 \sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=280

$$-\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}+\frac{2a\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}+\frac{2a\cos}{cf^3\sqrt{a-a}}$$

```
[Out] -2*a*x/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*arctanh(sin(
f*x+e))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*
cos(f*x+e)*ln(cos(f*x+e))/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-a*x^2*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*x
*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x^2*tan(f
*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.64, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {4700, 6873, 12, 6874, 4271, 3855, 4266, 2611, 2320, 6724, 3842, 4269, 3556, 4498}

$$\frac{2a\cos(e+fx)\log(\cos(e+fx))}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2a\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{2ax\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}+\frac{ax^2\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{ax^2\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*x)/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*A
rcTanh[Sin[e + f*x]]*Cos[e + f*x])/(c*f^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c +
 c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Log[Cos[e + f*x]])/(c*f^3*Sqrt[a - a*
Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) - (a*x^2*Sec[e + f*x])/(c*f*Sqrt[a
- a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (2*a*x*Sin[e + f*x])/(c*f^2*S
qrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x^2*Tan[e + f*x])/(c
*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
```

```
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x^2\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int a^2x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int (x^2\sec^3(e+fx)-2x^2\sec^2(e+fx)\tan(e+fx)+x^2\sec(e}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x^2\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x^2\sec(e+fx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+}}$$
$$= -\frac{ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax^2\sec(e+}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax^2\tan^{-1}\left(e^{i(e+fx)}\right)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$
$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$
$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$
$$= -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{2a\tanh^{-1}(\sin(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.03, size = 154, normalized size = 0.55

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a-a\sin(e+fx)}\left(2ifx+f^2x^2+2fx\cos(e+fx)-4\log\left(i+e^{i(e+fx)}\right)+\left(2ifx-4\log\left(i+e^{i(e+fx)}\right)\right)\sin(e+fx)\right)}{f^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((2*I)*f*
x + f^2*x^2 + 2*f*x*Cos[e + f*x] - 4*Log[I + E^(I*(e + f*x))] + ((2*I)*f*x
- 4*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f^3*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{a - a \sin \left(f x + e\right)}}{\left(c + c \sin \left(f x + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="f
ricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{- a \left(\sin \left(e + f x\right) - 1\right)}}{\left(c \left(\sin \left(e + f x\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**2*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1577 vs. $2(256) = 512$.
time = 0.70, size = 1577, normalized size = 5.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/16*sqrt(2)*(4*pi*c^(3/2)*e*(3*sqrt(2) + 4)*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 4*(pi - 2*f*x - 2*e)*c^(3/2)*e*(3*
sqrt(2) + 4)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/
4*e)^4 + sqrt(2)*pi^2*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*
pi + 1/4*f*x + 1/4*e)^4 - 2*sqrt(2)*pi*(pi - 2*f*x - 2*e)*c^(3/2)*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + sqrt(2)*(pi -
 2*f*x - 2*e)^2*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1
/4*f*x + 1/4*e)^4 + 4*sqrt(2)*c^(3/2)*e^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 16*sqrt(2)*c^(3/2)*log(4*(tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^4 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)/(tan(-1/8*p
i + 1/4*f*x + 1/4*e)^4 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 8*pi*c^(3/2)*
e*(3*sqrt(2) + 4)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^2 - 8*(pi - 2*f*x - 2*e)*c^(3/2)*e*(3*sqrt(2) + 4)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 16*pi*c^(3/2)*(sqr
t(2) + 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e
)^3 + 32*(pi - 2*f*x - 2*e)*c^(3/2)*(sqrt(2) + 2)*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 32*c^(3/2)*e*(sqrt(2) + 2)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + 16*(pi
 - 2*f*x - 2*e)*sqrt(c)*(3*sqrt(2) + 4)*abs(c)*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 32*pi*sqrt(c)*(sqrt(2) + 1)*abs(
c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - 6
4*sqrt(c)*e*(sqrt(2) + 1)*abs(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1
/8*pi + 1/4*f*x + 1/4*e)^3 + 2*sqrt(2)*pi^2*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 4*sqrt(2)*pi*(pi - 2*f*x -
2*e)*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/
4*e)^2 + 2*sqrt(2)*(pi - 2*f*x - 2*e)^2*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 8*sqrt(2)*c^(3/2)*e^2*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 32*sqrt(2)*c^
(3/2)*log(4*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 2*tan(-1/8*pi + 1/4*f*x + 1
/4*e)^2 + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 2*tan(-1/8*pi + 1/4*f*x +
1/4*e)^2 + 1))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x +
1/4*e)^2 + 4*pi*c^(3/2)*e*(3*sqrt(2) + 4)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) - 4*(pi - 2*f*x - 2*e)*c^(3/2)*e*(3*sqrt(2) + 4)*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + 16*pi*c^(3/2)*(sqrt(2) + 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
```

```
e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) - 32*(pi - 2*f*x - 2*e)*c^(3/2)*(sqrt(2)
 + 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) -
32*c^(3/2)*e*(sqrt(2) + 2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
+ 1/4*f*x + 1/4*e) - 16*(pi - 2*f*x - 2*e)*sqrt(c)*(3*sqrt(2) + 4)*abs(c)*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 32*pi*s
qrt(c)*(sqrt(2) + 1)*abs(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e) + 64*sqrt(c)*e*(sqrt(2) + 1)*abs(c)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) + sqrt(2)*pi^2*c^(3/2)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(2)*pi*(pi - 2*f*x - 2*e)*c^(3/2)*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + sqrt(2)*(pi - 2*f*x - 2*e)^2*c^(3/2)*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*sqrt(2)*c^(3/2)*e^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - 16*sqrt(2)*c^(3/2)*log(4*(tan(-1/8*pi + 1/4*f*x + 1/4
*e)^4 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1)/(tan(-1/8*pi + 1/4*f*x + 1/
4*e)^4 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)))*sqrt(a)/((c^3*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*
pi + 1/4*f*x + 1/4*e)^4 - 2*c^3*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan
(-1/8*pi + 1/4*f*x + 1/4*e)^2 + c^3*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{a-a\,\sin\left(e+f\,x\right)}}{{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
[Out] int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

3.183 $\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}-\frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}+\frac{\quad}{cf^2\sqrt{a-a\sin}}$$

```
[Out] -a/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-a*x*sec(f*x+e)/c/f/(
a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*sin(f*x+e)/c/f^2/(a-a*sin(f*
x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2
)/(c+c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {4700, 6873, 12, 6874, 4270, 4266, 2317, 2438, 3842, 3852, 8, 4498}

$$\frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}+\frac{ax\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}-\frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\,\sqrt{c\sin(e+fx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])) - (a*x*Sec[e
 + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*Sin[e
 + f*x])/(c*f^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]) + (a*x*T
an[e + f*x])/(c*f*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4498

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4700

```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
+ d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
```

```
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\cos(e+fx)\int x\sec^3(e+fx)(a-a\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{\cos(e+fx)\int a^2x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)(1-\sin(e+fx))^2\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int \left(x\sec^3(e+fx)-2x\sec^2(e+fx)\tan(e+fx)+x\sec(e+\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$
$$= \frac{(a\cos(e+fx))\int x\sec^3(e+fx)\,dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{(a\cos(e+fx))\int x\sec(e+fx)}{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c}}$$
$$= -\frac{a}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{iax\tan^{-1}\left(e^{i(e+fx)}\right)\text{co}}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+f}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$
$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ia\cos(e+fx)\text{Li}_2(-}{2cf^2\sqrt{a-a\sin(e+fx)}\sqrt{}}$$
$$= -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+f}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A]

time = 0.35, size = 150, normalized size = 0.88

$$-\frac{\left(\left(-1+fx\right)\cos\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}+fx\right)+\sin\left(\frac{e}{2}\right)+fx\sin\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}+fx\right)\right)\sqrt{c(1+\sin(e+fx))}\,\sqrt{a-a\sin(e+fx)}}{c^2f^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((((-1 + f*x)*Cos[e/2] + Cos[e/2 + f*x] + Sin[e/2] + f*x*Sin[e/2] - Sin[e/
2 + f*x])*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]])/(c^2*f^2*(Co
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a-a\sin\left(fx+e\right)}}{\left(c+c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

Fricas [A]
time = 1.36, size = 78, normalized size = 0.46

$$-\frac{\left(fx+\cos\left(fx+e\right)\right)\sqrt{-a\sin\left(fx+e\right)+a}\,\sqrt{c\sin\left(fx+e\right)+c}}{c^2f^2\cos\left(fx+e\right)\sin\left(fx+e\right)+c^2f^2\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fri
cas")
```

```
[Out] -(f*x + cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/(c
^2*f^2*cos(f*x + e)*sin(f*x + e) + c^2*f^2*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(x*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(155) = 310.
time = 0.50, size = 446, normalized size = 2.61

$$\frac{\sqrt{2}\left(\pi\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}-\left(\pi-2fx-2e\right)\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}-2\sqrt{c}\,e\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}+2\pi\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-2\left(\pi-2fx-2e\right)\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}-4\sqrt{c}\,e\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+8\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{3}+\pi\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\left(\pi-2fx-2e\right)\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-2\sqrt{c}\,e\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-8\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)\right)\sqrt{a}}{4\left(\sqrt{2}\,c^{2}f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{4}-2\sqrt{2}\,c^{2}f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^{2}+\sqrt{2}\,c^{2}f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="gia
c")
```

```
[Out] 1/4*sqrt(2)*(pi*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1
/4*f*x + 1/4*e)^4 - (pi - 2*f*x - 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 2*sqrt(c)*e*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 2*pi*sqrt(c)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*(pi - 2*f*x -
 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1
/4*e)^2 - 4*sqrt(c)*e*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4
*f*x + 1/4*e)^2 + 8*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^3 + pi*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (p
i - 2*f*x - 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sqrt(c)*e*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e))*sqrt(a)/((sqrt(2)*c^2*f*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 2*sqrt(2)*c^2*
f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + sq
rt(2)*c^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*f)
```

Mupad [B]
time = 3.32, size = 88, normalized size = 0.51

$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(2\,e+2\,f\,x\right)+2\,f\,x\,\cos\left(e+f\,x\right)+1-\cos\left(e+f\,x\right)\,2{}\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)}{c\,f^2\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(cos(2*e + 2*f*x) - cos(e + f*x)*2i - sin(2
*e + 2*f*x)*1i + 2*f*x*cos(e + f*x) + 1))/(c*f^2*(cos(2*e + 2*f*x) + 1)*(c*
(sin(e + f*x) + 1))^(1/2))
```

3.184 $\int \frac{z^2 \sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz$

Optimal. Leaf size=300

$$-\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}+\frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{1+\cos(z)}}+\frac{2iz\text{PolyLog}(2,-}{\sqrt{1-\cos(z)}\,\sqrt{}}$$

```
[Out] -1/3*I*z^3*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)-2*z^2*arctanh(exp(I*z))
*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+z^2*ln(1-exp(2*I*z))*sin(z)/(1-co
s(z))^(1/2)/(1+cos(z))^(1/2)+2*I*z*polylog(2,-exp(I*z))*sin(z)/(1-cos(z))^(
1/2)/(1+cos(z))^(1/2)-2*I*z*polylog(2,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+
cos(z))^(1/2)-I*z*polylog(2,exp(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^
(1/2)-2*polylog(3,-exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+2*pol
ylog(3,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+1/2*polylog(3,exp
(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4701, 6874, 3798, 2221, 2611, 2320, 6724, 4268}

$$\frac{2iz\text{Li}_2\left(-e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}-\frac{2iz\text{Li}_2\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}-\frac{iz\text{Li}_2\left(e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}-\frac{2\text{Li}_3\left(-e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}+\frac{2\text{Li}_3\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}+\frac{\text{Li}_3\left(e^{2iz}\right)\sin(z)}{2\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}-\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}+\frac{z^2\log\left(1-e^{2iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}-\frac{2z^2\sin(z)\tanh^{-1}\left(e^{iz}\right)}{\sqrt{1-\cos(z)}\,\sqrt{\cos(z)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] ((-1/3*I)*z^3*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*z^2*ArcTanh[
E^(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (z^2*Log[1 - E^((2*I
)*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + ((2*I)*z*PolyLog[2, -E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - ((2*I)*z*PolyLog[2, E^
(I*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (I*z*PolyLog[2, E^((2*
I)*z)]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) - (2*PolyLog[3, -E^(I*z)
]*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (2*PolyLog[3, E^(I*z)]*Sin[
z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]]) + (PolyLog[3, E^((2*I)*z)]*Sin[z])/
(2*Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4701

```
Int[(Cos[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(Cos[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_)*((g_.) + (h_.)*(x_))^(p_.), x_Symbol] :> Dist[a^IntPart[m]
*c^IntPart[m]*(a + b*Cos[e + f*x])^FracPart[m]*((c + d*Cos[e + f*x])^FracPa
rt[m]/Sin[e + f*x]^(2*FracPart[m])), Int[(g + h*x)^p*Sin[e + f*x]^(2*m)*(c
+ d*Cos[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] && I
GeQ[n - m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz &= \frac{\sin(z)\int z^2(1+\cos(z))\csc(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}\\
&= \frac{\sin(z)\int \left(z^2\cot(z)+z^2\csc(z)\right)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}\\
&= \frac{\sin(z)\int z^2\cot(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}+\frac{\sin(z)\int z^2\csc(z)\,dz}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}\\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{(2i\sin(z))\int \frac{e^{2iz}}{1-e \ldots}}{\sqrt{1-\cos(z)}\sqrt{1- \ldots}}\\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}+\frac{z^2\log\left(1-e^{2iz}\right) \ldots}{\sqrt{1-\cos(z)}\sqrt{1- \ldots}}\\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}+\frac{z^2\log\left(1-e^{2iz}\right) \ldots}{\sqrt{1-\cos(z)}\sqrt{1- \ldots}}\\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}+\frac{z^2\log\left(1-e^{2iz}\right) \ldots}{\sqrt{1-\cos(z)}\sqrt{1- \ldots}}\\
&= -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}-\frac{2z^2\tanh^{-1}\left(e^{iz}\right)\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}+\frac{z^2\log\left(1-e^{2iz}\right) \ldots}{\sqrt{1-\cos(z)}\sqrt{1- \ldots}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 85, normalized size = 0.28

$$\frac{\sqrt{1+\cos(z)}\left(-i\pi^3+iz^3+6z^2\log\left(1-e^{-iz}\right)+12iz\text{PolyLog}\left(2,e^{-iz}\right)+12\text{PolyLog}\left(3,e^{-iz}\right)\right)\tan\left(\frac{z}{2}\right)}{3\sqrt{1-\cos(z)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

```
[Out] (Sqrt[1 + Cos[z]]*((-I)*Pi^3 + I*z^3 + 6*z^2*Log[1 - E^((-I)*z)] + (12*I)*z
*PolyLog[2, E^((-I)*z)] + 12*PolyLog[3, E^((-I)*z)])*Tan[z/2])/(3*Sqrt[1 -
Cos[z]])
```

Maple [A]
time = 0.14, size = 154, normalized size = 0.51

method	result
risch	$\frac{(e^{iz}-1)\sqrt{(e^{iz}+1)^2 e^{-iz}}\, z^3}{3\sqrt{-(e^{iz}-1)^2 e^{-iz}}\,(e^{iz}+1)} + \frac{2i(e^{iz}-1)\sqrt{(e^{iz}+1)^2 e^{-iz}}\,\left(\frac{iz^3}{3}-z^2\ln(1-e^{iz})+2iz\,\text{polylog}(2,e^{iz})-2\,\text{polylog}(3,e^{iz}\right.}{\sqrt{-(e^{iz}-1)^2 e^{-iz}}\,(e^{iz}+1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*((exp(I*z)+1)^2*exp(-I*z
))^(1/2)/(exp(I*z)+1)*z^3+2*I/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1
)*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)*(1/3*I*z^3-z^2*ln(1-exp(I*z
))+2*I*z*polylog(2,exp(I*z))-2*polylog(3,exp(I*z)))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 0.19

$$\frac{1}{3}i\,z^3 + 2i\,z^2\arctan\left(\sin\left(z\right), -\cos\left(z\right)+1\right) - z^2\log\left(\cos\left(z\right)^2+\sin\left(z\right)^2-2\,\cos\left(z\right)+1\right) + 4i\,z\mathrm{Li}_2\left(e^{(i\,z)}\right) - 4\,\mathrm{Li}_3\left(e^{(i\,z)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="maxima")
```

```
[Out] 1/3*I*z^3 + 2*I*z^2*arctan2(sin(z), -cos(z) + 1) - z^2*log(cos(z)^2 + sin(z
)^2 - 2*cos(z) + 1) + 4*I*z*dilog(e^(I*z)) - 4*polylog(3, e^(I*z))
```

Fricas [A]
time = 1.38, size = 75, normalized size = 0.25

$$z^2\log\left(-\cos\left(z\right)+i\,\sin\left(z\right)+1\right) + z^2\log\left(-\cos\left(z\right)-i\,\sin\left(z\right)+1\right) - 2i\,z\mathrm{Li}_2(\cos\left(z\right)+i\,\sin\left(z\right)) + 2i\,z\mathrm{Li}_2(\cos\left(z\right)-i\,\sin\left(z\right)) + 2\,\text{polylog}(3,\cos\left(z\right)+i\,\sin\left(z\right)) + 2\,\text{polylog}(3,\cos\left(z\right)-i\,\sin\left(z\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="fricas")
```

```
[Out] z^2*log(-cos(z) + I*sin(z) + 1) + z^2*log(-cos(z) - I*sin(z) + 1) - 2*I*z*d
ilog(cos(z) + I*sin(z)) + 2*I*z*dilog(cos(z) - I*sin(z)) + 2*polylog(3, cos
(z) + I*sin(z)) + 2*polylog(3, cos(z) - I*sin(z))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{z^2\sqrt{\cos\left(z\right)+1}}{\sqrt{1-\cos\left(z\right)}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**2*(1+cos(z))**(1/2)/(1-cos(z))**(1/2),z)
[Out] Integral(z**2*sqrt(cos(z) + 1)/sqrt(1 - cos(z)), z)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="giac")
[Out] integrate(z^2*sqrt(cos(z) + 1)/sqrt(-cos(z) + 1), z)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{z^2\,\sqrt{\cos\left(z\right)+1}}{\sqrt{1-\cos\left(z\right)}}\,dz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2),z)
[Out] int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2), z)
```

3.185 $\int (a + a\cos(x))(A + B\sec(x))\,dx$

Optimal. Leaf size=18

$$a(A+B)x + aB\tanh^{-1}(\sin(x)) + aA\sin(x)$$

```
[Out] a*(A+B)*x+a*B*arctanh(sin(x))+a*A*sin(x)
```

Rubi [A]
time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2907, 3047, 3102, 2814, 3855}

$$ax(A+B) + aA\sin(x) + aB\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

```
[Out] a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&& !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(x))(A+B\sec(x))\,dx &= \int (a+a\cos(x))(B+A\cos(x))\sec(x)\,dx\\ &= \int \left(aB+(aA+aB)\cos(x)+aA\cos^2(x)\right)\sec(x)\,dx\\ &= aA\sin(x)+\int (aB+a(A+B)\cos(x))\sec(x)\,dx\\ &= a(A+B)x+aA\sin(x)+(aB)\int \sec(x)\,dx\\ &= a(A+B)x+aB\tanh^{-1}(\sin(x))+aA\sin(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.
time = 0.02, size = 51, normalized size = 2.83

$$aAx+aBx-aB\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+aB\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+aA\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])*(A + B*Sec[x]),x]
[Out] a*A*x + a*B*x - a*B*Log[Cos[x/2] - Sin[x/2]] + a*B*Log[Cos[x/2] + Sin[x/2]]
 + a*A*Sin[x]
```

Maple [A]
time = 0.12, size = 24, normalized size = 1.33

method	result	size
default	$aAx+Ba\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+aA\sin\left(x\right)+Bax$	24
risch	$aAx+Bax-\frac{iaA\,e^{ix}}{2}+\frac{iaA\,e^{-ix}}{2}+Ba\ln\left(e^{ix}+i\right)-Ba\ln\left(e^{ix}-i\right)$	55
norman	$\frac{(aA+Ba)x+(aA+Ba)x\left(\tan^2\left(\frac{x}{2}\right)\right)+2aA\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+Ba\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-Ba\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*A*x+B*a*ln(sec(x)+tan(x))+a*A*sin(x)+B*a*x
```

Maxima [**A**]
time = 0.26, size = 23, normalized size = 1.28

$$Aax + Bax + Ba\log\left(\sec\left(x\right) + \tan\left(x\right)\right) + Aa\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] A*a*x + B*a*x + B*a*log(sec(x) + tan(x)) + A*a*sin(x)
```

Fricas [**A**]
time = 1.97, size = 32, normalized size = 1.78

$$(A+B)ax + \frac{1}{2}\,Ba\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\,Ba\log\left(-\sin\left(x\right) + 1\right) + Aa\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] (A + B)*a*x + 1/2*B*a*log(sin(x) + 1) - 1/2*B*a*log(-sin(x) + 1) + A*a*sin(
x)
```

Sympy [**A**]
time = 1.25, size = 27, normalized size = 1.50

$$Aax + Aa\sin\left(x\right) + Bax + Ba\log\left(\tan\left(x\right) + \sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x)
```

```
[Out] A*a*x + A*a*sin(x) + B*a*x + B*a*log(tan(x) + sec(x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(18) = 36$.
time = 0.41, size = 51, normalized size = 2.83

$$Ba\log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right) - Ba\log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right) + (Aa + Ba)x + \frac{2\,Aa\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a*log(abs(tan(1/2*x) + 1)) - B*a*log(abs(tan(1/2*x) - 1)) + (A*a + B*a)*x
 + 2*A*a*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 2.48, size = 54, normalized size = 3.00

$$2 A a \operatorname{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2 B a \operatorname{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2 B a \operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + A a \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))*(A + B/cos(x)),x)
```

```
[Out] 2*A*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atanh
(sin(x/2)/cos(x/2)) + A*a*sin(x)
```

3.186 $\int (a+a\cos(x))^2(A+B\sec(x))\,dx$

Optimal. Leaf size=57

$$\frac{1}{2}a^2(3A+4B)x+a^2B\tanh^{-1}(\sin(x))+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)$$

```
[Out] 1/2*a^2*(3*A+4*B)*x+a^2*B*arctanh(sin(x))+1/2*a^2*(3*A+2*B)*sin(x)+1/2*A*(a
^2+a^2*cos(x))*sin(x)
```

Rubi [A]
time = 0.14, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2907, 3055, 3047, 3102, 2814, 3855}

$$\frac{1}{2}a^2x(3A+4B)+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)+a^2B\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(3*A + 4*B)*x)/2 + a^2*B*ArcTanh[Sin[x]] + (a^2*(3*A + 2*B)*Sin[x])/2
+ (A*(a^2 + a^2*Cos[x])*Sin[x])/2
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(x))^2(A+B\sec(x))\,dx &= \int (a+a\cos(x))^2(B+A\cos(x))\sec(x)\,dx \\
&= \frac{1}{2}A\big(a^2+a^2\cos(x)\big)\sin(x)+\frac{1}{2}\int (a+a\cos(x))(2aB+a(3A+2B)\operatorname{co} \\
&= \frac{1}{2}A\big(a^2+a^2\cos(x)\big)\sin(x)+\frac{1}{2}\int \big(2a^2B+\big(2a^2B+a^2(3A+2B)\big)\cos \\
&= \frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\big(a^2+a^2\cos(x)\big)\sin(x)+\frac{1}{2}\int \big(2a^2B+a^2(\\
&= \frac{1}{2}a^2(3A+4B)x+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\big(a^2+a^2\cos(x)\big)\sin(x)- \\
&= \frac{1}{2}a^2(3A+4B)x+a^2B\tanh^{-1}(\sin(x))+\frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A(a
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 1.18

$$\frac{1}{4}a^2\left(6Ax+8Bx-4B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+4B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+4(2A+B)\sin(x)+A\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

```
[Out] (a^2*(6*A*x + 8*B*x - 4*B*Log[Cos[x/2] - Sin[x/2]] + 4*B*Log[Cos[x/2] + Sin
[x/2]] + 4*(2*A + B)*Sin[x] + A*Sin[2*x]))/4
```

Maple [A]
time = 0.13, size = 56, normalized size = 0.98

method	result
default	$a^2Ax + a^2B\ln(\sec(x) + \tan(x)) + 2a^2A\sin(x) + 2a^2Bx + a^2A\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) + a^2B\sin(x)$
risch	$\frac{3a^2Ax}{2} + 2a^2Bx - iA\,e^{ix}a^2 - \frac{iB\,e^{ix}a^2}{2} + iA\,e^{-ix}a^2 + \frac{iB\,e^{-ix}a^2}{2} + a^2B\ln(e^{ix} + i) - a^2B\ln(e^{ix} - i) +$
norman	$\frac{\left(\frac{3}{2}a^2A+2a^2B\right)x+\left(3a^2A+2a^2B\right)\left(\tan^3\left(\frac{x}{2}\right)\right)+\left(5a^2A+2a^2B\right)\tan\left(\frac{x}{2}\right)+\left(\frac{3}{2}a^2A+2a^2B\right)x\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(3a^2A+4a^2B\right)x\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2} + a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^2*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*A*x+a^2*B*ln(sec(x)+tan(x))+2*a^2*A*sin(x)+2*a^2*B*x+a^2*A*(1/2*cos(x)*
sin(x)+1/2*x)+a^2*B*sin(x)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{1}{4}Aa^2(2x + \sin(2x)) + Aa^2x + 2Ba^2x + Ba^2\log(\sec(x) + \tan(x)) + 2Aa^2\sin(x) + Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/4*A*a^2*(2*x + sin(2*x)) + A*a^2*x + 2*B*a^2*x + B*a^2*log(sec(x) + tan(x
)) + 2*A*a^2*sin(x) + B*a^2*sin(x)
```

Fricas [A]
time = 2.82, size = 60, normalized size = 1.05

$$\frac{1}{2}(3A + 4B)a^2x + \frac{1}{2}Ba^2\log(\sin(x) + 1) - \frac{1}{2}Ba^2\log(-\sin(x) + 1) + \frac{1}{2}\left(Aa^2\cos(x) + 2(2A + B)a^2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*A + 4*B)*a^2*x + 1/2*B*a^2*log(sin(x) + 1) - 1/2*B*a^2*log(-sin(x) +
 1) + 1/2*(A*a^2*cos(x) + 2*(2*A + B)*a^2)*sin(x)
```

Sympy [A]
time = 2.11, size = 61, normalized size = 1.07

$$\frac{3Aa^2x}{2} + 2Aa^2\sin(x) + \frac{Aa^2\sin(2x)}{4} + 2Ba^2x + Ba^2\log(\tan(x) + \sec(x)) + Ba^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**2*(A+B*sec(x)),x)
```

```
[Out] 3*A*a**2*x/2 + 2*A*a**2*sin(x) + A*a**2*sin(2*x)/4 + 2*B*a**2*x + B*a**2*lo
g(tan(x) + sec(x)) + B*a**2*sin(x)
```

Giac [A]
time = 0.40, size = 100, normalized size = 1.75

$$Ba^2 \log \left(\left| \tan \left(\frac{1}{2} x \right) + 1 \right| \right) - Ba^2 \log \left(\left| \tan \left(\frac{1}{2} x \right) - 1 \right| \right) + \frac{1}{2} \left(3 Aa^2 + 4 Ba^2 \right) x + \frac{3 Aa^2 \tan \left(\frac{1}{2} x \right)^3 + 2 Ba^2 \tan \left(\frac{1}{2} x \right)^3 + 5 Aa^2 \tan \left(\frac{1}{2} x \right) + 2 Ba^2 \tan \left(\frac{1}{2} x \right)}{\left(\tan \left(\frac{1}{2} x \right)^2 + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^2*log(abs(tan(1/2*x) + 1)) - B*a^2*log(abs(tan(1/2*x) - 1)) + 1/2*(3*A*
a^2 + 4*B*a^2)*x + (3*A*a^2*tan(1/2*x)^3 + 2*B*a^2*tan(1/2*x)^3 + 5*A*a^2*t
an(1/2*x) + 2*B*a^2*tan(1/2*x))/(tan(1/2*x)^2 + 1)^2
```

Mupad [B]
time = 2.46, size = 403, normalized size = 7.07

$$\frac{\left(3 A a^2 + 2 B a^2\right) \tan\left(\frac{x}{2}\right)^3 + \left(5 A a^2 + 2 B a^2\right) \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4 + 2 \tan\left(\frac{x}{2}\right)^2 + 1} + a^2 \operatorname{atan}\left(\frac{216 A^3 a^6 \tan\left(\frac{x}{2}\right)}{216 A^3 a^6 + 864 A^2 B a^6 + 1248 A B^2 a^6 + 640 B^3 a^6} + \frac{640 B^3 a^6 \tan\left(\frac{x}{2}\right)}{216 A^3 a^6 + 864 A^2 B a^6 + 1248 A B^2 a^6 + 640 B^3 a^6} + \frac{1248 A B^2 a^6 \tan\left(\frac{x}{2}\right)}{216 A^3 a^6 + 864 A^2 B a^6 + 1248 A B^2 a^6 + 640 B^3 a^6} + \frac{864 A^2 B a^6 \tan\left(\frac{x}{2}\right)}{216 A^3 a^6 + 864 A^2 B a^6 + 1248 A B^2 a^6 + 640 B^3 a^6}\right) (3 A + 4 B) + 2 B a^2 \operatorname{atanh}\left(\frac{320 B^3 a^6 \tan\left(\frac{x}{2}\right)}{144 A^2 B a^6 + 384 A B^2 a^6 + 320 B^3 a^6} + \frac{384 A B^2 a^6 \tan\left(\frac{x}{2}\right)}{144 A^2 B a^6 + 384 A B^2 a^6 + 320 B^3 a^6} + \frac{144 A^2 B a^6 \tan\left(\frac{x}{2}\right)}{144 A^2 B a^6 + 384 A B^2 a^6 + 320 B^3 a^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^2*(A + B/cos(x)),x)
```

```
[Out] (tan(x/2)^3*(3*A*a^2 + 2*B*a^2) + tan(x/2)*(5*A*a^2 + 2*B*a^2))/(2*tan(x/2)
^2 + tan(x/2)^4 + 1) + a^2*atan((216*A^3*a^6*tan(x/2))/(216*A^3*a^6 + 640*B
^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (640*B^3*a^6*tan(x/2))/(216*A^3*
a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (1248*A*B^2*a^6*tan(x
/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (864*A^
2*B*a^6*tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a
^6))*(3*A + 4*B) + 2*B*a^2*atanh((320*B^3*a^6*tan(x/2))/(320*B^3*a^6 + 384*
A*B^2*a^6 + 144*A^2*B*a^6) + (384*A*B^2*a^6*tan(x/2))/(320*B^3*a^6 + 384*A*
B^2*a^6 + 144*A^2*B*a^6) + (144*A^2*B*a^6*tan(x/2))/(320*B^3*a^6 + 384*A*B^
2*a^6 + 144*A^2*B*a^6))
```

3.187 $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

Optimal. Leaf size=75

$$\frac{1}{2}a^3(5A+7B)x+a^3B\tanh^{-1}(\sin(x))+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)$$

```
[Out] 1/2*a^3*(5*A+7*B)*x+a^3*B*arctanh(sin(x))+5/2*a^3*(A+B)*sin(x)+1/3*a*A*(a+a
*cos(x))^2*sin(x)+1/6*(5*A+3*B)*(a^3+a^3*cos(x))*sin(x)
```

Rubi [A]
time = 0.20, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2907, 3055, 3047, 3102, 2814, 3855}

$$\frac{1}{2}a^3x(5A+7B)+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{6}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)+a^3B\tanh^{-1}(\sin(x))+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(5*A + 7*B)*x)/2 + a^3*B*ArcTanh[Sin[x]] + (5*a^3*(A + B)*Sin[x])/2 +
(a*A*(a + a*Cos[x])^2*Sin[x])/3 + ((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])/6
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx = \int (a+a\cos(x))^3(B+A\cos(x))\sec(x)\,dx$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{3}\int (a+a\cos(x))^2(3aB+a(5A+3B)\mathrm{co}$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int$$
$$= \frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)+\frac{1}{6}\int$$
$$= \frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}(5A+3B)\left(a^3+a\right.$$
$$= \frac{1}{2}a^3(5A+7B)x+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x)+\frac{1}{6}$$
$$= \frac{1}{2}a^3(5A+7B)x+a^3B\tanh^{-1}(\sin(x))+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 1.07

$$\frac{1}{12}a^3\left(30Ax+42Bx-12B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+12B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+9(5A+4B)\sin(x)+3(3A+B)\sin(2x)+A\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

```
[Out] (a^3*(30*A*x + 42*B*x - 12*B*Log[Cos[x/2] - Sin[x/2]] + 12*B*Log[Cos[x/2] +
 Sin[x/2]] + 9*(5*A + 4*B)*Sin[x] + 3*(3*A + B)*Sin[2*x] + A*Sin[3*x]))/12
```

Maple [A]
time = 0.15, size = 87, normalized size = 1.16

method	result
default	$A\,a^{3} x + B\,a^{3} \ln \left(\sec \left(x \right)+\tan \left(x \right)\right)+3 A\,a^{3} \sin \left(x \right)+3 B\,a^{3} x +3 A\,a^{3} \left(\frac{\cos \left(x \right) \sin \left(x \right)}{2}+\frac{x}{2}\right)+3 B\,a^{3} \mathrm{si}$
risch	$\frac{5 A\,a^{3} x}{2}+\frac{7 B\,a^{3} x}{2}-\frac{15 i A\,{\mathrm e}^{i x} a^{3}}{8}-\frac{3 i B\,{\mathrm e}^{i x} a^{3}}{2}+\frac{15 i A\,{\mathrm e}^{-i x} a^{3}}{8}+\frac{3 i B\,{\mathrm e}^{-i x} a^{3}}{2}+B\,a^{3} \ln \left({\mathrm e}^{i x}+i\right)-B\,a^{3} \ln \left({\mathrm e}^{i x}-i\right.$
norman	$\frac{\left(\frac{5}{2} A\,a^{3}+\frac{7}{2} B\,a^{3}\right) x +\left(\frac{40}{3} A\,a^{3}+12 B\,a^{3}\right) \left(\tan^{3}\left(\frac{x}{2}\right)\right)+\left(5 A\,a^{3}+5 B\,a^{3}\right) \left(\tan^{5}\left(\frac{x}{2}\right)\right)+\left(11 A\,a^{3}+7 B\,a^{3}\right) \tan \left(\frac{x}{2}\right)+\left(\frac{5}{2} A\,a^{3}+\frac{7}{2} B\,a^{3}\right) x \left(\tan^{6}\left(\frac{x}{2}\right.\right.}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^3*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] A*a^3*x+B*a^3*ln(sec(x)+tan(x))+3*A*a^3*sin(x)+3*B*a^3*x+3*A*a^3*(1/2*cos(x
)*sin(x)+1/2*x)+3*B*a^3*sin(x)+1/3*A*a^3*(2+cos(x)^2)*sin(x)+B*a^3*(1/2*cos
(x)*sin(x)+1/2*x)
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.12

$$-\frac{1}{3} \, {\left(\sin\left(x\right)^{3} - 3 \, \sin\left(x\right)\right)} A a^{3} + \frac{3}{4} \, A a^{3} {\left(2 \, x + \sin\left(2 \, x\right)\right)} + \frac{1}{4} \, B a^{3} {\left(2 \, x + \sin\left(2 \, x\right)\right)} + A a^{3} x + 3 \, B a^{3} x + B a^{3} \log\left(\sec\left(x\right) + \tan\left(x\right)\right) + 3 \, A a^{3} \sin\left(x\right) + 3 \, B a^{3} \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))*A*a^3 + 3/4*A*a^3*(2*x + sin(2*x)) + 1/4*B*a^3*(
2*x + sin(2*x)) + A*a^3*x + 3*B*a^3*x + B*a^3*log(sec(x) + tan(x)) + 3*A*a^
3*sin(x) + 3*B*a^3*sin(x)
```

Fricas [A]
time = 2.26, size = 77, normalized size = 1.03

$$\frac{1}{2} \, {\left(5 \, A + 7 \, B\right)} a^{3} x + \frac{1}{2} \, B a^{3} \log\left(\sin\left(x\right) + 1\right) - \frac{1}{2} \, B a^{3} \log\left(-\sin\left(x\right) + 1\right) + \frac{1}{6} \, {\left(2 \, A a^{3} \cos\left(x\right)^{2} + 3 \, {\left(3 \, A + B\right)} a^{3} \cos\left(x\right) + 2 \, {\left(11 \, A + 9 \, B\right)} a^{3}\right)} \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(5*A + 7*B)*a^3*x + 1/2*B*a^3*log(sin(x) + 1) - 1/2*B*a^3*log(-sin(x) +
 1) + 1/6*(2*A*a^3*cos(x)^2 + 3*(3*A + B)*a^3*cos(x) + 2*(11*A + 9*B)*a^3)*
sin(x)
```

Sympy [A]
time = 4.65, size = 92, normalized size = 1.23

$$\frac{5Aa^3x}{2}-\frac{Aa^3\sin^3{(x)}}{3}+4Aa^3\sin{(x)}+\frac{3Aa^3\sin{(2x)}}{4}+\frac{7Ba^3x}{2}+Ba^3\log{(\tan{(x)}+\sec{(x)})}+\frac{Ba^3\sin{(x)}\cos{(x)}}{2}+3Ba^3\sin{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**3*(A+B*sec(x)),x)
```

```
[Out] 5*A*a**3*x/2 - A*a**3*sin(x)**3/3 + 4*A*a**3*sin(x) + 3*A*a**3*sin(2*x)/4 +
 7*B*a**3*x/2 + B*a**3*log(tan(x) + sec(x)) + B*a**3*sin(x)*cos(x)/2 + 3*B*
a**3*sin(x)
```

Giac [A]
time = 0.42, size = 125, normalized size = 1.67

$$Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\left(5\,Aa^3+7\,Ba^3\right)x+\frac{15\,Aa^3\tan\left(\frac{1}{2}x\right)^5+15\,Ba^3\tan\left(\frac{1}{2}x\right)^5+40\,Aa^3\tan\left(\frac{1}{2}x\right)^3+36\,Ba^3\tan\left(\frac{1}{2}x\right)^3+33\,Aa^3\tan\left(\frac{1}{2}x\right)+21\,Ba^3\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] B*a^3*log(abs(tan(1/2*x) + 1)) - B*a^3*log(abs(tan(1/2*x) - 1)) + 1/2*(5*A*
a^3 + 7*B*a^3)*x + 1/3*(15*A*a^3*tan(1/2*x)^5 + 15*B*a^3*tan(1/2*x)^5 + 40*
A*a^3*tan(1/2*x)^3 + 36*B*a^3*tan(1/2*x)^3 + 33*A*a^3*tan(1/2*x) + 21*B*a^3
*tan(1/2*x))/(tan(1/2*x)^2 + 1)^3
```

Mupad [B]
time = 2.48, size = 431, normalized size = 5.75

$$\frac{\left(5\,A\,a^3+5\,B\,a^3\right)\tan\left(\frac{x}{2}\right)^5+\left(\frac{40\,A\,a^3}{3}+12\,B\,a^3\right)\tan\left(\frac{x}{2}\right)^3+\left(11\,A\,a^3+7\,B\,a^3\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1}+a^3\,\mathrm{atan}\left(\frac{1000\,A^3\,a^9\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{2968\,B^3\,a^9\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{6040\,A\,B^2\,a^9\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{4200\,A^2\,B\,a^9\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}\right)\left(5\,A+7\,B\right)+2\,B\,a^3\,\mathrm{atanh}\left(\frac{848\,B^3\,a^9\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}+\frac{1120\,A\,B^2\,a^9\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}+\frac{400\,A^2\,B\,a^9\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^3*(A + B/cos(x)),x)
```

```
[Out] (tan(x/2)^5*(5*A*a^3 + 5*B*a^3) + tan(x/2)^3*((40*A*a^3)/3 + 12*B*a^3) + ta
n(x/2)*(11*A*a^3 + 7*B*a^3))/(3*tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1)
 + a^3*atan((1000*A^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B
^2*a^9 + 4200*A^2*B*a^9) + (2968*B^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3
*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (6040*A*B^2*a^9*tan(x/2))/(1000*A
^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (4200*A^2*B*a^9*
tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9))*
(5*A + 7*B) + 2*B*a^3*atanh((848*B^3*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^
2*a^9 + 400*A^2*B*a^9) + (1120*A*B^2*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^
2*a^9 + 400*A^2*B*a^9) + (400*A^2*B*a^9*tan(x/2))/(848*B^3*a^9 + 1120*A*B^2
*a^9 + 400*A^2*B*a^9))
```

3.188 $\int (a+a\cos(x))^4(A+B\sec(x))\,dx$

Optimal. Leaf size=104

$$\frac{1}{8}a^4(35A+48B)x+a^4B\tanh^{-1}(\sin(x))+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+\right.$$

```
[Out] 1/8*a^4*(35*A+48*B)*x+a^4*B*arctanh(sin(x))+5/8*a^4*(7*A+8*B)*sin(x)+1/4*a*
A*(a+a*cos(x))^3*sin(x)+1/12*(7*A+4*B)*(a^2+a^2*cos(x))^2*sin(x)+1/24*(35*A
+32*B)*(a^4+a^4*cos(x))*sin(x)
```

Rubi [A]
time = 0.28, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2907, 3055, 3047, 3102, 2814, 3855}

$$\frac{1}{8}a^4x(35A+48B)+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{24}(35A+32B)\sin(x)\left(a^4\cos(x)+a^4\right)+a^4B\tanh^{-1}(\sin(x))+\frac{1}{12}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2+\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(35*A + 48*B)*x)/8 + a^4*B*ArcTanh[Sin[x]] + (5*a^4*(7*A + 8*B)*Sin[x]
)/8 + (a*A*(a + a*Cos[x])^3*Sin[x])/4 + ((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*S
in[x])/12 + ((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/24
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(x))^4(A+B\sec(x))\,dx &= \int (a+a\cos(x))^4(B+A\cos(x))\sec(x)\,dx \\
&= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{4}\int (a+a\cos(x))^3(4aB+a(7A+4B)\,\mathrm{co} \\
&= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+ \\
&= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+ \\
&= \frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x)+ \\
&= \frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x)+\frac{1}{12}(7A+4B)\left(a^2\right. \\
&= \frac{1}{8}a^4(35A+48B)x+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x \\
&= \frac{1}{8}a^4(35A+48B)x+a^4B\tanh^{-1}(\sin(x))+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}a
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 97, normalized size = 0.93

$$\frac{1}{96}a^4\left(420Ax+576Bx-96B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+96B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+24(28A+27B)\sin(x)+24(7A+4B)\sin(2x)+32A\sin(3x)+8B\sin(3x)+3A\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

```
[Out] (a^4*(420*A*x + 576*B*x - 96*B*Log[Cos[x/2] - Sin[x/2]] + 96*B*Log[Cos[x/2]
 + Sin[x/2]] + 24*(28*A + 27*B)*Sin[x] + 24*(7*A + 4*B)*Sin[2*x] + 32*A*Sin
[3*x] + 8*B*Sin[3*x] + 3*A*Sin[4*x]))/96
```

Maple [A]
time = 0.17, size = 124, normalized size = 1.19

method	result
default	$A\,a^4x+B\,a^4\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+4A\,a^4\sin\left(x\right)+4B\,a^4x+6A\,a^4\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+6B\,a^4\sin$
risch	$\frac{35A\,a^4x}{8}+6B\,a^4x-\frac{7iA\,e^{ix}a^4}{2}-\frac{27iB\,e^{ix}a^4}{8}+\frac{7iA\,e^{-ix}a^4}{2}+\frac{27iB\,e^{-ix}a^4}{8}+B\,a^4\ln\left(e^{ix}+i\right)-B\,a^4\ln\left(e^{ix}-\right.$
norman	$\frac{\left(\frac{35}{4}A\,a^4+10B\,a^4\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+\left(\frac{35}{8}A\,a^4+6B\,a^4\right)x+\left(\frac{93}{4}A\,a^4+18B\,a^4\right)\tan\left(\frac{x}{2}\right)+\left(\frac{385}{12}A\,a^4+\frac{106}{3}B\,a^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+\left(\frac{511}{12}A\,a^4+\frac{130}{3}B\,a^4\right.}{(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^4*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] A*a^4*x+B*a^4*ln(sec(x)+tan(x))+4*A*a^4*sin(x)+4*B*a^4*x+6*A*a^4*(1/2*cos(x
)*sin(x)+1/2*x)+6*B*a^4*sin(x)+4/3*A*a^4*(2+cos(x)^2)*sin(x)+4*B*a^4*(1/2*c
os(x)*sin(x)+1/2*x)+A*a^4*(1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x)+1/3*B*a^
4*(2+cos(x)^2)*sin(x)
```

Maxima [A]
time = 0.27, size = 118, normalized size = 1.13

$$-\frac{4}{3}\left(\sin\left(x\right)^3-3\sin\left(x\right)\right)Aa^4-\frac{1}{3}\left(\sin\left(x\right)^3-3\sin\left(x\right)\right)Ba^4+\frac{1}{32}Aa^4(12x+\sin\left(4x\right)+8\sin\left(2x\right))+\frac{3}{2}Aa^4(2x+\sin\left(2x\right))+Ba^4(2x+\sin\left(2x\right))+Aa^4x+4Ba^4x+Ba^4\log\left(\sec\left(x\right)+\tan\left(x\right)\right)+4Aa^4\sin\left(x\right)+6Ba^4\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] -4/3*(sin(x)^3 - 3*sin(x))*A*a^4 - 1/3*(sin(x)^3 - 3*sin(x))*B*a^4 + 1/32*A
*a^4*(12*x + sin(4*x) + 8*sin(2*x)) + 3/2*A*a^4*(2*x + sin(2*x)) + B*a^4*(2
*x + sin(2*x)) + A*a^4*x + 4*B*a^4*x + B*a^4*log(sec(x) + tan(x)) + 4*A*a^4
*sin(x) + 6*B*a^4*sin(x)
```

Fricas [A]
time = 2.22, size = 89, normalized size = 0.86

$$\frac{1}{8}\left(35A+48B\right)a^4x+\frac{1}{2}Ba^4\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}Ba^4\log\left(-\sin\left(x\right)+1\right)+\frac{1}{24}\left(6Aa^4\cos\left(x\right)^3+8\left(4A+B\right)a^4\cos\left(x\right)^2+3\left(27A+16B\right)a^4\cos\left(x\right)+160\left(A+B\right)a^4\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="fricas")

[Out] 1/8*(35*A + 48*B)*a^4*x + 1/2*B*a^4*log(sin(x) + 1) - 1/2*B*a^4*log(-sin(x) + 1) + 1/24*(6*A*a^4*cos(x)^3 + 8*(4*A + B)*a^4*cos(x)^2 + 3*(27*A + 16*B)*a^4*cos(x) + 160*(A + B)*a^4)*sin(x)

Sympy [A]
time = 9.82, size = 116, normalized size = 1.12

$$\frac{35Aa^4x}{8} - \frac{4Aa^4\sin^3{(x)}}{3} + 8Aa^4\sin{(x)} + \frac{7Aa^4\sin{(2x)}}{4} + \frac{Aa^4\sin{(4x)}}{32} + 6Ba^4x + Ba^4\log{(\tan{(x)} + \sec{(x)})} - \frac{Ba^4\sin^3{(x)}}{3} + 2Ba^4\sin{(x)}\cos{(x)} + 7Ba^4\sin{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))**4*(A+B*sec(x)),x)

[Out] 35*A*a**4*x/8 - 4*A*a**4*sin(x)**3/3 + 8*A*a**4*sin(x) + 7*A*a**4*sin(2*x)/4 + A*a**4*sin(4*x)/32 + 6*B*a**4*x + B*a**4*log(tan(x) + sec(x)) - B*a**4*sin(x)**3/3 + 2*B*a**4*sin(x)*cos(x) + 7*B*a**4*sin(x)

Giac [A]
time = 0.42, size = 149, normalized size = 1.43

$$Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right) - Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right) + \frac{1}{8}(35Aa^4 + 48Ba^4)x + \frac{105Aa^4\tan\left(\frac{1}{2}x\right)^7 + 120Ba^4\tan\left(\frac{1}{2}x\right)^7 + 385Aa^4\tan\left(\frac{1}{2}x\right)^5 + 424Ba^4\tan\left(\frac{1}{2}x\right)^5 + 511Aa^4\tan\left(\frac{1}{2}x\right)^3 + 520Ba^4\tan\left(\frac{1}{2}x\right)^3 + 279Aa^4\tan\left(\frac{1}{2}x\right) + 216Ba^4\tan\left(\frac{1}{2}x\right)}{12\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="giac")

[Out] B*a^4*log(abs(tan(1/2*x) + 1)) - B*a^4*log(abs(tan(1/2*x) - 1)) + 1/8*(35*A*a^4 + 48*B*a^4)*x + 1/12*(105*A*a^4*tan(1/2*x)^7 + 120*B*a^4*tan(1/2*x)^7 + 385*A*a^4*tan(1/2*x)^5 + 424*B*a^4*tan(1/2*x)^5 + 511*A*a^4*tan(1/2*x)^3 + 520*B*a^4*tan(1/2*x)^3 + 279*A*a^4*tan(1/2*x) + 216*B*a^4*tan(1/2*x))/(tan(1/2*x)^2 + 1)^4

Mupad [B]
time = 2.52, size = 460, normalized size = 4.42

$$\frac{\left(\frac{35Aa^4}{4}+10Ba^4\right)\tan\left(\frac{x}{2}\right)^7+\left(\frac{385Aa^4}{12}+\frac{106Ba^4}{3}\right)\tan\left(\frac{x}{2}\right)^5+\left(\frac{511Aa^4}{12}+\frac{130Ba^4}{3}\right)\tan\left(\frac{x}{2}\right)^3+\left(\frac{93Aa^4}{4}+18Ba^4\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^8+4\tan\left(\frac{x}{2}\right)^6+6\tan\left(\frac{x}{2}\right)^4+4\tan\left(\frac{x}{2}\right)^2+1} + \frac{a^4\,\mathrm{atan}\left(\frac{42875A^3a^{12}\tan\left(\frac{x}{2}\right)}{8\left(\frac{42875A^3a^{12}}{8}+22050A^2Ba^{12}+30520AB^2a^{12}+14208B^3a^{12}\right)}+\frac{14208B^3a^{12}\tan\left(\frac{x}{2}\right)}{\frac{42875A^3a^{12}}{8}+22050A^2Ba^{12}+30520AB^2a^{12}+14208B^3a^{12}}+\frac{30520AB^2a^{12}\tan\left(\frac{x}{2}\right)}{\frac{42875A^3a^{12}}{8}+22050A^2Ba^{12}+30520AB^2a^{12}+14208B^3a^{12}}+\frac{22050A^2Ba^{12}\tan\left(\frac{x}{2}\right)}{\frac{42875A^3a^{12}}{8}+22050A^2Ba^{12}+30520AB^2a^{12}+14208B^3a^{12}}\right)(35A+48B)}{4} + 2Ba^4\,\mathrm{atanh}\left(\frac{2368B^3a^{12}\tan\left(\frac{x}{2}\right)}{1225A^2Ba^{12}+3360AB^2a^{12}+2368B^3a^{12}}+\frac{3360AB^2a^{12}\tan\left(\frac{x}{2}\right)}{1225A^2Ba^{12}+3360AB^2a^{12}+2368B^3a^{12}}+\frac{1225A^2Ba^{12}\tan\left(\frac{x}{2}\right)}{1225A^2Ba^{12}+3360AB^2a^{12}+2368B^3a^{12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*cos(x))^4*(A + B/cos(x)),x)

[Out] (tan(x/2)^7*((35*A*a^4)/4 + 10*B*a^4) + tan(x/2)^5*((385*A*a^4)/12 + (106*B*a^4)/3) + tan(x/2)^3*((511*A*a^4)/12 + (130*B*a^4)/3) + tan(x/2)*((93*A*a^4)/4 + 18*B*a^4))/(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8 + 1) + (a^4*atan((42875*A^3*a^12*tan(x/2))/(8*((42875*A^3*a^12)/8 + 14208*B

```
^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12)) + (14208*B^3*a^12*tan(x/2))
/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12
) + (30520*A*B^2*a^12*tan(x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 3052
0*A*B^2*a^12 + 22050*A^2*B*a^12) + (22050*A^2*B*a^12*tan(x/2))/((42875*A^3*
a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12))*(35*A + 48
*B))/4 + 2*B*a^4*atanh((2368*B^3*a^12*tan(x/2))/(2368*B^3*a^12 + 3360*A*B^2
*a^12 + 1225*A^2*B*a^12) + (3360*A*B^2*a^12*tan(x/2))/(2368*B^3*a^12 + 3360
*A*B^2*a^12 + 1225*A^2*B*a^12) + (1225*A^2*B*a^12*tan(x/2))/(2368*B^3*a^12
+ 3360*A*B^2*a^12 + 1225*A^2*B*a^12))
```

3.189 $\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=25

$$\frac{B\tanh^{-1}(\sin(x))}{a}+\frac{(A-B)\sin(x)}{a+a\cos(x)}$$

[Out] B*arctanh(sin(x))/a+(A-B)*sin(x)/(a+a*cos(x))

Rubi [A]
time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2907, 3057, 12, 3855}

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[x])/(a + a*Cos[x]),x]

[Out] (B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*Cos[x])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{a+a\cos(x)}\,dx\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{\int aB\sec(x)\,dx}{a^2}\\ &= \frac{(A-B)\sin(x)}{a+a\cos(x)}+\frac{B\int \sec(x)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(x))}{a}+\frac{(A-B)\sin(x)}{a+a\cos(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.
time = 0.06, size = 71, normalized size = 2.84

$$-\frac{2\cos\left(\frac{x}{2}\right)\left(B\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(-A+B)\sin\left(\frac{x}{2}\right)\right)}{a(1+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x]),x]
[Out] (-2*Cos[x/2]*(B*Cos[x/2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2
]]) + (-A + B)*Sin[x/2]))/(a*(1 + Cos[x]))
```

Maple [A]
time = 0.12, size = 38, normalized size = 1.52

method	result	size
default	$\frac{A\tan\left(\frac{x}{2}\right)-B\tan\left(\frac{x}{2}\right)-B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}$	38
norman	$\frac{(A-B)\tan\left(\frac{x}{2}\right)}{a}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}$	40
risch	$\frac{2iA}{(e^{ix}+1)a}-\frac{2iB}{(e^{ix}+1)a}+\frac{B\ln(e^{ix}+i)}{a}-\frac{B\ln(e^{ix}-i)}{a}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(A*tan(1/2*x)-B*tan(1/2*x)-B*ln(tan(1/2*x)-1)+B*ln(tan(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.
time = 0.27, size = 63, normalized size = 2.52

$$B\left(\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}-\frac{\sin(x)}{a(\cos(x)+1)}\right)+\frac{A\sin(x)}{a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] B*(log(sin(x)/(cos(x) + 1) + 1)/a - log(sin(x)/(cos(x) + 1) - 1)/a - sin(x)
/(a*(cos(x) + 1))) + A*sin(x)/(a*(cos(x) + 1))
```

Fricas [A]
time = 4.81, size = 47, normalized size = 1.88

$$\frac{(B\cos(x)+B)\log(\sin(x)+1)-(B\cos(x)+B)\log(-\sin(x)+1)+2(A-B)\sin(x)}{2(a\cos(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*log(sin(x) + 1) - (B*cos(x) + B)*log(-sin(x) + 1) + 2*(
A - B)*sin(x))/(a*cos(x) + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A}{\cos(x)+1}\,dx+\int\frac{B\sec(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x)
```

```
[Out] (Integral(A/(cos(x) + 1), x) + Integral(B*sec(x)/(cos(x) + 1), x))/a
```

Giac [A]
time = 0.44, size = 46, normalized size = 1.84

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a}+\frac{A\tan\left(\frac{1}{2}x\right)-B\tan\left(\frac{1}{2}x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a - B*log(abs(tan(1/2*x) - 1))/a + (A*tan(1/2*x)
 - B*tan(1/2*x))/a
```

Mupad [B]
time = 2.36, size = 25, normalized size = 1.00

$$\frac{2B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{\tan\left(\frac{x}{2}\right)\,\left(A-B\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x)),x)
[Out] (2*B*atanh(tan(x/2)))/a + (tan(x/2)*(A - B))/a
```

3.190 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx$

Optimal. Leaf size=48

$$\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}$$

```
[Out] B*arctanh(sin(x))/a^2+1/3*(A-4*B)*sin(x)/a^2/(cos(x)+1)+1/3*(A-B)*sin(x)/(a
+a*cos(x))^2
```

Rubi [A]
time = 0.13, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2907, 3057, 12, 3855}

$$\frac{(A-4B)\sin(x)}{3a^2(\cos(x)+1)}+\frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^2 + ((A - 4*B)*Sin[x])/(3*a^2*(1 + Cos[x])) + ((A - B
)*Sin[x])/(3*(a + a*Cos[x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx\\
&= \frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{\int \frac{(3aB+a(A-B)\cos(x))\sec(x)}{a+a\cos(x)}\,dx}{3a^2}\\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{\int 3a^2B\sec(x)\,dx}{3a^4}\\
&= \frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}+\frac{B\int \sec(x)\,dx}{a^2}\\
&= \frac{B\tanh^{-1}(\sin(x))}{a^2}+\frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 76, normalized size = 1.58

$$\frac{-12B\cos^4\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(2A-5B+(A-4B)\cos(x))\sin(x)}{3a^2(1+\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
[Out] (-12*B*Cos[x/2]^4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + (
2*A - 5*B + (A - 4*B)*Cos[x])*Sin[x])/(3*a^2*(1 + Cos[x])^2)
```

Maple [A]
time = 0.12, size = 58, normalized size = 1.21

method	result	size
default	$\frac{\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)A}{3}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)B}{3}+A\tan\left(\frac{x}{2}\right)-3B\tan\left(\frac{x}{2}\right)-2B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+2B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{2a^2}$	58
norman	$\frac{\frac{(A-B)\left(\tan^3\left(\frac{x}{2}\right)\right)}{6a}+\frac{(-3B+A)\tan\left(\frac{x}{2}\right)}{2a}}{a}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^2}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}$	62

risch	$-\frac{2i(3B\,e^{2ix}-3A\,e^{ix}+9B\,e^{ix}-A+4B)}{3(e^{ix}+1)^3a^2}+\frac{B\ln(e^{ix}+i)}{a^2}-\frac{B\ln(e^{ix}-i)}{a^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*(1/3*tan(1/2*x)^3*A-1/3*tan(1/2*x)^3*B+A*tan(1/2*x)-3*B*tan(1/2*x)-
2*B*ln(tan(1/2*x)-1)+2*B*ln(tan(1/2*x)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.
time = 0.28, size = 93, normalized size = 1.94

$$-\frac{1}{6}B\left(\frac{\frac{9\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^2}\right)+\frac{A\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="maxima")
```

```
[Out] -1/6*B*((9*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2 - 6*log(sin(x
)/(cos(x) + 1) + 1)/a^2 + 6*log(sin(x)/(cos(x) + 1) - 1)/a^2) + 1/6*A*(3*si
n(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2
```

Fricas [A]
time = 3.62, size = 85, normalized size = 1.77

$$\frac{3\left(B\cos(x)^2+2B\cos(x)+B\right)\log(\sin(x)+1)-3\left(B\cos(x)^2+2B\cos(x)+B\right)\log(-\sin(x)+1)+2\left((A-4B)\cos(x)+2A-5B\right)\sin(x)}{6\left(a^2\cos(x)^2+2a^2\cos(x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(sin(x) + 1) - 3*(B*cos(x)^2 + 2*B*
cos(x) + B)*log(-sin(x) + 1) + 2*((A - 4*B)*cos(x) + 2*A - 5*B)*sin(x))/(a^
2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^2(x)+2\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos^2(x)+2\cos(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**2,x)
```

```
[Out] (Integral(A/(cos(x)**2 + 2*cos(x) + 1), x) + Integral(B*sec(x)/(cos(x)**2 +
 2*cos(x) + 1), x))/a**2
```

Giac [A]
time = 0.41, size = 77, normalized size = 1.60

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^2}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}x\right)^3-Ba^4\tan\left(\frac{1}{2}x\right)^3+3\,Aa^4\tan\left(\frac{1}{2}x\right)-9\,Ba^4\tan\left(\frac{1}{2}x\right)}{6\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^2 - B*log(abs(tan(1/2*x) - 1))/a^2 + 1/6*(A*a^
4*tan(1/2*x)^3 - B*a^4*tan(1/2*x)^3 + 3*A*a^4*tan(1/2*x) - 9*B*a^4*tan(1/2*
x))/a^6
```

Mupad [B]
time = 2.35, size = 50, normalized size = 1.04

$$\tan\left(\frac{x}{2}\right)\,\left(\frac{A-B}{2\,a^2}-\frac{B}{a^2}\right)+\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2}+\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^2,x)
```

```
[Out] tan(x/2)*((A - B)/(2*a^2) - B/a^2) + (tan(x/2)^3*(A - B))/(6*a^2) + (2*B*at
anh(tan(x/2)))/a^2
```

3.191 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx$

Optimal. Leaf size=75

$$\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2}+\frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}$$

```
[Out] B*arctanh(sin(x))/a^3+1/5*(A-B)*sin(x)/(a+a*cos(x))^3+1/15*(2*A-7*B)*sin(x)
/a/(a+a*cos(x))^2+2/15*(A-11*B)*sin(x)/(a^3+a^3*cos(x))
```

Rubi [A]
time = 0.20, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2907, 3057, 12, 3855}

$$\frac{2(A-11B)\sin(x)}{15\left(a^3\cos(x)+a^3\right)}+\frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(2A-7B)\sin(x)}{15a(a\cos(x)+a)^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^3 + ((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((2*A - 7
*B)*Sin[x])/(15*a*(a + a*Cos[x])^2) + (2*(A - 11*B)*Sin[x])/(15*(a^3 + a^3*
Cos[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{\int \frac{(5aB+2a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx}{5a^2}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2}+\frac{\int \frac{(15a^2B+a^2(2A-7B)\cos(x))\sec(x)}{a+a\cos(x)}\,dx}{15a^4}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2}+\frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}+\frac{\int 15a^3B\sec(x)\,dx}{15a^6}\\
&= \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2}+\frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}+\frac{B\int \sec(x)\,dx}{a^3}\\
&= \frac{B\tanh^{-1}(\sin(x))}{a^3}+\frac{(A-B)\sin(x)}{5(a+a\cos(x))^3}+\frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2}+\frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 88, normalized size = 1.17

$$\frac{-120B\cos^6\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(8A-43B+(6A-51B)\cos(x)+(A-11B)\cos(2x))\sin(x)}{15a^3(1+\cos(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

```
[Out] (-120*B*Cos[x/2]^6*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
(8*A - 43*B + (6*A - 51*B)*Cos[x] + (A - 11*B)*Cos[2*x])*Sin[x])/(15*a^3*(1
 + Cos[x])^3)
```

Maple [A]
time = 0.14, size = 76, normalized size = 1.01

method	result	size

default	$\frac{4B\ln(\tan(\frac{x}{2})+1)+A\tan(\frac{x}{2})-7B\tan(\frac{x}{2})+\frac{(\tan^5(\frac{x}{2}))A}{5}-\frac{(\tan^5(\frac{x}{2}))B}{5}+\frac{2(\tan^3(\frac{x}{2}))A}{3}-\frac{4(\tan^3(\frac{x}{2}))B}{3}-4B\ln(\tan(\frac{x}{2})-1)}{4a^3}$	76
norman	$\frac{\frac{(A-7B)\tan(\frac{x}{2})}{4a}+\frac{(A-B)(\tan^5(\frac{x}{2}))}{20a}+\frac{(-2B+A)(\tan^3(\frac{x}{2}))}{6a}}{a^2}+\frac{B\ln(\tan(\frac{x}{2})+1)}{a^3}-\frac{B\ln(\tan(\frac{x}{2})-1)}{a^3}$	78
risch	$-\frac{2i(15B\,e^{4ix}+75B\,e^{3ix}-20A\,e^{2ix}+145B\,e^{2ix}-10A\,e^{ix}+95B\,e^{ix}-2A+22B)}{15(e^{ix}+1)^5a^3}+\frac{B\ln(e^{ix}+i)}{a^3}-\frac{B\ln(e^{ix}-i)}{a^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(4*B*ln(tan(1/2*x)+1)+A*tan(1/2*x)-7*B*tan(1/2*x)+1/5*tan(1/2*x)^5*
A-1/5*tan(1/2*x)^5*B+2/3*tan(1/2*x)^3*A-4/3*tan(1/2*x)^3*B-4*B*ln(tan(1/2*x
)-1))
```

Maxima [A]
time = 0.28, size = 119, normalized size = 1.59

$$-\frac{1}{60}B\left(\frac{\frac{105\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^3}\right)+\frac{A\left(\frac{15\sin(x)}{\cos(x)+1}+\frac{10\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}\right)}{60\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="maxima")
```

```
[Out] -1/60*B*((105*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5
/(cos(x) + 1)^5)/a^3 - 60*log(sin(x)/(cos(x) + 1) + 1)/a^3 + 60*log(sin(x)/
(cos(x) + 1) - 1)/a^3) + 1/60*A*(15*sin(x)/(cos(x) + 1) + 10*sin(x)^3/(cos(
x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3
```

Fricas [A]
time = 3.00, size = 122, normalized size = 1.63

$$\frac{15\left(B\cos(x)^3+3B\cos(x)^2+3B\cos(x)+B\right)\log(\sin(x)+1)-15\left(B\cos(x)^3+3B\cos(x)^2+3B\cos(x)+B\right)\log(-\sin(x)+1)+2\left(2(A-11B)\cos(x)^2+3(2A-17B)\cos(x)+7A-32B\right)\sin(x)}{30\left(a^3\cos(x)^3+3a^3\cos(x)^2+3a^3\cos(x)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(sin(x) + 1) - 15*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(A - 1
1*B)*cos(x)^2 + 3*(2*A - 17*B)*cos(x) + 7*A - 32*B)*sin(x))/(a^3*cos(x)^3 +
 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx+\int\frac{B\sec(x)}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**3,x)
```

```
[Out] (Integral(A/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x) + Integral(B*sec(x
)/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x))/a**3
```

Giac [A]
time = 0.42, size = 102, normalized size = 1.36

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^3}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}x\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}x\right)^5+10\,Aa^{12}\tan\left(\frac{1}{2}x\right)^3-20\,Ba^{12}\tan\left(\frac{1}{2}x\right)^3+15\,Aa^{12}\tan\left(\frac{1}{2}x\right)-105\,Ba^{12}\tan\left(\frac{1}{2}x\right)}{60\,a^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^3 - B*log(abs(tan(1/2*x) - 1))/a^3 + 1/60*(3*A
*a^12*tan(1/2*x)^5 - 3*B*a^12*tan(1/2*x)^5 + 10*A*a^12*tan(1/2*x)^3 - 20*B*
a^12*tan(1/2*x)^3 + 15*A*a^12*tan(1/2*x) - 105*B*a^12*tan(1/2*x))/a^15
```

Mupad [B]
time = 2.37, size = 92, normalized size = 1.23

$$\tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{12\,a^3}+\frac{A-3\,B}{12\,a^3}\right)+\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{4\,a^3}+\frac{A-3\,B}{4\,a^3}-\frac{A+3\,B}{4\,a^3}\right)+\frac{\tan\left(\frac{x}{2}\right)^5\,(A-B)}{20\,a^3}+\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^3,x)
```

```
[Out] tan(x/2)^3*((A - B)/(12*a^3) + (A - 3*B)/(12*a^3)) + tan(x/2)*((A - B)/(4*a
^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)) + (tan(x/2)^5*(A - B))/(20*a^3
) + (2*B*atanh(tan(x/2)))/a^3
```

3.192 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

Optimal. Leaf size=96

$$\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2}+\frac{2(3A-80B)\sin(x)}{105a^4(1+\cos(x))}+\frac{(A-B)\sin(x)}{7(a+a\cos(x))^4}+\frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3}$$

```
[Out] B*arctanh(sin(x))/a^4+1/105*(6*A-55*B)*sin(x)/a^4/(cos(x)+1)^2+2/105*(3*A-8
0*B)*sin(x)/a^4/(cos(x)+1)+1/7*(A-B)*sin(x)/(a+a*cos(x))^4+1/35*(3*A-10*B)*
sin(x)/a/(a+a*cos(x))^3
```

Rubi [A]
time = 0.28, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2907, 3057, 12, 3855}

$$\frac{2(3A-80B)\sin(x)}{105a^4(\cos(x)+1)}+\frac{(6A-55B)\sin(x)}{105a^4(\cos(x)+1)^2}+\frac{B\tanh^{-1}(\sin(x))}{a^4}+\frac{(3A-10B)\sin(x)}{35a(a\cos(x)+a)^3}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[x]])/a^4 + ((6*A - 55*B)*Sin[x])/(105*a^4*(1 + Cos[x])^2) +
(2*(3*A - 80*B)*Sin[x])/(105*a^4*(1 + Cos[x])) + ((A - B)*Sin[x])/(7*(a + a
*Cos[x])^4) + ((3*A - 10*B)*Sin[x])/(35*a*(a + a*Cos[x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^4}\,dx \\
&= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{\int \frac{(7aB+3a(A-B)\cos(x))\sec(x)}{(a+a\cos(x))^3}\,dx}{7a^2} \\
&= \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{(35a^2B+2a^2(3A-10B)\cos(x))\sec(x)}{(a+a\cos(x))^2}\,dx}{35a^4} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{\int \frac{(105a^3B+a^3(6A-55B}{a+a\cos(}}{105a^6} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x}{105\left(a^4+a^4\cos(x)\right.} \\
&= \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} + \frac{2(3A-80B)\sin(x}{105\left(a^4+a^4\cos(x)\right.} \\
&= \frac{B\tanh^{-1}(\sin(x))}{a^4} + \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3} +
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 104, normalized size = 1.08

$$\frac{-3360B\cos^8\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(96A-1055B+(87A-1480B)\cos(x)+(24A-535B)\cos(2x)+3A\cos(3x)-80B\cos(3x))\sin(x)}{210a^4(1+\cos(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
[Out] (-3360*B*Cos[x/2]^8*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
 (96*A - 1055*B + (87*A - 1480*B)*Cos[x] + (24*A - 535*B)*Cos[2*x] + 3*A*Co
s[3*x] - 80*B*Cos[3*x])*Sin[x])/(210*a^4*(1 + Cos[x])^4)
```

Maple [A]
time = 0.16, size = 93, normalized size = 0.97

method	result
default	$\frac{\frac{\left(\tan^{7}\left(\frac{x}{2}\right)\right)A}{7}-\frac{\left(\tan^{7}\left(\frac{x}{2}\right)\right)B}{7}+8B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+A\tan\left(\frac{x}{2}\right)-15B\tan\left(\frac{x}{2}\right)+\frac{3\left(\tan^{5}\left(\frac{x}{2}\right)\right)A}{5}-\left(\tan^{5}\left(\frac{x}{2}\right)\right)B+\left(\tan^{3}\left(\frac{x}{2}\right)\right)A-\frac{11\left(\tan^{3}\left(\frac{x}{2}\right)\right)B}{3}}{8a^{4}}$
norman	$\frac{\frac{(A-15B)\tan\left(\frac{x}{2}\right)}{8a}+\frac{(A-B)\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{56a}+\frac{(3A-11B)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{24a}+\frac{(3A-5B)\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{40a}}{a^{3}}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^{4}}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^{4}}$
risch	$-\frac{2i\left(105B\,e^{6ix}+735B\,e^{5ix}+2170B\,e^{4ix}-210A\,e^{3ix}+3430B\,e^{3ix}-126A\,e^{2ix}+2625B\,e^{2ix}-42A\,e^{ix}+1015B\,e^{ix}-6A+160B\right)}{105\left(e^{ix}+1\right)^{7}a^{4}}+\frac{B\ln(}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^4*(1/7*tan(1/2*x)^7*A-1/7*tan(1/2*x)^7*B+8*B*ln(tan(1/2*x)+1)+A*tan(1
/2*x)-15*B*tan(1/2*x)+3/5*tan(1/2*x)^5*A-tan(1/2*x)^5*B+tan(1/2*x)^3*A-11/3
*tan(1/2*x)^3*B-8*B*ln(tan(1/2*x)-1))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.49

$$-\frac{1}{168}B\left(\frac{\frac{315\sin(x)}{\cos(x)+1}+\frac{77\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{21\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{3\sin(x)^{7}}{(\cos(x)+1)^{7}}}{a^{4}}-\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^{4}}+\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^{4}}\right)+\frac{A\left(\frac{35\sin(x)}{\cos(x)+1}+\frac{35\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{21\sin(x)^{5}}{(\cos(x)+1)^{5}}+\frac{5\sin(x)^{7}}{(\cos(x)+1)^{7}}\right)}{280\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="maxima")
```

```
[Out] -1/168*B*((315*sin(x)/(cos(x) + 1) + 77*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)
^5/(cos(x) + 1)^5 + 3*sin(x)^7/(cos(x) + 1)^7)/a^4 - 168*log(sin(x)/(cos(x)
 + 1) + 1)/a^4 + 168*log(sin(x)/(cos(x) + 1) - 1)/a^4) + 1/280*A*(35*sin(x)
/(cos(x) + 1) + 35*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)^5/(cos(x) + 1)^5 + 5
*sin(x)^7/(cos(x) + 1)^7)/a^4
```

Fricas [A]
time = 5.25, size = 158, normalized size = 1.65

$$\frac{105\left(B\cos(x)^{4}+4B\cos(x)^{3}+6B\cos(x)^{2}+4B\cos(x)+B\right)\log(\sin(x)+1)-105\left(B\cos(x)^{4}+4B\cos(x)^{3}+6B\cos(x)^{2}+4B\cos(x)+B\right)\log(-\sin(x)+1)+2\left(2(3A-80B)\cos(x)^{3}+(24A-535B)\cos(x)^{2}+(39A-620B)\cos(x)+36A-260B\right)\sin(x)}{210\left(a^{4}\cos(x)^{4}+4a^{4}\cos(x)^{3}+6a^{4}\cos(x)^{2}+4a^{4}\cos(x)+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="fricas")
```

```
[Out] 1/210*(105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log(
sin(x) + 1) - 105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) +
B)*log(-sin(x) + 1) + 2*(2*(3*A - 80*B)*cos(x)^3 + (24*A - 535*B)*cos(x)^2
+ (39*A - 620*B)*cos(x) + 36*A - 260*B)*sin(x))/(a^4*cos(x)^4 + 4*a^4*cos(x
)^3 + 6*a^4*cos(x)^2 + 4*a^4*cos(x) + a^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx + \int \frac{B\sec(x)}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**4,x)
```

```
[Out] (Integral(A/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x) + In
tegral(B*sec(x)/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x))
/a**4
```

Giac [**A**]
time = 0.43, size = 126, normalized size = 1.31

$$\frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^4} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}x\right)^7 - 15\,Ba^{24}\tan\left(\frac{1}{2}x\right)^7 + 63\,Aa^{24}\tan\left(\frac{1}{2}x\right)^5 - 105\,Ba^{24}\tan\left(\frac{1}{2}x\right)^5 + 105\,Aa^{24}\tan\left(\frac{1}{2}x\right)^3 - 385\,Ba^{24}\tan\left(\frac{1}{2}x\right)^3 + 105\,Aa^{24}\tan\left(\frac{1}{2}x\right) - 1575\,Ba^{24}\tan\left(\frac{1}{2}x\right)}{840\,a^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="giac")
```

```
[Out] B*log(abs(tan(1/2*x) + 1))/a^4 - B*log(abs(tan(1/2*x) - 1))/a^4 + 1/840*(15
*A*a^24*tan(1/2*x)^7 - 15*B*a^24*tan(1/2*x)^7 + 63*A*a^24*tan(1/2*x)^5 - 10
5*B*a^24*tan(1/2*x)^5 + 105*A*a^24*tan(1/2*x)^3 - 385*B*a^24*tan(1/2*x)^3 +
 105*A*a^24*tan(1/2*x) - 1575*B*a^24*tan(1/2*x))/a^28
```

Mupad [**B**]
time = 2.34, size = 140, normalized size = 1.46

$$\tan\left(\frac{x}{2}\right)\left(\frac{A-B}{8\,a^4} - \frac{3\,B}{4\,a^4} + \frac{2\,A-4\,B}{8\,a^4} - \frac{2\,A+4\,B}{8\,a^4}\right) + \tan\left(\frac{x}{2}\right)^5\left(\frac{A-B}{40\,a^4} + \frac{2\,A-4\,B}{40\,a^4}\right) + \tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{24\,a^4} - \frac{B}{4\,a^4} + \frac{2\,A-4\,B}{24\,a^4}\right) + \frac{\tan\left(\frac{x}{2}\right)^7(A-B)}{56\,a^4} + \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^4,x)
```

```
[Out] tan(x/2)*((A - B)/(8*a^4) - (3*B)/(4*a^4) + (2*A - 4*B)/(8*a^4) - (2*A + 4*
B)/(8*a^4)) + tan(x/2)^5*((A - B)/(40*a^4) + (2*A - 4*B)/(40*a^4)) + tan(x/
2)^3*((A - B)/(24*a^4) - B/(4*a^4) + (2*A - 4*B)/(24*a^4)) + (tan(x/2)^7*(A
 - B))/(56*a^4) + (2*B*atanh(tan(x/2)))/a^4
```

3.193 $\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx$

Optimal. Leaf size=98

$$2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{5}aA(a+$$

```
[Out] 2*a^(5/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/5*a*A*(a+a*cos(x))
^(3/2)*sin(x)+2/15*a^3*(32*A+35*B)*sin(x)/(a+a*cos(x))^(1/2)+2/15*a^2*(8*A+
5*B)*sin(x)*(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2907, 3055, 3060, 2852, 212}

$$2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a\cos(x)+a}}+\frac{2}{15}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^3*(32*A + 3
5*B)*Sin[x])/(15*Sqrt[a + a*Cos[x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[x]
]*Sin[x])/15 + (2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx &= \int (a+a\cos(x))^{5/2}(B+A\cos(x))\sec(x)\,dx \\
&= \frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x)+\frac{2}{5}\int (a+a\cos(x))^{3/2}\left(\frac{5aB}{2}+\frac{1}{2}a(8A\right. \\
&= \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x \\
&= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+ \\
&= \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\,\sin(x)+ \\
&= 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}}+\frac{2}{15}a
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 78, normalized size = 0.80

$$\frac{1}{30}a^2\sqrt{a(1+\cos(x))}\,\sec\left(\frac{x}{2}\right)\left(30\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)+2(89A+80B+2(14A+5B)\cos(x)+3A\cos(2x))\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(30*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]]
+ 2*(89*A + 80*B + 2*(14*A + 5*B)*Cos[x] + 3*A*Cos[2*x])*Sin[x/2]))/30
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(80) = 160$.
time = 0.52, size = 230, normalized size = 2.35

method	result
default	$\frac{a^{\frac{3}{2}} \cos\left(\frac{x}{2}\right) \sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a} \left(24 A \sqrt{2}\, \sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a}\, \sqrt{a}\, \left(\sin^4\left(\frac{x}{2}\right)\right) - 20 \sqrt{a}\, \sqrt{2}\, \sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a}\, (4A+B)\left(\sin^2\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(5/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(24*A*2^(1/2)*(sin(1/2*x)^2*
a)^(1/2)*a^(1/2)*sin(1/2*x)^4-20*a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*(4*
A+B)*sin(1/2*x)^2+120*A*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2)+90*B*2^(1/2)
*(sin(1/2*x)^2*a)^(1/2)*a^(1/2)+15*B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2
)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a+15*B*ln(4/(2*co
s(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1
/2)+2*a))*a)/sin(1/2*x)/(a*cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 43, normalized size = 0.44

$$\frac{1}{30} \left(3 \sqrt{2}\, a^2 \sin\left(\frac{5}{2} x\right) + 25 \sqrt{2}\, a^2 \sin\left(\frac{3}{2} x\right) + 150 \sqrt{2}\, a^2 \sin\left(\frac{1}{2} x\right)\right) A \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*x) + 25*sqrt(2)*a^2*sin(3/2*x) + 150*sqrt(2)*a^
2*sin(1/2*x))*A*sqrt(a)
```

Fricas [A]
time = 2.51, size = 123, normalized size = 1.26

$$\frac{15 \left(B a^2 \cos(x) + B a^2\right) \sqrt{a} \log\left(\frac{a \cos(x)^3 - 7 a \cos(x)^2 - 4 \sqrt{a \cos(x) + a}\, \sqrt{a}\, (\cos(x) - 2) \sin(x) + 8 a}{\cos(x)^3 + \cos(x)^2}\right) + 4 \left(3 A a^2 \cos(x)^2 + (14 A + 5 B) a^2 \cos(x) + (43 A + 40 B) a^2\right) \sqrt{a \cos(x) + a}\, \sin(x)}{30 (\cos(x) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(B*a^2*cos(x) + B*a^2)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*
sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)
) + 4*(3*A*a^2*cos(x)^2 + (14*A + 5*B)*a^2*cos(x) + (43*A + 40*B)*a^2)*sqrt
(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(5/2)*(A+B*sec(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 134, normalized size = 1.37

$$\frac{1}{30}\sqrt{2}\left(48\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^5 - 160\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^3 - 40\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^3 - 15\,\sqrt{2}\,Ba^2\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right) + 240\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right) + 180\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(48*A*a^2*sgn(cos(1/2*x))*sin(1/2*x)^5 - 160*A*a^2*sgn(cos(1/2
*x))*sin(1/2*x)^3 - 40*B*a^2*sgn(cos(1/2*x))*sin(1/2*x)^3 - 15*sqrt(2)*B*a^
2*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos
(1/2*x)) + 240*A*a^2*sgn(cos(1/2*x))*sin(1/2*x) + 180*B*a^2*sgn(cos(1/2*x))
*sin(1/2*x))*sqrt(a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,\cos(x))^{5/2}\left(A+\frac{B}{\cos(x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(5/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(5/2)*(A + B/cos(x)), x)
```

3.194 $\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx$

Optimal. Leaf size=72

$$2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\sin(x)$$

```
[Out] 2*a^(3/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/3*a^2*(4*A+3*B)*si
n(x)/(a+a*cos(x))^(1/2)+2/3*a*A*sin(x)*(a+a*cos(x))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2907, 3055, 3060, 2852, 212}

$$2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a\cos(x)+a}}+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

```
[Out] 2*a^(3/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a^2*(4*A + 3*
B)*Sin[x])/(3*Sqrt[a + a*Cos[x]]) + (2*a*A*Sqrt[a + a*Cos[x]]*Sin[x])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx &= \int (a+a\cos(x))^{3/2}(B+A\cos(x))\sec(x)\,dx\\ &= \frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x)+\frac{2}{3}\int \sqrt{a+a\cos(x)}\left(\frac{3aB}{2}+\frac{1}{2}a(4A\right.\\ &= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x)+(aB)\int \sqrt{a+}\\ &= \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\sqrt{a+a\cos(x)}\,\sin(x)-\left(2a^2B\right)\text{Subst}\\ &= 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}}+\frac{2}{3}aA\end{aligned}$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 0.86

$$\frac{1}{3}a\sqrt{a(1+\cos(x))}\,\sec\left(\frac{x}{2}\right)\left(3\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)+2(5A+3B+A\cos(x))\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(3*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2
*(5*A + 3*B + A*Cos[x])*Sin[x/2]))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(58) = 116$.
time = 0.44, size = 201, normalized size = 2.79

method	result
default	$\frac{\sqrt{a}\,\cos\left(\frac{x}{2}\right)\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}\,\left(-4A\sqrt{2}\,\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}\,\sqrt{a}\,\left(\sin^2\left(\frac{x}{2}\right)\right)+12A\sqrt{2}\,\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}\,\sqrt{a}+6B\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(3/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(-4*A*2^(1/2)*(sin(1/2*x)^2*a
)^(1/2)*a^(1/2)*sin(1/2*x)^2+12*A*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2)+6*
B*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2)+3*B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(
a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a+3*B*ln(
4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^
2*a)^(1/2)+2*a))*a)/sin(1/2*x)/(a*cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.55, size = 26, normalized size = 0.36

$$\frac{1}{3}\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,x\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,x\right)\right)A\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*x) + 9*sqrt(2)*a*sin(1/2*x))*A*sqrt(a)
```

Fricas [A]
time = 4.82, size = 99, normalized size = 1.38

$$\frac{3\,(Ba\cos{(x)}+Ba)\sqrt{a}\,\log\left(\frac{a\cos{(x)}^3-7\,a\cos{(x)}^2-4\,\sqrt{a\cos{(x)}+a}\,\sqrt{a}\,(\cos{(x)}-2)\sin{(x)}+8\,a}{\cos{(x)}^3+\cos{(x)}^2}\right)+4\,(Aa\cos{(x)}+(5\,A+3\,B)a)\sqrt{a\cos{(x)}+a}\,\sin{(x)}}{6\,(\cos{(x)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(B*a*cos(x) + B*a)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a
*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*
(A*a*cos(x) + (5*A + 3*B)*a)*sqrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\cos(x)+1)\right)^{\frac{3}{2}}\left(A+B\sec(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(3/2)*(A+B*sec(x)),x)
[Out] Integral((a*(cos(x) + 1))**(3/2)*(A + B*sec(x)), x)
```

Giac [**A**]
time = 0.41, size = 92, normalized size = 1.28

$$-\frac{1}{6}\sqrt{2}\left(8\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^{3}+3\,\sqrt{2}\,Ba\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-24\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)-12\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="giac")
[Out] -1/6*sqrt(2)*(8*A*a*sgn(cos(1/2*x))*sin(1/2*x)^3 + 3*sqrt(2)*B*a*log(abs(-2
*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos(1/2*x)) - 2
4*A*a*sgn(cos(1/2*x))*sin(1/2*x) - 12*B*a*sgn(cos(1/2*x))*sin(1/2*x))*sqrt(
a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,\cos\left(x\right)\right)}^{3/2}\,\left(A+\frac{B}{\cos\left(x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(3/2)*(A + B/cos(x)),x)
[Out] int((a + a*cos(x))^(3/2)*(A + B/cos(x)), x)
```

3.195 $\int \sqrt{a+a\cos(x)}\,(A+B\sec(x))\,dx$

Optimal. Leaf size=44

$$2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))*a^(1/2)+2*a*A*sin(x)/(a+a*co
s(x))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2907, 3060, 2852, 212}

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] 2*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a*A*Sin[x])/S
qrt[a + a*Cos[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(x)}\,(A+B\sec(x))\,dx &= \int \sqrt{a+a\cos(x)}\,(B+A\cos(x))\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}+B\int \sqrt{a+a\cos(x)}\,\sec(x)\,dx\\ &= \frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}-(2aB)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\\ &= 2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.07

$$\sqrt{a(1+\cos(x))}\,\sec\left(\frac{x}{2}\right)\left(\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)+2A\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

```
[Out] Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*A*Si
n[x/2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(36) = 72$.
time = 0.42, size = 154, normalized size = 3.50

method	result

default	$\frac{\cos\left(\frac{x}{2}\right)\sqrt{\left(\sin^{2}\left(\frac{x}{2}\right)\right)a}\,\left(2A\sqrt{2}\,\sqrt{\left(\sin^{2}\left(\frac{x}{2}\right)\right)a}\,\sqrt{a}+B\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{x}{2}\right)-\sqrt{a}\,\sqrt{2}\,\sqrt{\left(\sin^{2}\left(\frac{x}{2}\right)\right)a}-2a\right)}{2\cos\left(\frac{x}{2}\right)-\sqrt{2}}\right.\right.}{\sqrt{a}\,\sin\left(\frac{x}{2}\right)\sqrt{a\left(\cos^{2}\left(\frac{x}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(x))^(1/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(2*A*2^(1/2)*(sin(1/2*x)^2*a)^(
1/2)*a^(1/2)+B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2
^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1
/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a)/sin(1/2*x)/(
a*cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 13, normalized size = 0.30

$$2\sqrt{2}\,A\sqrt{a}\,\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*A*sqrt(a)*sin(1/2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(36) = 72.
time = 2.96, size = 81, normalized size = 1.84

$$\frac{(B\cos(x)+B)\sqrt{a}\,\log\left(\frac{a\cos(x)^{3}-7\,a\cos(x)^{2}-4\,\sqrt{a\cos(x)+a}\,\sqrt{a}\,(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^{3}+\cos(x)^{2}}\right)+4\,\sqrt{a\cos(x)+a}\,A\sin(x)}{2\,(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x
) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*sqrt(a
*cos(x) + a)*A*sin(x))/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\cos(x)+1\right)}\,\left(A+B\sec(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))**(1/2)*(A+B*sec(x)),x)
```

```
[Out] Integral(sqrt(a*(cos(x) + 1))*(A + B*sec(x)), x)
```

Giac [A]
time = 0.42, size = 61, normalized size = 1.39

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\,B\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-4A\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) +
4*sin(1/2*x)))*sgn(cos(1/2*x)) - 4*A*sgn(cos(1/2*x))*sin(1/2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a+a\,\cos\left(x\right)}\,\left(A+\frac{B}{\cos\left(x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cos(x))^(1/2)*(A + B/cos(x)),x)
```

```
[Out] int((a + a*cos(x))^(1/2)*(A + B/cos(x)), x)
```

3.196 $\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx$

Optimal. Leaf size=68

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(1/2)+(A-B)*arctanh(1/2*si
n(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2907, 3064, 2728, 212, 2852}

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/Sqrt[a] + (Sqrt[2]*(A -
B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx &= \int \frac{(B+A\cos(x))\sec(x)}{\sqrt{a+a\cos(x)}}\,dx\\ &= \frac{B\int \sqrt{a+a\cos(x)}\,\sec(x)\,dx}{a} - (-A+B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx\\ &= -\left((2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)\right) - (2B)\text{Subst}\left(\int \frac{}{a}\right.\\ &= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}} + \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 0.76

$$\frac{2\left((A-B)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)\right)\cos\left(\frac{x}{2}\right)}{\sqrt{a(1+\cos(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
[Out] (2*((A - B)*ArcTanh[Sin[x/2]] + Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]])*Cos[x/
2])/Sqrt[a*(1 + Cos[x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(53) = 106$.
time = 0.46, size = 194, normalized size = 2.85

method	result
default	$\frac{\cos\left(\frac{x}{2}\right)\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a}\left(\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a}+4a}{\cos\left(\frac{x}{2}\right)}\right) A-\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right) a}+4a}{\cos\left(\frac{x}{2}\right)}\right) B+B\ln\left(\right.\right.}{\sqrt{a}\ \sin\left(\frac{x}{2}\right)\sqrt{a\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(sin(1/
2*x)^2*a)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(sin(1/2*x)^2*a)^(1/
2)+a))*B+B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/
2)*(sin(1/2*x)^2*a)^(1/2)-2*a))+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*co
s(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)))/a^(1/2)/sin(1/2*x)/(
a*cos(1/2*x)^2)^(1/2)
```

Maxima [A]
time = 0.54, size = 58, normalized size = 0.85

$$\frac{\left(\sqrt{2}\ \log\left(\cos\left(\frac{1}{2}\,x\right)^2+\sin\left(\frac{1}{2}\,x\right)^2+2\,\sin\left(\frac{1}{2}\,x\right)+1\right)-\sqrt{2}\ \log\left(\cos\left(\frac{1}{2}\,x\right)^2+\sin\left(\frac{1}{2}\,x\right)^2-2\,\sin\left(\frac{1}{2}\,x\right)+1\right)\right)A}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 + 2*sin(1/2*x) + 1) - sqrt(2)*
log(cos(1/2*x)^2 + sin(1/2*x)^2 - 2*sin(1/2*x) + 1))*A/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.
time = 3.68, size = 116, normalized size = 1.71

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\ \log\left(-\frac{\cos(x)^2+\frac{2\sqrt{2}\ \sqrt{a\cos\left(x\right)+a}\ \sin(x)}{\sqrt{a}}-2\,\cos(x)-3}{\cos(x)^2+2\,\cos(x)+1}\right)-B\sqrt{a}\ \log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos\left(x\right)+a}\ \sqrt{a}\,(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)
*sin(x)/sqrt(a) - 2*cos(x) - 3)/(cos(x)^2 + 2*cos(x) + 1)) - B*sqrt(a)*log(
(a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(
x) + 8*a)/(cos(x)^3 + cos(x)^2)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(x)}}{\sqrt{a(\cos{(x)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral((A + B*sec(x))/sqrt(a*(cos(x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(53) = 106.
time = 0.44, size = 120, normalized size = 1.76

$$\frac{\sqrt{2}\,(A\sqrt{a} - B\sqrt{a})\log\left(\sin\left(\frac{1}{2}x\right) + 1\right)}{2\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{\sqrt{2}\,(A\sqrt{a} - B\sqrt{a})\log\left(-\sin\left(\frac{1}{2}x\right) + 1\right)}{2\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} + \frac{B\log\left(\left|\frac{1}{2}\sqrt{2} + \sin\left(\frac{1}{2}x\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{B\log\left(\left|-\frac{1}{2}\sqrt{2} + \sin\left(\frac{1}{2}x\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*x) + 1)/(a*sgn(cos(1/2*x)))
 - 1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*x) + 1)/(a*sgn(cos(1/2*
x))) + B*log(abs(1/2*sqrt(2) + sin(1/2*x)))/(sqrt(a)*sgn(cos(1/2*x))) - B*l
og(abs(-1/2*sqrt(2) + sin(1/2*x)))/(sqrt(a)*sgn(cos(1/2*x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(x)}}{\sqrt{a + a\,\cos\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(1/2),x)
```

```
[Out] int((A + B/cos(x))/(a + a*cos(x))^(1/2), x)
```

3.197 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{3/2}}+\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(3/2)+1/2*(A-B)*sin(x)/(a+
a*cos(x))^(3/2)+1/4*(A-5*B)*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))
^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2907, 3057, 3064, 2728, 212, 2852}

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a\cos(x)+a}}\right)}{2\sqrt{2}\,a^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{3/2}}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(3/2) + ((A - 5*B)*Arc
Tanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(2*Sqrt[2]*a^(3/2)) +
((A - B)*Sin[x])/(2*(a + a*Cos[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{\int \frac{\left(2aB+\frac{1}{2}a(A-B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{2a^2}$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} + \frac{(A-5B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{4a} + \frac{B\int \sqrt{a+a\cos(x)}\,\sec(x)\,}{a^2}$$
$$= \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}} - \frac{(A-5B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{2a} - \frac{(2B)\text{Su}}{}$$
$$= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{3/2}} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{2\sqrt{2}\,a^{3/2}} + \frac{(A}{2(a}$$

Mathematica [**A**]
time = 0.27, size = 73, normalized size = 0.79

$$\frac{(A-5B)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right) + 4\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right) + \frac{1}{2}(A-B)\sin(x)}{(a(1+\cos(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x
/2]]*Cos[x/2]^3 + ((A - B)*Sin[x])/2)/(a*(1 + Cos[x]))^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(71) = 142.
time = 0.50, size = 270, normalized size = 2.93

method	result
default	$\frac{\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}\left(A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}+4a}{\cos\left(\frac{x}{2}\right)}\right)\left(\cos^2\left(\frac{x}{2}\right)\right)a-5B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}+4a}{\cos\left(\frac{x}{2}\right)}\right)a(c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(A*2^(1/2)*ln(2*(2*a^(1/2)*(s
in(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a-5*B*2^(1/2)*ln(2*(2*a^
(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^2+4*B*ln(-4/(2*c
os(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(
1/2)-2*a))*a*cos(1/2*x)^2+4*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/
2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a*cos(1/2*x)^2+A*2^(1/2)*
(sin(1/2*x)^2*a)^(1/2)*a^(1/2)-B*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2))/si
n(1/2*x)/(a*cos(1/2*x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9153 vs. $2(71) = 142$.
time = 1.09, size = 9153, normalized size = 99.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(32*(cos(3/2*x)*sin(2*x) + cos(2*x)*sin(3/2*x) + cos(x)*sin(3/2*x) + co
s(3/2*x)*sin(x))*cos(3*x)^2 + 96*(cos(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(2*
x) - (3*cos(x) + 1)*sin(3/2*x) - cos(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/2*x
) + 3*cos(3/2*x)*sin(x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 96*(c
os(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/2*x) - co
s(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*cos(2/3*ar
ctan2(sin(3/2*x), cos(3/2*x)))^2 - 32*(cos(3/2*x)*sin(2*x) + cos(2*x)*sin(3
/2*x) + cos(x)*sin(3/2*x) + cos(3/2*x)*sin(x))*sin(3*x)^2 + 32*(6*cos(x) +
1)*cos(2*x)*sin(3/2*x) + 96*cos(2*x)^2*sin(3/2*x) + 96*sin(2*x)^2*sin(3/2*x
) + 96*(cos(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/
2*x) - cos(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*s
in(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 96*(cos(3/2*x)*sin(3*x) + 3*cos
(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/2*x) - cos(3*x)*sin(3/2*x) - 3*cos(
2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*sin(2/3*arctan2(sin(3/2*x), cos(3/2*
x)))^2 + 32*(2*(3*cos(x) + 1)*cos(2*x)*sin(3/2*x) + 3*cos(2*x)^2*sin(3/2*x)
 + 3*sin(2*x)^2*sin(3/2*x) + 2*(3*sin(3/2*x)*sin(x) + cos(3/2*x))*sin(2*x)
+ (3*cos(x)^2 + 3*sin(x)^2 + 2*cos(x))*sin(3/2*x) + 2*cos(3/2*x)*sin(x))*co
s(3*x) - 4*(6*(sin(2*x) + sin(x))*sin(3*x)^2 + sin(3*x)^3 + (2*(3*cos(2*x)
+ 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2
*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos
(x) + 1)*sin(3*x) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2
+ 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin
(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2
 + 6*cos(x) + 1)*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x))) + 3*(2*(3*cos(2*x
) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos
(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(
2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(2/3*arctan2(si
```

```
n(3/2*x), cos(3/2*x))))*cos(5/3*arctan2(sin(3/2*x), cos(3/2*x))) - 4*(8*cos
(3*x)^2*sin(3/2*x) - 72*cos(2*x)^2*sin(3/2*x) - 144*cos(2*x)*cos(x)*sin(3/2
*x) - 8*sin(3*x)^2*sin(3/2*x) - 72*sin(2*x)^2*sin(3/2*x) - 16*(3*cos(3/2*x)
*sin(2*x) + 3*cos(3/2*x)*sin(x) - sin(3/2*x))*cos(3*x) - 48*(cos(3/2*x)*sin
(3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/2*x) - cos(3*x)*sin(3/
2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*cos(2/3*arctan2(sin(3/2
*x), cos(3/2*x))) - 16*(cos(3*x)*cos(3/2*x) + 3*sin(2*x)*sin(3/2*x) + 3*sin
(3/2*x)*sin(x) + cos(3/2*x))*sin(3*x) - 48*(3*sin(3/2*x)*sin(x) + cos(3/2*x
))*sin(2*x) - 8*(9*cos(x)^2 + 9*sin(x)^2 - 1)*sin(3/2*x) - 48*cos(3/2*x)*si
n(x) + 3*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x)
 + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x)
 + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) +
 1)*sin(1/3*arctan2(sin(3/2*x), cos(3/2*x))))*cos(4/3*arctan2(sin(3/2*x), c
os(3/2*x))) - 4*(8*cos(3*x)^2*sin(3/2*x) - 72*cos(2*x)^2*sin(3/2*x) - 144*c
os(2*x)*cos(x)*sin(3/2*x) - 8*sin(3*x)^2*sin(3/2*x) - 72*sin(2*x)^2*sin(3/2
*x) - 16*(3*cos(3/2*x)*sin(2*x) + 3*cos(3/2*x)*sin(x) - sin(3/2*x))*cos(3*x
) - 16*(cos(3*x)*cos(3/2*x) + 3*sin(2*x)*sin(3/2*x) + 3*sin(3/2*x)*sin(x) +
 cos(3/2*x))*sin(3*x) - 48*(3*sin(3/2*x)*sin(x) + cos(3/2*x))*sin(2*x) - 8*
(9*cos(x)^2 + 9*sin(x)^2 - 1)*sin(3/2*x) - 48*cos(3/2*x)*sin(x) + 3*(2*(3*c
os(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) +
 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 +
9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*sin(1/3*arct
an2(sin(3/2*x), cos(3/2*x))))*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x))) + 4*
(6*(sin(2*x) + sin(x))*sin(3*x)^2 + sin(3*x)^3 + (2*(3*cos(2*x) + 3*cos(x)
+ 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*c
os(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*si
n(3*x))*cos(1/3*arctan2(sin(3/2*x), cos(3/2*x))) + (2*(3*cos(2*x) + 3*cos(x
) + 2)*cos(3*x)^3 + cos(3*x)^4 + 6*(sin(2*x) + sin(x))*sin(3*x)^3 + sin(3*x
)^4 + 3*(6*(cos(x) + 1)*cos(2*x) + 3*cos(2*x)^2 + 3*cos(x)^2 + 3*sin(2*x)^2
 + 6*sin(2*x)*sin(x) + 3*sin(x)^2 + 6*cos(x) + 2)*cos(3*x)^2 + 9*(2*(3*cos(
2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*
cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*s
in(2*x)^2 + 18*sin(2*x)*sin(x) + 9*sin(x)^2 + 6*cos(x) + 1)*cos(4/3*arctan2
(sin(3/2*x), cos(3/2*x)))^2 + 9*(2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + c
os(3*x)^2 + 6*(3*cos(x) + 1)*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 6*(sin(
2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) +
9*sin(x)^2 + 6*cos(x) + 1)*cos(2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + (2*
(3*cos(2*x) + 3*cos(x) + 2)*cos(3*x) + 2*cos(3*x)^2 + 6*(3*cos(x) + 1)*cos(
2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(x) + 9*si
n(x)^2 + 6*cos(x) + 2)*sin(3*x)^2 + 9*(2*(3*cos...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(71) = 142$.

time = 2.93, size = 187, normalized size = 2.03

$$-\frac{\sqrt{2}\left((A-5B)\cos(x)^2+2(A-5B)\cos(x)+A-5B\right)\sqrt{a}\log\left(-\frac{a\cos(x)^2+2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{a}\sin(x)-2a\cos(x)-3a}{\cos(x)^2+2\cos(x)+1}\right)-4\left(B\cos(x)^2+2B\cos(x)+B\right)\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8a}{\cos(x)^3+\cos(x)^2}\right)-4\sqrt{a\cos(x)+a}(A-B)\sin(x)}{8\left(a^2\cos(x)^2+2a^2\cos(x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*((A - 5*B)*cos(x)^2 + 2*(A - 5*B)*cos(x) + A - 5*B)*sqrt(a)*l
og(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x)
- 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 4*(B*cos(x)^2 + 2*B*cos(x) + B)*sqrt(a)
*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)
*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) - 4*sqrt(a*cos(x) + a)*(A - B)*sin(x)
)/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(x)}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(3/2),x)
```

```
[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(71) = 142.
time = 0.43, size = 158, normalized size = 1.72

$$\frac{B\log\left(\frac{\left|-2\sqrt{2}-4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}-4\sin\left(\frac{1}{2}x\right)\right|}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}+\frac{\sqrt{2}\left(A\sqrt{a}-5B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}x\right)+1\right)}{8a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}-5B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}x\right)+1\right)}{8a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}-\frac{\sqrt{2}\left(A\sqrt{a}\sin\left(\frac{1}{2}x\right)-B\sqrt{a}\sin\left(\frac{1}{2}x\right)\right)}{4\left(\sin\left(\frac{1}{2}x\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

```
[Out] B*log(abs(-2*sqrt(2) - 4*sin(1/2*x))/abs(2*sqrt(2) - 4*sin(1/2*x)))/(a^(3/2
)*sgn(cos(1/2*x))) + 1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log(sin(1/2*x) +
 1)/(a^2*sgn(cos(1/2*x))) - 1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log(-sin(
1/2*x) + 1)/(a^2*sgn(cos(1/2*x))) - 1/4*sqrt(2)*(A*sqrt(a)*sin(1/2*x) - B*s
qrt(a)*sin(1/2*x))/((sin(1/2*x)^2 - 1)*a^2*sgn(cos(1/2*x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(x)}}{{(a+a\,\cos(x))}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(3/2),x)
```

```
[Out] int((A + B/cos(x))/(a + a*cos(x))^(3/2), x)
```

3.198 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

Optimal. Leaf size=120

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{5/2}}+\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a+a\cos(x)}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}}+\frac{(3A-1}{16a(a+}$$

```
[Out] 2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)+1/4*(A-B)*sin(x)/(a+
a*cos(x))^(5/2)+1/16*(3*A-11*B)*sin(x)/a/(a+a*cos(x))^(3/2)+1/32*(3*A-43*B)
*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2907, 3057, 3064, 2728, 212, 2852}

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{2}\,\sqrt{a\cos(x)+a}}\right)}{16\sqrt{2}\,a^{5/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{a^{5/2}}+\frac{(3A-11B)\sin(x)}{16a(a\cos(x)+a)^{3/2}}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/a^(5/2) + ((3*A - 43*B)*
ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(16*Sqrt[2]*a^(5/2)
) + ((A - B)*Sin[x])/(4*(a + a*Cos[x])^(5/2)) + ((3*A - 11*B)*Sin[x])/(16*a
*(a + a*Cos[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \int \frac{(B+A\cos(x))\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}} + \frac{\int \frac{\left(4aB+\frac{3}{2}a(A-B)\cos(x)\right)\sec(x)}{(a+a\cos(x))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}} + \frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}} + \frac{\int \frac{\left(8a^2B+\frac{1}{4}a^2(3A-11B)\cos(x)\right)\sec(x)}{\sqrt{a+a\cos(x)}}\,dx}{8a^4}$$

$$= \frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}} + \frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}} + \frac{(3A-43B)\int \frac{1}{\sqrt{a+a\cos(x)}}\,dx}{32a^2} +$$

$$= \frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}} + \frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}} - \frac{(3A-43B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\right.}{16a^2}$$

$$= \frac{2B\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{5/2}} + \frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{16\sqrt{2}\,a^{5/2}} +$$

Mathematica [A]
time = 0.37, size = 95, normalized size = 0.79

$$\frac{2(3A-43B)\tanh^{-1}\left(\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right)+64\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right)+(7A-15B+3A\cos(x)-11B\cos(x))\tan\left(\frac{x}{2}\right)}{16a(a(1+\cos(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

```
[Out] (2*(3*A - 43*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 64*Sqrt[2]*B*ArcTanh[Sqrt[2]
*Sin[x/2]]*Cos[x/2]^3 + (7*A - 15*B + 3*A*Cos[x] - 11*B*Cos[x])*Tan[x/2])/(
16*a*(a*(1 + Cos[x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(95) = 190.
time = 0.60, size = 322, normalized size = 2.68

method	result

default	$\frac{\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}\left(3A\ln\left(\frac{4\sqrt{a}\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}+4a}{\cos\left(\frac{x}{2}\right)}\right)\sqrt{2}\left(\cos^4\left(\frac{x}{2}\right)\right)a-43B\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{\left(\sin^2\left(\frac{x}{2}\right)\right)a}+4a}{\cos\left(\frac{x}{2}\right)}\right)a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(x))/(a+a*cos(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*x)^3*(sin(1/2*x)^2*a)^(1/2)*(3*A*ln(2*(2*a^(1/2)*(sin(
1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*2^(1/2)*cos(1/2*x)^4*a-43*B*2^(1/2)*ln(2
*(2*a^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^4+32*B*ln(
-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)
^2*a)^(1/2)-2*a))*a*cos(1/2*x)^4+32*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2
)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a*cos(1/2*x)^4+3*
A*a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*cos(1/2*x)^2-11*B*a^(1/2)*2^(1/2)*
(sin(1/2*x)^2*a)^(1/2)*cos(1/2*x)^2+2*A*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1
/2)-2*B*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2))/sin(1/2*x)/(a*cos(1/2*x)^2)
^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50224 vs. $2(95) = 190$.
time = 23.51, size = 50224, normalized size = 418.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(512*((2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(4*x) + 2*cos(5
/2*x)*sin(3*x) + (2*cos(2*x) + cos(x))*sin(5/2*x) + cos(4*x)*sin(5/2*x) + 2
*cos(3*x)*sin(5/2*x))*cos(5*x)^2 + 2560*(5*(2*sin(2*x) + sin(x))*cos(5/2*x)
 + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x) - (
10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4*x)*s
in(5/2*x) - 10*cos(3*x)*sin(5/2*x))*cos(8/5*arctan2(sin(5/2*x), cos(5/2*x))
)^2 + 10240*(5*(2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x) + 5*c
os(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x) + 5*cos(x) + 1)*
sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x) - 10*cos(3*x)*sin(
5/2*x))*cos(6/5*arctan2(sin(5/2*x), cos(5/2*x)))^2 + 10240*(5*(2*sin(2*x) +
 sin(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(
5/2*x)*sin(3*x) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/
2*x) - 5*cos(4*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*cos(4/5*arctan2(sin(
5/2*x), cos(5/2*x)))^2 + 2560*(5*(2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2
```

*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x
) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x)
- 10*cos(3*x)*sin(5/2*x))*cos(2/5*arctan2(sin(5/2*x), cos(5/2*x)))^2 - 512*
((2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(4*x) + 2*cos(5/2*x)*sin(
3*x) + (2*cos(2*x) + cos(x))*sin(5/2*x) + cos(4*x)*sin(5/2*x) + 2*cos(3*x)*
sin(5/2*x))*sin(5*x)^2 + 2560*cos(4*x)^2*sin(5/2*x) + 1024*(20*cos(2*x) + 1
0*cos(x) + 1)*cos(3*x)*sin(5/2*x) + 10240*cos(3*x)^2*sin(5/2*x) + 2560*sin(
4*x)^2*sin(5/2*x) + 10240*sin(3*x)^2*sin(5/2*x) + 2560*(5*(2*sin(2*x) + sin
(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*
x)*sin(3*x) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x)
 - 5*cos(4*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*sin(8/5*arctan2(sin(5/2*
x), cos(5/2*x)))^2 + 10240*(5*(2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)
*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x) +
 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x) - 1
0*cos(3*x)*sin(5/2*x))*sin(6/5*arctan2(sin(5/2*x), cos(5/2*x)))^2 + 10240*(
5*(2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin
(4*x) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) -
cos(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*sin(4
/5*arctan2(sin(5/2*x), cos(5/2*x)))^2 + 2560*(5*(2*sin(2*x) + sin(x))*cos(5
/2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x
) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4
*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*sin(2/5*arctan2(sin(5/2*x), cos(5/
2*x)))^2 + 512*(5*cos(4*x)^2*sin(5/2*x) + 4*(10*cos(2*x) + 5*cos(x) + 1)*co
s(3*x)*sin(5/2*x) + 20*cos(3*x)^2*sin(5/2*x) + 5*sin(4*x)^2*sin(5/2*x) + 20
*sin(3*x)^2*sin(5/2*x) + 2*((10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) + 10*co
s(3*x)*sin(5/2*x))*cos(4*x) + 2*(2*sin(2*x) + sin(x))*cos(5/2*x) + 2*(5*(2*
sin(2*x) + sin(x))*sin(5/2*x) + 10*sin(3*x)*sin(5/2*x) + cos(5/2*x))*sin(4*
x) + 4*(5*(2*sin(2*x) + sin(x))*sin(5/2*x) + cos(5/2*x))*sin(3*x) + (4*(5*c
os(x) + 1)*cos(2*x) + 20*cos(2*x)^2 + 5*cos(x)^2 + 20*sin(2*x)^2 + 20*sin(2
*x)*sin(x) + 5*sin(x)^2 + 2*cos(x))*sin(5/2*x))*cos(5*x) + 512*((20*cos(2*x
) + 10*cos(x) + 1)*sin(5/2*x) + 20*cos(3*x)*sin(5/2*x))*cos(4*x) + 512*(2*s
in(2*x) + sin(x))*cos(5/2*x) - 12*(10*(sin(4*x) + 2*sin(3*x) + 2*sin(2*x) +
 sin(x))*sin(5*x)^2 + sin(5*x)^3 + (2*(5*cos(4*x) + 10*cos(3*x) + 10*cos(2*
x) + 5*cos(x) + 1)*cos(5*x) + cos(5*x)^2 + 10*(10*cos(3*x) + 10*cos(2*x) +
5*cos(x) + 1)*cos(4*x) + 25*cos(4*x)^2 + 20*(10*cos(2*x) + 5*cos(x) + 1)*co
s(3*x) + 100*cos(3*x)^2 + 20*(5*cos(x) + 1)*cos(2*x) + 100*cos(2*x)^2 + 25*
cos(x)^2 + 50*(2*sin(3*x) + 2*sin(2*x) + sin(x))*sin(4*x) + 25*sin(4*x)^2 +
 100*(2*sin(2*x) + sin(x))*sin(3*x) + 100*sin(3*x)^2 + 100*sin(2*x)^2 + 100
*sin(2*x)*sin(x) + 25*sin(x)^2 + 10*cos(x) + 1)*sin(5*x) + 5*(2*(5*cos(4*x)
 + 10*cos(3*x) + 10*cos(2*x) + 5*cos(x) + 1)*cos(5*x) + cos(5*x)^2 + 10*(10
*cos(3*x) + 10*cos(2*x) + 5*cos(x) + 1)*cos(4*x) + 25*cos(4*x)^2 + 20*(10*c
os(2*x) + 5*cos(x) + 1)*cos(3*x) + 100*cos(3*x)^2 + 20*(5*cos(x) + 1)*cos(2
*x) + 100*cos(2*x)^2 + 25*cos(x)^2 + 10*(sin(4*x) + 2*sin(3*x) + 2*sin(2*x)
 + sin(x))*sin(5*x) + sin(5*x)^2 + 50*(2*sin(3*x) + 2*sin(2*x) + sin(x))*si
n(4*x) + 25*sin(4*x)^2 + 100*(2*sin(2*x) + sin(x))*sin(3*x) + 100*sin(3*x)^

```
2 + 100*sin(2*x)^2 + 100*sin(2*x)*sin(x) + 25*sin(x)^2 + 10*cos(x) + 1)*sin
(8/5*arctan2(sin(5/2*x), cos(5/2*x))) + 10*(2*(5*cos(4*x) + 10*cos(3*x) + 1
0*cos(2*x) + 5*cos(x) + 1)*cos(5*x) + cos(5*x)^2 + 10*(10*cos(3*x) + 10*cos
(2*x) + 5*cos(x) + 1)*cos(4*x) + 25*cos(4*x)^2 + 20*(10*cos(2*x) + 5*cos(x)
 + 1)*cos(3*x) + 100*cos(3*x)^2 + 20*(5*cos(x) + 1)*cos(2*x) + 100*cos(2*x)
^2 + 25*cos(x)^2 + 10*(sin(4*x) + 2*sin(3*x) + 2*sin(2*x) + sin(x))*sin(5*x
) + sin(5*x)^2 + 50*(2*sin(3*x) + 2*sin(2*x) + sin(x))*sin(4*x) + 25*sin(4*
x)^2 + 100*(2*sin(2*x) + sin(x))*sin(3*x) + 100...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(95) = 190$.
time = 3.46, size = 234, normalized size = 1.95

$$-\frac{\sqrt{2}\left((3A-43B)\cos(x)^3+3(3A-43B)\cos(x)^2+3(3A-43B)\cos(x)+3A-43B\right)\sqrt{a}\log\left(-\frac{a\cos(x)^2+2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{a}\sin(x)-2a\cos(x)-3a}{\cos(x)^2+2\cos(x)+1}\right)-32\left(B\cos(x)^3+3B\cos(x)^2+3B\cos(x)+B\right)\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8a}{\cos(x)^3+\cos(x)^2}\right)-4\left((3A-11B)\cos(x)+7A-15B\right)\sqrt{a\cos(x)+a}\sin(x)}{64\left(a^3\cos(x)^3+3a^3\cos(x)^2+3a^3\cos(x)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*((3*A - 43*B)*cos(x)^3 + 3*(3*A - 43*B)*cos(x)^2 + 3*(3*A -
43*B)*cos(x) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(
x) + a)*sqrt(a)*sin(x) - 2*a*cos(x) - 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 32*
(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*
cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^
3 + cos(x)^2)) - 4*((3*A - 11*B)*cos(x) + 7*A - 15*B)*sqrt(a*cos(x) + a)*si
n(x))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(x)}{(a(\cos(x)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))**(5/2),x)
```

```
[Out] Integral((A + B*sec(x))/(a*(cos(x) + 1))**(5/2), x)
```

Giac [A]
time = 0.44, size = 183, normalized size = 1.52

$$-\frac{B\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}+\frac{\sqrt{2}\left(3A\sqrt{a}-43B\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}x\right)+1\right)}{64a^3\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}-\frac{\sqrt{2}\left(3A\sqrt{a}-43B\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}x\right)+1\right)}{64a^3\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}-\frac{3\sqrt{2}A\sin\left(\frac{1}{2}x\right)^3-11\sqrt{2}B\sin\left(\frac{1}{2}x\right)^3-5\sqrt{2}A\sin\left(\frac{1}{2}x\right)+13\sqrt{2}B\sin\left(\frac{1}{2}x\right)}{32\left(\sin\left(\frac{1}{2}x\right)^2-1\right)^2a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="giac")
```

```
[Out] -B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))/(a^(5/
2)*sgn(cos(1/2*x))) + 1/64*sqrt(2)*(3*A*sqrt(a) - 43*B*sqrt(a))*log(sin(1/2
*x) + 1)/(a^3*sgn(cos(1/2*x))) - 1/64*sqrt(2)*(3*A*sqrt(a) - 43*B*sqrt(a))*
log(-sin(1/2*x) + 1)/(a^3*sgn(cos(1/2*x))) - 1/32*(3*sqrt(2)*A*sin(1/2*x)^3
 - 11*sqrt(2)*B*sin(1/2*x)^3 - 5*sqrt(2)*A*sin(1/2*x) + 13*sqrt(2)*B*sin(1/
2*x))/((sin(1/2*x)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(x)}}{(a+a\,\cos(x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(x))/(a + a*cos(x))^(5/2),x)
```

```
[Out] int((A + B/cos(x))/(a + a*cos(x))^(5/2), x)
```

3.199 $\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx$

Optimal. Leaf size=25

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

[Out] ln(a+b*sin(x))/b-x*cos(x)/(a+b*sin(x))

Rubi [A]

time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4688, 2747, 31}

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]

[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4688

```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sin[(c_.) + (d_.)*(x_)]))/((a_) + (
b_.)*Sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(-B)*(e + f*x)*(Cos[c +
d*x]/(a*d*(a + b*Sin[c + d*x]))), x] + Dist[B*(f/(a*d)), Int[Cos[c + d*x]/(
a + b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*
A - b*B, 0]
```

Rubi steps

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = -\frac{x\cos(x)}{a+b\sin(x)} + \int \frac{\cos(x)}{a+b\sin(x)}\,dx$$
$$= -\frac{x\cos(x)}{a+b\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}$$
$$= \frac{\log(a+b\sin(x))}{b} - \frac{x\cos(x)}{a+b\sin(x)}$$

Mathematica [A]
time = 0.07, size = 25, normalized size = 1.00

$$\frac{\log(a+b\sin(x))}{b} - \frac{x\cos(x)}{a+b\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

```
[Out] Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.61, size = 73, normalized size = 2.92

method	result	size
risch	$-\frac{2ix}{b} - \frac{2x\left(ib+a\,e^{ix}\right)}{b\left(b\,e^{2ix}-b+2ia\,e^{ix}\right)} + \frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{b}$	73
norman	$\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)-x}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)} + \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{b} - \frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*sin(x))/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/b*x-2*x*(I*b+a*exp(I*x))/b/(b*exp(2*I*x)-b+2*I*a*exp(I*x))+1/b*ln(exp(
2*I*x)+2*I*a/b*exp(I*x)-1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.48, size = 35, normalized size = 1.40

$$-\frac{bx\cos{(x)}-(b\sin{(x)}+a)\log{(b\sin{(x)}+a)}}{b^2\sin{(x)}+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(b*x*cos(x) - (b*sin(x) + a)*log(b*sin(x) + a))/(b^2*sin(x) + a*b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(25) = 50.
time = 0.48, size = 283, normalized size = 11.32

$$\frac{4\,bx\tan\left(\frac{1}{2}\,x\right)^2+a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4+4\,ab\tan\left(\frac{1}{2}\,x\right)^3+2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+2\,b\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4+4\,ab\tan\left(\frac{1}{2}\,x\right)^3+2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)-4\,bx+a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4+4\,ab\tan\left(\frac{1}{2}\,x\right)^3+2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\tan\left(\frac{1}{2}\,x\right)^2+1}\right)}{2\left(ab\tan\left(\frac{1}{2}\,x\right)^2+2\,b^2\tan\left(\frac{1}{2}\,x\right)+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*x*tan(1/2*x)^2 + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 +
2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*
x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*b*log(4*(a^2*tan(1/2*x)^4 + 4*
a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*
x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 4*b*x + a*log(4
*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/
2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b
*tan(1/2*x)^2 + 2*b^2*tan(1/2*x) + a*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,\left(b+a\,\sin\left(x\right)\right)}{{\left(a+b\,\sin\left(x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + a*sin(x)))/(a + b*sin(x))^2,x)
```

```
[Out] int((x*(b + a*sin(x)))/(a + b*sin(x))^2, x)
```

3.200 $\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx$

Optimal. Leaf size=24

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

```
[Out] ln(a+b*cos(x))/b+x*sin(x)/(a+b*cos(x))
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4689, 2747, 31}

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

```
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4689

```
Int[((Cos[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cos[(c_.
) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[B*(e + f*x)*(Sin[c + d*x
]/(a*d*(a + b*Cos[c + d*x]))), x] - Dist[B*(f/(a*d)), Int[Sin[c + d*x]/(a +
 b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A -
 b*B, 0]
```

Rubi steps

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{x\sin(x)}{a+b\cos(x)} - \int \frac{\sin(x)}{a+b\cos(x)}\,dx$$
$$= \frac{x\sin(x)}{a+b\cos(x)} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}$$
$$= \frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 1.00

$$\frac{\log(a+b\cos(x))}{b} + \frac{x\sin(x)}{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
[Out] Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 67, normalized size = 2.79

method	result	size
risch	$-\frac{2ix}{b}+\frac{2ix\left(a\,e^{ix}+b\right)}{b\left(b\,e^{2ix}+2a\,e^{ix}+b\right)}+\frac{\ln\left(e^{2ix}+1+\frac{2a\,e^{ix}}{b}\right)}{b}$	67
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2x\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}+\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+a+b\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b+a*cos(x))/(a+b*cos(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*I/b*x+2*I*x*(a*exp(I*x)+b)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)+1/b*ln(exp(2*
I*x)+1+2*a/b*exp(I*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.69, size = 36, normalized size = 1.50

$$\frac{bx\sin(x) + (b\cos(x) + a)\log(-b\cos(x) - a)}{b^2\cos(x) + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) + (b*cos(x) + a)*log(-b*cos(x) - a))/(b^2*cos(x) + a*b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(24) = 48.
time = 0.51, size = 397, normalized size = 16.54

$$\frac{a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4-2ab\tan\left(\frac{1}{2}x\right)^4+b^2\tan\left(\frac{1}{2}x\right)^4+2a^2\tan\left(\frac{1}{2}x\right)^2-2b^2\tan\left(\frac{1}{2}x\right)^2+a^2+2ab+b^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-b\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4-2ab\tan\left(\frac{1}{2}x\right)^4+b^2\tan\left(\frac{1}{2}x\right)^4+2a^2\tan\left(\frac{1}{2}x\right)^2-2b^2\tan\left(\frac{1}{2}x\right)^2+a^2+2ab+b^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2+8bx\tan\left(\frac{1}{2}x\right)+a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4-2ab\tan\left(\frac{1}{2}x\right)^4+b^2\tan\left(\frac{1}{2}x\right)^4+2a^2\tan\left(\frac{1}{2}x\right)^2-2b^2\tan\left(\frac{1}{2}x\right)^2+a^2+2ab+b^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)+b\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4-2ab\tan\left(\frac{1}{2}x\right)^4+b^2\tan\left(\frac{1}{2}x\right)^4+2a^2\tan\left(\frac{1}{2}x\right)^2-2b^2\tan\left(\frac{1}{2}x\right)^2+a^2+2ab+b^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)}{2\left(ab\tan\left(\frac{1}{2}x\right)^2-b^2\tan\left(\frac{1}{2}x\right)^2+ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*
a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 +
2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1
/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2
+ 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*b*x*ta
n(1/2*x) + a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^
4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x
)^4 + 2*tan(1/2*x)^2 + 1)) + b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4
 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b
 + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 - b^2*tan(1
/2*x)^2 + a*b + b^2)
```

Mupad [B]
time = 2.74, size = 68, normalized size = 2.83

$$\frac{\ln\left(b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}\right)}{b}-\frac{x\,2\mathrm{i}}{b}+\frac{x\,2\mathrm{i}+\frac{a\,x\,\mathrm{e}^{x\,1\mathrm{i}}\,2\mathrm{i}}{b}}{b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + a*cos(x)))/(a + b*cos(x))^2,x)
```

```
[Out] log(b + 2*a*exp(x*1i) + b*exp(x*2i))/b - (x*2i)/b + (x*2i + (a*x*exp(x*1i)*
2i)/b)/(b + 2*a*exp(x*1i) + b*exp(x*2i))
```

3.201 $\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx$

Optimal. Leaf size=8

$$-x+2\tan(x)$$

```
[Out] -x+2*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3250, 3254, 3852, 8}

$$2\tan(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

```
[Out] -x + 2*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = -x+2\int \frac{1}{1-\sin^2(x)}\,dx$$
$$= -x+2\int \sec^2(x)\,dx$$
$$= -x-2\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)$$
$$= -x+2\tan(x)$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-x+2\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
[Out] -x + 2*Tan[x]
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.38

method	result	size
default	$2\tan(x)-\arctan(\tan(x))$	11
risch	$-x+\frac{4i}{e^{2ix}+1}$	17
norman	$\frac{x+x\left(\tan^2\left(\frac{x}{2}\right)\right)-8\left(\tan^3\left(\frac{x}{2}\right)\right)-4\left(\tan^5\left(\frac{x}{2}\right)\right)-x\left(\tan^4\left(\frac{x}{2}\right)\right)-x\left(\tan^6\left(\frac{x}{2}\right)\right)-4\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x)^2)/(1-sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 2*tan(x)-arctan(tan(x))
```

Maxima [A]
time = 0.50, size = 8, normalized size = 1.00

$$-x+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="maxima")
[Out] -x + 2*tan(x)
```

Fricas [A]
time = 2.45, size = 15, normalized size = 1.88

$$-\frac{x\cos\left(x\right)-2\,\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - 2*sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(5) = 10$.
time = 0.39, size = 41, normalized size = 5.12

$$-\frac{x\tan^{2}\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}+\frac{x}{\tan^{2}\left(\frac{x}{2}\right)-1}-\frac{4\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)**2)/(1-sin(x)**2),x)
```

```
[Out] -x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 4*tan(x/2)/(tan(x/
2)**2 - 1)
```

Giac [A]
time = 0.40, size = 8, normalized size = 1.00

$$-x+2\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="giac")
```

```
[Out] -x + 2*tan(x)
```

Mupad [B]
time = 2.31, size = 8, normalized size = 1.00

$$2\tan(x)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x)^2 + 1)/(sin(x)^2 - 1),x)
```

```
[Out] 2*tan(x) - x
```

3.202 $\int \frac{1-\sin^2(x)}{1+\sin^2(x)} \, dx$

Optimal. Leaf size=36

$$-x+\sqrt{2}\,x+\sqrt{2}\,\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

```
[Out] -x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3250, 3260, 209}

$$\sqrt{2}\,\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)+\sqrt{2}\,x-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

```
[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = -x+2\int \frac{1}{1+\sin^2(x)}\,dx$$
$$= -x+2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right)$$
$$= -x+\sqrt{2}\,x+\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.67

$$-2\left(\frac{x}{2}-\frac{\text{ArcTan}\left(\sqrt{2}\,\tan(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
[Out] -2*(x/2 - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2])
```

Maple [A]
time = 0.12, size = 18, normalized size = 0.50

method	result	size
default	$\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)-\arctan\left(\tan(x)\right)$	18
risch	$-x+\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x)^2)/(1+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctan(tan(x)*2^(1/2))-arctan(tan(x))
```

Maxima [A]
time = 0.49, size = 15, normalized size = 0.42

$$\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

Fricas [A]
time = 2.37, size = 35, normalized size = 0.97

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(x\right)^{2}-2\,\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(32) = 64$.
time = 23.50, size = 248, normalized size = 6.89

$$-\frac{22619537x}{15994428\sqrt{2}+22619537}-\frac{15994428\sqrt{2}\,x}{15994428\sqrt{2}+22619537}+\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}+\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}+\frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}+\frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)**2)/(1+sin(x)**2),x)
```

```
[Out] -22619537*x/(15994428*sqrt(2) + 22619537) - 15994428*sqrt(2)*x/(15994428*sq
rt(2) + 22619537) + 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqr
t(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619537
) + 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*f
loor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619537) + 9369319*sqrt(2)*sqrt
(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)
/pi))/(15994428*sqrt(2) + 22619537) + 13250218*sqrt(2*sqrt(2) + 3)*(atan(ta
n(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2)
+ 22619537)
```

Giac [A]
time = 0.40, size = 49, normalized size = 1.36

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)}{\sqrt{2}\,\cos\left(2\,x\right)+\sqrt{2}-2\,\cos\left(2\,x\right)+2}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sq
rt(2) - 2*cos(2*x) + 2))) - x
```

Mupad [B]
time = 2.32, size = 26, normalized size = 0.72

$$\sqrt{2}\,\left(x-\mathrm{atan}\left(\tan\left(x\right)\right)\right)-x+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x)^2 - 1)/(sin(x)^2 + 1),x)
[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan(2^(1/2)*tan(x))
```

3.203 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-x - 2\cot(x)$$

[Out] -x-2*cot(x)

Rubi [A]
time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3250, 3254, 3852, 8}

$$-x - 2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x+2\int \frac{1}{1-\cos^2(x)}\,dx$$
$$= -x+2\int \csc^2(x)\,dx$$
$$= -x-2\text{Subst}(\int 1\,dx, x, \cot(x))$$
$$= -x-2\cot(x)$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Maple [A]
time = 0.09, size = 13, normalized size = 1.62

method	result	size
default	$-\arctan(\tan(x))-\frac{2}{\tan(x)}$	13
risch	$-x-\frac{4i}{e^{2ix}-1}$	17
norman	$\frac{-1+\tan^4(\frac{x}{2})+\tan^6(\frac{x}{2})-(\tan^2(\frac{x}{2}))-x\tan(\frac{x}{2})-2x(\tan^3(\frac{x}{2}))-x(\tan^5(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2\tan(\frac{x}{2})}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1-cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(tan(x))-2/tan(x)
```

Maxima [A]
time = 0.48, size = 10, normalized size = 1.25

$$-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

```
[Out] -x - 2/tan(x)
```

Fricas [A]
time = 2.22, size = 15, normalized size = 1.88

$$-\frac{x\sin(x) + 2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

Sympy [A]
time = 0.32, size = 12, normalized size = 1.50

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1-cos(x)**2),x)
[Out] -x + tan(x/2) - 1/tan(x/2)
```

Giac [A]
time = 0.41, size = 16, normalized size = 2.00

$$-x - \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

Mupad [B]
time = 2.29, size = 8, normalized size = 1.00

$$-x - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x)
[Out] - x - 2*cot(x)
```

3.204 $\int \frac{1-\cos^2(x)}{1+\cos^2(x)} \, dx$

Optimal. Leaf size=37

$$-x+\sqrt{2}\,x-\sqrt{2}\,\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

```
[Out] -x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3250, 3260, 209}

$$-\sqrt{2}\,\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)+\sqrt{2}\,x-x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

```
[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3250

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3260

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = -x+2\int \frac{1}{1+\cos^2(x)}\,dx$$
$$= -x-2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \cot(x)\right)$$
$$= -x+\sqrt{2}\,x-\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.62

$$2\left(-\frac{x}{2}+\frac{\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

```
[Out] 2*(-1/2*x + ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2])
```

Maple [A]
time = 0.11, size = 19, normalized size = 0.51

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\tan\left(x\right)\sqrt{2}}{2}\right)$	19
risch	$-x+\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}+2\sqrt{2}+3\right)}{2}-\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}-2\sqrt{2}+3\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x)^2)/(1+cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(tan(x))+2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.43

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(x\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

Fricas [A]
time = 2.61, size = 35, normalized size = 0.95

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

Sympy [A]
time = 0.82, size = 61, normalized size = 1.65

$$-x+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)**2)/(1+cos(x)**2),x)
```

```
[Out] -x + sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi)) + sqr
t(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))
```

Giac [A]
time = 0.40, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

Mupad [B]
time = 2.29, size = 27, normalized size = 0.73

$$\sqrt{2}\,\left(x-\operatorname{atan}(\tan(x))\right)-x+\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2 - 1)/(cos(x)^2 + 1),x)
```

```
[Out] 2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan((2^(1/2)*tan(x))/2)
```

3.205 $\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=14

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

```
[Out] c*x/d^2-sin(x)/d
```

Rubi [A]
time = 0.09, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {4482, 3095, 2717}

$$\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (c*x)/d^2 - Sin[x]/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3095

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx &= \int \frac{\frac{c^2}{d^2}-\cos^2(x)}{c+d\cos(x)}\,dx \\
&= -\frac{\int(-c+d\cos(x))\,dx}{d^2} \\
&= \frac{cx}{d^2}-\frac{\int\cos(x)\,dx}{d} \\
&= \frac{cx}{d^2}-\frac{\sin(x)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$
\frac{cx}{d^2}-\frac{\sin(x)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (c*x)/d^2 - Sin[x]/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.13, size = 31, normalized size = 2.21

method	result	size
risch	$\frac{cx}{d^2}-\frac{\sin(x)}{d}$	15
default	$\frac{-\frac{2d\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}+2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$	31
norman	$\frac{\frac{cx}{d}+\frac{cx\left(\tan^4\left(\frac{x}{2}\right)\right)}{d}-2\left(\tan^3\left(\frac{x}{2}\right)\right)+\frac{2cx\left(\tan^2\left(\frac{x}{2}\right)\right)}{d}-2\tan\left(\frac{x}{2}\right)}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(-d*tan(1/2*x)/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.28, size = 13, normalized size = 0.93

$$\frac{cx - d\sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] (c*x - d*sin(x))/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(10) = 20$.
time = 34.40, size = 61, normalized size = 4.36

$$\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} + \frac{cx}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} - \frac{2d\tan\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right) + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) -
 2*d*tan(x/2)/(d**2*tan(x/2)**2 + d**2)
```

Giac [A]
time = 0.38, size = 26, normalized size = 1.86

$$\frac{cx}{d^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] c*x/d^2 - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d)
```

Mupad [B]
time = 2.47, size = 13, normalized size = 0.93

$$\frac{c\,x - d\,\sin(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2 + c^2/d^2 - 1)/(c + d*cos(x)),x)
[Out] (c*x - d*sin(x))/d^2
```

3.206 $\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=105

$$\frac{bcx}{d^2}+\frac{2a\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

```
[Out] b*c*x/d^2-b*sin(x)/d+2*a*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))/(c-d)^(
1/2)/(c+d)^(1/2)-2*b*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))*(c-d)^(1/2)
*(c+d)^(1/2)/d^2
```

Rubi [A]
time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4486, 2738, 211, 2774, 2814}

$$\frac{2a\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}+\frac{bcx}{d^2}-\frac{b\sin(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(Sqrt[c - d]
*Sqrt[c + d]) - (2*b*Sqrt[c - d]*Sqrt[c + d]*ArcTan[(Sqrt[c - d]*Tan[x/2])/
Sqrt[c + d]])/d^2 - (b*Sin[x])/d
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx &= \int \left(\frac{a}{c+d\cos(x)}+\frac{b\sin^2(x)}{c+d\cos(x)}\right)dx\\
&= a\int \frac{1}{c+d\cos(x)}\,dx+b\int \frac{\sin^2(x)}{c+d\cos(x)}\,dx\\
&= -\frac{b\sin(x)}{d}+(2a)\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)-\frac{b\int \frac{-d-c\cos(x)}{c+d\cos(x)}\,dx}{d}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{b\sin(x)}{d}+\frac{(b(-c^2+d^2))\int \frac{1}{c+d\cos(x)}\,dx}{d^2}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{b\sin(x)}{d}+\frac{(2b(-c^2+d^2))\,\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx,\right.}{d^2}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}}-\frac{2b\sqrt{c-d}\,\sqrt{c+d}\,\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.70

$$\frac{bcx-\frac{2\left(ad^2+b\left(-c^2+d^2\right)\right)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}-bd\sin(x)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (b*c*x - (2*(a*d^2 + b*(-c^2 + d^2))*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 +
 d^2]])/Sqrt[-c^2 + d^2] - b*d*Sin[x])/d^2
```

Maple [A]
time = 0.23, size = 88, normalized size = 0.84

method	result
default	$\frac{2\left(a\,d^{2}-c^{2} b+b\,d^{2}\right) \arctan\left(\frac{\left(c-d\right) \tan\left(\frac{x}{2}\right)}{\sqrt{\left(c+d\right)\left(c-d\right)}}\right)}{d^{2} \sqrt{\left(c+d\right)\left(c-d\right)}}+\frac{2 b \left(-\frac{d \tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}+c \arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{d^{2}}$
risch	$\frac{b c x}{d^{2}}+\frac{i b\,{\mathrm e}^{i x}}{2 d}-\frac{i b\,{\mathrm e}^{-i x}}{2 d}-\frac{\ln\left({\mathrm e}^{i x}+\frac{i c^{2}-i d^{2}+\sqrt{-c^{2}+d^{2}}\, c}{d \sqrt{-c^{2}+d^{2}}}\right) a}{\sqrt{-c^{2}+d^{2}}}+\frac{\ln\left({\mathrm e}^{i x}+\frac{i c^{2}-i d^{2}+\sqrt{-c^{2}+d^{2}}\, c}{d \sqrt{-c^{2}+d^{2}}}\right) c^{2} b}{\sqrt{-c^{2}+d^{2}}\, d^{2}}-\frac{\ln\left({\mathrm e}^{i x}+\frac{i c^{2}-}{}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*d^2-b*c^2+b*d^2)/d^2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d
)*(c-d))^(1/2))+2*b/d^2*(-d*tan(1/2*x)/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.46, size = 254, normalized size = 2.42

$$\left[\frac{(bc^2-(a+b)d^2)\sqrt{-c^2+d^2}\,\log\left(\frac{2\,cd\cos(x)+(2\,c^2-d^2)\cos(x)^2+2\,\sqrt{-c^2+d^2}\,(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+2\,(bc^3-bcd^2)x-2\,(bc^2d-bd^3)\sin(x)}{2\,(c^2d^2-d^4)},\ -\frac{(bc^2-(a+b)d^2)\sqrt{c^2-d^2}\,\arctan\left(-\frac{c\cos(x)+d}{\sqrt{c^2-d^2}\,\sin(x)}\right)-(bc^3-bcd^2)x+(bc^2d-bd^3)\sin(x)}{c^2d^2-d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log((2*c*d*cos(x) + (2*c^2 - d
^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d^2
```

```
*cos(x)^2 + 2*c*d*cos(x) + c^2)) + 2*(b*c^3 - b*c*d^2)*x - 2*(b*c^2*d - b*d
^3)*sin(x))/(c^2*d^2 - d^4), -((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*arctan
(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x))) - (b*c^3 - b*c*d^2)*x + (b*c^2*d
 - b*d^3)*sin(x))/(c^2*d^2 - d^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2608 vs. $2(92) = 184$.
time = 68.56, size = 2608, normalized size = 24.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Piecewise((zoo*(-a*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - a*log(
tan(x/2) - 1)/(tan(x/2)**2 + 1) + a*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)
**2 + 1) + a*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)/(tan(x/2)**2 + 1) + b*log(t
an(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + b*log(tan(x/2) + 1)/(tan(x/2)*
*2 + 1) - 2*b*tan(x/2)/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(d, 0)), (a*tan(x/2
)**2/(d*tan(x/2)**3 + d*tan(x/2)) + a/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*ta
n(x/2)**3/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*tan(x/2)/(d*tan(x/2)**3 + d*ta
n(x/2)) - 2*b*tan(x/2)**2/(d*tan(x/2)**3 + d*tan(x/2)), Eq(c, -d)), ((a*x +
 b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2)/c, Eq(d, 0)), (a*ta
n(x/2)**3/(d*tan(x/2)**2 + d) + a*tan(x/2)/(d*tan(x/2)**2 + d) + b*x*tan(x/
2)**2/(d*tan(x/2)**2 + d) + b*x/(d*tan(x/2)**2 + d) - 2*b*tan(x/2)/(d*tan(x
/2)**2 + d), Eq(c, d)), (a*d**2*log(-sqrt(-c/(c - d) - d/(c - d)) + tan(x/2
))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*sq
rt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2
 - d**3*sqrt(-c/(c - d) - d/(c - d))) + a*d**2*log(-sqrt(-c/(c - d) - d/(c -
 d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*
sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**
2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - a*d**2*log(sqrt(-c/(c - d) - d/(c
 - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)
**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d
))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - a*d**2*log(sqrt(-c/(c
 - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*x*sqrt(-c/(c
 - d) - d/(c - d))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)
**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d
))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*x*sqrt(-c/(c -
 d) - d/(c - d))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*d**2*
sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**
2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c**2*log(-sqrt(-c/(c - d) - d/(c
```

```
 - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c**2*log(-sqrt(-c/
(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x
/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c
 - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**2*log(sqrt(-c
/(c - d) - d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(
c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c
 - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*c**
2*log(sqrt(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/
(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(
c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c*
d*x*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c
 - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c
 - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*c*d*
x*sqrt(-c/(c - d) - d/(c - d))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2
)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c -
d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - 2*b*c*d*sqrt(-c/(c -
 d) - d/(c - d))*tan(x/2)/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2
+ c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*t
an(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + 2*b*d**2*sqrt(-c/(c - d)
 - d/(c - d))*tan(x/2)/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)**2 + c*
d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d))*tan(x
/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*d**2*log(-sqrt(-c/(c - d) -
 d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*ta
n(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/
(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + b*d**2*log(-sqr
t(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*
tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) -
d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - b*d**2*log(sq
rt(-c/(c - d) - d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d)
 - d/(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(
-c/(c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) -
b*d**2*log(sqrt(-c/(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d)
 - d/(c - d))*tan(x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt
(-c/(c - d) - d/(c - d))*tan(x/2)**2 - d**3*sqr...
```

Giac [A]
time = 0.41, size = 110, normalized size = 1.05

$$\frac{bcx}{d^2} - \frac{2\,b\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)d} + \frac{2\,(bc^2-ad^2-bd^2)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,x\right)-d\tan\left(\frac{1}{2}\,x\right)}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] b*c*x/d^2 - 2*b*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d) + 2*(b*c^2 - a*d^2 - b*d^
2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*ta
n(1/2*x))/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2)
```

Mupad [B]
time = 4.04, size = 2429, normalized size = 23.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + d*cos(x)),x)
```

```
[Out] (b*c^2*d*sin(x))/(d^4 - c^2*d^2) - (b*d^3*sin(x))/(d^4 - c^2*d^2) - (2*b*c^
3*atan(sin(x/2)/cos(x/2)))/(d^4 - c^2*d^2) - (a*d^2*atan((a^2*d^7*sin(x/2)*
(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*c^7*sin(
x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c*d
^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*1i
- b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c*d^6*sin(x/2)*(d^2 - c^2)^
(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^3*d^4*sin(x/2)
*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*c^3
*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)
*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d^2*sin(x/2)*(d^2
 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c*d^4*sin(x/
2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^2
*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)
*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^4*d^3*sin(x/2)*(d^2
 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*d^8*cos(
x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*a^2*c^2*d^6*cos(x/2) + a^2
*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^2*c^4*d^4*cos(x/2) - b^2*c
^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*c^4*d^4*cos(x/2) - 2*a*b*c
^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^2*d^2) + (b*c^2*atan((a^2*
d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i -
 b^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)
*1i - a^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(x/2)*(d^2 - c
^2)^(1/2)*1i - b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c*d^6*sin(x/2)
*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^3
*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)
*2i + b^2*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*sin(x/2)*(d^2
 - c^2)^(1/2)*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d^2*
sin(x/2)*(d^2 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b
*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)
*2i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*sin(x/2)*(d^2
 - c^2)^(3/2)*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^4*d^3*
sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/
```

```
(a^2*d^8*cos(x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*a^2*c^2*d^6*c
os(x/2) + a^2*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^2*c^4*d^4*cos
(x/2) - b^2*c^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*c^4*d^4*cos(x
/2) - 2*a*b*c^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^2*d^2) - (b*d
^2*atan((a^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^
2)^(3/2)*2i - b^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2
 - c^2)^(1/2)*1i - a^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(
x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c
*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)
*1i - a^2*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2
 - c^2)^(1/2)*2i + b^2*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*
sin(x/2)*(d^2 - c^2)^(1/2)*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i +
 b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(
1/2)*2i - a*b*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2
 - c^2)^(1/2)*2i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*
sin(x/2)*(d^2 - c^2)^(3/2)*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i +
 a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^
2)^(1/2)*2i)/(a^2*d^8*cos(x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*
a^2*c^2*d^6*cos(x/2) + a^2*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^
2*c^4*d^4*cos(x/2) - b^2*c^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*
c^4*d^4*cos(x/2) - 2*a*b*c^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^
2*d^2) + (2*b*c*d^2*atan(sin(x/2)/cos(x/2)))/(d^4 - c^2*d^2)
```

3.207 $\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}\,c}-\frac{(a+2b)\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}\,c}$$

```
[Out] -b*x/c+1/2*(a+2*b)*x/c*2^(1/2)-1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+cos(x)^2
+2^(1/2)))/c*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {12, 1180, 209}

$$-\frac{(a+2b)\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}+\frac{x(a+2b)}{\sqrt{2}\,c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) - ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Cos[x]^2)])/(Sqrt[2]*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx=\text{Subst}\left(\int \frac{a+(a+b)x^2}{c\left(2+3x^2+x^4\right)}\,dx,x,\tan(x)\right)$$
$$=\frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{2+3x^2+x^4}\,dx,x,\tan(x)\right)}{c}$$
$$=-\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)}{c}+\frac{(a+2b)\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)}{c}$$
$$=-\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}\,c}-\frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}\,c}$$

Mathematica [A]
time = 0.09, size = 34, normalized size = 0.60

$$-\frac{bx}{c}-\frac{(-a-2b)\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

```
[Out] -((b*x)/c) - ((-a - 2*b)*ArcTan[Tan[x]/Sqrt[2]])/(Sqrt[2]*c)
```

Maple [A]
time = 0.23, size = 30, normalized size = 0.53

method	result
default	$\frac{-b\arctan(\tan(x))+\frac{(a+2b)\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{2}}{c}$
risch	$-\frac{bx}{c}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)a}{4c}+\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)b}{2c}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)a}{4c}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+c*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-b*arctan(tan(x))+1/2*(a+2*b)*2^(1/2)*arctan(1/2*tan(x)*2^(1/2)))
```

Maxima [A]
time = 0.48, size = 29, normalized size = 0.51

$$\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(x\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(1/2*sqrt(2)*tan(x))/c - b*x/c
```

Fricas [A]
time = 3.84, size = 45, normalized size = 0.79

$$-\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*s
in(x))) + 4*b*x)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.
time = 4.42, size = 143, normalized size = 2.51

$$\frac{\sqrt{2}\,a\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}+\frac{\sqrt{2}\,a\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}-\frac{bx}{c}+\frac{\sqrt{2}\,b\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{c}+\frac{\sqrt{2}\,b\left(\operatorname{atan}\left(\sqrt{2}\,\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+c*cos(x)**2),x)
```

```
[Out] sqrt(2)*a*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) +
sqrt(2)*a*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) -
b*x/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/
c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/c
```

Giac [A]
time = 0.40, size = 62, normalized size = 1.09

$$\frac{\sqrt{2}\,(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,x)-\sin(2\,x)}{\sqrt{2}\,\cos(2\,x)+\sqrt{2}\,-\cos(2\,x)+1}\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*c
os(2*x) + sqrt(2) - cos(2*x) + 1)))/c - b*x/c
```

Mupad [B]
time = 2.43, size = 242, normalized size = 4.25

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^3\tan(x)}{2\,(a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3)}+\frac{2\,\sqrt{2}\,b^3\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}+\frac{5\,\sqrt{2}\,a\,b^2\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}+\frac{3\,\sqrt{2}\,a^2\,b\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}\right)(a+2\,b)}{2\,c}-\frac{b\,\mathrm{atan}\left(\frac{4\,b^3\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3}+\frac{8\,a\,b^2\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3}+\frac{2\,a^2\,b\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + c*cos(x)^2),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*a^3*tan(x))/(2*(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3)) +
 (2*2^(1/2)*b^3*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (5*2^(1/2)*a*b
^2*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (3*2^(1/2)*a^2*b*tan(x))/(1
0*a*b^2 + 6*a^2*b + a^3 + 4*b^3))*(a + 2*b))/(2*c) - (b*atan((4*b^3*tan(x))
/(8*a*b^2 + 2*a^2*b + 4*b^3) + (8*a*b^2*tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)
 + (2*a^2*b*tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)))/c
```

3.208 $\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

```
[Out] b*x/c-a*cot(x)/c
```

Rubi [A]
time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {464, 211}

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{a+(a+b)x^2}{x^2\left(c+cx^2\right)}\,dx,x,\tan(x)\right)\\ &= -\frac{a\cot(x)}{c}+b\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\tan(x)\right)\\ &= \frac{bx}{c}-\frac{a\cot(x)}{c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{bx}{c}-\frac{a\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

```
[Out] (b*x)/c - (a*Cot[x])/c
```

Maple [A]
time = 0.13, size = 18, normalized size = 1.20

method	result	size
default	$\frac{b\arctan(\tan(x))-\frac{a}{\tan(x)}}{c}$	18
risch	$\frac{bx}{c}-\frac{2ia}{(e^{2ix}-1)c}$	24
norman	$\frac{\frac{bx\tan\left(\frac{x}{2}\right)}{c}+\frac{bx\left(\tan^5\left(\frac{x}{2}\right)\right)}{c}-\frac{a}{2c}-\frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{2c}+\frac{a\left(\tan^4\left(\frac{x}{2}\right)\right)}{2c}+\frac{a\left(\tan^6\left(\frac{x}{2}\right)\right)}{2c}+\frac{2bx\left(\tan^3\left(\frac{x}{2}\right)\right)}{c}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\tan\left(\frac{x}{2}\right)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c-c*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(b*arctan(tan(x))-a/tan(x))
```

Maxima [A]
time = 0.49, size = 17, normalized size = 1.13

$$\frac{bx}{c}-\frac{a}{c\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] b*x/c - a/(c*tan(x))
```

Fricas [A]
time = 2.64, size = 19, normalized size = 1.27

$$\frac{bx\sin(x)-a\cos(x)}{c\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] (b*x*sin(x) - a*cos(x))/(c*sin(x))
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.46, size = 24, normalized size = 1.60

$$\frac{a\tan\left(\frac{x}{2}\right)}{2c} - \frac{a}{2c\tan\left(\frac{x}{2}\right)} + \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c-c*cos(x)**2),x)
```

```
[Out] a*tan(x/2)/(2*c) - a/(2*c*tan(x/2)) + b*x/c
```

Giac [**A**]
time = 0.38, size = 29, normalized size = 1.93

$$\frac{bx}{c} + \frac{a\tan\left(\frac{1}{2}\,x\right)}{2\,c} - \frac{a}{2\,c\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="giac")
```

```
[Out] b*x/c + 1/2*a*tan(1/2*x)/c - 1/2*a/(c*tan(1/2*x))
```

Mupad [**B**]
time = 2.31, size = 13, normalized size = 0.87

$$\frac{b\,x - a\cot(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c - c*cos(x)^2),x)
```

```
[Out] (b*x - a*cot(x))/c
```

3.209 $\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{bx}{d}+\frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}$$

```
[Out] -b*x/d+(a*d+b*(c+d))*arctan(c^(1/2)*tan(x)/(c+d)^(1/2))/d/c^(1/2)/(c+d)^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {536, 209, 211}

$$\frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)\,(c+d+cx^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(ad+b(c+d))\text{Subst}\left(\int \frac{1}{c+d+cx^2}\,dx, x, \tan(x)\right)}{d}$$
$$= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,d\sqrt{c+d}}$$

<u>**Mathematica**</u> [A]
time = 0.12, size = 47, normalized size = 0.96

$$\frac{-bx+\frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\,\sqrt{c+d}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

<u>**Maple**</u> [A]
time = 0.30, size = 44, normalized size = 0.90

method	result
default	$\frac{(ad+cb+db)\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)\,c}}\right)}{d\sqrt{(c+d)\,c}} - \frac{b\arctan(\tan(x))}{d}$
risch	$-\frac{bx}{d} - \frac{\ln\left(e^{2ix}+\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)a}{2\sqrt{-c^2-cd}} - \frac{\ln\left(e^{2ix}+\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}}{\sqrt{-c^2-cd}\,d}\right.}{2\sqrt{-c^2-cd}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)/(c+d*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] (a*d+b*c+b*d)/d/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))-b/d*arctan
(tan(x))
```

Maxima [A]
time = 0.48, size = 40, normalized size = 0.82

$$-\frac{bx}{d}+\frac{(bc+(a+b)d)\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -b*x/d + (b*c + (a + b)*d)*arctan(c*tan(x)/sqrt((c + d)*c))/(sqrt((c + d)*c
)*d)
```

Fricas [A]
time = 4.04, size = 228, normalized size = 4.65

$$\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\,\log\left(\frac{(8c^2+8cd+d^2)\cos(x)^4-2(4c^2+3cd)\cos(x)^2+4((2c+d)\cos(x)^3-c\cos(x))\sqrt{-c^2-cd}\,\sin(x)+c^2}{d^2\cos(x)^4+2cd\cos(x)^2+c^2}\right)+4(bc^2+bcd)x}{4(c^2d+cd^2)},-\frac{(bc+(a+b)d)\sqrt{c^2+cd}\,\arctan\left(\frac{(2c+d)\cos(x)^2-c}{2\sqrt{c^2+cd}\,\cos(x)\sin(x)}\right)+2(bc^2+bcd)x}{2(c^2d+cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 3*c*d)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - c*cos(x))*sqrt(-c
^2 - c*d)*sin(x) + c^2)/(d^2*cos(x)^4 + 2*c*d*cos(x)^2 + c^2)) + 4*(b*c^2 +
 b*c*d)*x)/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(
1/2*((2*c + d)*cos(x)^2 - c)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 +
b*c*d)*x)/(c^2*d + c*d^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)/(c+d*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.18

$$-\frac{bx}{d}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(c) + arctan(c*tan(x)/sqrt(c^2 + c*d)))*(
b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

Mupad [B]
time = 2.85, size = 1987, normalized size = 40.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)/(c + d*cos(x)^2),x)
```

```
[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^
2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i
 + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d -
c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan
(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*
d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i
 + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)
^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*
(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*
d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2
*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2
*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(-
c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x
)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*
d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i
 + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)
^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(-
 c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*
d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*
7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)
^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c
*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*
tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/
(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^
2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6
*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*d*atan((a^2*d^3
*tan(x)*(- c*d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^
```

```
2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i
 + a^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)
^(1/2)*1i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c
*d - c^2)^(1/2)*1i + b^2*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*ta
n(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a
^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)
^(1/2)*4i + b^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c
*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*ta
n(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b
*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/
2)*6i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4
 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^
2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d -
c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)
```

3.210 $\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=13

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

[Out] c*x/d^2+cos(x)/d

Rubi [A]
time = 0.08, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4482, 3095, 2718}

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

[In] Int[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]

[Out] (c*x)/d^2 + Cos[x]/d

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3095

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[C/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*
Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
EqQ[A*b^2 + a^2*C, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx = \int \frac{\frac{c^2}{d^2}-\sin^2(x)}{c+d\sin(x)}\,dx$$
$$= -\frac{\int(-c+d\sin(x))\,dx}{d^2}$$
$$= \frac{cx}{d^2}-\frac{\int\sin(x)\,dx}{d}$$
$$= \frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
[Out] (c*x)/d^2 + Cos[x]/d
```

Maple [A]
time = 0.16, size = 26, normalized size = 2.00

method	result	size
risch	$\frac{cx}{d^2}+\frac{\cos(x)}{d}$	14
default	$\frac{\frac{2d}{1+\tan^2\left(\frac{x}{2}\right)}+2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$	26
norman	$\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{cx}{d}+\frac{cx\left(\tan^4\left(\frac{x}{2}\right)\right)}{d}+\frac{2cx\left(\tan^2\left(\frac{x}{2}\right)\right)}{d}+2}{d\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(d/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 1.80, size = 12, normalized size = 0.92

$$\frac{cx + d\cos(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] (c*x + d*cos(x))/d^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(10) = 20$.
time = 60.73, size = 56, normalized size = 4.31

$$\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} + \frac{cx}{d^2\tan^2\left(\frac{x}{2}\right) + d^2} + \frac{2d}{d^2\tan^2\left(\frac{x}{2}\right) + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c**2/d**2+cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2) +
 2*d/(d**2*tan(x/2)**2 + d**2)
```

Giac [**A**]
time = 0.38, size = 22, normalized size = 1.69

$$\frac{cx}{d^2} + \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] c*x/d^2 + 2/((tan(1/2*x)^2 + 1)*d)
```

Mupad [**B**]
time = 2.46, size = 13, normalized size = 1.00

$$\frac{\cos(x)}{d} + \frac{c\,x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + c^2/d^2 - 1)/(c + d*sin(x)),x)
```

```
[Out] cos(x)/d + (c*x)/d^2
```

3.211 $\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=100

$$\frac{bcx}{d^2}+\frac{2a\text{ArcTan}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\,\text{ArcTan}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{b\cos(x)}{d}$$

```
[Out] b*c*x/d^2+b*cos(x)/d+2*a*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))/(c^2-d^2)
^(1/2)-2*b*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))*(c^2-d^2)^(1/2)/d^2
```

Rubi [A]
time = 0.17, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4486, 2739, 632, 210, 2774, 2814}

$$\frac{2a\text{ArcTan}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\,\text{ArcTan}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (b*c*x)/d^2 + (2*a*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2
] - (2*b*Sqrt[c^2 - d^2]*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/d^2 + (b
*Cos[x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx &= \int \left(\frac{a}{c+d\sin(x)}+\frac{b\cos^2(x)}{c+d\sin(x)}\right)dx\\
&= a\int \frac{1}{c+d\sin(x)}\,dx+b\int \frac{\cos^2(x)}{c+d\sin(x)}\,dx\\
&= \frac{b\cos(x)}{d}+(2a)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)+\frac{b\int \frac{d+c\sin(x)}{c+d\sin(x)}\,dx}{d}\\
&= \frac{bcx}{d^2}+\frac{b\cos(x)}{d}-(4a)\text{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx,x,2d+2c\tan\left(\frac{x}{2}\right)\right)-\frac{(b(c^2-\ldots}{\ldots}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{b\cos(x)}{d}-\frac{\left(2b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx,x,\tan\ldots\right.}{d^2}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{b\cos(x)}{d}+\frac{\left(4b\left(c^2-d^2\right)\right)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx,x,2\ldots\right.}{d^2}\\
&= \frac{bcx}{d^2}+\frac{2a\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\,\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{b\cos(x)}{d}
\end{aligned}
$$

Mathematica [**A**]

time = 0.13, size = 72, normalized size = 0.72

$$\frac{\frac{2(ad^2+b(-c^2+d^2))\operatorname{ArcTan}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b(cx+d\cos(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] ((2*(a*d^2 + b*(-c^2 + d^2))*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt
[c^2 - d^2] + b*(c*x + d*Cos[x]))/d^2
```

Maple [A]
time = 0.19, size = 86, normalized size = 0.86

method	result
default	$\frac{2\left(a\,d^{2}-c^{2}b+b\,d^{2}\right)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{d^{2}\sqrt{c^{2}-d^{2}}}+\frac{2b\left(\frac{d}{1+\tan^{2}\left(\frac{x}{2}\right)}+c\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{d^{2}}$
risch	$\frac{bcx}{d^{2}}+\frac{b\,e^{ix}}{2d}+\frac{b\,e^{-ix}}{2d}-\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^{2}+d^{2}}\,c-c^{2}+d^{2}}{d\sqrt{-c^{2}+d^{2}}}\right)a}{\sqrt{-c^{2}+d^{2}}}+\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^{2}+d^{2}}\,c-c^{2}+d^{2}}{d\sqrt{-c^{2}+d^{2}}}\right)c^{2}b}{\sqrt{-c^{2}+d^{2}}\,d^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*d^2-b*c^2+b*d^2)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(
c^2-d^2)^(1/2))+2*b/d^2*(d/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.02, size = 262, normalized size = 2.62

$$\left[\frac{(bc^2-(a+b)d^2)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(x)^2-2cd\sin(x)-c^2-d^2+2(c\cos(x)\sin(x)+d\cos(x))\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)+2(bc^3-bcd^2)x+2(bc^2d-bd^3)\cos(x)}{2(c^2d^2-d^4)},\ \frac{(bc^2-(a+b)d^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(x)+d}{\sqrt{c^2-d^2}\cos(x)}\right)+(bc^3-bcd^2)x+(bc^2d-bd^3)\cos(x)}{c^2d^2-d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 -
2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))
/(d^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2)) + 2*(b*c^3 - b*c*d^2)*x + 2*(b*
c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4), ((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d
^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x))) + (b*c^3 - b*c*d^2)*x
+ (b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(85) = 170$.

time = 156.12, size = 2035, normalized size = 20.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Piecewise((zoo*(a*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) + a*log(tan(x
/2))/(tan(x/2)**2 + 1) + b*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) + b*
log(tan(x/2))/(tan(x/2)**2 + 1) + 2*b/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(d,
0)), (2*a*sqrt(d**2)*tan(x/2)**2/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt
(d**2)*tan(x/2)**2 - d*sqrt(d**2)) + 2*a*sqrt(d**2)/(d**2*tan(x/2)**3 + d**
2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2)) + b*d*x*tan(x/2)**2/(
d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2))
 + b*d*x/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*s
qrt(d**2)) + 2*b*d*tan(x/2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2
)*tan(x/2)**2 - d*sqrt(d**2)) - b*x*sqrt(d**2)*tan(x/2)**3/(d**2*tan(x/2)**
3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2)) - b*x*sqrt(d**
2)*tan(x/2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 -
d*sqrt(d**2)) - 2*b*sqrt(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d
**2)*tan(x/2)**2 - d*sqrt(d**2)), Eq(c, -sqrt(d**2))), (-2*a*sqrt(d**2)*tan
(x/2)**2/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*s
qrt(d**2)) - 2*a*sqrt(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**2
)*tan(x/2)**2 + d*sqrt(d**2)) + b*d*x*tan(x/2)**2/(d**2*tan(x/2)**3 + d**2*
tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*sqrt(d**2)) + b*d*x/(d**2*tan(x/2)*
*3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*sqrt(d**2)) + 2*b*d*tan(x
/2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*sqrt(d
**2)) + b*x*sqrt(d**2)*tan(x/2)**3/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sq
rt(d**2)*tan(x/2)**2 + d*sqrt(d**2)) + b*x*sqrt(d**2)*tan(x/2)/(d**2*tan(x/
2)**3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*sqrt(d**2)) + 2*b*sqrt
(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*sqr
t(d**2)), Eq(c, sqrt(d**2))), ((a*x + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 + b
*sin(x)*cos(x)/2)/c, Eq(d, 0)), ((a*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2
```

```
+ 1) + a*log(tan(x/2))/(tan(x/2)**2 + 1) + b*log(tan(x/2))*tan(x/2)**2/(tan
(x/2)**2 + 1) + b*log(tan(x/2))/(tan(x/2)**2 + 1) + 2*b/(tan(x/2)**2 + 1))/
d, Eq(c, 0)), (a*d**2*log(tan(x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(x/2)**
2/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + a*d**2*
log(tan(x/2) + d/c - sqrt(-c**2 + d**2)/c)/(d**2*sqrt(-c**2 + d**2)*tan(x/2
)**2 + d**2*sqrt(-c**2 + d**2)) - a*d**2*log(tan(x/2) + d/c + sqrt(-c**2 +
d**2)/c)*tan(x/2)**2/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2
 + d**2)) - a*d**2*log(tan(x/2) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*sqrt(-c
**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) - b*c**2*log(tan(x/2) +
d/c - sqrt(-c**2 + d**2)/c)*tan(x/2)**2/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**
2 + d**2*sqrt(-c**2 + d**2)) - b*c**2*log(tan(x/2) + d/c - sqrt(-c**2 + d**
2)/c)/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + b*c
**2*log(tan(x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(x/2)**2/(d**2*sqrt(-c**2
 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + b*c**2*log(tan(x/2) + d/c
 + sqrt(-c**2 + d**2)/c)/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-
c**2 + d**2)) + b*c*x*sqrt(-c**2 + d**2)*tan(x/2)**2/(d**2*sqrt(-c**2 + d**
2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + b*c*x*sqrt(-c**2 + d**2)/(d**2*
sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + b*d**2*log(tan(
x/2) + d/c - sqrt(-c**2 + d**2)/c)*tan(x/2)**2/(d**2*sqrt(-c**2 + d**2)*tan
(x/2)**2 + d**2*sqrt(-c**2 + d**2)) + b*d**2*log(tan(x/2) + d/c - sqrt(-c**
2 + d**2)/c)/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)
) - b*d**2*log(tan(x/2) + d/c + sqrt(-c**2 + d**2)/c)*tan(x/2)**2/(d**2*sqr
t(-c**2 + d**2)*tan(x/2)**2 + d**2*sqrt(-c**2 + d**2)) - b*d**2*log(tan(x/2
) + d/c + sqrt(-c**2 + d**2)/c)/(d**2*sqrt(-c**2 + d**2)*tan(x/2)**2 + d**2
*sqrt(-c**2 + d**2)) + 2*b*d*sqrt(-c**2 + d**2)/(d**2*sqrt(-c**2 + d**2)*ta
n(x/2)**2 + d**2*sqrt(-c**2 + d**2)), True))
```

Giac [**A**]
time = 0.40, size = 93, normalized size = 0.93

$$\frac{bcx}{d^2} - \frac{2\left(bc^2 - ad^2 - bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2}\,d^2} + \frac{2\,b}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] b*c*x/d^2 - 2*(b*c^2 - a*d^2 - b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2) + 2*b/((tan
(1/2*x)^2 + 1)*d)
```

Mupad [**B**]
time = 5.15, size = 1646, normalized size = 16.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(x)^2)/(c + d*sin(x)),x)

[Out] (2*b)/(d*(tan(x/2)^2 + 1)) - (atan((((-(c + d)*(c - d))^(1/2)*((32*b^2*c^4)/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2) - ((-(c + d)*(c - d))^(1/2)*((32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 - (32*b^2*c^4)/d + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 - ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2))/((64*(b^3*c^2*d^2 - a*b^2*c^4 - b^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*c^2*d^2))/d^2 + (64*tan(x/2)*(2*b^3*c^3*d^2 - 2*b^3*c^5 + 2*a*b^2*c^3*d^2))/d^3 + ((-(c + d)*(c - d))^(1/2)*((32*b^2*c^4)/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2) + ((-(c + d)*(c - d))^(1/2)*((32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 - (32*b^2*c^4)/d + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 - ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2)))*(-(c + d)*(c - d))^(1/2)*(a*d^2 - b*c^2 + b*d^2)*2i)/(d^4 - c^2*d^2) + (2*b*c*atan((64*b^3*c^2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)/d^2 - (128*a*b^2*c^4)/d^2) - (64*b^3*c^4*tan(x/2))/(64*b^3*c^2*d^2 - 128*a*b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2*d^2 + 64*a^2*b*c^2*d^2) - (128*a*b^2*c^4*tan(x/2))/(64*b^3*c^2*d^2 - 128*a*b^2*c^4 - 64*b^3*c^4 + 128*a*b^2*c^2*d^2 + 64*a^2*b*c^2*d^2) + (128*a*b^2*c^2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)/d^2 - (128*a*b^2*c^4)/d^2) + (64*a^2*b*c^2*tan(x/2))/(64*b^3*c^2 + 128*a*b^2*c^2 + 64*a^2*b*c^2 - (64*b^3*c^4)/d^2 - (128*a*b^2*c^4)/d^2)))/d^2

3.212 $\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx$

Optimal. Leaf size=56

$$-\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}\,c}+\frac{(a+2b)\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}\,c}$$

```
[Out] -b*x/c+1/2*(a+2*b)*x/c*2^(1/2)+1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+sin(x)^2
+2^(1/2)))/c*2^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1180, 211}

$$\frac{(a+2b)\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}\,c}+\frac{x(a+2b)}{\sqrt{2}\,c}-\frac{bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

```
[Out] -((b*x)/c) + ((a + 2*b)*x)/(Sqrt[2]*c) + ((a + 2*b)*ArcTan[(Cos[x]*Sin[x])/
(1 + Sqrt[2] + Sin[x]^2)])/(Sqrt[2]*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{a+b+ax^2}{c+3cx^2+2cx^4}\,dx, x, \tan(x)\right)$$
$$= -\left((2b)\text{Subst}\left(\int \frac{1}{2c+2cx^2}\,dx, x, \tan(x)\right)\right) + (a+2b)\text{Subst}\left(\int \frac{1}{c+2cx^2}\,dx, x, \tan(x\right.$$
$$= -\frac{bx}{c} + \frac{(a+2b)x}{\sqrt{2}\,c} + \frac{(a+2b)\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}\,c}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.55

$$-\frac{bx}{c} + \frac{(a+2b)\text{ArcTan}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
[Out] -((b*x)/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)
```

Maple [A]
time = 0.20, size = 29, normalized size = 0.52

method	result
default	$\frac{\frac{(a+2b)\sqrt{2}\arctan\left(\tan(x)\sqrt{2}\right)}{2}-b\arctan(\tan(x))}{c}$
risch	$-\frac{bx}{c}+\frac{i\sqrt{2}\ln\left(e^{2ix}-2\sqrt{2}-3\right)a}{4c}+\frac{i\sqrt{2}\ln\left(e^{2ix}-2\sqrt{2}-3\right)b}{2c}-\frac{i\sqrt{2}\ln\left(e^{2ix}+2\sqrt{2}-3\right)a}{4c}-\frac{i\sqrt{2}\ln\left(e^{2ix}+2\sqrt{\right.}}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+c*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/c*(1/2*(a+2*b)*2^(1/2)*arctan(tan(x)*2^(1/2))-b*arctan(tan(x)))
```

Maxima [A]
time = 0.50, size = 28, normalized size = 0.50

$$\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\sqrt{2}\tan\left(x\right)\right)}{2\,c} - \frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*arctan(sqrt(2)*tan(x))/c - b*x/c
```

Fricas [A]
time = 1.13, size = 45, normalized size = 0.80

$$-\frac{\sqrt{2}\,(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\,\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)
*sin(x))) + 4*b*x)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(49) = 98$.
time = 25.92, size = 520, normalized size = 9.29

$$\frac{54608393\sqrt{2}\,a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{77227930a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{9369319\sqrt{2}\,a\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{13250218a\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}-\frac{45239074bx}{31988856\sqrt{2}\,c+45239074c}-\frac{31988856\sqrt{2}\,bx}{31988856\sqrt{2}\,c+45239074c}+\frac{109216786\sqrt{2}\,b\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{154455860b\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{18738638\sqrt{2}\,b\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}+\frac{26500436b\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan(\frac{x}{2})}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}\,c+45239074c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+c*sin(x)**2),x)
```

```
[Out] 54608393*sqrt(2)*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))
+ pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 77227930*a
*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 -
pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 9369319*sqrt(2)*a*sqrt(2*sqr
t(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/
(31988856*sqrt(2)*c + 45239074*c) + 13250218*a*sqrt(2*sqrt(2) + 3)*(atan(ta
n(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*
c + 45239074*c) - 45239074*b*x/(31988856*sqrt(2)*c + 45239074*c) - 31988856
*sqrt(2)*b*x/(31988856*sqrt(2)*c + 45239074*c) + 109216786*sqrt(2)*b*sqrt(3
 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/p
i))/(31988856*sqrt(2)*c + 45239074*c) + 154455860*b*sqrt(3 - 2*sqrt(2))*(at
an(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqr
t(2)*c + 45239074*c) + 18738638*sqrt(2)*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2
)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 4
5239074*c) + 26500436*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) +
 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c)
```

Giac [A]
time = 0.42, size = 62, normalized size = 1.11

$$\frac{\sqrt{2}\,(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,x)-2\,\sin(2\,x)}{\sqrt{2}\,\cos(2\,x)+\sqrt{2}-2\,\cos(2\,x)+2}\right)\right)}{2\,c}-\frac{bx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)
*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))/c - b*x/c
```

Mupad [B]
time = 2.39, size = 249, normalized size = 4.45

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{4\sqrt{2}\,a^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{16\sqrt{2}\,b^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{40\sqrt{2}\,ab^2\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{24\sqrt{2}\,a^2b\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}\right)(a+2b)}{2c}-\frac{b\,\mathrm{atan}\left(\frac{8b^3\tan(x)}{4a^2b+16ab^2+8b^3}+\frac{16ab^2\tan(x)}{4a^2b+16ab^2+8b^3}+\frac{4a^2b\tan(x)}{4a^2b+16ab^2+8b^3}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c + c*sin(x)^2),x)
```

```
[Out] (2^(1/2)*atan((4*2^(1/2)*a^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3)
 + (16*2^(1/2)*b^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (40*2^(
1/2)*a*b^2*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (24*2^(1/2)*a^2
*b*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3))*(a + 2*b))/(2*c) - (b*at
an((8*b^3*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3) + (16*a*b^2*tan(x))/(16*a*b^
2 + 4*a^2*b + 8*b^3) + (4*a^2*b*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3)))/c
```

3.213 $\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{bx}{c}+\frac{a\tan(x)}{c}$$

[Out] b*x/c+a*tan(x)/c

Rubi [A]

time = 0.04, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3254, 3091, 8}

$$\frac{a\tan(x)}{c}+\frac{bx}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]

[Out] (b*x)/c + (a*Tan[x])/c

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx &= \frac{\int (a+b\cos^2(x))\sec^2(x)\,dx}{c}\\ &= \frac{a\tan(x)}{c}+\frac{b\int 1\,dx}{c}\\ &= \frac{bx}{c}+\frac{a\tan(x)}{c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{bx}{c}+\frac{a\tan(x)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]

[Out] (b*x)/c + (a*Tan[x])/c

Maple [A]
time = 0.13, size = 15, normalized size = 1.07

method	result	size
default	$\frac{a\tan(x)+b\arctan(\tan(x))}{c}$	15
risch	$\frac{bx}{c}+\frac{2ia}{(e^{2ix}+1)c}$	24
norman	$\frac{\frac{bx\left(\tan^4\left(\frac{x}{2}\right)\right)}{c}+\frac{bx\left(\tan^6\left(\frac{x}{2}\right)\right)}{c}-\frac{2a\tan\left(\frac{x}{2}\right)}{c}-\frac{4a\left(\tan^3\left(\frac{x}{2}\right)\right)}{c}-\frac{2a\left(\tan^5\left(\frac{x}{2}\right)\right)}{c}-\frac{bx}{c}-\frac{bx\left(\tan^2\left(\frac{x}{2}\right)\right)}{c}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(x)^2)/(c-c*sin(x)^2),x,method=_RETURNVERBOSE)

[Out] 1/c*(a*tan(x)+b*arctan(tan(x)))

Maxima [A]
time = 0.52, size = 14, normalized size = 1.00

$$\frac{bx}{c}+\frac{a\tan\left(x\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="maxima")

[Out] b*x/c + a*tan(x)/c

Fricas [A]
time = 2.32, size = 18, normalized size = 1.29

$$\frac{bx\cos\left(x\right)+a\sin\left(x\right)}{c\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="fricas")

```
[Out] (b*x*cos(x) + a*sin(x))/(c*cos(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.
time = 0.48, size = 51, normalized size = 3.64

$$-\frac{2a\tan\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right)-c}+\frac{bx\tan^2\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right)-c}-\frac{bx}{c\tan^2\left(\frac{x}{2}\right)-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c-c*sin(x)**2),x)
```

```
[Out] -2*a*tan(x/2)/(c*tan(x/2)**2 - c) + b*x*tan(x/2)**2/(c*tan(x/2)**2 - c) - b
*x/(c*tan(x/2)**2 - c)
```

Giac [A]
time = 0.40, size = 23, normalized size = 1.64

$$\frac{b\arctan\left(\frac{|c|\tan(x)}{c}\right)}{|c|}+\frac{a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(abs(c)*tan(x)/c)/abs(c) + a*tan(x)/c
```

Mupad [B]
time = 2.31, size = 12, normalized size = 0.86

$$\frac{b\,x+a\tan(x)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c - c*sin(x)^2),x)
```

```
[Out] (b*x + a*tan(x))/c
```

3.214 $\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$-\frac{bx}{d}+\frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}$$

```
[Out] -b*x/d+(a*d+b*(c+d))*arctan((c+d)^(1/2)*tan(x)/c^(1/2))/d/c^(1/2)/(c+d)^(1/
2)
```

Rubi [A]
time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {536, 209, 211}

$$\frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}-\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] -((b*x)/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt
[c]*d*Sqrt[c + d])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{a+b+ax^2}{(1+x^2)\,(c+(c+d)x^2)}\,dx, x, \tan(x)\right)$$

$$= -\frac{b\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{d} + \frac{(-ac+(a+b)(c+d))\text{Subst}\left(\int \frac{1}{c+(c+d)x^2}\,dx, x, \tan(\right.}{d}$$

$$= -\frac{bx}{d} + \frac{(ad+b(c+d))\tan^{-1}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,d\sqrt{c+d}}$$

Mathematica [A]
time = 0.12, size = 47, normalized size = 0.96

$$\frac{-bx + \frac{(ad+b(c+d))\text{ArcTan}\left(\frac{\sqrt{c+d}\,\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{c+d}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

```
[Out] (-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt[c]
*Sqrt[c + d]))/d
```

Maple [A]
time = 0.29, size = 46, normalized size = 0.94

method	result
default	$-\frac{b\arctan(\tan(x))}{d} + \frac{(ad+cb+db)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)\,c}}\right)}{d\sqrt{(c+d)\,c}}$
risch	$-\frac{bx}{d} - \frac{\ln\left(e^{2ix} - \frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)a}{2\sqrt{-c^2-cd}} - \frac{\ln\left(e^{2ix} - \frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-c}\right.}{2\sqrt{-c^2-cd}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)^2)/(c+d*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -b/d*arctan(tan(x))+(a*d+b*c+b*d)/d/((c+d)*c)^(1/2)*arctan((c+d)*tan(x)/((c
+d)*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.86

$$-\frac{bx}{d}+\frac{(bc+(a+b)d)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -b*x/d + (b*c + (a + b)*d)*arctan((c + d)*tan(x)/sqrt((c + d)*c))/(sqrt((c
+ d)*c)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.
time = 1.02, size = 255, normalized size = 5.20

$$\left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\,\log\left(\frac{(8c^2+8cd+d^2)\cos(x)^4-2(4c^2+5cd+d^2)\cos(x)^2+4\left((2c+d)\cos(x)^3-(c+d)\cos(x)\right)\sqrt{-c^2-cd}\,\sin(x)+c^2+2cd+d^2}{d^2\cos(x)^4-2(cd+d^2)\cos(x)^2+c^2+2cd+d^2}\right)+4(bc^2+bcd)x}{4(c^2d+cd^2)},\ -\frac{(bc+(a+b)d)\sqrt{c^2+cd}\,\arctan\left(\frac{(2c+d)\cos(x)^2-c-d}{2\sqrt{c^2+cd}\,\cos(x)\sin(x)}\right)+2(bc^2+bcd)x}{2(c^2d+cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x)
^4 - 2*(4*c^2 + 5*c*d + d^2)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - (c + d)*cos
(x))*sqrt(-c^2 - c*d)*sin(x) + c^2 + 2*c*d + d^2)/(d^2*cos(x)^4 - 2*(c*d +
d^2)*cos(x)^2 + c^2 + 2*c*d + d^2)) + 4*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^2),
 -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^2 - c
 - d)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x)/(c^2*d + c*d^
2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)**2)/(c+d*sin(x)**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 70, normalized size = 1.43

$$-\frac{bx}{d}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c+2d)+\arctan\left(\frac{c\tan(x)+d\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="giac")
```

```
[Out] -b*x/d + (pi*floor(x/pi + 1/2)*sgn(2*c + 2*d) + arctan((c*tan(x) + d*tan(x)
)/sqrt(c^2 + c*d)))*(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

Mupad [B]
time = 2.95, size = 1774, normalized size = 36.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x)^2)/(c + d*sin(x)^2),x)
```

```
[Out] - (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)
*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^
(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d
- c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*tan(x
)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a^2*c^
```

```
2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2
)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x)*(- c*
d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d^2*tan(x
)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^
4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*
2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2*c^2*d^4 +
2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 +
 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2
)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*d*x)/(c*d^2 + c^2*d)
```

3.215 $\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$

Optimal. Leaf size=74

$$\frac{2(ac^2+bd^2)\,\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

```
[Out] -b*d*arctanh(sin(x))/c^2+2*(a*c^2+b*d^2)*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d
)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tan(x)/c
```

Rubi [A]
time = 0.17, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4319, 3135, 3080, 3855, 2738, 211}

$$\frac{2(ac^2+bd^2)\,\text{ArcTan}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\,\sqrt{c+d}}-\frac{bd\tanh^{-1}(\sin(x))}{c^2}+\frac{b\tan(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(c^2*Sqrt[c
- d]*Sqrt[c + d]) - (b*d*ArcTanh[Sin[x]])/c^2 + (b*Tan[x])/c
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4319

```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Cos[a + b*x]^2)/Cos[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx &= \int \frac{(b+a\cos^2(x))\sec^2(x)}{c+d\cos(x)}\,dx \\
&= \frac{b\tan(x)}{c} + \frac{\int \frac{(-bd+ac\cos(x))\sec(x)}{c+d\cos(x)}\,dx}{c} \\
&= \frac{b\tan(x)}{c} - \frac{(bd)\int \sec(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\cos(x)}\,dx \\
&= -\frac{bd\tanh^{-1}(\sin(x))}{c^2} + \frac{b\tan(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+d+(c-d)x^2}\,dx, x, \text{ta}\right. \\
&= \frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}} - \frac{bd\tanh^{-1}(\sin(x))}{c^2} + \frac{b\tan(x)}{c}
\end{aligned}
$$

Mathematica [A]

time = 0.34, size = 98, normalized size = 1.32

$$\frac{-\frac{2(ac^2+bd^2)\tanh^{-1}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}+bd\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+bc\tan(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

```
[Out] ((-2*(a*c^2 + b*d^2)*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c^
2 + d^2] + b*d*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + b*c*
Tan[x])/c^2
```

Maple [**A**]
time = 0.32, size = 106, normalized size = 1.43

method	result
default	$-\frac{b}{c\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{db\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{c^2}-\frac{b}{c\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{db\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{c^2}+\frac{2\left(a\,c^2+b\,d^2\right)\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{c^2\sqrt{(c+d)(c-d)}}$
risch	$\frac{2ib}{c\left(e^{2ix}+1\right)}-\frac{\ln\left(e^{ix}+\frac{ic^2-id^2+\sqrt{-c^2+d^2}\,c}{d\sqrt{-c^2+d^2}}\right)a}{\sqrt{-c^2+d^2}}-\frac{\ln\left(e^{ix}+\frac{ic^2-id^2+\sqrt{-c^2+d^2}\,c}{d\sqrt{-c^2+d^2}}\right)b\,d^2}{\sqrt{-c^2+d^2}\,c^2}+\frac{\ln\left(e^{ix}+\frac{-ic^2+id^2+\sqrt{-c^2}}{\sqrt{-c^2+}}\right)}{\sqrt{-c^2+d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(x)^2)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/c/(tan(1/2*x)-1)+d*b/c^2*ln(tan(1/2*x)-1)-b/c/(tan(1/2*x)+1)-d*b/c^2*ln(
tan(1/2*x)+1)+2*(a*c^2+b*d^2)/c^2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*
x)/((c+d)*(c-d))^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(64) = 128$.
time = 2.76, size = 318, normalized size = 4.30

$$\left[-\frac{(ac^2+bd^2)\sqrt{-c^2+d^2}\cos(x)\log\left(\frac{2cd\cos(x)+(2c^2-d^2)\cos(x)^2+2\sqrt{-c^2+d^2}(c\cos(x)+d)\sin(x)-c^2+2d^2}{d^2\cos(x)^2+2cd\cos(x)+c^2}\right)+(bc^2d-bd^3)\cos(x)\log(\sin(x)+1)-(bc^2d-bd^3)\cos(x)\log(-\sin(x)+1)-2(bc^3-bcd^2)\sin(x)}{2(c^4-c^2d^2)\cos(x)}, \frac{2(ac^2+bd^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\cos(x)+d}{\sqrt{c^2-d^2}\sin(x)}\right)\cos(x)-(bc^2d-bd^3)\cos(x)\log(\sin(x)+1)+(bc^2d-bd^3)\cos(x)\log(-\sin(x)+1)+2(bc^3-bcd^2)\sin(x)}{2(c^4-c^2d^2)\cos(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*cos(x)*log((2*c*d*cos(x) + (2*c^2 -
 d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d
^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + (b*c^2*d - b*d^3)*cos(x)*log(sin(x) +
1) - (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) - 2*(b*c^3 - b*c*d^2)*sin(x)
)/((c^4 - c^2*d^2)*cos(x)), 1/2*(2*(a*c^2 + b*d^2)*sqrt(c^2 - d^2)*arctan(-
(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x)))*cos(x) - (b*c^2*d - b*d^3)*cos(x)*
log(sin(x) + 1) + (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) + 2*(b*c^3 - b*
c*d^2)*sin(x))/((c^4 - c^2*d^2)*cos(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)**2)/(c+d*cos(x)),x)
```

```
[Out] Integral((a + b*sec(x)**2)/(c + d*cos(x)), x)
```

Giac [A]
time = 0.42, size = 125, normalized size = 1.69

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{c^2}+\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{c^2}-\frac{2b\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2-1\right)c}-\frac{2(ac^2+bd^2)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}x\right)-d\tan\left(\frac{1}{2}x\right)}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x) + 1))/c^2 + b*d*log(abs(tan(1/2*x) - 1))/c^2 - 2*b*
tan(1/2*x)/((tan(1/2*x)^2 - 1)*c) - 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*tan(1/2*x))/sqrt(c^2 - d^2
)))/(sqrt(c^2 - d^2)*c^2)
```

Mupad [B]
time = 3.51, size = 1302, normalized size = 17.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(x)^2)/(c + d*cos(x)),x)
```

```
[Out] (b*c^3*sin(x))/(c^4*cos(x) - c^2*d^2*cos(x)) - (b*c*d^2*sin(x))/(c^4*cos(x)
 - c^2*d^2*cos(x)) + (2*b*d^3*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x)
- c^2*d^2*cos(x)) + (a*c^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
 + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2)))*cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) + (b*d^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^
2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i
 + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 - c^2)
^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*sin(x/2
)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i - b^2*c^
3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2
)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin(x/2)*(d^
2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c^3*d^4
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*2i
+ a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2) + a^2*c^4*d^
4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2*b^2*c^4*d^4*
cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*a*b*c^4*d^4*co
s(x/2) + 2*a*b*c^6*d^2*cos(x/2)))*cos(x)*(d^2 - c^2)^(1/2)*2i)/(c^4*cos(x)
- c^2*d^2*cos(x)) - (2*b*c^2*d*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x)
 - c^2*d^2*cos(x))
```

3.216 $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

Optimal. Leaf size=72

$$\frac{2(ac^2+bd^2)\,\text{ArcTan}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

```
[Out] b*d*arctanh(cos(x))/c^2-b*cot(x)/c+2*(a*c^2+b*d^2)*arctan((d+c*tan(1/2*x))/
(c^2-d^2)^(1/2))/c^2/(c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4318, 3135, 3080, 3855, 2739, 632, 210}

$$\frac{2(ac^2+bd^2)\,\text{ArcTan}\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\tanh^{-1}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/(c^2*Sqrt[c^2
- d^2]) + (b*d*ArcTanh[Cos[x]])/c^2 - (b*Cot[x])/c
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
 - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4318

```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Sin[a + b*x]^2)/Sin[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx = \int \frac{\csc^2(x)\,(b+a\sin^2(x))}{c+d\sin(x)}\,dx$$
$$= -\frac{b\cot(x)}{c} + \frac{\int \frac{\csc(x)(-bd+ac\sin(x))}{c+d\sin(x)}\,dx}{c}$$
$$= -\frac{b\cot(x)}{c} - \frac{(bd)\int \csc(x)\,dx}{c^2} + \left(a+\frac{bd^2}{c^2}\right)\int \frac{1}{c+d\sin(x)}\,dx$$
$$= \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c} + \left(2\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\left(\frac{x}{2}\right.\right.$$
$$= \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c} - \left(4\left(a+\frac{bd^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{-4(c^2-d^2)-x^2}\,dx, x, 2d\right.$$
$$= \frac{2\left(a+\frac{bd^2}{c^2}\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{bd\tanh^{-1}(\cos(x))}{c^2} - \frac{b\cot(x)}{c}$$

Mathematica [A]
time = 0.40, size = 102, normalized size = 1.42

$$\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2(ac^2+bd^2)\text{ArcTan}\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)\sin(x)}{\sqrt{c^2-d^2}} - b\left(c\cos(x)+d\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)\right)\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

```
[Out] (Csc[x/2]*Sec[x/2]*((2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d
^2]]*Sin[x])/Sqrt[c^2 - d^2] - b*(c*Cos[x] + d*(-Log[Cos[x/2]] + Log[Sin[x/
2]])*Sin[x])))/(2*c^2)
```

Maple [A]
time = 0.34, size = 90, normalized size = 1.25

method	result
default	$\frac{b\tan\left(\frac{x}{2}\right)}{2c} - \frac{b}{2c\tan\left(\frac{x}{2}\right)} - \frac{db\ln\left(\tan\left(\frac{x}{2}\right)\right)}{c^2} + \frac{(4a\,c^2+4b\,d^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{2c^2\sqrt{c^2-d^2}}$
risch	$-\frac{2ib}{c(e^{2ix}-1)} - \frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{d\sqrt{-c^2+d^2}}\right)a}{\sqrt{-c^2+d^2}} - \frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{d\sqrt{-c^2+d^2}}\right)b\,d^2}{\sqrt{-c^2+d^2}\,c^2} + \frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-}{d\sqrt{-c^2+d}}\right.}{\sqrt{-c^2+d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*csc(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b/c*tan(1/2*x)-1/2*b/c/tan(1/2*x)-d*b/c^2*ln(tan(1/2*x))+1/2/c^2*(4*a*c
^2+4*b*d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2)
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.
time = 1.93, size = 332, normalized size = 4.61

$$\left[-\frac{(ac^2+bd^2)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(x)^2-2cd\sin(x)-c^2-d^2+2(c\cos(x)\sin(x)+d\cos(x))\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)\sin(x)-(bc^2d-bd^3)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+(bc^2d-bd^3)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2(bc^3-bcd^2)\cos(x)}{2(c^4-c^2d^2)\sin(x)},-\frac{2(ac^2+bd^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(x)+d}{\sqrt{c^2-d^2}\cos(x)}\right)\sin(x)-(bc^2d-bd^3)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+(bc^2d-bd^3)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+2(bc^3-bcd^2)\cos(x)}{2(c^4-c^2d^2)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*d
*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d^2
*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2))*sin(x) - (b*c^2*d - b*d^3)*log(1/2*c
os(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(
b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x)), -1/2*(2*(a*c^2 + b*d^2)*
sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x)))*sin(x) - (
b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*
cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x))
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)**2)/(c+d*sin(x)),x)
```

```
[Out] Integral((a + b*csc(x)**2)/(c + d*sin(x)), x)
```

Giac [A]
time = 0.40, size = 110, normalized size = 1.53

$$-\frac{bd\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{c^2}+\frac{b\tan\left(\frac{1}{2}x\right)}{2c}+\frac{2\left(ac^2+bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,c^2}+\frac{2\,bd\tan\left(\frac{1}{2}x\right)-bc}{2\,c^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

```
[Out] -b*d*log(abs(tan(1/2*x)))/c^2 + 1/2*b*tan(1/2*x)/c + 2*(a*c^2 + b*d^2)*(pi*
floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/
(sqrt(c^2 - d^2)*c^2) + 1/2*(2*b*d*tan(1/2*x) - b*c)/(c^2*tan(1/2*x))
```

Mupad [B]
time = 2.83, size = 463, normalized size = 6.43

$$\frac{b\,d^3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-b\,c^2\,d\,\ln\left(\tan\left(\frac{x}{2}\right)\right)+a\,c^2\,\mathrm{atan}\left(\frac{a\,c^3\,\sqrt{d^2-c^2}\,1\mathrm{i}+b\,d^3\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,4\mathrm{i}+b\,c\,d^2\,\sqrt{d^2-c^2}\,2\mathrm{i}+a\,c^2\,d\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,2\mathrm{i}-b\,c^2\,d\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,1\mathrm{i}}{4\,b\,d^4\,\tan\left(\frac{x}{2}\right)-a\,c^4\,\tan\left(\frac{x}{2}\right)+a\,c^3\,d+2\,b\,c\,d^3-b\,c^3\,d+2\,a\,c^2\,d^2\,\tan\left(\frac{x}{2}\right)-3\,b\,c^2\,d^2\,\tan\left(\frac{x}{2}\right)}\right)\,\sqrt{d^2-c^2}\,2\mathrm{i}+b\,d^2\,\mathrm{atan}\left(\frac{a\,c^3\,\sqrt{d^2-c^2}\,1\mathrm{i}+b\,d^3\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,4\mathrm{i}+b\,c\,d^2\,\sqrt{d^2-c^2}\,2\mathrm{i}+a\,c^2\,d\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,2\mathrm{i}-b\,c^2\,d\,\tan\left(\frac{x}{2}\right)\,\sqrt{d^2-c^2}\,1\mathrm{i}}{4\,b\,d^4\,\tan\left(\frac{x}{2}\right)-a\,c^4\,\tan\left(\frac{x}{2}\right)+a\,c^3\,d+2\,b\,c\,d^3-b\,c^3\,d+2\,a\,c^2\,d^2\,\tan\left(\frac{x}{2}\right)-3\,b\,c^2\,d^2\,\tan\left(\frac{x}{2}\right)}\right)\,\sqrt{d^2-c^2}\,2\mathrm{i}}{c^4-c^2\,d^2}-\frac{b\,c^3-b\,c\,d^2}{c^4\,\tan\left(x\right)-c^2\,d^2\,\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/sin(x)^2)/(c + d*sin(x)),x)
```

```
[Out] (b*d^3*log(tan(x/2)) + a*c^2*atan((a*c^3*(d^2 - c^2)^(1/2)*1i + b*d^3*tan(x
/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(1/2)*2i + a*c^2*d*tan(x/2)*
(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*1i)/(4*b*d^4*tan(
x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*c^3*d + 2*a*c^2*d^2*tan(x/2
) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i + b*d^2*atan((a*c^3*(d^2 -
c^2)^(1/2)*1i + b*d^3*tan(x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(
1/2)*2i + a*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c
^2)^(1/2)*1i)/(4*b*d^4*tan(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*
c^3*d + 2*a*c^2*d^2*tan(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i
- b*c^2*d*log(tan(x/2)))/(c^4 - c^2*d^2) - (b*c^3 - b*c*d^2)/(c^4*tan(x) -
c^2*d^2*tan(x))
```

3.217 $\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx$

Optimal. Leaf size=136

$$-\frac{\cos^{1+n}\left(c+dx-\tan^{-1}(a,b)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan^{-1}(a,b)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^n}{d(1+n)\sqrt{\sin^2\left(c+dx-\tan^{-1}(a,b)\right)}}$$

```
[Out] -cos(c+d*x-arctan(a,b))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(c+
d*x-arctan(a,b))^2)*(a*cos(d*x+c)+b*sin(d*x+c))^n*sin(c+d*x-arctan(a,b))/d/
(1+n)/(((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^n)/(sin(c+d*x-arctan(a
,b))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3157, 2722}

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c+dx\right)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2\left(c+dx-\tan^{-1}(a,b)\right)\right)}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x - ArcTan[a, b]]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3
 + n)/2, Cos[c + d*x - ArcTan[a, b]]^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^n
*Sin[c + d*x - ArcTan[a, b]])/(d*(1 + n)*((a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2])^n*Sqrt[Sin[c + d*x - ArcTan[a, b]]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx = \left((a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\right.$$

$$= -\frac{\cos^{1+n}\left(c+dx-\tan^{-1}(a,b)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\tan^{-1}\right.\right.}{d(1+n}$$

Mathematica [A]
time = 0.18, size = 94, normalized size = 0.69

$$-\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\cos^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^n\sin^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)^{-\frac{1}{2}-\frac{n}{2}}\sin\left(2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
[Out] -1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[a/b]]^2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[a/b]]^2)^(-1/2 -
n/2)*Sin[2*(c + d*x + ArcTan[a/b])])/d
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int (a\cos(dx+c)+b\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
[Out] int((a*cos(d*x+c)+b*sin(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^n,x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^n, x)
```

3.218 $\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{13^{n/2}\cos^{1+n}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{d(1+n)\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}$$

```
[Out] -13^(1/2*n)*cos(c+d*x-arctan(3/2))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/
2*n],cos(c+d*x-arctan(3/2))^2)*sin(c+d*x-arctan(3/2))/d/(1+n)/(sin(c+d*x-ar
ctan(3/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3156, 2722}

$$-\frac{13^{n/2}\sin\left(-\text{ArcTan}\left(\frac{3}{2}\right)+c+dx\right)\cos^{n+1}\left(-\text{ArcTan}\left(\frac{3}{2}\right)+c+dx\right)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}{d(n+1)\sqrt{\sin^2\left(-\text{ArcTan}\left(\frac{3}{2}\right)+c+dx\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

```
[Out] -((13^(n/2)*Cos[c + d*x - ArcTan[3/2]]^(1 + n)*Hypergeometric2F1[1/2, (1 +
n)/2, (3 + n)/2, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/
(d*(1 + n)*Sqrt[Sin[c + d*x - ArcTan[3/2]]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = 13^{n/2}\int \cos^n\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)dx$$

$$= -\frac{13^{n/2}\cos^{1+n}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2\left(c+dx-\right.\right.}{d(1+n)\sqrt{\sin^2\left(c+dx-\tan^{-1}\left(\right.\right.}}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.93

$$-\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\cos^2\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)(2\cos(c+dx)+3\sin(c+dx))^n\sin^2\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)^{-\frac{1}{2}-\frac{n}{2}}\sin\left(2\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[2/3]]^2]*
(2*Cos[c + d*x] + 3*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[2/3]]^2)^(-1/2 -
n/2)*Sin[2*(c + d*x + ArcTan[2/3])])/d
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

```
[Out] int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin(c+dx)+2\cos(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**n,x)
```

```
[Out] Integral((3*sin(c + d*x) + 2*cos(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,\cos(c+d\,x)+3\,\sin(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^n,x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^n, x)
```

3.219 $\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$

Optimal. Leaf size=127

$$-\frac{(a^2+b^2)^3\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{3(a^2+b^2)\,(b\cos(c$$

[Out] -(a^2+b^2)^3*(b*cos(d*x+c)-a*sin(d*x+c))/d+(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))^3/d-3/5*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))^5/d+1/7*(b*cos(d*x+c)-a*sin(d*x+c))^7/d

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$,
Rules used = {3151, 200}

$$-\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))^5}{5d}+\frac{(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))^3}{d}-\frac{(a^2+b^2)^3\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^7}{7d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]

[Out] -(((a^2 + b^2)^3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + ((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x])^3)/d - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])^5)/(5*d) + (b*Cos[c + d*x] - a*Sin[c + d*x])^7/(7*d)

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx=-\frac{\text{Subst}\left(\int\left(a^2+b^2-x^2\right)^3\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$=-\frac{\text{Subst}\left(\int\left(a^6\left(1+\frac{3a^4b^2+3a^2b^4+b^6}{a^6}\right)-3a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)x^2+3a^2\left(1\right.\right.\right.}{d}$$

$$=-\frac{(a^2+b^2)^3\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(a^2+b^2)^2\,(b\cos(c+$$

Mathematica [A]
time = 0.72, size = 246, normalized size = 1.94

$$\frac{-1225b(a^2+b^2)^3\cos(c+dx)+245b(-3a^2+b^2)(a^2+b^2)^2\cos(3(c+dx))-49b(5a^6-5a^4b^2-9a^2b^4+b^6)\cos(5(c+dx))+5b(-7a^6+35a^4b^2-21a^2b^4+b^6)\cos(7(c+dx))+1225a(a^2+b^2)^3\sin(c+dx)+245a(a^2-3b^2)(a^2+b^2)^2\sin(3(c+dx))+49a(a^6-9a^4b^2-5a^2b^4+5b^6)\sin(5(c+dx))+5a(a^6-21a^4b^2+35a^2b^4-7b^6)\sin(7(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

```
[Out] (-1225*b*(a^2 + b^2)^3*Cos[c + d*x] + 245*b*(-3*a^2 + b^2)*(a^2 + b^2)^2*Co
s[3*(c + d*x)] - 49*b*(5*a^6 - 5*a^4*b^2 - 9*a^2*b^4 + b^6)*Cos[5*(c + d*x)
] + 5*b*(-7*a^6 + 35*a^4*b^2 - 21*a^2*b^4 + b^6)*Cos[7*(c + d*x)] + 1225*a*
(a^2 + b^2)^3*Sin[c + d*x] + 245*a*(a^2 - 3*b^2)*(a^2 + b^2)^2*Sin[3*(c + d
*x)] + 49*a*(a^6 - 9*a^4*b^2 - 5*a^2*b^4 + 5*b^6)*Sin[5*(c + d*x)] + 5*a*(a
^6 - 21*a^4*b^2 + 35*a^2*b^4 - 7*b^6)*Sin[7*(c + d*x)])/(2240*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(123) = 246$.
time = 0.59, size = 321, normalized size = 2.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*b^7*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+b^6*a*sin(d*x+c)^7+21*a^2*b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(
d*x+c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3)+35*b^4*a^3*(-1/7*sin(d*x+c)^3*cos
(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+35
*a^4*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+21*b^2*a^5*(-1/
7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+
c))-a^6*b*cos(d*x+c)^7+1/7*a^7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(123) = 246$.
time = 0.26, size = 257, normalized size = 2.02

$$-\frac{35\,a^6b\cos(dx+c)^7-35\,ab^6\sin(dx+c)^7+\left(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c)\right)a^7-7\left(15\,\sin(dx+c)^7-42\,\sin(dx+c)^5+35\,\sin(dx+c)^3\right)a^5b^2-35\left(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5\right)a^4b^3+35\left(5\,\sin(dx+c)^7-7\,\sin(dx+c)^5\right)a^3b^4+7\left(15\,\cos(dx+c)^7-42\,\cos(dx+c)^5+35\,\cos(dx+c)^3\right)a^2b^5-\left(5\,\cos(dx+c)^7-21\,\cos(dx+c)^5+35\,\cos(dx+c)^3-35\,\cos(dx+c)\right)b^7}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="maxima")
```

```
[Out] -1/35*(35*a^6*b*cos(d*x + c)^7 - 35*a*b^6*sin(d*x + c)^7 + (5*sin(d*x + c)^
7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^7 - 7*(15*si
n(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5*b^2 - 35*(5*cos(d
*x + c)^7 - 7*cos(d*x + c)^5)*a^4*b^3 + 35*(5*sin(d*x + c)^7 - 7*sin(d*x +
c)^5)*a^3*b^4 + 7*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^
```

```
3)*a^2*b^5 - (5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35
*cos(d*x + c))*b^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(123) = 246$.
time = 1.32, size = 257, normalized size = 2.02

$$-\frac{35\,b^{7}\cos\left(dx+c\right)+5\,\left(7\,a^{6}b-35\,a^{4}b^{3}+21\,a^{2}b^{5}-b^{7}\right)\cos\left(dx+c\right)^{7}+7\,\left(35\,a^{4}b^{3}-42\,a^{2}b^{5}+3\,b^{7}\right)\cos\left(dx+c\right)^{5}+35\,\left(7\,a^{2}b^{5}-b^{7}\right)\cos\left(dx+c\right)^{3}-\left(16\,a^{7}+56\,a^{5}b^{2}+70\,a^{3}b^{4}+35\,ab^{6}+5\,\left(a^{7}-21\,a^{5}b^{2}+35\,a^{3}b^{4}-7\,ab^{6}\right)\cos\left(dx+c\right)^{6}+\left(6\,a^{7}+21\,a^{5}b^{2}-280\,a^{3}b^{4}+105\,ab^{6}\right)\cos\left(dx+c\right)^{4}+\left(8\,a^{7}+28\,a^{5}b^{2}+35\,a^{3}b^{4}-105\,ab^{6}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^7*cos(d*x + c) + 5*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*co
s(d*x + c)^7 + 7*(35*a^4*b^3 - 42*a^2*b^5 + 3*b^7)*cos(d*x + c)^5 + 35*(7*a
^2*b^5 - b^7)*cos(d*x + c)^3 - (16*a^7 + 56*a^5*b^2 + 70*a^3*b^4 + 35*a*b^6
 + 5*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^6 + (6*a^7 + 21
*a^5*b^2 - 280*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^4 + (8*a^7 + 28*a^5*b^2 +
35*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(110) = 220$.
time = 0.69, size = 461, normalized size = 3.63

$$\begin{cases} \frac{16a^{7}\sin^{7}\left(c+dx\right)}{35d}+\frac{8a^{7}\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{2a^{7}\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}+\frac{a^{7}\sin\left(c+dx\right)\cos^{6}\left(c+dx\right)}{d}-\frac{a^{6}b\cos^{7}\left(c+dx\right)}{d}+\frac{8a^{5}b^{2}\sin^{7}\left(c+dx\right)}{5d}+\frac{28a^{5}b^{2}\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{7a^{5}b^{2}\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{7a^{4}b^{3}\sin^{2}\left(c+dx\right)\cos^{5}\left(c+dx\right)}{d}-\frac{2a^{4}b^{3}\cos^{7}\left(c+dx\right)}{d}+\frac{2a^{3}b^{4}\sin^{7}\left(c+dx\right)}{d}+\frac{7a^{3}b^{4}\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{7a^{2}b^{5}\sin^{4}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{d}-\frac{28a^{2}b^{5}\sin^{2}\left(c+dx\right)\cos^{5}\left(c+dx\right)}{5d}-\frac{8a^{2}b^{5}\cos^{7}\left(c+dx\right)}{5d}+\frac{ab^{6}\sin^{7}\left(c+dx\right)}{d}-\frac{b^{7}\sin^{6}\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{2b^{7}\sin^{4}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{d}-\frac{8b^{7}\sin^{2}\left(c+dx\right)\cos^{5}\left(c+dx\right)}{5d}-\frac{16b^{7}\cos^{7}\left(c+dx\right)}{35d} & \text{for}\: d \neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**7,x)
```

```
[Out] Piecewise((16*a**7*sin(c + d*x)**7/(35*d) + 8*a**7*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**7*sin(c + d*x)**3*cos(c + d*x)**4/d + a**7*sin(c + d*x
)*cos(c + d*x)**6/d - a**6*b*cos(c + d*x)**7/d + 8*a**5*b**2*sin(c + d*x)**
7/(5*d) + 28*a**5*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 7*a**5*b**2*
sin(c + d*x)**3*cos(c + d*x)**4/d - 7*a**4*b**3*sin(c + d*x)**2*cos(c + d*x
)**5/d - 2*a**4*b**3*cos(c + d*x)**7/d + 2*a**3*b**4*sin(c + d*x)**7/d + 7*
a**3*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 7*a**2*b**5*sin(c + d*x)**4*c
os(c + d*x)**3/d - 28*a**2*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 8*a
**2*b**5*cos(c + d*x)**7/(5*d) + a*b**6*sin(c + d*x)**7/d - b**7*sin(c + d*
x)**6*cos(c + d*x)/d - 2*b**7*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**7*si
n(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*b**7*cos(c + d*x)**7/(35*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(123) = 246$.
time = 0.64, size = 316, normalized size = 2.49

$$-\frac{\left(7\,a^{6}b-35\,a^{4}b^{3}+21\,a^{2}b^{5}-b^{7}\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{7\,\left(5\,a^{6}b-5\,a^{4}b^{3}-9\,a^{2}b^{5}+b^{7}\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{7\,\left(3\,a^{6}b+5\,a^{4}b^{3}+a^{2}b^{5}-b^{7}\right)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{35\,\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)}{64\,d}+\frac{\left(a^{7}-21\,a^{5}b^{2}+35\,a^{3}b^{4}-7\,ab^{6}\right)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{7\,\left(a^{7}-9\,a^{5}b^{2}-5\,a^{3}b^{4}+5\,ab^{6}\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{7\,\left(a^{7}-a^{5}b^{2}-5\,a^{3}b^{4}-3\,ab^{6}\right)\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{35\,\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="giac")
```

```
[Out] -1/448*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(7*d*x + 7*c)/d - 7/320
*(5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*cos(5*d*x + 5*c)/d - 7/64*(3*a^6*b
 + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(3*d*x + 3*c)/d - 35/64*(a^6*b + 3*a^4*b^3
 + 3*a^2*b^5 + b^7)*cos(d*x + c)/d + 1/448*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 -
 7*a*b^6)*sin(7*d*x + 7*c)/d + 7/320*(a^7 - 9*a^5*b^2 - 5*a^3*b^4 + 5*a*b^6
)*sin(5*d*x + 5*c)/d + 7/64*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*sin(3*d*x
 + 3*c)/d + 35/64*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(d*x + c)/d
```

Mupad [B]
time = 6.16, size = 422, normalized size = 3.32

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(70\,a^6\,b-140\,a^4\,b^3+224\,a^2\,b^5\right)-2\,a^7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{424\,a^7}{35}+\frac{912\,a^5\,b^2}{5}-192\,a^3\,b^4+128\,a\,b^6\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(42\,a^6\,b-56\,a^4\,b^3+\frac{336\,a^2\,b^5}{5}+\frac{96\,b^7}{5}\right)+2\,a^6\,b+\frac{32\,b^7}{35}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{86\,a^7}{5}-\frac{224\,a^5\,b^2}{5}+224\,a^3\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{86\,a^7}{5}-\frac{224\,a^5\,b^2}{5}+224\,a^3\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(28\,a^4\,b^3+\frac{112\,a^2\,b^5}{5}+\frac{32\,b^7}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(280\,a^4\,b^3-112\,a^2\,b^5+32\,b^7\right)+\frac{16\,a^2\,b^5}{5}+4\,a^4\,b^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a^7+56\,a^5\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(4\,a^7+56\,a^5\,b^2\right)-2\,a^7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+140\,a^4\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+14\,a^6\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^7,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^8*(70*a^6*b + 224*a^2*b^5 - 140*a^4*b^3) - 2*a^7*tan(c
/2 + (d*x)/2)^13 - tan(c/2 + (d*x)/2)^7*(128*a*b^6 + (424*a^7)/35 - 192*a^3
*b^4 + (912*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^4*(42*a^6*b + (96*b^7)/5 + (33
6*a^2*b^5)/5 - 56*a^4*b^3) + 2*a^6*b + (32*b^7)/35 - tan(c/2 + (d*x)/2)^5*(
(86*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) - tan(c/2 + (d*x)/2)^9*((86*a^7
)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^2*((32*b^7)/5 + (
112*a^2*b^5)/5 + 28*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(32*b^7 - 112*a^2*b^5 +
 280*a^4*b^3) + (16*a^2*b^5)/5 + 4*a^4*b^3 - tan(c/2 + (d*x)/2)^3*(4*a^7 +
56*a^5*b^2) - tan(c/2 + (d*x)/2)^11*(4*a^7 + 56*a^5*b^2) - 2*a^7*tan(c/2 +
(d*x)/2) + 140*a^4*b^3*tan(c/2 + (d*x)/2)^10 + 14*a^6*b*tan(c/2 + (d*x)/2)^
12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.220 $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

Optimal. Leaf size=161

$$\frac{5}{16}\left(a^2+b^2\right)^3 x-\frac{5\left(a^2+b^2\right)^2\left(b\cos(c+dx)-a\sin(c+dx)\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}{16d}-\frac{5\left(a^2+b^2\right)\left(b\cos\right.}{}$$

```
[Out] 5/16*(a^2+b^2)^3*x-5/16*(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+
c)+b*sin(d*x+c))/d-5/24*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)
+b*sin(d*x+c))^3/d-1/6*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+
c))^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3152, 8}

$$-\frac{5\left(a^2+b^2\right)\left(b\cos(c+dx)-a\sin(c+dx)\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^3}{24d}-\frac{5\left(a^2+b^2\right)^2\left(b\cos(c+dx)-a\sin(c+dx)\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}{16d}+\frac{5}{16}x\left(a^2+b^2\right)^3-\frac{\left(b\cos(c+dx)-a\sin(c+dx)\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^5}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (5*(a^2 + b^2)^3*x)/16 - (5*(a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(16*d) - (5*(a^2 + b^2)*(b*Cos[c + d*x]
 - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(24*d) - ((b*Cos[c
+ d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d} + \frac{1}{6}$$
$$= -\frac{5(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+}{24d}$$
$$= -\frac{5(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+}{16d}$$
$$= \frac{5}{16}\left(a^2+b^2\right)^3 x - \frac{5(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c-}{16d}$$

Mathematica [A]
time = 0.53, size = 192, normalized size = 1.19

$$\frac{60(a^2+b^2)^3(c+dx)-90ab(a^2+b^2)^2\cos(2(c+dx))-36ab(a^4-b^4)\cos(4(c+dx))-2ab(3a^4-10a^2b^2+3b^4)\cos(6(c+dx))+45(a^2-b^2)(a^2+b^2)^2\sin(2(c+dx))+9(a^6-5a^4b^2-5a^2b^4+b^6)\sin(4(c+dx))+(a^6-15a^4b^2+15a^2b^4-b^6)\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

```
[Out] (60*(a^2 + b^2)^3*(c + d*x) - 90*a*b*(a^2 + b^2)^2*Cos[2*(c + d*x)] - 36*a*
b*(a^4 - b^4)*Cos[4*(c + d*x)] - 2*a*b*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Cos[6*(
c + d*x)] + 45*(a^2 - b^2)*(a^2 + b^2)^2*Sin[2*(c + d*x)] + 9*(a^6 - 5*a^4*
b^2 - 5*a^2*b^4 + b^6)*Sin[4*(c + d*x)] + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 -
b^6)*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.51, size = 285, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^6*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5
/16*d*x+5/16*c)+a*b^5*sin(d*x+c)^6+15*a^2*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)
^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+
20*a^3*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+15*a^4*b^2*(-
1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1
/16*d*x+1/16*c)-a^5*b*cos(d*x+c)^6+a^6*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.28, size = 238, normalized size = 1.48

$$-\frac{192\,a^5b\cos(dx+c)^6-192\,ab^5\sin(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^6-15\left(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(4\,dx+4\,c)\right)a^4b^2+320\left(2\,\sin(dx+c)^6-3\,\sin(dx+c)^4\right)a^3b^3+15\left(4\,\sin(2\,dx+2\,c)^3-12\,dx-12\,c+3\,\sin(4\,dx+4\,c)\right)a^2b^4-\left(4\,\sin(2\,dx+2\,c)^3+60\,dx+60\,c+9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)b^6}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/192*(192*a^5*b*cos(d*x + c)^6 - 192*a*b^5*sin(d*x + c)^6 + (4*sin(2*d*x
+ 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^6 -
15*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4*b^2 + 32
0*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3*b^3 + 15*(4*sin(2*d*x + 2*c)^3
- 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2*b^4 - (4*sin(2*d*x + 2*c)^3 + 60*
d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^6)/d
```

Fricas [A]
time = 1.43, size = 219, normalized size = 1.36

$$-\frac{144\,ab^5\cos\left(dx+c\right)^2+16\,(3\,a^5b-10\,a^3b^3+3\,ab^5)\cos\left(dx+c\right)^6+48\,(5\,a^3b^3-3\,ab^5)\cos\left(dx+c\right)^4-15\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)dx-\left(8\,(a^6-15\,a^4b^2+15\,a^2b^4-b^6)\cos\left(dx+c\right)^5+2\,(5\,a^6+15\,a^4b^2-105\,a^2b^4+13\,b^6)\cos\left(dx+c\right)^3+3\,(5\,a^6+15\,a^4b^2+15\,a^2b^4-11\,b^6)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/48*(144*a*b^5*cos(d*x + c)^2 + 16*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(d
*x + c)^6 + 48*(5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 - 15*(a^6 + 3*a^4*b^2 +
 3*a^2*b^4 + b^6)*d*x - (8*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*cos(d*x +
c)^5 + 2*(5*a^6 + 15*a^4*b^2 - 105*a^2*b^4 + 13*b^6)*cos(d*x + c)^3 + 3*(5*
a^6 + 15*a^4*b^2 + 15*a^2*b^4 - 11*b^6)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(151) = 302$.
time = 0.50, size = 770, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Piecewise((5*a**6*x*sin(c + d*x)**6/16 + 15*a**6*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**6*x*cos(c
+ d*x)**6/16 + 5*a**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**6*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**6*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a**5*b*cos(c + d*x)**6/d + 15*a**4*b**2*x*sin(c + d*x)**6/16 + 45*a**4*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 45*a**4*b**2*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 15*a**4*b**2*x*cos(c + d*x)**6/16 + 15*a**4*b**2*sin(c
+ d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*b**2*sin(c + d*x)**3*cos(c + d*x)**3
/(2*d) - 15*a**4*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 5*a**3*b**3*sin
(c + d*x)**2*cos(c + d*x)**4/d - 5*a**3*b**3*cos(c + d*x)**6/(3*d) + 15*a**
2*b**4*x*sin(c + d*x)**6/16 + 45*a**2*b**4*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 45*a**2*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 15*a**2*b**4*x*c
os(c + d*x)**6/16 + 15*a**2*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*
*2*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 15*a**2*b**4*sin(c + d*x)*c
```

```
os(c + d*x)**5/(16*d) + a*b**5*sin(c + d*x)**6/d + 5*b**6*x*sin(c + d*x)**6
/16 + 15*b**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**6*x*sin(c + d*x)
**2*cos(c + d*x)**4/16 + 5*b**6*x*cos(c + d*x)**6/16 - 11*b**6*sin(c + d*x)
**5*cos(c + d*x)/(16*d) - 5*b**6*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*
b**6*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c
))**6, True))
```

Giac [A]
time = 0.51, size = 235, normalized size = 1.46

$$\frac{5}{16}\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)x-\frac{\left(3\,a^{5}b-10\,a^{3}b^{3}+3\,ab^{5}\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{3\,\left(a^{5}b-ab^{5}\right)\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{15\,\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\cos\left(2\,dx+2\,c\right)}{32\,d}+\frac{\left(a^{6}-15\,a^{4}b^{2}+15\,a^{2}b^{4}-b^{6}\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,\left(a^{6}-5\,a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{15\,\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] 5/16*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 1/96*(3*a^5*b - 10*a^3*b^3 + 3
*a*b^5)*cos(6*d*x + 6*c)/d - 3/16*(a^5*b - a*b^5)*cos(4*d*x + 4*c)/d - 15/3
2*(a^5*b + 2*a^3*b^3 + a*b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^6 - 15*a^4*b^2
+ 15*a^2*b^4 - b^6)*sin(6*d*x + 6*c)/d + 3/64*(a^6 - 5*a^4*b^2 - 5*a^2*b^4
+ b^6)*sin(4*d*x + 4*c)/d + 15/64*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*sin(2*d*x
 + 2*c)/d
```

Mupad [B]
time = 4.12, size = 519, normalized size = 3.22

$$\frac{5\,\mathrm{atan}\left(\frac{5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^3}{8\,\left(\frac{5\,a^6}{8}+\frac{5\,b^6}{8}+\frac{15\,a^2\,b^4}{8}+\frac{15\,a^4\,b^2}{8}\right)}\right)\,{\left(a^2+b^2\right)}^3}{8\,d}-\frac{5\,\left(\mathrm{atan}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^3}{8\,d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(64\,a\,b^5+40\,a^5\,b-\frac{160\,a^3\,b^3}{3}\right)-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,b^6}{8}-\frac{11\,a^6}{8}+\frac{15\,a^2\,b^4}{8}+\frac{15\,a^4\,b^2}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{5\,b^6}{8}-\frac{11\,a^6}{8}+\frac{15\,a^2\,b^4}{8}+\frac{15\,a^4\,b^2}{8}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{5\,a^6}{24}+\frac{85\,b^6}{24}+\frac{85\,a^2\,b^4}{8}-\frac{235\,a^4\,b^2}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{5\,a^6}{24}+\frac{85\,b^6}{24}+\frac{85\,a^2\,b^4}{8}-\frac{235\,a^4\,b^2}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{15\,a^6}{4}-\frac{33\,b^6}{4}+\frac{285\,a^2\,b^4}{4}-\frac{195\,a^4\,b^2}{4}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{15\,a^6}{4}-\frac{33\,b^6}{4}+\frac{285\,a^2\,b^4}{4}-\frac{195\,a^4\,b^2}{4}\right)+80\,a^3\,b^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+80\,a^3\,b^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+12\,a^5\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+12\,a^5\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

```
[Out] (5*atan((5*tan(c/2 + (d*x)/2)*(a^2 + b^2)^3)/(8*((5*a^6)/8 + (5*b^6)/8 + (1
5*a^2*b^4)/8 + (15*a^4*b^2)/8)))*(a^2 + b^2)^3)/(8*d) - (5*(atan(tan(c/2 +
(d*x)/2)) - (d*x)/2)*(a^2 + b^2)^3)/(8*d) + (tan(c/2 + (d*x)/2)^6*(64*a*b^5
 + 40*a^5*b - (160*a^3*b^3)/3) - tan(c/2 + (d*x)/2)*((5*b^6)/8 - (11*a^6)/8
 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^11*((5*b^6)/8 - (1
1*a^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) - tan(c/2 + (d*x)/2)^3*((5*a^6)
/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^
9*((5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 +
 (d*x)/2)^5*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4*b^2)/4) -
 tan(c/2 + (d*x)/2)^7*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4
*b^2)/4) + 80*a^3*b^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*b^3*tan(c/2 + (d*x)/2)^
8 + 12*a^5*b*tan(c/2 + (d*x)/2)^2 + 12*a^5*b*tan(c/2 + (d*x)/2)^10)/(d*(6*t
an(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 1
5*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 +
1))
```

3.221 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$-\frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(b\cos(c+dx)-a}{5d}$$

```
[Out] -(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*si
n(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

Rubi [A]

time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3151, 200}

$$\frac{2(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int (a^2+b^2-x^2)^2\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right)\,dx,x,b\cos(c\right.}{d}$$

$$= -\frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)(b\cos(c+}{3}$$

Mathematica [A]

time = 0.34, size = 156, normalized size = 1.66

$$\frac{-150b(a^2+b^2)^2\cos(c+dx)+25b(-3a^4-2a^2b^2+b^4)\cos(3(c+dx))-3b(5a^4-10a^2b^2+b^4)\cos(5(c+dx))+150a(a^2+b^2)^2\sin(c+dx)+25a(a^4-2a^2b^2-3b^4)\sin(3(c+dx))+3a(a^4-10a^2b^2+5b^4)\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A]

time = 0.44, size = 175, normalized size = 1.86

method	result
derivativedivides	$\frac{-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10b^3a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)}{}$
default	$\frac{-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10b^3a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)}{}$
norman	$-\frac{30b\,a^4+40b^3a^2+16b^5}{15d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{40b^3a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10b\,a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(40b^3a^2+16b^5\right.}{}$
risch	$-\frac{5b\cos(dx+c)a^4}{8d}-\frac{5b^3\cos(dx+c)a^2}{4d}-\frac{5b^5\cos(dx+c)}{8d}+\frac{5a^5\sin(dx+c)}{8d}+\frac{5a^3\sin(dx+c)b^2}{4d}+\frac{5a\sin(dx+c)b^4}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*b^3*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-b*a^4*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]

time = 0.26, size = 172, normalized size = 1.83

$$-\frac{a^4b\cos\left(dx+c\right)^5}{d}+\frac{ab^4\sin\left(dx+c\right)^5}{d}+\frac{\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^5}{15\,d}-\frac{2\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)a^3b^2}{3\,d}+\frac{2\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^2b^3}{3\,d}-\frac{\left(3\,\cos\left(dx+c\right)^5-10\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)b^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

Fricas [A]
time = 2.20, size = 155, normalized size = 1.65

$$-\frac{15\,b^{5}\cos\left(dx+c\right)+3\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{5}+10\,\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{3}-\left(8\,a^{5}+20\,a^{3}b^{2}+15\,ab^{4}+3\,\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}+2\,\left(2\,a^{5}+5\,a^{3}b^{2}-15\,ab^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(82) = 164$.
time = 0.32, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^{5}\sin^{5}\left(c+dx\right)}{15d}+\frac{4a^{5}\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a^{5}\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{a^{4}b\cos^{5}\left(c+dx\right)}{d}+\frac{4a^{3}b^{2}\sin^{5}\left(c+dx\right)}{3d}+\frac{10a^{3}b^{2}\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}-\frac{10a^{2}b^{3}\sin^{2}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{3d}-\frac{4a^{2}b^{3}\cos^{5}\left(c+dx\right)}{3d}+\frac{ab^{4}\sin^{5}\left(c+dx\right)}{d}-\frac{b^{5}\sin^{4}\left(c+dx\right)\cos\left(c+dx\right)}{d}-\frac{4b^{5}\sin^{2}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{3d}-\frac{8b^{5}\cos^{5}\left(c+dx\right)}{15d} & \text{for}\: d \neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(90) = 180$.
time = 0.48, size = 187, normalized size = 1.99

$$-\frac{\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\,\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)}{8\,d}+\frac{\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,\left(a^{5}-2\,a^{3}b^{2}-3\,ab^{4}\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
```

```
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

Mupad [B]
time = 2.72, size = 248, normalized size = 2.64

$$\frac{2\left(\frac{3\sin(c+dx)\,a^5\cos(c+dx)^4}{2}+2\sin\left(c+dx\right)\,a^5\cos\left(c+dx\right)^2+4\sin\left(c+dx\right)\,a^5-\frac{15\,a^4\,b\cos(c+dx)^5}{2}-15\sin\left(c+dx\right)\,a^3\,b^2\cos\left(c+dx\right)^4+5\sin\left(c+dx\right)\,a^3\,b^2\cos\left(c+dx\right)^2+10\sin\left(c+dx\right)\,a^3\,b^2+15\,a^2\,b^3\cos\left(c+dx\right)^5-25\,a^2\,b^3\cos\left(c+dx\right)^3+\frac{15\sin(c+dx)\,a\,b^4\cos(c+dx)^4}{2}-15\sin\left(c+dx\right)\,a\,b^4\cos\left(c+dx\right)^2+\frac{15\sin(c+dx)\,a\,b^4}{2}-\frac{3\,b^5\cos(c+dx)^5}{2}+5\,b^5\cos\left(c+dx\right)^3-\frac{15\,b^5\cos(c+dx)}{2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 - (
3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x)^
2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(c +
 d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b^4*s
in(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*cos(c +
 d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b^4*cos
(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.222 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$\frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}-\frac{(b\cos(c+dx)-a$$

```
[Out] 3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b
*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^
3/d
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3152, 8}

$$-\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\\
&= -\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c-}{8d}\\
&= \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+}{8d}
\end{aligned}$$

Mathematica [A]

time = 0.28, size = 107, normalized size = 0.99

$$\frac{12(a^2+b^2)^2(c+dx)-16ab(a^2+b^2)\cos(2(c+dx))-4ab(a^2-b^2)\cos(4(c+dx))+8(a^4-b^4)\sin(2(c+dx))+(a^4-6a^2b^2+b^4)\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]

time = 0.39, size = 153, normalized size = 1.42

method	result
derivativedivides	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4(dx+c)\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(}{8}\right.}{d}$
default	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4(dx+c)\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(}{8}\right.}{d}$
risch	$\frac{3a^4x}{8}+\frac{3a^2b^2x}{4}+\frac{3b^4x}{8}-\frac{a^3b\cos(4dx+4c)}{8d}+\frac{a\,b^3\cos(4dx+4c)}{8d}+\frac{\sin(4dx+4c)a^4}{32d}-\frac{3\sin(4dx+4c)a^2b^2}{16d}+\sin(4$
norman	$\frac{(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d
*x+c)+1/8*d*x+1/8*c)-a^3*b*cos(d*x+c)^4+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]

time = 0.27, size = 136, normalized size = 1.26

$$-\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b^2}{16\,d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))b^4}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
```

```
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

Fricas [A]
time = 1.10, size = 121, normalized size = 1.12

$$-\frac{16\,ab^3\cos\left(dx+c\right)^2+8\left(a^3b-ab^3\right)\cos\left(dx+c\right)^4-3\left(a^4+2\,a^2b^2+b^4\right)dx-\left(2\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3+\left(3\,a^4+6\,a^2b^2-5\,b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.
time = 0.23, size = 381, normalized size = 3.53

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8}+\frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^4x\cos^4(c+dx)}{8}+\frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{a^3b\cos^4(c+dx)}{d}+\frac{3a^2b^2x\sin^4(c+dx)}{4}+\frac{3a^2b^2x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3a^2b^2x\cos^4(c+dx)}{4}+\frac{3a^2b^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{ab^3\sin^4(c+dx)}{d}+\frac{3b^4x\sin^4(c+dx)}{8}+\frac{3b^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3b^4x\cos^4(c+dx)}{8}-\frac{5b^4\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3b^4\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for}\ d\neq 0 \\ x(a\cos(c)+b\sin(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**
3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2
*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))**4, True))
```

Giac [A]
time = 0.44, size = 122, normalized size = 1.13

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(4\,dx+4\,c\right)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(a^4-b^4\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 3.46, size = 320, normalized size = 2.96

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,\left(\frac{3\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)}\right)\,{\left(a^2+b^2\right)}^2}{4\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b^4}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)+8\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{3\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (3
*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 - (5
*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4 - (
21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2
)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*a^3*b
*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c/2 + (
d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

3.223 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}$$

```
[Out] -(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3/
d
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3151}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(}{3d}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 81, normalized size = 1.40

$$\frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c+dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]

[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)

Maple [A]
time = 0.00, size = 75, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\,b^2\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\,b^2\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\,b^2\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}+$
norman	$\frac{-\frac{6a^2b+4b^3}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(a^2+6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x+c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))

Maxima [A]
time = 0.27, size = 84, normalized size = 1.45

$$-\frac{a^2b\cos(dx+c)^3}{d}+\frac{ab^2\sin(dx+c)^3}{d}-\frac{\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3}{3d}+\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)b^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d

Fricas [A]
time = 3.36, size = 77, normalized size = 1.33

$$-\frac{3\,b^3\cos(dx+c)+\left(3\,a^2b-b^3\right)\cos(dx+c)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos(dx+c)^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.
time = 0.13, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^2b\cos^3(c+dx)}{d} + \frac{ab^2\sin^3(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

Giac [A]
time = 0.42, size = 91, normalized size = 1.57

$$-\frac{(3a^2b - b^3)\cos(3dx + 3c)}{12d} - \frac{3(a^2b + b^3)\cos(dx + c)}{4d} + \frac{(a^3 - 3ab^2)\sin(3dx + 3c)}{12d} + \frac{3(a^3 + ab^2)\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

Mupad [B]
time = 2.48, size = 104, normalized size = 1.79

$$\frac{\frac{\sin(c+dx)\,a^3\cos(c+dx)^2}{3} + \frac{2\sin(c+dx)\,a^3}{3} - a^2b\cos(c+dx)^3 - \sin(c+dx)\,ab^2\cos(c+dx)^2 + \sin(c+dx)\,ab^2 + \frac{b^3\cos(c+dx)^3}{3} - b^3\cos(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.224 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))
/d
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3152, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx&=-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}(\\&=\frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(\end{aligned}$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.95

$$\frac{2(a^2+b^2)\,(c+dx)-2ab\cos(2(c+dx))+(a^2-b^2)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.34, size = 70, normalized size = 1.27

method	result
risch	$\frac{a^2 x}{2}+\frac{b^2 x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab(\cos^2(dx+c))+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab(\cos^2(dx+c))+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x+\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+(a^2+b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(a^2-b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-a*b*cos(d*x+c)^2+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 68, normalized size = 1.24

$$-\frac{ab\cos\left(dx+c\right)^2}{d}+\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

Fricas [A]
time = 2.56, size = 52, normalized size = 0.95

$$-\frac{2\,ab\cos\left(dx+c\right)^2-(a^2+b^2)dx-(a^2-b^2)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.
time = 0.09, size = 128, normalized size = 2.33

$$\begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{ab\sin^2(c+dx)}{d} + \frac{b^2x\sin^2(c+dx)}{2} + \frac{b^2x\cos^2(c+dx)}{2} - \frac{b^2\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) + a*b*sin(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

Giac [A]
time = 0.40, size = 50, normalized size = 0.91

$$\frac{1}{2}(a^2 + b^2)x - \frac{ab\cos(2\,dx + 2\,c)}{2\,d} + \frac{(a^2 - b^2)\sin(2\,dx + 2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

Mupad [B]
time = 2.42, size = 63, normalized size = 1.15

$$\frac{a^2\,x}{2} + \frac{b^2\,x}{2} + \frac{a^2\,\sin(2\,c + 2\,d\,x)}{4\,d} - \frac{b^2\,\sin(2\,c + 2\,d\,x)}{4\,d} - \frac{a\,b\,\cos(2\,c + 2\,d\,x)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*x
))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

3.225 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2717, 2718}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.92

$$-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.16, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50
meijerg	$\frac{\left(\sqrt{\pi}\,\cos(c)a+\sqrt{\pi}\,\sin(c)b\right)\sin(dx)}{\sqrt{\pi}\,d}+\frac{\left(\sqrt{\pi}\,\cos(c)b-\sqrt{\pi}\,\sin(c)a\right)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+b*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Fricas [A]
time = 3.30, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [A]
time = 0.06, size = 31, normalized size = 1.29

$$a\left(\begin{cases}\frac{\sin\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\cos\left(c\right) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\sin\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

Giac [A]
time = 0.40, size = 24, normalized size = 1.00

$$-\frac{b\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Mupad [B]
time = 2.32, size = 38, normalized size = 1.58

$$-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.226 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

```
[Out] -arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3153, 212}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A]
time = 0.00, size = 43, normalized size = 0.91

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.70

$$-\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(43) = 86$.
time = 2.18, size = 131, normalized size = 2.79

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(1/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.44, size = 74, normalized size = 1.57

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B]
time = 2.80, size = 39, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

3.227 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{3154\}$

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A]
time = 0.59, size = 21, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{1}{db(a+b\tan(dx+c))}$	21
default	$-\frac{1}{db(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}$	47
norman	$\frac{\frac{1}{bd}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.66

$$-\frac{1}{(b^2\tan(dx+c)+ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

Fricas [A]
time = 2.41, size = 57, normalized size = 1.78

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 20, normalized size = 0.62

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Mupad [B]
time = 2.34, size = 47, normalized size = 1.47

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
[Out] (2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)
/2)^2))
```

3.228 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3155, 3153, 212}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2
)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\,(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+d}}{2\,(a^2+b^2)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\,(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^2} - \text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,d\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\,(a^2+b^2)^{3/2}\,d} - \frac{b\cos(c+dx)-a\sin(c+}{2\,(a^2+b^2)\,d(a\cos(c+dx)+b\sin}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 132, normalized size = 1.28

$$\frac{(a^2+b^2)\,(-b\cos(c+dx)+a\sin(c+dx))+2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))^2}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.65, size = 191, normalized size = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a\left(a^2+b^2\right)}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a\left(a^2+b^2\right)}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{ia\,e^{3i(dx+c)}+b\,e^{3i(dx+c)}-ia\,e^{i(dx+c)}+b\,e^{i(dx+c)}}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^3+ia\,b^2-a^2}{\left(a^2+b^2\right)}\right.}{2(a^2+b^2)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/a/(a^2+b^2)*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(95) = 190$.
time = 0.48, size = 326, normalized size = 3.17

$$-\frac{\frac{2\left(a^2b-\frac{\left(a^3-2\,ab^2\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{\left(a^2b-2\,b^3\right)\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}-\frac{\left(a^3+2\,ab^2\right)\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}\right)}{a^6+a^4b^2+\frac{4\left(a^5b+a^3b^3\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{2\left(a^6-a^4b^2-2\,a^2b^4\right)\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}-\frac{4\left(a^5b+a^3b^3\right)\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{\left(a^6+a^4b^2\right)\sin\left(dx+c\right)^4}{\left(\cos\left(dx+c\right)+1\right)^4}}+\frac{\log\left(\frac{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(95) = 190$.
time = 1.58, size = 294, normalized size = 2.85

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2\left(a^2b+b^3\right)\cos\left(dx+c\right)+2\left(a^3+ab^2\right)\sin\left(dx+c\right)}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^4b^2+2\,a^2b^4+b^6\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
```

```
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(95) = 190.
time = 0.45, size = 221, normalized size = 2.15

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2b\right)}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B]
time = 4.55, size = 260, normalized size = 2.52

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\mathrm{atanh}\left(\frac{\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.229 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\sin(c+dx)}{3a\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] 1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+
2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3155, 3154}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -1/3*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*S
in[c + d*x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin}}{3\left(a^2+b^2\right.}\\ &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\,\mathrm{s}}{3a\left(a^2+b^2\right)d(a\,\mathrm{co}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 85, normalized size = 0.87

$$\frac{-ab\cos(3(c+dx))+(2a^2+b^2+(a^2-b^2)\cos(2(c+dx)))\sin(c+dx)}{3a(a^2+b^2)d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.73, size = 64, normalized size = 0.65

method	result	size
derivativedivides	$\frac{-\frac{1}{b^3(a+b\tan(dx+c))}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}}{d}$	64
default	$\frac{-\frac{1}{b^3(a+b\tan(dx+c))}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}}{d}$	64
risch	$\frac{4i\left(3ia\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+ia-b\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3 d(ia+b)^2}$	82
norman	$\frac{\frac{1}{3bd}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{3bd}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{db}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3/(a+b*tan(d*x+c))-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3+a/b^3/(a+
b*tan(d*x+c))^2)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.87

$$-\frac{3\,b^2\tan(dx+c)^2+3\,ab\tan(dx+c)+a^2+b^2}{3\left(b^6\tan(dx+c)^3+3\,ab^5\tan(dx+c)^2+3\,a^2b^4\tan(dx+c)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(94) = 188$.
time = 1.96, size = 217, normalized size = 2.21

$$-\frac{2\,(3\,a^2b-b^3)\cos{(dx+c)}^3-3\,(a^2b-b^3)\cos{(dx+c)}-\left(a^3+3\,ab^2+2\,(a^3-3\,ab^2)\cos{(dx+c)}^2\right)\sin{(dx+c)}}{3\left((a^7-a^5b^2-5\,a^3b^4-3\,ab^6)d\cos{(dx+c)}^3+3\,(a^5b^2+2\,a^3b^4+ab^6)d\cos{(dx+c)}+\left((3\,a^6b+5\,a^4b^3+a^2b^5-b^7)d\cos{(dx+c)}^2+(a^4b^3+2\,a^2b^5+b^7)d\right)\sin{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 50, normalized size = 0.51

$$-\frac{3\,b^2\tan{(dx+c)}^2+3\,ab\tan{(dx+c)}+a^2+b^2}{3\,(b\tan{(dx+c)}+a)^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

Mupad [**B**]
time = 3.12, size = 222, normalized size = 2.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2-2\,b^2)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(12\,a\,b^2-3\,a^3)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(12\,a\,b^2-3\,a^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(12\,a^2\,b-8\,b^3)+a^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x)
/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(c/
2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(
c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x
)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(
c/2 + (d*x)/2)^5))
```

3.230 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

Optimal. Leaf size=156

$$-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8\left(a^2+b^2\right)^{5/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+dx)-a}{8\left(a^2+b^2\right)^2d(a\cos(c+dx}$$

```
[Out] -3/8*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+1/4*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^4
-3/8*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^
2
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3155, 3153, 212}

$$-\frac{3(b\cos(c+dx)-a\sin(c+dx))}{8d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] (-3*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(8*(a^2 + b
^2)^(5/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(4*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) - (3*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(8*(a^2
 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
```

```
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c}}{4\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+d}{8\left(a^2+b^2\right)^2 d(a\cos}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{3(b\cos(c+d}{8\left(a^2+b^2\right)^2 d(a\cos}$$

$$= -\frac{3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8\left(a^2+b^2\right)^{5/2}d}-\frac{b\cos(c+dx)-a\sin(c-}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\,\mathrm{s}}$$

Mathematica [A]
time = 0.86, size = 157, normalized size = 1.01

$$\frac{\frac{6\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{-11b\left(a^2+b^2\right)\cos(c+dx)+\left(-9a^2b+3b^3\right)\cos(3(c+dx))+2a\left(7a^2+b^2+3\left(a^2-3b^2\right)\cos(2(c+dx))\right)\sin(c+dx)}{4\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

```
[Out] ((6*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
 (-11*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*
a*(7*a^2 + b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(4*(a^2 +
b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(146) = 292$.
time = 1.05, size = 514, normalized size = 3.29

method	result
risch	$\frac{3ia^3 e^{i(dx+c)}+9ia\,b^2 e^{7i(dx+c)}-9a^2 b\,e^{7i(dx+c)}+3b^3 e^{7i(dx+c)}-3ia^3 e^{7i(dx+c)}+11ia^3 e^{3i(dx+c)}-11a^2 b\,e^{5i(dx+c)}-11b^3 e^{5i(dx}}{4(-ia+b)^2\left(b\,e^{2i(dx+}\right.}$

derivativedivides	$-\frac{2\left(-\frac{\left(5a^4+16a^2b^2+8b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\left(a^4+2a^2b^2+b^4\right)}+\frac{3b\left(a^4+16a^2b^2+8b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(3a^6-36a^4b^2+56a^2b^4+32b^6\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8a^3\left(a^4+2a^2b^2+b^4\right)}}{}$
default	$-\frac{2\left(-\frac{\left(5a^4+16a^2b^2+8b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a\left(a^4+2a^2b^2+b^4\right)}+\frac{3b\left(a^4+16a^2b^2+8b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(3a^6-36a^4b^2+56a^2b^4+32b^6\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8a^3\left(a^4+2a^2b^2+b^4\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/8*(5*a^4+16*a^2*b^2+8*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/
2*c)^7+3/8*b*(a^4+16*a^2*b^2+8*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2
*c)^6-1/8/a^3*(3*a^6-36*a^4*b^2+56*a^2*b^4+32*b^6)/(a^4+2*a^2*b^2+b^4)*tan(
1/2*d*x+1/2*c)^5+1/8/a^4*b*(15*a^6-114*a^4*b^2-8*a^2*b^4+16*b^6)/(a^4+2*a^2
*b^2+b^4)*tan(1/2*d*x+1/2*c)^4-1/8/a^3*(3*a^6+84*a^4*b^2-56*a^2*b^4-32*b^6)
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-1/8*b*(23*a^4-64*a^2*b^2-24*b^4)/
a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^2-1/8*(5*a^4-24*a^2*b^2-8*b^4)/a
/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/8*b*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+
b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^4+3/4/(a^4+2*a^2*b^
2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(
1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(146) = 292$.

time = 0.53, size = 822, normalized size = 5.27

$$-\frac{\frac{2\left(5a^6b+2a^4b^3-\frac{(5a^7-24a^5b^2-8a^3b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(23a^6b-64a^4b^3-24a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(3a^7+84a^5b^2-56a^3b^4-32ab^6)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(15a^6b-114a^4b^3-8a^2b^5+16b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{(3a^7-36a^5b^2+56a^3b^4+32ab^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3(a^6b+16a^4b^3+8a^2b^5)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(5a^7+16a^5b^2+8a^3b^4)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^{12}+2a^{10}b^2+a^8b^4+\frac{8(a^{11}b+2a^9b^3+a^7b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{4(a^{12}-4a^{10}b^2-11a^8b^4-6a^6b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8(3a^{11}b+2a^9b^3-5a^7b^5-4a^5b^7)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(3a^{12}-18a^{10}b^2-37a^8b^4-8a^6b^6+8a^4b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{8(3a^{11}b+2a^9b^3-5a^7b^5-4a^5b^7)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4(a^{12}-4a^{10}b^2-11a^8b^4-6a^6b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8(a^{11}b+2a^9b^3+a^7b^5)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{(a^{12}+2a^{10}b^2+a^8b^4)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{3\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(5*a^6*b + 2*a^4*b^3 - (5*a^7 - 24*a^5*b^2 - 8*a^3*b^4)*sin(d*x + c
)/(cos(d*x + c) + 1) - (23*a^6*b - 64*a^4*b^3 - 24*a^2*b^5)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - (3*a^7 + 84*a^5*b^2 - 56*a^3*b^4 - 32*a*b^6)*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + (15*a^6*b - 114*a^4*b^3 - 8*a^2*b^5 + 16*b^7
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - (3*a^7 - 36*a^5*b^2 + 56*a^3*b^4 +
32*a*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*(a^6*b + 16*a^4*b^3 + 8*a
^2*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - (5*a^7 + 16*a^5*b^2 + 8*a^3*b
^4)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^12 + 2*a^10*b^2 + a^8*b^4 + 8*(
a^11*b + 2*a^9*b^3 + a^7*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*(a^12 - 4
*a^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8
*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4*a^5*b^7)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 2*(3*a^12 - 18*a^10*b^2 - 37*a^8*b^4 - 8*a^6*b^6 + 8*a^4*b^8)*si
```

```
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 8*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4
*a^5*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*(a^12 - 4*a^10*b^2 - 11*a
^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*(a^11*b + 2*a^9
*b^3 + a^7*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + (a^12 + 2*a^10*b^2 +
a^8*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 3*log((b - a*sin(d*x + c)/(
cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1)
 - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(146) = 292$.
time = 2.27, size = 544, normalized size = 3.49

$$-\frac{6\,(3\,a^{4}b+2\,a^{2}b^{3}-b^{5})\cos(dx+c)^{3}-3\,\left((a^{4}-6\,a^{2}b^{2}+b^{4})\cos(dx+c)^{4}+b^{4}+2\,(3\,a^{2}b^{2}-b^{4})\cos(dx+c)^{2}+4\,(ab^{3}\cos(dx+c)+(a^{3}b-ab^{3})\cos(dx+c)^{3})\sin(dx+c)\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-2\,(4\,a^{4}b-a^{2}b^{3}-5\,b^{5})\cos(dx+c)-2\,\left(2\,a^{5}+7\,a^{3}b^{2}+5\,ab^{4}+3\,(a^{5}-2\,a^{3}b^{2}-3\,ab^{4})\cos(dx+c)^{2}\right)\sin(dx+c)}{16\,\left((a^{10}-3\,a^{8}b^{2}-14\,a^{6}b^{4}-14\,a^{4}b^{6}-3\,a^{2}b^{8}+b^{10})d\cos(dx+c)^{4}+2\,(3\,a^{8}b^{2}+8\,a^{6}b^{4}+6\,a^{4}b^{6}-b^{10})d\cos(dx+c)^{2}+(a^{6}b^{4}+3\,a^{4}b^{6}+3\,a^{2}b^{8}+b^{10})d+4\,\left((a^{9}b+2\,a^{7}b^{3}-2\,a^{3}b^{7}-ab^{9})d\cos(dx+c)^{3}+(a^{7}b^{3}+3\,a^{5}b^{5}+3\,a^{3}b^{7}+ab^{9})d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(3*a^4*b + 2*a^2*b^3 - b^5)*cos(d*x + c)^3 - 3*((a^4 - 6*a^2*b^2 +
 b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^3*
cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^2 + b^2
)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(4*a^4*b - a^
2*b^3 - 5*b^5)*cos(d*x + c) - 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + 3*(a^5 - 2*a
^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 - 3*a^8*b^2 - 14*a^6
*b^4 - 14*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^4 + 2*(3*a^8*b^2 + 8*a
^6*b^4 + 6*a^4*b^6 - b^10)*d*cos(d*x + c)^2 + (a^6*b^4 + 3*a^4*b^6 + 3*a^2*
b^8 + b^10)*d + 4*((a^9*b + 2*a^7*b^3 - 2*a^3*b^7 - a*b^9)*d*cos(d*x + c)^3
 + (a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(146) = 292$.
time = 0.46, size = 588, normalized size = 3.77

$$-\frac{\frac{3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{(a^{4}+2\,a^{2}b^{2}+b^{4})\sqrt{a^{2}+b^{2}}}-\frac{2\left(5\,a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+16\,a^{5}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+8\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-48\,a^{4}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-24\,a^{2}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3\,a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-36\,a^{5}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+56\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+32\,ab^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+114\,a^{4}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a^{2}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,b^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3\,a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+84\,a^{5}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-56\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-32\,ab^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+23\,a^{6}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-64\,a^{4}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a^{2}b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,a^{7}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^{5}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^{6}b-2\,a^{4}b^{3}\right)}{(a^{8}+2\,a^{6}b^{2}+a^{4}b^{4})\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*
sqrt(a^2 + b^2)) - 2*(5*a^7*tan(1/2*d*x + 1/2*c)^7 + 16*a^5*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 8*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*a^6*b*tan(1/2*d*x + 1/2*
c)^6 - 48*a^4*b^3*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^
6 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 36*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 56*a
^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a^6*b*
tan(1/2*d*x + 1/2*c)^4 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b^5*tan
(1/2*d*x + 1/2*c)^4 - 16*b^7*tan(1/2*d*x + 1/2*c)^4 + 3*a^7*tan(1/2*d*x + 1
/2*c)^3 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 - 32*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 23*a^6*b*tan(1/2*d*x + 1/2*c)^2 -
64*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 + 5*a
^7*tan(1/2*d*x + 1/2*c) - 24*a^5*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^4*tan(1
/2*d*x + 1/2*c) - 5*a^6*b - 2*a^4*b^3)/((a^8 + 2*a^6*b^2 + a^4*b^4)*(a*tan(
1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^4))/d
```

Mupad [B]
time = 6.09, size = 719, normalized size = 4.61

$$-\frac{\frac{5a^2b+2b^3}{4\left(a^4+2a^2b^2+b^4\right)}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(a^4b+16a^2b^3+8b^5\right)}{4a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-23a^4b+64a^2b^3+24b^5\right)}{4a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-5a^4+24a^2b^2+8b^4\right)}{4a\left(a^4+2a^2b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(3a^6-36a^4b^2+56a^2b^4+32b^6\right)}{4a^3\left(a^4+2a^2b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(3a^6+84a^4b^2-56a^2b^4-32b^6\right)}{4a^3\left(a^4+2a^2b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(5a^4+16a^2b^2+8b^4\right)}{4a\left(a^4+2a^2b^2+b^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(5a^2b+2b^3\right)\left(3a^4-24a^2b^2+8b^4\right)}{4a^4\left(a^4+2a^2b^2+b^4\right)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(6a^4-48a^2b^2+16b^4\right)+a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+a^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4a^4-24a^2b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(4a^4-24a^2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(32ab^3-24a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(32ab^3-24a^3b\right)+8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}+\frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+a^4b\,1\mathrm{i}-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2+a^2b^3\,2\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)ab^4+b^5\,1\mathrm{i}}{\left(a^2+b^2\right)^{5/2}}\right)3\mathrm{i}}{4d\left(a^2+b^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (atan((a^4*b*1i + b^5*1i + a^2*b^3*2i - a^5*tan(c/2 + (d*x)/2)*1i - a*b^4*t
an(c/2 + (d*x)/2)*1i - a^3*b^2*tan(c/2 + (d*x)/2)*2i)/(a^2 + b^2)^(5/2))*3i
)/(4*d*(a^2 + b^2)^(5/2)) - ((5*a^2*b + 2*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2))
+ (3*tan(c/2 + (d*x)/2)^6*(a^4*b + 8*b^5 + 16*a^2*b^3))/(4*a^2*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^2*(24*b^5 - 23*a^4*b + 64*a^2*b^3))/(4*a
^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)*(8*b^4 - 5*a^4 + 24*a^2*b
^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a^6 + 32*b^6
+ 56*a^2*b^4 - 36*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d
*x)/2)^3*(3*a^6 - 32*b^6 - 56*a^2*b^4 + 84*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*
a^2*b^2)) - (tan(c/2 + (d*x)/2)^7*(5*a^4 + 8*b^4 + 16*a^2*b^2))/(4*a*(a^4 +
 b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^4*(5*a^2*b + 2*b^3)*(3*a^4 + 8*b^4
 - 24*a^2*b^2))/(4*a^4*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(
6*a^4 + 16*b^4 - 48*a^2*b^2) + a^4*tan(c/2 + (d*x)/2)^8 + a^4 - tan(c/2 + (
d*x)/2)^2*(4*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^4 - 24*a^2*b^2)
+ tan(c/2 + (d*x)/2)^3*(32*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^5*(32*a*b
^3 - 24*a^3*b) + 8*a^3*b*tan(c/2 + (d*x)/2) - 8*a^3*b*tan(c/2 + (d*x)/2)^7)
)
```

3.231 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx$

Optimal. Leaf size=151

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15\left(a^2+b^2\right)^2d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{}{15a\left(a^2+b^2\right)}$$

```
[Out] 1/5*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^5-
4/15*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^
3+8/15*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3155, 3154}

$$\frac{8\sin(c+dx)}{15ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{4(b\cos(c+dx)-a\sin(c+dx))}{15d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] -1/5*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^5) - (4*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(15*(a^2 + b^2)^2*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (8*Sin[c + d*x])/(15*a*(a^2 + b^2)^
2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{4\int \frac{1}{(a\cos(c+dx)+b\sin}}{5\left(a^2+b^2\right.}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{4(b\cos(c+}{15\left(a^2+b^2\right)^2 d(a\,\mathrm{c}}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{4(b\cos(c+}{15\left(a^2+b^2\right)^2 d(a\,\mathrm{c}}$$

Mathematica [A]
time = 0.37, size = 182, normalized size = 1.21

$$\frac{-10ab(a^2+b^2)\cos(3(c+dx))+(-4a^3b+4ab^3)\cos(5(c+dx))+10a^4\sin(c+dx)+20a^2b^2\sin(c+dx)+10b^4\sin(c+dx)+5a^4\sin(3(c+dx))-5b^4\sin(3(c+dx))+a^4\sin(5(c+dx))-6a^2b^2\sin(5(c+dx))+b^4\sin(5(c+dx))}{30a\left(a^2+b^2\right)^2 d(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

```
[Out] (-10*a*b*(a^2 + b^2)*Cos[3*(c + d*x)] + (-4*a^3*b + 4*a*b^3)*Cos[5*(c + d*x
)] + 10*a^4*Sin[c + d*x] + 20*a^2*b^2*Sin[c + d*x] + 10*b^4*Sin[c + d*x] +
5*a^4*Sin[3*(c + d*x)] - 5*b^4*Sin[3*(c + d*x)] + a^4*Sin[5*(c + d*x)] - 6*
a^2*b^2*Sin[5*(c + d*x)] + b^4*Sin[5*(c + d*x)])/(30*a*(a^2 + b^2)^2*d*(a*C
os[c + d*x] + b*Sin[c + d*x])^5)
```

Maple [A]
time = 1.05, size = 125, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{2a}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{3b^5(a+b\tan(dx+c))^3}-\frac{1}{b^5(a+b\tan(dx+c))}+\frac{\left(a^2+b^2\right)a}{b^5(a+b\tan(dx+c))^4}-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b\tan(dx+c))^5}}{d}$
default	$\frac{\frac{2a}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{3b^5(a+b\tan(dx+c))^3}-\frac{1}{b^5(a+b\tan(dx+c))}+\frac{\left(a^2+b^2\right)a}{b^5(a+b\tan(dx+c))^4}-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b\tan(dx+c))^5}}{d}$
risch	$\frac{16i\left(-20iab\,\mathrm{e}^{4i(dx+c)}+10a^2\mathrm{e}^{4i(dx+c)}-10b^2\mathrm{e}^{4i(dx+c)}+5a^2\mathrm{e}^{2i(dx+c)}+5b^2\mathrm{e}^{2i(dx+c)}+2iab+a^2-b^2\right)}{15d(-ib+a)^3\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)^5}$
norman	$\frac{\frac{1}{5bd}+\frac{2\left(32a^3-16a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d\,a^4}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{5bd}+\frac{\left(6a^3-16a\,b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3b}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a/b^5/(a+b*tan(d*x+c))^2-1/3*(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))^3-1/
b^5/(a+b*tan(d*x+c))+(a^2+b^2)*a/b^5/(a+b*tan(d*x+c))^4-1/5*(a^4+2*a^2*b^2+
b^4)/b^5/(a+b*tan(d*x+c))^5)
```

Maxima [A]
time = 0.31, size = 174, normalized size = 1.15

$$-\frac{15\,b^{4}\tan\left(dx+c\right)^{4}+30\,ab^{3}\tan\left(dx+c\right)^{3}+3\,a^{4}+a^{2}b^{2}+3\,b^{4}+10\,\left(3\,a^{2}b^{2}+b^{4}\right)\tan\left(dx+c\right)^{2}+5\,\left(3\,a^{3}b+ab^{3}\right)\tan\left(dx+c\right)}{15\,\left(b^{10}\tan\left(dx+c\right)^{5}+5\,ab^{9}\tan\left(dx+c\right)^{4}+10\,a^{2}b^{8}\tan\left(dx+c\right)^{3}+10\,a^{3}b^{7}\tan\left(dx+c\right)^{2}+5\,a^{4}b^{6}\tan\left(dx+c\right)+a^{5}b^{5}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 3*a^4 + a^2*b^2 +
3*b^4 + 10*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 5*(3*a^3*b + a*b^3)*tan(d*x +
 c))/((b^10*tan(d*x + c)^5 + 5*a*b^9*tan(d*x + c)^4 + 10*a^2*b^8*tan(d*x +
c)^3 + 10*a^3*b^7*tan(d*x + c)^2 + 5*a^4*b^6*tan(d*x + c) + a^5*b^5)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(145) = 290.
time = 4.18, size = 441, normalized size = 2.92

$$-\frac{8\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{5}-20\,\left(a^{4}b-6\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{3}-5\,\left(a^{4}b+6\,a^{2}b^{3}-3\,b^{5}\right)\cos\left(dx+c\right)-\left(3\,a^{5}+10\,a^{3}b^{2}+15\,ab^{4}+8\,\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}+4\,\left(a^{5}+10\,a^{3}b^{2}-15\,ab^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{15\,\left(\left(a^{11}-7\,a^{9}b^{2}-22\,a^{7}b^{4}-14\,a^{5}b^{6}+5\,a^{3}b^{8}+5\,ab^{10}\right)d\cos\left(dx+c\right)^{5}+10\,\left(a^{9}b^{2}+2\,a^{7}b^{4}-2\,a^{3}b^{8}-ab^{10}\right)d\cos\left(dx+c\right)^{3}+5\,\left(a^{7}b^{4}+3\,a^{5}b^{6}+3\,a^{3}b^{8}+ab^{10}\right)d\cos\left(dx+c\right)+\left(\left(5\,a^{10}b+5\,a^{8}b^{3}-14\,a^{6}b^{5}-22\,a^{4}b^{7}-7\,a^{2}b^{9}+b^{11}\right)d\cos\left(dx+c\right)^{4}+2\,\left(5\,a^{8}b^{3}+14\,a^{6}b^{5}+12\,a^{4}b^{7}+2\,a^{2}b^{9}-b^{11}\right)d\cos\left(dx+c\right)^{2}+\left(a^{6}b^{5}+3\,a^{4}b^{7}+3\,a^{2}b^{9}+b^{11}\right)d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5 - 20*(a^4*b - 6*a^2*b^
3 + b^5)*cos(d*x + c)^3 - 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cos(d*x + c) - (3*a
^5 + 10*a^3*b^2 + 15*a*b^4 + 8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4
+ 4*(a^5 + 10*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 7*
a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^10)*d*cos(d*x + c)^5
+ 10*(a^9*b^2 + 2*a^7*b^4 - 2*a^3*b^8 - a*b^10)*d*cos(d*x + c)^3 + 5*(a^7*b
^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((5*a^10*b + 5*a^8*b^
3 - 14*a^6*b^5 - 22*a^4*b^7 - 7*a^2*b^9 + b^11)*d*cos(d*x + c)^4 + 2*(5*a^8
*b^3 + 14*a^6*b^5 + 12*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^6*
b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 118, normalized size = 0.78

$$-\frac{15\,b^{4}\tan\left(dx+c\right)^{4}+30\,ab^{3}\tan\left(dx+c\right)^{3}+30\,a^{2}b^{2}\tan\left(dx+c\right)^{2}+10\,b^{4}\tan\left(dx+c\right)^{2}+15\,a^{3}b\tan\left(dx+c\right)+5\,ab^{3}\tan\left(dx+c\right)+3\,a^{4}+a^{2}b^{2}+3\,b^{4}}{15\,\left(b\tan\left(dx+c\right)+a\right)^{5}b^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 30*a^2*b^2*tan(d*x
 + c)^2 + 10*b^4*tan(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 5*a*b^3*tan(d*x +
 c) + 3*a^4 + a^2*b^2 + 3*b^4)/((b*tan(d*x + c) + a)^5*b^5*d)
```

Mupad [B]
time = 5.15, size = 470, normalized size = 3.11

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}-\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(7a^2b-6b^3\right)}{3a^4}+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(7a^2b-6b^3\right)}{3a^4}-\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^2-6b^2\right)}{3a^3}-\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(a^2-6b^2\right)}{3a^3}+\frac{8b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2}-\frac{8b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{a^2}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(29a^4-112a^2b^2+24b^4\right)}{15a^5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(10a^5-120a^3b^2+80ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(40a^4b-80a^2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(40a^4b-80a^2b^3\right)-a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(10a^5-120a^3b^2+80ab^4\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(60a^4b-160a^2b^3+32b^5\right)+a^5-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(5a^5-40a^3b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(5a^5-40a^3b^2\right)+10a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+10a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^9)/a + (2*tan(c/2 + (d*x)/2))/a - (8*tan(c/2 + (d*x)
/2)^4*(7*a^2*b - 6*b^3))/(3*a^4) + (8*tan(c/2 + (d*x)/2)^6*(7*a^2*b - 6*b^3
))/(3*a^4) - (8*tan(c/2 + (d*x)/2)^3*(a^2 - 6*b^2))/(3*a^3) - (8*tan(c/2 +
(d*x)/2)^7*(a^2 - 6*b^2))/(3*a^3) + (8*b*tan(c/2 + (d*x)/2)^2)/a^2 - (8*b*t
an(c/2 + (d*x)/2)^8)/a^2 + (4*tan(c/2 + (d*x)/2)^5*(29*a^4 + 24*b^4 - 112*a
^2*b^2))/(15*a^5))/(d*(tan(c/2 + (d*x)/2)^4*(80*a*b^4 + 10*a^5 - 120*a^3*b^
2) - tan(c/2 + (d*x)/2)^3*(40*a^4*b - 80*a^2*b^3) - tan(c/2 + (d*x)/2)^7*(4
0*a^4*b - 80*a^2*b^3) - a^5*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(8
0*a*b^4 + 10*a^5 - 120*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 -
 160*a^2*b^3) + a^5 - tan(c/2 + (d*x)/2)^2*(5*a^5 - 40*a^3*b^2) + tan(c/2 +
 (d*x)/2)^8*(5*a^5 - 40*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2) + 10*a^4*b*t
an(c/2 + (d*x)/2)^9))
```

3.232 $\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=186

$$-\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\,\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}-\frac{2(b\cos(c+dx)-a\sin(c+dx)$$

```
[Out] -2/7*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(5/2)/d-10/21*
(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+1
0/21*(a^2+b^2)^2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2
*d*x-1/2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))
*((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(
d*x+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3152, 3157, 2720}

$$\frac{10(a^2+b^2)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\,\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-10*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*
Sin[c + d*x]])/(21*d) - (2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x
] + b*Sin[c + d*x])^(5/2))/(7*d) + (10*(a^2 + b^2)^2*EllipticF[(c + d*x - A
rcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(21*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] && !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && !(GeQ[n, 1] || LeQ[n, -1]) && !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/}}{7d}$$

$$= -\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\,\sqrt{a\cos(c+dx)+b\,\mathrm{si}}}{21d}$$

$$= -\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\,\sqrt{a\cos(c+dx)+b\,\mathrm{si}}}{21d}$$

$$= -\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\,\sqrt{a\cos(c+dx)+b\,\mathrm{si}}}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.31, size = 205, normalized size = 1.10

$$\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,(-23b(a^2+b^2)\cos(c+dx)+(-9a^2b+3b^3)\cos(3(c+dx))+2a(13a^2+7b^2+3(a^2-3b^2)\cos(2(c+dx)))\sin(c+dx))+\frac{20(a^2+b^2)^2\sqrt{\cos^2\left(c+dx+\mathrm{ArcTan}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\mathrm{ArcTan}\left(\frac{a}{b}\right)\right)\right)\tan\left(c+dx+\mathrm{ArcTan}\left(\frac{a}{b}\right)\right)}{\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{a}{b}\right)\right)}}}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(-23*b*(a^2 + b^2)*Cos[c + d*x] + (-
9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(13*a^2 + 7*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(c + d*x)])*Sin[c + d*x]) + (20*(a^2 + b^2)^2*Sqrt[Cos[c + d*x + ArcTa
n[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^
2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTa
n[a/b]]])/(42*d)
```

Maple [A]
time = 0.64, size = 185, normalized size = 0.99

method	result
default	$-\frac{(a^2+b^2)^2\left(-6(\sin^5(dx+c-\arctan(-a,b)))+5\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\ \sqrt{2\sin(dx+c-\mathrm{arctan}}\right.}{21\cos(dx+c-\mathrm{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*(a^2+b^2)^2*(-6*sin(d*x+c-arctan(-a,b))^5+5*(-sin(d*x+c-arctan(-a,b))
+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)
*EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))-4*sin(d*x+c-arct
an(-a,b))^3+10*sin(d*x+c-arctan(-a,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-
arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 261, normalized size = 1.40

$$-\frac{5\sqrt{2}\left(i a^3-a^2b+i ab^2-b^3\right)\sqrt{a-i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(a^2+2i ab-b^2\right)}{a^2+b^2},0,\cos(dx+c)+i\sin(dx+c)\right)+5\sqrt{2}\left(-i a^3-a^2b-i ab^2-b^3\right)\sqrt{a+i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(a^2-2i ab-b^2\right)}{a^2+b^2},0,\cos(dx+c)-i\sin(dx+c)\right)+2\left(3\left(3a^2b-b^3\right)\cos(dx+c)^3-\left(a^2b-8b^3\right)\cos(dx+c)-\left(5a^3+8ab^2+3\left(a^3-3ab^2\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{21d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/21*(5*sqrt(2)*(I*a^3 - a^2*b + I*a*b^2 - b^3)*sqrt(a - I*b)*weierstrassP
Inverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x +
 c)) + 5*sqrt(2)*(-I*a^3 - a^2*b - I*a*b^2 - b^3)*sqrt(a + I*b)*weierstrass
PInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x
+ c)) + 2*(3*(3*a^2*b - b^3)*cos(d*x + c)^3 - (a^2*b - 8*b^3)*cos(d*x + c)
- (5*a^3 + 8*a*b^2 + 3*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a
*cos(d*x + c) + b*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

3.233 $\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=131

$$-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}+\frac{6(a^2+b^2)\,E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|\,2\right.}{5d\sqrt{\frac{a\cos(c+dx)+}{\sqrt{a^2+}}}}$$

```
[Out] -2/5*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(3/2)/d+6/5*(a
^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d
*x+c)+b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3152, 3157, 2719}

$$\frac{6(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/
2))/(5*d) + (6*(a^2 + b^2)*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*
Cos[c + d*x] + b*Sin[c + d*x]])/(5*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])
/Sqrt[a^2 + b^2]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/}}{5d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/}}{5d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/}}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.21, size = 256, normalized size = 1.95

$$\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,(6a(a^2+b^2)-2ab^2\cos(2(c+dx))+b(a^2-b^2)\sin(2(c+dx))) - \frac{3(a^2+b^2)^2\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\left(b\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)+\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}\left(2a\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)-b\sin\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)\right)}{\left(a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)^{3/2}\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(6*a*(a^2 + b^2) - 2*a*b^2*Cos[2*(c
+ d*x)] + b*(a^2 - b^2)*Sin[2*(c + d*x)]) - (3*(a^2 + b^2)^2*Cos[c + d*x -
ArcTan[b/a]]*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan
[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] + Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(
2*a*Cos[c + d*x - ArcTan[b/a]] - b*Sin[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1
 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a
]]^2]))/(5*b*d)
```

Maple [A]
time = 0.48, size = 250, normalized size = 1.91

method	result

default	$-\frac{(a^2+b^2)^{\frac{3}{2}}\left(6\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)+1}\,\sqrt{2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\left(\sqrt{\sin}\,(dx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(a^2+b^2)^(3/2)*(6*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arc
tan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticE((-sin(d*x+c-arc
tan(-a,b))+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*si
n(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-si
n(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^4+2*s
in(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*
(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 247, normalized size = 1.89

$$-\frac{3\sqrt{2}\,(-i\,a^2-i\,b^2)\sqrt{a-i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\,(i\,a^2+i\,b^2)\sqrt{a+i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(2\,ab\cos\left(dx+c\right)^2-(a^2-b^2)\cos\left(dx+c\right)\sin\left(dx+c\right)-ab\right)\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(2)*(-I*a^2 - I*b^2)*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2*
I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(
a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(I*a^2 + I*b^2)*
sqrt(a + I*b)*weierstrassZeta(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, weie
rstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(2*a*b*cos(d*x + c)^2 - (a^2 - b^2)*cos(d*x + c)*sin(d*x
+ c) - a*b)*sqrt(a*cos(d*x + c) + b*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

3.234 $\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}+\frac{2(a^2+b^2)\,F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|\,2\right)}{3d\sqrt{a\cos(c+dx)+}}$$

```
[Out] -2/3*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+2/3*(a
^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(
d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3152, 3157, 2720}

$$\frac{2(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|\,2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*C
os[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*d*Sqrt[a*Cos[c + d*x] +
b*Sin[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.95, size = 143, normalized size = 1.09

$$\frac{2\left((-b\cos(c+dx)+a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}+\frac{(a^2+b^2)\sqrt{\cos^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)\right)\tan\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}{\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*
x]] + ((a^2 + b^2)*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/S
qrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*d)
```

Maple [A]
time = 0.42, size = 165, normalized size = 1.26

method	result

default	$-\frac{(a^2+b^2)\left(\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\left(\sqrt{\sin}\,(dx+c\right.\right.}{3\cos(dx+c-\arctan(-a,b))\sqrt{\mathrm{si}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a^2+b^2)*((-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b
))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a,b
))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^3+2*sin(d*x+c-arctan(-a,
b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 159, normalized size = 1.21

$$\frac{\sqrt{2}\,\sqrt{a-i\,b}\,(-i\,a+b)\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos(dx+c)+i\,\sin(dx+c)\right)+\sqrt{2}\,\sqrt{a+i\,b}\,(i\,a+b)\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\cos(dx+c)-i\,\sin(dx+c)\right)-2\,\sqrt{a\cos(dx+c)+b\sin(dx+c)}\,(b\cos(dx+c)-a\sin(dx+c))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(a - I*b)*(-I*a + b)*weierstrassPInverse(-4*(a^2 + 2*I*a*b
 - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(a + I
*b)*(I*a + b)*weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0,
cos(d*x + c) - I*sin(d*x + c)) - 2*sqrt(a*cos(d*x + c) + b*sin(d*x + c))*(b
*cos(d*x + c) - a*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a \cos(c + d x) + b \sin(c + d x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

3.235 $\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

```
[Out] 2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan
(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*x+c)+
b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3157, 2719}

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/(d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}\,dx}{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|\,2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.80, size = 268, normalized size = 3.57

$$\frac{\cos\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)\left(-b(a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)\right)\sin\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)+\sqrt{\sin^2\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)}\left(-2a(a^2+b^2)\cos\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)+2a^2\sqrt{1+\frac{b^2}{a^2}}\sqrt{a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)}\sqrt{a\cos(c+dx)+b\sin(c+dx)}+b(a^2+b^2)\sin\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)\right)\right)}{bd\left(a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)\right)^{3/2}\sqrt{\sin^2\left(c+dx-\operatorname{ArcTan}\left(\frac{b}{a}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (Cos[c + d*x - ArcTan[b/a]]*(-(b*(a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]) + Sqrt[S
in[c + d*x - ArcTan[b/a]]^2]*(-2*a*(a^2 + b^2)*Cos[c + d*x - ArcTan[b/a]] +
 2*a^2*Sqrt[1 + b^2/a^2]*Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
]]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + b*(a^2 + b^2)*Sin[c + d*x - ArcT
an[b/a]])))/(b*d*(a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqr
t[Sin[c + d*x - ArcTan[b/a]]^2])
```

Maple [A]
time = 1.52, size = 163, normalized size = 2.17

method	result
default	$-\frac{\sqrt{a^2+b^2}\,\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)+1}\,\sqrt{2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\left(\sqrt{\sin}\right.}{\cos(dx+c-\arctan(-}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(a^2+b^2)^(1/2)*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,
b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*(2*EllipticE((-sin(d*x+c-arctan(
```

```
-a,b))+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1
/2*2^(1/2)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/
2))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 163, normalized size = 2.17

$$\frac{i\sqrt{2}\sqrt{a-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(a^2+2i\,ab-b^2\right)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\left(a^2+2i\,ab-b^2\right)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-i\sqrt{2}\sqrt{a+ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(a^2-2i\,ab-b^2\right)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\left(a^2-2i\,ab-b^2\right)}{a^2+b^2},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^
2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x
 + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(a + I*b)*weierstrassZeta(-4*(a^2
- 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^
2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)
[Out] int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

3.236 $\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] 2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan
(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(d*x+c)
+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3157, 2720}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c
+ d*x])/Sqrt[a^2 + b^2]])/(d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}}\,d}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$= \frac{2F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 92, normalized size = 1.23

$$\frac{2\sqrt{\cos^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)\right)\tan\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}{d\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/(d*Sqrt[Sqrt[1 +
a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]])
```

Maple [A]
time = 0.43, size = 124, normalized size = 1.65

method	result
default	$-\frac{\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)+1}\,\sqrt{2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\left(\sqrt{\sin}\,(dx+c-\arctan\right.}{\cos(dx+c-\arctan(-a,b))\sqrt{\sin\left(dx+c-\arctan\left(-a,b\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*sin
(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1/2
*2^(1/2))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 128, normalized size = 1.71

$$\frac{\sqrt{2}\,\sqrt{a-i\,b}\,(-i\,a+b)\text{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\,\sqrt{a+i\,b}\,(i\,a+b)\text{weierstrassPInverse}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)}{(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a - I*b)*(-I*a + b)*weierstrassPInverse(-4*(a^2 + 2*I*a*b - b
^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(a + I*b)*
(I*a + b)*weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(
d*x + c) - I*sin(d*x + c)))/((a^2 + b^2)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

3.237 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{(a^2+b^2)\,d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

```
[Out] -2*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2
)-2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arct
an(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*x+c
)+b*sin(d*x+c))^(1/2)/(a^2+b^2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1
/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3157, 2719}

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{d\,(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\,(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2 + b^2)*d*Sqrt[a*Cos[c + d*x] +
 b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[
c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c +
 d*x])/Sqrt[a^2 + b^2]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\int \sqrt{a\cos(c+dx\ldots}}{a^2\ldots}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{\sqrt{a\cos(c+dx)\ldots}}{(a^2\ldots}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx-\ldots\right.}{(a^2+\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.24, size = 219, normalized size = 1.59

$$\frac{-\frac{2b\cos(c+dx)}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}+\frac{2a\sin(c+dx)}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\sqrt{a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}\tan\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)+\frac{\sqrt{a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)\tan\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}{\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}}}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((-2*b*Cos[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + (2*a*Sin[c + d
*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] - Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c
 + d*x - ArcTan[b/a]]]*Tan[c + d*x - ArcTan[b/a]] + (Sqrt[a*Sqrt[1 + b^2/a^
2]*Cos[c + d*x - ArcTan[b/a]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
 + d*x - ArcTan[b/a]]^2]*Tan[c + d*x - ArcTan[b/a]])/Sqrt[Sin[c + d*x - Arc
Tan[b/a]]^2])/((a^2 + b^2)*d)
```

Maple [A]
time = 0.45, size = 232, normalized size = 1.68

method	result

default	$\frac{2\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)+1}\,\sqrt{2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\,\left(\sqrt{\sin}\,(dx+c-\arctan(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*s
in(d*x+c-arctan(-a,b))^(1/2)*EllipticE((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1
/2*2^(1/2))-(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2
)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a,b))+1
)^(1/2),1/2*2^(1/2))-2*cos(d*x+c-arctan(-a,b))^2)/(a^2+b^2)^(1/2)/cos(d*x+c
-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 278, normalized size = 2.01

$$\frac{\left(-i\sqrt{2}\,a\cos\left(dx+c\right)-i\sqrt{2}\,b\sin\left(dx+c\right)\right)\sqrt{a-i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+\left(i\sqrt{2}\,a\cos\left(dx+c\right)+i\sqrt{2}\,b\sin\left(dx+c\right)\right)\sqrt{a+i\,b}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2-2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-2\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}\left(b\cos\left(dx+c\right)-a\sin\left(dx+c\right)\right)}{(a^3+ab^2)d\cos\left(dx+c\right)+(a^2b+b^3)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] ((-I*sqrt(2)*a*cos(d*x + c) - I*sqrt(2)*b*sin(d*x + c))*sqrt(a - I*b)*weier
strassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*
(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c))) + (I*
sqrt(2)*a*cos(d*x + c) + I*sqrt(2)*b*sin(d*x + c))*sqrt(a + I*b)*weierstras
sZeta(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2
- 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(a
*cos(d*x + c) + b*sin(d*x + c))*(b*cos(d*x + c) - a*sin(d*x + c)))/((a^3 +
a*b^2)*d*cos(d*x + c) + (a^2*b + b^3)*d*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

3.238 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{2F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+}{\sqrt{a^2+b^2}}}}{3\left(a^2+b^2\right)d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

```
[Out] -2/3*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(3
/2)+2/3*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*
arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(
d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3157, 2720}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}F\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a
*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*(a^2 + b^2)*d*Sqrt[a*C
os[c + d*x] + b*Sin[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\int\frac{}{\sqrt{a\cos(c+}}}{3}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\sqrt{\frac{a\cos(c+d}{\sqrt{}}}}{3}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{2F\left(\frac{1}{2}(c+dx\right.}{3\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.25, size = 145, normalized size = 1.02

$$\frac{2\left(\frac{-b\cos(c+dx)+a\sin(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}+\frac{\sqrt{\cos^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)\right)\tan\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}{\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin\left(c+dx+\text{ArcTan}\left(\frac{a}{b}\right)\right)}}\right)}{3\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x])^
(3/2) + (Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {
5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1
 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*(a^2 + b^2)*d)
```

Maple [A]
time = 0.46, size = 182, normalized size = 1.28

method	result

default	$-\frac{\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\left(\sin^{\frac{5}{2}}(dx+c-\arctan(-a,b\right.}{3(a^2+b^2)\sin(dx+c-\arctan(-a,b))^2\cos(dx+c-\arctan(-a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(a^2+b^2)*((-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b
))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(5/2)*EllipticF((-sin(d*x+c-arctan(-a,b
))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^3+2*sin(d*x+c-arctan(-a,
b)))/sin(d*x+c-arctan(-a,b))^2/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a
,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 406, normalized size = 2.86

$$\frac{\left(\sqrt{2}\,(-i a^3+a^2b+i ab^2-b^3)\cos(dx+c)^2-2\sqrt{2}\,(i a^2b-ab^2)\cos(dx+c)\sin(dx+c)+\sqrt{2}\,(-i ab^2+b^3)\right)\sqrt{a-i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4(a^2+2i ab-b^2)}{a^2+b^2},0,\cos(dx+c)+i\sin(dx+c)\right)+\left(\sqrt{2}\,(i a^3+a^2b-i ab^2-b^3)\cos(dx+c)^2-2\sqrt{2}\,(-i a^2b-ab^2)\cos(dx+c)\sin(dx+c)+\sqrt{2}\,(i ab^2+b^3)\right)\sqrt{a+i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4(a^2-2i ab-b^2)}{a^2+b^2},0,\cos(dx+c)-i\sin(dx+c)\right)-2\left((a^2b+b^3)\cos(dx+c)-(a^3+ab^2)\sin(dx+c)\right)\sqrt{a\cos(dx+c)+b\sin(dx+c)}}{3\left((a^6+a^4b^2-a^2b^4-b^6)d\cos(dx+c)^2+2(a^5b+2a^3b^3+ab^5)d\cos(dx+c)\sin(dx+c)+(a^4b^2+2a^2b^4+b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((sqrt(2)*(-I*a^3 + a^2*b + I*a*b^2 - b^3)*cos(d*x + c)^2 - 2*sqrt(2)*(
I*a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-I*a*b^2 + b^3))*sqrt
(a - I*b)*weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(
d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(I*a^3 + a^2*b - I*a*b^2 - b^3)*cos(d
*x + c)^2 - 2*sqrt(2)*(-I*a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + sqrt(2
)*(I*a*b^2 + b^3))*sqrt(a + I*b)*weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^
2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c)) - 2*((a^2*b + b^3)*cos(d*
x + c) - (a^3 + a*b^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*x + c)))
/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 +
 a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

3.239 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)^2 d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+d\right.}{5(}$$

```
[Out] -2/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(5
/2)-6/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c
))^(1/2)-6/5*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x
-1/2*arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*
cos(d*x+c)+b*sin(d*x+c))^(1/2)/(a^2+b^2)^2/d/((a*cos(d*x+c)+b*sin(d*x+c))/(
a^2+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3157, 2719}

$$-\frac{6\sqrt{a\cos(c+dx)+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\left(a^2+b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{6(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)^2\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x])^(5/2)) - (6*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2
)^2*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - (6*EllipticE[(c + d*x - ArcT
an[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])/(5*(a^2 + b^2)^2*d*S
qrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3157

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n, Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2 +
 b^2, 0] || EqQ[a^2 + b^2, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}+\frac{3\int \frac{}{(a\cos(c+dx)+}}{5\left(a^2\right.}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(}{5\left(a^2+b^2\right)^2 d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(}{5\left(a^2+b^2\right)^2 d}$$

$$= -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^{5/2}}-\frac{6(b\cos(}{5\left(a^2+b^2\right)^2 d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.76, size = 277, normalized size = 1.41

$$\frac{-\frac{2\left(3a^2\cos^3(c+dx)-ab\sin(c+dx)+6ab\cos^2(c+dx)\sin(c+dx)+b^2\cos(c+dx)\left(1+3\sin^2(c+dx)\right)\right)}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}+\frac{\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\left(3b\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)-3\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}\left(-2a\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)+b\sin\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)\right)}{\left(a\sqrt{1+\frac{b^2}{a^2}}\cos\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)\right)^{3/2}\sqrt{\sin^2\left(c+dx-\text{ArcTan}\left(\frac{b}{a}\right)\right)}}}{5b\left(a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((-2*(3*a^2*Cos[c + d*x]^3 - a*b*Sin[c + d*x] + 6*a*b*Cos[c + d*x]^2*Sin[c
+ d*x] + b^2*Cos[c + d*x]*(1 + 3*Sin[c + d*x]^2)))/(a*Cos[c + d*x] + b*Sin[
c + d*x])^(5/2) + (Cos[c + d*x - ArcTan[b/a]]*(3*b*HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] - 3
*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*Cos[c + d*x - ArcTan[b/a]] + b*Si
n[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]
])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*(a^2 + b^2)*d)
```

Maple [A]
time = 0.47, size = 297, normalized size = 1.51

method	result
default	$\frac{\sqrt{a^2+b^2}\left(6\sqrt{-\sin\left(dx+c-\arctan\left(-a,b\right)\right)+1}\ \sqrt{2\sin\left(dx+c-\arctan\left(-a,b\right)\right)+2}\ \left(\sin^{\frac{7}{2}}\left(dx+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a^2+b^2)^(1/2)*(6*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arct
an(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(7/2)*EllipticE((-sin(d*x+c-arct
an(-a,b))+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin
(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(7/2)*EllipticF((-sin
(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c-arctan(-a,b))^5-4*si
n(d*x+c-arctan(-a,b))^3-2*sin(d*x+c-arctan(-a,b)))/sin(d*x+c-arctan(-a,b))^
3/(a^4+2*a^2*b^2+b^4)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2
+b^2)^(1/2))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 552, normalized size = 2.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*(-3*I*sqrt(2)*a*b^2*cos(d*x + c) + sqrt(2)*(-I*a^3 + 3*I*a*b^2)*cos(
d*x + c)^3 + (-I*sqrt(2)*b^3 + sqrt(2)*(-3*I*a^2*b + I*b^3)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a^2
+ b^2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos
(d*x + c) + I*sin(d*x + c))) + 3*(3*I*sqrt(2)*a*b^2*cos(d*x + c) + sqrt(2)*
(I*a^3 - 3*I*a*b^2)*cos(d*x + c)^3 + (I*sqrt(2)*b^3 + sqrt(2)*(3*I*a^2*b -
```

```
I*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a + I*b)*weierstrassZeta(-4*(a^2
- 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^
2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(3*a^2*b - b^3)*c
os(d*x + c)^3 - (5*a^2*b - 4*b^3)*cos(d*x + c) - (a^3 + 4*a*b^2 + 3*(a^3 -
3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*x + c)
))/((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2
*a^3*b^4 + a*b^6)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d
*cos(d*x + c)^2 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

3.240 $\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx$

Optimal. Leaf size=120

$$\frac{130\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{21d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}$$

[Out] 130/21*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-2/7*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)/d-130/21*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {3152, 3156, 2720}

$$\frac{130\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{21d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]

[Out] (130*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(21*d) - (130*(3*Cos[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(21*d) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2))/(7*d)

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{5}}{7d}$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+d}}{21d}$$

$$= -\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+d}}{21d}$$

$$= \frac{130\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{21d} - \frac{130(3\cos(c+dx)-2\,\mathrm{si}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.36, size = 153, normalized size = 1.28

$$\frac{-\sqrt{2\cos(c+dx)+3\sin(c+dx)}\,(897\cos(c+dx)+27\cos(3(c+dx))-598\sin(c+dx)+138\sin(3(c+dx)))+260\ 13^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sec\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\sqrt{-\left(\left(-1+\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\right)}\sqrt{1+\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)}}{42d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

```
[Out] (-(Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(897*Cos[c + d*x] + 27*Cos[3*(c +
d*x)] - 598*Sin[c + d*x] + 138*Sin[3*(c + d*x)])) + 260*13^(3/4)*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcT
an[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[2/3]
])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(42*d)
```

Maple [A]
time = 0.60, size = 128, normalized size = 1.07

method	result
default	$\frac{338\left(\cos^4\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\right)\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}{7}+\frac{845\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\mathrm{arc}\right.}}{\cos\left(dx+c+\mathrm{ar}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (338/7*cos(d*x+c+arctan(2/3))^4*sin(d*x+c+arctan(2/3))+845/21*(sin(d*x+c+ar
ctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(
```

```
2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))-2704/2
1*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+c+arctan(2/3))/(
13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.52, size = 118, normalized size = 0.98

$$\frac{(130i+195)\sqrt{3i+2}\,\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-(130i-195)\sqrt{2}\,\sqrt{-3i+2}\,\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-2\left(27\cos\left(dx+c\right)^{3}+46\left(3\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)+204\cos\left(dx+c\right)\right)\sqrt{2\cos\left(dx+c\right)+3\sin\left(dx+c\right)}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*((130*I + 195)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20/
13, 0, cos(d*x + c) - I*sin(d*x + c)) - (130*I - 195)*sqrt(2)*sqrt(-3*I + 2
)*weierstrassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)) -
 2*(27*cos(d*x + c)^3 + 46*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) + 204*cos(d*
x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,\cos(c+d\,x)+3\,\sin(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

3.241 $\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d}$$

```
[Out] 78/5*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*
x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-
2/5*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3152, 3156, 2719\}$

$$\frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3}}{5d}$$
$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3}}{5d}$$
$$= \frac{78\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d} - \frac{2(3\cos(c+dx)-2\sin(c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.64, size = 199, normalized size = 2.65

$$\frac{\sqrt{2\cos(c+dx)+3\sin(c+dx)}\,(52-12\cos(2(c+dx))-5\sin(2(c+dx)))-\frac{13\sqrt[4]{13}\left(4\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)-3\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}{\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}-\frac{39\sqrt[4]{13}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{\sqrt{-\left(\left(-1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}\sqrt{1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(52 - 12*Cos[2*(c + d*x)] - 5*Sin[2*
(c + d*x)]) - (13*13^(1/4)*(4*Cos[c + d*x - ArcTan[3/2]] - 3*Sin[c + d*x -
ArcTan[3/2]]))/Sqrt[Cos[c + d*x - ArcTan[3/2]]] - (39*13^(1/4)*Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcT
an[3/2]])/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/
2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]]))/(5*d)
```

Maple [A]
time = 0.50, size = 174, normalized size = 2.32

method	result
default	$-\frac{13\sqrt{13}\left(6\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -13/5*13^(1/2)*(6*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3
))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((sin(d*x+c+arctan(2/3
))+1)^(1/2),1/2*2^(1/2))-3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+a
rctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+a
rctan(2/3))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c+arctan(2/3))^4+2*sin(d*x+c+ar
```

```
ctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 111, normalized size = 1.48

$$\frac{-39i\,\sqrt{3i+2}\,\sqrt{2}\,\mathrm{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+39i\,\sqrt{2}\,\sqrt{-3i+2}\,\mathrm{weierstrassZeta}\left(-\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-2\,\left(12\,\cos\left(dx+c\right)^{2}+5\,\cos\left(dx+c\right)\sin\left(dx+c\right)-6\right)\sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-39*I*sqrt(3*I + 2)*sqrt(2)*weierstrassZeta(48/13*I + 20/13, 0, weiers
trassPInverse(48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c))) + 39*I*sq
rt(2)*sqrt(-3*I + 2)*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPInver
se(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*(12*cos(d*x + c
)^2 + 5*cos(d*x + c)*sin(d*x + c) - 6)*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)
))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

3.242 $\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{2\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}$$

```
[Out] 2/3*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x
-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d-2
/3*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3152, 3156, 2720}

$$\frac{2\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*d) - (2*(3*Cos[c +
d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}$$

$$= \frac{2\ 13^{3/4}F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{3d} - \frac{2(3\cos(c+dx)-2\sin(c-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.24, size = 133, normalized size = 1.77

$$\frac{2(-3\cos(c+dx)+2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}+2\ 13^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sec\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\sqrt{-\left(\left(-1+\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)}\sqrt{1+\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*(-3*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]
 + 2*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3
]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*S
in[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(3*d)
```

Maple [A]
time = 0.41, size = 108, normalized size = 1.44

method	result
default	$\dfrac{\frac{13\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\right.}}{3}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (13/3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*
(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),
1/2*2^(1/2))-26/3*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+
c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 96, normalized size = 1.28

$$\frac{(2i+3)\sqrt{3i+2}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos(dx+c)-i\sin(dx+c)\right)-(2i-3)\sqrt{2}\sqrt{-3i+2}\,\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos(dx+c)+i\sin(dx+c)\right)-2\left(3\cos(dx+c)-2\sin(dx+c)\right)\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*((2*I + 3)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20/13, 0
, cos(d*x + c) - I*sin(d*x + c)) - (2*I - 3)*sqrt(2)*sqrt(-3*I + 2)*weierst
rassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(3*cos
(d*x + c) - 2*sin(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (2\cos(c+d\,x)+3\sin(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

3.243 $\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

```
[Out] 2*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
```

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3156, 2719}

$$\frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx &= \sqrt[4]{13}\int \sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}\,dx\\ &= \frac{2\sqrt[4]{13}\,E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.63, size = 184, normalized size = 6.81

$$\frac{-4\sqrt[4]{13}\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}+4\sqrt{2\cos(c+dx)+3\sin(c+dx)}+\frac{3\sqrt[4]{13}\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}-\frac{3\sqrt[4]{13}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{\sqrt{-\left(\left(-1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}\sqrt{1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (-4*13^(1/4)*Sqrt[Cos[c + d*x - ArcTan[3/2]]] + 4*Sqrt[2*Cos[c + d*x] + 3*S
in[c + d*x]] + (3*13^(1/4)*Sin[c + d*x - ArcTan[3/2]])/Sqrt[Cos[c + d*x - A
rcTan[3/2]]] - (3*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d
*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c + d*x
- ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[
3/2]]]))/(3*d)
```

Maple [A]
time = 0.74, size = 112, normalized size = 4.15

method	result
default	$-\frac{\sqrt{13}\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\sqrt{-\sin\left(dx+c+a\right.}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{}}$
risch	$-\frac{i\sqrt{2}\sqrt{-\left(3i\,e^{2i(dx+c)}-2\,e^{2i(dx+c)}-2-3i\right)e^{-i(dx+c)}}}{d}+\frac{(12+5i)\left(\frac{\left(-\frac{4}{2197}+\frac{6i}{2197}\right)\left(-507i\,e^{2i(dx+c)}+33\right.}{\sqrt{e^{i(dx+c)}\left(-507i\,e^{2i(dx+c)}+338\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -13^(1/2)*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1
/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*(2*EllipticE((sin(d*x+c+arctan(2/3))+1)
^(1/2),1/2*2^(1/2))-EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))
)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 63, normalized size = 2.33

$$\frac{-i\sqrt{3i+2}\sqrt{2}\,\mathrm{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+i\sqrt{2}\sqrt{-3i+2}\,\mathrm{weierstrassZeta}\left(-\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(3*I + 2)*sqrt(2)*weierstrassZeta(48/13*I + 20/13, 0, weierstrassPI
nverse(48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c))) + I*sqrt(2)*sqrt
(-3*I + 2)*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPInverse(-48/13*
I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3\sin\left(c+dx\right)+2\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

```
[Out] int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

3.244 $\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}$$

```
[Out] 2/13*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*
x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3156, 2720}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx &= \frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)}}\,dx}{\sqrt[4]{13}}\\ &= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{13}\,d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.08, size = 88, normalized size = 3.26

$$\frac{2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sec\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\sqrt{-\left(\left(-1+\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)\right)}\sqrt{1+\sin\left(c+dx+\text{ArcTan}\left(\frac{2}{3}\right)\right)}}{\sqrt[4]{13}\,d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

```
[Out] (2*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c
 + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x
+ ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(13^(1/4)*d)
```

Maple [A]
time = 0.38, size = 85, normalized size = 3.15

method	result
default	$\frac{\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\right.}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(
d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^
(1/2))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 58, normalized size = 2.15

$$\frac{(2i+3)\sqrt{3i+2}\,\sqrt{2}\,\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-(2i-3)\sqrt{2}\,\sqrt{-3i+2}\,\text{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)}{13\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/13*((2*I + 3)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20/13,
0, cos(d*x + c) - I*sin(d*x + c)) - (2*I - 3)*sqrt(2)*sqrt(-3*I + 2)*weiers
trassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{2\cos(c+d\,x)+3\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

3.245 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}} dx$

Optimal. Leaf size=73

$$-\frac{2E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

```
[Out] -2/13*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
-2/13*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3156, 2719}

$$-\frac{2E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d) - (2*(3*Cos[c + d
*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{1}{13}\int \sqrt{2\cos(c+dx)+3}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{\int\sqrt{\cos\left(c+dx-\tan^{-1}\right.}}{13^{3/4}}$$

$$= -\frac{2E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|\,2\right)}{13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+}{13d\sqrt{2\cos(c+dx)+3\sin(c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.80, size = 190, normalized size = 2.60

$$\frac{\frac{4\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}{13^{3/4}} - \frac{2\cos(c+dx)}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{3\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{13^{3/4}\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}} + \frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{13^{3/4}\sqrt{-\left(\left(-1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}\sqrt{1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) - (2*Cos[c + d*x])/Sqrt[2*Co
s[c + d*x] + 3*Sin[c + d*x]] - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqr
t[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-(
(-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos
[c + d*x - ArcTan[3/2]]]))/(3*d)
```

Maple [A]
time = 0.43, size = 162, normalized size = 2.22

method	result
default	$\frac{\sqrt{13}\left(2\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\sqrt{-\sin\left(dx+c+\text{a}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*13^(1/2)*(2*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3)
)+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((sin(d*x+c+arctan(2/3)
)+1)^(1/2),1/2*2^(1/2))-(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arct
```

```
an(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+arct
an(2/3))+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+arctan
(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 158, normalized size = 2.16

$$\frac{\sqrt{3i+2}\left(2i\sqrt{2}\cos(dx+c)+3i\sqrt{2}\sin(dx+c)\right)\text{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos(dx+c)-i\sin(dx+c)\right)\right)+\sqrt{-3i+2}\left(-2i\sqrt{2}\cos(dx+c)-3i\sqrt{2}\sin(dx+c)\right)\text{weierstrassZeta}\left(-\frac{48}{13}i+\frac{20}{13},0,\text{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos(dx+c)+i\sin(dx+c)\right)\right)-2\left(3\cos(dx+c)-2\sin(dx+c)\right)\sqrt{2\cos(dx+c)+3\sin(dx+c)}}{13\left(2d\cos(dx+c)+3d\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*(sqrt(3*I + 2)*(2*I*sqrt(2)*cos(d*x + c) + 3*I*sqrt(2)*sin(d*x + c))*w
eierstrassZeta(48/13*I + 20/13, 0, weierstrassPInverse(48/13*I + 20/13, 0,
cos(d*x + c) - I*sin(d*x + c))) + sqrt(-3*I + 2)*(-2*I*sqrt(2)*cos(d*x + c)
 - 3*I*sqrt(2)*sin(d*x + c))*weierstrassZeta(-48/13*I + 20/13, 0, weierstra
ssPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*(3*cos(
d*x + c) - 2*sin(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/(2*d*cos(
d*x + c) + 3*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\sin(c+dx)+2\cos(c+dx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((3*sin(c + d*x) + 2*cos(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

3.246 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}$$

```
[Out] 2/507*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
-2/39*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3156, 2720}

$$\frac{2F\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}} + \frac{1}{39}\int \frac{1}{\sqrt{2\cos(c+dx)+}}$$

$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}\right.}}}{39\sqrt[4]{13}}$$

$$= \frac{2F\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|2\right)}{39\sqrt[4]{13}\,d} - \frac{2(3\cos(c+dx)-2\sin(c+dx}{39d(2\cos(c+dx)+3\sin(c+dx}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.54, size = 157, normalized size = 2.09

$$\frac{-78\cos(c+dx)+52\sin(c+dx)+\sqrt{2}\,13^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\right)\sec\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)(2\cos(c+dx)+3\sin(c+dx))^{3/2}\sqrt{1+\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)}\sqrt{-1+\cos\left(2\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)\right)+2\sin\left(c+dx+\mathrm{ArcTan}\left(\frac{2}{3}\right)\right)}}{507d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-78*Cos[c + d*x] + 52*Sin[c + d*x] + Sqrt[2]*13^(3/4)*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*(
2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2)*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]]
*Sqrt[-1 + Cos[2*(c + d*x + ArcTan[2/3])] + 2*Sin[c + d*x + ArcTan[2/3]]])/
(507*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.42, size = 118, normalized size = 1.57

method	result
default	$\frac{\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right.\right.}}{39\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/39/sin(d*x+c+arctan(2/3))*((sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c
+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c
+arctan(2/3))+1)^(1/2),1/2*2^(1/2))*sin(d*x+c+arctan(2/3))-2*cos(d*x+c+arct
an(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.32, size = 189, normalized size = 2.52

$$-\frac{\sqrt{3i+2}\left(-(10i+15)\sqrt{2}\cos\left(dx+c\right)^2+(24i+36)\sqrt{2}\cos\left(dx+c\right)\sin\left(dx+c\right)+(18i+27)\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+\sqrt{-3i+2}\left((10i-15)\sqrt{2}\cos\left(dx+c\right)^2-(24i-36)\sqrt{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-(18i-27)\sqrt{2}\right)\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-26\left(3\cos\left(dx+c\right)-2\sin\left(dx+c\right)\right)\sqrt{2\cos\left(dx+c\right)+3\sin\left(dx+c\right)}}{507\left(5d\cos\left(dx+c\right)^2-12d\cos\left(dx+c\right)\sin\left(dx+c\right)-9d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/507*(sqrt(3*I + 2)*(-(10*I + 15)*sqrt(2)*cos(d*x + c)^2 + (24*I + 36)*sq
rt(2)*cos(d*x + c)*sin(d*x + c) + (18*I + 27)*sqrt(2))*weierstrassPInverse(
48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(-3*I + 2)*((10*I
- 15)*sqrt(2)*cos(d*x + c)^2 - (24*I - 36)*sqrt(2)*cos(d*x + c)*sin(d*x + c
) - (18*I - 27)*sqrt(2))*weierstrassPInverse(-48/13*I + 20/13, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 26*(3*cos(d*x + c) - 2*sin(d*x + c))*sqrt(2*cos(d*x
 + c) + 3*sin(d*x + c)))/(5*d*cos(d*x + c)^2 - 12*d*cos(d*x + c)*sin(d*x +
c) - 9*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

3.247 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{6E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

```
[Out] -6/845*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/
d-2/65*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)-6/84
5*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3155, 3156, 2719}

$$-\frac{6E\left(\frac{1}{2}\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d}-\frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(65*13^(3/4)*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x]))/(65*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2)) -
(6*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(845*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c
+ d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3156

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[(a^2 + b^2)^(n/2), Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2 +
```

```
b^2, 0]
```

Rubi steps

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}+\frac{3}{65}\int \frac{1}{(2\cos(c+dx)+3}$$
$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6(3\cos(c+dx)-2\,\mathrm{si}}{845d\sqrt{2\cos(c+dx)+3}}$$
$$= -\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{6(3\cos(c+dx)-2\,\mathrm{si}}{845d\sqrt{2\cos(c+dx)+3}}$$
$$= -\frac{6E\left(\frac{1}{2}\left(c+dx-\tan^{-1}\left(\frac{3}{2}\right)\right)\middle|\,2\right)}{65\ 13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+}{65d(2\cos(c+dx)+3\sin(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.50, size = 224, normalized size = 1.87

$$\frac{\frac{4\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}{13^{3/4}}+\frac{-33\cos(c+dx)+5\cos(3(c+dx))-4(\sin(c+dx)+3\sin(3(c+dx)))}{2(2\cos(c+dx)+3\sin(c+dx))^{5/2}}-\frac{3\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{13^{3/4}\sqrt{\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}+\frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\sin\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}{13^{3/4}\sqrt{-\left(\left(-1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)\right)}\sqrt{1+\cos\left(c+dx-\text{ArcTan}\left(\frac{3}{2}\right)\right)}}}{65d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) + (-33*Cos[c + d*x] + 5*Cos[
3*(c + d*x)] - 4*(Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(2*(2*Cos[c + d*x] +
3*Sin[c + d*x])^(5/2)) - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[Cos[
c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
+ d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-((-1 +
Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d
*x - ArcTan[3/2]]]))/(65*d)
```

Maple [A]
time = 0.50, size = 205, normalized size = 1.71

method	result
default	$\frac{\sqrt{13}\left(6\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\sqrt{-\sin\left(dx+c+\mathrm{a}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/845*13^(1/2)/sin(d*x+c+arctan(2/3))^2*(6*(sin(d*x+c+arctan(2/3))+1)^(1/2)
*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*
x+c+arctan(2/3))^2*EllipticE((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))-
3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-si
n(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticF((sin(d*x+c+a
rctan(2/3))+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c+arctan(2/3))^4-4*sin(d*x+c+ar
ctan(2/3))^2-2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 264, normalized size = 2.20

$$\frac{3\sqrt{3i+2}\left(46i\sqrt{2}\cos\left(dx+c\right)^3+9\left(-i\sqrt{2}\cos\left(dx+c\right)^2-3i\sqrt{2}\right)\sin\left(dx+c\right)-54i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+3\sqrt{-3i+2}\left(-46i\sqrt{2}\cos\left(dx+c\right)^3+9\left(i\sqrt{2}\cos\left(dx+c\right)^2+3i\sqrt{2}\right)\sin\left(dx+c\right)+54i\sqrt{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassZeta}\left(-\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(-\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+2\left(27\cos\left(dx+c\right)^3+2\left(69\cos\left(dx+c\right)^2-40\right)\sin\left(dx+c\right)+48\cos\left(dx+c\right)\right)\sqrt{2\cos\left(dx+c\right)+3\sin\left(dx+c\right)}}{845\left(46\,d\cos\left(dx+c\right)^3-54\,d\cos\left(dx+c\right)-9\left(d\cos\left(dx+c\right)^2+3\,d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/845*(3*sqrt(3*I + 2)*(46*I*sqrt(2)*cos(d*x + c)^3 + 9*(-I*sqrt(2)*cos(d*x
 + c)^2 - 3*I*sqrt(2))*sin(d*x + c) - 54*I*sqrt(2)*cos(d*x + c))*weierstras
sZeta(48/13*I + 20/13, 0, weierstrassPInverse(48/13*I + 20/13, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 3*sqrt(-3*I + 2)*(-46*I*sqrt(2)*cos(d*x + c)^3 + 9
*(I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2))*sin(d*x + c) + 54*I*sqrt(2)*cos(d
*x + c))*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPInverse(-48/13*I
+ 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) + 2*(27*cos(d*x + c)^3 + 2*(69*
cos(d*x + c)^2 - 40)*sin(d*x + c) + 48*cos(d*x + c))*sqrt(2*cos(d*x + c) +
3*sin(d*x + c)))/(46*d*cos(d*x + c)^3 - 54*d*cos(d*x + c) - 9*(d*cos(d*x +
c)^2 + 3*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
```

```
[Out] int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

3.248 $\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

```
[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
[Out] ((-I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.97

$$-\frac{i(a(\cos(c+dx)+i\sin(c+dx)))^n}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
[Out] ((-I)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*n)
```

Maple [A]
time = 0.52, size = 31, normalized size = 0.97

method	result
derivativedivides	$-\frac{i(a\cos(dx+c)+ia\sin(dx+c))^{n}}{dn}$
default	$-\frac{i(a\cos(dx+c)+ia\sin(dx+c))^{n}}{dn}$
norman	$-\frac{i\,\mathrm{e}^{n\ln\left(\frac{a\left(1-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2ia\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{dn}$
risch	$-\frac{i\,\mathrm{e}^{\frac{n\left(-i\operatorname{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)^{3}\pi+i\operatorname{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)^{2}\operatorname{csgn}(ia)\pi+i\operatorname{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)^{2}\operatorname{csgn}\left(i\mathrm{e}^{i(dx+c)}\right)\pi-i\operatorname{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)\operatorname{csgn}(ia)\operatorname{cs}}{2}}}{dn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.48, size = 59, normalized size = 1.84

$$-\frac{i\,a^{n}e^{\left(-n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)+n\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)\right)}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -I*a^n*e^(-n*log(sin(d*x + c)/(cos(d*x + c) + 1) + I) + n*log(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))/(d*n)
```

Fricas [A]
time = 1.80, size = 23, normalized size = 0.72

$$-\frac{i\,e^{(i\,dnx+i\,cn+n\log(a))}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

Sympy [A]
time = 0.12, size = 42, normalized size = 1.31

$$\begin{cases} x & \text{for}\ d=0 \wedge n=0 \\ x(ia\sin(c)+a\cos(c))^{n} & \text{for}\ d=0 \\ x & \text{for}\ n=0 \\ -\frac{i(ia\sin(c+dx)+a\cos(c+dx))^{n}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*sin(c) + a*cos(c))**n, Eq(d, 0)
), (x, Eq(n, 0)), (-I*(I*a*sin(c + d*x) + a*cos(c + d*x))**n/(d*n), True))
```

Giac [A]
time = 0.49, size = 23, normalized size = 0.72

$$-\frac{i\,e^{(i\,dnx+i\,cn+n\log(a))}}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i})^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n, x)
```

3.249 $\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

```
[Out] -1/4*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^4/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-1/4*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx = -\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Mathematica [A]
time = 0.11, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

```
[Out] ((-1/4*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(27) = 54$.
time = 0.49, size = 151, normalized size = 4.87

method	result
risch	$-\frac{i a^4 e^{4i(dx+c)}}{4d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-ia^4(\sin^4(dx+c))-6a^4\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(c}{8}\right.}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-ia^4(\sin^4(dx+c))-6a^4\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(c}{8}\right.}{d}$
norman	$\frac{\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{14a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16ia^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ia^4\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-I*a^
4*sin(d*x+c)^4-6*a^4*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c
)+1/8*d*x+1/8*c)-I*a^4*cos(d*x+c)^4+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(25) = 50$.
time = 0.26, size = 132, normalized size = 4.26

$$-\frac{i\,a^4\cos(dx+c)^4}{d}-\frac{i\,a^4\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a^4}{32\,d}-\frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^4}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -I*a^4*cos(d*x + c)^4/d - I*a^4*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 1/32*(12*d*x + 12*c + sin(4*d*
x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d - 3/16*(4*d*x + 4*c - sin(4*d*x + 4*c)
)*a^4/d
```

Fricas [A]
time = 3.31, size = 17, normalized size = 0.55

$$-\frac{i\,a^4e^{(4i\,dx+4i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*I*a^4*e^(4*I*d*x + 4*I*c)/d
```

Sympy [A]
time = 0.07, size = 36, normalized size = 1.16

$$\begin{cases} -\frac{ia^4e^{4ic}e^{4idx}}{4d} & \text{for } d \neq 0 \\ a^4xe^{4ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-I*a**4*exp(4*I*c)*exp(4*I*d*x)/(4*d), Ne(d, 0)), (a**4*x*exp(4*
I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.
time = 0.44, size = 52, normalized size = 1.68

$$-\frac{i\,a^4e^{(4i\,dx+4i\,c)}}{8\,d}-\frac{i\,a^4e^{(-4i\,dx-4i\,c)}}{8\,d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*I*a^4*e^(4*I*d*x + 4*I*c)/d - 1/8*I*a^4*e^(-4*I*d*x - 4*I*c)/d + 1/4*a
^4*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 2.55, size = 84, normalized size = 2.71

$$-\frac{2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,4{}\mathrm{i}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4{}\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

```
[Out] -(2*a^4*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 1))/(d*(tan(c/2 + (d*x)/
2)^3*4i - 6*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)*4i + tan(c/2 + (d*x)/
2)^4 + 1))
```

3.250 $\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

```
[Out] -1/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3150}

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.
time = 0.44, size = 76, normalized size = 2.45

method	result	size
risch	$-\frac{ia^3 e^{3i(dx+c)}}{3d}$	19
derivativedivides	$\frac{\frac{ia^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-ia^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76
default	$\frac{\frac{ia^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-ia^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76
norman	$\frac{\frac{4ia^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2ia^3}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6ia^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*I*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)-a^3*sin(d*x+c)^3-I*a^3*cos(d*x+c
)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(25) = 50$.
time = 0.28, size = 83, normalized size = 2.68

$$-\frac{i\,a^3\cos\left(dx+c\right)^3}{d}-\frac{a^3\sin\left(dx+c\right)^3}{d}-\frac{i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^3}{3\,d}-\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -I*a^3*cos(d*x + c)^3/d - a^3*sin(d*x + c)^3/d - 1/3*I*(cos(d*x + c)^3 - 3*
cos(d*x + c))*a^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d
```

Fricas [A]
time = 2.27, size = 17, normalized size = 0.55

$$-\frac{i\,a^3 e^{(3i\,dx+3i\,c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A]
time = 0.07, size = 36, normalized size = 1.16

$$\begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } d \neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3*
I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.
time = 0.41, size = 52, normalized size = 1.68

$$-\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{6\,d}-\frac{i\,a^3e^{(-3i\,dx-3i\,c)}}{6\,d}+\frac{a^3\sin\left(3\,dx+3\,c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*I*a^3*e^(3*I*d*x + 3*I*c)/d - 1/6*I*a^3*e^(-3*I*d*x - 3*I*c)/d + 1/3*a
^3*sin(3*d*x + 3*c)/d
```

Mupad [B]
time = 2.47, size = 66, normalized size = 2.13

$$-\frac{2\,a^3\,\left(3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{3\,d\,\left(-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}+3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
+ (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.251 $\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx$

Optimal. Leaf size=31

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

```
[Out] -1/2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx = -\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(27) = 54$.
time = 0.40, size = 73, normalized size = 2.35

method	result	size
risch	$-\frac{i a^2 e^{2 i (d x+c)}}{2 d}$	19
derivativedivides	$\frac{-a^2 \left(-\frac{\cos(d x+c) \sin(d x+c)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^2 \left(\cos^2(d x+c)\right)+a^2 \left(\frac{\cos(d x+c) \sin(d x+c)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^2 \left(-\frac{\cos(d x+c) \sin(d x+c)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^2 \left(\cos^2(d x+c)\right)+a^2 \left(\frac{\cos(d x+c) \sin(d x+c)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
norman	$\frac{\frac{2 a^2 \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{2 a^2 \left(\tan^3\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4 i a^2 \left(\tan^2\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-I*a^2*cos(d*x+c)^2+a^2
*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.
time = 0.28, size = 69, normalized size = 2.23

$$-\frac{i a^2 \cos(dx+c)^2}{d}+\frac{(2 dx+2 c+\sin(2 dx+2 c))a^2}{4 d}-\frac{(2 dx+2 c-\sin(2 dx+2 c))a^2}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -I*a^2*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d - 1/4*
(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d
```

Fricas [A]
time = 3.63, size = 17, normalized size = 0.55

$$-\frac{i a^2 e^{(2 i dx+2 i c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A]
time = 0.06, size = 36, normalized size = 1.16

$$\begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2*
I*c), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.
time = 0.41, size = 52, normalized size = 1.68

$$-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{4\,d}-\frac{i\,a^2e^{(-2i\,dx-2i\,c)}}{4\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*I*a^2*e^(2*I*d*x + 2*I*c)/d - 1/4*I*a^2*e^(-2*I*d*x - 2*I*c)/d + 1/2*a
^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.42, size = 44, normalized size = 1.42

$$-\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2{}\mathrm{i}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)*2i + tan(c/2 + (d*x)/2)^
2 - 1))
```

3.252 $\int (a\cos(c+dx)+ia\sin(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {2717, 2718}

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+ia\sin(c+dx))\,dx &= (ia)\int \sin(c+dx)\,dx+a\int \cos(c+dx)\,dx\\ &= -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.96

$$-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{ia\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

```
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.16, size = 26, normalized size = 1.00

method	result	size
risch	$-\frac{i a\,\mathrm{e}^{i(dx+c)}}{d}$	17
derivativedivides	$\frac{a\sin(dx+c)-ia\cos(dx+c)}{d}$	24
default	$-\frac{ia\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	26
norman	$\frac{\frac{2ia\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	51
meijerg	$\frac{\left(i\sqrt{\pi}\,\sin(c)a+\sqrt{\pi}\,\cos(c)a\right)\sin(dx)}{\sqrt{\pi}\,d}+\frac{\left(i\sqrt{\pi}\,\cos(c)a-\sqrt{\pi}\,\sin(c)a\right)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+I*a*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.92

$$-\frac{i\,a\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Fricas [A]
time = 3.08, size = 15, normalized size = 0.58

$$-\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*e^(I*d*x + I*c)/d
```

Sympy [A]
time = 0.05, size = 26, normalized size = 1.00

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

```
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Giac [A]
time = 0.41, size = 24, normalized size = 0.92

$$-\frac{i\,a\cos\left(dx+c\right)}{d}+\frac{a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Mupad [B]
time = 2.39, size = 20, normalized size = 0.77

$$\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + a*sin(c + d*x)*1i,x)
```

```
[Out] (2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

3.253 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3150}

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.00

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Maple [A]
time = 0.56, size = 23, normalized size = 0.79

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
norman	$\frac{-\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a/(-I+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.00

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A]
time = 3.26, size = 17, normalized size = 0.59

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for}\ ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

Giac [A]
time = 0.43, size = 21, normalized size = 0.72

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

Mupad [B]
time = 2.39, size = 25, normalized size = 0.86

$$\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.254 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] 1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 1.00

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.74

method	result	size
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{2d\,a^{2}}$	19
derivativedivides	$\frac{i}{d\,a^{2}(i\tan(dx+c)+1)}$	23
default	$\frac{i}{d\,a^{2}(i\tan(dx+c)+1)}$	23
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4i\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] I/d/a^2/(I*tan(d*x+c)+1)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.71

$$\frac{1}{(a^{2}\tan\left(dx+c\right)-i\,a^{2})d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

Fricas [A]
time = 2.54, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [A]
time = 0.07, size = 44, normalized size = 1.42

$$\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{2a^{2}d} & \text{for}\ a^{2}de^{2ic}\neq 0\\ \frac{xe^{-2ic}}{a^{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(a**2*d*exp(2*I*c), 0)
), (x*exp(-2*I*c)/a**2, True))
```

Giac [A]
time = 0.40, size = 30, normalized size = 0.97

$$-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

Mupad [B]
time = 2.41, size = 31, normalized size = 1.00

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

3.255 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] 1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(27) = 54$.
time = 0.62, size = 57, normalized size = 1.84

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{3a^3d}$	19
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
default	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
norman	$\frac{-\frac{4i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2i}{3ad}+\frac{6i\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^2}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(1/(-I+tan(1/2*d*x+1/2*c))-4/3/(-I+tan(1/2*d*x+1/2*c))^3+2*I/(-I+ta
n(1/2*d*x+1/2*c))^2)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.94

$$\frac{i\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A]
time = 3.50, size = 17, normalized size = 0.55

$$\frac{i\,e^{\left(-3i\,dx-3i\,c\right)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [A]
time = 0.07, size = 44, normalized size = 1.42

$$\begin{cases}\frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for}\ a^3de^{3ic}\neq 0\\ \frac{xe^{-3ic}}{a^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(a**3*d*exp(3*I*c), 0)
), (x*exp(-3*I*c)/a**3, True))
```

Giac [A]
time = 0.42, size = 36, normalized size = 1.16

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}{3\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B]
time = 2.46, size = 68, normalized size = 2.19

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^{3}\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.256 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx$

Optimal. Leaf size=31

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

```
[Out] 1/4*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^4
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 1.00

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
[Out] (I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

Maple [A]
time = 0.68, size = 36, normalized size = 1.16

method	result
risch	$\frac{i e^{-4 i (d x+c)}}{4 a^{4} d}$
derivativedivides	$\frac{-\frac{i}{(\tan (d x+c)-i)^{2}}-\frac{1}{\tan (d x+c)-i}}{d a^{4}}$
default	$\frac{-\frac{i}{(\tan (d x+c)-i)^{2}}-\frac{1}{\tan (d x+c)-i}}{d a^{4}}$
norman	$\frac{\frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{a d}-\frac{14\left(\tan ^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}+\frac{14\left(\tan ^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}-\frac{2\left(\tan ^{7}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d a}-\frac{8 i\left(\tan ^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}-\frac{8 i\left(\tan ^{6}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}+\frac{16 i\left(\tan ^{4}\left(\right.\right.}{a d}}{a^{3}\left(1+\tan ^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d/a^4*(-I/(tan(d*x+c)-I)^2-1/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.94

$$\frac{i \cos (4 d x+4 c)+\sin (4 d x+4 c)}{4 a^{4} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] 1/4*(I*cos(4*d*x + 4*c) + sin(4*d*x + 4*c))/(a^4*d)
```

Fricas [A]
time = 3.26, size = 17, normalized size = 0.55

$$\frac{i e^{(-4 i d x-4 i c)}}{4 a^{4} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [A]
time = 0.07, size = 44, normalized size = 1.42

$$\begin{cases} \frac{i e^{-4ic} e^{-4idx}}{4a^{4}d} & \text{for}\ a^{4} d e^{4ic} \neq 0 \\ \frac{x e^{-4ic}}{a^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((I*exp(-4*I*c)*exp(-4*I*d*x)/(4*a**4*d), Ne(a**4*d*exp(4*I*c), 0)
), (x*exp(-4*I*c)/a**4, True))
```

Giac [A]
time = 0.43, size = 44, normalized size = 1.42

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{4}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

Mupad [B]
time = 2.56, size = 91, normalized size = 2.94

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1\mathrm{i}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,6\mathrm{i}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2*1i - 1i))/(a^4*d*(4*tan(c/2 +
(d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (d*x)/2) + tan(c/2 + (d*
x)/2)^4*1i + 1i))
```

3.257 $\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

```
[Out] -2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3150}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = -\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))/d
```

Maple [A]
time = 0.41, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{2i(a\cos(dx+c)+ia\sin(dx+c))^{\frac{5}{2}}}{5d}$	28
default	$-\frac{2i(a\cos(dx+c)+ia\sin(dx+c))^{\frac{5}{2}}}{5d}$	28
risch	$-\frac{2ia^2\sqrt{a\,e^{i(dx+c)}}\,e^{2i(dx+c)}}{5d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.50, size = 51, normalized size = 1.55

$$-\frac{2i\,a^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*a^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(5/2))
```

Fricas [A]
time = 2.33, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 17, normalized size = 0.52

$$-\frac{2i\, a^{\frac{5}{2}} e^{\left(\frac{5}{2} i\, dx+\frac{5}{2} i\, c\right)}}{5\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

Mupad [B]
time = 0.43, size = 35, normalized size = 1.06

$$-\frac{a^2\, \mathrm{e}^{c\, 2\mathrm{i}}\, \mathrm{e}^{d\, x\, 2\mathrm{i}}\, \sqrt{a\, \mathrm{e}^{c\, 1\mathrm{i}}\, \mathrm{e}^{d\, x\, 1\mathrm{i}}}\, 2\mathrm{i}}{5\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
```

```
[Out] -(a^2*exp(c*2i)*exp(d*x*2i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(5*d)
```

3.258 $\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3150}

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))/d
```

Maple [A]
time = 0.38, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{2i(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$	28
default	$-\frac{2i(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$	28
risch	$-\frac{2ia\sqrt{a\,\mathrm{e}^{i(dx+c)}}\;\mathrm{e}^{i(dx+c)}}{3d}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.49, size = 51, normalized size = 1.55

$$-\frac{2i\,a^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*a^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(3/2))
```

Fricas [A]
time = 2.67, size = 17, normalized size = 0.52

$$-\frac{2i\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*I*a^(3/2)*e^(3/2*I*d*x + 3/2*I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.43, size = 25, normalized size = 0.76

$$-\frac{2i\left(a\cos\left(dx+c\right)+i\,a\sin\left(dx+c\right)\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] -2/3*I*(a*cos(d*x + c) + I*a*sin(d*x + c))^(3/2)/d
```

Mupad [B]
time = 2.38, size = 33, normalized size = 1.00

$$-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}\,\sqrt{a\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}}\,2\mathrm{i}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
[Out] -(a*exp(c*1i)*exp(d*x*1i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(3*d)
```

3.259 $\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

```
[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3150\}$

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])/d
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.97

$$-\frac{2i\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])/d
```

Maple [A]
time = 0.37, size = 28, normalized size = 0.90

method	result	size
risch	$-\frac{2i\sqrt{a\,\mathrm{e}^{i(dx+c)}}}{d}$	20
derivativedivides	$-\frac{2i\sqrt{a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)}}{d}$	28
default	$-\frac{2i\sqrt{a\cos\left(dx+c\right)+ia\sin\left(dx+c\right)}}{d}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.49, size = 51, normalized size = 1.65

$$-\frac{2i\sqrt{a}\sqrt{-\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+i}}{d\sqrt{\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(a)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I)/(d*sqrt(sin(d*x + c
)/(cos(d*x + c) + 1) + I))
```

Fricas [A]
time = 1.35, size = 17, normalized size = 0.55

$$-\frac{2i\sqrt{a}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

Giac [**A**]
time = 0.42, size = 25, normalized size = 0.81

$$-\frac{2i\sqrt{a\cos\left(dx+c\right)+i\,a\sin\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\ \cos\left(c+d\,x\right)+a\ \sin\left(c+d\,x\right)\ 1\mathrm{i}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

3.260 $\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

```
[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{3150\}$

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx = \frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.97

$$\frac{2i}{d\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

```
[Out] (2*I)/(d*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])
```

Maple [A]
time = 0.39, size = 28, normalized size = 0.90

method	result	size
risch	$\frac{2i}{\sqrt{a\,\mathrm{e}^{i(dx+c)}}\,d}$	20
derivativedivides	$\frac{2i}{d\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}$	28
default	$\frac{2i}{d\sqrt{a\cos(dx+c)+ia\sin(dx+c)}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.54, size = 51, normalized size = 1.65

$$\frac{2i\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{\sqrt{a}\,d\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I)/(sqrt(a)*d*sqrt(-sin(d*x + c)
/(cos(d*x + c) + 1) + I))
```

Fricas [A]
time = 0.96, size = 17, normalized size = 0.55

$$\frac{2i\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*I*e^(-1/2*I*d*x - 1/2*I*c)/(sqrt(a)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ia\sin(c+dx)+a\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

Giac [A]
time = 0.41, size = 25, normalized size = 0.81

$$\frac{2i}{\sqrt{a\cos\left(dx+c\right)+i\,a\sin\left(dx+c\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*I/(sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

3.261 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

```
[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3150}

$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{3d(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)/3)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 0.38, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2i}{3d(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}$	28
default	$\frac{2i}{3d(a\cos(dx+c)+ia\sin(dx+c))^{\frac{3}{2}}}$	28
risch	$\frac{2i\mathrm{e}^{-i(dx+c)}}{3a\sqrt{a\,\mathrm{e}^{i(dx+c)}}\,d}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.49, size = 51, normalized size = 1.55

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(a^(3/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(3/2))
```

Fricas [A]
time = 1.20, size = 17, normalized size = 0.52

$$\frac{2i\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*I*e^(-3/2*I*d*x - 3/2*I*c)/(a^(3/2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\sin(c+dx)+a\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-3/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.
time = 0.89, size = 65, normalized size = 1.97

$$-\frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*I*(tan(1/2*d*x + 1/2*c) + I)/((a*tan(1/2*d*x + 1/2*c) - I*a)*d*sqrt(-(
a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2), x)
```

3.262 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

```
[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3150}

$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{2i}{5d(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

```
[Out] ((2*I)/5)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 0.45, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2i}{5d(a\cos(dx+c)+ia\sin(dx+c))^{\frac{5}{2}}}$	28
default	$\frac{2i}{5d(a\cos(dx+c)+ia\sin(dx+c))^{\frac{5}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.
time = 0.51, size = 51, normalized size = 1.55

$$\frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(a^(5/2)*d*(-sin(d*x + c)
/(cos(d*x + c) + 1) + I)^(5/2))
```

Fricas [A]
time = 1.19, size = 17, normalized size = 0.52

$$\frac{2i\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*I*e^(-5/2*I*d*x - 5/2*I*c)/(a^(5/2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\sin\left(c+dx\right)+a\cos\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-5/2), x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(25) = 50.
time = 0.75, size = 67, normalized size = 2.03

$$\frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^2}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)^2d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*I*(tan(1/2*d*x + 1/2*c) + I)^2/((a*tan(1/2*d*x + 1/2*c) - I*a)^2*d*sqrt
(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
```

```
[Out] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2), x)
```

3.263 $\int (a\sec(x)+b\tan(x))^5\,dx$

Optimal. Leaf size=149

$$-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(1+\sin(x))-\frac{1}{8}a\left(7-\frac{3a^2}{b^2}\right)b^4\sin$$

```
[Out] -1/16*(a+b)^3*(3*a^2-9*a*b+8*b^2)*ln(1-sin(x))+1/16*(a-b)^3*(3*a^2+9*a*b+8*
b^2)*ln(1+sin(x))-1/8*a*(7-3*a^2/b^2)*b^4*sin(x)+1/4*sec(x)^4*(b+a*sin(x))*
(a+b*sin(x))^4+1/8*sec(x)^2*(a+b*sin(x))^2*(2*b*(a^2-2*b^2)+a*(3*a^2-5*b^2)
*sin(x))
```

Rubi [A]
time = 0.16, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4476, 2747, 753, 833, 788, 647, 31}

$$-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(\sin(x)+1)+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(a\left(3a^2-5b^2\right)\sin(x)+2b\left(a^2-2b^2\right)\right)-\frac{1}{8}ab^4\left(7-\frac{3a^2}{b^2}\right)\sin(x)+\frac{1}{4}\sec^4(x)(a\sin(x)+b)(a+b\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -1/16*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]]) + ((a - b)^3*(3*a
^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]])/16 - (a*(7 - (3*a^2)/b^2)*b^4*Sin[x])/
8 + (Sec[x]^4*(b + a*Sin[x])*(a + b*Sin[x])^4)/4 + (Sec[x]^2*(a + b*Sin[x])
^2*(2*b*(a^2 - 2*b^2) + a*(3*a^2 - 5*b^2)*Sin[x]))/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

```
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (a\sec(x)+b\tan(x))^5\,dx &= \int \sec^5(x)(a+b\sin(x))^5\,dx \\
&= b^5\text{Subst}\left(\int \frac{(a+x)^5}{(b^2-x^2)^3}\,dx, x, b\sin(x)\right) \\
&= \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 - \frac{1}{4}b^3\text{Subst}\left(\int \frac{(a+x)^3\left(-3a^2+4b^2+\right.}{(b^2-x^2)^2}\right. \\
&= \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(2b(a^2-2b^2)\right. \\
&= \frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x) + \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+ \\
&= \frac{1}{8}ab^2\left(3a^2-7b^2\right)\sin(x) + \frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4 + \frac{1}{8}\sec^2(x)(a+ \\
&= -\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x)) + \frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(149) = 298.
time = 0.89, size = 303, normalized size = 2.03

$$
-\frac{(a^2-b^2)^2\left((a+b)^3(3a^2-9ab+8b^2)\log(1-\sin(x))-(a-b)^3(3a^2+9ab+8b^2)\log(1+\sin(x))\right)-10ab^2(9a^6-6a^4b^2+8a^2b^4-3b^6)\sin(x)+8b^3(-15a^6-4a^4b^2-2a^2b^4+b^6)\sin^2(x)-10ab^4(9a^4+8a^2b^2-b^4)\sin^3(x)+4b^5(-9a^4-12a^2b^2+b^4)\sin^4(x)-2ab^6(3a^2+5b^2)\sin^5(x)+4(-a^2+b^2)\sec^4(x)(-b+a\sin(x))(a+b\sin(x))^6+2\sec^2(x)(a+b\sin(x))^6\left(6a^2b+2b^3-3a^3\sin(x)-5ab^2\sin(x)\right)}{16(a^2-b^2)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^5,x]
```

```
[Out] -1/16*((a^2 - b^2)^2*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]] - (
a - b)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]]) - 10*a*b^2*(9*a^6 - 6*a^4
*b^2 + 8*a^2*b^4 - 3*b^6)*Sin[x] + 8*b^3*(-15*a^6 - 4*a^4*b^2 - 2*a^2*b^4 +
 b^6)*Sin[x]^2 - 10*a*b^4*(9*a^4 + 8*a^2*b^2 - b^4)*Sin[x]^3 + 4*b^5*(-9*a^
4 - 12*a^2*b^2 + b^4)*Sin[x]^4 - 2*a*b^6*(3*a^2 + 5*b^2)*Sin[x]^5 + 4*(-a^2
 + b^2)*Sec[x]^4*(-b + a*Sin[x])*(a + b*Sin[x])^6 + 2*Sec[x]^2*(a + b*Sin[x
])^6*(6*a^2*b + 2*b^3 - 3*a^3*Sin[x] - 5*a*b^2*Sin[x]))/(a^2 - b^2)^2
```

Maple [A]
time = 0.33, size = 164, normalized size = 1.10

method	result
default	$a^5\left(-\left(-\frac{(\sec^3(x))}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{3\ln(\sec(x)+\tan(x))}{8}\right)+\frac{5b\,a^4}{4\cos(x)^4}+10a^3b^2\left(\frac{\sin^3(x)}{4\cos(x)^4}+\frac{\sin^3(x)}{8\cos(x)^2}+\frac{\sin(}{8}\right.$
risch	$ib^5x-\frac{70ia^3b^2e^{5ix}-25ia\,b^4e^{7ix}+3ia^5e^{7ix}+15ia\,b^4e^{5ix}+11ia^5e^{5ix}-70ia^3b^2e^{3ix}+80a^2b^3e^{6ix}+16b^5e^{6ix}-10ia^3b^2e^{7ix}-3ie^{ix}a^5+10i}{4(e^{2ix}+1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x)))+5/4*b*a^4/co
s(x)^4+10*a^3*b^2*(1/4*sin(x)^3/cos(x)^4+1/8*sin(x)^3/cos(x)^2+1/8*sin(x)-1
/8*ln(sec(x)+tan(x)))+5/2*b^3*a^2*sin(x)^4/cos(x)^4+5*a*b^4*(1/4*sin(x)^5/c
os(x)^4-1/8*sin(x)^5/cos(x)^2-1/8*sin(x)^3-3/8*sin(x)+3/8*ln(sec(x)+tan(x))
)+b^5*(1/4*tan(x)^4-1/2*tan(x)^2-ln(cos(x)))
```

Maxima [A]
time = 0.28, size = 204, normalized size = 1.37

$$\frac{5}{2}\,a^2b^3\tan\left(x\right)^4+\frac{5}{16}\,ab^4\left(\frac{2\,\left(5\,\sin\left(x\right)^3-3\,\sin\left(x\right)\right)}{\sin\left(x\right)^4-2\,\sin\left(x\right)^2+1}+3\,\log\left(\sin\left(x\right)+1\right)-3\,\log\left(\sin\left(x\right)-1\right)\right)-\frac{1}{16}\,a^5\left(\frac{2\,\left(3\,\sin\left(x\right)^3-5\,\sin\left(x\right)\right)}{\sin\left(x\right)^4-2\,\sin\left(x\right)^2+1}-3\,\log\left(\sin\left(x\right)+1\right)+3\,\log\left(\sin\left(x\right)-1\right)\right)+\frac{5}{8}\,a^3b^2\left(\frac{2\,\left(\sin\left(x\right)^3+\sin\left(x\right)\right)}{\sin\left(x\right)^4-2\,\sin\left(x\right)^2+1}-\log\left(\sin\left(x\right)+1\right)+\log\left(\sin\left(x\right)-1\right)\right)+\frac{1}{4}\,b^5\left(\frac{4\,\sin\left(x\right)^2-3}{\sin\left(x\right)^4-2\,\sin\left(x\right)^2+1}-2\,\log\left(\sin\left(x\right)^2-1\right)\right)+\frac{5\,a^4b}{4\,\left(\sin\left(x\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*a^2*b^3*tan(x)^4 + 5/16*a*b^4*(2*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*
sin(x)^2 + 1) + 3*log(sin(x) + 1) - 3*log(sin(x) - 1)) - 1/16*a^5*(2*(3*sin
(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 3*log(sin(x) + 1) + 3*log(s
in(x) - 1)) + 5/8*a^3*b^2*(2*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1
) - log(sin(x) + 1) + log(sin(x) - 1)) + 1/4*b^5*((4*sin(x)^2 - 3)/(sin(x)^
4 - 2*sin(x)^2 + 1) - 2*log(sin(x)^2 - 1)) + 5/4*a^4*b/(sin(x)^2 - 1)^2
```

Fricas [A]
time = 2.83, size = 166, normalized size = 1.11

$$\frac{\left(3\,a^5-10\,a^3b^2+15\,ab^4-8\,b^5\right)\cos\left(x\right)^4\log\left(\sin\left(x\right)+1\right)-\left(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5\right)\cos\left(x\right)^4\log\left(-\sin\left(x\right)+1\right)+20\,a^4b+40\,a^2b^3+4\,b^5-16\,\left(5\,a^2b^3+b^5\right)\cos\left(x\right)^2+2\,\left(2\,a^5+20\,a^3b^2+10\,ab^4+\left(3\,a^5-10\,a^3b^2-25\,ab^4\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{16\,\cos\left(x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cos(x)^4*log(sin(x) + 1) - (3
*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cos(x)^4*log(-sin(x) + 1) + 20*a^4*b
+ 40*a^2*b^3 + 4*b^5 - 16*(5*a^2*b^3 + b^5)*cos(x)^2 + 2*(2*a^5 + 20*a^3*b^
2 + 10*a*b^4 + (3*a^5 - 10*a^3*b^2 - 25*a*b^4)*cos(x)^2)*sin(x))/cos(x)^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(141) = 282$.
time = 4.23, size = 308, normalized size = 2.07

$$-\frac{3a^5\log\left(\sin\left(x\right)-1\right)}{16}+\frac{3a^5\log\left(\sin\left(x\right)+1\right)}{16}-\frac{3a^5\sin^3\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5a^5\sin\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5a^4b\sec^4\left(x\right)}{4}+\frac{5a^3b^2\log\left(\sin\left(x\right)-1\right)}{8}-\frac{5a^3b^2\log\left(\sin\left(x\right)+1\right)}{8}+\frac{10a^3b^2\sin^3\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{10a^3b^2\sin\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{5a^2b^3\tan^4\left(x\right)}{2}-\frac{15ab^4\log\left(\sin\left(x\right)-1\right)}{16}+\frac{15ab^4\log\left(\sin\left(x\right)+1\right)}{16}+\frac{25ab^4\sin^3\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}-\frac{15ab^4\sin\left(x\right)}{8\sin^4\left(x\right)-16\sin^2\left(x\right)+8}+\frac{b^5\log\left(\sec^2\left(x\right)\right)}{2}+\frac{b^5\sec^4\left(x\right)}{4}-b^5\sec^2\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**5,x)
```

```
[Out] -3*a**5*log(sin(x) - 1)/16 + 3*a**5*log(sin(x) + 1)/16 - 3*a**5*sin(x)**3/(
8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2
 + 8) + 5*a**4*b*sec(x)**4/4 + 5*a**3*b**2*log(sin(x) - 1)/8 - 5*a**3*b**2*
log(sin(x) + 1)/8 + 10*a**3*b**2*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
 + 10*a**3*b**2*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**2*b**3*tan(x
)**4/2 - 15*a*b**4*log(sin(x) - 1)/16 + 15*a*b**4*log(sin(x) + 1)/16 + 25*a
*b**4*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*a*b**4*sin(x)/(8*sin(
x)**4 - 16*sin(x)**2 + 8) + b**5*log(sec(x)**2)/2 + b**5*sec(x)**4/4 - b**5
*sec(x)**2
```

Giac [A]
time = 0.40, size = 178, normalized size = 1.19

$$\frac{1}{16}\left(3a^5 - 10a^3b^2 + 15ab^4 - 8b^5\right)\log\left(\sin\left(x\right) + 1\right) - \frac{1}{16}\left(3a^5 - 10a^3b^2 + 15ab^4 + 8b^5\right)\log\left(-\sin\left(x\right) + 1\right) + \frac{6b^5\sin\left(x\right)^4 - 3a^5\sin\left(x\right)^3 + 10a^3b^2\sin\left(x\right)^3 + 25ab^4\sin\left(x\right)^3 + 40a^2b^3\sin\left(x\right)^2 - 4b^5\sin\left(x\right)^2 + 5a^5\sin\left(x\right) + 10a^3b^2\sin\left(x\right) - 15ab^4\sin\left(x\right) + 10a^4b - 20a^2b^3}{8\left(\sin\left(x\right)^2 - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*log(sin(x) + 1) - 1/16*(3*a^5
- 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*log(-sin(x) + 1) + 1/8*(6*b^5*sin(x)^4 - 3
*a^5*sin(x)^3 + 10*a^3*b^2*sin(x)^3 + 25*a*b^4*sin(x)^3 + 40*a^2*b^3*sin(x)
^2 - 4*b^5*sin(x)^2 + 5*a^5*sin(x) + 10*a^3*b^2*sin(x) - 15*a*b^4*sin(x) +
10*a^4*b - 20*a^2*b^3)/(sin(x)^2 - 1)^2
```

Mupad [B]
time = 2.88, size = 272, normalized size = 1.83

$$\frac{\left(\frac{5a^5}{4} + \frac{5a^3b^2}{2} - \frac{15ab^4}{4}\right)\tan\left(\frac{x}{2}\right)^7 + \left(10a^4b - 2b^5\right)\tan\left(\frac{x}{2}\right)^6 + \left(\frac{3a^5}{4} + \frac{35a^3b^2}{2} + \frac{55ab^4}{4}\right)\tan\left(\frac{x}{2}\right)^5 + \left(40a^2b^3 + 8b^5\right)\tan\left(\frac{x}{2}\right)^4 + \left(\frac{3a^5}{4} + \frac{35a^3b^2}{2} + \frac{55ab^4}{4}\right)\tan\left(\frac{x}{2}\right)^3 + \left(10a^4b - 2b^5\right)\tan\left(\frac{x}{2}\right)^2 + \left(\frac{5a^5}{4} + \frac{5a^3b^2}{2} - \frac{15ab^4}{4}\right)\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^8 - 4\tan\left(\frac{x}{2}\right)^6 + 6\tan\left(\frac{x}{2}\right)^4 - 4\tan\left(\frac{x}{2}\right)^2 + 1} + b^5\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)\left(a + b\right)^3\left(3a^2 - 9ab + 8b^2\right)}{8} + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)\left(a - b\right)^3\left(\frac{3a^2}{8} + \frac{9ab}{8} + b^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^5,x)
```

```
[Out] (tan(x/2)^2*(10*a^4*b - 2*b^5) + tan(x/2)^6*(10*a^4*b - 2*b^5) + tan(x/2)^4
*(8*b^5 + 40*a^2*b^3) + tan(x/2)*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/2)
 + tan(x/2)^7*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/2) + tan(x/2)^3*((55*
a*b^4)/4 + (3*a^5)/4 + (35*a^3*b^2)/2) + tan(x/2)^5*((55*a*b^4)/4 + (3*a^5)
/4 + (35*a^3*b^2)/2))/(6*tan(x/2)^4 - 4*tan(x/2)^2 - 4*tan(x/2)^6 + tan(x/2
)^8 + 1) + b^5*log(tan(x/2)^2 + 1) - (log(tan(x/2) - 1)*(a + b)^3*(3*a^2 -
9*a*b + 8*b^2))/8 + log(tan(x/2) + 1)*(a - b)^3*((9*a*b)/8 + (3*a^2)/8 + b^
2)
```

3.264 $\int (a\sec(x)+b\tan(x))^4\,dx$

Optimal. Leaf size=100

$$b^4x+\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\cos(x)\sin(x)+\frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3-\frac{1}{3}\sec(x)(a+$$

```
[Out] b^4*x+4/3*a*b*(a^2-2*b^2)*cos(x)+1/3*b^2*(2*a^2-3*b^2)*cos(x)*sin(x)+1/3*se
c(x)^3*(b+a*sin(x))*(a+b*sin(x))^3-1/3*sec(x)*(a+b*sin(x))^2*(a*b-(2*a^2-3*
b^2)*sin(x))
```

Rubi [A]

time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4476, 2770, 2940, 2813}

$$\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3+b^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] b^4*x + (4*a*b*(a^2 - 2*b^2)*Cos[x])/3 + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x]
)/3 + (Sec[x]^3*(b + a*Sin[x])*(a + b*Sin[x])^3)/3 - (Sec[x]*(a + b*Sin[x])
^2*(a*b - (2*a^2 - 3*b^2)*Sin[x]))/3
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
```

```
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^4\,dx &= \int \sec^4(x)(a+b\sin(x))^4\,dx\\ &= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3-\frac{1}{3}\int \sec^2(x)(a+b\sin(x))^2\left(-2a^2+3b^2\right.\\ &= \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3-\frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b\right.\right.\\ &= b^4x+\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\cos(x)\sin(x)+\frac{1}{3}\sec^3(x)(b+a\sin\end{aligned}$$

Mathematica [A]
time = 0.22, size = 96, normalized size = 0.96

$$\frac{1}{12}\sec^3(x)\left(16a^3b-8ab^3+9b^4x\cos(x)-24ab^3\cos(2x)+3b^4x\cos(3x)+6a^4\sin(x)+18a^2b^2\sin(x)+2a^4\sin(3x)-6a^2b^2\sin(3x)-4b^4\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(16*a^3*b - 8*a*b^3 + 9*b^4*x*Cos[x] - 24*a*b^3*Cos[2*x] + 3*b^4*
x*Cos[3*x] + 6*a^4*Sin[x] + 18*a^2*b^2*Sin[x] + 2*a^4*Sin[3*x] - 6*a^2*b^2*
Sin[3*x] - 4*b^4*Sin[3*x]))/12
```

Maple [A]
time = 0.17, size = 96, normalized size = 0.96

method	result
default	$-a^4\left(-\frac{2}{3}-\frac{(\sec^2(x))}{3}\right)\tan(x)+\frac{4a^3b}{3\cos(x)^3}+\frac{2a^2b^2(\sin^3(x))}{\cos(x)^3}+4a\,b^3\left(\frac{\sin^4(x)}{3\cos(x)^3}-\frac{\sin^4(x)}{3\cos(x)}-\frac{(2+\sin^2(x))\cos(x)}{3}\right)$

risch	$b^4x-\frac{4(9ia^2b^2e^{4ix}+3ib^4e^{4ix}+6a\,b^3e^{5ix}-3ia^4e^{2ix}+3ib^4e^{2ix}-8a^3b\,e^{3ix}+4a\,b^3e^{3ix}-ia^4+3ia^2b^2+2ib^4+6a\,b^3e^{ix})}{3(e^{2ix}+1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -a^4*(-2/3-1/3*sec(x)^2)*tan(x)+4/3*a^3*b/cos(x)^3+2*a^2*b^2*sin(x)^3/cos(x
)^3+4*a*b^3*(1/3*sin(x)^4/cos(x)^3-1/3*sin(x)^4/cos(x)-1/3*(2+sin(x)^2)*cos
(x))+b^4*(1/3*tan(x)^3-tan(x)+x)
```

Maxima [A]
time = 0.48, size = 72, normalized size = 0.72

$$2\,a^2b^2\tan\left(x\right)^3+\frac{1}{3}\left(\tan\left(x\right)^3+3\,\tan\left(x\right)\right)a^4+\frac{1}{3}\left(\tan\left(x\right)^3+3\,x-3\,\tan\left(x\right)\right)b^4-\frac{4\left(3\,\cos\left(x\right)^2-1\right)ab^3}{3\,\cos\left(x\right)^3}+\frac{4\,a^3b}{3\,\cos\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] 2*a^2*b^2*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*a^4 + 1/3*(tan(x)^3 + 3*x -
3*tan(x))*b^4 - 4/3*(3*cos(x)^2 - 1)*a*b^3/cos(x)^3 + 4/3*a^3*b/cos(x)^3
```

Fricas [A]
time = 2.06, size = 80, normalized size = 0.80

$$\frac{3\,b^4x\cos\left(x\right)^3-12\,ab^3\cos\left(x\right)^2+4\,a^3b+4\,ab^3+\left(a^4+6\,a^2b^2+b^4+2\left(a^4-3\,a^2b^2-2\,b^4\right)\cos\left(x\right)^2\right)\sin\left(x\right)}{3\,\cos\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x*cos(x)^3 - 12*a*b^3*cos(x)^2 + 4*a^3*b + 4*a*b^3 + (a^4 + 6*a^
2*b^2 + b^4 + 2*(a^4 - 3*a^2*b^2 - 2*b^4)*cos(x)^2)*sin(x))/cos(x)^3
```

Sympy [A]
time = 2.42, size = 97, normalized size = 0.97

$$\frac{a^4\tan^3\left(x\right)}{3}+a^4\tan\left(x\right)+\frac{4a^3b\sec^3\left(x\right)}{3}+2a^2b^2\tan^3\left(x\right)+\frac{4ab^3\sec^3\left(x\right)}{3}-4ab^3\sec\left(x\right)+b^4x+\frac{b^4\sin^3\left(x\right)}{3\cos^3\left(x\right)}-\frac{b^4\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**4,x)
```

```
[Out] a**4*tan(x)**3/3 + a**4*tan(x) + 4*a**3*b*sec(x)**3/3 + 2*a**2*b**2*tan(x)*
*3 + 4*a*b**3*sec(x)**3/3 - 4*a*b**3*sec(x) + b**4*x + b**4*sin(x)**3/(3*co
s(x)**3) - b**4*sin(x)/cos(x)
```

Giac [A]
time = 0.42, size = 131, normalized size = 1.31

$$b^4x - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,x\right)^5 - 3\,b^4\tan\left(\frac{1}{2}\,x\right)^5 + 12\,a^3b\tan\left(\frac{1}{2}\,x\right)^4 - 2\,a^4\tan\left(\frac{1}{2}\,x\right)^3 + 24\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3 + 10\,b^4\tan\left(\frac{1}{2}\,x\right)^3 + 24\,ab^3\tan\left(\frac{1}{2}\,x\right)^2 + 3\,a^4\tan\left(\frac{1}{2}\,x\right) - 3\,b^4\tan\left(\frac{1}{2}\,x\right) + 4\,a^3b - 8\,ab^3\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] b^4*x - 2/3*(3*a^4*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a^3*b*tan(1/2*x)^
4 - 2*a^4*tan(1/2*x)^3 + 24*a^2*b^2*tan(1/2*x)^3 + 10*b^4*tan(1/2*x)^3 + 24
*a*b^3*tan(1/2*x)^2 + 3*a^4*tan(1/2*x) - 3*b^4*tan(1/2*x) + 4*a^3*b - 8*a*b
^3)/(tan(1/2*x)^2 - 1)^3
```

Mupad [B]
time = 2.53, size = 115, normalized size = 1.15

$$b^4\,x - \frac{\tan\left(\frac{x}{2}\right)\,\left(2\,a^4 - 2\,b^4\right) - \frac{16\,a\,b^3}{3} + \frac{8\,a^3\,b}{3} + \tan\left(\frac{x}{2}\right)^3\,\left(-\frac{4\,a^4}{3} + 16\,a^2\,b^2 + \frac{20\,b^4}{3}\right) + \tan\left(\frac{x}{2}\right)^5\,\left(2\,a^4 - 2\,b^4\right) + 16\,a\,b^3\tan\left(\frac{x}{2}\right)^2 + 8\,a^3\,b\tan\left(\frac{x}{2}\right)^4}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^4,x)
```

```
[Out] b^4*x - (tan(x/2)*(2*a^4 - 2*b^4) - (16*a*b^3)/3 + (8*a^3*b)/3 + tan(x/2)^3
*((20*b^4)/3 - (4*a^4)/3 + 16*a^2*b^2) + tan(x/2)^5*(2*a^4 - 2*b^4) + 16*a*
b^3*tan(x/2)^2 + 8*a^3*b*tan(x/2)^4)/(tan(x/2)^2 - 1)^3
```

3.265 $\int (a\sec(x)+b\tan(x))^3\,dx$

Optimal. Leaf size=75

$$-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{4}(a-b)^2(a+2b)\log(1+\sin(x))+\frac{1}{2}ab^2\sin(x)+\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2$$

```
[Out] -1/4*(a-2*b)*(a+b)^2*ln(1-sin(x))+1/4*(a-b)^2*(a+2*b)*ln(1+sin(x))+1/2*a*b^
2*sin(x)+1/2*sec(x)^2*(b+a*sin(x))*(a+b*sin(x))^2
```

Rubi [A]

time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4476, 2747, 753, 788, 647, 31}

$$\frac{1}{2}ab^2\sin(x)+\frac{1}{4}(a+2b)(a-b)^2\log(\sin(x)+1)-\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{2}\sec^2(x)(a\sin(x)+b)(a+b\sin(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] -1/4*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]]) + ((a - b)^2*(a + 2*b)*Log[1 + S
in[x]])/4 + (a*b^2*Sin[x])/2 + (Sec[x]^2*(b + a*Sin[x])*(a + b*Sin[x])^2)/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^3\,dx &= \int \sec^3(x)(a+b\sin(x))^3\,dx\\ &= b^3\text{Subst}\left(\int \frac{(a+x)^3}{\left(b^2-x^2\right)^2}\,dx,x,b\sin(x)\right)\\ &= \frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2-\frac{1}{2}b\text{Subst}\left(\int \frac{(a+x)\left(-a^2+2b^2+ax\right)}{b^2-x^2}\right.\\ &= \frac{1}{2}ab^2\sin(x)+\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2+\frac{1}{2}b\text{Subst}\left(\int \frac{-ab^2-a(-}{b}\right.\\ &= \frac{1}{2}ab^2\sin(x)+\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2+\frac{1}{4}\left((a-2b)(a+b)^2\right)\text{Sub}\\ &= -\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x))+\frac{1}{4}(a-b)^2(a+2b)\log(1+\sin(x))+\frac{1}{2}ab^2\end{aligned}$$

Mathematica [A]
time = 0.44, size = 123, normalized size = 1.64

$$\frac{\left(a^2-b^2\right)\left((a-2b)(a+b)^2\log(1-\sin(x))-(a-b)^2(a+2b)\log(1+\sin(x))\right)+2a^4b\sec^2(x)-2a\left(a^4+2a^2b^2-3b^4\right)\sec(x)\tan(x)+\left(-8a^4b+4a^2b^3+2b^5\right)\tan^2(x)}{4\left(-a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^3,x]
```

```
[Out] ((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]] - (a - b)^2*(a + 2*b)*Log
[1 + Sin[x]]) + 2*a^4*b*Sec[x]^2 - 2*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sec[x]*Tan
[x] + (-8*a^4*b + 4*a^2*b^3 + 2*b^5)*Tan[x]^2)/(4*(-a^2 + b^2))
```

Maple [A]
time = 0.16, size = 74, normalized size = 0.99

method	result
default	$a^3\left(\frac{\sec(x)\tan(x)}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}\right)+\frac{3a^2b}{2\cos(x)^2}+3a\,b^2\left(\frac{\sin^3(x)}{2\cos(x)^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+\tan(x))}{2}\right)+b^3\left(\frac{(\tan^2(x$
risch	$-ix\,b^3+\frac{-ia^3e^{3ix}-3ia\,b^2e^{3ix}+ie^{ix}a^3+3ie^{ix}a\,b^2+6a^2b\,e^{2ix}+2b^3e^{2ix}}{(e^{2ix}+1)^2}+\frac{\ln(e^{ix}+i)a^3}{2}-\frac{3\ln(e^{ix}+i)a\,b^2}{2}+\ln\left(e^{ix}+i\right)b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(1/2*sec(x)*tan(x)+1/2*ln(sec(x)+tan(x)))+3/2*a^2*b/cos(x)^2+3*a*b^2*(1
/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x)))+b^3*(1/2*tan(x)^2+ln
(cos(x)))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 1.27

$$\frac{3}{2}a^2b\tan(x)^2-\frac{3}{4}ab^2\left(\frac{2\sin(x)}{\sin(x)^2-1}+\log(\sin(x)+1)-\log(\sin(x)-1)\right)-\frac{1}{4}a^3\left(\frac{2\sin(x)}{\sin(x)^2-1}-\log(\sin(x)+1)+\log(\sin(x)-1)\right)-\frac{1}{2}b^3\left(\frac{1}{\sin(x)^2-1}-\log(\sin(x)^2-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*b*tan(x)^2 - 3/4*a*b^2*(2*sin(x)/(sin(x)^2 - 1) + log(sin(x) + 1) -
 log(sin(x) - 1)) - 1/4*a^3*(2*sin(x)/(sin(x)^2 - 1) - log(sin(x) + 1) + lo
g(sin(x) - 1)) - 1/2*b^3*(1/(sin(x)^2 - 1) - log(sin(x)^2 - 1))
```

Fricas [A]
time = 3.06, size = 85, normalized size = 1.13

$$\frac{(a^3-3\,ab^2+2\,b^3)\cos(x)^2\log(\sin(x)+1)-(a^3-3\,ab^2-2\,b^3)\cos(x)^2\log(-\sin(x)+1)+6\,a^2b+2\,b^3+2\,(a^3+3\,ab^2)\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(x)^2*log(sin(x) + 1) - (a^3 - 3*a*b^2 - 2*
b^3)*cos(x)^2*log(-sin(x) + 1) + 6*a^2*b + 2*b^3 + 2*(a^3 + 3*a*b^2)*sin(x)
)/cos(x)^2
```

Sympy [A]
time = 2.91, size = 122, normalized size = 1.63

$$-\frac{a^3\log(\sin(x)-1)}{4}+\frac{a^3\log(\sin(x)+1)}{4}-\frac{a^3\sin(x)}{2\sin^2(x)-2}+\frac{3a^2b\sec^2(x)}{2}+\frac{3ab^2\log(\sin(x)-1)}{4}-\frac{3ab^2\log(\sin(x)+1)}{4}-\frac{3ab^2\sin(x)}{2\sin^2(x)-2}-\frac{b^3\log(\sec^2(x))}{2}+\frac{b^3\sec^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**3,x)
```

```
[Out] -a**3*log(sin(x) - 1)/4 + a**3*log(sin(x) + 1)/4 - a**3*sin(x)/(2*sin(x)**2
 - 2) + 3*a**2*b*sec(x)**2/2 + 3*a*b**2*log(sin(x) - 1)/4 - 3*a*b**2*log(si
n(x) + 1)/4 - 3*a*b**2*sin(x)/(2*sin(x)**2 - 2) - b**3*log(sec(x)**2)/2 + b
**3*sec(x)**2/2
```

Giac [A]
time = 0.41, size = 86, normalized size = 1.15

$$\frac{1}{4}\left(a^{3}-3\,ab^{2}+2\,b^{3}\right)\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\left(a^{3}-3\,ab^{2}-2\,b^{3}\right)\log\left(-\sin\left(x\right)+1\right)-\frac{b^{3}\sin\left(x\right)^{2}+a^{3}\sin\left(x\right)+3\,ab^{2}\sin\left(x\right)+3\,a^{2}b}{2\left(\sin\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^3 - 3*a*b^2 + 2*b^3)*log(sin(x) + 1) - 1/4*(a^3 - 3*a*b^2 - 2*b^3)*l
og(-sin(x) + 1) - 1/2*(b^3*sin(x)^2 + a^3*sin(x) + 3*a*b^2*sin(x) + 3*a^2*b
)/(sin(x)^2 - 1)
```

Mupad [B]
time = 2.50, size = 126, normalized size = 1.68

$$\frac{\left(a^{3}+3\,a\,b^{2}\right)\,\tan\left(\frac{x}{2}\right)^{3}+\left(6\,a^{2}\,b+2\,b^{3}\right)\,\tan\left(\frac{x}{2}\right)^{2}+\left(a^{3}+3\,a\,b^{2}\right)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{4}-2\tan\left(\frac{x}{2}\right)^{2}+1}-b^{3}\,\ln\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)\,\left(a+b\right)^{2}\left(a-2\,b\right)}{2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)\,\left(a-b\right)^{2}\left(a+2\,b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^3,x)
```

```
[Out] (tan(x/2)^2*(6*a^2*b + 2*b^3) + tan(x/2)*(3*a*b^2 + a^3) + tan(x/2)^3*(3*a*
b^2 + a^3))/(tan(x/2)^4 - 2*tan(x/2)^2 + 1) - b^3*log(tan(x/2)^2 + 1) - (lo
g(tan(x/2) - 1)*(a + b)^2*(a - 2*b))/2 + (log(tan(x/2) + 1)*(a - b)^2*(a +
2*b))/2
```

3.266 $\int (a\sec(x)+b\tan(x))^2\,dx$

Optimal. Leaf size=27

$$-b^2x+ab\cos(x)+\sec(x)(b+a\sin(x))(a+b\sin(x))$$

```
[Out] -b^2*x+a*b*cos(x)+sec(x)*(b+a*sin(x))*(a+b*sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {4476, 2770, 2718}

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^2,x]
```

```
[Out] -(b^2*x) + a*b*Cos[x] + Sec[x]*(b + a*Sin[x])*(a + b*Sin[x])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\sec(x)+b\tan(x))^2\,dx = \int \sec^2(x)(a+b\sin(x))^2\,dx$$
$$= \sec(x)(b+a\sin(x))(a+b\sin(x)) - \int \left(b^2+ab\sin(x)\right)\,dx$$
$$= -b^2x+\sec(x)(b+a\sin(x))(a+b\sin(x)) - (ab)\int \sin(x)\,dx$$
$$= -b^2x+ab\cos(x)+\sec(x)(b+a\sin(x))(a+b\sin(x))$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.93

$$-b^2\text{ArcTan}(\tan(x))+2ab\sec(x)+\left(a^2+b^2\right)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^2,x]
[Out] -(b^2*ArcTan[Tan[x]]) + 2*a*b*Sec[x] + (a^2 + b^2)*Tan[x]
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.96

method	result	size
default	$a^2\tan(x)+\frac{2ab}{\cos(x)}+b^2(\tan(x)-x)$	26
risch	$-b^2x+\frac{4ab\,e^{ix}+2ia^2+2ib^2}{e^{2ix}+1}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)+b*tan(x))^2,x,method=_RETURNVERBOSE)
[Out] a^2*tan(x)+2*a*b/cos(x)+b^2*(tan(x)-x)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.96

$$-b^2(x-\tan(x))+a^2\tan(x)+\frac{2\,ab}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
[Out] -b^2*(x - tan(x)) + a^2*tan(x) + 2*a*b/cos(x)
```

Fricas [A]
time = 2.40, size = 29, normalized size = 1.07

$$-\frac{b^2x\cos\left(x\right)-2\,ab-\left(a^2+b^2\right)\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
[Out] -(b^2*x*cos(x) - 2*a*b - (a^2 + b^2)*sin(x))/cos(x)
```

Sympy [A]
time = 0.71, size = 22, normalized size = 0.81

$$a^2\tan\left(x\right)+2ab\sec\left(x\right)+b^2(-x+\tan\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))**2,x)
[Out] a**2*tan(x) + 2*a*b*sec(x) + b**2*(-x + tan(x))
```

Giac [A]
time = 0.41, size = 40, normalized size = 1.48

$$-b^2x-\frac{2\left(a^2\tan\left(\frac{1}{2}\,x\right)+b^2\tan\left(\frac{1}{2}\,x\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,x\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)+b*tan(x))^2,x, algorithm="giac")
[Out] -b^2*x - 2*(a^2*tan(1/2*x) + b^2*tan(1/2*x) + 2*a*b)/(tan(1/2*x)^2 - 1)
```

Mupad [B]
time = 2.38, size = 40, normalized size = 1.48

$$-\frac{4\,a\,b+\tan\left(\frac{x}{2}\right)\,\left(2\,a^2+2\,b^2\right)}{\tan\left(\frac{x}{2}\right)^2-1}-b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(x) + a/cos(x))^2,x)
[Out] - (4*a*b + tan(x/2)*(2*a^2 + 2*b^2))/(tan(x/2)^2 - 1) - b^2*x
```

3.267 $\int (a\sec(x) + b\tan(x))\,dx$

Optimal. Leaf size=12

$$a\tanh^{-1}(\sin(x)) - b\log(\cos(x))$$

```
[Out] a*arctanh(sin(x))-b*ln(cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3855, 3556}

$$a\tanh^{-1}(\sin(x)) - b\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Sec[x] + b*Tan[x],x]
```

```
[Out] a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sec(x) + b\tan(x))\,dx &= a\int \sec(x)\,dx + b\int \tan(x)\,dx\\ &= a\tanh^{-1}(\sin(x)) - b\log(\cos(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.
time = 0.01, size = 42, normalized size = 3.50

$$-b\log(\cos(x)) - a\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + a\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sec[x] + b*Tan[x],x]
```

```
[Out] -(b*Log[Cos[x]]) - a*Log[Cos[x/2] - Sin[x/2]] + a*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.08, size = 16, normalized size = 1.33

method	result	size
default	$a\ln(\sec(x)+\tan(x))-b\ln(\cos(x))$	16
norman	$a\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-a\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{b\ln(\tan^2(x)+1)}{2}$	31
risch	$a\ln(\mathrm{e}^{ix}+i)-a\ln(\mathrm{e}^{ix}-i)+ibx-b\ln(\mathrm{e}^{2ix}+1)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sec(x)+b*tan(x),x,method=_RETURNVERBOSE)
[Out] a*ln(sec(x)+tan(x))-b*ln(cos(x))
```

Maxima [A]
time = 0.27, size = 14, normalized size = 1.17

$$a\log(\sec(x)+\tan(x))+b\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="maxima")
[Out] a*log(sec(x) + tan(x)) + b*log(sec(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.
time = 3.05, size = 25, normalized size = 2.08

$$\frac{1}{2}(a-b)\log(\sin(x)+1)-\frac{1}{2}(a+b)\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="fricas")
[Out] 1/2*(a - b)*log(sin(x) + 1) - 1/2*(a + b)*log(-sin(x) + 1)
```

Sympy [A]
time = 0.03, size = 24, normalized size = 2.00

$$a\left(-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}\right)-b\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x)
```

```
[Out] a*(-log(sin(x) - 1)/2 + log(sin(x) + 1)/2) - b*log(cos(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.
time = 0.41, size = 34, normalized size = 2.83

$$\frac{1}{4}a\left(\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)\right)-b\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sec(x)+b*tan(x),x, algorithm="giac")
```

```
[Out] 1/4*a*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) - 2))) -
 b*log(abs(cos(x)))
```

Mupad [B]
time = 2.49, size = 37, normalized size = 3.08

$$b\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)\,(a+b)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)\,(a-b)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*tan(x) + a/cos(x),x)
```

```
[Out] b*log(tan(x/2)^2 + 1) - log(tan(x/2) - 1)*(a + b) + log(tan(x/2) + 1)*(a -
b)
```

3.268 $\int \frac{1}{a\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sin(x))}{b}$$

```
[Out] ln(a+b*sin(x))/b
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3238, 2747, 31}

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\sec(x)+b\tan(x)}\,dx &= \int \frac{\cos(x)}{a+b\sin(x)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sin(x)\right)}{b}\\ &= \frac{\log(a+b\sin(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a + b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Maple [A]
time = 0.16, size = 12, normalized size = 1.09

method	result	size
default	$\frac{\ln(a+b\sin(x))}{b}$	12
risch	$-\frac{ix}{b}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{b}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*sin(x))/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(11) = 22$.
time = 0.49, size = 50, normalized size = 4.55

$$\frac{\log\left(a + \frac{2b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b} - \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b - log(sin(x)
^2/(cos(x) + 1)^2 + 1)/b
```

Fricas [A]
time = 2.60, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sin\left(x\right) + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] log(b*sin(x) + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(8) = 16$.
time = 0.23, size = 65, normalized size = 5.91

$$\begin{cases} \tilde{\infty}\left(-\frac{\log\left(\tan^2\left(x\right)+1\right)}{2} + \log\left(\tan\left(x\right)\right)\right) & \text{for } a = 0 \wedge b = 0 \\ \frac{\tan\left(x\right)}{a\sec\left(x\right)} & \text{for } b = 0 \\ \frac{-\frac{\log\left(\tan^2\left(x\right)+1\right)}{2}+\log\left(\tan\left(x\right)\right)}{b} & \text{for } a = 0 \\ -\frac{\log\left(\tan^2\left(x\right)+1\right)}{2b} + \frac{\log\left(\sec\left(x\right)+\frac{b\tan\left(x\right)}{a}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x)**2 + 1)/2 + log(tan(x))), Eq(a, 0) & Eq(b, 0)),
 (tan(x)/(a*sec(x)), Eq(b, 0)), ((-log(tan(x)**2 + 1)/2 + log(tan(x)))/b, E
q(a, 0)), (-log(tan(x)**2 + 1)/(2*b) + log(sec(x) + b*tan(x)/a)/b, True))
```

Giac [A]
time = 0.42, size = 12, normalized size = 1.09

$$\frac{\log\left(\left|b\sin\left(x\right) + a\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b
```

Mupad [B]
time = 3.79, size = 55, normalized size = 5.00

$$\frac{2\,\mathrm{atanh}\left(\frac{b\,\left(2\,a^3\,\sin(x)+\frac{5\,a^2\,b}{2}-b^3-\frac{a^2\,b\,\cos(2\,x)}{2}\right)}{{\left(2\,a^2+\sin(x)\,a\,b-b^2\right)}^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x)),x)
```

```
[Out] (2*atanh((b*(2*a^3*sin(x) + (5*a^2*b)/2 - b^3 - (a^2*b*cos(2*x))/2))/(2*a^2
 - b^2 + a*b*sin(x))^2))/b
```

3.269 $\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$

Optimal. Leaf size=66

$$-\frac{x}{b^2}+\frac{2a\text{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}$$

```
[Out] -x/b^2-cos(x)/b/(a+b*sin(x))+2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b
^2/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4476, 2772, 2814, 2739, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-2),x]
```

```
[Out] -(x/b^2) + (2*a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b
^2]) - Cos[x]/(b*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx &= \int \frac{\cos^2(x)}{(a+b\sin(x))^2}\,dx\\
&= -\frac{\cos(x)}{b(a+b\sin(x))}-\frac{\int \frac{\sin(x)}{a+b\sin(x)}\,dx}{b}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}+\frac{a\int \frac{1}{a+b\sin(x)}\,dx}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}+\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{x}{b^2}-\frac{\cos(x)}{b(a+b\sin(x))}-\frac{(4a)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\frac{x}{2}\right)\right)}{b^2}\\
&= -\frac{x}{b^2}+\frac{2a\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 344 vs. 2(66) = 132.
time = 1.49, size = 344, normalized size = 5.21

$$
\frac{\cos(x)(1+\sin(x))\left(2a(a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(x))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(x))}{a+b}}}\right)\sqrt{1-\sin(x)}(a+b\sin(x))+\sqrt{a+b}\left(-2a\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{\frac{b(1+\sin(x))}{-a+b}}}{\sqrt{\frac{b-b\sin(x)}{a+b}}}\right)\sqrt{1-\sin(x)}(a+b\sin(x))-(-a+b)\sqrt{\frac{b-b\sin(x)}{a+b}}\left(\sqrt{a-b}(a+b)\sqrt{1-\sin(x)}\sqrt{-\frac{b(1+\sin(x))}{a-b}}+2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(x))}{a-b}}}{\sqrt{2}\sqrt{b}}\right)(a+b\sin(x))\right)\right)\right)}{(a-b)^{5/2}(a+b)^{3/2}\sqrt{1-\sin(x)}\left(-\frac{b(1+\sin(x))}{a-b}\right)^{3/2}\sqrt{\frac{b-b\sin(x)}{a+b}}(a+b\sin(x))}
$$

Antiderivative was successfully verified.

[In] Integrate[(a*Sec[x] + b*Tan[x])^(-2),x]

[Out] (Cos[x]*(1 + Sin[x])*(2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[x]))/(a + b))])]*Sqrt[1 - Sin[x]]*(a + b*Sin[x]) + Sqrt[a + b]*(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1 + Sin[x]))/(-a + b)]/Sqrt[(b - b*Sin[x])/(a + b)]]*Sqrt[1 - Sin[x]]*(a + b*Sin[x]) - (-a + b)*Sqrt[(b - b*Sin[x])/(a + b)]*(Sqrt[a - b]*(a + b)*Sqrt[1 - Sin[x]]*Sqrt[-((b*(1 + Sin[x]))/(a - b))] + 2*Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[x])))))/((a - b)^(5/2)*(a + b)^(3/2)*Sqrt[1 - Sin[x]]*(-((b*(1 + Sin[x]))/(a - b)))^(3/2)*Sqrt[(b - b*Sin[x])/(a + b)]*(a + b*Sin[x]))

Maple [A]
time = 0.35, size = 92, normalized size = 1.39

method	result	size
default	$\frac{\frac{2\left(-\frac{b^{2}\tan\left(\frac{x}{2}\right)}{a}-b\right)}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}}{b^{2}}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^{2}}$	92
risch	$-\frac{x}{b^{2}}-\frac{2\left(ib+a\,\mathrm{e}^{ix}\right)}{b^{2}\left(b\,\mathrm{e}^{2ix}-b+2ia\,\mathrm{e}^{ix}\right)}-\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{a\ln\left(\mathrm{e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}$	172

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sec(x)+b*tan(x))^2,x,method=_RETURNVERBOSE)

[Out] 2/b^2*((-b^2/a*tan(1/2*x)-b)/(a*tan(1/2*x)^2+2*b*tan(1/2*x)+a)+a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))-2/b^2*arctan(tan(1/2*x))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(60) = 120$.
time = 3.26, size = 308, normalized size = 4.67

$$\left[-\frac{2\,(a^2b-b^3)x\sin(x)+(ab\sin(x)+a^2)\sqrt{-a^2+b^2}\,\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right)+2\,(a^3-ab^2)x+2\,(a^2b-b^3)\cos(x)}{2\,(a^3b^2-ab^4+(a^2b^3-b^5)\sin(x))},\ -\frac{(a^2b-b^3)x\sin(x)+(ab\sin(x)+a^2)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+(a^3-ab^2)x+(a^2b-b^3)\cos(x)}{a^3b^2-ab^4+(a^2b^3-b^5)\sin(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log((
(2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*
cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a
^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*
sin(x)), -((a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(a^2 - b^2)*arct
an(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^3 - a*b^2)*x + (a^2*b - b
^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec(x)+b\tan(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**2,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-2), x)
```

Giac [A]
time = 0.40, size = 94, normalized size = 1.42

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}\,b^2}-\frac{x}{b^2}-\frac{2\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))*a/(sqrt(a^2 - b^2)*b^2) - x/b^2 - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)
^2 + 2*b*tan(1/2*x) + a)*a*b)
```

Mupad [B]
time = 2.81, size = 604, normalized size = 9.15

$$-\frac{x}{b^2}-\frac{\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{2}{b}}{a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{a\,\mathrm{atan}\left(\frac{\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}+\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)+\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)1i}{b^2\sqrt{b^2-a^2}}+\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}-\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)-\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)1i}{b^2\sqrt{b^2-a^2}}}{\frac{128\,a^2\tan\left(\frac{x}{2}\right)}{b^3}+\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}+\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)+\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}-\frac{a\left(\frac{32\,a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2\,a\,b^3-2\,a^3\,b\right)}{b^3}-\frac{a\left(32\,a\,b^2+64\,a^2\,b\tan\left(\frac{x}{2}\right)-\frac{a\left(32\,a^2\,b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3\,a\,b^7-2\,a^3\,b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}}\right)2i}{b^2\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^2,x)
```

```
[Out] - x/b^2 - ((2*tan(x/2))/a + 2/b)/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (a*ata
n(((a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 +
64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^
3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 - a^2)
^(1/2)) + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a
*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b
^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2
 - a^2)^(1/2)))/((128*a^2*tan(x/2))/b^3 + (a*((32*a^2)/b + (32*tan(x/2)*(2*
a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 +
 (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(
b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) - (a*((32*a^2)/b + (32*tan(x/2)
*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b
^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b
^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))*2i)/(b^2*(b^2 - a^2)^(1/2
))
```

3.270 $\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx$

Optimal. Leaf size=51

$$-\frac{\log(a+b\sin(x))}{b^3}+\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}$$

```
[Out] -ln(a+b*sin(x))/b^3+1/2*(a^2-b^2)/b^3/(a+b*sin(x))^2-2*a/b^3/(a+b*sin(x))
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4476, 2747, 711}

$$\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}-\frac{\log(a+b\sin(x))}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -(Log[a + b*Sin[x]]/b^3) + (a^2 - b^2)/(2*b^3*(a + b*Sin[x])^2) - (2*a)/(b^
3*(a + b*Sin[x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \int \frac{\cos^3(x)}{(a+b\sin(x))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{(a+x)^3}\,dx, x, b\sin(x)\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{-a^2+b^2}{(a+x)^3}+\frac{2a}{(a+x)^2}\right)\,dx, x, b\sin(x)\right)}{b^3}$$
$$= -\frac{\log(a+b\sin(x))}{b^3}+\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}$$

Mathematica [A]
time = 0.14, size = 67, normalized size = 1.31

$$-\frac{3a^2+b^2+2a^2\log(a+b\sin(x))+4ab(1+\log(a+b\sin(x)))\sin(x)+2b^2\log(a+b\sin(x))\sin^2(x)}{2b^3(a+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-3),x]
```

```
[Out] -1/2*(3*a^2 + b^2 + 2*a^2*Log[a + b*Sin[x]] + 4*a*b*(1 + Log[a + b*Sin[x]])
*Sin[x] + 2*b^2*Log[a + b*Sin[x]]*Sin[x]^2)/(b^3*(a + b*Sin[x])^2)
```

Maple [A]
time = 0.33, size = 50, normalized size = 0.98

method	result	size
default	$-\frac{\ln(a+b\sin(x))}{b^3}-\frac{2a}{b^3(a+b\sin(x))}-\frac{-a^2+b^2}{2b^3(a+b\sin(x))^2}$	50
risch	$\frac{ix}{b^3}-\frac{2i(3ia^2e^{2ix}+ib^2e^{2ix}+2bae^{3ix}-2abe^{ix})}{(be^{2ix}-b+2iae^{ix})^2b^3}-\frac{\ln\left(e^{2ix}+\frac{2iae^{ix}}{b}-1\right)}{b^3}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+b*sin(x))/b^3-2*a/b^3/(a+b*sin(x))-1/2*(-a^2+b^2)/b^3/(a+b*sin(x))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(49) = 98$.
time = 0.50, size = 201, normalized size = 3.94

$$\frac{2\left(\frac{(a^3+ab^2)\sin(x)}{\cos(x)+1}+\frac{(3a^2b+b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^3+ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(x)}{\cos(x)+1}+\frac{4a^3b^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^4b^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{2(a^4b^2+2a^2b^4)\sin(x)^2}{(\cos(x)+1)^2}}-\frac{\log\left(a+\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

```
[Out] 2*((a^3 + a*b^2)*sin(x)/(cos(x) + 1) + (3*a^2*b + b^3)*sin(x)^2/(cos(x) + 1
)^2 + (a^3 + a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(x)/(c
os(x) + 1) + 4*a^3*b^3*sin(x)^3/(cos(x) + 1)^3 + a^4*b^2*sin(x)^4/(cos(x) +
 1)^4 + 2*(a^4*b^2 + 2*a^2*b^4)*sin(x)^2/(cos(x) + 1)^2) - log(a + 2*b*sin(
x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b^3 + log(sin(x)^2/(cos(x) + 1
)^2 + 1)/b^3
```

Fricas [A]
time = 1.99, size = 83, normalized size = 1.63

$$\frac{4\,ab\sin\left(x\right)+3\,a^2+b^2-2\,\left(b^2\cos\left(x\right)^2-2\,ab\sin\left(x\right)-a^2-b^2\right)\log\left(b\sin\left(x\right)+a\right)}{2\,\left(b^5\cos\left(x\right)^2-2\,ab^4\sin\left(x\right)-a^2b^3-b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*sin(x) + 3*a^2 + b^2 - 2*(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^
2)*log(b*sin(x) + a))/(b^5*cos(x)^2 - 2*a*b^4*sin(x) - a^2*b^3 - b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(46) = 92$.
time = 1.26, size = 554, normalized size = 10.86

$$\begin{cases} \tilde{\infty}\left(\frac{\log\left(\tan^2(x)+1\right)}{2}-\log\left(\tan(x)\right)-\frac{1}{2\tan^2(x)}\right) & \text{for } a=0\wedge b=0 \\ \frac{\frac{\log\left(\tan^2(x)+1\right)}{2}-\log\left(\tan(x)\right)-\frac{1}{2\tan^2(x)}}{b^3} & \text{for } a=0 \\ \frac{\frac{2\tan^3(x)}{3\sec^3(x)}+\frac{\tan(x)}{\sec^3(x)}}{a^3} & \text{for } b=0 \\ \frac{a^2\log\left(\tan^2(x)+1\right)\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}-\frac{2a^2\log\left(\sec(x)+\frac{b\tan(x)}{a}\right)\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}-\frac{a^2\sec^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}+\frac{2ab\log\left(\tan^2(x)+1\right)\tan(x)\sec(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}-\frac{4ab\log\left(\sec(x)+\frac{b\tan(x)}{a}\right)\tan(x)\sec(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}+\frac{b^2\log\left(\tan^2(x)+1\right)\tan^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}-\frac{2b^2\log\left(\sec(x)+\frac{b\tan(x)}{a}\right)\tan^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}+\frac{b^2\tan^2(x)}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)}-\frac{b^2}{2a^2b^3\sec^2(x)+4ab^4\tan(x)\sec(x)+2b^5\tan^2(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**3,x)
```

```
[Out] Piecewise((zoo*(log(tan(x)**2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)**2)), Eq(a
, 0) & Eq(b, 0)), ((log(tan(x)**2 + 1)/2 - log(tan(x)) - 1/(2*tan(x)**2))/b
**3, Eq(a, 0)), ((2*tan(x)**3/(3*sec(x)**3) + tan(x)/sec(x)**3)/a**3, Eq(b,
 0)), (a**2*log(tan(x)**2 + 1)*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*
tan(x)*sec(x) + 2*b**5*tan(x)**2) - 2*a**2*log(sec(x) + b*tan(x)/a)*sec(x)*
*2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - a*
*2*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x
)**2) + 2*a*b*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a
*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) - 4*a*b*log(sec(x) + b*tan(x)/a)*ta
n(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)
**2) + b**2*log(tan(x)**2 + 1)*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*
tan(x)*sec(x) + 2*b**5*tan(x)**2) - 2*b**2*log(sec(x) + b*tan(x)/a)*tan(x)*
*2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b*
```

```
*2*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x
)**2) - b**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x
)**2), True))
```

Giac [A]
time = 0.44, size = 43, normalized size = 0.84

$$-\frac{\log\left(\left|b\sin\left(x\right)+a\right|\right)}{b^3}+\frac{3\,b\sin\left(x\right)^2+2\,a\sin\left(x\right)-b}{2\left(b\sin\left(x\right)+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*sin(x) + a))/b^3 + 1/2*(3*b*sin(x)^2 + 2*a*sin(x) - b)/((b*sin(x
) + a)^2*b^2)
```

Mupad [B]
time = 2.70, size = 106, normalized size = 2.08

$$\frac{2\,a^3\,b\,\sin\left(x\right)+3\,a^2\,b^2\sin\left(x\right)^2+2\,a\,b^3\,\sin\left(x\right)+b^4\sin\left(x\right)^2}{2\,a^4\,b^3+4\,a^3\,b^4\,\sin\left(x\right)+2\,a^2\,b^5\sin\left(x\right)^2}-\frac{2\,\mathrm{atanh}\left(\frac{b^2+a\,\sin\left(x\right)\,b}{2\,a^2+\sin\left(x\right)\,a\,b-b^2}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^3,x)
```

```
[Out] (b^4*sin(x)^2 + 3*a^2*b^2*sin(x)^2 + 2*a*b^3*sin(x) + 2*a^3*b*sin(x))/(2*a^
4*b^3 + 2*a^2*b^5*sin(x)^2 + 4*a^3*b^4*sin(x)) - (2*atanh((b^2 + a*b*sin(x)
)/(2*a^2 - b^2 + a*b*sin(x))))/b^3
```

3.271 $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

Optimal. Leaf size=156

$$\frac{x}{b^4}-\frac{a(2a^2-3b^2)\,\text{ArcTan}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}+\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\cos(x)\left(2(a^2-b^2)\right.}{2b^3\left(a^2-b^2\right)(a\ \ldots}$$

```
[Out] x/b^4-a*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^4/(a^2-b^2
)^(3/2)-1/3*cos(x)^3/b/(a+b*sin(x))^3+1/2*a*cos(x)^3/b/(a^2-b^2)/(a+b*sin(x
))^2+1/2*cos(x)*(2*a^2-2*b^2+a*b*sin(x))/b^3/(a^2-b^2)/(a+b*sin(x))
```

Rubi [A]

time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4476, 2772, 2943, 2942, 2814, 2739, 632, 210}

$$-\frac{a(2a^2-3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}}+\frac{a\cos^3(x)}{2b\left(a^2-b^2\right)(a+b\sin(x))^2}+\frac{\cos(x)\left(2(a^2-b^2)+ab\sin(x)\right)}{2b^3\left(a^2-b^2\right)(a+b\sin(x))}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] x/b^4 - (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^4*(
a^2 - b^2)^(3/2)) - Cos[x]^3/(3*b*(a + b*Sin[x])^3) + (a*Cos[x]^3)/(2*b*(a^
2 - b^2)*(a + b*Sin[x])^2) + (Cos[x]*(2*(a^2 - b^2) + a*b*Sin[x]))/(2*b^3*(
a^2 - b^2)*(a + b*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \int \frac{\cos^4(x)}{(a+b\sin(x))^4}\,dx$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} - \frac{\int \frac{\cos^2(x)\sin(x)}{(a+b\sin(x))^3}\,dx}{b}$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\int \frac{\cos^2(x)(2b+a\sin(x))}{(a+b\sin(x))^2}\,dx}{2b\,(a^2-b^2)}$$

$$= -\frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\cos(x)\,(2(a^2-b^2)+ab\,\mathrm{si}}{2b^3\,(a^2-b^2)\,(a+b\sin(}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\cos(x)\,(2(a^2-b^2)+}{2b^3\,(a^2-b^2)\,(a+b}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\cos(x)\,(2(a^2-b^2)+}{2b^3\,(a^2-b^2)\,(a+b}$$

$$= \frac{x}{b^4} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b\,(a^2-b^2)\,(a+b\sin(x))^2} + \frac{\cos(x)\,(2(a^2-b^2)+}{2b^3\,(a^2-b^2)\,(a+b}$$

$$= \frac{x}{b^4} - \frac{a(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4\,(a^2-b^2)^{3/2}} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3}{2b\,(a^2-b^2)\,(a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2661 vs. $2(156) = 312$.
time = 6.24, size = 2661, normalized size = 17.06

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-4),x]
```

```
[Out] (Sec[x]*(a + b*Sin[x])^4*(-1/3*(b*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)
*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a
*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^3) - ((a*b^3*(-(b/(a - b)) - (b*S
in[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(2*(a^2 - b^2
)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x]
)^2) - (-((((-3*a^2*b^5)/((a - b)^2*(a + b)^2) + (2*b^5*(3*a^2 - 2*b^2))/((
a - b)^2*(a + b)^2))*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a + b) -
 (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) +
 b^2/(a + b))*(a + b*Sin[x]))) - ((16*Sqrt[2]*b^6*(3*a^2 - 4*b^2)*(-(b/(a -
 b)) - (b*Sin[x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 +
```

```
((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((5*(1/(2*(1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b
/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*(((a - b)*(-(b/(
a - b)) - (b*Sin[x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) - (b*Sin[x])/(a
 - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a -
 b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x
])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b))
)/(2*b)])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[x])/(a - b))^3*(1 + ((a -
b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2)))/(5*(a - b)^2*(a + b)^4*
Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) + ((-((a*b^7*(6*a^2 - 7
*b^2))/((a - b)^3*(a + b)^3)) + (4*a*b^7*(3*a^2 - 4*b^2))/((a - b)^3*(a + b
)^3))*((4*Sqrt[2]*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(3/2)*Sqrt[b/(a + b)
- (b*Sin[x])/(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2
*b))^(5/2)*((3/(4*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))
^2) + (1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/2 + (
3*b^2*(((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/b - (Sqrt[2]*Sqrt[a -
b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*S
qrt[b])]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b
)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)])))/(8*(a - b)^2*(-(b/(a - b))
 - (b*Sin[x])/(a - b))^2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))
/(2*b))^2)))/(3*(a + b)*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])
 - ((-((a*b)/(a - b)) + b^2/(a - b))*(-(((-((a*b)/(a - b)) + b^2/(a - b))*(
-(((-((a*b)/(a + b)) - b^2/(a + b))*((2*Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Sq
rt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[a + b]*Sqrt[b/(a + b) - (b*Sin
[x])/(a + b)])])/(b*Sqrt[a + b]) - (2*Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*
ArcTanh[(Sqrt[-((a*b)/(a + b)) - b^2/(a + b)]*Sqrt[-(b/(a - b)) - (b*Sin[x]
)/(a - b)])/(Sqrt[-((a*b)/(a - b)) + b^2/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x
])/(a + b)])])/(b*Sqrt[-((a*b)/(a - b)) + b^2/(a - b)])))/b) + (2*Sqrt[2]*(
a - b)*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x])/
(a + b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(3/2)*((
Sqrt[b]*ArcSinh[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt
[2]*Sqrt[b])])/(Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]
*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(3/2)) + 1/(2*(1
 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b)))))/(b*(a + b)*Sqrt[
((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b])))/b) + (4*Sqrt[2]*Sqrt[-(b/(
a - b)) - (b*Sin[x])/(a - b)]*Sqrt[b/(a + b) - (b*Sin[x])/(a + b)]*(1 + ((a
 - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((3*Sqrt[b]*ArcSinh
[(Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])])/
(4*Sqrt[2]*Sqrt[a - b]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)]*(1 + ((a - b
)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)) + (3/(2*(1 + ((a - b)*
(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + ((a - b)*(-(b/(a - b)
) - (b*Sin[x])/(a - b)))/(2*b))^(-1))/4))/((a + b)*Sqrt[((a + b)*(b/(a + b)
 - (b*Sin[x])/(a + b)))/b])))/b))/b)/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/
(a + b) + b^2/(a + b))))/(2*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) +
b^2/(a + b))))/(3*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b
```

```
)))))/((1 - (a + b*Sin[x])/(a - b))^(3/2)*(1 - (a + b*Sin[x])/(a + b))^(3/2
)*(a*Sec[x] + b*Tan[x])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(143) = 286$.
time = 0.65, size = 358, normalized size = 2.29

method	result
default	$-\frac{2\left(\frac{-\frac{b^2\left(a^4-2a^2b^2+2b^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)}{2a\left(a^2-b^2\right)}-\frac{b\left(2a^6+3a^4b^2-4a^2b^4+4b^6\right)\left(\tan^4\left(\frac{x}{2}\right)\right)}{2\left(a^2-b^2\right)a^2}-\frac{b^2\left(18a^6-3a^4b^2-4a^2b^4+4b^6\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{3a^3\left(a^2-b^2\right)}-\frac{b\left(2a^6+8a^4b^2-7\right.}{(a}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)^3}\right.}{b^4}$
risch	$\frac{x}{b^4}-\frac{i(-54ib\,a^4e^{4ix}+27ib^3a^2e^{4ix}+12ib^5e^{4ix}-18b^2a^3e^{5ix}+15b^4a\,e^{5ix}+78ia^4b\,e^{2ix}-36ia^2b^3e^{2ix}-12ib^5e^{2ix}+44a^5e^{3ix}+34a^3b^2e^{3i}}{3(-ib\,e^{2ix}+ib+2a\,e^{ix})^3(a^2-b^2)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*((-1/2*b^2*(a^4-2*a^2*b^2+2*b^4)/a/(a^2-b^2)*tan(1/2*x)^5-1/2*b*(2*a
^6+3*a^4*b^2-4*a^2*b^4+4*b^6)/(a^2-b^2)/a^2*tan(1/2*x)^4-1/3/a^3*b^2*(18*a^
6-3*a^4*b^2-4*a^2*b^4+4*b^6)/(a^2-b^2)*tan(1/2*x)^3-b*(2*a^6+8*a^4*b^2-7*a^
2*b^4+2*b^6)/(a^2-b^2)/a^2*tan(1/2*x)^2-1/2*b^2*(11*a^4-8*a^2*b^2+2*b^4)/a/
(a^2-b^2)*tan(1/2*x)-1/6*(6*a^4-5*a^2*b^2+2*b^4)*b/(a^2-b^2))/(a*tan(1/2*x)
^2+2*b*tan(1/2*x)+a)^3+1/2*a*(2*a^2-3*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*
tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))+2/b^4*arctan(tan(1/2*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(143) = 286$.
time = 2.37, size = 931, normalized size = 5.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + 2*(11*a^4*b^3 - 19*a^
2*b^5 + 8*b^7)*cos(x)^3 + 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 -
 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^
5)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*
sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*
cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a
*b^6)*x - 12*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 6*(2*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a^5*b^2 -
 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*
b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 +
 a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b^11)*cos(x)^2)*sin(x)), -1/6*(18*
(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + (11*a^4*b^3 - 19*a^2*b^5 + 8*b^7
)*cos(x)^3 - 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 - 3*a^2*b^4)*c
os(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b^5)*cos(x)^2)*
sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) -
6*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 6*(a^6*b - 2*a^2*b^5 + b^7)*cos
(x) + 3*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3
+ a^2*b^5 + b^7)*x - (5*a^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7
*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*co
s(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 + a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b
^11)*cos(x)^2)*sin(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec\left(x\right) + b\tan\left(x\right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**4,x)
```

```
[Out] Integral((a*sec(x) + b*tan(x))**(-4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(143) = 286.
time = 0.42, size = 369, normalized size = 2.37

$$-\frac{\left(2\,a^{3} - 3\,ab^{2}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,x\right) + b}{\sqrt{a^{2} - b^{2}}}\right)\right)}{\left(a^{2}b^{4} - b^{6}\right)\sqrt{a^{2} - b^{2}}} + \frac{3\,a^{6}b\tan\left(\frac{1}{2}\,x\right)^{5} - 6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right)^{5} + 6\,a^{2}b^{5}\tan\left(\frac{1}{2}\,x\right)^{5} + 6\,a^{7}\tan\left(\frac{1}{2}\,x\right)^{4} + 9\,a^{5}b^{2}\tan\left(\frac{1}{2}\,x\right)^{4} - 12\,a^{3}b^{4}\tan\left(\frac{1}{2}\,x\right)^{4} + 12\,ab^{6}\tan\left(\frac{1}{2}\,x\right)^{4} + 36\,a^{6}b\tan\left(\frac{1}{2}\,x\right)^{3} - 6\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right)^{3} - 8\,a^{2}b^{5}\tan\left(\frac{1}{2}\,x\right)^{3} + 8\,b^{7}\tan\left(\frac{1}{2}\,x\right)^{3} + 12\,a^{7}\tan\left(\frac{1}{2}\,x\right)^{2} + 48\,a^{5}b^{2}\tan\left(\frac{1}{2}\,x\right)^{2} - 42\,a^{3}b^{4}\tan\left(\frac{1}{2}\,x\right)^{2} + 12\,ab^{6}\tan\left(\frac{1}{2}\,x\right)^{2} + 33\,a^{6}b\tan\left(\frac{1}{2}\,x\right) - 24\,a^{4}b^{3}\tan\left(\frac{1}{2}\,x\right) + 6\,a^{2}b^{5}\tan\left(\frac{1}{2}\,x\right) + 6\,a^{7} - 5\,a^{5}b^{2} + 2\,a^{3}b^{4}}{3\,\left(a^{5}b^{3} - a^{3}b^{5}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2} + 2\,b\tan\left(\frac{1}{2}\,x\right) + a\right)^{3}} + \frac{x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

```
[Out] -(2*a^3 - 3*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
+ b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/3*(3*a^6*b*tan
(1/2*x)^5 - 6*a^4*b^3*tan(1/2*x)^5 + 6*a^2*b^5*tan(1/2*x)^5 + 6*a^7*tan(1/2
*x)^4 + 9*a^5*b^2*tan(1/2*x)^4 - 12*a^3*b^4*tan(1/2*x)^4 + 12*a*b^6*tan(1/2
*x)^4 + 36*a^6*b*tan(1/2*x)^3 - 6*a^4*b^3*tan(1/2*x)^3 - 8*a^2*b^5*tan(1/2*
x)^3 + 8*b^7*tan(1/2*x)^3 + 12*a^7*tan(1/2*x)^2 + 48*a^5*b^2*tan(1/2*x)^2 -
 42*a^3*b^4*tan(1/2*x)^2 + 12*a*b^6*tan(1/2*x)^2 + 33*a^6*b*tan(1/2*x) - 24
*a^4*b^3*tan(1/2*x) + 6*a^2*b^5*tan(1/2*x) + 6*a^7 - 5*a^5*b^2 + 2*a^3*b^4)
/((a^5*b^3 - a^3*b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^3) + x/b^4
```

Mupad [B]
time = 6.37, size = 2782, normalized size = 17.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^4,x)
```

```
[Out] ((6*a^4 + 2*b^4 - 5*a^2*b^2)/(3*b^3*(a^2 - b^2)) + (tan(x/2)*(11*a^4 + 2*b^
4 - 8*a^2*b^2))/(a*b^2*(a^2 - b^2)) + (tan(x/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2)
)/(a*b^2*(a^2 - b^2)) + (tan(x/2)^4*(2*a^6 + 4*b^6 - 4*a^2*b^4 + 3*a^4*b^2)
)/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^2*(2*a^6 + 2*b^6 - 7*a^2*b^4 + 8*a^4*
b^2))/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^3*(3*a^2 + 2*b^2)*(6*a^4 + 2*b^4
- 5*a^2*b^2))/(3*a^3*b^2*(a^2 - b^2)))/(tan(x/2)^2*(12*a*b^2 + 3*a^3) + tan
(x/2)^4*(12*a*b^2 + 3*a^3) + tan(x/2)^3*(12*a^2*b + 8*b^3) + a^3 + a^3*tan(
x/2)^6 + 6*a^2*b*tan(x/2)^5 + 6*a^2*b*tan(x/2)) + (2*atan((48*a^3*b^3*tan(x
/2))/((176*a^3*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 -
 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*
a*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8)) - (64*a*b^5*tan(x/2))/((176*a^3*b^15
)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 - 2*a^2*b^10 + a^4*b
^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*a*b^17)/(b^12 - 2*a
^2*b^10 + a^4*b^8))))/b^4 + (a*atan(((a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)
^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4
*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13
- 2*a^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(
(8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*
tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*
b^9) - (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 +
a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11)
)/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/
2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3
*a^4*b^6 - a^6*b^4)))*1i)/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a
*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 +
4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b
^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3
```

```
*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))
/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8
*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13
+ 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*
a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2
 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*1i)/(2*(b^10 - 3*
a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/((16*(2*a^5 - 3*a^3*b^2))/(b^12 - 2*a^2*b^
10 + a^4*b^8) + (16*tan(x/2)*(8*a^6 + 12*a^2*b^4 - 20*a^4*b^2))/(b^13 - 2*a
^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4
*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x
/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*a^7*b^3))/(b^13 - 2*a^2*b^11 + a
^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^12 - 6
*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a^2
*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a*((8*(4
*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*ta
n(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*
b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(b^10 - 3
*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b
^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) + (a*(2*a^2 - 3*b^2)*(-
(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 -
 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8
*a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^10
 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13 -
2*a^2*b^11 + a^4*b^9) + (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^1
2 - 2*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b
^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(2*a^2 - 3*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^1
0 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))))/(2*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 -
a^6*b^4))))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(b^10 - 3*a^2*
b^8 + 3*a^4*b^6 - a^6*b^4)
```

3.272 $\int \frac{1}{(a\sec(x)+b\tan(x))^5} dx$

Optimal. Leaf size=101

$$\frac{\log(a+b\sin(x))}{b^5}-\frac{(a^2-b^2)^2}{4b^5(a+b\sin(x))^4}+\frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}$$

```
[Out] ln(a+b*sin(x))/b^5-1/4*(a^2-b^2)^2/b^5/(a+b*sin(x))^4+4/3*a*(a^2-b^2)/b^5/(
a+b*sin(x))^3+(-3*a^2+b^2)/b^5/(a+b*sin(x))^2+4*a/b^5/(a+b*sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4476, 2747, 711}

$$-\frac{(a^2-b^2)^2}{4b^5(a+b\sin(x))^4}+\frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}+\frac{\log(a+b\sin(x))}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] Log[a + b*Sin[x]]/b^5 - (a^2 - b^2)^2/(4*b^5*(a + b*Sin[x])^4) + (4*a*(a^2
- b^2))/(3*b^5*(a + b*Sin[x])^3) - (3*a^2 - b^2)/(b^5*(a + b*Sin[x])^2) + (
4*a)/(b^5*(a + b*Sin[x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = \int \frac{\cos^5(x)}{(a+b\sin(x))^5}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{(a+x)^5}\,dx, x, b\sin(x)\right)}{b^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(a+x)^5} - \frac{4(a^3-ab^2)}{(a+x)^4} + \frac{2(3a^2-b^2)}{(a+x)^3} - \frac{4a}{(a+x)^2} + \frac{1}{a+x}\right)\,dx, x, b\sin(x)\right)}{b^5}$$
$$= \frac{\log(a+b\sin(x))}{b^5} - \frac{(a^2-b^2)^2}{4b^5(a+b\sin(x))^4} + \frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3} - \frac{3a^2-b^2}{b^5(a+b\sin(x))^2}$$

Mathematica [A]
time = 0.27, size = 138, normalized size = 1.37

$$\frac{25a^4+2a^2b^2-3b^4+12a^4\log(a+b\sin(x))+8ab(11a^2+b^2+6a^2\log(a+b\sin(x)))\sin(x)+12b^2(9a^2+b^2+6a^2\log(a+b\sin(x)))\sin^2(x)+48ab^3(1+\log(a+b\sin(x)))\sin^3(x)+12b^4\log(a+b\sin(x))\sin^4(x)}{12b^5(a+b\sin(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x] + b*Tan[x])^(-5),x]
```

```
[Out] (25*a^4 + 2*a^2*b^2 - 3*b^4 + 12*a^4*Log[a + b*Sin[x]] + 8*a*b*(11*a^2 + b^
2 + 6*a^2*Log[a + b*Sin[x]])*Sin[x] + 12*b^2*(9*a^2 + b^2 + 6*a^2*Log[a + b
*Sin[x]])*Sin[x]^2 + 48*a*b^3*(1 + Log[a + b*Sin[x]])*Sin[x]^3 + 12*b^4*Log
[a + b*Sin[x]]*Sin[x]^4)/(12*b^5*(a + b*Sin[x])^4)
```

Maple [A]
time = 0.82, size = 102, normalized size = 1.01

method	result
default	$\frac{\ln(a+b\sin(x))}{b^5}+\frac{4a}{b^5(a+b\sin(x))}+\frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3}-\frac{a^4-2a^2b^2+b^4}{4b^5(a+b\sin(x))^4}-\frac{6a^2-2b^2}{2b^5(a+b\sin(x))^2}$
risch	$-\frac{ix}{b^5}+\frac{8iab^3e^{7ix}-\frac{176ia^3b\,e^{5ix}}{3}-\frac{88iab^3e^{5ix}}{3}-36a^2b^2e^{6ix}-4\,e^{6ix}b^4+\frac{176ia^3b\,e^{3ix}}{3}+\frac{88iab^3e^{3ix}}{3}+\frac{100a^4e^{4ix}}{3}+\frac{224a^2b^2e^{4ix}}{3}+4\,e^{4ix}b^4-8i}{(-ib\,e^{2ix}+ib+2a\,e^{ix})^4b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)+b*tan(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*sin(x))/b^5+4*a/b^5/(a+b*sin(x))+4/3*a*(a^2-b^2)/b^5/(a+b*sin(x))^3-
1/4*(a^4-2*a^2*b^2+b^4)/b^5/(a+b*sin(x))^4-1/2*(6*a^2-2*b^2)/b^5/(a+b*sin(x
))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(97) = 194.
time = 0.52, size = 483, normalized size = 4.78

$$-\frac{2\left(\frac{3(a^7-a^3b^4)\sin(x)}{\cos(x)+1}+\frac{3(7a^6b-3a^2b^5)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(9a^7+52a^5b^2-a^3b^4-12ab^6)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2(21a^6b+25a^4b^3-7a^2b^5-3b^7)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(9a^7+52a^5b^2-a^3b^4-12ab^6)\sin(x)^5}{(\cos(x)+1)^5}+\frac{3(7a^6b-3a^2b^5)\sin(x)^6}{(\cos(x)+1)^6}+\frac{3(a^7-a^3b^4)\sin(x)^7}{(\cos(x)+1)^7}\right)}{3\left(a^8b^4+\frac{8a^7b^5\sin(x)}{\cos(x)+1}+\frac{8a^7b^5\sin(x)^7}{(\cos(x)+1)^7}+\frac{a^8b^4\sin(x)^8}{(\cos(x)+1)^8}+\frac{4(a^8b^4+6a^6b^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{8(3a^7b^5+4a^5b^7)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2(3a^8b^4+24a^6b^6+8a^4b^8)\sin(x)^4}{(\cos(x)+1)^4}+\frac{8(3a^7b^5+4a^5b^7)\sin(x)^5}{(\cos(x)+1)^5}+\frac{4(a^8b^4+6a^6b^6)\sin(x)^6}{(\cos(x)+1)^6}\right)}+\frac{\log\left(a+\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^5}-\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(3*(a^7 - a^3*b^4)*sin(x)/(cos(x) + 1) + 3*(7*a^6*b - 3*a^2*b^5)*sin(x
)^2/(cos(x) + 1)^2 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^3/(co
s(x) + 1)^3 + 2*(21*a^6*b + 25*a^4*b^3 - 7*a^2*b^5 - 3*b^7)*sin(x)^4/(cos(x
) + 1)^4 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^5/(cos(x) + 1)^
5 + 3*(7*a^6*b - 3*a^2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3*(a^7 - a^3*b^4)*sin
(x)^7/(cos(x) + 1)^7)/(a^8*b^4 + 8*a^7*b^5*sin(x)/(cos(x) + 1) + 8*a^7*b^5*
sin(x)^7/(cos(x) + 1)^7 + a^8*b^4*sin(x)^8/(cos(x) + 1)^8 + 4*(a^8*b^4 + 6*
a^6*b^6)*sin(x)^2/(cos(x) + 1)^2 + 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^3/(cos(
x) + 1)^3 + 2*(3*a^8*b^4 + 24*a^6*b^6 + 8*a^4*b^8)*sin(x)^4/(cos(x) + 1)^4
+ 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^5/(cos(x) + 1)^5 + 4*(a^8*b^4 + 6*a^6*b^
6)*sin(x)^6/(cos(x) + 1)^6) + log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/
(cos(x) + 1)^2)/b^5 - log(sin(x)^2/(cos(x) + 1)^2 + 1)/b^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.
time = 3.15, size = 217, normalized size = 2.15

$$\frac{25\,a^4+110\,a^2b^2+9\,b^4-12\,(9\,a^2b^2+b^4)\cos(x)^2+12\,\left(b^4\cos(x)^4+a^4+6\,a^2b^2+b^4-2\,(3\,a^2b^2+b^4)\cos(x)^2-4\,\left(ab^3\cos(x)^2-a^3b-ab^3\right)\sin(x)\right)\log(b\sin(x)+a)-8\,\left(6\,ab^3\cos(x)^2-11\,a^3b-7\,ab^3\right)\sin(x)}{12\,\left(b^9\cos(x)^4+a^4b^5+6\,a^2b^7+b^9-2\,(3\,a^2b^7+b^9)\cos(x)^2-4\,\left(ab^8\cos(x)^2-a^3b^6-ab^8\right)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(25*a^4 + 110*a^2*b^2 + 9*b^4 - 12*(9*a^2*b^2 + b^4)*cos(x)^2 + 12*(b^
4*cos(x)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*cos(x)^2 - 4*(a*b^
3*cos(x)^2 - a^3*b - a*b^3)*sin(x))*log(b*sin(x) + a) - 8*(6*a*b^3*cos(x)^2
 - 11*a^3*b - 7*a*b^3)*sin(x))/(b^9*cos(x)^4 + a^4*b^5 + 6*a^2*b^7 + b^9 -
2*(3*a^2*b^7 + b^9)*cos(x)^2 - 4*(a*b^8*cos(x)^2 - a^3*b^6 - a*b^8)*sin(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(90) = 180$.
time = 7.66, size = 1787, normalized size = 17.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))**5,x)
```

```
[Out] Piecewise((zoo*(-log(tan(x)**2 + 1)/2 + log(tan(x)) + 1/(2*tan(x)**2) - 1/(
4*tan(x)**4)), Eq(a, 0) & Eq(b, 0)), ((-log(tan(x)**2 + 1)/2 + log(tan(x))
+ 1/(2*tan(x)**2) - 1/(4*tan(x)**4))/b**5, Eq(a, 0)), ((8*tan(x)**5/(15*sec
(x)**5) + 4*tan(x)**3/(3*sec(x)**5) + tan(x)/sec(x)**5)/a**5, Eq(b, 0)), (-
18*a**4*log(tan(x)**2 + 1)*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**
```

```
6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)*
*3*sec(x) + 36*b**9*tan(x)**4) + 36*a**4*log(sec(x) + b*tan(x)/a)*sec(x)**4
/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*t
an(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 20*
a**4*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 2
16*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*ta
n(x)**4) - 72*a**3*b*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*a**4*b**5*sec(
x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2
+ 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 144*a**3*b*log(sec(x)
+ b*tan(x)/a)*tan(x)*sec(x)**3/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(
x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec
(x) + 36*b**9*tan(x)**4) + 44*a**3*b*tan(x)*sec(x)**3/(36*a**4*b**5*sec(x)*
*4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 1
44*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 108*a**2*b**2*log(tan(x)*
*2 + 1)*tan(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*
sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x)
 + 36*b**9*tan(x)**4) + 216*a**2*b**2*log(sec(x) + b*tan(x)/a)*tan(x)**2*se
c(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2
*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4
) + 6*a**2*b**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*se
c(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) +
 36*b**9*tan(x)**4) - 72*a*b**3*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(36*a**
4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2
*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 144*a*b**3*
log(sec(x) + b*tan(x)/a)*tan(x)**3*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**
3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*ta
n(x)**3*sec(x) + 36*b**9*tan(x)**4) - 52*a*b**3*tan(x)**3*sec(x)/(36*a**4*b
**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*se
c(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 24*a*b**3*tan(
x)*sec(x)/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a*
*2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)*
*4) - 18*b**4*log(tan(x)**2 + 1)*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a*
*3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*t
an(x)**3*sec(x) + 36*b**9*tan(x)**4) + 36*b**4*log(sec(x) + b*tan(x)/a)*tan
(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*
b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4)
 - 28*b**4*tan(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)*
*3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b
**9*tan(x)**4) + 18*b**4*tan(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*
tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3
*sec(x) + 36*b**9*tan(x)**4) - 9*b**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b*
*6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)
**3*sec(x) + 36*b**9*tan(x)**4), True))
```

Giac [**A**]

time = 0.40, size = 91, normalized size = 0.90

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{b^5}-\frac{25\,b^3\sin\left(x\right)^4+52\,ab^2\sin\left(x\right)^3+42\,a^2b\sin\left(x\right)^2-12\,b^3\sin\left(x\right)^2+12\,a^3\sin\left(x\right)-8\,ab^2\sin\left(x\right)-2\,a^2b+3\,b^3}{12\,\left(b\sin\left(x\right)+a\right)^4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b^5 - 1/12*(25*b^3*sin(x)^4 + 52*a*b^2*sin(x)^3 + 42
*a^2*b*sin(x)^2 - 12*b^3*sin(x)^2 + 12*a^3*sin(x) - 8*a*b^2*sin(x) - 2*a^2*
b + 3*b^3)/((b*sin(x) + a)^4*b^4)
```

Mupad [B]
time = 3.84, size = 541, normalized size = 5.36

$$\frac{2\,\mathrm{atanh}\left(\frac{16\,a}{\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\tan\left(\frac{x}{2}\right)^2}{b^2}}+\frac{16\,a\tan\left(\frac{x}{2}\right)^2}{\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\tan\left(\frac{x}{2}\right)^2}{b^2}}+\frac{32\,a^2\tan\left(\frac{x}{2}\right)}{32\,a^2\tan\left(\frac{x}{2}\right)-16\,a\,b+\frac{32\,a^3}{b}+\frac{32\,a^3\tan\left(\frac{x}{2}\right)^2}{b}-16\,a\,b\tan\left(\frac{x}{2}\right)^2}\right)}{b^5}-\frac{\frac{2\tan\left(\frac{x}{2}\right)^2\left(7\,a^4-3\,b^4\right)}{a^2b^3}+\frac{2\tan\left(\frac{x}{2}\right)^6\left(7\,a^4-3\,b^4\right)}{a^2b^3}+\frac{2\tan\left(\frac{x}{2}\right)\left(a^4-b^4\right)}{a\,b^4}+\frac{2\tan\left(\frac{x}{2}\right)^7\left(a^4-b^4\right)}{a\,b^4}+\frac{4\tan\left(\frac{x}{2}\right)^4\left(21\,a^6+25\,a^4b^2-7\,a^2b^4-3\,b^6\right)}{3\,a^4b^3}+\frac{2\tan\left(\frac{x}{2}\right)^3\left(9\,a^6+52\,a^4b^2-a^2b^4-12\,b^6\right)}{3\,a^3b^4}+\frac{2\tan\left(\frac{x}{2}\right)^5\left(9\,a^6+52\,a^4b^2-a^2b^4-12\,b^6\right)}{3\,a^3b^4}}{\tan\left(\frac{x}{2}\right)^2\left(4\,a^4+24\,a^2b^2\right)+\tan\left(\frac{x}{2}\right)^6\left(4\,a^4+24\,a^2b^2\right)+\tan\left(\frac{x}{2}\right)^3\left(24\,a^3b+32\,a\,b^3\right)+\tan\left(\frac{x}{2}\right)^5\left(24\,a^3b+32\,a\,b^3\right)+\tan\left(\frac{x}{2}\right)^4\left(6\,a^4+48\,a^2b^2+16\,b^4\right)+a^4+a^4\tan\left(\frac{x}{2}\right)^8+8\,a^3b\tan\left(\frac{x}{2}\right)^7+8\,a^3b\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(x) + a/cos(x))^5,x)
```

```
[Out] (2*atanh((16*a)/((32*a^3)/b^2 - 16*a*tan(x/2)^2 - 16*a + (32*a^2*tan(x/2))/
b + (32*a^3*tan(x/2)^2)/b^2) + (16*a*tan(x/2)^2)/((32*a^3)/b^2 - 16*a*tan(x
/2)^2 - 16*a + (32*a^2*tan(x/2))/b + (32*a^3*tan(x/2)^2)/b^2) + (32*a^2*tan
(x/2))/(32*a^2*tan(x/2) - 16*a*b + (32*a^3)/b + (32*a^3*tan(x/2)^2)/b - 16*
a*b*tan(x/2)^2)))/b^5 - ((2*tan(x/2)^2*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*tan(
x/2)^6*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*tan(x/2)*(a^4 - b^4))/(a*b^4) + (2*t
an(x/2)^7*(a^4 - b^4))/(a*b^4) + (4*tan(x/2)^4*(21*a^6 - 3*b^6 - 7*a^2*b^4
+ 25*a^4*b^2))/(3*a^4*b^3) + (2*tan(x/2)^3*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a
^4*b^2))/(3*a^3*b^4) + (2*tan(x/2)^5*(9*a^6 - 12*b^6 - a^2*b^4 + 52*a^4*b^2
))/(3*a^3*b^4))/(tan(x/2)^2*(4*a^4 + 24*a^2*b^2) + tan(x/2)^6*(4*a^4 + 24*a
^2*b^2) + tan(x/2)^3*(32*a*b^3 + 24*a^3*b) + tan(x/2)^5*(32*a*b^3 + 24*a^3*
b) + tan(x/2)^4*(6*a^4 + 16*b^4 + 48*a^2*b^2) + a^4 + a^4*tan(x/2)^8 + 8*a^
3*b*tan(x/2)^7 + 8*a^3*b*tan(x/2))
```

3.273 $\int (\sec(x)+\tan(x))^5\,dx$

Optimal. Leaf size=30

$$-\log(1-\sin(x))+\frac{2}{(1-\sin(x))^2}-\frac{4}{1-\sin(x)}$$

```
[Out] -ln(1-sin(x))+2/(1-sin(x))^2-4/(1-sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {4476, 2746, 45}

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^5,x]
```

```
[Out] -Log[1 - Sin[x]] + 2/(1 - Sin[x])^2 - 4/(1 - Sin[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\sec(x)+\tan(x))^5\,dx &= \int \sec^5(x)(1+\sin(x))^5\,dx \\
&= \text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \sin(x)\right) \\
&= -\log(1-\sin(x))+\frac{2}{(1-\sin(x))^2}-\frac{4}{1-\sin(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 1.80

$$\tanh^{-1}(\sin(x))-\log(\cos(x))+\frac{5\sec^4(x)}{4}+\sec(x)\tan(x)-\sec^3(x)\tan(x)-\frac{\tan^2(x)}{2}+5\sec(x)\tan^3(x)+\frac{11\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^5,x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]] + (5*Sec[x]^4)/4 + Sec[x]*Tan[x] - Sec[x]^3*T
an[x] - Tan[x]^2/2 + 5*Sec[x]*Tan[x]^3 + (11*Tan[x]^4)/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(30) = 60.
time = 0.16, size = 106, normalized size = 3.53

method	result
risch	$ix+\frac{8i(-ie^{2ix}+e^{3ix}-e^{ix})}{(e^{ix}-i)^4}-2\ln(e^{ix}-i)$
default	$-\left(-\frac{(\sec^3(x))}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\ln(\sec(x)+\tan(x))+\frac{5}{4\cos(x)^4}+\frac{5(\sin^3(x))}{2\cos(x)^4}+\frac{5(\sin^3(x))}{4\cos(x)^2}-\frac{5\sin(x)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+ln(sec(x)+tan(x))+5/4/cos(x)^4+5/2*sin(x
)^3/cos(x)^4+5/4*sin(x)^3/cos(x)^2-5/8*sin(x)+5/2*sin(x)^4/cos(x)^4+5/4*sin
(x)^5/cos(x)^4-5/8*sin(x)^5/cos(x)^2-5/8*sin(x)^3+1/4*tan(x)^4-1/2*tan(x)^2
-ln(cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(26) = 52.
time = 0.31, size = 141, normalized size = 4.70

$$\frac{5}{2}\tan(x)^4+\frac{5(5\sin(x)^3-3\sin(x))}{8(\sin(x)^4-2\sin(x)^2+1)}-\frac{3\sin(x)^3-5\sin(x)}{8(\sin(x)^4-2\sin(x)^2+1)}+\frac{5(\sin(x)^3+\sin(x))}{4(\sin(x)^4-2\sin(x)^2+1)}+\frac{4\sin(x)^2-3}{4(\sin(x)^4-2\sin(x)^2+1)}+\frac{5}{4(\sin(x)^2-1)^2}-\frac{1}{2}\log(\sin(x)^2-1)+\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] 5/2*tan(x)^4 + 5/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/
8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4*(sin(x)^3 + sin
(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x
)^2 + 1) + 5/4/(sin(x)^2 - 1)^2 - 1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x) +
1) - 1/2*log(sin(x) - 1)
```

Fricas [A]
time = 2.03, size = 38, normalized size = 1.27

$$-\frac{\left(\cos\left(x\right)^{2}+2\,\sin\left(x\right)-2\right)\log\left(-\sin\left(x\right)+1\right)+4\,\sin\left(x\right)-2}{\cos\left(x\right)^{2}+2\,\sin\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="fricas")
```

```
[Out] -((cos(x)^2 + 2*sin(x) - 2)*log(-sin(x) + 1) + 4*sin(x) - 2)/(cos(x)^2 + 2*
sin(x) - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.
time = 4.16, size = 68, normalized size = 2.27

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}+\frac{\log\left(\sec^{2}\left(x\right)\right)}{2}+\frac{5\tan^{4}\left(x\right)}{2}+\frac{3\sec^{4}\left(x\right)}{2}-\sec^{2}\left(x\right)+\frac{32\sin^{3}\left(x\right)}{8\sin^{4}\left(x\right)-16\sin^{2}\left(x\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**5,x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 + log(sec(x)**2)/2 + 5*tan(x)**4/2 +
 3*sec(x)**4/2 - sec(x)**2 + 32*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(26) = 52$.
time = 0.41, size = 62, normalized size = 2.07

$$\frac{25\,\tan\left(\frac{1}{2}\,x\right)^{4}-100\,\tan\left(\frac{1}{2}\,x\right)^{3}+198\,\tan\left(\frac{1}{2}\,x\right)^{2}-100\,\tan\left(\frac{1}{2}\,x\right)+25}{6\left(\tan\left(\frac{1}{2}\,x\right)-1\right)^{4}}+\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] 1/6*(25*tan(1/2*x)^4 - 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 - 100*tan(1/2*x)
 + 25)/(tan(1/2*x) - 1)^4 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) -
1))
```

Mupad [B]
time = 2.44, size = 59, normalized size = 1.97

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{8\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4-4\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-4\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^5,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) + (8*tan(x/2)^2)/(6*tan(x/2)^2 -
4*tan(x/2) - 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.274 $\int (\sec(x)+\tan(x))^4\,dx$

Optimal. Leaf size=30

$$x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

```
[Out] x+2/3*cos(x)^3/(1-sin(x))^3-2*cos(x)/(1-sin(x))
```

Rubi [A]
time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4476, 2749, 2759, 8}

$$x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^4,x]
```

```
[Out] x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\sec(x)+\tan(x))^4\,dx &= \int \sec^4(x)(1+\sin(x))^4\,dx \\
&= \int \frac{\cos^4(x)}{(1-\sin(x))^4}\,dx \\
&= \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx \\
&= \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \frac{2\cos(x)}{1-\sin(x)} + \int 1\,dx \\
&= x + \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \frac{2\cos(x)}{1-\sin(x)}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.
time = 0.10, size = 64, normalized size = 2.13

$$
-\frac{-3(8+3x)\cos\left(\frac{x}{2}\right)+(16+3x)\cos\left(\frac{3x}{2}\right)+6(4+2x+x\cos(x))\sin\left(\frac{x}{2}\right)}{6\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^4,x]
```

```
[Out] -1/6*(-3*(8 + 3*x)*Cos[x/2] + (16 + 3*x)*Cos[(3*x)/2] + 6*(4 + 2*x + x*Cos[
x])*Sin[x/2])/(Cos[x/2] - Sin[x/2])^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.
time = 0.13, size = 71, normalized size = 2.37

method	result
risch	$x-\frac{8\left(-2-3i e^{ix}+3\,e^{2ix}\right)}{3\left(e^{ix}-i\right)^3}$
default	$-\left(-\frac{2}{3}-\frac{\left(\sec^2(x)\right)}{3}\right)\tan(x)+\frac{4}{3\cos(x)^3}+\frac{2\left(\sin^3(x)\right)}{\cos(x)^3}+\frac{4\left(\sin^4(x)\right)}{3\cos(x)^3}-\frac{4\left(\sin^4(x)\right)}{3\cos(x)}-\frac{4\left(2+\sin^2(x)\right)\cos(x)}{3}+\frac{\left(\tan^3(x\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)+4/3/cos(x)^3+2*sin(x)^3/cos(x)^3+4/3*sin(x)^4/c
os(x)^3-4/3*sin(x)^4/cos(x)-4/3*(2+sin(x)^2)*cos(x)+1/3*tan(x)^3-tan(x)+x
```

Maxima [A]
time = 0.50, size = 28, normalized size = 0.93

$$\frac{8}{3}\,\tan\left(x\right)^{3} + x - \frac{4\,\left(3\,\cos\left(x\right)^{2} - 1\right)}{3\,\cos\left(x\right)^{3}} + \frac{4}{3\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 8/3*tan(x)^3 + x - 4/3*(3*cos(x)^2 - 1)/cos(x)^3 + 4/3/cos(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.
time = 2.71, size = 61, normalized size = 2.03

$$\frac{(3\,x + 8)\cos\left(x\right)^{2} - (3\,x - 4)\cos\left(x\right) + ((3\,x - 8)\cos\left(x\right) + 6\,x - 4)\sin\left(x\right) - 6\,x - 4}{3\,\left(\cos\left(x\right)^{2} + (\cos\left(x\right) + 2)\sin\left(x\right) - \cos\left(x\right) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x + 8)*cos(x)^2 - (3*x - 4)*cos(x) + ((3*x - 8)*cos(x) + 6*x - 4)*s
in(x) - 6*x - 4)/(cos(x)^2 + (cos(x) + 2)*sin(x) - cos(x) - 2)
```

Sympy [A]
time = 2.36, size = 44, normalized size = 1.47

$$x + \frac{\sin^{3}\left(x\right)}{3\cos^{3}\left(x\right)} - \frac{\sin\left(x\right)}{\cos\left(x\right)} + \frac{7\tan^{3}\left(x\right)}{3} + \tan\left(x\right) + \frac{8\sec^{3}\left(x\right)}{3} - 4\sec\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x) + 7*tan(x)**3/3 + tan(x) + 8*se
c(x)**3/3 - 4*sec(x)
```

Giac [A]
time = 0.39, size = 20, normalized size = 0.67

$$x - \frac{8\,\left(3\,\tan\left(\frac{1}{2}\,x\right) - 1\right)}{3\,\left(\tan\left(\frac{1}{2}\,x\right) - 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] x - 8/3*(3*tan(1/2*x) - 1)/(tan(1/2*x) - 1)^3
```

Mupad [B]
time = 2.37, size = 20, normalized size = 0.67

$$x-\frac{8\tan\left(\frac{x}{2}\right)-\frac{8}{3}}{{\left(\tan\left(\frac{x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^4,x)
```

```
[Out] x - (8*tan(x/2) - 8/3)/(tan(x/2) - 1)^3
```

3.275 $\int (\sec(x)+\tan(x))^3\,dx$

Optimal. Leaf size=18

$$\log(1-\sin(x))+\frac{2}{1-\sin(x)}$$

[Out] ln(1-sin(x))+2/(1-sin(x))

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4476, 2746, 45}

$$\frac{2}{1-\sin(x)}+\log(1-\sin(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^3,x]

[Out] Log[1 - Sin[x]] + 2/(1 - Sin[x])

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))^3\,dx &= \int \sec^3(x)(1+\sin(x))^3\,dx\\ &= \text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, \sin(x)\right)\\ &= \log(1-\sin(x))+\frac{2}{1-\sin(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.72

$$-\tanh^{-1}(\sin(x))+\log(\cos(x))+\frac{3\sec^2(x)}{2}+2\sec(x)\tan(x)+\frac{\tan^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^3,x]
```

```
[Out] -ArcTanh[Sin[x]] + Log[Cos[x]] + (3*Sec[x]^2)/2 + 2*Sec[x]*Tan[x] + Tan[x]^
2/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.
time = 0.12, size = 45, normalized size = 2.50

method	result	size
risch	$-ix-\frac{4i e^{ix}}{(e^{ix}-i)^2}+2\ln(e^{ix}-i)$	35
default	$\frac{\sec(x)\tan(x)}{2}-\ln(\sec(x)+\tan(x))+\frac{3}{2\cos(x)^2}+\frac{3(\sin^3(x))}{2\cos(x)^2}+\frac{3\sin(x)}{2}+\frac{(\tan^2(x))}{2}+\ln(\cos(x))$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sec(x)*tan(x)-ln(sec(x)+tan(x))+3/2/cos(x)^2+3/2*sin(x)^3/cos(x)^2+3/2*
sin(x)+1/2*tan(x)^2+ln(cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(16) = 32.
time = 0.31, size = 52, normalized size = 2.89

$$\frac{3}{2}\tan(x)^2-\frac{2\sin(x)}{\sin(x)^2-1}-\frac{1}{2(\sin(x)^2-1)}+\frac{1}{2}\log(\sin(x)^2-1)-\frac{1}{2}\log(\sin(x)+1)+\frac{1}{2}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*tan(x)^2 - 2*sin(x)/(sin(x)^2 - 1) - 1/2/(sin(x)^2 - 1) + 1/2*log(sin(x
)^2 - 1) - 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1)
```

Fricas [A]
time = 3.18, size = 21, normalized size = 1.17

$$\frac{(\sin(x)-1)\log(-\sin(x)+1)-2}{\sin(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] ((sin(x) - 1)*log(-sin(x) + 1) - 2)/(sin(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.
time = 2.87, size = 44, normalized size = 2.44

$$\frac{\log(\sin(x)-1)}{2}-\frac{\log(\sin(x)+1)}{2}-\frac{\log(\sec^2(x))}{2}+2\sec^2(x)-\frac{4\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**3,x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/2 - log(sec(x)**2)/2 + 2*sec(x)**2 - 4*
sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(16) = 32$.
time = 0.39, size = 48, normalized size = 2.67

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2-10\tan\left(\frac{1}{2}x\right)+3}{\left(\tan\left(\frac{1}{2}x\right)-1\right)^2}-\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] -(3*tan(1/2*x)^2 - 10*tan(1/2*x) + 3)/(tan(1/2*x) - 1)^2 - log(tan(1/2*x)^2
 + 1) + 2*log(abs(tan(1/2*x) - 1))
```

Mupad [B]
time = 2.40, size = 43, normalized size = 2.39

$$2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+\frac{4\tan\left(\frac{x}{2}\right)}{{\tan\left(\frac{x}{2}\right)}^2-2\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^3,x)
```

```
[Out] 2*log(tan(x/2) - 1) - log(tan(x/2)^2 + 1) + (4*tan(x/2))/(tan(x/2)^2 - 2*ta
n(x/2) + 1)
```

3.276 $\int (\sec(x)+\tan(x))^2\,dx$

Optimal. Leaf size=16

$$-x+\frac{2\cos(x)}{1-\sin(x)}$$

[Out] -x+2*cos(x)/(1-sin(x))

Rubi [A]
time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {4476, 2749, 2759, 8}

$$\frac{2\cos(x)}{1-\sin(x)}-x$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^2,x]

[Out] -x + (2*Cos[x])/(1 - Sin[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\sec(x)+\tan(x))^2\,dx &= \int \sec^2(x)(1+\sin(x))^2\,dx \\
&= \int \frac{\cos^2(x)}{(1-\sin(x))^2}\,dx \\
&= \frac{2\cos(x)}{1-\sin(x)} - \int 1\,dx \\
&= -x + \frac{2\cos(x)}{1-\sin(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.88

$$-\text{ArcTan}(\tan(x)) + 2\sec(x) + 2\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^2,x]
```

```
[Out] -ArcTan[Tan[x]] + 2*Sec[x] + 2*Tan[x]
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.94

method	result	size
default	$2\tan(x) + \frac{2}{\cos(x)} - x$	15
risch	$-x + \frac{4}{e^{ix} - i}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*tan(x)+2/cos(x)-x
```

Maxima [A]
time = 0.51, size = 14, normalized size = 0.88

$$-x + \frac{2}{\cos(x)} + 2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -x + 2/cos(x) + 2*tan(x)
```

Fricas [A]
time = 1.55, size = 28, normalized size = 1.75

$$-\frac{(x-2)\cos(x)-(x+2)\sin(x)+x-2}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="fricas")
[Out] -((x - 2)*cos(x) - (x + 2)*sin(x) + x - 2)/(cos(x) - sin(x) + 1)
```

Sympy [A]
time = 0.66, size = 10, normalized size = 0.62

$$-x+2\tan(x)+2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))**2,x)
[Out] -x + 2*tan(x) + 2*sec(x)
```

Giac [A]
time = 0.41, size = 14, normalized size = 0.88

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)+tan(x))^2,x, algorithm="giac")
[Out] -x - 4/(tan(1/2*x) - 1)
```

Mupad [B]
time = 2.36, size = 14, normalized size = 0.88

$$-x-\frac{4}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^2,x)
[Out] - x - 4/(tan(x/2) - 1)
```

3.277 $\int (\sec(x) + \tan(x)) \, dx$

Optimal. Leaf size=13

$$-2\log\left(\cos\left(\frac{1}{4}(\pi+2x)\right)\right)$$

```
[Out] -2*ln(cos(1/4*Pi+1/2*x))
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3855, 3556}

$$\tanh^{-1}(\sin(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x] + Tan[x],x]
```

```
[Out] ArcTanh[Sin[x]] - Log[Cos[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\sec(x)+\tan(x))\,dx &= \int \sec(x)\,dx+\int \tan(x)\,dx\\ &= \tanh^{-1}(\sin(x))-\log(\cos(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.
time = 0.01, size = 38, normalized size = 2.92

$$-\log(\cos(x))-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x] + Tan[x],x]
[Out] -Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.00

method	result	size
default	$\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)-\ln\left(\cos\left(x\right)\right)$	13
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{\ln\left(\tan^{2}\left(x\right)+1\right)}{2}+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	27
risch	$\ln\left(\mathrm{e}^{ix}+i\right)-\ln\left(\mathrm{e}^{ix}-i\right)+ix-\ln\left(\mathrm{e}^{2ix}+1\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)+tan(x),x,method=_RETURNVERBOSE)
[Out] ln(sec(x)+tan(x))-ln(cos(x))
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.77

$$\log\left(\sec\left(x\right)+\tan\left(x\right)\right)+\log\left(\sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="maxima")
[Out] log(sec(x) + tan(x)) + log(sec(x))
```

Fricas [A]
time = 2.40, size = 9, normalized size = 0.69

$$-\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="fricas")
[Out] -log(-sin(x) + 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 1.54

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\log\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(cos(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 0.39, size = 31, normalized size = 2.38

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)+tan(x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 log(abs(cos(x)))
```

Mupad [B]
time = 2.40, size = 19, normalized size = 1.46

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x) + 1/cos(x),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

3.278 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=5

$$\log(1+\sin(x))$$

```
[Out] ln(1+sin(x))
```

Rubi [A]

time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3238, 2746, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.
time = 0.01, size = 16, normalized size = 3.20

$$2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.13, size = 6, normalized size = 1.20

method	result	size
default	$\ln(1+\sin(x))$	6
risch	$-ix+2\ln(e^{ix}+i)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(5) = 10$.
time = 0.27, size = 31, normalized size = 6.20

$$2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 1.57, size = 5, normalized size = 1.00

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.
time = 0.06, size = 17, normalized size = 3.40

$$\log\left(\tan\left(x\right) + \sec\left(x\right)\right) - \frac{\log\left(\tan^2\left(x\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5) = 10$.
time = 0.40, size = 22, normalized size = 4.40

$$-\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right) + 2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 2.78, size = 21, normalized size = 4.20

$$2\,\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

3.279 $\int \frac{1}{(\sec(x)+\tan(x))^2} dx$

Optimal. Leaf size=14

$$-x - \frac{2\cos(x)}{1+\sin(x)}$$

```
[Out] -x-2*cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4476, 2759, 8}

$$-x - \frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-2),x]
```

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx$$
$$= -\frac{2\cos(x)}{1+\sin(x)} - \int 1\,dx$$
$$= -x - \frac{2\cos(x)}{1+\sin(x)}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 1.93

$$-x + \frac{4\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-2),x]
[Out] -x + (4*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.17, size = 19, normalized size = 1.36

method	result	size
risch	$-x-\frac{4}{e^{ix}+i}$	17
default	$-\frac{4}{\tan\left(\frac{x}{2}\right)+1}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
[Out] -4/(tan(1/2*x)+1)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.49, size = 28, normalized size = 2.00

$$-\frac{4}{\frac{\sin(x)}{\cos(x)+1}+1} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="maxima")
[Out] -4/(sin(x)/(cos(x) + 1) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 1.72, size = 25, normalized size = 1.79

$$-\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="fricas")
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\tan(x)+\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**2,x)
[Out] Integral((tan(x) + sec(x))**(-2), x)
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.00

$$-x-\frac{4}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^2,x, algorithm="giac")
[Out] -x - 4/(tan(1/2*x) + 1)
```

Mupad [B]
time = 2.34, size = 14, normalized size = 1.00

$$-x-\frac{4}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^2,x)
[Out] - x - 4/(tan(x/2) + 1)
```

3.280 $\int \frac{1}{(\sec(x)+\tan(x))^3} dx$

Optimal. Leaf size=16

$$-\log(1+\sin(x)) - \frac{2}{1+\sin(x)}$$

```
[Out] -ln(1+sin(x))-2/(1+sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4476, 2746, 45}

$$-\frac{2}{\sin(x)+1} - \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-3),x]
```

```
[Out] -Log[1 + Sin[x]] - 2/(1 + Sin[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^3} dx = \int \frac{\cos^3(x)}{(1+\sin(x))^3} dx$$
$$= \text{Subst}\left(\int \frac{1-x}{(1+x)^2} dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right) dx, x, \sin(x)\right)$$
$$= -\log(1+\sin(x)) - \frac{2}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.02, size = 34, normalized size = 2.12

$$-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{2}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-3),x]
[Out] -2*Log[Cos[x/2] + Sin[x/2]] - 2/(Cos[x/2] + Sin[x/2])^2
```

Maple [A]
time = 0.19, size = 17, normalized size = 1.06

method	result	size
default	$-\ln\left(1+\sin\left(x\right)\right)-\frac{2}{1+\sin\left(x\right)}$	17
risch	$ix-\frac{4i\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+i\right)^{2}}-2\ln\left(\mathrm{e}^{ix}+i\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^3,x,method=_RETURNVERBOSE)
[Out] -ln(1+sin(x))-2/(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(16) = 32$.
time = 0.48, size = 64, normalized size = 4.00

$$\frac{4\sin\left(x\right)}{\left(\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}+\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)\left(\cos\left(x\right)+1\right)}-2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 4*sin(x)/((2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1
)) - 2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 1.44, size = 20, normalized size = 1.25

$$-\frac{(\sin(x)+1)\log(\sin(x)+1)+2}{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] -((sin(x) + 1)*log(sin(x) + 1) + 2)/(sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(14) = 28$.
time = 0.41, size = 301, normalized size = 18.81

$$-\frac{2\log(\tan(x)+\sec(x))\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}-\frac{4\log(\tan(x)+\sec(x))\tan(x)\sec(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}-\frac{2\log(\tan(x)+\sec(x))\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}+\frac{\log(\tan^2(x)+1)\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}+\frac{2\log(\tan^2(x)+1)\tan(x)\sec(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}+\frac{\log(\tan^2(x)+1)\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}+\frac{\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}-\frac{\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}-\frac{1}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**3,x)
```

```
[Out] -2*log(tan(x) + sec(x))*tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)
**2) - 4*log(tan(x) + sec(x))*tan(x)*sec(x)/(2*tan(x)**2 + 4*tan(x)*sec(x)
+ 2*sec(x)**2) - 2*log(tan(x) + sec(x))*sec(x)**2/(2*tan(x)**2 + 4*tan(x)*s
ec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*tan(x)**2/(2*tan(x)**2 + 4*tan(x)
*sec(x) + 2*sec(x)**2) + 2*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*tan(x)**2 +
4*tan(x)*sec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*sec(x)**2/(2*tan(x)**2
+ 4*tan(x)*sec(x) + 2*sec(x)**2) + tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x)
 + 2*sec(x)**2) - sec(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2) -
 1/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.
time = 0.41, size = 45, normalized size = 2.81

$$\frac{3\tan\left(\frac{1}{2}x\right)^2+10\tan\left(\frac{1}{2}x\right)+3}{\left(\tan\left(\frac{1}{2}x\right)+1\right)^2}+\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)-2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] (3*tan(1/2*x)^2 + 10*tan(1/2*x) + 3)/(tan(1/2*x) + 1)^2 + log(tan(1/2*x)^2
+ 1) - 2*log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 2.36, size = 41, normalized size = 2.56

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{4\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^3,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + (4*tan(x/2))/(2*tan(x/2) + tan(
x/2)^2 + 1)
```

3.281 $\int \frac{1}{(\sec(x)+\tan(x))^4} dx$

Optimal. Leaf size=26

$$x-\frac{2\cos^3(x)}{3(1+\sin(x))^3}+\frac{2\cos(x)}{1+\sin(x)}$$

```
[Out] x-2/3*cos(x)^3/(1+sin(x))^3+2*cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4476, 2759, 8}

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-4),x]
```

```
[Out] x - (2*Cos[x]^3)/(3*(1 + Sin[x])^3) + (2*Cos[x])/(1 + Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx &= \int \frac{\cos^4(x)}{(1+\sin(x))^4}\,dx \\ &= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} - \int \frac{\cos^2(x)}{(1+\sin(x))^2}\,dx \\ &= -\frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)} + \int 1\,dx \\ &= x - \frac{2\cos^3(x)}{3(1+\sin(x))^3} + \frac{2\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.
time = 0.06, size = 62, normalized size = 2.38

$$\frac{3(-8+3x)\cos\left(\frac{x}{2}\right)+(16-3x)\cos\left(\frac{3x}{2}\right)+6(-4+2x+x\cos(x))\sin\left(\frac{x}{2}\right)}{6\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-4),x]
[Out] (3*(-8 + 3*x)*Cos[x/2] + (16 - 3*x)*Cos[(3*x)/2] + 6*(-4 + 2*x + x*Cos[x])*
Sin[x/2])/(6*(Cos[x/2] + Sin[x/2])^3)
```

Maple [A]
time = 0.19, size = 29, normalized size = 1.12

method	result	size
default	$-\frac{16}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{8}{\left(\tan\left(\frac{x}{2}\right)+1\right)^2}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	29
risch	$x+\frac{-\frac{16}{3}+8i\,e^{ix}+8\,e^{2ix}}{\left(e^{ix}+i\right)^3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^4,x,method=_RETURNVERBOSE)
[Out] -16/3/(tan(1/2*x)+1)^3+8/(tan(1/2*x)+1)^2+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(24) = 48.
time = 0.48, size = 64, normalized size = 2.46

$$\frac{8\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 8/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x)
 + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.
time = 1.25, size = 63, normalized size = 2.42

$$\frac{(3x-8)\cos(x)^2-(3x+4)\cos(x)-((3x+8)\cos(x)+6x+4)\sin(x)-6x+4}{3\left(\cos(x)^2-(\cos(x)+2)\sin(x)-\cos(x)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x - 8)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 8)*cos(x) + 6*x + 4)*s
in(x) - 6*x + 4)/(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\tan(x)+\sec(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**4,x)
```

```
[Out] Integral((tan(x) + sec(x))**(-4), x)
```

Giac [A]
time = 0.41, size = 20, normalized size = 0.77

$$x+\frac{8\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] x + 8/3*(3*tan(1/2*x) + 1)/(tan(1/2*x) + 1)^3
```

Mupad [B]
time = 2.34, size = 19, normalized size = 0.73

$$x+\frac{8\tan\left(\frac{x}{2}\right)+\frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^4,x)
```

```
[Out] x + (8*tan(x/2) + 8/3)/(tan(x/2) + 1)^3
```

3.282 $\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx$

Optimal. Leaf size=22

$$\log(1+\sin(x))-\frac{2}{(1+\sin(x))^2}+\frac{4}{1+\sin(x)}$$

```
[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4476, 2746, 45}

$$\frac{4}{\sin(x)+1}-\frac{2}{(\sin(x)+1)^2}+\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-5),x]
```

```
[Out] Log[1 + Sin[x]] - 2/(1 + Sin[x])^2 + 4/(1 + Sin[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = \int \frac{\cos^5(x)}{(1+\sin(x))^5}\,dx$$
$$= \text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, \sin(x)\right)$$
$$= \log(1+\sin(x)) - \frac{2}{(1+\sin(x))^2}+\frac{4}{1+\sin(x)}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.77

$$2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{2+4\sin(x)}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-5),x]
[Out] 2*Log[Cos[x/2] + Sin[x/2]] + (2 + 4*Sin[x])/(Cos[x/2] + Sin[x/2])^4
```

Maple [A]
time = 0.22, size = 23, normalized size = 1.05

method	result	size
default	$\ln(1+\sin(x))-\frac{2}{(1+\sin(x))^2}+\frac{4}{1+\sin(x)}$	23
risch	$-ix+\frac{8i(i e^{2ix}+e^{3ix}-e^{ix})}{(e^{ix}+i)^4}+2\ln(e^{ix}+i)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x))^5,x,method=_RETURNVERBOSE)
[Out] ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(22) = 44.
time = 0.55, size = 92, normalized size = 4.18

$$-\frac{8\sin(x)^2}{\left(\frac{4\sin(x)}{\cos(x)+1}+\frac{6\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\sin(x)^3}{(\cos(x)+1)^3}+\frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)(\cos(x)+1)^2}+2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="maxima")
```

```
[Out] -8*sin(x)^2/((4*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 4*sin(x)^
3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)*(cos(x) + 1)^2) + 2*log(sin
(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [**A**]
time = 2.85, size = 35, normalized size = 1.59

$$\frac{\left(\cos\left(x\right)^{2}-2\,\sin\left(x\right)-2\right)\log\left(\sin\left(x\right)+1\right)-4\,\sin\left(x\right)-2}{\cos\left(x\right)^{2}-2\,\sin\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="fricas")
```

```
[Out] ((cos(x)^2 - 2*sin(x) - 2)*log(sin(x) + 1) - 4*sin(x) - 2)/(cos(x)^2 - 2*si
n(x) - 2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(20) = 40$.
time = 1.27, size = 1059, normalized size = 48.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))**5,x)
```

```
[Out] 36*log(tan(x) + sec(x))*tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 21
6*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(tan(
x) + sec(x))*tan(x)**3*sec(x)/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*ta
n(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 216*log(tan(x) +
 sec(x))*tan(x)**2*sec(x)**2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan
(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(tan(x) +
sec(x))*tan(x)*sec(x)**3/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)*
*2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 36*log(tan(x) + sec(x
))*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2
 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 18*log(tan(x)**2 + 1)*tan(x)**4/(
36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*
sec(x)**3 + 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)**3*sec(x)/(36*tan(
x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)*
*3 + 36*sec(x)**4) - 108*log(tan(x)**2 + 1)*tan(x)**2*sec(x)**2/(36*tan(x)*
*4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3
+ 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*tan(x)**4 + 14
4*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*se
c(x)**4) - 18*log(tan(x)**2 + 1)*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*se
c(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 28*
```

```
tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 +
144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 52*tan(x)**3*sec(x)/(36*tan(x)**4 +
144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*
sec(x)**4) + 18*tan(x)**2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)
**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 44*tan(x)*sec(x)**3/
(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)
*sec(x)**3 + 36*sec(x)**4) + 24*tan(x)*sec(x)/(36*tan(x)**4 + 144*tan(x)**3
*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) +
20*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2
 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 6*sec(x)**2/(36*tan(x)**4 + 144*t
an(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x
)**4) - 9/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 +
144*tan(x)*sec(x)**3 + 36*sec(x)**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.
time = 0.41, size = 64, normalized size = 2.91

$$-\frac{25\tan\left(\frac{1}{2}x\right)^{4}+100\tan\left(\frac{1}{2}x\right)^{3}+198\tan\left(\frac{1}{2}x\right)^{2}+100\tan\left(\frac{1}{2}x\right)+25}{6\left(\tan\left(\frac{1}{2}x\right)+1\right)^{4}}-\log\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x))^5,x, algorithm="giac")
```

```
[Out] -1/6*(25*tan(1/2*x)^4 + 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 + 100*tan(1/2*x
) + 25)/(tan(1/2*x) + 1)^4 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) +
 1))
```

Mupad [B]
time = 2.38, size = 61, normalized size = 2.77

$$2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)-\frac{8\tan\left(\frac{x}{2}\right)^{2}}{\tan\left(\frac{x}{2}\right)^{4}+4\tan\left(\frac{x}{2}\right)^{3}+6\tan\left(\frac{x}{2}\right)^{2}+4\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x))^5,x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1) - (8*tan(x/2)^2)/(4*tan(x/2) + 6*
tan(x/2)^2 + 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.283 $\int (a\cot(x)+b\csc(x))^5\,dx$

Optimal. Leaf size=152

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x)-\frac{1}{4}(b+a\cos(x))^4(a+b$$

```
[Out] 1/8*a^2*b*(7*a^2-3*b^2)*cos(x)+1/8*(b+a*cos(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^2
-3*b^2)*cos(x))*csc(x)^2-1/4*(b+a*cos(x))^4*(a+b*cos(x))*csc(x)^4+1/16*(a+b
)^3*(8*a^2-9*a*b+3*b^2)*ln(1-cos(x))+1/16*(a-b)^3*(8*a^2+9*a*b+3*b^2)*ln(co
s(x)+1)
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4477, 2747, 753, 833, 788, 647, 31}

$$\frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x)+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(\cos(x)+1)+\frac{1}{8}\csc^2(x)(a\cos(x)+b)^2\left(b\left(5a^2-3b^2\right)\cos(x)+2a\left(2a^2-b^2\right)\right)-\frac{1}{4}\csc^4(x)(a\cos(x)+b)^4(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (a^2*b*(7*a^2 - 3*b^2)*Cos[x])/8 + ((b + a*Cos[x])^2*(2*a*(2*a^2 - b^2) + b
*(5*a^2 - 3*b^2)*Cos[x])*Csc[x]^2)/8 - ((b + a*Cos[x])^4*(a + b*Cos[x])*Csc
[x]^4)/4 + ((a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Log[1 - Cos[x]])/16 + ((a - b
)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[1 + Cos[x]])/16
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
```

```
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^5\,dx = \int (b+a\cos(x))^5\csc^5(x)\,dx$$
$$= -\left(a^5\text{Subst}\left(\int \frac{(b+x)^5}{(a^2-x^2)^3}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{4}(b+a\cos(x))^4(a+b\cos(x))\csc^4(x) + \frac{1}{4}a^3\text{Subst}\left(\int \frac{(b+x)^3\,(4a^2-3b^2+\right.$$
$$= \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x)\right)\csc^2(x) - \frac{1}{4}(b+a\cos(x$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x) + \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x\right.$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x) + \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x\right.$$
$$= \frac{1}{8}a^2b\left(7a^2-3b^2\right)\cos(x) + \frac{1}{8}(b+a\cos(x))^2\left(2a\left(2a^2-b^2\right)+b\left(5a^2-3b^2\right)\cos(x\right.$$

Mathematica [A]
time = 0.51, size = 143, normalized size = 0.94

$$\frac{1}{64}\left(2(7a-3b)(a+b)^4\csc^2\left(\frac{x}{2}\right)-(a+b)^5\csc^4\left(\frac{x}{2}\right)+8(a-b)^3\left(8a^2+9ab+3b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right)+8(a+b)^3\left(8a^2-9ab+3b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)+2(a-b)^4(7a+3b)\sec^2\left(\frac{x}{2}\right)-(a-b)^5\sec^4\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^5,x]
```

```
[Out] (2*(7*a - 3*b)*(a + b)^4*Csc[x/2]^2 - (a + b)^5*Csc[x/2]^4 + 8*(a - b)^3*(8
*a^2 + 9*a*b + 3*b^2)*Log[Cos[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*L
og[Sin[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sec[x/2]^2 - (a - b)^5*Sec[x/2]^4)/6
4
```

Maple [A]
time = 0.26, size = 167, normalized size = 1.10

method	result
default	$b^5\left(\left(-\frac{(\csc^3(x))}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(-\cot(x)+\csc(x))}{8}\right)-\frac{5ab^4}{4\sin(x)^4}+10b^3a^2\left(-\frac{\cos^3(x)}{4\sin(x)^4}-\frac{\cos^3(x)}{8\sin(x)^2}-\right.$
risch	$-ia^5x-\frac{25a^4b\,e^{7ix}+10a^2b^3e^{7ix}-3b^5e^{7ix}+16a^5e^{6ix}+80b^2a^3e^{6ix}+15a^4b\,e^{5ix}+70a^2b^3e^{5ix}+11b^5e^{5ix}-16a^5e^{4ix}+80ab^4e^{4ix}+15a}{4(e^{2ix}-1)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] b^5*((-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(-cot(x)+csc(x)))-5/4*a*b^4/si
n(x)^4+10*b^3*a^2*(-1/4/sin(x)^4*cos(x)^3-1/8/sin(x)^2*cos(x)^3-1/8*cos(x)-
1/8*ln(-cot(x)+csc(x)))-5/2*a^3*b^2/sin(x)^4*cos(x)^4+5*b*a^4*(-1/4/sin(x)^
4*cos(x)^5+1/8/sin(x)^2*cos(x)^5+1/8*cos(x)^3+3/8*cos(x)+3/8*ln(-cot(x)+csc
(x)))+a^5*(-1/4*cot(x)^4+1/2*cot(x)^2+ln(sin(x)))
```

Maxima [A]
time = 0.33, size = 188, normalized size = 1.24

$$-\frac{5}{2}a^3b^2\cot(x)^4-\frac{5}{16}a^4b\left(\frac{2\left(5\cos(x)^3-3\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}+3\log(\cos(x)+1)-3\log(\cos(x)-1)\right)+\frac{1}{16}b^5\left(\frac{2\left(3\cos(x)^3-5\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}-3\log(\cos(x)+1)+3\log(\cos(x)-1)\right)-\frac{5}{8}a^2b^3\left(\frac{2\left(\cos(x)^3+\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}-\log(\cos(x)+1)+\log(\cos(x)-1)\right)+\frac{1}{4}a^5\left(\frac{4\sin(x)^2-1}{\sin(x)^4}+2\log\left(\sin(x)^2\right)\right)-\frac{5ab^4}{4\sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*a^3*b^2*cot(x)^4 - 5/16*a^4*b*(2*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2
*cos(x)^2 + 1) + 3*log(cos(x) + 1) - 3*log(cos(x) - 1)) + 1/16*b^5*(2*(3*co
s(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3*log(cos(x) + 1) + 3*log(
cos(x) - 1)) - 5/8*a^2*b^3*(2*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 +
1) - log(cos(x) + 1) + log(cos(x) - 1)) + 1/4*a^5*((4*sin(x)^2 - 1)/sin(x)^
4 + 2*log(sin(x)^2)) - 5/4*a*b^4/sin(x)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(142) = 284$.
time = 2.32, size = 292, normalized size = 1.92

$$\frac{12a^5+40a^3b^2-20ab^4-2\left(25a^4b+10a^2b^3-3b^5\right)\cos(x)^3-16\left(a^5+5a^3b^2\right)\cos(x)^2+10\left(3a^4b-2a^2b^3-b^5\right)\cos(x)+\left(8a^5-15a^4b+10a^2b^3-3b^5+\left(8a^5-15a^4b+10a^2b^3-3b^5\right)\cos(x)^4-2\left(8a^5-15a^4b+10a^2b^3-3b^5\right)\cos(x)^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(8a^5+15a^4b-10a^2b^3+3b^5+\left(8a^5+15a^4b-10a^2b^3+3b^5\right)\cos(x)^4-2\left(8a^5+15a^4b-10a^2b^3+3b^5\right)\cos(x)^2\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{16\left(\cos(x)^4-2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^5 + 40*a^3*b^2 - 20*a*b^4 - 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*co
s(x)^3 - 16*(a^5 + 5*a^3*b^2)*cos(x)^2 + 10*(3*a^4*b - 2*a^2*b^3 - b^5)*cos
(x) + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + (8*a^5 - 15*a^4*b + 10*a^2*b
^3 - 3*b^5)*cos(x)^4 - 2*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^2)*
log(1/2*cos(x) + 1/2) + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + (8*a^5 + 1
5*a^4*b - 10*a^2*b^3 + 3*b^5)*cos(x)^4 - 2*(8*a^5 + 15*a^4*b - 10*a^2*b^3 +
 3*b^5)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(141) = 282$.
time = 61.68, size = 308, normalized size = 2.03

$$-\frac{a^5\log\left(\csc^2(x)\right)}{2}-\frac{a^5\csc^4(x)}{4}+a^5\csc^2(x)+\frac{15a^4b\log(\cos(x)-1)}{16}-\frac{15a^4b\log(\cos(x)+1)}{16}-\frac{25a^4b\cos^3(x)}{8\cos^4(x)-16\cos^2(x)+8}+\frac{15a^4b\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{5a^3b^2\cot^4(x)}{2}-\frac{5a^2b^3\log(\cos(x)-1)}{8}+\frac{5a^2b^3\log(\cos(x)+1)}{8}-\frac{10a^2b^3\cos^3(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{10a^2b^3\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{5ab^4\csc^4(x)}{4}+\frac{3b^5\log(\cos(x)-1)}{16}-\frac{3b^5\log(\cos(x)+1)}{16}+\frac{3b^5\cos^3(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{5b^5\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**5,x)
```

```
[Out] -a**5*log(csc(x)**2)/2 - a**5*csc(x)**4/4 + a**5*csc(x)**2 + 15*a**4*b*log(
cos(x) - 1)/16 - 15*a**4*b*log(cos(x) + 1)/16 - 25*a**4*b*cos(x)**3/(8*cos(
x)**4 - 16*cos(x)**2 + 8) + 15*a**4*b*cos(x)/(8*cos(x)**4 - 16*cos(x)**2 +
8) - 5*a**3*b**2*cot(x)**4/2 - 5*a**2*b**3*log(cos(x) - 1)/8 + 5*a**2*b**3*
log(cos(x) + 1)/8 - 10*a**2*b**3*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8)
 - 10*a**2*b**3*cos(x)/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*a*b**4*csc(x)**
4/4 + 3*b**5*log(cos(x) - 1)/16 - 3*b**5*log(cos(x) + 1)/16 + 3*b**5*cos(x)
**3/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*b**5*cos(x)/(8*cos(x)**4 - 16*cos(
x)**2 + 8)
```

Giac [A]
time = 0.43, size = 169, normalized size = 1.11

$$\frac{1}{16}\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\log\left(\cos\left(x\right)+1\right)+\frac{1}{16}\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\log\left(-\cos\left(x\right)+1\right)+\frac{6\,a^5+20\,a^3b^2-10\,ab^4-\left(25\,a^4b+10\,a^2b^3-3\,b^5\right)\cos\left(x\right)^3-8\left(a^5+5\,a^3b^2\right)\cos\left(x\right)^2+5\left(3\,a^4b-2\,a^2b^3-b^5\right)\cos\left(x\right)}{8\left(\cos\left(x\right)+1\right)^2\left(\cos\left(x\right)-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(cos(x) + 1) + 1/16*(8*a^5
+ 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(-cos(x) + 1) + 1/8*(6*a^5 + 20*a^3*b^2
 - 10*a*b^4 - (25*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^3 - 8*(a^5 + 5*a^3*b^2
)*cos(x)^2 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cos(x))/((cos(x) + 1)^2*(cos(x)
- 1)^2)
```

Mupad [B]
time = 2.58, size = 174, normalized size = 1.14

$$\tan\left(\frac{x}{2}\right)^2\left(\frac{5\,\left(a+b\right)\,{\left(a-b\right)}^4}{32}+\frac{{\left(a-b\right)}^5}{32}\right)-\frac{\frac{5\,a\,b^4}{4}+\frac{5\,a^4\,b}{4}-{\tan\left(\frac{x}{2}\right)}^2\,\left(3\,a^5+10\,a^4\,b+10\,a^3\,b^2-5\,a\,b^4-2\,b^5\right)+\frac{a^5}{4}+\frac{b^5}{4}+\frac{5\,a^2\,b^3}{2}+\frac{5\,a^3\,b^2}{2}}{16\,{\tan\left(\frac{x}{2}\right)}^4}-a^5\,\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{{\tan\left(\frac{x}{2}\right)}^4\,{\left(a-b\right)}^5}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^5,x)
```

```
[Out] tan(x/2)^2*((5*(a + b)*(a - b)^4)/32 + (a - b)^5/32) - ((5*a*b^4)/4 + (5*a^
4*b)/4 - tan(x/2)^2*(10*a^4*b - 5*a*b^4 + 3*a^5 - 2*b^5 + 10*a^3*b^2) + a^5
/4 + b^5/4 + (5*a^2*b^3)/2 + (5*a^3*b^2)/2)/(16*tan(x/2)^4) - a^5*log(tan(x
/2)^2 + 1) + log(tan(x/2))*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a^2*b^3)/4)
 - (tan(x/2)^4*(a - b)^5)/64
```

3.284 $\int (a\cot(x)+b\csc(x))^4\,dx$

Optimal. Leaf size=101

$$a^4x+\frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x)-\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x)+\frac{4}{3}ab\left(2a^2-b^2\right)\text{si}$$

```
[Out] a^4*x+1/3*(b+a*cos(x))^2*(a*b+(3*a^2-2*b^2)*cos(x))*csc(x)-1/3*(b+a*cos(x))
^3*(a+b*cos(x))*csc(x)^3+4/3*a*b*(2*a^2-b^2)*sin(x)+1/3*a^2*(3*a^2-2*b^2)*c
os(x)*sin(x)
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4477, 2770, 2940, 2813}

$$a^4x+\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\sin(x)\cos(x)+\frac{1}{3}\csc(x)(a\cos(x)+b)^2\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)-\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] a^4*x + ((b + a*Cos[x])^2*(a*b + (3*a^2 - 2*b^2)*Cos[x])*Csc[x])/3 - ((b +
a*Cos[x])^3*(a + b*Cos[x])*Csc[x]^3)/3 + (4*a*b*(2*a^2 - b^2)*Sin[x])/3 + (
a^2*(3*a^2 - 2*b^2)*Cos[x]*Sin[x])/3
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
```

```
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))^4\,dx &= \int (b+a\cos(x))^4\csc^4(x)\,dx\\ &= -\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x)-\frac{1}{3}\int (b+a\cos(x))^2\left(3a^2-2b^2+ab\right.\\ &= \frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x)-\frac{1}{3}(b+a\cos(x))^3(a+b\cos\\ &= a^4x+\frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x)-\frac{1}{3}(b+a\cos(x))^3(a\end{aligned}$$

Mathematica [A]
time = 0.19, size = 95, normalized size = 0.94

$$-\frac{1}{12}\csc^3(x)\left(-8a^3b+16ab^3+6b^2\left(3a^2+b^2\right)\cos(x)+24a^3b\cos(2x)+4a^4\cos(3x)+6a^2b^2\cos(3x)-2b^4\cos(3x)-9a^4x\sin(x)+3a^4x\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^4,x]
```

```
[Out] -1/12*(Csc[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cos[x] + 24*a^3*
b*Cos[2*x] + 4*a^4*Cos[3*x] + 6*a^2*b^2*Cos[3*x] - 2*b^4*Cos[3*x] - 9*a^4*x
*Sin[x] + 3*a^4*x*Sin[3*x]))
```

Maple [A]
time = 0.11, size = 93, normalized size = 0.92

method	result
default	$b^4\left(-\frac{2}{3}-\frac{(\csc^2(x))}{3}\right)\cot(x)-\frac{4a\,b^3}{3\sin(x)^3}-\frac{2a^2b^2(\cos^3(x))}{\sin(x)^3}+4a^3b\left(-\frac{\cos^4(x)}{3\sin(x)^3}+\frac{\cos^4(x)}{3\sin(x)}+\frac{(2+\cos^2(x))\sin(x)}{3}\right)$

risch	$a^4x+\frac{4i(6a^3b\,e^{5ix}+3a^4e^{4ix}+9a^2b^2e^{4ix}-4a^3b\,e^{3ix}+8a\,b^3e^{3ix}-3a^4e^{2ix}+3\,e^{2ix}b^4+6a^3b\,e^{ix}+2a^4+3a^2b^2-b^4)}{3(e^{2ix}-1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] b^4*(-2/3-1/3*csc(x)^2)*cot(x)-4/3*a*b^3/sin(x)^3-2*a^2*b^2/sin(x)^3*cos(x)
^3+4*a^3*b*(-1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin
(x))+a^4*(-1/3*cot(x)^3+cot(x)+x)
```

Maxima [A]
time = 0.49, size = 80, normalized size = 0.79

$$-2\,a^2b^2\cot\left(x\right)^3+\frac{1}{3}\,a^4\left(3\,x+\frac{3\,\tan\left(x\right)^2-1}{\tan\left(x\right)^3}\right)+\frac{4\,\left(3\,\sin\left(x\right)^2-1\right)a^3b}{3\,\sin\left(x\right)^3}-\frac{\left(3\,\tan\left(x\right)^2+1\right)b^4}{3\,\tan\left(x\right)^3}-\frac{4\,ab^3}{3\,\sin\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*a^2*b^2*cot(x)^3 + 1/3*a^4*(3*x + (3*tan(x)^2 - 1)/tan(x)^3) + 4/3*(3*si
n(x)^2 - 1)*a^3*b/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)*b^4/tan(x)^3 - 4/3*a*b^3/
sin(x)^3
```

Fricas [A]
time = 1.99, size = 95, normalized size = 0.94

$$\frac{12\,a^3b\cos\left(x\right)^2-8\,a^3b+4\,ab^3+2\left(2\,a^4+3\,a^2b^2-b^4\right)\cos\left(x\right)^3-3\left(a^4-b^4\right)\cos\left(x\right)+3\left(a^4x\cos\left(x\right)^2-a^4x\right)\sin\left(x\right)}{3\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(12*a^3*b*cos(x)^2 - 8*a^3*b + 4*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)*co
s(x)^3 - 3*(a^4 - b^4)*cos(x) + 3*(a^4*x*cos(x)^2 - a^4*x)*sin(x))/((cos(x)
^2 - 1)*sin(x))
```

Sympy [A]
time = 19.59, size = 97, normalized size = 0.96

$$a^4x+\frac{a^4\cos\left(x\right)}{\sin\left(x\right)}-\frac{a^4\cos^3\left(x\right)}{3\sin^3\left(x\right)}-\frac{4a^3b\csc^3\left(x\right)}{3}+4a^3b\csc\left(x\right)-2a^2b^2\cot^3\left(x\right)-\frac{4ab^3\csc^3\left(x\right)}{3}-\frac{b^4\cot^3\left(x\right)}{3}-b^4\cot\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**4,x)
```

```
[Out] a**4*x + a**4*cos(x)/sin(x) - a**4*cos(x)**3/(3*sin(x)**3) - 4*a**3*b*csc(x
)**3/3 + 4*a**3*b*csc(x) - 2*a**2*b**2*cot(x)**3 - 4*a*b**3*csc(x)**3/3 - b
**4*cot(x)**3/3 - b**4*cot(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(93) = 186.

time = 0.39, size = 215, normalized size = 2.13

$$\frac{1}{24}a^4\tan\left(\frac{1}{2}x\right)^3-\frac{1}{6}a^3b\tan\left(\frac{1}{2}x\right)^3+\frac{1}{4}a^2b^2\tan\left(\frac{1}{2}x\right)^3-\frac{1}{6}ab^3\tan\left(\frac{1}{2}x\right)^3+\frac{1}{24}b^4\tan\left(\frac{1}{2}x\right)^3+a^4x-\frac{5}{8}a^4\tan\left(\frac{1}{2}x\right)+\frac{3}{2}a^3b\tan\left(\frac{1}{2}x\right)-\frac{3}{4}a^2b^2\tan\left(\frac{1}{2}x\right)-\frac{1}{2}ab^3\tan\left(\frac{1}{2}x\right)+\frac{3}{8}b^4\tan\left(\frac{1}{2}x\right)+\frac{15a^4\tan\left(\frac{1}{2}x\right)^2+36a^3b\tan\left(\frac{1}{2}x\right)^2+18a^2b^2\tan\left(\frac{1}{2}x\right)^2-12ab^3\tan\left(\frac{1}{2}x\right)^2-9b^4\tan\left(\frac{1}{2}x\right)^2-a^4-4a^3b-6a^2b^2-4ab^3-b^4}{24\tan\left(\frac{1}{2}x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*a^4*tan(1/2*x)^3 - 1/6*a^3*b*tan(1/2*x)^3 + 1/4*a^2*b^2*tan(1/2*x)^3 -
 1/6*a*b^3*tan(1/2*x)^3 + 1/24*b^4*tan(1/2*x)^3 + a^4*x - 5/8*a^4*tan(1/2*x
) + 3/2*a^3*b*tan(1/2*x) - 3/4*a^2*b^2*tan(1/2*x) - 1/2*a*b^3*tan(1/2*x) +
3/8*b^4*tan(1/2*x) + 1/24*(15*a^4*tan(1/2*x)^2 + 36*a^3*b*tan(1/2*x)^2 + 18
*a^2*b^2*tan(1/2*x)^2 - 12*a*b^3*tan(1/2*x)^2 - 9*b^4*tan(1/2*x)^2 - a^4 -
4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4)/tan(1/2*x)^3
```

Mupad [B]

time = 2.53, size = 127, normalized size = 1.26

$$a^4\,x-\frac{\frac{4\,a\,b^3}{3}+\frac{4\,a^3\,b}{3}-{\mathrm{tan}\left(\frac{x}{2}\right)}^2\,\left(5\,a^4+12\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3-3\,b^4\right)+\frac{a^4}{3}+\frac{b^4}{3}+2\,a^2\,b^2}{8\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3}-\mathrm{tan}\left(\frac{x}{2}\right)\,\left(\frac{\left(a+b\right)\,{\left(a-b\right)}^3}{2}+\frac{{\left(a-b\right)}^4}{8}\right)+\frac{{\mathrm{tan}\left(\frac{x}{2}\right)}^3\,{\left(a-b\right)}^4}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^4,x)
```

```
[Out] a^4*x - ((4*a*b^3)/3 + (4*a^3*b)/3 - tan(x/2)^2*(12*a^3*b - 4*a*b^3 + 5*a^4
 - 3*b^4 + 6*a^2*b^2) + a^4/3 + b^4/3 + 2*a^2*b^2)/(8*tan(x/2)^3) - tan(x/2
)*(((a + b)*(a - b)^3)/2 + (a - b)^4/8) + (tan(x/2)^3*(a - b)^4)/24
```

3.285 $\int (a\cot(x)+b\csc(x))^3\,dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(1+c$$

```
[Out] -1/2*a^2*b*cos(x)-1/2*(b+a*cos(x))^2*(a+b*cos(x))*csc(x)^2-1/4*(2*a-b)*(a+b
)^2*ln(1-cos(x))-1/4*(a-b)^2*(2*a+b)*ln(cos(x)+1)
```

Rubi [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {4477, 2747, 753, 788, 647, 31}

$$-\frac{1}{2}a^2b\cos(x)-\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x))-\frac{1}{4}(a-b)^2(2a+b)\log(\cos(x)+1)-\frac{1}{2}\csc^2(x)(a\cos(x)+b)^2(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] -1/2*(a^2*b*Cos[x]) - ((b + a*Cos[x])^2*(a + b*Cos[x])*Csc[x]^2)/2 - ((2*a
- b)*(a + b)^2*Log[1 - Cos[x]])/4 - ((a - b)^2*(2*a + b)*Log[1 + Cos[x]])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^3\,dx = \int (b+a\cos(x))^3\csc^3(x)\,dx$$
$$= -\left(a^3\text{Subst}\left(\int \frac{(b+x)^3}{(a^2-x^2)^2}\,dx, x, a\cos(x)\right)\right)$$
$$= -\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) + \frac{1}{2}a\text{Subst}\left(\int \frac{(b+x)\left(2a^2-b^2+bx\right.}{a^2-x^2}\right.$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) - \frac{1}{2}a\text{Subst}\left(\int \frac{-a^2b-}{}\right.$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) + \frac{1}{4}\left((2a-b)(a+b)^2\right)$$
$$= -\frac{1}{2}a^2b\cos(x) - \frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) - \frac{1}{4}(2a-b)(a+b)^2\log$$

Mathematica [A]
time = 0.21, size = 79, normalized size = 1.03

$$\frac{1}{8}\left(-(a+b)^3\csc^2\left(\frac{x}{2}\right)-4(a-b)^2(2a+b)\log\left(\cos\left(\frac{x}{2}\right)\right)-4(2a-b)(a+b)^2\log\left(\sin\left(\frac{x}{2}\right)\right)-(a-b)^3\sec^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^3,x]
```

```
[Out] (-((a + b)^3*Csc[x/2]^2) - 4*(a - b)^2*(2*a + b)*Log[Cos[x/2]] - 4*(2*a - b
)*(a + b)^2*Log[Sin[x/2]] - (a - b)^3*Sec[x/2]^2)/8
```

Maple [A]
time = 0.14, size = 80, normalized size = 1.04

method	result
default	$b^3\left(-\frac{\cot(x)\csc(x)}{2}+\frac{\ln(-\cot(x)+\csc(x))}{2}\right)-\frac{3ab^2}{2\sin(x)^2}+3a^2b\left(-\frac{\cos^3(x)}{2\sin(x)^2}-\frac{\cos(x)}{2}-\frac{\ln(-\cot(x)+\csc(x))}{2}\right)+a^3\Big(-$
risch	$ia^3x+\frac{3a^2b\,e^{3ix}+b^3e^{3ix}+2a^3e^{2ix}+6a\,b^2e^{2ix}+3a^2b\,e^{ix}+b^3e^{ix}}{(e^{2ix}-1)^2}-\ln\left(e^{ix}+1\right)a^3+\frac{3\ln(e^{ix}+1)a^2b}{2}-\frac{\ln(e^{ix}+1)b^3}{2}-\ln\big(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*(-1/2*cot(x)*csc(x)+1/2*ln(-cot(x)+csc(x)))-3/2*a*b^2/sin(x)^2+3*a^2*b*
(-1/2/sin(x)^2*cos(x)^3-1/2*cos(x)-1/2*ln(-cot(x)+csc(x)))+a^3*(-1/2*cot(x)
^2-ln(sin(x)))
```

Maxima [A]
time = 0.29, size = 87, normalized size = 1.13

$$-\frac{3}{2}\,ab^2\cot\left(x\right)^2+\frac{3}{4}\,a^2b\left(\frac{2\,\cos\left(x\right)}{\cos\left(x\right)^2-1}+\log\left(\cos\left(x\right)+1\right)-\log\left(\cos\left(x\right)-1\right)\right)+\frac{1}{4}\,b^3\left(\frac{2\,\cos\left(x\right)}{\cos\left(x\right)^2-1}-\log\left(\cos\left(x\right)+1\right)+\log\left(\cos\left(x\right)-1\right)\right)-\frac{1}{2}\,a^3\left(\frac{1}{\sin\left(x\right)^2}+\log\left(\sin\left(x\right)^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*cot(x)^2 + 3/4*a^2*b*(2*cos(x)/(cos(x)^2 - 1) + log(cos(x) + 1)
- log(cos(x) - 1)) + 1/4*b^3*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) + l
og(cos(x) - 1)) - 1/2*a^3*(1/sin(x)^2 + log(sin(x)^2))
```

Fricas [A]
time = 3.05, size = 128, normalized size = 1.66

$$\frac{2\,a^3+6\,ab^2+2\,(3\,a^2b+b^3)\cos\left(x\right)+\left(2\,a^3-3\,a^2b+b^3-(2\,a^3-3\,a^2b+b^3)\cos\left(x\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\left(2\,a^3+3\,a^2b-b^3-(2\,a^3+3\,a^2b-b^3)\cos\left(x\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{4\,\left(\cos\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^3 + 6*a*b^2 + 2*(3*a^2*b + b^3)*cos(x) + (2*a^3 - 3*a^2*b + b^3 -
(2*a^3 - 3*a^2*b + b^3)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^3 + 3*a^2*b
- b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^2
 - 1)
```

Sympy [A]
time = 8.31, size = 124, normalized size = 1.61

$$\frac{a^3\log\left(-\csc^2\left(x\right)\right)}{2}-\frac{a^3\csc^2\left(x\right)}{2}-\frac{3a^2b\log\left(\cos\left(x\right)-1\right)}{4}+\frac{3a^2b\log\left(\cos\left(x\right)+1\right)}{4}+\frac{3a^2b\cos\left(x\right)}{2\cos^2\left(x\right)-2}-\frac{3ab^2\csc^2\left(x\right)}{2}+\frac{b^3\log\left(\cos\left(x\right)-1\right)}{4}-\frac{b^3\log\left(\cos\left(x\right)+1\right)}{4}+\frac{b^3\cos\left(x\right)}{2\cos^2\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**3,x)
```

```
[Out] a**3*log(-csc(x)**2)/2 - a**3*csc(x)**2/2 - 3*a**2*b*log(cos(x) - 1)/4 + 3*
a**2*b*log(cos(x) + 1)/4 + 3*a**2*b*cos(x)/(2*cos(x)**2 - 2) - 3*a*b**2*csc
(x)**2/2 + b**3*log(cos(x) - 1)/4 - b**3*log(cos(x) + 1)/4 + b**3*cos(x)/(2
*cos(x)**2 - 2)
```

Giac [A]
time = 0.39, size = 86, normalized size = 1.12

$$-\frac{1}{4}\left(2\,a^{3}-3\,a^{2}b+b^{3}\right)\log\left(\cos\left(x\right)+1\right)-\frac{1}{4}\left(2\,a^{3}+3\,a^{2}b-b^{3}\right)\log\left(-\cos\left(x\right)+1\right)+\frac{a^{3}+3\,ab^{2}+\left(3\,a^{2}b+b^{3}\right)\cos\left(x\right)}{2\left(\cos\left(x\right)+1\right)\left(\cos\left(x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3 - 3*a^2*b + b^3)*log(cos(x) + 1) - 1/4*(2*a^3 + 3*a^2*b - b^3)*
log(-cos(x) + 1) + 1/2*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*cos(x))/((cos(x) +
1)*(cos(x) - 1))
```

Mupad [B]
time = 2.45, size = 82, normalized size = 1.06

$$a^{3}\,\ln\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)-\frac{\frac{a^{3}}{8}+\frac{3\,a^{2}\,b}{8}+\frac{3\,a\,b^{2}}{8}+\frac{b^{3}}{8}}{\tan\left(\frac{x}{2}\right)^{2}}-\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(a^{3}+\frac{3\,a^{2}\,b}{2}-\frac{b^{3}}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^{2}\,\left(a-b\right)^{3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^3,x)
```

```
[Out] a^3*log(tan(x/2)^2 + 1) - ((3*a*b^2)/8 + (3*a^2*b)/8 + a^3/8 + b^3/8)/tan(x
/2)^2 - log(tan(x/2))*((3*a^2*b)/2 + a^3 - b^3/2) - (tan(x/2)^2*(a - b)^3)/
8
```

3.286 $\int (a\cot(x)+b\csc(x))^2\,dx$

Optimal. Leaf size=29

$$-a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-ab\sin(x)$$

```
[Out] -a^2*x-(b+a*cos(x))*(a+b*cos(x))*csc(x)-a*b*sin(x)
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {4477, 2770, 2717}

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^2,x]
```

```
[Out] -(a^2*x) - (b + a*Cos[x])*(a + b*Cos[x])*Csc[x] - a*b*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2770

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Dist[1/(g^2*(p + 1)),
 Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a
^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g}
, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*p
] || IntegerQ[m])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (a\cot(x)+b\csc(x))^2\,dx=\int (b+a\cos(x))^2\csc^2(x)\,dx$$
$$=-(b+a\cos(x))(a+b\cos(x))\csc(x)-\int \left(a^2+ab\cos(x)\right)\,dx$$
$$=-a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-(ab)\int \cos(x)\,dx$$
$$=-a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-ab\sin(x)$$

Mathematica [A]
time = 0.10, size = 24, normalized size = 0.83

$$-\left(\left(a^2+b^2\right)\cot(x)\right)-a(ax+2b\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^2,x]
[Out] -((a^2 + b^2)*Cot[x]) - a*(a*x + 2*b*Csc[x])
```

Maple [A]
time = 0.09, size = 29, normalized size = 1.00

method	result	size
default	$-b^2\cot(x)-\frac{2ab}{\sin(x)}+a^2(-\cot(x)-x)$	29
risch	$-a^2x-\frac{2i\left(2ab\,e^{ix}+a^2+b^2\right)}{e^{2ix}-1}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(x)+b*csc(x))^2,x,method=_RETURNVERBOSE)
[Out] -b^2*cot(x)-2*a*b/sin(x)+a^2*(-cot(x)-x)
```

Maxima [A]
time = 0.48, size = 29, normalized size = 1.00

$$-a^2\left(x+\frac{1}{\tan(x)}\right)-\frac{2\,ab}{\sin(x)}-\frac{b^2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
[Out] -a^2*(x + 1/tan(x)) - 2*a*b/sin(x) - b^2/tan(x)
```

Fricas [A]
time = 4.99, size = 28, normalized size = 0.97

$$-\frac{a^2x\sin(x)+2\,ab+(a^2+b^2)\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*x*sin(x) + 2*a*b + (a^2 + b^2)*cos(x))/sin(x)
```

Sympy [A]
time = 1.52, size = 31, normalized size = 1.07

$$-a^2x-\frac{a^2\cos(x)}{\sin(x)}-2ab\csc(x)-b^2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))**2,x)
```

```
[Out] -a**2*x - a**2*cos(x)/sin(x) - 2*a*b*csc(x) - b**2*cot(x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.79

$$-a^2x+\frac{1}{2}\,a^2\tan\left(\frac{1}{2}\,x\right)-ab\tan\left(\frac{1}{2}\,x\right)+\frac{1}{2}\,b^2\tan\left(\frac{1}{2}\,x\right)-\frac{a^2+2\,ab+b^2}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] -a^2*x + 1/2*a^2*tan(1/2*x) - a*b*tan(1/2*x) + 1/2*b^2*tan(1/2*x) - 1/2*(a^
2 + 2*a*b + b^2)/tan(1/2*x)
```

Mupad [B]
time = 2.42, size = 30, normalized size = 1.03

$$-\frac{\cos(x)\,a^2+2\,a\,b+\cos(x)\,b^2}{\sin(x)}-a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(x) + a*cot(x))^2,x)
```

```
[Out] - (2*a*b + a^2*cos(x) + b^2*cos(x))/sin(x) - a^2*x
```

3.287 $\int (a\cot(x)+b\csc(x))\,dx$

Optimal. Leaf size=12

$$-b\tanh^{-1}(\cos(x))+a\log(\sin(x))$$

```
[Out] -b*arctanh(cos(x))+a*ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {3556, 3855}

$$a\log(\sin(x))-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[a*Cot[x] + b*Csc[x],x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) + a*Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cot(x)+b\csc(x))\,dx&=a\int \cot(x)\,dx+b\int \csc(x)\,dx\\&=-b\tanh^{-1}(\cos(x))+a\log(\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.
time = 0.01, size = 25, normalized size = 2.08

$$-b\log\left(\cos\left(\frac{x}{2}\right)\right)+b\log\left(\sin\left(\frac{x}{2}\right)\right)+a\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cot[x] + b*Csc[x],x]
```

[Out] -(b*Log[Cos[x/2]]) + b*Log[Sin[x/2]] + a*Log[Sin[x]]

Maple [A]
time = 0.08, size = 16, normalized size = 1.33

method	result	size
default	$a\ln(\sin(x)) - b\ln(\cot(x) + \csc(x))$	16
norman	$a\ln(\tan(x)) + b\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{a\ln(\tan^2(x)+1)}{2}$	24
risch	$-iax + a\ln(e^{2ix} - 1) - b\ln(e^{ix} + 1) + b\ln(e^{ix} - 1)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cot(x)+b*csc(x),x,method=_RETURNVERBOSE)
[Out] a*ln(sin(x))-b*ln(cot(x)+csc(x))
```

Maxima [A]
time = 0.27, size = 15, normalized size = 1.25

$$-b\log(\cot(x) + \csc(x)) + a\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="maxima")
[Out] -b*log(cot(x) + csc(x)) + a*log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 3.30, size = 27, normalized size = 2.25

$$\frac{1}{2}(a-b)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}(a+b)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="fricas")
[Out] 1/2*(a - b)*log(1/2*cos(x) + 1/2) + 1/2*(a + b)*log(-1/2*cos(x) + 1/2)
```

Sympy [A]
time = 0.03, size = 24, normalized size = 2.00

$$a\log(\sin(x)) + b\left(\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x)
```

```
[Out] a*log(sin(x)) + b*(log(cos(x) - 1)/2 - log(cos(x) + 1)/2)
```

Giac [A]
time = 0.41, size = 15, normalized size = 1.25

$$a\log\left(|\sin\left(x\right)|\right)+b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cot(x)+b*csc(x),x, algorithm="giac")
[Out] a*log(abs(sin(x))) + b*log(abs(tan(1/2*x)))
```

Mupad [B]
time = 2.41, size = 27, normalized size = 2.25

$$a\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-a\,\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/sin(x) + a*cot(x),x)
[Out] a*log(tan(x/2)) - a*log(tan(x/2)^2 + 1) + b*log(tan(x/2))
```

3.288 $\int \frac{1}{a\cot(x)+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(b+a\cos(x))}{a}$$

```
[Out] -ln(b+a*cos(x))/a
```

Rubi [A]

time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3239, 2747, 31}

$$-\frac{\log(a\cos(x)+b)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-1),x]
```

```
[Out] -(Log[b + a*Cos[x]]/a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cot(x)+b\csc(x)}\,dx &= \int \frac{\sin(x)}{b+a\cos(x)}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(x)\right)}{a}\\ &= -\frac{\log(b+a\cos(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(b+a\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-1),x]
```

```
[Out] -(Log[b + a*Cos[x]]/a)
```

Maple [A]
time = 0.15, size = 13, normalized size = 1.08

method	result	size
default	$-\frac{\ln(b+a\cos(x))}{a}$	13
risch	$\frac{ix}{a}-\frac{\ln\left(e^{2ix}+\frac{2b\,e^{ix}}{a}+1\right)}{a}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(b+a*cos(x))/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(12) = 24.
time = 0.48, size = 45, normalized size = 3.75

$$-\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(x)^2/(cos(x) + 1)
^2 + 1)/a
```

Fricas [A]
time = 1.91, size = 12, normalized size = 1.00

$$-\frac{\log\left(a\cos\left(x\right)+b\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="fricas")
```

```
[Out] -log(a*cos(x) + b)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\cot(x) + b\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x)
[Out] Integral(1/(a*cot(x) + b*csc(x)), x)
```

Giac [A]
time = 0.39, size = 13, normalized size = 1.08

$$-\frac{\log(|a\cos(x) + b|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="giac")
[Out] -log(abs(a*cos(x) + b))/a
```

Mupad [B]
time = 3.28, size = 36, normalized size = 3.00

$$\frac{\operatorname{atan}\left(\frac{a\sin\left(\frac{x}{2}\right)^2}{-1\mathrm{i}\,a\sin\left(\frac{x}{2}\right)^2 + a\,1\mathrm{i} + b\,1\mathrm{i}}\right)\,2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x)),x)
[Out] (atan((a*sin(x/2)^2)/(a*1i + b*1i - a*sin(x/2)^2*1i))*2i)/a
```

3.289 $\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$

Optimal. Leaf size=67

$$-\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sin(x)}{a(b+a\cos(x))}$$

```
[Out] -x/a^2+sin(x)/a/(b+a*cos(x))+2*b*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2)
)/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4477, 2772, 2814, 2738, 214}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}-\frac{x}{a^2}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-2),x]
```

```
[Out] -(x/a^2) + (2*b*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]) + Sin[x]/(a*(b + a*Cos[x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx &= \int \frac{\sin^2(x)}{(b+a\cos(x))^2}\,dx\\ &= \frac{\sin(x)}{a(b+a\cos(x))}-\frac{\int \frac{\cos(x)}{b+a\cos(x)}\,dx}{a}\\ &= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{b\int \frac{1}{b+a\cos(x)}\,dx}{a^2}\\ &= -\frac{x}{a^2}+\frac{\sin(x)}{a(b+a\cos(x))}+\frac{(2b)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{a^2}\\ &= -\frac{x}{a^2}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}}+\frac{\sin(x)}{a(b+a\cos(x))}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 71, normalized size = 1.06

$$-\frac{\frac{2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{bx+ax\cos(x)-a\sin(x)}{b+a\cos(x)}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-2),x]
```

```
[Out] -(((2*b*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (b*
x + a*x*Cos[x] - a*Sin[x])/(b + a*Cos[x]))/a^2)
```

Maple [A]
time = 0.25, size = 86, normalized size = 1.28

method	result	size
default	$\frac{-\frac{2a\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b}+\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a^2}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$	86
risch	$-\frac{x}{a^2}+\frac{2i\left(b\,\mathrm{e}^{ix}+a\right)}{a^2\left(a\,\mathrm{e}^{2ix}+2b\,\mathrm{e}^{ix}+a\right)}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{ia^2-ib^2+\sqrt{a^2-b^2}\,b}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2}-\frac{b\ln\left(\mathrm{e}^{ix}+\frac{-ia^2+ib^2+\sqrt{a^2-b^2}\,b}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2*(-a*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)+b/((a+b)*(a-b))^(1
/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2)))-2/a^2*arctan(tan(1/2*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(57) = 114.
time = 2.70, size = 307, normalized size = 4.58

$$\left[-\frac{2\,(a^3-ab^2)x\cos(x)-(ab\cos(x)+b^2)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(x)-(a^2-2\,b^2)\cos(x)^2+2\,\sqrt{a^2-b^2}\,(b\cos(x)+a)\sin(x)+2\,a^2-b^2}{a^2\cos(x)^2+2\,ab\cos(x)+b^2}\right)+2\,(a^2b-b^3)x-2\,(a^3-ab^2)\sin(x)}{2\,(a^4b-a^2b^3+(a^5-a^3b^2)\cos(x))},\ -\frac{(a^3-ab^2)x\cos(x)-(ab\cos(x)+b^2)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(x)+a)}{(a^2-b^2)\sin(x)}\right)+(a^2b-b^3)x-(a^3-ab^2)\sin(x)}{a^4b-a^2b^3+(a^5-a^3b^2)\cos(x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(a^2 - b^2)*log((2
*a*b*cos(x) - (a^2 - 2*b^2)*cos(x)^2 + 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin
(x) + 2*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 2*(a^2*b - b^3)*x
 - 2*(a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x)), -((a
^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + (a^2*b - b^3)*x - (a^3 - a*
b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**2,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-2), x)
```

Giac [A]
time = 0.39, size = 107, normalized size = 1.60

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2}-\frac{x}{a^2}-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2-a-b\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a^2) - x/a^2 - 2*tan(1/2*x)
/((a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)*a)
```

Mupad [B]
time = 3.04, size = 440, normalized size = 6.57

$$\frac{a^3\sin(x)+b^2\left(-a\sin(x)+\mathrm{atan}\left(\frac{-a^5\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\sin\left(\frac{x}{2}\right)\left(a^2-b^2\right)^{3/2}\,2\mathrm{i}+b^5\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,2\mathrm{i}+a^4b\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}-a^2b^3\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,3\mathrm{i}+a^3b^2\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}}{\cos\left(\frac{x}{2}\right)a^6-2\cos\left(\frac{x}{2}\right)a^4b^2+\cos\left(\frac{x}{2}\right)a^2b^4}\right)\sqrt{a^2-b^2}\,2\mathrm{i}\right)+a\,b\,\mathrm{atan}\left(\frac{-a^5\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\sin\left(\frac{x}{2}\right)\left(a^2-b^2\right)^{3/2}\,2\mathrm{i}+b^5\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,2\mathrm{i}+a^4b\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}-a^2b^3\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,3\mathrm{i}+a^3b^2\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}}{\cos\left(\frac{x}{2}\right)a^6-2\cos\left(\frac{x}{2}\right)a^4b^2+\cos\left(\frac{x}{2}\right)a^2b^4}\right)\cos(x)\sqrt{a^2-b^2}\,2\mathrm{i}}{\cos(x)\,a^5+a^4b-\cos(x)\,a^3b^2-a^2b^3}-\frac{2\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^2,x)
```

```
[Out] (a^3*sin(x) + b^2*(atan((b^3*sin(x/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(x/2)*(
a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1/2)*2i + a^4*b*sin(x/2)*(a
^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(x/
2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*cos(x/2) - 2*a^4*b^2*cos(x
/2)))*(a^2 - b^2)^(1/2)*2i - a*sin(x)) + a*b*atan((b^3*sin(x/2)*(a^2 - b^2)
^(3/2)*2i - a^5*sin(x/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1
/2)*2i + a^4*b*sin(x/2)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)
^(1/2)*3i + a^3*b^2*sin(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*
cos(x/2) - 2*a^4*b^2*cos(x/2)))*cos(x)*(a^2 - b^2)^(1/2)*2i)/(a^4*b - a^2*b
^3 + a^5*cos(x) - a^3*b^2*cos(x)) - (2*atan(sin(x/2)/cos(x/2)))/a^2
```

3.290 $\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$

Optimal. Leaf size=50

$$\frac{a^2-b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}+\frac{\log(b+a\cos(x))}{a^3}$$

```
[Out] 1/2*(a^2-b^2)/a^3/(b+a*cos(x))^2+2*b/a^3/(b+a*cos(x))+ln(b+a*cos(x))/a^3
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4477, 2747, 711}

$$\frac{2b}{a^3(a\cos(x)+b)}+\frac{\log(a\cos(x)+b)}{a^3}+\frac{a^2-b^2}{2a^3(a\cos(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 - b^2)/(2*a^3*(b + a*Cos[x])^2) + (2*b)/(a^3*(b + a*Cos[x])) + Log[b +
 a*Cos[x]]/a^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \int \frac{\sin^3(x)}{(b+a\cos(x))^3}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^2-x^2}{(b+x)^3}\,dx, x, a\cos(x)\right)}{a^3}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{-b-x}+\frac{a^2-b^2}{(b+x)^3}+\frac{2b}{(b+x)^2}\right)\,dx, x, a\cos(x)\right)}{a^3}$$
$$= \frac{a^2-b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}+\frac{\log(b+a\cos(x))}{a^3}$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 1.54

$$\frac{a^2+3b^2+a^2\log(b+a\cos(x))+2b^2\log(b+a\cos(x))+a^2\cos(2x)\log(b+a\cos(x))+4ab\cos(x)(1+\log(b+a\cos(x)))}{2a^3(b+a\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-3),x]
```

```
[Out] (a^2 + 3*b^2 + a^2*Log[b + a*Cos[x]] + 2*b^2*Log[b + a*Cos[x]] + a^2*Cos[2*
x]*Log[b + a*Cos[x]] + 4*a*b*Cos[x]*(1 + Log[b + a*Cos[x]]))/(2*a^3*(b + a*
Cos[x])^2)
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.98

method	result	size
default	$\frac{\ln(b+a\cos(x))}{a^3}-\frac{-a^2+b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}$	49
risch	$-\frac{ix}{a^3}+\frac{4ba\,e^{3ix}+2a^2e^{2ix}+6b^2e^{2ix}+4ab\,e^{ix}}{a^3(a\,e^{2ix}+2b\,e^{ix}+a)^2}+\frac{\ln\left(e^{2ix}+\frac{2b\,e^{ix}}{a}+1\right)}{a^3}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln(b+a*cos(x))/a^3-1/2*(-a^2+b^2)/a^3/(b+a*cos(x))^2+2*b/a^3/(b+a*cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(48) = 96.
time = 0.49, size = 177, normalized size = 3.54

$$\frac{2\left(ab+b^2+\frac{(a^2-2ab+b^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+a^4b-a^3b^2-a^2b^3-\frac{2(a^5-a^4b-a^3b^2+a^2b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^5-3a^4b+3a^3b^2-a^2b^3)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}-\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*b + b^2 + (a^2 - 2*a*b + b^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + a^4*b -
a^3*b^2 - a^2*b^3 - 2*(a^5 - a^4*b - a^3*b^2 + a^2*b^3)*sin(x)^2/(cos(x) +
1)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(x)^4/(cos(x) + 1)^4) + log
(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^3 - log(sin(x)^2/(cos(x) + 1)^2
 + 1)/a^3
```

Fricas [A]
time = 2.29, size = 70, normalized size = 1.40

$$\frac{4\,ab\cos\left(x\right)+a^2+3\,b^2+2\,\left(a^2\cos\left(x\right)^2+2\,ab\cos\left(x\right)+b^2\right)\log\left(a\cos\left(x\right)+b\right)}{2\,\left(a^5\cos\left(x\right)^2+2\,a^4b\cos\left(x\right)+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*cos(x) + a^2 + 3*b^2 + 2*(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)*log
(a*cos(x) + b))/(a^5*cos(x)^2 + 2*a^4*b*cos(x) + a^3*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\cot\left(x\right)+b\csc\left(x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**3,x)
```

```
[Out] Integral((a*cot(x) + b*csc(x))**(-3), x)
```

Giac [A]
time = 0.39, size = 45, normalized size = 0.90

$$\frac{\log\left(\left|a\cos\left(x\right)+b\right|\right)}{a^3}+\frac{4\,b\cos\left(x\right)+\frac{a^2+3\,b^2}{a}}{2\,\left(a\cos\left(x\right)+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

```
[Out] log(abs(a*cos(x) + b))/a^3 + 1/2*(4*b*cos(x) + (a^2 + 3*b^2)/a)/((a*cos(x)
+ b)^2*a^2)
```

Mupad [B]
time = 2.77, size = 311, normalized size = 6.22

$$\frac{\frac{2\,\left(b^2+a\,b\right)}{a^2\,\left(a-b\right)}+\frac{2\,\mathrm{tan}\left(\frac{x}{2}\right)^2\,\left(a-b\right)}{a^2}}{\mathrm{tan}\left(\frac{x}{2}\right)^4\,\left(a^2-2\,a\,b+b^2\right)+2\,a\,b-\mathrm{tan}\left(\frac{x}{2}\right)^2\,\left(2\,a^2-2\,b^2\right)+a^2+b^2}-\frac{2\,\mathrm{atanh}\left(\frac{32\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a^2}+\frac{32\,b^3\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a^3}+32}-\frac{64\,b\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{32\,a-32\,b+32\,b\,\mathrm{tan}\left(\frac{x}{2}\right)^2-\frac{32\,b^2}{a}+\frac{32\,b^3}{a^2}-\frac{64\,b^2\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a}+\frac{32\,b^3\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a^2}}+\frac{32\,b^2\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{32\,a^2-32\,a\,b-32\,b^2-64\,b^2\,\mathrm{tan}\left(\frac{x}{2}\right)^2+\frac{32\,b^3}{a}+\frac{32\,b^3\,\mathrm{tan}\left(\frac{x}{2}\right)^2}{a}+32\,a\,b\,\mathrm{tan}\left(\frac{x}{2}\right)^2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^3,x)
```

```
[Out] ((2*(a*b + b^2))/(a^2*(a - b)) + (2*tan(x/2)^2*(a - b))/a^2)/(tan(x/2)^4*(a
^2 - 2*a*b + b^2) + 2*a*b - tan(x/2)^2*(2*a^2 - 2*b^2) + a^2 + b^2) - (2*at
anh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*tan(x/2
)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (64*b*ta
n(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 - (64*
b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)/(32*a^2
- 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(x/2)^2)/a
+ 32*a*b*tan(x/2)^2)))/a^3
```

3.291 $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

Optimal. Leaf size=159

$$\frac{x}{a^4}-\frac{b(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2a^3\left(a^2-b^2\right)(b+a\cos(x))}+\frac{\sin^3(x)}{3a(b+a\cos(x))^3}+\frac{b}{2a\left(a^2-b^2\right.}$$

```
[Out] x/a^4-b*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/a^4/(a-b)
^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/a^3/(a^2-b^2)/(b+a*c
os(x))+1/3*sin(x)^3/a/(b+a*cos(x))^3+1/2*b*sin(x)^3/a/(a^2-b^2)/(b+a*cos(x)
)^2
```

Rubi [A]
time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4477, 2772, 2943, 2942, 2814, 2738, 214}

$$\frac{x}{a^4}+\frac{b\sin^3(x)}{2a\left(a^2-b^2\right)(a\cos(x)+b)^2}-\frac{b(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}}-\frac{\sin(x)\left(2(a^2-b^2)-ab\cos(x)\right)}{2a^3\left(a^2-b^2\right)(a\cos(x)+b)}+\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] x/a^4 - (b*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a^
4*(a - b)^(3/2)*(a + b)^(3/2)) - ((2*(a^2 - b^2) - a*b*Cos[x])*Sin[x])/(2*a
^3*(a^2 - b^2)*(b + a*Cos[x])) + Sin[x]^3/(3*a*(b + a*Cos[x])^3) + (b*Sin[x
]^3)/(2*a*(a^2 - b^2)*(b + a*Cos[x])^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \int \frac{\sin^4(x)}{(b+a\cos(x))^4}\,dx$$
$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} - \frac{\int \frac{\cos(x)\sin^2(x)}{(b+a\cos(x))^3}\,dx}{a}$$
$$= \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\,(a^2-b^2)\,(b+a\cos(x))^2} - \frac{\int \frac{(2a+b\cos(x))\sin^2(x)}{(b+a\cos(x))^2}\,dx}{2a\,(a^2-b^2)}$$
$$= -\frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2a^3\,(a^2-b^2)\,(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x)}{2a\,(a^2-b^2)\,(b+a\cos}$$
$$= \frac{x}{a^4} - \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2a^3\,(a^2-b^2)\,(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x}{2a\,(a^2-b^2)\,(b+}$$
$$= \frac{x}{a^4} - \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2a^3\,(a^2-b^2)\,(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3} + \frac{b\sin^3(x}{2a\,(a^2-b^2)\,(b+}$$
$$= \frac{x}{a^4} - \frac{b(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{(2(a^2-b^2)-ab\cos(x))\sin(x}{2a^3\,(a^2-b^2)\,(b+a\cos(x))}$$

Mathematica [A]
time = 0.35, size = 150, normalized size = 0.94

$$\frac{\left(2a(a^2-b^2)+7ab(b+a\cos(x)) - \frac{a(8a^2-11b^2)(b+a\cos(x))^2}{(a-b)(a+b)} + 6x(b+a\cos(x))^3\csc(x) - \frac{6b(-3a^2+2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(x))^3\csc(x)}{(a^2-b^2)^{3/2}}\right)\sin(x)}{6a^4(b+a\cos(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-4),x]
```

```
[Out] ((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cos[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cos[x
])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cos[x])^3*Csc[x] - (6*b*(-3*a^2 + 2*b^
2)*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[x])^3*Csc[x])/(a
^2 - b^2)^(3/2))*Sin[x])/(6*a^4*(b + a*Cos[x])^3)
```

Maple [A]
time = 0.34, size = 197, normalized size = 1.24

method	result

default	$-\frac{2\left(\frac{-\frac{\left(2a^3-a^2b-3a\,b^2+2b^3\right)a\left(\tan^5\left(\frac{x}{2}\right)\right)}{2(a+b)}+\frac{2a\left(5a^2-3b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\frac{\left(2a^3+a^2b-3a\,b^2-2b^3\right)a\tan\left(\frac{x}{2}\right)}{2(a-b)}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b\right)^3}+\frac{b\left(3a^2-2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{(a+b)}}\right.}{2\left(a^2-b^2\right)\sqrt{(a+b)\,(a}}\right.}{a^4}$
risch	$\frac{x}{a^4}-\frac{i\left(-15a^4b\,e^{5ix}+18a^2b^3e^{5ix}-12a^5e^{4ix}-27a^3b^2e^{4ix}+54a\,b^4e^{4ix}-48a^4b\,e^{3ix}+34a^2b^3e^{3ix}+44\,e^{3ix}b^5-12a^5e^{2ix}-36a^3b^2e^{2ix}+78a\right.}{3a^4\left(a\,e^{2ix}+2b\,e^{ix}+a\right)^3\left(-a^2+b^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^4*((-1/2*(2*a^3-a^2*b-3*a*b^2+2*b^3)*a/(a+b)*tan(1/2*x)^5+2/3*a*(5*a^2
-3*b^2)*tan(1/2*x)^3-1/2*(2*a^3+a^2*b-3*a*b^2-2*b^3)*a/(a-b)*tan(1/2*x))/(a
*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)^3+1/2*b*(3*a^2-2*b^2)/(a^2-b^2)/((a+b)*(a
-b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2)))+2/a^4*arctan(tan(
1/2*x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(141) = 282$.
time = 3.05, size = 878, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 36*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*x*cos(x)^2 + 36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) + 3*(3*a^2*
b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)*co
s(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(a^2 - b^2)*log((2*a*b*cos(x)
```

```
- (a^2 - 2*b^2)*cos(x)^2 - 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin(x) + 2*a^2
- b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 12*(a^4*b^3 - 2*a^2*b^5 + b^7
)*x + 2*(2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2 + 1
1*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x))/(
a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 + 3*(
a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*
cos(x)), 1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 18*(a^6*b - 2*a^4*
b^3 + a^2*b^5)*x*cos(x)^2 + 18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) - 3*(
3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b
^4)*cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + 6*(a^4*b^3 - 2*a^2*b^5
 + b^7)*x + (2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*b^2
 + 11*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*sin(x
))/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x)^3 +
 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 + a^5*b
^6)*cos(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 282, normalized size = 1.77

$$-\frac{(3\,a^2b-2\,b^3)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{-a^2+b^2}}+\frac{6\,a^4\tan\left(\frac{1}{2}\,x\right)^5-9\,a^3b\tan\left(\frac{1}{2}\,x\right)^5-6\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^5+15\,ab^3\tan\left(\frac{1}{2}\,x\right)^5-6\,b^4\tan\left(\frac{1}{2}\,x\right)^5-20\,a^4\tan\left(\frac{1}{2}\,x\right)^3+32\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3-12\,b^4\tan\left(\frac{1}{2}\,x\right)^3+6\,a^4\tan\left(\frac{1}{2}\,x\right)+9\,a^3b\tan\left(\frac{1}{2}\,x\right)-6\,a^2b^2\tan\left(\frac{1}{2}\,x\right)-15\,ab^3\tan\left(\frac{1}{2}\,x\right)-6\,b^4\tan\left(\frac{1}{2}\,x\right)}{3\,(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}\,x\right)^2-b\tan\left(\frac{1}{2}\,x\right)^2-a-b\right)^3}+\frac{x}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - 2*b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*x) - b*tan(1/2*x))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b
^2)) + 1/3*(6*a^4*tan(1/2*x)^5 - 9*a^3*b*tan(1/2*x)^5 - 6*a^2*b^2*tan(1/2*x
)^5 + 15*a*b^3*tan(1/2*x)^5 - 6*b^4*tan(1/2*x)^5 - 20*a^4*tan(1/2*x)^3 + 32
*a^2*b^2*tan(1/2*x)^3 - 12*b^4*tan(1/2*x)^3 + 6*a^4*tan(1/2*x) + 9*a^3*b*ta
n(1/2*x) - 6*a^2*b^2*tan(1/2*x) - 15*a*b^3*tan(1/2*x) - 6*b^4*tan(1/2*x))/(
(a^5 - a^3*b^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)^3) + x/a^4
```

Mupad [**B**]
time = 8.25, size = 3068, normalized size = 19.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/sin(x) + a*cot(x))^4,x)

[Out] (2*atan((((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))/a^4 - ((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))/a^4)/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 - 10*a^2*b^3 + 3*a^3*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))*1i)/a^4 + (((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))*1i)/a^4)))/a^4 + ((4*tan(x/2)^3*(5*a^2 - 3*b^2))/(3*a^3) - (tan(x/2)*(a + b)*(a*b - 2*a^2 + 2*b^2))/(a^3*b - a^4) + (tan(x/2)^5*(3*a*b^2 + a^2*b - 2*a^3 - 2*b^3))/(a^3*(a + b)))/(3*a*b^2 - tan(x/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3) + 3*a^2*b + tan(x/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(x/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + a^3 + b^3) + (b*atan(((b*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*b*tan(x/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*1i)/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)) + (b*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*b*tan(x/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12

```
*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a
^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^10 - a^4*b^6 +
 3*a^6*b^4 - 3*a^8*b^2)))*1i)/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))
/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 - 10*a^2*b^3 + 3*a^3*b^2))/(a^11*b + a^12
- a^9*b^3 - a^10*b^2) + (b*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*
tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a
^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*((8*(6*a^12*b - 4*a^13 + 4*
a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 -
 a^10*b^2) - (4*b*tan(x/2)*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a
^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((
a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))
*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^
4 - 3*a^8*b^2))))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)) - (b*(3*a^2
- 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^
5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 -
a^7*b^2) - (b*((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3
+ 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4*b*tan(x/2)*(3*a^2
- 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16
*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(
a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b
)^3)^(1/2))/(2*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))))/(2*(a^10 - a^4*b
^6 + 3*a^6*b^4 - 3*a^8*b^2))))*(3*a^2 - 2*b^2)*((a + b)^3*(a - b)^3)^(1/2)*
1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
```

3.292 $\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$

Optimal. Leaf size=100

$$\frac{(a^2-b^2)^2}{4a^5(b+a\cos(x))^4}+\frac{4b(a^2-b^2)}{3a^5(b+a\cos(x))^3}-\frac{a^2-3b^2}{a^5(b+a\cos(x))^2}-\frac{4b}{a^5(b+a\cos(x))}-\frac{\log(b+a\cos(x))}{a^5}$$

```
[Out] 1/4*(a^2-b^2)^2/a^5/(b+a*cos(x))^4+4/3*b*(a^2-b^2)/a^5/(b+a*cos(x))^3+(-a^2
+3*b^2)/a^5/(b+a*cos(x))^2-4*b/a^5/(b+a*cos(x))-ln(b+a*cos(x))/a^5
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4477, 2747, 711}

$$-\frac{4b}{a^5(a\cos(x)+b)}-\frac{\log(a\cos(x)+b)}{a^5}+\frac{(a^2-b^2)^2}{4a^5(a\cos(x)+b)^4}+\frac{4b(a^2-b^2)}{3a^5(a\cos(x)+b)^3}-\frac{a^2-3b^2}{a^5(a\cos(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] (a^2 - b^2)^2/(4*a^5*(b + a*Cos[x])^4) + (4*b*(a^2 - b^2))/(3*a^5*(b + a*Co
s[x])^3) - (a^2 - 3*b^2)/(a^5*(b + a*Cos[x])^2) - (4*b)/(a^5*(b + a*Cos[x])
) - Log[b + a*Cos[x]]/a^5
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = \int \frac{\sin^5(x)}{(b+a\cos(x))^5}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(a^2-x^2)^2}{(b+x)^5}\,dx, x, a\cos(x)\right)}{a^5}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a^2-b^2)^2}{(b+x)^5} - \frac{4b(-a^2+b^2)}{(b+x)^4} - \frac{2(a^2-3b^2)}{(b+x)^3} - \frac{4b}{(b+x)^2} + \frac{1}{b+x}\right)\,dx, x, a\cos(x)\right)}{a^5}$$
$$= \frac{(a^2-b^2)^2}{4a^5(b+a\cos(x))^4} + \frac{4b(a^2-b^2)}{3a^5(b+a\cos(x))^3} - \frac{a^2-3b^2}{a^5(b+a\cos(x))^2} - \frac{4b}{a^5(b+a\cos(x}$$

Mathematica [A]
time = 0.24, size = 138, normalized size = 1.38

$$-\frac{-3a^4+2a^2b^2+25b^4+12b^4\log(b+a\cos(x))+12a^4\cos^4(x)\log(b+a\cos(x))+48a^3b\cos^3(x)(1+\log(b+a\cos(x)))+12a^2\cos^2(x)\left(a^2+9b^2+6b^2\log(b+a\cos(x))\right)+8ab\cos(x)\left(a^2+11b^2+6b^2\log(b+a\cos(x))\right)}{12a^5(b+a\cos(x))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[x] + b*Csc[x])^(-5),x]
```

```
[Out] -1/12*(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cos[x]] + 12*a^4*Cos[
x]^4*Log[b + a*Cos[x]] + 48*a^3*b*Cos[x]^3*(1 + Log[b + a*Cos[x]]) + 12*a^2
*Cos[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cos[x]]) + 8*a*b*Cos[x]*(a^2 + 11*
b^2 + 6*b^2*Log[b + a*Cos[x]]))/(a^5*(b + a*Cos[x])^4)
```

Maple [A]
time = 0.32, size = 107, normalized size = 1.07

method	result
default	$\frac{4b(a^2-b^2)}{3a^5(b+a\cos(x))^3} - \frac{-a^4+2a^2b^2-b^4}{4a^5(b+a\cos(x))^4} - \frac{\ln(b+a\cos(x))}{a^5} - \frac{2a^2-6b^2}{2a^5(b+a\cos(x))^2} - \frac{4b}{a^5(b+a\cos(x))}$
risch	$\frac{ix}{a^5} - \frac{4(6a^3b\,e^{7ix}+3a^4e^{6ix}+27a^2b^2e^{6ix}+22a^3b\,e^{5ix}+44a\,b^3e^{5ix}+3a^4e^{4ix}+56a^2b^2e^{4ix}+25\,e^{4ix}b^4+22a^3b\,e^{3ix}+44a\,b^3e^{3ix}+3a^4e^{2ix}}{3a^5(a\,e^{2ix}+2b\,e^{ix}+a)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cot(x)+b*csc(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*b*(a^2-b^2)/a^5/(b+a*cos(x))^3-1/4*(-a^4+2*a^2*b^2-b^4)/a^5/(b+a*cos(x)
)^4-ln(b+a*cos(x))/a^5-1/2*(2*a^2-6*b^2)/a^5/(b+a*cos(x))^2-4*b/a^5/(b+a*co
s(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(96) = 192.
time = 0.54, size = 497, normalized size = 4.97

$$-\frac{2\left(5a^4b+10a^3b^2+2a^2b^3-6ab^4-3b^5+\frac{(3a^5-17a^4b-6a^3b^2+26a^2b^3+3ab^4-9b^5)\sin(x)^2}{(\cos(x)+1)^2}-\frac{3(4a^5-13a^4b+12a^3b^2+2a^2b^3-8ab^4+3b^5)\sin(x)^4}{(\cos(x)+1)^4}+\frac{3(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)\sin(x)^6}{(\cos(x)+1)^6}\right)}{3\left(a^{10}+2a^9b-a^8b^2-4a^7b^3-a^6b^4+2a^5b^5+a^4b^6-\frac{4(a^{10}-3a^8b^2+3a^6b^4-a^4b^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6(a^{10}-2a^9b-a^8b^2+4a^7b^3-a^6b^4-2a^5b^5+a^4b^6)\sin(x)^4}{(\cos(x)+1)^4}-\frac{4(a^{10}-4a^9b+5a^8b^2-5a^6b^4+4a^5b^5-a^4b^6)\sin(x)^6}{(\cos(x)+1)^6}+\frac{(a^{10}-6a^9b+15a^8b^2-20a^7b^3+15a^6b^4-6a^5b^5+a^4b^6)\sin(x)^8}{(\cos(x)+1)^8}\right)}-\frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

```
[Out] -2/3*(5*a^4*b + 10*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 - 3*b^5 + (3*a^5 - 17*a^4*
b - 6*a^3*b^2 + 26*a^2*b^3 + 3*a*b^4 - 9*b^5)*sin(x)^2/(cos(x) + 1)^2 - 3*(
4*a^5 - 13*a^4*b + 12*a^3*b^2 + 2*a^2*b^3 - 8*a*b^4 + 3*b^5)*sin(x)^4/(cos(
x) + 1)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*sin
(x)^6/(cos(x) + 1)^6)/(a^10 + 2*a^9*b - a^8*b^2 - 4*a^7*b^3 - a^6*b^4 + 2*a
^5*b^5 + a^4*b^6 - 4*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sin(x)^2/(cos
(x) + 1)^2 + 6*(a^10 - 2*a^9*b - a^8*b^2 + 4*a^7*b^3 - a^6*b^4 - 2*a^5*b^5
+ a^4*b^6)*sin(x)^4/(cos(x) + 1)^4 - 4*(a^10 - 4*a^9*b + 5*a^8*b^2 - 5*a^6*
b^4 + 4*a^5*b^5 - a^4*b^6)*sin(x)^6/(cos(x) + 1)^6 + (a^10 - 6*a^9*b + 15*a
^8*b^2 - 20*a^7*b^3 + 15*a^6*b^4 - 6*a^5*b^5 + a^4*b^6)*sin(x)^8/(cos(x) +
1)^8) - log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^5 + log(sin(x)^2/(co
s(x) + 1)^2 + 1)/a^5
```

Fricas [A]
time = 3.12, size = 166, normalized size = 1.66

$$-\frac{48\,a^{3}b\cos\left(x\right)^{3}-3\,a^{4}+2\,a^{2}b^{2}+25\,b^{4}+12\,\left(a^{4}+9\,a^{2}b^{2}\right)\cos\left(x\right)^{2}+8\,\left(a^{3}b+11\,ab^{3}\right)\cos\left(x\right)+12\,\left(a^{4}\cos\left(x\right)^{4}+4\,a^{3}b\cos\left(x\right)^{3}+6\,a^{2}b^{2}\cos\left(x\right)^{2}+4\,ab^{3}\cos\left(x\right)+b^{4}\right)\log\left(a\cos\left(x\right)+b\right)}{12\,\left(a^{9}\cos\left(x\right)^{4}+4\,a^{8}b\cos\left(x\right)^{3}+6\,a^{7}b^{2}\cos\left(x\right)^{2}+4\,a^{6}b^{3}\cos\left(x\right)+a^{5}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

```
[Out] -1/12*(48*a^3*b*cos(x)^3 - 3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*(a^4 + 9*a^2*b^2
)*cos(x)^2 + 8*(a^3*b + 11*a*b^3)*cos(x) + 12*(a^4*cos(x)^4 + 4*a^3*b*cos(x
)^3 + 6*a^2*b^2*cos(x)^2 + 4*a*b^3*cos(x) + b^4)*log(a*cos(x) + b))/(a^9*co
s(x)^4 + 4*a^8*b*cos(x)^3 + 6*a^7*b^2*cos(x)^2 + 4*a^6*b^3*cos(x) + a^5*b^4
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 93, normalized size = 0.93

$$-\frac{\log\left(\left|a\cos\left(x\right)+b\right|\right)}{a^{5}}-\frac{48\,a^{2}b\cos\left(x\right)^{3}+12\,\left(a^{3}+9\,ab^{2}\right)\cos\left(x\right)^{2}+8\,\left(a^{2}b+11\,b^{3}\right)\cos\left(x\right)-\frac{3\,a^{4}-2\,a^{2}b^{2}-25\,b^{4}}{a}}{12\,\left(a\cos\left(x\right)+b\right)^{4}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

```
[Out] -log(abs(a*cos(x) + b))/a^5 - 1/12*(48*a^2*b*cos(x)^3 + 12*(a^3 + 9*a*b^2)*
cos(x)^2 + 8*(a^2*b + 11*b^3)*cos(x) - (3*a^4 - 2*a^2*b^2 - 25*b^4)/a)/((a*
cos(x) + b)^4*a^4)
```

Mupad [B]
time = 3.68, size = 538, normalized size = 5.38

$$\frac{2\,\mathrm{atanh}\left(\frac{32\tan\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\tan\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a^2}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^3}+32}-\frac{64\,b\tan\left(\frac{x}{2}\right)^2}{32\,a-32\,b+32\,b\tan\left(\frac{x}{2}\right)^2-\frac{32\,b^2}{a}+\frac{32\,b^3}{a^2}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^2}}+\frac{32\,b^2\tan\left(\frac{x}{2}\right)^2}{32\,a^2-32\,a\,b-32\,b^2-64\,b^2\tan\left(\frac{x}{2}\right)^2+\frac{32\,b^3}{a}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a}+32\,a\,b\tan\left(\frac{x}{2}\right)^2}\right)}{a^5}-\frac{\frac{2\tan\left(\frac{x}{2}\right)^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{a^4}+\frac{2\,\left(5\,a^4\,b+10\,a^3\,b^2+2\,a^2\,b^3-6\,a\,b^4-3\,b^5\right)}{3\,a^4\,{\left(a-b\right)}^2}+\frac{2\tan\left(\frac{x}{2}\right)^4\,\left(-4\,a^3+5\,a^2\,b+2\,a\,b^2-3\,b^3\right)}{a^4}+\frac{2\tan\left(\frac{x}{2}\right)^2\,\left(3\,a^4-14\,a^3\,b-20\,a^2\,b^2+6\,a\,b^3+9\,b^4\right)}{3\,a^4\,\left(a-b\right)}}{4\,a\,b^3+4\,a^3\,b+{\tan\left(\frac{x}{2}\right)}^4\,\left(6\,a^4-12\,a^2\,b^2+6\,b^4\right)+{\tan\left(\frac{x}{2}\right)}^2\,\left(-4\,a^4-8\,a^3\,b+8\,a\,b^3+4\,b^4\right)-{\tan\left(\frac{x}{2}\right)}^6\,\left(4\,a^4-8\,a^3\,b+8\,a\,b^3-4\,b^4\right)+a^4+b^4+{\tan\left(\frac{x}{2}\right)}^8\,\left(a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4\right)+6\,a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/sin(x) + a*cot(x))^5,x)
```

```
[Out] (2*atanh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*ta
n(x/2)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (64
*b*tan(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 -
 (64*b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)/(32
*a^2 - 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(x/2)^
2)/a + 32*a*b*tan(x/2)^2)))/a^5 - ((2*tan(x/2)^6*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))/a^4 + (2*(5*a^4*b - 6*a*b^4 - 3*b^5 + 2*a^2*b^3 + 10*a^3*b^2))/(3*a^
4*(a - b)^2) + (2*tan(x/2)^4*(2*a*b^2 + 5*a^2*b - 4*a^3 - 3*b^3))/a^4 + (2*
tan(x/2)^2*(6*a*b^3 - 14*a^3*b + 3*a^4 + 9*b^4 - 20*a^2*b^2))/(3*a^4*(a - b
)))/(4*a*b^3 + 4*a^3*b + tan(x/2)^4*(6*a^4 + 6*b^4 - 12*a^2*b^2) + tan(x/2)
^2*(8*a*b^3 - 8*a^3*b - 4*a^4 + 4*b^4) - tan(x/2)^6*(8*a*b^3 - 8*a^3*b + 4*
a^4 - 4*b^4) + a^4 + b^4 + tan(x/2)^8*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^
2*b^2) + 6*a^2*b^2)
```

3.293 $\int (\cot(x)+\csc(x))^5\,dx$

Optimal. Leaf size=28

$$-\frac{2}{(1-\cos(x))^2}+\frac{4}{1-\cos(x)}+\log(1-\cos(x))$$

```
[Out] -2/(1-cos(x))^2+4/(1-cos(x))+ln(1-cos(x))
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4477, 2746, 45}

$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^5,x]
```

```
[Out] -2/(1 - Cos[x])^2 + 4/(1 - Cos[x]) + Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))^5\,dx &= \int (1+\cos(x))^5\csc^5(x)\,dx\\ &= -\text{Subst}\left(\int \frac{(1+x)^2}{(1-x)^3}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{1-x}-\frac{4}{(-1+x)^3}-\frac{4}{(-1+x)^2}\right)dx, x, \cos(x)\right)\\ &= -\frac{2}{(1-\cos(x))^2}+\frac{4}{1-\cos(x)}+\log(1-\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.14

$$2\csc^2\left(\frac{x}{2}\right)-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)+2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^5,x]
```

```
[Out] 2*Csc[x/2]^2 - Csc[x/2]^4/2 + 2*Log[Sin[x/2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.
time = 0.13, size = 105, normalized size = 3.75

method	result
risch	$-ix-\frac{8(\mathrm{e}^{3ix}-\mathrm{e}^{2ix}+\mathrm{e}^{ix})}{(\mathrm{e}^{ix}-1)^4}+2\ln(\mathrm{e}^{ix}-1)$
default	$\left(-\frac{(\csc^3(x))}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\ln(-\cot(x)+\csc(x))-\frac{5}{4\sin(x)^4}-\frac{5(\cos^3(x))}{2\sin(x)^4}-\frac{5(\cos^3(x))}{4\sin(x)^2}+\frac{5\cos(x)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+ln(-cot(x)+csc(x))-5/4/sin(x)^4-5/2/sin(x
)^4*cos(x)^3-5/4/sin(x)^2*cos(x)^3+5/8*cos(x)-5/2/sin(x)^4*cos(x)^4-5/4/sin
(x)^4*cos(x)^5+5/8/sin(x)^2*cos(x)^5+5/8*cos(x)^3-1/4*cot(x)^4+1/2*cot(x)^2
+ln(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(24) = 48.
time = 0.30, size = 125, normalized size = 4.46

$$-\frac{5}{2}\cot(x)^4-\frac{5\left(5\cos(x)^3-3\cos(x)\right)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{3\cos(x)^3-5\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}-\frac{5\left(\cos(x)^3+\cos(x)\right)}{4\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{4\sin(x)^2-1}{4\sin(x)^4}-\frac{5}{4\sin(x)^4}+\frac{1}{2}\log\left(\sin(x)^2\right)-\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -5/2*cot(x)^4 - 5/8*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1
/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 5/4*(cos(x)^3 + co
s(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 - 5/4/sin
(x)^4 + 1/2*log(sin(x)^2) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [A]
time = 3.08, size = 37, normalized size = 1.32

$$\frac{\left(\cos\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-4\,\cos\left(x\right)+2}{\cos\left(x\right)^{2}-2\,\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="fricas")
```

```
[Out] ((cos(x)^2 - 2*cos(x) + 1)*log(-1/2*cos(x) + 1/2) - 4*cos(x) + 2)/(cos(x)^2
 - 2*cos(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.
time = 82.11, size = 68, normalized size = 2.43

$$\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}-\frac{\log\left(\csc^{2}\left(x\right)\right)}{2}-\frac{5\cot^{4}\left(x\right)}{2}-\frac{3\csc^{4}\left(x\right)}{2}+\csc^{2}\left(x\right)-\frac{32\cos^{3}\left(x\right)}{8\cos^{4}\left(x\right)-16\cos^{2}\left(x\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**5,x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 - log(csc(x)**2)/2 - 5*cot(x)**4/2 -
3*csc(x)**4/2 + csc(x)**2 - 32*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8)
```

Giac [A]
time = 0.39, size = 22, normalized size = 0.79

$$-\frac{2\left(2\,\cos\left(x\right)-1\right)}{\left(\cos\left(x\right)-1\right)^{2}}+\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^5,x, algorithm="giac")
```

```
[Out] -2*(2*cos(x) - 1)/(cos(x) - 1)^2 + log(-cos(x) + 1)
```

Mupad [B]
time = 2.42, size = 34, normalized size = 1.21

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+\frac{{\tan\left(\frac{x}{2}\right)}^2-\frac{1}{2}}{{\tan\left(\frac{x}{2}\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^5,x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + (tan(x/2)^2 - 1/2)/tan(x/2)^4
```

3.294 $\int (\cot(x)+\csc(x))^4\,dx$

Optimal. Leaf size=30

$$x+\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}$$

```
[Out] x+2*sin(x)/(1-cos(x))-2/3*sin(x)^3/(1-cos(x))^3
```

Rubi [A]
time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {4477, 2749, 2759, 8}

$$x-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^4,x]
```

```
[Out] x + (2*Sin[x])/(1 - Cos[x]) - (2*Sin[x]^3)/(3*(1 - Cos[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))^4\,dx &= \int (1+\cos(x))^4\csc^4(x)\,dx\\ &= \int \frac{\sin^4(x)}{(1-\cos(x))^4}\,dx\\ &= -\frac{2\sin^3(x)}{3(1-\cos(x))^3}-\int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx\\ &= \frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\int 1\,dx\\ &= x+\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$x+\frac{8}{3}\cot\left(\frac{x}{2}\right)-\frac{2}{3}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^4,x]
[Out] x + (8*Cot[x/2])/3 - (2*Cot[x/2]*Csc[x/2]^2)/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(28) = 56.
time = 0.11, size = 68, normalized size = 2.27

method	result
risch	$x+\frac{8i\left(3\,e^{2ix}-3\,e^{ix}+2\right)}{3\left(e^{ix}-1\right)^3}$
default	$\left(-\frac{2}{3}-\frac{(\csc^2(x))}{3}\right)\cot(x)-\frac{4}{3\sin(x)^3}-\frac{2(\cos^3(x))}{\sin(x)^3}-\frac{4(\cos^4(x))}{3\sin(x)^3}+\frac{4(\cos^4(x))}{3\sin(x)}+\frac{4(2+\cos^2(x))\sin(x)}{3}-\frac{(\cot^3(x))}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^4,x,method=_RETURNVERBOSE)
[Out] (-2/3-1/3*csc(x)^2)*cot(x)-4/3/sin(x)^3-2/sin(x)^3*cos(x)^3-4/3/sin(x)^3*co
s(x)^4+4/3/sin(x)*cos(x)^4+4/3*(2+cos(x)^2)*sin(x)-1/3*cot(x)^3+cot(x)+x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

time = 0.49, size = 56, normalized size = 1.87

$$-2 \cot(x)^3 + x + \frac{4\left(3 \sin(x)^2 - 1\right)}{3 \sin(x)^3} - \frac{3 \tan(x)^2 + 1}{3 \tan(x)^3} + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3} - \frac{4}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*cot(x)^3 + x + 4/3*(3*sin(x)^2 - 1)/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)/tan(
x)^3 + 1/3*(3*tan(x)^2 - 1)/tan(x)^3 - 4/3/sin(x)^3
```

Fricas [A]
time = 1.64, size = 36, normalized size = 1.20

$$\frac{8 \cos(x)^2 + 3\left(x \cos(x) - x\right) \sin(x) + 4 \cos(x) - 4}{3\left(\cos(x) - 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(8*cos(x)^2 + 3*(x*cos(x) - x)*sin(x) + 4*cos(x) - 4)/((cos(x) - 1)*sin
(x))
```

Sympy [A]
time = 25.05, size = 44, normalized size = 1.47

$$x - \frac{7 \cot^3(x)}{3} - \cot(x) - \frac{8 \csc^3(x)}{3} + 4 \csc(x) + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**4,x)
```

```
[Out] x - 7*cot(x)**3/3 - cot(x) - 8*csc(x)**3/3 + 4*csc(x) + cos(x)/sin(x) - cos
(x)**3/(3*sin(x)**3)
```

Giac [A]
time = 0.40, size = 20, normalized size = 0.67

$$x + \frac{2\left(3 \tan\left(\frac{1}{2} x\right)^2 - 1\right)}{3 \tan\left(\frac{1}{2} x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^4,x, algorithm="giac")
```

```
[Out] x + 2/3*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3
```

Mupad [B]
time = 2.41, size = 16, normalized size = 0.53

$$-\frac{2\cot\left(\frac{x}{2}\right)^3}{3}+2\cot\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^4,x)
[Out] x + 2*cot(x/2) - (2*cot(x/2)^3)/3
```

3.295 $\int (\cot(x)+\csc(x))^3\,dx$

Optimal. Leaf size=20

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

```
[Out] -2/(1-cos(x))-ln(1-cos(x))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {4477, 2746, 45}

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^3,x]
```

```
[Out] -2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int (\cot(x)+\csc(x))^3\,dx = \int (1+\cos(x))^3\csc^3(x)\,dx$$
$$= -\text{Subst}\left(\int \frac{1+x}{(1-x)^2}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{2}{(-1+x)^2}+\frac{1}{-1+x}\right)dx, x, \cos(x)\right)$$
$$= -\frac{2}{1-\cos(x)} - \log(1-\cos(x))$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\csc^2\left(\frac{x}{2}\right) - 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^3,x]
```

```
[Out] -Csc[x/2]^2 - 2*Log[Sin[x/2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.
time = 0.10, size = 49, normalized size = 2.45

method	result	si
risch	$ix+\frac{4\,e^{ix}}{(e^{ix}-1)^2}-2\ln\left(e^{ix}-1\right)$	32
default	$-\frac{\cot(x)\csc(x)}{2}-\ln\left(-\cot(x)+\csc(x)\right)-\frac{3}{2\sin(x)^2}-\frac{3(\cos^3(x))}{2\sin(x)^2}-\frac{3\cos(x)}{2}-\frac{(\cot^2(x))}{2}-\ln\left(\sin(x)\right)$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cot(x)*csc(x)-ln(-cot(x)+csc(x))-3/2/sin(x)^2-3/2/sin(x)^2*cos(x)^3-3/
2*cos(x)-1/2*cot(x)^2-ln(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.27, size = 46, normalized size = 2.30

$$-\frac{3}{2}\cot(x)^2+\frac{2\cos(x)}{\cos(x)^2-1}-\frac{1}{2\sin(x)^2}-\frac{1}{2}\log\left(\sin(x)^2\right)+\frac{1}{2}\log\left(\cos(x)+1\right)-\frac{1}{2}\log\left(\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] -3/2*cot(x)^2 + 2*cos(x)/(cos(x)^2 - 1) - 1/2/sin(x)^2 - 1/2*log(sin(x)^2)
+ 1/2*log(cos(x) + 1) - 1/2*log(cos(x) - 1)
```

Fricas [A]
time = 2.06, size = 22, normalized size = 1.10

$$-\frac{(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{\cos(x)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="fricas")
```

```
[Out] -((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.
time = 9.29, size = 46, normalized size = 2.30

$$-\frac{\log(\cos(x)-1)}{2}+\frac{\log(\cos(x)+1)}{2}+\frac{\log\left(-\csc^2(x)\right)}{2}-2\csc^2(x)+\frac{4\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**3,x)
```

```
[Out] -log(cos(x) - 1)/2 + log(cos(x) + 1)/2 + log(-csc(x)**2)/2 - 2*csc(x)**2 +
4*cos(x)/(2*cos(x)**2 - 2)
```

Giac [A]
time = 0.39, size = 18, normalized size = 0.90

$$\frac{2}{\cos(x)-1}-\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^3,x, algorithm="giac")
```

```
[Out] 2/(cos(x) - 1) - log(-cos(x) + 1)
```

Mupad [B]
time = 2.39, size = 25, normalized size = 1.25

$$\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{\tan\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^3,x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2)) - 1/tan(x/2)^2
```

3.296 $\int (\cot(x)+\csc(x))^2\,dx$

Optimal. Leaf size=16

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

[Out] -x-2*sin(x)/(1-cos(x))

Rubi [A]
time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4477, 2749, 2759, 8}

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^2,x]

[Out] -x - (2*Sin[x])/(1 - Cos[x])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int (\cot(x)+\csc(x))^2\,dx &= \int (1+\cos(x))^2\csc^2(x)\,dx\\
&= \int \frac{\sin^2(x)}{(1-\cos(x))^2}\,dx\\
&= -\frac{2\sin(x)}{1-\cos(x)} - \int 1\,dx\\
&= -x - \frac{2\sin(x)}{1-\cos(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 0.75

$$-x-2\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^2,x]
```

```
[Out] -x - 2*Cot[x/2]
```

Maple [A]
time = 0.07, size = 15, normalized size = 0.94

method	result	size
default	$-2\cot\left(x\right)-\frac{2}{\sin\left(x\right)}-x$	15
risch	$-x-\frac{4i}{e^{ix}-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*cot(x)-2/sin(x)-x
```

Maxima [A]
time = 0.49, size = 16, normalized size = 1.00

$$-x-\frac{2}{\sin\left(x\right)}-\frac{2}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/sin(x) - 2/tan(x)
```

Fricas [A]
time = 2.32, size = 16, normalized size = 1.00

$$-\frac{x\sin\left(x\right)+2\,\cos\left(x\right)+2}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="fricas")
[Out] -(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

Sympy [A]
time = 1.45, size = 17, normalized size = 1.06

$$-x-\cot\left(x\right)-2\csc\left(x\right)-\frac{\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))**2,x)
[Out] -x - cot(x) - 2*csc(x) - cos(x)/sin(x)
```

Giac [A]
time = 0.40, size = 12, normalized size = 0.75

$$-x-\frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x))^2,x, algorithm="giac")
[Out] -x - 2/tan(1/2*x)
```

Mupad [B]
time = 2.40, size = 10, normalized size = 0.62

$$-x-2\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x) + 1/sin(x))^2,x)
[Out] - x - 2*cot(x/2)
```

3.297 $\int (\cot(x)+\csc(x))\,dx$

Optimal. Leaf size=9

$$-\tanh^{-1}(\cos(x))+\log(\sin(x))$$

```
[Out] -arctanh(cos(x))+ln(sin(x))
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3556, 3855}

$$\log(\sin(x))-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x] + Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\cot(x)+\csc(x))\,dx &= \int \cot(x)\,dx+\int \csc(x)\,dx\\ &= -\tanh^{-1}(\cos(x))+\log(\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 0.01, size = 20, normalized size = 2.22

$$-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x] + Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.44

method	result	size
default	$\ln(\sin(x)) - \ln(\cot(x) + \csc(x))$	13
norman	$-\frac{\ln(\tan^2(x)+1)}{2} + \ln(\tan(x)) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	19
risch	$-ix + \ln(e^{2ix} - 1) - \ln(e^{ix} + 1) + \ln(e^{ix} - 1)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)+csc(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))-ln(cot(x)+csc(x))
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.33

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="maxima")
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Fricas [A]
time = 2.79, size = 7, normalized size = 0.78

$$\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 0.03, size = 20, normalized size = 2.22

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + log(sin(x))
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.22

$$\log\left(|\sin\left(x\right)|\right) + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)+csc(x),x, algorithm="giac")
[Out] log(abs(sin(x))) + log(abs(tan(1/2*x)))
```

Mupad [B]
time = 2.41, size = 19, normalized size = 2.11

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x) + 1/sin(x),x)
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1)
```

3.298 $\int \frac{1}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=7

$$-\log(1+\cos(x))$$

[Out] -ln(cos(x)+1)

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3239, 2746, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Cot[x] + Csc[x])^(-1),x]

[Out] -Log[1 + Cos[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot(x)+\csc(x)}\,dx &= \int \frac{\sin(x)}{1+\cos(x)}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right) \\ &= -\log(1+\cos(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-1),x]
[Out] -2*Log[Cos[x/2]]
```

Maple [A]
time = 0.14, size = 8, normalized size = 1.14

method	result	size
default	$-\ln\left(1+\cos\left(x\right)\right)$	8
risch	$ix-2\ln\left(\mathrm{e}^{ix}+1\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+cos(x))
```

Maxima [A]
time = 0.49, size = 14, normalized size = 2.00

$$\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="maxima")
[Out] log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 3.07, size = 9, normalized size = 1.29

$$-\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -log(1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot(x) + \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x)
```

```
[Out] Integral(1/(cot(x) + csc(x)), x)
```

Giac [A]
time = 0.41, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -log(cos(x) + 1)
```

Mupad [B]
time = 2.86, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1)
```

3.299 $\int \frac{1}{(\cot(x)+\csc(x))^2} dx$

Optimal. Leaf size=14

$$-x + \frac{2\sin(x)}{1+\cos(x)}$$

```
[Out] -x+2*sin(x)/(cos(x)+1)
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4477, 2759, 8}

$$\frac{2\sin(x)}{\cos(x)+1} - x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-2),x]
```

```
[Out] -x + (2*Sin[x])/(1 + Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx$$
$$= \frac{2\sin(x)}{1+\cos(x)} - \int 1\,dx$$
$$= -x + \frac{2\sin(x)}{1+\cos(x)}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 0.86

$$-x + 2\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-2),x]
```

```
[Out] -x + 2*Tan[x/2]
```

Maple [A]
time = 0.15, size = 15, normalized size = 1.07

method	result	size
default	$2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$-x+\frac{4i}{e^{ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*tan(1/2*x)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.49, size = 23, normalized size = 1.64

$$\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1} - 2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="maxima")
```

```
[Out] 2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.43, size = 18, normalized size = 1.29

$$-\frac{x\cos\left(x\right)+x-2\,\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="fricas")
[Out] -(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\cot\left(x\right)+\csc\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**2,x)
[Out] Integral((cot(x) + csc(x))**(-2), x)
```

Giac [A]
time = 0.38, size = 10, normalized size = 0.71

$$-x+2\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^2,x, algorithm="giac")
[Out] -x + 2*tan(1/2*x)
```

Mupad [B]
time = 2.41, size = 10, normalized size = 0.71

$$2\tan\left(\frac{x}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^2,x)
[Out] 2*tan(x/2) - x
```

3.300 $\int \frac{1}{(\cot(x)+\csc(x))^3} dx$

Optimal. Leaf size=14

$$\frac{2}{1+\cos(x)}+\log(1+\cos(x))$$

```
[Out] 2/(cos(x)+1)+ln(cos(x)+1)
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4477, 2746, 45}

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-3),x]
```

```
[Out] 2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \int \frac{\sin^3(x)}{(1+\cos(x))^3}\,dx$$
$$= -\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, \cos(x)\right)$$
$$= \frac{2}{1+\cos(x)}+\log(1+\cos(x))$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.29

$$2\log\left(\cos\left(\frac{x}{2}\right)\right)+\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-3),x]
[Out] 2*Log[Cos[x/2]] + Sec[x/2]^2
```

Maple [A]
time = 0.18, size = 15, normalized size = 1.07

method	result	size
default	$\frac{2}{1+\cos(x)}+\ln(1+\cos(x))$	15
risch	$-ix+\frac{4\,e^{ix}}{(e^{ix}+1)^2}+2\ln(e^{ix}+1)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/(1+cos(x))+ln(1+cos(x))
```

Maxima [A]
time = 0.48, size = 28, normalized size = 2.00

$$\frac{\sin(x)^2}{(\cos(x)+1)^2}-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="maxima")
```

```
[Out] sin(x)^2/(cos(x) + 1)^2 - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [**A**]
time = 2.93, size = 21, normalized size = 1.50

$$\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="fricas")
[Out] ((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(\cot(x)+\csc(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**3,x)
[Out] Integral((cot(x) + csc(x))**(-3), x)
```

Giac [**A**]
time = 0.39, size = 14, normalized size = 1.00

$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^3,x, algorithm="giac")
[Out] 2/(cos(x) + 1) + log(cos(x) + 1)
```

Mupad [**B**]
time = 2.35, size = 18, normalized size = 1.29

$$\tan\left(\frac{x}{2}\right)^2-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^3,x)
[Out] tan(x/2)^2 - log(tan(x/2)^2 + 1)
```

3.301 $\int \frac{1}{(\cot(x)+\csc(x))^4} dx$

Optimal. Leaf size=26

$$x - \frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3}$$

```
[Out] x-2*sin(x)/(cos(x)+1)+2/3*sin(x)^3/(cos(x)+1)^3
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4477, 2759, 8}

$$x + \frac{2\sin^3(x)}{3(\cos(x)+1)^3} - \frac{2\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-4),x]
```

```
[Out] x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx &= \int \frac{\sin^4(x)}{(1+\cos(x))^4}\,dx \\ &= \frac{2\sin^3(x)}{3(1+\cos(x))^3} - \int \frac{\sin^2(x)}{(1+\cos(x))^2}\,dx \\ &= -\frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3} + \int 1\,dx \\ &= x - \frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.15

$$x - \frac{8}{3}\tan\left(\frac{x}{2}\right) + \frac{2}{3}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-4),x]
[Out] x - (8*Tan[x/2])/3 + (2*Sec[x/2]^2*Tan[x/2])/3
```

Maple [A]
time = 0.17, size = 23, normalized size = 0.88

method	result	size
default	$\frac{2\left(\tan^3\left(\frac{x}{2}\right)\right)}{3} - 2\tan\left(\frac{x}{2}\right) + 2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	23
risch	$x - \frac{8i\left(3\,e^{2ix}+3\,e^{ix}+2\right)}{3\left(e^{ix}+1\right)^3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^4,x,method=_RETURNVERBOSE)
[Out] 2/3*tan(1/2*x)^3-2*tan(1/2*x)+2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.49, size = 35, normalized size = 1.35

$$-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^3}{3\left(\cos(x)+1\right)^3} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="maxima")
```

```
[Out] -2*sin(x)/(cos(x) + 1) + 2/3*sin(x)^3/(cos(x) + 1)^3 + 2*arctan(sin(x)/(cos
(x) + 1))
```

Fricas [A]
time = 2.54, size = 40, normalized size = 1.54

$$\frac{3\,x\cos\left(x\right)^{2}+6\,x\cos\left(x\right)-4\,\left(2\,\cos\left(x\right)+1\right)\sin\left(x\right)+3\,x}{3\,\left(\cos\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*x*cos(x)^2 + 6*x*cos(x) - 4*(2*cos(x) + 1)*sin(x) + 3*x)/(cos(x)^2 +
 2*cos(x) + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 16, normalized size = 0.62

$$\frac{2}{3}\,\tan\left(\frac{1}{2}\,x\right)^{3}+x-2\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^4,x, algorithm="giac")
```

```
[Out] 2/3*tan(1/2*x)^3 + x - 2*tan(1/2*x)
```

Mupad [B]
time = 2.37, size = 16, normalized size = 0.62

$$\frac{2\tan\left(\frac{x}{2}\right)^{3}}{3}-2\tan\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^4,x)
```

```
[Out] x - 2*tan(x/2) + (2*tan(x/2)^3)/3
```

3.302 $\int \frac{1}{(\cot(x)+\csc(x))^5} \, dx$

Optimal. Leaf size=24

$$\frac{2}{(1+\cos(x))^2} - \frac{4}{1+\cos(x)} - \log(1+\cos(x))$$

```
[Out] 2/(cos(x)+1)^2-4/(cos(x)+1)-ln(cos(x)+1)
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4477, 2746, 45}

$$-\frac{4}{\cos(x)+1} + \frac{2}{(\cos(x)+1)^2} - \log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x])^(-5),x]
```

```
[Out] 2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx &= \int \frac{\sin^5(x)}{(1+\cos(x))^5}\,dx \\
&= -\text{Subst}\left(\int \frac{(1-x)^2}{(1+x)^3}\,dx, x, \cos(x)\right) \\
&= -\text{Subst}\left(\int \left(\frac{4}{(1+x)^3}-\frac{4}{(1+x)^2}+\frac{1}{1+x}\right)dx, x, \cos(x)\right) \\
&= \frac{2}{(1+\cos(x))^2}-\frac{4}{1+\cos(x)}-\log(1+\cos(x))
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.33

$$
-2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\sec^4\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x])^(-5),x]
[Out] -2*Log[Cos[x/2]] - 2*Sec[x/2]^2 + Sec[x/2]^4/2
```

Maple [A]
time = 0.20, size = 25, normalized size = 1.04

method	result	size
default	$\frac{2}{(1+\cos(x))^2}-\frac{4}{1+\cos(x)}-\ln(1+\cos(x))$	25
risch	$ix-\frac{8(e^{3ix}+e^{2ix}+e^{ix})}{(e^{ix}+1)^4}-2\ln(e^{ix}+1)$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)+csc(x))^5,x,method=_RETURNVERBOSE)
[Out] 2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))
```

Maxima [A]
time = 0.50, size = 39, normalized size = 1.62

$$
-\frac{\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^4}{2(\cos(x)+1)^4}+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="maxima")
```

```
[Out] -sin(x)^2/(cos(x) + 1)^2 + 1/2*sin(x)^4/(cos(x) + 1)^4 + log(sin(x)^2/(cos(
x) + 1)^2 + 1)
```

Fricas [A]
time = 5.47, size = 38, normalized size = 1.58

$$-\frac{\left(\cos\left(x\right)^2+2\,\cos\left(x\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+4\,\cos\left(x\right)+2}{\cos\left(x\right)^2+2\,\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="fricas")
[Out] -((cos(x)^2 + 2*cos(x) + 1)*log(1/2*cos(x) + 1/2) + 4*cos(x) + 2)/(cos(x)^2
 + 2*cos(x) + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))**5,x)
[Out] Timed out
```

Giac [A]
time = 0.40, size = 22, normalized size = 0.92

$$-\frac{2\left(2\,\cos\left(x\right)+1\right)}{\left(\cos\left(x\right)+1\right)^2}-\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)+csc(x))^5,x, algorithm="giac")
[Out] -2*(2*cos(x) + 1)/(cos(x) + 1)^2 - log(cos(x) + 1)
```

Mupad [B]
time = 2.41, size = 26, normalized size = 1.08

$$\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+1\right)-{\mathrm{tan}\left(\frac{x}{2}\right)}^2+\frac{{\mathrm{tan}\left(\frac{x}{2}\right)}^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x) + 1/sin(x))^5,x)
[Out] log(tan(x/2)^2 + 1) - tan(x/2)^2 + tan(x/2)^4/2
```

3.303 $\int (\csc(x) - \sin(x))^4 \, dx$

Optimal. Leaf size=44

$$\frac{35x}{8} + \frac{35\cot(x)}{8} - \frac{35\cot^3(x)}{24} + \frac{7}{8}\cos^2(x)\cot^3(x) + \frac{1}{4}\cos^4(x)\cot^3(x)$$

[Out] 35/8*x+35/8*cot(x)-35/24*cot(x)^3+7/8*cos(x)^2*cot(x)^3+1/4*cos(x)^4*cot(x)^3

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {296, 331, 209}

$$\frac{35x}{8} - \frac{35\cot^3(x)}{24} + \frac{35\cot(x)}{8} + \frac{1}{4}\cos^4(x)\cot^3(x) + \frac{7}{8}\cos^2(x)\cot^3(x)$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^4,x]

[Out] (35*x)/8 + (35*Cot[x])/8 - (35*Cot[x]^3)/24 + (7*Cos[x]^2*Cot[x]^3)/8 + (Cos[x]^4*Cot[x]^3)/4

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int (\csc(x)-\sin(x))^4\,dx &= \text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^3}\,dx,x,\tan(x)\right)\\
&= \frac{1}{4}\cos^4(x)\cot^3(x)+\frac{7}{4}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx,x,\tan(x)\right)\\
&= \frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{35}{24}\cot^3(x)+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)-\frac{35}{8}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,d\right.\\
&= \frac{35\cot(x)}{8}-\frac{35\cot^3(x)}{24}+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{}{1}\right.\\
&= \frac{35x}{8}+\frac{35\cot(x)}{8}-\frac{35\cot^3(x)}{24}+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.86

$$\frac{35x}{8}+\frac{10\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)+\frac{3}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^4,x]
```

```
[Out] (35*x)/8 + (10*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (3*Sin[2*x])/4 + Sin[4*x]/
32
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.89

method	result
default	$-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{35x}{8}+2\cos(x)\sin(x)+4\cot(x)+\left(-\frac{2}{3}-\frac{\left(\csc^2(x)\right)}{3}\right)\cot(x)$
risch	$\frac{35x}{8}-\frac{i e^{4ix}}{64}-\frac{3i e^{2ix}}{8}+\frac{3i e^{-2ix}}{8}+\frac{i e^{-4ix}}{64}+\frac{4i\left(6\,e^{4ix}-9\,e^{2ix}+5\right)}{3\left(e^{2ix}-1\right)^3}$
norman	$\frac{-\frac{1}{24}+\frac{35\left(\tan^2\left(\frac{x}{2}\right)\right)}{24}+\frac{63\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{35\left(\tan^6\left(\frac{x}{2}\right)\right)}{8}-\frac{35\left(\tan^8\left(\frac{x}{2}\right)\right)}{8}-\frac{63\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{8}-\frac{35\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{24}+\frac{35x\left(\tan^3\left(\frac{x}{2}\right)\right)}{8}+\frac{35}{}}{\tan\left(\frac{x}{2}\right)^3\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+35/8*x+2*cos(x)*sin(x)+4*cot(x)+(-2/3-1/3
*csc(x)^2)*cot(x)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.82

$$\frac{35}{8}\,x+\frac{4}{\tan\left(x\right)}-\frac{3\,\tan\left(x\right)^{2}+1}{3\,\tan\left(x\right)^{3}}+\frac{1}{32}\,\sin\left(4\,x\right)+\frac{3}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="maxima")
```

```
[Out] 35/8*x + 4/tan(x) - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/32*sin(4*x) + 3/4*sin
(2*x)
```

Fricas [A]
time = 2.77, size = 51, normalized size = 1.16

$$-\frac{6\,\cos\left(x\right)^{7}+21\,\cos\left(x\right)^{5}-140\,\cos\left(x\right)^{3}-105\,\left(x\cos\left(x\right)^{2}-x\right)\sin\left(x\right)+105\,\cos\left(x\right)}{24\,\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(x)^7 + 21*cos(x)^5 - 140*cos(x)^3 - 105*(x*cos(x)^2 - x)*sin(x
) + 105*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [A]
time = 4.96, size = 44, normalized size = 1.00

$$\frac{35x}{8}+2\sin\left(x\right)\cos\left(x\right)-\frac{\sin\left(2x\right)}{4}+\frac{\sin\left(4x\right)}{32}-\frac{\cot^{3}\left(x\right)}{3}-\cot\left(x\right)+\frac{4\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**4,x)
```

```
[Out] 35*x/8 + 2*sin(x)*cos(x) - sin(2*x)/4 + sin(4*x)/32 - cot(x)**3/3 - cot(x)
+ 4*cos(x)/sin(x)
```

Giac [A]
time = 0.39, size = 39, normalized size = 0.89

$$\frac{35}{8}\,x+\frac{11\,\tan\left(x\right)^{3}+13\,\tan\left(x\right)}{8\,\left(\tan\left(x\right)^{2}+1\right)^{2}}+\frac{9\,\tan\left(x\right)^{2}-1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^4,x, algorithm="giac")
```

```
[Out] 35/8*x + 1/8*(11*tan(x)^3 + 13*tan(x))/(tan(x)^2 + 1)^2 + 1/3*(9*tan(x)^2 -
 1)/tan(x)^3
```

Mupad [B]
time = 2.49, size = 59, normalized size = 1.34

$$\frac{\frac{\cos(x)^7}{4}+\frac{7\cos(x)^5}{8}-\frac{35\cos(x)^3}{6}+\frac{35\cos(x)}{8}}{\sin(x)-\cos(x)^2\sin(x)}-\frac{\frac{35x}{8}-\frac{35x\cos(x)^2}{8}}{\cos(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) - 1/sin(x))^4,x)
```

```
[Out] ((35*cos(x))/8 - (35*cos(x)^3)/6 + (7*cos(x)^5)/8 + cos(x)^7/4)/(sin(x) - c
os(x)^2*sin(x)) - ((35*x)/8 - (35*x*cos(x)^2)/8)/(cos(x)^2 - 1)
```

3.304 $\int (\csc(x)-\sin(x))^3\,dx$

Optimal. Leaf size=34

$$\frac{5}{2}\tanh^{-1}(\cos(x))-\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)$$

```
[Out] 5/2*arctanh(cos(x))-5/2*cos(x)-5/6*cos(x)^3-1/2*cos(x)^3*cot(x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.556$, Rules used = {4482, 2672, 294, 308, 212}

$$-\frac{5\cos^3(x)}{6}-\frac{5\cos(x)}{2}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^3,x]
[Out] (5*ArcTanh[Cos[x]])/2 - (5*Cos[x])/2 - (5*Cos[x]^3)/6 - (Cos[x]^3*Cot[x]^2)
/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (\csc(x)-\sin(x))^3\,dx &= \int \cos^3(x)\cot^3(x)\,dx \\
&= -\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(x)\right) \\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(x)\right) \\
&= -\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cos(x)\right) \\
&= -\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)+\frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right) \\
&= \frac{5}{2}\tanh^{-1}(\cos(x))-\frac{5\cos(x)}{2}-\frac{5\cos^3(x)}{6}-\frac{1}{2}\cos^3(x)\cot^2(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.79

$$-\frac{9\cos(x)}{4}-\frac{1}{12}\cos(3x)-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{5}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{5}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^3,x]
[Out] (-9*Cos[x])/4 - Cos[3*x]/12 - Csc[x/2]^2/8 + (5*Log[Cos[x/2]])/2 - (5*Log[S
in[x/2]])/2 + Sec[x/2]^2/8
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.94

method	result	size
default	$\frac{(2+\sin^2(x))\cos(x)}{3}-3\cos(x)-\frac{5\ln(-\cot(x)+\csc(x))}{2}-\frac{\cot(x)\csc(x)}{2}$	32
norman	$\frac{-\frac{1}{8}-\frac{25\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}-\frac{5\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{55\left(\tan^8\left(\frac{x}{2}\right)\right)}{24}+\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{8}}{\tan\left(\frac{x}{2}\right)^2\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$	60

risch	$-\frac{e^{3ix}}{24}-\frac{9\,e^{ix}}{8}-\frac{9\,e^{-ix}}{8}-\frac{e^{-3ix}}{24}+\frac{e^{3ix}+e^{ix}}{(e^{2ix}-1)^2}-\frac{5\ln(e^{ix}-1)}{2}+\frac{5\ln(e^{ix}+1)}{2}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/3*(2+sin(x)^2)*cos(x)-3*cos(x)-5/2*ln(-cot(x)+csc(x))-1/2*cot(x)*csc(x)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.09

$$-\frac{1}{3}\cos(x)^3+\frac{\cos(x)}{2\left(\cos(x)^2-1\right)}-2\cos(x)+\frac{5}{4}\log(\cos(x)+1)-\frac{5}{4}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="maxima")
[Out] -1/3*cos(x)^3 + 1/2*cos(x)/(cos(x)^2 - 1) - 2*cos(x) + 5/4*log(cos(x) + 1)
- 5/4*log(cos(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.
time = 2.07, size = 57, normalized size = 1.68

$$-\frac{4\cos(x)^5+20\cos(x)^3-15\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-30\cos(x)}{12\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="fricas")
[Out] -1/12*(4*cos(x)^5 + 20*cos(x)^3 - 15*(cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) +
 15*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 30*cos(x))/(cos(x)^2 - 1)
```

Sympy [A]
time = 2.12, size = 42, normalized size = 1.24

$$-\frac{5\log(\cos(x)-1)}{4}+\frac{5\log(\cos(x)+1)}{4}-\frac{\cos^3(x)}{3}-2\cos(x)+\frac{\cos(x)}{2\cos^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**3,x)
[Out] -5*log(cos(x) - 1)/4 + 5*log(cos(x) + 1)/4 - cos(x)**3/3 - 2*cos(x) + cos(x
)/(2*cos(x)**2 - 2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(26) = 52$.
time = 0.41, size = 99, normalized size = 2.91

$$\frac{\left(\frac{10\,(\cos(x)-1)}{\cos(x)+1}+1\right)(\cos(x)+1)}{8\,(\cos(x)-1)}-\frac{\cos(x)-1}{8\,(\cos(x)+1)}-\frac{2\left(\frac{12\,(\cos(x)-1)}{\cos(x)+1}-\frac{9\,(\cos(x)-1)^2}{(\cos(x)+1)^2}-7\right)}{3\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)^3}-\frac{5}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/8*(10*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) - 2/3*(12*(cos(x) - 1)/(cos(x) + 1) - 9*(cos(x) - 1)^
2/(cos(x) + 1)^2 - 7)/((cos(x) - 1)/(cos(x) + 1) - 1)^3 - 5/4*log(-(cos(x)
- 1)/(cos(x) + 1))
```

Mupad [B]
time = 2.49, size = 75, normalized size = 2.21

$$\frac{\tan\left(\frac{x}{2}\right)^2}{8}-\frac{\frac{49\tan\left(\frac{x}{2}\right)^6}{8}+\frac{67\tan\left(\frac{x}{2}\right)^4}{8}+\frac{121\tan\left(\frac{x}{2}\right)^2}{24}+\frac{1}{8}}{\tan\left(\frac{x}{2}\right)^8+3\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+\tan\left(\frac{x}{2}\right)^2}-\frac{5\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 1/sin(x))^3,x)
```

```
[Out] tan(x/2)^2/8 - ((121*tan(x/2)^2)/24 + (67*tan(x/2)^4)/8 + (49*tan(x/2)^6)/8
 + 1/8)/(tan(x/2)^2 + 3*tan(x/2)^4 + 3*tan(x/2)^6 + tan(x/2)^8) - (5*log(ta
n(x/2)))/2
```

3.305 $\int (\csc(x) - \sin(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} - \frac{3\cot(x)}{2} + \frac{1}{2}\cos^2(x)\cot(x)$$

```
[Out] -3/2*x-3/2*cot(x)+1/2*cos(x)^2*cot(x)
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {296, 331, 209}

$$-\frac{3x}{2} - \frac{3\cot(x)}{2} + \frac{1}{2}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[x])/2 + (Cos[x]^2*Cot[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int (\csc(x)-\sin(x))^2\,dx = \text{Subst}\left(\int \frac{1}{x^2(1+x^2)^2}\,dx,x,\tan(x)\right)$$
$$= \frac{1}{2}\cos^2(x)\cot(x)+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x^2)}\,dx,x,\tan(x)\right)$$
$$= -\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)$$
$$= -\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.82

$$-\frac{3x}{2}-\cot(x)-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^2,x]
[Out] (-3*x)/2 - Cot[x] - Sin[2*x]/4
```

Maple [A]
time = 0.08, size = 15, normalized size = 0.68

method	result	size
default	$-\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}-\cot(x)$	15
risch	$-\frac{3x}{2}+\frac{i\,e^{2ix}}{8}-\frac{i\,e^{-2ix}}{8}-\frac{2i}{e^{2ix}-1}$	33
norman	$\frac{-\frac{1}{2}-\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-\frac{3x\tan\left(\frac{x}{2}\right)}{2}-3x\left(\tan^3\left(\frac{x}{2}\right)\right)-\frac{3x\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*cos(x)*sin(x)-3/2*x-cot(x)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.73

$$-\frac{3}{2}\,x-\frac{1}{\tan\left(x\right)}-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/tan(x) - 1/4*sin(2*x)
```

Fricas [A]
time = 3.89, size = 20, normalized size = 0.91

$$\frac{\cos(x)^3 - 3\,x\sin(x) - 3\,\cos(x)}{2\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)^3 - 3*x*sin(x) - 3*cos(x))/sin(x)
```

Sympy [A]
time = 0.91, size = 15, normalized size = 0.68

$$-\frac{3x}{2} - \frac{\sin(2x)}{4} - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**2,x)
```

```
[Out] -3*x/2 - sin(2*x)/4 - cot(x)
```

Giac [A]
time = 0.40, size = 23, normalized size = 1.05

$$-\frac{3}{2}\,x - \frac{3\,\tan(x)^2 + 2}{2\left(\tan(x)^3 + \tan(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] -3/2*x - 1/2*(3*tan(x)^2 + 2)/(tan(x)^3 + tan(x))
```

Mupad [B]
time = 2.40, size = 21, normalized size = 0.95

$$-\frac{3\,x}{2} - \frac{\frac{3\,\cos(x)}{2} - \frac{\cos(x)^3}{2}}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) - 1/sin(x))^2,x)
```

```
[Out] - (3*x)/2 - ((3*cos(x))/2 - cos(x)^3/2)/sin(x)
```

3.306 $\int (\csc(x)-\sin(x))\,dx$

Optimal. Leaf size=8

$$-\tanh^{-1}(\cos(x))+\cos(x)$$

[Out] -arctanh(cos(x))+cos(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3855, 2718}

$$\cos(x)-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x] - Sin[x],x]

[Out] -ArcTanh[Cos[x]] + Cos[x]

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\csc(x)-\sin(x))\,dx &= \int \csc(x)\,dx-\int \sin(x)\,dx\\ &= -\tanh^{-1}(\cos(x))+\cos(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 0.00, size = 19, normalized size = 2.38

$$\cos(x)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x] - Sin[x],x]

[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]

Maple [A]
time = 0.05, size = 12, normalized size = 1.50

method	result	size
default	$\cos(x) - \ln(\cot(x) + \csc(x))$	12
norman	$\frac{2}{1+\tan^2(\frac{x}{2})} + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	19
risch	$-\ln(e^{ix}+1) + \ln(e^{ix}-1) + \frac{e^{ix}}{2} + \frac{e^{-ix}}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)-sin(x),x,method=_RETURNVERBOSE)

[Out] cos(x)-ln(cot(x)+csc(x))

Maxima [A]
time = 0.29, size = 11, normalized size = 1.38

$$\cos(x) - \log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x, algorithm="maxima")

[Out] cos(x) - log(cot(x) + csc(x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 3.42, size = 21, normalized size = 2.62

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x, algorithm="fricas")

[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.
time = 0.03, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)-sin(x),x)

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Giac [A]
time = 0.39, size = 9, normalized size = 1.12

$$\cos(x) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)-sin(x),x, algorithm="giac")
```

```
[Out] cos(x) + log(abs(tan(1/2*x)))
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x) - sin(x),x)
```

```
[Out] log(tan(x/2)) + cos(x)
```

3.307 $\int \frac{1}{\csc(x)-\sin(x)}\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

[Out] sec(x)

Rubi [A]
time = 0.01, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-1),x]

[Out] Sec[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\csc(x)-\sin(x)}\,dx &= \int \sec(x)\tan(x)\,dx \\ &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-1),x]
```

```
[Out] Sec[x]
```

Maple [A]
time = 0.13, size = 5, normalized size = 2.50

method	result	size
default	$\frac{1}{\cos(x)}$	5
norman	$-\frac{2}{\tan^2(\frac{x}{2})-1}$	13
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(2) = 4$.
time = 0.27, size = 17, normalized size = 8.50

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 - 1)
```

Fricas [A]
time = 2.50, size = 4, normalized size = 2.00

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-\sin(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x)
```

```
[Out] Integral(1/(-sin(x) + csc(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(2) = 4$.
time = 0.40, size = 17, normalized size = 8.50

$$\frac{2}{\frac{\cos(x)-1}{\cos(x)+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x)),x, algorithm="giac")
```

```
[Out] 2/((cos(x) - 1)/(cos(x) + 1) + 1)
```

Mupad [B]
time = 2.46, size = 12, normalized size = 6.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x)),x)
```

```
[Out] -2/(tan(x/2)^2 - 1)
```

3.308 $\int \frac{1}{(\csc(x)-\sin(x))^2} dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

[Out] 1/3*tan(x)^3

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(Csc[x] - Sin[x])^(-2),x]

[Out] Tan[x]^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(\csc(x)-\sin(x))^2} dx &= \text{Subst}\left(\int x^2 dx, x, \tan(x)\right) \\ &= \frac{\tan^3(x)}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[x] - Sin[x])^(-2),x]

[Out] Tan[x]^3/3

Maple [A]
time = 0.16, size = 7, normalized size = 0.88

method	result	size
default	$\frac{(\tan^3(x))}{3}$	7
norman	$-\frac{8(\tan^3(\frac{x}{2}))}{3(\tan^2(\frac{x}{2})-1)^3}$	19
risch	$-\frac{2i(3\,\mathrm{e}^{4ix}+1)}{3(\mathrm{e}^{2ix}+1)^3}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(x)^3
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 2.64, size = 14, normalized size = 1.75

$$-\frac{(\cos(x)^2-1)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-\sin(x)+\csc(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**2,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-2), x)
```

Giac [A]
time = 0.39, size = 6, normalized size = 0.75

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3
```

Mupad [B]
time = 2.41, size = 6, normalized size = 0.75

$$\frac{\tan(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^2,x)
```

```
[Out] tan(x)^3/3
```

3.309 $\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx$

Optimal. Leaf size=17

$$-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

```
[Out] -1/3*sec(x)^3+1/5*sec(x)^5
```

Rubi [A]

time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 14}

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3),x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx = \int \sec^3(x)\tan^3(x)\,dx$$
$$= \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \sec(x)\right)$$
$$= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

<u>**Mathematica**</u> [**A**]
time = 0.02, size = 17, normalized size = 1.00

$$-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3),x]
```

```
[Out] -1/3*Sec[x]^3 + Sec[x]^5/5
```

<u>**Maple**</u> [**A**]
time = 0.20, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{3\cos(x)^3}+\frac{1}{5\cos(x)^5}$	14
risch	$-\frac{8(5\,e^{7ix}-2\,e^{5ix}+5\,e^{3ix})}{15(e^{2ix}+1)^5}$	34
norman	$\frac{-2(\tan^8(\frac{x}{2}))-\frac{16(\tan^4(\frac{x}{2}))}{3}+\frac{2(\tan^2(\frac{x}{2}))}{3}+\frac{2(\tan^{10}(\frac{x}{2}))}{5}-\frac{2}{15}}{(\tan^2(\frac{x}{2})-1)^5}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/cos(x)^3+1/5/cos(x)^5
```

<u>**Maxima**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(13) = 26.
time = 0.28, size = 103, normalized size = 6.06

$$-\frac{4\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{15\,\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{15\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2}-\frac{10\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{10\,\sin(x)^6}{(\cos(x)+1)^6}-\frac{5\,\sin(x)^8}{(\cos(x)+1)^8}+\frac{\sin(x)^{10}}{(\cos(x)+1)^{10}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="maxima")
```

```
[Out] -4/15*(5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^4/(cos(x) + 1)^4 + 15*sin(x)^6/
(cos(x) + 1)^6 - 1)/(5*sin(x)^2/(cos(x) + 1)^2 - 10*sin(x)^4/(cos(x) + 1)^4
 + 10*sin(x)^6/(cos(x) + 1)^6 - 5*sin(x)^8/(cos(x) + 1)^8 + sin(x)^10/(cos(
x) + 1)^10 - 1)
```

Fricas [A]
time = 2.43, size = 14, normalized size = 0.82

$$-\frac{5\cos\left(x\right)^{2}-3}{15\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**3,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.
time = 0.41, size = 59, normalized size = 3.47

$$-\frac{4\left(\frac{5\left(\cos\left(x\right)-1\right)}{\cos\left(x\right)+1}-\frac{5\left(\cos\left(x\right)-1\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+\frac{15\left(\cos\left(x\right)-1\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+1\right)}{15\left(\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^3,x, algorithm="giac")
```

```
[Out] -4/15*(5*(cos(x) - 1)/(cos(x) + 1) - 5*(cos(x) - 1)^2/(cos(x) + 1)^2 + 15*(
cos(x) - 1)^3/(cos(x) + 1)^3 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^5
```

Mupad [B]
time = 2.55, size = 13, normalized size = 0.76

$$\frac{1}{5\cos\left(x\right)^{5}}-\frac{1}{3\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^3,x)
[Out] 1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

3.310 $\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^7(x)}{7}$$

```
[Out] 1/5*tan(x)^5+1/7*tan(x)^7
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] Tan[x]^5/5 + Tan[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx &= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,\tan(x)\right)\\ &= \frac{\tan^5(x)}{5}+\frac{\tan^7(x)}{7}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 0.01, size = 37, normalized size = 2.18

$$\frac{2\tan(x)}{35}+\frac{1}{35}\sec^2(x)\tan(x)-\frac{8}{35}\sec^4(x)\tan(x)+\frac{1}{7}\sec^6(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-4),x]
```

```
[Out] (2*Tan[x])/35 + (Sec[x]^2*Tan[x])/35 - (8*Sec[x]^4*Tan[x])/35 + (Sec[x]^6*T
an[x])/7
```

Maple [A]
time = 0.25, size = 14, normalized size = 0.82

method	result	size
default	$\frac{(\tan^5(x))}{5}+\frac{(\tan^7(x))}{7}$	14
norman	$\frac{-\frac{32\left(\tan^5\left(\frac{x}{2}\right)\right)}{5}-\frac{192\left(\tan^7\left(\frac{x}{2}\right)\right)}{35}-\frac{32\left(\tan^9\left(\frac{x}{2}\right)\right)}{5}}{\left(\tan^2\left(\frac{x}{2}\right)-1\right)^7}$	37
risch	$\frac{4i\left(35\,e^{10ix}-35\,e^{8ix}+70\,e^{6ix}-14\,e^{4ix}+7\,e^{2ix}+1\right)}{35\left(e^{2ix}+1\right)^7}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^4,x,method=_RETURNVERBOSE)
[Out] 1/5*tan(x)^5+1/7*tan(x)^7
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.76

$$\frac{1}{7}\,\tan\left(x\right)^{7}+\frac{1}{5}\,\tan\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="maxima")
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

Fricas [A]
time = 2.60, size = 26, normalized size = 1.53

$$\frac{\left(2\,\cos\left(x\right)^{6}+\cos\left(x\right)^{4}-8\,\cos\left(x\right)^{2}+5\right)\sin\left(x\right)}{35\,\cos\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="fricas")
[Out] 1/35*(2*cos(x)^6 + cos(x)^4 - 8*cos(x)^2 + 5)*sin(x)/cos(x)^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**4,x)
[Out] Integral((-sin(x) + csc(x))**(-4), x)
```

Giac [A]
time = 0.39, size = 13, normalized size = 0.76

$$\frac{1}{7}\tan(x)^{7}+\frac{1}{5}\tan(x)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^4,x, algorithm="giac")
```

```
[Out] 1/7*tan(x)^7 + 1/5*tan(x)^5
```

Mupad [B]
time = 2.57, size = 23, normalized size = 1.35

$$\frac{2\cos(x)^{2}\sin(x)^{5}+5\sin(x)^{5}}{35\cos(x)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^4,x)
```

```
[Out] (5*sin(x)^5 + 2*cos(x)^2*sin(x)^5)/(35*cos(x)^7)
```

3.311 $\int \frac{1}{(\csc(x)-\sin(x))^5} dx$

Optimal. Leaf size=25

$$\frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

```
[Out] 1/5*sec(x)^5-2/7*sec(x)^7+1/9*sec(x)^9
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 276}

$$\frac{\sec^9(x)}{9} - \frac{2\sec^7(x)}{7} + \frac{\sec^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5),x]
```

```
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \int \sec^5(x)\tan^5(x)\,dx$$
$$= \text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \sec(x)\right)$$
$$= \frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5),x]
[Out] Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

Maple [A]
time = 0.21, size = 20, normalized size = 0.80

method	result	size
default	$\frac{1}{9\cos(x)^9}+\frac{1}{5\cos(x)^5}-\frac{2}{7\cos(x)^7}$	20
risch	$\frac{\frac{32\,e^{13ix}}{5}-\frac{384\,e^{11ix}}{35}+\frac{6976\,e^{9ix}}{315}-\frac{384\,e^{7ix}}{35}+\frac{32\,e^{5ix}}{5}}{\left(e^{2ix}+1\right)^9}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^5,x,method=_RETURNVERBOSE)
[Out] 1/9/cos(x)^9+1/5/cos(x)^5-2/7/cos(x)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(19) = 38.
time = 0.29, size = 187, normalized size = 7.48

$$\frac{16\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}-\frac{126\sin(x)^6}{(\cos(x)+1)^6}-\frac{441\sin(x)^8}{(\cos(x)+1)^8}-\frac{315\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{210\sin(x)^{12}}{(\cos(x)+1)^{12}}-1\right)}{315\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}+\frac{84\sin(x)^6}{(\cos(x)+1)^6}-\frac{126\sin(x)^8}{(\cos(x)+1)^8}+\frac{126\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{84\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{36\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{9\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{\sin(x)^{18}}{(\cos(x)+1)^{18}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="maxima")
```

```
[Out] 16/315*(9*sin(x)^2/(cos(x) + 1)^2 - 36*sin(x)^4/(cos(x) + 1)^4 - 126*sin(x)
^6/(cos(x) + 1)^6 - 441*sin(x)^8/(cos(x) + 1)^8 - 315*sin(x)^10/(cos(x) + 1
)^10 - 210*sin(x)^12/(cos(x) + 1)^12 - 1)/(9*sin(x)^2/(cos(x) + 1)^2 - 36*s
in(x)^4/(cos(x) + 1)^4 + 84*sin(x)^6/(cos(x) + 1)^6 - 126*sin(x)^8/(cos(x)
+ 1)^8 + 126*sin(x)^10/(cos(x) + 1)^10 - 84*sin(x)^12/(cos(x) + 1)^12 + 36*
sin(x)^14/(cos(x) + 1)^14 - 9*sin(x)^16/(cos(x) + 1)^16 + sin(x)^18/(cos(x)
 + 1)^18 - 1)
```

Fricas [A]
time = 2.24, size = 20, normalized size = 0.80

$$\frac{63\cos(x)^4 - 90\cos(x)^2 + 35}{315\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(63*cos(x)^4 - 90*cos(x)^2 + 35)/cos(x)^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**5,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-5), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(19) = 38.
time = 0.41, size = 101, normalized size = 4.04

$$\frac{16\left(\frac{9(\cos(x)-1)}{\cos(x)+1} + \frac{36(\cos(x)-1)^2}{(\cos(x)+1)^2} - \frac{126(\cos(x)-1)^3}{(\cos(x)+1)^3} + \frac{441(\cos(x)-1)^4}{(\cos(x)+1)^4} - \frac{315(\cos(x)-1)^5}{(\cos(x)+1)^5} + \frac{210(\cos(x)-1)^6}{(\cos(x)+1)^6} + 1\right)}{315\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^5,x, algorithm="giac")
```

```
[Out] 16/315*(9*(cos(x) - 1)/(cos(x) + 1) + 36*(cos(x) - 1)^2/(cos(x) + 1)^2 - 12
6*(cos(x) - 1)^3/(cos(x) + 1)^3 + 441*(cos(x) - 1)^4/(cos(x) + 1)^4 - 315*(
cos(x) - 1)^5/(cos(x) + 1)^5 + 210*(cos(x) - 1)^6/(cos(x) + 1)^6 + 1)/((cos
(x) - 1)/(cos(x) + 1) + 1)^9
```

Mupad [B]
time = 2.94, size = 19, normalized size = 0.76

$$\frac{1}{5\cos(x)^5}-\frac{2}{7\cos(x)^7}+\frac{1}{9\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^5,x)
[Out] 1/(5*cos(x)^5) - 2/(7*cos(x)^7) + 1/(9*cos(x)^9)
```

3.312 $\int \frac{1}{(\csc(x)-\sin(x))^6} dx$

Optimal. Leaf size=25

$$\frac{\tan^7(x)}{7}+\frac{2\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}$$

```
[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{276\}$

$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx &= \text{Subst}\left(\int x^6\left(1+x^2\right)^2\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^6+2x^8+x^{10}\right)\,dx,x,\tan(x)\right)\\ &= \frac{\tan^7(x)}{7}+\frac{2\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.
time = 0.01, size = 57, normalized size = 2.28

$$-\frac{8\tan(x)}{693}-\frac{4}{693}\sec^2(x)\tan(x)-\frac{1}{231}\sec^4(x)\tan(x)+\frac{113}{693}\sec^6(x)\tan(x)-\frac{23}{99}\sec^8(x)\tan(x)+\frac{1}{11}\sec^{10}(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-6),x]
```

```
[Out] (-8*Tan[x])/693 - (4*Sec[x]^2*Tan[x])/693 - (Sec[x]^4*Tan[x])/231 + (113*Se
c[x]^6*Tan[x])/693 - (23*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

Maple [A]
time = 0.24, size = 20, normalized size = 0.80

method	result	size
default	$\frac{(\tan^{7}(x))}{7}+\frac{2(\tan^{9}(x))}{9}+\frac{(\tan^{11}(x))}{11}$	20
risch	$-\frac{16 i \left(462 \,{\mathrm e}^{16 i x}-1155 \,{\mathrm e}^{14 i x}+2541 \,{\mathrm e}^{12 i x}-2079 \,{\mathrm e}^{10 i x}+1485 \,{\mathrm e}^{8 i x}-297 \,{\mathrm e}^{6 i x}+55 \,{\mathrm e}^{4 i x}+11 \,{\mathrm e}^{2 i x}+1\right)}{693 \left({\mathrm e}^{2 i x}+1\right)^{11}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

Maxima [A]
time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{11}\,\tan\left(x\right)^{11}+\frac{2}{9}\,\tan\left(x\right)^{9}+\frac{1}{7}\,\tan\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 2.94, size = 40, normalized size = 1.60

$$-\frac{\left(8\,\cos\left(x\right)^{10}+4\,\cos\left(x\right)^{8}+3\,\cos\left(x\right)^{6}-113\,\cos\left(x\right)^{4}+161\,\cos\left(x\right)^{2}-63\right)\sin\left(x\right)}{693\,\cos\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="fricas")
```

```
[Out] -1/693*(8*cos(x)^10 + 4*cos(x)^8 + 3*cos(x)^6 - 113*cos(x)^4 + 161*cos(x)^2
 - 63)*sin(x)/cos(x)^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**6,x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-6), x)
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.76

$$\frac{1}{11}\tan\left(x\right)^{11}+\frac{2}{9}\tan\left(x\right)^{9}+\frac{1}{7}\tan\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^6,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

Mupad [B]
time = 2.91, size = 33, normalized size = 1.32

$$\frac{8\cos\left(x\right)^{4}\sin\left(x\right)^{7}+28\cos\left(x\right)^{2}\sin\left(x\right)^{7}+63\sin\left(x\right)^{7}}{693\cos\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) - 1/sin(x))^6,x)
```

```
[Out] (63*sin(x)^7 + 28*cos(x)^2*sin(x)^7 + 8*cos(x)^4*sin(x)^7)/(693*cos(x)^11)
```

3.313 $\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}$$

```
[Out] -1/7*sec(x)^7+1/3*sec(x)^9-3/11*sec(x)^11+1/13*sec(x)^13
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 276}

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7),x]
```

```
[Out] -1/7*Sec[x]^7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx &= \int \sec^7(x)\tan^7(x)\,dx \\
&= \text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \sec(x)\right) \\
&= \text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)\,dx, x, \sec(x)\right) \\
&= -\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$
-\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7),x]
[Out] -1/7*Sec[x]^7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

Maple [A]
time = 0.26, size = 26, normalized size = 0.79

method	result	size
default	$\frac{1}{3\cos(x)^9}-\frac{1}{7\cos(x)^7}-\frac{3}{11\cos(x)^{11}}+\frac{1}{13\cos(x)^{13}}$	26
risch	$-\frac{128\left(429\,e^{19ix}-1430\,e^{17ix}+3523\,e^{15ix}-4020\,e^{13ix}+3523\,e^{11ix}-1430\,e^{9ix}+429\,e^{7ix}\right)}{3003\left(e^{2ix}+1\right)^{13}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^7,x,method=_RETURNVERBOSE)
[Out] 1/3/cos(x)^9-1/7/cos(x)^7-3/11/cos(x)^11+1/13/cos(x)^13
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(25) = 50.
time = 0.29, size = 271, normalized size = 8.21

$$
-\frac{32\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2}-\frac{78\sin(x)^4}{(\cos(x)+1)^4}+\frac{286\sin(x)^6}{(\cos(x)+1)^6}+\frac{2288\sin(x)^8}{(\cos(x)+1)^8}+\frac{10296\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{16302\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{18018\sin(x)^{14}}{(\cos(x)+1)^{14}}+\frac{9009\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{3003\sin(x)^{18}}{(\cos(x)+1)^{18}}-1\right)}{3003\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2}-\frac{78\sin(x)^4}{(\cos(x)+1)^4}+\frac{286\sin(x)^6}{(\cos(x)+1)^6}-\frac{715\sin(x)^8}{(\cos(x)+1)^8}+\frac{1287\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{1716\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{1716\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{1287\sin(x)^{16}}{(\cos(x)+1)^{16}}+\frac{715\sin(x)^{18}}{(\cos(x)+1)^{18}}-\frac{286\sin(x)^{20}}{(\cos(x)+1)^{20}}+\frac{78\sin(x)^{22}}{(\cos(x)+1)^{22}}-\frac{13\sin(x)^{24}}{(\cos(x)+1)^{24}}+\frac{\sin(x)^{26}}{(\cos(x)+1)^{26}}-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="maxima")
```

```
[Out] -32/3003*(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin
(x)^6/(cos(x) + 1)^6 + 2288*sin(x)^8/(cos(x) + 1)^8 + 10296*sin(x)^10/(cos(
x) + 1)^10 + 16302*sin(x)^12/(cos(x) + 1)^12 + 18018*sin(x)^14/(cos(x) + 1)
^14 + 9009*sin(x)^16/(cos(x) + 1)^16 + 3003*sin(x)^18/(cos(x) + 1)^18 - 1)/
(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin(x)^6/(co
s(x) + 1)^6 - 715*sin(x)^8/(cos(x) + 1)^8 + 1287*sin(x)^10/(cos(x) + 1)^10
- 1716*sin(x)^12/(cos(x) + 1)^12 + 1716*sin(x)^14/(cos(x) + 1)^14 - 1287*si
n(x)^16/(cos(x) + 1)^16 + 715*sin(x)^18/(cos(x) + 1)^18 - 286*sin(x)^20/(co
s(x) + 1)^20 + 78*sin(x)^22/(cos(x) + 1)^22 - 13*sin(x)^24/(cos(x) + 1)^24
+ sin(x)^26/(cos(x) + 1)^26 - 1)
```

Fricas [A]
time = 4.18, size = 26, normalized size = 0.79

$$-\frac{429 \cos (x)^6-1001 \cos (x)^4+819 \cos (x)^2-231}{3003 \cos (x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*cos(x)^6 - 1001*cos(x)^4 + 819*cos(x)^2 - 231)/cos(x)^13
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(25) = 50.
time = 0.41, size = 143, normalized size = 4.33

$$-\frac{32 \left(\frac{13 (\cos(x)-1)}{\cos(x)+1}+\frac{78 (\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{286 (\cos(x)-1)^3}{(\cos(x)+1)^3}-\frac{2288 (\cos(x)-1)^4}{(\cos(x)+1)^4}+\frac{10296 (\cos(x)-1)^5}{(\cos(x)+1)^5}-\frac{16302 (\cos(x)-1)^6}{(\cos(x)+1)^6}+\frac{18018 (\cos(x)-1)^7}{(\cos(x)+1)^7}-\frac{9009 (\cos(x)-1)^8}{(\cos(x)+1)^8}+\frac{3003 (\cos(x)-1)^9}{(\cos(x)+1)^9}+1\right)}{3003 \left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^7,x, algorithm="giac")
```

```
[Out] -32/3003*(13*(cos(x) - 1)/(cos(x) + 1) + 78*(cos(x) - 1)^2/(cos(x) + 1)^2 +
 286*(cos(x) - 1)^3/(cos(x) + 1)^3 - 2288*(cos(x) - 1)^4/(cos(x) + 1)^4 + 1
0296*(cos(x) - 1)^5/(cos(x) + 1)^5 - 16302*(cos(x) - 1)^6/(cos(x) + 1)^6 +
```

```
18018*(cos(x) - 1)^7/(cos(x) + 1)^7 - 9009*(cos(x) - 1)^8/(cos(x) + 1)^8 +
3003*(cos(x) - 1)^9/(cos(x) + 1)^9 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^13
```

Mupad [B]
time = 3.54, size = 25, normalized size = 0.76

$$\frac{1}{3\cos(x)^9} - \frac{1}{7\cos(x)^7} - \frac{3}{11\cos(x)^{11}} + \frac{1}{13\cos(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x) - 1/sin(x))^7,x)
[Out] 1/(3*cos(x)^9) - 1/(7*cos(x)^7) - 3/(11*cos(x)^11) + 1/(13*cos(x)^13)
```

3.314 $\int (\csc(x)-\sin(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\sqrt{\cos(x)\cot(x)}\csc(x)+\frac{256}{35}$$

```
[Out] 8/7*cos(x)*cot(x)^2*(cos(x)*cot(x))^(1/2)+2/7*cos(x)^3*cot(x)^2*(cos(x)*cot
(x))^(1/2)-64/35*cot(x)*csc(x)*(cos(x)*cot(x))^(1/2)+256/35*sec(x)*(cos(x)*
cot(x))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4482, 4485, 2678, 2674, 2669}

$$\frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}+\frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{35}\cot(x)\csc(x)\sqrt{\cos(x)\cot(x)}+\frac{256}{35}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(7/2),x]
```

```
[Out] (8*Cos[x]*Cot[x]^2*Sqrt[Cos[x]*Cot[x]])/7 + (2*Cos[x]^3*Cot[x]^2*Sqrt[Cos[x
]*Cot[x]])/7 - (64*Cot[x]*Sqrt[Cos[x]*Cot[x]]*Csc[x])/35 + (256*Sqrt[Cos[x]
*Cot[x]]*Sec[x])/35
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{7/2}\,dx = \int (\cos(x)\cot(x))^{7/2}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(12\sqrt{\cos(x)\cot(x)}\right)\int \cos^{\frac{3}{2}}(x)\cot^{\frac{7}{2}}(x)\,dx}{7\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(32\sqrt{c\right.}}{}$$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot($$
$$= \frac{8}{7}\cos(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{7}\cos^3(x)\cot^2(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{35}\cot($$

Mathematica [A]
time = 0.06, size = 37, normalized size = 0.51

$$-\frac{1}{70}\sqrt{\cos(x)\cot(x)}\left(-512+115\cos^2(x)+5\cos(x)\cos(3x)+28\cot^2(x)\right)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(7/2),x]
```

```
[Out] -1/70*(Sqrt[Cos[x]*Cot[x]]*(-512 + 115*Cos[x]^2 + 5*Cos[x]*Cos[3*x] + 28*Co
t[x]^2)*Sec[x])
```

Maple [A]
time = 0.40, size = 40, normalized size = 0.55

method	result	size
default	$\frac{2\left(5\left(\cos^{6}\left(x\right)\right)+20\left(\cos^{4}\left(x\right)\right)-160\left(\cos^{2}\left(x\right)\right)+128\right)\sin\left(x\right)\left(\frac{\cos^{2}\left(x\right)}{\sin\left(x\right)}\right)^{\frac{7}{2}}}{35\cos\left(x\right)^{7}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(5*cos(x)^6+20*cos(x)^4-160*cos(x)^2+128)*sin(x)*(cos(x)^2/sin(x))^(7/
2)/cos(x)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(57) = 114.
time = 0.59, size = 578, normalized size = 7.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/280*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*
cos(x) + 1)^(1/4)*(((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 5
817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos
(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x) - 5817*sin(9/2
*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275*sin(1/2*x))*co
s(7/2*arctan2(sin(x), cos(x) - 1)) + (5*cos(21/2*x) + 105*cos(17/2*x) - 227
5*cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(
3/2*x) + 2275*cos(1/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*
x) + 5817*sin(9/2*x) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 22
75*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*cos(7/2*arctan2(sin(x)
, cos(x) + 1)) + ((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) + 581
7*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*cos(1
/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*x) + 5817*sin(9/2*x
) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 2275*sin(1/2*x))*cos(
7/2*arctan2(sin(x), cos(x) - 1)) - (5*cos(21/2*x) + 105*cos(17/2*x) - 2275*
cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/
2*x) + 2275*cos(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x)
 - 5817*sin(9/2*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275
*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*sin(7/2*arctan2(sin(x),
cos(x) + 1)))/(cos(x)^8 + sin(x)^8 + 4*(cos(x)^2 + 1)*sin(x)^6 - 4*cos(x)^6
 + 2*(3*cos(x)^4 + 2*cos(x)^2 + 3)*sin(x)^4 + 6*cos(x)^4 + 4*(cos(x)^6 - co
s(x)^4 - cos(x)^2 + 1)*sin(x)^2 - 4*cos(x)^2 + 1)
```

Fricas [A]
time = 3.28, size = 44, normalized size = 0.60

$$-\frac{2\left(5\cos\left(x\right)^{6}+20\cos\left(x\right)^{4}-160\cos\left(x\right)^{2}+128\right)\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}}{35\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="fricas")
[Out] -2/35*(5*cos(x)^6 + 20*cos(x)^4 - 160*cos(x)^2 + 128)*sqrt(cos(x)^2/sin(x))
/(cos(x)^3 - cos(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(7/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(7/2),x, algorithm="giac")
[Out] integrate((csc(x) - sin(x))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\sin\left(x\right)}-\sin\left(x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(7/2),x)
[Out] int((1/sin(x) - sin(x))^(7/2), x)
```

3.315 $\int (\csc(x)-\sin(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] -16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1/
2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4482, 4485, 2678, 2674, 2669}

$$\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(5/2),x]
```

```
[Out] (-16*Cot[x]*Sqrt[Cos[x]*Cot[x]])/15 + (2*Cos[x]^2*Cot[x]*Sqrt[Cos[x]*Cot[x]
])/5 - (64*Sqrt[Cos[x]*Cot[x]]*Tan[x])/15
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = \int (\cos(x)\cot(x))^{5/2}\,dx$$
$$= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{\left(8\sqrt{\cos(x)\cot(x)}\right)\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)\,dx}{5\sqrt{\cos(x)}\sqrt{\cot(x)}}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{\left(32\sqrt{\cos(x)\right.}}{}$$
$$= -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)} + \frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)} - \frac{64}{15}\sqrt{\cos(x)\,c}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.58

$$-\frac{2}{15}\sqrt{\cos(x)\cot(x)}\left(32+3\cos^2(x)+5\cot^2(x)\right)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

Maple [A]
time = 0.34, size = 34, normalized size = 0.68

method	result	size

default	$\frac{2\left(3\left(\cos^{4}(x)\right)+24\left(\cos^{2}(x)\right)-32\right)\left(\frac{\cos^{2}(x)}{\sin(x)}\right)^{\frac{5}{2}}\sin(x)}{15\cos(x)^{5}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*cos(x)^4+24*cos(x)^2-32)*(cos(x)^2/sin(x))^(5/2)*sin(x)/cos(x)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(38) = 76$.
time = 0.58, size = 427, normalized size = 8.54

$$-\frac{\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)+3\sin\left(\frac{15}{2}x\right)+105\sin\left(\frac{11}{2}x\right)-410\sin\left(\frac{7}{2}x\right)+3\sin\left(\frac{5}{2}x\right)+410\sin\left(\frac{3}{2}x\right)+105\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{15}{2}x\right)-105\sin\left(\frac{11}{2}x\right)+410\sin\left(\frac{7}{2}x\right)-3\sin\left(\frac{5}{2}x\right)-410\sin\left(\frac{3}{2}x\right)-105\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)-\left(\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{15}{2}x\right)-105\sin\left(\frac{11}{2}x\right)+410\sin\left(\frac{7}{2}x\right)-3\sin\left(\frac{5}{2}x\right)-410\sin\left(\frac{3}{2}x\right)-105\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(3\cos\left(\frac{15}{2}x\right)+105\cos\left(\frac{11}{2}x\right)-410\cos\left(\frac{7}{2}x\right)-3\cos\left(\frac{5}{2}x\right)+410\cos\left(\frac{3}{2}x\right)-105\cos\left(\frac{1}{2}x\right)+3\sin\left(\frac{15}{2}x\right)+105\sin\left(\frac{11}{2}x\right)-410\sin\left(\frac{7}{2}x\right)+3\sin\left(\frac{5}{2}x\right)+410\sin\left(\frac{3}{2}x\right)+105\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{5}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)}{60\left(\cos(x)^{4}+\sin(x)^{4}+2\left(\cos(x)^{2}+1\right)\sin(x)^{2}-2\cos(x)^{2}+1\right)\left(\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^{2}+\sin(x)^{2}-2\cos(x)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*sin
(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan2(s
in(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3
*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/
2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*sin
(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) + 1)) - (
(3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) + 410*cos(
3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*sin(7/2*x)
- 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2(sin(x), c
os(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2
*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 4
10*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*sin(5/2*arc
tan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)^4
 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4))
```

Fricas [A]
time = 2.93, size = 35, normalized size = 0.70

$$\frac{2\left(3\cos(x)^{4}+24\cos(x)^{2}-32\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(5/2),x, algorithm="giac")
[Out] integrate((csc(x) - sin(x))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\sin\left(x\right)}-\sin\left(x\right)\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(5/2),x)
[Out] int((1/sin(x) - sin(x))^(5/2), x)
```

3.316 $\int (\csc(x) - \sin(x))^{3/2} \, dx$

Optimal. Leaf size=31

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

```
[Out] 2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4482, 4485, 2678, 2669}

$$\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sec(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(3/2),x]
[Out] (2*Cos[x]*Sqrt[Cos[x]*Cot[x]])/3 - (8*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$
\begin{aligned}
\int (\csc(x)-\sin(x))^{3/2}\,dx &= \int (\cos(x)\cot(x))^{3/2}\,dx\\
&= \frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)\,dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}+\frac{\left(4\sqrt{\cos(x)\cot(x)}\right)\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}\,dx}{3\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 0.68

$$
\frac{2}{3}\left(-4+\cos^2(x)\right)\sqrt{\cos(x)\cot(x)}\sec(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(3/2),x]
```

```
[Out] (2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

Maple [A]
time = 0.30, size = 26, normalized size = 0.84

method	result	size
default	$\frac{2\left(\cos^2(x)-4\right)\left(\frac{\cos^2(x)}{\sin(x)}\right)^{\frac{3}{2}}\sin(x)}{3\cos(x)^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(cos(x)^2-4)*(cos(x)^2/sin(x))^(3/2)*sin(x)/cos(x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(23) = 46$.
time = 0.58, size = 314, normalized size = 10.13

$$
\frac{\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^{\frac{1}{4}}\left(\left(\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{9}{2}x\right)+15\sin\left(\frac{5}{2}x\right)-\sin\left(\frac{3}{2}x\right)-15\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{9}{2}x\right)-15\sin\left(\frac{5}{2}x\right)+\sin\left(\frac{3}{2}x\right)+15\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)+\left(\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{9}{2}x\right)-15\sin\left(\frac{5}{2}x\right)+\sin\left(\frac{3}{2}x\right)+15\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{9}{2}x\right)-15\cos\left(\frac{5}{2}x\right)-\cos\left(\frac{3}{2}x\right)+15\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{9}{2}x\right)+15\sin\left(\frac{5}{2}x\right)-\sin\left(\frac{3}{2}x\right)-15\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)\right)}{6\left(\cos(x)^4+\sin(x)^4+2\left(\cos(x)^2+1\right)\sin(x)^2-2\cos(x)^2+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos
(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) -
 sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan2(s
in(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2
*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2*arct
an2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((cos(9/2*
x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*sin(5/2*x
) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)) - (cos
(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x) + 15*sin(
5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(x) - 1)))*
sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 +
1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

Fricas [A]
time = 3.35, size = 23, normalized size = 0.74

$$\frac{2\left(\cos\left(x\right)^{2}-4\right)\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}}{3\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(3/2),x)
```

```
[Out] Integral((-sin(x) + csc(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\sin\left(x\right)}-\sin\left(x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(3/2),x)
[Out] int((1/sin(x) - sin(x))^(3/2), x)
```

3.317 $\int \sqrt{\csc(x) - \sin(x)}\, dx$

Optimal. Leaf size=13

$$2\sqrt{\cos(x)\cot(x)}\tan(x)$$

```
[Out] 2*(cos(x)*cot(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 4485, 2669}

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\csc(x) - \sin(x)}\, dx &= \int \sqrt{\cos(x)\cot(x)}\, dx\\
&= \frac{\sqrt{\cos(x)\cot(x)} \int \sqrt{\cos(x)}\sqrt{\cot(x)}\, dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\\
&= 2\sqrt{\cos(x)\cot(x)}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$2\sqrt{\cos(x)\cot(x)}\,\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x] - Sin[x]],x]
[Out] 2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

Maple [A]
time = 0.30, size = 20, normalized size = 1.54

method	result	size
default	$\frac{2\sin(x)\sqrt{\frac{\cos^{2}(x)}{\sin(x)}}}{\cos(x)}$	20
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-ix}}{\mathrm{e}^{2ix}-1}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*sin(x)*(cos(x)^2/sin(x))^(1/2)/cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.
time = 0.55, size = 188, normalized size = 14.46

$$\frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)-\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)-\sin\left(\frac{3}{2}x\right)-\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)+\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)\right)\sin\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)+1\right)\right)}{\left(\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^{2}+\sin(x)^{2}-2\cos(x)+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
[Out] (((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin(1
/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((c
os(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x), c
os(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(1/2*a
rctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((cos(x)
^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1
/4))
```

Fricas [A]
time = 3.28, size = 19, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}}\,\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sin(x) + \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(-sin(x) + csc(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(x) - sin(x)), x)
```

Mupad [B]
time = 2.48, size = 15, normalized size = 1.15

$$\frac{2\,|\cos(x)|}{\cos(x)\,\sqrt{\frac{1}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(x) - sin(x))^(1/2),x)
```

```
[Out] (2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

3.318 $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

Optimal. Leaf size=60

$$\frac{\text{ArcTan}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}-\frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

```
[Out] arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-arctan
h((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4482, 4485, 2681, 2644, 335, 304, 209, 212}

$$\frac{\cos(x)\text{ArcTan}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Csc[x] - Sin[x]],x]
```

```
[Out] (ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) - (ArcTa
nh[Sqrt[-Sin[x]]]*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \int \frac{1}{\sqrt{\cos(x)\cot(x)}}\,dx$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}\,dx}{\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\cos(x)\int \sec(x)\sqrt{-\sin(x)}\,dx}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{\cos(x)\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, -\sin(x)\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{(2\cos(x))\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{\cos(x)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{\cos(x)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

Mathematica [A]
time = 0.20, size = 44, normalized size = 0.73

$$-\frac{\left(\text{ArcTan}\left(\sqrt[4]{\sin^2(x)}\right)-\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\right)\sqrt{\cos(x)\cot(x)}\sin(x)\tan(x)}{\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Csc[x] - Sin[x]],x]
[Out] -(((ArcTan[(Sin[x]^2)^(1/4)] - ArcTanh[(Sin[x]^2)^(1/4)])*Sqrt[Cos[x]*Cot[x
]]*Sin[x]*Tan[x])/(Sin[x]^2)^(3/4))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(1/2),x)
```

```
[Out] int(1/(csc(x)-sin(x))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(48) = 96.
time = 3.52, size = 124, normalized size = 2.07

$$\frac{1}{2}\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)+\frac{1}{4}\log\left(\frac{\cos(x)^3-5\cos(x)^2-(\cos(x)^2+6\cos(x)+4)\sin(x)+4(\cos(x)^2-(\cos(x)+1)\sin(x)-1)\sqrt{\frac{\cos(x)^2}{\sin(x)}}-2\cos(x)+4}{\cos(x)^3+3\cos(x)^2-(\cos(x)^2-2\cos(x)-4)\sin(x)-2\cos(x)-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))) + 1/4*l
og((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) + 4*(cos(x)^2
- (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)^3
+ 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\sin(x)+\csc(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sin(x) + csc(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(csc(x) - sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\sin(x)} - \sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(1/2),x)
[Out] int(1/(1/sin(x) - sin(x))^(1/2), x)
```

3.319 $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\mathrm{ArcTan}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}}+\frac{\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] 1/2*sec(x)/(cos(x)*cot(x))^(1/2)+1/4*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x
))^(1/2)/(cos(x)*cot(x))^(1/2)+1/4*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x)
)^(1/2)/(cos(x)*cot(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {4482, 4485, 2677, 2681, 2644, 335, 218, 212, 209}

$$\frac{\sqrt{-\sin(x)}\cot(x)\mathrm{ArcTan}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}+\frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}}+\frac{\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-3/2),x]
```

```
[Out] Sec[x]/(2*Sqrt[Cos[x]*Cot[x]]) + (ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]
])/(4*Sqrt[Cos[x]*Cot[x]]) + (ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/
(4*Sqrt[Cos[x]*Cot[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{3/2}}\,dx$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} - \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\int \frac{\sec(x)}{\sqrt{-\sin(x)}}\,dx}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx, x, -\sin(x)\right)}{4\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{-\sin(x)}\right)}{2\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\left(\cot(x)\sqrt{-\sin(x)}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{-\sin(x)}\right)}{4\sqrt{\cos(x)\cot(x)}} + \cdot$$
$$= \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}} + \frac{\tanh^{-1}\left(\sqrt{-\sin}\right.}{4\sqrt{c}}$$

Mathematica [A]
time = 0.18, size = 60, normalized size = 0.75

$$\frac{-\mathrm{ArcTan}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)-\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)+2\sec(x)\sqrt[4]{\sin^2(x)}}{4\sqrt{\cos(x)\cot(x)}\sqrt[4]{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-3/2),x]
[Out] (-(ArcTan[(Sin[x]^2)^(1/4)]*Cos[x]) - ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] + 2*
Sec[x]*(Sin[x]^2)^(1/4))/(4*Sqrt[Cos[x]*Cot[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.58, size = 465, normalized size = 5.81

method	result
default	$\frac{(\cos(x)-1)\left(i(\cos^2(x))\sin(x)\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i(\cos(x)-1)}{\sin(x)}}\sqrt{-\frac{i\,c}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(cos(x)-1)*(I*cos(x)^2*sin(x)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-
1)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)+I*cos(x)^2*sin(x)*Elli
pticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(x)
+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)
/sin(x))^(1/2)-2*I*cos(x)^2*sin(x)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(
1/2),1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))
^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)-cos(x)^2*sin(x)*EllipticPi(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/s
in(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/
2)+cos(x)^2*sin(x)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2
)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)+2*cos(x)*2^(1/2)-2*2^(1/2))*cos(x)*(1
+cos(x))^2/sin(x)^5/(cos(x)^2/sin(x))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(60) = 120$.
time = 1.84, size = 152, normalized size = 1.90

$$\frac{2\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^3+\cos(x)^3\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}-2\cos(x)+4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)+8\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{16\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*cos
(x)^3 + cos(x)^3*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin
(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(
x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x
) - 4)) + 8*sqrt(cos(x)^2/sin(x))*sin(x))/cos(x)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x) + \csc(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(3/2),x)
[Out] Integral((-sin(x) + csc(x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="giac")
[Out] integrate((csc(x) - sin(x))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\sin(x)} - \sin(x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(3/2),x)
[Out] int(1/(1/sin(x) - sin(x))^(3/2), x)
```

3.320 $\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\text{ArcTan}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}+\frac{3\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}$$

```
[Out] -3/32*arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)+
3/32*arctanh((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-
3/16*tan(x)/(cos(x)*cot(x))^(1/2)+1/4*sec(x)^2*tan(x)/(cos(x)*cot(x))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4482, 4485, 2677, 2679, 2681, 2644, 335, 304, 209, 212}

$$-\frac{3\cos(x)\text{ArcTan}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}-\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}}+\frac{3\cos(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{32\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}+\frac{\tan(x)\sec^2(x)}{4\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) +
(3*ArcTanh[Sqrt[-Sin[x]]]*Cos[x])/(32*Sqrt[Cos[x]*Cot[x]]*Sqrt[-Sin[x]]) -
(3*Tan[x])/(16*Sqrt[Cos[x]*Cot[x]]) + (Sec[x]^2*Tan[x])/(4*Sqrt[Cos[x]*Cot[
x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
```

```
x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{5/2}}\,dx$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$
$$= \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)\sqrt{\cot(x)}}\,dx}{8\sqrt{\cos(x)\cot(x)}}$$
$$= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{\left(3\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{}{\sqrt{\cos(}}}{32\sqrt{\cos(x)\cot(x)}}$$
$$= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} - \frac{(3\cos(x))\int \sec(x)\sqrt{-\sin(x)}\,dx}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$
$$= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, -\right.}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(}}$$
$$= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{}\right.}{16\sqrt{\cos(x)\cot(x)}\sqrt{-\sin}}$$
$$= -\frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}} + \frac{(3\cos(x))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{}\right.}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin}}$$
$$= -\frac{3\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{3\tanh^{-1}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{3\,\text{t}}{16\sqrt{\cos}}$$

Mathematica [A]
time = 0.39, size = 69, normalized size = 0.70

$$-\frac{\sqrt{\cos(x)\cot(x)}\sin(x)\left(-3\text{ArcTan}\left(\sqrt[4]{\sin^2(x)}\right)+3\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)+(-5+3\cos(2x))\sec^4(x)\sin^2(x)^{3/4}\right)\tan(x)}{32\sin^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-5/2),x]
```

```
[Out] -1/32*(Sqrt[Cos[x]*Cot[x]]*Sin[x]*(-3*ArcTan[(Sin[x]^2)^(1/4)] + 3*ArcTanh[
(Sin[x]^2)^(1/4)] + (-5 + 3*Cos[2*x])*Sec[x]^4*(Sin[x]^2)^(3/4))*Tan[x])/(S
in[x]^2)^(3/4)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.47, size = 382, normalized size = 3.86

method	result
default	$\frac{(\cos(x)-1)\left(3i(\cos^4(x))\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i(\cos(x)-1)}{\sin(x)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{1}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(cos(x)-1)*(3*I*cos(x)^4*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x
)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*EllipticPi(((I*cos(x
)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*cos(x)^4*((-I*cos(x)+s
in(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/si
n(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(
1/2))+3*cos(x)^4*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/s
in(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*EllipticPi(((I*cos(x)+sin(x)-I)/s
in(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*cos(x)^4*((-I*cos(x)+sin(x)+I)/sin(x)
)^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*Ell
ipticPi(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*cos(x)^
3*2^(1/2)+6*cos(x)^2*2^(1/2)+8*cos(x)*2^(1/2)-8*2^(1/2))*cos(x)*(1+cos(x))^
2/sin(x)^5/(cos(x)^2/sin(x))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(75) = 150.
time = 4.37, size = 165, normalized size = 1.67

$$-\frac{6\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^5-3\cos(x)^5\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-1\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}-2\cos(x)+4}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-4}\right)-8\left(3\cos(x)^4-7\cos(x)^2+4\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{128\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/128*(6*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*c
os(x)^5 - 3*cos(x)^5*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)
*sin(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*
cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*c
os(x) - 4)) - 8*(3*cos(x)^4 - 7*cos(x)^2 + 4)*sqrt(cos(x)^2/sin(x)))/cos(x)
^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\sin(x)+\csc(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(5/2),x)
```

```
[Out] Integral((-sin(x) + csc(x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((csc(x) - sin(x))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\sin(x)}-\sin(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(5/2),x)
```

```
[Out] int(1/(1/sin(x) - sin(x))^(5/2), x)
```

3.321 $\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}-\frac{5\text{ArcTan}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\tanh^{-1}\left(\sqrt{-\sin(x)}\right.}{128\sqrt{\cos(}}$$

```
[Out] 5/192*sec(x)/(cos(x)*cot(x))^(1/2)-5/48*sec(x)^3/(cos(x)*cot(x))^(1/2)-5/12
8*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)-5/12
8*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)+1/6
*sec(x)^3*tan(x)^2/(cos(x)*cot(x))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4482, 4485, 2677, 2679, 2681, 2644, 335, 218, 212, 209}

$$-\frac{5\sqrt{-\sin(x)}\cot(x)\text{ArcTan}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}-\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}}+\frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}}-\frac{5\sqrt{-\sin(x)}\cot(x)\tanh^{-1}\left(\sqrt{-\sin(x)}\right)}{128\sqrt{\cos(x)\cot(x)}}+\frac{\tan^2(x)\sec^3(x)}{6\sqrt{\cos(x)\cot(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (5*Sec[x])/(192*Sqrt[Cos[x]*Cot[x]]) - (5*Sec[x]^3)/(48*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTan[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]]
) - (5*ArcTanh[Sqrt[-Sin[x]]]*Cot[x]*Sqrt[-Sin[x]])/(128*Sqrt[Cos[x]*Cot[x]
]) + (Sec[x]^3*Tan[x]^2)/(6*Sqrt[Cos[x]*Cot[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \int \frac{1}{(\cos(x)\cot(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)}\,dx}{\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\cos(x)\cot(x)}}$$

$$= -\frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\sqrt{\cot(x)}\right)\int \frac{\sqrt{\cot(x}}{\cos^{\frac{7}{2}}(x)}}{96\sqrt{\cos(x)\cot(x)}}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\sqrt{\cos(x)}\right.}{128}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} + \frac{\left(5\cot(x)\sqrt{-\right.}{128}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\right.}{}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\right.}{}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}} - \frac{\left(5\cot(x)\sqrt{-\right.}{}$$

$$= \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} - \frac{5\tan^{-1}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-}}{128\sqrt{\cos(x)\cot(x)}}$$

Mathematica [A]

time = 0.19, size = 74, normalized size = 0.63

$$\frac{15\text{ArcTan}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)+15\tanh^{-1}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)+2\sec(x)\left(5-52\sec^2(x)+32\sec^4(x)\right)\sqrt[4]{\sin^2(x)}}{384\sqrt{\cos(x)\cot(x)}\sqrt[4]{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sin[x])^(-7/2),x]
```

```
[Out] (15*ArcTan[(Sin[x]^2)^(1/4)]*Cos[x] + 15*ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] +
 2*Sec[x]*(5 - 52*Sec[x]^2 + 32*Sec[x]^4)*(Sin[x]^2)^(1/4))/(384*Sqrt[Cos[x
]*Cot[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.48, size = 487, normalized size = 4.13

method	result
default	$\frac{(\cos(x)-1)\left(30i\sin(x)\left(\cos^6(x)\right)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{-\frac{i(\cos(x)-1)}{\sin(x)}}\text{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)}{\sin(x)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(x)-sin(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768*(cos(x)-1)*(30*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(
(-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*EllipticF((
(I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-15*I*sin(x)*EllipticPi(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+I)/
sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/
2)*cos(x)^6-15*I*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*co
s(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*EllipticPi(((I*co
s(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*sin(x)*EllipticPi(((
I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I*cos(x)+sin(x)+
I)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^
(1/2)*cos(x)^6-15*sin(x)*cos(x)^6*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*c
os(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(cos(x)-1)/sin(x))^(1/2)*EllipticPi(((I*c
os(x)+sin(x)-I)/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+10*2^(1/2)*cos(x)^5-10
*2^(1/2)*cos(x)^4-104*cos(x)^3*2^(1/2)+104*cos(x)^2*2^(1/2)+64*cos(x)*2^(1/
2)-64*2^(1/2))*cos(x)*(1+cos(x))^2/sin(x)^7/(cos(x)^2/sin(x))^(7/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="maxima")
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

Fricas [A]
time = 1.81, size = 167, normalized size = 1.42

$$-\frac{30 \arctan\left(\frac{2 \sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}} \sin\left(x\right)}{\cos\left(x\right) \sin\left(x\right) - \cos\left(x\right)}\right) \cos\left(x\right)^{7} - 15 \cos\left(x\right)^{7} \log\left(\frac{\cos\left(x\right)^{3} - 5 \cos\left(x\right)^{2} - \left(\cos\left(x\right)^{2} + 6 \cos\left(x\right) + 4\right) \sin\left(x\right) + 4 \left(\cos\left(x\right)^{2} - \left(\cos\left(x\right) + 1\right) \sin\left(x\right) - 1\right) \sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}} - 2 \cos\left(x\right) + 4}{\cos\left(x\right)^{3} + 3 \cos\left(x\right)^{2} - \left(\cos\left(x\right)^{2} - 2 \cos\left(x\right) - 4\right) \sin\left(x\right) - 2 \cos\left(x\right) - 4}\right) - 8 \left(5 \cos\left(x\right)^{4} - 52 \cos\left(x\right)^{2} + 32\right) \sqrt{\frac{\cos\left(x\right)^{2}}{\sin\left(x\right)}} \sin\left(x\right)}{1536 \cos\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="fricas")
[Out] -1/1536*(30*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))
*cos(x)^7 - 15*cos(x)^7*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) +
 4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) -
 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) -
2*cos(x) - 4)) - 8*(5*cos(x)^4 - 52*cos(x)^2 + 32)*sqrt(cos(x)^2/sin(x))*si
n(x))/cos(x)^7
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))**(7/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="giac")
[Out] integrate((csc(x) - sin(x))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\sin\left(x\right)}-\sin\left(x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/sin(x) - sin(x))^(7/2),x)
[Out] int(1/(1/sin(x) - sin(x))^(7/2), x)
```

3.322 $\int (-\cos(x)+\sec(x))^4 \, dx$

Optimal. Leaf size=44

$$\frac{35x}{8}-\frac{35\tan(x)}{8}+\frac{35\tan^3(x)}{24}-\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)$$

```
[Out] 35/8*x-35/8*tan(x)+35/24*tan(x)^3-7/8*sin(x)^2*tan(x)^3-1/4*sin(x)^4*tan(x)
^3
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {294, 308, 209}

$$\frac{35x}{8}+\frac{35\tan^3(x)}{24}-\frac{35\tan(x)}{8}-\frac{1}{4}\sin^4(x)\tan^3(x)-\frac{7}{8}\sin^2(x)\tan^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (35*Tan[x])/8 + (35*Tan[x]^3)/24 - (7*Sin[x]^2*Tan[x]^3)/8 - (Si
n[x]^4*Tan[x]^3)/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int (-\cos(x)+\sec(x))^4\,dx &= \text{Subst}\left(\int \frac{x^8}{(1+x^2)^3}\,dx,x,\tan(x)\right) \\
&= -\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{7}{4}\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx,x,\tan(x)\right) \\
&= -\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx,x,\tan(x)\right) \\
&= -\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)\,dx \right. \\
&= -\frac{35\tan(x)}{8}+\frac{35\tan^3(x)}{24}-\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)+\frac{35}{8}\text{Subst}\left(\int \right. \\
&= \frac{35x}{8}-\frac{35\tan(x)}{8}+\frac{35\tan^3(x)}{24}-\frac{7}{8}\sin^2(x)\tan^3(x)-\frac{1}{4}\sin^4(x)\tan^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.86

$$
\frac{35x}{8}-\frac{3}{4}\sin(2x)+\frac{1}{32}\sin(4x)-\frac{10\tan(x)}{3}+\frac{1}{3}\sec^2(x)\tan(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^4,x]
```

```
[Out] (35*x)/8 - (3*Sin[2*x])/4 + Sin[4*x]/32 - (10*Tan[x])/3 + (Sec[x]^2*Tan[x])
/3
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.91

method	result
default	$\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{35x}{8}-2\cos(x)\sin(x)-4\tan(x)-\left(-\frac{2}{3}-\frac{(\sec^2(x))}{3}\right)\tan(x)$
risch	$\frac{35x}{8}-\frac{i e^{4ix}}{64}+\frac{3i e^{2ix}}{8}-\frac{3i e^{-2ix}}{8}+\frac{i e^{-4ix}}{64}-\frac{4i(6\,e^{4ix}+9\,e^{2ix}+5)}{3(e^{2ix}+1)^3}$
norman	$\frac{-\frac{35x}{8}+\frac{35\left(\tan^3\left(\frac{x}{2}\right)\right)}{6}-\frac{329\left(\tan^5\left(\frac{x}{2}\right)\right)}{12}-17\left(\tan^7\left(\frac{x}{2}\right)\right)-\frac{329\left(\tan^9\left(\frac{x}{2}\right)\right)}{12}+\frac{35\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{6}+\frac{35\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{4}-\frac{35x\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}+\frac{105x\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4\left(\tan^2\left(\frac{x}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+35/8*x-2*cos(x)*sin(x)-4*tan(x)-(-2/3-1/3*
sec(x)^2)*tan(x)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.59

$$\frac{1}{3}\,\tan\left(x\right)^{3}+\frac{35}{8}\,x+\frac{1}{32}\,\sin\left(4\,x\right)-\frac{3}{4}\,\sin\left(2\,x\right)-3\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x + 1/32*sin(4*x) - 3/4*sin(2*x) - 3*tan(x)
```

Fricas [A]
time = 1.25, size = 37, normalized size = 0.84

$$\frac{105\,x\cos\left(x\right)^{3}+\left(6\,\cos\left(x\right)^{6}-39\,\cos\left(x\right)^{4}-80\,\cos\left(x\right)^{2}+8\right)\sin\left(x\right)}{24\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*x*cos(x)^3 + (6*cos(x)^6 - 39*cos(x)^4 - 80*cos(x)^2 + 8)*sin(x))
/cos(x)^3
```

Sympy [A]
time = 6.09, size = 44, normalized size = 1.00

$$\frac{35x}{8}-2\sin\left(x\right)\cos\left(x\right)-\frac{4\sin\left(x\right)}{\cos\left(x\right)}+\frac{\sin\left(2x\right)}{4}+\frac{\sin\left(4x\right)}{32}+\frac{\tan^{3}\left(x\right)}{3}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**4,x)
```

```
[Out] 35*x/8 - 2*sin(x)*cos(x) - 4*sin(x)/cos(x) + sin(2*x)/4 + sin(4*x)/32 + tan
(x)**3/3 + tan(x)
```

Giac [A]
time = 0.39, size = 35, normalized size = 0.80

$$\frac{1}{3}\,\tan\left(x\right)^{3}+\frac{35}{8}\,x-\frac{13\,\tan\left(x\right)^{3}+11\,\tan\left(x\right)}{8\,\left(\tan\left(x\right)^{2}+1\right)^{2}}-3\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + 35/8*x - 1/8*(13*tan(x)^3 + 11*tan(x))/(tan(x)^2 + 1)^2 - 3*
tan(x)
```

Mupad [B]
time = 2.57, size = 80, normalized size = 1.82

$$\frac{35\,x}{8}+\frac{\frac{35\,{\mathrm{tan}\left(\frac{x}{2}\right)}^{13}}{4}+\frac{35\,{\mathrm{tan}\left(\frac{x}{2}\right)}^{11}}{6}-\frac{329\,{\mathrm{tan}\left(\frac{x}{2}\right)}^9}{12}-17\,{\mathrm{tan}\left(\frac{x}{2}\right)}^7-\frac{329\,{\mathrm{tan}\left(\frac{x}{2}\right)}^5}{12}+\frac{35\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3}{6}+\frac{35\,\mathrm{tan}\left(\frac{x}{2}\right)}{4}}{{\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2-1\right)}^3\,{\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - 1/cos(x))^4,x)
```

```
[Out] (35*x)/8 + ((35*tan(x/2))/4 + (35*tan(x/2)^3)/6 - (329*tan(x/2)^5)/12 - 17*
tan(x/2)^7 - (329*tan(x/2)^9)/12 + (35*tan(x/2)^11)/6 + (35*tan(x/2)^13)/4)
/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

3.323 $\int (-\cos(x)+\sec(x))^3\,dx$

Optimal. Leaf size=34

$$-\frac{5}{2}\tanh^{-1}(\sin(x))+\frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)$$

```
[Out] -5/2*arctanh(sin(x))+5/2*sin(x)+5/6*sin(x)^3+1/2*sin(x)^3*tan(x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4482, 2672, 294, 308, 212}

$$\frac{5\sin^3(x)}{6}+\frac{5\sin(x)}{2}+\frac{1}{2}\sin^3(x)\tan^2(x)-\frac{5}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^3,x]
```

```
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sin[x])/2 + (5*Sin[x]^3)/6 + (Sin[x]^3*Tan[x]^2
)/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^3\,dx &= \int \sin^3(x)\tan^3(x)\,dx\\
&= \text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\sin^3(x)\tan^2(x) - \frac{5}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(x)\right)\\
&= \frac{5\sin(x)}{2} + \frac{5\sin^3(x)}{6} + \frac{1}{2}\sin^3(x)\tan^2(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= -\frac{5}{2}\tanh^{-1}(\sin(x)) + \frac{5\sin(x)}{2} + \frac{5\sin^3(x)}{6} + \frac{1}{2}\sin^3(x)\tan^2(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.12

$$-\frac{5}{2}\tanh^{-1}(\sin(x)) + \frac{5}{2}\sec(x)\tan(x) - \frac{5}{3}\sin(x)\tan^2(x) - \frac{1}{3}\sin^3(x)\tan^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^3,x]
[Out] (-5*ArcTanh[Sin[x]])/2 + (5*Sec[x]*Tan[x])/2 - (5*Sin[x]*Tan[x]^2)/3 - (Sin
[x]^3*Tan[x]^2)/3
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.88

method	result	size
default	$-\frac{(2+\cos^2(x))\sin(x)}{3} + 3\sin(x) - \frac{5\ln(\sec(x)+\tan(x))}{2} + \frac{\sec(x)\tan(x)}{2}$	30
norman	$\frac{\frac{20(\tan^3(\frac{x}{2}))}{3} - \frac{22(\tan^5(\frac{x}{2}))}{3} + \frac{20(\tan^7(\frac{x}{2}))}{3} + 5(\tan^9(\frac{x}{2})) + 5\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^3(\tan^2(\frac{x}{2})-1)^2} + \frac{5\ln(\tan(\frac{x}{2})-1)}{2} - \frac{5\ln(\tan(\frac{x}{2})+1)}{2}$	80

risch	$\frac{i e^{3ix}}{24}-\frac{9 i e^{ix}}{8}+\frac{9 i e^{-ix}}{8}-\frac{i e^{-3ix}}{24}-\frac{i(e^{3ix}-e^{ix})}{(e^{2ix}+1)^2}+\frac{5\ln(e^{ix}-i)}{2}-\frac{5\ln(e^{ix}+i)}{2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2+cos(x)^2)*sin(x)+3*sin(x)-5/2*ln(sec(x)+tan(x))+1/2*sec(x)*tan(x)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.09

$$\frac{1}{3}\sin(x)^3-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{5}{4}\log(\sin(x)+1)+\frac{5}{4}\log(\sin(x)-1)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(si
n(x) - 1) + 2*sin(x)
```

Fricas [A]
time = 1.28, size = 49, normalized size = 1.44

$$-\frac{15\cos(x)^2\log(\sin(x)+1)-15\cos(x)^2\log(-\sin(x)+1)+2\left(2\cos(x)^4-14\cos(x)^2-3\right)\sin(x)}{12\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(x)^2*log(sin(x) + 1) - 15*cos(x)^2*log(-sin(x) + 1) + 2*(2*co
s(x)^4 - 14*cos(x)^2 - 3)*sin(x))/cos(x)^2
```

Sympy [A]
time = 2.26, size = 42, normalized size = 1.24

$$\frac{5\log(\sin(x)-1)}{4}-\frac{5\log(\sin(x)+1)}{4}+\frac{\sin^3(x)}{3}+2\sin(x)-\frac{\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**3,x)
```

```
[Out] 5*log(sin(x) - 1)/4 - 5*log(sin(x) + 1)/4 + sin(x)**3/3 + 2*sin(x) - sin(x)
/(2*sin(x)**2 - 2)
```

Giac [A]
time = 0.39, size = 39, normalized size = 1.15

$$\frac{1}{3}\sin(x)^3-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{5}{4}\log(\sin(x)+1)+\frac{5}{4}\log(-\sin(x)+1)+2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^3,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(-s
in(x) + 1) + 2*sin(x)
```

Mupad [B]
time = 2.46, size = 68, normalized size = 2.00

$$\frac{5\tan\left(\frac{x}{2}\right)^9+\frac{20\tan\left(\frac{x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{x}{2}\right)^3}{3}+5\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}-5\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) - 1/cos(x))^3,x)
```

```
[Out] (5*tan(x/2) + (20*tan(x/2)^3)/3 - (22*tan(x/2)^5)/3 + (20*tan(x/2)^7)/3 + 5
*tan(x/2)^9)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3) - 5*atanh(tan(x/2))
```

3.324 $\int (-\cos(x) + \sec(x))^2 \, dx$

Optimal. Leaf size=22

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

```
[Out] -3/2*x+3/2*tan(x)-1/2*sin(x)^2*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {294, 327, 209}

$$-\frac{3x}{2} + \frac{3\tan(x)}{2} - \frac{1}{2}\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^2,x]
```

```
[Out] (-3*x)/2 + (3*Tan[x])/2 - (Sin[x]^2*Tan[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int (-\cos(x)+\sec(x))^2\,dx &= \text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\tan(x)\right)\\ &= -\frac{1}{2}\sin^2(x)\tan(x)+\frac{3}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{3x}{2}+\frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.73

$$-\frac{3x}{2}+\frac{1}{4}\sin(2x)+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^2,x]
[Out] (-3*x)/2 + Sin[2*x]/4 + Tan[x]
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.59

method	result	size
default	$\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}+\tan(x)$	13
risch	$-\frac{3x}{2}-\frac{i\mathrm{e}^{2ix}}{8}+\frac{i\mathrm{e}^{-2ix}}{8}+\frac{2i}{\mathrm{e}^{2ix}+1}$	33
norman	$\frac{\frac{3x}{2}-2\left(\tan^3\left(\frac{x}{2}\right)\right)-3\left(\tan^5\left(\frac{x}{2}\right)\right)+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}-\frac{3x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*cos(x)*sin(x)-3/2*x+tan(x)
```

Maxima [A]
time = 0.30, size = 12, normalized size = 0.55

$$-\frac{3}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="maxima")
```

```
[Out] -3/2*x + 1/4*sin(2*x) + tan(x)
```

Fricas [A]
time = 2.04, size = 22, normalized size = 1.00

$$-\frac{3\,x\cos\left(x\right)-\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)}{2\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

Sympy [A]
time = 0.95, size = 14, normalized size = 0.64

$$-\frac{3x}{2}+\frac{\sin\left(2x\right)}{4}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**2,x)
```

```
[Out] -3*x/2 + sin(2*x)/4 + tan(x)
```

Giac [A]
time = 0.42, size = 18, normalized size = 0.82

$$-\frac{3}{2}\,x+\frac{\tan\left(x\right)}{2\left(\tan\left(x\right)^{2}+1\right)}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^2,x, algorithm="giac")
```

```
[Out] -3/2*x + 1/2*tan(x)/(tan(x)^2 + 1) + tan(x)
```

Mupad [B]
time = 2.41, size = 49, normalized size = 2.23

$$-\frac{3\,x}{2}-\frac{3\tan\left(\frac{x}{2}\right)^{5}+2\tan\left(\frac{x}{2}\right)^{3}+3\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\,\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - 1/cos(x))^2,x)
```

```
[Out] - (3*x)/2 - (3*tan(x/2) + 2*tan(x/2)^3 + 3*tan(x/2)^5)/((tan(x/2)^2 - 1)*(t
an(x/2)^2 + 1)^2)
```

3.325 $\int (-\cos(x)+\sec(x))\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(\sin(x))-\sin(x)$$

```
[Out] arctanh(sin(x))-sin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2717, 3855}

$$\tanh^{-1}(\sin(x))-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x] + Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (-\cos(x)+\sec(x))\,dx &= -\int \cos(x)\,dx+\int \sec(x)\,dx\\ &= \tanh^{-1}(\sin(x))-\sin(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(8) = 16.
time = 0.00, size = 37, normalized size = 4.62

$$-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x] + Sec[x],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x]
```

Maple [A]
time = 0.06, size = 12, normalized size = 1.50

method	result	size
default	$-\sin(x)+\ln(\sec(x)+\tan(x))$	12
norman	$-\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	34
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}+\ln(e^{ix}+i)-\ln(e^{ix}-i)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)+sec(x),x,method=_RETURNVERBOSE)
[Out] -sin(x)+ln(sec(x)+tan(x))
```

Maxima [A]
time = 0.30, size = 11, normalized size = 1.38

$$\log(\sec(x)+\tan(x))-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="maxima")
[Out] log(sec(x) + tan(x)) - sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.
time = 1.57, size = 21, normalized size = 2.62

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="fricas")
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.
time = 0.03, size = 19, normalized size = 2.38

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - sin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.
time = 0.37, size = 29, normalized size = 3.62

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)+sec(x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2)) -
 sin(x)
```

Mupad [B]
time = 2.30, size = 14, normalized size = 1.75

$$\ln\left(\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)-\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x) - cos(x),x)
```

```
[Out] log(tan(x/2 + pi/4)) - sin(x)
```

3.326 $\int \frac{1}{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -csc(x)
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\cos(x)+\sec(x)}\,dx &= \int \cot(x)\csc(x)\,dx\\ &= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\ &= -\csc(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-1),x]
```

```
[Out] -Csc[x]
```

Maple [A]
time = 0.12, size = 7, normalized size = 1.75

method	result	size
default	$-\frac{1}{\sin(x)}$	7
norman	$\frac{-\frac{1}{2}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}$	18
risch	$-\frac{2 i \,{\mathrm e}^{i x}}{{\mathrm e}^{2 i x}-1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(4) = 8$.
time = 0.28, size = 21, normalized size = 5.25

$$-\frac{\cos\left(x\right)+1}{2\,\sin\left(x\right)}-\frac{\sin\left(x\right)}{2\,(\cos\left(x\right)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + 1)/sin(x) - 1/2*sin(x)/(cos(x) + 1)
```

Fricas [A]
time = 2.11, size = 6, normalized size = 1.50

$$-\frac{1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\cos\left(x\right)-\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x)
```

```
[Out] -Integral(1/(cos(x) - sec(x)), x)
```

Giac [A]
time = 0.41, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x)),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

Mupad [B]
time = 2.37, size = 6, normalized size = 1.50

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x)),x)
```

```
[Out] -1/sin(x)
```

3.327 $\int \frac{1}{(-\cos(x)+\sec(x))^2} dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\cot^3(x)$$

```
[Out] -1/3*cot(x)^3
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {30}

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -1/3*Cot[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^2} dx &= \text{Subst}\left(\int \frac{1}{x^4} dx, x, \tan(x)\right)\\ &= -\frac{1}{3}\cot^3(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$-\frac{1}{3}\cot^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-2),x]
```

```
[Out] -1/3*Cot[x]^3
```

Maple [A]
time = 0.13, size = 7, normalized size = 0.88

method	result	size
default	$-\frac{1}{3\tan(x)^3}$	7
risch	$\frac{2i(3\,\mathrm{e}^{4ix}+1)}{3(\mathrm{e}^{2ix}-1)^3}$	22
norman	$\frac{-\frac{1}{24}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{8}-\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{24}}{\tan\left(\frac{x}{2}\right)^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/tan(x)^3
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$-\frac{1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="maxima")
```

```
[Out] -1/3/tan(x)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.
time = 2.35, size = 18, normalized size = 2.25

$$\frac{\cos(x)^3}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3/((cos(x)^2 - 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(-\cos(x)+\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**2,x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-2), x)
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.75

$$-\frac{1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^2,x, algorithm="giac")
```

```
[Out] -1/3/tan(x)^3
```

Mupad [B]
time = 2.45, size = 6, normalized size = 0.75

$$-\frac{\cot(x)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^2,x)
```

```
[Out] -cot(x)^3/3
```

3.328 $\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx$

Optimal. Leaf size=17

$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

```
[Out] 1/3*csc(x)^3-1/5*csc(x)^5
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4482, 2686, 14}

$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3),x]
```

```
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \int \cot^3(x)\csc^3(x)\,dx$$
$$= -\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \csc(x)\right)$$
$$= \frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3),x]
[Out] Csc[x]^3/3 - Csc[x]^5/5
```

Maple [A]
time = 0.15, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{5\sin(x)^5}+\frac{1}{3\sin(x)^3}$	14
risch	$-\frac{8i\left(5\,e^{7ix}+2\,e^{5ix}+5\,e^{3ix}\right)}{15\left(e^{2ix}-1\right)^5}$	35
norman	$\frac{-\frac{1}{160}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{96}+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{16}+\frac{\left(\tan^6\left(\frac{x}{2}\right)\right)}{16}+\frac{\left(\tan^8\left(\frac{x}{2}\right)\right)}{96}-\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{160}}{\tan\left(\frac{x}{2}\right)^5}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/5/sin(x)^5+1/3/sin(x)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(13) = 26.
time = 0.29, size = 73, normalized size = 4.29

$$\frac{\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{30\,\sin(x)^4}{(\cos(x)+1)^4}-3\right)(\cos(x)+1)^5}{480\,\sin(x)^5}+\frac{\sin(x)}{16\,(\cos(x)+1)}+\frac{\sin(x)^3}{96\,(\cos(x)+1)^3}-\frac{\sin(x)^5}{160\,(\cos(x)+1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(5*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^4/(cos(x) + 1)^4 - 3)*(cos(x)
+ 1)^5/sin(x)^5 + 1/16*sin(x)/(cos(x) + 1) + 1/96*sin(x)^3/(cos(x) + 1)^3 -
 1/160*sin(x)^5/(cos(x) + 1)^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 1.84, size = 28, normalized size = 1.65

$$-\frac{5\,\cos\left(x\right)^{2}-2}{15\,\left(\cos\left(x\right)^{4}-2\,\cos\left(x\right)^{2}+1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 2)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\cos^{3}\left(x\right)-3\cos^{2}\left(x\right)\sec\left(x\right)+3\cos\left(x\right)\sec^{2}\left(x\right)-\sec^{3}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**3,x)
```

```
[Out] -Integral(1/(cos(x)**3 - 3*cos(x)**2*sec(x) + 3*cos(x)*sec(x)**2 - sec(x)**
3), x)
```

Giac [A]
time = 0.39, size = 14, normalized size = 0.82

$$\frac{5\,\sin\left(x\right)^{2}-3}{15\,\sin\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^3,x, algorithm="giac")
```

```
[Out] 1/15*(5*sin(x)^2 - 3)/sin(x)^5
```

Mupad [B]
time = 2.38, size = 14, normalized size = 0.82

$$\frac{5\sin\left(x\right)^{2}-3}{15\sin\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x))^3,x)
[Out] (5*sin(x)^2 - 3)/(15*sin(x)^5)
```

3.329 $\int \frac{1}{(-\cos(x)+\sec(x))^4} \, dx$

Optimal. Leaf size=17

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^7(x)}{7}$$

```
[Out] -1/5*cot(x)^5-1/7*cot(x)^7
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] -1/5*Cot[x]^5 - Cot[x]^7/7
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^4} \, dx &= \text{Subst}\left(\int \left(\frac{1}{x^8}+\frac{1}{x^6}\right) \, dx, x, \tan(x)\right)\\ &= -\frac{1}{5}\cot^5(x)-\frac{\cot^7(x)}{7}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.
time = 0.02, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35}-\frac{1}{35}\cot(x)\csc^2(x)+\frac{8}{35}\cot(x)\csc^4(x)-\frac{1}{7}\cot(x)\csc^6(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-4),x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

Maple [A]
time = 0.15, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{5\tan(x)^5}-\frac{1}{7\tan(x)^7}$	14
risch	$\frac{4i(35\,e^{10ix}+35\,e^{8ix}+70\,e^{6ix}+14\,e^{4ix}+7\,e^{2ix}-1)}{35(e^{2ix}-1)^7}$	50
norman	$\frac{-\frac{1}{896}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{640}+\frac{\left(\tan^4\left(\frac{x}{2}\right)\right)}{128}-\frac{3\left(\tan^6\left(\frac{x}{2}\right)\right)}{128}+\frac{3\left(\tan^8\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{128}-\frac{\left(\tan^{12}\left(\frac{x}{2}\right)\right)}{640}+\frac{\left(\tan^{14}\left(\frac{x}{2}\right)\right)}{896}}{\tan\left(\frac{x}{2}\right)^7}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/tan(x)^5-1/7/tan(x)^7
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.82

$$-\frac{7\,\tan\left(x\right)^2+5}{35\,\tan\left(x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.
time = 1.24, size = 39, normalized size = 2.29

$$-\frac{2\,\cos\left(x\right)^7-7\,\cos\left(x\right)^5}{35\,\left(\cos\left(x\right)^6-3\,\cos\left(x\right)^4+3\,\cos\left(x\right)^2-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(-\cos\left(x\right)+\sec\left(x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**4,x)
[Out] Integral((-cos(x) + sec(x))**(-4), x)
```

Giac [A]
time = 0.41, size = 14, normalized size = 0.82

$$-\frac{7\,\tan\left(x\right)^{2}+5}{35\,\tan\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^4,x, algorithm="giac")
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Mupad [B]
time = 2.43, size = 16, normalized size = 0.94

$$\frac{\cos\left(x\right)^{5}\,\left(\cos\left(2\,x\right)-6\right)}{35\sin\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^4,x)
[Out] (cos(x)^5*(cos(2*x) - 6))/(35*sin(x)^7)
```

3.330 $\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

[Out] -1/5*csc(x)^5+2/7*csc(x)^7-1/9*csc(x)^9

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4482, 2686, 276}

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-5),x]

[Out] -1/5*Csc[x]^5 + (2*Csc[x]^7)/7 - Csc[x]^9/9

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = \int \cot^5(x)\csc^5(x)\,dx$$
$$= -\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx, x, \csc(x)\right)$$
$$= -\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5),x]
[Out] -1/5*Csc[x]^5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

Maple [A]
time = 0.17, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{1}{5\sin(x)^5}+\frac{2}{7\sin(x)^7}-\frac{1}{9\sin(x)^9}$	20
risch	$-\frac{32i\left(63\,e^{13ix}+108\,e^{11ix}+218\,e^{9ix}+108\,e^{7ix}+63\,e^{5ix}\right)}{315\left(e^{2ix}-1\right)^9}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^5,x,method=_RETURNVERBOSE)
[Out] -1/5/sin(x)^5+2/7/sin(x)^7-1/9/sin(x)^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(19) = 38$.
time = 0.28, size = 121, normalized size = 4.84

$$\frac{\left(\frac{45\sin(x)^2}{(\cos(x)+1)^2}+\frac{252\sin(x)^4}{(\cos(x)+1)^4}-\frac{420\sin(x)^6}{(\cos(x)+1)^6}-\frac{1890\sin(x)^8}{(\cos(x)+1)^8}-35\right)(\cos(x)+1)^9}{161280\sin(x)^9}-\frac{3\sin(x)}{256(\cos(x)+1)}-\frac{\sin(x)^3}{384(\cos(x)+1)^3}+\frac{\sin(x)^5}{640(\cos(x)+1)^5}+\frac{\sin(x)^7}{3584(\cos(x)+1)^7}-\frac{\sin(x)^9}{4608(\cos(x)+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="maxima")
```

[Out] 1/161280*(45*sin(x)^2/(cos(x) + 1)^2 + 252*sin(x)^4/(cos(x) + 1)^4 - 420*sin(x)^6/(cos(x) + 1)^6 - 1890*sin(x)^8/(cos(x) + 1)^8 - 35)*(cos(x) + 1)^9/sin(x)^9 - 3/256*sin(x)/(cos(x) + 1) - 1/384*sin(x)^3/(cos(x) + 1)^3 + 1/640*sin(x)^5/(cos(x) + 1)^5 + 1/3584*sin(x)^7/(cos(x) + 1)^7 - 1/4608*sin(x)^9/(cos(x) + 1)^9

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 1.35, size = 46, normalized size = 1.84

$$-\frac{63\,\cos(x)^4-36\,\cos(x)^2+8}{315\,\left(\cos(x)^8-4\,\cos(x)^6+6\,\cos(x)^4-4\,\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="fricas")

[Out] -1/315*(63*cos(x)^4 - 36*cos(x)^2 + 8)/((cos(x)^8 - 4*cos(x)^6 + 6*cos(x)^4 - 4*cos(x)^2 + 1)*sin(x))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\cos^5(x)-5\cos^4(x)\sec(x)+10\cos^3(x)\sec^2(x)-10\cos^2(x)\sec^3(x)+5\cos(x)\sec^4(x)-\sec^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))**5,x)

[Out] -Integral(1/(cos(x)**5 - 5*cos(x)**4*sec(x) + 10*cos(x)**3*sec(x)**2 - 10*cos(x)**2*sec(x)**3 + 5*cos(x)*sec(x)**4 - sec(x)**5), x)

Giac [A]
time = 0.40, size = 20, normalized size = 0.80

$$-\frac{63\,\sin(x)^4-90\,\sin(x)^2+35}{315\,\sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-cos(x)+sec(x))^5,x, algorithm="giac")

[Out] -1/315*(63*sin(x)^4 - 90*sin(x)^2 + 35)/sin(x)^9

Mupad [B]
time = 2.42, size = 20, normalized size = 0.80

$$-\frac{63\sin(x)^4-90\sin(x)^2+35}{315\sin(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(cos(x) - 1/cos(x))^5,x)

[Out] -(63*sin(x)^4 - 90*sin(x)^2 + 35)/(315*sin(x)^9)

3.331 $\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{7}\cot^7(x) - \frac{2\cot^9(x)}{9} - \frac{\cot^{11}(x)}{11}$$

```
[Out] -1/7*cot(x)^7-2/9*cot(x)^9-1/11*cot(x)^11
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {276}

$$-\frac{1}{11}\cot^{11}(x) - \frac{2\cot^9(x)}{9} - \frac{\cot^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] -1/7*Cot[x]^7 - (2*Cot[x]^9)/9 - Cot[x]^11/11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-\cos(x)+\sec(x))^6} \, dx &= \text{Subst}\left(\int \frac{(1+x^2)^2}{x^{12}} \, dx, x, \tan(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{x^{12}} + \frac{2}{x^{10}} + \frac{1}{x^8}\right) \, dx, x, \tan(x)\right) \\
&= -\frac{1}{7}\cot^7(x) - \frac{2\cot^9(x)}{9} - \frac{\cot^{11}(x)}{11}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.
time = 0.02, size = 57, normalized size = 2.28

$$\frac{8\cot(x)}{693} + \frac{4}{693}\cot(x)\csc^2(x) + \frac{1}{231}\cot(x)\csc^4(x) - \frac{113}{693}\cot(x)\csc^6(x) + \frac{23}{99}\cot(x)\csc^8(x) - \frac{1}{11}\cot(x)\csc^{10}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-6),x]
```

```
[Out] (8*Cot[x])/693 + (4*Cot[x]*Csc[x]^2)/693 + (Cot[x]*Csc[x]^4)/231 - (113*Cot
[x]*Csc[x]^6)/693 + (23*Cot[x]*Csc[x]^8)/99 - (Cot[x]*Csc[x]^10)/11
```

Maple [A]
time = 0.18, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{1}{7\tan(x)^{7}}-\frac{1}{11\tan(x)^{11}}-\frac{2}{9\tan(x)^{9}}$	20
risch	$\frac{16i\left(462\,e^{16ix}+1155\,e^{14ix}+2541\,e^{12ix}+2079\,e^{10ix}+1485\,e^{8ix}+297\,e^{6ix}+55\,e^{4ix}-11\,e^{2ix}+1\right)}{693\left(e^{2ix}-1\right)^{11}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/tan(x)^7-1/11/tan(x)^11-2/9/tan(x)^9
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.80

$$-\frac{99\tan\left(x\right)^{4}+154\tan\left(x\right)^{2}+63}{693\tan\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="maxima")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(19) = 38$.
time = 2.49, size = 57, normalized size = 2.28

$$\frac{8\cos\left(x\right)^{11}-44\cos\left(x\right)^{9}+99\cos\left(x\right)^{7}}{693\left(\cos\left(x\right)^{10}-5\cos\left(x\right)^{8}+10\cos\left(x\right)^{6}-10\cos\left(x\right)^{4}+5\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(8*cos(x)^11 - 44*cos(x)^9 + 99*cos(x)^7)/((cos(x)^10 - 5*cos(x)^8 +
10*cos(x)^6 - 10*cos(x)^4 + 5*cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 20, normalized size = 0.80

$$-\frac{99\,\tan\left(x\right)^{4}+154\,\tan\left(x\right)^{2}+63}{693\,\tan\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^6,x, algorithm="giac")
```

```
[Out] -1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

Mupad [B]
time = 2.45, size = 46, normalized size = 1.84

$$-\frac{80\cos\left(x\right)^{7}-18\cos\left(x\right)^{7}\left(2\cos\left(x\right)^{2}-1\right)+\cos\left(x\right)^{7}\left(2\left(2\cos\left(x\right)^{2}-1\right)^{2}-1\right)}{693\sin\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x) - 1/cos(x))^6,x)
```

```
[Out] -(80*cos(x)^7 - 18*cos(x)^7*(2*cos(x)^2 - 1) + cos(x)^7*(2*(2*cos(x)^2 - 1)
^2 - 1))/(693*sin(x)^11)
```

3.332 $\int \frac{1}{(-\cos(x)+\sec(x))^7} \, dx$

Optimal. Leaf size=33

$$\frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3\csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13}$$

[Out] 1/7*csc(x)^7-1/3*csc(x)^9+3/11*csc(x)^11-1/13*csc(x)^13

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 2686, 276}

$$-\frac{1}{13}\csc^{13}(x) + \frac{3\csc^{11}(x)}{11} - \frac{\csc^9(x)}{3} + \frac{\csc^7(x)}{7}$$

Antiderivative was successfully verified.

[In] Int[(-Cos[x] + Sec[x])^(-7),x]

[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \int \cot^7(x)\csc^7(x)\,dx$$
$$= -\text{Subst}\left(\int x^6\left(-1+x^2\right)^3\,dx, x, \csc(x)\right)$$
$$= -\text{Subst}\left(\int \left(-x^6+3x^8-3x^{10}+x^{12}\right)\,dx, x, \csc(x)\right)$$
$$= \frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3\csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3\csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7),x]
[Out] Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

Maple [A]
time = 0.21, size = 26, normalized size = 0.79

method	result	size
default	$\frac{1}{7\sin(x)^7} - \frac{1}{13\sin(x)^{13}} - \frac{1}{3\sin(x)^9} + \frac{3}{11\sin(x)^{11}}$	26
risch	$-\frac{128i\left(429\,e^{19ix}+1430\,e^{17ix}+3523\,e^{15ix}+4020\,e^{13ix}+3523\,e^{11ix}+1430\,e^{9ix}+429\,e^{7ix}\right)}{3003\left(e^{2ix}-1\right)^{13}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^7,x,method=_RETURNVERBOSE)
[Out] 1/7/sin(x)^7-1/13/sin(x)^13-1/3/sin(x)^9+3/11/sin(x)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(25) = 50$.
time = 0.29, size = 169, normalized size = 5.12

$$\frac{\left(\frac{273\,\sin(x)^2}{(\cos(x)+1)^2} + \frac{2002\,\sin(x)^4}{(\cos(x)+1)^4} - \frac{2574\,\sin(x)^6}{(\cos(x)+1)^6} - \frac{9009\,\sin(x)^8}{(\cos(x)+1)^8} + \frac{15015\,\sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{60060\,\sin(x)^{12}}{(\cos(x)+1)^{12}} - 231\right)(\cos(x)+1)^{13}}{24600576\,\sin(x)^{13}} + \frac{5\,\sin(x)}{2048\,(\cos(x)+1)} + \frac{5\,\sin(x)^3}{8192\,(\cos(x)+1)^3} - \frac{3\,\sin(x)^5}{8192\,(\cos(x)+1)^5} - \frac{3\,\sin(x)^7}{28672\,(\cos(x)+1)^7} + \frac{\sin(x)^9}{12288\,(\cos(x)+1)^9} + \frac{\sin(x)^{11}}{90112\,(\cos(x)+1)^{11}} - \frac{\sin(x)^{13}}{106496\,(\cos(x)+1)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="maxima")
```

```
[Out] 1/24600576*(273*sin(x)^2/(cos(x) + 1)^2 + 2002*sin(x)^4/(cos(x) + 1)^4 - 25
74*sin(x)^6/(cos(x) + 1)^6 - 9009*sin(x)^8/(cos(x) + 1)^8 + 15015*sin(x)^10
/(cos(x) + 1)^10 + 60060*sin(x)^12/(cos(x) + 1)^12 - 231)*(cos(x) + 1)^13/s
in(x)^13 + 5/2048*sin(x)/(cos(x) + 1) + 5/8192*sin(x)^3/(cos(x) + 1)^3 - 3/
8192*sin(x)^5/(cos(x) + 1)^5 - 3/28672*sin(x)^7/(cos(x) + 1)^7 + 1/12288*si
n(x)^9/(cos(x) + 1)^9 + 1/90112*sin(x)^11/(cos(x) + 1)^11 - 1/106496*sin(x)
^13/(cos(x) + 1)^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.
time = 4.98, size = 64, normalized size = 1.94

$$-\frac{429 \cos(x)^6 - 286 \cos(x)^4 + 104 \cos(x)^2 - 16}{3003 \left(\cos(x)^{12} - 6 \cos(x)^{10} + 15 \cos(x)^8 - 20 \cos(x)^6 + 15 \cos(x)^4 - 6 \cos(x)^2 + 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*cos(x)^6 - 286*cos(x)^4 + 104*cos(x)^2 - 16)/((cos(x)^12 - 6*c
os(x)^10 + 15*cos(x)^8 - 20*cos(x)^6 + 15*cos(x)^4 - 6*cos(x)^2 + 1)*sin(x)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**7,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 26, normalized size = 0.79

$$\frac{429 \sin(x)^6 - 1001 \sin(x)^4 + 819 \sin(x)^2 - 231}{3003 \sin(x)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^7,x, algorithm="giac")
```

```
[Out] 1/3003*(429*sin(x)^6 - 1001*sin(x)^4 + 819*sin(x)^2 - 231)/sin(x)^13
```

Mupad [B]
time = 2.51, size = 109, normalized size = 3.30

$$-\frac{\cot\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\cot\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\cot\left(\frac{x}{2}\right)^{9}}{12288} - \frac{3\cot\left(\frac{x}{2}\right)^{7}}{28672} - \frac{3\cot\left(\frac{x}{2}\right)^{5}}{8192} + \frac{5\cot\left(\frac{x}{2}\right)^{3}}{8192} + \frac{5\cot\left(\frac{x}{2}\right)}{2048} - \frac{\tan\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\tan\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\tan\left(\frac{x}{2}\right)^{9}}{12288} - \frac{3\tan\left(\frac{x}{2}\right)^{7}}{28672} - \frac{3\tan\left(\frac{x}{2}\right)^{5}}{8192} + \frac{5\tan\left(\frac{x}{2}\right)^{3}}{8192} + \frac{5\tan\left(\frac{x}{2}\right)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x) - 1/cos(x))^7,x)
```

```
[Out] (5*cot(x/2))/2048 + (5*tan(x/2))/2048 + (5*cot(x/2)^3)/8192 - (3*cot(x/2)^5
)/8192 - (3*cot(x/2)^7)/28672 + cot(x/2)^9/12288 + cot(x/2)^11/90112 - cot(
x/2)^13/106496 + (5*tan(x/2)^3)/8192 - (3*tan(x/2)^5)/8192 - (3*tan(x/2)^7)
/28672 + tan(x/2)^9/12288 + tan(x/2)^11/90112 - tan(x/2)^13/106496
```

3.333 $\int (-\cos(x)+\sec(x))^{7/2}\,dx$

Optimal. Leaf size=73

$$-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{7}\sin^3(x)$$

```
[Out] -256/35*csc(x)*(sin(x)*tan(x))^(1/2)+64/35*sec(x)*(sin(x)*tan(x))^(1/2)*tan
(x)-8/7*sin(x)*(sin(x)*tan(x))^(1/2)*tan(x)^2-2/7*sin(x)^3*(sin(x)*tan(x))^
(1/2)*tan(x)^2
```

Rubi [A]

time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {4482, 4485, 2678, 2674, 2669}

$$-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\tan(x)\sec(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(7/2),x]
```

```
[Out] (-256*Csc[x]*Sqrt[Sin[x]*Tan[x]])/35 + (64*Sec[x]*Tan[x]*Sqrt[Sin[x]*Tan[x]
])/35 - (8*Sin[x]*Tan[x]^2*Sqrt[Sin[x]*Tan[x]])/7 - (2*Sin[x]^3*Tan[x]^2*Sq
rt[Sin[x]*Tan[x]])/7
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \int (\sin(x)\tan(x))^{7/2}\,dx$$
$$= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(12\sqrt{\sin(x)\tan(x)}\right)\int \sin^{\frac{3}{2}}(x)\tan^{\frac{7}{2}}(x$$
$$= -\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} + \frac{(3}{}$$
$$= \frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{7}\sin$$
$$= -\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)} + \frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{8}{7}\sin(x)$$

Mathematica [A]
time = 0.15, size = 37, normalized size = 0.51

$$\frac{1}{70}\sec(x)\sqrt{\sin(x)\tan(x)}\,(-512\cot(x)-5\cos(x)(-23\sin(x)+\sin(3x))+28\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(7/2),x]
[Out] (Sec[x]*Sqrt[Sin[x]*Tan[x]]*(-512*Cot[x] - 5*Cos[x]*(-23*Sin[x] + Sin[3*x])
 + 28*Tan[x]))/70
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(57) = 114$.

time = 0.50, size = 603, normalized size = 8.26

method	result	size
default	Expression too large to display	603

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*(cos(x)-1)^2*(-105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2
)^(1/2)-1)/sin(x)^2)+105*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x
)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)
^(1/2)-1)/sin(x)^2)-315*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(
x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2
)^(1/2)-1)/sin(x)^2)+315*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x
)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)
^(1/2)-1)/sin(x)^2)+20*cos(x)^6-315*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*l
n(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/
(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+315*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*
ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(
1+cos(x))^2)^(1/2)-1)/sin(x)^2)-105*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(
-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1
+cos(x))^2)^(1/2)-1)/sin(x)^2)+105*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-
(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+co
s(x))^2)^(1/2)-1)/sin(x)^2)-140*cos(x)^4-420*cos(x)^2+28)*cos(x)*(1+cos(x))
^2*(-(cos(x)^2-1)/cos(x))^(7/2)/sin(x)^11
```

Maxima [A]

time = 0.51, size = 82, normalized size = 1.12

$$\frac{128\left(\frac{7\sin(x)^4}{(\cos(x)+1)^4}-\frac{7\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{2\sin(x)^{14}}{(\cos(x)+1)^{14}}-2\right)}{35\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] 128/35*(7*sin(x)^4/(cos(x) + 1)^4 - 7*sin(x)^10/(cos(x) + 1)^10 + 2*sin(x)^
14/(cos(x) + 1)^14 - 2)/((sin(x)/(cos(x) + 1) + 1)^(7/2)*(-sin(x)/(cos(x) +
 1) + 1)^(7/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(7/2))
```

Fricas [A]
time = 3.14, size = 44, normalized size = 0.60

$$\frac{2\left(5\cos\left(x\right)^{6}-35\cos\left(x\right)^{4}-105\cos\left(x\right)^{2}+7\right)\sqrt{-\frac{\cos\left(x\right)^{2}-1}{\cos\left(x\right)}}}{35\cos\left(x\right)^{2}\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(x)^6 - 35*cos(x)^4 - 105*cos(x)^2 + 7)*sqrt(-(cos(x)^2 - 1)/cos
(x))/(cos(x)^2*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((-cos(x) + sec(x))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(x\right)}-\cos\left(x\right)\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(7/2),x)
```

```
[Out] int((1/cos(x) - cos(x))^(7/2), x)
```

3.334 $\int (-\cos(x)+\sec(x))^{5/2}\,dx$

Optimal. Leaf size=50

$$\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*s
in(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.454$, Rules used = {4482, 4485, 2678, 2674, 2669}

$$-\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(5/2),x]
```

```
[Out] (64*Cot[x]*Sqrt[Sin[x]*Tan[x]])/15 + (16*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15 - (
2*Sin[x]^2*Tan[x]*Sqrt[Sin[x]*Tan[x]])/5
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = \int (\sin(x)\tan(x))^{5/2}\,dx$$
$$= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$
$$= -\frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} + \frac{\left(8\sqrt{\sin(x)\tan(x)}\right)\int \sqrt{\sin(x)}\,\tan^{\frac{5}{2}}(x$$
$$= \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{\left(32\sqrt{\sin(x$$
$$= \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{$$

Mathematica [**A**]
time = 0.06, size = 29, normalized size = 0.58

$$\frac{2}{15}\left(5+3\cos^2(x)+32\cot^2(x)\right)\tan(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(5/2),x]
```

```
[Out] (2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(38) = 76.
time = 0.40, size = 321, normalized size = 6.42

method	result

default	$-\frac{(\cos(x)-1)^2\left(6(\cos^4(x))-15(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}\ \ln\left(-\frac{2(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-(\cos^2(x))+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos}}}{\sin(x)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(cos(x)-1)^2*(6*cos(x)^4-15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(
-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)*ln(-2
*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+c
os(x))^2)^(1/2)-1)/sin(x)^2)-15*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^
2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-co
s(x)/(1+cos(x))^2)^(1/2)+15*cos(x)*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)
^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(
x)/(1+cos(x))^2)^(1/2)-60*cos(x)^2-10)*cos(x)*(1+cos(x))^2*(-(cos(x)^2-1)/c
os(x))^(5/2)/sin(x)^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.
time = 0.50, size = 82, normalized size = 1.64

$$-\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] -32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^10
/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) + 1
) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

Fricas [A]
time = 3.02, size = 38, normalized size = 0.76

$$-\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*s
in(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(5/2),x, algorithm="giac")
[Out] integrate((-cos(x) + sec(x))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cos\left(x\right)}-\cos\left(x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(5/2),x)
[Out] int((1/cos(x) - cos(x))^(5/2), x)
```

3.335 $\int (-\cos(x)+\sec(x))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] 8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {4482, 4485, 2678, 2669}

$$\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(3/2),x]
```

```
[Out] (8*Csc[x]*Sqrt[Sin[x]*Tan[x]])/3 - (2*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int (-\cos(x)+\sec(x))^{3/2}\,dx &= \int (\sin(x)\tan(x))^{3/2}\,dx\\
&= \frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\
&= -\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}+\frac{\left(4\sqrt{\sin(x)\tan(x)}\right)\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}\,dx}{3\sqrt{\sin(x)}\sqrt{\tan(x)}}\\
&= \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.74

$$\frac{2}{3}\left(-1+4\csc^2(x)\right)\sin(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(3/2),x]
[Out] (2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(23) = 46$.
time = 0.21, size = 584, normalized size = 18.84

method	result	size
default	Expression too large to display	584

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(cos(x)-1)^2*(3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2
*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1
/2)-1)/sin(x)^2)-3*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-
cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)
-1)/sin(x)^2)+9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-c
os(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-
1)/sin(x)^2)-9*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos(
x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/
sin(x)^2)+9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-2*(2*cos(x)^2*(-cos(x)/
```

```
(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin
(x)^2)-9*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)*ln(-(2*cos(x)^2*(-cos(x)/(1+co
s(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2
)+3*ln(-2*(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-co
s(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)-3*ln(-(2
*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(
x))^2)^(1/2)-1)/sin(x)^2)*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(x)^3+12*cos(x)
)*(1+cos(x))^2*(-(cos(x)^2-1)/cos(x))^(3/2)/sin(x)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.
time = 0.49, size = 57, normalized size = 1.84

$$-\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
[Out] -8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(x
)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

Fricas [A]
time = 2.75, size = 26, normalized size = 0.84

$$\frac{2\left(\cos(x)^2+3\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{3\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
[Out] 2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(-\cos(x)+\sec(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))**(3/2),x)
[Out] Integral((-cos(x) + sec(x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(3/2),x, algorithm="giac")
[Out] integrate((-cos(x) + sec(x))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cos\left(x\right)}-\cos\left(x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(3/2),x)
[Out] int((1/cos(x) - cos(x))^(3/2), x)
```

3.336 $\int \sqrt{-\cos(x)+\sec(x)}\,dx$

Optimal. Leaf size=13

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

```
[Out] -2*cot(x)*(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4482, 4485, 2669}

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{-\cos(x)+\sec(x)}\,dx &= \int \sqrt{\sin(x)\tan(x)}\,dx\\
&= \frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}\,dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\\
&= -2\cot(x)\sqrt{\sin(x)\tan(x)}
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 13, normalized size = 1.00

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] -2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(11) = 22$.
time = 0.23, size = 174, normalized size = 13.38

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{-\frac{(e^{2ix}-1)^2 e^{-ix}}{e^{2ix}+1}}\,(e^{2ix}+1)}{e^{2ix}-1}$
default	$\frac{(\cos(x)-1)\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}+\ln\left(-\frac{2\left(2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-1\right)}{\sin(x)^2}\right)-\right.}{2\sin(x)^3\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sec(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(x)-1)*(4*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)+ln(-2*(2*cos(x)^2*(-c
os(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-
1)/sin(x)^2)-ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)
-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+4*(-cos(x)/(1+cos(x))^2)^(1/2)
)*cos(x)*(-(cos(x)^2-1)/cos(x))^(1/2)/sin(x)^3/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.
time = 0.50, size = 57, normalized size = 4.38

$$\frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\,\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\,\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

[Out] 2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))

Fricas [A]
time = 1.73, size = 22, normalized size = 1.69

$$-\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\cos(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))**(1/2),x)

[Out] Integral(sqrt(-cos(x) + sec(x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(11) = 22.
time = 0.43, size = 46, normalized size = 3.54

$$-\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)\right)\mathrm{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-cos(x)+sec(x))^(1/2),x, algorithm="giac")

[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x))*sgn(cos(x))/((sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 1)

Mupad [B]
time = 2.42, size = 20, normalized size = 1.54

$$-\frac{2\sin(x)}{\sqrt{\frac{1}{\cos(x)}}\sqrt{1-\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x) - cos(x))^(1/2),x)
[Out] -(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

3.337 $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

Optimal. Leaf size=52

$$\frac{\operatorname{ArcTan}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-arctanh(cos(
x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4482, 4485, 2681, 2645, 335, 304, 209, 212}

$$\frac{\sin(x)\operatorname{ArcTan}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (ArcTanh
[Sqrt[Cos[x]]]*Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \int \frac{1}{\sqrt{\sin(x)\tan(x)}}\,dx$$
$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(x)\tan(x)}}$$
$$= \frac{\sin(x)\int \sqrt{\cos(x)}\csc(x)\,dx}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\sin(x)\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(x)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{(2\sin(x))\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\sin(x)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.83

$$\frac{\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(x)}\right) - \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\right)\cos(x)\cot(x)\sqrt{\sin(x)\tan(x)}}{\cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Cos[x] + Sec[x]],x]
```

```
[Out] ((ArcTan[(Cos[x]^2)^(1/4)] - ArcTanh[(Cos[x]^2)^(1/4)])*Cos[x]*Cot[x]*Sqrt[
Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(40) = 80$.
time = 0.21, size = 105, normalized size = 2.02

method	result

default	$-\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}\right)+\ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-\left(\cos^2(x)\right)+2\cos(x)-2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-1}{\sin(x)^2}\right)\right)(1+\cos(}{2\sin(x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+ln(-(2*cos(x)^2*(-cos(x)/(1+
cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)
^2))*(1+cos(x))*(-cos(x)/(1+cos(x))^2)^(1/2)*((1-cos(x)^2)/cos(x))^(1/2)/si
n(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-cos(x) + sec(x)), x)
```

Fricas [A]
time = 2.92, size = 72, normalized size = 1.38

$$-\frac{1}{2}\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)+\frac{1}{2}\log\left(\frac{(\cos(x)+1)\sin(x)-2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x))) +
1/2*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos
(x) - 1)*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-cos(x) + sec(x)), x)
```

Giac [A]
time = 0.45, size = 35, normalized size = 0.67

$$\frac{1}{2}\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right)-\frac{1}{2}\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(tan(1/2*x)^2) - 1/2*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x
)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{1}{\cos\left(x\right)}-\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(1/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(1/2), x)
```

3.338 $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\operatorname{ArcTan}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] -1/2*csc(x)/(sin(x)*tan(x))^(1/2)+1/4*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1
/2)/(sin(x)*tan(x))^(1/2)+1/4*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(si
n(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {4482, 4485, 2677, 2681, 2645, 335, 218, 212, 209}

$$\frac{\sin(x)\operatorname{ArcTan}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}}+\frac{\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] -1/2*Csc[x]/Sqrt[Sin[x]*Tan[x]] + (ArcTan[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos
[x]]*Sqrt[Sin[x]*Tan[x]]) + (ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(4*Sqrt[Cos[x]]*
Sqrt[Sin[x]*Tan[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{3/2}}\,dx$$
$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{3}{2}}(x)}\,dx}{4\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} - \frac{\sin(x)\int \frac{\csc(x)}{\sqrt{\cos(x)}}\,dx}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-x^2)}\,dx, x, \cos(x)\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(x)}\right)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\sin(x)\text{Subst}}{4\sqrt{\cos}}$$
$$= -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\tanh^{-1}\left(\sqrt{\cos(x)}\right)\text{s}}{4\sqrt{\cos(x)}\sqrt{\sin(x)\text{ta}}}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 0.78

$$\frac{\cot(x)\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(x)}\right) + \tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right) - 2\sqrt[4]{\cos^2(x)}\csc^2(x)\right)\sqrt{\sin(x)\tan(x)}}{4\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-3/2),x]
```

```
[Out] (Cot[x]*(ArcTan[(Cos[x]^2)^(1/4)] + ArcTanh[(Cos[x]^2)^(1/4)] - 2*(Cos[x]^2
)^(1/4)*Csc[x]^2)*Sqrt[Sin[x]*Tan[x]])/(4*(Cos[x]^2)^(1/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(52) = 104.
time = 0.23, size = 265, normalized size = 3.68

method	result
default	$-\frac{(\cos(x)-1)\left(8(\cos^2(x))\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}+16\cos(x)\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}+(\cos^2(x))\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}}\right)-(\cos^2(x))\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(cos(x)-1)*(8*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/2)+16*cos(x)*(-cos(x)
/(1+cos(x))^2)^(3/2)+cos(x)^2*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-cos(
x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos
(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+8*(-cos(x)/(1+cos(x))^2)^(3/2)+4*cos(x
)*(-cos(x)/(1+cos(x))^2)^(1/2)-4*(-cos(x)/(1+cos(x))^2)^(1/2)-arctan(1/2/(-
cos(x)/(1+cos(x))^2)^(1/2))+ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-co
s(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2))/cos(x)/sin(x)/
(-(cos(x)^2-1)/cos(x))^(3/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.
time = 3.37, size = 119, normalized size = 1.65

$$-\frac{(\cos(x)^2-1)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-(\cos(x)^2-1)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-4\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{8(\cos(x)^2-1)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x)
- 1)*sin(x)))*sin(x) - (cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-(c
os(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*sqrt(-(cos(x
)^2 - 1)/cos(x))*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\cos\left(x\right)+\sec\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(3/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-3/2), x)
```

Giac [A]
time = 0.47, size = 95, normalized size = 1.32

$$-\frac{\tan\left(\frac{1}{2}\,x\right)^{2}}{16\,\left(\sqrt{-\tan\left(\frac{1}{2}\,x\right)^{4}+1}-1\right)}+\frac{1}{8}\,\sqrt{-\tan\left(\frac{1}{2}\,x\right)^{4}+1}+\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^{4}+1}-1}{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}+\frac{1}{8}\,\arcsin\left(\tan\left(\frac{1}{2}\,x\right)^{2}\right)+\frac{1}{8}\,\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^{4}+1}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*tan(1/2*x)^2/(sqrt(-tan(1/2*x)^4 + 1) - 1) + 1/8*sqrt(-tan(1/2*x)^4 +
 1) + 1/16*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1/8*arcsin(tan(1/2*
x)^2) + 1/8*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos\left(x\right)}-\cos\left(x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(3/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(3/2), x)
```

3.339 $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}-\frac{3\text{ArcTan}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{3\tanh^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

```
[Out] 3/16*cot(x)/(sin(x)*tan(x))^(1/2)-1/4*cot(x)*csc(x)^2/(sin(x)*tan(x))^(1/2)
-3/32*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)+3/32*a
rctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {4482, 4485, 2677, 2679, 2681, 2645, 335, 304, 209, 212}

$$-\frac{3\sin(x)\text{ArcTan}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}}+\frac{3\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] (3*Cot[x])/(16*Sqrt[Sin[x]*Tan[x]]) - (Cot[x]*Csc[x]^2)/(4*Sqrt[Sin[x]*Tan[
x]]) - (3*ArcTan[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]
) + (3*ArcTanh[Sqrt[Cos[x]]]*Sin[x])/(32*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
```

```
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{5/2}}\,dx$$

$$= \frac{\left(\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)\sqrt{\tan(x)}}\,dx}{8\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{\left(3\sqrt{\sin(x)}\sqrt{\tan(x)}\right)\int \frac{}{\sqrt{\sin(x}}}{32\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{(3\sin(x))\int \sqrt{\cos(x)}\csc(x)\,dx}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos\right.}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\text{co}}\right.}{16\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} + \frac{(3\sin(x))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\text{co}}\right.}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{3\tan^{-1}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{3}{32}$$

Mathematica [A]
time = 0.50, size = 73, normalized size = 0.80

$$-\frac{\cot(x)\left(3\text{ArcTan}\left(\sqrt[4]{\cos^2(x)}\right)\cos(x)-3\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)\cos(x)+\cos^2(x)^{3/4}(5+3\cos(2x))\cot(x)\csc^3(x)\right)\sqrt{\sin(x)\tan(x)}}{32\cos^2(x)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-5/2),x]
```

```
[Out] -1/32*(Cot[x]*(3*ArcTan[(Cos[x]^2)^(1/4)]*Cos[x] - 3*ArcTanh[(Cos[x]^2)^(1/
4)]*Cos[x] + (Cos[x]^2)^(3/4)*(5 + 3*Cos[2*x])*Cot[x]*Csc[x]^3)*Sqrt[Sin[x]
*Tan[x]])/(Cos[x]^2)^(3/4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(67) = 134$.
time = 0.22, size = 454, normalized size = 4.99

method	result
default	$\frac{\left(24\left(\cos^3(x)\right)\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}+40\left(\cos^2(x)\right)\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}-12\left(\cos^3(x)\right)\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-3\left(\cos^3(x)\right)\ln\left(-\frac{2\left(\cos^2(x)\right)\sqrt{-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*(24*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(3/2)+40*cos(x)^2*(-cos(x)/(1+cos(
x))^2)^(3/2)-12*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)-3*cos(x)^3*ln(-(2*cos
(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^
2)^(1/2)-1)/sin(x)^2)-3*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+8
*cos(x)*(-cos(x)/(1+cos(x))^2)^(3/2)+24*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/
2)+3*cos(x)^2*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x
)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)^2*arctan(1/2/(-cos(x
)/(1+cos(x))^2)^(1/2))-8*(-cos(x)/(1+cos(x))^2)^(3/2)-12*cos(x)*(-cos(x)/(1
+cos(x))^2)^(1/2)+3*cos(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos
(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+3*cos(x)*arctan(
1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-3*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^
(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)-3*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)/cos(x)^2/(-(cos(x)^2-1)/cos(x))
^(5/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(67) = 134$.

time = 3.05, size = 147, normalized size = 1.62

$$\frac{3\left(\cos(x)^4-2\cos(x)^2+1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-4\left(3\cos(x)^4+\cos(x)^2\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{64\left(\cos(x)^4-2\cos(x)^2+1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos(x)^4 - 2*cos(x)^2 + 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*c
os(x)/((cos(x) - 1)*sin(x)))*sin(x) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(((c
os(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*si
n(x)))*sin(x) - 4*(3*cos(x)^4 + cos(x)^2)*sqrt(-(cos(x)^2 - 1)/cos(x)))/((c
os(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(5/2),x)
```

```
[Out] Integral((-cos(x) + sec(x))**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.
time = 0.49, size = 151, normalized size = 1.66

$$\frac{\left(\frac{4\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}+1\right)\tan\left(\frac{1}{2}x\right)^4}{256\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}-\frac{1}{64}\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}\left(\tan\left(\frac{1}{2}x\right)^2-2\right)-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{64\tan\left(\frac{1}{2}x\right)^2}-\frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{256\tan\left(\frac{1}{2}x\right)^4}-\frac{3}{64}\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right)+\frac{3}{64}\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

```
[Out] 1/256*(4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1)*tan(1/2*x)^4/(sqrt
(-tan(1/2*x)^4 + 1) - 1)^2 - 1/64*sqrt(-tan(1/2*x)^4 + 1)*(tan(1/2*x)^2 - 2
) - 1/64*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 1/256*(sqrt(-tan(1/2*
x)^4 + 1) - 1)^2/tan(1/2*x)^4 - 3/64*arcsin(tan(1/2*x)^2) + 3/64*log(-(sqrt
(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)}-\cos(x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(5/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(5/2), x)
```

3.340 $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}}-\frac{5\text{ArcTan}\left(\sqrt{\cos(x)}\right)\sin(x)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{5\tanh^{-1}\left(\right.}{128\sqrt{\cos(x}$$

```
[Out] -5/192*csc(x)/(sin(x)*tan(x))^(1/2)+5/48*csc(x)^3/(sin(x)*tan(x))^(1/2)-1/6
*cot(x)^2*csc(x)^3/(sin(x)*tan(x))^(1/2)-5/128*arctan(cos(x)^(1/2))*sin(x)/
cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-5/128*arctanh(cos(x)^(1/2))*sin(x)/cos(x
)^(1/2)/(sin(x)*tan(x))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4482, 4485, 2677, 2679, 2681, 2645, 335, 218, 212, 209}

$$-\frac{5\sin(x)\text{ArcTan}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}}-\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}-\frac{5\sin(x)\tanh^{-1}\left(\sqrt{\cos(x)}\right)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}-\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] (-5*Csc[x])/(192*Sqrt[Sin[x]*Tan[x]]) + (5*Csc[x]^3)/(48*Sqrt[Sin[x]*Tan[x]
]) - (Cot[x]^2*Csc[x]^3)/(6*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTan[Sqrt[Cos[x]]]*
Sin[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]]) - (5*ArcTanh[Sqrt[Cos[x]]]*S
in[x])/(128*Sqrt[Cos[x]]*Sqrt[Sin[x]*Tan[x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \int \frac{1}{(\sin(x)\tan(x))^{7/2}}\,dx$$

$$= \frac{\left(\sqrt{\sin(x)}\,\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)}\,dx}{\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{\left(5\sqrt{\sin(x)}\,\sqrt{\tan(x)}\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{3}{2}}(x)}\,dx}{12\sqrt{\sin(x)\tan(x)}}$$

$$= \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin(x)}\,\sqrt{\tan(x)}\right)\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{7}{2}}(x)}}{96\sqrt{\sin(x)\tan(x)}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{\left(5\sqrt{\sin}\right.}{}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} + \frac{(5\sin}{128\sqrt{\mathrm{co}}}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x)}{1}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x)}{64}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{(5\sin(x)}{128}$$

$$= -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}} + \frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{5\tan^{-1}}{128\sqrt{\mathrm{co}}}$$

Mathematica [A]

time = 0.27, size = 74, normalized size = 0.67

$$-\frac{\cot(x)\left(15\mathrm{ArcTan}\left(\sqrt[4]{\cos^2(x)}\right)+15\tanh^{-1}\left(\sqrt[4]{\cos^2(x)}\right)+2\sqrt[4]{\cos^2(x)}\,\csc^2(x)\left(5-52\csc^2(x)+32\csc^4(x)\right)\right)\sqrt{\sin(x)\tan(x)}}{384\sqrt[4]{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sec[x])^(-7/2),x]
```

```
[Out] -1/384*(Cot[x]*(15*ArcTan[(Cos[x]^2)^(1/4)] + 15*ArcTanh[(Cos[x]^2)^(1/4)]
+ 2*(Cos[x]^2)^(1/4)*Csc[x]^2*(5 - 52*Csc[x]^2 + 32*Csc[x]^4))*Sqrt[Sin[x]*
Tan[x]])/(Cos[x]^2)^(1/4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(82) = 164$.
time = 0.24, size = 494, normalized size = 4.49

method	result
default	$\frac{\left(56(\cos^4(x))\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}-16(\cos^3(x))\left(-\frac{\cos(x)}{(1+\cos(x))^2}\right)^{\frac{3}{2}}-15(\cos^4(x))\ln\left(-\frac{2(\cos^2(x))\sqrt{-\frac{\cos(x)}{(1+\cos(x))^2}}-(\cos^2(x))+2\cos(x}{\sin(x)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(x)+sec(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768*(56*cos(x)^4*(-cos(x)/(1+cos(x))^2)^(3/2)-16*cos(x)^3*(-cos(x)/(1+cos
(x))^2)^(3/2)-15*cos(x)^4*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(
x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+15*cos(x)^4*arcta
n(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))-192*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(3/
2)+76*cos(x)^3*(-cos(x)/(1+cos(x))^2)^(1/2)+30*cos(x)^3*ln(-(2*cos(x)^2*(-c
os(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-
1)/sin(x)^2)-30*cos(x)^3*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2))+16*cos(x)
*(-cos(x)/(1+cos(x))^2)^(3/2)-148*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)+136
*(-cos(x)/(1+cos(x))^2)^(3/2)+196*cos(x)*(-cos(x)/(1+cos(x))^2)^(1/2)-30*co
s(x)*ln(-(2*cos(x)^2*(-cos(x)/(1+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos
(x)/(1+cos(x))^2)^(1/2)-1)/sin(x)^2)+30*cos(x)*arctan(1/2/(-cos(x)/(1+cos(x
))^2)^(1/2))-60*(-cos(x)/(1+cos(x))^2)^(1/2)+15*ln(-(2*cos(x)^2*(-cos(x)/(1
+cos(x))^2)^(1/2)-cos(x)^2+2*cos(x)-2*(-cos(x)/(1+cos(x))^2)^(1/2)-1)/sin(x
)^2)-15*arctan(1/2/(-cos(x)/(1+cos(x))^2)^(1/2)))*sin(x)^3/(cos(x)-1)/cos(x
)^3/(-(cos(x)^2-1)/cos(x))^(7/2)/(-cos(x)/(1+cos(x))^2)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((-cos(x) + sec(x))^(-7/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.
time = 3.05, size = 171, normalized size = 1.55

$$\frac{15 \left(\cos\left(x\right)^{6}-3 \cos\left(x\right)^{4}+3 \cos\left(x\right)^{2}-1\right) \arctan\left(\frac{2 \sqrt{-\frac{\cos\left(x\right)^{2}-1}{\cos\left(x\right)}} \cos\left(x\right)}{\left(\cos\left(x\right)-1\right) \sin\left(x\right)}\right) \sin\left(x\right)+15 \left(\cos\left(x\right)^{6}-3 \cos\left(x\right)^{4}+3 \cos\left(x\right)^{2}-1\right) \log\left(\frac{\left(\cos\left(x\right)+1\right) \sin\left(x\right)-2 \sqrt{-\frac{\cos\left(x\right)^{2}-1}{\cos\left(x\right)}} \cos\left(x\right)}{\left(\cos\left(x\right)-1\right) \sin\left(x\right)}\right) \sin\left(x\right)+4 \left(5 \cos\left(x\right)^{5}+42 \cos\left(x\right)^{3}-15 \cos\left(x\right)\right) \sqrt{-\frac{\cos\left(x\right)^{2}-1}{\cos\left(x\right)}}}{768 \left(\cos\left(x\right)^{6}-3 \cos\left(x\right)^{4}+3 \cos\left(x\right)^{2}-1\right) \sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/768*(15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2
 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 15*(cos(x)^6 - 3*cos(x
)^4 + 3*cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos
(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) + 4*(5*cos(x)^5 + 42*cos(x)^3 -
15*cos(x))*sqrt(-(cos(x)^2 - 1)/cos(x)))/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1)*sin(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(82) = 164.
time = 0.54, size = 210, normalized size = 1.91

$$-\frac{\left(\frac{3 \left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)}{\tan\left(\frac{1}{2} x\right)^{2}}-\frac{27 \left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)^{2}}{\tan\left(\frac{1}{2} x\right)^{4}}+1\right) \tan\left(\frac{1}{2} x\right)^{6}}{3072 \left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)^{3}}+\frac{1}{768} \sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1} \left(\left(2 \tan\left(\frac{1}{2} x\right)^{2}-3\right) \tan\left(\frac{1}{2} x\right)^{2}-14\right)-\frac{9 \left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)}{1024 \tan\left(\frac{1}{2} x\right)^{2}}+\frac{\left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)^{2}}{1024 \tan\left(\frac{1}{2} x\right)^{4}}+\frac{\left(\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1\right)^{3}}{3072 \tan\left(\frac{1}{2} x\right)^{6}}-\frac{5}{256} \arcsin\left(\tan\left(\frac{1}{2} x\right)^{2}\right)-\frac{5}{256} \log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2} x\right)^{4}+1}-1}{\tan\left(\frac{1}{2} x\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

```
[Out] -1/3072*(3*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 27*(sqrt(-tan(1/2*x
)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1)*tan(1/2*x)^6/(sqrt(-tan(1/2*x)^4 + 1) - 1
)^3 + 1/768*sqrt(-tan(1/2*x)^4 + 1)*((2*tan(1/2*x)^2 - 3)*tan(1/2*x)^2 - 14
```

```
) - 9/1024*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1/1024*(sqrt(-tan(1
/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1/3072*(sqrt(-tan(1/2*x)^4 + 1) - 1)^3/t
an(1/2*x)^6 - 5/256*arcsin(tan(1/2*x)^2) - 5/256*log(-(sqrt(-tan(1/2*x)^4 +
 1) - 1)/tan(1/2*x)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{1}{\cos(x)} - \cos(x)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x) - cos(x))^(7/2),x)
```

```
[Out] int(1/(1/cos(x) - cos(x))^(7/2), x)
```

3.341 $\int (\sin(x)+\tan(x))^4\,dx$

Optimal. Leaf size=55

$$-\frac{61x}{8}-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan(x)+2\sec(x)\tan(x)+\frac{\tan^3(x)}{3}$$

```
[Out] -61/8*x-2*arctanh(sin(x))+19/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)-4/3*sin(x)
^3+5*tan(x)+2*sec(x)*tan(x)+1/3*tan(x)^3
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.286$, Rules used = {4482, 2788, 2717, 2715, 8, 2713, 3855, 3852, 3853}

$$-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)-2\tanh^{-1}(\sin(x))+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (-61*x)/8 - 2*ArcTanh[Sin[x]] + (19*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
- (4*Sin[x]^3)/3 + 5*Tan[x] + 2*Sec[x]*Tan[x] + Tan[x]^3/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (\sin(x)+\tan(x))^4\,dx = \int (1+\cos(x))^4\tan^4(x)\,dx$$
$$= \int \left(-10-4\cos(x)+4\cos^2(x)+4\cos^3(x)+\cos^4(x)-4\sec(x)+4\sec^2(x)+4\,\mathrm{s}\right.$$
$$= -10x-4\int\cos(x)\,dx+4\int\cos^2(x)\,dx+4\int\cos^3(x)\,dx-4\int\sec(x)\,dx+4\int$$
$$= -10x-4\tanh^{-1}(\sin(x))-4\sin(x)+2\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+2\sec($$
$$= -8x-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\tan$$
$$= -\frac{61x}{8}-2\tanh^{-1}(\sin(x))+\frac{19}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)-\frac{4\sin^3(x)}{3}+5\,\mathrm{t}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.
time = 0.15, size = 129, normalized size = 2.35

$$\frac{1}{768}\sec^3(x)\left(-72\cos(x)\left(61x-16\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+16\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)-24\cos(3x)\left(61x-16\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+16\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+1395\sin(x)+672\sin(2x)+1265\sin(3x)+129\sin(5x)+32\sin(6x)+3\sin(7x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^4,x]
```

```
[Out] (Sec[x]^3*(-72*Cos[x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Cos[x/2]
 + Sin[x/2]]) - 24*Cos[3*x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Co
s[x/2] + Sin[x/2]]) + 1395*Sin[x] + 672*Sin[2*x] + 1265*Sin[3*x] + 129*Sin[
5*x] + 32*Sin[6*x] + 3*Sin[7*x]))/768
```

Maple [A]
time = 0.07, size = 66, normalized size = 1.20

method	result
default	$\frac{(\tan^3(x))}{3}-\tan(x)-\frac{61x}{8}+\frac{2(\sin^5(x))}{\cos(x)^2}+\frac{2(\sin^3(x))}{3}+2\sin(x)-2\ln(\sec(x)+\tan(x))+\frac{6(\sin^5(x))}{\cos(x)}+\frac{23}{}$
risch	$-\frac{61x}{8}-\frac{ie^{4ix}}{64}-\frac{5ie^{2ix}}{8}+\frac{ie^{ix}}{2}-\frac{ie^{-ix}}{2}+\frac{5ie^{-2ix}}{8}+\frac{ie^{-4ix}}{64}-\frac{4i(3e^{5ix}-6e^{4ix}-15e^{2ix}-3e^{ix}-7)}{3(e^{2ix}+1)^3}+2\ln(e^{ix}-i)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*tan(x)^3-tan(x)-61/8*x+2*sin(x)^5/cos(x)^2+2/3*sin(x)^3+2*sin(x)-2*ln(s
ec(x)+tan(x))+6*sin(x)^5/cos(x)+23/4*(sin(x)^3+3/2*sin(x))*cos(x)
```

Maxima [A]
time = 0.48, size = 68, normalized size = 1.24

$$-\frac{4}{3}\sin(x)^3+\frac{1}{3}\tan(x)^3-\frac{61}{8}x-\frac{2\sin(x)}{\sin(x)^2-1}+\frac{3\tan(x)}{\tan(x)^2+1}-\log(\sin(x)+1)+\log(\sin(x)-1)+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)+5\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] -4/3*sin(x)^3 + 1/3*tan(x)^3 - 61/8*x - 2*sin(x)/(sin(x)^2 - 1) + 3*tan(x)/
(tan(x)^2 + 1) - log(sin(x) + 1) + log(sin(x) - 1) + 1/32*sin(4*x) - 1/4*si
n(2*x) + 5*tan(x)
```

Fricas [A]
time = 2.64, size = 78, normalized size = 1.42

$$-\frac{183x\cos(x)^3+24\cos(x)^3\log(\sin(x)+1)-24\cos(x)^3\log(-\sin(x)+1)-(6\cos(x)^6+32\cos(x)^5+57\cos(x)^4-32\cos(x)^3+112\cos(x)^2+48\cos(x)+8)\sin(x)}{24\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(183*x*cos(x)^3 + 24*cos(x)^3*log(sin(x) + 1) - 24*cos(x)^3*log(-sin(
x) + 1) - (6*cos(x)^6 + 32*cos(x)^5 + 57*cos(x)^4 - 32*cos(x)^3 + 112*cos(x
)^2 + 48*cos(x) + 8)*sin(x))/cos(x)^3
```

Sympy [A]
time = 2.91, size = 90, normalized size = 1.64

$$-\frac{61x}{8}+\log(\sin(x)-1)-\log(\sin(x)+1)-\frac{4\sin^3(x)}{3}+\frac{6\sin^3(x)}{\cos(x)}+\frac{\sin^3(x)}{3\cos^3(x)}+9\sin(x)\cos(x)-\frac{\sin(x)}{\cos(x)}-\frac{\sin(2x)}{4}+\frac{\sin(4x)}{32}-\frac{4\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**4,x)
```

```
[Out] -61*x/8 + log(sin(x) - 1) - log(sin(x) + 1) - 4*sin(x)**3/3 + 6*sin(x)**3/c
os(x) + sin(x)**3/(3*cos(x)**3) + 9*sin(x)*cos(x) - sin(x)/cos(x) - sin(2*x
)/4 + sin(4*x)/32 - 4*sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(45) = 90$.
time = 2.69, size = 1375, normalized size = 25.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/24*(8*tan(1/2*x)^10*tan(x)^5 - 183*x*tan(1/2*x)^10*tan(x)^2 - 24*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^10*tan(x)^2
+ 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)
^10*tan(x)^2 + 128*tan(1/2*x)^10*tan(x)^3 + 8*tan(1/2*x)^8*tan(x)^5 - 183*x
*tan(1/2*x)^10 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^10 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)
^2 + 1))*tan(1/2*x)^10 + 180*tan(1/2*x)^10*tan(x) - 183*x*tan(1/2*x)^8*tan(
x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1
/2*x)^8*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^8*tan(x)^2 + 96*tan(1/2*x)^9*tan(x)^2 + 128*tan(1/2*x)^8*
tan(x)^3 - 16*tan(1/2*x)^6*tan(x)^5 - 183*x*tan(1/2*x)^8 - 24*log(2*(tan(1/
2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 24*log(2*(tan
(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 96*tan(1/2
*x)^9 + 180*tan(1/2*x)^8*tan(x) + 366*x*tan(1/2*x)^6*tan(x)^2 + 48*log(2*(t
an(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6*tan(x)^2 -
 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^
6*tan(x)^2 + 128*tan(1/2*x)^7*tan(x)^2 - 256*tan(1/2*x)^6*tan(x)^3 - 16*tan
```

```
(1/2*x)^4*tan(x)^5 + 366*x*tan(1/2*x)^6 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/
2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 48*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 + 128*tan(1/2*x)^7 - 360*tan(
1/2*x)^6*tan(x) + 366*x*tan(1/2*x)^4*tan(x)^2 + 48*log(2*(tan(1/2*x)^2 + 2*
tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 - 48*log(2*(tan(1
/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 + 108
8*tan(1/2*x)^5*tan(x)^2 - 256*tan(1/2*x)^4*tan(x)^3 + 8*tan(1/2*x)^2*tan(x)
^5 + 366*x*tan(1/2*x)^4 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^4 - 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(ta
n(1/2*x)^2 + 1))*tan(1/2*x)^4 + 1088*tan(1/2*x)^5 - 360*tan(1/2*x)^4*tan(x)
 - 183*x*tan(1/2*x)^2*tan(x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)
/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan
(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 + 128*tan(1/2*x)^3*t
an(x)^2 + 128*tan(1/2*x)^2*tan(x)^3 + 8*tan(x)^5 - 183*x*tan(1/2*x)^2 - 24*
log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 +
24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2
 + 128*tan(1/2*x)^3 + 180*tan(1/2*x)^2*tan(x) - 183*x*tan(x)^2 - 24*log(2*(
tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 24*log(2*(t
an(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(x)^2 + 96*tan(1/2*x
)*tan(x)^2 + 128*tan(x)^3 - 183*x - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1)) + 96*tan(1/2*x) + 180*tan(x))/(tan(1/2*x)^10*tan(x)^2 + tan(1
/2*x)^10 + tan(1/2*x)^8*tan(x)^2 + tan(1/2*x)^8 - 2*tan(1/2*x)^6*tan(x)^2 -
 2*tan(1/2*x)^6 - 2*tan(1/2*x)^4*tan(x)^2 - 2*tan(1/2*x)^4 + tan(1/2*x)^2*t
an(x)^2 + tan(1/2*x)^2 + tan(x)^2 + 1) + 1/32*sin(4*x)
```

Mupad [B]
time = 2.54, size = 88, normalized size = 1.60

$$-\frac{61\,x}{8}-4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-\frac{\frac{45\,{\tan\left(\frac{x}{2}\right)}^{13}}{4}+\frac{29\,{\tan\left(\frac{x}{2}\right)}^{11}}{6}-\frac{455\,{\tan\left(\frac{x}{2}\right)}^{9}}{12}-15\,{\tan\left(\frac{x}{2}\right)}^{7}+\frac{179\,{\tan\left(\frac{x}{2}\right)}^{5}}{4}+\frac{31\,{\tan\left(\frac{x}{2}\right)}^{3}}{2}+\frac{77\,\tan\left(\frac{x}{2}\right)}{4}}{{\left({\tan\left(\frac{x}{2}\right)}^2-1\right)}^3\,{\left({\tan\left(\frac{x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^4,x)
```

```
[Out] - (61*x)/8 - 4*atanh(tan(x/2)) - ((77*tan(x/2))/4 + (31*tan(x/2)^3)/2 + (17
9*tan(x/2)^5)/4 - 15*tan(x/2)^7 - (455*tan(x/2)^9)/12 + (29*tan(x/2)^11)/6
+ (45*tan(x/2)^13)/4)/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

3.342 $\int (\sin(x)+\tan(x))^3\,dx$

Optimal. Leaf size=38

$$2\cos(x)+\frac{3\cos^2(x)}{2}+\frac{\cos^3(x)}{3}-2\log(\cos(x))+3\sec(x)+\frac{\sec^2(x)}{2}$$

```
[Out] 2*cos(x)+3/2*cos(x)^2+1/3*cos(x)^3-2*ln(cos(x))+3*sec(x)+1/2*sec(x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {4482, 2786, 76}

$$\frac{\cos^3(x)}{3}+\frac{3\cos^2(x)}{2}+2\cos(x)+\frac{\sec^2(x)}{2}+3\sec(x)-2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^3,x]
```

```
[Out] 2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]^
2/2
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (\sin(x)+\tan(x))^3\,dx = \int (1+\cos(x))^3\tan^3(x)\,dx$$
$$= -\text{Subst}\left(\int \frac{(1-x)(1+x)^4}{x^3}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(-2+\frac{1}{x^3}+\frac{3}{x^2}+\frac{2}{x}-3x-x^2\right)\,dx, x, \cos(x)\right)$$
$$= 2\cos(x)+\frac{3\cos^2(x)}{2}+\frac{\cos^3(x)}{3}-2\log(\cos(x))+3\sec(x)+\frac{\sec^2(x)}{2}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.05

$$\frac{9\cos(x)}{4}+\frac{3}{4}\cos(2x)+\frac{1}{12}\cos(3x)-2\log(\cos(x))+3\sec(x)+\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^3,x]
```

```
[Out] (9*Cos[x])/4 + (3*Cos[2*x])/4 + Cos[3*x]/12 - 2*Log[Cos[x]] + 3*Sec[x] + Se
c[x]^2/2
```

Maple [A]
time = 0.07, size = 39, normalized size = 1.03

method	result	size
default	$\frac{(\tan^2(x))}{2}-2\ln(\cos(x))+\frac{3(\sin^4(x))}{\cos(x)}+\frac{8(2+\sin^2(x))\cos(x)}{3}-\frac{3(\sin^2(x))}{2}$	39
risch	$2ix+\frac{e^{3ix}}{24}+\frac{3e^{2ix}}{8}+\frac{9e^{ix}}{8}+\frac{9e^{-ix}}{8}+\frac{3e^{-2ix}}{8}+\frac{e^{-3ix}}{24}+\frac{6e^{3ix}+2e^{2ix}+6e^{ix}}{(e^{2ix}+1)^2}-2\ln(e^{2ix}+1)$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x)^2-2*ln(cos(x))+3*sin(x)^4/cos(x)+8/3*(2+sin(x)^2)*cos(x)-3/2*sin
(x)^2
```

Maxima [A]
time = 0.26, size = 42, normalized size = 1.11

$$\frac{1}{3}\cos(x)^3-\frac{3}{2}\sin(x)^2-\frac{1}{2(\sin(x)^2-1)}+\frac{3}{\cos(x)}+2\cos(x)-\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - 3/2*sin(x)^2 - 1/2/(sin(x)^2 - 1) + 3/cos(x) + 2*cos(x) - lo
g(sin(x)^2 - 1)
```

Fricas [A]
time = 1.97, size = 47, normalized size = 1.24

$$\frac{4 \cos(x)^5 + 18 \cos(x)^4 + 24 \cos(x)^3 - 24 \cos(x)^2 \log(-\cos(x)) - 9 \cos(x)^2 + 36 \cos(x) + 6}{12 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(x)^5 + 18*cos(x)^4 + 24*cos(x)^3 - 24*cos(x)^2*log(-cos(x)) - 9
*cos(x)^2 + 36*cos(x) + 6)/cos(x)^2
```

Sympy [A]
time = 3.60, size = 46, normalized size = 1.21

$$-3\log(\cos(x)) - \frac{\log(\sec^2(x))}{2} + \frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + \frac{3}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**3,x)
```

```
[Out] -3*log(cos(x)) - log(sec(x)**2)/2 + cos(x)**3/3 + 3*cos(x)**2/2 + 2*cos(x)
+ sec(x)**2/2 + 3/cos(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(32) = 64.
time = 0.56, size = 173, normalized size = 4.55

$$\frac{\tan\left(\frac{1}{2}x\right)^4 \tan(x)^4 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4 \tan(x)^2 - 10\tan\left(\frac{1}{2}x\right)^4 \tan(x)^2 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4 - 8\tan\left(\frac{1}{2}x\right)^4 - 3\tan\left(\frac{1}{2}x\right)^2 \tan(x)^2 - \tan(x)^4 + 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan(x)^2 - 3\tan\left(\frac{1}{2}x\right)^2 - 11\tan(x)^2 + 2\log\left(\frac{4}{\tan(x)^2+1}\right) - 13}{2\left(\tan\left(\frac{1}{2}x\right)^4 \tan(x)^2 + \tan\left(\frac{1}{2}x\right)^4 - \tan(x)^2 - 1\right)} + \frac{1}{12}\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(tan(1/2*x)^4*tan(x)^4 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4*tan(x)^2
- 10*tan(1/2*x)^4*tan(x)^2 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4 - 8*tan(1
/2*x)^4 - 3*tan(1/2*x)^2*tan(x)^2 - tan(x)^4 + 2*log(4/(tan(x)^2 + 1))*tan(
x)^2 - 3*tan(1/2*x)^2 - 11*tan(x)^2 + 2*log(4/(tan(x)^2 + 1)) - 13)/(tan(1/
2*x)^4*tan(x)^2 + tan(1/2*x)^4 - tan(x)^2 - 1) + 1/12*cos(3*x)
```

Mupad [B]
time = 2.45, size = 65, normalized size = 1.71

$$4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^2\right) + \frac{-4\tan\left(\frac{x}{2}\right)^8 - 4\tan\left(\frac{x}{2}\right)^6 + \frac{20\tan\left(\frac{x}{2}\right)^4}{3} + \frac{20\tan\left(\frac{x}{2}\right)^2}{3} + \frac{32}{3}}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^2 \left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^3,x)
```

```
[Out] 4*atanh(tan(x/2)^2) + ((20*tan(x/2)^2)/3 + (20*tan(x/2)^4)/3 - 4*tan(x/2)^6
 - 4*tan(x/2)^8 + 32/3)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3)
```

3.343 $\int (\sin(x) + \tan(x))^2 \, dx$

Optimal. Leaf size=25

$$-\frac{x}{2} + 2\tanh^{-1}(\sin(x)) - 2\sin(x) - \frac{1}{2}\cos(x)\sin(x) + \tan(x)$$

```
[Out] -1/2*x+2*arctanh(sin(x))-2*sin(x)-1/2*cos(x)*sin(x)+tan(x)
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4482, 2788, 2717, 2715, 8, 3855, 3852}

$$-\frac{x}{2} - 2\sin(x) + \tan(x) + 2\tanh^{-1}(\sin(x)) - \frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^2,x]
```

```
[Out] -1/2*x + 2*ArcTanh[Sin[x]] - 2*Sin[x] - (Cos[x]*Sin[x])/2 + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int (\sin(x)+\tan(x))^2\,dx &= \int (1+\cos(x))^2\tan^2(x)\,dx\\ &= \int \left(-2\cos(x)-\cos^2(x)+2\sec(x)+\sec^2(x)\right)\,dx\\ &= -(2\int \cos(x)\,dx)+2\int \sec(x)\,dx-\int \cos^2(x)\,dx+\int \sec^2(x)\,dx\\ &= 2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)-\frac{\int 1\,dx}{2}-\text{Subst}(\int 1\,dx,x,-\tan(x\\ &= -\frac{x}{2}+2\tanh^{-1}(\sin(x))-2\sin(x)-\frac{1}{2}\cos(x)\sin(x)+\tan(x)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.
time = 0.07, size = 60, normalized size = 2.40

$$-\frac{x}{2}-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-2\sin(x)-\frac{1}{8}\sec(x)\sin(3x)+\frac{7\tan(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^2,x]
[Out] -1/2*x - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]
 - (Sec[x]*Sin[3*x])/8 + (7*Tan[x])/8
```

Maple [**A**]
time = 0.06, size = 25, normalized size = 1.00

method	result	size
default	$\tan(x) - \frac{x}{2} - 2\sin(x) + 2\ln(\sec(x) + \tan(x)) - \frac{\cos(x)\sin(x)}{2}$	25
risch	$-\frac{x}{2} + \frac{i e^{2ix}}{8} + i e^{ix} - i e^{-ix} - \frac{i e^{-2ix}}{8} + \frac{2i}{e^{2ix}+1} + 2\ln(e^{ix} + i) - 2\ln(e^{ix} - i)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] tan(x)-1/2*x-2*sin(x)+2*ln(sec(x)+tan(x))-1/2*cos(x)*sin(x)
```

Maxima [A]
time = 0.47, size = 28, normalized size = 1.12

$$-\frac{1}{2}x + \log(\sin(x) + 1) - \log(\sin(x) - 1) - \frac{1}{4}\sin(2x) - 2\sin(x) + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + log(sin(x) + 1) - log(sin(x) - 1) - 1/4*sin(2*x) - 2*sin(x) + tan(
x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(21) = 42.
time = 2.06, size = 44, normalized size = 1.76

$$-\frac{x\cos(x) - 2\cos(x)\log(\sin(x) + 1) + 2\cos(x)\log(-\sin(x) + 1) + (\cos(x)^2 + 4\cos(x) - 2)\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x) - 2*cos(x)*log(sin(x) + 1) + 2*cos(x)*log(-sin(x) + 1) + (co
s(x)^2 + 4*cos(x) - 2)*sin(x))/cos(x)
```

Sympy [A]
time = 0.96, size = 31, normalized size = 1.24

$$-\frac{x}{2} - \log(\sin(x) - 1) + \log(\sin(x) + 1) - 2\sin(x) - \frac{\sin(2x)}{4} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))**2,x)
```

```
[Out] -x/2 - log(sin(x) - 1) + log(sin(x) + 1) - 2*sin(x) - sin(2*x)/4 + tan(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(21) = 42.
time = 0.44, size = 177, normalized size = 7.08

$$\frac{1}{2}\,x - \frac{x \tan\left(\frac{1}{2}\,x\right)^{2} - \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2} + \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2} - \tan\left(\frac{1}{2}\,x\right)^{2}\tan\left(x\right) + x - \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}+2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right) + \log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right) + 4\,\tan\left(\frac{1}{2}\,x\right) - \tan\left(x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1} - \frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - (x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^2 - tan(1/2*x)^2*tan(x) + x - log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) +
 1)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x) - tan(x))/(tan(1/2*x)^2 + 1) - 1/4*s
in(2*x)
```

Mupad [B]
time = 2.42, size = 61, normalized size = 2.44

$$4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-\frac{x}{2}+\frac{5\,{\tan\left(\frac{x}{2}\right)}^5+6\,{\tan\left(\frac{x}{2}\right)}^3-3\,\tan\left(\frac{x}{2}\right)}{-{\tan\left(\frac{x}{2}\right)}^6-{\tan\left(\frac{x}{2}\right)}^4+{\tan\left(\frac{x}{2}\right)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + tan(x))^2,x)
```

```
[Out] 4*atanh(tan(x/2)) - x/2 + (6*tan(x/2)^3 - 3*tan(x/2) + 5*tan(x/2)^5)/(tan(x
/2)^2 - tan(x/2)^4 - tan(x/2)^6 + 1)
```

3.344 $\int (\sin(x) + \tan(x))\, dx$

Optimal. Leaf size=10

$$-\cos(x) - \log(\cos(x))$$

[Out] -cos(x)-ln(cos(x))

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2718, 3556}

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x] + Tan[x],x]

[Out] -Cos[x] - Log[Cos[x]]

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (\sin(x) + \tan(x))\, dx &= \int \sin(x)\, dx + \int \tan(x)\, dx \\ &= -\cos(x) - \log(\cos(x)) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\cos(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x] + Tan[x],x]

```
[Out] -Cos[x] - Log[Cos[x]]
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$-\cos(x)-\ln(\cos(x))$	11
risch	$ix-\ln(e^{2ix}+1)-\cos(x)$	20
norman	$-\frac{2}{1+\tan^2(\frac{x}{2})}+\frac{\ln(\tan^2(x)+1)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)+tan(x),x,method=_RETURNVERBOSE)
[Out] -cos(x)-ln(cos(x))
```

Maxima [A]
time = 0.26, size = 8, normalized size = 0.80

$$-\cos(x)+\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="maxima")
[Out] -cos(x) + log(sec(x))
```

Fricas [A]
time = 1.63, size = 12, normalized size = 1.20

$$-\cos(x)-\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="fricas")
[Out] -cos(x) - log(-cos(x))
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$-\log(\cos(x))-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x)
[Out] -log(cos(x)) - cos(x)
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.10

$$-\cos(x) - \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)+tan(x),x, algorithm="giac")
```

```
[Out] -cos(x) - log(abs(cos(x)))
```

Mupad [B]
time = 2.40, size = 22, normalized size = 2.20

$$2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^2\right) - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x) + tan(x),x)
```

```
[Out] 2*atanh(tan(x/2)^2) - 2/(tan(x/2)^2 + 1)
```

3.345 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)-\frac{\csc^2(x)}{2}$$

```
[Out] -1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4482, 2785, 2686, 30, 2691, 3855}

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\cos(x)}\,dx\\ &= -\int \cot^2(x)\csc(x)\,dx+\int \cot(x)\csc^2(x)\,dx\\ &= \frac{1}{2}\cot(x)\csc(x)+\frac{1}{2}\int \csc(x)\,dx-\text{Subst}\Big(\int x\,dx,x,\csc(x)\Big)\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)-\frac{\csc^2(x)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.46

$$-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\sec^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
[Out] -1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

Maple [A]
time = 0.10, size = 24, normalized size = 1.00

method	result	size
default	$-\frac{1}{2(1+\cos(x))}-\frac{\ln(1+\cos(x))}{4}+\frac{\ln(\cos(x)-1)}{4}$	24
risch	$-\frac{e^{ix}}{(e^{ix}+1)^2}-\frac{\ln(e^{ix}+1)}{2}+\frac{\ln(e^{ix}-1)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(1+cos(x))-1/4*ln(1+cos(x))+1/4*ln(cos(x)-1)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 1.04

$$-\frac{\sin(x)^2}{4(\cos(x)+1)^2}+\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.41, size = 35, normalized size = 1.46

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sin(x)+\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] Integral(1/(sin(x) + tan(x)), x)
```

Giac [A]
time = 0.41, size = 28, normalized size = 1.17

$$\frac{\cos(x)-1}{4(\cos(x)+1)}+\frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B]
time = 2.47, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x)),x)
[Out] log(tan(x/2))/2 - tan(x/2)^2/4
```

3.346 $\int \frac{1}{(\sin(x)+\tan(x))^2} \, dx$

Optimal. Leaf size=33

$$-\frac{1}{3}\cot^3(x) - \frac{2\cot^5(x)}{5} - \frac{2\csc^3(x)}{3} + \frac{2\csc^5(x)}{5}$$

```
[Out] -1/3*cot(x)^3-2/5*cot(x)^5-2/3*csc(x)^3+2/5*csc(x)^5
```

Rubi [A]
time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4482, 2790, 2687, 30, 2686, 14}

$$-\frac{2}{5}\cot^5(x) - \frac{\cot^3(x)}{3} + \frac{2\csc^5(x)}{5} - \frac{2\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -1/3*Cot[x]^3 - (2*Cot[x]^5)/5 - (2*Csc[x]^3)/3 + (2*Csc[x]^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx &= \int \frac{\cot^2(x)}{(1+\cos(x))^2}\,dx\\ &= \int \left(\cot^4(x)\csc^2(x)-2\cot^3(x)\csc^3(x)+\cot^2(x)\csc^4(x)\right)\,dx\\ &= -\left(2\int \cot^3(x)\csc^3(x)\,dx\right)+\int \cot^4(x)\csc^2(x)\,dx+\int \cot^2(x)\csc^4(x)\,dx\\ &= 2\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(x)\right)+\text{Subst}\left(\int x^4\,dx,x,-\cot(x)\right)+\text{Subst}\\ &= -\frac{1}{5}\cot^5(x)+2\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(x)\right)+\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,\right.\\ &= -\frac{1}{3}\cot^3(x)-\frac{2\cot^5(x)}{5}-\frac{2\csc^3(x)}{3}+\frac{2\csc^5(x)}{5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.73

$$-\frac{1}{8}\cot\left(\frac{x}{2}\right)-\frac{7}{120}\tan\left(\frac{x}{2}\right)-\frac{11}{120}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)+\frac{1}{40}\sec^4\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-2),x]
```

```
[Out] -1/8*Cot[x/2] - (7*Tan[x/2])/120 - (11*Sec[x/2]^2*Tan[x/2])/120 + (Sec[x/2]
^4*Tan[x/2])/40
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.97

method	result	size

default	$\frac{\left(\tan^5\left(\frac{x}{2}\right)\right)}{40}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)}{24}-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{1}{8\tan\left(\frac{x}{2}\right)}$	32
risch	$-\frac{2i\left(15\,\mathrm{e}^{4ix}+20\,\mathrm{e}^{3ix}+20\,\mathrm{e}^{2ix}+4\,\mathrm{e}^{ix}+1\right)}{15\left(\mathrm{e}^{ix}+1\right)^5\left(\mathrm{e}^{ix}-1\right)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/40*tan(1/2*x)^5-1/24*tan(1/2*x)^3-1/8*tan(1/2*x)-1/8/tan(1/2*x)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 1.36

$$-\frac{\cos\left(x\right)+1}{8\,\sin\left(x\right)}-\frac{\sin\left(x\right)}{8\left(\cos\left(x\right)+1\right)}-\frac{\sin\left(x\right)^3}{24\left(\cos\left(x\right)+1\right)^3}+\frac{\sin\left(x\right)^5}{40\left(\cos\left(x\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="maxima")
[Out] -1/8*(cos(x) + 1)/sin(x) - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x)
+ 1)^3 + 1/40*sin(x)^5/(cos(x) + 1)^5
```

Fricas [A]
time = 1.13, size = 34, normalized size = 1.03

$$-\frac{\cos\left(x\right)^3+2\,\cos\left(x\right)^2+8\,\cos\left(x\right)+4}{15\left(\cos\left(x\right)^2+2\,\cos\left(x\right)+1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="fricas")
[Out] -1/15*(cos(x)^3 + 2*cos(x)^2 + 8*cos(x) + 4)/((cos(x)^2 + 2*cos(x) + 1)*sin
(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\sin\left(x\right)+\tan\left(x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**2,x)
[Out] Integral((sin(x) + tan(x))**(-2), x)
```

Giac [A]
time = 0.40, size = 31, normalized size = 0.94

$$\frac{1}{40}\tan\left(\frac{1}{2}x\right)^{5}-\frac{1}{24}\tan\left(\frac{1}{2}x\right)^{3}-\frac{1}{8\tan\left(\frac{1}{2}x\right)}-\frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] 1/40*tan(1/2*x)^5 - 1/24*tan(1/2*x)^3 - 1/8/tan(1/2*x) - 1/8*tan(1/2*x)
```

Mupad [B]
time = 2.39, size = 40, normalized size = 1.21

$$-\frac{8\cos\left(\frac{x}{2}\right)^{6}-4\cos\left(\frac{x}{2}\right)^{4}+14\cos\left(\frac{x}{2}\right)^{2}-3}{120\cos\left(\frac{x}{2}\right)^{5}\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^2,x)
```

```
[Out] -(14*cos(x/2)^2 - 4*cos(x/2)^4 + 8*cos(x/2)^6 - 3)/(120*cos(x/2)^5*sin(x/2)
)
```

3.347 $\int \frac{1}{(\sin(x)+\tan(x))^3} \, dx$

Optimal. Leaf size=60

$$\frac{1}{32}\tanh^{-1}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))}$$

```
[Out] 1/32*arctanh(cos(x))-1/32/(1-cos(x))-1/16/(cos(x)+1)^4+1/6/(cos(x)+1)^3-3/3
2/(cos(x)+1)^2-1/16/(cos(x)+1)
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4482, 2786, 90, 213}

$$-\frac{1}{32(1-\cos(x))} - \frac{1}{16(\cos(x)+1)} - \frac{3}{32(\cos(x)+1)^2} + \frac{1}{6(\cos(x)+1)^3} - \frac{1}{16(\cos(x)+1)^4} + \frac{1}{32}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1
+ Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \int \frac{\cot^3(x)}{(1+\cos(x))^3}\,dx$$
$$= -\text{Subst}\left(\int \frac{x^3}{(1-x)^2(1+x)^5}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{32(-1+x)^2} - \frac{1}{4(1+x)^5} + \frac{1}{2(1+x)^4} - \frac{3}{16(1+x)^3} - \frac{1}{16(1+x)^{\cdot}}\right.\right.$$
$$= -\frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{}{16(}$$
$$= \frac{1}{32}\tanh^{-1}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{}{32(}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.38

$$-\frac{1}{64}\csc^2\left(\frac{x}{2}\right) + \frac{1}{32}\log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{32}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) - \frac{3}{128}\sec^4\left(\frac{x}{2}\right) + \frac{1}{48}\sec^6\left(\frac{x}{2}\right) - \frac{1}{256}\sec^8\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-3),x]
```

```
[Out] -1/64*Csc[x/2]^2 + Log[Cos[x/2]]/32 - Log[Sin[x/2]]/32 - Sec[x/2]^2/32 - (3
*Sec[x/2]^4)/128 + Sec[x/2]^6/48 - Sec[x/2]^8/256
```

Maple [A]
time = 0.13, size = 56, normalized size = 0.93

method	result	si
default	$-\frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))} + \frac{\ln(1+\cos(x))}{64} + \frac{1}{32\cos(x)-32} - \frac{\ln(\cos(x)-1)}{64}$	5
risch	$-\frac{3\,e^{9ix}+18\,e^{8ix}-88\,e^{7ix}-162\,e^{6ix}-310\,e^{5ix}-162\,e^{4ix}-88\,e^{3ix}+18\,e^{2ix}+3\,e^{ix}}{48(e^{ix}+1)^8(e^{ix}-1)^2} - \frac{\ln(e^{ix}-1)}{32} + \frac{\ln(e^{ix}+1)}{32}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/32/(1+cos(x))^2-1/16/(1+cos(x))+1/64*
ln(1+cos(x))+1/32/(cos(x)-1)-1/64*ln(cos(x)-1)
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.22

$$-\frac{(\cos(x)+1)^2}{64\sin(x)^2} - \frac{\sin(x)^2}{32(\cos(x)+1)^2} + \frac{\sin(x)^4}{64(\cos(x)+1)^4} + \frac{\sin(x)^6}{192(\cos(x)+1)^6} - \frac{\sin(x)^8}{256(\cos(x)+1)^8} - \frac{1}{32}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="maxima")
```

```
[Out] -1/64*(cos(x) + 1)^2/sin(x)^2 - 1/32*sin(x)^2/(cos(x) + 1)^2 + 1/64*sin(x)^
4/(cos(x) + 1)^4 + 1/192*sin(x)^6/(cos(x) + 1)^6 - 1/256*sin(x)^8/(cos(x) +
 1)^8 - 1/32*log(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(46) = 92$.
time = 1.09, size = 130, normalized size = 2.17

$$-\frac{6 \cos(x)^4 + 18 \cos(x)^3 - 50 \cos(x)^2 - 3 (\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3 (\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 54 \cos(x) - 16}{192 (\cos(x)^5 + 3 \cos(x)^4 + 2 \cos(x)^3 - 2 \cos(x)^2 - 3 \cos(x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(x)^4 + 18*cos(x)^3 - 50*cos(x)^2 - 3*(cos(x)^5 + 3*cos(x)^4 +
 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x)^
5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(-1/2*cos(x) +
1/2) - 54*cos(x) - 16)/(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3
*cos(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x) + \tan(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**3,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.
time = 0.41, size = 95, normalized size = 1.58

$$\frac{\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)(\cos(x) + 1)}{64 (\cos(x) - 1)} + \frac{\cos(x) - 1}{32 (\cos(x) + 1)} + \frac{(\cos(x) - 1)^2}{64 (\cos(x) + 1)^2} - \frac{(\cos(x) - 1)^3}{192 (\cos(x) + 1)^3} - \frac{(\cos(x) - 1)^4}{256 (\cos(x) + 1)^4} - \frac{1}{64} \log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^3,x, algorithm="giac")
```

```
[Out] 1/64*((cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/32*(cos(
x) - 1)/(cos(x) + 1) + 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/192*(cos(x) -
 1)^3/(cos(x) + 1)^3 - 1/256*(cos(x) - 1)^4/(cos(x) + 1)^4 - 1/64*log(-(cos
(x) - 1)/(cos(x) + 1))
```

Mupad [B]
time = 2.39, size = 48, normalized size = 0.80

$$\frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{32} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32} + \frac{\tan\left(\frac{x}{2}\right)^6}{192} - \frac{\tan\left(\frac{x}{2}\right)^8}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^3,x)
```

```
[Out] tan(x/2)^4/64 - 1/(64*tan(x/2)^2) - tan(x/2)^2/32 - log(tan(x/2))/32 + tan(
x/2)^6/192 - tan(x/2)^8/256
```

3.348 $\int \frac{1}{(\sin(x)+\tan(x))^4} dx$

Optimal. Leaf size=65

$$-\frac{1}{5}\cot^5(x)-\frac{9\cot^7(x)}{7}-\frac{16\cot^9(x)}{9}-\frac{8\cot^{11}(x)}{11}-\frac{4\csc^5(x)}{5}+\frac{16\csc^7(x)}{7}-\frac{20\csc^9(x)}{9}+\frac{8\csc^{11}(x)}{11}$$

```
[Out] -1/5*cot(x)^5-9/7*cot(x)^7-16/9*cot(x)^9-8/11*cot(x)^11-4/5*csc(x)^5+16/7*c
sc(x)^7-20/9*csc(x)^9+8/11*csc(x)^11
```

Rubi [A]
time = 0.14, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {4482, 2790, 2687, 14, 2686, 276}

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] -1/5*Cot[x]^5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*Cs
c[x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx &= \int \frac{\cot^4(x)}{(1+\cos(x))^4}\,dx\\ &= \int \left(\cot^8(x)\csc^4(x)-4\cot^7(x)\csc^5(x)+6\cot^6(x)\csc^6(x)-4\cot^5(x)\csc^7(x)+\right.\\ &= -\left(4\int \cot^7(x)\csc^5(x)\,dx\right)-4\int \cot^5(x)\csc^7(x)\,dx+6\int \cot^6(x)\csc^6(x)\,dx+\\ &= 4\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx,x,\csc(x)\right)+4\text{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx,x,\csc(\right.\\ &= 4\text{Subst}\left(\int \left(-x^4+3x^6-3x^8+x^{10}\right)\,dx,x,\csc(x)\right)+4\text{Subst}\left(\int \left(x^6-2x^8+x\right.\right.\\ &= -\frac{1}{5}\cot^5(x)-\frac{9\cot^7(x)}{7}-\frac{16\cot^9(x)}{9}-\frac{8\cot^{11}(x)}{11}-\frac{4\csc^5(x)}{5}+\frac{16\csc^7(x)}{7}-\end{aligned}$$

Mathematica [A]
time = 0.02, size = 129, normalized size = 1.98

$$\frac{1}{96}\cot\left(\frac{x}{2}\right)-\frac{1}{384}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)-\frac{2749\tan\left(\frac{x}{2}\right)}{110880}-\frac{2033\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{443520}+\frac{179\sec^4\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{73920}+\frac{641\sec^6\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{88704}-\frac{7\sec^8\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{1584}+\frac{\sec^{10}\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{1408}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-4),x]
```

```
[Out] Cot[x/2]/96 - (Cot[x/2]*Csc[x/2]^2)/384 - (2749*Tan[x/2])/110880 - (2033*Se
c[x/2]^2*Tan[x/2])/443520 + (179*Sec[x/2]^4*Tan[x/2])/73920 + (641*Sec[x/2]
^6*Tan[x/2])/88704 - (7*Sec[x/2]^8*Tan[x/2])/1584 + (Sec[x/2]^10*Tan[x/2])/
1408
```

Maple [A]
time = 0.14, size = 64, normalized size = 0.98

method	result	size
default	$\frac{\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{1408}-\frac{\left(\tan^{9}\left(\frac{x}{2}\right)\right)}{1152}-\frac{3\left(\tan^{7}\left(\frac{x}{2}\right)\right)}{896}+\frac{3\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{640}+\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{128}-\frac{3\tan\left(\frac{x}{2}\right)}{128}+\frac{1}{128\tan\left(\frac{x}{2}\right)}-\frac{1}{384\tan\left(\frac{x}{2}\right)^{3}}$	64
risch	$\frac{4i\left(3465\,\mathrm{e}^{10ix}+5544\,\mathrm{e}^{9ix}+10857\,\mathrm{e}^{8ix}+5280\,\mathrm{e}^{7ix}+4818\,\mathrm{e}^{6ix}+176\,\mathrm{e}^{5ix}+2794\,\mathrm{e}^{4ix}+1952\,\mathrm{e}^{3ix}+1525\,\mathrm{e}^{2ix}+488\,\mathrm{e}^{ix}+61\right)}{3465\left(\mathrm{e}^{ix}+1\right)^{11}\left(\mathrm{e}^{ix}-1\right)^{3}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1408*tan(1/2*x)^11-1/1152*tan(1/2*x)^9-3/896*tan(1/2*x)^7+3/640*tan(1/2*x
)^5+1/128*tan(1/2*x)^3-3/128*tan(1/2*x)+1/128/tan(1/2*x)-1/384/tan(1/2*x)^3
```

Maxima [A]
time = 0.26, size = 97, normalized size = 1.49

$$\frac{\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)^3}{384\sin(x)^3}-\frac{3\sin(x)}{128(\cos(x)+1)}+\frac{\sin(x)^3}{128(\cos(x)+1)^3}+\frac{3\sin(x)^5}{640(\cos(x)+1)^5}-\frac{3\sin(x)^7}{896(\cos(x)+1)^7}-\frac{\sin(x)^9}{1152(\cos(x)+1)^9}+\frac{\sin(x)^{11}}{1408(\cos(x)+1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="maxima")
```

```
[Out] 1/384*(3*sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^3/sin(x)^3 - 3/128*sin(x
)/(cos(x) + 1) + 1/128*sin(x)^3/(cos(x) + 1)^3 + 3/640*sin(x)^5/(cos(x) + 1
)^5 - 3/896*sin(x)^7/(cos(x) + 1)^7 - 1/1152*sin(x)^9/(cos(x) + 1)^9 + 1/14
08*sin(x)^11/(cos(x) + 1)^11
```

Fricas [A]
time = 1.41, size = 78, normalized size = 1.20

$$\frac{122\cos(x)^7+488\cos(x)^6+549\cos(x)^5-244\cos(x)^4-64\cos(x)^3+144\cos(x)^2+128\cos(x)+32}{3465\left(\cos(x)^6+4\cos(x)^5+5\cos(x)^4-5\cos(x)^2-4\cos(x)-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="fricas")
```

```
[Out] 1/3465*(122*cos(x)^7 + 488*cos(x)^6 + 549*cos(x)^5 - 244*cos(x)^4 - 64*cos(
x)^3 + 144*cos(x)^2 + 128*cos(x) + 32)/((cos(x)^6 + 4*cos(x)^5 + 5*cos(x)^4
 - 5*cos(x)^2 - 4*cos(x) - 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(\sin(x)+\tan(x))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))**4,x)
```

```
[Out] Integral((sin(x) + tan(x))**(-4), x)
```

Giac [A]
time = 0.41, size = 65, normalized size = 1.00

$$\frac{1}{1408}\tan\left(\frac{1}{2}x\right)^{11}-\frac{1}{1152}\tan\left(\frac{1}{2}x\right)^{9}-\frac{3}{896}\tan\left(\frac{1}{2}x\right)^{7}+\frac{3}{640}\tan\left(\frac{1}{2}x\right)^{5}+\frac{1}{128}\tan\left(\frac{1}{2}x\right)^{3}+\frac{3\tan\left(\frac{1}{2}x\right)^{2}-1}{384\tan\left(\frac{1}{2}x\right)^{3}}-\frac{3}{128}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x))^4,x, algorithm="giac")
```

```
[Out] 1/1408*tan(1/2*x)^11 - 1/1152*tan(1/2*x)^9 - 3/896*tan(1/2*x)^7 + 3/640*tan
(1/2*x)^5 + 1/128*tan(1/2*x)^3 + 1/384*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 -
3/128*tan(1/2*x)
```

Mupad [B]
time = 2.45, size = 87, normalized size = 1.34

$$-\frac{15616\cos\left(\frac{x}{2}\right)^{14}-23424\cos\left(\frac{x}{2}\right)^{12}+5856\cos\left(\frac{x}{2}\right)^{10}+976\cos\left(\frac{x}{2}\right)^{8}+7296\cos\left(\frac{x}{2}\right)^{6}-7440\cos\left(\frac{x}{2}\right)^{4}+2590\cos\left(\frac{x}{2}\right)^{2}-315}{443520\left(\cos\left(\frac{x}{2}\right)^{11}\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)^{13}\sin\left(\frac{x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x) + tan(x))^4,x)
```

```
[Out] -(2590*cos(x/2)^2 - 7440*cos(x/2)^4 + 7296*cos(x/2)^6 + 976*cos(x/2)^8 + 58
56*cos(x/2)^10 - 23424*cos(x/2)^12 + 15616*cos(x/2)^14 - 315)/(443520*(cos(
x/2)^11*sin(x/2) - cos(x/2)^13*sin(x/2)))
```

3.349 $\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=74

$$\frac{cCx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] c*C*x/(b^2+c^2)-b*C*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*s
in(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3216, 3153, 212}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] - (b*C*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{cCx}{b^2+c^2} - \frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2} + A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx$$
$$= \frac{cCx}{b^2+c^2} - \frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2} - A\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx, x, c\cos(x)\right.$$
$$= \frac{cCx}{b^2+c^2} - \frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} - \frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A]
time = 0.15, size = 68, normalized size = 0.92

$$\frac{2A\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{C(cx-b\log(b\cos(x)+c\sin(x)))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (C*(c*x
- b*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

Maple [A]
time = 0.23, size = 117, normalized size = 1.58

method	result
default	$\frac{-bC\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)-\frac{2\left(-A\,b^2-A\,c^2\right)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}}{b^2+c^2} + \frac{2C\left(\frac{b\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+c\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^2+c^2}$
risch	$\frac{iCx}{ic-b} + \frac{2iCx\,b^3}{b^4+2b^2c^2+c^4} + \frac{2iCx\,c^2b}{b^4+2b^2c^2+c^4} - \frac{\ln\left(\mathrm{e}^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A\left(b^2+c^2\right)}\right)bC}{b^2+c^2} + \frac{\ln\left(\mathrm{e}^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A\left(b^2+c^2\right)}\right)\sqrt{}}{b^2+c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(-1/2*b*C*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-(-A*b^2-A*c^2)/(b
^2+c^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2)))+2*C/(b^2+c
^2)*(1/2*b*ln(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(70) = 140$.
time = 0.48, size = 153, normalized size = 2.07

$$C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} - \frac{b\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right) - \frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^
2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 1.28, size = 144, normalized size = 1.95

$$\frac{2\,Ccx - Cb\log\left(2\,bc\cos(x)\sin(x) + (b^2-c^2)\cos(x)^2 + c^2\right) + \sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)}{2\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*c*x - C*b*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.49, size = 634, normalized size = 8.57

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx\right) & \text{for } b=0 \wedge c=0 \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx}{c} & \text{for } b=0 \\ -\frac{2A}{2ic\sin(x)+2c\cos(x)} + \frac{iCx\sin(x)}{2ic\sin(x)+2c\cos(x)} + \frac{Cx\cos(x)}{2ic\sin(x)+2c\cos(x)} - \frac{C\sin(x)}{2ic\sin(x)+2c\cos(x)} & \text{for } b=-ic \\ -\frac{2A}{-2ic\sin(x)+2c\cos(x)} - \frac{iCx\sin(x)}{-2ic\sin(x)+2c\cos(x)} + \frac{Cx\cos(x)}{-2ic\sin(x)+2c\cos(x)} - \frac{C\sin(x)}{-2ic\sin(x)+2c\cos(x)} & \text{for } b=ic \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} - \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Cb\sqrt{b^2+c^2}\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} - \frac{Cb\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} - \frac{Cb\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ccx\sqrt{b^2+c^2}}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x
/2)) + C*x)/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) + I*C*x*sin(x)/
(2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) - C*si
n(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x) + 2*c*
```

```
cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(-2*I*c*si
n(x) + 2*c*cos(x)) - C*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)), Eq(b, I*c)), (-
A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) +
c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/
(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) - c
/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2))
 + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2)
 + c**2*sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b*
*2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(
tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b
**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)
/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c
**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

Giac [A]
time = 0.44, size = 131, normalized size = 1.77

$$\frac{Ccx}{b^2+c^2}+\frac{Cb\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{b^2+c^2}-\frac{Cb\log\left(\left|b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right|\right)}{b^2+c^2}-\frac{A\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - C*b*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

Mupad [B]
time = 7.02, size = 695, normalized size = 9.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] (C*log(tan(x/2) + 1i))/(b - c*1i) - log(- 32*A*C^2*b^2 - ((C*b^3 - A*((b^2
+ c^2)^3)^(1/2) + C*b*c^2)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^
2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((C*b^3 - A*((b^2 +
 c^2)^3)^(1/2) + C*b*c^2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3
 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b +
c*tan(x/2))*(C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2))/(b^2 + c^2)))/(b^2
 + c^2)^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b
 + 2*A*C*c))*((C*b)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2)
```

```
- log(- 32*A*C^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(64*A^2
*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*
C*b*c) + 64*A*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2)*(32*A*b^
4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2)
- 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/
2) + C*b^3 + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2))/(b^2 + c^2)^2 - 32*A^2
*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b + 2*A*C*c))*((C*b)/(b^2 + c^2) +
(A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + (C*log(tan(x/2) - 1i)*1i)/(b*1i
- c)
```

3.350 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

```
[Out] -c*C*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(b*C-A*c*
cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3233, 3153, 212}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*C*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx &= \frac{bC-Ac\cos(x)+Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}+\frac{(cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= \frac{bC-Ac\cos(x)+Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(cC)\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(x)\right)}{b^2+c^2}\\ &= -\frac{cC\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 82, normalized size = 1.09

$$\frac{2cC\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{b^2C+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*c*C*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2
*C + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.23, size = 108, normalized size = 1.44

method	result	size
default	$\frac{-\frac{2\left(A\,b^2+A\,c^2+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}-\frac{2Cb}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2Cc\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$	108
risch	$-\frac{2i\left(-iAb-bC\,\mathrm{e}^{ix}+Ac\right)}{(-ib+c)(ib+c)\left(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib\right)}+\frac{cC\ln\left(\mathrm{e}^{ix}+\frac{ib^3+ib\,c^2-b^2c-c^3}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{cC\ln\left(\mathrm{e}^{ix}-\frac{ib^3+ib\,c^2-b^2c-c^3}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$	175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(A*b^2+A*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)-C*b/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*C*c/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 146, normalized size = 1.95

$$-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1}-\frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)-\frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1
))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x
)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(73) = 146.
time = 1.46, size = 200, normalized size = 2.67

$$\frac{2Cb^3+2Cbc^2+(Cbc\cos(x)+Cc^2\sin(x))\sqrt{b^2+c^2}\log\left(-\frac{2bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2b^2-c^2+2\sqrt{b^2+c^2}(c\cos(x)-b\sin(x))}{2bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)-2(Ab^2c+Ac^3)\cos(x)+2(Ab^3+Abc^2)\sin(x)}{2((b^5+2b^3c^2+bc^4)\cos(x)+(b^4c+2b^2c^3+c^5)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 + 2*C*b*c^2 + (C*b*c*cos(x) + C*c^2*sin(x))*sqrt(b^2 + c^2)*lo
g(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 +
 c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*
b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.44, size = 130, normalized size = 1.73

$$-\frac{Cc\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}x\right)+Cbc\tan\left(\frac{1}{2}x\right)+Ac^2\tan\left(\frac{1}{2}x\right)+Cb^2\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -C*c*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + C*b*c*
tan(1/2*x) + A*c^2*tan(1/2*x) + C*b^2)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

Mupad [B]
time = 2.58, size = 105, normalized size = 1.40

$$\frac{\frac{2\,C\,b}{b^2+c^2}+\frac{2\,\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+C\,b\,c+A\,c^2\right)}{b\,\left(b^2+c^2\right)}}{-b\,{\tan\left(\frac{x}{2}\right)}^2+2\,c\,\tan\left(\frac{x}{2}\right)+b}-\frac{2\,C\,c\,\mathrm{atanh}\left(\frac{2\,c-2\,b\,\tan\left(\frac{x}{2}\right)}{2\,\sqrt{b^2+c^2}}\right)}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*C*b)/(b^2 + c^2) + (2*tan(x/2)*(A*b^2 + A*c^2 + C*b*c))/(b*(b^2 + c^2))
)/(b + 2*c*tan(x/2) - b*tan(x/2)^2) - (2*C*c*atanh((2*c - 2*b*tan(x/2))/(2*
(b^2 + c^2)^(1/2))))/(b^2 + c^2)^(3/2)
```

3.351 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)^2}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2\left(b\cos(x)+c\sin(x)\right)}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(b*
C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c^2*C*cos(x)+b*c
*C*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3236, 3232, 3153, 212}

$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2\left(b\cos(x)+c\sin(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -1/2*(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) +
 (b*C - A*c*Cos[x] + A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(c^2*C*Cos[x] - b*c*C*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
```

```
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3236

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx &= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2cC+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2(b^2+c^2)}\\ &= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x)-bcC\sin(x)}{(b^2+c^2)^2(b\cos(x)+c\sin(x))}+\frac{A\int \frac{}{b\cos(x)}}{2(b^2}\\ &= \frac{bC-Ac\cos(x)+Ab\sin(x)}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x)-bcC\sin(x)}{(b^2+c^2)^2(b\cos(x)+c\sin(x))}-\frac{A\text{Subst}(\int}{}\\ &= -\frac{A\tanh^{-1}\left(\frac{\frac{c\cos(x)-b\sin(x)}{}}{\sqrt{b^2+c^2}}\right)}{2(b^2+c^2)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}-\frac{c^2C\cos(x}{(b^2+c^2)^2(b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 132, normalized size = 1.14

$$\frac{2Ab\sqrt{b^2+c^2}\,\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)(b\cos(x)+c\sin(x))^2+(b^2+c^2)\left(-Abc\cos(x)+Ab^2\sin(x)+2c^2C\sin^2(x)+bC(b+c\sin(2x))\right)}{2b(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*b*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x]
 + c*Sin[x])^2 + (b^2 + c^2)*(-(A*b*c*Cos[x]) + A*b^2*Sin[x] + 2*c^2*C*Sin[
x]^2 + b*C*(b + c*Sin[2*x])))/(2*b*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Si
n[x])^2)
```

Maple [A]
time = 0.29, size = 177, normalized size = 1.53

method	result
default	$-\frac{2\left(-\frac{A\left(b^{2}+2c^{2}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b}-\frac{\left(A\,b^{2}c-2A\,c^{3}+2C\,b^{3}+2Cb\,c^{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b^{2}}-\frac{A\left(b^{2}-2c^{2}\right)\tan\left(\frac{x}{2}\right)}{2\left(b^{2}+c^{2}\right)b}+\frac{Ac}{2b^{2}+2c^{2}}\right)}{\left(b\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}+\frac{A\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2}{2\sqrt{b^{2}+c}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{i\left(-A\,c^{2}\mathrm{e}^{ix}-2Cbc-A\,b^{2}\mathrm{e}^{ix}-A\,c^{2}\mathrm{e}^{3ix}-2iAbc\,\mathrm{e}^{3ix}+A\,b^{2}\mathrm{e}^{3ix}+2iC\,b^{2}\mathrm{e}^{2ix}+2iC\,c^{2}\mathrm{e}^{2ix}-2iC\,c^{2}\right)}{\left(-ic\,\mathrm{e}^{2ix}+b\,\mathrm{e}^{2ix}+ic+b\right)^{2}\left(ic+b\right)\left(-ic+b\right)^{2}}+\frac{A\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{2\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*A*(b^2+2*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*C*b^
3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*A*(b^2-2*c^2)/(b^2+c^2)/b*tan(1
/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3
/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(109) = 218$.
time = 0.50, size = 338, normalized size = 2.91

$$-\frac{1}{2}\,A\left(\frac{2\left(b^{2}c-\frac{\left(b^{3}-2\,bc^{2}\right)\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{\left(b^{2}c-2\,c^{3}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}-\frac{\left(b^{3}+2\,bc^{2}\right)\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}\right)}{b^{6}+b^{4}c^{2}+\frac{4\left(b^{5}c+b^{3}c^{3}\right)\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{2\left(b^{6}-b^{4}c^{2}-2\,b^{2}c^{4}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}-\frac{4\left(b^{5}c+b^{3}c^{3}\right)\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+\frac{\left(b^{6}+b^{4}c^{2}\right)\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}}+\frac{\log\left(\frac{c-\frac{b\sin\left(x\right)}{\cos\left(x\right)+1}+\sqrt{b^{2}+c^{2}}}{c-\frac{b\sin\left(x\right)}{\cos\left(x\right)+1}-\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)+\frac{2\,C\sin\left(x\right)^{2}}{\left(b^{3}+\frac{4\,b^{2}c\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{4\,b^{2}c\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+\frac{b^{3}\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}-\frac{2\left(b^{3}-2\,bc^{2}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}\right)\left(\cos\left(x\right)+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(
x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x
)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(109) = 218$.
time = 1.28, size = 279, normalized size = 2.41

$$-\frac{8\,Cbc^{2}\cos\left(x\right)^{2}-2\,Cb^{3}-6\,Cbc^{2}-\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\cos\left(x\right)^{2}\right)\sqrt{b^{2}+c^{2}}\,\log\left(-\frac{2\,bc\cos\left(x\right)\sin\left(x\right)+\left(b^{2}-c^{2}\right)\cos\left(x\right)^{2}-2\,b^{2}-c^{2}+2\,\sqrt{b^{2}+c^{2}}\left(c\cos\left(x\right)-b\sin\left(x\right)\right)}{2\,bc\cos\left(x\right)\sin\left(x\right)+\left(b^{2}-c^{2}\right)\cos\left(x\right)^{2}+c^{2}}\right)+2\left(Ab^{2}c+Ac^{3}\right)\cos\left(x\right)-2\left(Ab^{3}+Abc^{2}+2\left(Cb^{2}c-Cc^{3}\right)\cos\left(x\right)\right)\sin\left(x\right)}{4\left(b^{4}c^{2}+2\,b^{2}c^{4}+c^{6}+\left(b^{6}+b^{4}c^{2}-b^{2}c^{4}-c^{6}\right)\cos\left(x\right)^{2}+2\left(b^{5}c+2\,b^{3}c^{3}+bc^{5}\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*C*b*c^2*cos(x)^2 - 2*C*b^3 - 6*C*b*c^2 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(C*b^2*c - C*c^3)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 199, normalized size = 1.72

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,(b^2+c^2)^{\frac{3}{2}}}+\frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3+2\,Cb^3\tan\left(\frac{1}{2}\,x\right)^2+Ab^2c\tan\left(\frac{1}{2}\,x\right)^2+2\,Cbc^2\tan\left(\frac{1}{2}\,x\right)^2-2\,Ac^3\tan\left(\frac{1}{2}\,x\right)^2+Ab^3\tan\left(\frac{1}{2}\,x\right)-2\,Abc^2\tan\left(\frac{1}{2}\,x\right)-Ab^2c}{(b^4+b^2c^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 + 2*A
*b*c^2*tan(1/2*x)^3 + 2*C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*C*b*c
^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1
/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

Mupad [B]
time = 2.86, size = 227, normalized size = 1.96

$$\frac{\frac{{\mathrm{tan}\left(\frac{x}{2}\right)}^3\,\left(A\,b^2+2\,A\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{{\mathrm{tan}\left(\frac{x}{2}\right)}^2\,\left(2\,C\,b^3+A\,b^2\,c+2\,C\,b\,c^2-2\,A\,c^3\right)}{b^2\,\left(b^2+c^2\right)}+\frac{\mathrm{tan}\left(\frac{x}{2}\right)\,\left(A\,b^2-2\,A\,c^2\right)}{b\,\left(b^2+c^2\right)}}{b^2-{\mathrm{tan}\left(\frac{x}{2}\right)}^2\,\left(2\,b^2-4\,c^2\right)+b^2\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4+4\,b\,c\,\mathrm{tan}\left(\frac{x}{2}\right)-4\,b\,c\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3}+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1{}\mathrm{i}+c^3\,1{}\mathrm{i}-b\,\mathrm{tan}\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(A*b^2 + 2*A*c^2))/(b*(b^2 + c^2)) - (A*c)/(b^2 + c^2) + (tan(
x/2)^2*(2*C*b^3 - 2*A*c^3 + A*b^2*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(
x/2)*(A*b^2 - 2*A*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2)
+ b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i +
 c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/
2)
```

3.352 $\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=73

$$\frac{bBx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] b*B*x/(b^2+c^2)+B*c*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*s
in(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3217, 3153, 212}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sq
rt[b^2 + c^2] + (B*c*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(x)}\,dx \\
&= \frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)\right. \\
&= \frac{bBx}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 0.92

$$
\frac{2A\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{B(bx+c\log(b\cos(x)+c\sin(x)))}{b^2+c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (B*(b*x
+ c*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

Maple [A]
time = 0.24, size = 117, normalized size = 1.60

method	result
default	$\frac{Bc\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)-\frac{2\left(-Ab^2-Ac^2\right)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}}{b^2+c^2}+\frac{2B\left(-\frac{c\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+b\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^2+c^2}$
risch	$-\frac{Bx}{ic-b}+\frac{2iBx\,b^2c}{-b^4-2b^2c^2-c^4}+\frac{2iBx\,c^3}{-b^4-2b^2c^2-c^4}+\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A\left(b^2+c^2\right)}\right)Bc}{b^2+c^2}+\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A\left(b^2+c^2\right)}\right.}{b^2+c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(1/2*B*c*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-(-A*b^2-A*c^2)/(b^
2+c^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2)))+2*B/(b^2+c^
2)*(-1/2*c*ln(1+tan(1/2*x)^2)+b*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(69) = 138$.

time = 0.49, size = 153, normalized size = 2.10

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)-\frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^
2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(69) = 138$.

time = 1.50, size = 143, normalized size = 1.96

$$\frac{2\,Bbx+Bc\log\left(2\,bc\cos\left(x\right)\sin\left(x\right)+\left(b^2-c^2\right)\cos\left(x\right)^2+c^2\right)+\sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)}{2\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*B*b*x + B*c*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2
- c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^
2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

Sympy [C] Result contains complex when optimal does not.

time = 20.70, size = 673, normalized size = 9.22

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)\right) & \text{for } b=0\wedge c=0 \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)}{c} & \text{for } b=0 \\ -\frac{2A}{2ic\sin(x)+2c\cos(x)}-\frac{Bx\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iBx\cos(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iB\sin(x)}{2ic\sin(x)+2c\cos(x)} & \text{for } b=-ic \\ -\frac{2A}{-2ic\sin(x)+2c\cos(x)}-\frac{Bx\sin(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iBx\cos(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iB\sin(x)}{-2ic\sin(x)+2c\cos(x)} & \text{for } b=ic \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bbx\sqrt{b^2+c^2}}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Bc\sqrt{b^2+c^2}\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bc\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bc\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2)))
, Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(
tan(x/2)))/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) - B*x*sin(x)/(2*
I*c*sin(x) + 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*s
```

```
in(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x) + 2*c
*cos(x)) - B*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*s
in(x) + 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)), Eq(b, I*c)),
 (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2)
 + c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/
b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2)
- c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**
2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c*
*2) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c
**2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)
/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*
log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sq
rt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c
**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

Giac [A]
time = 0.46, size = 131, normalized size = 1.79

$$\frac{Bbx}{b^2+c^2}-\frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{Bc\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2}-\frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + B*c*log(abs(b*t
an(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x) -
 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/sq
rt(b^2 + c^2)
```

Mupad [B]
time = 6.53, size = 692, normalized size = 9.48

$$\ln\left(32A^2Bb^2-32AB^2b^2-\frac{\left(A\sqrt{(b^2+c^2)^3}+Bc^3+Bb^2c\right)\left(32b\tan\left(\frac{x}{2}\right)(A^2b^2-A^2c^2+4ABc^2+B^2b^2-3B^2c^2)-64A^2b^2c-32B^2b^2c+\frac{\left(A\sqrt{(b^2+c^2)^3}+Bc^3+Bb^2c\right)\left(32Ab^4+32Bb^4+32Ab^2c^2-64Bb^2c^2+32bc\tan\left(\frac{x}{2}\right)(2Ab^2+2Ac^2+4Bb^2+Bc^2)+\frac{96bc\left(b+c\tan\left(\frac{x}{2}\right)\right)\left(A\sqrt{(b^2+c^2)^3}+Bc^3+Bb^2c\right)}{b^2+c^2}\right)}{(b^2+c^2)^2}+64ABb^2c\right)}{(b^2+c^2)^2}+32Bbc\tan\left(\frac{x}{2}\right)(A-B)^2\right)\left(\frac{Bc}{b^2+c^2}+\frac{A\sqrt{(b^2+c^2)^3}}{(b^2+c^2)^2}\right)+\ln\left(32A^2Bb^2-32AB^2b^2-\frac{\left(Bc^3-A\sqrt{(b^2+c^2)^3}+Bb^2c\right)\left(32b\tan\left(\frac{x}{2}\right)(A^2b^2-A^2c^2+4ABc^2+B^2b^2-3B^2c^2)-64A^2b^2c-32B^2b^2c+\frac{\left(Bc^3-A\sqrt{(b^2+c^2)^3}+Bb^2c\right)\left(32Ab^4+32Bb^4+32Ab^2c^2-64Bb^2c^2+32bc\tan\left(\frac{x}{2}\right)(2Ab^2+2Ac^2+4Bb^2+Bc^2)+\frac{96bc\left(b+c\tan\left(\frac{x}{2}\right)\right)\left(Bc^3-A\sqrt{(b^2+c^2)^3}+Bb^2c\right)}{b^2+c^2}\right)}{(b^2+c^2)^2}+64ABb^2c\right)}{(b^2+c^2)^2}+32Bbc\tan\left(\frac{x}{2}\right)(A-B)^2\right)\left(\frac{Bc}{b^2+c^2}-\frac{A\sqrt{(b^2+c^2)^3}}{(b^2+c^2)^2}\right)-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1{}i\right)1{}i}{b+c\,1{}i}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1{}i\right)}{c+b\,1{}i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^2
*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) -
64*A^2*b^2*c - 32*B^2*b^2*c + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^2*c)*
(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b
^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^
3)^(1/2) + B*c^3 + B*b^2*c))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*b^2*c))/(
b^2 + c^2)^2 + 32*B*b*c*tan(x/2)*(A - B)^2)*((B*c)/(b^2 + c^2) + (A*((b^2 +
```

```
 c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((B*c^3
- A*((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^
2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) - 64*A^2*b^2*c - 32*B^2*b^2*c + ((B*c^3 - A*
((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B
*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*
(b + c*tan(x/2))*(B*c^3 - A*((b^2 + c^2)^3)^(1/2) + B*b^2*c))/(b^2 + c^2)))
/(b^2 + c^2)^2 + 64*A*B*b^2*c))/(b^2 + c^2)^2 + 32*B*b*c*tan(x/2)*(A - B)^2
)*((B*c)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) - (B*log(ta
n(x/2) - 1i)*1i)/(b + c*1i) - (B*log(tan(x/2) + 1i))/(b*1i + c)
```

3.353 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=76

$$-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

```
[Out] -b*B*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B*c-A*c
*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A]

time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3234, 3153, 212}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((b*B*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2)) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx &= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}+\frac{(bB)\int\frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB)\mathrm{Subst}\left(\int\frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)-b\sin(\right.}{b^2+c^2}\\ &= -\frac{bB\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 82, normalized size = 1.08

$$\frac{2bB\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{-bBc+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
[Out] (2*b*B*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (-(b
*B*c) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.23, size = 109, normalized size = 1.43

method	result	size
default	$\frac{-\frac{2\left(A\,b^2+A\,c^2-B\,c^2\right)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}+\frac{2Bc}{b^2+c^2}}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2bB\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$	109
risch	$-\frac{2i\left(Ac-iAb+Bc\,\mathrm{e}^{ix}\right)}{(-ib+c)(ib+c)(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib)}+\frac{bB\ln\left(\mathrm{e}^{ix}+\frac{ib^3+ib\,c^2-b^2c-c^3}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{bB\ln\left(\mathrm{e}^{ix}-\frac{ib^3+ib\,c^2-b^2c-c^3}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*(-(A*b^2+A*c^2-B*c^2)/b/(b^2+c^2)*tan(1/2*x)+B*c/(b^2+c^2))/(b*tan(1/2*x)
^2-2*c*tan(1/2*x)-b)+2*b*B/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)
/(b^2+c^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(72) = 144$.
time = 0.49, size = 156, normalized size = 2.05

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2\left(b^3c+bc^3\right)\sin(x)}{\cos(x)+1}-\frac{\left(b^4+b^2c^2\right)\sin(x)^2}{\left(\cos(x)+1\right)^2}}\right)-\frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(72) = 144$.
time = 1.12, size = 201, normalized size = 2.64

$$-\frac{2\,Bb^2c+2\,Bc^3-\left(Bb^2\cos\left(x\right)+Bbc\sin\left(x\right)\right)\sqrt{b^2+c^2}\,\log\left(-\frac{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,\left(c\cos(x)-b\sin(x)\right)}{2\,bc\cos(x)\sin(x)+\left(b^2-c^2\right)\cos(x)^2+c^2}\right)+2\left(Ab^2c+Ac^3\right)\cos\left(x\right)-2\left(Ab^3+Abc^2\right)\sin\left(x\right)}{2\left(\left(b^5+2\,b^3c^2+bc^4\right)\cos\left(x\right)+\left(b^4c+2\,b^2c^3+c^5\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*b^2*c + 2*B*c^3 - (B*b^2*cos(x) + B*b*c*sin(x))*sqrt(b^2 + c^2)*l
og(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2
+ c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 +
 c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2
*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.45, size = 132, normalized size = 1.74

$$-\frac{Bb\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^{2}+c^{2}}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^{2}+c^{2}}\right|}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{2\left(Ab^{2}\tan\left(\frac{1}{2}\,x\right)+Ac^{2}\tan\left(\frac{1}{2}\,x\right)-Bc^{2}\tan\left(\frac{1}{2}\,x\right)-Bbc\right)}{\left(b^{3}+bc^{2}\right)\left(b\tan\left(\frac{1}{2}\,x\right)^{2}-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -B*b*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + A*c^2*
tan(1/2*x) - B*c^2*tan(1/2*x) - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

Mupad [B]
time = 2.62, size = 126, normalized size = 1.66

$$-\frac{\frac{2\,B\,c}{b^{2}+c^{2}}-\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^{2}+A\,c^{2}-B\,c^{2}\right)}{b\,\left(b^{2}+c^{2}\right)}}{-b\tan\left(\frac{x}{2}\right)^{2}+2\,c\tan\left(\frac{x}{2}\right)+b}+\frac{B\,b\,\mathrm{atan}\left(\frac{b^{2}\,c\,1\mathrm{i}+c^{3}\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,\left(b^{2}+c^{2}\right)\,1\mathrm{i}}{\left(b^{2}+c^{2}\right)^{3/2}}\right)\,2\mathrm{i}}{\left(b^{2}+c^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (B*b*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2)
)*2i)/(b^2 + c^2)^(3/2) - ((2*B*c)/(b^2 + c^2) - (2*tan(x/2)*(A*b^2 + A*c^2
 - B*c^2))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

3.354 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-B
*c-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-b*B*c*cos(x)+b^
2*B*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3237, 3232, 3153, 212}

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -1/2*(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) -
 (B*c + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) -
(b*B*c*Cos[x] - b^2*B*Sin[x])/((b^2 + c^2)^2*(b*Cos[x] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
```

```
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3237

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2
- b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x
], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 -
 b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2bB+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}$$
$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}+\frac{A\int \frac{}{b\cos}}{2(b}$$
$$= -\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}-\frac{A\text{Subst}}{}$$
$$= -\frac{A\tanh^{-1}\left(\frac{\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}}{}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{bBc\cos(}{\left(b^2+c^2\right)^2(b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 118, normalized size = 1.02

$$\frac{2A\sqrt{b^2+c^2}\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)(b\cos(x)+c\sin(x))^2+\left(b^2+c^2\right)\left(-Ac\cos(x)-Bc\cos(2x)+b(A+2B\cos(x))\sin(x)\right)}{2(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x] +
 c*Sin[x])^2 + (b^2 + c^2)*(-(A*c*Cos[x]) - B*c*Cos[2*x] + b*(A + 2*B*Cos[x
])*Sin[x]))/(2*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A]
time = 0.28, size = 204, normalized size = 1.76

method	result
default	$-\frac{2\left(-\frac{\left(A\,b^{2}+2A\,c^{2}-2B\,b^{2}-2B\,c^{2}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b}-\frac{c\left(A\,b^{2}-2A\,c^{2}+2B\,b^{2}+2B\,c^{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b^{2}}-\frac{\left(A\,b^{2}-2A\,c^{2}+2B\,b^{2}+2B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{2\left(b^{2}+c^{2}\right)b}+\frac{Ac}{2b^{2}+2c^{2}}\right)}{\left(b\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}$
risch	$\frac{-A\,b^{2}\mathrm{e}^{3ix}+A\,c^{2}\mathrm{e}^{3ix}+2iAbc\,\mathrm{e}^{3ix}+2B\,b^{2}\mathrm{e}^{2ix}+2B\,c^{2}\mathrm{e}^{2ix}+A\,b^{2}\mathrm{e}^{ix}+A\,c^{2}\mathrm{e}^{ix}+2B\,b^{2}+2iBbc}{\left(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib\right)^{2}(-ib+c)(ib+c)^{2}}+\frac{A\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{2\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*c*(A*
b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^2+
2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^2))/(b*tan(1/2*x)^2-
2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2
+c^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(108) = 216$.
time = 0.49, size = 366, normalized size = 3.16

$$-\frac{1}{2}\,A\left(\frac{2\left(b^{2}c-\frac{(b^{3}-2\,bc^{2})\sin(x)}{\cos(x)+1}-\frac{(b^{2}c-2\,c^{3})\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{(b^{3}+2\,bc^{2})\sin(x)^{3}}{(\cos(x)+1)^{3}}\right)}{b^{6}+b^{4}c^{2}+\frac{4\,(b^{5}c+b^{3}c^{3})\sin(x)}{\cos(x)+1}-\frac{2\,(b^{6}-b^{4}c^{2}-2\,b^{2}c^{4})\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{4\,(b^{5}c+b^{3}c^{3})\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{(b^{6}+b^{4}c^{2})\sin(x)^{4}}{(\cos(x)+1)^{4}}}+\frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^{2}+c^{2}}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^{2}+c^{2}}}\right)}{(b^{2}+c^{2})^{\frac{3}{2}}}\right)+\frac{2\,B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{b\sin(x)^{3}}{(\cos(x)+1)^{3}}\right)}{b^{4}+\frac{4\,b^{3}c\sin(x)}{\cos(x)+1}-\frac{4\,b^{3}c\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{b^{4}\sin(x)^{4}}{(\cos(x)+1)^{4}}-\frac{2\,(b^{4}-2\,b^{2}c^{2})\sin(x)^{2}}{(\cos(x)+1)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(
x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/
(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2
/(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(108) = 216$.
time = 0.88, size = 279, normalized size = 2.41

$$-\frac{8\,Bb^{2}c\cos\left(x\right)^{2}-2\,Bb^{2}c+2\,Bc^{3}-\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\cos\left(x\right)^{2}\right)\sqrt{b^{2}+c^{2}}\,\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^{2}-c^{2})\cos(x)^{2}-2\,b^{2}-c^{2}+2\,\sqrt{b^{2}+c^{2}}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^{2}-c^{2})\cos(x)^{2}+c^{2}}\right)+2\left(Ab^{2}c+Ac^{3}\right)\cos\left(x\right)-2\left(Ab^{3}+Abc^{2}+2\left(Bb^{3}-Bbc^{2}\right)\cos\left(x\right)\right)\sin\left(x\right)}{4\left(b^{4}c^{2}+2\,b^{2}c^{4}+c^{6}+\left(b^{6}+b^{4}c^{2}-b^{2}c^{4}-c^{6}\right)\cos\left(x\right)^{2}+2\left(b^{5}c+2\,b^{3}c^{3}+bc^{5}\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*B*b^2*c*cos(x)^2 - 2*B*b^2*c + 2*B*c^3 - (2*A*b*c*cos(x)*sin(x) + A
*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(
x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c + A*c
^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 - B*b*c^2)*cos(x))*sin(x))/(b^4*
c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c
 + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(108) = 216.
time = 0.44, size = 245, normalized size = 2.11

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,(b^2+c^2)^{\frac{3}{2}}}+\frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3-2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3-2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3+Ab^2c\tan\left(\frac{1}{2}\,x\right)^2+2\,Bb^2c\tan\left(\frac{1}{2}\,x\right)^2-2\,Ac^3\tan\left(\frac{1}{2}\,x\right)^2+2\,Bc^3\tan\left(\frac{1}{2}\,x\right)^2+Ab^3\tan\left(\frac{1}{2}\,x\right)+2\,Bb^3\tan\left(\frac{1}{2}\,x\right)-2\,Abc^2\tan\left(\frac{1}{2}\,x\right)+2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)-Ab^2c}{(b^4+b^2c^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + A*b^2
*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*t
an(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x)
+ 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b)^2)
```

Mupad [**B**]
time = 2.88, size = 251, normalized size = 2.16

$$\frac{\frac{\tan\left(\frac{x}{2}\right)\,\left(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,B\,c^3-2\,A\,c^3+A\,b^2\,c+2\,B\,b^2\,c\right)}{b^2\,\left(b^2+c^2\right)}+\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(A\,b^2+2\,A\,c^2-2\,B\,b^2-2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{b^2-{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,b^2-4\,c^2\right)+b^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,b\,c\,\tan\left(\frac{x}{2}\right)-4\,b\,c\,{\tan\left(\frac{x}{2}\right)}^3}+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1{}\mathrm{i}+c^3\,1{}\mathrm{i}-b\,\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,1{}\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(
b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + A*b^2*c + 2*B*b^2*c))/(b^2*(b
^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 - 2*B*c^2))/(b*(b^2 + c
^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) -
 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i
)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

3.355 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$

Optimal. Leaf size=246

$$\frac{35}{8}\left(b^2+c^2\right)^2 x-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}+\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35\left(b^2+c^2\right)\left(c\cos(d+ex)-b\sin(d+ex)\right.}{}$$

```
[Out] 35/8*(b^2+c^2)^2*x-35/8*c*(b^2+c^2)^(3/2)*cos(e*x+d)/e+35/8*b*(b^2+c^2)^(3/
2)*sin(e*x+d)/e-35/24*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c
*sin(e*x+d)+(b^2+c^2)^(1/2))/e-7/12*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(
1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2/e-1/4*(c*cos(e*x+d)-b*si
n(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3/e
```

Rubi [A]
time = 0.11, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3192, 2717, 2718}

$$\frac{35b\left(b^2+c^2\right)^{3/2}\sin(d+ex)}{8e}-\frac{35c\left(b^2+c^2\right)^{3/2}\cos(d+ex)}{8e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}-\frac{7\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{12e}-\frac{35\left(b^2+c^2\right)(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{24e}+\frac{35}{8}x\left(b^2+c^2\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (35*(b^2 + c^2)^2*x)/8 - (35*c*(b^2 + c^2)^(3/2)*Cos[d + e*x])/(8*e) + (35*
b*(b^2 + c^2)^(3/2)*Sin[d + e*x])/(8*e) - (35*(b^2 + c^2)*(c*Cos[d + e*x] -
 b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(24*e
) - (7*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d +
e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx &= -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(\right.}{4e}\\ &= -\frac{7\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c\right.}{12e}\\ &= -\frac{35(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+\right.}{24e}\\ &= \frac{35}{8}\left(b^2+c^2\right)^2 x-\frac{35(b^2+c^2)\,(c\cos(d+ex)-b\sin(d}{}\\ &= \frac{35}{8}\left(b^2+c^2\right)^2 x-\frac{35c(b^2+c^2)^{3/2}\cos(d+ex)}{8e}+\frac{35b(b}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.04, size = 238, normalized size = 0.97

$$\frac{420(b^2+c^2)^2(d+ex)-672(b-ic)(b+ic)c\sqrt{b^2+c^2}\cos(d+ex)-336bc(b^2+c^2)\cos(2(d+ex))+32c(-3b^2+c^2)\sqrt{b^2+c^2}\cos(3(d+ex))-12bc(b^2-c^2)\cos(4(d+ex))+672b(b-ic)(b+ic)\sqrt{b^2+c^2}\sin(d+ex)+168(b^4-c^4)\sin(2(d+ex))+32b(b^2-3c^2)\sqrt{b^2+c^2}\sin(3(d+ex))+3(b^4-6b^2c^2+c^4)\sin(4(d+ex))}{96e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (420*(b^2 + c^2)^2*(d + e*x) - 672*(b - I*c)*(b + I*c)*c*Sqrt[b^2 + c^2]*Co
s[d + e*x] - 336*b*c*(b^2 + c^2)*Cos[2*(d + e*x)] + 32*c*(-3*b^2 + c^2)*Sqr
t[b^2 + c^2]*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 672*b
*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*Sin[d + e*x] + 168*(b^4 - c^4)*Sin[2*(
d + e*x)] + 32*b*(b^2 - 3*c^2)*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 3*(b^4 -
6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(222) = 444$.
time = 0.55, size = 514, normalized size = 2.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(b^4*(e*x+d)+c^4*(e*x+d)+2*b^2*c^2*(e*x+d)-b^3*c*cos(e*x+d)^4+6*b^2*c^2
*(-1/4*sin(e*x+d)*cos(e*x+d)^3+1/8*cos(e*x+d)*sin(e*x+d)+1/8*e*x+1/8*d)+c^3
*b*sin(e*x+d)^4-6*b^3*c*cos(e*x+d)^2-6*c^3*b*cos(e*x+d)^2+4*(b^2+c^2)^(1/2)
*b*c^2*sin(e*x+d)-4*(b^2+c^2)^(1/2)*b^2*c*cos(e*x+d)+4/3*(b^2+c^2)^(1/2)*b^
3*(2+cos(e*x+d)^2)*sin(e*x+d)+6*b^2*c^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+
1/2*d)+4*(b^2+c^2)^(1/2)*b^3*sin(e*x+d)-4/3*(b^2+c^2)^(1/2)*c^3*(2+sin(e*x+
d)^2)*cos(e*x+d)+6*b^2*c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-4*(b^
2+c^2)^(1/2)*c^3*cos(e*x+d)+b^4*(1/4*(cos(e*x+d)^3+3/2*cos(e*x+d))*sin(e*x+
d)+3/8*e*x+3/8*d)+6*b^4*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+c^4*(-1/4
*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)+6*c^4*(-1/2*cos(e*
x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-4*(b^2+c^2)^(1/2)*b^2*c*cos(e*x+d)^3+4*(b^2+
c^2)^(1/2)*b*c^2*sin(e*x+d)^3)
```

Maxima [A]
time = 0.27, size = 363, normalized size = 1.48

$$-b^{3}c\cos\left(xe+d\right)^{4}e^{(-1)}+bc^{3}e^{(-1)}\sin\left(xe+d\right)^{4}+\frac{1}{32}\left(12\,xe+12\,d+\sin\left(4\,xe+4\,d\right)+8\,\sin\left(2\,xe+2\,d\right)\right)b^{4}e^{(-1)}+\frac{3}{16}\left(4\,xe+4\,d-\sin\left(4\,xe+4\,d\right)\right)b^{2}c^{2}e^{(-1)}+\frac{1}{32}\left(12\,xe+12\,d+\sin\left(4\,xe+4\,d\right)-8\,\sin\left(2\,xe+2\,d\right)\right)c^{4}e^{(-1)}+\left(b^{2}+c^{2}\right)^{2}x-4\left(c\cos\left(xe+d\right)e^{(-1)}-be^{(-1)}\sin\left(xe+d\right)\right)\left(b^{2}+c^{2}\right)^{\frac{3}{2}}-\frac{3}{2}\left(4\,bc\cos\left(xe+d\right)^{2}e^{(-1)}-\left(2\,xe+2\,d+\sin\left(2\,xe+2\,d\right)\right)b^{2}e^{(-1)}-\left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)c^{2}e^{(-1)}\right)\left(b^{2}+c^{2}\right)-\frac{4}{3}\left(3\,b^{2}c\cos\left(xe+d\right)^{3}e^{(-1)}-3\,bc^{2}e^{(-1)}\sin\left(xe+d\right)^{3}+\left(\sin\left(xe+d\right)^{3}-3\,\sin\left(xe+d\right)\right)b^{3}e^{(-1)}-\left(\cos\left(xe+d\right)^{3}-3\,\cos\left(xe+d\right)\right)c^{3}e^{(-1)}\right)\sqrt{b^{2}+c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxim
a")
```

```
[Out] -b^3*c*cos(x*e + d)^4*e^(-1) + b*c^3*e^(-1)*sin(x*e + d)^4 + 1/32*(12*x*e +
 12*d + sin(4*x*e + 4*d) + 8*sin(2*x*e + 2*d))*b^4*e^(-1) + 3/16*(4*x*e + 4
*d - sin(4*x*e + 4*d))*b^2*c^2*e^(-1) + 1/32*(12*x*e + 12*d + sin(4*x*e + 4
*d) - 8*sin(2*x*e + 2*d))*c^4*e^(-1) + (b^2 + c^2)^2*x - 4*(c*cos(x*e + d)*
e^(-1) - b*e^(-1)*sin(x*e + d))*(b^2 + c^2)^(3/2) - 3/2*(4*b*c*cos(x*e + d)
^2*e^(-1) - (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1) - (2*x*e + 2*d - si
n(2*x*e + 2*d))*c^2*e^(-1))*(b^2 + c^2) - 4/3*(3*b^2*c*cos(x*e + d)^3*e^(-1
) - 3*b*c^2*e^(-1)*sin(x*e + d)^3 + (sin(x*e + d)^3 - 3*sin(x*e + d))*b^3*e
^(-1) - (cos(x*e + d)^3 - 3*cos(x*e + d))*c^3*e^(-1))*sqrt(b^2 + c^2)
```

Fricas [A]
time = 0.99, size = 230, normalized size = 0.93

$$-\frac{1}{24}\left(24\left(b^{3}c-bc^{3}\right)\cos\left(xe+d\right)^{4}+48\left(3\,b^{3}c+4\,bc^{3}\right)\cos\left(xe+d\right)^{2}-105\left(b^{4}+2\,b^{2}c^{2}+c^{4}\right)xe-3\left(2\left(b^{4}-6\,b^{2}c^{2}+c^{4}\right)\cos\left(xe+d\right)^{3}+\left(27\,b^{4}+6\,b^{2}c^{2}-29\,c^{4}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)+32\left(\left(3\,b^{2}c-c^{3}\right)\cos\left(xe+d\right)^{3}+3\left(b^{2}c+2\,c^{3}\right)\cos\left(xe+d\right)-\left(5\,b^{3}+6\,bc^{2}+\left(b^{3}-3\,bc^{2}\right)\cos\left(xe+d\right)^{2}\right)\sin\left(xe+d\right)\right)\sqrt{b^{2}+c^{2}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="frica
s")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(x*e + d)^4 + 48*(3*b^3*c + 4*b*c^3)*cos(x*e +
 d)^2 - 105*(b^4 + 2*b^2*c^2 + c^4)*x*e - 3*(2*(b^4 - 6*b^2*c^2 + c^4)*cos(
x*e + d)^3 + (27*b^4 + 6*b^2*c^2 - 29*c^4)*cos(x*e + d))*sin(x*e + d) + 32*
((3*b^2*c - c^3)*cos(x*e + d)^3 + 3*(b^2*c + 2*c^3)*cos(x*e + d) - (5*b^3 +
```

```
6*b*c^2 + (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d))*sqrt(b^2 + c^2))*e
^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(235) = 470$.
time = 0.65, size = 857, normalized size = 3.48

$$\begin{cases} \frac{3b^4x\sin^4(d+ex)}{8} + \frac{3b^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + 3b^4x\sin^2(d+ex) + \frac{3b^4x\cos^4(d+ex)}{8} + 3b^4x\cos^2(d+ex) + b^4x + \frac{3b^4\sin^3(d+ex)\cos(d+ex)}{8e} + \frac{5b^4\sin(d+ex)\cos^3(d+ex)}{8e} + \frac{3b^4\sin(d+ex)\cos(d+ex)}{e} + \frac{6b^3c\sin^2(d+ex)}{e} - \frac{b^3c\cos^4(d+ex)}{e} + \frac{8b^3\sqrt{b^2+c^2}\sin^3(d+ex)}{3e} + \frac{4b^3\sqrt{b^2+c^2}\sin(d+ex)\cos^2(d+ex)}{e} + \frac{4b^3\sqrt{b^2+c^2}\sin(d+ex)}{e} + \frac{3b^2c^2x\sin^4(d+ex)}{4} + \frac{3b^2c^2x\sin^2(d+ex)\cos^2(d+ex)}{2} + 6b^2c^2x\sin^2(d+ex) + \frac{3b^2c^2x\cos^4(d+ex)}{4} + 6b^2c^2x\cos^2(d+ex) + 2b^2c^2x + \frac{3b^2c^2\sin^3(d+ex)\cos(d+ex)}{4e} - \frac{3b^2c^2\sin(d+ex)\cos^3(d+ex)}{4e} - \frac{4b^2c\sqrt{b^2+c^2}\cos^3(d+ex)}{e} - \frac{4b^2c\sqrt{b^2+c^2}\cos(d+ex)}{e} + \frac{bc^3\sin^4(d+ex)}{e} + \frac{6bc^3\sin^2(d+ex)}{e} + \frac{4bc^2\sqrt{b^2+c^2}\sin^3(d+ex)}{e} + \frac{4bc^2\sqrt{b^2+c^2}\sin(d+ex)}{e} + \frac{3c^4x\sin^4(d+ex)}{8} + \frac{3c^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + 3c^4x\sin^2(d+ex) + \frac{3c^4x\cos^4(d+ex)}{8} + 3c^4x\cos^2(d+ex) + c^4x - \frac{5c^4\sin^3(d+ex)\cos(d+ex)}{8e} - \frac{3c^4\sin(d+ex)\cos^3(d+ex)}{8e} - \frac{3c^4\sin(d+ex)\cos(d+ex)}{e} - \frac{4c^3\sqrt{b^2+c^2}\sin^2(d+ex)\cos(d+ex)}{e} - \frac{8c^3\sqrt{b^2+c^2}\cos^3(d+ex)}{3e} - \frac{4c^3\sqrt{b^2+c^2}\cos(d+ex)}{e} & \text{for}\ e \neq 0 \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2+c^2}\right)^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Piecewise((3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*b**4*x*sin(d + e*x)**2 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*x*
cos(d + e*x)**2 + b**4*x + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5*b*
*4*sin(d + e*x)*cos(d + e*x)**3/(8*e) + 3*b**4*sin(d + e*x)*cos(d + e*x)/e
+ 6*b**3*c*sin(d + e*x)**2/e - b**3*c*cos(d + e*x)**4/e + 8*b**3*sqrt(b**2
+ c**2)*sin(d + e*x)**3/(3*e) + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)*cos(d
 + e*x)**2/e + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*b**2*c**2*x*sin(
d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 6*b**2*c*
*2*x*sin(d + e*x)**2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 6*b**2*c**2*x*cos(
d + e*x)**2 + 2*b**2*c**2*x + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e*x)/(4*e
) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 4*b**2*c*sqrt(b**2 + c
**2)*cos(d + e*x)**3/e - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x)/e + b*c**3
*sin(d + e*x)**4/e + 6*b*c**3*sin(d + e*x)**2/e + 4*b*c**2*sqrt(b**2 + c**2
)*sin(d + e*x)**3/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*c**4*x*
sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 + 3*c**4*x*s
in(d + e*x)**2 + 3*c**4*x*cos(d + e*x)**4/8 + 3*c**4*x*cos(d + e*x)**2 + c*
*4*x - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(
d + e*x)**3/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)/e - 4*c**3*sqrt(b**2 +
 c**2)*sin(d + e*x)**2*cos(d + e*x)/e - 8*c**3*sqrt(b**2 + c**2)*cos(d + e*
x)**3/(3*e) - 4*c**3*sqrt(b**2 + c**2)*cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos
(d) + c*sin(d) + sqrt(b**2 + c**2))**4, True))
```

Giac [A]
time = 0.51, size = 287, normalized size = 1.17

$$\frac{35}{8}\left(b^4 + 2b^2c^2 + c^4\right)x - \frac{\left(b^3c - bc^3\right)\cos(4ex + 4d)}{8e} - \frac{\left(3\sqrt{b^2+c^2}\,b^2c - \sqrt{b^2+c^2}\,c^3\right)\cos(3ex + 3d)}{3e} - \frac{7\left(b^3c + bc^3\right)\cos(2ex + 2d)}{2e} - \frac{7\left(\sqrt{b^2+c^2}\,b^2c + \sqrt{b^2+c^2}\,c^3\right)\cos(ex + d)}{e} + \frac{\left(b^4 - 6b^2c^2 + c^4\right)\sin(4ex + 4d)}{32e} + \frac{\left(\sqrt{b^2+c^2}\,b^3 - 3\sqrt{b^2+c^2}\,bc^2\right)\sin(3ex + 3d)}{3e} + \frac{7\left(b^4 - c^4\right)\sin(2ex + 2d)}{4e} + \frac{7\left(\sqrt{b^2+c^2}\,b^3 + \sqrt{b^2+c^2}\,bc^2\right)\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac"
)
```

```
[Out] 35/8*(b^4 + 2*b^2*c^2 + c^4)*x - 1/8*(b^3*c - b*c^3)*cos(4*e*x + 4*d)/e - 1
/3*(3*sqrt(b^2 + c^2)*b^2*c - sqrt(b^2 + c^2)*c^3)*cos(3*e*x + 3*d)/e - 7/2
*(b^3*c + b*c^3)*cos(2*e*x + 2*d)/e - 7*(sqrt(b^2 + c^2)*b^2*c + sqrt(b^2 +
```

```
c^2)*c^3)*cos(e*x + d)/e + 1/32*(b^4 - 6*b^2*c^2 + c^4)*sin(4*e*x + 4*d)/e
 + 1/3*(sqrt(b^2 + c^2)*b^3 - 3*sqrt(b^2 + c^2)*b*c^2)*sin(3*e*x + 3*d)/e +
 7/4*(b^4 - c^4)*sin(2*e*x + 2*d)/e + 7*(sqrt(b^2 + c^2)*b^3 + sqrt(b^2 + c
^2)*b*c^2)*sin(e*x + d)/e
```

Mupad [B]
time = 7.32, size = 522, normalized size = 2.12

$$\frac{35\,\mathrm{atan}\left(\frac{35\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,{\left(b^2+c^2\right)}^2}{4\left(\frac{35\,b^4}{4}+\frac{35\,b^2\,c^2}{2}+\frac{35\,c^4}{4}\right)}\right)\,{\left(b^2+c^2\right)}^2}{4\,e}-\frac{35\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,{\left(b^2+c^2\right)}^2}{4\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\left(16\,b^3+8\,b\,c^2\right)\,\sqrt{b^2+c^2}+\frac{29\,b^4}{4}-\frac{27\,c^4}{4}-\frac{3\,b^2\,c^2}{2}\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6\,\left(24\,b\,c^3+32\,b^3\,c-\left(32\,b^2\,c+8\,c^3\right)\,\sqrt{b^2+c^2}\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(64\,b\,c^3+48\,b^3\,c-\left(48\,b^2\,c+40\,c^3\right)\,\sqrt{b^2+c^2}\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(24\,b\,c^3+32\,b^3\,c-\left(32\,b^2\,c+\frac{136\,c^3}{3}\right)\,\sqrt{b^2+c^2}\right)-\left(16\,b^2\,c+\frac{40\,c^3}{3}\right)\,\sqrt{b^2+c^2}+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^7\,\left(\left(16\,b^3+8\,b\,c^2\right)\,\sqrt{b^2+c^2}-\frac{29\,b^4}{4}+\frac{27\,c^4}{4}+\frac{3\,b^2\,c^2}{2}\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(\left(\frac{112\,b^3}{3}+56\,b\,c^2\right)\,\sqrt{b^2+c^2}+\frac{21\,b^4}{4}-\frac{35\,c^4}{4}+\frac{21\,b^2\,c^2}{2}\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(\left(\frac{112\,b^3}{3}+56\,b\,c^2\right)\,\sqrt{b^2+c^2}-\frac{21\,b^4}{4}+\frac{35\,c^4}{4}-\frac{21\,b^2\,c^2}{2}\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^8+4\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+6\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+4\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

```
[Out] (35*atan((35*tan(d/2 + (e*x)/2)*(b^2 + c^2)^2)/(4*((35*b^4)/4 + (35*c^4)/4
+ (35*b^2*c^2)/2)))*(b^2 + c^2)^2)/(4*e) - (35*(atan(tan(d/2 + (e*x)/2)) -
(e*x)/2)*(b^2 + c^2)^2)/(4*e) + (tan(d/2 + (e*x)/2)*((8*b*c^2 + 16*b^3)*(b^
2 + c^2)^(1/2) + (29*b^4)/4 - (27*c^4)/4 - (3*b^2*c^2)/2) + tan(d/2 + (e*x)
/2)^6*(24*b*c^3 + 32*b^3*c - (32*b^2*c + 8*c^3)*(b^2 + c^2)^(1/2)) + tan(d/
2 + (e*x)/2)^4*(64*b*c^3 + 48*b^3*c - (48*b^2*c + 40*c^3)*(b^2 + c^2)^(1/2)
) + tan(d/2 + (e*x)/2)^2*(24*b*c^3 + 32*b^3*c - (32*b^2*c + (136*c^3)/3)*(b
^2 + c^2)^(1/2)) - (16*b^2*c + (40*c^3)/3)*(b^2 + c^2)^(1/2) + tan(d/2 + (e
*x)/2)^7*((8*b*c^2 + 16*b^3)*(b^2 + c^2)^(1/2) - (29*b^4)/4 + (27*c^4)/4 +
(3*b^2*c^2)/2) + tan(d/2 + (e*x)/2)^3*((56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)
^(1/2) + (21*b^4)/4 - (35*c^4)/4 + (21*b^2*c^2)/2) + tan(d/2 + (e*x)/2)^5*(
(56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)^(1/2) - (21*b^4)/4 + (35*c^4)/4 - (21*
b^2*c^2)/2))/(e*(4*tan(d/2 + (e*x)/2)^2 + 6*tan(d/2 + (e*x)/2)^4 + 4*tan(d/
2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^8 + 1))
```

3.356 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$

Optimal. Leaf size=178

$$\frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5c(b^2+c^2)\cos(d+ex)}{2e}+\frac{5b(b^2+c^2)\sin(d+ex)}{2e}-5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex)$$

```
[Out] 5/2*(b^2+c^2)^(3/2)*x-5/2*c*(b^2+c^2)*cos(e*x+d)/e+5/2*b*(b^2+c^2)*sin(e*x+
d)/e-5/6*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*
x+d)+(b^2+c^2)^(1/2))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin
(e*x+d)+(b^2+c^2)^(1/2))^2/e
```

Rubi [A]
time = 0.07, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3192, 2717, 2718}

$$\frac{5b(b^2+c^2)\sin(d+ex)}{2e}-\frac{5c(b^2+c^2)\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}-\frac{5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{6e}+\frac{5}{2}x\left(b^2+c^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (5*(b^2 + c^2)^(3/2)*x)/2 - (5*c*(b^2 + c^2)*Cos[d + e*x])/(2*e) + (5*b*(b^
2 + c^2)*Sin[d + e*x])/(2*e) - (5*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d
 + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c*C
os[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^2)/(3*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
```

```
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d\right.}{3e}$$

$$= -\frac{5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}\right.}{6e}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x - \frac{5\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+}{}$$

$$= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x - \frac{5c(b^2+c^2)\cos(d+ex)}{2e} + \frac{5b(b^2+c^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 163, normalized size = 0.92

$$\frac{30(b-ic)(b+ic)\sqrt{b^2+c^2}\,(d+ex)-45c(b^2+c^2)\cos(d+ex)-18bc\sqrt{b^2+c^2}\,\cos(2(d+ex))+c(-3b^2+c^2)\cos(3(d+ex))+45b(b^2+c^2)\sin(d+ex)+9(b^2-c^2)\sqrt{b^2+c^2}\,\sin(2(d+ex))+b(b^2-3c^2)\sin(3(d+ex))}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (30*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*(d + e*x) - 45*c*(b^2 + c^2)*Cos[d
+ e*x] - 18*b*c*Sqrt[b^2 + c^2]*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(
d + e*x)] + 45*b*(b^2 + c^2)*Sin[d + e*x] + 9*(b^2 - c^2)*Sqrt[b^2 + c^2]*S
in[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

Maple [A]
time = 0.40, size = 250, normalized size = 1.40

method	result
risch	$\frac{5\left(b^2+c^2\right)^{\frac{3}{2}}x}{2}-\frac{15c\cos(ex+d)b^2}{4e}-\frac{15c^3\cos(ex+d)}{4e}+\frac{15b^3\sin(ex+d)}{4e}+\frac{15b\sin(ex+d)c^2}{4e}-\frac{c\cos(3ex+3d)b^2}{4e}+$
derivativedivides	$\frac{\frac{b^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-b^2c\left(\cos^3(ex+d)\right)+3\sqrt{b^2+c^2}\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+c^2b\left(\sin^3(ex+d)\right)-3\sqrt{b}}{}$
default	$\frac{\frac{b^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-b^2c\left(\cos^3(ex+d)\right)+3\sqrt{b^2+c^2}\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+c^2b\left(\sin^3(ex+d)\right)-3\sqrt{b}}{}$

norman	$\frac{\left(-16c^3+12\sqrt{b^2+c^2}\,bc-12b^2c\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{\left(-3\sqrt{b^2+c^2}\,b^2+3\sqrt{b^2+c^2}\,c^2+8b^3+6c^2b\right)\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-b^2*c*cos(e*x+d)^3+3*(b^2+c^2)^(1/
2)*b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+c^2*b*sin(e*x+d)^3-3*(b^2+
c^2)^(1/2)*b*c*cos(e*x+d)^2+3*b^3*sin(e*x+d)+3*c^2*b*sin(e*x+d)-1/3*c^3*(2+
sin(e*x+d)^2)*cos(e*x+d)+3*(b^2+c^2)^(1/2)*c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+
1/2*e*x+1/2*d)-3*b^2*c*cos(e*x+d)-3*c^3*cos(e*x+d)+(b^2+c^2)^(1/2)*b^2*(e*x
+d)+(b^2+c^2)^(1/2)*c^2*(e*x+d))
```

Maxima [A]
time = 0.26, size = 211, normalized size = 1.19

$$-b^2c\cos\left(xe+d\right)^3e^{(-1)}+bc^2e^{(-1)}\sin\left(xe+d\right)^3-\frac{1}{3}\left(\sin\left(xe+d\right)^3-3\,\sin\left(xe+d\right)\right)b^3e^{(-1)}+\frac{1}{3}\left(\cos\left(xe+d\right)^3-3\,\cos\left(xe+d\right)\right)c^3e^{(-1)}+\left(b^2+c^2\right)^{\frac{3}{2}}x-3\left(c\cos\left(xe+d\right)e^{(-1)}-be^{(-1)}\sin\left(xe+d\right)\right)\left(b^2+c^2\right)-\frac{3}{4}\left(4\,bc\cos\left(xe+d\right)^2e^{(-1)}-\left(2\,xe+2\,d+\sin\left(2\,xe+2\,d\right)\right)b^2e^{(-1)}-\left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)c^2e^{(-1)}\right)\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxim
a")
```

```
[Out] -b^2*c*cos(x*e + d)^3*e^(-1) + b*c^2*e^(-1)*sin(x*e + d)^3 - 1/3*(sin(x*e +
 d)^3 - 3*sin(x*e + d))*b^3*e^(-1) + 1/3*(cos(x*e + d)^3 - 3*cos(x*e + d))*
c^3*e^(-1) + (b^2 + c^2)^(3/2)*x - 3*(c*cos(x*e + d)*e^(-1) - b*e^(-1)*sin(
x*e + d))*(b^2 + c^2) - 3/4*(4*b*c*cos(x*e + d)^2*e^(-1) - (2*x*e + 2*d + s
in(2*x*e + 2*d))*b^2*e^(-1) - (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1))*
sqrt(b^2 + c^2)
```

Fricas [A]
time = 0.80, size = 152, normalized size = 0.85

$$-\frac{1}{6}\left(2\left(3\,b^2c-c^3\right)\cos\left(xe+d\right)^3+6\left(3\,b^2c+4\,c^3\right)\cos\left(xe+d\right)-2\left(11\,b^3+12\,bc^2+\left(b^3-3\,bc^2\right)\cos\left(xe+d\right)^2\right)\sin\left(xe+d\right)+3\left(6\,bc\cos\left(xe+d\right)^2-5\left(b^2+c^2\right)xe-3\left(b^2-c^2\right)\cos\left(xe+d\right)\sin\left(xe+d\right)\right)\sqrt{b^2+c^2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="frica
s")
```

```
[Out] -1/6*(2*(3*b^2*c - c^3)*cos(x*e + d)^3 + 6*(3*b^2*c + 4*c^3)*cos(x*e + d) -
 2*(11*b^3 + 12*b*c^2 + (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d) + 3*(6
*b*c*cos(x*e + d)^2 - 5*(b^2 + c^2)*x*e - 3*(b^2 - c^2)*cos(x*e + d)*sin(x*
e + d))*sqrt(b^2 + c^2))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(168) = 336$.
time = 0.39, size = 415, normalized size = 2.33

$$\begin{cases} \frac{2b^3\sin^3(d+ex)}{3e} + \frac{b^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{3b^3\sin(d+ex)}{e} - \frac{b^2c\cos^3(d+ex)}{e} - \frac{3b^2c\cos(d+ex)}{e} + \frac{3b^2x\sqrt{b^2+c^2}\sin^2(d+ex)}{2} + \frac{3b^2x\sqrt{b^2+c^2}\cos^2(d+ex)}{2} + b^2x\sqrt{b^2+c^2} + \frac{3b^2\sqrt{b^2+c^2}\sin(d+ex)\cos(d+ex)}{2e} + \frac{bc^2\sin^3(d+ex)}{e} + \frac{3bc^2\sin(d+ex)}{e} + \frac{3bc\sqrt{b^2+c^2}\sin^2(d+ex)}{e} - \frac{c^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{2c^3\cos^3(d+ex)}{3e} - \frac{3c^3\cos(d+ex)}{e} + \frac{3c^2x\sqrt{b^2+c^2}\sin^2(d+ex)}{2} + \frac{3c^2x\sqrt{b^2+c^2}\cos^2(d+ex)}{2} + c^2x\sqrt{b^2+c^2} - \frac{3c^2\sqrt{b^2+c^2}\sin(d+ex)\cos(d+ex)}{2e} & \text{for } e \neq 0 \\ x\left(b\cos(d)+c\sin(d)+\sqrt{b^2+c^2}\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Piecewise((2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2
/e + 3*b**3*sin(d + e*x)/e - b**2*c*cos(d + e*x)**3/e - 3*b**2*c*cos(d + e*
x)/e + 3*b**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*b**2*x*sqrt(b**2 +
c**2)*cos(d + e*x)**2/2 + b**2*x*sqrt(b**2 + c**2) + 3*b**2*sqrt(b**2 + c**
2)*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c**2*sin(d + e*x)**3/e + 3*b*c**2*si
n(d + e*x)/e + 3*b*c*sqrt(b**2 + c**2)*sin(d + e*x)**2/e - c**3*sin(d + e*x
)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e) - 3*c**3*cos(d + e*x)/e
+ 3*c**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*c**2*x*sqrt(b**2 + c**2)
*cos(d + e*x)**2/2 + c**2*x*sqrt(b**2 + c**2) - 3*c**2*sqrt(b**2 + c**2)*si
n(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b*
*2 + c**2))**3, True))
```

Giac [A]
time = 0.44, size = 199, normalized size = 1.12

$$(b^2+c^2)^{\frac{3}{2}}x - \frac{3\sqrt{b^2+c^2}\,bc\cos(2ex+2d)}{2e} + \frac{3}{2}\left(\sqrt{b^2+c^2}\,b^2 + \sqrt{b^2+c^2}\,c^2\right)x - \frac{(3b^2c-c^3)\cos(3ex+3d)}{12e} - \frac{15(b^2c+c^3)\cos(ex+d)}{4e} + \frac{(b^3-3bc^2)\sin(3ex+3d)}{12e} + \frac{3\left(\sqrt{b^2+c^2}\,b^2 - \sqrt{b^2+c^2}\,c^2\right)\sin(2ex+2d)}{4e} + \frac{15(b^3+bc^2)\sin(ex+d)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac"
)
```

```
[Out] (b^2 + c^2)^(3/2)*x - 3/2*sqrt(b^2 + c^2)*b*c*cos(2*e*x + 2*d)/e + 3/2*(sqr
t(b^2 + c^2)*b^2 + sqrt(b^2 + c^2)*c^2)*x - 1/12*(3*b^2*c - c^3)*cos(3*e*x
+ 3*d)/e - 15/4*(b^2*c + c^3)*cos(e*x + d)/e + 1/12*(b^3 - 3*b*c^2)*sin(3*e
*x + 3*d)/e + 3/4*(sqrt(b^2 + c^2)*b^2 - sqrt(b^2 + c^2)*c^2)*sin(2*e*x + 2
*d)/e + 15/4*(b^3 + b*c^2)*sin(e*x + d)/e
```

Mupad [B]
time = 7.32, size = 261, normalized size = 1.47

$$\frac{5x(b^2+c^2)^{3/2}}{2} - \frac{8b^2c - \tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left((3b^2-3c^2)\sqrt{b^2+c^2}+6bc^2+8b^3\right) - \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(\frac{40b^3}{3}+20bc^2\right) + \frac{22c^3}{3} - \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^5\left(6bc^2-(3b^2-3c^2)\sqrt{b^2+c^2}+8b^3\right) + \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4\left(12b^2c+6c^3-12bc\sqrt{b^2+c^2}\right) + \tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(12b^2c+16c^3-12bc\sqrt{b^2+c^2}\right)}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^6+3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4+3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

```
[Out] (5*x*(b^2 + c^2)^(3/2))/2 - (8*b^2*c - tan(d/2 + (e*x)/2)*((3*b^2 - 3*c^2)*
(b^2 + c^2)^(1/2) + 6*b*c^2 + 8*b^3) - tan(d/2 + (e*x)/2)^3*(20*b*c^2 + (40
```

```
*b^3)/3) + (22*c^3)/3 - tan(d/2 + (e*x)/2)^5*(6*b*c^2 - (3*b^2 - 3*c^2)*(b^
2 + c^2)^(1/2) + 8*b^3) + tan(d/2 + (e*x)/2)^4*(12*b^2*c + 6*c^3 - 12*b*c*(
b^2 + c^2)^(1/2)) + tan(d/2 + (e*x)/2)^2*(12*b^2*c + 16*c^3 - 12*b*c*(b^2 +
 c^2)^(1/2)))/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2
 + (e*x)/2)^6 + 1))
```

3.357 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$

Optimal. Leaf size=116

$$\frac{3}{2}\left(b^2+c^2\right)x-\frac{3c\sqrt{b^2+c^2}\,\cos(d+ex)}{2e}+\frac{3b\sqrt{b^2+c^2}\,\sin(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}\right.}{2e}$$

[Out] 3/2*(b^2+c^2)*x-3/2*c*cos(e*x+d)*(b^2+c^2)^(1/2)/e+3/2*b*sin(e*x+d)*(b^2+c^2)^(1/2)/e-1/2*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))/e

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {3192, 2717, 2718}

$$\frac{3b\sqrt{b^2+c^2}\,\sin(d+ex)}{2e}-\frac{3c\sqrt{b^2+c^2}\,\cos(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{2e}+\frac{3}{2}x\left(b^2+c^2\right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]

[Out] (3*(b^2 + c^2)*x)/2 - (3*c*Sqrt[b^2 + c^2]*Cos[d + e*x])/(2*e) + (3*b*Sqrt[b^2 + c^2]*Sin[d + e*x])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/(2*e)

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2 dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(\right.}{2e}$$

$$=\frac{3}{2}\left(b^2+c^2\right)x-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2\right.}{2}$$

$$=\frac{3}{2}\left(b^2+c^2\right)x-\frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e}+\frac{3b\sqrt{b^2+c}}{}$$

Mathematica [A]
time = 0.16, size = 111, normalized size = 0.96

$$\frac{6b^2d+6c^2d+6b^2ex+6c^2ex-8c\sqrt{b^2+c^2}\cos(d+ex)-2bc\cos(2(d+ex))+8b\sqrt{b^2+c^2}\sin(d+ex)+b^2\sin(2(d+ex))-c^2\sin(2(d+ex))}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (6*b^2*d + 6*c^2*d + 6*b^2*e*x + 6*c^2*e*x - 8*c*Sqrt[b^2 + c^2]*Cos[d + e*
x] - 2*b*c*Cos[2*(d + e*x)] + 8*b*Sqrt[b^2 + c^2]*Sin[d + e*x] + b^2*Sin[2*
(d + e*x)] - c^2*Sin[2*(d + e*x)])/(4*e)
```

Maple [A]
time = 0.29, size = 124, normalized size = 1.07

method	result
risch	$\frac{3b^2x}{2}+\frac{3xc^2}{2}-\frac{2c\cos(ex+d)\sqrt{b^2+c^2}}{e}+\frac{2b\sin(ex+d)\sqrt{b^2+c^2}}{e}-\frac{bc\cos(2ex+2d)}{2e}+\frac{\sin(2ex+2d)b^2}{4e}-$
derivativedivides	$\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\left(\cos^2(ex+d)\right)+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}\,b\sin(ex+d)-2\sqrt{}}{e}$
default	$\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\left(\cos^2(ex+d)\right)+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}\,b\sin(ex+d)-2\sqrt{}}{e}$
norman	$\frac{\left(\frac{3b^2}{2}+\frac{3c^2}{2}\right)x-\frac{4c\sqrt{b^2+c^2}}{e}+\left(\frac{3b^2}{2}+\frac{3c^2}{2}\right)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{\left(4\sqrt{b^2+c^2}\,b+b^2-c^2\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}+\left(3b^2+3c^2\right)x\left(\right.}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-c*b*cos(e*x+d)^2+c^2*(-1
/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+2*(b^2+c^2)^(1/2)*b*sin(e*x+d)-2*(b
^2+c^2)^(1/2)*c*cos(e*x+d)+b^2*(e*x+d)+c^2*(e*x+d))
```

Maxima [A]
time = 0.26, size = 115, normalized size = 0.99

$$-bc\cos(xe+d)^2 e^{(-1)} + \frac{1}{4}(2xe + 2d + \sin(2xe + 2d))b^2 e^{(-1)} + \frac{1}{4}(2xe + 2d - \sin(2xe + 2d))c^2 e^{(-1)} + b^2 x + c^2 x - 2\left(c\cos(xe+d)e^{(-1)} - be^{(-1)}\sin(xe+d)\right)\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxim
a")
```

```
[Out] -b*c*cos(x*e + d)^2*e^(-1) + 1/4*(2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1
) + 1/4*(2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1) + b^2*x + c^2*x - 2*(c*
cos(x*e + d)*e^(-1) - b*e^(-1)*sin(x*e + d))*sqrt(b^2 + c^2)
```

Fricas [A]
time = 0.97, size = 86, normalized size = 0.74

$$-\frac{1}{2}\left(2bc\cos(xe+d)^2 - 3(b^2+c^2)xe - (b^2-c^2)\cos(xe+d)\sin(xe+d) + 4\sqrt{b^2+c^2}(c\cos(xe+d) - b\sin(xe+d))\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="frica
s")
```

```
[Out] -1/2*(2*b*c*cos(x*e + d)^2 - 3*(b^2 + c^2)*x*e - (b^2 - c^2)*cos(x*e + d)*s
in(x*e + d) + 4*sqrt(b^2 + c^2)*(c*cos(x*e + d) - b*sin(x*e + d)))*e^(-1)
```

Sympy [A]
time = 0.15, size = 192, normalized size = 1.66

$$\begin{cases} \frac{b^2 x\sin^2(d+ex)}{2} + \frac{b^2 x\cos^2(d+ex)}{2} + b^2 x + \frac{b^2\sin(d+ex)\cos(d+ex)}{2e} + \frac{bc\sin^2(d+ex)}{e} + \frac{2b\sqrt{b^2+c^2}\sin(d+ex)}{e} + \frac{c^2 x\sin^2(d+ex)}{2} + \frac{c^2 x\cos^2(d+ex)}{2} + c^2 x - \frac{c^2\sin(d+ex)\cos(d+ex)}{2e} - \frac{2c\sqrt{b^2+c^2}\cos(d+ex)}{e} & \text{for } e \neq 0 \\ x\left(b\cos(d) + c\sin(d) + \sqrt{b^2+c^2}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Piecewise((b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*x + b
**2*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c*sin(d + e*x)**2/e + 2*b*sqrt(b**2
 + c**2)*sin(d + e*x)/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)**2
/2 + c**2*x - c**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*c*sqrt(b**2 + c**2)*
cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2))**2,
 True))
```

Giac [A]
time = 0.41, size = 92, normalized size = 0.79

$$\frac{3}{2}(b^2+c^2)x - \frac{bc\cos(2ex+2d)}{2e} - \frac{2\sqrt{b^2+c^2}c\cos(ex+d)}{e} + \frac{2\sqrt{b^2+c^2}b\sin(ex+d)}{e} + \frac{(b^2-c^2)\sin(2ex+2d)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac"
)
```

```
[Out] 3/2*(b^2 + c^2)*x - 1/2*b*c*cos(2*e*x + 2*d)/e - 2*sqrt(b^2 + c^2)*c*cos(e*
x + d)/e + 2*sqrt(b^2 + c^2)*b*sin(e*x + d)/e + 1/4*(b^2 - c^2)*sin(2*e*x +
 2*d)/e
```

Mupad [B]
time = 3.08, size = 100, normalized size = 0.86

$$\frac{b^2\sin\left(2\,d+2\,e\,x\right)-c^2\sin\left(2\,d+2\,e\,x\right)+16\,c\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\sqrt{b^2+c^2}+8\,b\sin\left(d+e\,x\right)\sqrt{b^2+c^2}+4\,b\,c\sin\left(d+e\,x\right)^2+6\,b^2\,e\,x+6\,c^2\,e\,x}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

```
[Out] (b^2*sin(2*d + 2*e*x) - c^2*sin(2*d + 2*e*x) + 16*c*sin(d/2 + (e*x)/2)^2*(b
^2 + c^2)^(1/2) + 8*b*sin(d + e*x)*(b^2 + c^2)^(1/2) + 4*b*c*sin(d + e*x)^2
 + 6*b^2*e*x + 6*c^2*e*x)/(4*e)
```

3.358 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx$

Optimal. Leaf size=37

$$\sqrt{b^2+c^2}\,x-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}$$

```
[Out] -c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2717, 2718}

$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx &= \sqrt{b^2+c^2}\,x+b\int \cos(d+ex)\,dx+c\int \sin(d+ex)\,dx\\ &= \sqrt{b^2+c^2}\,x-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.97

$$\frac{\sqrt{b^2+c^2}\,ex-c\cos(d+ex)+b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
[Out] (Sqrt[b^2 + c^2]*e*x - c*Cos[d + e*x] + b*Sin[d + e*x])/e
```

Maple [A]
time = 0.14, size = 36, normalized size = 0.97

method	result	size
default	$-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}+x\sqrt{b^2+c^2}$	36
derivativedivides	$\frac{(ex+d)\sqrt{b^2+c^2}+b\sin(ex+d)-c\cos(ex+d)}{e}$	38
norman	$\frac{x\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{2c}{e}+\frac{2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

Maxima [A]
time = 0.25, size = 35, normalized size = 0.95

$$-c\cos(xe+d)\,e^{(-1)}+be^{(-1)}\sin(xe+d)+\sqrt{b^2+c^2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="maxima")
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + sqrt(b^2 + c^2)*x
```

Fricas [A]
time = 0.88, size = 36, normalized size = 0.97

$$\left(\sqrt{b^2+c^2}\,xe-c\cos(xe+d)+b\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="fricas")
[Out] (sqrt(b^2 + c^2)*x*e - c*cos(x*e + d) + b*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.06, size = 42, normalized size = 1.14

$$b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)+x\sqrt{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2),x)
```

```
[Out] b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewise((-c
os(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + x*sqrt(b**2 + c**2)
```

Giac [A]
time = 0.40, size = 35, normalized size = 0.95

$$\sqrt{b^2+c^2}\,x-\frac{c\cos\left(ex+d\right)}{e}+\frac{b\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2 + c^2)*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

Mupad [B]
time = 2.67, size = 48, normalized size = 1.30

$$x\sqrt{b^2+c^2}-\frac{2\,c-2\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2),x)
```

```
[Out] x*(b^2 + c^2)^(1/2) - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)
^2 + 1))
```

3.359 $\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=49

$$-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] (-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{3193\}$

$$-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] -((c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 1.00

$$\frac{-c+\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-c + Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Maple [A]
time = 0.33, size = 50, normalized size = 1.02

method	result
derivativedivides	$-\frac{2\left(\sqrt{b^2+c^2}+b\right)}{e\,c^2\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{\sqrt{b^2+c^2}}{c}+\frac{b}{c}\right)}$
default	$-\frac{2\left(\sqrt{b^2+c^2}+b\right)}{e\,c^2\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{\sqrt{b^2+c^2}}{c}+\frac{b}{c}\right)}$
risch	$\frac{2ib}{\left(i\sqrt{b^2+c^2}\,c+b^2 e^{i(ex+d)}+c^2 e^{i(ex+d)}+\sqrt{b^2+c^2}\,b\right)e}-\frac{2c}{\left(i\sqrt{b^2+c^2}\,c+b^2 e^{i(ex+d)}+c^2 e^{i(ex+d)}+\sqrt{b^2+c\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/e*((b^2+c^2)^(1/2)+b)/c^2/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [A]
time = 0.26, size = 41, normalized size = 0.84

$$-\frac{2\,e^{(-1)}}{c-\frac{\left(b-\sqrt{b^2+c^2}\right)\sin(xe+d)}{\cos(xe+d)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="maxim
a")
```

```
[Out] -2*e^(-1)/(c - (b - sqrt(b^2 + c^2))*sin(x*e + d)/(cos(x*e + d) + 1))
```

Fricas [A]
time = 0.76, size = 81, normalized size = 1.65

$$-\frac{b^2+c^2-\sqrt{b^2+c^2}\,(b\cos(xe+d)+c\sin(xe+d))}{(b^2c+c^3)\cos(xe+d)\,e-(b^3+bc^2)e\sin(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="frica
s")
```

```
[Out] -(b^2 + c^2 - sqrt(b^2 + c^2)*(b*cos(x*e + d) + c*sin(x*e + d)))/((b^2*c +
c^3)*cos(x*e + d)*e - (b^3 + b*c^2)*e*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 43, normalized size = 0.88

$$-\frac{2\left(b+\sqrt{b^2+c^2}\right)}{\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="giac"
)
```

```
[Out] -2*(b + sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))*c*
e)
```

Mupad [B]
time = 2.82, size = 38, normalized size = 0.78

$$\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left(b+\sqrt{b^2+c^2}+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2)),x)
```

```
[Out] (2*tan(d/2 + (e*x)/2))/(e*(b + (b^2 + c^2)^(1/2) + c*tan(d/2 + (e*x)/2)))
```

3.360 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{-c\cos(d+ex)+b\sin(d+ex)}{3\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{3c\sqrt{b^2+c^2}\,e(c\cos(d+ex)-b\sin(d+ex))}$$

```
[Out] 1/3*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^2+1/3*(-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)
-b*sin(e*x+d))/(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3195, 3193}

$$-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\,\sin(d+ex)}{3ce\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/3*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
+ b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/
(3*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{3\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

Mathematica [A]
time = 0.19, size = 98, normalized size = 0.76

$$\frac{-2c\sqrt{b^2+c^2}+2bc\cos^3(d+ex)+2c^2\sin(d+ex)+c^2\cos^2(d+ex)\sin(d+ex)+b^2\sin^3(d+ex)}{3ce(c\cos(d+ex)-b\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
[Out] (-2*c*Sqrt[b^2 + c^2] + 2*b*c*Cos[d + e*x]^3 + 2*c^2*Sin[d + e*x] + c^2*Cos
[d + e*x]^2*Sin[d + e*x] + b^2*Sin[d + e*x]^3)/(3*c*e*(c*Cos[d + e*x] - b*S
in[d + e*x])^3)
```

Maple [A]
time = 0.54, size = 233, normalized size = 1.81

method	result
risch	$\frac{2\left(i\sqrt{b^2+c^2}\,c+3b^2 e^{i(ex+d)}+3c^2 e^{i(ex+d)}+\sqrt{b^2+c^2}\,b\right)(ib^2-ic^2-2cb)}{3\left(i\sqrt{b^2+c^2}\,c+b^2 e^{i(ex+d)}+c^2 e^{i(ex+d)}+\sqrt{b^2+c^2}\,b\right)^3 e}$
derivativedivides	$\frac{2\left(\sqrt{b^2+c^2}+b\right)\left(-\frac{\left(\sqrt{b^2+c^2}+b\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^2}-\frac{\left(2b^2+c^2+2\sqrt{b^2+c^2}\,b\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c^3}-\frac{2\left(2\sqrt{b^2+c^2}\right.}{}\right.}{e\,c^2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{2\sqrt{b^2+c^2}\,\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2b\sqrt{b^2+c^2}}{c^2}+\frac{2b^2}{c^2}+1\right)\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right.}$
default	$\frac{2\left(\sqrt{b^2+c^2}+b\right)\left(-\frac{\left(\sqrt{b^2+c^2}+b\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^2}-\frac{\left(2b^2+c^2+2\sqrt{b^2+c^2}\,b\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c^3}-\frac{2\left(2\sqrt{b^2+c^2}\right.}{}\right.}{e\,c^2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{2\sqrt{b^2+c^2}\,\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+\frac{2b\sqrt{b^2+c^2}}{c^2}+\frac{2b^2}{c^2}+1\right)\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x,method=_RETURNVERBOSE
)
```

```
[Out] 2/e*((b^2+c^2)^(1/2)+b)/c^2*(-((b^2+c^2)^(1/2)+b)/c^2*tan(1/2*d+1/2*e*x)^2-
1/c^3*(2*b^2+c^2+2*(b^2+c^2)^(1/2)*b)*tan(1/2*d+1/2*e*x)-2/3*(2*(b^2+c^2)^(
1/2)*b^2+(b^2+c^2)^(1/2)*c^2+2*b^3+2*c^2*b)/c^4)/(tan(1/2*d+1/2*e*x)^2+2/c*
(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2/c*b*tan(1/2*d+1/2*e*x)+2/c^2*b*(b^2+c^
2)^(1/2)+2/c^2*b^2+1)/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.67, size = 204, normalized size = 1.58

$$-\frac{3\,b^3\cos\left(xe+d\right)-\left(b^3-3\,bc^2\right)\cos\left(xe+d\right)^3+\left(3\,b^2c+2\,c^3-\left(3\,b^2c-c^3\right)\cos\left(xe+d\right)^2\right)\sin\left(xe+d\right)-2\left(b^2+c^2\right)^{\frac{3}{2}}}{3\left(\left(3\,b^4c+2\,b^2c^3-c^5\right)\cos\left(xe+d\right)^3e-3\left(b^4c+b^2c^3\right)\cos\left(xe+d\right)e-\left(\left(b^5-2\,b^3c^2-3\,bc^4\right)\cos\left(xe+d\right)^2e-\left(b^5+b^3c^2\right)e\right)\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fri
cas")
```

```
[Out] -1/3*(3*b^3*cos(x*e + d) - (b^3 - 3*b*c^2)*cos(x*e + d)^3 + (3*b^2*c + 2*c^
3 - (3*b^2*c - c^3)*cos(x*e + d)^2)*sin(x*e + d) - 2*(b^2 + c^2)^(3/2))/((3
*b^4*c + 2*b^2*c^3 - c^5)*cos(x*e + d)^3*e - 3*(b^4*c + b^2*c^3)*cos(x*e +
d)*e - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*cos(x*e + d)^2*e - (b^5 + b^3*c^2)*e)*s
in(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 158, normalized size = 1.22

$$-\frac{2\left(8\,b^4+10\,b^2c^2+2\,c^4+3\left(2\,b^2c^2+c^4+2\,\sqrt{b^2+c^2}\,bc^2\right)\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+3\left(4\,b^3c+3\,bc^3+\left(4\,b^2c+c^3\right)\sqrt{b^2+c^2}\right)\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\left(4\,b^3+3\,bc^2\right)\sqrt{b^2+c^2}\right)}{3\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)^3c^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="gia
c")
```

```
[Out] -2/3*(8*b^4 + 10*b^2*c^2 + 2*c^4 + 3*(2*b^2*c^2 + c^4 + 2*sqrt(b^2 + c^2)*b
*c^2)*tan(1/2*e*x + 1/2*d)^2 + 3*(4*b^3*c + 3*b*c^3 + (4*b^2*c + c^3)*sqrt(
b^2 + c^2))*tan(1/2*e*x + 1/2*d) + 2*(4*b^3 + 3*b*c^2)*sqrt(b^2 + c^2))/((c
*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))^3*c^3*e)
```

Mupad [B]
time = 3.67, size = 274, normalized size = 2.12

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(\frac{4\,b^2+2\,c^2}{c^4}+\frac{4\,b\,\sqrt{b^2+c^2}}{c^4}\right)+\frac{\frac{16\,b^4}{3}+\frac{20\,b^2\,c^2}{3}+\frac{4\,c^4}{3}}{c^6}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{8\,b^3+6\,b\,c^2}{c^5}+\frac{\left(8\,b^2+2\,c^2\right)\,\sqrt{b^2+c^2}}{c^5}\right)+\frac{\left(\frac{16\,b^3}{3}+4\,b\,c^2\right)\,\sqrt{b^2+c^2}}{c^6}}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{6\,b^2+3\,c^2}{c^2}+\frac{6\,b\,\sqrt{b^2+c^2}}{c^2}\right)+\frac{4\,b^3+3\,b\,c^2}{c^3}+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(\frac{3\,\sqrt{b^2+c^2}}{c}+\frac{3\,b}{c}\right)+\frac{\left(4\,b^2+c^2\right)\,\sqrt{b^2+c^2}}{c^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^2*((4*b^2 + 2*c^2)/c^4 + (4*b*(b^2 + c^2)^(1/2))/c^4)
+ ((16*b^4)/3 + (4*c^4)/3 + (20*b^2*c^2)/3)/c^6 + tan(d/2 + (e*x)/2)*((6*b*
c^2 + 8*b^3)/c^5 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^5) + ((4*b*c^2 + (
16*b^3)/3)*(b^2 + c^2)^(1/2))/c^6)/(e*(tan(d/2 + (e*x)/2)*((6*b^2 + 3*c^2)/
c^2 + (6*b*(b^2 + c^2)^(1/2))/c^2) + (3*b*c^2 + 4*b^3)/c^3 + tan(d/2 + (e*x
)/2)^3 + tan(d/2 + (e*x)/2)^2*((3*(b^2 + c^2)^(1/2))/c + (3*b)/c) + ((4*b^2
 + c^2)*(b^2 + c^2)^(1/2))/c^3))
```

3.361 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

Optimal. Leaf size=191

$$\frac{-c\cos(d+ex)+b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15\left(b^2+c^2\right)e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(\right.}$$

```
[Out] 1/5*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^3-2/15*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e
*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/15*(c-sin(e*x+d)*(b^2+c^2)^(1/2))/c
/(b^2+c^2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A]
time = 0.09, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3195, 3193}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{15e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-\frac{2\left(c-\sqrt{b^2+c^2}\sin(d+ex)\right)}{15ce\left(b^2+c^2\right)\left(c\cos(d+ex)-b\sin(d+ex)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/5*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
+ b*Cos[d + e*x] + c*Sin[d + e*x])^3) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]
))/(15*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2)
 - (2*(c - Sqrt[b^2 + c^2]*Sin[d + e*x]))/(15*c*(b^2 + c^2)*e*(c*Cos[d + e*
x] - b*Sin[d + e*x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{5\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 420 vs. 2(191) = 382.
time = 1.87, size = 420, normalized size = 2.20

$$\frac{-76b^4c-152b^2c^3-76c^5+90bc(b^2+c^2)^{3/2}\cos(d+ex)+20c(-b^4+c^4)\cos(2(d+ex))+10b^3c\sqrt{b^2+c^2}\cos(3(d+ex))+10bc^3\sqrt{b^2+c^2}\cos(3(d+ex))-4b^3c\sqrt{b^2+c^2}\cos(5(d+ex))+4bc^3\sqrt{b^2+c^2}\cos(5(d+ex))+10b^4\sqrt{b^2+c^2}\sin(d+ex)+110b^2c^2\sqrt{b^2+c^2}\sin(d+ex)+100c^4\sqrt{b^2+c^2}\sin(d+ex)-40b^3c^2\sin(2(d+ex))-40bc^4\sin(2(d+ex))-5b^4\sqrt{b^2+c^2}\sin(3(d+ex))+5c^4\sqrt{b^2+c^2}\sin(3(d+ex))+b^4\sqrt{b^2+c^2}\sin(5(d+ex))-6b^2c^2\sqrt{b^2+c^2}\sin(5(d+ex))+c^4\sqrt{b^2+c^2}\sin(5(d+ex))}{120c(b^2+c^2)e(c\cos(d+ex)-b\sin(d+ex))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] (-76*b^4*c - 152*b^2*c^3 - 76*c^5 + 90*b*c*(b^2 + c^2)^(3/2)*Cos[d + e*x] +
 20*c*(-b^4 + c^4)*Cos[2*(d + e*x)] + 10*b^3*c*Sqrt[b^2 + c^2]*Cos[3*(d + e
*x)] + 10*b*c^3*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 4*b^3*c*Sqrt[b^2 + c^2]*
Cos[5*(d + e*x)] + 4*b*c^3*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 10*b^4*Sqrt[b
^2 + c^2]*Sin[d + e*x] + 110*b^2*c^2*Sqrt[b^2 + c^2]*Sin[d + e*x] + 100*c^4
*Sqrt[b^2 + c^2]*Sin[d + e*x] - 40*b^3*c^2*Sin[2*(d + e*x)] - 40*b*c^4*Sin[
2*(d + e*x)] - 5*b^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 5*c^4*Sqrt[b^2 + c^
2]*Sin[3*(d + e*x)] + b^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] - 6*b^2*c^2*Sqrt
[b^2 + c^2]*Sin[5*(d + e*x)] + c^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)])/(120*c
*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^5)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(177) = 354.
time = 0.90, size = 496, normalized size = 2.60

method	result
risch	$\frac{4\left(5i\sqrt{b^2+c^2}\,b^2c\,e^{i(ex+d)}+5i\sqrt{b^2+c^2}\,c^3e^{i(ex+d)}+10b^4e^{2i(ex+d)}+20b^2c^2e^{2i(ex+d)}+10c^4e^{2i(ex+d)}+2ib^3c+2i\right.}{15e\left(i\sqrt{b^2+c^2}\,c+b^2e^{i(ex+d)}+c^2e^{i(ex+d)}\right.}$

derivativedivides	$-\frac{2\left(4\sqrt{b^{2}+c^{2}}\, b^{2}+\sqrt{b^{2}+c^{2}}\, c^{2}+4b^{3}+3c^{2}b\right)\left(\tan^{4}\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^{2}}-\frac{4\left(8\sqrt{b^{2}+c^{2}}\, b^{3}+4\sqrt{b^{2}+c^{2}}\, b\, c^{2}+8b^{4}+8b^{2}c^{2}+c\right.}{c^{3}}$
default	$-\frac{2\left(4\sqrt{b^{2}+c^{2}}\, b^{2}+\sqrt{b^{2}+c^{2}}\, c^{2}+4b^{3}+3c^{2}b\right)\left(\tan^{4}\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^{2}}-\frac{4\left(8\sqrt{b^{2}+c^{2}}\, b^{3}+4\sqrt{b^{2}+c^{2}}\, b\, c^{2}+8b^{4}+8b^{2}c^{2}+c\right.}{c^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x,method=_RETURNVERBOSE
)
```

```
[Out] 2/e/c^4*(-(4*(b^2+c^2)^(1/2)*b^2+(b^2+c^2)^(1/2)*c^2+4*b^3+3*c^2*b)/c^2*tan
(1/2*d+1/2*e*x)^4-2*(8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^(1/2)*b*c^2+8*b^4+8*
b^2*c^2+c^4)/c^3*tan(1/2*d+1/2*e*x)^3-4/3*(24*(b^2+c^2)^(1/2)*b^4+20*(b^2+c
^2)^(1/2)*b^2*c^2+2*(b^2+c^2)^(1/2)*c^4+24*b^5+32*b^3*c^2+9*c^4*b)/c^4*tan(
1/2*d+1/2*e*x)^2-2/3*(48*b^6+76*b^4*c^2+31*b^2*c^4+2*c^6+48*(b^2+c^2)^(1/2)
*b^5+52*(b^2+c^2)^(1/2)*b^3*c^2+11*(b^2+c^2)^(1/2)*b*c^4)/c^5*tan(1/2*d+1/2
*e*x)-1/15/c^6*(192*(b^2+c^2)^(1/2)*b^6+256*(b^2+c^2)^(1/2)*b^4*c^2+96*b^2*
(b^2+c^2)^(1/2)*c^4+7*c^6*(b^2+c^2)^(1/2)+192*b^7+352*b^5*c^2+200*b^3*c^4+3
5*c^6*b))/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan(1/2*d+1/2*e*x)+2/c*
b*tan(1/2*d+1/2*e*x)+2/c^2*b*(b^2+c^2)^(1/2)+2/c^2*b^2+1)^2/(tan(1/2*d+1/2*
e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(186) = 372$.
time = 1.36, size = 511, normalized size = 2.68

$$-\frac{7\,b^{6}+26\,b^{4}c^{2}+31\,b^{2}c^{4}+12\,c^{6}+5\,(b^{6}+b^{4}c^{2}-b^{2}c^{4}-c^{6})\cos(xe+d)^{2}+10\,(b^{5}c+2\,b^{3}c^{3}+bc^{5})\cos(xe+d)\sin(xe+d)-\left(2\,(b^{5}-10\,b^{3}c^{2}+5\,bc^{4})\cos(xe+d)^{5}-5\,(b^{5}-6\,b^{3}c^{2}+bc^{4})\cos(xe+d)^{3}+5\,(3\,b^{5}+3\,b^{3}c^{2}+2\,bc^{4})\cos(xe+d)+\left(15\,b^{4}c+25\,b^{2}c^{3}+12\,c^{5}+2\,(5\,b^{4}c-10\,b^{2}c^{3}+c^{5})\cos(xe+d)^{4}-(15\,b^{4}c-10\,b^{2}c^{3}-c^{5})\cos(xe+d)^{2}\right)\sin(xe+d)\right)\sqrt{b^{2}+c^{2}}}{15\left((5\,b^{8}c-14\,b^{4}c^{5}-8\,b^{2}c^{7}+c^{9})\cos(xe+d)^{5}e-10\,(b^{8}c+b^{6}c^{3}-b^{4}c^{5}-b^{2}c^{7})\cos(xe+d)^{3}e+5\,(b^{8}c+2\,b^{6}c^{3}+b^{4}c^{5})\cos(xe+d)e-\left((b^{9}-8\,b^{7}c^{2}-14\,b^{5}c^{4}+5\,bc^{8})\cos(xe+d)^{4}e-2\,(b^{9}-3\,b^{7}c^{2}-9\,b^{5}c^{4}-5\,b^{3}c^{6})\cos(xe+d)^{2}e+(b^{9}+2\,b^{7}c^{2}+b^{5}c^{4})e\right)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fri
cas")
```

```
[Out] -1/15*(7*b^6 + 26*b^4*c^2 + 31*b^2*c^4 + 12*c^6 + 5*(b^6 + b^4*c^2 - b^2*c^
4 - c^6)*cos(x*e + d)^2 + 10*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(x*e + d)*sin(x
*e + d) - (2*(b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(x*e + d)^5 - 5*(b^5 - 6*b^3*c
^2 + b*c^4)*cos(x*e + d)^3 + 5*(3*b^5 + 3*b^3*c^2 + 2*b*c^4)*cos(x*e + d) +
 (15*b^4*c + 25*b^2*c^3 + 12*c^5 + 2*(5*b^4*c - 10*b^2*c^3 + c^5)*cos(x*e +
 d)^4 - (15*b^4*c - 10*b^2*c^3 - c^5)*cos(x*e + d)^2)*sin(x*e + d))*sqrt(b^
2 + c^2))/((5*b^8*c - 14*b^4*c^5 - 8*b^2*c^7 + c^9)*cos(x*e + d)^5*e - 10*(
b^8*c + b^6*c^3 - b^4*c^5 - b^2*c^7)*cos(x*e + d)^3*e + 5*(b^8*c + 2*b^6*c^
3 + b^4*c^5)*cos(x*e + d)*e - ((b^9 - 8*b^7*c^2 - 14*b^5*c^4 + 5*b*c^8)*cos
(x*e + d)^4*e - 2*(b^9 - 3*b^7*c^2 - 9*b^5*c^4 - 5*b^3*c^6)*cos(x*e + d)^2*
e + (b^9 + 2*b^7*c^2 + b^5*c^4)*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 342, normalized size = 1.79

$$-\frac{2\left(192\,b^{7}+352\,b^{5}c^{2}+200\,b^{3}c^{4}+35\,bc^{6}+15\left(4\,b^{3}c^{4}+3\,bc^{6}+\left(4\,b^{2}c^{4}+c^{6}\right)\sqrt{b^{2}+c^{2}}\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^{4}+30\left(8\,b^{4}c^{3}+8\,b^{2}c^{5}+c^{7}+4\left(2\,b^{3}c^{3}+bc^{5}\right)\sqrt{b^{2}+c^{2}}\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^{3}+20\left(24\,b^{5}c^{2}+32\,b^{3}c^{4}+9\,bc^{6}+2\left(12\,b^{4}c^{2}+10\,b^{2}c^{4}+c^{6}\right)\sqrt{b^{2}+c^{2}}\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^{2}+10\left(48\,b^{6}c+76\,b^{4}c^{3}+31\,b^{2}c^{5}+2\,c^{7}+\left(48\,b^{5}c+52\,b^{3}c^{3}+11\,bc^{5}\right)\sqrt{b^{2}+c^{2}}\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+\left(192\,b^{6}+256\,b^{4}c^{2}+96\,b^{2}c^{4}+7\,c^{6}\right)\sqrt{b^{2}+c^{2}}\right)}{15\left(c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b+\sqrt{b^{2}+c^{2}}\right)^{5}c^{5}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="gia
c")
```

```
[Out] -2/15*(192*b^7 + 352*b^5*c^2 + 200*b^3*c^4 + 35*b*c^6 + 15*(4*b^3*c^4 + 3*b
*c^6 + (4*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^4 + 30*(8*b^
4*c^3 + 8*b^2*c^5 + c^7 + 4*(2*b^3*c^3 + b*c^5)*sqrt(b^2 + c^2))*tan(1/2*e*
x + 1/2*d)^3 + 20*(24*b^5*c^2 + 32*b^3*c^4 + 9*b*c^6 + 2*(12*b^4*c^2 + 10*b
^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^2 + 10*(48*b^6*c + 76*b
^4*c^3 + 31*b^2*c^5 + 2*c^7 + (48*b^5*c + 52*b^3*c^3 + 11*b*c^5)*sqrt(b^2 +
 c^2))*tan(1/2*e*x + 1/2*d) + (192*b^6 + 256*b^4*c^2 + 96*b^2*c^4 + 7*c^6)*
sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))^5*c^5*e)
```

Mupad [**B**]
time = 8.12, size = 592, normalized size = 3.10

$$-\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(\frac{32\,b^4+32\,b^2\,c^2+4\,c^4}{c^7}+\frac{\left(32\,b^3+16\,b\,c^2\right)\,\sqrt{b^2+c^2}}{c^7}\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(\frac{8\,b^3+6\,b\,c^2}{c^6}+\frac{\left(8\,b^2+2\,c^2\right)\,\sqrt{b^2+c^2}}{c^6}\right)+\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{64\,b^6+\frac{304\,b^4\,c^2}{3}+\frac{124\,b^2\,c^4}{3}+\frac{8\,c^6}{3}}{c^9}+\frac{\sqrt{b^2+c^2}\,\left(64\,b^5+\frac{208\,b^3\,c^2}{3}+\frac{44\,b\,c^4}{3}\right)}{c^9}\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(\frac{64\,b^5+\frac{256\,b^3\,c^2}{3}+24\,b\,c^4}{c^8}+\frac{\sqrt{b^2+c^2}\,\left(64\,b^4+\frac{160\,b^2\,c^2}{3}+\frac{16\,c^4}{3}\right)}{c^8}\right)+\frac{\frac{128\,b^7}{5}+\frac{704\,b^5\,c^2}{15}+\frac{80\,b^3\,c^4}{3}+\frac{14\,b\,c^6}{3}}{c^{10}}+\frac{\sqrt{b^2+c^2}\,\left(\frac{128\,b^6}{5}+\frac{512\,b^4\,c^2}{15}+\frac{64\,b^2\,c^4}{5}+\frac{14\,c^6}{15}\right)}{c^{10}}}{e\,\left(\frac{16\,b^5+20\,b^3\,c^2+5\,b\,c^4}{c^5}+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(\frac{40\,b^3+30\,b\,c^2}{c^3}+\frac{\left(40\,b^2+10\,c^2\right)\,\sqrt{b^2+c^2}}{c^3}\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(\frac{20\,b^2+10\,c^2}{c^2}+\frac{20\,b\,\sqrt{b^2+c^2}}{c^2}\right)+\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{40\,b^4+40\,b^2\,c^2+5\,c^4}{c^4}+\frac{\left(40\,b^3+20\,b\,c^2\right)\,\sqrt{b^2+c^2}}{c^4}\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(\frac{5\,\sqrt{b^2+c^2}}{c}+\frac{5\,b}{c}\right)+\frac{\sqrt{b^2+c^2}\,\left(16\,b^4+12\,b^2\,c^2+c^4\right)}{c^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^3*((32*b^4 + 4*c^4 + 32*b^2*c^2)/c^7 + ((16*b*c^2 + 32
*b^3)*(b^2 + c^2)^(1/2))/c^7) + tan(d/2 + (e*x)/2)^4*((6*b*c^2 + 8*b^3)/c^6
 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^6) + tan(d/2 + (e*x)/2)*((64*b^6 +
 (8*c^6)/3 + (124*b^2*c^4)/3 + (304*b^4*c^2)/3)/c^9 + ((b^2 + c^2)^(1/2)*((
44*b*c^4)/3 + 64*b^5 + (208*b^3*c^2)/3))/c^9) + tan(d/2 + (e*x)/2)^2*((24*b
*c^4 + 64*b^5 + (256*b^3*c^2)/3)/c^8 + ((b^2 + c^2)^(1/2)*(64*b^4 + (16*c^4
)/3 + (160*b^2*c^2)/3))/c^8) + ((14*b*c^6)/3 + (128*b^7)/5 + (80*b^3*c^4)/3
 + (704*b^5*c^2)/15)/c^10 + ((b^2 + c^2)^(1/2)*((128*b^6)/5 + (14*c^6)/15 +
 (64*b^2*c^4)/5 + (512*b^4*c^2)/15))/c^10)/(e*((5*b*c^4 + 16*b^5 + 20*b^3*c
^2)/c^5 + tan(d/2 + (e*x)/2)^2*((30*b*c^2 + 40*b^3)/c^3 + ((40*b^2 + 10*c^2
)*(b^2 + c^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^5 + tan(d/2 + (e*x)/2)^3*((2
0*b^2 + 10*c^2)/c^2 + (20*b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 + (e*x)/2)*((
40*b^4 + 5*c^4 + 40*b^2*c^2)/c^4 + ((20*b*c^2 + 40*b^3)*(b^2 + c^2)^(1/2))/
c^4) + tan(d/2 + (e*x)/2)^4*((5*(b^2 + c^2)^(1/2))/c + (5*b)/c) + ((b^2 + c
^2)^(1/2)*(16*b^4 + c^4 + 12*b^2*c^2))/c^5))
```

3.362 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

Optimal. Leaf size=259

$$\frac{-c\cos(d+ex)+b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35\left(b^2+c^2\right)e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

```
[Out] 1/7*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^4-3/35*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e
*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3-2/35*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2
+c^2)^(3/2)/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/35*(c-sin(e*x
+d)*(b^2+c^2)^(1/2))/c/(b^2+c^2)^(3/2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

Rubi [A]
time = 0.13, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3195, 3193}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)^{3/2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{35e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}-\frac{2\left(c-\sqrt{b^2+c^2}\sin(d+ex)\right)}{35ce\left(b^2+c^2\right)^{3/2}(c\cos(d+ex)-b\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] -1/7*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
+ b*Cos[d + e*x] + c*Sin[d + e*x])^4) - (3*(c*Cos[d + e*x] - b*Sin[d + e*x]
))/(35*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3)
 - (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*(b^2 + c^2)^(3/2)*e*(Sqrt[b^2
+ c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(c - Sqrt[b^2 + c^2]*Sin[
d + e*x]))/(35*c*(b^2 + c^2)^(3/2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

Rule 3193

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{7\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+\right.}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 533 vs. $2(259) = 518$.
time = 1.36, size = 533, normalized size = 2.06

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] (832*b^4*c*Sqrt[b^2 + c^2] + 1664*b^2*c^3*Sqrt[b^2 + c^2] + 832*c^5*Sqrt[b^
2 + c^2] - 1190*b*c*(b^2 + c^2)^2*Cos[d + e*x] + 448*c*Sqrt[b^2 + c^2]*(b^4
 - c^4)*Cos[2*(d + e*x)] - 112*b^5*c*Cos[3*(d + e*x)] + 56*b^3*c^3*Cos[3*(d
 + e*x)] + 168*b*c^5*Cos[3*(d + e*x)] + 28*b^5*c*Cos[5*(d + e*x)] - 28*b*c^
5*Cos[5*(d + e*x)] - 6*b^5*c*Cos[7*(d + e*x)] + 20*b^3*c^3*Cos[7*(d + e*x)]
 - 6*b*c^5*Cos[7*(d + e*x)] - 35*b^6*Sin[d + e*x] - 1295*b^4*c^2*Sin[d + e*
x] - 2485*b^2*c^4*Sin[d + e*x] - 1225*c^6*Sin[d + e*x] + 896*b^3*c^2*Sqrt[b
^2 + c^2]*Sin[2*(d + e*x)] + 896*b*c^4*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 2
1*b^6*Sin[3*(d + e*x)] - 189*b^4*c^2*Sin[3*(d + e*x)] - 161*b^2*c^4*Sin[3*(
d + e*x)] + 49*c^6*Sin[3*(d + e*x)] - 7*b^6*Sin[5*(d + e*x)] + 35*b^4*c^2*S
in[5*(d + e*x)] + 35*b^2*c^4*Sin[5*(d + e*x)] - 7*c^6*Sin[5*(d + e*x)] + b^
6*Sin[7*(d + e*x)] - 15*b^4*c^2*Sin[7*(d + e*x)] + 15*b^2*c^4*Sin[7*(d + e*
x)] - c^6*Sin[7*(d + e*x)])/(1120*c*(b^2 + c^2)*e*(-(c*Cos[d + e*x]) + b*Si
n[d + e*x])^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(237) = 474$.
time = 1.57, size = 823, normalized size = 3.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x,method=_RETURNVERBOSE
)
```

```
[Out] -2/e/c^6*((8*(b^2+c^2)^(1/2)*b^3+4*(b^2+c^2)^(1/2)*b*c^2+8*b^4+8*b^2*c^2+c^
4)/c^2*tan(1/2*d+1/2*e*x)^6+3*(16*(b^2+c^2)^(1/2)*b^4+12*(b^2+c^2)^(1/2)*b^
2*c^2+(b^2+c^2)^(1/2)*c^4+16*b^5+20*b^3*c^2+5*c^4*b)/c^3*tan(1/2*d+1/2*e*x)
^5+2*(80*(b^2+c^2)^(1/2)*b^5+84*(b^2+c^2)^(1/2)*b^3*c^2+17*(b^2+c^2)^(1/2)*
b*c^4+80*b^6+124*b^4*c^2+49*b^2*c^4+3*c^6)/c^4*tan(1/2*d+1/2*e*x)^4+2*(160*
b^7+288*b^5*c^2+150*b^3*c^4+20*c^6*b+160*(b^2+c^2)^(1/2)*b^6+208*(b^2+c^2)^
(1/2)*b^4*c^2+66*b^2*(b^2+c^2)^(1/2)*c^4+3*c^6*(b^2+c^2)^(1/2))/c^5*tan(1/2
*d+1/2*e*x)^3+3/5*(640*b^7*(b^2+c^2)^(1/2)+992*(b^2+c^2)^(1/2)*b^5*c^2+440*
b^3*(b^2+c^2)^(1/2)*c^4+50*b*c^6*(b^2+c^2)^(1/2)+640*b^8+1312*b^6*c^2+856*b
^4*c^4+186*b^2*c^6+7*c^8)/c^6*tan(1/2*d+1/2*e*x)^2+1/5*(1280*b^9+2944*b^7*c
^2+2288*b^5*c^4+676*b^3*c^6+57*b*c^8+1280*(b^2+c^2)^(1/2)*b^8+2304*(b^2+c^2
)^(1/2)*b^6*c^2+1296*(b^2+c^2)^(1/2)*b^4*c^4+236*(b^2+c^2)^(1/2)*b^2*c^6+7*
(b^2+c^2)^(1/2)*c^8)/c^7*tan(1/2*d+1/2*e*x)+4/35*(640*(b^2+c^2)^(1/2)*b^9+1
312*(b^2+c^2)^(1/2)*b^7*c^2+896*(b^2+c^2)^(1/2)*b^5*c^4+238*(b^2+c^2)^(1/2)
*b^3*c^6+21*(b^2+c^2)^(1/2)*b*c^8+640*b^10+1632*b^8*c^2+1472*b^6*c^4+562*b^
4*c^6+85*b^2*c^8+3*c^10)/c^8)/(tan(1/2*d+1/2*e*x)^2+2/c*(b^2+c^2)^(1/2)*tan
(1/2*d+1/2*e*x)+2/c*b*tan(1/2*d+1/2*e*x)+2/c^2*b*(b^2+c^2)^(1/2)+2/c^2*b^2+
1)^3/(tan(1/2*d+1/2*e*x)+1/c*(b^2+c^2)^(1/2)+b/c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="max
ima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(249) = 498$.
time = 2.02, size = 766, normalized size = 2.96

$$\frac{2\left(b^{7}-21 b^{5} c^{2}+35 b^{3} c^{4}-7 b c^{6}\right) \cos (x e+d)^{7}-7\left(b^{7}-15 b^{5} c^{2}+15 b^{3} c^{4}-b c^{6}\right) \cos (x e+d)^{5}-14\left(5 b^{5} c^{2}-5 b^{3} c^{4}-2 b c^{6}\right) \cos (x e+d)^{3}-7\left(5 b^{7}+15 b^{5} c^{2}+20 b^{3} c^{4}+8 b c^{6}\right) \cos (x e+d)-\left(35 b^{6} c+105 b^{4} c^{3}+112 b^{2} c^{5}+40 c^{7}-2\left(7 b^{6} c-35 b^{4} c^{3}+21 b^{2} c^{5}-c^{7}\right) \cos (x e+d)^{6}+\left(35 b^{6} c-105 b^{4} c^{3}+21 b^{2} c^{5}+c^{7}\right) \cos (x e+d)^{4}+2\left(35 b^{4} c^{3}+7 b^{2} c^{5}-4 c^{7}\right) \cos (x e+d)^{2}\right) \sin (x e+d)+4\left(3 b^{6}+16 b^{4} c^{2}+23 b^{2} c^{4}+10 c^{6}+7\left(b^{6}+b^{4} c^{2}-b^{2} c^{4}-c^{6}\right) \cos (x e+d)^{2}+14\left(b^{5} c+2 b^{3} c^{3}+b c^{5}\right) \cos (x e+d) \sin (x e+d)\right) \sqrt{b^{2}+c^{2}}}{35\left(\left(7 b^{10} c-21 b^{8} c^{3}-42 b^{6} c^{5}+6 b^{4} c^{7}+19 b^{2} c^{9}-c^{11}\right) \cos (x e+d)^{7} e-7\left(3 b^{10} c-4 b^{8} c^{3}-14 b^{6} c^{5}-4 b^{4} c^{7}+3 b^{2} c^{9}\right) \cos (x e+d)^{5} e+7\left(3 b^{10} c+b^{8} c^{3}-7 b^{6} c^{5}-5 b^{4} c^{7}\right) \cos (x e+d)^{3} e-7\left(b^{10} c+2 b^{8} c^{3}+b^{6} c^{5}\right) \cos (x e+d) e-\left(\left(b^{11}-19 b^{9} c^{2}-6 b^{7} c^{4}+42 b^{5} c^{6}+21 b^{3} c^{8}-7 b c^{10}\right) \cos (x e+d)^{6} e-\left(3 b^{11}-36 b^{9} c^{2}-46 b^{7} c^{4}+28 b^{5} c^{6}+35 b^{3} c^{8}\right) \cos (x e+d)^{4} e+3\left(b^{11}-5 b^{9} c^{2}-13 b^{7} c^{4}-7 b^{5} c^{6}\right) \cos (x e+d)^{2} e-\left(b^{11}+2 b^{9} c^{2}+b^{7} c^{4}\right) e\right) \sin (x e+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fri
cas")
```

```
[Out] 1/35*(2*(b^7 - 21*b^5*c^2 + 35*b^3*c^4 - 7*b*c^6)*cos(x*e + d)^7 - 7*(b^7 -
 15*b^5*c^2 + 15*b^3*c^4 - b*c^6)*cos(x*e + d)^5 - 14*(5*b^5*c^2 - 5*b^3*c^
4 - 2*b*c^6)*cos(x*e + d)^3 - 7*(5*b^7 + 15*b^5*c^2 + 20*b^3*c^4 + 8*b*c^6)
*cos(x*e + d) - (35*b^6*c + 105*b^4*c^3 + 112*b^2*c^5 + 40*c^7 - 2*(7*b^6*c
 - 35*b^4*c^3 + 21*b^2*c^5 - c^7)*cos(x*e + d)^6 + (35*b^6*c - 105*b^4*c^3
+ 21*b^2*c^5 + c^7)*cos(x*e + d)^4 + 2*(35*b^4*c^3 + 7*b^2*c^5 - 4*c^7)*cos
(x*e + d)^2)*sin(x*e + d) + 4*(3*b^6 + 16*b^4*c^2 + 23*b^2*c^4 + 10*c^6 + 7
*(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x*e + d)^2 + 14*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(x*e + d)*sin(x*e + d))*sqrt(b^2 + c^2))/((7*b^10*c - 21*b^8*c^3 -
 42*b^6*c^5 + 6*b^4*c^7 + 19*b^2*c^9 - c^11)*cos(x*e + d)^7*e - 7*(3*b^10*c
 - 4*b^8*c^3 - 14*b^6*c^5 - 4*b^4*c^7 + 3*b^2*c^9)*cos(x*e + d)^5*e + 7*(3*
b^10*c + b^8*c^3 - 7*b^6*c^5 - 5*b^4*c^7)*cos(x*e + d)^3*e - 7*(b^10*c + 2*
b^8*c^3 + b^6*c^5)*cos(x*e + d)*e - ((b^11 - 19*b^9*c^2 - 6*b^7*c^4 + 42*b^
5*c^6 + 21*b^3*c^8 - 7*b*c^10)*cos(x*e + d)^6*e - (3*b^11 - 36*b^9*c^2 - 46
*b^7*c^4 + 28*b^5*c^6 + 35*b^3*c^8)*cos(x*e + d)^4*e + 3*(b^11 - 5*b^9*c^2
- 13*b^7*c^4 - 7*b^5*c^6)*cos(x*e + d)^2*e - (b^11 + 2*b^9*c^2 + b^7*c^4)*e
)*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(238) = 476.
time = 0.73, size = 593, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="gia
c")
```

```
[Out] -2/35*(2560*b^10 + 6528*b^8*c^2 + 5888*b^6*c^4 + 2248*b^4*c^6 + 340*b^2*c^8
 + 12*c^10 + 35*(8*b^4*c^6 + 8*b^2*c^8 + c^10 + 4*(2*b^3*c^6 + b*c^8)*sqrt(
b^2 + c^2))*tan(1/2*e*x + 1/2*d)^6 + 105*(16*b^5*c^5 + 20*b^3*c^7 + 5*b*c^9
 + (16*b^4*c^5 + 12*b^2*c^7 + c^9)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^5
+ 70*(80*b^6*c^4 + 124*b^4*c^6 + 49*b^2*c^8 + 3*c^10 + (80*b^5*c^4 + 84*b^3
```

```
*c^6 + 17*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^4 + 70*(160*b^7*c^3
+ 288*b^5*c^5 + 150*b^3*c^7 + 20*b*c^9 + (160*b^6*c^3 + 208*b^4*c^5 + 66*b^
2*c^7 + 3*c^9)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^3 + 21*(640*b^8*c^2 +
1312*b^6*c^4 + 856*b^4*c^6 + 186*b^2*c^8 + 7*c^10 + 2*(320*b^7*c^2 + 496*b^
5*c^4 + 220*b^3*c^6 + 25*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^2 + 7
*(1280*b^9*c + 2944*b^7*c^3 + 2288*b^5*c^5 + 676*b^3*c^7 + 57*b*c^9 + (1280
*b^8*c + 2304*b^6*c^3 + 1296*b^4*c^5 + 236*b^2*c^7 + 7*c^9)*sqrt(b^2 + c^2)
)*tan(1/2*e*x + 1/2*d) + 4*(640*b^9 + 1312*b^7*c^2 + 896*b^5*c^4 + 238*b^3*
c^6 + 21*b*c^8)*sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 +
c^2))^7*c^7*e)
```

Mupad [B]
time = 12.31, size = 1004, normalized size = 3.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

```
[Out] -(tan(d/2 + (e*x)/2)^6*((16*b^4 + 2*c^4 + 16*b^2*c^2)/c^8 + ((8*b*c^2 + 16*
b^3)*(b^2 + c^2)^(1/2))/c^8) + tan(d/2 + (e*x)/2)*(((114*b*c^8)/5 + 512*b^9
 + (1352*b^3*c^6)/5 + (4576*b^5*c^4)/5 + (5888*b^7*c^2)/5)/c^13 + ((b^2 + c
^2)^(1/2)*(512*b^8 + (14*c^8)/5 + (472*b^2*c^6)/5 + (2592*b^4*c^4)/5 + (460
8*b^6*c^2)/5))/c^13) + ((1024*b^10)/7 + (24*c^10)/35 + (136*b^2*c^8)/7 + (4
496*b^4*c^6)/35 + (11776*b^6*c^4)/35 + (13056*b^8*c^2)/35)/c^14 + tan(d/2 +
 (e*x)/2)^2*((768*b^8 + (42*c^8)/5 + (1116*b^2*c^6)/5 + (5136*b^4*c^4)/5 +
(7872*b^6*c^2)/5)/c^12 + ((b^2 + c^2)^(1/2)*(60*b*c^6 + 768*b^7 + 528*b^3*c
^4 + (5952*b^5*c^2)/5))/c^12) + tan(d/2 + (e*x)/2)^3*((80*b*c^6 + 640*b^7 +
 600*b^3*c^4 + 1152*b^5*c^2)/c^11 + ((b^2 + c^2)^(1/2)*(640*b^6 + 12*c^6 +
264*b^2*c^4 + 832*b^4*c^2))/c^11) + tan(d/2 + (e*x)/2)^4*((320*b^6 + 12*c^6
 + 196*b^2*c^4 + 496*b^4*c^2)/c^10 + ((b^2 + c^2)^(1/2)*(68*b*c^4 + 320*b^5
 + 336*b^3*c^2))/c^10) + tan(d/2 + (e*x)/2)^5*((30*b*c^4 + 96*b^5 + 120*b^3
*c^2)/c^9 + ((b^2 + c^2)^(1/2)*(96*b^4 + 6*c^4 + 72*b^2*c^2))/c^9) + ((b^2
+ c^2)^(1/2)*((24*b*c^8)/5 + (1024*b^9)/7 + (272*b^3*c^6)/5 + (1024*b^5*c^4
)/5 + (10496*b^7*c^2)/35))/c^14)/(e*(tan(d/2 + (e*x)/2)^3*((280*b^4 + 35*c^
4 + 280*b^2*c^2)/c^4 + ((140*b*c^2 + 280*b^3)*(b^2 + c^2)^(1/2))/c^4) + tan
(d/2 + (e*x)/2)^4*((105*b*c^2 + 140*b^3)/c^3 + ((140*b^2 + 35*c^2)*(b^2 + c
^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^7 + tan(d/2 + (e*x)/2)*((224*b^6 + 7*c
^6 + 126*b^2*c^4 + 336*b^4*c^2)/c^6 + ((b^2 + c^2)^(1/2)*(42*b*c^4 + 224*b^
5 + 224*b^3*c^2))/c^6) + tan(d/2 + (e*x)/2)^5*((42*b^2 + 21*c^2)/c^2 + (42*
b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 + (e*x)/2)^6*((7*(b^2 + c^2)^(1/2))/c +
 (7*b)/c) + tan(d/2 + (e*x)/2)^2*((105*b*c^4 + 336*b^5 + 420*b^3*c^2)/c^5 +
 ((b^2 + c^2)^(1/2)*(336*b^4 + 21*c^4 + 252*b^2*c^2))/c^5) + (7*b*c^6 + 64*
b^7 + 56*b^3*c^4 + 112*b^5*c^2)/c^7 + ((b^2 + c^2)^(1/2)*(64*b^6 + c^6 + 24
*b^2*c^4 + 80*b^4*c^2))/c^7))
```

3.363 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$4a\left(5a^2+3c^2\right)x-\frac{4c(15a^2+4c^2)\cos(d+ex)}{3e}+\frac{4a(15a^2+4c^2)\sin(d+ex)}{3e}-\frac{20(ac\cos(d+ex)-a^2\sin(d+ex)$$

```
[Out] 4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e+4/3*a*(15*a^2+4*c^2)*
sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)-a^2*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x
+d))/e-8/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

Rubi [A]

time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3199, 3225, 2717, 2718}

$$\frac{4a(15a^2+4c^2)\sin(d+ex)}{3e}-\frac{4c(15a^2+4c^2)\cos(d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20(ac\cos(d+ex)-a^2\sin(d+ex))\left(a\cos(d+ex)+a+c\sin(d+ex)\right)}{3e}-\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) + (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] - a^2*Sin[d + e*
x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] - a*S
in[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex)-a\sin(d+ex))(a+a\cos(d+ex)+c\sin}{3e}$$

$$= -\frac{20(ac\cos(d+ex)-a^2\sin(d+ex))\left(a+a\cos(d+ex)+c\right.}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{20(ac\cos(d+ex)-a^2\sin(d+ex))\left(a+\right.}{3e}$$

$$= 4a\left(5a^2+3c^2\right)x - \frac{4c(15a^2+4c^2)\cos(d+ex)}{3e} + \frac{4a(15a^2+}{}$$

Mathematica [A]

time = 0.30, size = 135, normalized size = 0.86

$$\frac{2\left(6a(5a^2+3c^2)(d+ex)-9c(5a^2+c^2)\cos(d+ex)-18a^2c\cos(2(d+ex))+c(-3a^2+c^2)\cos(3(d+ex))+9a(5a^2+c^2)\sin(d+ex)+9a(a^2-c^2)\sin(2(d+ex))+a(a^2-3c^2)\sin(3(d+ex))\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] - 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] + 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] + a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A]

time = 0.26, size = 177, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{8a^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-8a^2c\left(\cos^3(ex+d)\right)+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a\,c^2\left(\sin^3(ex+d)\right)-24a^2c\left(\cos^2\right.}{}$
default	$\frac{\frac{8a^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-8a^2c\left(\cos^3(ex+d)\right)+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a\,c^2\left(\sin^3(ex+d)\right)-24a^2c\left(\cos^2\right.}{}$

risch	$20a^3x + 12a\,c^2x - \frac{30c\cos(ex+d)a^2}{e} - \frac{6c^3\cos(ex+d)}{e} + \frac{30a^3\sin(ex+d)}{e} + \frac{6a\sin(ex+d)c^2}{e} - \frac{2c\cos(3ex+3d}{e}$
norman	$\frac{-\frac{192a^2c+32c^3}{3e}+4a(5a^2+3c^2)x-\frac{32c^3\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{64a(5a^2+3c^2)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{3e}+\frac{8a(5a^2+3c^2)\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+12a(5a}{\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/e*(1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)^3+3*a^3*(1/2*cos(
e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+a*c^2*sin(e*x+d)^3-3*a^2*c*cos(e*x+d)^2+3*
a^3*sin(e*x+d)-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a*c^2*(-1/2*cos(e*x+d)
*sin(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)+a^3*(e*x+d))
```

Maxima [A]
time = 0.28, size = 195, normalized size = 1.24

$$-8a^2c\cos(xe+d)^3e^{(-1)} + 8ac^2e^{(-1)}\sin(xe+d)^3 - \frac{8}{3}\left(\sin(xe+d)^3 - 3\sin(xe+d)\right)a^3e^{(-1)} + \frac{8}{3}\left(\cos(xe+d)^3 - 3\cos(xe+d)\right)c^3e^{(-1)} + 8a^3x - 24\left(c\cos(xe+d)e^{(-1)} - ae^{(-1)}\sin(xe+d)\right)a^2 - 6\left(4ac\cos(xe+d)^2e^{(-1)} - (2xe+2d+\sin(2xe+2d))a^2e^{(-1)} - (2xe+2d-\sin(2xe+2d))c^2e^{(-1)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(x*e + d)^3*e^(-1) + 8*a*c^2*e^(-1)*sin(x*e + d)^3 - 8/3*(sin(x
*e + d)^3 - 3*sin(x*e + d))*a^3*e^(-1) + 8/3*(cos(x*e + d)^3 - 3*cos(x*e +
d))*c^3*e^(-1) + 8*a^3*x - 24*(c*cos(x*e + d)*e^(-1) - a*e^(-1)*sin(x*e + d
))*a^2 - 6*(4*a*c*cos(x*e + d)^2*e^(-1) - (2*x*e + 2*d + sin(2*x*e + 2*d))*
a^2*e^(-1) - (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1))*a
```

Fricas [A]
time = 1.48, size = 140, normalized size = 0.89

$$-\frac{4}{3}\left(18a^2c\cos(xe+d)^2 + 2(3a^2c-c^3)\cos(xe+d)^3 - 3(5a^3+3ac^2)xe + 6(3a^2c+c^3)\cos(xe+d) - \left(22a^3+6ac^2+2(a^3-3ac^2)\cos(xe+d)^2+9(a^3-ac^2)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*c*cos(x*e + d)^2 + 2*(3*a^2*c - c^3)*cos(x*e + d)^3 - 3*(5*a^3
 + 3*a*c^2)*x*e + 6*(3*a^2*c + c^3)*cos(x*e + d) - (22*a^3 + 6*a*c^2 + 2*(a
^3 - 3*a*c^2)*cos(x*e + d)^2 + 9*(a^3 - a*c^2)*cos(x*e + d))*sin(x*e + d))*
e^(-1)
```

Sympy [A]
time = 0.17, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x + \frac{16a^3\sin^3(d+ex)}{3e} + \frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} + \frac{24a^3\sin(d+ex)}{e} + \frac{24a^2c\sin^2(d+ex)}{e} - \frac{8a^2c\cos^3(d+ex)}{e} - \frac{24a^2c\cos(d+ex)}{e} + 12ac^2x\sin^2(d+ex) + 12ac^2x\cos^2(d+ex) + \frac{8ac^2\sin^3(d+ex)}{e} - \frac{12ac^2\sin(d+ex)\cos(d+ex)}{e} - \frac{8c^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{16c^3\cos^3(d+ex)}{3e} & \text{for}\ e \neq 0 \\ x(2a\cos(d)+2a+2c\sin(d))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 16*a**3*sin(d + e*x)**3/(3*e) + 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e + 24*a**3*sin(d + e*x)/e + 24*a**2*c*si
n(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 + 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(2*a*cos(d)
+ 2*a + 2*c*sin(d))**3, True))
```

Giac [A]
time = 0.43, size = 151, normalized size = 0.96

$$-\frac{12\,a^2c\cos\left(2\,ex+2\,d\right)}{e}+4\,\left(5\,a^3+3\,ac^2\right)x-\frac{2\,\left(3\,a^2c-c^3\right)\cos\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\,\left(5\,a^2c+c^3\right)\cos\left(ex+d\right)}{e}+\frac{2\,\left(a^3-3\,ac^2\right)\sin\left(3\,ex+3\,d\right)}{3\,e}+\frac{6\,\left(a^3-ac^2\right)\sin\left(2\,ex+2\,d\right)}{e}+\frac{6\,\left(5\,a^3+ac^2\right)\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*c*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*c^2)*x - 2/3*(3*a^2*c - c^3)*
cos(3*e*x + 3*d)/e - 6*(5*a^2*c + c^3)*cos(e*x + d)/e + 2/3*(a^3 - 3*a*c^2)
*sin(3*e*x + 3*d)/e + 6*(a^3 - a*c^2)*sin(2*e*x + 2*d)/e + 6*(5*a^3 + a*c^2
)*sin(e*x + d)/e
```

Mupad [B]
time = 2.57, size = 239, normalized size = 1.52

$$20\,a^3\,x-\frac{32\,c^3\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4}{e}+\frac{64\,c^3\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6}{3\,e}+12\,a\,c^2\,x-\frac{64\,a^2\,c\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6}{e}+\frac{40\,a^3\,\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e}+\frac{80\,a^3\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{3\,e}+\frac{64\,a^3\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{3\,e}+\frac{16\,a\,c^2\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e}-\frac{64\,a\,c^2\,{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e}+\frac{24\,a\,c^2\,\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] 20*a^3*x - (32*c^3*cos(d/2 + (e*x)/2)^4)/e + (64*c^3*cos(d/2 + (e*x)/2)^6)/
(3*e) + 12*a*c^2*x - (64*a^2*c*cos(d/2 + (e*x)/2)^6)/e + (40*a^3*cos(d/2 +
(e*x)/2)*sin(d/2 + (e*x)/2))/e + (80*a^3*cos(d/2 + (e*x)/2)^3*sin(d/2 + (e*
x)/2))/(3*e) + (64*a^3*cos(d/2 + (e*x)/2)^5*sin(d/2 + (e*x)/2))/(3*e) + (16
*a*c^2*cos(d/2 + (e*x)/2)^3*sin(d/2 + (e*x)/2))/e - (64*a*c^2*cos(d/2 + (e*
x)/2)^5*sin(d/2 + (e*x)/2))/e + (24*a*c^2*cos(d/2 + (e*x)/2)*sin(d/2 + (e*x
)/2))/e
```

3.364 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2(3a^2+c^2)\,x-\frac{6ac\cos(d+ex)}{e}+\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+a\cos(d+ex)+c\sin(d+ex))}{e}$$

```
[Out] 2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e+6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)-a*sin
(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))/e
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3199, 2717, 2718}

$$2x(3a^2+c^2)+\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e + (6*a^2*Sin[d + e*x])/e - (2*(c
*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))^2 \, dx = -\frac{2(c\cos(d + ex) - a\sin(d + ex))(a + a\cos(d + ex) + c\sin}{e}$$
$$= 2(3a^2 + c^2)\, x - \frac{2(c\cos(d + ex) - a\sin(d + ex))(a + a\cos(}{e}$$
$$= 2(3a^2 + c^2)\, x - \frac{6ac\cos(d + ex)}{e} + \frac{6a^2\sin(d + ex)}{e} - \frac{2(c\cos}{}$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 1.14

$$4\left(\frac{(3a^2 + c^2)(d + ex)}{2e} - \frac{2ac\cos(d + ex)}{e} - \frac{ac\cos(2(d + ex))}{2e} + \frac{2a^2\sin(d + ex)}{e} + \frac{(a^2 - c^2)\sin(2(d + ex))}{4e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e - (a*c*Cos[2*(d
 + e*x)])/(2*e) + (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A]
time = 0.21, size = 101, normalized size = 1.25

method	result
risch	$6a^2x + 2x\,c^2 - \frac{8ac\cos(ex+d)}{e} + \frac{8a^2\sin(ex+d)}{e} - \frac{2ac\cos(2ex+2d)}{e} + \frac{\sin(2ex+2d)a^2}{e} - \frac{\sin(2ex+2d)c^2}{e}$
derivativedivides	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ac(\cos^2(ex+d))+8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+}{e}$
default	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ac(\cos^2(ex+d))+8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+}{e}$
norman	$\frac{(6a^2+2c^2)x+(6a^2+2c^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(12a^2+4c^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{16ac}{e}+\frac{4(3a^2+c^2)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{4(5a^2-c}{}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/e*(a^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-a*c*cos(e*x+d)^2+2*a^2*s
in(e*x+d)+c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-2*a*c*cos(e*x+d)+a
^2*(e*x+d))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.25

$$-4\,ac\cos(xe+d)^2\,e^{(-1)} + (2\,xe + 2\,d + \sin(2\,xe + 2\,d))a^2e^{(-1)} + (2\,xe + 2\,d - \sin(2\,xe + 2\,d))c^2e^{(-1)} + 4\,a^2x - 8\,(c\cos(xe + d)\,e^{(-1)} - ae^{(-1)}\sin(xe + d))a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -4*a*c*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d + sin(2*x*e + 2*d))*a^2*e^(-1)
+ (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1) + 4*a^2*x - 8*(c*cos(x*e + d)
*e^(-1) - a*e^(-1)*sin(x*e + d))*a
```

Fricas [A]
time = 1.57, size = 75, normalized size = 0.93

$$-2\left(2\,ac\cos\left(xe+d\right)^2+4\,ac\cos\left(xe+d\right)-\left(3\,a^2+c^2\right)xe-\left(4\,a^2+\left(a^2-c^2\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*c*cos(x*e + d)^2 + 4*a*c*cos(x*e + d) - (3*a^2 + c^2)*x*e - (4*a^2
+ (a^2 - c^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.
time = 0.11, size = 170, normalized size = 2.10

$$\begin{cases} 2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x+\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^2\sin\left(d+ex\right)}{e}+\frac{4ac\sin^2\left(d+ex\right)}{e}-\frac{8ac\cos\left(d+ex\right)}{e}+2c^2x\sin^2\left(d+ex\right)+2c^2x\cos^2\left(d+ex\right)-\frac{2c^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e} & \text{for } e\neq 0 \\ x\left(2a\cos\left(d\right)+2a+2c\sin\left(d\right)\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*sin(d + e*x)/e + 4*a*c*sin(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*cos(d
) + 2*a + 2*c*sin(d))**2, True))
```

Giac [A]
time = 0.42, size = 78, normalized size = 0.96

$$2\left(3\,a^2+c^2\right)x-\frac{2\,ac\cos\left(2\,ex+2\,d\right)}{e}-\frac{8\,ac\cos\left(ex+d\right)}{e}+\frac{8\,a^2\sin\left(ex+d\right)}{e}+\frac{\left(a^2-c^2\right)\sin\left(2\,ex+2\,d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2 + c^2)*x - 2*a*c*cos(2*e*x + 2*d)/e - 8*a*c*cos(e*x + d)/e + 8*a^2
*sin(e*x + d)/e + (a^2 - c^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 3.21, size = 96, normalized size = 1.19

$$\frac{x\,\left(12\,a^2+4\,c^2\right)}{2}+\frac{\left(12\,a^2+4\,c^2\right)\,{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3+\left(20\,a^2-4\,c^2\right)\,\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)-16\,a\,c}{e\,\left({\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+2\,{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*c^2))/2 + (tan(d/2 + (e*x)/2)^3*(12*a^2 + 4*c^2) - 16*a*c +
tan(d/2 + (e*x)/2)*(20*a^2 - 4*c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 +
 (e*x)/2)^4 + 1))
```

3.365 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$2ax-\frac{2c\cos(d+ex)}{e}+\frac{2a\sin(d+ex)}{e}$$

[Out] 2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2717, 2718}

$$\frac{2a\sin(d+ex)}{e}+2ax-\frac{2c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

[In] Int[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]

[Out] 2*a*x - (2*c*Cos[d + e*x])/e + (2*a*Sin[d + e*x])/e

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a+2a\cos(d+ex)+2c\sin(d+ex))\,dx &= 2ax+(2a)\int\cos(d+ex)\,dx+(2c)\int\sin(d+ex)\,dx\\ &= 2ax-\frac{2c\cos(d+ex)}{e}+\frac{2a\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.83

$$2ax-\frac{2c\cos(d)\cos(ex)}{e}+\frac{2a\cos(ex)\sin(d)}{e}+\frac{2a\cos(d)\sin(ex)}{e}+\frac{2c\sin(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e + (2*a*Cos[e*x]*Sin[d])/e + (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

Maple [A]
time = 0.09, size = 30, normalized size = 1.03

method	result	size
default	$2ax-\frac{2c\cos(ex+d)}{e}+\frac{2a\sin(ex+d)}{e}$	30
risch	$2ax-\frac{2c\cos(ex+d)}{e}+\frac{2a\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a+2a\sin(ex+d)-2c\cos(ex+d)}{e}$	31
norman	$\frac{\frac{4c\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+2ax+\frac{4a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}+2ax\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.00

$$-2\,c\cos\left(xe+d\right)e^{(-1)}+2\,ae^{(-1)}\sin\left(xe+d\right)+2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) + 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

Fricas [A]
time = 1.43, size = 29, normalized size = 1.00

$$2\left(axe-c\cos\left(xe+d\right)+a\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*x*e - c*cos(x*e + d) + a*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.06, size = 39, normalized size = 1.34

$$2ax+2a\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\: e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)+2c\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\: e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x + 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A]
time = 0.38, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,c\cos\left(ex + d\right)}{e} + \frac{2\,a\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e + 2*a*sin(e*x + d)/e
```

Mupad [B]
time = 2.43, size = 29, normalized size = 1.00

$$2\,a\,x - \frac{2\,c\,\cos\left(d+e\,x\right)}{e} + \frac{2\,a\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

```
[Out] 2*a*x - (2*c*cos(d + e*x))/e + (2*a*sin(d + e*x))/e
```

3.366 $\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] 1/2*ln(a+c*tan(1/2*e*x+1/2*d))/c/e
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3203, 31}

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{4a+4cx}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= \frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.
time = 0.04, size = 57, normalized size = 2.28

$$\frac{1}{2}\left(-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{ce}+\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}{ce}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-(Log[Cos[(d + e*x)/2]]/(c*e)) + Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/
2]]/(c*e))/2
```

Maple [A]
time = 0.31, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce}$	23
default	$\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce}$	23
norman	$\frac{\ln\left(4c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+4a\right)}{2ce}$	26
risch	$\frac{\ln\left(e^{i(ex+d)}-\frac{ic+a}{ic-a}\right)}{2ce}-\frac{\ln\left(e^{i(ex+d)}+1\right)}{2ce}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(a+c*tan(1/2*d+1/2*e*x))/c/e
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.20

$$\frac{e^{(-1)}\log\left(a+\frac{c\sin(xe+d)}{\cos(xe+d)+1}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-1)*log(a + c*sin(x*e + d)/(cos(x*e + d) + 1))/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(22) = 44$.
time = 1.71, size = 62, normalized size = 2.48

$$\frac{\left(\log\left(ac\sin\left(xe+d\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2+\frac{1}{2}\left(a^2-c^2\right)\cos\left(xe+d\right)\right)-\log\left(\frac{1}{2}\,\cos\left(xe+d\right)+\frac{1}{2}\right)\right)e^{(-1)}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(x*e + d
)) - log(1/2*cos(x*e + d) + 1/2))*e^(-1)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.
time = 0.64, size = 63, normalized size = 2.52

$$\begin{cases} \frac{x}{2a\cos(d)+2a} & \text{for } c = 0 \wedge e = 0 \\ \frac{x}{2a\cos(d)+2a+2c\sin(d)} & \text{for } e = 0 \\ \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2ae} & \text{for } c = 0 \\ \frac{\log\left(\frac{a}{c}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] Piecewise((x/(2*a*cos(d) + 2*a), Eq(c, 0) & Eq(e, 0)), (x/(2*a*cos(d) + 2*a
 + 2*c*sin(d)), Eq(e, 0)), (tan(d/2 + e*x/2)/(2*a*e), Eq(c, 0)), (log(a/c +
 tan(d/2 + e*x/2))/(2*c*e), True))
```

Giac [A]
time = 0.40, size = 23, normalized size = 0.92

$$\frac{\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{2\,ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/(c*e)
```

Mupad [B]
time = 2.82, size = 22, normalized size = 0.88

$$\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}{2\,c\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
```

```
[Out] log(a + c*tan(d/2 + (e*x)/2))/(2*c*e)
```

3.367 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] -1/4*a*ln(a+c*tan(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)+a*sin(e*x+d))/c^
2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3208, 12, 3203, 31}

$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/4*(a*Log[a + c*Tan[(d + e*x)/2]])/(c^3*e) - (c*Cos[d + e*x] - a*Sin[d +
e*x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}+\frac{\int -\frac{}{2a+2a\cos(d+}}{4}\\ &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}-\frac{a\int \frac{}{2a+2a\cos(d+}}{2}\\ &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}-\frac{a\text{Subst}\left(\int \frac{1}{4a+4}\right.}{}\\ &= -\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin}{4c^2e(a+a\cos(d+ex)+}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 115, normalized size = 1.53

$$\frac{2a\left(\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)+\frac{c(a^2+c^2)\sin\left(\frac{1}{2}(d+ex)\right)}{a\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}+c\tan\left(\frac{1}{2}(d+ex)\right)}{8c^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*(Log[Cos[(d + e*x)/2]] - Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]])
 + (c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x
)/2])) + c*Tan[(d + e*x)/2])/(8*c^3*e)
```

Maple [A]
time = 0.39, size = 68, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2c^2}-\frac{a\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}-\frac{a^2+c^2}{2c^3\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}}{4e}$	68
default	$\frac{\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2c^2}-\frac{a\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}-\frac{a^2+c^2}{2c^3\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}}{4e}$	68
norman	$\frac{-\frac{2a^2+c^2}{8c^3e}+\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{8ce}}{a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{a\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^3e}$	78

risch	$-\frac{i\left(-ia\,\mathrm{e}^{i(ex+d)}-ia+c\right)}{2c^{2}e\left(c\,\mathrm{e}^{2i(ex+d)}+ia\,\mathrm{e}^{2i(ex+d)}-c+2ia\,\mathrm{e}^{i(ex+d)}+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(ex+d)}+1\right)}{4c^{3}e}-\frac{a\ln\left(\mathrm{e}^{i(ex+d)}-\frac{ic+a}{ic-a}\right)}{4c^{3}e}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e*(1/2/c^2*tan(1/2*d+1/2*e*x)-1/c^3*a*ln(a+c*tan(1/2*d+1/2*e*x))-1/2/c^
3*(a^2+c^2)/(a+c*tan(1/2*d+1/2*e*x)))
```

Maxima [A]
time = 0.27, size = 95, normalized size = 1.27

$$-\frac{1}{8}\left(\frac{a^{2}+c^{2}}{ac^{3}+\frac{c^{4}\sin(xe+d)}{\cos(xe+d)+1}}+\frac{2\,a\log\left(a+\frac{c\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^{3}}-\frac{\sin\left(xe+d\right)}{c^{2}(\cos\left(xe+d\right)+1)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a^2 + c^2)/(a*c^3 + c^4*sin(x*e + d)/(cos(x*e + d) + 1)) + 2*a*log(a
 + c*sin(x*e + d)/(cos(x*e + d) + 1))/c^3 - sin(x*e + d)/(c^2*(cos(x*e + d)
 + 1)))*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(73) = 146.
time = 1.63, size = 168, normalized size = 2.24

$$-\frac{2\,c^{2}\cos\left(xe+d\right)-2\,ac\sin\left(xe+d\right)+\left(a^{2}\cos\left(xe+d\right)+ac\sin\left(xe+d\right)+a^{2}\right)\log\left(ac\sin\left(xe+d\right)+\frac{1}{2}\,a^{2}+\frac{1}{2}\,c^{2}+\frac{1}{2}\,(a^{2}-c^{2})\cos\left(xe+d\right)\right)-\left(a^{2}\cos\left(xe+d\right)+ac\sin\left(xe+d\right)+a^{2}\right)\log\left(\frac{1}{2}\,\cos\left(xe+d\right)+\frac{1}{2}\right)}{8\,(ac^{3}\cos\left(xe+d\right)e+c^{4}e\sin\left(xe+d\right)+ac^{3}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*c^2*cos(x*e + d) - 2*a*c*sin(x*e + d) + (a^2*cos(x*e + d) + a*c*sin
(x*e + d) + a^2)*log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)
*cos(x*e + d)) - (a^2*cos(x*e + d) + a*c*sin(x*e + d) + a^2)*log(1/2*cos(x*
e + d) + 1/2))/(a*c^3*cos(x*e + d)*e + c^4*e*sin(x*e + d) + a*c^3*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

[Out] Timed out

Giac [A]
time = 0.43, size = 83, normalized size = 1.11

$$-\frac{\frac{2\,a\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{c^3}-\frac{\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{c^2}-\frac{2\,ac\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a^2-c^2}{\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right)c^3}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*a*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^3 - tan(1/2*e*x + 1/2*d)/c
^2 - (2*a*c*tan(1/2*e*x + 1/2*d) + a^2 - c^2)/((c*tan(1/2*e*x + 1/2*d) + a)
*c^3))/e
```

Mupad [B]
time = 2.48, size = 79, normalized size = 1.05

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{8\,c^2\,e}-\frac{a\,\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}{4\,c^3\,e}-\frac{a^2+c^2}{c\,e\,\left(8\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,c^3+8\,a\,c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] tan(d/2 + (e*x)/2)/(8*c^2*e) - (a*log(a + c*tan(d/2 + (e*x)/2)))/(4*c^3*e)
- (a^2 + c^2)/(c*e*(8*a*c^2 + 8*c^3*tan(d/2 + (e*x)/2)))
```

3.368 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$\frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{16c^4e(a+a\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] 1/16*(3*a^2+c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)+a*sin
(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)-a^2*sin
(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 3232, 3203, 31}

$$\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{16c^4e(a\cos(d+ex)+a+c\sin(d+ex))}+\frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((3*a^2 + c^2)*Log[a + c*Tan[(d + e*x)/2]])/(16*c^5*e) - (c*Cos[d + e*x] -
a*Sin[d + e*x])/(16*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*
c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(16*c^4*e*(a + a*Cos[d + e*x] + c*Sin[d
 + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{\int \frac{-4a+2a\cos(}{(2a+2a\cos(d}}{} \\ &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(}{16c^4e(a+a} \\ &= -\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(}{16c^4e(a+a} \\ &= \frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex}{16c^2e(a+a\cos(d}\end{aligned}$$

Mathematica [A]
time = 2.14, size = 186, normalized size = 1.39

$$-\frac{4(3a^2+c^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-4(3a^2+c^2)\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)-c^2\sec^2\left(\frac{1}{2}(d+ex)\right)+\frac{c^2(a^2+c^2)}{\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6c(a^2+c^2)\sin\left(\frac{1}{2}(d+ex)\right)}{a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)}+6ac\tan\left(\frac{1}{2}(d+ex)\right)}{64c^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/64*(4*(3*a^2 + c^2)*Log[Cos[(d + e*x)/2]] - 4*(3*a^2 + c^2)*Log[a*Cos[(d
 + e*x)/2] + c*Sin[(d + e*x)/2]] - c^2*Sec[(d + e*x)/2]^2 + (c^2*(a^2 + c^2
))/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^2 + (6*c*(a^2 + c^2)*Sin[(d +
e*x)/2])/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]) + 6*a*c*Tan[(d + e*x)/2]
)/(c^5*e)
```

Maple [A]
time = 0.50, size = 131, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{-\frac{c\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}+3a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{4c^4}+\frac{\left(6a^2+2c^2\right)\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^5}-\frac{a^4+2a^2c^2+c^4}{8c^5\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{a\left(a^2+c^2\right)}{c^5\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}}{8e}$
default	$\frac{-\frac{-\frac{c\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}+3a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{4c^4}+\frac{\left(6a^2+2c^2\right)\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^5}-\frac{a^4+2a^2c^2+c^4}{8c^5\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{a\left(a^2+c^2\right)}{c^5\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}}{8e}$
norman	$\frac{\frac{18a^4+6a^2c^2-c^4}{64c^5e}+\frac{\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)}{64ce}-\frac{a\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{16c^2e}+\frac{\left(3a^2+c^2\right)a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8c^4e}}{\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{\left(3a^2+c^2\right)\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{16c^5e}$
risch	$\frac{3ia^3e^{3i(ex+d)}+ia\,c^2e^{3i(ex+d)}+9ia^3e^{2i(ex+d)}+3a^2c\,e^{3i(ex+d)}+3ia\,c^2e^{2i(ex+d)}+c^3e^{3i(ex+d)}+9ia^3e^{i(ex+d)}-ia\,c^2e^{i(ex+d)}}{8\left(c\,e^{2i(ex+d)}+ia\,e^{2i(ex+d)}-c+2ia\,e^{i(ex+d)}+ia\right)^2c^4e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/e*(-1/4/c^4*(-1/2*c*tan(1/2*d+1/2*e*x)^2+3*a*tan(1/2*d+1/2*e*x))+1/4*(6
*a^2+2*c^2)/c^5*ln(a+c*tan(1/2*d+1/2*e*x))-1/8/c^5*(a^4+2*a^2*c^2+c^4)/(a+c
*tan(1/2*d+1/2*e*x))^2+a/c^5*(a^2+c^2)/(a+c*tan(1/2*d+1/2*e*x)))
```

Maxima [A]
time = 0.29, size = 201, normalized size = 1.50

$$\frac{1}{64}\left(\frac{7\,a^4+6\,a^2c^2-c^4+\frac{8\,(a^3c+ac^3)\sin(xe+d)}{\cos(xe+d)+1}}{a^2c^5+\frac{2\,ac^6\sin(xe+d)}{\cos(xe+d)+1}+\frac{c^7\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}-\frac{\frac{6\,a\sin(xe+d)}{\cos(xe+d)+1}-\frac{c\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}{c^4}+\frac{4\,(3\,a^2+c^2)\log\left(a+\frac{c\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*((7*a^4 + 6*a^2*c^2 - c^4 + 8*(a^3*c + a*c^3)*sin(x*e + d)/(cos(x*e +
d) + 1))/(a^2*c^5 + 2*a*c^6*sin(x*e + d)/(cos(x*e + d) + 1) + c^7*sin(x*e +
 d)^2/(cos(x*e + d) + 1)^2) - (6*a*sin(x*e + d)/(cos(x*e + d) + 1) - c*sin(
x*e + d)^2/(cos(x*e + d) + 1)^2)/c^4 + 4*(3*a^2 + c^2)*log(a + c*sin(x*e +
d)/(cos(x*e + d) + 1))/c^5)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(133) = 266$.
time = 1.62, size = 457, normalized size = 3.41

$$\frac{12\,a^2c^2\cos(xe+d)^2-6\,a^2c^2+2\,(3\,a^2c^2-c^4)\cos(xe+d)+(3\,a^4+4\,a^2c^2+c^4+(3\,a^4-2\,a^2c^2-c^4)\cos(xe+d)^2+2\,(3\,a^4+a^2c^2)\cos(xe+d)+2\,(3\,a^3c+ac^3+(3\,a^3c+ac^3)\cos(xe+d))\sin(xe+d))\log\left(ac\sin(xe+d)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2+\frac{1}{2}\,(a^2-c^2)\cos(xe+d)\right)-(3\,a^4+4\,a^2c^2+c^4+(3\,a^4-2\,a^2c^2-c^4)\cos(xe+d)^2+2\,(3\,a^4+a^2c^2)\cos(xe+d)+2\,(3\,a^3c+ac^3+(3\,a^3c+ac^3)\cos(xe+d))\sin(xe+d))\log\left(\frac{1}{2}\cos(xe+d)+\frac{1}{2}\right)-2\,(3\,a^3c-ac^3+3\,(a^3c-ac^3)\cos(xe+d))\sin(xe+d)}{32\,\left(2\,a^2c^5\cos(xe+d)\,e+(a^2c^5-c^7)\cos(xe+d)^2\,e+(a^2c^5+c^7)e+2\,(ac^6\cos(xe+d)\,e+ac^6e)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(x*e + d)^2 - 6*a^2*c^2 + 2*(3*a^2*c^2 - c^4)*cos(x*e +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(x*e + d)^2 +
 2*(3*a^4 + a^2*c^2)*cos(x*e + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^3)*
cos(x*e + d))*sin(x*e + d))*log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 + 1/2*
(a^2 - c^2)*cos(x*e + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(x*e + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(x*e + d) + 2*(3*a^3*c + a*c^
3 + (3*a^3*c + a*c^3)*cos(x*e + d))*sin(x*e + d))*log(1/2*cos(x*e + d) + 1/
2) - 2*(3*a^3*c - a*c^3 + 3*(a^3*c - a*c^3)*cos(x*e + d))*sin(x*e + d))/(2*
a^2*c^5*cos(x*e + d)*e + (a^2*c^5 - c^7)*cos(x*e + d)^2*e + (a^2*c^5 + c^7)
*e + 2*(a*c^6*cos(x*e + d)*e + a*c^6*e)*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 164, normalized size = 1.22

$$\frac{\frac{4\,\left(3\,a^{2}+c^{2}\right)\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{c^{5}}+\frac{c^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-6\,ac^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{c^{6}}-\frac{18\,a^{2}c^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+6\,c^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+28\,a^{3}c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+4\,ac^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+11\,a^{4}+c^{4}}{\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right)^{2}c^{5}}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^5 + (c^3*tan(1
/2*e*x + 1/2*d)^2 - 6*a*c^2*tan(1/2*e*x + 1/2*d))/c^6 - (18*a^2*c^2*tan(1/2
*e*x + 1/2*d)^2 + 6*c^4*tan(1/2*e*x + 1/2*d)^2 + 28*a^3*c*tan(1/2*e*x + 1/2
*d) + 4*a*c^3*tan(1/2*e*x + 1/2*d) + 11*a^4 + c^4)/((c*tan(1/2*e*x + 1/2*d)
 + a)^2*c^5))/e
```

Mupad [B]
time = 2.48, size = 162, normalized size = 1.21

$$\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2}{64\,c^3\,e}+\frac{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a^3+4\,a\,c^2\right)+\frac{7\,a^4+6\,a^2\,c^2-c^4}{2\,c}}{e\,\left(32\,a^2\,c^4+64\,a\,c^5\,\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)+32\,c^6\,{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\right)}-\frac{3\,a\,\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,c^4\,e}+\frac{\ln\left(a+c\,\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(3\,a^2+c^2\right)}{16\,c^5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] tan(d/2 + (e*x)/2)^2/(64*c^3*e) + (tan(d/2 + (e*x)/2)*(4*a*c^2 + 4*a^3) + (
7*a^4 - c^4 + 6*a^2*c^2)/(2*c))/(e*(32*c^6*tan(d/2 + (e*x)/2)^2 + 32*a^2*c^
4 + 64*a*c^5*tan(d/2 + (e*x)/2))) - (3*a*tan(d/2 + (e*x)/2))/(32*c^4*e) + (
log(a + c*tan(d/2 + (e*x)/2))*(3*a^2 + c^2))/(16*c^5*e)
```

3.369 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=207

$$-\frac{a(5a^2+3c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3}+\frac{5(ac\cos(d+ex)-}{96c^4e(a+a\cos(d+e}$$

```
[Out] -1/32*a*(5*a^2+3*c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)+
a*sin(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)-a^
2*sin(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)
*cos(e*x+d)+a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a+a*cos(e*x+d)+c*sin(e*x+d)
)
```

Rubi [A]
time = 0.17, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3208, 3235, 3232, 3203, 31}

$$\frac{5(ac\cos(d+ex)-a^2\sin(d+ex))}{96c^4e(a\cos(d+ex)+a+c\sin(d+ex))^2}-\frac{a(5a^2+3c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c(15a^2+4c^2)\cos(d+ex)-a(15a^2+4c^2)\sin(d+ex)}{96c^6e(a\cos(d+ex)+a+c\sin(d+ex))}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] -1/32*(a*(5*a^2 + 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])/(c^7*e) - (c*Cos[d +
e*x] - a*Sin[d + e*x])/(48*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^3) +
 (5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(96*c^4*e*(a + a*Cos[d + e*x] +
c*Sin[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] - a*(15*a^2 + 4*c^2)*
Sin[d + e*x])/(96*c^6*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3}+\frac{\int \frac{-6a+4a\cos(}{(2a+2a\cos(d}}{}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3}+\frac{5(ac\cos(}{96c^4e(a+a}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3}+\frac{5(ac\cos(}{96c^4e(a+a}$$

$$= -\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3}+\frac{5(ac\cos(}{96c^4e(a+a}$$

$$= -\frac{a(5a^2+3c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\cos(d}{48c^2e(a+a\,c}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 492 vs. 2(207) = 414.

time = 1.17, size = 492, normalized size = 2.38

$$\frac{\cos\left(\frac{1}{2}(d+ex)\right)\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(192(5a^3+3ac^2)\cos^3\left(\frac{1}{2}(d+ex)\right)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)^3-192(5a^3+3ac^2)\cos^3\left(\frac{1}{2}(d+ex)\right)\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)^3+\text{[illegible]}\right)}{384c^7e(a+a\cos(d+ex)+c\sin(d+ex))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Cos[(d + e*x)/2]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])*(192*(5*a^3 + 3
*a*c^2)*Cos[(d + e*x)/2]^3*Log[Cos[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Si
n[(d + e*x)/2])^3 - 192*(5*a^3 + 3*a*c^2)*Cos[(d + e*x)/2]^3*Log[a*Cos[(d +
 e*x)/2] + c*Sin[(d + e*x)/2]]*(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^3
+ (c*(150*a^5*c + 130*a^3*c^3 + 24*a*c^5 + 3*a*c*(25*a^4 + 25*a^2*c^2 - 4*c
^4)*Cos[d + e*x] - 6*(25*a^5*c + 15*a^3*c^3 + 4*a*c^5)*Cos[2*(d + e*x)] - 7
5*a^5*c*Cos[3*(d + e*x)] - 35*a^3*c^3*Cos[3*(d + e*x)] - 4*a*c^5*Cos[3*(d +
 e*x)] + 150*a^6*Sin[d + e*x] + 255*a^4*c^2*Sin[d + e*x] + 129*a^2*c^4*Sin[
d + e*x] + 12*c^6*Sin[d + e*x] + 120*a^6*Sin[2*(d + e*x)] + 72*a^4*c^2*Sin[
2*(d + e*x)] + 36*a^2*c^4*Sin[2*(d + e*x)] + 30*a^6*Sin[3*(d + e*x)] - 37*a
^4*c^2*Sin[3*(d + e*x)] - 27*a^2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*
x)]))/a))/(384*c^7*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^4)
```

Maple [A]
time = 0.68, size = 221, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)c^2}{3}-2a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)c+10a^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+3c^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8c^6}-\frac{a\left(5a^2+3c^2\right)\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^7}-\frac{a^6+3a^4c^2+3c}{24c^7\left(a+c\tan\right(}$ over $16e$
default	$\frac{\frac{\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)c^2}{3}-2a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)c+10a^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+3c^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8c^6}-\frac{a\left(5a^2+3c^2\right)\ln\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^7}-\frac{a^6+3a^4c^2+3c}{24c^7\left(a+c\tan\right(}$ over $16e$
norman	$\frac{-\frac{110a^6+66a^4c^2+3a^2c^4+c^6}{384c^7e}+\frac{\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)}{384ce}-\frac{a\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{128c^2e}+\frac{\left(5a^2+3c^2\right)\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{128c^3e}-\frac{3\left(20a^4+12a^2c^2+c^4\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right.\right.}{128c^5e}}{\left(a+c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}$
risch	$i\left(9iac^4e^{5i(ex+d)}-130ia^3c^2e^{3i(ex+d)}-24iac^4e^{3i(ex+d)}-60ia^3c^2e^{2i(ex+d)}+60ia^3c^2e^{i(ex+d)}+15iac^4e^{i(ex+d)}-12iac^4e^{2i(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/e*(1/8/c^6*(1/3*tan(1/2*d+1/2*e*x)^3*c^2-2*a*tan(1/2*d+1/2*e*x)^2*c+10
*a^2*tan(1/2*d+1/2*e*x)+3*c^2*tan(1/2*d+1/2*e*x))-1/2*a/c^7*(5*a^2+3*c^2)*l
n(a+c*tan(1/2*d+1/2*e*x))-1/24/c^7*(a^6+3*a^4*c^2+3*a^2*c^4+c^6)/(a+c*tan(1
/2*d+1/2*e*x))^3+3/8*a/c^7*(a^4+2*a^2*c^2+c^4)/(a+c*tan(1/2*d+1/2*e*x))^2-1
/8*(15*a^4+18*a^2*c^2+3*c^4)/c^7/(a+c*tan(1/2*d+1/2*e*x)))
```

Maxima [A]
time = 0.30, size = 324, normalized size = 1.57

$$-\frac{1}{384}\left(\frac{37\,a^6+39\,a^4c^2+3\,a^2c^4+c^6+\frac{9\,(9\,a^5c+10\,a^3c^3+ac^5)\sin(xe+d)}{\cos(xe+d)+1}+\frac{9\,(5\,a^4c^2+6\,a^2c^4+c^6)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}{a^3c^7+\frac{3\,a^2c^8\sin(xe+d)}{\cos(xe+d)+1}+\frac{3\,ac^9\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{c^{10}\sin(xe+d)^3}{(\cos(xe+d)+1)^3}}+\frac{\frac{6\,ac\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{c^2\sin(xe+d)^3}{(\cos(xe+d)+1)^3}-\frac{3\,(10\,a^2+3\,c^2)\sin(xe+d)}{\cos(xe+d)+1}}{c^6}+\frac{12\,(5\,a^3+3\,ac^2)\log\left(a+\frac{c\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^7}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((37*a^6 + 39*a^4*c^2 + 3*a^2*c^4 + c^6 + 9*(9*a^5*c + 10*a^3*c^3 +
a*c^5)*sin(x*e + d)/(cos(x*e + d) + 1) + 9*(5*a^4*c^2 + 6*a^2*c^4 + c^6)*si
n(x*e + d)^2/(cos(x*e + d) + 1)^2)/(a^3*c^7 + 3*a^2*c^8*sin(x*e + d)/(cos(x
*e + d) + 1) + 3*a*c^9*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + c^10*sin(x*e +
 d)^3/(cos(x*e + d) + 1)^3) + (6*a*c*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 -
c^2*sin(x*e + d)^3/(cos(x*e + d) + 1)^3 - 3*(10*a^2 + 3*c^2)*sin(x*e + d)/(
cos(x*e + d) + 1))/c^6 + 12*(5*a^3 + 3*a*c^2)*log(a + c*sin(x*e + d)/(cos(x
*e + d) + 1))/c^7)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(207) = 414$.
time = 1.65, size = 825, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 - 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(x*e
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(x*e + d)^2 + 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(x*e + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 + (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(x*e + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
x*e + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(x*e + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(x*e + d)^2 + 6*(
5*a^5*c + 3*a^3*c^3)*cos(x*e + d))*sin(x*e + d))*log(a*c*sin(x*e + d) + 1/2
*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(x*e + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 + (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(x*e + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(x*e + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(x
*e + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(x*e + d)^2 + 6*(5*a^5*c + 3*a^3*c^3)*cos(x*e + d))*sin(x*e + d))*log
(1/2*cos(x*e + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c -
 41*a^3*c^3 - 12*a*c^5)*cos(x*e + d)^2 + 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*c
os(x*e + d))*sin(x*e + d))/((a^3*c^7 - 3*a*c^9)*cos(x*e + d)^3*e + 3*(a^3*c
^7 - a*c^9)*cos(x*e + d)^2*e + 3*(a^3*c^7 + a*c^9)*cos(x*e + d)*e + (a^3*c^
7 + 3*a*c^9)*e + (6*a^2*c^8*cos(x*e + d)*e + (3*a^2*c^8 - c^10)*cos(x*e + d
)^2*e + (3*a^2*c^8 + c^10)*e)*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 291, normalized size = 1.41

$$-\frac{\frac{12\left(5a^3+3ac^2\right)\log\left(\left|c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a\right|\right)}{c^7}-\frac{110a^3c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+66ac^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+285a^4c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+144a^2c^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-9c^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+249a^5c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+108a^3c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-9ac^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+73a^6+27a^4c^2-3a^2c^4-c^6}{\left(c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a\right)^3c^7}-\frac{c^8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-6ac^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+30a^2c^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+9c^8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{c^{12}}}{384e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^7 - (11
0*a^3*c^3*tan(1/2*e*x + 1/2*d)^3 + 66*a*c^5*tan(1/2*e*x + 1/2*d)^3 + 285*a^
4*c^2*tan(1/2*e*x + 1/2*d)^2 + 144*a^2*c^4*tan(1/2*e*x + 1/2*d)^2 - 9*c^6*t
an(1/2*e*x + 1/2*d)^2 + 249*a^5*c*tan(1/2*e*x + 1/2*d) + 108*a^3*c^3*tan(1/
2*e*x + 1/2*d) - 9*a*c^5*tan(1/2*e*x + 1/2*d) + 73*a^6 + 27*a^4*c^2 - 3*a^2
*c^4 - c^6)/((c*tan(1/2*e*x + 1/2*d) + a)^3*c^7) - (c^8*tan(1/2*e*x + 1/2*d
)^3 - 6*a*c^7*tan(1/2*e*x + 1/2*d)^2 + 30*a^2*c^6*tan(1/2*e*x + 1/2*d) + 9*
c^8*tan(1/2*e*x + 1/2*d))/c^12)/e
```

Mupad [B]
time = 2.53, size = 260, normalized size = 1.26

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,c^4\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{3}{128\,c^4}+\frac{5\,a^2}{64\,c^6}\right)}{e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(27\,a^5+30\,a^3\,c^2+3\,a\,c^4\right)+\frac{37\,a^6+39\,a^4\,c^2+3\,a^2\,c^4+c^6}{3\,c}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(15\,a^4\,c+18\,a^2\,c^3+3\,c^5\right)}{e\,\left(128\,a^3\,c^6+384\,a^2\,c^7\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+384\,a\,c^8\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+128\,c^9\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\right)}-\frac{a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,c^5\,e}-\frac{\ln\left(a+c\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(5\,a^3+3\,a\,c^2\right)}{32\,c^7\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

```
[Out] tan(d/2 + (e*x)/2)^3/(384*c^4*e) + (tan(d/2 + (e*x)/2)*(3/(128*c^4) + (5*a^
2)/(64*c^6)))/e - (tan(d/2 + (e*x)/2)*(3*a*c^4 + 27*a^5 + 30*a^3*c^2) + (37
*a^6 + c^6 + 3*a^2*c^4 + 39*a^4*c^2)/(3*c) + tan(d/2 + (e*x)/2)^2*(15*a^4*c
 + 3*c^5 + 18*a^2*c^3))/(e*(128*c^9*tan(d/2 + (e*x)/2)^3 + 128*a^3*c^6 + 38
4*a^2*c^7*tan(d/2 + (e*x)/2) + 384*a*c^8*tan(d/2 + (e*x)/2)^2)) - (a*tan(d/
2 + (e*x)/2)^2)/(64*c^5*e) - (log(a + c*tan(d/2 + (e*x)/2))*(3*a*c^2 + 5*a^
3))/(32*c^7*e)
```

3.370 $\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ae}$$

```
[Out] 1/2*ln(1+tan(1/2*e*x+1/2*d))/a/e
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3203, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(2*a*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{4a+4ax}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ae}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.
time = 0.02, size = 50, normalized size = 2.17

$$\frac{-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}+\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{e}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] (-(Log[Cos[(d + e*x)/2]]/e) + Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/e)/(
2*a)
```

Maple [A]
time = 0.31, size = 21, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ae}$	21
default	$\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ae}$	21
norman	$\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ae}$	21
risch	$\frac{\ln\left(e^{i(ex+d)}+i\right)}{2ae}-\frac{\ln\left(e^{i(ex+d)}+1\right)}{2ae}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1+tan(1/2*d+1/2*e*x))/a/e
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.26

$$\frac{e^{(-1)}\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-1)*log(sin(x*e + d)/(cos(x*e + d) + 1) + 1)/a
```

Fricas [A]
time = 1.27, size = 32, normalized size = 1.39

$$-\frac{\left(\log\left(\frac{1}{2}\cos\left(xe+d\right)+\frac{1}{2}\right)-\log\left(\sin\left(xe+d\right)+1\right)\right)e^{(-1)}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(1/2*cos(x*e + d) + 1/2) - log(sin(x*e + d) + 1))*e^(-1)/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.
time = 0.32, size = 36, normalized size = 1.57

$$\begin{cases} \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2ae} & \text{for } e \neq 0 \\ \frac{x}{2a\sin(d)+2a\cos(d)+2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((log(tan(d/2 + e*x/2) + 1)/(2*a*e), Ne(e, 0)), (x/(2*a*sin(d) + 2
*a*cos(d) + 2*a), True))
```

Giac [A]
time = 0.41, size = 21, normalized size = 0.91

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right|\right)}{2\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*e*x + 1/2*d) + 1))/(a*e)
```

Mupad [B]
time = 2.49, size = 20, normalized size = 0.87

$$\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{2\,a\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x)),x)
```

```
[Out] log(tan(d/2 + (e*x)/2) + 1)/(2*a*e)
```

3.371 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}$$

```
[Out] -1/4*ln(1+tan(1/2*e*x+1/2*d))/a^2/e+1/4*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^3
+a^3*cos(e*x+d)+a^3*sin(e*x+d))
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3208, 12, 3203, 31}

$$-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/4*Log[1 + Tan[(d + e*x)/2]]/(a^2*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/
(4*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} + \frac{\int -\frac{}{2a+2a\cos(\ldots}}{}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} - \frac{\int \frac{}{2a+2a\cos(d+\ldots}}{}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)} - \frac{\text{Subst}\left(\int \frac{1}{4a+\ldots}\right.}{}$$

$$= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^2e} - \frac{a\cos(d+ex)-a\sin(\ldots}{4e\left(a^3+a^3\cos(d+ex)+a^3\ldots\right.}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 1.24

$$\frac{2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+\frac{2\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)}+\tan\left(\frac{1}{2}(d+ex)\right)}{8a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*Log[Cos[(d + e*x)/2]] - 2*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + (2*
Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + Tan[(d + e*x)/2])
/(8*a^2*e)
```

Maple [A]
time = 0.30, size = 48, normalized size = 0.64

method	result	size
derivativedivides	$\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-2\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{2}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}}{8e\,a^2}$	48
default	$\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-2\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{2}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}}{8e\,a^2}$	48
norman	$\frac{\frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{8ae}-\frac{3}{8ae}}{a\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a^2e}$	67

risch	$\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\left(\mathrm{e}^{i(ex+d)}+1+i\right)}{a^{2} e\left(i \mathrm{e}^{i(ex+d)}+\mathrm{e}^{2 i(ex+d)}+i+\mathrm{e}^{i(ex+d)}\right)}-\frac{\ln\left(\mathrm{e}^{i(ex+d)}+i\right)}{4 a^{2} e}+\frac{\ln\left(\mathrm{e}^{i(ex+d)}+1\right)}{4 a^{2} e}$	101

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)

[Out] 1/8/e/a^2*(tan(1/2*d+1/2*e*x)-2*ln(1+tan(1/2*d+1/2*e*x))-2/(1+tan(1/2*d+1/2*e*x)))

Maxima [A]
time = 0.28, size = 85, normalized size = 1.13

$$-\frac{1}{8}\left(\frac{2}{a^{2}+\frac{a^{2} \sin(xe+d)}{\cos(xe+d)+1}}+\frac{2 \log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{a^{2}}-\frac{\sin(xe+d)}{a^{2}(\cos(xe+d)+1)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")

[Out] -1/8*(2/(a^2 + a^2*sin(x*e + d)/(cos(x*e + d) + 1)) + 2*log(sin(x*e + d)/(cos(x*e + d) + 1) + 1)/a^2 - sin(x*e + d)/(a^2*(cos(x*e + d) + 1)))*e^(-1)

Fricas [A]
time = 1.73, size = 113, normalized size = 1.51

$$\frac{(\cos(xe+d)+\sin(xe+d)+1)\log\left(\frac{1}{2}\cos(xe+d)+\frac{1}{2}\right)-(\cos(xe+d)+\sin(xe+d)+1)\log(\sin(xe+d)+1)-2\cos(xe+d)+2\sin(xe+d)}{8\left(a^{2}\cos(xe+d)e+a^{2}e\sin(xe+d)+a^{2}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")

[Out] 1/8*((cos(x*e + d) + sin(x*e + d) + 1)*log(1/2*cos(x*e + d) + 1/2) - (cos(x*e + d) + sin(x*e + d) + 1)*log(sin(x*e + d) + 1) - 2*cos(x*e + d) + 2*sin(x*e + d))/(a^2*cos(x*e + d)*e + a^2*e*sin(x*e + d) + a^2*e)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(65) = 130$.
time = 0.91, size = 168, normalized size = 2.24

$$\begin{cases}-\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^{2}e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^{2}e}-\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{8a^{2}e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^{2}e}+\frac{\tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^{2}e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^{2}e}-\frac{3}{8a^{2}e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^{2}e} & \text{for}\ e\neq 0\\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^{2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**2,x)

```
[Out] Piecewise((-2*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(8*a**2*e*tan(d/2
+ e*x/2) + 8*a**2*e) - 2*log(tan(d/2 + e*x/2) + 1)/(8*a**2*e*tan(d/2 + e*x/
2) + 8*a**2*e) + tan(d/2 + e*x/2)**2/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e)
 - 3/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*
a*cos(d) + 2*a)**2, True))
```

Giac [A]
time = 0.40, size = 65, normalized size = 0.87

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^2\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right)}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^2 - tan(1/2*e*x + 1/2*d)/a^2 -
 2*tan(1/2*e*x + 1/2*d)/(a^2*(tan(1/2*e*x + 1/2*d) + 1)))/e
```

Mupad [B]
time = 2.45, size = 59, normalized size = 0.79

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{8\,a^2\,e}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^2\,e}-\frac{1}{4\,a^2\,e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] tan(d/2 + (e*x)/2)/(8*a^2*e) - log(tan(d/2 + (e*x)/2) + 1)/(4*a^2*e) - 1/(4
*a^2*e*(tan(d/2 + (e*x)/2) + 1))
```

3.372 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=123

$$\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^3e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}$$

```
[Out] 1/4*ln(1+tan(1/2*e*x+1/2*d))/a^3/e+1/16*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^2
+a^2*cos(e*x+d)+a^2*sin(e*x+d))^2+3/16*(cos(e*x+d)-sin(e*x+d))/e/(a^3+a^3*c
os(e*x+d)+a^3*sin(e*x+d))
```

Rubi [A]

time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 3232, 3203, 31}

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^3e}+\frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] Log[1 + Tan[(d + e*x)/2]]/(4*a^3*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])/(16
*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) + (3*(Cos[d + e*x] - Sin[
d + e*x]))/(16*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{\int\frac{-4a+2a\cos}{(2a+2a\cos}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\mathrm{co}}{16e\left(a^3+\right.}$$

$$= -\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2}+\frac{3(\mathrm{co}}{16e\left(a^3+\right.}$$

$$= \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^3e}-\frac{a\cos(d+ex)-a\sin(}{16e\left(a^2+a^2\cos(d+ex)+a^2\right.}$$

Mathematica [A]
time = 0.41, size = 135, normalized size = 1.10

$$\frac{\sec^2\left(\frac{1}{2}(d+ex)\right)+2\left(-8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)-\frac{1}{\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}-\frac{6\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)}-3\tan\left(\frac{1}{2}(d+ex)\right)\right)}{64a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sec[(d + e*x)/2]^2 + 2*(-8*Log[Cos[(d + e*x)/2]] + 8*Log[Cos[(d + e*x)/2]
+ Sin[(d + e*x)/2]] - (Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^(-2) - (6*Sin[(
d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - 3*Tan[(d + e*x)/2]))/(
64*a^3*e)
```

Maple [A]
time = 0.38, size = 78, normalized size = 0.63

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}-3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{8}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{2}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}}{32e\,a^3}$	78
default	$\frac{\frac{\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}-3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{8}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{2}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}}{32e\,a^3}$	78
norman	$\frac{-\frac{\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)}{16ae}+\frac{\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)}{64ae}+\frac{23}{64ae}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2ae}}{a^2\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a^3e}$	103
risch	$\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\left(6i\mathrm{e}^{2i(ex+d)}+4\,\mathrm{e}^{3i(ex+d)}+8i\mathrm{e}^{i(ex+d)}+6\,\mathrm{e}^{2i(ex+d)}-3+3i\right)}{a^3e\left(i\mathrm{e}^{i(ex+d)}+\mathrm{e}^{2i(ex+d)}+i+\mathrm{e}^{i(ex+d)}\right)^2}+\frac{\ln\left(\mathrm{e}^{i(ex+d)}+i\right)}{4a^3e}-\frac{\ln\left(\mathrm{e}^{i(ex+d)}+1\right)}{4a^3e}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/e/a^3*(1/2*tan(1/2*d+1/2*e*x)^2-3*tan(1/2*d+1/2*e*x)+8*ln(1+tan(1/2*d+
1/2*e*x))+8/(1+tan(1/2*d+1/2*e*x))-2/(1+tan(1/2*d+1/2*e*x))^2)
```

Maxima [A]

time = 0.29, size = 157, normalized size = 1.28

$$\frac{1}{64}\left(\frac{4\left(\frac{4\sin(xe+d)}{\cos(xe+d)+1}+3\right)}{a^3+\frac{2a^3\sin(xe+d)}{\cos(xe+d)+1}+\frac{a^3\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}-\frac{\frac{6\sin(xe+d)}{\cos(xe+d)+1}-\frac{\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}{a^3}+\frac{16\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{a^3}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(4*sin(x*e + d)/(cos(x*e + d) + 1) + 3)/(a^3 + 2*a^3*sin(x*e + d)/(
cos(x*e + d) + 1) + a^3*sin(x*e + d)^2/(cos(x*e + d) + 1)^2) - (6*sin(x*e +
 d)/(cos(x*e + d) + 1) - sin(x*e + d)^2/(cos(x*e + d) + 1)^2)/a^3 + 16*log(
sin(x*e + d)/(cos(x*e + d) + 1) + 1)/a^3)*e^(-1)
```

Fricas [A]

time = 1.31, size = 161, normalized size = 1.31

$$\frac{6\cos(xe+d)^2-4\left((\cos(xe+d)+1)\sin(xe+d)+\cos(xe+d)+1\right)\log\left(\frac{1}{2}\cos(xe+d)+\frac{1}{2}\right)+4\left((\cos(xe+d)+1)\sin(xe+d)+\cos(xe+d)+1\right)\log(\sin(xe+d)+1)+2\cos(xe+d)-2\sin(xe+d)-3}{32\left(a^3\cos(xe+d)e+a^3e+(a^3\cos(xe+d)e+a^3e)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(x*e + d)^2 - 4*((cos(x*e + d) + 1)*sin(x*e + d) + cos(x*e + d)
+ 1)*log(1/2*cos(x*e + d) + 1/2) + 4*((cos(x*e + d) + 1)*sin(x*e + d) + cos
(x*e + d) + 1)*log(sin(x*e + d) + 1) + 2*cos(x*e + d) - 2*sin(x*e + d) - 3)
/(a^3*cos(x*e + d)*e + a^3*e + (a^3*cos(x*e + d)*e + a^3*e)*sin(x*e + d))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(112) = 224$.

time = 3.14, size = 423, normalized size = 3.44

$$\begin{cases} \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{32\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} - \frac{4\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{32\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{23}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} & \text{for } e \neq 0 \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((16*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(64*a**3*e*tan(
d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*log(tan(d/2
 + e*x/2) + 1)*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e
*tan(d/2 + e*x/2) + 64*a**3*e) + 16*log(tan(d/2 + e*x/2) + 1)/(64*a**3*e*ta
n(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + tan(d/2 + e*
x/2)**4/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a
**3*e) - 4*tan(d/2 + e*x/2)**3/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*
tan(d/2 + e*x/2) + 64*a**3*e) + 32*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*
x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 23/(64*a**3*e*tan(d/2
+ e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e), Ne(e, 0)), (x/(2*a*
sin(d) + 2*a*cos(d) + 2*a)**3, True))
```

Giac [A]

time = 0.43, size = 102, normalized size = 0.83

$$\frac{\frac{16\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)}{a^3} - \frac{4\left(6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+3\right)}{a^3\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right)^2} + \frac{a^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-6a^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^6}}{64e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(16*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^3 - 4*(6*tan(1/2*e*x + 1/2*d)
^2 + 8*tan(1/2*e*x + 1/2*d) + 3)/(a^3*(tan(1/2*e*x + 1/2*d) + 1)^2) + (a^3*
tan(1/2*e*x + 1/2*d)^2 - 6*a^3*tan(1/2*e*x + 1/2*d))/a^6)/e
```

Mupad [B]

time = 2.45, size = 90, normalized size = 0.73

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^3\,e} + \frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^3\,e} - \frac{3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,a^3\,e} + \frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{4}+\frac{3}{16}}{a^3\,e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] tan(d/2 + (e*x)/2)^2/(64*a^3*e) + log(tan(d/2 + (e*x)/2) + 1)/(4*a^3*e) - (
3*tan(d/2 + (e*x)/2))/(32*a^3*e) + (tan(d/2 + (e*x)/2)/4 + 3/16)/(a^3*e*(ta
n(d/2 + (e*x)/2) + 1)^2)
```

3.373 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=168

$$-\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2+a^2\cos(d+ex)+a^2\sin(d\right.}$$

```
[Out] -1/4*ln(1+tan(1/2*e*x+1/2*d))/a^4/e+1/48*(-cos(e*x+d)+sin(e*x+d))/a/e/(a+a*
cos(e*x+d)+a*sin(e*x+d))^3+5/96*(cos(e*x+d)-sin(e*x+d))/e/(a^2+a^2*cos(e*x+
d)+a^2*sin(e*x+d))^2-19/96*(a*cos(e*x+d)-a*sin(e*x+d))/e/(a^5+a^5*cos(e*x+d
)+a^5*sin(e*x+d))
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3208, 3235, 3232, 3203, 31}

$$-\frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5\sin(d+ex)+a^5\cos(d+ex)+a^5\right)}-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^4e}+\frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] -1/4*Log[1 + Tan[(d + e*x)/2]]/(a^4*e) - (Cos[d + e*x] - Sin[d + e*x])/(48*
a*e*(a + a*Cos[d + e*x] + a*Sin[d + e*x])^3) + (5*(Cos[d + e*x] - Sin[d + e
*x]))/(96*e*(a^2 + a^2*Cos[d + e*x] + a^2*Sin[d + e*x])^2) - (19*(a*Cos[d +
 e*x] - a*Sin[d + e*x]))/(96*e*(a^5 + a^5*Cos[d + e*x] + a^5*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{\int \frac{-6a+4a\cos(\cdots}{(2a+2a\cos(d\cdots}}{\cdots}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos\cdots}{96e\,(a^2+a^2\cdots}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos\cdots}{96e\,(a^2+a^2\cdots}$$

$$= -\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(\cos\cdots}{96e\,(a^2+a^2\cdots}$$

$$= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\cos(d+ex)-\sin(d\cdots}{48ae(a+a\cos(d+ex)+a\cdots}$$

Mathematica [A]
time = 0.71, size = 247, normalized size = 1.47

$$\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)}{64a^4e} + \frac{\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3} + \frac{5}{192a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{19\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)} + \frac{19\tan\left(\frac{1}{2}(d+ex)\right)}{192a^4e} + \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)\tan\left(\frac{1}{2}(d+ex)\right)}{384a^4e}$$

Antiderivative was successfully verified.

[In] Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]

[Out] Log[Cos[(d + e*x)/2]]/(4*a^4*e) - Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(4*a^4*e) - Sec[(d + e*x)/2]^2/(64*a^4*e) + Sin[(d + e*x)/2]/(96*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3) + 5/(192*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^2) + (19*Sin[(d + e*x)/2])/(96*a^4*e*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])) + (19*Tan[(d + e*x)/2])/(192*a^4*e) + (Sec[(d + e*x)/2]^2*Tan[(d + e*x)/2])/(384*a^4*e)

Maple [A]

time = 0.42, size = 106, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+13\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-32\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{36}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{12}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{}{3(1+\mathrm{t}}}{128e\,a^4}$
default	$\frac{\frac{\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+13\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-32\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{36}{1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}+\frac{12}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{}{3(1+\mathrm{t}}}{128e\,a^4}$
norman	$\frac{\frac{\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)}{16ae}-\frac{\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)}{128ae}+\frac{\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)}{384ae}-\frac{15}{32ae}-\frac{147\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{128ae}-\frac{99\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{128ae}}{a^3\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}-\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a^4e}$
risch	$\frac{\left(-\frac{1}{96}+\frac{i}{96}\right)\left(60i e^{4i(ex+d)}+24\,e^{5i(ex+d)}+152i e^{3i(ex+d)}+60\,e^{4i(ex+d)}+111i e^{2i(ex+d)}-111\,e^{2i(ex+d)}-19-19i-90\,e^{i(ex+d)}\right.}{e\,a^4\left(i e^{i(ex+d)}+e^{2i(ex+d)}+i+e^{i(ex+d)}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)

[Out] 1/128/e/a^4*(1/3*tan(1/2*d+1/2*e*x)^3-2*tan(1/2*d+1/2*e*x)^2+13*tan(1/2*d+1/2*e*x)-32*ln(1+tan(1/2*d+1/2*e*x))-36/(1+tan(1/2*d+1/2*e*x))+12/(1+tan(1/2*d+1/2*e*x))^2-8/3/(1+tan(1/2*d+1/2*e*x))^3)

Maxima [A]

time = 0.29, size = 225, normalized size = 1.34

$$-\frac{1}{384}\left(\frac{4\left(\frac{45\sin(xe+d)}{\cos(xe+d)+1}+\frac{27\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+20\right)}{a^4+\frac{3a^4\sin(xe+d)}{\cos(xe+d)+1}+\frac{3a^4\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{a^4\sin(xe+d)^3}{(\cos(xe+d)+1)^3}}-\frac{\frac{39\sin(xe+d)}{\cos(xe+d)+1}-\frac{6\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{\sin(xe+d)^3}{(\cos(xe+d)+1)^3}}{a^4}+\frac{96\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{a^4}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")

[Out] -1/384*(4*(45*sin(x*e + d)/(cos(x*e + d) + 1) + 27*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + 20)/(a^4 + 3*a^4*sin(x*e + d)/(cos(x*e + d) + 1) + 3*a^4*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + a^4*sin(x*e + d)^3/(cos(x*e + d) + 1)^3) - (39*sin(x*e + d)/(cos(x*e + d) + 1) - 6*sin(x*e + d)^2/(cos(x*e + d) + 1)

```
^2 + sin(x*e + d)^3/(cos(x*e + d) + 1)^3)/a^4 + 96*log(sin(x*e + d)/(cos(x*
e + d) + 1) + 1)/a^4)*e^(-1)
```

Fricas [A]
time = 1.81, size = 265, normalized size = 1.58

$$\frac{38 \cos\left(xe+d\right)^{3}+66 \cos\left(xe+d\right)^{2}+24 \left(\cos\left(xe+d\right)^{3}-\left(\cos\left(xe+d\right)^{2}+3 \cos\left(xe+d\right)+2\right) \sin\left(xe+d\right)-3 \cos\left(xe+d\right)-2\right) \log\left(\frac{1}{2} \cos\left(xe+d\right)+\frac{1}{2}\right)-24 \left(\cos\left(xe+d\right)^{3}-\left(\cos\left(xe+d\right)^{2}+3 \cos\left(xe+d\right)+2\right) \sin\left(xe+d\right)-3 \cos\left(xe+d\right)-2\right) \log\left(\sin\left(xe+d\right)+1\right)+\left(38 \cos\left(xe+d\right)^{2}-35\right) \sin\left(xe+d\right)-3 \cos\left(xe+d\right)-33}{192 \left(a^{4} \cos\left(xe+d\right)^{3} e-3 a^{4} \cos\left(xe+d\right) e-2 a^{4} e-\left(a^{4} \cos\left(xe+d\right)^{2} e+3 a^{4} \cos\left(xe+d\right) e+2 a^{4} e\right) \sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(38*cos(x*e + d)^3 + 66*cos(x*e + d)^2 + 24*(cos(x*e + d)^3 - (cos(x*
e + d)^2 + 3*cos(x*e + d) + 2)*sin(x*e + d) - 3*cos(x*e + d) - 2)*log(1/2*c
os(x*e + d) + 1/2) - 24*(cos(x*e + d)^3 - (cos(x*e + d)^2 + 3*cos(x*e + d)
+ 2)*sin(x*e + d) - 3*cos(x*e + d) - 2)*log(sin(x*e + d) + 1) + (38*cos(x*e
 + d)^2 - 35)*sin(x*e + d) - 3*cos(x*e + d) - 33)/(a^4*cos(x*e + d)^3*e - 3
*a^4*cos(x*e + d)*e - 2*a^4*e - (a^4*cos(x*e + d)^2*e + 3*a^4*cos(x*e + d)*
e + 2*a^4*e)*sin(x*e + d))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(156) = 312$.
time = 11.63, size = 792, normalized size = 4.71

$$\begin{cases} -\frac{96 \log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right) \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{288 \log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right) \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{288 \log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right) \tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{96 \log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}+\frac{\tan^{6}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{3 \tan^{5}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}+\frac{24 \tan^{4}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{297 \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{441 \tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e}-\frac{180}{384 a^{4} e \tan^{3}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan^{2}\left(\frac{d}{2}+\frac{ex}{2}\right)+1152 a^{4} e \tan\left(\frac{d}{2}+\frac{ex}{2}\right)+384 a^{4} e} & \text{for}\: e \neq 0 \\ \frac{x}{\left(2 a \sin\left(d\right)+2 a \cos\left(d\right)+2 a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

```
[Out] Piecewise((-96*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**3/(384*a**4*e*ta
n(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 +
 e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(
384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**
4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2
+ e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2
+ 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 96*log(tan(d/2 + e*x/2) + 1)
/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a
**4*e*tan(d/2 + e*x/2) + 384*a**4*e) + tan(d/2 + e*x/2)**6/(384*a**4*e*tan(
d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e
*x/2) + 384*a**4*e) - 3*tan(d/2 + e*x/2)**5/(384*a**4*e*tan(d/2 + e*x/2)**3
 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**
4*e) + 24*tan(d/2 + e*x/2)**4/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e
*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 297*tan
(d/2 + e*x/2)**2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*
x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 441*tan(d/2 + e*x/2)
/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a
**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 180/(384*a**4*e*tan(d/2 + e*x/2)**3
```

```
+ 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4
*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**4, True))
```

Giac [A]
time = 0.43, size = 132, normalized size = 0.79

$$-\frac{\frac{96 \log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right|\right)}{a^4}-\frac{4\left(44\,\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+105\,\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+87\,\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+24\right)}{a^4\left(\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right)^3}-\frac{a^8\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-6\,a^8\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+39\,a^8\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{a^{12}}}{384\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(96*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^4 - 4*(44*tan(1/2*e*x + 1/2
*d)^3 + 105*tan(1/2*e*x + 1/2*d)^2 + 87*tan(1/2*e*x + 1/2*d) + 24)/(a^4*(ta
n(1/2*e*x + 1/2*d) + 1)^3) - (a^8*tan(1/2*e*x + 1/2*d)^3 - 6*a^8*tan(1/2*e*
x + 1/2*d)^2 + 39*a^8*tan(1/2*e*x + 1/2*d))/a^12)/e
```

Mupad [B]
time = 2.44, size = 161, normalized size = 0.96

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,a^4\,e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^4\,e}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^4\,e}+\frac{13\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{128\,a^4\,e}-\frac{9\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+15\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+\frac{20}{3}}{e\left(32\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+96\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+96\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+32\,a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

```
[Out] tan(d/2 + (e*x)/2)^3/(384*a^4*e) - tan(d/2 + (e*x)/2)^2/(64*a^4*e) - log(ta
n(d/2 + (e*x)/2) + 1)/(4*a^4*e) + (13*tan(d/2 + (e*x)/2))/(128*a^4*e) - (15
*tan(d/2 + (e*x)/2) + 9*tan(d/2 + (e*x)/2)^2 + 20/3)/(e*(96*a^4*tan(d/2 + (
e*x)/2)^2 + 32*a^4*tan(d/2 + (e*x)/2)^3 + 32*a^4 + 96*a^4*tan(d/2 + (e*x)/2
)))
```

3.374 $\int (2a-2a\cos (d+ex)+2c\sin (d+ex))^3\,dx$

Optimal. Leaf size=157

$$4a\left(5a^2+3c^2\right)x-\frac{4c(15a^2+4c^2)\cos (d+ex)}{3e}-\frac{4a(15a^2+4c^2)\sin (d+ex)}{3e}-\frac{20(ac\cos (d+ex)+a^2\sin (d+e$$

```
[Out] 4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e-4/3*a*(15*a^2+4*c^2)*
sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)+a^2*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x
+d))/e-8/3*(c*cos(e*x+d)+a*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

Rubi [A]

time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3199, 3225, 2717, 2718}

$$-\frac{4a(15a^2+4c^2)\sin (d+ex)}{3e}-\frac{4c(15a^2+4c^2)\cos (d+ex)}{3e}+4ax\left(5a^2+3c^2\right)-\frac{20(a^2\sin (d+ex)+ac\cos (d+ex))\left(a(-\cos (d+ex))+a+c\sin (d+ex)\right)}{3e}-\frac{8(a\sin (d+ex)+c\cos (d+ex))(a(-\cos (d+ex))+a+c\sin (d+ex))^2}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*c^2)*x - (4*c*(15*a^2 + 4*c^2)*Cos[d + e*x])/(3*e) - (4*a*(1
5*a^2 + 4*c^2)*Sin[d + e*x])/(3*e) - (20*(a*c*Cos[d + e*x] + a^2*Sin[d + e*
x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/(3*e) - (8*(c*Cos[d + e*x] + a*S
in[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = -\frac{8(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(
3e}$$
$$= -\frac{20(ac\cos(d+ex)+a^2\sin(d+ex))\,(a-a\cos(d+ex)+c\,s
3e}$$
$$= 4a(5a^2+3c^2)\,x-\frac{20(ac\cos(d+ex)+a^2\sin(d+ex))\,(a-a
3e}$$
$$= 4a(5a^2+3c^2)\,x-\frac{4c(15a^2+4c^2)\cos(d+ex)}{3e}-\frac{4a(15a^2+4$$

Mathematica [A]
time = 0.33, size = 136, normalized size = 0.87

$$\frac{2(6a(5a^2+3c^2)\,(d+ex)-9c(5a^2+c^2)\cos(d+ex)+18a^2c\cos(2(d+ex))+c(-3a^2+c^2)\cos(3(d+ex))-9a(5a^2+c^2)\sin(d+ex)+9a(a^2-c^2)\sin(2(d+ex))-a(a^2-3c^2)\sin(3(d+ex)))}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] + 18*a^2
*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] - 9*a*(5*a^2 + c^2)
*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] - a*(a^2 - 3*c^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A]
time = 0.26, size = 178, normalized size = 1.13

method	result
derivativedivides	$\frac{-\frac{8a^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-8a^2c\left(\cos^3(ex+d)\right)+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a\,c^2\left(\sin^3(ex+d)\right)+24a^2c\left(\cos^2(\right.$
default	$\frac{-\frac{8a^3\left(2+\cos^2(ex+d)\right)\sin(ex+d)}{3}-8a^2c\left(\cos^3(ex+d)\right)+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a\,c^2\left(\sin^3(ex+d)\right)+24a^2c\left(\cos^2(\right.$

risch	$20a^3x + 12a\,c^2x - \frac{30c\cos(ex+d)a^2}{e} - \frac{6c^3\cos(ex+d)}{e} - \frac{30a^3\sin(ex+d)}{e} - \frac{6a\sin(ex+d)c^2}{e} - \frac{2c\cos(3ex+\cdots}{e}$
norman	$\frac{-\frac{192a^2c+32c^3}{3e}+4a(5a^2+3c^2)x-\frac{192a^2c\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}-\frac{(192a^2c+32c^3)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}-\frac{8a(5a^2+3c^2)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}-64a(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/e*(-1/3*a^3*(2+cos(e*x+d)^2)*sin(e*x+d)-a^2*c*cos(e*x+d)^3+3*a^3*(1/2*cos
(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-a*c^2*sin(e*x+d)^3+3*a^2*c*cos(e*x+d)^2-3
*a^3*sin(e*x+d)-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a*c^2*(-1/2*cos(e*x+d
)*sin(e*x+d)+1/2*e*x+1/2*d)-3*a^2*c*cos(e*x+d)+a^3*(e*x+d))
```

Maxima [A]
time = 0.26, size = 192, normalized size = 1.22

$$-8\,a^2c\cos(xe+d)^3\,e^{(-1)} - 8\,ac^2e^{(-1)}\sin(xe+d)^3 + \frac{8}{3}\left(\sin(xe+d)^3 - 3\,\sin(xe+d)\right)a^3e^{(-1)} + \frac{8}{3}\left(\cos(xe+d)^3 - 3\,\cos(xe+d)\right)c^3e^{(-1)} + 8\,a^3x - 24\left(c\cos(xe+d)\,e^{(-1)} + ae^{(-1)}\sin(xe+d)\right)a^2 + 6\left(4\,ac\cos(xe+d)^2\,e^{(-1)} + (2\,xe + 2\,d + \sin(2\,xe + 2\,d))a^2e^{(-1)} + (2\,xe + 2\,d - \sin(2\,xe + 2\,d))c^2e^{(-1)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a^2*c*cos(x*e + d)^3*e^(-1) - 8*a*c^2*e^(-1)*sin(x*e + d)^3 + 8/3*(sin(x
*e + d)^3 - 3*sin(x*e + d))*a^3*e^(-1) + 8/3*(cos(x*e + d)^3 - 3*cos(x*e +
d))*c^3*e^(-1) + 8*a^3*x - 24*(c*cos(x*e + d)*e^(-1) + a*e^(-1)*sin(x*e + d
))*a^2 + 6*(4*a*c*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d + sin(2*x*e + 2*d))*
a^2*e^(-1) + (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1))*a
```

Fricas [A]
time = 1.68, size = 140, normalized size = 0.89

$$\frac{4}{3}\left(18\,a^2c\cos(xe+d)^2 - 2\left(3\,a^2c - c^3\right)\cos(xe+d)^3 + 3\left(5\,a^3 + 3\,ac^2\right)xe - 6\left(3\,a^2c + c^3\right)\cos(xe+d) - \left(22\,a^3 + 6\,ac^2 + 2\left(a^3 - 3\,ac^2\right)\cos(xe+d)^2 - 9\left(a^3 - ac^2\right)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*c*cos(x*e + d)^2 - 2*(3*a^2*c - c^3)*cos(x*e + d)^3 + 3*(5*a^3
+ 3*a*c^2)*x*e - 6*(3*a^2*c + c^3)*cos(x*e + d) - (22*a^3 + 6*a*c^2 + 2*(a^
3 - 3*a*c^2)*cos(x*e + d)^2 - 9*(a^3 - a*c^2)*cos(x*e + d))*sin(x*e + d))*e
^(-1)
```

Sympy [A]
time = 0.16, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x - \frac{16a^3\sin^3(d+ex)}{3e} - \frac{8a^3\sin(d+ex)\cos^2(d+ex)}{e} + \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} - \frac{24a^3\sin(d+ex)}{e} - \frac{24a^2c\sin^2(d+ex)}{e} - \frac{8a^2c\cos^3(d+ex)}{e} - \frac{24a^2c\cos(d+ex)}{e} + 12ac^2x\sin^2(d+ex) + 12ac^2x\cos^2(d+ex) - \frac{8ac^2\sin^3(d+ex)}{e} - \frac{12ac^2\sin(d+ex)\cos(d+ex)}{e} - \frac{8c^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{16c^3\cos^3(d+ex)}{3e} & \text{for } e \neq 0 \\ x(-2a\cos(d) + 2a + 2c\sin(d))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise(((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 16*a**3*sin(d + e*x)**3/(3*e) - 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e +
12*a**3*sin(d + e*x)*cos(d + e*x)/e - 24*a**3*sin(d + e*x)/e - 24*a**2*c*si
n(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e + 1
2*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 - 8*a*c**2*sin(d +
 e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e*x)**2
*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(-2*a*cos(d)
 + 2*a + 2*c*sin(d))**3, True))
```

Giac [A]
time = 0.43, size = 151, normalized size = 0.96

$$\frac{12\,a^2c\cos\left(2\,ex+2\,d\right)}{e}+4\left(5\,a^3+3\,ac^2\right)x-\frac{2\left(3\,a^2c-c^3\right)\cos\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\left(5\,a^2c+c^3\right)\cos\left(ex+d\right)}{e}-\frac{2\left(a^3-3\,ac^2\right)\sin\left(3\,ex+3\,d\right)}{3\,e}+\frac{6\left(a^3-ac^2\right)\sin\left(2\,ex+2\,d\right)}{e}-\frac{6\left(5\,a^3+ac^2\right)\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*c*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*c^2)*x - 2/3*(3*a^2*c - c^3)*c
os(3*e*x + 3*d)/e - 6*(5*a^2*c + c^3)*cos(e*x + d)/e - 2/3*(a^3 - 3*a*c^2)*
sin(3*e*x + 3*d)/e + 6*(a^3 - a*c^2)*sin(2*e*x + 2*d)/e - 6*(5*a^3 + a*c^2)
*sin(e*x + d)/e
```

Mupad [B]
time = 3.21, size = 258, normalized size = 1.64

$$\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(5\,a^2+3\,c^2\right)}{40\,a^3+24\,a\,c^2}\right)\,\left(5\,a^2+3\,c^2\right)}{e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(40\,a^3+24\,a\,c^2\right)+64\,a^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\left(24\,a\,c^2-88\,a^3\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(\frac{320\,a^3}{3}+64\,a\,c^2\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(192\,a^2\,c+32\,c^3\right)+\frac{32\,c^3}{3}+192\,a^2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}-\frac{8\,a\,\left(5\,a^2+3\,c^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*c^2))/(24*a*c^2 + 40*a^3))*(5*
a^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)*(24*a*c^2 + 40*a^3) + 64*a^2*c - tan(
d/2 + (e*x)/2)^5*(24*a*c^2 - 88*a^3) + tan(d/2 + (e*x)/2)^3*(64*a*c^2 + (32
0*a^3)/3) + tan(d/2 + (e*x)/2)^2*(192*a^2*c + 32*c^3) + (32*c^3)/3 + 192*a^
2*c*tan(d/2 + (e*x)/2)^4)/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)
^4 + tan(d/2 + (e*x)/2)^6 + 1)) - (8*a*(5*a^2 + 3*c^2)*(atan(tan(d/2 + (e*x
)/2)) - (e*x)/2))/e
```

3.375 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2(3a^2+c^2)\,x-\frac{6ac\cos(d+ex)}{e}-\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin}{e}$$

[Out] 2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e-6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)+a*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))/e

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3199, 2717, 2718}

$$2x(3a^2+c^2)-\frac{6a^2\sin(d+ex)}{e}-\frac{6ac\cos(d+ex)}{e}-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]

[Out] 2*(3*a^2 + c^2)*x - (6*a*c*Cos[d + e*x])/e - (6*a^2*Sin[d + e*x])/e - (2*(c*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))/e

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = -\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex)}{e}$$
$$= 2\left(3a^2+c^2\right)x - \frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d}{e}$$
$$= 2\left(3a^2+c^2\right)x - \frac{6ac\cos(d+ex)}{e} - \frac{6a^2\sin(d+ex)}{e} - \frac{2(c\cos(}{}$$

Mathematica [A]
time = 0.12, size = 92, normalized size = 1.14

$$4\left(\frac{(3a^2+c^2)(d+ex)}{2e} - \frac{2ac\cos(d+ex)}{e} + \frac{ac\cos(2(d+ex))}{2e} - \frac{2a^2\sin(d+ex)}{e} + \frac{(a^2-c^2)\sin(2(d+ex))}{4e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e + (a*c*Cos[2*(d
 + e*x)])/(2*e) - (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A]
time = 0.21, size = 100, normalized size = 1.23

method	result
risch	$6a^2x+2x\,c^2-\frac{8ac\cos(ex+d)}{e}-\frac{8a^2\sin(ex+d)}{e}+\frac{2ac\cos(2ex+2d)}{e}+\frac{\sin(2ex+2d)a^2}{e}-\frac{\sin(2ex+2d)c^2}{e}$
derivativedivides	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4ac\left(\cos^2(ex+d)\right)-8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
default	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4ac\left(\cos^2(ex+d)\right)-8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
norman	$\frac{(6a^2+2c^2)x+(6a^2+2c^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(12a^2+4c^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{16ac\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}-\frac{4\left(3a^2+c^2\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/e*(a^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+a*c*cos(e*x+d)^2-2*a^2*s
in(e*x+d)+c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-2*a*c*cos(e*x+d)+a
^2*(e*x+d))
```

Maxima [A]
time = 0.27, size = 100, normalized size = 1.23

$$4\,ac\cos\left(xe+d\right)^2e^{(-1)} + \left(2\,xe+2\,d+\sin\left(2\,xe+2\,d\right)\right)a^2e^{(-1)} + \left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)c^2e^{(-1)} + 4\,a^2x - 8\,\left(c\cos\left(xe+d\right)e^{(-1)} + ae^{(-1)}\sin\left(xe+d\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a*c*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d + sin(2*x*e + 2*d))*a^2*e^(-1) +
 (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1) + 4*a^2*x - 8*(c*cos(x*e + d)*
e^(-1) + a*e^(-1)*sin(x*e + d))*a
```

Fricas [A]
time = 1.64, size = 75, normalized size = 0.93

$$2\left(2\,ac\cos\left(xe+d\right)^{2}-4\,ac\cos\left(xe+d\right)+\left(3\,a^{2}+c^{2}\right)xe-\left(4\,a^{2}-\left(a^{2}-c^{2}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*c*cos(x*e + d)^2 - 4*a*c*cos(x*e + d) + (3*a^2 + c^2)*x*e - (4*a^2 -
 (a^2 - c^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.
time = 0.12, size = 170, normalized size = 2.10

$$\begin{cases}2a^{2}x\sin^{2}\left(d+ex\right)+2a^{2}x\cos^{2}\left(d+ex\right)+4a^{2}x+\frac{2a^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{8a^{2}\sin\left(d+ex\right)}{e}-\frac{4ac\sin^{2}\left(d+ex\right)}{e}-\frac{8ac\cos\left(d+ex\right)}{e}+2c^{2}x\sin^{2}\left(d+ex\right)+2c^{2}x\cos^{2}\left(d+ex\right)-\frac{2c^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\left(-2a\cos\left(d\right)+2a+2c\sin\left(d\right)\right)^{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x +
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*sin(d + e*x)/e - 4*a*c*sin(d +
 e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*cos
(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*cos(
d) + 2*a + 2*c*sin(d))**2, True))
```

Giac [A]
time = 0.41, size = 78, normalized size = 0.96

$$2\left(3\,a^{2}+c^{2}\right)x+\frac{2\,ac\cos\left(2\,ex+2\,d\right)}{e}-\frac{8\,ac\cos\left(ex+d\right)}{e}-\frac{8\,a^{2}\sin\left(ex+d\right)}{e}+\frac{\left(a^{2}-c^{2}\right)\sin\left(2\,ex+2\,d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2 + c^2)*x + 2*a*c*cos(2*e*x + 2*d)/e - 8*a*c*cos(e*x + d)/e - 8*a^2
*sin(e*x + d)/e + (a^2 - c^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 2.50, size = 84, normalized size = 1.04

$$\frac{a^2 \sin(2d+2ex) - 8a^2 \sin(d+ex) - c^2 \sin(2d+2ex) + 16ac\sin\left(\frac{d}{2}+\frac{ex}{2}\right)^2 - 4ac\sin(d+ex)^2 + 6a^2ex + 2c^2ex}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (a^2*sin(2*d + 2*e*x) - 8*a^2*sin(d + e*x) - c^2*sin(2*d + 2*e*x) + 16*a*c*
sin(d/2 + (e*x)/2)^2 - 4*a*c*sin(d + e*x)^2 + 6*a^2*e*x + 2*c^2*e*x)/e
```

3.376 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$2ax - \frac{2c\cos(d+ex)}{e} - \frac{2a\sin(d+ex)}{e}$$

```
[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2717, 2718}

$$-\frac{2a\sin(d+ex)}{e} + 2ax - \frac{2c\cos(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx &= 2ax - (2a)\int \cos(d+ex)\,dx + (2c)\int \sin(d+ex)\,dx \\ &= 2ax - \frac{2c\cos(d+ex)}{e} - \frac{2a\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.83

$$2ax - \frac{2c\cos(d)\cos(ex)}{e} - \frac{2a\cos(ex)\sin(d)}{e} - \frac{2a\cos(d)\sin(ex)}{e} + \frac{2c\sin(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*c*Cos[d]*Cos[e*x])/e - (2*a*Cos[e*x]*Sin[d])/e - (2*a*Cos[d]*Sin
[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.03

method	result	size
default	$2ax-\frac{2c\cos(ex+d)}{e}-\frac{2a\sin(ex+d)}{e}$	30
risch	$2ax-\frac{2c\cos(ex+d)}{e}-\frac{2a\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a-2c\cos(ex+d)-2a\sin(ex+d)}{e}$	32
norman	$\frac{\frac{4c\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+2ax-\frac{4a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}+2ax\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.00

$$-2\,c\cos\left(xe+d\right)e^{(-1)}-2\,ae^{(-1)}\sin\left(xe+d\right)+2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -2*c*cos(x*e + d)*e^(-1) - 2*a*e^(-1)*sin(x*e + d) + 2*a*x
```

Fricas [A]
time = 1.72, size = 30, normalized size = 1.03

$$2\left(axe-c\cos\left(xe+d\right)-a\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*x*e - c*cos(x*e + d) - a*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.06, size = 39, normalized size = 1.34

$$2ax-2a\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)+2c\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*P
iecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A]
time = 0.39, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,c\cos\left(ex+d\right)}{e} - \frac{2\,a\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*x - 2*c*cos(e*x + d)/e - 2*a*sin(e*x + d)/e
```

Mupad [B]
time = 2.43, size = 29, normalized size = 1.00

$$2\,a\,x - \frac{2\,c\,\cos\left(d+e\,x\right)}{e} - \frac{2\,a\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

```
[Out] 2*a*x - (2*c*cos(d + e*x))/e - (2*a*sin(d + e*x))/e
```

3.377 $\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

Optimal. Leaf size=25

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

```
[Out] -1/2*ln(a+c*cot(1/2*e*x+1/2*d))/c/e
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3200, 31}

$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] -1/2*Log[a + c*Cot[(d + e*x)/2]]/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3200

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Dist[-f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{2a+2cx}\,dx,x,\cot\left(\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= -\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 2.00

$$\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Sin[(d + e*x)/2]] - Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])/(2*c
*e)
```

Maple [A]
time = 0.32, size = 40, normalized size = 1.60

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}+\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}}{2e}$	40
default	$\frac{-\frac{\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}+\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}}{2e}$	40
norman	$\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce}-\frac{\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce}$	42
risch	$-\frac{\ln\left(e^{i(ex+d)}+\frac{ic-a}{ic+a}\right)}{2ce}+\frac{\ln\left(e^{i(ex+d)}-1\right)}{2ce}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(-1/c*ln(c+a*tan(1/2*d+1/2*e*x))+1/c*ln(tan(1/2*d+1/2*e*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.
time = 0.29, size = 57, normalized size = 2.28

$$-\frac{1}{2}\left(\frac{\log\left(c+\frac{a\sin(xe+d)}{\cos(xe+d)+1}\right)}{c}-\frac{\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)}{c}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(c + a*sin(x*e + d)/(cos(x*e + d) + 1))/c - log(sin(x*e + d)/(cos(
x*e + d) + 1))/c)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(22) = 44.
time = 1.66, size = 62, normalized size = 2.48

$$-\frac{\left(\log\left(ac\sin\left(xe+d\right)+\frac{1}{2}a^2+\frac{1}{2}c^2-\frac{1}{2}\left(a^2-c^2\right)\cos\left(xe+d\right)\right)-\log\left(-\frac{1}{2}\cos\left(xe+d\right)+\frac{1}{2}\right)\right)e^{(-1)}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(x*e +
d)) - log(-1/2*cos(x*e + d) + 1/2))*e^(-1)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(20) = 40$.
time = 0.67, size = 95, normalized size = 3.80

$$\begin{cases} \frac{\tilde{\infty}x}{\sin(d)} & \text{for } a = 0 \wedge c = 0 \wedge e = 0 \\ \frac{x}{-2a\cos(d)+2a+2c\sin(d)} & \text{for } e = 0 \\ -\frac{1}{2ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)} & \text{for } c = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{for } a = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}\right)}{2ce} + \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), (x/(-2*a*cos(d) +
 2*a + 2*c*sin(d)), Eq(e, 0)), (-1/(2*a*e*tan(d/2 + e*x/2)), Eq(c, 0)), (lo
g(tan(d/2 + e*x/2))/(2*c*e), Eq(a, 0)), (-log(tan(d/2 + e*x/2) + c/a)/(2*c*
e) + log(tan(d/2 + e*x/2))/(2*c*e), True))
```

Giac [A]
time = 0.42, size = 41, normalized size = 1.64

$$-\frac{\frac{\log\left(\left|a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c\right|\right)}{c} - \frac{\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)\right|\right)}{c}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(a*tan(1/2*e*x + 1/2*d) + c))/c - log(abs(tan(1/2*e*x + 1/2*d)
))/c)/e
```

Mupad [B]
time = 2.62, size = 26, normalized size = 1.04

$$-\frac{\operatorname{atanh}\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{c}+1\right)}{ce}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
```

```
[Out] -atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)/(c*e)
```

3.378 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=75

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] 1/4*a*ln(a+c*cot(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)-a*sin(e*x+d))/c^2
/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 12, 3200, 31}

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*x
])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3200

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Dist[-f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))}+\frac{\int -\frac{2}{2a-2a\cos(d+ex)}}{4c} \\ &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))}-\frac{a\int \frac{1}{2a-2a\cos(d+ex)}}{2c} \\ &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))}+\frac{a\text{Subst}\left(\int \frac{1}{2a+2cx}\right.}{} \\ &= \frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)+a\sin(d}{4c^2e(a-a\cos(d+ex)+c\sin}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(75) = 150.
time = 0.31, size = 229, normalized size = 3.05

$$-\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(\cos(d+ex)\left(a^2+2c^2-2a^2\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)+2a^2\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)+a\left(a\left(-1+2\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-2\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)+c\left(1+2\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-2\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)\sin(d+ex)\right)\right)}{4c^3e(a-a\cos(d+ex)+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/4*(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(Cos[d + e
*x]*(a^2 + 2*c^2 - 2*a^2*Log[Sin[(d + e*x)/2]] + 2*a^2*Log[c*Cos[(d + e*x)/
2] + a*Sin[(d + e*x)/2]]) + a*(a*(-1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c*Co
s[(d + e*x)/2] + a*Sin[(d + e*x)/2]]) + c*(1 + 2*Log[Sin[(d + e*x)/2]] - 2*
Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])*Sin[d + e*x])))/(c^3*e*(a - a
*Cos[d + e*x] + c*Sin[d + e*x])^2)
```

Maple [A]
time = 0.41, size = 88, normalized size = 1.17

method	result	size
derivativedivides	$\frac{-\frac{a^2+c^2}{2c^2a\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{a\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}-\frac{1}{2c^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{a\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}}{4e}$	88
default	$\frac{-\frac{a^2+c^2}{2c^2a\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{a\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}-\frac{1}{2c^2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}-\frac{a\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c^3}}{4e}$	88

norman	$\frac{-\frac{1}{8ce}-\frac{\left(2a^{2}+c^{2}\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a\,c^{2}e}}{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^{3}e}+\frac{a\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4c^{3}e}$	109
risch	$-\frac{i\left(-ia\,e^{i(ex+d)}+ia+c\right)}{2c^{2}e\left(c\,e^{2i(ex+d)}-ia\,e^{2i(ex+d)}-c+2ia\,e^{i(ex+d)}-ia\right)}+\frac{a\ln\left(e^{i(ex+d)}+\frac{ic-a}{ic+a}\right)}{4c^{3}e}-\frac{a\ln\left(e^{i(ex+d)}-1\right)}{4c^{3}e}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e*(-1/2*(a^2+c^2)/c^2/a/(c+a*tan(1/2*d+1/2*e*x))+1/c^3*a*ln(c+a*tan(1/2
*d+1/2*e*x))-1/2/c^2/tan(1/2*d+1/2*e*x)-1/c^3*a*ln(tan(1/2*d+1/2*e*x)))
```

Maxima [A]
time = 0.30, size = 146, normalized size = 1.95

$$-\frac{1}{8}\left(\frac{ac+\frac{(2\,a^{2}+c^{2})\sin(xe+d)}{\cos(xe+d)+1}}{\frac{ac^{3}\sin(xe+d)}{\cos(xe+d)+1}+\frac{a^{2}c^{2}\sin(xe+d)^{2}}{(\cos(xe+d)+1)^{2}}}-\frac{2\,a\log\left(c+\frac{a\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^{3}}+\frac{2\,a\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^{3}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/8*((a*c + (2*a^2 + c^2)*sin(x*e + d)/(cos(x*e + d) + 1))/(a*c^3*sin(x*e
+ d)/(cos(x*e + d) + 1) + a^2*c^2*sin(x*e + d)^2/(cos(x*e + d) + 1)^2) - 2*
a*log(c + a*sin(x*e + d)/(cos(x*e + d) + 1))/c^3 + 2*a*log(sin(x*e + d)/(co
s(x*e + d) + 1))/c^3)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(75) = 150.
time = 1.90, size = 176, normalized size = 2.35

$$\frac{2\,c^{2}\cos\left(xe+d\right)+2\,ac\sin\left(xe+d\right)+\left(a^{2}\cos\left(xe+d\right)-ac\sin\left(xe+d\right)-a^{2}\right)\log\left(ac\sin\left(xe+d\right)+\frac{1}{2}\,a^{2}+\frac{1}{2}\,c^{2}-\frac{1}{2}\,\left(a^{2}-c^{2}\right)\cos\left(xe+d\right)\right)-\left(a^{2}\cos\left(xe+d\right)-ac\sin\left(xe+d\right)-a^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(xe+d\right)+\frac{1}{2}\right)}{8\,\left(ac^{3}\cos\left(xe+d\right)e-c^{4}e\sin\left(xe+d\right)-ac^{3}e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*c^2*cos(x*e + d) + 2*a*c*sin(x*e + d) + (a^2*cos(x*e + d) - a*c*sin(
x*e + d) - a^2)*log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*
cos(x*e + d)) - (a^2*cos(x*e + d) - a*c*sin(x*e + d) - a^2)*log(-1/2*cos(x*
e + d) + 1/2))/(a*c^3*cos(x*e + d)*e - c^4*e*sin(x*e + d) - a*c^3*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 110, normalized size = 1.47

$$\frac{\frac{2\,a\log\left(\left|a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c\right|\right)}{c^3}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right|\right)}{c^3}-\frac{2\,a^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+ac}{\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right)ac^2}}{8\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*a*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/c^3 - 2*a*log(abs(tan(1/2*e*x
 + 1/2*d)))/c^3 - (2*a^2*tan(1/2*e*x + 1/2*d) + c^2*tan(1/2*e*x + 1/2*d) +
a*c)/((a*tan(1/2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))*a*c^2))/e
```

Mupad [B]
time = 2.55, size = 91, normalized size = 1.21

$$\frac{a\,\mathrm{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)}{2\,c^3\,e}-\frac{\frac{1}{c}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a^2+c^2\right)}{a\,c^2}}{e\,\left(8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+8\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

```
[Out] (a*atanh((2*a*tan(d/2 + (e*x)/2))/c + 1))/(2*c^3*e) - (1/c + (tan(d/2 + (e*
x)/2)*(2*a^2 + c^2))/(a*c^2))/(e*(8*c*tan(d/2 + (e*x)/2) + 8*a*tan(d/2 + (e
*x)/2)^2))
```

3.379 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=134

$$-\frac{(3a^2+c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(d+ex)+\ldots}{16c^4e(a-a\cos(d+e\ldots}$$

```
[Out] -1/16*(3*a^2+c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)-a*si
n(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)+a^2*si
n(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 3232, 3200, 31}

$$\frac{3(a^2\sin(d+ex)+ac\cos(d+ex))}{16c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{(3a^2+c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/16*((3*a^2 + c^2)*Log[a + c*Cot[(d + e*x)/2]])/(c^5*e) - (c*Cos[d + e*x]
 + a*Sin[d + e*x])/(16*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*
(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(16*c^4*e*(a - a*Cos[d + e*x] + c*Si
n[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3200

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Dist[-f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2}+\frac{\int \frac{-4a-2a\cos(d\cdots}{(2a-2a\cos(d+\cdots}}{\cdots}\\ &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(d\cdots}{16c^4e(a-a\,c\cdots}\\ &= -\frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2}+\frac{3(ac\cos(d\cdots}{16c^4e(a-a\,c\cdots}\\ &= -\frac{(3a^2+c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e}-\frac{c\cos(d+e\cdots}{16c^2e(a-a\cos(d\cdots}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 350, normalized size = 2.61

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(c^2(-ia+c)(ia+c)\sin^2\left(\frac{1}{2}(d+ex)\right)-6a(a^2+c^2)\sin^3\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)-c^2\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^2+4(3a^2+c^2)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)\sin^2\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^2-4(3a^2+c^2)\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\sin^2\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^2+3ac\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^2\sin(d+ex)\right)}{8c^5e(a-a\cos(d+ex)+c\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(c^2*((-I)*a +
c)*(I*a + c)*Sin[(d + e*x)/2]^2 - 6*a*(a^2 + c^2)*Sin[(d + e*x)/2]^3*(c*Cos
[(d + e*x)/2] + a*Sin[(d + e*x)/2]) - c^2*(c*Cos[(d + e*x)/2] + a*Sin[(d +
e*x)/2])^2 + 4*(3*a^2 + c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Co
s[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 - 4*(3*a^2 + c^2)*Log[c*Cos[(d + e*x
)/2] + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a*Sin[(
d + e*x)/2])^2 + 3*a*c*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2*Sin[d +
e*x]))/(8*c^5*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3)
```

Maple [A]
time = 0.55, size = 176, normalized size = 1.31

method	result
derivativedivides	$\frac{-\frac{-3a^4-2a^2c^2+c^4}{4a^2c^4\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{-a^4-2a^2c^2-c^4}{8a^2c^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{\left(3a^2+c^2\right)\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^5}-\frac{1}{8c^3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2}+\frac{\left(6a^2+2c^2\right)\ln\left(\tan \cdots\right.}{4c^5}\cdots}{8e}$
default	$\frac{-\frac{-3a^4-2a^2c^2+c^4}{4a^2c^4\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{-a^4-2a^2c^2-c^4}{8a^2c^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{\left(3a^2+c^2\right)\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^5}-\frac{1}{8c^3\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2}+\frac{\left(6a^2+2c^2\right)\ln\left(\tan \cdots\right.}{4c^5}\cdots}{8e}$
norman	$\frac{-\frac{1}{64ce}+\frac{a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{16c^2e}-\frac{\left(18a^4+6a^2c^2-c^4\right)\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{64c^5e}-\frac{\left(3a^2+c^2\right)a\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{8c^4e}}{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}+\frac{(3a^2+c^2)\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{16c^5e}-\cdots$
risch	$\frac{-3ia^3e^{3i(ex+d)}-iac^2e^{3i(ex+d)}+9ia^3e^{2i(ex+d)}+3a^2c\,e^{3i(ex+d)}+3iac^2e^{2i(ex+d)}+c^3e^{3i(ex+d)}-9ia^3e^{i(ex+d)}+iac^2e^{i(ex\cdots}{8\left(c\,e^{2i(ex+d)}-ia\,e^{2i(ex+d)}-c+2ia\,e^{i(ex+d)}-ia\right)^2c^4e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/e*(-1/4*(-3*a^4-2*a^2*c^2+c^4)/a^2/c^4/(c+a*tan(1/2*d+1/2*e*x))-1/8*(-a
^4-2*a^2*c^2-c^4)/a^2/c^3/(c+a*tan(1/2*d+1/2*e*x))^2-1/2*(3*a^2+c^2)/c^5*ln
(c+a*tan(1/2*d+1/2*e*x))-1/8/c^3/tan(1/2*d+1/2*e*x)^2+1/4*(6*a^2+2*c^2)/c^5
*ln(tan(1/2*d+1/2*e*x))+3/4/c^4*a/tan(1/2*d+1/2*e*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(137) = 274$.
time = 0.28, size = 280, normalized size = 2.09

$$-\frac{1}{64}\left(\frac{a^2c^3-\frac{4a^3c^2\sin(xe+d)}{\cos(xe+d)+1}-\frac{(18a^4c+6a^2c^3-c^5)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{2(6a^5+2a^3c^2-ac^4)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}}{\frac{a^2c^6\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{2a^3c^5\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{a^4c^4\sin(xe+d)^4}{(\cos(xe+d)+1)^4}}+\frac{4(3a^2+c^2)\log\left(c+\frac{a\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^5}-\frac{4(3a^2+c^2)\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*c^3 - 4*a^3*c^2*sin(x*e + d)/(cos(x*e + d) + 1) - (18*a^4*c + 6
*a^2*c^3 - c^5)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - 2*(6*a^5 + 2*a^3*c^2
- a*c^4)*sin(x*e + d)^3/(cos(x*e + d) + 1)^3)/(a^2*c^6*sin(x*e + d)^2/(cos(
x*e + d) + 1)^2 + 2*a^3*c^5*sin(x*e + d)^3/(cos(x*e + d) + 1)^3 + a^4*c^4*s
in(x*e + d)^4/(cos(x*e + d) + 1)^4) + 4*(3*a^2 + c^2)*log(c + a*sin(x*e + d
)/(cos(x*e + d) + 1))/c^5 - 4*(3*a^2 + c^2)*log(sin(x*e + d)/(cos(x*e + d)
+ 1))/c^5)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(137) = 274$.
time = 1.46, size = 462, normalized size = 3.45

$$\frac{12a^2c^2\cos(xe+d)^2-6a^2c^2-2(3a^2c^2-c^4)\cos(xe+d)+\left(3a^4+4a^2c^2+c^4+(3a^4-2a^2c^2-c^4)\cos(xe+d)^2-2(3a^4+a^2c^2)\cos(xe+d)+2(3a^3c+ac^3-(3a^3c+ac^3)\cos(xe+d))\sin(xe+d)\right)\log\left(ac\sin(xe+d)+\frac{1}{2}a^2+\frac{1}{2}c^2-\frac{1}{2}(a^2-c^2)\cos(xe+d)\right)-\left(3a^4+4a^2c^2+c^4+(3a^4-2a^2c^2-c^4)\cos(xe+d)^2-2(3a^4+a^2c^2)\cos(xe+d)+2(3a^3c+ac^3-(3a^3c+ac^3)\cos(xe+d))\sin(xe+d)\right)\log\left(-\frac{1}{2}\cos(xe+d)+\frac{1}{2}\right)-2(3a^3c-ac^3-3(a^3c-ac^3)\cos(xe+d))\sin(xe+d)}{32\left(2a^2c^5\cos(xe+d)e-(a^2c^5-c^7)\cos(xe+d)^2e-(a^2c^5+c^7)e+2(ac^6\cos(xe+d)e-ac^6e)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*c^2*cos(x*e + d)^2 - 6*a^2*c^2 - 2*(3*a^2*c^2 - c^4)*cos(x*e +
 d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(x*e + d)^2 -
 2*(3*a^4 + a^2*c^2)*cos(x*e + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^3)*
cos(x*e + d))*sin(x*e + d))*log(a*c*sin(x*e + d) + 1/2*a^2 + 1/2*c^2 - 1/2*
(a^2 - c^2)*cos(x*e + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 -
 c^4)*cos(x*e + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(x*e + d) + 2*(3*a^3*c + a*c^
3 - (3*a^3*c + a*c^3)*cos(x*e + d))*sin(x*e + d))*log(-1/2*cos(x*e + d) + 1
/2) - 2*(3*a^3*c - a*c^3 - 3*(a^3*c - a*c^3)*cos(x*e + d))*sin(x*e + d))/(2
*a^2*c^5*cos(x*e + d)*e - (a^2*c^5 - c^7)*cos(x*e + d)^2*e - (a^2*c^5 + c^7
)*e + 2*(a*c^6*cos(x*e + d)*e - a*c^6*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 229, normalized size = 1.71

$$\frac{\frac{4\,(3\,a^2+c^2)\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right|\right)}{c^5}-\frac{4\,(3\,a^3+ac^2)\log\left(\left|a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c\right|\right)}{ac^5}+\frac{12\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+4\,a^3c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-2\,ac^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+18\,a^4c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+6\,a^2c^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2-c^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+4\,a^3c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-a^2c^3}{\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right)^2a^2c^4}}{64\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 + c^2)*log(abs(tan(1/2*e*x + 1/2*d)))/c^5 - 4*(3*a^3 + a*c^2
)*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/(a*c^5) + (12*a^5*tan(1/2*e*x + 1/2*
d)^3 + 4*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 - 2*a*c^4*tan(1/2*e*x + 1/2*d)^3 +
18*a^4*c*tan(1/2*e*x + 1/2*d)^2 + 6*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 - c^5*ta
n(1/2*e*x + 1/2*d)^2 + 4*a^3*c^2*tan(1/2*e*x + 1/2*d) - a^2*c^3)/((a*tan(1/
2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))^2*a^2*c^4))/e
```

Mupad [**B**]
time = 4.47, size = 186, normalized size = 1.39

$$\frac{\frac{2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}-\frac{1}{2\,c}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(6\,a^4+2\,a^2\,c^2-c^4\right)}{a\,c^4}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(18\,a^4+6\,a^2\,c^2-c^4\right)}{2\,a^2\,c^3}}{e\,\left(32\,a^2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+64\,a\,c\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3+32\,c^2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\right)}-\frac{\mathrm{atanh}\left(\frac{2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)\,\left(3\,a^2+c^2\right)}{8\,c^5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

```
[Out] ((2*a*tan(d/2 + (e*x)/2))/c^2 - 1/(2*c) + (tan(d/2 + (e*x)/2)^3*(6*a^4 - c^
4 + 2*a^2*c^2))/(a*c^4) + (tan(d/2 + (e*x)/2)^2*(18*a^4 - c^4 + 6*a^2*c^2))
/(2*a^2*c^3))/(e*(32*a^2*tan(d/2 + (e*x)/2)^4 + 32*c^2*tan(d/2 + (e*x)/2)^2
 + 64*a*c*tan(d/2 + (e*x)/2)^3)) - (atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)*(
3*a^2 + c^2))/(8*c^5*e)
```

3.380 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=207

$$\frac{a(5a^2+3c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3}+\frac{5(ac\cos(d+ex)+\cdots}{96c^4e(a-a\cos(d+ex)\cdots}$$

```
[Out] 1/32*a*(5*a^2+3*c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)-a
*sin(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)+a^2
*sin(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^2)*
cos(e*x+d)-a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.16, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3208, 3235, 3232, 3200, 31}

$$\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{96c^4e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}+\frac{a(5a^2+3c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e}-\frac{a(15a^2+4c^2)\sin(d+ex)+c(15a^2+4c^2)\cos(d+ex)}{96c^6e(a(-\cos(d+ex))+a+c\sin(d+ex))}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(5*a^2 + 3*c^2)*Log[a + c*Cot[(d + e*x)/2]])/(32*c^7*e) - (c*Cos[d + e*x
] + a*Sin[d + e*x])/(48*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5
*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(96*c^4*e*(a - a*Cos[d + e*x] + c*S
in[d + e*x])^2) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] + a*(15*a^2 + 4*c^2)*Sin
[d + e*x])/(96*c^6*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3200

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Dist[-f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
```

```
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{\int \frac{-6a-4a\cos(}{(2a-2a\cos(d}}{}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(}{96c^4e(a-a}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(}{96c^4e(a-a}$$

$$= -\frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} + \frac{5(ac\cos(}{96c^4e(a-a}$$

$$= \frac{a(5a^2+3c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+}{48c^2e(a-a\cos}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 494 vs. 2(207) = 414.

time = 0.93, size = 494, normalized size = 2.39

$$\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(150a^6+130a^4c^2+24a^2c^4-225a^6\cos(d+ex)-255a^4c^2\cos(d+ex)-42a^2c^4\cos(d+ex)-24c^6\cos(d+ex)+90a^6\cos(2(d+ex))+174a^4c^2\cos(2(d+ex))-15a^6\cos(3(d+ex))-49a^4c^2\cos(3(d+ex))+18a^2c^4\cos(3(d+ex))+8c^6\cos(3(d+ex))-192(5a^3+3ac^2)\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)\sin^3\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^3+192(5a^3+3ac^2)\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\sin^3\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)^3+75a^5c\sin(d+ex)+75a^3c^3\sin(d+ex)-12ac^5\sin(d+ex)-60a^5c\sin(2(d+ex))-156a^3c^3\sin(2(d+ex))-12ac^5\sin(2(d+ex))+15a^5c\sin(3(d+ex))+79a^3c^3\sin(3(d+ex))+20ac^5\sin(3(d+ex))\right)}{384c^7e(a-a\cos(d+ex)+c\sin(d+ex))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

```
[Out] (Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(150*a^6 + 130*
a^4*c^2 + 24*a^2*c^4 - 225*a^6*Cos[d + e*x] - 255*a^4*c^2*Cos[d + e*x] - 42
*a^2*c^4*Cos[d + e*x] - 24*c^6*Cos[d + e*x] + 90*a^6*Cos[2*(d + e*x)] + 174
*a^4*c^2*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] - 49*a^4*c^2*Cos[3*(d +
 e*x)] + 18*a^2*c^4*Cos[3*(d + e*x)] + 8*c^6*Cos[3*(d + e*x)] - 192*(5*a^3
+ 3*a*c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a
*Sin[(d + e*x)/2])^3 + 192*(5*a^3 + 3*a*c^2)*Log[c*Cos[(d + e*x)/2] + a*Sin
[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])
^3 + 75*a^5*c*Sin[d + e*x] + 75*a^3*c^3*Sin[d + e*x] - 12*a*c^5*Sin[d + e*x
] - 60*a^5*c*Sin[2*(d + e*x)] - 156*a^3*c^3*Sin[2*(d + e*x)] - 12*a*c^5*Sin
[2*(d + e*x)] + 15*a^5*c*Sin[3*(d + e*x)] + 79*a^3*c^3*Sin[3*(d + e*x)] + 2
0*a*c^5*Sin[3*(d + e*x)]))/(384*c^7*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])
^4)
```

Maple [A]

time = 0.70, size = 253, normalized size = 1.22

method	result
derivativedivides	$\frac{-\frac{4a^6+6a^4c^2-2c^6}{16a^3c^5\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{10a^6+9a^4c^2+c^6}{8c^6a^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a^6+3a^4c^2+3a^2c^4+c^6}{24a^3c^4\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}+\frac{a(5a^2+3c^2)\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^7}-\frac{}{24c^4}}{16e}$
default	$\frac{-\frac{4a^6+6a^4c^2-2c^6}{16a^3c^5\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{10a^6+9a^4c^2+c^6}{8c^6a^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a^6+3a^4c^2+3a^2c^4+c^6}{24a^3c^4\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}+\frac{a(5a^2+3c^2)\ln\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2c^7}-\frac{}{24c^4}}{16e}$
norman	$\frac{-\frac{1}{384ce}+\frac{a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{128c^2e}+\frac{(50a^6+30a^4c^2+c^6)\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{384c^7e}-\frac{(5a^2+3c^2)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{128c^3e}+\frac{a(15a^4+9a^2c^2)\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{64c^6e}-}{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(c+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^3}$
risch	$i(9iac^4e^{5i(ex+d)}-130ia^3c^2e^{3i(ex+d)}-24iac^4e^{3i(ex+d)}+60ia^3c^2e^{2i(ex+d)}+60ia^3c^2e^{i(ex+d)}+15iac^4e^{i(ex+d)}+12iac^4e^{2i}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/e*(-1/16*(4*a^6+6*a^4*c^2-2*c^6)/a^3/c^5/(c+a*tan(1/2*d+1/2*e*x))^2-1/
8*(10*a^6+9*a^4*c^2+c^6)/c^6/a^3/(c+a*tan(1/2*d+1/2*e*x))-1/24*(a^6+3*a^4*c
^2+3*a^2*c^4+c^6)/a^3/c^4/(c+a*tan(1/2*d+1/2*e*x))^3+1/2*a*(5*a^2+3*c^2)/c^
7*ln(c+a*tan(1/2*d+1/2*e*x))-1/24/c^4/tan(1/2*d+1/2*e*x)^3-1/8*(10*a^2+3*c^
2)/c^6/tan(1/2*d+1/2*e*x)+1/4/c^5*a/tan(1/2*d+1/2*e*x)^2-1/2*a*(5*a^2+3*c^2
)/c^7*ln(tan(1/2*d+1/2*e*x)))
```

Maxima [A]
time = 0.30, size = 403, normalized size = 1.95

$$-\frac{1}{384}\left(\frac{a^3c^5 - \frac{3a^4c^4\sin(xe+d)}{\cos(xe+d)+1} + \frac{3(5a^5c^3+3a^3c^5)\sin(xe+d)^2}{(\cos(xe+d)+1)^2} + \frac{(110a^6c^2+66a^4c^4+3a^2c^6+c^8)\sin(xe+d)^3}{(\cos(xe+d)+1)^3} + \frac{3(50a^7c+30a^5c^3+ac^7)\sin(xe+d)^4}{(\cos(xe+d)+1)^4} + \frac{3(20a^8+12a^6c^2+a^2c^6)\sin(xe+d)^5}{(\cos(xe+d)+1)^5}}{\frac{a^3c^9\sin(xe+d)^3}{(\cos(xe+d)+1)^3} + \frac{3a^4c^8\sin(xe+d)^4}{(\cos(xe+d)+1)^4} + \frac{3a^5c^7\sin(xe+d)^5}{(\cos(xe+d)+1)^5} + \frac{a^6c^6\sin(xe+d)^6}{(\cos(xe+d)+1)^6}} - \frac{12(5a^3+3ac^2)\log\left(c+\frac{a\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^7} + \frac{12(5a^3+3ac^2)\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)}{c^7}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/384*((a^3*c^5 - 3*a^4*c^4*sin(x*e + d)/(cos(x*e + d) + 1) + 3*(5*a^5*c^3
 + 3*a^3*c^5)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + (110*a^6*c^2 + 66*a^4*c
^4 + 3*a^2*c^6 + c^8)*sin(x*e + d)^3/(cos(x*e + d) + 1)^3 + 3*(50*a^7*c + 3
0*a^5*c^3 + a*c^7)*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + 3*(20*a^8 + 12*a^6
*c^2 + a^2*c^6)*sin(x*e + d)^5/(cos(x*e + d) + 1)^5)/(a^3*c^9*sin(x*e + d)^
3/(cos(x*e + d) + 1)^3 + 3*a^4*c^8*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + 3*
a^5*c^7*sin(x*e + d)^5/(cos(x*e + d) + 1)^5 + a^6*c^6*sin(x*e + d)^6/(cos(x
*e + d) + 1)^6) - 12*(5*a^3 + 3*a*c^2)*log(c + a*sin(x*e + d)/(cos(x*e + d)
 + 1))/c^7 + 12*(5*a^3 + 3*a*c^2)*log(sin(x*e + d)/(cos(x*e + d) + 1))/c^7)
*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(213) = 426$.
time = 1.89, size = 830, normalized size = 4.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*c^2 + 6*a^2*c^4 + 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(x*e
+ d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(x*e + d)^2 - 6*(5*a^4*c^2 - 2*a^2*c^
4 - 2*c^6)*cos(x*e + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 12*a
^4*c^2 - 9*a^2*c^4)*cos(x*e + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(
x*e + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(x*e + d) + (15*a^5*c + 1
4*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(x*e + d)^2 - 6*(
5*a^5*c + 3*a^3*c^3)*cos(x*e + d))*sin(x*e + d))*log(a*c*sin(x*e + d) + 1/2
*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(x*e + d)) + 3*(5*a^6 + 18*a^4*c^2 + 9*
a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(x*e + d)^3 + 3*(5*a^6 - 2*a^
4*c^2 - 3*a^2*c^4)*cos(x*e + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(x
*e + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^
5)*cos(x*e + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(x*e + d))*sin(x*e + d))*log
(-1/2*cos(x*e + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5 + (15*a^5*c
- 41*a^3*c^3 - 12*a*c^5)*cos(x*e + d)^2 - 3*(10*a^5*c - 9*a^3*c^3 - a*c^5)*
cos(x*e + d))*sin(x*e + d))/((a^3*c^7 - 3*a*c^9)*cos(x*e + d)^3*e - 3*(a^3*
c^7 - a*c^9)*cos(x*e + d)^2*e + 3*(a^3*c^7 + a*c^9)*cos(x*e + d)*e - (a^3*c
```

```
^7 + 3*a*c^9)*e + (6*a^2*c^8*cos(x*e + d)*e - (3*a^2*c^8 - c^10)*cos(x*e +
d)^2*e - (3*a^2*c^8 + c^10)*e)*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 347, normalized size = 1.68

$$-\frac{\frac{12\,(5\,a^3+3\,a\,c^2)\log\left(\left|\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)\right|\right)}{c^7}-\frac{12\,(5\,a^4+3\,a^2c^2)\log\left(\left|a\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)+c\right|\right)}{a c^7}+\frac{60\,a^8\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^5+36\,a^6c^2\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^5+3\,a^2c^6\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^5+150\,a^7c\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^4+90\,a^5c^3\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^4+3\,a c^7\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^4+110\,a^6c^2\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^3+66\,a^4c^4\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^3+3\,a^2c^6\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^3+c^8\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^3+15\,a^5c^3\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^2+9\,a^3c^5\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^2-3\,a^4c^4\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)+a^3c^5}{\left(a\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,e x+\frac{1}{2}\,d\right)\right)^3a^3c^6}}{384\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(tan(1/2*e*x + 1/2*d)))/c^7 - 12*(5*a^4
 + 3*a^2*c^2)*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/(a*c^7) + (60*a^8*tan(1/
2*e*x + 1/2*d)^5 + 36*a^6*c^2*tan(1/2*e*x + 1/2*d)^5 + 3*a^2*c^6*tan(1/2*e*
x + 1/2*d)^5 + 150*a^7*c*tan(1/2*e*x + 1/2*d)^4 + 90*a^5*c^3*tan(1/2*e*x +
1/2*d)^4 + 3*a*c^7*tan(1/2*e*x + 1/2*d)^4 + 110*a^6*c^2*tan(1/2*e*x + 1/2*d
)^3 + 66*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 + 3*a^2*c^6*tan(1/2*e*x + 1/2*d)^3
+ c^8*tan(1/2*e*x + 1/2*d)^3 + 15*a^5*c^3*tan(1/2*e*x + 1/2*d)^2 + 9*a^3*c^
5*tan(1/2*e*x + 1/2*d)^2 - 3*a^4*c^4*tan(1/2*e*x + 1/2*d) + a^3*c^5)/((a*ta
n(1/2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))^3*a^3*c^6))/e
```

Mupad [B]
time = 6.05, size = 301, normalized size = 1.45

$$\frac{a\,\mathrm{atanh}\left(\frac{a\,\left(c+2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(5\,a^2+3\,c^2\right)}{c\,\left(5\,a^3+3\,a\,c^2\right)}\right)\,\left(5\,a^2+3\,c^2\right)}{16\,c^7\,e}-\frac{\frac{1}{3\,c}-\frac{a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(5\,a^2+3\,c^2\right)}{c^3}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(110\,a^6+66\,a^4\,c^2+3\,a^2\,c^4+c^6\right)}{3\,a^3\,c^4}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(20\,a^6+12\,a^4\,c^2+c^6\right)}{a\,c^6}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(50\,a^6+30\,a^4\,c^2+c^6\right)}{a^2\,c^5}}{e\,\left(128\,a^3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+384\,a^2\,c\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5+384\,a\,c^2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+128\,c^3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

```
[Out] (a*atanh((a*(c + 2*a*tan(d/2 + (e*x)/2))*(5*a^2 + 3*c^2))/(c*(3*a*c^2 + 5*a
^3)))*(5*a^2 + 3*c^2))/(16*c^7*e) - (1/(3*c) - (a*tan(d/2 + (e*x)/2))/c^2 +
 (tan(d/2 + (e*x)/2)^2*(5*a^2 + 3*c^2))/c^3 + (tan(d/2 + (e*x)/2)^3*(110*a^
6 + c^6 + 3*a^2*c^4 + 66*a^4*c^2))/(3*a^3*c^4) + (tan(d/2 + (e*x)/2)^5*(20*
a^6 + c^6 + 12*a^4*c^2))/(a*c^6) + (tan(d/2 + (e*x)/2)^4*(50*a^6 + c^6 + 30
*a^4*c^2))/(a^2*c^5))/(e*(128*a^3*tan(d/2 + (e*x)/2)^6 + 128*c^3*tan(d/2 +
(e*x)/2)^3 + 384*a*c^2*tan(d/2 + (e*x)/2)^4 + 384*a^2*c*tan(d/2 + (e*x)/2)^
5))
```

3.381 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$4a\left(5a^2+3b^2\right)x-\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e}-\frac{8(a+b\cos(d+ex)+a\sin(d+$$

```
[Out] 4*a*(5*a^2+3*b^2)*x-4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*
sin(e*x+d)/e-8/3*(a+b*cos(e*x+d)+a*sin(e*x+d))^2*(a*cos(e*x+d)-b*sin(e*x+d)
)/e-20/3*(a+b*cos(e*x+d)+a*sin(e*x+d))*(a^2*cos(e*x+d)-a*b*sin(e*x+d))/e
```

Rubi [A]

time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3199, 3225, 2717, 2718}

$$\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e}-\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+4ax(5a^2+3b^2)-\frac{20(a^2\cos(d+ex)-ab\sin(d+ex))(a\sin(d+ex)+a+b\cos(d+ex))}{3e}-\frac{8(a\cos(d+ex)-b\sin(d+ex))(a\sin(d+ex)+a+b\cos(d+ex))^2}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*b^2)*x - (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) - (8*(a + b*Cos[d + e*x] + a*Sin[d + e*x
])^2*(a*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e) - (20*(a + b*Cos[d + e*x] + a
*Sin[d + e*x])*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(3*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\begin{aligned}\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx &= -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(\\ &= -\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(\\ &= 4a(5a^2+3b^2)\,x-\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos\\ &= 4a(5a^2+3b^2)\,x-\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+\frac{4b(15a^2+4\end{aligned}$$

Mathematica [A]

time = 0.34, size = 135, normalized size = 0.86

$$\frac{2(6a(5a^2+3b^2)(d+ex)-9a(5a^2+b^2)\cos(d+ex)-18a^2b\cos(2(d+ex))+a(a^2-3b^2)\cos(3(d+ex))+9b(5a^2+b^2)\sin(d+ex)-9a(a^2-b^2)\sin(2(d+ex))+b(-3a^2+b^2)\sin(3(d+ex)))}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) - 9*a*(5*a^2 + b^2)*Cos[d + e*x] - 18*a^2
*b*Cos[2*(d + e*x)] + a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A]

time = 0.26, size = 177, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{8b^3(2+\cos^2(ex+d))\sin(ex+d)}{3}-8(\cos^3(ex+d))ab^2+24ab^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b(\sin^3(ex+d))-24(\cos^2(ex+$
default	$\frac{\frac{8b^3(2+\cos^2(ex+d))\sin(ex+d)}{3}-8(\cos^3(ex+d))ab^2+24ab^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b(\sin^3(ex+d))-24(\cos^2(ex+$

risch	$20a^3x + 12a\,b^2x - \frac{30a^3\cos(ex+d)}{e} - \frac{6a\cos(ex+d)b^2}{e} + \frac{30b\sin(ex+d)a^2}{e} + \frac{6b^3\sin(ex+d)}{e} + \frac{2a^3\cos(3ex+$
norman	$\frac{(20a^3+12a\,b^2)x+(20a^3+12a\,b^2)x\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(60a^3+36a\,b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(60a^3+36a\,b^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-cos(e*x+d)^3*a*b^2+3*a*b^2*(1/2*co
s(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+a^2*b*sin(e*x+d)^3-3*cos(e*x+d)^2*a^2*b+
3*a^2*b*sin(e*x+d)-1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a^3*(-1/2*cos(e*x+
d)*sin(e*x+d)+1/2*e*x+1/2*d)-3*a^3*cos(e*x+d)+a^3*(e*x+d))
```

Maxima [A]
time = 0.27, size = 195, normalized size = 1.24

$$-8\,ab^2\cos(xe+d)^3\,e^{(-1)} + 8\,a^2be^{(-1)}\sin(xe+d)^3 + \frac{8}{3}\left(\cos(xe+d)^3 - 3\,\cos(xe+d)\right)a^3e^{(-1)} - \frac{8}{3}\left(\sin(xe+d)^3 - 3\,\sin(xe+d)\right)b^3e^{(-1)} + 8\,a^3x - 24\left(a\cos(xe+d)\,e^{(-1)} - be^{(-1)}\sin(xe+d)\right)a^2 - 6\left(4\,ab\cos(xe+d)^2\,e^{(-1)} - (2\,xe+2\,d-\sin(2\,xe+2\,d))a^2e^{(-1)} - (2\,xe+2\,d+\sin(2\,xe+2\,d))b^2e^{(-1)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -8*a*b^2*cos(x*e + d)^3*e^(-1) + 8*a^2*b*e^(-1)*sin(x*e + d)^3 + 8/3*(cos(x
*e + d)^3 - 3*cos(x*e + d))*a^3*e^(-1) - 8/3*(sin(x*e + d)^3 - 3*sin(x*e +
d))*b^3*e^(-1) + 8*a^3*x - 24*(a*cos(x*e + d)*e^(-1) - b*e^(-1)*sin(x*e + d
))*a^2 - 6*(4*a*b*cos(x*e + d)^2*e^(-1) - (2*x*e + 2*d - sin(2*x*e + 2*d))*
a^2*e^(-1) - (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1))*a
```

Fricas [A]
time = 1.26, size = 133, normalized size = 0.85

$$-\frac{4}{3}\left(18\,a^2b\cos(xe+d)^2 + 24\,a^3\cos(xe+d) - 2\left(a^3-3\,ab^2\right)\cos(xe+d)^3 - 3\left(5\,a^3+3\,ab^2\right)xe - \left(24\,a^2b + 4\,b^3 - 2\left(3\,a^2b - b^3\right)\cos(xe+d)^2 - 9\left(a^3-ab^2\right)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -4/3*(18*a^2*b*cos(x*e + d)^2 + 24*a^3*cos(x*e + d) - 2*(a^3 - 3*a*b^2)*cos
(x*e + d)^3 - 3*(5*a^3 + 3*a*b^2)*x*e - (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^
3)*cos(x*e + d)^2 - 9*(a^3 - a*b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.17, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x - \frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} - \frac{16a^3\cos^3(d+ex)}{3e} - \frac{24a^3\cos(d+ex)}{e} + \frac{8a^2b\sin^3(d+ex)}{e} + \frac{24a^2b\sin^2(d+ex)}{e} + \frac{24a^2b\sin^2(d+ex)}{e} + \frac{24a^2b\sin(d+ex)}{e} + 12ab^2x\sin^2(d+ex) + 12ab^2x\cos^2(d+ex) + \frac{12ab^2\sin(d+ex)\cos(d+ex)}{e} - \frac{8ab^2\cos^3(d+ex)}{e} + \frac{16b^3\sin^3(d+ex)}{3e} + \frac{8b^3\sin(d+ex)\cos^2(d+ex)}{e} & \text{for } e \neq 0 \\ x(2a\sin(d) + 2a + 2b\cos(d))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 - 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e - 16*a**3*cos(d + e*x)**3/(3*e) - 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e - 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(2*a*sin(d)
+ 2*a + 2*b*cos(d))**3, True))
```

Giac [A]
time = 0.42, size = 151, normalized size = 0.96

$$-\frac{12\,a^2 b\cos\left(2\,ex+2\,d\right)}{e}+4\left(5\,a^3+3\,ab^2\right)x+\frac{2\left(a^3-3\,ab^2\right)\cos\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\left(5\,a^3+ab^2\right)\cos\left(ex+d\right)}{e}-\frac{2\left(3\,a^2b-b^3\right)\sin\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\left(a^3-ab^2\right)\sin\left(2\,ex+2\,d\right)}{e}+\frac{6\left(5\,a^2b+b^3\right)\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -12*a^2*b*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*b^2)*x + 2/3*(a^3 - 3*a*b^2)*
cos(3*e*x + 3*d)/e - 6*(5*a^3 + a*b^2)*cos(e*x + d)/e - 2/3*(3*a^2*b - b^3)
*sin(3*e*x + 3*d)/e - 6*(a^3 - a*b^2)*sin(2*e*x + 2*d)/e + 6*(5*a^2*b + b^3
)*sin(e*x + d)/e
```

Mupad [B]
time = 3.60, size = 292, normalized size = 1.86

$$\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\left(24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)-16\,a\,b^2+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(96\,a^2\,b-128\,a^3\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(160\,a^2\,b+\frac{32\,b^3}{3}\right)-\frac{176\,a^3}{3}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(-24\,a^3+48\,a^2\,b+24\,a\,b^2+16\,b^3\right)}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}+\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(5\,a^2+3\,b^2\right)}{40\,a^3+24\,a\,b^2}\right)\left(5\,a^2+3\,b^2\right)}{e}-\frac{8\,a\left(5\,a^2+3\,b^2\right)\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] (tan(d/2 + (e*x)/2)^5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) - tan(d/2 + (
e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) - 16*a*b^2 + tan(d/2 + (e*x)/2)^2*
(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) - (176
*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3
*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
 + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*
(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)
/2))/e
```

3.382 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2\left(3a^2+b^2\right)x-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

```
[Out] 2*(3*a^2+b^2)*x-6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e-2*(a+b*cos(e*x+d)+a*s
in(e*x+d))*(a*cos(e*x+d)-b*sin(e*x+d))/e
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3199, 2717, 2718}

$$2x\left(3a^2+b^2\right)-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x - (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e - (2*(a
 + b*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - b*Sin[d + e*x]))/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a + 2b\cos(d+ex) + 2a\sin(d+ex))^2 \, dx = -\frac{2(a + b\cos(d+ex) + a\sin(d+ex))(a\cos(d+ex) - b\sin(d
}{e}$$
$$= 2(3a^2 + b^2)\, x - \frac{2(a + b\cos(d+ex) + a\sin(d+ex))(a\cos(d}{e}$$
$$= 2(3a^2 + b^2)\, x - \frac{6a^2\cos(d+ex)}{e} + \frac{6ab\sin(d+ex)}{e} - \frac{2(a+b}{}$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 1.14

$$4\left(\frac{(3a^2+b^2)(d+ex)}{2e} - \frac{2a^2\cos(d+ex)}{e} - \frac{ab\cos(2(d+ex))}{2e} + \frac{2ab\sin(d+ex)}{e} - \frac{(a^2-b^2)\sin(2(d+ex))}{4e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) - (2*a^2*Cos[d + e*x])/e - (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A]
time = 0.24, size = 101, normalized size = 1.25

method	result
risch	$6a^2x + 2b^2x - \frac{8a^2\cos(ex+d)}{e} + \frac{8ab\sin(ex+d)}{e} - \frac{2ab\cos(2ex+2d)}{e} - \frac{\sin(2ex+2d)a^2}{e} + \frac{\sin(2ex+2d)b^2}{e}$
derivativedivides	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4(\cos^2(ex+d))ab+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a^2\cos(ex+d}{e}$
default	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4(\cos^2(ex+d))ab+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a^2\cos(ex+d}{e}$
norman	$\frac{(6a^2+2b^2)x+(6a^2+2b^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(12a^2+4b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{16a^2\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}-\frac{4\left(a^2-4ab-b^2\right)\tan\left(\frac{d}{2}+\right.}{e}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/e*(b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-cos(e*x+d)^2*a*b+2*a*b*s
in(e*x+d)+a^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-2*a^2*cos(e*x+d)+a
^2*(e*x+d))
```

Maxima [A]
time = 0.28, size = 101, normalized size = 1.25

$$-4\,ab\cos(xe+d)^2\,e^{(-1)} + (2\,xe + 2\,d - \sin(2\,xe + 2\,d))a^2e^{(-1)} + (2\,xe + 2\,d + \sin(2\,xe + 2\,d))b^2e^{(-1)} + 4\,a^2x - 8\,\left(a\cos(xe+d)\,e^{(-1)} - be^{(-1)}\sin(xe+d)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -4*a*b*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d - sin(2*x*e + 2*d))*a^2*e^(-1)
+ (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1) + 4*a^2*x - 8*(a*cos(x*e + d)
*e^(-1) - b*e^(-1)*sin(x*e + d))*a
```

Fricas [A]
time = 1.28, size = 76, normalized size = 0.94

$$-2\left(2\,ab\cos\left(xe+d\right)^{2}+4\,a^{2}\cos\left(xe+d\right)-\left(3\,a^{2}+b^{2}\right)xe-\left(4\,ab-\left(a^{2}-b^{2}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -2*(2*a*b*cos(x*e + d)^2 + 4*a^2*cos(x*e + d) - (3*a^2 + b^2)*x*e - (4*a*b
- (a^2 - b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(80) = 160$.
time = 0.10, size = 170, normalized size = 2.10

$$\begin{cases} 2a^{2}x\sin^{2}\left(d+ex\right)+2a^{2}x\cos^{2}\left(d+ex\right)+4a^{2}x-\frac{2a^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{8a^{2}\cos\left(d+ex\right)}{e}+\frac{4ab\sin^{2}\left(d+ex\right)}{e}+\frac{8ab\sin\left(d+ex\right)}{e}+2b^{2}x\sin^{2}\left(d+ex\right)+2b^{2}x\cos^{2}\left(d+ex\right)+\frac{2b^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\left(2a\sin\left(d\right)+2a+2b\cos\left(d\right)\right)^{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x -
 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*cos(d + e*x)/e + 4*a*b*sin(d +
 e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos
(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*sin(d
) + 2*a + 2*b*cos(d))**2, True))
```

Giac [A]
time = 0.41, size = 79, normalized size = 0.98

$$2\left(3\,a^{2}+b^{2}\right)x-\frac{2\,ab\cos\left(2\,ex+2\,d\right)}{e}-\frac{8\,a^{2}\cos\left(ex+d\right)}{e}+\frac{8\,ab\sin\left(ex+d\right)}{e}-\frac{\left(a^{2}-b^{2}\right)\sin\left(2\,ex+2\,d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2 + b^2)*x - 2*a*b*cos(2*e*x + 2*d)/e - 8*a^2*cos(e*x + d)/e + 8*a*b
*sin(e*x + d)/e - (a^2 - b^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 3.72, size = 127, normalized size = 1.57

$$\frac{x\,\left(12\,a^2+4\,b^2\right)}{2}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(16\,a\,b-16\,a^2\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(4\,a^2+16\,a\,b-4\,b^2\right)-16\,a^2+\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(-4\,a^2+16\,a\,b+4\,b^2\right)}{e\,\left({\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+2\,{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + tan(d/2
+ (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - 16*a^2 + tan(d/2 + (e*x)/2)*(16*a*b
 - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

3.383 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$2ax-\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}$$

```
[Out] 2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2717, 2718}

$$-\frac{2a\cos(d+ex)}{e}+2ax+\frac{2b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
[Out] 2*a*x - (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx &= 2ax+(2a)\int \sin(d+ex)\,dx+(2b)\int \cos(d+ex)\,dx\\ &= 2ax-\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.83

$$2ax-\frac{2a\cos(d)\cos(ex)}{e}+\frac{2b\cos(ex)\sin(d)}{e}+\frac{2b\cos(d)\sin(ex)}{e}+\frac{2a\sin(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x - (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin[e*x])/e + (2*a*Sin[d]*Sin[e*x])/e
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.03

method	result	size
default	$2ax - \frac{2a\cos(ex+d)}{e} + \frac{2b\sin(ex+d)}{e}$	30
risch	$2ax - \frac{2a\cos(ex+d)}{e} + \frac{2b\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a-2a\cos(ex+d)+2b\sin(ex+d)}{e}$	31
norman	$\frac{\frac{4a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+2ax+2ax\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{4b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Maxima [A]
time = 0.26, size = 29, normalized size = 1.00

$$-2\,a\cos\left(xe+d\right)e^{(-1)}+2\,be^{(-1)}\sin\left(xe+d\right)+2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] -2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

Fricas [A]
time = 1.27, size = 29, normalized size = 1.00

$$2\left(axe-a\cos\left(xe+d\right)+b\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*x*e - a*cos(x*e + d) + b*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.05, size = 39, normalized size = 1.34

$$2ax+2a\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\sin\left(d\right) & \text{otherwise}\end{cases}\right)+2b\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\cos\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x + 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 1.00

$$2\,ax - \frac{2\,a\cos\left(ex + d\right)}{e} + \frac{2\,b\sin\left(ex + d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*x - 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

Mupad [B]
time = 2.44, size = 29, normalized size = 1.00

$$2\,a\,x - \frac{2\,a\,\cos\left(d + e\,x\right)}{e} + \frac{2\,b\,\sin\left(d + e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x),x)
```

```
[Out] 2*a*x - (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

3.384 $\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

```
[Out] -1/2*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3202, 31}

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -1/2*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3202

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Di
st[-f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \cot\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= -\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(33) = 66.
time = 0.06, size = 93, normalized size = 2.82

$$\frac{1}{2}\left(\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{be}-\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)-b\sin\left(\frac{1}{2}(d+ex)\right)\right)}{be}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] (Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(b*e) - Log[a*Cos[(d + e*x)/2] +
b*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2] - b*Sin[(d + e*x)/2]]/(b*e))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.
time = 0.32, size = 66, normalized size = 2.00

method	result	size
norman	$\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be}-\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}{2be}$	57
risch	$\frac{\ln\left(e^{i(ex+d)}+i\right)}{2be}-\frac{\ln\left(e^{i(ex+d)}+\frac{ia+b}{ib+a}\right)}{2be}$	57
derivativedivides	$\frac{\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b}+\frac{(-a+b)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}{b(a-b)}}{2e}$	66
default	$\frac{\frac{\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b}+\frac{(-a+b)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}{b(a-b)}}{2e}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(1/b*ln(1+tan(1/2*d+1/2*e*x))+(-a+b)/b/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-
b*tan(1/2*d+1/2*e*x)+a+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(25) = 50$.
time = 0.27, size = 69, normalized size = 2.09

$$-\frac{1}{2}\left(\frac{\log\left(-a-b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{b}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(-a - b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b - log(sin(x*e
 + d)/(cos(x*e + d) + 1) + 1)/b)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.
time = 1.22, size = 56, normalized size = 1.70

$$-\frac{\left(\log\left(2\,ab\cos\left(xe+d\right)+a^{2}+b^{2}+\left(a^{2}-b^{2}\right)\sin\left(xe+d\right)\right)-\log\left(\sin\left(xe+d\right)+1\right)\right)e^{(-1)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] -1/4*(log(2*a*b*cos(x*e + d) + a^2 + b^2 + (a^2 - b^2)*sin(x*e + d)) - log(
sin(x*e + d) + 1))*e^(-1)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(24) = 48$.
time = 0.96, size = 107, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be} & \text{for } a = b \\ \frac{x}{2a\sin(d)+2a+2b\cos(d)} & \text{for } e = 0 \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ae} & \text{for } b = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2be} - \frac{\log\left(\frac{a}{a-b}+\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d/2 + e*
x/2) + 1)/(2*b*e), Eq(a, b)), (x/(2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0))
, (-1/(a*e*tan(d/2 + e*x/2) + a*e), Eq(b, 0)), (log(tan(d/2 + e*x/2) + 1)/(
2*b*e) - log(a/(a - b) + b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.
time = 0.44, size = 79, normalized size = 2.39

$$\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a+2\,|b|\right|}\right)}{2\,e|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*a
bs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*ab
s(b)))/(e*abs(b))
```

Mupad [B]
time = 2.83, size = 33, normalized size = 1.00

$$-\frac{\operatorname{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x)),x)
[Out] -atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

3.385 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}$$

```
[Out] 1/4*a*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(-a*cos(e*x+d)+b*sin(e*x+
d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 12, 3202, 31}

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*Si
n[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3202

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Di
st[-f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}+\frac{\int -\frac{}{2a+2b\cos(d+}}{4}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}-\frac{a\int \frac{}{2a+2b\cos(d+}}{2}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}+\frac{a\text{Subst}\left(\int \frac{1}{2a+2}\right.}{}\\ &= \frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin}{4b^2e(a+b\cos(d+ex)+}\end{aligned}$$

Mathematica [A]

time = 0.43, size = 162, normalized size = 1.95

$$\frac{-a\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+a\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+\frac{b\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)}+\frac{b(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)}}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] (-(a*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]) + a*Log[(a + b)*Cos[(d + e*x
)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] +
 Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos
[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

Maple [A]

time = 0.47, size = 122, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{1}{b^2\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b^3}-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}+\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right.\right.}{b^3}}{4e}$
default	$\frac{-\frac{1}{b^2\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}-\frac{a\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b^3}-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}+\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right.\right.}{b^3}}{4e}$
risch	$\frac{i\left(ia+b+a\,e^{i(ex+d)}\right)}{2b^2e\left(-ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+ia+2a\,e^{i(ex+d)}+b\right)}+\frac{a\ln\left(e^{i(ex+d)}+\frac{ia+b}{ib+a}\right)}{4b^3e}-\frac{a\ln\left(e^{i(ex+d)}+i\right)}{4b^3e}$

norman	$\frac{-\frac{a^2+ab+b^2}{4a\,b^2e}+\frac{(a^2-ab+b^2)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a\,b^2e}}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)}-\frac{a\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4b^3e}+\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{e}{2}\right.\right.}{4b^3e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e*(-1/b^2/(1+tan(1/2*d+1/2*e*x))-a/b^3*ln(1+tan(1/2*d+1/2*e*x))-(a^2+b^
2)/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)+a/b^3*ln(a*tan
(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(76) = 152$.
time = 0.28, size = 194, normalized size = 2.34

$$-\frac{1}{4}\left(\frac{2\left(a^2+\frac{(a^2-ab+b^2)\sin(xe+d)}{\cos(xe+d)+1}\right)}{a^2b^2-b^4+\frac{2\,(a^2b^2-ab^3)\sin(xe+d)}{\cos(xe+d)+1}+\frac{(a^2b^2-2\,ab^3+b^4)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}-\frac{a\log\left(-a-b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b^3}+\frac{a\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{b^3}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + (a^2 - a*b + b^2)*sin(x*e + d)/(cos(x*e + d) + 1))/(a^2*b^2
- b^4 + 2*(a^2*b^2 - a*b^3)*sin(x*e + d)/(cos(x*e + d) + 1) + (a^2*b^2 - 2*
a*b^3 + b^4)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2) - a*log(-a - b - (a - b)*
sin(x*e + d)/(cos(x*e + d) + 1))/b^3 + a*log(sin(x*e + d)/(cos(x*e + d) + 1
) + 1)/b^3)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(76) = 152$.
time = 2.94, size = 162, normalized size = 1.95

$$-\frac{2\,ab\cos(xe+d)-2\,b^2\sin(xe+d)-(ab\cos(xe+d)+a^2\sin(xe+d)+a^2)\log(2\,ab\cos(xe+d)+a^2+b^2+(a^2-b^2)\sin(xe+d))+(ab\cos(xe+d)+a^2\sin(xe+d)+a^2)\log(\sin(xe+d)+1)}{8\,(b^4\cos(xe+d)\,e+ab^3e\sin(xe+d)+ab^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*cos(x*e + d) - 2*b^2*sin(x*e + d) - (a*b*cos(x*e + d) + a^2*sin
(x*e + d) + a^2)*log(2*a*b*cos(x*e + d) + a^2 + b^2 + (a^2 - b^2)*sin(x*e +
 d)) + (a*b*cos(x*e + d) + a^2*sin(x*e + d) + a^2)*log(sin(x*e + d) + 1))/(
b^4*cos(x*e + d)*e + a*b^3*e*sin(x*e + d) + a*b^3*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(73) = 146$.
time = 0.43, size = 187, normalized size = 2.25

$$-\frac{\frac{2\left(a^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-ab\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a^{2}\right)}{(ab^{2}-b^{3})\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a+2\,|b|\right|}\right)}{b^{2}|b|}}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(a^2*tan(1/2*e*x + 1/2*d) - a*b*tan(1/2*e*x + 1/2*d) + b^2*tan(1/2*
e*x + 1/2*d) + a^2)/((a*b^2 - b^3)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*
x + 1/2*d)^2 + 2*a*tan(1/2*e*x + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/2*e
*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*
x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*abs(b)))/(b^2*abs(b)))/e
```

Mupad [B]
time = 2.72, size = 126, normalized size = 1.52

$$\frac{a\,\mathrm{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{2\,b^3\,e}-\frac{\frac{a^2}{b^2\,(a-b)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(a^2-a\,b+b^2)}{b^2\,(a-b)}}{e\,\left((2\,a-2\,b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+4\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+2\,a+2\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

```
[Out] (a*atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e) - (a^2/(b^2
*(a - b)) + (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/(e*(2*a +
 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) + 4*a*tan(d/2 + (e*x)/2)))
```

3.386 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{(3a^2+b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{3(a^2\cos(d+ex)-}{16b^4e(a+b\cos(d+ex}$$

```
[Out] -1/16*(3*a^2+b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(-a*cos(e*x+
d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+3/16*(a^2*cos(e*x+d)
-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

Rubi [A]
time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 3232, 3202, 31}

$$\frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{16b^4e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{(3a^2+b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/16*((3*a^2 + b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(b^5*e) - (a*Cos
[d + e*x] - b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x])
^2) + (3*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*
x] + a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3202

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Di
st[-f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{\int\frac{-4a+2b\cos(\cdots}{(2a+2b\cos(d\cdots}}{\cdots}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{3(a^2\cos(\cdots}{16b^4e(a+b\cdots}\\ &= -\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2}+\frac{3(a^2\cos(\cdots}{16b^4e(a+b\cdots}\\ &= -\frac{(3a^2+b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e}-\frac{a\cos(d+\cdots}{16b^2e(a+b\cos\cdots}\end{aligned}$$

Mathematica [A]
time = 1.76, size = 255, normalized size = 1.80

$$-\frac{-2(3a^2+b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+2(3a^2+b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+\frac{b^2}{\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6ab\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)}-\frac{b^2(a^2+b^2)}{\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6ab(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)}}{32b^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/32*(-2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 2*(3*a^2
 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + b^2/(Cos
[(d + e*x)/2] + Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*
x)/2] + Sin[(d + e*x)/2]) - (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (
a - b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*
((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(b^5*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(130) = 260$.
time = 0.66, size = 264, normalized size = 1.86

method	result
derivativedivides	$\frac{-\frac{1}{2b^3\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{-3a-b}{2b^4\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{\left(3a^2+b^2\right)\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2b^5}+\frac{\left(-3a^3+3a^2b-a\,b^2+b^3\right)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\right.\right.}{2b^5(a-b)}}{8e}$
default	$\frac{-\frac{1}{2b^3\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}-\frac{-3a-b}{2b^4\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}+\frac{\left(3a^2+b^2\right)\ln\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2b^5}+\frac{\left(-3a^3+3a^2b-a\,b^2+b^3\right)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\right.\right.}{2b^5(a-b)}}{8e}$
risch	$-\frac{i\left(3a^2b\,e^{3i(ex+d)}+b^3e^{3i(ex+d)}+9a^3e^{2i(ex+d)}+3a\,b^2e^{2i(ex+d)}-3ia^3e^{3i(ex+d)}-ia\,b^2e^{3i(ex+d)}+9a^2b\,e^{i(ex+d)}-b^3e^{i(ex+d)}\right.}{8\left(-ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+ia+2a\,e^{i(ex+d)}+b\right)^2b^4e}$
norman	$\frac{\frac{9a^5+18b\,a^4+12a^3b^2+6b^3a^2+a\,b^4}{16b^4e\left(3a^2-b^2\right)}-\frac{\left(9a^5-9b\,a^4+6a^3b^2-6b^3a^2+3a\,b^4+b^5\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{8b^4e\left(3a^2-b^2\right)}+\frac{\left(9a^5+9b\,a^4+6a^3b^2+6b^3a^2+3a\,b^4-b^5\right)}{8b^4e\left(3a^2-b^2\right)}}{\left(1+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a+b\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/e*(-1/2/b^3/(1+tan(1/2*d+1/2*e*x))^2-1/2*(-3*a-b)/b^4/(1+tan(1/2*d+1/2*
e*x))+1/2*(3*a^2+b^2)/b^5*ln(1+tan(1/2*d+1/2*e*x))+1/2*(-3*a^3+3*a^2*b-a*b^
2+b^3)/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)-1/2*(-a^
4-2*a^2*b^2-b^4)/b^3/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b
)^2-1/2*(-3*a^4+4*a^3*b-2*a^2*b^2+4*a*b^3+b^4)/b^4/(a-b)^2/(a*tan(1/2*d+1/2
*e*x)-b*tan(1/2*d+1/2*e*x)+a+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(136) = 272$.
time = 0.30, size = 510, normalized size = 3.59

$$\frac{1}{16}\left(\frac{2\left(3a^5-4a^3b^2-ab^4+\frac{(9a^5-9a^4b-2a^3b^2+2a^2b^3-5ab^4+b^5)\sin(xe+d)}{\cos(xe+d)+1}+\frac{(9a^5-18a^4b+12a^3b^2-6a^2b^3+ab^4)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{(3a^5-9a^4b+10a^3b^2-6a^2b^3+ab^4+b^5)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}\right)}{a^4b^4-2a^2b^6+b^8+\frac{4(a^4b^4-a^3b^5-a^2b^6+ab^7)\sin(xe+d)}{\cos(xe+d)+1}+\frac{2(3a^4b^4-6a^3b^5+2a^2b^6+2ab^7-b^8)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{4(a^4b^4-3a^3b^5+3a^2b^6-ab^7)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{(a^4b^4-4a^3b^5+6a^2b^6-4ab^7+b^8)\sin(xe+d)^4}{(\cos(xe+d)+1)^4}}-\frac{(3a^2+b^2)\log\left(-a-b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b^5}+\frac{(3a^2+b^2)\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{b^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 + (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*b
^3 - 5*a*b^4 + b^5)*sin(x*e + d)/(cos(x*e + d) + 1) + (9*a^5 - 18*a^4*b + 1
2*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + (3*a^5
 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(x*e + d)^3/(cos(x*e
+ d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 + 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
a*b^7)*sin(x*e + d)/(cos(x*e + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*b
^6 + 2*a*b^7 - b^8)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + 4*(a^4*b^4 - 3*a^
3*b^5 + 3*a^2*b^6 - a*b^7)*sin(x*e + d)^3/(cos(x*e + d) + 1)^3 + (a^4*b^4 -
 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(x*e + d)^4/(cos(x*e + d) + 1)^4
) - (3*a^2 + b^2)*log(-a - b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(x*e + d)/(cos(x*e + d) + 1) + 1)/b^5)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(136) = 272$.
time = 3.25, size = 444, normalized size = 3.13

$$\frac{12\,a^{2}b^{2}\cos\left(xe+d\right)^{2}-6\,a^{2}b^{2}+2\,\left(3\,a^{3}b-ab^{3}\right)\cos\left(xe+d\right)-\left(6\,a^{4}+2\,a^{2}b^{2}-\left(3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)\cos\left(xe+d\right)^{2}+2\,\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)+2\,\left(3\,a^{4}+a^{2}b^{2}+\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)\log\left(2\,ab\cos\left(xe+d\right)+a^{2}+b^{2}+\left(a^{2}-b^{2}\right)\sin\left(xe+d\right)\right)+\left(6\,a^{4}+2\,a^{2}b^{2}-\left(3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)\cos\left(xe+d\right)^{2}+2\,\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)+2\,\left(3\,a^{4}+a^{2}b^{2}+\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)\log\left(\sin\left(xe+d\right)+1\right)-2\,\left(3\,a^{2}b^{2}-b^{4}-3\,\left(a^{3}b-ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)}{32\,\left(2\,ab^{6}\cos\left(xe+d\right)e+2\,a^{2}b^{5}e-\left(a^{2}b^{5}-b^{7}\right)\cos\left(xe+d\right)^{2}e+2\,\left(ab^{6}\cos\left(xe+d\right)e+a^{2}b^{5}e\right)\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(12*a^2*b^2*cos(x*e + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(x*e +
 d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(x*e + d)^2 + 2*(3*
a^3*b + a*b^3)*cos(x*e + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(x*
e + d))*sin(x*e + d))*log(2*a*b*cos(x*e + d) + a^2 + b^2 + (a^2 - b^2)*sin(
x*e + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(x*e + d)^2 +
 2*(3*a^3*b + a*b^3)*cos(x*e + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*
cos(x*e + d))*sin(x*e + d))*log(sin(x*e + d) + 1) - 2*(3*a^2*b^2 - b^4 - 3*
(a^3*b - a*b^3)*cos(x*e + d))*sin(x*e + d))/(2*a*b^6*cos(x*e + d)*e + 2*a^2
*b^5*e - (a^2*b^5 - b^7)*cos(x*e + d)^2*e + 2*(a*b^6*cos(x*e + d)*e + a^2*b
^5*e)*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(130) = 260$.
time = 0.44, size = 458, normalized size = 3.23

$$\frac{\frac{2\,\left(3\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-9\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+10\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-6\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+b^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+9\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-18\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+12\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-6\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+9\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-9\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-5\,ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+3\,a^{5}-4\,a^{3}b^{2}-ab^{4}\right)}{\left(a^{2}b^{4}-2\,ab^{5}+b^{6}\right)\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a+b\right)^{2}}+\frac{\left(3\,a^{2}+b^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a+2\,|b|\right|}\right)}{b^{4}|b|}}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 10
*a^3*b^2*tan(1/2*e*x + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + a*b^4*
tan(1/2*e*x + 1/2*d)^3 + b^5*tan(1/2*e*x + 1/2*d)^3 + 9*a^5*tan(1/2*e*x + 1
/2*d)^2 - 18*a^4*b*tan(1/2*e*x + 1/2*d)^2 + 12*a^3*b^2*tan(1/2*e*x + 1/2*d)
^2 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^2 + a*b^4*tan(1/2*e*x + 1/2*d)^2 + 9*a^
5*tan(1/2*e*x + 1/2*d) - 9*a^4*b*tan(1/2*e*x + 1/2*d) - 2*a^3*b^2*tan(1/2*e
```

```
*x + 1/2*d) + 2*a^2*b^3*tan(1/2*e*x + 1/2*d) - 5*a*b^4*tan(1/2*e*x + 1/2*d)
 + b^5*tan(1/2*e*x + 1/2*d) + 3*a^5 - 4*a^3*b^2 - a*b^4)/((a^2*b^4 - 2*a*b^
5 + b^6)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan(1/2
*e*x + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*e*x + 1/2*d)
- 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) -
 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*abs(b)))/(b^4*abs(b)))/e
```

Mupad [B]
time = 6.45, size = 360, normalized size = 2.54

$$-\frac{\frac{-3\,a^5+4\,a^3\,b^2+a\,b^4}{2\,b^4\,{\left(a-b\right)}^2}+\frac{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(-9\,a^5+9\,a^4\,b+2\,a^3\,b^2-2\,a^2\,b^3+5\,a\,b^4-b^5\right)}{2\,b^4\,{\left(a-b\right)}^2}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(-3\,a^4+6\,a^3\,b-4\,a^2\,b^2+2\,a\,b^3+b^4\right)}{2\,b^4\,\left(a-b\right)}-\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(9\,a^5-18\,a^4\,b+12\,a^3\,b^2-6\,a^2\,b^3+a\,b^4\right)}{2\,b^4\,{\left(a-b\right)}^2}}{e\,\left(8\,a\,b+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(24\,a^2-8\,b^2\right)-{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(16\,a\,b-16\,a^2\right)+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(4\,a^2-8\,a\,b+4\,b^2\right)+4\,a^2+4\,b^2+\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(16\,a^2+16\,b\,a\right)\right)}-\frac{\mathrm{atanh}\left(\frac{2\,a+\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,b}\right)\,\left(3\,a^2+b^2\right)}{8\,b^5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

```
[Out] - ((a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)*(5*a
*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*a^3*b^2))/(2*b^4*(a - b)^2) +
(tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*b^4
*(a - b)) - (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3 + 1
2*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2 - 8
*b^2) - tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4*a^
2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 + tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2)))
 - (atanh((2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b
^5*e)
```

3.387 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$\frac{a(5a^2+3b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3}+\frac{5(a^2\cos(d+ex)}{96b^4e(a+b\cos(d+}$$

```
[Out] 1/32*a*(5*a^2+3*b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(-a*cos(e
*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^3+5/96*(a^2*cos(e*x
+d)-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+1/96*(-a*(15*a^2+
4*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)+a*sin(
e*x+d))
```

Rubi [A]

time = 0.17, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3208, 3235, 3232, 3202, 31}

$$\frac{5(a^2\cos(d+ex)-ab\sin(d+ex))}{96b^4e(a\sin(d+ex)+a+b\cos(d+ex))^2}+\frac{a(5a^2+3b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}-\frac{a(15a^2+4b^2)\cos(d+ex)-b(15a^2+4b^2)\sin(d+ex)}{96b^6e(a\sin(d+ex)+a+b\cos(d+ex))}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(32*b^7*e) - (a*Co
s[d + e*x] - b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]
)^3) + (5*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d + e
*x] + a*Sin[d + e*x])^2) - (a*(15*a^2 + 4*b^2)*Cos[d + e*x] - b*(15*a^2 + 4
*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3202

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Di
st[-f/e, Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+}{(2a+2b\cos(d+e}}{}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(a^2\cos(d}{96b^4e(a+b\cos}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(a^2\cos(d}{96b^4e(a+b\cos}$$

$$= -\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(a^2\cos(d}{96b^4e(a+b\cos}$$

$$= \frac{a(5a^2+3b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} - \frac{a\cos(d+}{48b^2e(a+b\cos(}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 632 vs. 2(215) = 430.

time = 1.34, size = 632, normalized size = 2.94

$$\frac{-12a(5a^2+3b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+12a(5a^2+3b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+\frac{b\left(150a^6+130a^4b^2+24a^2b^4-3a^2(25a^4-50a^3b+5a^2b^2-30ab^3+4b^4)\cos(d+ex)-6a^2(15a^4+20a^3b+9a^2b^2+2ab^3-2b^4)\cos(2(d+ex))+15a^6\cos(3(d+ex))-30a^5b\cos(3(d+ex))-41a^4b^2\cos(3(d+ex))-38a^3b^3\cos(3(d+ex))-12a^2b^4\cos(3(d+ex))-8ab^5\cos(3(d+ex))+225a^6\sin(d+ex)+75a^5b\sin(d+ex)+180a^4b^2\sin(d+ex)+15a^3b^3\sin(d+ex)+27a^2b^4\sin(d+ex)+12ab^5\sin(d+ex)+12b^6\sin(d+ex)-60a^6\sin(2(d+ex))+120a^5b\sin(2(d+ex))+54a^4b^2\sin(2(d+ex))+102a^3b^3\sin(2(d+ex))+6a^2b^4\sin(2(d+ex))+6ab^5\sin(2(d+ex))-15a^6\sin(3(d+ex))-45a^5b\sin(3(d+ex))-4a^4b^2\sin(3(d+ex))+3a^3b^3\sin(3(d+ex))+15a^2b^4\sin(3(d+ex))+4ab^5\sin(3(d+ex))+4b^6\sin(3(d+ex))\right)}{(a+b)\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}}{384b^7e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (-12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 12*a*(5*a
^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*(
150*a^6 + 130*a^4*b^2 + 24*a^2*b^4 - 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2 -
 30*a*b^3 + 4*b^4)*Cos[d + e*x] - 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2*
a*b^3 - 2*b^4)*Cos[2*(d + e*x)] + 15*a^6*Cos[3*(d + e*x)] - 30*a^5*b*Cos[3*
(d + e*x)] - 41*a^4*b^2*Cos[3*(d + e*x)] - 38*a^3*b^3*Cos[3*(d + e*x)] - 12
*a^2*b^4*Cos[3*(d + e*x)] - 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x]
 + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e*
x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
- 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*(
d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*a
*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)]
 - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Sin
[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]))/((a + b
)*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (a -
b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

Maple [A]
time = 1.15, size = 398, normalized size = 1.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/e*(-1/3/b^4/(1+tan(1/2*d+1/2*e*x))^3-1/2*(-2*a-b)/b^5/(1+tan(1/2*d+1/2
*e*x))^2-1/2*(5*a^2+2*a*b+2*b^2)/b^6/(1+tan(1/2*d+1/2*e*x))-1/2*a*(5*a^2+3*
b^2)/b^7*ln(1+tan(1/2*d+1/2*e*x))+1/2*(5*a^3-5*a^2*b+3*a*b^2-3*b^3)*a/b^7/(
a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)-1/3*(a^6+3*a^4*b^2+3
*a^2*b^4+b^6)/b^4/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^3
-1/2*(2*a^6-3*a^5*b+3*a^4*b^2-6*a^3*b^3-3*a*b^5-b^6)/b^5/(a-b)^3/(a*tan(1/2
*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)+a+b)^2-1/2*(5*a^6-12*a^5*b+12*a^4*b^2-12*a
^3*b^3+9*a^2*b^4+2*b^6)/b^6/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e
*x)+a+b))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 988 vs. 2(210) = 420.
time = 0.34, size = 988, normalized size = 4.60

$$-\frac{1}{96}\left(\frac{2\left(15a^8-31a^6b^2+9a^4b^4+15a^2b^6+\frac{3(25a^8-25a^7b-25a^6b^2+25a^5b^3-13a^4b^4+13a^3b^5+11a^2b^6-5ab^7+2b^8)\sin(xe+d)}{\cos(xe+d)+1}+\frac{6(25a^8-50a^7b+20a^6b^2+10a^5b^3-17a^4b^4+24a^3b^5-10a^2b^6+2ab^7)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{2(75a^8-225a^7b+250a^6b^2-150a^5b^3+63a^4b^4+11a^3b^5-24a^2b^6+6ab^7-2b^8)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{3(25a^8-100a^7b+165a^6b^2-160a^5b^3+115a^4b^4-60a^3b^5+19a^2b^6-4ab^7)\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+\frac{3(5a^8-25a^7b+53a^6b^2-65a^5b^3+55a^4b^4-35a^3b^5+17a^2b^6-7ab^7+2b^8)\sin(xe+d)^5}{(\cos(xe+d)+1)^5}\right)}{a^6b^6-3a^4b^8+3a^2b^{10}-b^{12}+\frac{6(a^6b^6-a^5b^7-2a^4b^8+2a^3b^9+a^2b^{10}-ab^{11})\sin(xe+d)}{\cos(xe+d)+1}+\frac{3(5a^6b^6-10a^5b^7-a^4b^8+12a^3b^9-5a^2b^{10}-2ab^{11}+b^{12})\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{4(5a^6b^6-15a^5b^7+12a^4b^8+4a^3b^9-9a^2b^{10}+3ab^{11})\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{3(5a^6b^6-20a^5b^7+29a^4b^8-16a^3b^9-a^2b^{10}+4ab^{11}-b^{12})\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+\frac{6(a^6b^6-5a^5b^7+10a^4b^8-10a^3b^9+5a^2b^{10}-ab^{11})\sin(xe+d)^5}{(\cos(xe+d)+1)^5}+\frac{(a^6b^6-6a^5b^7+15a^4b^8-20a^3b^9+15a^2b^{10}-6ab^{11}+b^{12})\sin(xe+d)^6}{(\cos(xe+d)+1)^6}}-\frac{3(5a^3+3ab^2)\log\left(-a-b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b^7}+\frac{3(5a^3+3ab^2)\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)}{b^7}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 + 3*(25*a^8 - 25*a^7
*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b
^7 + 2*b^8)*sin(x*e + d)/(cos(x*e + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6
*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(x*e
 + d)^2/(cos(x*e + d) + 1)^2 + 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^
5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(x*e + d
)^3/(cos(x*e + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^
3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(x*e + d)^4/(cos(x*
e + d) + 1)^4 + 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4
- 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(x*e + d)^5/(cos(x*e + d) +
 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 + 6*(a^6*b^6 - a^5*b^7 - 2*
a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(x*e + d)/(cos(x*e + d) + 1) +
3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 +
b^12)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*
a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(x*e + d)^3/(cos(x*e + d) +
 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*
a*b^11 - b^12)*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + 6*(a^6*b^6 - 5*a^5*b^7
 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(x*e + d)^5/(cos(x*e +
 d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
 6*a*b^11 + b^12)*sin(x*e + d)^6/(cos(x*e + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2
)*log(-a - b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b^7 + 3*(5*a^3 + 3*
a*b^2)*log(sin(x*e + d)/(cos(x*e + d) + 1) + 1)/b^7)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(210) = 420$.

time = 3.20, size = 763, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] 1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(x*
e + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(x*e + d)^2 - 6*(10*a^5*b - 9*a^3*b
^3 - 2*a*b^5)*cos(x*e + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3
 - 3*a*b^5)*cos(x*e + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(x*e + d)
^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 -
 12*a^4*b^2 - 9*a^2*b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e +
 d))*sin(x*e + d))*log(2*a*b*cos(x*e + d) + a^2 + b^2 + (a^2 - b^2)*sin(x*e
 + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(x*e
+ d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3*
a^3*b^3)*cos(x*e + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2*
b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e + d))*sin(x*e + d))*l
og(sin(x*e + d) + 1) + 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 - 3*
```

```
a^2*b^4 - 4*b^6)*cos(x*e + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(x*e
+ d))*sin(x*e + d))/(6*a^2*b^8*cos(x*e + d)*e + 4*a^3*b^7*e - (3*a^2*b^8 -
b^10)*cos(x*e + d)^3*e - 3*(a^3*b^7 - a*b^9)*cos(x*e + d)^2*e + (6*a^2*b^8*
cos(x*e + d)*e + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*cos(x*e + d)^2*e)*sin(x*
e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(201) = 402.
time = 0.46, size = 957, normalized size = 4.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*a^8*tan(1/2*e*x + 1/2*d)^5 - 75*a^7*b*tan(1/2*e*x + 1/2*d)^5 +
 159*a^6*b^2*tan(1/2*e*x + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*e*x + 1/2*d)^5 +
165*a^4*b^4*tan(1/2*e*x + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*e*x + 1/2*d)^5 + 5
1*a^2*b^6*tan(1/2*e*x + 1/2*d)^5 - 21*a*b^7*tan(1/2*e*x + 1/2*d)^5 + 6*b^8*
tan(1/2*e*x + 1/2*d)^5 + 75*a^8*tan(1/2*e*x + 1/2*d)^4 - 300*a^7*b*tan(1/2*
e*x + 1/2*d)^4 + 495*a^6*b^2*tan(1/2*e*x + 1/2*d)^4 - 480*a^5*b^3*tan(1/2*e
*x + 1/2*d)^4 + 345*a^4*b^4*tan(1/2*e*x + 1/2*d)^4 - 180*a^3*b^5*tan(1/2*e*
x + 1/2*d)^4 + 57*a^2*b^6*tan(1/2*e*x + 1/2*d)^4 - 12*a*b^7*tan(1/2*e*x + 1
/2*d)^4 + 150*a^8*tan(1/2*e*x + 1/2*d)^3 - 450*a^7*b*tan(1/2*e*x + 1/2*d)^3
 + 500*a^6*b^2*tan(1/2*e*x + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*e*x + 1/2*d)^3
+ 126*a^4*b^4*tan(1/2*e*x + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*e*x + 1/2*d)^3 -
48*a^2*b^6*tan(1/2*e*x + 1/2*d)^3 + 12*a*b^7*tan(1/2*e*x + 1/2*d)^3 - 4*b^8
*tan(1/2*e*x + 1/2*d)^3 + 150*a^8*tan(1/2*e*x + 1/2*d)^2 - 300*a^7*b*tan(1/
2*e*x + 1/2*d)^2 + 120*a^6*b^2*tan(1/2*e*x + 1/2*d)^2 + 60*a^5*b^3*tan(1/2*
e*x + 1/2*d)^2 - 102*a^4*b^4*tan(1/2*e*x + 1/2*d)^2 + 144*a^3*b^5*tan(1/2*e
*x + 1/2*d)^2 - 60*a^2*b^6*tan(1/2*e*x + 1/2*d)^2 + 12*a*b^7*tan(1/2*e*x +
1/2*d)^2 + 75*a^8*tan(1/2*e*x + 1/2*d) - 75*a^7*b*tan(1/2*e*x + 1/2*d) - 75
*a^6*b^2*tan(1/2*e*x + 1/2*d) + 75*a^5*b^3*tan(1/2*e*x + 1/2*d) - 39*a^4*b^
4*tan(1/2*e*x + 1/2*d) + 39*a^3*b^5*tan(1/2*e*x + 1/2*d) + 33*a^2*b^6*tan(1
/2*e*x + 1/2*d) - 15*a*b^7*tan(1/2*e*x + 1/2*d) + 6*b^8*tan(1/2*e*x + 1/2*d
) + 15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3
```

```
*a*b^8 - b^9)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + 2*a*ta
n(1/2*e*x + 1/2*d) + a + b)^3) + 3*(5*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*e*
x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*abs(b)))/(b^6*abs(b)))/e
```

Mupad [B]
time = 7.22, size = 730, normalized size = 3.40

$$\frac{a\,\mathrm{atanh}\left(\frac{a\,\left(2\,a+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)\right)\,\left(5\,a^2+3\,b^2\right)}{2\,b\,\left(5\,a^3+3\,a\,b^2\right)}\right)\,\left(5\,a^2+3\,b^2\right)}{16\,b^7\,e}-\frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,{\left(a-b\right)}^3}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+10\,a^5\,b^3-17\,a^4\,b^4+24\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7\right)}{b^6\,{\left(a-b\right)}^3}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(25\,a^7-75\,a^6\,b+90\,a^5\,b^2-70\,a^4\,b^3+45\,a^3\,b^4-15\,a^2\,b^5+4\,a\,b^6\right)}{2\,b^6\,{\left(a-b\right)}^2}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(75\,a^8-225\,a^7\,b+250\,a^6\,b^2-150\,a^5\,b^3+63\,a^4\,b^4+11\,a^3\,b^5-24\,a^2\,b^6+6\,a\,b^7-2\,b^8\right)}{3\,b^6\,{\left(a-b\right)}^3}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(5\,a^6-15\,a^5\,b+18\,a^4\,b^2-14\,a^3\,b^3+9\,a^2\,b^4-3\,a\,b^5+2\,b^6\right)}{2\,b^6\,\left(a-b\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(25\,a^8-25\,a^7\,b-25\,a^6\,b^2+25\,a^5\,b^3-13\,a^4\,b^4+13\,a^3\,b^5+11\,a^2\,b^6-5\,a\,b^7+2\,b^8\right)}{2\,b^6\,{\left(a-b\right)}^3}}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6\,\left(8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3\right)-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(-120\,a^3-120\,a^2\,b+24\,a\,b^2+24\,b^3\right)-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(-120\,a^3+120\,a^2\,b+24\,a\,b^2-24\,b^3\right)+24\,a\,b^2+24\,a^2\,b-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(96\,a\,b^2-160\,a^3\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(48\,a^3+96\,a^2\,b+48\,a\,b^2\right)+8\,a^3+8\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

```
[Out] (a*atanh((a*(2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(3
*a*b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e) - ((15*a^8 + 15*a^2*b^6 + 9*a
^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^2*(2*a*b^7 - 5
0*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*a^4*b^4 + 10*a^5*b^3 + 20*a
^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4*(4*a*b^6 - 75*a^6*b + 25*a
^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a^5*b^2))/(2*b^6*(a - b)^2)
+ (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 75*a^8 - 2*b^8 - 24*a^2*b^6
+ 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^6*b^2))/(3*b^6*(a - b)^3) +
 (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b^5 + 2*b^6 + 9*a^2*b^4 - 14
*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) + (tan(d/2 + (e*x)/2)*(25*a^8 - 25*
a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^5 - 13*a^4*b^4 + 25*a^5*b^3
 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 + (e*x)/2)^5*(48*a*b^2 - 96*
a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3
) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 - 120*a^2*b - 120*a^3 + 24*b^3) - tan(d/
2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*b - 120*a^3 - 24*b^3) + 24*a*b^2 + 24*a^
2*b - tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3) + tan(d/2 + (e*x)/2)*(48*a*
b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3))
```

3.388 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$

Optimal. Leaf size=157

$$4a\left(5a^2+3b^2\right)x+\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e}+\frac{8(a+b\cos(d+ex)-a\sin(d+$$

```
[Out] 4*a*(5*a^2+3*b^2)*x+4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)*
sin(e*x+d)/e+8/3*(a+b*cos(e*x+d)-a*sin(e*x+d))^2*(a*cos(e*x+d)+b*sin(e*x+d)
)/e+20/3*(a+b*cos(e*x+d)-a*sin(e*x+d))*(a^2*cos(e*x+d)+a*b*sin(e*x+d))/e
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3199, 3225, 2717, 2718}

$$\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e}+\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+4ax\left(5a^2+3b^2\right)+\frac{20(a^2\cos(d+ex)+ab\sin(d+ex))\,(a(-\sin(d+ex))+a+b\cos(d+ex))}{3e}+\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

```
[Out] 4*a*(5*a^2 + 3*b^2)*x + (4*a*(15*a^2 + 4*b^2)*Cos[d + e*x])/(3*e) + (4*b*(1
5*a^2 + 4*b^2)*Sin[d + e*x])/(3*e) + (8*(a + b*Cos[d + e*x] - a*Sin[d + e*x
])^2*(a*Cos[d + e*x] + b*Sin[d + e*x]))/(3*e) + (20*(a + b*Cos[d + e*x] - a
*Sin[d + e*x])*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(3*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx = \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+b\sin(d}{3e}$$
$$= \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+b\sin(d}{3e}$$
$$= 4a(5a^2+3b^2)\,x + \frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos}{3e}$$
$$= 4a(5a^2+3b^2)\,x + \frac{4a(15a^2+4b^2)\cos(d+ex)}{3e} + \frac{4b(15a^2+4}{}$$

Mathematica [A]

time = 0.32, size = 136, normalized size = 0.87

$$\frac{2(6a(5a^2+3b^2)(d+ex)+9a(5a^2+b^2)\cos(d+ex)+18a^2b\cos(2(d+ex))-a(a^2-3b^2)\cos(3(d+ex))+9b(5a^2+b^2)\sin(d+ex)-9a(a^2-b^2)\sin(2(d+ex))+b(-3a^2+b^2)\sin(3(d+ex)))}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

```
[Out] (2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) + 9*a*(5*a^2 + b^2)*Cos[d + e*x] + 18*a^2
*b*Cos[2*(d + e*x)] - a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2)*
Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3*(d
 + e*x)]))/(3*e)
```

Maple [A]

time = 0.26, size = 176, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{8b^3(2+\cos^2(ex+d))\sin(ex+d)}{3}+8(\cos^3(ex+d))\,a\,b^2+24a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b(\sin^3(ex+d))+24(\cos^2(ex+}{}$
default	$\frac{\frac{8b^3(2+\cos^2(ex+d))\sin(ex+d)}{3}+8(\cos^3(ex+d))\,a\,b^2+24a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b(\sin^3(ex+d))+24(\cos^2(ex+}{}$

risch	$20a^3x+12a\,b^2x+\frac{30a^3\cos(ex+d)}{e}+\frac{6a\cos(ex+d)b^2}{e}+\frac{30b\sin(ex+d)a^2}{e}+\frac{6b^3\sin(ex+d)}{e}-\frac{2a^3\cos(3ex+}{3e}$
norman	$\frac{(20a^3+12a\,b^2)x+(20a^3+12a\,b^2)x\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(60a^3+36a\,b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(60a^3+36a\,b^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/e*(1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)+cos(e*x+d)^3*a*b^2+3*a*b^2*(1/2*co
s(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+a^2*b*sin(e*x+d)^3+3*cos(e*x+d)^2*a^2*b+
3*a^2*b*sin(e*x+d)+1/3*a^3*(2+sin(e*x+d)^2)*cos(e*x+d)+3*a^3*(-1/2*cos(e*x+
d)*sin(e*x+d)+1/2*e*x+1/2*d)+3*a^3*cos(e*x+d)+a^3*(e*x+d))
```

Maxima [A]
time = 0.30, size = 192, normalized size = 1.22

$$8\,ab^2\cos(xe+d)^3e^{(-1)}+8\,a^2be^{(-1)}\sin(xe+d)^3-\frac{8}{3}\left(\cos(xe+d)^3-3\cos(xe+d)\right)a^3e^{(-1)}-\frac{8}{3}\left(\sin(xe+d)^3-3\sin(xe+d)\right)b^3e^{(-1)}+8\,a^3x+24\left(a\cos(xe+d)e^{(-1)}+be^{(-1)}\sin(xe+d)\right)a^2+6\left(4\,ab\cos(xe+d)^2e^{(-1)}+(2\,xe+2\,d-\sin(2\,xe+2\,d))a^2e^{(-1)}+(2\,xe+2\,d+\sin(2\,xe+2\,d))b^2e^{(-1)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] 8*a*b^2*cos(x*e + d)^3*e^(-1) + 8*a^2*b*e^(-1)*sin(x*e + d)^3 - 8/3*(cos(x*
e + d)^3 - 3*cos(x*e + d))*a^3*e^(-1) - 8/3*(sin(x*e + d)^3 - 3*sin(x*e + d
))*b^3*e^(-1) + 8*a^3*x + 24*(a*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d)
)*a^2 + 6*(4*a*b*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d - sin(2*x*e + 2*d))*a
^2*e^(-1) + (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1))*a
```

Fricas [A]
time = 3.07, size = 132, normalized size = 0.84

$$\frac{4}{3}\left(18\,a^2b\cos(xe+d)^2+24\,a^3\cos(xe+d)-2\left(a^3-3\,ab^2\right)\cos(xe+d)^3+3\left(5\,a^3+3\,ab^2\right)xe+\left(24\,a^2b+4\,b^3-2\left(3\,a^2b-b^3\right)\cos(xe+d)^2-9\left(a^3-ab^2\right)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] 4/3*(18*a^2*b*cos(x*e + d)^2 + 24*a^3*cos(x*e + d) - 2*(a^3 - 3*a*b^2)*cos(
x*e + d)^3 + 3*(5*a^3 + 3*a*b^2)*x*e + (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b^3
)*cos(x*e + d)^2 - 9*(a^3 - a*b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.18, size = 291, normalized size = 1.85

$$\begin{cases} 12a^3x\sin^2(d+ex)+12a^3x\cos^2(d+ex)+8a^3x+\frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e}-\frac{12a^3\sin(d+ex)\cos(d+ex)}{e}+\frac{16a^3\cos^3(d+ex)}{3e}+\frac{24a^3\cos(d+ex)}{e}+\frac{8a^2b\sin^3(d+ex)}{e}-\frac{24a^2b\sin^2(d+ex)}{e}+\frac{24a^2b\sin(d+ex)}{e}+12ab^2x\sin^2(d+ex)+12ab^2x\cos^2(d+ex)+\frac{12ab^2\sin(d+ex)\cos(d+ex)}{e}+\frac{8ab^2\cos^3(d+ex)}{e}+\frac{16b^3\sin^3(d+ex)}{3e}+\frac{8b^3\sin(d+ex)\cos^2(d+ex)}{e} & \text{for}\ e\neq 0 \\ x(-2a\sin(d)+2a+2b\cos(d))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*x
 + 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e*x
)/e + 16*a**3*cos(d + e*x)**3/(3*e) + 24*a**3*cos(d + e*x)/e + 8*a**2*b*sin
(d + e*x)**3/e - 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e + 1
2*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*sin(d
+ e*x)*cos(d + e*x)/e + 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d + e*x)**
3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(-2*a*sin(d)
 + 2*a + 2*b*cos(d))**3, True))
```

Giac [A]
time = 0.42, size = 151, normalized size = 0.96

$$\frac{12\,a^2b\cos\left(2\,ex+2\,d\right)}{e}+4\left(5\,a^3+3\,ab^2\right)x-\frac{2\left(a^3-3\,ab^2\right)\cos\left(3\,ex+3\,d\right)}{3\,e}+\frac{6\left(5\,a^3+ab^2\right)\cos\left(ex+d\right)}{e}-\frac{2\left(3\,a^2b-b^3\right)\sin\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\left(a^3-ab^2\right)\sin\left(2\,ex+2\,d\right)}{e}+\frac{6\left(5\,a^2b+b^3\right)\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 12*a^2*b*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*b^2)*x - 2/3*(a^3 - 3*a*b^2)*c
os(3*e*x + 3*d)/e + 6*(5*a^3 + a*b^2)*cos(e*x + d)/e - 2/3*(3*a^2*b - b^3)*
sin(3*e*x + 3*d)/e - 6*(a^3 - a*b^2)*sin(2*e*x + 2*d)/e + 6*(5*a^2*b + b^3)
*sin(e*x + d)/e
```

Mupad [B]
time = 3.44, size = 292, normalized size = 1.86

$$\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3\right)+16\,a\,b^2-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(96\,a^2\,b-128\,a^3\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(160\,a^2\,b+\frac{32\,b^3}{3}\right)+\frac{176\,a^3}{3}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(-24\,a^3+48\,a^2\,b+24\,a\,b^2+16\,b^3\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}+\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(5\,a^2+3\,b^2\right)}{40\,a^3+24\,a\,b^2}\right)\,\left(5\,a^2+3\,b^2\right)}{e}-\frac{8\,a\,\left(5\,a^2+3\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

```
[Out] (tan(d/2 + (e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^5
*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) + 16*a*b^2 - tan(d/2 + (e*x)/2)^2*
(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) + (176
*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(e*(3
*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
 + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*a^3))*
(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2)) - (e*x)
/2))/e
```

3.389 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx$

Optimal. Leaf size=81

$$2(3a^2+b^2)\,x+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}+\frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\sin}{e}$$

```
[Out] 2*(3*a^2+b^2)*x+6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e+2*(a+b*cos(e*x+d)-a*s
in(e*x+d))*(a*cos(e*x+d)+b*sin(e*x+d))/e
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3199, 2717, 2718}

$$2x(3a^2+b^2)+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e}+\frac{2(a(-\sin(d+ex))+a+b\cos(d+ex))(a\cos(d+ex)+b\sin(d+ex))}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 2*(3*a^2 + b^2)*x + (6*a^2*Cos[d + e*x])/e + (6*a*b*Sin[d + e*x])/e + (2*(a
 + b*Cos[d + e*x] - a*Sin[d + e*x])*(a*Cos[d + e*x] + b*Sin[d + e*x]))/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\sin(d-}{e}$$
$$= 2(3a^2+b^2)\,x + \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d}{e}$$
$$= 2(3a^2+b^2)\,x + \frac{6a^2\cos(d+ex)}{e} + \frac{6ab\sin(d+ex)}{e} + \frac{2(a+b}{}$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 1.14

$$4\left(\frac{(3a^2+b^2)(d+ex)}{2e}+\frac{2a^2\cos(d+ex)}{e}+\frac{ab\cos(2(d+ex))}{2e}+\frac{2ab\sin(d+ex)}{e}-\frac{(a^2-b^2)\sin(2(d+ex))}{4e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

```
[Out] 4*(((3*a^2 + b^2)*(d + e*x))/(2*e) + (2*a^2*Cos[d + e*x])/e + (a*b*Cos[2*(d
 + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])/(
4*e))
```

Maple [A]
time = 0.23, size = 100, normalized size = 1.23

method	result
risch	$6a^2x+2b^2x+\frac{8a^2\cos(ex+d)}{e}+\frac{8ab\sin(ex+d)}{e}+\frac{2ab\cos(2ex+2d)}{e}-\frac{\sin(2ex+2d)a^2}{e}+\frac{\sin(2ex+2d)b^2}{e}$
derivativedivides	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4(\cos^2(ex+d))ab+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2\cos(ex+d}{e}$
default	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4(\cos^2(ex+d))ab+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2\cos(ex+d}{e}$
norman	$\frac{(6a^2+2b^2)x+(6a^2+2b^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(12a^2+4b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{16a^2\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}-\frac{4\left(a^2-4ab-b^2\right)\tan\left(\frac{d}{2}+\frac{e}{}\right.}{e}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/e*(b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+cos(e*x+d)^2*a*b+2*a*b*s
in(e*x+d)+a^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+2*a^2*cos(e*x+d)+a
^2*(e*x+d))
```

Maxima [A]
time = 0.28, size = 100, normalized size = 1.23

$$4\,ab\cos\left(xe+d\right)^2e^{(-1)}+\left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)a^2e^{(-1)}+\left(2\,xe+2\,d+\sin\left(2\,xe+2\,d\right)\right)b^2e^{(-1)}+4\,a^2x+8\,\left(a\cos\left(xe+d\right)e^{(-1)}+be^{(-1)}\sin\left(xe+d\right)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 4*a*b*cos(x*e + d)^2*e^(-1) + (2*x*e + 2*d - sin(2*x*e + 2*d))*a^2*e^(-1) +
 (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1) + 4*a^2*x + 8*(a*cos(x*e + d)*
e^(-1) + b*e^(-1)*sin(x*e + d))*a
```

Fricas [A]
time = 3.06, size = 74, normalized size = 0.91

$$2\left(2\,ab\cos\left(xe+d\right)^{2}+4\,a^{2}\cos\left(xe+d\right)+\left(3\,a^{2}+b^{2}\right)xe+\left(4\,ab-\left(a^{2}-b^{2}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 2*(2*a*b*cos(x*e + d)^2 + 4*a^2*cos(x*e + d) + (3*a^2 + b^2)*x*e + (4*a*b -
 (a^2 - b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(80) = 160$.
time = 0.10, size = 170, normalized size = 2.10

$$\begin{cases} 2a^{2}x\sin^{2}\left(d+ex\right)+2a^{2}x\cos^{2}\left(d+ex\right)+4a^{2}x-\frac{2a^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^{2}\cos\left(d+ex\right)}{e}-\frac{4ab\sin^{2}\left(d+ex\right)}{e}+\frac{8ab\sin\left(d+ex\right)}{e}+2b^{2}x\sin^{2}\left(d+ex\right)+2b^{2}x\cos^{2}\left(d+ex\right)+\frac{2b^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e} & \text{for}\ e\neq 0\\ x\left(-2a\sin\left(d\right)+2a+2b\cos\left(d\right)\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x -
 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*cos(d + e*x)/e - 4*a*b*sin(d +
 e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*cos
(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*sin(
d) + 2*a + 2*b*cos(d))**2, True))
```

Giac [A]
time = 0.42, size = 79, normalized size = 0.98

$$2\left(3\,a^{2}+b^{2}\right)x+\frac{2\,ab\cos\left(2\,ex+2\,d\right)}{e}+\frac{8\,a^{2}\cos\left(ex+d\right)}{e}+\frac{8\,ab\sin\left(ex+d\right)}{e}-\frac{\left(a^{2}-b^{2}\right)\sin\left(2\,ex+2\,d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 2*(3*a^2 + b^2)*x + 2*a*b*cos(2*e*x + 2*d)/e + 8*a^2*cos(e*x + d)/e + 8*a*b
*sin(e*x + d)/e - (a^2 - b^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 3.74, size = 128, normalized size = 1.58

$$\frac{x\left(12a^2+4b^2\right)}{2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3\left(4a^2+16ab-4b^2\right)-\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2\left(16ab-16a^2\right)+16a^2+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(-4a^2+16ab+4b^2\right)}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

```
[Out] (x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - t
an(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + 16*a^2 + tan(d/2 + (e*x)/2)*(16*a*b
 - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

3.390 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx$

Optimal. Leaf size=29

$$2ax+\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}$$

```
[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {2717, 2718}

$$\frac{2a\cos(d+ex)}{e}+2ax+\frac{2b\sin(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx &= 2ax-(2a)\int \sin(d+ex)\,dx+(2b)\int \cos(d+ex)\,dx\\ &= 2ax+\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.83

$$2ax+\frac{2a\cos(d)\cos(ex)}{e}+\frac{2b\cos(ex)\sin(d)}{e}+\frac{2b\cos(d)\sin(ex)}{e}-\frac{2a\sin(d)\sin(ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

```
[Out] 2*a*x + (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Sin
[e*x])/e - (2*a*Sin[d]*Sin[e*x])/e
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.03

method	result	size
derivativedivides	$\frac{2(ex+d)a+2b\sin(ex+d)+2a\cos(ex+d)}{e}$	30
default	$2ax+\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
risch	$2ax+\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
norman	$\frac{2ax+2ax\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{4b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}-\frac{4a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.00

$$2\,a\cos\left(xe+d\right)e^{(-1)}+2\,be^{(-1)}\sin\left(xe+d\right)+2\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="maxima")
```

```
[Out] 2*a*cos(x*e + d)*e^(-1) + 2*b*e^(-1)*sin(x*e + d) + 2*a*x
```

Fricas [A]
time = 2.64, size = 28, normalized size = 0.97

$$2\left(axe+a\cos\left(xe+d\right)+b\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="fricas")
```

```
[Out] 2*(a*x*e + a*cos(x*e + d) + b*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.06, size = 39, normalized size = 1.34

$$2ax-2a\left(\begin{cases}-\frac{\cos\left(d+ex\right)}{e} & \text{for}\: e\neq 0\\x\sin\left(d\right) & \text{otherwise}\end{cases}\right)+2b\left(\begin{cases}\frac{\sin\left(d+ex\right)}{e} & \text{for}\: e\neq 0\\x\cos\left(d\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

```
[Out] 2*a*x - 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b*
Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

Giac [A]
time = 0.39, size = 29, normalized size = 1.00

$$2\,ax+\frac{2\,a\cos\left(ex+d\right)}{e}+\frac{2\,b\sin\left(ex+d\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="giac")
```

```
[Out] 2*a*x + 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

Mupad [B]
time = 2.44, size = 29, normalized size = 1.00

$$2\,a\,x+\frac{2\,a\,\cos\left(d+e\,x\right)}{e}+\frac{2\,b\,\sin\left(d+e\,x\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x),x)
```

```
[Out] 2*a*x + (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

3.391 $\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx$

Optimal. Leaf size=33

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

```
[Out] 1/2*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3201, 31}

$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3201

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{2a+2bx}\,dx, x, \tan\left(\frac{\pi}{4}+\frac{1}{2}(d+ex)\right)\right)}{e}\\ &= \frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 0.07, size = 96, normalized size = 2.91

$$-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2be}+\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)-a\sin\left(\frac{1}{2}(d+ex)\right)+b\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2be}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

```
[Out] -1/2*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]]/(b*e) + Log[a*Cos[(d + e*x)/2
] + b*Cos[(d + e*x)/2] - a*Sin[(d + e*x)/2] + b*Sin[(d + e*x)/2]]/(2*b*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(25) = 50$.
time = 0.33, size = 59, normalized size = 1.79

method	result	size
derivativedivides	$\frac{\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{b}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{b}}{2e}$	59
default	$\frac{\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{b}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{b}}{2e}$	59
norman	$-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be}+\frac{\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{2be}$	61
risch	$\frac{\ln\left(e^{i(ex+d)}+\frac{ia-b}{ib-a}\right)}{2be}-\frac{\ln\left(e^{i(ex+d)}-i\right)}{2be}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(1/b*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/b*ln(tan(1/2
*d+1/2*e*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.
time = 0.28, size = 65, normalized size = 1.97

$$\frac{1}{2}\left(\frac{\log\left(a+b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b}-\frac{\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}-1\right)}{b}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a + b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b - log(sin(x*e +
 d)/(cos(x*e + d) + 1) - 1)/b)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.
time = 2.76, size = 59, normalized size = 1.79

$$\frac{\left(\log\left(2\,ab\cos\left(xe+d\right)+a^2+b^2-\left(a^2-b^2\right)\sin\left(xe+d\right)\right)-\log\left(-\sin\left(xe+d\right)+1\right)\right)e^{(-1)}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(2*a*b*cos(x*e + d) + a^2 + b^2 - (a^2 - b^2)*sin(x*e + d)) - log(-
sin(x*e + d) + 1))*e^(-1)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(22) = 44.
time = 1.01, size = 109, normalized size = 3.30

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} & \text{for } a = b \\ \frac{x}{-2a\sin(d)+2a+2b\cos(d)} & \text{for } e = 0 \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ae} & \text{for } b = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} + \frac{\log\left(-\frac{a}{a-b}-\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-log(tan(d/2 + e
*x/2) - 1)/(2*b*e), Eq(a, b)), (x/(-2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0
)), (-1/(a*e*tan(d/2 + e*x/2) - a*e), Eq(b, 0)), (-log(tan(d/2 + e*x/2) - 1
)/(2*b*e) + log(-a/(a - b) - b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(25) = 50.
time = 0.42, size = 79, normalized size = 2.39

$$\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a+2\,|b|\right|}\right)}{2\,e|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*a
bs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*ab
s(b)))/(e*abs(b))
```

Mupad [B]
time = 2.74, size = 32, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x)),x)
[Out] atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

3.392 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))}$$

```
[Out] -1/4*a*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(a*cos(e*x+d)+b*sin(e*x+
d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 12, 3201, 31}

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

```
[Out] -1/4*(a*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(b^3*e) + (a*Cos[d + e*x] + b
*Sin[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3201

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} + \frac{\int -\frac{2a}{2a+2b\cos(d+ex}}{4b}$$
$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\int \frac{1}{2a+2b\cos(d+ex}}{2b}$$
$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))} - \frac{a\text{Subst}\left(\int \frac{1}{2a+2bx}\right.}{}$$
$$= -\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e} + \frac{a\cos(d+ex)+b\sin}{4b^2e(a+b\cos(d+ex)}$$

Mathematica [A]
time = 0.53, size = 166, normalized size = 2.00

$$\frac{a\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)-a\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+\frac{b\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)}+\frac{b(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)}}{4b^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
[Out] (a*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - a*Log[(a + b)*Cos[(d + e*x)/2
] + (-a + b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] - S
in[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Cos[(
d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

Maple [A]
time = 0.45, size = 130, normalized size = 1.57

method	result
derivativedivides	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}-\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{d}{2}+\right.\right.}{b^3}}{4e}$
default	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}-\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{d}{2}+\right.\right.}{b^3}}{4e}$
risch	$\frac{i\left(-ia+b+a\,e^{i(ex+d)}\right)}{2b^2e\left(ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}-ia+2a\,e^{i(ex+d)}+b\right)}+\frac{a\ln\left(e^{i(ex+d)}-i\right)}{4b^3e}-\frac{a\ln\left(e^{i(ex+d)}+\frac{ia-b}{ib-a}\right)}{4b^3e}$

norman	$\frac{\frac{a^2+ab+b^2}{4a\,b^2e}-\frac{\left(a^2-ab+b^2\right)\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{4a\,b^2e}}{\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}+\frac{a\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{4b^3e}-\frac{a\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{e}{}\right.\right.}{4b^3e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e*(-(a^2+b^2)/b^2/(a-b)/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)
-a/b^3*ln(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/b^2/(tan(1/2*d+1
/2*e*x)-1)+a/b^3*ln(tan(1/2*d+1/2*e*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.
time = 0.28, size = 191, normalized size = 2.30

$$\frac{1}{4}\left(\frac{2\left(a^2-\frac{(a^2-ab+b^2)\sin(xe+d)}{\cos(xe+d)+1}\right)}{a^2b^2-b^4-\frac{2\,(a^2b^2-ab^3)\sin(xe+d)}{\cos(xe+d)+1}+\frac{(a^2b^2-2\,ab^3+b^4)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}-\frac{a\log\left(a+b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b^3}+\frac{a\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}-1\right)}{b^3}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2 - (a^2 - a*b + b^2)*sin(x*e + d)/(cos(x*e + d) + 1))/(a^2*b^2 -
 b^4 - 2*(a^2*b^2 - a*b^3)*sin(x*e + d)/(cos(x*e + d) + 1) + (a^2*b^2 - 2*a
*b^3 + b^4)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2) - a*log(a + b - (a - b)*si
n(x*e + d)/(cos(x*e + d) + 1))/b^3 + a*log(sin(x*e + d)/(cos(x*e + d) + 1)
- 1)/b^3)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.
time = 2.33, size = 168, normalized size = 2.02

$$\frac{2\,ab\cos(xe+d)+2\,b^2\sin(xe+d)-(ab\cos(xe+d)-a^2\sin(xe+d)+a^2)\log\left(2\,ab\cos(xe+d)+a^2+b^2-(a^2-b^2)\sin(xe+d)\right)+(ab\cos(xe+d)-a^2\sin(xe+d)+a^2)\log(-\sin(xe+d)+1)}{8\,(b^4\cos(xe+d)\,e-ab^3e\sin(xe+d)+ab^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*b*cos(x*e + d) + 2*b^2*sin(x*e + d) - (a*b*cos(x*e + d) - a^2*sin(
x*e + d) + a^2)*log(2*a*b*cos(x*e + d) + a^2 + b^2 - (a^2 - b^2)*sin(x*e +
d)) + (a*b*cos(x*e + d) - a^2*sin(x*e + d) + a^2)*log(-sin(x*e + d) + 1))/(
b^4*cos(x*e + d)*e - a*b^3*e*sin(x*e + d) + a*b^3*e)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.

time = 0.43, size = 189, normalized size = 2.28

$$-\frac{\frac{2\left(a^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-ab\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-a^2\right)}{(ab^2-b^3)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2a-2|b|\right|}{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2a+2|b|\right|}\right)}{b^2|b|}}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(a^2*tan(1/2*e*x + 1/2*d) - a*b*tan(1/2*e*x + 1/2*d) + b^2*tan(1/2*
e*x + 1/2*d) - a^2)/((a*b^2 - b^3)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*
x + 1/2*d)^2 - 2*a*tan(1/2*e*x + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/2*e
*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*
x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*abs(b)))/(b^2*abs(b)))/e
```

Mupad [B]

time = 2.74, size = 126, normalized size = 1.52

$$\frac{\frac{a^2}{b^2(a-b)}-\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(a^2-ab+b^2)}{b^2(a-b)}}{e\left((2a-2b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2-4a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2a+2b\right)}-\frac{a\,\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(2a-2b)}{2}}{b}\right)}{2b^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

```
[Out] (a^2/(b^2*(a - b)) - (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/
(e*(2*a + 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) - 4*a*tan(d/2 + (e*x)/2)))
 - (a*atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e)
```

3.393 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx$

Optimal. Leaf size=142

$$\frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2}-\frac{3(a^2\cos(d+ex)+ab}{16b^4e(a+b\cos(d+ex)}$$

```
[Out] 1/16*(3*a^2+b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(a*cos(e*x+d)
+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2-3/16*(a^2*cos(e*x+d)+a
*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

Rubi [A]
time = 0.08, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3208, 3232, 3201, 31}

$$-\frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{16b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{16b^5e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] ((3*a^2 + b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(16*b^5*e) + (a*Cos[d
+ e*x] + b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x])^2)
 - (3*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(16*b^4*e*(a + b*Cos[d + e*x]
- a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3201

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
```

```
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} + \frac{\int \frac{-4a+2b\cos(d+}{(2a+2b\cos(d+e}}{}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3(a^2\cos(d}{16b^4e(a+b\cos}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3(a^2\cos(d}{16b^4e(a+b\cos}$$

$$= \frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} + \frac{a\cos(d+}{16b^2e(a+b\cos(}$$

Mathematica [**A**]
time = 1.97, size = 261, normalized size = 1.84

$$-\frac{2(3a^2+b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)-2(3a^2+b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)-\frac{b^2}{\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6ab\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)}+\frac{b^2(a^2+b^2)}{\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)^2}+\frac{6ab(a^2+b^2)\sin\left(\frac{1}{2}(d+ex)\right)}{(a+b)\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)}}{32b^5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

```
[Out] -1/32*(2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 2*(3*a^2
+ b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] - b^2/(Cos
[(d + e*x)/2] - Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d + e*
x)/2] - Sin[(d + e*x)/2]) + (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2] + (
-a + b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)
*((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(b^5*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(130) = 260$.
time = 0.64, size = 275, normalized size = 1.94

method	result
derivativedivides	$\frac{\frac{(3a^3-3a^2b+a\,b^2-b^3)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{2b^5(a-b)}-\frac{a^4+2a^2b^2+b^4}{2b^3(a-b)^2\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)^2}-\frac{-3a^4}{2b^4(a-b)^2\left(a\,\mathrm{ta}\right.}}{8e}$
default	$\frac{\frac{(3a^3-3a^2b+a\,b^2-b^3)\ln\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}{2b^5(a-b)}-\frac{a^4+2a^2b^2+b^4}{2b^3(a-b)^2\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)^2}-\frac{-3a^4}{2b^4(a-b)^2\left(a\,\mathrm{ta}\right.}}{8e}$
risch	$-\frac{i(3a^2b\,e^{3i(ex+d)}+b^3e^{3i(ex+d)}+9a^3e^{2i(ex+d)}+3a\,b^2e^{2i(ex+d)}+3ia^3e^{3i(ex+d)}+ia\,b^2e^{3i(ex+d)}+9a^2b\,e^{i(ex+d)}-b^3e^{i(ex+d)}}{8\left(ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}-ia+2a\,e^{i(ex+d)}+b\right)^2b^4e}$
norman	$\frac{-\frac{9a^5+18b\,a^4+12a^3b^2+6b^3a^2+a\,b^4}{16b^4e(3a^2-b^2)}+\frac{(9a^5+9b\,a^4+6a^3b^2+6b^3a^2+3a\,b^4-b^5)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8b^4e(3a^2-b^2)}-\frac{(9a^5-9b\,a^4+6a^3b^2-6b^3a^2+3a\,b^4+b^5)\left(\right.}{8b^4e(3a^2-b^2)}}{\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)^2\left(a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-a-b\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/e*(1/2*(3*a^3-3*a^2*b+a*b^2-b^3)/b^5/(a-b)*ln(a*tan(1/2*d+1/2*e*x)-b*ta
n(1/2*d+1/2*e*x)-a-b)-1/2*(a^4+2*a^2*b^2+b^4)/b^3/(a-b)^2/(a*tan(1/2*d+1/2*
e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^2-1/2*(-3*a^4+4*a^3*b-2*a^2*b^2+4*a*b^3+b^4)
/b^4/(a-b)^2/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)+1/2/b^3/(tan(1
/2*d+1/2*e*x)-1)^2-1/2*(-3*a-b)/b^4/(tan(1/2*d+1/2*e*x)-1)+1/2/b^5*(-3*a^2-
b^2)*ln(tan(1/2*d+1/2*e*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(136) = 272$.

time = 0.30, size = 508, normalized size = 3.58

$$-\frac{1}{16}\left(\frac{2\left(3a^5-4a^3b^2-ab^4-\frac{(9a^5-9a^4b-2a^3b^2+2a^2b^3-5ab^4+b^5)\sin(xe+d)}{\cos(xe+d)+1}+\frac{(9a^5-18a^4b+12a^3b^2-6a^2b^3+ab^4)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{(3a^5-9a^4b+10a^3b^2-6a^2b^3+ab^4+b^5)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}\right)}{a^4b^4-2a^2b^6+b^8-\frac{4(a^4b^4-a^3b^5-a^2b^6+ab^7)\sin(xe+d)}{\cos(xe+d)+1}+\frac{2(3a^4b^4-6a^3b^5+2a^2b^6+2ab^7-b^8)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{4(a^4b^4-3a^3b^5+3a^2b^6-ab^7)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{(a^4b^4-4a^3b^5+6a^2b^6-4ab^7+b^8)\sin(xe+d)^4}{(\cos(xe+d)+1)^4}}-\frac{(3a^2+b^2)\log\left(a+b-\frac{(a-b)\sin(xe+d)}{\cos(xe+d)+1}\right)}{b^5}+\frac{(3a^2+b^2)\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}-1\right)}{b^5}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 - (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 - 5*a*b^4 + b^5)*sin(x*e + d)/(cos(x*e + d) + 1) + (9*a^5 - 18*a^4*b +
12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - (3*a^
5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(x*e + d)^3/(cos(x*e
 + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 - 4*(a^4*b^4 - a^3*b^5 - a^2*b^6 +
 a*b^7)*sin(x*e + d)/(cos(x*e + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2*a^2*
b^6 + 2*a*b^7 - b^8)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - 4*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*sin(x*e + d)^3/(cos(x*e + d) + 1)^3 + (a^4*b^4
- 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(x*e + d)^4/(cos(x*e + d) + 1)^
4) - (3*a^2 + b^2)*log(a + b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b^5
 + (3*a^2 + b^2)*log(sin(x*e + d)/(cos(x*e + d) + 1) - 1)/b^5)*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(136) = 272$.

time = 2.75, size = 447, normalized size = 3.15

$$-\frac{12\,a^{2}b^{2}\cos\left(xe+d\right)^{2}-6\,a^{2}b^{2}+2\,\left(3\,a^{3}b-ab^{3}\right)\cos\left(xe+d\right)-\left(6\,a^{4}+2\,a^{2}b^{2}-\left(3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)\cos\left(xe+d\right)^{2}+2\,\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)-2\,\left(3\,a^{4}+a^{2}b^{2}+\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)\log\left(2\,ab\cos\left(xe+d\right)+a^{2}+b^{2}-\left(a^{2}-b^{2}\right)\sin\left(xe+d\right)\right)+\left(6\,a^{4}+2\,a^{2}b^{2}-\left(3\,a^{4}-2\,a^{2}b^{2}-b^{4}\right)\cos\left(xe+d\right)^{2}+2\,\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)-2\,\left(3\,a^{4}+a^{2}b^{2}+\left(3\,a^{3}b+ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)\log\left(-\sin\left(xe+d\right)+1\right)+2\,\left(3\,a^{2}b^{2}-b^{4}-3\,\left(a^{3}b-ab^{3}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)}{32\,\left(2\,ab^{6}\cos\left(xe+d\right)e+2\,a^{2}b^{5}e-\left(a^{2}b^{5}-b^{7}\right)\cos\left(xe+d\right)^{2}e-2\,\left(ab^{6}\cos\left(xe+d\right)e+a^{2}b^{5}e\right)\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(12*a^2*b^2*cos(x*e + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(x*e
+ d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(x*e + d)^2 + 2*(3
*a^3*b + a*b^3)*cos(x*e + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos(x
*e + d))*sin(x*e + d))*log(2*a*b*cos(x*e + d) + a^2 + b^2 - (a^2 - b^2)*sin
(x*e + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(x*e + d)^2
+ 2*(3*a^3*b + a*b^3)*cos(x*e + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)
*cos(x*e + d))*sin(x*e + d))*log(-sin(x*e + d) + 1) + 2*(3*a^2*b^2 - b^4 -
3*(a^3*b - a*b^3)*cos(x*e + d))*sin(x*e + d))/(2*a*b^6*cos(x*e + d)*e + 2*a
^2*b^5*e - (a^2*b^5 - b^7)*cos(x*e + d)^2*e - 2*(a*b^6*cos(x*e + d)*e + a^2
*b^5*e)*sin(x*e + d))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(130) = 260$.

time = 0.43, size = 458, normalized size = 3.23

$$\frac{\frac{2\,\left(3\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-9\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+10\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-6\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+b^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-9\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+18\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-12\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+6\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+9\,a^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-9\,a^{4}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a^{3}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a^{2}b^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-5\,ab^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b^{5}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-3\,a^{5}+4\,a^{3}b^{2}+ab^{4}\right)}{\left(a^{2}b^{4}-2\,ab^{5}+b^{6}\right)\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a+b\right)^{2}}+\frac{\left(3\,a^{2}+b^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a-2\,\left|b\right|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a+2\,\left|b\right|\right|}\right)}{b^{4}\left|b\right|}}{16\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 10
*a^3*b^2*tan(1/2*e*x + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + a*b^4*
tan(1/2*e*x + 1/2*d)^3 + b^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^5*tan(1/2*e*x + 1
/2*d)^2 + 18*a^4*b*tan(1/2*e*x + 1/2*d)^2 - 12*a^3*b^2*tan(1/2*e*x + 1/2*d)
^2 + 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^2 - a*b^4*tan(1/2*e*x + 1/2*d)^2 + 9*a^
5*tan(1/2*e*x + 1/2*d) - 9*a^4*b*tan(1/2*e*x + 1/2*d) - 2*a^3*b^2*tan(1/2*e
```

```
*x + 1/2*d) + 2*a^2*b^3*tan(1/2*e*x + 1/2*d) - 5*a*b^4*tan(1/2*e*x + 1/2*d)
 + b^5*tan(1/2*e*x + 1/2*d) - 3*a^5 + 4*a^3*b^2 + a*b^4)/((a^2*b^4 - 2*a*b^
5 + b^6)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 - 2*a*tan(1/2
*e*x + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*e*x + 1/2*d)
- 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) -
 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*abs(b)))/(b^4*abs(b)))/e
```

Mupad [B]
time = 6.48, size = 361, normalized size = 2.54

$$\frac{\mathrm{atanh}\left(\frac{2\,a-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,b}\right)\,\left(3\,a^2+b^2\right)}{8\,b^5\,e}-\frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(-9\,a^5+9\,a^4\,b+2\,a^3\,b^2-2\,a^2\,b^3+5\,a\,b^4-b^5\right)}{2\,b^4\,{\left(a-b\right)}^2}-\frac{-3\,a^5+4\,a^3\,b^2+a\,b^4}{2\,b^4\,{\left(a-b\right)}^2}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(-3\,a^4+6\,a^3\,b-4\,a^2\,b^2+2\,a\,b^3+b^4\right)}{2\,b^4\,\left(a-b\right)}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(9\,a^5-18\,a^4\,b+12\,a^3\,b^2-6\,a^2\,b^3+a\,b^4\right)}{2\,b^4\,{\left(a-b\right)}^2}}{e\,\left(8\,a\,b+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(24\,a^2-8\,b^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(16\,a\,b-16\,a^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(4\,a^2-8\,a\,b+4\,b^2\right)+4\,a^2+4\,b^2-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(16\,a^2+16\,b\,a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

```
[Out] (atanh((2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b^5*
e) - ((tan(d/2 + (e*x)/2)*(5*a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*
a^3*b^2))/(2*b^4*(a - b)^2) - (a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2)
 + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*
b^4*(a - b)) + (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3
+ 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^2
- 8*b^2) + tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4*(4
*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 - tan(d/2 + (e*x)/2)*(16*a*b + 16*a^2
)))
```

3.394 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

Optimal. Leaf size=215

$$-\frac{a(5a^2+3b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3}-\frac{5(a^2\cos(d+ex}{96b^4e(a+b\cos(d}$$

```
[Out] -1/32*a*(5*a^2+3*b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(a*cos(e
*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^3-5/96*(a^2*cos(e*x
+d)+a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2+1/96*(a*(15*a^2+4
*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)-a*sin(e
*x+d))
```

Rubi [A]
time = 0.16, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3208, 3235, 3232, 3201, 31}

$$-\frac{5(a^2\cos(d+ex)+ab\sin(d+ex))}{96b^4e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{a(5a^2+3b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{32b^7e}+\frac{b(15a^2+4b^2)\sin(d+ex)+a(15a^2+4b^2)\cos(d+ex)}{96b^6e(a(-\sin(d+ex))+a+b\cos(d+ex))}+\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] -1/32*(a*(5*a^2 + 3*b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(b^7*e) + (a
*Cos[d + e*x] + b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e
*x])^3) - (5*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(96*b^4*e*(a + b*Cos[d
+ e*x] - a*Sin[d + e*x])^2) + (a*(15*a^2 + 4*b^2)*Cos[d + e*x] + b*(15*a^2
+ 4*b^2)*Sin[d + e*x])/(96*b^6*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3201

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Di
st[f/e, Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
```

```
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx = \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} + \frac{\int \frac{-6a+4b\cos(d+ex}{(2a+2b\cos(d+ex)}}{12}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5(a^2\cos(d+}{96b^4e(a+b\cos}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5(a^2\cos(d+}{96b^4e(a+b\cos}$$

$$= \frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3} - \frac{5(a^2\cos(d+}{96b^4e(a+b\cos}$$

$$= -\frac{a(5a^2+3b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} + \frac{a\cos(d}{48b^2e(a+b\cos}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 636 vs. 2(215) = 430.

time = 1.36, size = 636, normalized size = 2.96

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

```
[Out] (12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 12*a*(5*a^
2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b*(
-150*a^6 - 130*a^4*b^2 - 24*a^2*b^4 + 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^2
- 30*a*b^3 + 4*b^4)*Cos[d + e*x] + 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2 + 2
*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] + 30*a^5*b*Cos[3
*(d + e*x)] + 41*a^4*b^2*Cos[3*(d + e*x)] + 38*a^3*b^3*Cos[3*(d + e*x)] + 1
2*a^2*b^4*Cos[3*(d + e*x)] + 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d + e*x
] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Sin[d + e
*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[d + e*x]
 - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b^2*Sin[2*
(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d + e*x)] + 6*
a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[3*(d + e*x)
] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 15*a^2*b^4*Si
n[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e*x)]))/((a +
b)*(Cos[(d + e*x)/2] - Sin[(d + e*x)/2])^3*((a + b)*Cos[(d + e*x)/2] + (-a
+ b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(201) = 402.
time = 1.05, size = 410, normalized size = 1.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/e*(-1/2*(5*a^3-5*a^2*b+3*a*b^2-3*b^3)*a/b^7/(a-b)*ln(a*tan(1/2*d+1/2*e
*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/3*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/b^4/(a-b)^3
/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)^3-1/2*(-2*a^6+3*a^5*b-3*a^
4*b^2+6*a^3*b^3+3*a*b^5+b^6)/b^5/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+
1/2*e*x)-a-b)^2-1/2*(5*a^6-12*a^5*b+12*a^4*b^2-12*a^3*b^3+9*a^2*b^4+2*b^6)/
b^6/(a-b)^3/(a*tan(1/2*d+1/2*e*x)-b*tan(1/2*d+1/2*e*x)-a-b)-1/3/b^4/(tan(1/
2*d+1/2*e*x)-1)^3-1/2*(2*a+b)/b^5/(tan(1/2*d+1/2*e*x)-1)^2-1/2*(5*a^2+2*a*b
+2*b^2)/b^6/(tan(1/2*d+1/2*e*x)-1)+1/2*a*(5*a^2+3*b^2)/b^7*ln(tan(1/2*d+1/2
*e*x)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 984 vs. 2(210) = 420.
time = 0.34, size = 984, normalized size = 4.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 - 3*(25*a^8 - 25*a^7*
b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*b^
7 + 2*b^8)*sin(x*e + d)/(cos(x*e + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a^6*
b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(x*e
+ d)^2/(cos(x*e + d) + 1)^2 - 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 150*a^5
*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(x*e + d)
^3/(cos(x*e + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*a^5*b^3
 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(x*e + d)^4/(cos(x*e
 + d) + 1)^4 - 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*a^4*b^4 -
 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(x*e + d)^5/(cos(x*e + d) +
1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 - 6*(a^6*b^6 - a^5*b^7 - 2*a
^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(x*e + d)/(cos(x*e + d) + 1) + 3
*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 - 2*a*b^11 + b
^12)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - 4*(5*a^6*b^6 - 15*a^5*b^7 + 12*a
^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(x*e + d)^3/(cos(x*e + d) +
1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9 - a^2*b^10 + 4*a
*b^11 - b^12)*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 - 6*(a^6*b^6 - 5*a^5*b^7
+ 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(x*e + d)^5/(cos(x*e +
d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b^9 + 15*a^2*b^10 -
6*a*b^11 + b^12)*sin(x*e + d)^6/(cos(x*e + d) + 1)^6) - 3*(5*a^3 + 3*a*b^2)
*log(a + b - (a - b)*sin(x*e + d)/(cos(x*e + d) + 1))/b^7 + 3*(5*a^3 + 3*a*
b^2)*log(sin(x*e + d)/(cos(x*e + d) + 1) - 1)/b^7)*e^(-1)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(210) = 420$.
time = 2.14, size = 769, normalized size = 3.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(x
*e + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(x*e + d)^2 - 6*(10*a^5*b - 9*a^3*
b^3 - 2*a*b^5)*cos(x*e + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^
3 - 3*a*b^5)*cos(x*e + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(x*e + d
)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6
- 12*a^4*b^2 - 9*a^2*b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e
+ d))*sin(x*e + d))*log(2*a*b*cos(x*e + d) + a^2 + b^2 - (a^2 - b^2)*sin(x*
e + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(x*e
 + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3
*a^3*b^3)*cos(x*e + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^2 - 9*a^2
```

```
*b^4)*cos(x*e + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(x*e + d))*sin(x*e + d))*
log(-sin(x*e + d) + 1) - 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*a^4*b^2 -
3*a^2*b^4 - 4*b^6)*cos(x*e + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*b^5)*cos(x*
e + d))*sin(x*e + d))/(6*a^2*b^8*cos(x*e + d)*e + 4*a^3*b^7*e - (3*a^2*b^8
- b^10)*cos(x*e + d)^3*e - 3*(a^3*b^7 - a*b^9)*cos(x*e + d)^2*e - (6*a^2*b^
8*cos(x*e + d)*e + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*cos(x*e + d)^2*e)*sin(
x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

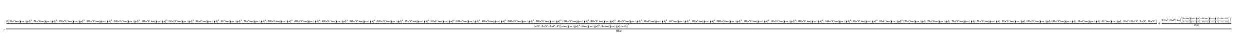
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(201) = 402.
time = 0.47, size = 957, normalized size = 4.45



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*a^8*tan(1/2*e*x + 1/2*d)^5 - 75*a^7*b*tan(1/2*e*x + 1/2*d)^5 +
 159*a^6*b^2*tan(1/2*e*x + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*e*x + 1/2*d)^5 +
165*a^4*b^4*tan(1/2*e*x + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*e*x + 1/2*d)^5 + 5
1*a^2*b^6*tan(1/2*e*x + 1/2*d)^5 - 21*a*b^7*tan(1/2*e*x + 1/2*d)^5 + 6*b^8*
tan(1/2*e*x + 1/2*d)^5 - 75*a^8*tan(1/2*e*x + 1/2*d)^4 + 300*a^7*b*tan(1/2*
e*x + 1/2*d)^4 - 495*a^6*b^2*tan(1/2*e*x + 1/2*d)^4 + 480*a^5*b^3*tan(1/2*e
*x + 1/2*d)^4 - 345*a^4*b^4*tan(1/2*e*x + 1/2*d)^4 + 180*a^3*b^5*tan(1/2*e*
x + 1/2*d)^4 - 57*a^2*b^6*tan(1/2*e*x + 1/2*d)^4 + 12*a*b^7*tan(1/2*e*x + 1
/2*d)^4 + 150*a^8*tan(1/2*e*x + 1/2*d)^3 - 450*a^7*b*tan(1/2*e*x + 1/2*d)^3
 + 500*a^6*b^2*tan(1/2*e*x + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*e*x + 1/2*d)^3
+ 126*a^4*b^4*tan(1/2*e*x + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*e*x + 1/2*d)^3 -
48*a^2*b^6*tan(1/2*e*x + 1/2*d)^3 + 12*a*b^7*tan(1/2*e*x + 1/2*d)^3 - 4*b^8
*tan(1/2*e*x + 1/2*d)^3 - 150*a^8*tan(1/2*e*x + 1/2*d)^2 + 300*a^7*b*tan(1/
2*e*x + 1/2*d)^2 - 120*a^6*b^2*tan(1/2*e*x + 1/2*d)^2 - 60*a^5*b^3*tan(1/2*
e*x + 1/2*d)^2 + 102*a^4*b^4*tan(1/2*e*x + 1/2*d)^2 - 144*a^3*b^5*tan(1/2*e
*x + 1/2*d)^2 + 60*a^2*b^6*tan(1/2*e*x + 1/2*d)^2 - 12*a*b^7*tan(1/2*e*x +
1/2*d)^2 + 75*a^8*tan(1/2*e*x + 1/2*d) - 75*a^7*b*tan(1/2*e*x + 1/2*d) - 75
*a^6*b^2*tan(1/2*e*x + 1/2*d) + 75*a^5*b^3*tan(1/2*e*x + 1/2*d) - 39*a^4*b^
4*tan(1/2*e*x + 1/2*d) + 39*a^3*b^5*tan(1/2*e*x + 1/2*d) + 33*a^2*b^6*tan(1
```

```
/2*e*x + 1/2*d) - 15*a*b^7*tan(1/2*e*x + 1/2*d) + 6*b^8*tan(1/2*e*x + 1/2*d
) - 15*a^8 + 31*a^6*b^2 - 9*a^4*b^4 - 15*a^2*b^6)/((a^3*b^6 - 3*a^2*b^7 + 3
*a*b^8 - b^9)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 - 2*a*ta
n(1/2*e*x + 1/2*d) + a + b)^3) + 3*(5*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*e*
x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*abs(b)))/(b^6*abs(b)))/e
```

Mupad [B]
time = 7.09, size = 731, normalized size = 3.40

$$\frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,{\left(a-b\right)}^3}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+10\,a^5\,b^3-17\,a^4\,b^4+24\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7\right)}{b^6\,{\left(a-b\right)}^3}+\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(25\,a^7-75\,a^6\,b+90\,a^5\,b^2-70\,a^4\,b^3+45\,a^3\,b^4-15\,a^2\,b^5+4\,a\,b^6\right)}{2\,b^6\,{\left(a-b\right)}^2}-\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(75\,a^8-225\,a^7\,b+250\,a^6\,b^2-150\,a^5\,b^3+63\,a^4\,b^4+11\,a^3\,b^5-24\,a^2\,b^6+6\,a\,b^7-2\,b^8\right)}{3\,b^6\,{\left(a-b\right)}^3}-\frac{{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(5\,a^6-15\,a^5\,b+18\,a^4\,b^2-14\,a^3\,b^3+9\,a^2\,b^4-3\,a\,b^5+2\,b^6\right)}{2\,b^6\,\left(a-b\right)}-\frac{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(25\,a^8-25\,a^7\,b-25\,a^6\,b^2+25\,a^5\,b^3-13\,a^4\,b^4+13\,a^3\,b^5+11\,a^2\,b^6-5\,a\,b^7+2\,b^8\right)}{2\,b^6\,{\left(a-b\right)}^3}}{e\,\left({\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6\,\left(8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3\right)-{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(48\,a^3-96\,a^2\,b+48\,a\,b^2\right)-{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(-120\,a^3-120\,a^2\,b+24\,a\,b^2+24\,b^3\right)-{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(-120\,a^3+120\,a^2\,b+24\,a\,b^2-24\,b^3\right)+24\,a\,b^2+24\,a^2\,b+{\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(96\,a\,b^2-160\,a^3\right)-\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(48\,a^3+96\,a^2\,b+48\,a\,b^2\right)+8\,a^3+8\,b^3\right)}-\frac{a\,\mathrm{atanh}\left(\frac{a\,\left(2\,a-\mathrm{tan}\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)\right)\,\left(5\,a^2+3\,b^2\right)}{2\,b\,\left(5\,a^3+3\,a\,b^2\right)}\right)\,\left(5\,a^2+3\,b^2\right)}{16\,b^7\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^4,x)
```

```
[Out] ((15*a^8 + 15*a^2*b^6 + 9*a^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/
2 + (e*x)/2)^2*(2*a*b^7 - 50*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*
a^4*b^4 + 10*a^5*b^3 + 20*a^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4
*(4*a*b^6 - 75*a^6*b + 25*a^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a
^5*b^2))/(2*b^6*(a - b)^2) - (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 7
5*a^8 - 2*b^8 - 24*a^2*b^6 + 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^
6*b^2))/(3*b^6*(a - b)^3) - (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b
^5 + 2*b^6 + 9*a^2*b^4 - 14*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) - (tan(d
/2 + (e*x)/2)*(25*a^8 - 25*a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^
5 - 13*a^4*b^4 + 25*a^5*b^3 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 +
 (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) - tan(d/2 + (e*x)/2)^5*(4
8*a*b^2 - 96*a^2*b + 48*a^3) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 - 120*a^2*b -
 120*a^3 + 24*b^3) - tan(d/2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*b - 120*a^3 -
 24*b^3) + 24*a*b^2 + 24*a^2*b + tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3)
- tan(d/2 + (e*x)/2)*(48*a*b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3)) - (a*
atanh((a*(2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(3*a*
b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e)
```

3.395 $\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$

Optimal. Leaf size=260

$$\frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{5ac(10a^2+11(b^2+c^2))\cos(d+ex)}{24e}+\frac{5ab(10a^2+11(b^2+c^2))\sin$$

```
[Out] 1/8*(8*a^4+24*a^2*(b^2+c^2)+3*(b^2+c^2)^2)*x-5/24*a*c*(10*a^2+11*b^2+11*c^2
)*cos(e*x+d)/e+5/24*a*b*(10*a^2+11*b^2+11*c^2)*sin(e*x+d)/e-7/12*(a*c*cos(e
*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^2/e-1/4*(c*cos(e*x+d)-b
*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^3/e-1/24*(a+b*cos(e*x+d)+c*sin(e
*x+d))*(c*(26*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(26*a^2+9*b^2+9*c^2)*sin(e*x+d)
)/e
```

Rubi [A]
time = 0.27, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3199, 3225, 2717, 2718}

$$\frac{5ab(10a^2+11(b^2+c^2))\sin(d+ex)}{24e}-\frac{5ac(10a^2+11(b^2+c^2))\cos(d+ex)}{24e}-\frac{(c(26a^2+9(b^2+c^2))\cos(d+ex)-b(26a^2+9(b^2+c^2))\sin(d+ex))\,(a+b\cos(d+ex)+c\sin(d+ex))}{24e}+\frac{1}{8}x\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}-\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x)/8 - (5*a*c*(10*a^2 + 11*
(b^2 + c^2))*Cos[d + e*x])/(24*e) + (5*a*b*(10*a^2 + 11*(b^2 + c^2))*Sin[d
+ e*x])/(24*e) - (7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^2)/(12*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])^3)/(4*e) - ((a + b*Cos[d + e*x] + c*Sin[d +
e*x])*(c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(26*a^2 + 9*(b^2 + c^2))
*Sin[d + e*x]))/(24*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
```

```
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex}{4e}$$
$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d}{12e}$$
$$= -\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d}{12e}$$
$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{7(ac\cos(d+ex)-ab}{}$$
$$= \frac{1}{8}\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)x-\frac{5ac(10a^2+11(b^2+c}{24e}$$

Mathematica [A]
time = 0.79, size = 237, normalized size = 0.91

$$\frac{12\left(8a^4+24a^2(b^2+c^2)+3(b^2+c^2)^2\right)(d+ex)-96ac(4a^2+3(b^2+c^2))\cos(d+ex)-48bc(6a^2+b^2+c^2)\cos(2(d+ex))+32ac(-3b^2+c^2)\cos(3(d+ex))-12bc(b^2-c^2)\cos(4(d+ex))+96ab(4a^2+3(b^2+c^2))\sin(d+ex)+24(b^2-c^2)(6a^2+b^2+c^2)\sin(2(d+ex))+32ab(b^2-3c^2)\sin(3(d+ex))+3(b^4-6b^2c^2+c^4)\sin(4(d+ex))}{96e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*(d + e*x) - 96*a*c*(4*a^
2 + 3*(b^2 + c^2))*Cos[d + e*x] - 48*b*c*(6*a^2 + b^2 + c^2)*Cos[2*(d + e*x
)] + 32*a*c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d +
 e*x)] + 96*a*b*(4*a^2 + 3*(b^2 + c^2))*Sin[d + e*x] + 24*(b^2 - c^2)*(6*a^
2 + b^2 + c^2)*Sin[2*(d + e*x)] + 32*a*b*(b^2 - 3*c^2)*Sin[3*(d + e*x)] + 3
*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

Maple [A]
time = 0.30, size = 335, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(a^4*(e*x+d)-6*a^2*b*c*cos(e*x+d)^2-4*a*b^2*c*cos(e*x+d)^3+4*a*b*c^2*si
n(e*x+d)^3-b^3*c*cos(e*x+d)^4+6*b^2*c^2*(-1/4*sin(e*x+d)*cos(e*x+d)^3+1/8*c
os(e*x+d)*sin(e*x+d)+1/8*e*x+1/8*d)+c^3*b*sin(e*x+d)^4+4*a^3*b*sin(e*x+d)-4
*cos(e*x+d)*a^3*c+6*a^2*b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+6*a^2
*c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)+4/3*a*b^3*(2+cos(e*x+d)^2)*
sin(e*x+d)-4/3*a*c^3*(2+sin(e*x+d)^2)*cos(e*x+d)+b^4*(1/4*(cos(e*x+d)^3+3/2
*cos(e*x+d))*sin(e*x+d)+3/8*e*x+3/8*d)+c^4*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+
d))*cos(e*x+d)+3/8*e*x+3/8*d))
```

Maxima [A]
time = 0.28, size = 339, normalized size = 1.30

$$-b^3c\cos(xe+d)^4e^{(-1)} + bc^3e^{(-1)}\sin(xe+d)^4 + \frac{1}{32}(12xe+12d+\sin(4xe+4d)+8\sin(2xe+2d))b^4e^{(-1)} + \frac{3}{16}(4xe+4d-\sin(4xe+4d))b^2c^2e^{(-1)} + \frac{1}{32}(12xe+12d+\sin(4xe+4d)-8\sin(2xe+2d))c^4e^{(-1)} + a^4x - 4(c\cos(xe+d)e^{(-1)} - be^{(-1)}\sin(xe+d))a^3 - \frac{3}{2}(4bc\cos(xe+d)^2e^{(-1)} - (2xe+2d+\sin(2xe+2d))b^2e^{(-1)} - (2xe+2d-\sin(2xe+2d))c^2e^{(-1)})a^2 - \frac{4}{3}(3b^2c\cos(xe+d)^3e^{(-1)} - 3bc^2e^{(-1)}\sin(xe+d)^3 + (\sin(xe+d)^3 - 3\sin(xe+d))b^3e^{(-1)} - (\cos(xe+d)^3 - 3\cos(xe+d))c^3e^{(-1)})a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] -b^3*c*cos(x*e + d)^4*e^(-1) + b*c^3*e^(-1)*sin(x*e + d)^4 + 1/32*(12*x*e +
 12*d + sin(4*x*e + 4*d) + 8*sin(2*x*e + 2*d))*b^4*e^(-1) + 3/16*(4*x*e + 4
*d - sin(4*x*e + 4*d))*b^2*c^2*e^(-1) + 1/32*(12*x*e + 12*d + sin(4*x*e + 4
*d) - 8*sin(2*x*e + 2*d))*c^4*e^(-1) + a^4*x - 4*(c*cos(x*e + d)*e^(-1) - b
*e^(-1)*sin(x*e + d))*a^3 - 3/2*(4*b*c*cos(x*e + d)^2*e^(-1) - (2*x*e + 2*d
 + sin(2*x*e + 2*d))*b^2*e^(-1) - (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-
1))*a^2 - 4/3*(3*b^2*c*cos(x*e + d)^3*e^(-1) - 3*b*c^2*e^(-1)*sin(x*e + d)^
3 + (sin(x*e + d)^3 - 3*sin(x*e + d))*b^3*e^(-1) - (cos(x*e + d)^3 - 3*cos(
x*e + d))*c^3*e^(-1))*a
```

Fricas [A]
time = 2.59, size = 263, normalized size = 1.01

$$-\frac{1}{24}\left(24(b^3c - bc^3)\cos(xe+d)^4 + 32(3ab^2c - ac^3)\cos(xe+d)^3 + 48(3a^2bc + bc^3)\cos(xe+d)^2 - 3(8a^4 + 24a^2b^2 + 3b^4 + 3c^4 + 6(4a^2+b^2)c^2)xe + 96(a^3c + ac^3)\cos(xe+d) - (96a^3b + 64ab^3 + 96abc^2 + 6(b^4 - 6b^2c^2 + c^4)\cos(xe+d)^3 + 32(ab^3 - 3abc^2)\cos(xe+d)^2 + 3(24a^2b^2 + 3b^4 - 5c^4 - 6(4a^2 - b^2)c^2)\cos(xe+d))\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(24*(b^3*c - b*c^3)*cos(x*e + d)^4 + 32*(3*a*b^2*c - a*c^3)*cos(x*e +
 d)^3 + 48*(3*a^2*b*c + b*c^3)*cos(x*e + d)^2 - 3*(8*a^4 + 24*a^2*b^2 + 3*b
^4 + 3*c^4 + 6*(4*a^2 + b^2)*c^2)*x*e + 96*(a^3*c + a*c^3)*cos(x*e + d) - (
96*a^3*b + 64*a*b^3 + 96*a*b*c^2 + 6*(b^4 - 6*b^2*c^2 + c^4)*cos(x*e + d)^3
```

```
 + 32*(a*b^3 - 3*a*b*c^2)*cos(x*e + d)^2 + 3*(24*a^2*b^2 + 3*b^4 - 5*c^4 -
6*(4*a^2 - b^2)*c^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(253) = 506$.
time = 0.32, size = 682, normalized size = 2.62

$$\begin{cases} a^4x + \frac{4a^3b\sin(d+ex)}{e} - \frac{4a^3c\cos(d+ex)}{e} + 3a^2b^2x\sin^2(d+ex) + 3a^2b^2x\cos^2(d+ex) + \frac{3a^2b^2\sin(d+ex)\cos(d+ex)}{e} + \frac{6a^2bc\sin^2(d+ex)}{e} + 3a^2c^2x\sin^2(d+ex) + 3a^2c^2x\cos^2(d+ex) - \frac{3a^2c^2\sin(d+ex)\cos(d+ex)}{e} + \frac{8ab^3\sin^3(d+ex)}{3e} + \frac{4ab^3\sin(d+ex)\cos^2(d+ex)}{e} - \frac{4ab^2c\cos^3(d+ex)}{e} + \frac{4abc^2\sin^3(d+ex)}{e} - \frac{4ac^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{8ac^3\cos^3(d+ex)}{3e} + \frac{3b^4x\sin^4(d+ex)}{8} + \frac{3b^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + \frac{3b^4x\cos^4(d+ex)}{8} + \frac{3b^4\sin^3(d+ex)\cos(d+ex)}{8e} + \frac{5b^4\sin(d+ex)\cos^3(d+ex)}{8e} - \frac{b^3c\cos^4(d+ex)}{e} + \frac{3b^2c^2x\sin^4(d+ex)}{4} + \frac{3b^2c^2x\sin^2(d+ex)\cos^2(d+ex)}{2} + \frac{3b^2c^2x\cos^4(d+ex)}{4} + \frac{3b^2c^2\sin^3(d+ex)\cos(d+ex)}{4e} - \frac{3b^2c^2\sin(d+ex)\cos^3(d+ex)}{4e} + \frac{bc^3\sin^4(d+ex)}{e} + \frac{3c^4x\sin^4(d+ex)}{8} + \frac{3c^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + \frac{3c^4x\cos^4(d+ex)}{8} - \frac{5c^4\sin^3(d+ex)\cos(d+ex)}{8e} - \frac{3c^4\sin(d+ex)\cos^3(d+ex)}{8e} & \text{for } e \neq 0 \\ x(a + b\cos(d) + c\sin(d))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(d + e*x)/e - 4*a**3*c*cos(d + e*x)/e + 3*a
**2*b**2*x*sin(d + e*x)**2 + 3*a**2*b**2*x*cos(d + e*x)**2 + 3*a**2*b**2*si
n(d + e*x)*cos(d + e*x)/e + 6*a**2*b*c*sin(d + e*x)**2/e + 3*a**2*c**2*x*si
n(d + e*x)**2 + 3*a**2*c**2*x*cos(d + e*x)**2 - 3*a**2*c**2*sin(d + e*x)*co
s(d + e*x)/e + 8*a*b**3*sin(d + e*x)**3/(3*e) + 4*a*b**3*sin(d + e*x)*cos(d
 + e*x)**2/e - 4*a*b**2*c*cos(d + e*x)**3/e + 4*a*b*c**2*sin(d + e*x)**3/e
- 4*a*c**3*sin(d + e*x)**2*cos(d + e*x)/e - 8*a*c**3*cos(d + e*x)**3/(3*e)
+ 3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x)**2/4 +
 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5
*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - b**3*c*cos(d + e*x)**4/e + 3*b**
2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/
2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 3*b**2*c**2*sin(d + e*x)**3*cos(d + e
*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) + b*c**3*sin(d +
 e*x)**4/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d +
e*x)**2/4 + 3*c**4*x*cos(d + e*x)**4/8 - 5*c**4*sin(d + e*x)**3*cos(d + e*x
)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e), Ne(e, 0)), (x*(a + b*c
os(d) + c*sin(d))**4, True))
```

Giac [**A**]
time = 0.45, size = 286, normalized size = 1.10

$$\frac{1}{8}\left(8a^4 + 24a^2b^2 + 3b^4 + 24a^2c^2 + 6b^2c^2 + 3c^4\right)x - \frac{(b^3c - bc^3)\cos(4ex + 4d)}{8e} - \frac{(3ab^2c - ac^3)\cos(3ex + 3d)}{3e} - \frac{(6a^2bc + b^3c + bc^3)\cos(2ex + 2d)}{2e} - \frac{(4a^3c + 3ab^2c + 3ac^3)\cos(ex + d)}{e} + \frac{(b^4 - 6b^2c^2 + c^4)\sin(4ex + 4d)}{32e} + \frac{(ab^3 - 3abc^2)\sin(3ex + 3d)}{3e} + \frac{(6a^2b^2 + b^4 - 6a^2c^2 - c^4)\sin(2ex + 2d)}{4e} + \frac{(4a^3b + 3ab^3 + 3abc^2)\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 24*a^2*c^2 + 6*b^2*c^2 + 3*c^4)*x - 1/8*(
b^3*c - b*c^3)*cos(4*e*x + 4*d)/e - 1/3*(3*a*b^2*c - a*c^3)*cos(3*e*x + 3*d
)/e - 1/2*(6*a^2*b*c + b^3*c + b*c^3)*cos(2*e*x + 2*d)/e - (4*a^3*c + 3*a*b
^2*c + 3*a*c^3)*cos(e*x + d)/e + 1/32*(b^4 - 6*b^2*c^2 + c^4)*sin(4*e*x + 4
*d)/e + 1/3*(a*b^3 - 3*a*b*c^2)*sin(3*e*x + 3*d)/e + 1/4*(6*a^2*b^2 + b^4 -
 6*a^2*c^2 - c^4)*sin(2*e*x + 2*d)/e + (4*a^3*b + 3*a*b^3 + 3*a*b*c^2)*sin(
e*x + d)/e
```

Mupad [B]
time = 3.37, size = 376, normalized size = 1.45

$$\frac{6b^4\sin(2d+2ex)+\frac{3b^4\sin(4d+4ex)}{4}-6c^4\sin(2d+2ex)+\frac{3c^4\sin(4d+4ex)}{4}+8ac^3\cos(3d+3ex)-12bc^3\cos(2d+2ex)-12b^3c\cos(2d+2ex)+3bc^3\cos(4d+4ex)-3b^3c\cos(4d+4ex)+8ab^3\sin(3d+3ex)+36a^2b^2\sin(2d+2ex)-36a^2c^2\sin(2d+2ex)-\frac{9b^2c^2\sin(4d+4ex)}{2}-72ac^3\cos(d+ex)-96a^3c\cos(d+ex)+72ab^3\sin(d+ex)+96a^3b\sin(d+ex)+24a^4ex+9b^4ex+9c^4ex-72ab^2c\cos(d+ex)+72abc^2\sin(d+ex)-72a^2bc\cos(2d+2ex)-24ab^2c\cos(3d+3ex)-24abc^2\sin(3d+3ex)+72a^2b^2ex+72a^2c^2ex+18b^2c^2ex}{24e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)
```

```
[Out] (6*b^4*sin(2*d + 2*e*x) + (3*b^4*sin(4*d + 4*e*x))/4 - 6*c^4*sin(2*d + 2*e*
x) + (3*c^4*sin(4*d + 4*e*x))/4 + 8*a*c^3*cos(3*d + 3*e*x) - 12*b*c^3*cos(2
*d + 2*e*x) - 12*b^3*c*cos(2*d + 2*e*x) + 3*b*c^3*cos(4*d + 4*e*x) - 3*b^3*
c*cos(4*d + 4*e*x) + 8*a*b^3*sin(3*d + 3*e*x) + 36*a^2*b^2*sin(2*d + 2*e*x)
 - 36*a^2*c^2*sin(2*d + 2*e*x) - (9*b^2*c^2*sin(4*d + 4*e*x))/2 - 72*a*c^3*
cos(d + e*x) - 96*a^3*c*cos(d + e*x) + 72*a*b^3*sin(d + e*x) + 96*a^3*b*sin
(d + e*x) + 24*a^4*e*x + 9*b^4*e*x + 9*c^4*e*x - 72*a*b^2*c*cos(d + e*x) +
72*a*b*c^2*sin(d + e*x) - 72*a^2*b*c*cos(2*d + 2*e*x) - 24*a*b^2*c*cos(3*d
+ 3*e*x) - 24*a*b*c^2*sin(3*d + 3*e*x) + 72*a^2*b^2*e*x + 72*a^2*c^2*e*x +
18*b^2*c^2*e*x)/(24*e)
```

3.396 $\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$

Optimal. Leaf size=170

$$\frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x-\frac{c(11a^2+4(b^2+c^2))\cos(d+ex)}{6e}+\frac{b(11a^2+4(b^2+c^2))\sin(d+ex)}{6e}-\frac{5(ac\cos(d+ex)}{}$$

```
[Out] 1/2*a*(2*a^2+3*b^2+3*c^2)*x-1/6*c*(11*a^2+4*b^2+4*c^2)*cos(e*x+d)/e+1/6*b*(
11*a^2+4*b^2+4*c^2)*sin(e*x+d)/e-5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*c
os(e*x+d)+c*sin(e*x+d))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c
*sin(e*x+d))^2/e
```

Rubi [A]
time = 0.13, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3199, 3225, 2717, 2718}

$$\frac{b(11a^2+4(b^2+c^2))\sin(d+ex)}{6e}-\frac{c(11a^2+4(b^2+c^2))\cos(d+ex)}{6e}+\frac{1}{2}ax\left(2a^2+3\left(b^2+c^2\right)\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}-\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*x)/2 - (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x])
/(6*e) + (b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*e) - (5*(a*c*Cos[d +
e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))/(6*e) - ((c
*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3
*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+e}{3e}$$
$$= -\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d}{6e}$$
$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x-\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a}{6e}$$
$$= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x-\frac{c(11a^2+4(b^2+c^2))\cos(d+ex)}{6e}+\frac{b}{}$$

Mathematica [A]

time = 0.32, size = 144, normalized size = 0.85

$$\frac{6a(2a^2+3(b^2+c^2))(d+ex)-9c(4a^2+b^2+c^2)\cos(d+ex)-18abc\cos(2(d+ex))+c(-3b^2+c^2)\cos(3(d+ex))+9b(4a^2+b^2+c^2)\sin(d+ex)+9a(b^2-c^2)\sin(2(d+ex))+b(b^2-3c^2)\sin(3(d+ex))}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

```
[Out] (6*a*(2*a^2 + 3*(b^2 + c^2))*(d + e*x) - 9*c*(4*a^2 + b^2 + c^2)*Cos[d + e*
x] - 18*a*b*c*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 9*b*(4
*a^2 + b^2 + c^2)*Sin[d + e*x] + 9*a*(b^2 - c^2)*Sin[2*(d + e*x)] + b*(b^2
- 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

Maple [A]

time = 0.25, size = 177, normalized size = 1.04

method	result
derivativedivides	$\frac{a^3(ex+d)+3a^2b\sin(ex+d)-3a^2c\cos(ex+d)+3a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-3abc\left(\cos^2(ex+d)\right)+3a\,c^2\left(-\frac{\cos(ex+}{}\right.}{e}$

default	$\frac{a^3(ex+d)+3a^2b\sin(ex+d)-3a^2c\cos(ex+d)+3a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-3abc\left(\cos^2(ex+d)\right)+3a\,c^2\left(-\frac{\cos(ex+d)\,\mathrm{s}}{2}\right.}{e}$
risch	$a^3x+\frac{3a\,b^2x}{2}+\frac{3a\,c^2x}{2}-\frac{3c\cos(ex+d)a^2}{e}-\frac{3c\cos(ex+d)b^2}{4e}-\frac{3c^3\cos(ex+d)}{4e}+\frac{3b\sin(ex+d)a^2}{e}+\frac{3b^3\sin(ex+}{4e}$
norman	$\frac{(a^3+\frac{3}{2}a\,b^2+\frac{3}{2}a\,c^2)x+(a^3+\frac{3}{2}a\,b^2+\frac{3}{2}a\,c^2)x\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2+\frac{9}{2}a\,c^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(3a^3+\frac{9}{2}a\,b^2+\frac{9}{2}a\,c^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(a^3*(e*x+d)+3*a^2*b*sin(e*x+d)-3*a^2*c*cos(e*x+d)+3*a*b^2*(1/2*cos(e*x
+d)*sin(e*x+d)+1/2*e*x+1/2*d)-3*a*b*c*cos(e*x+d)^2+3*a*c^2*(-1/2*cos(e*x+d)
*sin(e*x+d)+1/2*e*x+1/2*d)+1/3*b^3*(2+cos(e*x+d)^2)*sin(e*x+d)-b^2*c*cos(e*
x+d)^3+c^2*b*sin(e*x+d)^3-1/3*c^3*(2+sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A]
time = 0.27, size = 193, normalized size = 1.14

$$-b^2c\cos(xe+d)^3e^{(-1)}+bc^2e^{(-1)}\sin(xe+d)^3-\frac{1}{3}\left(\sin(xe+d)^3-3\sin(xe+d)\right)b^3e^{(-1)}+\frac{1}{3}\left(\cos(xe+d)^3-3\cos(xe+d)\right)c^3e^{(-1)}+a^3x-3\left(c\cos(xe+d)e^{(-1)}-be^{(-1)}\sin(xe+d)\right)a^2-\frac{3}{4}\left(4bc\cos(xe+d)^2e^{(-1)}-(2xe+2d+\sin(2xe+2d))b^2e^{(-1)}-(2xe+2d-\sin(2xe+2d))c^2e^{(-1)}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] -b^2*c*cos(x*e + d)^3*e^(-1) + b*c^2*e^(-1)*sin(x*e + d)^3 - 1/3*(sin(x*e +
 d)^3 - 3*sin(x*e + d))*b^3*e^(-1) + 1/3*(cos(x*e + d)^3 - 3*cos(x*e + d))*
c^3*e^(-1) + a^3*x - 3*(c*cos(x*e + d)*e^(-1) - b*e^(-1)*sin(x*e + d))*a^2
- 3/4*(4*b*c*cos(x*e + d)^2*e^(-1) - (2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e
^(-1) - (2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1))*a
```

Fricas [A]
time = 2.70, size = 153, normalized size = 0.90

$$-\frac{1}{6}\left(18abc\cos(xe+d)^2+2\left(3b^2c-c^3\right)\cos(xe+d)^3-3\left(2a^3+3ab^2+3ac^2\right)xe+6\left(3a^2c+c^3\right)\cos(xe+d)-\left(18a^2b+4b^3+6bc^2+2\left(b^3-3bc^2\right)\cos(xe+d)^2+9\left(ab^2-ac^2\right)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a*b*c*cos(x*e + d)^2 + 2*(3*b^2*c - c^3)*cos(x*e + d)^3 - 3*(2*a^3
 + 3*a*b^2 + 3*a*c^2)*x*e + 6*(3*a^2*c + c^3)*cos(x*e + d) - (18*a^2*b + 4*
b^3 + 6*b*c^2 + 2*(b^3 - 3*b*c^2)*cos(x*e + d)^2 + 9*(a*b^2 - a*c^2)*cos(x*
e + d))*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.17, size = 294, normalized size = 1.73

$$\begin{cases} a^3x+\frac{3a^2b\sin(d+ex)}{e}-\frac{3a^2c\cos(d+ex)}{e}+\frac{3ab^2x\sin^2(d+ex)}{2}+\frac{3ab^2x\cos^2(d+ex)}{2}+\frac{3ab^2\sin(d+ex)\cos(d+ex)}{2e}+\frac{3abc\sin^2(d+ex)}{e}+\frac{3ac^2x\sin^2(d+ex)}{2}+\frac{3ac^2x\cos^2(d+ex)}{2}-\frac{3ac^2\sin(d+ex)\cos(d+ex)}{2e}+\frac{2b^3\sin^3(d+ex)}{3e}+\frac{b^3\sin(d+ex)\cos^2(d+ex)}{e}-\frac{b^2c\cos^3(d+ex)}{e}+\frac{bc^2\sin^3(d+ex)}{e}-\frac{c^3\sin^2(d+ex)\cos(d+ex)}{e}-\frac{2c^3\cos^3(d+ex)}{3e} & \text{for } e\neq 0 \\ x(a+b\cos(d)+c\sin(d))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(d + e*x)/e - 3*a**2*c*cos(d + e*x)/e + 3*a
*b**2*x*sin(d + e*x)**2/2 + 3*a*b**2*x*cos(d + e*x)**2/2 + 3*a*b**2*sin(d +
 e*x)*cos(d + e*x)/(2*e) + 3*a*b*c*sin(d + e*x)**2/e + 3*a*c**2*x*sin(d + e
*x)**2/2 + 3*a*c**2*x*cos(d + e*x)**2/2 - 3*a*c**2*sin(d + e*x)*cos(d + e*x
)/(2*e) + 2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**2/
e - b**2*c*cos(d + e*x)**3/e + b*c**2*sin(d + e*x)**3/e - c**3*sin(d + e*x)
**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(a + b*cos
(d) + c*sin(d))**3, True))
```

Giac [A]
time = 0.44, size = 167, normalized size = 0.98

$$-\frac{3\,abc\cos\left(2\,ex+2\,d\right)}{2\,e}+\frac{1}{2}\left(2\,a^{3}+3\,ab^{2}+3\,ac^{2}\right)x-\frac{\left(3\,b^{2}c-c^{3}\right)\cos\left(3\,ex+3\,d\right)}{12\,e}-\frac{3\left(4\,a^{2}c+b^{2}c+c^{3}\right)\cos\left(ex+d\right)}{4\,e}+\frac{\left(b^{3}-3\,bc^{2}\right)\sin\left(3\,ex+3\,d\right)}{12\,e}+\frac{3\left(ab^{2}-ac^{2}\right)\sin\left(2\,ex+2\,d\right)}{4\,e}+\frac{3\left(4\,a^{2}b+b^{3}+bc^{2}\right)\sin\left(ex+d\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*b*c*cos(2*e*x + 2*d)/e + 1/2*(2*a^3 + 3*a*b^2 + 3*a*c^2)*x - 1/12*(3
*b^2*c - c^3)*cos(3*e*x + 3*d)/e - 3/4*(4*a^2*c + b^2*c + c^3)*cos(e*x + d)
/e + 1/12*(b^3 - 3*b*c^2)*sin(3*e*x + 3*d)/e + 3/4*(a*b^2 - a*c^2)*sin(2*e*
x + 2*d)/e + 3/4*(4*a^2*b + b^3 + b*c^2)*sin(e*x + d)/e
```

Mupad [B]
time = 3.70, size = 333, normalized size = 1.96

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(2\,a^2+3\,b^2+3\,c^2\right)}{2\,a^3+3\,a\,b^2+3\,a\,c^2}\right)\left(2\,a^2+3\,b^2+3\,c^2\right)}{e}-\frac{a\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\left(2\,a^2+3\,b^2+3\,c^2\right)}{e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(12\,a^2\,c-12\,b\,a\,c+4\,c^3\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(12\,a^2\,b+\frac{4\,b^3}{3}+8\,b\,c^2\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(6\,a^2\,b+3\,a\,b^2-3\,a\,c^2+2\,b^3\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\left(6\,c\,a^2-12\,c\,a\,b+6\,c\,b^2\right)+6\,a^2\,c+2\,b^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\left(6\,a^2\,b-3\,a\,b^2+3\,a\,c^2+2\,b^3\right)+\frac{4\,c^3}{3}}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

```
[Out] (a*atan((a*tan(d/2 + (e*x)/2)*(2*a^2 + 3*b^2 + 3*c^2))/(3*a*b^2 + 3*a*c^2 +
 2*a^3))*(2*a^2 + 3*b^2 + 3*c^2))/e - (a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/
2)*(2*a^2 + 3*b^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)^2*(12*a^2*c + 4*c^3 - 1
2*a*b*c) - tan(d/2 + (e*x)/2)^3*(12*a^2*b + 8*b*c^2 + (4*b^3)/3) - tan(d/2
+ (e*x)/2)*(3*a*b^2 + 6*a^2*b - 3*a*c^2 + 2*b^3) + tan(d/2 + (e*x)/2)^4*(6*
a^2*c + 6*b^2*c - 12*a*b*c) + 6*a^2*c + 2*b^2*c - tan(d/2 + (e*x)/2)^5*(6*a
^2*b - 3*a*b^2 + 3*a*c^2 + 2*b^3) + (4*c^3)/3)/(e*(3*tan(d/2 + (e*x)/2)^2 +
 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
```

3.397 $\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}\left(2a^2+b^2+c^2\right)x-\frac{3ac\cos(d+ex)}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c$$

```
[Out] 1/2*(2*a^2+b^2+c^2)*x-3/2*a*c*cos(e*x+d)/e+3/2*a*b*sin(e*x+d)/e-1/2*(c*cos(
e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))/e
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3199, 2717, 2718}

$$\frac{1}{2}x\left(2a^2+b^2+c^2\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}+\frac{3ab\sin(d+ex)}{2e}-\frac{3ac\cos(d+ex)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*x)/2 - (3*a*c*Cos[d + e*x])/(2*e) + (3*a*b*Sin[d + e*x
])/(2*e) - ((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d
 + e*x]))/(2*e)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = -\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+e}{2e}$$
$$= \frac{1}{2}\left(2a^2+b^2+c^2\right)x - \frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(}{2e}$$
$$= \frac{1}{2}\left(2a^2+b^2+c^2\right)x - \frac{3ac\cos(d+ex)}{2e} + \frac{3ab\sin(d+ex)}{2e} - \frac{(c\,co}{}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.85

$$\frac{2(2a^2+b^2+c^2)(d+ex)-8ac\cos(d+ex)-2bc\cos(2(d+ex))+8ab\sin(d+ex)+(b^2-c^2)\sin(2(d+ex))}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2 + c^2)*(d + e*x) - 8*a*c*Cos[d + e*x] - 2*b*c*Cos[2*(d + e*
x)] + 8*a*b*Sin[d + e*x] + (b^2 - c^2)*Sin[2*(d + e*x)])/(4*e)
```

Maple [A]
time = 0.21, size = 99, normalized size = 1.09

method	result
risch	$a^2x+\frac{b^2x}{2}+\frac{x\,c^2}{2}-\frac{2ac\cos(ex+d)}{e}+\frac{2ab\sin(ex+d)}{e}-\frac{bc\cos(2ex+2d)}{2e}+\frac{\sin(2ex+2d)b^2}{4e}-\frac{\sin(2ex+2d)c^2}{4e}$
derivativedivides	$\frac{a^2(ex+d)+2ab\sin(ex+d)-2ac\cos(ex+d)+b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\left(\cos^2(ex+d)\right)+c^2\left(-\frac{\cos(ex+d)\sin(ex+d}{2}\right.}{e}$
default	$\frac{a^2(ex+d)+2ab\sin(ex+d)-2ac\cos(ex+d)+b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\left(\cos^2(ex+d)\right)+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}\right.}{e}$
norman	$\frac{\left(a^2+\frac{b^2}{2}+\frac{c^2}{2}\right)x+\left(a^2+\frac{b^2}{2}+\frac{c^2}{2}\right)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{4ac\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{\left(4ab-b^2+c^2\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{\left(4ab+b^2-c^2\right)\tan(}{e}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(a^2*(e*x+d)+2*a*b*sin(e*x+d)-2*a*c*cos(e*x+d)+b^2*(1/2*cos(e*x+d)*sin(
e*x+d)+1/2*e*x+1/2*d)-c*b*cos(e*x+d)^2+c^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*
e*x+1/2*d))
```

Maxima [A]
time = 0.27, size = 102, normalized size = 1.12

$$-bc\cos(xe+d)^2e^{(-1)}+\frac{1}{4}(2\,xe+2\,d+\sin(2\,xe+2\,d))b^2e^{(-1)}+\frac{1}{4}(2\,xe+2\,d-\sin(2\,xe+2\,d))c^2e^{(-1)}+a^2x-2\left(c\cos(xe+d)\,e^{(-1)}-be^{(-1)}\sin(xe+d)\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] -b*c*cos(x*e + d)^2*e^(-1) + 1/4*(2*x*e + 2*d + sin(2*x*e + 2*d))*b^2*e^(-1
) + 1/4*(2*x*e + 2*d - sin(2*x*e + 2*d))*c^2*e^(-1) + a^2*x - 2*(c*cos(x*e
+ d)*e^(-1) - b*e^(-1)*sin(x*e + d))*a
```

Fricas [A]
time = 2.91, size = 77, normalized size = 0.85

$$-\frac{1}{2}\left(2\,bc\cos\left(xe+d\right)^{2}+4\,ac\cos\left(xe+d\right)-\left(2\,a^{2}+b^{2}+c^{2}\right)xe-\left(4\,ab+\left(b^{2}-c^{2}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*cos(x*e + d)^2 + 4*a*c*cos(x*e + d) - (2*a^2 + b^2 + c^2)*x*e -
 (4*a*b + (b^2 - c^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.10, size = 162, normalized size = 1.78

$$\begin{cases} a^{2}x+\frac{2ab\sin\left(d+ex\right)}{e}-\frac{2ac\cos\left(d+ex\right)}{e}+\frac{b^{2}x\sin^{2}\left(d+ex\right)}{2}+\frac{b^{2}x\cos^{2}\left(d+ex\right)}{2}+\frac{b^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}+\frac{bc\sin^{2}\left(d+ex\right)}{e}+\frac{c^{2}x\sin^{2}\left(d+ex\right)}{2}+\frac{c^{2}x\cos^{2}\left(d+ex\right)}{2}-\frac{c^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e} & \text{for}\ e\neq 0 \\ x\left(a+b\cos\left(d\right)+c\sin\left(d\right)\right)^{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sin(d + e*x)/e - 2*a*c*cos(d + e*x)/e + b**2*x*si
n(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*sin(d + e*x)*cos(d + e*x)
/(2*e) + b*c*sin(d + e*x)**2/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d +
e*x)**2/2 - c**2*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(a + b*cos(
d) + c*sin(d))**2, True))
```

Giac [A]
time = 0.41, size = 81, normalized size = 0.89

$$\frac{1}{2}\left(2\,a^{2}+b^{2}+c^{2}\right)x-\frac{bc\cos\left(2\,ex+2\,d\right)}{2\,e}-\frac{2\,ac\cos\left(ex+d\right)}{e}+\frac{2\,ab\sin\left(ex+d\right)}{e}+\frac{\left(b^{2}-c^{2}\right)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2 + c^2)*x - 1/2*b*c*cos(2*e*x + 2*d)/e - 2*a*c*cos(e*x + d)
/e + 2*a*b*sin(e*x + d)/e + 1/4*(b^2 - c^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 3.78, size = 125, normalized size = 1.37

$$\frac{x\left(2\,a^{2}+b^{2}+c^{2}\right)}{2}-\frac{\left(b^{2}-4\,a\,b-c^{2}\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^{3}+\left(4\,a\,c-4\,b\,c\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^{2}+\left(-b^{2}-4\,a\,b+c^{2}\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+4\,a\,c}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^{4}+2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

```
[Out] (x*(2*a^2 + b^2 + c^2))/2 - (4*a*c + tan(d/2 + (e*x)/2)^2*(4*a*c - 4*b*c) -
 tan(d/2 + (e*x)/2)*(4*a*b + b^2 - c^2) - tan(d/2 + (e*x)/2)^3*(4*a*b - b^2
 + c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

3.398 $\int (a+b\cos (d+ex)+c\sin (d+ex))\,dx$

Optimal. Leaf size=27

$$ax-\frac{c\cos (d+ex)}{e}+\frac{b\sin (d+ex)}{e}$$

```
[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2717, 2718}

$$ax+\frac{b\sin (d+ex)}{e}-\frac{c\cos (d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

```
[Out] a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos (d+ex)+c\sin (d+ex))\,dx &= ax+b\int \cos (d+ex)\,dx+c\int \sin (d+ex)\,dx\\ &= ax-\frac{c\cos (d+ex)}{e}+\frac{b\sin (d+ex)}{e}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.81

$$ax-\frac{c\cos (d)\cos (ex)}{e}+\frac{b\cos (ex)\sin (d)}{e}+\frac{b\cos (d)\sin (ex)}{e}+\frac{c\sin (d)\sin (ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
[Out] a*x - (c*Cos[d]*Cos[e*x])/e + (b*Cos[e*x]*Sin[d])/e + (b*Cos[d]*Sin[e*x])/e
 + (c*Sin[d]*Sin[e*x])/e
```

Maple [A]
time = 0.09, size = 28, normalized size = 1.04

method	result	size
default	$ax-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}$	28
risch	$ax-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}$	28
derivativedivides	$\frac{(ex+d)a+b\sin(ex+d)-c\cos(ex+d)}{e}$	30
norman	$\frac{ax+ax\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{2c\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}}{1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(e*x+d)+c*sin(e*x+d),x,method=_RETURNVERBOSE)
[Out] a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.00

$$-c\cos(xe+d)\,e^{(-1)}+be^{(-1)}\sin(xe+d)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="maxima")
[Out] -c*cos(x*e + d)*e^(-1) + b*e^(-1)*sin(x*e + d) + a*x
```

Fricas [A]
time = 2.64, size = 28, normalized size = 1.04

$$(axe-c\cos(xe+d)+b\sin(xe+d))e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="fricas")
[Out] (a*x*e - c*cos(x*e + d) + b*sin(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.05, size = 34, normalized size = 1.26

$$ax+b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

```
[Out] a*x + b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewi
se((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

Giac [A]
time = 0.39, size = 27, normalized size = 1.00

$$ax - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="giac")
```

```
[Out] a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

Mupad [B]
time = 2.51, size = 40, normalized size = 1.48

$$a\,x - \frac{2\,c - 2\,b\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)}{e\,\left(\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cos(d + e*x) + c*sin(d + e*x),x)
```

```
[Out] a*x - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)^2 + 1))
```

3.399 $\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=61

$$\frac{2\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

```
[Out] 2*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/e/(a^2-b^2-c^2)^
(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3203, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (2*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2
- b^2 - c^2]*e)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= -\frac{4\text{Subst}\left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2}\,dx, x, 2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$
$$= \frac{2\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.93

$$-\frac{2\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(Sqrt[-
a^2 + b^2 + c^2]*e)
```

Maple [A]
time = 0.33, size = 61, normalized size = 1.00

method	result
derivativedivides	$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$
default	$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$
risch	$-\frac{\ln\left(e^{i(ex+d)}+\frac{iac\sqrt{-a^2+b^2+c^2}+ia^2b-ib^3-ib\,c^2+ab\sqrt{-a^2+b^2+c^2}-a^2c+b^2c+c^3}{(b^2+c^2)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}\,e}+\frac{\ln\left(e^{i(ex+d)}+\frac{ic}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^
2-c^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.
time = 2.33, size = 448, normalized size = 7.34

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,e^{(-1)}\log\left(-\frac{a^2b^2-2\,b^4-c^4-(a^2+3\,b^2)c^2-(2\,a^2b^2-b^4-2\,a^2c^2+c^4)\cos(xe+d)^2-2\,(ab^3+abc^2)\cos(xe+d)-2\,(ab^2c+ac^3-(bc^3-(2\,a^2b-b^3)c)\cos(xe+d))\sin(xe+d)+2\left(2\,abc\cos(xe+d)^2-abc+(b^2c+c^3)\cos(xe+d)-(b^3+bc^2+(ab^2-ac^2)\cos(xe+d))\sin(xe+d)\right)\sqrt{-a^2+b^2+c^2}}{2\,ab\cos(xe+d)+(b^2-c^2)\cos(xe+d)^2+a^2+c^2+2\,(bc\cos(xe+d)+ac)\sin(xe+d)}\right)}{2\,(a^2-b^2-c^2)}, \frac{\arctan\left(-\frac{(ab\cos(xe+d)+ac\sin(xe+d)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(xe+d)+(a^2b-b^3-bc^2)\sin(xe+d)}\right)e^{(-1)}}{\sqrt{a^2-b^2-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + c^2)*e^(-1)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*
b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x*e + d)^2 - 2*(a*b^3 +
a*b*c^2)*cos(x*e + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*co
s(x*e + d))*sin(x*e + d) + 2*(2*a*b*c*cos(x*e + d)^2 - a*b*c + (b^2*c + c^3
)*cos(x*e + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x*e + d))*sin(x*e + d))
*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 +
 a^2 + c^2 + 2*(b*c*cos(x*e + d) + a*c)*sin(x*e + d)))/(a^2 - b^2 - c^2), a
rctan(-(a*b*cos(x*e + d) + a*c*sin(x*e + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c
^2)/((c^3 - (a^2 - b^2)*c)*cos(x*e + d) + (a^2*b - b^3 - b*c^2)*sin(x*e + d
)))*e^(-1)/sqrt(a^2 - b^2 - c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. $2(46) = 92$.
time = 91.74, size = 3179, normalized size = 52.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

```
[Out] Piecewise((log(b/c + tan(d/2 + e*x/2))/(c*e), Eq(a, b)), (x/(a + b*cos(d) +
 c*sin(d)), Eq(e, 0)), (32*b**5/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e +
 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*
```

```
x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt
(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*
c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/
2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 32*b**4*
sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqr
t(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4
*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*
tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b*
*2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c
**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 40*b**3*c**2/(32*b**6*
e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*
x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20
*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*
c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6
*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c*
*5*e*sqrt(b**2 + c**2)) + 24*b**2*c**2*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2
 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48
*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**
3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*ta
n(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e
*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt
(b**2 + c**2)) + 10*b*c**4/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b
**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2)
- 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2
 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*
e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e
*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 2*c**4*sqrt(b
**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2
 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*s
qrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/
2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c
**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*
tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)), Eq(a, -sqrt(b**2 + c**2))), (
32*b**5/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c
**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(
b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 +
e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2)
 - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(
d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) - 32*b**4*sqrt(b**2 + c**2)/(32*b*
*6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 +
 e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) -
 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b*
*2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e
- 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) +
 c**5*e*sqrt(b**2 + c**2)) + 40*b**3*c**2/(32*b**6*e*tan(d/2 + e*x/2) - 16*
```

```
b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*ta
n(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c
**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2)
+ 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c*
*2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2))
- 24*b**2*c**2*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e
- 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e
*x/2) + 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqr
t(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2
*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d
/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) + c**5*e*sqrt(b**2 + c**2)) + 10*b*c*
*4/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*
tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c*e*sqrt(b**2
+ c**2) - 20*b**3*c**3*e - 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2
) + 18*b**2*c**4*e*tan(d/2 + e*x/2) + 12*b**2*c**3*e*sqrt(b**2 + c**2) - 5*
b*c**5*e - 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan(d/2 +
 e*x/2) + c**5*e*sqrt(b**2 + c**2)) - 2*c**4*sqrt(b**2 + c**2)/(32*b**6*e*t
an(d/2 + e*x/2) - 16*b**5*c*e - 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2
) + 48*b**4*c**2*e*tan(d/2 + e*x/2) + 16*b**4*c...
```

Giac [A]
time = 0.41, size = 89, normalized size = 1.46

$$-\frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*e
*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2
 - b^2 - c^2)*e)
```

Mupad [B]
time = 4.01, size = 75, normalized size = 1.23

$$\frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2\,\sqrt{a^2-b^2-c^2}}\right)}{e\,\sqrt{a^2-b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x)),x)
```

```
[Out] (2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*(a^
2 - b^2 - c^2)^(1/2))))/(e*(a^2 - b^2 - c^2)^(1/2))
```

3.400 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=121

$$\frac{2a\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}\,e}+\frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))}$$

```
[Out] 2*a*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)/e+(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e
*x+d))
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3208, 12, 3203, 632, 210}

$$\frac{2a\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\,(a^2-b^2-c^2)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\,(a^2-b^2-c^2)\,(a+b\cos(d+ex)+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 - c^2]])/((a^2 -
b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b^2 - c^2)*
e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{\int \frac{\cdots}{a+b\cos(\cdots}}{-a\cdots}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{a\int \frac{\cdots}{a+b\cos\cdots}}{a\cdots}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))} + \frac{(2a)\mathrm{Subs}\cdots}{\cdots}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))} - \frac{(4a)\mathrm{Subs}\cdots}{\cdots}$$

$$= \frac{2a\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}\,e} + \frac{c\cos(d+ex)-b\,\mathrm{s}\cdots}{(a^2-b^2-c^2)\,e(a+b\cos(d+\cdots}$$

Mathematica [A]
time = 0.27, size = 116, normalized size = 0.96

$$\frac{\frac{2a\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} + \frac{ac+(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

```
[Out] ((2*a*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2
 + b^2 + c^2)^(3/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2
)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e
```

Maple [A]
time = 0.51, size = 203, normalized size = 1.68

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(a b-b^{2}-c^{2}\right) \tan\left(\frac{d}{2}+\frac{e x}{2}\right)}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}+\frac{2 a c}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}}{a\left(\tan^{2}\left(\frac{d}{2}+\frac{e x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{d}{2}+\frac{e x}{2}\right)\right)+2 c \tan\left(\frac{d}{2}+\frac{e x}{2}\right)+a+b}+\frac{2 a \arctan\left(\frac{2(a-b) \tan\left(\frac{d}{2}+\frac{e x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}}{e}$
default	$\frac{\frac{-\frac{2\left(a b-b^{2}-c^{2}\right) \tan\left(\frac{d}{2}+\frac{e x}{2}\right)}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}+\frac{2 a c}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}}{a\left(\tan^{2}\left(\frac{d}{2}+\frac{e x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{d}{2}+\frac{e x}{2}\right)\right)+2 c \tan\left(\frac{d}{2}+\frac{e x}{2}\right)+a+b}+\frac{2 a \arctan\left(\frac{2(a-b) \tan\left(\frac{d}{2}+\frac{e x}{2}\right)+2 c}{2 \sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}}{e}$
risch	$-\frac{2 i\left(-i a\,e^{i(e x+d)}-i b+c\right)}{\left(-a^{2}+b^{2}+c^{2}\right) e\left(c\,e^{2 i(e x+d)}+i b\,e^{2 i(e x+d)}-c+2 i a\,e^{i(e x+d)}+i b\right)}-\frac{a \ln\left(e^{i(e x+d)}+\frac{i a c \sqrt{-a^{2}+b^{2}+c^{2}}+i a^{2} b-i b^{3}}{\left(b^{2}+c^{2}\right) \sqrt{}}\right.}{\sqrt{-a^{2}+b^{2}+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(2*(-(a*b-b^2-c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*d+1/2*e*x)
+a*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d
+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b)+2*a/(a^2-b^2-c^2)^(3/2)*arctan(1/2*
(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(119) = 238.
time = 3.14, size = 853, normalized size = 7.05

$$\left[\frac{(ab\cos(xe+d)+ac\sin(xe+d)+a^{2})\sqrt{-a^{2}+b^{2}+c^{2}}\log\left(\frac{a^{2}b^{2}-2b^{4}-c^{4}-(a^{2}+3b^{2})c^{2}-(2a^{2}b^{2}-b^{4}-2a^{2}c^{2}+c^{4})\cos(xe+d)^{2}-2(ab^{3}+abc^{2})\cos(xe+d)-2(ab^{2}c+ac^{3}-(bc^{3}-(2a^{2}b-b^{3})c)\cos(xe+d))\sin(xe+d)-2\left(2abc\cos(xe+d)^{2}-abc+(b^{2}c+c^{3})\cos(xe+d)-(b^{3}+bc^{2}+(ab^{2}-ac^{2})\cos(xe+d))\sin(xe+d)\right)\sqrt{-a^{2}+b^{2}+c^{2}}}{2ab\cos(xe+d)+(b^{2}-c^{2})\cos(xe+d)^{2}+a^{2}+c^{2}+2(bc\cos(xe+d)+ac)\sin(xe+d)}\right)-2(c^{3}-(a^{2}-b^{2})c)\cos(xe+d)-2(a^{2}b-b^{3}-bc^{2})\sin(xe+d)}{2((a^{4}b-2a^{2}b^{3}+b^{5}+bc^{4}-2(a^{2}b-b^{3})c^{2})\cos(xe+d)e+(c^{5}-2(a^{2}-b^{2})c^{3}+(a^{4}-2a^{2}b^{2}+b^{4})c)e\sin(xe+d)+(a^{5}-2a^{3}b^{2}+ab^{4}+ac^{4}-2(a^{3}-ab^{2})c^{2})e)},\ \frac{(ab\cos(xe+d)+ac\sin(xe+d)+a^{2})\sqrt{a^{2}-b^{2}-c^{2}}\arctan\left(-\frac{(ab\cos(xe+d)+ac\sin(xe+d)+b^{2}+c^{2})\sqrt{a^{2}-b^{2}-c^{2}}}{(c^{3}-(a^{2}-b^{2})c)\cos(xe+d)+(a^{2}b-b^{3}-bc^{2})\sin(xe+d)}\right)-(c^{3}-(a^{2}-b^{2})c)\cos(xe+d)-(a^{2}b-b^{3}-bc^{2})\sin(xe+d)}{(a^{4}b-2a^{2}b^{3}+b^{5}+bc^{4}-2(a^{2}b-b^{3})c^{2})\cos(xe+d)e+(c^{5}-2(a^{2}-b^{2})c^{3}+(a^{4}-2a^{2}b^{2}+b^{4})c)e\sin(xe+d)+(a^{5}-2a^{3}b^{2}+ab^{4}+ac^{4}-2(a^{3}-ab^{2})c^{2})e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(x*e + d) + a*c*sin(x*e + d) + a^2)*sqrt(-a^2 + b^2 + c^2)*lo
g((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2
 + c^4)*cos(x*e + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(x*e + d) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x*e + d))*sin(x*e + d) - 2*(2*a*b*c*c
os(x*e + d)^2 - a*b*c + (b^2*c + c^3)*cos(x*e + d) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(x*e + d))*sin(x*e + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x*e
 + d) + (b^2 - c^2)*cos(x*e + d)^2 + a^2 + c^2 + 2*(b*c*cos(x*e + d) + a*c)
*sin(x*e + d))) - 2*(c^3 - (a^2 - b^2)*c)*cos(x*e + d) - 2*(a^2*b - b^3 - b
*c^2)*sin(x*e + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2
)*cos(x*e + d)*e + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*
sin(x*e + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e),
((a*b*cos(x*e + d) + a*c*sin(x*e + d) + a^2)*sqrt(a^2 - b^2 - c^2)*arctan(-
(a*b*cos(x*e + d) + a*c*sin(x*e + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c
^3 - (a^2 - b^2)*c)*cos(x*e + d) + (a^2*b - b^3 - b*c^2)*sin(x*e + d))) - (
c^3 - (a^2 - b^2)*c)*cos(x*e + d) - (a^2*b - b^3 - b*c^2)*sin(x*e + d))/((a
^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*cos(x*e + d)*e + (c^5
 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(x*e + d) + (a^5 - 2
*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cos\left(d + ex\right) + c\sin\left(d + ex\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

```
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-2), x)
```

Giac [A]
time = 0.41, size = 214, normalized size = 1.77

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)a}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{ab\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-ac}{\left(a^3-a^2b-ab^2+b^3-ac^2+bc^2\right)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a+b\right)}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*a/(a^2 -
 b^2 - c^2)^(3/2) + (a*b*tan(1/2*e*x + 1/2*d) - b^2*tan(1/2*e*x + 1/2*d) -
```

```
c^2*tan(1/2*e*x + 1/2*d) - a*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b*c^2
)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e*x +
1/2*d) + a + b)))/e
```

Mupad [B]
time = 3.06, size = 195, normalized size = 1.61

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)}{e\,{\left(-a^2+b^2+c^2\right)}^{3/2}}-\frac{\frac{2\,a\,c}{\left(a-b\right)\,\left(-a^2+b^2+c^2\right)}+\frac{2\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(b^2-a\,b+c^2\right)}{\left(a-b\right)\,\left(-a^2+b^2+c^2\right)}}{e\,\left(\left(a-b\right)\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+2\,c\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

```
[Out] (2*a*atanh((tan(d/2 + (e*x)/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2
 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2))))/(e*(b^2 - a^2 + c^2)^(3/2)) -
((2*a*c)/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(d/2 + (e*x)/2)*(b^2 - a*b + c
^2))/((a - b)*(b^2 - a^2 + c^2)))/(e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b)
+ 2*c*tan(d/2 + (e*x)/2)))
```

3.401 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=197

$$\frac{(2a^2+b^2+c^2)\,\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}\,e}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^2}+\frac{}{2\,(a^2-}$$

```
[Out] (2*a^2+b^2+c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a
^2-b^2-c^2)^(5/2)/e+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*co
s(e*x+d)+c*sin(e*x+d))^2+3/2*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^
2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))
```

Rubi [A]
time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3208, 3232, 3203, 632, 210}

$$\frac{(2a^2+b^2+c^2)\,\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\,(a^2-b^2-c^2)^{5/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2e\,(a^2-b^2-c^2)^2\,(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\,(a^2-b^2-c^2)\,(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2 + c^2)*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2 -
 c^2]])/((a^2 - b^2 - c^2)^(5/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*
(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + (3*(a*c*Cos[
d + e*x] - a*b*Sin[d + e*x]))/(2*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
```

```
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} - \frac{\int \frac{-2a+b}{(a+b\cos}}{2}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{}{2\left(a^2-\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{}{2\left(a^2-\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{}{2\left(a^2-\right.}$$

$$= \frac{\left(2a^2+b^2+c^2\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}e} + \frac{c\cos(}{2\left(a^2-b^2-c^2\right)e(}$$

Mathematica [A]

time = 0.67, size = 200, normalized size = 1.02

$$\frac{-\frac{2(2a^2+b^2+c^2)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}+\frac{ac+(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)(a+b\cos(d+ex)+c\sin(d+ex))^2}-\frac{c(2a^2+b^2+c^2)+3a(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)^2(a+b\cos(d+ex)+c\sin(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2 + c^2)*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 +
 b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(
b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c*(2*a^2 +
 b^2 + c^2) + 3*a*(b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)^2*(a + b*
Cos[d + e*x] + c*Sin[d + e*x])))/(2*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(188) = 376$.
time = 0.85, size = 611, normalized size = 3.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(2*(-1/2*(4*a^3*b-7*a^2*b^2-5*a^2*c^2+2*a*b^3+2*a*b*c^2+b^4+3*b^2*c^2+2
*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*d+1/2*e*x)^
3+1/2*c*(4*a^4-12*a^3*b+13*a^2*b^2+7*a^2*c^2-6*a*b^3-6*a*b*c^2+b^4-b^2*c^2-
2*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*
d+1/2*e*x)^2-1/2*(4*a^4*b-5*a^3*b^2-11*a^3*c^2-3*a^2*b^3+3*a^2*b*c^2+5*a*b^
4+7*a*b^2*c^2+2*a*c^4-b^5+b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b
^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x)+1/2*c*(4*a^4-3*a^2*b^2-a^2*c
^2-b^4-b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)
)/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a+b
)^2+(2*a^2+b^2+c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^
2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(194) = 388$.

time = 3.16, size = 1997, normalized size = 10.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*a*b*c^3 - 12*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(x*e + d)^2 - (2*a^4
+ a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)*c
os(x*e + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(x*e + d) + 2*(a*c^3 + (2*
a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(x*e + d))*sin(x*e + d))*sq
rt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a
^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x*e + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(x*
e + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x*e + d))*sin
(x*e + d) + 2*(2*a*b*c*cos(x*e + d)^2 - a*b*c + (b^2*c + c^3)*cos(x*e + d)
- (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x*e + d))*sin(x*e + d))*sqrt(-a^2 + b^
2 + c^2))/(2*a*b*cos(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 + a^2 + c^2 + 2*
(b*c*cos(x*e + d) + a*c)*sin(x*e + d))) - 6*(a^3*b - a*b^3)*c + 2*(c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(x*e + d) - 2*(4*a^4*b
- 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^
3*c^2 + a*c^4)*cos(x*e + d))*sin(x*e + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*
b^4 + 2*b^6)*c^2)*cos(x*e + d)^2*e + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*c
os(x*e + d)*e + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b
^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*
c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*c)*cos(x*e + d)*e + (a*c^7 - 3*(a^3 - a*b^2)*c^5 +
 3*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)
*e)*sin(x*e + d)), 1/2*(3*a*b*c^3 - 6*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(x*e
 + d)^2 + (2*a^4 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2
*a^2*c^2 - c^4)*cos(x*e + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(x*e + d)
 + 2*(a*c^3 + (2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(x*e + d))
*sin(x*e + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x*e + d) + a*c*sin(x*
e + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x*e +
d) + (a^2*b - b^3 - b*c^2)*sin(x*e + d))) - 3*(a^3*b - a*b^3)*c + (c^5 - (5
*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(x*e + d) - (4*a^4*b -
5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2 - a*b^4 - a^3*
c^2 + a*c^4)*cos(x*e + d))*sin(x*e + d))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4 - (a^6 - 3*a^2*b^
4 + 2*b^6)*c^2)*cos(x*e + d)^2*e + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7
 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^3 + a*b^5)*c^2)*cos
(x*e + d)*e + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - c^8 + (2*a^2 - 3*b^2
```

```
)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b^6)*c^2)*e - 2*((b*c^
7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b^5)*c^3 - (a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*c)*cos(x*e + d)*e + (a*c^7 - 3*(a^3 - a*b^2)*c^5 + 3
*(a^5 - 2*a^3*b^2 + a*b^4)*c^3 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c)*e
)*sin(x*e + d))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 856 vs. 2(188) = 376.
time = 0.45, size = 856, normalized size = 4.35

$$-\frac{\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(2a^2+b^2+c^2)}{(a^4-2a^2b^2+b^4-2a^2c^2+2b^2c^2+c^4)\sqrt{a^2-b^2-c^2}}+\frac{4a^4b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-11a^3b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+9a^2b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-ab^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-5a^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+7a^2bc^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+ab^2c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-3b^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+2ac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-2bc^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-4a^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+12a^3bc\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-13a^2b^2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+6ab^3c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-7a^2c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+6abc^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+b^2c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+4a^4b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-5a^3b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-3a^2b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+5ab^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-11a^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+3a^2bc^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+7ab^2c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+2ac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+2bc^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-4a^4c+3a^2b^2c+b^4c+a^2c^3+b^2c^3}{(a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-2a^4c^2+4a^3bc^2-4ab^3c^2+2b^4c^2+a^2c^4-2abc^4+b^2c^4)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a+b\right)^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*e*
x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*(2*a^2 + b
^2 + c^2)/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 -
 b^2 - c^2)) + (4*a^4*b*tan(1/2*e*x + 1/2*d)^3 - 11*a^3*b^2*tan(1/2*e*x + 1
/2*d)^3 + 9*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 - a*b^4*tan(1/2*e*x + 1/2*d)^3 -
 b^5*tan(1/2*e*x + 1/2*d)^3 - 5*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 + 7*a^2*b*c^
2*tan(1/2*e*x + 1/2*d)^3 + a*b^2*c^2*tan(1/2*e*x + 1/2*d)^3 - 3*b^3*c^2*tan
(1/2*e*x + 1/2*d)^3 + 2*a*c^4*tan(1/2*e*x + 1/2*d)^3 - 2*b*c^4*tan(1/2*e*x
+ 1/2*d)^3 - 4*a^4*c*tan(1/2*e*x + 1/2*d)^2 + 12*a^3*b*c*tan(1/2*e*x + 1/2*
d)^2 - 13*a^2*b^2*c*tan(1/2*e*x + 1/2*d)^2 + 6*a*b^3*c*tan(1/2*e*x + 1/2*d)
^2 - b^4*c*tan(1/2*e*x + 1/2*d)^2 - 7*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 + 6*a*
b*c^3*tan(1/2*e*x + 1/2*d)^2 + b^2*c^3*tan(1/2*e*x + 1/2*d)^2 + 2*c^5*tan(1
/2*e*x + 1/2*d)^2 + 4*a^4*b*tan(1/2*e*x + 1/2*d) - 5*a^3*b^2*tan(1/2*e*x +
1/2*d) - 3*a^2*b^3*tan(1/2*e*x + 1/2*d) + 5*a*b^4*tan(1/2*e*x + 1/2*d) - b^
5*tan(1/2*e*x + 1/2*d) - 11*a^3*c^2*tan(1/2*e*x + 1/2*d) + 3*a^2*b*c^2*tan(
1/2*e*x + 1/2*d) + 7*a*b^2*c^2*tan(1/2*e*x + 1/2*d) + b^3*c^2*tan(1/2*e*x +
 1/2*d) + 2*a*c^4*tan(1/2*e*x + 1/2*d) + 2*b*c^4*tan(1/2*e*x + 1/2*d) - 4*a
^4*c + 3*a^2*b^2*c + b^4*c + a^2*c^3 + b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 +
 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^
2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*e*x + 1/2*d)^2 -
b*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e*x + 1/2*d) + a + b)^2))/e
```

Mupad [B]

time = 6.06, size = 700, normalized size = 3.55

$$-\frac{\frac{-4\,a^4\,c+3\,a^2\,b^2\,c+a^2\,c^3+b^4\,c+b^2\,c^3}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a^4\,b-5\,a^3\,b^2-11\,a^3\,c^2-3\,a^2\,b^3+3\,a^2\,b\,c^2+5\,a\,b^4+7\,a\,b^2\,c^2+2\,a\,c^4-b^5+b^3\,c^2+2\,b\,c^4\right)}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(-4\,a^4\,c+12\,a^3\,b\,c-13\,a^2\,b^2\,c-7\,a^2\,c^3+6\,a\,b^3\,c+6\,a\,b\,c^3-b^4\,c+b^2\,c^3+2\,c^5\right)}{{\left(a-b\right)}^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(4\,a^3\,b-7\,a^2\,b^2-5\,a^2\,c^2+2\,a\,b^3+2\,a\,b\,c^2+b^4+3\,b^2\,c^2+2\,c^4\right)}{\left(a-b\right)\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}}{e\,\left(2\,a\,b+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(4\,a\,c-4\,b\,c\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2+4\,c^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a\,c+4\,b\,c\right)\right)}-\frac{\mathrm{atanh}\left(\frac{a^4\,c-2\,a^2\,b^2\,c-2\,a^2\,c^3+b^4\,c+2\,b^2\,c^3+c^5}{{\left(-a^2+b^2+c^2\right)}^{5/2}}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}{2\,{\left(-a^2+b^2+c^2\right)}^{5/2}}\right)\,\left(2\,a^2+b^2+c^2\right)}{e\,{\left(-a^2+b^2+c^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

```
[Out] - ((b^4*c - 4*a^4*c + a^2*c^3 + b^2*c^3 + 3*a^2*b^2*c)/((a - b)^2*(a^4 + b^
4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)*(5*a*b^
4 + 4*a^4*b + 2*a*c^4 + 2*b*c^4 - b^5 - 3*a^2*b^3 - 5*a^3*b^2 - 11*a^3*c^2
+ b^3*c^2 + 7*a*b^2*c^2 + 3*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2
*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*c^5 - b^4*c - 4*a
^4*c - 7*a^2*c^3 + b^2*c^3 - 13*a^2*b^2*c + 6*a*b*c^3 + 6*a*b^3*c + 12*a^3*
b*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (
tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 4*a^3*b + b^4 + 2*c^4 - 7*a^2*b^2 - 5*a^2*c
^2 + 3*b^2*c^2 + 2*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*
c^2 + 2*b^2*c^2)))/(e*(2*a*b + tan(d/2 + (e*x)/2)^3*(4*a*c - 4*b*c) + tan(d
/2 + (e*x)/2)^2*(2*a^2 - 2*b^2 + 4*c^2) + tan(d/2 + (e*x)/2)^4*(a^2 - 2*a*b
 + b^2) + a^2 + b^2 + tan(d/2 + (e*x)/2)*(4*a*c + 4*b*c))) - (atanh((a^4*c
+ b^4*c + c^5 - 2*a^2*c^3 + 2*b^2*c^3 - 2*a^2*b^2*c)/(b^2 - a^2 + c^2)^(5/2
) + (tan(d/2 + (e*x)/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^
2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*a^2 + b^2 + c^2))/(e*(b^2 -
 a^2 + c^2)^(5/2))
```

3.402 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=292

$$\frac{a(2a^2+3(b^2+c^2))\,\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{7/2}\,e}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^3}+\frac{}{6}$$

```
[Out] a*(2*a^2+3*b^2+3*c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(7/2)/e+1/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(
a+b*cos(e*x+d)+c*sin(e*x+d))^3+5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2
-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^2+1/6*(c*(11*a^2+4*b^2+4*c^2)*cos(e
*x+d)-b*(11*a^2+4*b^2+4*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+
c*sin(e*x+d))
```

Rubi [A]
time = 0.28, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3208, 3235, 3232, 3203, 632, 210}

$$\frac{a(2a^2+3(b^2+c^2))\,\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{e\,(a^2-b^2-c^2)^{7/2}}+\frac{c(11a^2+4(b^2+c^2))\cos(d+ex)-b(11a^2+4(b^2+c^2))\sin(d+ex)}{6e\,(a^2-b^2-c^2)^3\,(a+b\cos(d+ex)+c\sin(d+ex))}+\frac{5(ac\cos(d+ex)-ab\sin(d+ex))}{6e\,(a^2-b^2-c^2)^2\,(a+b\cos(d+ex)+c\sin(d+ex))^2}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\,(a^2-b^2-c^2)\,(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[a^2 -
 b^2 - c^2]])/((a^2 - b^2 - c^2)^(7/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x
])/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (5*(a*
c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(6*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^2) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos[d + e*x] - b*(
11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/(6*(a^2 - b^2 - c^2)^3*e*(a + b*Cos[d
 + e*x] + c*Sin[d + e*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} - \frac{\int \frac{-3a+\cdots}{(a+b\cdots}}{\cdots}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{\cdots}{6\left(a^2-\cdots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{\cdots}{6\left(a^2-\cdots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{\cdots}{6\left(a^2-\cdots\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^3} + \frac{\cdots}{6\left(a^2-\cdots\right.}$$

$$= \frac{a(2a^2+3(b^2+c^2))\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{7/2}e} + \frac{\cdots}{3\left(a^2-b^2-\cdots\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 606 vs. $2(292) = 584$.

time = 1.42, size = 606, normalized size = 2.08

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

```
[Out] ((24*a*(2*a^2 + 3*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(7/2) + (44*a^5*c + 82*a^3*b^2*c + 2
4*a*b^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 + 24*a*c^5 + 30*a^2*b*c*(2*a^2 + 3*(b
^2 + c^2))*Cos[d + e*x] - 6*a*c*(-2*b^4 + 2*b^2*c^2 + 4*c^4 + a^2*(7*b^2 +
11*c^2))*Cos[2*(d + e*x)] - 22*a^2*b^3*c*Cos[3*(d + e*x)] - 8*b^5*c*Cos[3*(
d + e*x)] - 22*a^2*b*c^3*Cos[3*(d + e*x)] - 16*b^3*c^3*Cos[3*(d + e*x)] - 8
*b*c^5*Cos[3*(d + e*x)] + 72*a^4*b^2*Sin[d + e*x] - 9*a^2*b^4*Sin[d + e*x]
+ 12*b^6*Sin[d + e*x] + 132*a^4*c^2*Sin[d + e*x] + 72*a^2*b^2*c^2*Sin[d + e
*x] + 36*b^4*c^2*Sin[d + e*x] + 81*a^2*c^4*Sin[d + e*x] + 36*b^2*c^4*Sin[d
+ e*x] + 12*c^6*Sin[d + e*x] + 54*a^3*b^3*Sin[2*(d + e*x)] + 6*a*b^5*Sin[2*
(d + e*x)] + 78*a^3*b*c^2*Sin[2*(d + e*x)] + 48*a*b^3*c^2*Sin[2*(d + e*x)]
+ 42*a*b*c^4*Sin[2*(d + e*x)] + 11*a^2*b^4*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(
d + e*x)] + 4*b^4*c^2*Sin[3*(d + e*x)] - 11*a^2*c^4*Sin[3*(d + e*x)] - 4*b^
2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*x)])/(b*(-a^2 + b^2 + c^2)^3*(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^3))/(24*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. $2(284) = 568$.
time = 1.72, size = 1656, normalized size = 5.67

method	result	size
risch	Expression too large to display	1655
derivativedivides	Expression too large to display	1656
default	Expression too large to display	1656

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(2*(-1/2*(6*a^5*b-15*a^4*b^2-9*a^4*c^2+11*a^3*b^3+9*a^3*b*c^2-3*a^2*b^4
+3*a^2*b^2*c^2+6*a^2*c^4+3*a*b^5+3*a*b^3*c^2-2*b^6-6*b^4*c^2-6*b^2*c^4-2*c^
6)/(a^6-3*a^4*b^2-3*a^4*c^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2
-3*b^2*c^4-c^6)/(a-b)*tan(1/2*d+1/2*e*x)^5+1/2*c*(6*a^6-30*a^5*b+57*a^4*b^2
+27*a^4*c^2-55*a^3*b^3-45*a^3*b*c^2+33*a^2*b^4+21*a^2*b^2*c^2-12*a^2*c^4-15
*a*b^5-15*a*b^3*c^2+4*b^6+12*b^4*c^2+12*b^2*c^4+4*c^6)/(a^6-3*a^4*b^2-3*a^4
*c^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2-3*b^2*c^4-c^6)/(a^2-2*
a*b+b^2)*tan(1/2*d+1/2*e*x)^4-1/3*(18*a^7*b-54*a^6*b^2-54*a^6*c^2+38*a^5*b^
3+120*a^5*b*c^2+30*a^4*b^4-81*a^4*b^2*c^2-21*a^4*c^4-50*a^3*b^5+61*a^3*b^3*
c^2+81*a^3*b*c^4+22*a^2*b^6-87*a^2*b^4*c^2-105*a^2*b^2*c^4+4*a^2*c^6-6*a*b^
7+39*a*b^5*c^2+51*a*b^3*c^4+6*a*b*c^6+2*b^8+2*b^6*c^2-6*b^4*c^4-10*b^2*c^6-
4*c^8)/(a^6-3*a^4*b^2-3*a^4*c^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4
*c^2-3*b^2*c^4-c^6)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d+1/2*e*x)^3+c*(6*a^7
-18*a^6*b+18*a^5*b^2+20*a^5*c^2-2*a^4*b^3-22*a^4*b*c^2-14*a^3*b^4-7*a^3*b^2
*c^2-3*a^3*c^4+18*a^2*b^5+6*a^2*b^3*c^2-12*a^2*b*c^4-10*a*b^6-3*a*b^4*c^2+9
*a*b^2*c^4+2*a*c^6+2*b^7+6*b^5*c^2+6*b^3*c^4+2*b*c^6)/(a^6-3*a^4*b^2-3*a^4*
c^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2-3*b^2*c^4-c^6)/(a^2-2*a
*b+b^2)/(a-b)*tan(1/2*d+1/2*e*x)^2-1/2*(6*a^7*b-9*a^6*b^2-27*a^6*c^2-7*a^5*
b^3+9*a^5*b*c^2+16*a^4*b^4+30*a^4*b^2*c^2+4*a^4*c^4-4*a^3*b^5+14*a^3*b*c^4-
5*a^2*b^6+3*a^2*b^4*c^2+6*a^2*b^2*c^4-2*a^2*c^6+5*a*b^7-9*a*b^5*c^2-18*a*b^
3*c^4-4*a*b*c^6-2*b^8-6*b^6*c^2-6*b^4*c^4-2*b^2*c^6)/(a^6-3*a^4*b^2-3*a^4*c
^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2-3*b^2*c^4-c^6)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d+1/2*e*x)+1/6*a*c*(18*a^6-21*a^4*b^2-5*a^4*c^2-12
*a^2*b^4-16*a^2*b^2*c^2+2*a^2*c^4+15*b^6+21*b^4*c^2+6*b^2*c^4)/(a^6-3*a^4*b
^2-3*a^4*c^2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2-3*b^2*c^4-c^6)
/(a^3-3*a^2*b+3*a*b^2-b^3))/(a*tan(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+
2*c*tan(1/2*d+1/2*e*x)+a+b)^3+a*(2*a^2+3*b^2+3*c^2)/(a^6-3*a^4*b^2-3*a^4*c^
2+3*a^2*b^4+6*a^2*b^2*c^2+3*a^2*c^4-b^6-3*b^4*c^2-3*b^2*c^4-c^6)/(a^2-b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*d+1/2*e*x)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1969 vs. $2(293) = 586$.
time = 3.59, size = 4135, normalized size = 14.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*b*c^5 + 12*(4*a^3*b + a*b^3)*c^3 + 2*(4*c^7 + (7*a^2 - 4*b^2)*c^
5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^6)
*c)*cos(x*e + d)^3 - 12*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b - 8*a
^3*b^3 - a*b^5)*c)*cos(x*e + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 + (2*a
^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(x*e + d)^3 + 9*(a
^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)*cos(x
*e + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)*c^2)*c
os(x*e + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3 - 3*a*b
^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(x*e + d)^2 + 3*(2*a^5 + 3*a^3*b^2)
*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(x*e + d))*sin(x*e + d))*
sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*
a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x*e + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(x
*e + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x*e + d))*si
n(x*e + d) - 2*(2*a*b*c*cos(x*e + d)^2 - a*b*c + (b^2*c + c^3)*cos(x*e + d)
 - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x*e + d))*sin(x*e + d))*sqrt(-a^2 + b
^2 + c^2))/(2*a*b*cos(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 + a^2 + c^2 + 2
*(b*c*cos(x*e + d) + a*c)*sin(x*e + d))) - 6*(9*a^5*b - 8*a^3*b^3 - a*b^5)*
c - 6*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15*a^
4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(x*e + d) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a
^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^2
 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 - (
33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(x*e + d)^2 + 3*(9*a^5*b^2 - 8*a^3*b
^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(x*e + d
))*sin(x*e + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*b
*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6*
(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*
b^5 - b^9)*c^2)*cos(x*e + d)^3*e + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10 - a*c^10 + (4*a^3 - 3*a*b^2)*c^8 - 2*(3*a^5 - 4*a^3*b^2 + a
```

```
*b^4)*c^6 + 2*(2*a^7 - 3*a^5*b^2 + a*b^6)*c^4 - (a^9 - 6*a^5*b^4 + 8*a^3*b^
6 - 3*a*b^8)*c^2)*cos(x*e + d)^2*e + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9 + b*c^10 - (3*a^2*b - 4*b^3)*c^8 + 2*(a^4*b - 4*a^2*b^3 +
 3*b^5)*c^6 + 2*(a^6*b - 3*a^2*b^5 + 2*b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*
a^4*b^5 - b^9)*c^2)*cos(x*e + d)*e + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8 + 3*a*c^10 - (11*a^3 - 12*a*b^2)*c^8 + 2*(7*a^5 - 16*a^3*b^2
+ 9*a*b^4)*c^6 - 6*(a^7 - 4*a^5*b^2 + 5*a^3*b^4 - 2*a*b^6)*c^4 - (a^9 - 6*a
^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*c^2)*e - ((c^11 - (4*a^2 - b^2)*c^9 + 6*(a^4
- b^4)*c^7 - 2*(2*a^6 + 3*a^4*b^2 - 12*a^2*b^4 + 7*b^6)*c^5 + (a^8 + 8*a^6*
b^2 - 30*a^4*b^4 + 32*a^2*b^6 - 11*b^8)*c^3 - 3*(a^8*b^2 - 4*a^6*b^4 + 6*a^
4*b^6 - 4*a^2*b^8 + b^10)*c)*cos(x*e + d)^2*e - 6*(a*b*c^9 - 4*(a^3*b - a*b
^3)*c^7 + 6*(a^5*b - 2*a^3*b^3 + a*b^5)*c^5 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*c^3 + (a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*c)*c
os(x*e + d)*e - (c^11 - (a^2 - 4*b^2)*c^9 - 6*(a^4 - b^4)*c^7 + 2*(7*a^6 -
12*a^4*b^2 + 3*a^2*b^4 + 2*b^6)*c^5 - (11*a^8 - 32*a^6*b^2 + 30*a^4*b^4 - 8
*a^2*b^6 - b^8)*c^3 + 3*(a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8
)*c)*e)*sin(x*e + d)), 1/6*(3*a*b*c^5 + 6*(4*a^3*b + a*b^3)*c^3 + (4*c^7 +
(7*a^2 - 4*b^2)*c^5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 -
7*a^2*b^4 - 4*b^6)*c)*cos(x*e + d)^3 - 6*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3
 - (9*a^5*b - 8*a^3*b^3 - a*b^5)*c)*cos(x*e + d)^2 + 3*(2*a^6 + 3*a^4*b^2 +
 9*a^2*c^4 + (2*a^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(
x*e + d)^3 + 9*(a^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 -
 3*a^2*c^4)*cos(x*e + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b
+ 3*a*b^3)*c^2)*cos(x*e + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5
 + 2*(a^3 - 3*a*b^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(x*e + d)^2 + 3*(2
*a^5 + 3*a^3*b^2)*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(x*e + d
))*sin(x*e + d))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x*e + d) + a*c*sin(
x*e + d) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x*e
+ d) + (a^2*b - b^3 - b*c^2)*sin(x*e + d))) - 3*(9*a^5*b - 8*a^3*b^3 - a*b^
5)*c - 3*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^3 - (6*a^6 - 15
*a^4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(x*e + d) - (18*a^6*b - 23*a^4*b^3 + 7*
a^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - 7*a^2*b^3 + 10*b^5)*c^
2 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7 + 12*b*c^6 + (21*a^2*b + 20*b^3)*c^4 -
(33*a^4*b - 14*a^2*b^3 - 4*b^5)*c^2)*cos(x*e + d)^2 + 3*(9*a^5*b^2 - 8*a^3*
b^4 - a*b^6 + a*c^6 + (8*a^3 + a*b^2)*c^4 - (9*a^5 + a*b^4)*c^2)*cos(x*e +
d))*sin(x*e + d))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 - 3*
b*c^10 + (12*a^2*b - 11*b^3)*c^8 - 2*(9*a^4*b - 16*a^2*b^3 + 7*b^5)*c^6 + 6
*(2*a^6*b - 5*a^4*b^3 + 4*a^2*b^5 - b^7)*c^4 - (3*a^8*b - 8*a^6*b^3 + 6*a^4
*b^5 - b^9)*c^2)*cos(x*e + d)^3*e + 3*(a^9*b^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2558 vs. $2(284) = 568$.
time = 0.51, size = 2558, normalized size = 8.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/
sqrt(a^2 - b^2 - c^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 3*a^4*c^2 + 6
*a^2*b^2*c^2 - 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 - c^6)*sqrt(a^2 - b^2 - c^
2)) + (18*a^7*b*tan(1/2*e*x + 1/2*d)^5 - 81*a^6*b^2*tan(1/2*e*x + 1/2*d)^5
+ 141*a^5*b^3*tan(1/2*e*x + 1/2*d)^5 - 120*a^4*b^4*tan(1/2*e*x + 1/2*d)^5 +
 60*a^3*b^5*tan(1/2*e*x + 1/2*d)^5 - 33*a^2*b^6*tan(1/2*e*x + 1/2*d)^5 + 21
*a*b^7*tan(1/2*e*x + 1/2*d)^5 - 6*b^8*tan(1/2*e*x + 1/2*d)^5 - 27*a^6*c^2*t
an(1/2*e*x + 1/2*d)^5 + 81*a^5*b*c^2*tan(1/2*e*x + 1/2*d)^5 - 72*a^4*b^2*c^
2*tan(1/2*e*x + 1/2*d)^5 + 18*a^3*b^3*c^2*tan(1/2*e*x + 1/2*d)^5 - 27*a^2*b
^4*c^2*tan(1/2*e*x + 1/2*d)^5 + 45*a*b^5*c^2*tan(1/2*e*x + 1/2*d)^5 - 18*b^
6*c^2*tan(1/2*e*x + 1/2*d)^5 + 18*a^4*c^4*tan(1/2*e*x + 1/2*d)^5 - 36*a^3*b
*c^4*tan(1/2*e*x + 1/2*d)^5 + 36*a*b^3*c^4*tan(1/2*e*x + 1/2*d)^5 - 18*b^4*
c^4*tan(1/2*e*x + 1/2*d)^5 - 6*a^2*c^6*tan(1/2*e*x + 1/2*d)^5 + 12*a*b*c^6*
tan(1/2*e*x + 1/2*d)^5 - 6*b^2*c^6*tan(1/2*e*x + 1/2*d)^5 - 18*a^7*c*tan(1/
2*e*x + 1/2*d)^4 + 108*a^6*b*c*tan(1/2*e*x + 1/2*d)^4 - 261*a^5*b^2*c*tan(1
/2*e*x + 1/2*d)^4 + 336*a^4*b^3*c*tan(1/2*e*x + 1/2*d)^4 - 264*a^3*b^4*c*ta
n(1/2*e*x + 1/2*d)^4 + 144*a^2*b^5*c*tan(1/2*e*x + 1/2*d)^4 - 57*a*b^6*c*ta
n(1/2*e*x + 1/2*d)^4 + 12*b^7*c*tan(1/2*e*x + 1/2*d)^4 - 81*a^5*c^3*tan(1/2
*e*x + 1/2*d)^4 + 216*a^4*b*c^3*tan(1/2*e*x + 1/2*d)^4 - 198*a^3*b^2*c^3*ta
n(1/2*e*x + 1/2*d)^4 + 108*a^2*b^3*c^3*tan(1/2*e*x + 1/2*d)^4 - 81*a*b^4*c^
3*tan(1/2*e*x + 1/2*d)^4 + 36*b^5*c^3*tan(1/2*e*x + 1/2*d)^4 + 36*a^3*c^5*t
an(1/2*e*x + 1/2*d)^4 - 36*a^2*b*c^5*tan(1/2*e*x + 1/2*d)^4 - 36*a*b^2*c^5*
tan(1/2*e*x + 1/2*d)^4 + 36*b^3*c^5*tan(1/2*e*x + 1/2*d)^4 - 12*a*c^7*tan(1
/2*e*x + 1/2*d)^4 + 12*b*c^7*tan(1/2*e*x + 1/2*d)^4 + 36*a^7*b*tan(1/2*e*x
+ 1/2*d)^3 - 108*a^6*b^2*tan(1/2*e*x + 1/2*d)^3 + 76*a^5*b^3*tan(1/2*e*x +
1/2*d)^3 + 60*a^4*b^4*tan(1/2*e*x + 1/2*d)^3 - 100*a^3*b^5*tan(1/2*e*x + 1/
2*d)^3 + 44*a^2*b^6*tan(1/2*e*x + 1/2*d)^3 - 12*a*b^7*tan(1/2*e*x + 1/2*d)^
3 + 4*b^8*tan(1/2*e*x + 1/2*d)^3 - 108*a^6*c^2*tan(1/2*e*x + 1/2*d)^3 + 240
*a^5*b*c^2*tan(1/2*e*x + 1/2*d)^3 - 162*a^4*b^2*c^2*tan(1/2*e*x + 1/2*d)^3
+ 122*a^3*b^3*c^2*tan(1/2*e*x + 1/2*d)^3 - 174*a^2*b^4*c^2*tan(1/2*e*x + 1/
2*d)^3 + 78*a*b^5*c^2*tan(1/2*e*x + 1/2*d)^3 + 4*b^6*c^2*tan(1/2*e*x + 1/2*
d)^3 - 42*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 + 162*a^3*b*c^4*tan(1/2*e*x + 1/2*
```

d)^3 - 210*a^2*b^2*c^4*tan(1/2*e*x + 1/2*d)^3 + 102*a*b^3*c^4*tan(1/2*e*x +
 1/2*d)^3 - 12*b^4*c^4*tan(1/2*e*x + 1/2*d)^3 + 8*a^2*c^6*tan(1/2*e*x + 1/2
*d)^3 + 12*a*b*c^6*tan(1/2*e*x + 1/2*d)^3 - 20*b^2*c^6*tan(1/2*e*x + 1/2*d)
^3 - 8*c^8*tan(1/2*e*x + 1/2*d)^3 - 36*a^7*c*tan(1/2*e*x + 1/2*d)^2 + 108*a
^6*b*c*tan(1/2*e*x + 1/2*d)^2 - 108*a^5*b^2*c*tan(1/2*e*x + 1/2*d)^2 + 12*a
^4*b^3*c*tan(1/2*e*x + 1/2*d)^2 + 84*a^3*b^4*c*tan(1/2*e*x + 1/2*d)^2 - 108
*a^2*b^5*c*tan(1/2*e*x + 1/2*d)^2 + 60*a*b^6*c*tan(1/2*e*x + 1/2*d)^2 - 12*
b^7*c*tan(1/2*e*x + 1/2*d)^2 - 120*a^5*c^3*tan(1/2*e*x + 1/2*d)^2 + 132*a^4
*b*c^3*tan(1/2*e*x + 1/2*d)^2 + 42*a^3*b^2*c^3*tan(1/2*e*x + 1/2*d)^2 - 36*
a^2*b^3*c^3*tan(1/2*e*x + 1/2*d)^2 + 18*a*b^4*c^3*tan(1/2*e*x + 1/2*d)^2 -
36*b^5*c^3*tan(1/2*e*x + 1/2*d)^2 + 18*a^3*c^5*tan(1/2*e*x + 1/2*d)^2 + 72*
a^2*b*c^5*tan(1/2*e*x + 1/2*d)^2 - 54*a*b^2*c^5*tan(1/2*e*x + 1/2*d)^2 - 36
*b^3*c^5*tan(1/2*e*x + 1/2*d)^2 - 12*a*c^7*tan(1/2*e*x + 1/2*d)^2 - 12*b*c^
7*tan(1/2*e*x + 1/2*d)^2 + 18*a^7*b*tan(1/2*e*x + 1/2*d) - 27*a^6*b^2*tan(1
/2*e*x + 1/2*d) - 21*a^5*b^3*tan(1/2*e*x + 1/2*d) + 48*a^4*b^4*tan(1/2*e*x
+ 1/2*d) - 12*a^3*b^5*tan(1/2*e*x + 1/2*d) - 15*a^2*b^6*tan(1/2*e*x + 1/2*d
) + 15*a*b^7*tan(1/2*e*x + 1/2*d) - 6*b^8*tan(1/2*e*x + 1/2*d) - 81*a^6*c^2
*tan(1/2*e*x + 1/2*d) + 27*a^5*b*c^2*tan(1/2*e*x + 1/2*d) + 90*a^4*b^2*c^2*
tan(1/2*e*x + 1/2*d) + 9*a^2*b^4*c^2*tan(1/2*e*x + 1/2*d) - 27*a*b^5*c^2*ta
n(1/2*e*x + 1/2*d) - 18*b^6*c^2*tan(1/2*e*x + 1/2*d) + 12*a^4*c^4*tan(1/2*e
*x + 1/2*d) + 42*a^3*b*c^4*tan(1/2*e*x + 1/2*d) + 18*a^2*b^2*c^4*tan(1/2*e*
x + 1/2*d) - 54*a*b^3*c^4*tan(1/2*e*x + 1/2*d) - 18*b^4*c^4*tan(1/2*e*x + 1
/2*d) - 6*a^2*c^6*tan(1/2*e*x + 1/2*d) - 12*a*b*c^6*tan(1/2*e*x + 1/2*d) -
6*b^2*c^6*tan(1/2*e*x + 1/2*d) - 18*a^7*c + 21*a^5*b^2*c + 12*a^3*b^4*c - 1
5*a*b^6*c + 5*a^5*c^3 + 16*a^3*b^2*c^3 - 21*a*b^4*c^3 - 2*a^3*c^5 - 6*a*b^2
*c^5)/((a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a
*b^8 + b^9 - 3*a^7*c^2 + 9*a^6*b*c^2 - 3*a^5*b^2*c^2 - 15*a^4*b^3*c^2 + 15*
a^3*b^4*c^2 + 3*a^2*b^5*c^2 - 9*a*b^6*c^2 + 3*b^7*c^2 + 3*a^5*c^4 - 9*a^4*b
*c^4 + 6*a^3*b^2*c^4 + 6*a^2*b^3*c^4 - 9*a*b^4*c^4 + 3*b^5*c^4 - a^3*c^6 +
3*a^2*b*c^6 - 3*a*b^2*c^6 + b^3*c^6)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*
e*x + 1/2*d)^2 + 2*c*tan(1/2*e*x + 1/2*d) + a + b)^3))/e

Mupad [B]
time = 4.81, size = 1946, normalized size = 6.66

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)

[Out] (a*atanh((a*(2*a^2 + 3*b^2 + 3*c^2)*(2*b^6*c - 2*a^6*c + 2*c^7 - 6*a^2*c^5
+ 6*a^4*c^3 + 6*b^2*c^5 + 6*b^4*c^3 - 6*a^2*b^4*c + 6*a^4*b^2*c - 12*a^2*b^
2*c^3))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)) + (a*tan(d/
2 + (e*x)/2)*(2*a - 2*b)*(2*a^2 + 3*b^2 + 3*c^2)*(b^6 - a^6 + c^6 - 3*a^2*b
^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*

c^2))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)))*(2*a^2 + 3*b ^2 + 3*c^2))/(e*(b^2 - a^2 + c^2)^(7/2)) - ((18*a^7*c + 2*a^3*c^5 - 5*a^5*c ^3 + 6*a*b^2*c^5 + 21*a*b^4*c^3 - 12*a^3*b^4*c - 21*a^5*b^2*c - 16*a^3*b^2* c^3 + 15*a*b^6*c)/(3*(a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3 *a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (tan(d/2 + (e*x)/2)*(2*b^8 - 6*a^7*b - 5*a*b^7 + 5*a^2*b^6 + 4*a^3*b^5 - 16*a^4*b^4 + 7*a^5*b^3 + 9*a^6*b^2 + 2*a^2*c^6 - 4*a^4*c^4 + 27*a^6*c^2 + 2*b^2*c^6 + 6 *b^4*c^4 + 6*b^6*c^2 + 18*a*b^3*c^4 + 9*a*b^5*c^2 - 14*a^3*b*c^4 - 9*a^5*b* c^2 - 6*a^2*b^2*c^4 - 3*a^2*b^4*c^2 - 30*a^4*b^2*c^2 + 4*a*b*c^6))/((a - b) ^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2 *c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^4*(6*a^6*c + 4*b^6 *c + 4*c^7 - 12*a^2*c^5 + 27*a^4*c^3 + 12*b^2*c^5 + 12*b^4*c^3 - 15*a*b^3*c ^3 + 33*a^2*b^4*c - 45*a^3*b*c^3 - 55*a^3*b^3*c + 57*a^4*b^2*c + 21*a^2*b^2 *c^3 - 15*a*b^5*c - 30*a^5*b*c))/((a - b)^2*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) - (2*tan(d/2 + (e*x)/2)^3*(18*a^7*b - 6*a*b^7 + 2*b^8 - 4*c^8 + 22*a^2*b^6 - 50*a^3*b^5 + 30*a^4*b^4 + 38*a^5*b^3 - 54*a^6*b^2 + 4*a^2*c^6 - 21*a^4*c ^4 - 54*a^6*c^2 - 10*b^2*c^6 - 6*b^4*c^4 + 2*b^6*c^2 + 51*a*b^3*c^4 + 39*a* b^5*c^2 + 81*a^3*b*c^4 + 120*a^5*b*c^2 - 105*a^2*b^2*c^4 - 87*a^2*b^4*c^2 + 61*a^3*b^3*c^2 - 81*a^4*b^2*c^2 + 6*a*b*c^6))/(3*(a - b)^3*(b^6 - a^6 + c^ 6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) - (tan(d/2 + (e*x)/2)^5*(3*a*b^5 + 6*a^5*b - 2*b^6 - 2*c^6 - 3*a^2*b^4 + 11*a^3*b^3 - 15*a^4*b^2 + 6*a^2*c^4 - 9*a^4*c^2 - 6*b^2*c^4 - 6*b^4*c^2 + 3*a*b^3*c^2 + 9*a^3*b*c^2 + 3*a^2*b^2*c^2))/((a - b)*(b^6 - a ^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^ 4*c^2 - 6*a^2*b^2*c^2)) + (2*tan(d/2 + (e*x)/2)^2*(2*a*c^7 + 6*a^7*c + 2*b* c^7 + 2*b^7*c - 3*a^3*c^5 + 20*a^5*c^3 + 6*b^3*c^5 + 6*b^5*c^3 + 9*a*b^2*c^ 5 - 3*a*b^4*c^3 - 12*a^2*b*c^5 + 18*a^2*b^5*c - 14*a^3*b^4*c - 22*a^4*b*c^3 - 2*a^4*b^3*c + 18*a^5*b^2*c + 6*a^2*b^3*c^3 - 7*a^3*b^2*c^3 - 10*a*b^6*c - 18*a^6*b*c))/((a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2* c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)))/(e*(tan(d/2 + (e *x)/2)^3*(12*a^2*c - 12*b^2*c + 8*c^3) + tan(d/2 + (e*x)/2)*(6*a^2*c + 6*b^ 2*c + 12*a*b*c) + tan(d/2 + (e*x)/2)^2*(3*a^2*b - 3*a*b^2 + 12*a*c^2 + 12*b *c^2 + 3*a^3 - 3*b^3) - tan(d/2 + (e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 12*a*c^2 + 12*b*c^2 - 3*a^3 - 3*b^3) + tan(d/2 + (e*x)/2)^5*(6*a^2*c + 6*b^2*c - 12* a*b*c) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 + tan(d/2 + (e*x)/2)^6*(3*a*b^2 - 3* a^2*b + a^3 - b^3)))

3.403 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=185

$$\frac{796\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}+\frac{64F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34}}{\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] -2/5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)/e-32/1
5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+64*(cos
(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))
*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))
/e/(2+34^(1/2))^(1/2)+796/15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/
3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.19, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3199, 3225, 3228, 3197, 2732, 3205, 2740}

$$\frac{64F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}+\frac{796\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}-\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}-\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

```
[Out] (796*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(15*e) + (64*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[
34]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (32*(5*Cos[d + e*x] - 3*Sin[d + e*x])*S
qrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(15*e) - (2*(5*Cos[d + e*x] - 3*S
in[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3205

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))(2+3\cos(d+ex)+5\sin(d+\cdots}{5e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin\cdots}}{15e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin\cdots}}{15e}$$

$$= -\frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin\cdots}}{15e}$$

$$= \frac{796\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 4.00, size = 399, normalized size = 2.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-2388*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 2*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]]*(550*Cos[d + e*x] + 3*(-398 + 75*Cos[2*(d + e*x)] -
 110*Sin[d + e*x] + 40*Sin[2*(d + e*x)])) + 1276*Sqrt[10/3]*AppellF1[1/2, 1
/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]),
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*
x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*
x + ArcTan[3/5]]] + (1990*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e
*x - ArcTan[5/3]]/Sqrt[34]] - (1990*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2
, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] +
17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*
Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[
5/3]]])/(75*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.65, size = 821, normalized size = 4.44

method	result	size
default	Expression too large to display	821

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-68*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*
(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+a
rctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))
+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+10
36/17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)
*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+
arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5)
)+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+1
20*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e
*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))
+1)/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/
(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+68/5*34^(1/2)
*sin(e*x+d+arctan(3/5))^4-116/15*34^(1/2)*sin(e*x+d+arctan(3/5))^2+1904/15*
sin(e*x+d+arctan(3/5))^3-1904/15*sin(e*x+d+arctan(3/5))-88/15*34^(1/2)+424/
17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((si
n(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-
1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34
^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+184*((
17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x
+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(3
4^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2
)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-240/17*34^(1/2)*(-17
*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arcta
n(3/5))+1)/(-34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^
(1/2)+17))^(1/2)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+
17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2)))/cos(e*x+d+arctan(3/5)
)/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(x*e + d) + 5*sin(x*e + d) + 2)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 193, normalized size = 1.04

$$-\frac{2}{765}\left(-(1677i+2795)\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)+(1677i-2795)\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)+10149i\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)\right)-10149i\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)\right)+51\left(90\cos(xe+d)^2+6(8\cos(xe+d)-11)\sin(xe+d)+110\cos(xe+d)-45\right)\sqrt{3\cos(xe+d)+5\sin(xe+d)+2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/765*(-(1677*I + 2795)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*
I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) -
 10/51*I + 2/17) + (1677*I - 2795)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInver
se(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin
(x*e + d) + 10/51*I + 2/17) + 10149*I*sqrt(5*I + 3)*sqrt(2)*weierstrassZeta
(860/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(86
0/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e
+ d) - 10/51*I + 2/17)) - 10149*I*sqrt(-5*I + 3)*sqrt(2)*weierstrassZeta(-8
60/289*I + 1376/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/
289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d
) + 10/51*I + 2/17)) + 51*(90*cos(x*e + d)^2 + 6*(8*cos(x*e + d) - 11)*sin(
x*e + d) + 110*cos(x*e + d) - 45)*sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2)
)*e^(-1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

3.404 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{16\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e}+\frac{20F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right.\right.}{\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] -2/3*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+20*(
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/
2))/e/(2+34^(1/2))^(1/2)+16/3*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5
/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3199, 3228, 3197, 2732, 3205, 2740}

$$\frac{20F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}+\frac{16\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e}-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

```
[Out] (16*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/(3*e) + (20*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(Sqrt[2 + Sqrt[34]]*e) - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*Sqrt
[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3205

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin}}{3e}$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin}}{3e}$$

$$= -\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin}}{3e}$$

$$= \frac{16\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 2.46, size = 349, normalized size = 2.51

$$\frac{2\left(-60\sqrt{30}F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\sin\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)+\left(-15(30\cos(d+ex)+15\cos(2(d+ex))-18\sin(d+ex)+8\sin(2(d+ex)))+23\sqrt{30}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}\sec\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\right)\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}\right)}{45e\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

```
[Out] (2*(-60*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x
 - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]
])/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-15*(30*Cos[d + e*x] + 15
*Cos[2*(d + e*x)] - 18*Sin[d + e*x] + 8*Sin[2*(d + e*x)]) + 23*Sqrt[30]*App
ellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 +
 Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqr
t[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]
]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]]
)*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2]))/(45*e*Sqrt[2 + Sqrt[34]*Cos[d + e*x
- ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 806, normalized size = 5.80

method	result	size
default	Expression too large to display	806

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (120/17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/
2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+
d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/
5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))
+16*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(
e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5)
)+1)/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))
/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))-8*34^(1/2)*(
-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d
+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/
(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34
^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+68/3*sin(e*x+d+ar
ctan(3/5))^3-68/3*sin(e*x+d+arctan(3/5))+4/3*34^(1/2)*sin(e*x+d+arctan(3/5)
)^2-4/3*34^(1/2)+76/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))
^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin
(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan
```

```
(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1
/2))*34^(1/2)+36*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)
*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d
+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))
+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-4
0/17*34^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)
*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/
5))+34^(1/2))/(34^(1/2)+17))^(1/2)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34
^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2)))/cos
(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(x*e + d) + 5*sin(x*e + d) + 2)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 170, normalized size = 1.22

$$\frac{1}{153}\left((159i+265)\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)-(159i-265)\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)-408i\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)\right)+408i\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)\right)-102(5\cos(xe+d)-3\sin(xe+d))\sqrt{3\cos(xe+d)+5\sin(xe+d)+2}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/153*((159*I + 265)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I +
1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/
51*I + 2/17) - (159*I - 265)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(-86
0/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e +
 d) + 10/51*I + 2/17) - 408*I*sqrt(5*I + 3)*sqrt(2)*weierstrassZeta(860/289
*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I
+ 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 1
0/51*I + 2/17)) + 408*I*sqrt(-5*I + 3)*sqrt(2)*weierstrassZeta(-860/289*I +
 1376/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/289*I + 13
76/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*
I + 2/17)) - 102*(5*cos(x*e + d) - 3*sin(x*e + d))*sqrt(3*cos(x*e + d) + 5*
sin(x*e + d) + 2))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (3 \cos (d+e\,x)+5 \sin (d+e\,x)+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
```

```
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

3.405 $\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3197, 2732}

$$\frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/e
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx &= \int \sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}\,dx\\ &= \frac{2\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 1.78, size = 326, normalized size = 7.24

$$\frac{-15\sqrt{30}\,F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\sin\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)+\left(-75\cos(d+ex)+45\sin(d+ex)+2\sqrt{30}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}\sec\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\right)\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}{15e\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (-15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x -
ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/
(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-75*Cos[d + e*x] + 45*Sin[d
+ e*x] + 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x
 + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]
])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[
d + e*x + ArcTan[3/5]]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/(15*e*Sqrt[2 +
 Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.74, size = 461, normalized size = 10.24

method	result
default	$\frac{2\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+1}{-\sqrt{34}+17}}\left(15\sqrt{34}\,\sqrt{-\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{-\sqrt{34}+17}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*
x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(15*34^(1/2)*(-(17*sin(e*x+d+arct
an(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3
/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2)
)-17*34^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*
EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((
-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+2*((17*sin(e*x+d+arctan(3/5))+34^(1/2))
/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(
1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2)+34*(-(17
*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin
(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(
1/2)+17))^(1/2))+34*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1
```

```
/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*
(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2)))/cos(e*x+d+arctan(3/5))/(34^(1/2)*s
in(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 127, normalized size = 2.82

$$\frac{1}{51}\left((3i+5)\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)-(3i-5)\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)-51i\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)\right)+51i\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassZeta}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/51*((3*I + 5)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I + 1376/
867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I
+ 2/17) - (3*I - 5)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(-860/289*I +
 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/
51*I + 2/17) - 51*I*sqrt(5*I + 3)*sqrt(2)*weierstrassZeta(860/289*I + 1376/
867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I + 1376/867
, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2
/17)) + 51*I*sqrt(-5*I + 3)*sqrt(2)*weierstrassZeta(-860/289*I + 1376/867,
5480/132651*I - 12056/14739, weierstrassPInverse(-860/289*I + 1376/867, 548
0/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*I + 2/17)))
*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5\sin(d+ex)+3\cos(d+ex)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
```

```
[Out] int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

3.406 $\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx$

Optimal. Leaf size=45

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))/e/(2+34^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3205, 2740}

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 +
Sqrt[34]]*e)
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3205

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)}}\,dx$$
$$= \frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 0.21, size = 128, normalized size = 2.84

$$\frac{\sqrt{\frac{2}{15}}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/15]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTa
n[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 +
 Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*S
qrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/e
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 152, normalized size = 3.38

method	result
default	$\frac{2\left(\sqrt{34}+17\right)\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+1}{-\sqrt{34}+17}}\,\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}}{17\cos\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\sqrt{\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/17*(34^(1/2)+17)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/
2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x
+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5
))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))
/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 63, normalized size = 1.40

$$\frac{1}{34}\left((3i+5)\sqrt{5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)-i\sin(xe+d)-\frac{10}{51}i+\frac{2}{17}\right)-(3i-5)\sqrt{-5i+3}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-\frac{860}{289}i+\frac{1376}{867},\frac{5480}{132651}i-\frac{12056}{14739},\cos(xe+d)+i\sin(xe+d)+\frac{10}{51}i+\frac{2}{17}\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/34*((3*I + 5)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I + 1376/
867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I
+ 2/17) - (3*I - 5)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(-860/289*I +
 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/
51*I + 2/17))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

3.407 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} dx$

Optimal. Leaf size=94

$$-\frac{\sqrt{2+\sqrt{34}}\, E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}$$

```
[Out] 1/15*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-1/1
5*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3207, 3197, 2732}

$$-\frac{\sqrt{2+\sqrt{34}}\, E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] -1/15*(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqr
t[34]))/15])/e - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d +
 e*x] + 5*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
```

```
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+3\cos}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}} - \frac{1}{30}\int \sqrt{2+\sqrt{34}}$$

$$= -\frac{\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 4.32, size = 390, normalized size = 4.15

$$\frac{18\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}-68\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}+\frac{20(5+17\sin(d+ex))}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}-2\sqrt{30}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}-\frac{15\sin\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{\sqrt{\frac{1}{17}+\frac{\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{\sqrt{34}}}}+\frac{15\sqrt{30}F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\csc\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}}{450e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (18*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 68*Sqrt[2 + 3*Cos[d + e
*x] + 5*Sin[d + e*x]] + (20*(5 + 17*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x]
+ 5*Sin[d + e*x]] - 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*
Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x +
ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e
*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] - (15*Sin[d
 + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]]/Sqrt[34]] + (
15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - Ar
cTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(1
7 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2
])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]])/(450*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 425, normalized size = 4.52

method	result

default	$-\frac{\sqrt{34}\left(255\,\mathrm{EllipticE}\left(\sqrt{\frac{17\sin(ex+d+\arctan(\frac{3}{5}))+\sqrt{34}}{\sqrt{34}+17}}, i\sqrt{\frac{\sqrt{34}+17}{-\sqrt{34}+17}}\right)\sqrt{\left(17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4335*34^(1/2)*(255*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(
1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*((17*sin(e*x+d+ar
ctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((17*sin(e*x+
d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(
-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)-2
55*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)
^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((sin(e*x
+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(3
4^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2
)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-289*((34^(1/2)*sin(e
*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*sin(e*x+d+arctan(3/5))
^2+289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
)*17^(1/2)/((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(
3/5))^2)^(1/2)/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(
1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(x*e + d) + 5*sin(x*e + d) + 2)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 303, normalized size = 3.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1530*(sqrt(5*I + 3)*(-(9*I + 15)*sqrt(2)*cos(x*e + d) - (15*I + 25)*sqrt(
2)*sin(x*e + d) - (6*I + 10)*sqrt(2))*weierstrassPInverse(860/289*I + 1376/
```

```
867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I
+ 2/17) + sqrt(-5*I + 3)*((9*I - 15)*sqrt(2)*cos(x*e + d) + (15*I - 25)*sqr
t(2)*sin(x*e + d) + (6*I - 10)*sqrt(2))*weierstrassPInverse(-860/289*I + 13
76/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*
I + 2/17) - 51*sqrt(5*I + 3)*(-3*I*sqrt(2)*cos(x*e + d) - 5*I*sqrt(2)*sin(x
*e + d) - 2*I*sqrt(2))*weierstrassZeta(860/289*I + 1376/867, -5480/132651*I
 - 12056/14739, weierstrassPInverse(860/289*I + 1376/867, -5480/132651*I -
12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2/17)) - 51*sqrt(-5*
I + 3)*(3*I*sqrt(2)*cos(x*e + d) + 5*I*sqrt(2)*sin(x*e + d) + 2*I*sqrt(2))*
weierstrassZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, weierst
rassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e +
d) + I*sin(x*e + d) + 10/51*I + 2/17)) - 102*(5*cos(x*e + d) - 3*sin(x*e +
d))*sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2))/(3*cos(x*e + d)*e + 5*e*sin(
x*e + d) + 2*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\sin\left(d+ex\right)+3\cos\left(d+ex\right)+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

3.408 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=187

$$\frac{4\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e}+\frac{F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{45\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] 1/45*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)+4/6
75*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)+1/45*(
cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/
2))/e/(2+34^(1/2))^(1/2)+4/675*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(
5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.14, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3208, 3235, 3228, 3197, 2732, 3205, 2740}

$$\frac{F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{45\sqrt{2+\sqrt{34}}\,e}+\frac{4\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e}+\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34
]))/15])/(675*e) + EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))
/15]/(45*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(45*e*(2
 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) + (4*(5*Cos[d + e*x] - 3*Sin[d +
 e*x]))/(675*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3205

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{1}{45}\int \frac{-3+\frac{3}{2}\ldots}{(2+3\cos\ldots}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d\ldots}{675e\sqrt{2+3\cos\ldots}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d\ldots}{675e\sqrt{2+3\cos\ldots}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}} + \frac{4(5\cos(d\ldots}{675e\sqrt{2+3\cos\ldots}}$$

$$= \frac{4\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e} + \ldots$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 2.32, size = 430, normalized size = 2.30

$$\frac{-24\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}+\frac{272}{3}\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}+\frac{100(5+17\sin(d+ex))}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}-\frac{10(115+136\sin(d+ex))}{3\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}+23\sqrt{\frac{10}{3}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\text{ArcTan}\left(\frac{3}{5}\right)\right)}+\frac{20\sin\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{\sqrt{\frac{1}{17}+\frac{\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{\sqrt{34}}}}-\frac{20\sqrt{30}F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\csc\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)\sqrt{\sin^2\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)}}}{6750e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-24*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (272*Sqrt[2 + 3*Cos[d
+ e*x] + 5*Sin[d + e*x]])/3 + (100*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*
x] + 5*Sin[d + e*x])^(3/2) - (10*(115 + 136*Sin[d + e*x]))/(3*Sqrt[2 + 3*Co
s[d + e*x] + 5*Sin[d + e*x]]) + 23*Sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3/2,
(Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17
*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5
]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5
]]] + (20*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]
]/Sqrt[34]] - (20*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*C
os[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - A
rcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x -
 ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(6750*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.65, size = 586, normalized size = 3.13

method	result
default	$\frac{17\sqrt{-\left(-\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos^2\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\right)}\,\left(2\sqrt{34}\,\sin(ex+d+\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 17/2*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
/(17*sin(e*x+d+arctan(3/5))+34^(1/2))^2*(2*34^(1/2)*sin(e*x+d+arctan(3/5))+
17*sin(e*x+d+arctan(3/5))^2+2)*(-1/765*34^(1/2)*(-(-34^(1/2)*sin(e*x+d+arct
an(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+arctan(3/5))+1/17*34
^(1/2))^2+136/675*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(-(-289*sin(e*x+d+arcta
n(3/5))-17*34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2)+46/675*(-1+1/
17*34^(1/2))*((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2)*(
(-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(
3/5))+17)/(-34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(
e*x+d+arctan(3/5))^2)^(1/2)*EllipticF(((-17*sin(e*x+d+arctan(3/5))-34^(1/2)
)/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+8/675*34^(1
/2)*(-1+1/17*34^(1/2))*((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17
))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)*((17*sin(e*x
+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arctan(3/5
))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-1/17*34^(1/2)-1)*EllipticE(((-17*s
in(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34
^(1/2)+17))^(1/2))+EllipticF(((-17*sin(e*x+d+arctan(3/5))-34^(1/2))/(-34^(1
/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))))/cos(e*x+d+arctan(3
/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((3*cos(x*e + d) + 5*sin(x*e + d) + 2)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 470, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")

[Out] 1/137700*(53*sqrt(5*I + 3)*((48*I + 80)*sqrt(2)*cos(x*e + d)^2 + 10*(-(9*I + 15)*sqrt(2)*cos(x*e + d) - (6*I + 10)*sqrt(2))*sin(x*e + d) - (36*I + 60)*sqrt(2)*cos(x*e + d) - (87*I + 145)*sqrt(2))*weierstrassPInverse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2/17) + 53*sqrt(-5*I + 3)*(-(48*I - 80)*sqrt(2)*cos(x*e + d)^2 + 10*((9*I - 15)*sqrt(2)*cos(x*e + d) + (6*I - 10)*sqrt(2))*sin(x*e + d) + (36*I - 60)*sqrt(2)*cos(x*e + d) + (87*I - 145)*sqrt(2))*weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*I + 2/17) + 408*sqrt(5*I + 3)*(-16*I*sqrt(2)*cos(x*e + d)^2 + 10*(3*I*sqrt(2)*cos(x*e + d) + 2*I*sqrt(2))*sin(x*e + d) + 12*I*sqrt(2)*cos(x*e + d) + 29*I*sqrt(2))*weierstrassZeta(860/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2/17)) + 408*sqrt(-5*I + 3)*(16*I*sqrt(2)*cos(x*e + d)^2 + 10*(-3*I*sqrt(2)*cos(x*e + d) - 2*I*sqrt(2))*sin(x*e + d) - 12*I*sqrt(2)*cos(x*e + d) - 29*I*sqrt(2))*weierstrassZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*I + 2/17)) - 204*(120*cos(x*e + d)^2 + (64*cos(x*e + d) + 21)*sin(x*e + d) - 35*cos(x*e + d) - 60)*sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2))/(16*cos(x*e + d)^2*e - 12*cos(x*e + d)*e - 10*(3*cos(x*e + d)*e + 2*e)*sin(x*e + d) - 29*e)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(5\sin\left(d+ex\right)+3\cos\left(d+ex\right)+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)

[Out] Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-5/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

3.409 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{199\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{101250e}-\frac{8F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{\right.\right.}}{3375\sqrt{2+\sqrt{34}}\,e}$$

```
[Out] 1/75*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2)+8/3
375*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)-199/1
01250*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-8/3
375*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arct
an(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2)
)^(1/2))/e/(2+34^(1/2))^(1/2)-199/101250*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3)
)^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1
/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.18, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3208, 3235, 3228, 3197, 2732, 3205, 2740}

$$-\frac{8F\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3375\sqrt{2+\sqrt{34}}\,e}-\frac{199\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\text{ArcTan}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{101250e}-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{3375e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (-199*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt
[34]))/15])/(101250*e) - (8*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - S
qrt[34]))/15])/(3375*Sqrt[2 + Sqrt[34]]*e) - (5*Cos[d + e*x] - 3*Sin[d + e*
x])/(75*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2)) + (8*(5*Cos[d + e*x]
 - 3*Sin[d + e*x]))/(3375*e*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2)) -
(199*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(101250*e*Sqrt[2 + 3*Cos[d + e*x] +
 5*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3197

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

Rule 3205

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
```

```
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{1}{75}\int \frac{-5+\frac{9}{2}}{(2+3\,\mathrm{c}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(}{3375e(2+3\,\mathrm{c}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(}{3375e(2+3\,\mathrm{c}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(}{3375e(2+3\,\mathrm{c}}$$

$$= -\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}} + \frac{8(5\cos(}{3375e(2+3\,\mathrm{c}}$$

$$= -\frac{199\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right.\right.}{101250e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 2.75, size = 436, normalized size = 1.87

$$\frac{-13532\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}+\frac{597(12+43\cos(d+ex)+15\sin(d+ex))}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)}}+\frac{27000(5+17\sin(d+ex))}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}-\frac{300(305+272\sin(d+ex))}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}+\frac{20(1595+3383\sin(d+ex))}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}-638\sqrt{30}\,F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{\sqrt{34}+17\sin\left(d+ex+\mathrm{ArcTan}\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\mathrm{ArcTan}\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\mathrm{ArcTan}\left(\frac{3}{5}\right)\right)}\sec\left(d+ex+\mathrm{ArcTan}\left(\frac{3}{5}\right)\right)\sqrt{2+\sqrt{34}\sin\left(d+ex+\mathrm{ArcTan}\left(\frac{3}{5}\right)\right)}+\frac{2985\sqrt{30}\,F_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{\sqrt{34}+17\cos\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\csc\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)\sqrt{\sin^2\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)}}{\sqrt{2+\sqrt{34}\cos\left(d+ex-\mathrm{ArcTan}\left(\frac{5}{3}\right)\right)}}}{3037500e}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (-13532*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (597*(12 + 43*Cos[d + e
*x] + 15*Sin[d + e*x]))/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (27
000*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2) - (3
00*(305 + 272*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2) +
(20*(1595 + 3383*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] -
 638*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + Arc
Tan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17
 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]
```

```
*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (2985*Sqrt[30]*AppellF1[-1
/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt
[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d +
e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Co
s[d + e*x - ArcTan[5/3]]])/(3037500*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.68, size = 631, normalized size = 2.71

method	result
default	$\frac{17\sqrt{-\left(-\sqrt{34}\ \sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\left(\cos^2\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\right)}\ \left(51\sqrt{34}\ (\sin^2(ex+d+\mathrm{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 17/4*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
/(17*sin(e*x+d+arctan(3/5))+34^(1/2))^3*(51*34^(1/2)*sin(e*x+d+arctan(3/5))
^2+289*sin(e*x+d+arctan(3/5))^3+2*34^(1/2)+102*sin(e*x+d+arctan(3/5)))*(-2/
1275*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)
/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^3+16/57375*34^(1/2)*(-(-34^(1/2)*si
n(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+arctan(3
/5))+1/17*34^(1/2))^2-6766/50625*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(-(-289*
sin(e*x+d+arctan(3/5))-17*34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5))^2)^(1/2
)-1276/50625*(1/17*34^(1/2)+1)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1
/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(1/2)*((-17*
sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arct
an(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF(((17*sin(e*x+d+arctan
(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1
/2))-398/50625*34^(1/2)*(1/17*34^(1/2)+1)*((17*sin(e*x+d+arctan(3/5))+34^(1
/2))/(34^(1/2)+17))^(1/2)*((17*sin(e*x+d+arctan(3/5))+17)/(-34^(1/2)+17))^(
1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)/(-(-34^(1/2)*sin
(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-1/17*34^(1/2)+1)*
EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(
-34^(1/2)+17)*(34^(1/2)+17))^(1/2))-EllipticF(((17*sin(e*x+d+arctan(3/5))+3
4^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))))/c
os(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="maxima")

[Out] integrate((3*cos(x*e + d) + 5*sin(x*e + d) + 2)^(-7/2), x)

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 630, normalized size = 2.70

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="fricas")

[Out] 1/10327500*(559*sqrt(5*I + 3)*(-(594*I + 990)*sqrt(2)*cos(x*e + d)^3 - (288*I + 480)*sqrt(2)*cos(x*e + d)^2 + 5*((6*I + 10)*sqrt(2)*cos(x*e + d)^2 + (108*I + 180)*sqrt(2)*cos(x*e + d) + (111*I + 185)*sqrt(2))*sin(x*e + d) + (783*I + 1305)*sqrt(2)*cos(x*e + d) + (474*I + 790)*sqrt(2))*weierstrassPInverse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2/17) + 559*sqrt(-5*I + 3)*((594*I - 990)*sqrt(2)*cos(x*e + d)^3 + (288*I - 480)*sqrt(2)*cos(x*e + d)^2 + 5*(-(6*I - 10)*sqrt(2)*cos(x*e + d)^2 - (108*I - 180)*sqrt(2)*cos(x*e + d) - (111*I - 185)*sqrt(2))*sin(x*e + d) - (783*I - 1305)*sqrt(2)*cos(x*e + d) - (474*I - 790)*sqrt(2))*weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*I + 2/17) + 10149*sqrt(5*I + 3)*(198*I*sqrt(2)*cos(x*e + d)^3 + 96*I*sqrt(2)*cos(x*e + d)^2 + 5*(-2*I*sqrt(2)*cos(x*e + d)^2 - 36*I*sqrt(2)*cos(x*e + d) - 37*I*sqrt(2))*sin(x*e + d) - 261*I*sqrt(2)*cos(x*e + d) - 158*I*sqrt(2))*weierstrassZeta(860/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(x*e + d) - I*sin(x*e + d) - 10/51*I + 2/17)) + 10149*sqrt(-5*I + 3)*(-198*I*sqrt(2)*cos(x*e + d)^3 - 96*I*sqrt(2)*cos(x*e + d)^2 + 5*(2*I*sqrt(2)*cos(x*e + d)^2 + 36*I*sqrt(2)*cos(x*e + d) + 37*I*sqrt(2))*sin(x*e + d) + 261*I*sqrt(2)*cos(x*e + d) + 158*I*sqrt(2))*weierstrassZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(x*e + d) + I*sin(x*e + d) + 10/51*I + 2/17)) + 102*(1990*cos(x*e + d)^3 + 16680*cos(x*e + d)^2 + (39402*cos(x*e + d)^2 + 8896*cos(x*e + d) - 19923)*sin(x*e + d) + 15295*cos(x*e + d) - 8340)*sqrt(3*cos(x*e + d) + 5*sin(x*e + d) + 2))/(198*cos(x*e + d)^3*e + 96*cos(x*e + d)^2*e - 261*cos(x*e + d)*e - 5*(2*cos(x*e + d)^2*e + 36*cos(x*e + d)*e + 37*e)*sin(x*e + d) - 158*e)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2),x)
```

```
[Out] int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2), x)
```

3.410 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=347

$$-\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2)/e-16/1
5*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+2/1
5*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2
*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^
(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*
x+d))^(1/2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-16/
15*a*(a^2-b^2-c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1
/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2
)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d
))/(a+(b^2+c^2)^(1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3199, 3225, 3228, 3198, 2732, 3206, 2740}

$$-\frac{16a(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2(23a^2+9(b^2+c^2))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}-\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[
d + e*x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x
] + c*Sin[d + e*x])^(3/2))/(5*e) + (2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d
 + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]])/(15*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin
[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d +
 e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(15*e*Sqrt[a + b
*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d}{5e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c}}{15e}$$

$$= -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c}}{15e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.46, size = 3767, normalized size = 10.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((2*b*(23*a^2 + 9*b^2 + 9*c^2))/
(15*c) - (22*a*c*Cos[d + e*x])/15 - (2*b*c*Cos[2*(d + e*x)])/5 + (22*a*b*Si
```

n[d + e*x])/15 + ((b^2 - c^2)*Sin[2*(d + e*x)])/5))/e + (2*a^3*AppellF1[1/2
, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sq
rt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]
*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqr
t[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2]
)]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqr
t[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-
a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (34*a*b^2*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])
/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/
c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2
/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c
*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c
*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*c*e) + (34*a*c*App
ellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b
/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 +
 b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1
+ b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2
] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 +
c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sq
rt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*e) + (23*a^2*
b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))),
 -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
 c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[
(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])
])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^
2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*e) + (3*b^4*(-((c*AppellF1
[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[
1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*
x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*C
os[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*

```
x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
 + e*x - ArcTan[c/b]]]))/(5*c*e) + (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))
]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)
/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^
2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]
]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]])) - ((2*b*(a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]]
)/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[
c/b]]]))/(15*e) + (6*b^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*S
qrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b
*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sq
rt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^
2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2269 vs. $2(394) = 788$.
time = 1.22, size = 2270, normalized size = 6.54

method	result	size
default	Expression too large to display	2270

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)^(3/2)*(-2/5
/(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))*(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(e*x+d-arc
tan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c
^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/
2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*si
n(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^
(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-a
```

```
rctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arcta
n(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2
)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-Elli
pticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/
2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a*b^2+3*a*c^2)*(
-2/3/(b^2+c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+
1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1
)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b
^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)
*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2)
)/(-a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1
)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*(
(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(
e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((
-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))
+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^
(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^
(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+6*a^2*(b
^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b
,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(
1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1
/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^
2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^
2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(
e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a
+(b^2+c^2)^(1/2)))^(1/2)))+2*a^3*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b
,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,
c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e*x+d-arctan(
-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c
))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b
,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2
)))^(1/2)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e
*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(x*e + d) + c*sin(x*e + d) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 1608, normalized size = 4.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
[Out] 1/45*(sqrt(2)*(-I*a^3*b + 33*I*a*b^3 + 33*I*a*b*c^2 + 33*a*c^3 - (a^3 - 33*
a*b^2)*c)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*
c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4),
 -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2
)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b
^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d
) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2)) + sqrt(2)*(I*a^3*b - 33*I*
a*b^3 - 33*I*a*b*c^2 + 33*a*c^3 - (a^3 - 33*a*b^2)*c)*sqrt(b - I*c)*weierst
rassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(
4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 2
7*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c
^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*
(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e
+ d))/(b^2 + c^2)) - 3*sqrt(2)*(23*I*a^2*b^2 + 9*I*b^4 + 9*I*c^4 + I*(23*a^
2 + 18*b^2)*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a
^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^
4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*
b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 +
3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 -
4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 +
 c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9
*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6
 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x
*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2))) - 3*sqrt(2)*(-23*I*
a^2*b^2 - 9*I*b^4 - 9*I*c^4 - I*(23*a^2 + 18*b^2)*c^2)*sqrt(b - I*c)*weiers
trassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a
^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a
*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2
+ 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierst
rassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(
4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 2
7*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c
^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*
(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e
```

```
+ d))/(b^2 + c^2))) + 6*(3*b^3*c + 3*b*c^3 - 6*(b^3*c + b*c^3)*cos(x*e + d)
^2 - 11*(a*b^2*c + a*c^3)*cos(x*e + d) + (11*a*b^3 + 11*a*b*c^2 + 3*(b^4 -
c^4)*cos(x*e + d))*sin(x*e + d))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + a))
*e^(-1)/(b^2 + c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\cos(d + e\,x) + c\,\sin(d + e\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
```

```
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

3.411 $\int (a+b\cos (d+ex)+c\sin (d+ex))^{3/2}\,dx$

Optimal. Leaf size=283

$$-\frac{2(c\cos (d+ex)-b\sin (d+ex))\sqrt{a+b\cos (d+ex)+c\sin (d+ex)}}{3e}+\frac{8aE\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b}}{a+\sqrt{}}\right.}{3e\sqrt{\frac{a+b\cos}{}}}$$

```
[Out] -2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+8/3*
a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+b*c
os(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(a^2-b^2-c^2)*(cos(1
/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*E
llipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(
1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3199, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2(a^2-b^2-c^2)\sqrt{\frac{a+b\cos (d+ex)+c\sin (d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{a+b\cos (d+ex)+c\sin (d+ex)}}+\frac{8a\sqrt{a+b\cos (d+ex)+c\sin (d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\sqrt{\frac{a+b\cos (d+ex)+c\sin (d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{2(c\cos (d+ex)-b\sin (d+ex))\sqrt{a+b\cos (d+ex)+c\sin (d+ex)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(3*e) + (8*a*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2]
)/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e*Sq
rt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2
- b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a +
 Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2
 + c^2])])/(3*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(}{3e}$$

$$= -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(}{3e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.22, size = 2190, normalized size = 7.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*b*Sin[d + e*x])/3)*Sqrt[a + b*C
os[d + e*x] + c*Sin[d + e*x]])/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a
 + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*
x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Si
n[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2
)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
 a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e
*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*S
```

```
in[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] +
 c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^
2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
- a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Ar
cTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d +
e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*
Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
+ c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c
^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1/2, -
1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1
+ c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[
d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^
2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(
b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b
]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/
b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b
]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTa
n[c/b]]]))/(3*c*e) + (4*a*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(
b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/
b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b
^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*S
qrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a +
b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x -
ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1493 vs. $2(333) = 666$.
time = 0.63, size = 1494, normalized size = 5.28

method	result	size
default	Expression too large to display	1494

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*((b^2+c^2)*(-2/3/(b^2+
```

```
c^2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^
2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2
)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1
/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d
-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2
+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d
-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arct
an(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^
2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^
2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-Ell
ipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1
/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+4*a*(b^2+c^2)^(1/2
)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^
2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+
c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)
*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2)
)/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan
(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(
1/2)))^(1/2)))+2*a^2*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+
c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*s
in(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^
2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))/
cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(
-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x*e + d) + c*sin(x*e + d) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 2.05, size = 1494, normalized size = 5.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(I*a^2*b + 3*I*b^3 + 3*I*b*c^2 + 3*c^3 + (a^2 + 3*b^2)*c)*sqrt
(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^
3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*
b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a
^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2
*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 +
 I*c^2)*sin(x*e + d))/(b^2 + c^2)) + sqrt(2)*(-I*a^2*b - 3*I*b^3 - 3*I*b*c^
2 + 3*c^3 + (a^2 + 3*b^2)*c)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b
^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
+ 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2
*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a
*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2
 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e + d))/(b^2 + c^2)) - 12*s
qrt(2)*(I*a*b^2 + I*a*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3
*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^
2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*
a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*
c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2
- 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2
*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*
(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^
4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 +
c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2))) - 12*sqrt
(2)*(-I*a*b^2 - I*a*c^2)*sqrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b
^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*
c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^
3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)
/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 -
3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b
^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4
*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)
*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^
2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e + d))/(b^2 + c^2))) - 6*((b^2*
c + c^3)*cos(x*e + d) - (b^3 + b*c^2)*sin(x*e + d))*sqrt(b*cos(x*e + d) + c
*sin(x*e + d) + a))*e^(-1)/(b^2 + c^2)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\cos(d + e\,x) + c\,\sin(d + e\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

3.412 $\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=108

$$\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+b*c
os(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$,
Rules used = {3198, 2732}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d +
e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

```
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b}\cdots}{\cdots}}}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+\cdots}{a+\sqrt{b^2+c^2}}}}$$

$$= \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+e\cdots}}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.17, size = 1408, normalized size = 13.04

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*b*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(c*e) + (2*a*AppellF1[1/2,
1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*
c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]
*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[
(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a
+ c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (b^2*(-((c*AppellF1[
-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]
])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 +
c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1
 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/
(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
 - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Co
s[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*
```

```
Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x
 - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d
+ e*x - ArcTan[c/b]]]))/(c*e) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar
cTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d +
e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*
Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*
Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])
/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d +
 e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[
1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(137) = 274$.

time = 1.28, size = 691, normalized size = 6.40

method	result
default	$-\frac{2\left(a+\sqrt{b^2+c^2}\right)\sqrt{\frac{\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+a}{a+\sqrt{b^2+c^2}}}\sqrt{\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}{-a+\sqrt{b^2+c^2}}}\sqrt{-\frac{(\sin(ex}{}}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b^2+c^2)^(1/2)*(a+(b^2+c^2)^(1/2))*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-
b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(
1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)*((b^2+c^2)^(1/2)*cos(e*x+d-arctan(-b,c))^2*
sin(e*x+d-arctan(-b,c))+a*cos(e*x+d-arctan(-b,c))^2)^(1/2)*((b^2+c^2)^(1/2)
*EllipticF(((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))*sin(e*x+d-arctan(-b,c))+a/(
a+(b^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)
)-(b^2+c^2)^(1/2)*EllipticE(((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))*sin(e*x+d-
arctan(-b,c))+a/(a+(b^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c
^2)^(1/2)))^(1/2))-EllipticF(((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))*sin(e*x+d
-arctan(-b,c))+a/(a+(b^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+
c^2)^(1/2)))^(1/2))*a+EllipticE(((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))*sin(e*
x+d-arctan(-b,c))+a/(a+(b^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b
^2+c^2)^(1/2)))^(1/2))*a)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e
*x+d-arctan(-b,c))+a))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan
(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/
2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(x*e + d) + c*sin(x*e + d) + a), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 1378, normalized size = 12.76

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*(I*a*b + a*c)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2)) + sqrt(2)*(-I*a*b + a*c)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e + d))/(b^2 + c^2)) - 3*sqrt(2)*(I*b^2 + I*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2))) - 3*sqrt(2)*(-I*b^2 - I*c^2)*sqrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I

```
*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8
*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6
*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 +
3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I
*b^2 - I*c^2)*sin(x*e + d))/(b^2 + c^2))))*e^(-1)/(b^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos{(d + ex)} + c\sin{(d + ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
[Out] Integral(sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
[Out] int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

3.413 $\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=108

$$\frac{2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2
)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(
1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3206, 2740}

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
+ c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int \frac{}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2}}{}}}}{\sqrt{a+b\cos(d+ex)+c\sin(d+}}$$

$$= \frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex}{a+\sqrt{b}}}}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 0.43, size = 285, normalized size = 2.64

$$\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{a+\sqrt{1+\frac{b^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)}{a-\sqrt{1+\frac{b^2}{c^2}}\,c},\frac{a+\sqrt{1+\frac{b^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)}{a+\sqrt{1+\frac{b^2}{c^2}}\,c}\right)\sec\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)\sqrt{-\frac{\sqrt{1+\frac{b^2}{c^2}}\,c\left(-1+\sin\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)\right)}{a+\sqrt{1+\frac{b^2}{c^2}}\,c}}\sqrt{\frac{\sqrt{1+\frac{b^2}{c^2}}\,c\left(1+\sin\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)\right)}{-a+\sqrt{1+\frac{b^2}{c^2}}\,c}}\sqrt{a+\sqrt{1+\frac{b^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{b}{c}\right)\right)}}{\sqrt{1+\frac{b^2}{c^2}}\,ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcT
an[b/c]])/(a - Sqrt[1 + b^2/c^2]*c), (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
 ArcTan[b/c]])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[-
((Sqrt[1 + b^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[b/c]]))/(a + Sqrt[1 + b^2/
c^2]*c))]*Sqrt[(Sqrt[1 + b^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[b/c]]))/(-a +
 Sqrt[1 + b^2/c^2]*c)]*Sqrt[a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/
c]]])/(Sqrt[1 + b^2/c^2]*c*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(137) = 274$.
time = 0.34, size = 295, normalized size = 2.73

method	result
default	$\frac{2\left(a+\sqrt{b^2+c^2}\right)\sqrt{\frac{\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+a}{a+\sqrt{b^2+c^2}}}\sqrt{\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}{-a+\sqrt{b^2+c^2}}}\sqrt{-\frac{(\sin(ex+d}{}}}{\sqrt{b^2+c^2}\,\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a+(b^2+c^2)^(1/2))*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+
c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)
^(1/2)))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b
^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))/(b
^2+c^2)^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin
(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(x*e + d) + c*sin(x*e + d) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 509, normalized size = 4.71

$$\frac{\left(\sqrt{2}\,\sqrt{b+i\,c}\,(i\,b+c)\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2b^2-3\,b^4-4\,a^2c^2+6i\,bc^3+3\,c^4-2i\,(4\,a^2b-3\,b^3)c)}{3\,(b^4+2\,b^2c^2+c^4)},-\frac{8\,(8\,a^3b^3-9\,ab^5+27\,abc^4-9i\,ac^5+2i\,(4\,a^3+9\,ab^2)c^3-6\,(4\,a^3b-3\,ab^3)c^2-3i\,(8\,a^3b^2-9\,ab^4)c)}{27\,(b^6+3\,b^4c^2+3\,b^2c^4+c^6)},\frac{2\,ab-2i\,ac+3\,(b^2+c^2)\cos(xe+d)-3\,(i\,b^2+i\,c^2)\sin(xe+d)}{3\,(b^2+c^2)}\right)+\sqrt{2}\,\sqrt{b-i\,c}\,(-i\,b+c)\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2b^2-3\,b^4-4\,a^2c^2-6i\,bc^3+3\,c^4+2i\,(4\,a^2b-3\,b^3)c)}{3\,(b^4+2\,b^2c^2+c^4)},-\frac{8\,(8\,a^3b^3-9\,ab^5+27\,abc^4+9i\,ac^5-2i\,(4\,a^3+9\,ab^2)c^3-6\,(4\,a^3b-3\,ab^3)c^2+3i\,(8\,a^3b^2-9\,ab^4)c)}{27\,(b^6+3\,b^4c^2+3\,b^2c^4+c^6)},\frac{2\,ab+2i\,ac+3\,(b^2+c^2)\cos(xe+d)-3\,(-i\,b^2-i\,c^2)\sin(xe+d)}{3\,(b^2+c^2)}\right)\right)e^{(-1)}}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(b + I*c)*(I*b + c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4
 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^
2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3
+ 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(
b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*co
s(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2)) + sqrt(2)*sqrt(b
- I*c)*(-I*b + c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 -
6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27
*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3
- 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2
 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*
(-I*b^2 - I*c^2)*sin(x*e + d))/(b^2 + c^2)))*e^(-1)/(b^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\cos{\left(d + ex \right)} + c\sin{\left(d + ex \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
```

```
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

3.414 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{(a^2-b^2-c^2)\,e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] 2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))
^(1/2)+2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2
*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^
2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/(a
^2-b^2-c^2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3207, 3198, 2732}

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\,(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\,(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d
 + e*x] + c*Sin[d + e*x]]) + (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sq
rt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]])/((a^2 - b^2 - c^2)*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
```

```
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\int \sqrt{a-\cdots}}{\cdots}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{2E\left(\frac{1}{2}(\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.33, size = 1540, normalized size = 8.28

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2))/(b*c*(-a^2 + b
^2 + c^2)) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(b*(-a^2 + b^2
 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e - (2*a*AppellF1[1/2, 1/2
, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1
+ b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Si
```

```
n[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))
*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^
2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c
*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)*e) - (b^2
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e
*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -(
(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(
-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^
2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x
- ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/
b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])]))
 - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2)
- (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 +
 c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)*e) - (c*(-((c
*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b
*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a
/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcT
an[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*C
os[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^
2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2
*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*S
in[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b
^2]*Cos[d + e*x - ArcTan[c/b]]]))/((-a^2 + b^2 + c^2)*e)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2644 vs. $2(213) = 426$.
time = 1.02, size = 2645, normalized size = 14.22

method	result	size
default	Expression too large to display	2645

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1
/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(3/2)*(b^2*
sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))*(cos
(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2
))^(1/2)*(b^2*sin(e*x+d-arctan(-b,c))^2+c^2*sin(e*x+d-arctan(-b,c))^2-a^2)/
(b^2*sin(e*x+d-arctan(-b,c))^2+c^2*sin(e*x+d-arctan(-b,c))^2+2*(b^2+c^2)^(1
```

/2)*a*sin(e*x+d-arctan(-b,c))+a^2)/(a*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)-sin(e*x+d-arctan(-b,c))*(cos(e*x+d
-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/
2))*(-(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(c
os(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c
^2))^(1/2)+a*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1
/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arcta
n(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan
(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*Elliptic
F((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),(
(-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(-(b^2+c^2)^(3/2)+2*(b^
2+c^2)^(1/2)*b^2+2*(b^2+c^2)^(1/2)*c^2)/(2*a^2-2*b^2-2*c^2)*(1/(b^2+c^2)^(1
/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^
(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1
/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b
^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(e*
x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(
b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))
+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^
(1/2)))+1/2*(b^2+c^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-a
rctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)/a*EllipticPi((((b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2
+c^2)^(1/2)*a-1)*(b^2+c^2)^(1/2)/a,((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2
)))^(1/2))+(b^2+c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)/(cos(e*x+d-arc
tan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+1/(a^2-b^2-
c^2)*(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b
^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^
2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(
e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a
+(b^2+c^2)^(1/2)))^(1/2))+1/(a^2-b^2-c^2)*(b^2+c^2)*(1/(b^2+c^2)^(1/2)*a+1)
*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((
sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-s
in(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e
x+d-arctan(-b,c))^2((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*((-
1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+
a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(
1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(
1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))-(1/2*b^2+1

```
/2*c^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d
-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(
b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b
^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/a*EllipticPi((((b^2+c^2)^(1/2)*
sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1
/2)*a-1)*(b^2+c^2)^(1/2)/a,((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2
)))/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(x*e + d) + c*sin(x*e + d) + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.56, size = 1762, normalized size = 9.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
[Out] -1/3*((sqrt(2)*(-I*a*b^2 - a*b*c)*cos(x*e + d) + sqrt(2)*(-I*a*b*c - a*c^2)
*sin(x*e + d) + sqrt(2)*(-I*a^2*b - a^2*c))*sqrt(b + I*c)*weierstrassPInver
se(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b -
3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
- 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(
8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2
*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2
+ c^2)) + (sqrt(2)*(I*a*b^2 - a*b*c)*cos(x*e + d) + sqrt(2)*(I*a*b*c - a*c^
2)*sin(x*e + d) + sqrt(2)*(I*a^2*b - a^2*c))*sqrt(b - I*c)*weierstrassPInve
rse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b -
 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*
(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b +
2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e + d))/(b^
2 + c^2)) - 3*(sqrt(2)*(-I*b^3 - I*b*c^2)*cos(x*e + d) + sqrt(2)*(-I*b^2*c
- I*c^3)*sin(x*e + d) + sqrt(2)*(-I*a*b^2 - I*a*c^2))*sqrt(b + I*c)*weierst
rassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^
2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*
```

```
b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 -
 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstr
assPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4
*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27
*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^
2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(
2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e +
d))/(b^2 + c^2))) - 3*(sqrt(2)*(I*b^3 + I*b*c^2)*cos(x*e + d) + sqrt(2)*(I*
b^2*c + I*c^3)*sin(x*e + d) + sqrt(2)*(I*a*b^2 + I*a*c^2))*sqrt(b - I*c)*we
ierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*
(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 +
27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*
c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), wei
erstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2
*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5
 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^
3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6),
1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(
x*e + d))/(b^2 + c^2))) - 6*((b^2*c + c^3)*cos(x*e + d) - (b^3 + b*c^2)*sin
(x*e + d))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + a))/((a^2*b^3 - b^5 - b*c
^4 + (a^2*b - 2*b^3)*c^2)*cos(x*e + d)*e - (c^5 - (a^2 - 2*b^2)*c^3 - (a^2*
b^2 - b^4)*c)*e*sin(x*e + d) + (a^3*b^2 - a*b^4 - a*c^4 + (a^3 - 2*a*b^2)*c
^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cos\left(d + ex\right) + c\sin\left(d + ex\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

3.415 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=382

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{3\left(a^2-b^2-c^2\right)^2e\sqrt{a+b\cos(d+ex)+c\sin(d+ex}}$$

```
[Out] 2/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d
))^(3/2)+8/3*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x
+d)+c*sin(e*x+d))^(1/2)+8/3*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b
,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c
*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^2/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2
+c^2)^(1/2)))^(1/2)-2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/
2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2
^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(
e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*
x+d))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3e\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(3/2)) + (8*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/
(3*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (8*a*
EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c
^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*(a^2 - b^2 - c^2)^2*e*S
qrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*Elli
pticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(3*(a^
2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} - \frac{2\int}{}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{}{3\left(a^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{}{3\left(a^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{}{3\left(a^2\right.}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{}{3\left(a^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.47, size = 2408, normalized size = 6.30

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]

[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((8*a*(b^2 + c^2))/(3*b*c*(a^2 - b^2 - c^2)^2) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(3*a^2*c + b^2*c + c^3 + 4*a*b^2*Sin[d + e*x] + 4*a*c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^2*e) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*(-a^2 + b^2 + c^2)^2*e) + (4*a*b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(3*c*(-a^2 + b^2 + c^2)^2*e) + (4*a*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^

```
2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2
/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b
^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b
*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - Arc
Tan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d +
e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1
 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
- ArcTan[c/b]]]))/(3*(-a^2 + b^2 + c^2)^2*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3641 vs. $2(428) = 856$.
time = 4.91, size = 3642, normalized size = 9.53

method	result	size
default	Expression too large to display	3642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1
/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(b^2*sin(e*x+d-arctan
(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))*(cos(e*x+d-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*(b^4*si
n(e*x+d-arctan(-b,c))^4+2*b^2*c^2*sin(e*x+d-arctan(-b,c))^4+c^4*sin(e*x+d-a
rctan(-b,c))^4-2*a^2*b^2*sin(e*x+d-arctan(-b,c))^2-2*a^2*c^2*sin(e*x+d-arct
an(-b,c))^2+a^4)/(b^4*sin(e*x+d-arctan(-b,c))^3+2*b^2*c^2*sin(e*x+d-arctan(
-b,c))^3+c^4*sin(e*x+d-arctan(-b,c))^3+3*(b^2+c^2)^(1/2)*a*b^2*sin(e*x+d-ar
ctan(-b,c))^2+3*(b^2+c^2)^(1/2)*a*c^2*sin(e*x+d-arctan(-b,c))^2+3*a^2*b^2*s
in(e*x+d-arctan(-b,c))+3*a^2*c^2*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2)*a^
3)/(2*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a
))^(1/2)*sin(e*x+d-arctan(-b,c))*a*b^2+2*(cos(e*x+d-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*sin(e*x+d-arctan(-b,c))*a*c^2-(
cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+
c^2))^(1/2)*sin(e*x+d-arctan(-b,c))^2*b^2-(cos(e*x+d-arctan(-b,c))^2*((b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(e*x+d-arctan(-b,
c))^2*c^2-(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))+a)*(b^2+c^2))^(1/2)*a^2)*(-1/4/a/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)*(cos(e*x+
d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1
/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1
/2))+1/3/(a^2-b^2-c^2)/(b^2+c^2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)/(sin(e*x+d-arctan(-b,c))+1/(b
^2+c^2)^(1/2)*a)^2-4/3*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^2
*a/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(
b^2+c^2))^(1/2)+2*(-1/24*(b^2+c^2)^(1/2)/(a^2-b^2-c^2)+2/3*a^2*(b^2+c^2)^(1
```

```
/2)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+
c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)
^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*EllipticF((((b^2+c^2)^(1
/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1
/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(13*a^2*b^2+13*a^2*c^2+3*b^4+6*b^2*c^2+
3*c^4)/(24*a^5-48*a^3*b^2-48*a^3*c^2+24*a*b^4+48*a*b^2*c^2+24*a*c^4)*(1/(b^
2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)
^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(
1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a)*(b^2+c^2))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/
2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/
2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)
^(1/2)))^(1/2)))-1/8*(5*a^2*b^2+5*a^2*c^2-b^4-2*b^2*c^2-c^4)/a^2/(a^2-b^2-c
^2)/(b^2+c^2)^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arc
tan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+
c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c
^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*EllipticPi((((b^2+c^2)^(1
/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2
)^(1/2)*a-1)*(b^2+c^2)^(1/2)/a,((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^
(1/2))+1/4*(b^2+c^2)/a/(a^2-b^2-c^2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*si
n(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1/2))+1/3/(a^2-b^2-c^2)/(b^2+c^2)^(1/2)*
(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/
2)/(sin(e*x+d-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2+4/3*(b^2+c^2)^(1/2)*cos(
e*x+d-arctan(-b,c))^2/(a^2-b^2-c^2)^2*a/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+a))^(1/2)+2*(-7/24/(a^2-b^2-c^2)+2/3*a^2/(
a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(
-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)
^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(e*x+d-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-a
rctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c
^2)^(1/2)))^(1/2))+2*(1/8/a/(a^2-b^2-c^2)*(b^2+c^2)^(1/2)+2/3*a*(b^2+c^2)^(
1/2)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-a
rctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^
2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e*x...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x*e + d) + c*sin(x*e + d) + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.20, size = 2844, normalized size = 7.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(I*a^2*b^3 + 3*I*b^5 - I*a^2*b*c^2 - a^2*c^3 - 3*I*b*c^4 - 3*
c^5 + (a^2*b^2 + 3*b^4)*c)*cos(x*e + d)^2 - 2*sqrt(2)*(-I*a^3*b^2 - 3*I*a*b
^4 - 3*I*a*b^2*c^2 - 3*a*b*c^3 - (a^3*b + 3*a*b^3)*c)*cos(x*e + d) - 2*(sqr
t(2)*(-3*I*b^2*c^3 - 3*b*c^4 - (a^2*b + 3*b^3)*c^2 - I*(a^2*b^2 + 3*b^4)*c)
*cos(x*e + d) + sqrt(2)*(-3*I*a*b*c^3 - 3*a*c^4 - (a^3 + 3*a*b^2)*c^2 - I*(
a^3*b + 3*a*b^3)*c))*sin(x*e + d) + sqrt(2)*(I*a^4*b + 3*I*a^2*b^3 + 3*I*b*
c^4 + 3*c^5 + (4*a^2 + 3*b^2)*c^3 + I*(4*a^2*b + 3*b^3)*c^2 + (a^4 + 3*a^2*
b^2)*c))*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c
^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4),
-8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)
*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^
4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d)
 - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2)) + (sqrt(2)*(-I*a^2*b^3 - 3*
I*b^5 + I*a^2*b*c^2 - a^2*c^3 + 3*I*b*c^4 - 3*c^5 + (a^2*b^2 + 3*b^4)*c)*co
s(x*e + d)^2 - 2*sqrt(2)*(I*a^3*b^2 + 3*I*a*b^4 + 3*I*a*b^2*c^2 - 3*a*b*c^3
 - (a^3*b + 3*a*b^3)*c)*cos(x*e + d) - 2*(sqrt(2)*(3*I*b^2*c^3 - 3*b*c^4 -
(a^2*b + 3*b^3)*c^2 + I*(a^2*b^2 + 3*b^4)*c)*cos(x*e + d) + sqrt(2)*(3*I*a*
b*c^3 - 3*a*c^4 - (a^3 + 3*a*b^2)*c^2 + I*(a^3*b + 3*a*b^3)*c))*sin(x*e + d
) + sqrt(2)*(-I*a^4*b - 3*I*a^2*b^3 - 3*I*b*c^4 + 3*c^5 + (4*a^2 + 3*b^2)*c
^3 - I*(4*a^2*b + 3*b^3)*c^2 + (a^4 + 3*a^2*b^2)*c))*sqrt(b - I*c)*weierstr
assPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4
*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27
*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^
2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(
2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e +
 d))/(b^2 + c^2)) - 12*(sqrt(2)*(I*a*b^4 - I*a*c^4)*cos(x*e + d)^2 + 2*sqrt
(2)*(I*a^2*b^3 + I*a^2*b*c^2)*cos(x*e + d) + 2*(sqrt(2)*(I*a*b^3*c + I*a*b*
c^3)*cos(x*e + d) + sqrt(2)*(I*a^2*b^2*c + I*a^2*c^3))*sin(x*e + d) + sqrt(
```

```
2)*(I*a^3*b^2 + I*a*c^4 + I*(a^3 + a*b^2)*c^2))*sqrt(b + I*c)*weierstrassZe
ta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b -
3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
- 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(
8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPIn
verse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b
 - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c
^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*
I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b
- 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b
^2 + c^2))) - 12*(sqrt(2)*(-I*a*b^4 + I*a*c^4)*cos(x*e + d)^2 + 2*sqrt(2)*(
-I*a^2*b^3 - I*a^2*b*c^2)*cos(x*e + d) + 2*(sqrt(2)*(-I*a*b^3*c - I*a*b*c^3
)*cos(x*e + d) + sqrt(2)*(-I*a^2*b^2*c - I*a^2*c^3))*sin(x*e + d) + sqrt(2)
*(-I*a^3*b^2 - I*a*c^4 - I*(a^3 + a*b^2)*c^2))*sqrt(b - I*c)*weierstrassZet
a(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3
*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 +
 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8
*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInv
erse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b
- 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^
4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I
*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b +
 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*sin(x*e + d))/(b
^2 + c^2))) - 6*(4*a*b^3*c + 4*a*b*c^3 - 8*(a*b^3*c + a*b*c^3)*cos(x*e + d)
^2 + (c^5 - (5*a^2 - 2*b^2)*c^3 - (5*a^2*b^2 - b^4)*c)*cos(x*e + d) + (5*a^
2*b^3 - b^5 - b*c^4 + (5*a^2*b - 2*b^3)*c^2 + 4*(a*b^4 - a*c^4)*cos(x*e + d
))*sin(x*e + d))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + a))/((a^4*b^4 - 2*a
^2*b^6 + b^8 - c^8 + 2*(a^2 - b^2)*c^6 - (a^4 - 2*a^2*b^2)*c^4 - 2*(a^2*b^4
 - b^6)*c^2)*cos(x*e + d)^2*e + 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7 + a*b*c^6 -
(2*a^3*b - 3*a*b^3)*c^4 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*c^2)*cos(x*e + d)*e
 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6 + c^8 - (a^2 - 3*b^2)*c^6 - (a^4 + a^2*b^
2 - 3*b^4)*c^4 + (a^6 - 3*a^4*b^2 + a^2*b^4 + b^6)*c^2)*e + 2*((b*c^7 - (2*
a^2*b - 3*b^3)*c^5 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^3 + (a^4*b^3 - 2*a^2*b^5
 + b^7)*c)*cos(x*e + d)*e + (a*c^7 - (2*a^3 - 3*a*b^2)*c^5 + (a^5 - 4*a^3*b
^2 + 3*a*b^4)*c^3 + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*c)*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

3.416 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

Optimal. Leaf size=490

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\left(a^2-b^2-c^2\right)e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}+\frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{15\left(a^2-b^2-c^2\right)^2e(a+b\cos(d+ex)+c\sin(d+e$$

```
[Out] 2/5*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d
))^(5/2)+16/15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e
*x+d)+c*sin(e*x+d))^(3/2)+2/15*(c*(23*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(23*a^2
+9*b^2+9*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(
1/2)+2/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(
b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+
c*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^3/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^
2+c^2)^(1/2)))^(1/2)-16/15*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,
c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c
*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)^2/e/(a+b*cos(e*x+d)+c
*sin(e*x+d))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$-\frac{16a\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15e\left(a^2-b^2-c^2\right)^3\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)\right)}{15e\left(a^2-b^2-c^2\right)^3\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}+\frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{15e\left(a^2-b^2-c^2\right)^2(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])^(5/2)) + (16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))
/(15*(a^2 - b^2 - c^2)^2*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (
2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a +
Sqrt[b^2 + c^2])]) - (16*a*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])
/(a + Sqrt[b^2 + c^2])])/(15*(a^2 - b^2 - c^2)^2*e*Sqrt[a + b*Cos[d + e*x]
+ c*Sin[d + e*x]]) + (2*(c*(23*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(23*a^
```

```
2 + 9*(b^2 + c^2))*Sin[d + e*x]))/(15*(a^2 - b^2 - c^2)^3*e*Sqrt[a + b*Cos[
d + e*x] + c*Sin[d + e*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} - \frac{2\int}{}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15}$$

$$= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} + \frac{}{15}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.68, size = 4116, normalized size = 8.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

```
[Out] (Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2)*(23*a^2 + 9*b^2
 + 9*c^2))/(15*b*c*(-a^2 + b^2 + c^2)^3) + (2*(a*c + b^2*Sin[d + e*x] + c^2
*Sin[d + e*x]))/(5*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x
])^3) - (2*(5*a^2*c + 3*b^2*c + 3*c^3 + 8*a*b^2*Sin[d + e*x] + 8*a*c^2*Sin[
d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x])
^2) + (2*(15*a^3*c + 17*a*b^2*c + 17*a*c^3 + 23*a^2*b^2*Sin[d + e*x] + 9*b^
4*Sin[d + e*x] + 23*a^2*c^2*Sin[d + e*x] + 18*b^2*c^2*Sin[d + e*x] + 9*c^4*
Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^3*(a + b*Cos[d + e*x] + c*Sin[d + e
*x]))))/e - (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*
Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c)
)*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2
/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(
c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]
])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2
/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
- a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Ar
cTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d +
e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*
Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
+ c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c
^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*c*Appel
lF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c
]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b
^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 +
b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
- c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^
2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt
[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/
c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(15*Sqrt[1 + b^2/c^2]*(-a^2 + b^2 + c
^2)^3*e) - (23*a^2*b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqr
t[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/
```

```
(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 +
c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a
 + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 +
c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTa
n[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*c*(-a^2 +
 b^2 + c^2)^3*e) - (3*b^4*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sq
rt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*
Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]
])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2
 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqr
t[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2
 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*
Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - Ar
cTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(5*c*(-a^2
 + b^2 + c^2)^3*e) - (23*a^2*c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a +
 b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 -
a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x
- ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt
[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])
]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt
[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a
 + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x
 - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 +
c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]]))/(15*(
-a^2 + b^2 + c^2)^3*e) - (6*b^2*c*(-((c*AppellF...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5027 vs. $2(535) = 1070$.
time = 20.10, size = 5028, normalized size = 10.26

method	result	size
default	Expression too large to display	5028

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(x*e + d) + c*sin(x*e + d) + a)^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.00, size = 5008, normalized size = 10.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/45*((sqrt(2)*(-I*a^3*b^4 + 33*I*a*b^6 - 99*I*a*b^2*c^4 - 99*a*b*c^5 + 3*(
a^3*b - 22*a*b^3)*c^3 + 3*I*(a^3*b^2 - 22*a*b^4)*c^2 - (a^3*b^3 - 33*a*b^5)
*c)*cos(x*e + d)^3 - 3*sqrt(2)*(I*a^4*b^3 - 33*I*a^2*b^5 - I*a^4*b*c^2 - a^
4*c^3 + 33*I*a^2*b*c^4 + 33*a^2*c^5 + (a^4*b^2 - 33*a^2*b^4)*c)*cos(x*e + d
)^2 - 3*sqrt(2)*(I*a^5*b^2 - 33*I*a^3*b^4 - 33*I*a*b^2*c^4 - 33*a*b*c^5 - (
32*a^3*b + 33*a*b^3)*c^3 - I*(32*a^3*b^2 + 33*a*b^4)*c^2 + (a^5*b - 33*a^3*
b^3)*c)*cos(x*e + d) + (sqrt(2)*(-33*I*a*b*c^5 - 33*a*c^6 + (a^3 + 66*a*b^2
)*c^4 + I*(a^3*b + 66*a*b^3)*c^3 - 3*(a^3*b^2 - 33*a*b^4)*c^2 - 3*I*(a^3*b^
3 - 33*a*b^5)*c)*cos(x*e + d)^2 - 6*sqrt(2)*(-33*I*a^2*b^2*c^3 - 33*a^2*b*c
^4 + (a^4*b - 33*a^2*b^3)*c^2 + I*(a^4*b^2 - 33*a^2*b^4)*c)*cos(x*e + d) +
sqrt(2)*(33*I*a*b*c^5 + 33*a*c^6 + (98*a^3 + 33*a*b^2)*c^4 + I*(98*a^3*b +
33*a*b^3)*c^3 - 3*(a^5 - 33*a^3*b^2)*c^2 - 3*I*(a^5*b - 33*a^3*b^3)*c))*sin
(x*e + d) + sqrt(2)*(-I*a^6*b + 33*I*a^4*b^3 + 99*I*a^2*b*c^4 + 99*a^2*c^5
+ 3*(10*a^4 + 33*a^2*b^2)*c^3 + 3*I*(10*a^4*b + 33*a^2*b^3)*c^2 - (a^6 - 33
*a^4*b^2)*c))*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*
a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c
^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a
*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 +
 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x*e
 + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2 + c^2)) + (sqrt(2)*(I*a^3*b^4
- 33*I*a*b^6 + 99*I*a*b^2*c^4 - 99*a*b*c^5 + 3*(a^3*b - 22*a*b^3)*c^3 - 3*I
*(a^3*b^2 - 22*a*b^4)*c^2 - (a^3*b^3 - 33*a*b^5)*c)*cos(x*e + d)^3 - 3*sqrt
(2)*(-I*a^4*b^3 + 33*I*a^2*b^5 + I*a^4*b*c^2 - a^4*c^3 - 33*I*a^2*b*c^4 + 3
3*a^2*c^5 + (a^4*b^2 - 33*a^2*b^4)*c)*cos(x*e + d)^2 - 3*sqrt(2)*(-I*a^5*b^
2 + 33*I*a^3*b^4 + 33*I*a*b^2*c^4 - 33*a*b*c^5 - (32*a^3*b + 33*a*b^3)*c^3
+ I*(32*a^3*b^2 + 33*a*b^4)*c^2 + (a^5*b - 33*a^3*b^3)*c)*cos(x*e + d) + (s
qrt(2)*(33*I*a*b*c^5 - 33*a*c^6 + (a^3 + 66*a*b^2)*c^4 - I*(a^3*b + 66*a*b^
3)*c^3 - 3*(a^3*b^2 - 33*a*b^4)*c^2 + 3*I*(a^3*b^3 - 33*a*b^5)*c)*cos(x*e +
```

```
d)^2 - 6*sqrt(2)*(33*I*a^2*b^2*c^3 - 33*a^2*b*c^4 + (a^4*b - 33*a^2*b^3)*c
^2 - I*(a^4*b^2 - 33*a^2*b^4)*c)*cos(x*e + d) + sqrt(2)*(-33*I*a*b*c^5 + 33
*a*c^6 + (98*a^3 + 33*a*b^2)*c^4 - I*(98*a^3*b + 33*a*b^3)*c^3 - 3*(a^5 - 3
3*a^3*b^2)*c^2 + 3*I*(a^5*b - 33*a^3*b^3)*c))*sin(x*e + d) + sqrt(2)*(I*a^6
*b - 33*I*a^4*b^3 - 99*I*a^2*b*c^4 + 99*a^2*c^5 + 3*(10*a^4 + 33*a^2*b^2)*c
^3 - 3*I*(10*a^4*b + 33*a^2*b^3)*c^2 - (a^6 - 33*a^4*b^2)*c))*sqrt(b - I*c)
*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4
 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a
*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*
a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^
6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(-I*b^2 - I*c^2)*
sin(x*e + d))/(b^2 + c^2)) - 3*(sqrt(2)*(23*I*a^2*b^5 + 9*I*b^7 - 27*I*b*c^
6 - 3*I*(23*a^2*b + 15*b^3)*c^4 - I*(46*a^2*b^3 + 9*b^5)*c^2)*cos(x*e + d)^
3 + 3*sqrt(2)*(23*I*a^3*b^4 + 9*I*a*b^6 + 9*I*a*b^4*c^2 - 9*I*a*c^6 - I*(23
*a^3 + 9*a*b^2)*c^4)*cos(x*e + d)^2 + 3*sqrt(2)*(23*I*a^4*b^3 + 9*I*a^2*b^5
 + 9*I*b*c^6 + 2*I*(16*a^2*b + 9*b^3)*c^4 + I*(23*a^4*b + 41*a^2*b^3 + 9*b^
5)*c^2)*cos(x*e + d) + (sqrt(2)*(-9*I*c^7 - I*(23*a^2 - 9*b^2)*c^5 + I*(46*
a^2*b^2 + 45*b^4)*c^3 + 3*I*(23*a^2*b^4 + 9*b^6)*c)*cos(x*e + d)^2 + 6*sqrt
(2)*(9*I*a*b*c^5 + I*(23*a^3*b + 18*a*b^3)*c^3 + I*(23*a^3*b^3 + 9*a*b^5)*c
)*cos(x*e + d) + sqrt(2)*(9*I*c^7 + 2*I*(25*a^2 + 9*b^2)*c^5 + I*(69*a^4 +
77*a^2*b^2 + 9*b^4)*c^3 + 3*I*(23*a^4*b^2 + 9*a^2*b^4)*c))*sin(x*e + d) + s
qrt(2)*(23*I*a^5*b^2 + 9*I*a^3*b^4 + 27*I*a*c^6 + 6*I*(13*a^3 + 9*a*b^2)*c^
4 + I*(23*a^5 + 87*a^3*b^2 + 27*a*b^4)*c^2))*sqrt(b + I*c)*weierstrassZeta(
4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b
^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9
*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a
^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInver
se(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b -
3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
- 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(
8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2
*I*a*c + 3*(b^2 + c^2)*cos(x*e + d) - 3*(I*b^2 + I*c^2)*sin(x*e + d))/(b^2
+ c^2))) - 3*(sqrt(2)*(-23*I*a^2*b^5 - 9*I*b^7 + 27*I*b*c^6 + 3*I*(23*a^2*b
 + 15*b^3)*c^4 + I*(46*a^2*b^3 + 9*b^5)*c^2)*cos(x*e + d)^3 + 3*sqrt(2)*(-2
3*I*a^3*b^4 - 9*I*a*b^6 - 9*I*a*b^4*c^2 + 9*I*a*c^6 + I*(23*a^3 + 9*a*b^2)*
c^4)*cos(x*e + d)^2 + 3*sqrt(2)*(-23*I*a^4*b^3 - 9*I*a^2*b^5 - 9*I*b*c^6 -
2*I*(16*a^2*b + 9*b^3)*c^4 - I*(23*a^4*b + 41*a^2*b^3 + 9*b^5)*c^2)*cos(x*e
 + d) + (sqrt(2)*(9*I*c^7 + I*(23*a^2 - 9*b^2)*c^5 - I*(46*a^2*b^2 + 45*b^4
)*c^3 - 3*I*(23*a^2*b^4 + 9*b^6)*c)*cos(x*e + d...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2),x)
```

```
[Out] int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2), x)
```

3.417 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+}}{3e}$$

```
[Out] -2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-320/
3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-16/3*(3
*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3192, 3191}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}-\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{3e}-\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*
x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(5+4\cos(d+ex)+3\sin(d+}{5e}$$
$$= -\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(}}{3e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{16(3\cos(d+ex)-}{}$$

Mathematica [A]
time = 0.68, size = 130, normalized size = 0.94

$$-\frac{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\left(3750\cos\left(\frac{1}{2}(d+ex)\right)+1625\cos\left(\frac{3}{2}(d+ex)\right)+3\left(79\cos\left(\frac{5}{2}(d+ex)\right)-3750\sin\left(\frac{1}{2}(d+ex)\right)-375\sin\left(\frac{3}{2}(d+ex)\right)+3\sin\left(\frac{5}{2}(d+ex)\right)\right)\right)}{30e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] -1/30*((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(3750*Cos[(d + e*x)/2] +
 1625*Cos[(3*(d + e*x))/2] + 3*(79*Cos[(5*(d + e*x))/2] - 3750*Sin[(d + e*x
)/2] - 375*Sin[(3*(d + e*x))/2] + 3*Sin[(5*(d + e*x))/2])))/(e*(3*Cos[(d +
e*x)/2] + Sin[(d + e*x)/2])^5)
```

Maple [A]
time = 0.34, size = 74, normalized size = 0.53

method	result	size
default	$\frac{50\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(3\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+14\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+43\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(3*sin(e*x+d+arc
tan(4/3))^2+14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(5+5*sin(e
*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")

[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) + 5)^(5/2), x)

Fricas [A]
time = 1.33, size = 114, normalized size = 0.82

$$-\frac{2\left(237\cos\left(xe+d\right)^{3}+931\cos\left(xe+d\right)^{2}+9\left(\cos\left(xe+d\right)^{2}-62\cos\left(xe+d\right)-344\right)\sin\left(xe+d\right)+1166\cos\left(xe+d\right)+472\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}}{15\left(3\cos\left(xe+d\right)e+e\sin\left(xe+d\right)+3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")

[Out] -2/15*(237*cos(x*e + d)^3 + 931*cos(x*e + d)^2 + 9*(cos(x*e + d)^2 - 62*cos(x*e + d) - 344)*sin(x*e + d) + 1166*cos(x*e + d) + 472)*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5)/(3*cos(x*e + d)*e + e*sin(x*e + d) + 3*e)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")

[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)

[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)

3.418 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+e
$$

```
[Out] -40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/3
*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3192, 3191}

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*x]
 + 3*Sin[d + e*x]])/(3*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx &= -\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(
\\ &= -\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 104, normalized size = 1.12

$$\frac{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\left(-45\cos\left(\frac{1}{2}(d+ex)\right)-13\cos\left(\frac{3}{2}(d+ex)\right)+9\left(15\sin\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{3}{2}(d+ex)\right)\right)\right)}{3e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] ((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(-45*Cos[(d + e*x)/2] - 13*Cos
[(3*(d + e*x))/2] + 9*(15*Sin[(d + e*x)/2] + Sin[(3*(d + e*x))/2])))/(3*e*(
3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3)
```

Maple [A]
time = 0.27, size = 60, normalized size = 0.65

method	result	size
default	$\frac{50\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(sin(e*x+d+arcta
n(4/3))+5)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) + 5)^(3/2), x)
```

Fricas [A]
time = 1.72, size = 92, normalized size = 0.99

$$-\frac{2\left(13\cos\left(xe+d\right)^{2}-9\left(\cos\left(xe+d\right)+8\right)\sin\left(xe+d\right)+29\cos\left(xe+d\right)+16\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}}{3\left(3\cos\left(xe+d\right)e+e\sin\left(xe+d\right)+3e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(13*cos(x*e + d)^2 - 9*(cos(x*e + d) + 8)*sin(x*e + d) + 29*cos(x*e +
d) + 16)*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5)/(3*cos(x*e + d)*e + e*si
n(x*e + d) + 3*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (4\cos(d+e\,x)+3\sin(d+e\,x)+5)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

3.419 $\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

```
[Out] -2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3191\}$

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d
 + e*x]])
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A]
time = 0.05, size = 75, normalized size = 1.70

$$-\frac{2\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])/(e*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
```

Maple [A]
time = 0.26, size = 50, normalized size = 1.14

method	result
default	$\frac{10\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$
risch	$-\frac{5i\sqrt{2}\,\sqrt{10+8\cos\left(ex+d\right)+6\sin\left(ex+d\right)}\,\sqrt{\left(4-3i\right)\left(25\,\mathrm{e}^{3i(ex+d)}+7\,\mathrm{e}^{i(ex+d)}+24i\mathrm{e}^{i(ex+d)}+3}{\left(25\,\mathrm{e}^{2i(ex+d)}+7+24i+30i\mathrm{e}^{i(ex+d)}+40\,\mathrm{e}^{i(ex+d)}\right)e\sqrt{\left(100-75i\right)\left(25\,\mathrm{e}^{3i(ex+d)}+7\,\mathrm{e}^{i(ex+d)}+24}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4
/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5), x)
```

Fricas [A]
time = 2.01, size = 70, normalized size = 1.59

$$-\frac{2\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}\,\left(\cos\left(xe+d\right)-3\sin\left(xe+d\right)+1\right)}{3\cos\left(xe+d\right)e+e\sin\left(xe+d\right)+3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5)*(cos(x*e + d) - 3*sin(x*e + d)
 + 1)/(3*cos(x*e + d)*e + e*sin(x*e + d) + 3*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

Mupad [B]
time = 0.31, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{5}\,\left(3\,\cos\left(d+e\,x\right)-4\,\sin\left(d+e\,x\right)\right)}{5\,e\,\sqrt{\cos\left(d-\mathrm{atan}\left(\frac{3}{4}\right)+e\,x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
```

```
[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*
x) + 1)^(1/2))
```

3.420 $\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

```
[Out] 1/5*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(
1/2))*10^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3194, 2728, 212}

$$\frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x
 - ArcTan[3/4]]])])/e
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

$$= \frac{\sqrt{\frac{2}{5}}\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 101, normalized size = 2.10

$$-\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\,\text{ArcTan}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(-1+3\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
[Out] ((-2/5 - (6*I)/5)*Sqrt[4/5 + (3*I)/5]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (
3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

Maple [A]
time = 0.64, size = 77, normalized size = 1.60

method	result

default	$-\frac{\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)}{5\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(1+sin(e*x+d+arctan(4/3)))*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2
)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))/cos(e*x+d+arct
an(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(39) = 78$.
time = 2.03, size = 158, normalized size = 3.29

$$\frac{1}{10}\sqrt{5}\sqrt{2}e^{(-1)}\log\left(-\frac{9\cos\left(xe+d\right)^{2}+\left(13\cos\left(xe+d\right)-6\right)\sin\left(xe+d\right)+2\left(\sqrt{5}\sqrt{2}\cos\left(xe+d\right)-3\sqrt{5}\sqrt{2}\sin\left(xe+d\right)+\sqrt{5}\sqrt{2}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}-33\cos\left(xe+d\right)-42}{9\cos\left(xe+d\right)^{2}+\left(13\cos\left(xe+d\right)+14\right)\sin\left(xe+d\right)+27\cos\left(xe+d\right)+18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*e^(-1)*log(-(9*cos(x*e + d)^2 + (13*cos(x*e + d) - 6)*
sin(x*e + d) + 2*(sqrt(5)*sqrt(2)*cos(x*e + d) - 3*sqrt(5)*sqrt(2)*sin(x*e
+ d) + sqrt(5)*sqrt(2))*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5) - 33*cos(
x*e + d) - 42)/(9*cos(x*e + d)^2 + (13*cos(x*e + d) + 14)*sin(x*e + d) + 27
*cos(x*e + d) + 18))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2), x)
```

3.421 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

```
[Out] 1/10*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/1
00*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1
/2))*10^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3195, 3194, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{10\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*Co
s[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}+\frac{1}{20}\int \frac{}{\sqrt{5+4}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}+\frac{1}{20}\int \frac{}{\sqrt{5+5}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{10-x}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e}-\frac{}{10e(5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 154, normalized size = 1.60

$$-\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)-(1-i)\sqrt{20+15i}\,\text{ArcTan}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(-1+3\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2\right)}{e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ((-1/250 + I/125)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((5 + 10*I)*(Cos[
(d + e*x)/2] - 3*Sin[(d + e*x)/2]) - (1 - I)*Sqrt[20 + 15*I]*ArcTan[(1/10 +
 (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/
2] + Sin[(d + e*x)/2])^2))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Maple [A]
time = 0.32, size = 117, normalized size = 1.22

method	result
default	$-\frac{\left(\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5}\cdots}{}\right.\right.}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right.\right.\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/100*(10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))
*sin(e*x+d+arctan(4/3))+10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)
^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin(e*x+d+arcta
n(4/3))+5)^(1/2)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/
e
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) + 5)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(85) = 170$.
time = 1.31, size = 297, normalized size = 3.09

$$\frac{\left(9\sqrt{10}\cos\left(xe+d\right)^2+\left(13\sqrt{10}\cos\left(xe+d\right)+14\sqrt{10}\right)\sin\left(xe+d\right)+27\sqrt{10}\cos\left(xe+d\right)+18\sqrt{10}\right)\log\left(-\frac{9\cos(xe+d)^2+(13\cos(xe+d)-6)\sin(xe+d)+2\left(\sqrt{10}\cos(xe+d)-3\sqrt{10}\sin(xe+d)+\sqrt{10}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}-33\cos(xe+d)-42}{9\cos(xe+d)^2+(13\cos(xe+d)+14)\sin(xe+d)+27\cos(xe+d)+18}\right)-20\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}\left(\cos\left(xe+d\right)-3\sin\left(xe+d\right)+1\right)}{200\left(9\cos\left(xe+d\right)^2e+27\cos\left(xe+d\right)e+\left(13\cos\left(xe+d\right)e+14e\right)\sin\left(xe+d\right)+18e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/200*((9*sqrt(10)*cos(x*e + d)^2 + (13*sqrt(10)*cos(x*e + d) + 14*sqrt(10)
)*sin(x*e + d) + 27*sqrt(10)*cos(x*e + d) + 18*sqrt(10))*log(-(9*cos(x*e +
```

```
d)^2 + (13*cos(x*e + d) - 6)*sin(x*e + d) + 2*(sqrt(10)*cos(x*e + d) - 3*sq
rt(10)*sin(x*e + d) + sqrt(10))*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5) -
 33*cos(x*e + d) - 42)/(9*cos(x*e + d)^2 + (13*cos(x*e + d) + 14)*sin(x*e +
 d) + 27*cos(x*e + d) + 18)) - 20*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5)
*(cos(x*e + d) - 3*sin(x*e + d) + 1))/(9*cos(x*e + d)^2*e + 27*cos(x*e + d)
*e + (13*cos(x*e + d)*e + 14*e)*sin(x*e + d) + 18*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

3.422 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$\frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{3(3}{400e(5}$$

[Out] 1/20*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/400*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+3/4000*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A]
time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3195, 3194, 2728, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{400\sqrt{10}\,e}-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[3/4]]])])/(400*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(400*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} + \frac{3}{40}\int \frac{}{(5+4\,\mathrm{c}}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d}{400e(5+4\cos}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d}{400e(5+4\cos}$$

$$= -\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d}{400e(5+4\cos}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e} - \frac{}{20e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 180, normalized size = 1.27

$$-\frac{\left(\frac{1}{20000}-\frac{i}{10000}\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((-6+6i)\sqrt{20+15i}\,\mathrm{ArcTan}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(-1+3\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^4+(5+10i)\left(55\cos\left(\frac{1}{2}(d+ex)\right)+39\cos\left(\frac{3}{2}(d+ex)\right)-165\sin\left(\frac{1}{2}(d+ex)\right)-27\sin\left(\frac{3}{2}(d+ex)\right)\right)\right)}{e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((-1/20000 + I/10000)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((-6 + 6*I)*S
qrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d
+ e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^4 + (5 + 10*I)*(55*Cos[
(d + e*x)/2] + 39*Cos[(3*(d + e*x))/2] - 165*Sin[(d + e*x)/2] - 27*Sin[(3*(
d + e*x))/2])))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

Maple [A]
time = 0.35, size = 190, normalized size = 1.34

method	result
default	$-\frac{\left(3\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)\left(\sin^{2}\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+6\sqrt{10}\,\operatorname{arctanh}\left(\sqrt{\right.\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4000*(3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/
2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4
/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctanh(1/10*(-5*
sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))+6*(-5*sin(e*x+d+arctan(4/3))+5)^(
1/2)*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin
(e*x+d+arctan(4/3))+5)^(1/2)/(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4/
3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) + 5)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(130) = 260.
time = 1.71, size = 378, normalized size = 2.66

$$\frac{3\left(3\sqrt{10}\cos\left(xe+d\right)^{3}-111\sqrt{10}\cos\left(xe+d\right)^{2}-\left(79\sqrt{10}\cos\left(xe+d\right)^{2}+202\sqrt{10}\cos\left(xe+d\right)+124\sqrt{10}\right)\sin\left(xe+d\right)-246\sqrt{10}\cos\left(xe+d\right)-132\sqrt{10}\right)\log\left(-\frac{9\cos(xe+d)^{2}+(13\cos(xe+d)-6)\sin(xe+d)+2\left(\sqrt{10}\cos(xe+d)-3\sqrt{10}\sin(xe+d)+\sqrt{10}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}-33\cos(xe+d)-42}{9\cos(xe+d)^{2}+(13\cos(xe+d)+14)\sin(xe+d)+27\cos(xe+d)+18}\right)+20\left(39\cos\left(xe+d\right)^{2}-3\left(9\cos\left(xe+d\right)+32\right)\sin\left(xe+d\right)+47\cos\left(xe+d\right)+8\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)+5}}{8000\left(3\cos\left(xe+d\right)^{3}e-111\cos\left(xe+d\right)^{2}e-246\cos\left(xe+d\right)e-\left(79\cos\left(xe+d\right)^{2}e+202\cos\left(xe+d\right)e+124e\right)\sin\left(xe+d\right)-132e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/8000*(3*(3*sqrt(10)*cos(x*e + d)^3 - 111*sqrt(10)*cos(x*e + d)^2 - (79*sq
rt(10)*cos(x*e + d)^2 + 202*sqrt(10)*cos(x*e + d) + 124*sqrt(10))*sin(x*e +
 d) - 246*sqrt(10)*cos(x*e + d) - 132*sqrt(10))*log(-(9*cos(x*e + d)^2 + (1
3*cos(x*e + d) - 6)*sin(x*e + d) + 2*(sqrt(10)*cos(x*e + d) - 3*sqrt(10)*si
n(x*e + d) + sqrt(10))*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5) - 33*cos(x
*e + d) - 42)/(9*cos(x*e + d)^2 + (13*cos(x*e + d) + 14)*sin(x*e + d) + 27*
cos(x*e + d) + 18)) + 20*(39*cos(x*e + d)^2 - 3*(9*cos(x*e + d) + 32)*sin(x
*e + d) + 47*cos(x*e + d) + 8)*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) + 5))/(
3*cos(x*e + d)^3*e - 111*cos(x*e + d)^2*e - 246*cos(x*e + d)*e - (79*cos(x*
e + d)^2*e + 202*cos(x*e + d)*e + 124*e)*sin(x*e + d) - 132*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3\sin\left(d + ex\right) + 4\cos\left(d + ex\right) + 5\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-5/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

3.423 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx$

Optimal. Leaf size=185

$$\frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d}}{7e}$$

```
[Out] 24/7*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-2/7
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)/e+6400/7*
(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-320/7*(3
*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A]
time = 0.07, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3192, 3191}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}+\frac{24(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{7e}-\frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{7e}+\frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

```
[Out] (6400*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(7*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) - (320*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
 e*x] + 3*Sin[d + e*x]])/(7*e) + (24*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5
+ 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(7*e) - (2*(3*Cos[d + e*x] - 4*Si
n[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))/(7*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\,\mathrm{s}}{7e}$$
$$= \frac{24(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\,\mathrm{si}}{7e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)\,-}}{7e}$$
$$= \frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{320(3\cos(d+e}{}$$

Mathematica [A]
time = 2.09, size = 151, normalized size = 0.82

$$\frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\left(91875\cos\left(\frac{1}{2}(d+ex)\right)-11025\cos\left(\frac{3}{2}(d+ex)\right)-147\cos\left(\frac{5}{2}(d+ex)\right)+249\cos\left(\frac{7}{2}(d+ex)\right)+30625\sin\left(\frac{1}{2}(d+ex)\right)-15925\sin\left(\frac{3}{2}(d+ex)\right)+3871\sin\left(\frac{5}{2}(d+ex)\right)-307\sin\left(\frac{7}{2}(d+ex)\right)\right)}{28e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2)*(91875*Cos[(d + e*x)/2] - 110
25*Cos[(3*(d + e*x))/2] - 147*Cos[(5*(d + e*x))/2] + 249*Cos[(7*(d + e*x))/
2] + 30625*Sin[(d + e*x)/2] - 15925*Sin[(3*(d + e*x))/2] + 3871*Sin[(5*(d +
 e*x))/2] - 307*Sin[(7*(d + e*x))/2]))/(28*e*(Cos[(d + e*x)/2] - 3*Sin[(d +
 e*x)/2])^7)
```

Maple [A]
time = 0.34, size = 86, normalized size = 0.46

method	result
default	$\frac{250\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(5\left(\sin^3\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)-27\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+71\sin\left(ex+d\right.\right.}{7\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 250/7*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(5*sin(e*x+d+ar
ctan(4/3))^3-27*sin(e*x+d+arctan(4/3))^2+71*sin(e*x+d+arctan(4/3))-177)/cos
(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) - 5)^(7/2), x)
```

Fricas [A]
time = 1.99, size = 136, normalized size = 0.74

$$-\frac{2\,\left(249\,\cos\left(xe+d\right)^{4}+51\,\cos\left(xe+d\right)^{3}-3042\,\cos\left(xe+d\right)^{2}-\left(307\,\cos\left(xe+d\right)^{3}-1782\,\cos\left(xe+d\right)^{2}+2860\,\cos\left(xe+d\right)-1392\right)\sin\left(xe+d\right)+10068\,\cos\left(xe+d\right)+12912\right)\sqrt{4\,\cos\left(xe+d\right)+3\,\sin\left(xe+d\right)-5}}{7\,\left(\cos\left(xe+d\right)e-3\,e\sin\left(xe+d\right)+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/7*(249*cos(x*e + d)^4 + 51*cos(x*e + d)^3 - 3042*cos(x*e + d)^2 - (307*c
os(x*e + d)^3 - 1782*cos(x*e + d)^2 + 2860*cos(x*e + d) - 1392)*sin(x*e + d
) + 10068*cos(x*e + d) + 12912)*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) - 5)/(
cos(x*e + d)*e - 3*e*sin(x*e + d) + e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2), x)
```

3.424 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

Optimal. Leaf size=139

$$-\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{3e}$$

```
[Out] -2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-320
/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)+16/3*
(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3192, 3191}

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}+\frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}-\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-320*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*
Sin[d + e*x]]) + (16*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d +
e*x] + 3*Sin[d + e*x]])/(3*e) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 +
4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin}{5e}$$
$$= \frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin}}{3e}$$
$$= -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} + \frac{16(3\cos(d+ex}{}$$

Mathematica [A]
time = 0.55, size = 127, normalized size = 0.91

$$\frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\left(11250\cos\left(\frac{1}{2}(d+ex)\right)-1125\cos\left(\frac{3}{2}(d+ex)\right)-9\cos\left(\frac{5}{2}(d+ex)\right)+3750\sin\left(\frac{1}{2}(d+ex)\right)-1625\sin\left(\frac{3}{2}(d+ex)\right)+237\sin\left(\frac{5}{2}(d+ex)\right)\right)}{30e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

```
[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(11250*Cos[(d + e*x)/2] - 112
5*Cos[(3*(d + e*x))/2] - 9*Cos[(5*(d + e*x))/2] + 3750*Sin[(d + e*x)/2] - 1
625*Sin[(3*(d + e*x))/2] + 237*Sin[(5*(d + e*x))/2]))/(30*e*(Cos[(d + e*x)/
2] - 3*Sin[(d + e*x)/2])^5)
```

Maple [A]
time = 0.37, size = 74, normalized size = 0.53

method	result	size
default	$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(3\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)-14\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+43\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(3*sin(e*x+d+arc
tan(4/3))^2-14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(-5+5*sin(
e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) - 5)^(5/2), x)
```

Fricas [**A**]
time = 1.35, size = 114, normalized size = 0.82

$$-\frac{2\left(9\cos\left(xe+d\right)^{3}+567\cos\left(xe+d\right)^{2}-\left(237\cos\left(xe+d\right)^{2}-694\cos\left(xe+d\right)+472\right)\sin\left(xe+d\right)-2538\cos\left(xe+d\right)-3096\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}}{15\left(\cos\left(xe+d\right)e-3e\sin\left(xe+d\right)+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(9*cos(x*e + d)^3 + 567*cos(x*e + d)^2 - (237*cos(x*e + d)^2 - 694*co
s(x*e + d) + 472)*sin(x*e + d) - 2538*cos(x*e + d) - 3096)*sqrt(4*cos(x*e +
 d) + 3*sin(x*e + d) - 5)/(cos(x*e + d)*e - 3*e*sin(x*e + d) + e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

3.425 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

Optimal. Leaf size=93

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+}}{3e}$$

```
[Out] 40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/3
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

Rubi [A]

time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3192, 3191}

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

```
[Out] (40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x
] + 3*Sin[d + e*x]])/(3*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx=-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\,\mathrm{s}}}{3e}$$

$$=\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-}{}$$

Mathematica [A]
time = 0.25, size = 103, normalized size = 1.11

$$\frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\left(135\cos\left(\frac{1}{2}(d+ex)\right)-9\cos\left(\frac{3}{2}(d+ex)\right)+45\sin\left(\frac{1}{2}(d+ex)\right)-13\sin\left(\frac{3}{2}(d+ex)\right)\right)}{3e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]

[Out] ((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(135*Cos[(d + e*x)/2] - 9*Cos[(3*(d + e*x))/2] + 45*Sin[(d + e*x)/2] - 13*Sin[(3*(d + e*x))/2]))/(3*e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^3)

Maple [A]
time = 0.26, size = 60, normalized size = 0.65

method	result	size
default	$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)

[Out] 50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-5)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) - 5)^(3/2), x)

Fricas [A]
time = 3.96, size = 91, normalized size = 0.98

$$\frac{2\left(9\cos(xe+d)^2+(13\cos(xe+d)-16)\sin(xe+d)-63\cos(xe+d)-72\right)\sqrt{4\cos(xe+d)+3\sin(xe+d)-5}}{3\left(\cos(xe+d)e-3e\sin(xe+d)+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")

```
[Out] 2/3*(9*cos(x*e + d)^2 + (13*cos(x*e + d) - 16)*sin(x*e + d) - 63*cos(x*e +
d) - 72)*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) - 5)/(cos(x*e + d)*e - 3*e*si
n(x*e + d) + e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (3\sin{(d+ex)}+4\cos{(d+ex)}-5)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

```
[Out] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

3.426 $\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

Optimal. Leaf size=44

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

```
[Out] -2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3191\}$

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

```
[Out] (-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[
d + e*x]])
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx = -\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.70

$$\frac{2\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] (2*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin [d + e*x]])/(e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))

Maple [A]
time = 0.28, size = 50, normalized size = 1.14

method	result
default	$\frac{10\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$
risch	$\frac{i\sqrt{-10+8\cos\left(ex+d\right)+6\sin\left(ex+d\right)}\,\sqrt{\left(-4+3i\right)\left(30i\mathrm{e}^{2i(ex+d)}-25\,\mathrm{e}^{3i(ex+d)}-24i\mathrm{e}^{i(ex+d)}+40}{\left(30i\mathrm{e}^{i(ex+d)}-25\,\mathrm{e}^{2i(ex+d)}-7-24i+40\,\mathrm{e}^{i(ex+d)}\right)e\sqrt{\left(4-3i\right)\left(25\,\mathrm{e}^{3i(ex+d)}-30i\mathrm{e}^{2i(ex+d)}-40}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)

[Out] 10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(4 /3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(x*e + d) + 3*sin(x*e + d) - 5), x)

Fricas [A]
time = 2.70, size = 68, normalized size = 1.55

$$\frac{2\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}\,\left(3\cos\left(xe+d\right)+\sin\left(xe+d\right)+3\right)}{\cos\left(xe+d\right)e-3\,e\sin\left(xe+d\right)+e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(4*cos(x*e + d) + 3*sin(x*e + d) - 5)*(3*cos(x*e + d) + sin(x*e + d) + 3)/(cos(x*e + d)*e - 3*e*sin(x*e + d) + e)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

Mupad [B]
time = 0.42, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{5}\,\left(3\,\cos\left(d+e\,x\right)-4\,\sin\left(d+e\,x\right)\right)}{5\,e\,\sqrt{\cos\left(d-\mathrm{atan}\left(\frac{3}{4}\right)+e\,x\right)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
```

```
[Out] -(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e*
x) - 1)^(1/2))
```

3.427 $\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{\frac{2}{5}}\,\text{ArcTan}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

[Out] -1/5*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1/2))*10^(1/2)/e

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3194, 2728, 210}

$$-\frac{\sqrt{\frac{2}{5}}\,\text{ArcTan}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]

[Out] -((Sqrt[2/5]*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3/4]]])])/e)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-10-x^2}\,dx, x, -\frac{5\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{-5+5\cos\left(d+ex-\tan^{-1}\right.}}\right.}{e}$$

$$= -\frac{\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 99, normalized size = 2.02

$$\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\tanh^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(3+\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
[Out] ((2/5 + (6*I)/5)*Sqrt[-4/5 - (3*I)/5]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 -
 (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))/
(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

Maple [A]
time = 0.84, size = 77, normalized size = 1.57

method	result

default	$\frac{(\sin(ex+d+\arctan(\frac{4}{3}))-1)\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\right.\right.}}{10}\right.}{5\cos(ex+d+\arctan(\frac{4}{3}))\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(sin(e*x+d+arctan(4/3))-1)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2)
*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))/cos(e*x+d+arctan
(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(x*e + d) + 3*sin(x*e + d) - 5), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(39) = 78.
time = 2.76, size = 93, normalized size = 1.90

$$\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(-\frac{\left(3\sqrt{5}\sqrt{2}\cos\left(xe+d\right)+\sqrt{5}\sqrt{2}\sin\left(xe+d\right)+3\sqrt{5}\sqrt{2}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}}{10\left(\cos\left(xe+d\right)-3\sin\left(xe+d\right)+1\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arctan(-1/10*(3*sqrt(5)*sqrt(2)*cos(x*e + d) + sqrt(5)*
sqrt(2)*sin(x*e + d) + 3*sqrt(5)*sqrt(2))*sqrt(4*cos(x*e + d) + 3*sin(x*e +
 d) - 5)/(cos(x*e + d) - 3*sin(x*e + d) + 1))*e^(-1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{3\sin\left(d+ex\right)+4\cos\left(d+ex\right)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
[Out] Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2), x)
```

3.428 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{\text{ArcTan}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

```
[Out] 1/10*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/1
00*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1
/2))*10^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3195, 3194, 2728, 210}

$$\frac{\text{ArcTan}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{10\sqrt{10}\,e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[3
/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4*C
os[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{}{\sqrt{-5}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} - \frac{1}{20}\int \frac{}{\sqrt{-5}}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}} + \frac{\text{Subst}\left(\int \frac{}{-1}\right)}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}\,e} + \frac{}{10e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 152, normalized size = 1.58

$$\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((-1+i)\sqrt{-20-15i}\tanh^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(3+\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^2+(5+10i)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)}{e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]

[Out] ((1/250 - I/125)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((-1 + I)*Sqrt[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^2 + (5 + 10*I)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))

Maple [A]
time = 0.32, size = 118, normalized size = 1.23

method	result
default	$\frac{\left(-\sqrt{10}\, \arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\, \arctan\left(\frac{\sqrt{-5\sin}}{}\right.\right.}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/100*(-10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")

[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) - 5)^(-3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(85) = 170.
time = 2.76, size = 233, normalized size = 2.43

$$-\frac{\left(13\sqrt{10}\cos\left(xe+d\right)^{2}-9\left(\sqrt{10}\cos\left(xe+d\right)-2\sqrt{10}\right)\sin\left(xe+d\right)-\sqrt{10}\cos\left(xe+d\right)-14\sqrt{10}\right)\arctan\left(-\frac{\left(3\sqrt{10}\cos(xe+d)+\sqrt{10}\sin(xe+d)+3\sqrt{10}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}}{10\left(\cos(xe+d)-3\sin(xe+d)+1\right)}\right)+10\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}\left(3\cos\left(xe+d\right)+\sin\left(xe+d\right)+3\right)}{100\left(13\cos\left(xe+d\right)^{2}e-\cos\left(xe+d\right)e-9\left(\cos\left(xe+d\right)e-2e\right)\sin\left(xe+d\right)-14e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")

[Out] -1/100*((13*sqrt(10)*cos(x*e + d)^2 - 9*(sqrt(10)*cos(x*e + d) - 2*sqrt(10))*sin(x*e + d) - sqrt(10)*cos(x*e + d) - 14*sqrt(10))*arctan(-1/10*(3*sqrt(

```
10)*cos(x*e + d) + sqrt(10)*sin(x*e + d) + 3*sqrt(10))*sqrt(4*cos(x*e + d)
+ 3*sin(x*e + d) - 5)/(cos(x*e + d) - 3*sin(x*e + d) + 1)) + 10*sqrt(4*cos(
x*e + d) + 3*sin(x*e + d) - 5)*(3*cos(x*e + d) + sin(x*e + d) + 3))/(13*cos
(x*e + d)^2*e - cos(x*e + d)*e - 9*(cos(x*e + d)*e - 2*e)*sin(x*e + d) - 14
*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}-5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(-t
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

3.429 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{3\text{ArcTan}\left(\frac{\sin\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\text{ArcTan}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}-\frac{}{40}$$

```
[Out] 1/20*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/4
00*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)-3/400
0*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^(1/
2))*10^(1/2)/e
```

Rubi [A]

time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3195, 3194, 2728, 210}

$$-\frac{3\text{ArcTan}\left(\frac{\sin\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\text{ArcTan}\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{400\sqrt{10}\,e}-\frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}+\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcT
an[3/4]]])])/(400*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5
 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)) - (3*(3*Cos[d + e*x] - 4*Sin[d +
 e*x]))/(400*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3}{40}\int \frac{}{(-5+}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos}{400e(-5+4}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos}{400e(-5+4}$$

$$= \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos}{400e(-5+4}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sin\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\tan^{-1}\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}\,e} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 178, normalized size = 1.25

$$\frac{\left(\frac{1}{10000}+\frac{i}{20000}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((6+6i)\sqrt{-20-15i}\tanh^{-1}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(3+\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^4+(10-5i)\left(165\cos\left(\frac{1}{2}(d+ex)\right)-27\cos\left(\frac{3}{2}(d+ex)\right)+55\sin\left(\frac{1}{2}(d+ex)\right)-39\sin\left(\frac{3}{2}(d+ex)\right)\right)\right)}{e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

```
[Out] ((1/10000 + I/20000)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((6 + 6*I)*Sqr
t[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d +
e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^4 + (10 - 5*I)*(165*Cos[(
d + e*x)/2] - 27*Cos[(3*(d + e*x))/2] + 55*Sin[(d + e*x)/2] - 39*Sin[(3*(d
+ e*x))/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

Maple [A]
time = 0.36, size = 190, normalized size = 1.34

method	result
default	$-\frac{\left(-3\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)\left(\sin^2\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)+6\sqrt{10}\,\arctan\left(\sqrt{\ }\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4000*(-3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/
2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/
3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+6*(-5*sin(e*x+d+arctan(4/3))-
5)^(1/2)*sin(e*x+d+arctan(4/3))-3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan
(4/3))-5)^(1/2)*10^(1/2))-14*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e
*x+d+arctan(4/3))-5)^(1/2)/(sin(e*x+d+arctan(4/3))-1)/cos(e*x+d+arctan(4/3)
)/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((4*cos(x*e + d) + 3*sin(x*e + d) - 5)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(130) = 260$.
time = 2.35, size = 311, normalized size = 2.19

$$\frac{3\left(79\sqrt{10}\cos\left(xe+d\right)^{3}-123\sqrt{10}\cos\left(xe+d\right)^{2}+3\left(\sqrt{10}\cos\left(xe+d\right)^{2}+38\sqrt{10}\cos\left(xe+d\right)-44\sqrt{10}\right)\sin\left(xe+d\right)-78\sqrt{10}\cos\left(xe+d\right)+124\sqrt{10}\right)\arctan\left(-\frac{\left(3\sqrt{10}\cos(xe+d)+\sqrt{10}\sin(xe+d)+3\sqrt{10}\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}}{10\left(\cos(xe+d)-3\sin(xe+d)+1\right)}\right)+10\left(27\cos\left(xe+d\right)^{2}+\left(39\cos\left(xe+d\right)-8\right)\sin\left(xe+d\right)-69\cos\left(xe+d\right)-96\right)\sqrt{4\cos\left(xe+d\right)+3\sin\left(xe+d\right)-5}}{4000\left(79\cos\left(xe+d\right)^{3}e-123\cos\left(xe+d\right)^{2}e-78\cos\left(xe+d\right)e+3\left(\cos\left(xe+d\right)^{2}e+38\cos\left(xe+d\right)e-44e\right)\sin\left(xe+d\right)+124e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4000*(3*(79*sqrt(10)*cos(x*e + d)^3 - 123*sqrt(10)*cos(x*e + d)^2 + 3*(sq
rt(10)*cos(x*e + d)^2 + 38*sqrt(10)*cos(x*e + d) - 44*sqrt(10))*sin(x*e + d
) - 78*sqrt(10)*cos(x*e + d) + 124*sqrt(10))*arctan(-1/10*(3*sqrt(10)*cos(x
*e + d) + sqrt(10)*sin(x*e + d) + 3*sqrt(10))*sqrt(4*cos(x*e + d) + 3*sin(x
*e + d) - 5)/(cos(x*e + d) - 3*sin(x*e + d) + 1)) + 10*(27*cos(x*e + d)^2 +
 (39*cos(x*e + d) - 8)*sin(x*e + d) - 69*cos(x*e + d) - 96)*sqrt(4*cos(x*e
+ d) + 3*sin(x*e + d) - 5))/(79*cos(x*e + d)^3*e - 123*cos(x*e + d)^2*e - 7
8*cos(x*e + d)*e + 3*(cos(x*e + d)^2*e + 38*cos(x*e + d)*e - 44*e)*sin(x*e
+ d) + 124*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

```
[Out] Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(-t
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

```
[Out] int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

3.430 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx$

Optimal. Leaf size=258

$$-\frac{256(b^2+c^2)^{3/2}\left(c\cos(d+ex)-b\sin(d+ex)\right)}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{64(b^2+c^2)\left(c\cos(d+ex)-b\sin(d+ex)\right)\sqrt{\sqrt{b^2+c^2}+}}{35e}$$

```
[Out] -24/35*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^(3/2)/e-2/7*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c
*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2)/e-256/35*(b^2+c^2)^(3/2)*(c*cos(e*x+d)-b
*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-64/35*(b^2
+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2
))^(1/2)/e
```

Rubi [A]
time = 0.13, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3192, 3191}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}-\frac{24\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{35e}-\frac{64(b^2+c^2)(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{35e}-\frac{256(b^2+c^2)^{3/2}(c\cos(d+ex)-b\sin(d+ex))}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
```

```
[Out] (-256*(b^2 + c^2)^(3/2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(35*e*Sqrt[Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (64*(b^2 + c^2)*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e
*x]])/(35*e) - (24*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[
b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))/(35*e) - (2*(c*Cos[d +
 e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])
^(5/2))/(7*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\,\mathrm{c}\right.}{7e}$$

$$= -\frac{24\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2\right.}}{35e}$$

$$= -\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2}}}{35e}$$

$$= -\frac{256(b^2+c^2)^{3/2}\,(c\cos(d+ex)-b\sin(d+ex))}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 47.14, size = 11888, normalized size = 46.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
[Out] Result too large to show
```

Maple [A]
time = 0.46, size = 306, normalized size = 1.19

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)(\sin(ex+d-\arctan(-b,c))-1)(5b^4(\sin^3(ex+d-\arctan(-b,c)))+10b^2c^2(\sin^3(ex+d-\arctan(-b,c)))+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/35*(sin(e*x+d-arctan(-b,c))+1)*(sin(e*x+d-arctan(-b,c))-1)*(5*b^4*sin(e*x
+d-arctan(-b,c))^3+10*b^2*c^2*sin(e*x+d-arctan(-b,c))^3+5*c^4*sin(e*x+d-arc
tan(-b,c))^3+27*b^4*sin(e*x+d-arctan(-b,c))^2+54*b^2*c^2*sin(e*x+d-arctan(-
b,c))^2+27*c^4*sin(e*x+d-arctan(-b,c))^2+71*b^4*sin(e*x+d-arctan(-b,c))+142
*b^2*c^2*sin(e*x+d-arctan(-b,c))+71*c^4*sin(e*x+d-arctan(-b,c))+177*b^4+354
*b^2*c^2+177*c^4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2
*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.20, size = 283, normalized size = 1.10

$$\frac{2 \left(5 \left(b^4-6 b^2c^2+c^4\right) \cos \left(xe+d\right)^4-177 b^4-310 b^2c^2-128 c^4+2 \left(22 b^4+15 b^2c^2-27 c^4\right) \cos \left(xe+d\right)^2+4 \left(5 \left(b^3c-bc^3\right) \cos \left(xe+d\right)^3+\left(22 b^3c+27 bc^3\right) \cos \left(xe+d\right)\right) \sin \left(xe+d\right)+2 \left(11 \left(b^3-3 bc^2\right) \cos \left(xe+d\right)^3+\left(53 b^3+86 bc^2\right) \cos \left(xe+d\right)+\left(53 b^2c+64 c^3+11 \left(3 b^2c-c^3\right) \cos \left(xe+d\right)^2\right) \sin \left(xe+d\right)\right) \sqrt{b^2+c^2}\right) \sqrt{b \cos \left(xe+d\right)+c \sin \left(xe+d\right)+\sqrt{b^2+c^2}}}{35 \left(c \cos \left(xe+d\right) e-be \sin \left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="f
ricas")
```

```
[Out] 2/35*(5*(b^4 - 6*b^2*c^2 + c^4)*cos(x*e + d)^4 - 177*b^4 - 310*b^2*c^2 - 12
8*c^4 + 2*(22*b^4 + 15*b^2*c^2 - 27*c^4)*cos(x*e + d)^2 + 4*(5*(b^3*c - b*c
^3)*cos(x*e + d)^3 + (22*b^3*c + 27*b*c^3)*cos(x*e + d))*sin(x*e + d) + 2*(
11*(b^3 - 3*b*c^2)*cos(x*e + d)^3 + (53*b^3 + 86*b*c^2)*cos(x*e + d) + (53*
b^2*c + 64*c^3 + 11*(3*b^2*c - c^3)*cos(x*e + d)^2)*sin(x*e + d))*sqrt(b^2
+ c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))/(c*cos(x*e
+ d)*e - b*e*sin(x*e + d))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Evaluation time: 0.44sym2poly/r2sym(const gen & e,const index_m & i,const
 vecte
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2), x)
```

3.431 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

Optimal. Leaf size=190

$$-\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}}}{15e}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)
)^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin
(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^
(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.09, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3192, 3191}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[Sqrt[b^2 + c
^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (16*Sqrt[b^2 + c^2]*(c*Cos[d + e*
x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]
])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\,c\right.}{5e}$$

$$=-\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2}}}{15e}$$

$$=-\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 38.67, size = 11771, normalized size = 61.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
[Out] Result too large to show
```

Maple [A]
time = 0.34, size = 200, normalized size = 1.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)(3b^2(\sin^2(ex+d-\arctan(-b,c)))+3c^2(\sin^2(ex+d-\arctan}{15\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d}{\sqrt{b^2+c^2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVERBO
SE)
[Out] 2/15*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1
)*(3*b^2*sin(e*x+d-arctan(-b,c))^2+3*c^2*sin(e*x+d-arctan(-b,c))^2+14*b^2*s
in(e*x+d-arctan(-b,c))+14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.07, size = 202, normalized size = 1.06

$$\frac{2 \left(3 \left(b^3-3 b c^2\right) \cos \left(xe+d\right)^3+\left(29 b^3+38 b c^2\right) \cos \left(xe+d\right)+\left(29 b^2 c+32 c^3+3 \left(3 b^2 c-c^3\right) \cos \left(xe+d\right)^2\right) \sin \left(xe+d\right)+\left(22 b c \cos \left(xe+d\right) \sin \left(xe+d\right)+11 \left(b^2-c^2\right) \cos \left(xe+d\right)^2-43 b^2-32 c^2\right) \sqrt{b^2+c^2}\right) \sqrt{b \cos \left(xe+d\right)+c \sin \left(xe+d\right)+\sqrt{b^2+c^2}}}{15 \left(c \cos \left(xe+d\right) e-b e \sin \left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(x*e + d)^3 + (29*b^3 + 38*b*c^2)*cos(x*e + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(x*e + d)^2)*sin(x*e + d) + (22*
b*c*cos(x*e + d)*sin(x*e + d) + 11*(b^2 - c^2)*cos(x*e + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^
2))/(c*cos(x*e + d)*e - b*e*sin(x*e + d))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

3.432 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+e}}{3e}$$

```
[Out] -8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+
d)+(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*s
in(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A]

time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3192, 3191}

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\,c}}{3e}$$

$$=-\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 21.85, size = 11679, normalized size = 92.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
[Out] Result too large to show
```

Maple [A]
time = 0.37, size = 126, normalized size = 1.00

method	result	size
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)(b^2+c^2)(\sin(ex+d-\arctan(-b,c))-1)(\sin(ex+d-\arctan(-b,c))+5)}{3\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVERBO
SE)
[Out] 2/3*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))-1)*(sin(
e*x+d-arctan(-b,c))+5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.60, size = 136, normalized size = 1.08

$$\frac{2\left(2\,bc\cos\left(xe+d\right)\sin\left(xe+d\right)+\left(b^2-c^2\right)\cos\left(xe+d\right)^2-5\,b^2-4\,c^2+4\,\sqrt{b^2+c^2}\left(b\cos\left(xe+d\right)+c\sin\left(xe+d\right)\right)\right)\sqrt{b\cos\left(xe+d\right)+c\sin\left(xe+d\right)+\sqrt{b^2+c^2}}}{3\left(c\cos\left(xe+d\right)e-be\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 - 5*b^2 -
 4*c^2 + 4*sqrt(b^2 + c^2)*(b*cos(x*e + d) + c*sin(x*e + d)))*sqrt(b*cos(x*
e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))/(c*cos(x*e + d)*e - b*e*sin(x*e
+ d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

3.433 $\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=55

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] -2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3191}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x]])
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 32.02, size = 11586, normalized size = 210.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.
time = 0.36, size = 113, normalized size = 2.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}{\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$
risch	$-\frac{\sqrt{2\sqrt{b^2+c^2}+2b\cos(ex+d)+2c\sin(ex+d)}\,\sqrt{-ic\,e^{3i(ex+d)}+b\,e^{3i(ex+d)}+2\sqrt{b^2+c^2}\,e^{2i(e}}}{\left(-ic\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+2\sqrt{b^2+c^2}\,e\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1)/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.31, size = 88, normalized size = 1.60

$$\frac{2\sqrt{b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}}\left(b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}\right)}{c\cos(xe+d)e-be\sin(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))*(b*cos(x*e + d) +
 c*sin(x*e + d) - sqrt(b^2 + c^2))/(c*cos(x*e + d)*e - b*e*sin(x*e + d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

3.434 $\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

```
[Out] arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(1/2)
+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3194, 2728, 212}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*S
qrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b^2
 + c^2)^(1/4)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{2\sqrt{b^2+c^2}-x^2}\,dx, x, -\frac{\sqrt{b^2+}}{\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2}}}\right.}{e}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c\right.}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex\right.}}\right.}{\sqrt[4]{b^2+c^2}\,e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 51.14, size = 63264, normalized size = 718.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(75) = 150$.
time = 1.23, size = 172, normalized size = 1.95

method	result
default	$-\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}\,\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+}}}{}\right.}{(b^2+c^2)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan}{\sqrt{b^2+c^2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -(sin(e*x+d-arctan(-b,c))+1)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))
^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctanh(1/2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arc
tan(-b,c))-1))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2
*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/
2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(82) = 164$.
time = 3.64, size = 366, normalized size = 4.16

$$\frac{\sqrt{2}\,e^{(-1)}\log\left(\frac{(3\,b^2c-c^3)\cos(xe+d)^3+(b^2c+4\,c^3)\cos(xe+d)-\left(3\,b^3+4\,bc^2+(b^3-3\,bc^2)\cos(xe+d)^2\right)\sin(xe+d)+\frac{2\sqrt{2}\left(2\left(b^3+bc^2\right)\cos(xe+d)+2\left(b^2c+c^3\right)\sin(xe+d)-\left(2\,bc\cos(xe+d)\sin(xe+d)+\left(b^2-c^2\right)\cos(xe+d)^2+b^2+2\,c^2\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos\left(xe+d\right)+c\sin\left(xe+d\right)+\sqrt{b^2+c^2}}}{\left(b^2+c^2\right)^{\frac{1}{4}}}-4\left(2\,bc\cos(xe+d)^2-(b^2-c^2)\cos(xe+d)\sin(xe+d)-bc\right)\sqrt{b^2+c^2}}{3\,b^2c\cos(xe+d)-(3\,b^2c-c^3)\cos(xe+d)^3-\left(b^3-(b^3-3\,bc^2)\cos(xe+d)^2\right)\sin(xe+d)}\right)}{2\left(b^2+c^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(2)*e^(-1)*log(((3*b^2*c - c^3)*cos(x*e + d)^3 + (b^2*c + 4*c^3)*co
s(x*e + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d
) + 2*sqrt(2)*(2*(b^3 + b*c^2)*cos(x*e + d) + 2*(b^2*c + c^3)*sin(x*e + d)
- (2*b*c*cos(x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 + b^2 + 2*c
^2)*sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2)
)/(b^2 + c^2)^(1/4) - 4*(2*b*c*cos(x*e + d)^2 - (b^2 - c^2)*cos(x*e + d)*si
n(x*e + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(x*e + d) - (3*b^2*c - c^3)*
cos(x*e + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d)))/(b^2
 + c^2)^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

3.435 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e}-\frac{c\cos(d+ex)-b\sin(}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+e\right.}$$

```
[Out] 1/4*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(
1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(1/
2)+1/2*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e
*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3195, 3194, 2728, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\left(b^2+c^2\right)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d-\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d-\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d-\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\right.}}\right.}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

[Out] $Aborted

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(137) = 274$.

time = 0.42, size = 350, normalized size = 2.19

method	result
default	$-\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\ \sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\ \sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/4/(b^2+c^2)^(7/4)*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(
1/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2)
)^(1/2)*(b^2+c^2)^(3/4)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arc
tan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arct
anh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^
(1/2)/(b^2+c^2)^(1/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^
(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-a
rctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(147) = 294$.

time = 3.10, size = 683, normalized size = 4.27

$$-\frac{\left(3\sqrt{2}\,b^2c\cos(xe+d)-\sqrt{2}\,(3b^2c-c^3)\cos(xe+d)^3-\left(\sqrt{2}\,b^3-\sqrt{2}\,(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)\right)(b^2+c^2)^{\frac{1}{4}}\log\left(\frac{(3b^2c-c^3)\cos(xe+d)^3+(b^2c+4c^3)\cos(xe+d)-\left(3b^3+4bc^2+(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)-2\left(2\sqrt{2}\,bc\cos(xe+d)\sin(xe+d)+\sqrt{2}\,(b^2-c^2)\cos(xe+d)^2+\sqrt{2}\,(b^2+2c^2)-2\left(\sqrt{2}\,b\cos(xe+d)+\sqrt{2}\,c\sin(xe+d)\right)\sqrt{b^2+c^2}\right)(b^2+c^2)^{\frac{1}{4}}\sqrt{b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}}-4\left(2bc\cos(xe+d)^2-(b^2-c^2)\cos(xe+d)\sin(xe+d)-bc\right)\sqrt{b^2+c^2}}{3b^2c\cos(xe+d)-(3b^2c-c^3)\cos(xe+d)^3-\left(b^3-(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)}\right)+4\left(2(b^3+bc^2)\cos(xe+d)+2(b^2c+c^3)\sin(xe+d)-\left(2bc\cos(xe+d)\sin(xe+d)+(b^2-c^2)\cos(xe+d)^2+b^2+2c^2\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}}}{8\left((3b^4c+2b^2c^3-c^5)\cos(xe+d)^3e-3(b^4c+b^2c^3)\cos(xe+d)e-\left((b^5-2b^3c^2-3bc^4)\cos(xe+d)^2e-(b^5+b^3c^2)e\right)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] -1/8*((3*sqrt(2)*b^2*c*cos(x*e + d) - sqrt(2)*(3*b^2*c - c^3)*cos(x*e + d)^
3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d))*(b
^2 + c^2)^(1/4)*log(((3*b^2*c - c^3)*cos(x*e + d)^3 + (b^2*c + 4*c^3)*cos(x
*e + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d) -
 2*(2*sqrt(2)*b*c*cos(x*e + d)*sin(x*e + d) + sqrt(2)*(b^2 - c^2)*cos(x*e +
 d)^2 + sqrt(2)*(b^2 + 2*c^2) - 2*(sqrt(2)*b*cos(x*e + d) + sqrt(2)*c*sin(x
*e + d))*sqrt(b^2 + c^2))*(b^2 + c^2)^(1/4)*sqrt(b*cos(x*e + d) + c*sin(x*e
 + d) + sqrt(b^2 + c^2)) - 4*(2*b*c*cos(x*e + d)^2 - (b^2 - c^2)*cos(x*e +
d)*sin(x*e + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(x*e + d) - (3*b^2*c -
c^3)*cos(x*e + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d)))
 + 4*(2*(b^3 + b*c^2)*cos(x*e + d) + 2*(b^2*c + c^3)*sin(x*e + d) - (2*b*c*
cos(x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 + b^2 + 2*c^2)*sqrt(
b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2)))/((3*b^
4*c + 2*b^2*c^3 - c^5)*cos(x*e + d)^3*e - 3*(b^4*c + b^2*c^3)*cos(x*e + d)*
e - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*cos(x*e + d)^2*e - (b^5 + b^3*c^2)*e)*sin(
x*e + d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

3.436 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=226

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e}-\frac{c\cos(d+ex)-b\sin(\ldots}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+\ldots\right.}$$

```
[Out] 3/32*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^
(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2^(1
/2)+1/4*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(
e*x+d)+(b^2+c^2)^(1/2))^(5/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/
(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3195, 3194, 2728, 212}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sq
rt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2]
*(b^2 + c^2)^(5/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^2
]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Cos[
d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(Sqrt[b^2 + c^2] + b*Cos[d +
e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

$$= -\frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d\right.}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c\right.}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos\left(d+e\right.}}\right.}{16\sqrt{2}\,\left(b^2+c^2\right)^{5/4}e}$$

Mathematica [F]

time = 180.05, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.43, size = 350, normalized size = 1.55

method	result
default	$\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\,\sqrt{2}}{2(b^2+c^2)^{\frac{1}{4}}}\right)(b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/4*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+
d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+2
*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*(b^2+c^2)
^(3/4)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c
^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*(-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(
1/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^(1/2)/(b^2+c^2)^(
5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-ar
ctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(210) = 420$.
time = 3.50, size = 942, normalized size = 4.17

$$\frac{\frac{3\sqrt{\frac{1}{2}}\left(5b^4c\cos(xe+d)+(5b^4c-10b^2c^3+c^5)\cos(xe+d)^5-10(b^4c-b^2c^3)\cos(xe+d)^3-\left(b^5+(b^5-10b^3c^2+5bc^4)\cos(xe+d)^4-2(b^5-5b^3c^2)\cos(xe+d)^2\right)\sin(xe+d)\right)\log\left(\frac{(3b^2c-c^3)\cos(xe+d)^3+(b^2c+4c^3)\cos(xe+d)-\left(3b^3+4bc^2+(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)+\frac{4\sqrt{\frac{1}{2}}\left(2(b^3+bc^2)\cos(xe+d)+2(b^2c+c^3)\sin(xe+d)-\left(2bc\cos(xe+d)\sin(xe+d)+(b^2-c^2)\cos(xe+d)^2+b^2+2c^2\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}}}{(b^2+c^2)^{\frac{1}{4}}}-4\left(2bc\cos(xe+d)^2-(b^2-c^2)\cos(xe+d)\sin(xe+d)-bc\right)\sqrt{b^2+c^2}}{3b^2c\cos(xe+d)-(3b^2c-c^3)\cos(xe+d)^3-\left(b^3-(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)}\right)}{(b^2+c^2)^{\frac{1}{4}}}+2\left(3(b^4-6b^2c^2+c^4)\cos(xe+d)^4-7b^4-26b^2c^2-16c^4-6(2b^4-3b^2c^2-c^4)\cos(xe+d)^2+12\left((b^3c-bc^3)\cos(xe+d)^3-(2b^3c+bc^3)\cos(xe+d)\right)\sin(xe+d)-2\left((b^3-3bc^2)\cos(xe+d)^3-3(3b^3+2bc^2)\cos(xe+d)-\left(9b^2c+8c^3-(3b^2c-c^3)\cos(xe+d)^2\right)\sin(xe+d)\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}}}{32\left((5b^6c-5b^4c^3-9b^2c^5+c^7)\cos(xe+d)^5e-10(b^6c-b^2c^5)\cos(xe+d)^3e+5(b^6c+b^4c^3)\cos(xe+d)e-\left((b^7-9b^5c^2-5b^3c^4+5bc^6)\cos(xe+d)^4e-2(b^7-4b^5c^2-5b^3c^4)\cos(xe+d)^2e+(b^7+b^5c^2)e\right)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/32*(3*sqrt(1/2)*(5*b^4*c*cos(x*e + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(
x*e + d)^5 - 10*(b^4*c - b^2*c^3)*cos(x*e + d)^3 - (b^5 + (b^5 - 10*b^3*c^2
 + 5*b*c^4)*cos(x*e + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(x*e + d)^2)*sin(x*e +
d))*log(((3*b^2*c - c^3)*cos(x*e + d)^3 + (b^2*c + 4*c^3)*cos(x*e + d) - (3
*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d) + 4*sqrt(1/2)
*(2*(b^3 + b*c^2)*cos(x*e + d) + 2*(b^2*c + c^3)*sin(x*e + d) - (2*b*c*cos(
x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 + b^2 + 2*c^2)*sqrt(b^2
+ c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))/(b^2 + c^2)
^(1/4) - 4*(2*b*c*cos(x*e + d)^2 - (b^2 - c^2)*cos(x*e + d)*sin(x*e + d) -
b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(x*e + d) - (3*b^2*c - c^3)*cos(x*e + d)^
3 - (b^3 - (b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d)))/(b^2 + c^2)^(1/4)
 + 2*(3*(b^4 - 6*b^2*c^2 + c^4)*cos(x*e + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^
4 - 6*(2*b^4 - 3*b^2*c^2 - c^4)*cos(x*e + d)^2 + 12*((b^3*c - b*c^3)*cos(x*
e + d)^3 - (2*b^3*c + b*c^3)*cos(x*e + d))*sin(x*e + d) - 2*((b^3 - 3*b*c^2
)*cos(x*e + d)^3 - 3*(3*b^3 + 2*b*c^2)*cos(x*e + d) - (9*b^2*c + 8*c^3 - (3
*b^2*c - c^3)*cos(x*e + d)^2)*sin(x*e + d))*sqrt(b^2 + c^2))*sqrt(b*cos(x*e
 + d) + c*sin(x*e + d) + sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^
5 + c^7)*cos(x*e + d)^5*e - 10*(b^6*c - b^2*c^5)*cos(x*e + d)^3*e + 5*(b^6*
c + b^4*c^3)*cos(x*e + d)*e - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*cos(
x*e + d)^4*e - 2*(b^7 - 4*b^5*c^2 - 5*b^3*c^4)*cos(x*e + d)^2*e + (b^7 + b^
5*c^2)*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

3.437 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} d$

Optimal. Leaf size=196

$$-\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}+\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+}}}{15e}$$

```
[Out] -2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2)
)^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin
(e*x+d)-(b^2+c^2)^(1/2))^(1/2)+16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^
(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A]

time = 0.09, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {3192, 3191}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}+\frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{15e}-\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] (-64*(b^2 + c^2)*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(15*e*Sqrt[-Sqrt[b^2 +
c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) + (16*Sqrt[b^2 + c^2]*(c*Cos[d + e
*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*
x]])/(15*e) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Co
s[d + e*x] + c*Sin[d + e*x])^(3/2))/(5*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\right.}{5e}$$

$$= \frac{16\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2}}}{15e}$$

$$= -\frac{64(b^2+c^2)(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 50.97, size = 11602, normalized size = 59.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 0.41, size = 204, normalized size = 1.04

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))+1)(3b^2(\sin^2(ex+d-\arctan(-b,c)))+3c^2(\sin^2(ex+d-\arctan(-}{15\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-}{\sqrt{b^2+c^2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2/15*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1
)*(3*b^2*sin(e*x+d-arctan(-b,c))^2+3*c^2*sin(e*x+d-arctan(-b,c))^2-14*b^2*s
in(e*x+d-arctan(-b,c))-14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos(e*
x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))
-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 2.80, size = 205, normalized size = 1.05

$$\frac{2\left(3\left(b^{3}-3\,bc^{2}\right)\cos\left(xe+d\right)^{3}+\left(29\,b^{3}+38\,bc^{2}\right)\cos\left(xe+d\right)+\left(29\,b^{2}c+32\,c^{3}+3\left(3\,b^{2}c-c^{3}\right)\cos\left(xe+d\right)^{2}\right)\sin\left(xe+d\right)-\left(22\,bc\cos\left(xe+d\right)\sin\left(xe+d\right)+11\left(b^{2}-c^{2}\right)\cos\left(xe+d\right)^{2}-43\,b^{2}-32\,c^{2}\right)\sqrt{b^{2}+c^{2}}\right)\sqrt{b\cos\left(xe+d\right)+c\sin\left(xe+d\right)-\sqrt{b^{2}+c^{2}}}}{15\left(c\cos\left(xe+d\right)e-be\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="f
ricas")
```

```
[Out] 2/15*(3*(b^3 - 3*b*c^2)*cos(x*e + d)^3 + (29*b^3 + 38*b*c^2)*cos(x*e + d) +
 (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(x*e + d)^2)*sin(x*e + d) - (22*
b*c*cos(x*e + d)*sin(x*e + d) + 11*(b^2 - c^2)*cos(x*e + d)^2 - 43*b^2 - 32
*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) - sqrt(b^2 + c^
2))/(c*cos(x*e + d)*e - b*e*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \cos(d+e x) + c \sin(d+e x) - \sqrt{b^2+c^2}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

3.438 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}$

Optimal. Leaf size=130

$$\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d}}{3e}$$

```
[Out] 8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+d
)-(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*si
n(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3192, 3191}

$$\frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

```
[Out] (8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d +
e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rule 3192

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[a*((2*n - 1)/n), Int[(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e}
, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2} dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+}}{3e}$$

$$= \frac{8\sqrt{b^2+c^2}\,(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 27.52, size = 11512, normalized size = 88.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
[Out] Result too large to show
```

Maple [A]
time = 0.38, size = 130, normalized size = 1.00

method	result	size
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)(b^2+c^2)(\sin(ex+d-\arctan(-b,c))+1)(\sin(ex+d-\arctan(-b,c))-5)}{3\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\,e}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVERBO
SE)
[Out] 2/3*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))+1)*(sin(
e*x+d-arctan(-b,c))-5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c)
)+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.41, size = 138, normalized size = 1.06

$$\frac{2\left(2\,bc\cos\left(xe+d\right)\sin\left(xe+d\right)+\left(b^2-c^2\right)\cos\left(xe+d\right)^2-5\,b^2-4\,c^2-4\,\sqrt{b^2+c^2}\left(b\cos\left(xe+d\right)+c\sin\left(xe+d\right)\right)\right)\sqrt{b\cos\left(xe+d\right)+c\sin\left(xe+d\right)-\sqrt{b^2+c^2}}}{3\left(c\cos\left(xe+d\right)e-be\sin\left(xe+d\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="f
ricas")
```

```
[Out] 2/3*(2*b*c*cos(x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d)^2 - 5*b^2 -
 4*c^2 - 4*sqrt(b^2 + c^2)*(b*cos(x*e + d) + c*sin(x*e + d)))*sqrt(b*cos(x*
e + d) + c*sin(x*e + d) - sqrt(b^2 + c^2))/(c*cos(x*e + d)*e - b*e*sin(x*e
+ d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

3.439 $\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

Optimal. Leaf size=57

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

```
[Out] -2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {3191}

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] (-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d +
 e*x] + c*Sin[d + e*x]])
```

Rule 3191

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 17.45, size = 5053, normalized size = 88.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(53) = 106.
time = 0.36, size = 117, normalized size = 2.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))+1)}{\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\;e}$
risch	$\frac{\sqrt{2}\,\sqrt{-2\sqrt{b^2+c^2}+2b\cos(ex+d)+2c\sin(ex+d)}\,(ib+c)\left(i\sqrt{b^2+c^2}\,c+b^2 e^{i(ex+d)}+c^2 e^{i(ex+d)}+\sqrt{b^2 \right.}{\left(ic\,e^{2i(ex+d)}-b\,e^{2i(ex+d)}+2\sqrt{b^2+c^2}\,e^{i(ex+d)}-ic-b\right)(b^2+c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] 2*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1)/c
os(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-
b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.94, size = 88, normalized size = 1.54

$$\frac{2\left(b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}}}{c\cos(xe+d)\,e-be\sin(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="f
ricas")
```

```
[Out] 2*(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) +
 c*sin(x*e + d) - sqrt(b^2 + c^2))/(c*cos(x*e + d)*e - b*e*sin(x*e + d))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

3.440 $\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos(d+ex-\tan^{-1}(b,c))}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

```
[Out] -arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^(1/2
)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3194, 2728, 210}

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\,\cos(-\tan^{-1}(b,c)+d+ex)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*
Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]])])/((b
^2 + c^2)^(1/4)*e))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
```

```
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(\right.}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{-2\sqrt{b^2+c^2}-x^2}\,dx, x, -\frac{}{\sqrt{-\sqrt{b^2+c^2}}}\right.}{e}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos(}}\right.}{\sqrt[4]{b^2+c^2}\,e}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 55.27, size = 61904, normalized size = 680.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(78) = 156$.
time = 0.97, size = 175, normalized size = 1.92

method	result
default	$\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))+1)}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-\sqrt{b^2+c}}}{}\right.}{(b^2+c^2)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(}{\sqrt{b^2+c^2}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] (sin(e*x+d-arctan(-b,c))-1)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1))^
(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctan(1/2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arcta
n(-b,c))+1))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^2*s
in(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2)
)^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.61, size = 112, normalized size = 1.23

$$\frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos\left(xe+d\right)+c\sin\left(xe+d\right)-\sqrt{b^2+c^2}}}{2\left(b^2+c^2\right)^{\frac{1}{4}}\left(c\cos(xe+d)-b\sin(xe+d)\right)}\right)e^{(-1)}}{\left(b^2+c^2\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] sqrt(2)*arctan(-1/2*sqrt(2)*(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c
^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(
1/4)*(c*cos(x*e + d) - b*sin(x*e + d))))*e^(-1)/(b^2 + c^2)^(1/4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Ar
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\cos(d+e\,x)+c\sin(d+e\,x)-\sqrt{b^2+c^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

3.441 $\displaystyle \int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=164

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\,\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\,(b^2+c^2)^{3/4}\,e}+\frac{c\cos(d+ex)-b\sin}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d\right.}$$

```
[Out] 1/4*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^(
1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(1/
2)+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(
1/2))^(3/2)/(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3195, 3194, 2728, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\,\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}\,e\,(b^2+c^2)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt[
b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2
 + c^2)^(3/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(
-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(d+ex-\tan^{-1}(b,c}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos(d+e}\right.}{2\sqrt{2}\,(b^2+c^2)^{3/4}\,e}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

```
[Out] $Aborted
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(141) = 282$.
time = 0.44, size = 363, normalized size = 2.21

method	result
default	$\frac{\left(-\sin(ex+d-\arctan(-b,c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)(b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/4/(b^2+c^2)^(7/4)*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1
/4))*(b^2+c^2)+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))
^(1/2)*(b^2+c^2)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arcta
n(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan
(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/
2)/(b^2+c^2)^(1/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1))^(1/
2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arct
an(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(151) = 302$.
time = 2.80, size = 467, normalized size = 2.85

$$\frac{\left(3\sqrt{2}\,b^2c\cos(xe+d)-\sqrt{2}\,(3b^2c-c^3)\cos(xe+d)^3-\left(\sqrt{2}\,b^3-\sqrt{2}\,(b^3-3bc^2)\cos(xe+d)^2\right)\sin(xe+d)\right)(b^2+c^2)^{\frac{1}{4}}\arctan\left(-\frac{(b^2+c^2)^{\frac{1}{4}}\sqrt{b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}}\left(\left(\sqrt{2}\,b\cos(xe+d)+\sqrt{2}\,c\sin(xe+d)\right)\sqrt{b^2+c^2}+\sqrt{2}\,(b^2+c^2)\right)}{2\left((b^2c+c^3)\cos(xe+d)-(b^3+bc^2)\sin(xe+d)\right)}\right)-2\left(2(b^3+bc^2)\cos(xe+d)+2(b^2c+c^3)\sin(xe+d)+\left(2bc\cos(xe+d)\sin(xe+d)+(b^2-c^2)\cos(xe+d)^2+b^2+2c^2\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}}}{4\left((3b^4c+2b^2c^3-c^5)\cos(xe+d)^3e-3(b^4c+b^2c^3)\cos(xe+d)e-\left((b^5-2b^3c^2-3bc^4)\cos(xe+d)^2e-(b^5+b^3c^2)e\right)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"fricas")
```

```
[Out] 1/4*((3*sqrt(2)*b^2*c*cos(x*e + d) - sqrt(2)*(3*b^2*c - c^3)*cos(x*e + d)^3
 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(x*e + d)^2)*sin(x*e + d))*(b^
2 + c^2)^(1/4)*arctan(-1/2*(b^2 + c^2)^(1/4)*sqrt(b*cos(x*e + d) + c*sin(x*
e + d) - sqrt(b^2 + c^2))*((sqrt(2)*b*cos(x*e + d) + sqrt(2)*c*sin(x*e + d)
)*sqrt(b^2 + c^2) + sqrt(2)*(b^2 + c^2))/((b^2*c + c^3)*cos(x*e + d) - (b^3
 + b*c^2)*sin(x*e + d))) - 2*(2*(b^3 + b*c^2)*cos(x*e + d) + 2*(b^2*c + c^3
)*sin(x*e + d) + (2*b*c*cos(x*e + d)*sin(x*e + d) + (b^2 - c^2)*cos(x*e + d
)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) -
sqrt(b^2 + c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*cos(x*e + d)^3*e - 3*(b^4*c
+ b^2*c^3)*cos(x*e + d)*e - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*cos(x*e + d)^2*e -
 (b^5 + b^3*c^2)*e)*sin(x*e + d))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos{(d+ex)}+c\sin{(d+ex)}-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

3.442 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$-\frac{3\mathrm{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e}+\frac{c\cos(d+ex)-b}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos\right.}$$

```
[Out] -3/32*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)
^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2^(
1/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e*x+d)+c*sin(e*x+d
)-(b^2+c^2)^(1/2))^(3/2)+1/4*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*
sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2)/(b^2+c^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3195, 3194, 2728, 210}

$$-\frac{3\mathrm{ArcTan}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}\right)}{16\sqrt{2}\,e\left(b^2+c^2\right)^{5/4}}-\frac{3(c\cos(d+ex)-b\sin(d+ex))}{16e\left(b^2+c^2\right)\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] (-3*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-S
qrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(16*Sqrt[2
]*(b^2 + c^2)^(5/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^
2]*e*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(c*Co
s[d + e*x] - b*Sin[d + e*x]))/(16*(b^2 + c^2)*e*(-Sqrt[b^2 + c^2] + b*Cos[d
 + e*x] + c*Sin[d + e*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3194

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rule 3195

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)),
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}\,e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin\right.}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt[4]{b^2+c^2}\,\sin(d+ex-\tan^{-}}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,\cos(d}}\right.}{16\sqrt{2}\,(b^2+c^2)^{5/4}\,e}$$

Mathematica [F]

time = 180.03, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

```
[Out] $Aborted
```

Maple [**A**]
time = 0.44, size = 363, normalized size = 1.56

method	result
default	$-\frac{\left(-\sin(ex+d-\arctan(-b,c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/4*(-sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)+
2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*(b^2+c^2
)^(3/4)+2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c
^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctan(1/2*(-(b^2+c^2
)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1
/4))*c^2)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1))^(1/2)/(b^2+c^2)^(5
/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arc
tan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(216) = 432$.
time = 3.01, size = 690, normalized size = 2.97

$$\frac{\frac{3\sqrt{\frac{1}{2}}\left(5\,b^4c\cos(xe+d)+(5\,b^4c-10\,b^2c^3+c^5)\cos(xe+d)^5-10\,(b^4c-b^2c^3)\cos(xe+d)^3-\left(b^5+(b^5-10\,b^3c^2+5\,bc^4)\cos(xe+d)^4-2\,(b^5-5\,b^3c^2)\cos(xe+d)^2\right)\sin(xe+d)\right)\arctan\left(-\frac{\sqrt{\frac{1}{2}}\left(b\cos(xe+d)+c\sin(xe+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}}}{(b^2+c^2)^{\frac{1}{4}}(c\cos(xe+d)-b\sin(xe+d))}\right)}{(b^2+c^2)^{\frac{1}{4}}}+\left(3\,(b^4-6\,b^2c^2+c^4)\cos(xe+d)^4-7\,b^4-26\,b^2c^2-16\,c^4-6\,(2\,b^4-3\,b^2c^2-c^4)\cos(xe+d)^2+12\,((b^3c-bc^3)\cos(xe+d)^3-(2\,b^3c+bc^3)\cos(xe+d))\sin(xe+d)+2\,\left((b^3-3\,bc^2)\cos(xe+d)^3-3\,(3\,b^3+2\,bc^2)\cos(xe+d)-\left(9\,b^2c+8\,c^3-(3\,b^2c-c^3)\cos(xe+d)^2\right)\sin(xe+d)\right)\sqrt{b^2+c^2}\right)\sqrt{b\cos(xe+d)+c\sin(xe+d)-\sqrt{b^2+c^2}}}{16\,\left((5\,b^6c-5\,b^4c^3-9\,b^2c^5+c^7)\cos(xe+d)^5e-10\,(b^6c-b^2c^5)\cos(xe+d)^3e+5\,(b^6c+b^4c^3)\cos(xe+d)e-\left((b^7-9\,b^5c^2-5\,b^3c^4+5\,bc^6)\cos(xe+d)^4e-2\,(b^7-4\,b^5c^2-5\,b^3c^4)\cos(xe+d)^2e+(b^7+b^5c^2)e\right)\sin(xe+d)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"fricas")
```

```
[Out] 1/16*(3*sqrt(1/2)*(5*b^4*c*cos(x*e + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos(
x*e + d)^5 - 10*(b^4*c - b^2*c^3)*cos(x*e + d)^3 - (b^5 + (b^5 - 10*b^3*c^2
 + 5*b*c^4)*cos(x*e + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(x*e + d)^2)*sin(x*e +
d))*arctan(-sqrt(1/2)*(b*cos(x*e + d) + c*sin(x*e + d) + sqrt(b^2 + c^2))*s
qrt(b*cos(x*e + d) + c*sin(x*e + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(1/4)*(
c*cos(x*e + d) - b*sin(x*e + d))))/(b^2 + c^2)^(1/4) + (3*(b^4 - 6*b^2*c^2
+ c^4)*cos(x*e + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^4 - 6*(2*b^4 - 3*b^2*c^2
- c^4)*cos(x*e + d)^2 + 12*((b^3*c - b*c^3)*cos(x*e + d)^3 - (2*b^3*c + b*c
^3)*cos(x*e + d))*sin(x*e + d) + 2*((b^3 - 3*b*c^2)*cos(x*e + d)^3 - 3*(3*b
^3 + 2*b*c^2)*cos(x*e + d) - (9*b^2*c + 8*c^3 - (3*b^2*c - c^3)*cos(x*e + d
)^2)*sin(x*e + d))*sqrt(b^2 + c^2))*sqrt(b*cos(x*e + d) + c*sin(x*e + d) -
sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^5 + c^7)*cos(x*e + d)^5*e
 - 10*(b^6*c - b^2*c^5)*cos(x*e + d)^3*e + 5*(b^6*c + b^4*c^3)*cos(x*e + d)
*e - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*cos(x*e + d)^4*e - 2*(b^7 - 4
*b^5*c^2 - 5*b^3*c^4)*cos(x*e + d)^2*e + (b^7 + b^5*c^2)*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\cos\left(d+e\,x\right)+c\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
[Out] int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

3.443 $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=101

$$\frac{cx}{b^2+c^2}-\frac{2ac\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] c*x/(b^2+c^2)-b*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*c*arctan((c+(a-b)*tan
(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3216, 3203, 632, 210}

$$-\frac{2ac\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x)/(b^2 + c^2) - (2*a*c*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c
^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[a + b*Cos[x] + c*Sin[x]]
)/(b^2 + c^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(ac)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(2ac)\text{Subst}\left(\int \frac{1}{a+b+2cx+(a-b)x^2}\,dx,x\right.}{b^2+c^2}\\ &= \frac{cx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{(4ac)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2-c^2)-x^2}\,dx,x\right.}{b^2+c^2}\\ &= \frac{cx}{b^2+c^2}-\frac{2ac\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 80, normalized size = 0.79

$$\frac{cx+\frac{2ac\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}-b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*x + (2*a*c*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[
-a^2 + b^2 + c^2] - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [A]
time = 0.34, size = 176, normalized size = 1.74

method	result

default	$\frac{\frac{4\left(-ab+b^2\right)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{4\left(-ac-cb-\frac{\left(-ab+b^2\right)c}{a-b}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{2b^2+2c^2}+\frac{2b\ln(1+\tan^2}{}$
risch	$\frac{ix}{ic-b}+\frac{2ix\,a^2b}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}-\frac{2ix\,b^3}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}-\frac{2ixb\,c^2}{a^2b^2+a^2c^2-b^4-2b^2c^2-c^4}-\frac{\ln\left(e^{ix}-\frac{-a^2bc-ic^2a^2-i\sqrt{a^4}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*b^2+2*c^2)*(1/2*(-a*b+b^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*
tan(1/2*x)+a+b)+(-a*c-c*b-(-a*b+b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/
2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))+4/(2*b^2+2*c^2)*(1/2*b*ln(
1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(95) = 190.
time = 2.78, size = 579, normalized size = 5.73

$$\left[-\frac{\sqrt{-a^2+b^2+c^2}\,ac\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^3)c)\cos(x))\sin(x)-2\left(2abc\cos(x)^2-abc+(b^2c+c^3)\cos(x)-(b^3+bc^2+(ab^2-ac^2)\cos(x))\sin(x)\right)\sqrt{-a^2+b^2+c^2}}{2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)}\right)+2(c^3-(a^2-b^2)c)x+(a^2b-b^3-bc^2)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)},-\frac{2\sqrt{a^2-b^2-c^2}\,ac\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)+2(c^3-(a^2-b^2)c)x+(a^2b-b^3-bc^2)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2 + c^2)*a*c*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2
)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*
cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) -
2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2
)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(c^3 - (a^2 - b^
2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
```

```
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^
2)*c^2), -1/2*(2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)
*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)
*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 160, normalized size = 1.58

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)ac}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{cx}{b^2+c^2}-\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{b\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*a*c/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2)
) + c*x/(b^2 + c^2) - b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2)
```

Mupad [B]
time = 11.44, size = 950, normalized size = 9.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] log(tan(x/2) + 1i)/(b - c*1i) + (log(tan(x/2) - 1i)*1i)/(b*1i - c) + (log(6
4*tan(x/2)*(a - b)^2 - ((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))
*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2) +
((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*b*c^3 - 32*a*c^3 -
 64*b^3*c + 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2 - 2*b^3) + 128*a
*b^2*c - 64*a^2*b*c + (32*(a - b)*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c
^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2)
```

```
 - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4
*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c
^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*(a^2 - c^2) -
b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) - (log(
64*tan(x/2)*(a - b)^2 + ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)
)*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2) +
 ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*a*c^3 - 32*b*c^3
+ 64*b^3*c - 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2 - 2*b^3) - 128*
a*b^2*c + 64*a^2*b*c + (32*(a - b)*(b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 +
c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2
) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c -
4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 +
c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b^3 - b*(a^2 -
c^2) + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

3.444 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)} dx$

Optimal. Leaf size=22

$$\frac{x}{2} - \log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)$$

```
[Out] 1/2*x-ln(cos(1/2*x)+sin(1/2*x))
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3216, 3203, 31}

$$\frac{x}{2} - \frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right) + 1\right) - \frac{1}{2}\log(\sin(x) + \cos(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$\frac{x}{2}-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.10, size = 27, normalized size = 1.23

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\ln\left(e^{ix}+i\right)$	20
default	$-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	27
norman	$\frac{\frac{x}{2}+\frac{x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}}{1+\tan^2\left(\frac{x}{2}\right)}-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] -ln(tan(1/2*x)+1)+1/2*ln(1+tan(1/2*x)^2)+arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.
time = 0.48, size = 41, normalized size = 1.86

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.20, size = 11, normalized size = 0.50

$$\frac{1}{2}x - \frac{1}{2}\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A]
time = 0.11, size = 22, normalized size = 1.00

$$\frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right) + 1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

Giac [A]
time = 0.42, size = 25, normalized size = 1.14

$$\frac{1}{2}x + \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 2.79, size = 34, normalized size = 1.55

$$-\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x) + sin(x) + 1),x)
[Out] log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(1
/2 + 1i/2)
```

3.445 $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=97

$$\frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)+b*ln(c+a*cos(x)+b*sin(x))/(a^2+b^2)+2*a*c*arctanh((b-(a-c)*ta
n(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3238, 3217, 3203, 632, 212}

$$\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x] + b*Sin[x]
])/(a^2 + b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{\cos(x)}{c+a\cos(x)+b\sin(x)}\,dx \\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(ac)\int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} \\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}-\frac{(2ac)\text{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx,\right.}{a^2+b^2} \\
&= \frac{ax}{a^2+b^2}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}+\frac{(4ac)\text{Subst}\left(\int \frac{1}{4(a^2+b^2-c^2)-x^2}\,dx,x,\right.}{a^2+b^2} \\
&= \frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 79, normalized size = 0.81

$$\frac{ax+\frac{2ac\tanh^{-1}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}+b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] + b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Maple [A]
time = 0.28, size = 171, normalized size = 1.76

method	result
default	$\frac{\frac{2(ab-cb)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a-c\right)}{2a-2c}+\frac{2\left(ac-b^2+\frac{(ab-cb)b}{a-c}\right)\arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}}{a^2+b^2}+\frac{-b\ln\left(1+\tan^2\left(\frac{x}{2}\right.\right.}{}$
risch	$-\frac{x}{ib-a}-\frac{2ix\,a^2b}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}-\frac{2ix\,b^3}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}+\frac{2ix\,c^2b}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}+\frac{\ln\left(e^{ix}-\frac{-iab\,c^2-a^2c^2+i\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)*(1/2*(a*b-b*c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1
/2*x)-a-c)+(a*c-b^2+(a*b-b*c)*b/(a-c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(
a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2)))+2/(a^2+b^2)*(-1/2*b*ln(1+tan(1/
2*x)^2)+a*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(93) = 186$.
time = 3.01, size = 553, normalized size = 5.70

$$\left[\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^2b+b^3)c-(a^3b+ab^3-2abc^2)\cos(x))\sin(x)+2\left(2abc\cos(x)^2-abc+(a^2b+b^3)\cos(x)-(a^3+ab^2+(a^2-b^2)c\cos(x))\sin(x)\right)\sqrt{a^2+b^2-c^2}}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)}\right)+2(a^3+ab^2-ac^2)x+(a^2b+b^3-bc^2)\log\left(2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)\right)}{2(a^4+2a^2b^2+b^4-(a^2+b^2)c^2)},-\frac{2\sqrt{-a^2-b^2+c^2}\,ac\arctan\left(\frac{(ac\cos(x)+bc\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^2b+b^3-bc^2)\cos(x)-(a^3+ab^2-ac^2)\sin(x)}\right)-2(a^3+ab^2-ac^2)x-(a^2b+b^3-bc^2)\log\left(2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)\right)}{2(a^4+2a^2b^2+b^4-(a^2+b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*
c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*c
```

```
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
+ b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x + (
a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 +
2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) + b*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2)*log(2*a*c
*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\tan(x)+c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(1/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A]
time = 0.45, size = 158, normalized size = 1.63

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2c)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a+c\right)}{a^2+b^2}-\frac{b\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*ta
n(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1
/2*x) + a + c)/(a^2 + b^2) - b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

Mupad [B]
time = 13.03, size = 988, normalized size = 10.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*tan(x) + c/cos(x)),x)
```

```
[Out] (log(32*a*c - 32*c^2 + 32*b*tan(x/2)*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*t
an(x/2)*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 + a*c
*(a^2 + b^2 - c^2)^(1/2))*(32*a^4 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 - 32
```

```
*b^2*c^2 + 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(4*a^2 - 4*a*c + b^2) + (32*(
a - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2)
+ 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 2*b
^2*c^2*tan(x/2) - 2*a^3*c*tan(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c*tan(x/
2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(a^
2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((a^2 + b^2)*(a^2 + b^2 -
 c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^
2 - c^2) + (a^2 + b^2 - c^2)^2) - log(tan(x/2) + 1i)/(a*1i + b) - (log(tan(
x/2) - 1i)*1i)/(a + b*1i) + (log(32*a*c - 32*c^2 + 32*b*tan(x/2)*(a - c) +
((32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) -
 ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(32*a^4 - 64*a^3*c -
64*a^2*b^2 + 32*a^2*c^2 - 32*b^2*c^2 + 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(
4*a^2 - 4*a*c + b^2) + (32*(a - c)*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 -
c^2)^(1/2))*(3*b^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*tan(x/2
) + 2*a^2*c^2*tan(x/2) - 2*b^2*c^2*tan(x/2) - 2*a^3*c*tan(x/2) - 4*a*b*c^2
+ a^2*b*c - 2*a*b^2*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 +
b^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)
))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 -
 c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2)
```

3.446 $\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] -2*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3244, 3203, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3244

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
```

```
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx &= \int \frac{1}{c+a\cos(x)+b\sin(x)}\,dx\\
&= 2\text{Subst}\left(\int \frac{1}{a+c+2bx+(-a+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\left(4\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx, x, 2b+2(-a+c)\tan\left(\frac{x}{2}\right)\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.98

$$
-\frac{2\tanh^{-1}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
[Out] (-2*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
- c^2]
```

Maple [A]
time = 0.19, size = 53, normalized size = 1.04

method	result
default	$-\frac{2\arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$
risch	$\frac{\ln\left(e^{ix}+\frac{icb\sqrt{a^2+b^2-c^2}+ia^3+ia\,b^2-ia\,c^2+ac\sqrt{a^2+b^2-c^2}-a^2b-b^3+c^2b}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} - \frac{\ln\left(e^{ix}+\frac{icb\sqrt{a^2+b^2-c^2}-ia^3-i}{\left(a^2+b\right.}\right.}{\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^
(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(47) = 94$.
time = 3.03, size = 349, normalized size = 6.84

$$\left[\frac{\log\left(-\frac{2\,a^4+3\,a^2b^2+b^4-(a^2-b^2)c^2+2\,(a^3+ab^2)c\cos(x)-(a^4-b^4-2\,(a^2-b^2)c^2)\cos(x)^2+2\,((a^2b+b^3)c-(a^3b+ab^3-2\,abc^2)\cos(x))\sin(x)-2\,(2\,abc\cos(x)^2-abc+(a^2b+b^3)\cos(x)-(a^3+ab^2+(a^2-b^2)c\cos(x))\sin(x))\sqrt{a^2+b^2-c^2}}{2\,ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2\,(ab\cos(x)+bc)\sin(x)}\right)}{2\,\sqrt{a^2+b^2-c^2}},\ \frac{\sqrt{-a^2-b^2+c^2}\arctan\left(\frac{(ac\cos(x)+bc\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^2b+b^3-bc^2)\cos(x)-(a^3+ab^2-ac^2)\sin(x)}\right)}{a^2+b^2-c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*co
s(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^2
*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x
) + b*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((a*c
*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*
c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{a + b\tan(x) + c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A]
time = 0.50, size = 73, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2c)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2*c) + arctan((a*tan(1/2*x) - c*tan(
1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

Mupad [B]
time = 2.78, size = 47, normalized size = 0.92

$$-\frac{2\operatorname{atanh}\left(\frac{b-\frac{\tan\left(\frac{x}{2}\right)(2a-2c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*tan(x) + c/cos(x))),x)
```

```
[Out] -(2*atanh((b - (tan(x/2)*(2*a - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 + b
^2 - c^2)^(1/2)
```

3.447 $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

Optimal. Leaf size=142

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c}+\frac{b\log\left(a+c+2b\tan\left(\frac{x}{2}\right)-(a-c)\,t\right.}{b^2-c^2}$$

```
[Out] -ln(1-tan(1/2*x))/(b+c)-ln(1+tan(1/2*x))/(b-c)+b*ln(a+c+2*b*tan(1/2*x)-(a-c
)*tan(1/2*x)^2)/(b^2-c^2)-2*a*c*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^
(1/2))/(b^2-c^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {4482, 1089, 648, 632, 212, 642, 647, 31}

$$-\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)}{b^2-c^2}-\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(b - (a - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) - Log[1 - Tan[x/2]]/(b + c) - Log[1 + Tan[x/2]]/(b
- c) + (b*Log[a + c + 2*b*Tan[x/2] - (a - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1089

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*((-b)*
C*d + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*((-b)*C*d + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = \int \frac{\sec(x)}{c+a\cos(x)+b\sin(x)}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)\,(a+c+2bx-(a-c)x^2)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{4c-4bx}{1-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{2\,(b^2-c^2)} - \frac{\text{Subst}\left(\int \frac{-4b^2+(-a+c)^2-(a+c)^2-4b(-a+c)x}{a+c+2bx+(-a+c)x^2}\,dx,\right.}{2\,(b^2-c^2)}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c} + \frac{b\text{Subst}\left(\int \frac{}{a+c}\right.}{}$$
$$= -\frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a+c+2b\tan\left(\frac{x}{2}\right)-(a-c\right.}{b^2-c^2}$$
$$= -\frac{2ac\tanh^{-1}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} - \frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} +$$

Mathematica [A]

time = 0.21, size = 120, normalized size = 0.85

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} + (b-c)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + (b+c)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) - b\log(c+a\cos(x)+b\sin(x))}{(-b+c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] + (b - c)*Log[Cos[x/2] - Sin[x/2]] + (b + c)*Log[Cos[x/2] + Sin[
x/2]] - b*Log[c + a*Cos[x] + b*Sin[x]])/((-b + c)*(b + c))
```

Maple [A]

time = 0.40, size = 180, normalized size = 1.27

method	result
default	$-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2b+2c} - \frac{2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{-2c+2b} + \frac{\frac{2(ab-cb)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-c\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a-c\right)}{2a-2c} + \frac{2\left(-ac-b^2+\frac{(ab-cb)b}{a-c}\right)\arctan\left(\frac{}{2\sqrt{}}\right.}{\sqrt{-a^2-b^2+}}}{(b-c)(b+c)}$
risch	$-\frac{2ix\,a^2b}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4} - \frac{2ix\,b^3}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4} + \frac{2ix\,c^2b}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4} + \frac{ix}{b-c} + \frac{ix}{b+c} + \frac{\ln\left(e^{ix}+\frac{iab\,c^2+a^2c^2+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(2*b+2*c)*ln(tan(1/2*x)-1)-2/(-2*c+2*b)*ln(tan(1/2*x)+1)+2/(b-c)/(b+c)*(
1/2*(a*b-b*c)/(a-c)*ln(a*tan(1/2*x)^2-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)+(-
a*c-b^2+(a*b-b*c)*b/(a-c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2
*x)-2*b)/(-a^2-b^2+c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 6.06, size = 663, normalized size = 4.67

$$\left[-\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2\left((a^2b+b^3)c-(a^3b+ab^3-2abc^2)\cos(x)\right)\sin(x)+2\left(2abc\cos(x)^2-abc+(a^2b+b^3)\cos(x)-(a^3+ab^2+(a^2-b^2)c\cos(x))\sin(x)\right)\sqrt{a^2+b^2-c^2}}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)}\right)-(a^2b+b^3-bc^2)\log\left(2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)\right)+(a^2b+b^3-bc^2-c^3+(a^2+b^2)c)\log(\sin(x)+1)+(a^2b+b^3-bc^2+c^3-(a^2+b^2)c)\log(-\sin(x)+1)}{2(a^2b^2+b^4+c^4-(a^2+2b^2)c^2)},\ \frac{2\sqrt{-a^2-b^2+c^2}\,ac\arctan\left(\frac{(ac\cos(x)+bc\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^2b+b^3-bc^2)\cos(x)-(a^3+ab^2-ac^2)\sin(x)}\right)+(a^2b+b^3-bc^2)\log\left(2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+bc)\sin(x)\right)-(a^2b+b^3-bc^2-c^3+(a^2+b^2)c)\log(\sin(x)+1)-(a^2b+b^3-bc^2+c^3-(a^2+b^2)c)\log(-\sin(x)+1)}{2(a^2b^2+b^4+c^4-(a^2+2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)
*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2
 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) - (a^2*b + b^3 - b*c^2)*log(2*
a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)
) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) + (a^2*b +
b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 -
 (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) +
 b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(
x) - (a^3 + a*b^2 - a*c^2)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x
) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x)) - (a^2*
b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) - (a^2*b + b^3 - b*c
```

```
^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 + c^4 - (a^2 + 2
*b^2)*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{a + b\tan(x) + c\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+c*sec(x)+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x) + c*sec(x)), x)
```

Giac [A]
time = 0.43, size = 161, normalized size = 1.13

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2a+2c)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\,(b^2-c^2)}+\frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2b\tan\left(\frac{1}{2}x\right)+a+c\right)}{b^2-c^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{b-c}-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{b+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*tan
(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(b^2
 - c^2) - log(abs(tan(1/2*x) + 1))/(b - c) - log(abs(tan(1/2*x) - 1))/(b +
c)
```

Mupad [B]
time = 11.44, size = 977, normalized size = 6.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*tan(x) + c/cos(x))),x)
```

```
[Out] (log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2 + b^3 + a*c*
(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a^2
- 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2
))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^2 - 96*a*b^2*c +
 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)*(a^2*b - b*c^2 +
 b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b^3 - 3*b*c^3 + 3*b
^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c^2*tan(x/2) - 2*a*c
^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2)))/((b^2 - c^2)*(a^2
+ b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 -
```

```
c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(
a^2 + b^2 - c^2)) - log(tan(x/2) - 1)/(b + c) - log(tan(x/2) + 1)/(b - c) +
 (log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2 + b^3 - a*c
*(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a^2
 - 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/
2))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^2 - 96*a*b^2*c
+ 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)*(a^2*b - b*c^2
+ b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b^3 - 3*b*c^3 + 3*
b^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c^2*tan(x/2) - 2*a*
c^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2)))/((b^2 - c^2)*(a^2
 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2
- c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*
(a^2 + b^2 - c^2))
```

3.448 $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}+\frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\ldots}{b+\ldots}\right.}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+\ldots}$$

```
[Out] -2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(
e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*(cos(1/2*d+1/2*e*x-1/2*arc
tan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*d+1
/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*
sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^
2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+
(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(e*x+d)^(3
/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

Rubi [A]

time = 0.30, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3246, 3199, 3228, 3198, 2732, 3206, 2740}

$$\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2}+\frac{8b(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])
^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/
2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x
- ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*C
os[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] +
c*Tan[d + e*x])^(3/2))/(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3246

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
), Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx = \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c}{\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex)}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.32, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

```
[Out] (((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*a*Sin[d + e*x])/3)*(a + b*Sec[d
 + e*x] + c*Tan[d + e*x])^(3/2))/(e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x]
```

```
+ c*Sin[d + e*x])) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^
2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^
2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqr
t[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c
]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*
x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*b^2*AppellF1[1/2
, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sq
rt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]
*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2
]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqr
t[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2]
)]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-
b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(Sqrt[1 + a^2/c^2]*c*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^(3/2)) + (2*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*
Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(
c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]
])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d +
 e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*e*Sec[d + e*x]^(3/2)*(b
 + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*a^2*b*(-((c*AppellF1[-1/2,
-1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*
Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2
/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a
^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a
*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
- ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*c*e*Sec[d
+ e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (4*b*c*(-((c*Ap
pellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcT
an[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sq
rt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a
*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sq
```

```
rt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[
c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[
d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/
a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]
*Cos[d + e*x - ArcTan[c/a]]])*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/
(3*e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 8.42, size = 70673, normalized size = 190.49

method	result	size
default	Expression too large to display	70673

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(x*e + d) + c*tan(x*e + d) + a)^(3/2)/sec(x*e + d)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.08, size = 1518, normalized size = 4.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/9*((-3*I*a^3 - I*a*b^2 - 3*I*a*c^2 + 3*c^3 + (3*a^2 + b^2)*c)*sqrt(2*a -
2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3
- 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b -
 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*
```

```
b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^
4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I
*c^2)*sin(x*e + d))/(a^2 + c^2)) + (3*I*a^3 + I*a*b^2 + 3*I*a*c^2 + 3*c^3 +
 (3*a^2 + b^2)*c)*sqrt(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2
*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^
2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a
^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)
*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 12*(-I*a^2*b
 - I*b*c^2)*sqrt(2*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b
^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^
4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b
^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3
*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 +
4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
 c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b +
4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6
+ 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*
e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 12*(I*a^2*b + I*b
*c^2)*sqrt(2*a + 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2
 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/
27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^
3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c
^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*
c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)
*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^
4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d)
 - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 6*((a^2*c + c^3)*cos(x*e
 + d)^2 - (a^3 + a*c^2)*cos(x*e + d)*sin(x*e + d))*sqrt((a*cos(x*e + d) + c
*sin(x*e + d) + b)/cos(x*e + d))/sqrt(cos(x*e + d)))*e^(-1)/(a^2 + c^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec(d + ex) + c\tan(d + ex))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2)/sec(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2)/sec(d + e*x)**(3/2),
x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2),x)
```

```
[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2), x)
```

3.449 $\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{e\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/e/sec(e*x
+d)^(1/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3246, 3198, 2732}

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

```
[Out] (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
```

```
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3246

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
), Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{b+a\cos(d+ex)+c}}{\sqrt{\sec(d+ex)}\sqrt{b+a\cos(d+ex)+c\sin(d+ex}}$$

$$= \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a}}{}}}{\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c}{b+\sqrt{a^2+c}}}}$$

$$= \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex}}{e\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+e}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.19, size = 1580, normalized size = 13.39

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

```
[Out] (2*a*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]) +
(2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/
(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 +
```

```
c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt
[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a
/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[a + b*Sec[d + e*x] + c*
Tan[d + e*x]])/(Sqrt[1 + a^2/c^2]*c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Cos[d +
 e*x] + c*Sin[d + e*x]]) + (a^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b
+ a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
 b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTa
n[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x
 - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2]
)]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqr
t[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-
b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*
x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
 c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sqrt[
a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*Co
s[d + e*x] + c*Sin[d + e*x]]) + (c*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - Ar
cTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d +
e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*
Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a
^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*
Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])
/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[
1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]])*Sq
rt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]*Sqrt[b + a*C
os[d + e*x] + c*Sin[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 2.33, size = 45041, normalized size = 381.70

method	result	size
risch	Expression too large to display	2660
default	Expression too large to display	45041

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(x*e + d) + c*tan(x*e + d) + a)/sqrt(sec(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.51, size = 1374, normalized size = 11.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*((-I*a*b + b*c)*sqrt(2*a - 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9
*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)
*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^
2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (I*a*b +
b*c)*sqrt(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*
c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)
*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^
4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d)
 - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 3*(-I*a^2 - I*c^2)*sqrt(2
*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3
 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b
- 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3
*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c
^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*
c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5
*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*
a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^
2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2
 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 3*(I*a^2 + I*c^2)*sqrt(2*a + 2*I*c)
*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 -
2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3
 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^
3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6),
```

```
weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4
 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*
b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a
*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6
), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*si
n(x*e + d))/(a^2 + c^2))))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sec(d + ex) + c\tan(d + ex)}}{\sqrt{\sec(d + ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2)/sec(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))/sqrt(sec(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + c\tan(d + e\,x) + \frac{b}{\cos(d + e\,x)}}}{\sqrt{\frac{1}{\cos(d + e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2),x)
```

```
[Out] int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2), x)
```

3.450 $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(1/2)*((b+a*cos(e*x+d)+c*sin(e*x+d
))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3246, 3206, 2740}

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3246

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
), Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\left(\sqrt{\sec(d+ex)}\,\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\right)\int \frac{}{\sqrt{b-}}}{\sqrt{a+b\sec(d+ex)+c\tan(d+}}$$

$$= \frac{\left(\sqrt{\sec(d+ex)}\,\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{}{\sqrt{\frac{}{b}}}}{\sqrt{a+b\sec(d+ex)}}$$

$$= \frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\,\sqrt{}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 0.72, size = 339, normalized size = 2.87

$$\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{b+\sqrt{1+\frac{a^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)}{b-\sqrt{1+\frac{a^2}{c^2}}\,c},\frac{b+\sqrt{1+\frac{a^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)}{b+\sqrt{1+\frac{a^2}{c^2}}\,c}\right)\sqrt{\sec(d+ex)}\,\sec\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\,\sqrt{-\frac{\sqrt{1+\frac{a^2}{c^2}}\,c\left(-1+\sin\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)\right)}{b+\sqrt{1+\frac{a^2}{c^2}}\,c}}\,\sqrt{\frac{\sqrt{1+\frac{a^2}{c^2}}\,c\left(1+\sin\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)\right)}{-b+\sqrt{1+\frac{a^2}{c^2}}\,c}}\,\sqrt{b+\sqrt{1+\frac{a^2}{c^2}}\,c\sin\left(d+ex+\text{ArcTan}\left(\frac{a}{c}\right)\right)}}{\sqrt{1+\frac{a^2}{c^2}}\,ce\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcT
an[a/c]])/(b - Sqrt[1 + a^2/c^2]*c), (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
 ArcTan[a/c]])/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[Sec[d + e*x]]*Sec[d + e*x +
ArcTan[a/c]]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]*Sqrt[-((Sqrt[1 + a^2
/c^2]*c*(-1 + Sin[d + e*x + ArcTan[a/c]]))/(b + Sqrt[1 + a^2/c^2]*c))]*Sqrt
[(Sqrt[1 + a^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[a/c]]))/(-b + Sqrt[1 + a^2/
c^2]*c)]*Sqrt[b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]]])/(Sqrt[1
+ a^2/c^2]*c*e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.30, size = 1133, normalized size = 9.60

method	result
default	$-\frac{4\,\mathrm{EllipticF}\left(\sqrt{-\frac{\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}+c\cos(ex+d)-a\sin(ex+d)+b\sin(ex+d)+\sqrt{a^2-b^2+c^2}+c\right)\left(ia-ib-\sqrt{}\right.}{\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-c\cos(ex+d)+a\sin(ex+d)-b\sin(ex+d)+\sqrt{a^2-b^2+c^2}-c\right)\left(ia-ib+\sqrt{}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/e*EllipticF((-(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)+c*cos(e*x+d)-a*sin(e*x+d)+
b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)*(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(cos(e*
x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x+d)+(a^2-b^2+c^
2)^(1/2)-c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2),((I*a-I*b+(a^2-b^2+c^2)^
(1/2)-c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I
*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*(1/cos(e*x+d))^(1/2)*((b+a*cos(e*x+d)
+c*sin(e*x+d))/cos(e*x+d))^(1/2)*((I*cos(e*x+d)+I-sin(e*x+d))/(cos(e*x+d)*(
a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x+d)+(a^2-b^2+c^2)^(1/
2)-c)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2)*((I*
cos(e*x+d)+sin(e*x+d)+I)/(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin
(e*x+d)-b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-
I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*(-(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)+c*cos(e
*x+d)-a*sin(e*x+d)+b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)*(I*a-I*b-(a^2-b^2+c^
2)^(1/2)+c)/(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin
(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*(cos(
e*x+d)+1)^2*cos(e*x+d)*(cos(e*x+d)-1)^2*(-I*a^2-I*c^2-I*a*(a^2-b^2+c^2)^(1/
2)*sin(e*x+d)+I*cos(e*x+d)*(a^2-b^2+c^2)^(1/2)*c-I*b*c*sin(e*x+d)-I*cos(e*x
+d)*c^2-I*cos(e*x+d)*a*b+I*a*b+I*c*(a^2-b^2+c^2)^(1/2)+I*cos(e*x+d)*b^2+I*b
*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*a*c*sin(e*x+d)-cos(e*x+d)*(a^2-b^2+c^2)^(
1/2)*b+a*c*cos(e*x+d)-a^2*sin(e*x+d)+sin(e*x+d)*b^2-a*(a^2-b^2+c^2)^(1/2)+c
*b)/sin(e*x+d)^4/(b+a*cos(e*x+d)+c*sin(e*x+d))/(I*a-I*b-(a^2-b^2+c^2)^(1/2)
+c)/(a^2-b^2+c^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(sec(x*e + d))/sqrt(b*sec(x*e + d) + c*tan(x*e + d) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.12, size = 507, normalized size = 4.30

$$\frac{\left(\sqrt{2\,a-2i\,c}\,(-i\,a+c)\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^4-4\,a^2b^2+4\,b^2c^2+6i\,ac^3-3\,c^4+2i\,(3\,a^3-4\,ab^2)c)}{3\,(a^4+2\,a^2c^2+c^4)},\frac{8\,(9\,a^5b-8\,a^3b^3-27\,abc^4-9i\,bc^5+2i\,(9\,a^2b+4\,b^3)c^3-6\,(3\,a^3b-4\,ab^3)c^2+3i\,(9\,a^4b-8\,a^2b^3)c)}{27\,(a^6+3\,a^4c^2+3\,a^2c^4+c^6)},\frac{2\,ab+2i\,bc+3\,(a^2+c^2)\cos(xe+d)-3\,(-i\,a^2-i\,c^2)\sin(xe+d)}{3\,(a^2+c^2)}\right)+\sqrt{2\,a+2i\,c}\,(i\,a+c)\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^4-4\,a^2b^2+4\,b^2c^2-6i\,ac^3-3\,c^4-2i\,(3\,a^3-4\,ab^2)c)}{3\,(a^4+2\,a^2c^2+c^4)},\frac{8\,(9\,a^5b-8\,a^3b^3-27\,abc^4+9i\,bc^5-2i\,(9\,a^2b+4\,b^3)c^3-6\,(3\,a^3b-4\,ab^3)c^2-3i\,(9\,a^4b-8\,a^2b^3)c)}{27\,(a^6+3\,a^4c^2+3\,a^2c^4+c^6)},\frac{2\,ab-2i\,bc+3\,(a^2+c^2)\cos(xe+d)-3\,(i\,a^2+i\,c^2)\sin(xe+d)}{3\,(a^2+c^2)}\right)\right)e^{(-1)}}{a^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] (sqrt(2*a - 2*I*c)*(-I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 +
 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
+ c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b +
 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6
 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x
*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2*a + 2*I*c)
*(I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*
c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5
*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*
a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^
2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2
+ I*c^2)*sin(x*e + d))/(a^2 + c^2)))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(d + e*x))/sqrt(a + b*sec(d + e*x) + c*tan(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(d+e\,x)}}}{\sqrt{a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
[Out] int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

3.451 $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2E\Big(\frac{1}{2}(d+ex-\tan^{-1}(}{(a^2-b^2+c^2)\,e\sqrt{\frac{b+}{}}}$$

```
[Out] -2*sec(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d
))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*sec(e*x+d)^(3/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)/e/
((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c
*tan(e*x+d))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$,
Rules used = {3246, 3207, 3198, 2732}

$$-\frac{2\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^2 E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))}{e(a^2-b^2+c^2)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (-2*Sec[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e
*x])^2)/((a^2 - b^2 + c^2)*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3246

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
), Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{\left(\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\right)\int \frac{}{(b+a\cos}}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 6.30, size = 1732, normalized size = 7.22

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1
- b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d +
e*x]^(3/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
^(3/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/
c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^
2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])/
(Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*
x])^(3/2)) - (a^2*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^
(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[
d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))
), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a
^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1
 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 +
c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqr
t[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2
])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 +
c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqr
t[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(c*(a^2 - b^2 + c^2)*e*(a + b*
Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (c*Sec[d + e*x]^(3/2)*(b + a*Cos[d
+ e*x] + c*Sin[d + e*x])^(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
 a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 -
b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan
[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x
- ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt
[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])
]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt
[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b
 + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x
 - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/((a^2
 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.99, size = 44834, normalized size = 186.81

method	result	size
default	Expression too large to display	44834

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(x*e + d)^(3/2)/(b*sec(x*e + d) + c*tan(x*e + d) + a)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.59, size = 1752, normalized size = 7.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*((I*a*b^2 - b^2*c + (I*a^2*b - a*b*c)*cos(x*e + d) + (I*a*b*c - b*c^2)*
sin(x*e + d))*sqrt(2*a - 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2
 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^
2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b
 + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a
^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos
(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (-I*a*b^2 - b^2
*c + (-I*a^2*b - a*b*c)*cos(x*e + d) + (-I*a*b*c - b*c^2)*sin(x*e + d))*sqr
t(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*
I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9
*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6
*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 +
```

```
3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*
a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 3*(I*a^2*b + I*b*c^2 + (I*a^3 + I
*a*c^2)*cos(x*e + d) + (I*a^2*c + I*c^3)*sin(x*e + d))*sqrt(2*a - 2*I*c)*we
ierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I
*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 -
27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*
c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), wei
erstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 +
2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3
 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^
3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6),
1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(
x*e + d))/(a^2 + c^2))) - 3*(-I*a^2*b - I*b*c^2 + (-I*a^3 - I*a*c^2)*cos(x*
e + d) + (-I*a^2*c - I*c^3)*sin(x*e + d))*sqrt(2*a + 2*I*c)*weierstrassZeta
(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a
*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 +
9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*
a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInve
rse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 -
4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4
 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*
(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b -
2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2
 + c^2))) - 6*((a^2*c + c^3)*cos(x*e + d)^2 - (a^3 + a*c^2)*cos(x*e + d)*si
n(x*e + d))*sqrt((a*cos(x*e + d) + c*sin(x*e + d) + b)/cos(x*e + d))/sqrt(c
os(x*e + d)))/((a^5 - a^3*b^2 + a*c^4 + (2*a^3 - a*b^2)*c^2)*cos(x*e + d)*e
 + (c^5 + (2*a^2 - b^2)*c^3 + (a^4 - a^2*b^2)*c)*e*sin(x*e + d) + (a^4*b -
a^2*b^3 + b*c^4 + (2*a^2*b - b^3)*c^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Integral(sec(d + e*x)**(3/2)/(a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2),
x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm ="giac")

[Out] Timed out

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}{\left(a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)

[Out] int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)

3.452 $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$-\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{3\left(a^2-b^2+c^2\right)e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8\sec^{\frac{5}{2}}(d+ex)(bc\cos(d}{3\left(a^2-b^2\right.}$$

```
[Out] -2/3*sec(e*x+d)^(5/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x
+d))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*sec(e*x+d)^(5/
2)*(b*c*cos(e*x+d)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2
+c^2)^2/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*sec(e*x+d)^(5/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/((b
+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c*ta
n(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(5/2)*(b+a*cos(e
*x+d)+c*sin(e*x+d))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(
1/2)/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

Rubi [A]
time = 0.35, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3246, 3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\frac{2\sec^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^3E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8\sec^{\frac{5}{2}}(d+ex)(bc\cos(d+ex)-ab\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))^2}{3e\left(a^2-b^2+c^2\right)^2(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}-\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))}{3e\left(a^2-b^2+c^2\right)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (-2*Sec[d + e*x]^(5/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*Sec[d + e*x]^(5/2)*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a
*Cos[d + e*x] + c*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
```

```
 + c^2])/(b + Sqrt[a^2 + c^2])]*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*
Sin[d + e*x])^2*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 +
c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rule 3246

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Dist[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
), Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{\left(\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\right)\int \frac{}{(b+a\cos}}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

$$= -\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.34, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

```
[Out] (Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2) - (
2*(a^2*c + 3*b^2*c + c^3 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*a*(a^2 - b^2 + c^2)^2*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2,
-((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(
```

```
1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x +
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d
+ e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x
])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*
x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2
)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(
a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]
)/(3*Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[
d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/
c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/
c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[
1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d +
 e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqr
t[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b
 + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x +
 ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[
d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(Sqrt[1 + a^2/c^2]
*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*
c*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Arc
Tan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqr
t[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sq
rt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*S
qrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^
2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*S
qrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/
(-b + c*Sqrt[(a^2 + c^2)/c^2])])/(3*Sqrt[1 + a^2/c^2]*(a^2 - b^2 + c^2)^2*e
*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (4*a^2*b*Sec[d + e*x]^(5/2)
*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*(-((c*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d
+ e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))
]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)
/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^
2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]]
]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - Arc
Tan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 + c^2/a^
2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - ArcTan[c/a]]
)/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[
c/a]]]))/(3*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(
5/2)) + (4*b*c*Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/
2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d +
 e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))),
-((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]
*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 +
```

```
c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*
x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2
)/a^2]*Cos[d + e*x - ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(
a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])]
)) - ((2*a*(b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2
) - (c*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]))/Sqrt[b + a*Sqrt[1
 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 2.40, size = 171436, normalized size = 348.45

method	result	size
default	Expression too large to display	171436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(x*e + d)^(5/2)/(b*sec(x*e + d) + c*tan(x*e + d) + a)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.35, size = 2812, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] 1/9*((-3*I*a^3*b^2 - I*a*b^4 - 3*I*a*c^4 + 3*c^5 + (3*a^2 + 4*b^2)*c^3 - I*
(3*a^3 + 4*a*b^2)*c^2 + (-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3*I
*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 + (3*a^2*b^2 + b^4)*c
- 2*(3*I*a^4*b + I*a^2*b^3 + 3*I*a^2*b*c^2 - 3*a*b*c^3 - (3*a^3*b + a*b^3)*
```

c)*cos(x*e + d) - 2*(3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 + I*(3*a^3 *b + a*b^3)*c + (3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 + I*(3*a^4 + a ^2*b^2)*c)*cos(x*e + d))*sin(x*e + d))*sqrt(2*a - 2*I*c)*weierstrassPInvers e(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4* a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9 *a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2* I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (3*I*a^3*b^2 + I*a*b^4 + 3*I*a*c^4 + 3*c^5 + (3*a^2 + 4*b^2)*c^3 + I*(3*a^3 + 4*a*b^2)*c^2 + (3*I*a^5 + I*a^3*b^2 - I*a*b^2*c^2 - b^2*c^3 - 3*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 + (3*a^2*b^2 + b^4) *c - 2*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*c^2 - 3*a*b*c^3 - (3*a^3*b + a*b ^3)*c)*cos(x*e + d) - 2*(-3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 - I*(3*a^3*b + a*b^3)*c + (-3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 - I*(3*a ^4 + a^2*b^2)*c)*cos(x*e + d))*sin(x*e + d))*sqrt(2*a + 2*I*c)*weierstrassP Inverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^ 3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b *c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a* b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/ (a^2 + c^2)) - 12*(-I*a^2*b^3 - I*b*c^4 - I*(a^2*b + b^3)*c^2 + (-I*a^4*b + I*b*c^4)*cos(x*e + d)^2 + 2*(-I*a^3*b^2 - I*a*b^2*c^2)*cos(x*e + d) + 2*(- I*a^2*b^2*c - I*b^2*c^3 + (-I*a^3*b*c - I*a*b*c^3)*cos(x*e + d))*sin(x*e + d))*sqrt(2*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27 *(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^ 2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8 /27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c ^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4* c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 12*(I*a^2*b^3 + I*b*c^4 + I*(a^2*b + b^3)*c^2 + (I*a^4*b - I*b*c^4)*cos(x*e + d)^2 + 2*(I*a^3*b^2 + I*a*b^2*c^2)*cos(x*e + d) + 2*(I*a^2*b^2*c + I*b^2*c^3 + (I*a^3*b*c + I*a*b *c^3)*cos(x*e + d))*sin(x*e + d))*sqrt(2*a + 2*I*c)*weierstrassZeta(-4/3*(3 *a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c) /(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^ 5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3 *(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2) *c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b *c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))

```
) + 6*(8*(a^3*b*c + a*b*c^3)*cos(x*e + d)^3 - (c^5 + (2*a^2 - 5*b^2)*c^3 +
(a^4 - 5*a^2*b^2)*c)*cos(x*e + d)^2 - 4*(a^3*b*c + a*b*c^3)*cos(x*e + d) -
(4*(a^4*b - b*c^4)*cos(x*e + d)^2 - (a^5 - 5*a^3*b^2 + a*c^4 + (2*a^3 - 5*a
*b^2)*c^2)*cos(x*e + d))*sin(x*e + d))*sqrt((a*cos(x*e + d) + c*sin(x*e + d
) + b)/cos(x*e + d))/sqrt(cos(x*e + d)))/((a^8 - 2*a^6*b^2 + a^4*b^4 - c^8
- 2*(a^2 - b^2)*c^6 + (2*a^2*b^2 - b^4)*c^4 + 2*(a^6 - a^4*b^2)*c^2)*cos(x*
e + d)^2*e + 2*(a^7*b - 2*a^5*b^3 + a^3*b^5 + a*b*c^6 + (3*a^3*b - 2*a*b^3)
*c^4 + (3*a^5*b - 4*a^3*b^3 + a*b^5)*c^2)*cos(x*e + d)*e + (a^6*b^2 - 2*a^4
*b^4 + a^2*b^6 + c^8 + (3*a^2 - b^2)*c^6 + (3*a^4 - a^2*b^2 - b^4)*c^4 + (a
^6 + a^4*b^2 - 3*a^2*b^4 + b^6)*c^2)*e + 2*((a*c^7 + (3*a^3 - 2*a*b^2)*c^5
+ (3*a^5 - 4*a^3*b^2 + a*b^4)*c^3 + (a^7 - 2*a^5*b^2 + a^3*b^4)*c)*cos(x*e
+ d)*e + (b*c^7 + (3*a^2*b - 2*b^3)*c^5 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^3 +
 (a^6*b - 2*a^4*b^3 + a^2*b^5)*c)*e)*sin(x*e + d))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorithm
="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(d+e\,x)}\right)^{5/2}}{\left(a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2),x)
[Out] int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2), x)
```

3.453 $\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

Optimal. Leaf size=371

$$-\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))}+\frac{8b\cos^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}\right.}{3e(b+a}$$

```
[Out] -2/3*cos(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*x
+d))^(3/2)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*cos(e*x+d)^(3/2)*(cos(1/2*
d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*Elli
pticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c
^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/(b+a*cos(e*x+d)+c*
sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*cos(e*x+d)^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/
cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a
,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+
c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2
)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

Rubi [A]
time = 0.26, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3242, 3199, 3228, 3198, 2732, 3206, 2740}

$$\frac{2(a^2-b^2+c^2)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a\cos(d+ex)+b+c\sin(d+ex))^2}+\frac{8b\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a\cos(d+ex)+b+c\sin(d+ex))\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(a\cos(d+ex)+b+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] (-2*Cos[d + e*x]^(3/2)*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + b*Sec[d + e*x
] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])) + (8
*b*Cos[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3
*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*Cos[d + e*x]^(3/2)*
EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c
^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a
+ b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*e*(b + a*Cos[d + e*x] + c*Sin[
d + e*x])^2)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3242

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \frac{\left(\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex)\right.}{(b+a\cos(d+ex)}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(}{3e(b+a\cos(d+ex)+}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(}{3e(b+a\cos(d+ex)+}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(}{3e(b+a\cos(d+ex)+}$$

$$= -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(}{3e(b+a\cos(d+ex)+}$$

Mathematica [F]
time = 173.16, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

```
[Out] Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2), x
]
```

Maple [C] Result contains complex when optimal does not.
time = 1.64, size = 70663, normalized size = 190.47

method	result	size
default	Expression too large to display	70663

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((b*sec(x*e + d) + c*tan(x*e + d) + a)^(3/2)*cos(x*e + d)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.00, size = 1513, normalized size = 4.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-3*I*a^3 - I*a*b^2 - 3*I*a*c^2 + 3*c^3 + (3*a^2 + b^2)*c)*sqr
t(a - I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*
c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5
*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*
a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^
2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2
 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2)*(3*I*a^3 + I*a*b^2 + 3*I*a*c
^2 + 3*c^3 + (3*a^2 + b^2)*c)*sqrt(a + I*c)*weierstrassPInverse(-4/3*(3*a^4
 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^
4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 -
```

```
2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^
2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^
2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 12*s
qrt(2)*(-I*a^2*b - I*b*c^2)*sqrt(a - I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9
*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)
*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 -
4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I
*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b
^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 +
 c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 12*sq
rt(2)*(I*a^2*b + I*b*c^2)*sqrt(a + I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2
*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^
2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a
^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*
a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2
*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(
9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3
)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c
^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 6*((a^2*
c + c^3)*cos(x*e + d) - (a^3 + a*c^2)*sin(x*e + d))*sqrt((a*cos(x*e + d) +
c*sin(x*e + d) + b)/cos(x*e + d))*sqrt(cos(x*e + d)))*e^(-1)/(a^2 + c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos{(d+e\,x)}^{3/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos{(d+e\,x)}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
[Out] int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

3.454 $\int \sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}$

Optimal. Leaf size=118

$$\frac{2\sqrt{\cos(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d)
)^(1/2)/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3242, 3198, 2732}

$$\frac{2\sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

```
[Out] (2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*S
qrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3242

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \frac{\left(\sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+\right.}{\sqrt{b+a\cos(d+ex}}$$

$$= \frac{\left(\sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+\right.}{\sqrt{\frac{b+a}{}}}$$

$$= \frac{2\sqrt{\cos(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{}}{b+\sqrt{}}\right.}{e\sqrt{\frac{b+a\cos(d+e}{b+\sqrt{}}}}$$

Mathematica [**F**]
time = 13.71, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(d+ex)}\,\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
[Out] Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]], x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 1.10, size = 44690, normalized size = 378.73

method	result	size
risch	Expression too large to display	2660
default	Expression too large to display	44690

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(b*sec(x*e + d) + c*tan(x*e + d) + a)*sqrt(cos(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.34, size = 1378, normalized size = 11.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a*b + b*c)*sqrt(a - I*c)*weierstrassPInverse(-4/3*(3*a^4 -
 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
+ 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*
I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*
b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2
+ c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(
2)*(I*a*b + b*c)*sqrt(a + I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2
+ 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b
+ 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^
6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(
x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 3*sqrt(2)*(-I*a^2
 - I*c^2)*sqrt(a - I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2
 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/
27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^
3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c
^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*
```

```
c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)
*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^
4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d)
 - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 3*sqrt(2)*(I*a^2 + I*c^
2)*sqrt(a + I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*
a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a
^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(
3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*
a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6
*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(
9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 -
6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 +
 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I
*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,\sqrt{\cos(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)**(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(d+e\,x)}\,\sqrt{a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
[Out] int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

3.455 $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$

Optimal. Leaf size=118

$$\frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(
1/2)))^(1/2)/e/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3242, 3206, 2740}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3242

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}\int \frac{}{\sqrt{b+a\,c}}}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)}}$$

$$= \frac{\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{}{\sqrt{\overline{b+\sqrt{}}}}}{\sqrt{\cos(d+ex)}\sqrt{a+b\,s}}$$

$$= \frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{b}}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 506, normalized size = 4.29

$$\frac{4\left(ia-ib+c+\sqrt{a^2-b^2+c^2}\right)F\left(\text{ArcSin}\left(\sqrt{\frac{\left(-ia+ib+c+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{ia-ib+c+\sqrt{a^2-b^2+c^2}}}\right)|\frac{b+i\sqrt{a^2-b^2+c^2}}{b-i\sqrt{a^2-b^2+c^2}}\right)\sqrt{\frac{\left(-ia+ib+c+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{ia-ib+c+\sqrt{a^2-b^2+c^2}}}(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(-c+\sqrt{a^2-b^2+c^2}+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(-ia+ib-c+\sqrt{a^2-b^2+c^2}\right)\left(-i+\tan\left(\frac{1}{2}(d+ex)\right)\right)}}\sqrt{-\frac{i\left(c+\sqrt{a^2-b^2+c^2}+(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(ia-ib+c+\sqrt{a^2-b^2+c^2}\right)\left(-i+\tan\left(\frac{1}{2}(d+ex)\right)\right)}}}{\left(a+i\left(ib+c+\sqrt{a^2-b^2+c^2}\right)\right)e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),
x]
```

```
[Out] (4*(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[(((-I)*a +
 I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a -
I*b + c + Sqrt[a^2 - b^2 + c^2])]], (b + I*Sqrt[a^2 - b^2 + c^2])/(b - I*Sq
rt[a^2 - b^2 + c^2])]*Sqrt[(((-I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]*(Cos
[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(-c + Sqrt[a^2 - b^2 + c^2] + (a - b
)*Tan[(d + e*x)/2]))/(((-I)*a + I*b - c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[
(d + e*x)/2]))]*Sqrt[((-I)*(c + Sqrt[a^2 - b^2 + c^2] + (-a + b)*Tan[(d + e
*x)/2]))/((I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))]
```

```
)/((a + I*(I*b + c + Sqrt[a^2 - b^2 + c^2]))*e*Sqrt[Cos[d + e*x]]*Sqrt[a +
b*Sec[d + e*x] + c*Tan[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.18, size = 1125, normalized size = 9.53

method	result
default	$-\frac{4\sqrt{\frac{b+a\cos(ex+d)+c\sin(ex+d)}{\cos(ex+d)}}\sqrt{\frac{(i\cos(ex+d)+i-\sin(ex+d))(a-b)\sqrt{a^2-b^2+}}{\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-c\cos(ex+d)+a\sin(ex+d)-b\sin(ex+d)+\sqrt{a^2-b^2+}\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -4/e*((b+a*cos(e*x+d)+c*sin(e*x+d))/cos(e*x+d))^(1/2)*((I*cos(e*x+d)+I-sin(
e*x+d))/(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x
+d)+(a^2-b^2+c^2)^(1/2)-c)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-I*b-(a^2-b^2+c^2)
^(1/2)-c))^(1/2)*((I*cos(e*x+d)+sin(e*x+d)+I)/(cos(e*x+d)*(a^2-b^2+c^2)^(1/
2)-c*cos(e*x+d)+a*sin(e*x+d)-b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)*(a-b)*(a^2
-b^2+c^2)^(1/2)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*(-(cos(e*x+d)*(a^2-b
^2+c^2)^(1/2)+c*cos(e*x+d)-a*sin(e*x+d)+b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)
*(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+
d)+a*sin(e*x+d)-b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*a-I*b+(a^2-b^2+c^2)^
(1/2)+c))^(1/2)*(cos(e*x+d)+1)^2*EllipticF((-(cos(e*x+d)*(a^2-b^2+c^2)^(1/2
)+c*cos(e*x+d)-a*sin(e*x+d)+b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)+c)*(I*a-I*b-(a
^2-b^2+c^2)^(1/2)+c)/(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-c*cos(e*x+d)+a*sin(e*x
+d)-b*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1
/2),((I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c)/(I*a-I
*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*cos(e*x+d
)^(1/2)*(cos(e*x+d)-1)^2*(I*a*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*a^2+I*c^2-I*
b*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)-I*c*(a^2-b^2+c^2)^(1/2)-I*cos(e*x+d)*(a^2-
b^2+c^2)^(1/2)*c+I*b*c*sin(e*x+d)+I*cos(e*x+d)*c^2+I*cos(e*x+d)*a*b+cos(e*x
+d)*(a^2-b^2+c^2)^(1/2)*b-I*a*b-I*cos(e*x+d)*b^2-I*a*c*sin(e*x+d)+a^2*sin(e
*x+d)-sin(e*x+d)*b^2-a*c*cos(e*x+d)+a*(a^2-b^2+c^2)^(1/2)-c*b)/sin(e*x+d)^4
/(b+a*cos(e*x+d)+c*sin(e*x+d))/(-I*a+I*b+(a^2-b^2+c^2)^(1/2)-c)/(a^2-b^2+c^
2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(x*e + d) + c*tan(x*e + d) + a)*sqrt(cos(x*e + d))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 509, normalized size = 4.31

$$\frac{\left(\sqrt{2}\,\sqrt{a-i\,c}\,(-i\,a+c)\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^4-4\,a^2b^2+4\,b^2c^2+6i\,ac^3-3\,c^4+2i\,(3\,a^3-4\,ab^2)c)}{3\,(a^4+2\,a^2c^2+c^4)},\frac{8\,(9\,a^5b-8\,a^3b^3-27\,abc^4-9i\,bc^5+2i\,(9\,a^2b+4\,b^3)c^3-6\,(3\,a^3b-4\,ab^3)c^2+3i\,(9\,a^4b-8\,a^2b^3)c)}{27\,(a^6+3\,a^4c^2+3\,a^2c^4+c^6)},\frac{2\,ab+2i\,bc+3\,(a^2+c^2)\cos(xe+d)-3\,(-i\,a^2-i\,c^2)\sin(xe+d)}{3\,(a^2+c^2)}\right)+\sqrt{2}\,\sqrt{a+i\,c}\,(i\,a+c)\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^4-4\,a^2b^2+4\,b^2c^2-6i\,ac^3-3\,c^4-2i\,(3\,a^3-4\,ab^2)c)}{3\,(a^4+2\,a^2c^2+c^4)},\frac{8\,(9\,a^5b-8\,a^3b^3-27\,abc^4+9i\,bc^5-2i\,(9\,a^2b+4\,b^3)c^3-6\,(3\,a^3b-4\,ab^3)c^2-3i\,(9\,a^4b-8\,a^2b^3)c)}{27\,(a^6+3\,a^4c^2+3\,a^2c^4+c^6)},\frac{2\,ab-2i\,bc+3\,(a^2+c^2)\cos(xe+d)-3\,(i\,a^2+i\,c^2)\sin(xe+d)}{3\,(a^2+c^2)}\right)\right)e^{(-1)}}{a^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a - I*c)*(-I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b
^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2
*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/
(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*c
os(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2)*sqrt(
a + I*c)*(I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2
- 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/2
7*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3
 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^
2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3
*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,\sqrt{\cos(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d))),
 x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(d+e\,x\right)}\,\sqrt{a+c\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)),x)
```

```
[Out] int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)), x)
```

3.456 $\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c\right.}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{b+a\,c}{}}}$$

```
[Out] -2*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^2)/
e/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)/e/cos(e*x+d)^(3/2)/
((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c
*tan(e*x+d))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3242, 3207, 3198, 2732}

$$-\frac{2(a\cos(d+ex)+b+c\sin(d+ex))^2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\,(a^2-b^2+c^2)\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))}{e\,(a^2-b^2+c^2)\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/((a^2 - b^2 + c^2)*e*Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e
*x])^(3/2)) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/
(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)/((a^2 - b^2
 + c^2)*e*Cos[d + e*x]^(3/2)*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3242

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}\int \frac{}{(b+a\cos(}}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex}$$

Mathematica [F]
time = 18.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.87, size = 44824, normalized size = 186.77

method	result	size
default	Expression too large to display	44824

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(x*e + d) + c*tan(x*e + d) + a)^(3/2)*cos(x*e + d)^(3/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.37, size = 1779, normalized size = 7.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 1/3*((sqrt(2)*(I*a^2*b - a*b*c)*cos(x*e + d) + sqrt(2)*(I*a*b*c - b*c^2)*si
n(x*e + d) + sqrt(2)*(I*a*b^2 - b^2*c))*sqrt(a - I*c)*weierstrassPInverse(-
4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b
^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*
I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^
```

```
4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b
*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c
^2)) + (sqrt(2)*(-I*a^2*b - a*b*c)*cos(x*e + d) + sqrt(2)*(-I*a*b*c - b*c^2
)*sin(x*e + d) + sqrt(2)*(-I*a*b^2 - b^2*c))*sqrt(a + I*c)*weierstrassPInve
rse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 -
4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4
 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*
(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b -
2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2
 + c^2)) - 3*(sqrt(2)*(I*a^3 + I*a*c^2)*cos(x*e + d) + sqrt(2)*(I*a^2*c + I
*c^3)*sin(x*e + d) + sqrt(2)*(I*a^2*b + I*b*c^2))*sqrt(a - I*c)*weierstrass
Zeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 -
 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^
4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I
*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassP
Inverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^
3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b
*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 +
3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*
b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))
/(a^2 + c^2))) - 3*(sqrt(2)*(-I*a^3 - I*a*c^2)*cos(x*e + d) + sqrt(2)*(-I*a
^2*c - I*c^3)*sin(x*e + d) + sqrt(2)*(-I*a^2*b - I*b*c^2))*sqrt(a + I*c)*we
ierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I
*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 -
27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*
c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), wei
erstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 -
2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3
 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^
3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6),
1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x
*e + d))/(a^2 + c^2))) - 6*((a^2*c + c^3)*cos(x*e + d) - (a^3 + a*c^2)*sin(
x*e + d))*sqrt((a*cos(x*e + d) + c*sin(x*e + d) + b)/cos(x*e + d))*sqrt(cos
(x*e + d)))/((a^5 - a^3*b^2 + a*c^4 + (2*a^3 - a*b^2)*c^2)*cos(x*e + d)*e +
 (c^5 + (2*a^2 - b^2)*c^3 + (a^4 - a^2*b^2)*c)*e*sin(x*e + d) + (a^4*b - a^
2*b^3 + b*c^4 + (2*a^2*b - b^3)*c^2)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(d+e\,x)^{3/2}\,\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)),x)
```

```
[Out] int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)), x)
```

3.457 $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$-\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8(bc\cos(d+ex)-ab\sin(d+ex))(b}{3(a^2-b^2+c^2)^2\,e\cos^{\frac{5}{2}}(d+ex)(a+}$$

```
[Out] -2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^2
)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*(b*c*cos(e*x+d
)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)^2/e/cos(e*x
+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/cos(e*x+d)^(5/2)/((b
+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x+d)+c*ta
n(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d
))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2
)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

Rubi [A]

time = 0.34, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3242, 3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a\cos(d+ex)+b+c\sin(d+ex))^2F\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\,(a^2-b^2+c^2)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8b(a\cos(d+ex)+b+c\sin(d+ex))^3E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\,(a^2-b^2+c^2)^2\cos^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}+\frac{8(bc\cos(d+ex)-ab\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))^2}{3e\,(a^2-b^2+c^2)^2\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b+c\sin(d+ex))}{3e\,(a^2-b^2+c^2)\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)),x]
```

```
[Out] (-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(b + a*Cos[d + e*x] + c*Sin[d + e*x])
)/(3*(a^2 - b^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d +
 e*x])^(5/2)) + (8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(b + a*Cos[d + e*x
] + c*Sin[d + e*x])^2)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*(a + b*S
ec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (8*b*EllipticE[(d + e*x - ArcTan[a,
c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*S
in[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*Cos[d + e*x]^(5/2)*Sqrt[(b + a*Cos
[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]*(a + b*Sec[d + e*x] + c*
Tan[d + e*x])^(5/2)) + (2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*Sqr
```

t[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && NeQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rule 3242

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
, Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \frac{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}\int \frac{}{(b+a\cos(d+}}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex}$$

$$= -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+}{3(a^2-b^2+c^2)\,e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex}$$

Mathematica [**F**]
time = 32.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
),x]
```

```
[Out] Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
), x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 1.75, size = 169879, normalized size = 345.28

method	result	size
default	Expression too large to display	169879

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((b*sec(x*e + d) + c*tan(x*e + d) + a)^(5/2)*cos(x*e + d)^(5/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.11, size = 2857, normalized size = 5.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3*I*a*c^4 - 3
*c^5 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 - 2*sqrt(2)*(3*I*a^4*b + I*a^2*b
^3 + 3*I*a^2*b*c^2 - 3*a*b*c^3 - (3*a^3*b + a*b^3)*c)*cos(x*e + d) - 2*(sqr
t(2)*(3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 + I*(3*a^4 + a^2*b^2)*c)*
cos(x*e + d) + sqrt(2)*(3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 + I*(3*
a^3*b + a*b^3)*c))*sin(x*e + d) + sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4 - 3*I*a*c
^4 + 3*c^5 + (3*a^2 + 4*b^2)*c^3 - I*(3*a^3 + 4*a*b^2)*c^2 + (3*a^2*b^2 + b
^4)*c))*sqrt(a - I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c
^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*
c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4
*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d)
- 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (sqrt(2)*(3*I*a^5 + I*a^3
*b^2 - I*a*b^2*c^2 - b^2*c^3 - 3*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos
```

(x*e + d)^2 - 2*sqrt(2)*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*c^2 - 3*a*b*c^3
 - (3*a^3*b + a*b^3)*c)*cos(x*e + d) - 2*(sqrt(2)*(-3*I*a^2*c^3 - 3*a*c^4 -
 (3*a^3 + a*b^2)*c^2 - I*(3*a^4 + a^2*b^2)*c)*cos(x*e + d) + sqrt(2)*(-3*I*
a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 - I*(3*a^3*b + a*b^3)*c))*sin(x*e +
 d) + sqrt(2)*(3*I*a^3*b^2 + I*a*b^4 + 3*I*a*c^4 + 3*c^5 + (3*a^2 + 4*b^2)*
c^3 + I*(3*a^3 + 4*a*b^2)*c^2 + (3*a^2*b^2 + b^4)*c))*sqrt(a + I*c)*weierst
rassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*
(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 2
7*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c
^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*
(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e +
 d))/(a^2 + c^2)) - 12*(sqrt(2)*(-I*a^4*b + I*b*c^4)*cos(x*e + d)^2 + 2*sqr
t(2)*(-I*a^3*b^2 - I*a*b^2*c^2)*cos(x*e + d) + 2*(sqrt(2)*(-I*a^3*b*c - I*a
*b*c^3)*cos(x*e + d) + sqrt(2)*(-I*a^2*b^2*c - I*b^2*c^3))*sin(x*e + d) + s
qrt(2)*(-I*a^2*b^3 - I*b*c^4 - I*(a^2*b + b^3)*c^2))*sqrt(a - I*c)*weierstr
assZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^
3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b
*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 +
3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstra
ssPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3
*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*
a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2
 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2
*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e +
d))/(a^2 + c^2))) - 12*(sqrt(2)*(I*a^4*b - I*b*c^4)*cos(x*e + d)^2 + 2*sqrt
(2)*(I*a^3*b^2 + I*a*b^2*c^2)*cos(x*e + d) + 2*(sqrt(2)*(I*a^3*b*c + I*a*b*
c^3)*cos(x*e + d) + sqrt(2)*(I*a^2*b^2*c + I*b^2*c^3))*sin(x*e + d) + sqrt(
2)*(I*a^2*b^3 + I*b*c^4 + I*(a^2*b + b^3)*c^2))*sqrt(a + I*c)*weierstrassZe
ta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4
*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4
+ 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(
9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPIn
verse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3
- 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c
^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*
I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b
- 2*I*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a
^2 + c^2))) - 6*(4*a^3*b*c + 4*a*b*c^3 - 8*(a^3*b*c + a*b*c^3)*cos(x*e + d)
^2 + (c^5 + (2*a^2 - 5*b^2)*c^3 + (a^4 - 5*a^2*b^2)*c)*cos(x*e + d) - (a^5
- 5*a^3*b^2 + a*c^4 + (2*a^3 - 5*a*b^2)*c^2 - 4*(a^4*b - b*c^4)*cos(x*e + d
))*sin(x*e + d))*sqrt((a*cos(x*e + d) + c*sin(x*e + d) + b)/cos(x*e + d))*s
qrt(cos(x*e + d)))/((a^8 - 2*a^6*b^2 + a^4*b^4 - c^8 - 2*(a^2 - b^2)*c^6 +
(2*a^2*b^2 - b^4)*c^4 + 2*(a^6 - a^4*b^2)*c^2)*cos(x*e + d)^2*e + 2*(a^7*b
- 2*a^5*b^3 + a^3*b^5 + a*b*c^6 + (3*a^3*b - 2*a*b^3)*c^4 + (3*a^5*b - 4*a^
3*b^3 + a*b^5)*c^2)*cos(x*e + d)*e + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6 + c^8 +

```
 (3*a^2 - b^2)*c^6 + (3*a^4 - a^2*b^2 - b^4)*c^4 + (a^6 + a^4*b^2 - 3*a^2*b
^4 + b^6)*c^2)*e + 2*((a*c^7 + (3*a^3 - 2*a*b^2)*c^5 + (3*a^5 - 4*a^3*b^2 +
 a*b^4)*c^3 + (a^7 - 2*a^5*b^2 + a^3*b^4)*c)*cos(x*e + d)*e + (b*c^7 + (3*a
^2*b - 2*b^3)*c^5 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^3 + (a^6*b - 2*a^4*b^3 +
a^2*b^5)*c)*e)*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(d+e\,x\right)^{5/2}\,{\left(a+c\,\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)),x)
```

```
[Out] int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)), x)
```

3.458 $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=98

$$\frac{ax}{a^2+b^2}+\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}-\frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

```
[Out] a*x/(a^2+b^2)-b*ln(c+b*cos(x)+a*sin(x))/(a^2+b^2)+2*a*c*arctanh((a-(b-c)*ta
n(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3239, 3216, 3203, 632, 212}

$$\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 -
c^2]])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x] + a*Sin[x]
])/(a^2 + b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3239

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \int \frac{\sin(x)}{c+b\cos(x)+a\sin(x)}\,dx$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2} - \frac{(ac)\int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx}{a^2+b^2}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2} - \frac{(2ac)\text{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx\right.}{a^2+b^2}$$

$$= \frac{ax}{a^2+b^2} - \frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2} + \frac{(4ac)\text{Subst}\left(\int \frac{1}{4(a^2+b^2-c^2)-x^2}\,dx, x\right.}{a^2+b^2}$$

$$= \frac{ax}{a^2+b^2} + \frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} - \frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A]
time = 0.16, size = 80, normalized size = 0.82

$$\frac{ax + \frac{2ac\tanh^{-1}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} - b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

```
[Out] (a*x + (2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[
a^2 + b^2 - c^2] - b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

Maple [A]
time = 0.29, size = 176, normalized size = 1.80

method	result
default	$\frac{\frac{4\left(b^2-cb\right)\ln\left(-b\left(\tan^2\left(\frac{x}{2}\right)\right)+c\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b+c\right)}{-2b+2c}+\frac{4\left(-ab-ac-\frac{\left(b^2-cb\right)a}{-b+c}\right)\arctan\left(\frac{2(-b+c)\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}}{2a^2+2b^2}+\frac{2b\ln\left(1+\tan^2\left(\right.\right.}{}$ …
risch	$\frac{x}{ib+a}+\frac{2ix\,a^2b}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}+\frac{2ix\,b^3}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}-\frac{2ix\,c^2b}{a^4+2a^2b^2-a^2c^2+b^4-b^2c^2}-\frac{\ln\left(e^{ix}-\frac{-ic^2a^2-ab\,c^2+i\sqrt{a^4}\right.}{}}{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(2*a^2+2*b^2)*(1/2*(b^2-b*c)/(-b+c)*ln(-b*tan(1/2*x)^2+c*tan(1/2*x)^2+2*a
*tan(1/2*x)+b+c)+(-a*b-a*c-(b^2-b*c)*a/(-b+c))/(-a^2-b^2+c^2)^(1/2)*arctan(
1/2*(2*(-b+c)*tan(1/2*x)+2*a)/(-a^2-b^2+c^2)^(1/2)))+4/(2*a^2+2*b^2)*(1/2*b
*ln(1+tan(1/2*x)^2)+a*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.
time = 3.55, size = 555, normalized size = 5.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*
c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 +
 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*b
*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*c
```

```
os(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
+ a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)*x - (
a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 +
2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2), -1
/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^
2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*
c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2)*log(2*b*c
*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/
(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cot(x) + c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x)
[Out] Integral(1/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A]
time = 0.41, size = 158, normalized size = 1.61

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}-\frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2a\tan\left(\frac{1}{2}x\right)+b+c\right)}{a^2+b^2}+\frac{b\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*ta
n(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 + c
^2)) + a*x/(a^2 + b^2) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1
/2*x) + b + c)/(a^2 + b^2) + b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

Mupad [B]
time = 13.76, size = 965, normalized size = 9.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c/sin(x) + b*cot(x)),x)
[Out] log(tan(x/2) - 1i)/(a*1i + b) + (log(tan(x/2) + 1i)*1i)/(a + b*1i) - (log(-
 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2
))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x/2)*(b - c)*(b*c + a^2 - c^2)
```

```
+ ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(64*a*b^3 - 32*a^3*b
 + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b - 2*a^2*c + 2*b^2*c - 2*b^3) + 64*
a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 -
 c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + a^3*c + 3*a^2*b^2*tan(x/
2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(x/2) - 2*b^3*c*tan(x/2) - 4*a*b*c^2
 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 +
 b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2)
+ b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^2) + (a^2 + b^2 -
 c^2)^2) - (log(- 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3 - a*c*(a^2
+ b^2 - c^2)^(1/2))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x/2)*(b - c)*(
b*c + a^2 - c^2) + ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(64
*a*b^3 - 32*a^3*b + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b - 2*a^2*c + 2*b^2
*c - 2*b^3) + 64*a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b - b*c^2 + b^3 -
 a*c*(a^2 + b^2 - c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + a^3*c +
 3*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(x/2) - 2*b^3*c*tan
(x/2) - 4*a*b*c^2 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2
- c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2))
)*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^
2) + (a^2 + b^2 - c^2)^2)
```

3.459 $\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

```
[Out] -2*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3245, 3203, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2 -
 c^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
```

```
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx &= \int \frac{1}{c+b\cos(x)+a\sin(x)}\,dx \\
&= 2\text{Subst}\left(\int \frac{1}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) \\
&= -\left(4\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2-c^2\right)-x^2}\,dx, x, 2a+2(-b+c)\tan\left(\frac{x}{2}\right)\right)\right) \\
&= -\frac{2\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
[Out] (-2*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
 - c^2]
```

Maple [A]
time = 0.16, size = 53, normalized size = 1.04

method	result
default	$\frac{2\arctan\left(\frac{2(-b+c)\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$
risch	$-\frac{i\ln\left(e^{ix}+\frac{ica\sqrt{-a^2-b^2+c^2}+ia^3+ia\,b^2-ia\,c^2+cb\sqrt{-a^2-b^2+c^2}+a^2b+b^3-c^2b}{\left(a^2+b^2\right)\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}+\frac{i\ln\left(e^{ix}+\frac{ica\sqrt{-a^2-b^2+}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(-b+c)*tan(1/2*x)+2*a)/(-a^2-b^2+c^2)^
(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(47) = 94.
time = 3.87, size = 349, normalized size = 6.84

$$\left[\frac{\log\left(-\frac{a^4+3a^2b^2+2b^4+(a^2-b^2)c^2+2(a^2b+b^3)c\cos(x)+(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^3+ab^2)c-(a^3b+ab^3-2abc^2)\cos(x))\sin(x)-2\left(2abc\cos(x)^2-abc+(a^3+ab^2)\cos(x)-(a^2b+b^3-(a^2-b^2)c\cos(x))\sin(x)\right)\sqrt{a^2+b^2-c^2}}{2bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2(ab\cos(x)+ac)\sin(x)}\right)}{2\sqrt{a^2+b^2-c^2}}, \frac{\sqrt{-a^2-b^2+c^2}\arctan\left(\frac{(bc\cos(x)+ac\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^3+ab^2-ac^2)\cos(x)-(a^2b+b^3-bc^2)\sin(x)}\right)}{a^2+b^2-c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*co
s(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a^3
*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (a^3
 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^2 +
b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x
) + a*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arctan((b*c
*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*
c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A]
time = 0.43, size = 73, normalized size = 1.43

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2b-2c)+\arctan\left(\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*b - 2*c) + arctan((b*tan(1/2*x) - c*tan(
1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

Mupad [B]
time = 2.79, size = 47, normalized size = 0.92

$$-\frac{2\,\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{x}{2}\right)\,(2\,b-2\,c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a + c/sin(x) + b*cot(x))),x)
```

```
[Out] -(2*atanh((a - (tan(x/2)*(2*b - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 + b
^2 - c^2)^(1/2)
```

3.460 $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

Optimal. Leaf size=120

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

```
[Out] ln(tan(1/2*x))/(b+c)-b*ln(b+c+2*a*tan(1/2*x)-(b-c)*tan(1/2*x)^2)/(b^2-c^2)-
2*a*c*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(b^2-c^2)/(a^2+b^2-
c^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4482, 12, 1642, 648, 632, 212, 642}

$$-\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] (-2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2)
*Sqrt[a^2 + b^2 - c^2]) + Log[Tan[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tan[x/
2] - (b - c)*Tan[x/2]^2])/(b^2 - c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \int \frac{\csc(x)}{c+b\cos(x)+a\sin(x)}\,dx$$

$$= 2\text{Subst}\left(\int \frac{1+x^2}{2x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \text{Subst}\left(\int \frac{1+x^2}{x\left(b+c+2ax+(-b+c)x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{(b+c)x}+\frac{2(-a+bx)}{(b+c)\left(b+c+2ax-(b-c)x^2\right)}\right)\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}+\frac{2\text{Subst}\left(\int \frac{-a+bx}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b+c}$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\text{Subst}\left(\int \frac{2a+2(-b+c)x}{b+c+2ax+(-b+c)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{b^2-c^2}+\frac{(2ac)\text{Subst}\left(\int \frac{}{b}\right.}{}$$

$$= \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{(4ac)\text{Subst}\left(\right.}{}$$

$$= -\frac{2ac\tanh^{-1}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)\right.}{b^2-c^2}$$

Mathematica [A]
time = 0.20, size = 104, normalized size = 0.87

$$\frac{\frac{2ac\tanh^{-1}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}-(b+c)\log\left(\cos\left(\frac{x}{2}\right)\right)+(-b+c)\log\left(\sin\left(\frac{x}{2}\right)\right)+b\log(c+b\cos(x)+a\sin(x))}{(-b+c)(b+c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

```
[Out] ((2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
b^2 - c^2] - (b + c)*Log[Cos[x/2]] + (-b + c)*Log[Sin[x/2]] + b*Log[c + b*C
os[x] + a*Sin[x]])/((-b + c)*(b + c))
```

Maple [A]
time = 0.36, size = 123, normalized size = 1.02

method	result
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}+\frac{\frac{b\ln\left(-b\left(\tan^2\left(\frac{x}{2}\right)\right)+c\left(\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b+c\right)}{-b+c}+\frac{\left(-2a-\frac{2ba}{-b+c}\right)\arctan\left(\frac{2(-b+c)\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}}{b+c}$
risch	$\frac{2ixa^2b}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4}+\frac{2ixb^3}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4}-\frac{2ixc^2b}{a^2b^2-a^2c^2+b^4-2b^2c^2+c^4}-\frac{ix}{b-c}-\frac{ix}{b+c}-\frac{\ln\left(e^{ix}-\frac{-ic^2a^2-abc^2}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(1/2*x))/(b+c)+1/(b+c)*(b/(-b+c)*ln(-b*tan(1/2*x)^2+c*tan(1/2*x)^2+2*
a*tan(1/2*x)+b+c)+(-2*a-2*b*a/(-b+c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(-
b+c)*tan(1/2*x)+2*a)/(-a^2-b^2+c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(110) = 220$.
time = 7.68, size = 669, normalized size = 5.58

$$\left[-\frac{\sqrt{a^2+b^2-c^2}\,ac\log\left(\frac{a^4+3a^2b^2+2b^4+(a^2-b^2)c^2+2(a^2b+b^3)c\cos(x)+(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^3+ab^2)c-(a^3b+ab^3-2abc^2)\cos(x))\sin(x)+2(2abc\cos(x)^2-abc+(a^3+ab^2)\cos(x)-(a^2b+b^3-(a^2-b^2)c\cos(x))\sin(x))\sqrt{a^2+b^2-c^2}}{2bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2(ab\cos(x)+ac)\sin(x)}\right)+(a^2b+b^3-bc^2)\log\left(2bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2(ab\cos(x)+ac)\sin(x)\right)-(a^2b+b^3-bc^2-c^3+(a^2+b^2)c)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(a^2b+b^3-bc^2+c^3-(a^2+b^2)c)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a^2b^2+b^4+c^4-(a^2+2b^2)c^2)}, \frac{2\sqrt{-a^2-b^2+c^2}\,ac\arctan\left(\frac{(bc\cos(x)+ac\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^3+ab^2-ac^2)\cos(x)-(a^2b+b^3-bc^2)\sin(x)}\right)-(a^2b+b^3-bc^2)\log\left(2bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2(ab\cos(x)+ac)\sin(x)\right)+(a^2b+b^3-bc^2-c^3+(a^2+b^2)c)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+(a^2b+b^3-bc^2+c^3-(a^2+b^2)c)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2(a^2b^2+b^4+c^4-(a^2+2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)
*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
+ 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*a*
b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*
cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2
 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + (a^2*b + b^3 - b*c^2)*log(2*
b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x)
) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) - (a^
2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^2 +
 b^4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((
b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 -
 a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x))) - (a^2*b + b^3 - b*c^2)*log
(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin
(x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/2) +
(a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(a^2*b^
2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b\cot(x) + c\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)+c*csc(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cot(x) + c*csc(x)), x)
```

Giac [A]
time = 0.45, size = 142, normalized size = 1.18

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}\,(b^2-c^2)} - \frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2a\tan\left(\frac{1}{2}x\right)+b+c\right)}{b^2-c^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{b+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*tan
(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 - c^
2)) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b + c)/(b^2
 - c^2) + log(abs(tan(1/2*x)))/(b + c)
```

Mupad [B]
time = 8.70, size = 531, normalized size = 4.42

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}-\frac{\ln\left(2\,a-2\,b\,\tan\left(\frac{x}{2}\right)-\frac{\left(\tan\left(\frac{x}{2}\right)\,\left(-8\,a^2-8\,b^2+6\,b\,c+2\,c^2\right)-4\,a\,c+\frac{2\,\left(b-c\right)\,\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\,\left(4\,\tan\left(\frac{x}{2}\right)\,a^2+a\,b+a\,c+3\,\tan\left(\frac{x}{2}\right)\,b^2-3\,\tan\left(\frac{x}{2}\right)\,c^2\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}\right)\,\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}\right)\,\left(b\,\left(a^2-c^2\right)+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}-\frac{\ln\left(2\,a-2\,b\,\tan\left(\frac{x}{2}\right)-\frac{\left(\tan\left(\frac{x}{2}\right)\,\left(-8\,a^2-8\,b^2+6\,b\,c+2\,c^2\right)-4\,a\,c+\frac{2\,\left(b-c\right)\,\left(a^2\,b-b\,c^2+b^3-a\,c\,\sqrt{a^2+b^2-c^2}\right)\,\left(4\,\tan\left(\frac{x}{2}\right)\,a^2+a\,b+a\,c+3\,\tan\left(\frac{x}{2}\right)\,b^2-3\,\tan\left(\frac{x}{2}\right)\,c^2\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}\right)\,\left(a^2\,b-b\,c^2+b^3-a\,c\,\sqrt{a^2+b^2-c^2}\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}\right)\,\left(b\,\left(a^2-c^2\right)+b^3-a\,c\,\sqrt{a^2+b^2-c^2}\right)}{\left(b^2-c^2\right)\,\left(a^2+b^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + c/sin(x) + b*cot(x))),x)
```

```
[Out] log(tan(x/2))/(b + c) - (log(2*a - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2
 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^
2 - c^2)^(1/2))*(a*b + a*c + 4*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/
2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2
 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c
*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)) - (log(2*a - 2*b
*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)
*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(a*b + a*c + 4*a^2*tan
(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*
(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^
2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2
)*(a^2 + b^2 - c^2))
```

3.461 $\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx$

Optimal. Leaf size=21

$$x + 2\text{ArcTan}\left(\frac{\cos(x) - \sin(x)}{2 + \cos(x) + \sin(x)}\right)$$

```
[Out] x+2*arctan((cos(x)-sin(x))/(2+cos(x)+sin(x)))
```

Rubi [A]

time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3245, 3203, 632, 210}

$$2\text{ArcTan}\left(\frac{\cos(x) - \sin(x)}{\sin(x) + \cos(x) + 2}\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

```
[Out] x + 2*ArcTan[(Cos[x] - Sin[x])/(2 + Cos[x] + Sin[x])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
```

```
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx &= \int \frac{1}{3+2\cos(x)+2\sin(x)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{5+4x+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, 4+2\tan\left(\frac{x}{2}\right)\right)\right)\\ &= x+2\tan^{-1}\left(\frac{\cos(x)-\sin(x)}{2+\cos(x)+\sin(x)}\right)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.
time = 0.02, size = 51, normalized size = 2.43

$$-\text{ArcTan}\left(\frac{\cos\left(\frac{x}{2}\right)}{2\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}\right)+\text{ArcTan}\left(\sec\left(\frac{x}{2}\right)\left(2\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
[Out] -ArcTan[Cos[x/2]/(2*Cos[x/2] + Sin[x/2])] + ArcTan[Sec[x/2]*(2*Cos[x/2] + S
in[x/2])]
```

Maple [**A**]
time = 0.11, size = 10, normalized size = 0.48

method	result	size
default	$2\arctan\left(2+\tan\left(\frac{x}{2}\right)\right)$	10
risch	$-i\ln\left(e^{ix}+\frac{1}{2}+\frac{i}{2}\right)+i\ln\left(e^{ix}+1+i\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(2+2*cot(x)+3*csc(x)),x,method=_RETURNVERBOSE)
[Out] 2*arctan(2+tan(1/2*x))
```

Maxima [**A**]
time = 0.48, size = 14, normalized size = 0.67

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1) + 2)
```

Fricas [A]
time = 2.78, size = 24, normalized size = 1.14

$$-\arctan\left(-\frac{3\,\cos\left(x\right)+3\,\sin\left(x\right)+4}{\cos\left(x\right)-\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="fricas")
```

```
[Out] -arctan(-(3*cos(x) + 3*sin(x) + 4)/(cos(x) - sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{2\cot\left(x\right)+3\csc\left(x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x)
```

```
[Out] Integral(csc(x)/(2*cot(x) + 3*csc(x) + 2), x)
```

Giac [A]
time = 0.42, size = 22, normalized size = 1.05

$$2\,\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor+2\,\arctan\left(\tan\left(\frac{1}{2}\,x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="giac")
```

```
[Out] 2*pi*floor(1/2*x/pi + 1/2) + 2*arctan(tan(1/2*x) + 2)
```

Mupad [B]
time = 3.14, size = 9, normalized size = 0.43

$$2\,\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(2*cot(x) + 3/sin(x) + 2)),x)
```

```
[Out] 2*atan(tan(x/2) + 2)
```

3.462 $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=371

$$\frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}+\frac{2(a^2-b^2+c^2)\,(a}{}$$

```
[Out] -2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(a*cos(e*x+d)-c*sin(e*x+d))/e/csc(
e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*x+
d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/
2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c
^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))/e/csc(e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*
sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arc
tan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1
/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/csc(e*x+d)^(3
/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

Rubi [A]
time = 0.29, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3247, 3199, 3228, 3198, 2732, 3206, 2740}

$$\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2}+\frac{8b(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2(a\cos(d+ex)-c\sin(d+ex))(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}{3e\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(
3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(a*Cos[d +
e*x] - c*Sin[d + e*x]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3247

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
), Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\int(b+c\cos(d+ex)+}{\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex)}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex)}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex)}$$

$$= \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan\right.}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos}{}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.31, size = 2490, normalized size = 6.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

```
[Out] ((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((8*b*c)/(3*a) - (2*a*Cos[d +
e*x])/3 + (2*c*Sin[d + e*x])/3))/(e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x]
```

```
+ a*Sin[d + e*x])) + (4*a*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*(-(
(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b +
 Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b
/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]
*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]
*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 +
c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - (
(2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a
*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(3/2)) + (4*b*c^2*(a + c*Cot[d + e*x] + b*Csc[d + e*
x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*
Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt
[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos
[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^
2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c
*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)
/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^
2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + S
qrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*a*e*Csc[d + e*x]^(3/2)*
(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*a*AppellF1[1/2, 1/2, 1/2,
 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^
2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*
(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt
[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/
a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^
2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*Sqrt[1 +
 c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
 + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e
*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -(
(b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(
-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2
)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[
d + e*x] + a*Sin[d + e*x])^(3/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((
b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1
```

```
 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + Arc
Tan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*(a + c*Co
t[d + e*x] + b*Csc[d + e*x])^(3/2)*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[
(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b +
 a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + A
rcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d
+ e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*a*Sqrt[1 + c^2/a^
2]*e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 5.53, size = 20867, normalized size = 56.25

method	result	size
default	Expression too large to display	20867

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(x*e + d) + b*csc(x*e + d) + a)^(3/2)/csc(x*e + d)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.66, size = 1525, normalized size = 4.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/9*((3*I*a^3 + I*a*b^2 + 3*I*a*c^2 - 3*c^3 - (3*a^2 + b^2)*c)*sqrt(-2*I*a
- 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 -
 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*
b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3
```

```
*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2
*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2
+ I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (-3*I*a^3 - I*a*b^2 - 3*I*a*c^2 - 3*c
^3 - (3*a^2 + b^2)*c)*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*
a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2
*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 +
2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*
b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2
+ c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + 12*(a
^2*b + b*c^2)*sqrt(-2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4
*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b
 + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a
^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b
^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9
*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)
*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c
^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 12*(a^2*
b + b*c^2)*sqrt(2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2
*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4)
, -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*
b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 +
3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 +
4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
 c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b
 + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a
^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos
(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 6*(a^2*c + c^3
 - (a^2*c + c^3)*cos(x*e + d)^2 - (a^3 + a*c^2)*cos(x*e + d)*sin(x*e + d))*
sqrt((c*cos(x*e + d) + a*sin(x*e + d) + b)/sin(x*e + d))/sqrt(sin(x*e + d))
)*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\csc(d + ex) + c\cot(d + ex))^{\frac{3}{2}}}{\csc^{\frac{3}{2}}(d + ex)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/csc(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2)/csc(d + e*x)**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2),x)
```

```
[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2), x)
```

3.463 $\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/e/csc(e*x
+d)^(1/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3247, 3198, 2732}

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
```

```
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3247

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
), Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int \sqrt{b+c\cos(d+ex)+ \cdots}}{\sqrt{\csc(d+ex)}\sqrt{b+c\cos(d+ex)+a\sin(d+ \cdots}}$$

$$= \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\cdots}}{\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+\cdots}{b+\sqrt{a^2+\cdots}}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,\cdots\right.}{e\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+\cdots}{b+\sqrt{a^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.15, size = 1580, normalized size = 13.39

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

```
[Out] (2*c*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])/(a*e*Sqrt[Csc[d + e*x]]) +
(a*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -1/
2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^
2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d
+ e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))
```

```
]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)
/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^
2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]
]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Arc
Tan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]
*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]]
)/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]]]))/(e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) +
 (c^2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2,
-1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 +
 a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos
[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*
c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c
^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[
(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/
c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x -
ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c
^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/
c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcT
an[a/c]]]))/(a*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x
]]) + (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d +
e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -
((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*
(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]
*Sec[d + e*x + ArcTan[c/a]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c
^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
+ a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c
^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt
[(a^2 + c^2)/a^2])])/(a*Sqrt[1 + c^2/a^2]*e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*C
os[d + e*x] + a*Sin[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 1.77, size = 12367, normalized size = 104.81

method	result	size
risch	Expression too large to display	1862
default	Expression too large to display	12367

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(x*e + d) + b*csc(x*e + d) + a)/sqrt(csc(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.77, size = 1368, normalized size = 11.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*((I*a*b - b*c)*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^
2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a
^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2
*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b
^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2
+ c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (-I*a*
b - b*c)*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b
^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^
4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b +
4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6
+ 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*
e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + 3*(a^2 + c^2)*sqrt
(-2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c
^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*
a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*
I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3
*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6
*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*
(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^
3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c
^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) -
 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 3*(a^2 + c^2)*sqrt(2*I*a -
 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^
4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*
I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b
 - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 +
```

```
c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 -
3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b
- 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*
a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*
c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 -
I*c^2)*sin(x*e + d))/(a^2 + c^2))))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\csc(d + ex) + c\cot(d + ex)}}{\sqrt{\csc(d + ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/csc(e*x+d)**(1/2),x)
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))/sqrt(csc(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + c\cot(d + e\,x) + \frac{b}{\sin(d + e\,x)}}}{\sqrt{\frac{1}{\sin(d + e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2),x)
[Out] int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2), x)
```

3.464 $\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{\csc(d+ex)}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*csc(e*x+d)^(1/2)*((b+c*cos(e*x+d)+a*sin(e*x+d
))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3247, 3206, 2740}

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

```
[Out] (2*Sqrt[Csc[d + e*x]]*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c
^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b +
 Sqrt[a^2 + c^2])])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3247

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
), Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{\left(\sqrt{\csc(d+ex)}\,\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\right)\int \frac{}{\sqrt{b+}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+e}}$$

$$= \frac{\left(\sqrt{\csc(d+ex)}\,\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\right)\int \frac{}{\sqrt{\frac{}{b-}}}}{\sqrt{a+c\cot(d+ex)+}}$$

$$= \frac{2\sqrt{\csc(d+ex)}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b}{}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 0.52, size = 339, normalized size = 2.87

$$\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)}{b-a\sqrt{1+\frac{c^2}{a^2}}},\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)}{b+a\sqrt{1+\frac{c^2}{a^2}}}\right)\sqrt{\csc(d+ex)}\sec\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\sqrt{-\frac{a\sqrt{1+\frac{c^2}{a^2}}\left(-1+\sin\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)\right)}{b+a\sqrt{1+\frac{c^2}{a^2}}}}\sqrt{\frac{a\sqrt{1+\frac{c^2}{a^2}}\left(1+\sin\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)\right)}{-b+a\sqrt{1+\frac{c^2}{a^2}}}}\sqrt{b+a\sqrt{1+\frac{c^2}{a^2}}\sin\left(d+ex+\text{ArcTan}\left(\frac{c}{a}\right)\right)}}{a\sqrt{1+\frac{c^2}{a^2}}\,e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

```
[Out] (2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcT
an[c/a]])/(b - a*Sqrt[1 + c^2/a^2]), (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x +
 ArcTan[c/a]])/(b + a*Sqrt[1 + c^2/a^2])]*Sqrt[Csc[d + e*x]]*Sec[d + e*x +
ArcTan[c/a]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]*Sqrt[-((a*Sqrt[1 + c
^2/a^2]*(-1 + Sin[d + e*x + ArcTan[c/a]]))/(b + a*Sqrt[1 + c^2/a^2]))]*Sqrt
[(a*Sqrt[1 + c^2/a^2]*(1 + Sin[d + e*x + ArcTan[c/a]]))/(-b + a*Sqrt[1 + c^
2/a^2])]*Sqrt[b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]]])/(a*Sqrt[
1 + c^2/a^2]*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 2.03, size = 701, normalized size = 5.94

method	result
default	$-\frac{4i\,\mathrm{EllipticF}\left(\sqrt{-\frac{(i\sin(ex+d)+\cos(ex+d))\left(-ib+ic+\sqrt{a^2-b^2+c^2}+a\right)}{ib-ic+\sqrt{a^2-b^2+c^2}+a}},\sqrt{\frac{\left(ib-ic+\sqrt{a^2-b^2+c^2}+a\right)\left(ib-\right.}{\left(-ib+ic+\sqrt{a^2-b^2+c^2}+a\right)\left(-ib\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4*I/e*EllipticF((-(I*sin(e*x+d)+cos(e*x+d))*(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+
a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*
(I*b-I*c+(a^2-b^2+c^2)^(1/2)-a)/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)/(-I*b+I*c+
(a^2-b^2+c^2)^(1/2)-a))^(1/2))*(1/sin(e*x+d))^(1/2)*((b+c*cos(e*x+d)+a*sin(
e*x+d))/sin(e*x+d))^(1/2)*(I*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-a*cos(e*x+d)-b
*sin(e*x+d)+c*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-a)/(I*cos(e*x+d)+sin(e*x+d)+I)
/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)-a))^(1/2)*(I*(cos(e*x+d)*(a^2-b^2+c^2)^(1/2)
+a*cos(e*x+d)+b*sin(e*x+d)-c*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)+a)/(I*cos(e*x+d
)+sin(e*x+d)+I)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(-(I*sin(e*x+d)+cos(
e*x+d))*(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(
1/2)*(cos(e*x+d)+1)^2*(cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*
a*sin(e*x+d)-I*b*cos(e*x+d)+I*c*cos(e*x+d)-cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-b
*sin(e*x+d)+c*sin(e*x+d)-a*cos(e*x+d))/sin(e*x+d)^3/(b+c*cos(e*x+d)+a*sin(e
*x+d))/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(csc(x*e + d))/sqrt(c*cot(x*e + d) + b*csc(x*e + d) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 511, normalized size = 4.33

$$\frac{\left((i a - c)\sqrt{-2 i a - 2 c}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(3 a^4-4 a^2 b^2+4 b^2 c^2+6 i a c^3-3 c^4+2 i\left(3 a^3-4 a b^2\right) c\right)}{3\left(a^4+2 a^2 c^2+c^4\right)}, -\frac{8\left(-9 i a^5 b+8 i a^3 b^3+27 i a b c^4-9 b c^5+2\left(9 a^2 b+4 b^3\right) c^3+6 i\left(3 a^3 b-4 a b^3\right) c^2+3\left(9 a^4 b-8 a^2 b^3\right) c\right)}{27\left(a^6+3 a^4 c^2+3 a^2 c^4+c^6\right)}, \frac{-2 i a b+2 b c+3\left(a^2+c^2\right)\cos(xe+d)-3\left(i a^2+i c^2\right)\sin(xe+d)}{3\left(a^2+c^2\right)}\right) + \sqrt{2 i a-2 c}\,(-i a-c)\,\mathrm{weierstrassPInverse}\left(\frac{4\left(3 a^4-4 a^2 b^2+4 b^2 c^2-6 i a c^3-3 c^4-2 i\left(3 a^3-4 a b^2\right) c\right)}{3\left(a^4+2 a^2 c^2+c^4\right)}, -\frac{8\left(9 i a^5 b-8 i a^3 b^3-27 i a b c^4-9 b c^5+2\left(9 a^2 b+4 b^3\right) c^3-6 i\left(3 a^3 b-4 a b^3\right) c^2+3\left(9 a^4 b-8 a^2 b^3\right) c\right)}{27\left(a^6+3 a^4 c^2+3 a^2 c^4+c^6\right)}, \frac{2 i a b+2 b c+3\left(a^2+c^2\right)\cos(xe+d)-3\left(-i a^2-i c^2\right)\sin(xe+d)}{3\left(a^2+c^2\right)}\right)\right) e^{(-1)}}{a^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="fricas")
```

```
[Out] ((I*a - c)*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 +
4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
 c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*
b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*c
os(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2*I*a - 2
*c)*(-I*a - c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I
*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9
*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 -
 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2
+ 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(
-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)))*e^(-1)/(a^2 + c^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(d + e*x))/sqrt(a + b*csc(d + e*x) + c*cot(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sin(d+e\,x)}}}{\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
[Out] int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

3.465 $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

Optimal. Leaf size=240

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}-\frac{2\csc^{\frac{3}{2}}(d+e}{}$$

```
[Out] -2*csc(e*x+d)^(3/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d
))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)-2*csc(e*x+d)^(3/2)*(
cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,
a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(
b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^2)/e/
(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+
c^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3247, 3207, 3198, 2732}

$$-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\,(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2\csc^{\frac{3}{2}}(d+ex)(a\cos(d+ex)-c\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))}{e\,(a^2-b^2+c^2)\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

```
[Out] (-2*Csc[d + e*x]^(3/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2
 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*Csc[d + e*x]^(3/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(
(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3247

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
), Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = \frac{\left(\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\right)\int \frac{}{(b+c\,\mathrm{co}}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

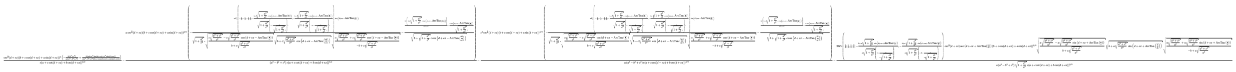
$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= -\frac{2\csc^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sqrt{\frac{b}{}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 6.29, size = 1732, normalized size = 7.22



Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
[Out] (Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*((-2*(a^2 + c^2
))/(a*c*(a^2 - b^2 + c^2)) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x])
)/(c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (a*Csc[d + e*x]^(3/2)*(b + c*Cos[d +
 e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b +
Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(
Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x -
 ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[
(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]
*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[
(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b
+ c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
- ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/((a^2
- b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (c^2*Csc[d +
e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2,
 -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(S
qrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2
]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^
2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(
a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b +
c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - Ar
cTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d +
 e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1
+ a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
 - ArcTan[a/c]]]))/(a*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e
*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*
Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2
])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c
^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(3/2)*Sec[d + e*x +
ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*Sqrt[(a*Sqrt[(a^2
+ c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sq
rt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan
[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x
 + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)*Sqrt
[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.88, size = 12477, normalized size = 51.99

method	result	size
default	Expression too large to display	12477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(x*e + d)^(3/2)/(c*cot(x*e + d) + b*csc(x*e + d) + a)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.96, size = 1739, normalized size = 7.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*((-I*a*b^2 + b^2*c + (-I*a*b*c + b*c^2)*cos(x*e + d) + (-I*a^2*b + a*b*
c)*sin(x*e + d))*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*
b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2
*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(
9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3
)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 +
c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (I*a*b^2
 + b^2*c + (I*a*b*c + b*c^2)*cos(x*e + d) + (I*a^2*b + a*b*c)*sin(x*e + d))
*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 -
 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/2
7*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c
^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*
```

```
c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) -
 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 3*(a^2*b + b*c^2 + (a^2*c
+ c^3)*cos(x*e + d) + (a^3 + a*c^2)*sin(x*e + d))*sqrt(-2*I*a - 2*c)*weiers
trassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a
^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 +
 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)
*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weie
rstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*
I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3
*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*
a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)
, 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*si
n(x*e + d))/(a^2 + c^2))) - 3*(a^2*b + b*c^2 + (a^2*c + c^3)*cos(x*e + d) +
 (a^3 + a*c^2)*sin(x*e + d))*sqrt(2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 -
 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
+ 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5
 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a
^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*
a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/
(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*
b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b
- 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c +
 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))
) + 6*(a^2*c + c^3 - (a^2*c + c^3)*cos(x*e + d)^2 - (a^3 + a*c^2)*cos(x*e +
 d)*sin(x*e + d))*sqrt((c*cos(x*e + d) + a*sin(x*e + d) + b)/sin(x*e + d))/
sqrt(sin(x*e + d)))/((c^5 + (2*a^2 - b^2)*c^3 + (a^4 - a^2*b^2)*c)*cos(x*e
+ d)*e + (a^5 - a^3*b^2 + a*c^4 + (2*a^3 - a*b^2)*c^2)*e*sin(x*e + d) + (a^
4*b - a^2*b^3 + b*c^4 + (2*a^2*b - b^3)*c^2)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+b\csc(d+ex)+c\cot(d+ex))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2),x)
```

```
[Out] Integral(csc(d + e*x)**(3/2)/(a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}{\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
```

```
[Out] int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

3.466 $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

Optimal. Leaf size=492

$$\frac{8b\csc^{\frac{5}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^3}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}+\frac{2\csc^{\frac{5}{2}}(d+ex)\cdots}{\cdots}$$

[Out] -2/3*csc(e*x+d)^(5/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*csc(e*x+d)^(5/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-b^2+c^2)^2/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*b*csc(e*x+d)^(5/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*csc(e*x+d)^(5/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)

Rubi [A]
time = 0.34, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3247, 3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\frac{2\csc^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^3E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e\left(a^2-b^2+c^2\right)^2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8\csc^{\frac{5}{2}}(d+ex)(ab\cos(d+ex)-bc\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))^2}{3e\left(a^2-b^2+c^2\right)^2(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}-\frac{2\csc^{\frac{5}{2}}(d+ex)(a\cos(d+ex)-c\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))}{3e\left(a^2-b^2+c^2\right)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]

[Out] (8*b*Csc[d + e*x]^(5/2)*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*Csc[d + e*x]^(5/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) - (2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 +

```
c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (8*Csc[d + e*x]^(5/2)
*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*
x]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rule 3247

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Dist[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
), Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \frac{\left(\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\right)\int \frac{}{(b+c\,\mathrm{co}}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= -\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+}$$

$$= \frac{8b\csc^{\frac{5}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b}{3(a^2-b^2+c^2)^2\,e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sqrt{\frac{b}{-}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.32, size = 2708, normalized size = 5.50

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

```
[Out] (Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3*((8*b*(a^2 + c^
2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d +
 e*x]))/(3*c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2) - (
2*(a^3 + 3*a*b^2 + a*c^2 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x]))/(3
*c*(a^2 - b^2 + c^2)^2*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*a*b*Csc[d + e*x]^(5/2)*(b + c*Cos
[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((
```

b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*b*c^2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*a*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*a*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(3*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])/(a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (2*c^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2

```
/a^2]*Sin[d + e*x + ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 +
c^2/a^2]))))]*Csc[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[c/a]]*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(5/2)*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2 +
c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*Sqrt[b
 + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]]]*Sqrt[(a*Sqrt[(a^2 +
c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Sin[d + e*x + ArcTan[c/a]])/(-b + a*Sqr
t[(a^2 + c^2)/a^2])])/(3*a*(a^2 - b^2 + c^2)^2*Sqrt[1 + c^2/a^2]*e*(a + c*C
ot[d + e*x] + b*Csc[d + e*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.63, size = 64203, normalized size = 130.49

method	result	size
default	Expression too large to display	64203

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(csc(x*e + d)^(5/2)/(c*cot(x*e + d) + b*csc(x*e + d) + a)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.39, size = 2820, normalized size = 5.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/9*((3*I*a^5 + 4*I*a^3*b^2 + I*a*b^4 - 3*(a^2 + b^2)*c^3 + 3*I*(a^3 + a*b
^2)*c^2 + (-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3*I*a*c^4 - 3*c^5
 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 - (3*a^4 + 4*a^2*b^2 + b^4)*c - 2*(-
3*I*a*b*c^3 + 3*b*c^4 + (3*a^2*b + b^3)*c^2 - I*(3*a^3*b + a*b^3)*c)*cos(x*
```

e + d) - 2*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*c^2 + 3*a*b*c^3 + (3*a^3*b + a*b^3)*c + (-3*I*a^2*c^3 + 3*a*c^4 + (3*a^3 + a*b^2)*c^2 - I*(3*a^4 + a^2*b^2)*c)*cos(x*e + d))*sin(x*e + d))*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (-3*I*a^5 - 4*I*a^3*b^2 - I*a*b^4 - 3*(a^2 + b^2)*c^3 - 3*I*(a^3 + a*b^2)*c^2 + (3*I*a^5 + I*a^3*b^2 - I*a*b^2*c^2 - b^2*c^3 - 3*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 - (3*a^4 + 4*a^2*b^2 + b^4)*c - 2*(3*I*a*b*c^3 + 3*b*c^4 + (3*a^2*b + b^3)*c^2 + I*(3*a^3*b + a*b^3)*c)*cos(x*e + d) - 2*(3*I*a^4*b + I*a^2*b^3 + 3*I*a^2*b*c^2 + 3*a*b*c^3 + (3*a^3*b + a*b^3)*c + (3*I*a^2*c^3 + 3*a*c^4 + (3*a^3 + a*b^2)*c^2 + I*(3*a^4 + a^2*b^2)*c)*cos(x*e + d))*sin(x*e + d))*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + 12*(a^4*b + a^2*b^3 + (a^2*b + b^3)*c^2 - (a^4*b - b*c^4)*cos(x*e + d)^2 + 2*(a^2*b^2*c + b^2*c^3)*cos(x*e + d) + 2*(a^3*b^2 + a*b^2*c^2 + (a^3*b*c + a*b*c^3)*cos(x*e + d))*sin(x*e + d))*sqrt(-2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 12*(a^4*b + a^2*b^3 + (a^2*b + b^3)*c^2 - (a^4*b - b*c^4)*cos(x*e + d)^2 + 2*(a^2*b^2*c + b^2*c^3)*cos(x*e + d) + 2*(a^3*b^2 + a*b^2*c^2 + (a^3*b*c + a*b*c^3)*cos(x*e + d))*sin(x*e + d))*sqrt(2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 6*(c^5 + (2*a^2 - 5*b^2)*c^3 - 4*(a^4

```
*b - b*c^4)*cos(x*e + d)^3 - (c^5 + (2*a^2 - 5*b^2)*c^3 + (a^4 - 5*a^2*b^2)
*c)*cos(x*e + d)^2 + (a^4 - 5*a^2*b^2)*c + 4*(a^4*b - b*c^4)*cos(x*e + d) -
 (4*a^3*b*c + 4*a*b*c^3 - 8*(a^3*b*c + a*b*c^3)*cos(x*e + d)^2 + (a^5 - 5*a
^3*b^2 + a*c^4 + (2*a^3 - 5*a*b^2)*c^2)*cos(x*e + d))*sin(x*e + d))*sqrt((c
*cos(x*e + d) + a*sin(x*e + d) + b)/sin(x*e + d))/sqrt(sin(x*e + d)))/((a^8
 - 2*a^6*b^2 + a^4*b^4 - c^8 - 2*(a^2 - b^2)*c^6 + (2*a^2*b^2 - b^4)*c^4 +
2*(a^6 - a^4*b^2)*c^2)*cos(x*e + d)^2*e - 2*(b*c^7 + (3*a^2*b - 2*b^3)*c^5
+ (3*a^4*b - 4*a^2*b^3 + b^5)*c^3 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*c)*cos(x*
e + d)*e - (a^8 - a^6*b^2 - a^4*b^4 + a^2*b^6 + (a^2 + b^2)*c^6 + (3*a^4 +
a^2*b^2 - 2*b^4)*c^4 + (3*a^6 - a^4*b^2 - 3*a^2*b^4 + b^6)*c^2)*e - 2*((a*c
^7 + (3*a^3 - 2*a*b^2)*c^5 + (3*a^5 - 4*a^3*b^2 + a*b^4)*c^3 + (a^7 - 2*a^5
*b^2 + a^3*b^4)*c)*cos(x*e + d)*e + (a^7*b - 2*a^5*b^3 + a^3*b^5 + a*b*c^6
+ (3*a^3*b - 2*a*b^3)*c^4 + (3*a^5*b - 4*a^3*b^3 + a*b^5)*c^2)*e)*sin(x*e +
 d))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)**(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorithm
="giac")
[Out] integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2), x
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{5/2}}{\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2),x)
[Out] int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2), x)
```

3.467 $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

Optimal. Leaf size=371

$$\frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sin^{\frac{3}{2}}(d+ex)}{3e(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}+\frac{2(a^2-b^2+$$

```
[Out] -2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2)*(a*cos(e*x+d)-c*s
in(e*x+d))/e/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*x+
d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/
2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c
^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sin(e*x+d)^(3/2)/e/(b+c*cos(e*x+d)+a*
sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)+2/3*(
a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arc
tan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1
/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2)
)*sin(e*x+d)^(3/2)*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2
)/e/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

Rubi [A]
time = 0.26, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3243, 3199, 3228, 3198, 2732, 3206, 2740}

$$\frac{2(a^2-b^2+c^2)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a\sin(d+ex)+b+c\cos(d+ex))^2}+\frac{8b\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a\sin(d+ex)+b+c\cos(d+ex))\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2\sin^{\frac{3}{2}}(d+ex)(a\cos(d+ex)-c\sin(d+ex))(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}{3e(a\sin(d+ex)+b+c\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] (8*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*EllipticE[(d + e*x - ArcTa
n[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2))/
(3*e*(b + c*Cos[d + e*x] + a*Sin[d + e*x])*Sqrt[(b + c*Cos[d + e*x] + a*Sin
[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*(a + c*Cot[d + e*
x] + b*Csc[d + e*x])^(3/2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^
2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(3*e*(b + c*Cos[d + e*x] + a*Si
n[d + e*x])^2) - (2*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x
]^(3/2)*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*e*(b + c*Cos[d + e*x] + a*Sin
[d + e*x]))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3199

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Dist[1/n, Int[Simp[n*a^2 + (n
 - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x]
, x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
```

```
b - a*B, 0]
```

Rule 3243

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx = \frac{\left((a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+e\right.}{(b+c\cos(d+ex)+}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+}{3e(b+c\cos(d+ex)+a\,\mathrm{s}}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+}{3e(b+c\cos(d+ex)+a\,\mathrm{s}}$$

$$= -\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+}{3e(b+c\cos(d+ex)+a\,\mathrm{s}}$$

$$= \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+\right.}{3e(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{}}$$

Mathematica [**F**]
time = 91.44, size = 0, normalized size = 0.00

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

```
[Out] Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2), x
]
```

Maple [C] Result contains complex when optimal does not.
time = 1.12, size = 20858, normalized size = 56.22

method	result	size
default	Expression too large to display	20858

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((c*cot(x*e + d) + b*csc(x*e + d) + a)^(3/2)*sin(x*e + d)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.03, size = 1513, normalized size = 4.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/9*(sqrt(2)*(3*a^3 + a*b^2 + 3*a*c^2 + 3*I*c^3 + I*(3*a^2 + b^2)*c)*sqrt(I
*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3
- 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5
*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(
3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^
2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2
 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2)*(3*a^3 + a*b^2 + 3*a*c^2 - 3
*I*c^3 - I*(3*a^2 + b^2)*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 -
 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
+ 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5
```

```
 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a
^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a
^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 12
*sqrt(2)*(I*a^2*b + I*b*c^2)*sqrt(I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 +
2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*
b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4
 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^
4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*
c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b -
8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c +
3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)))
- 12*sqrt(2)*(-I*a^2*b - I*b*c^2)*sqrt(-I*a + c)*weierstrassZeta(4/3*(3*a^4
 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^
4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c
^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8
*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(
3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c
)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 -
9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*
b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c
 + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2
))) - 6*((a^3 + a*c^2)*cos(x*e + d) - (a^2*c + c^3)*sin(x*e + d))*sqrt((c*c
os(x*e + d) + a*sin(x*e + d) + b)/sin(x*e + d))*sqrt(sin(x*e + d)))*e^(-1)/
(a^2 + c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)*sin(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(d+e\,x\right)^{3/2}\,{\left(a+c\,\mathrm{cot}\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
[Out] int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

3.468 $\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}$

Optimal. Leaf size=118

$$\frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sin(d+ex)}}{e\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d
)^(1/2)/e/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3243, 3198, 2732}

$$\frac{2\sqrt{\sin(d+ex)}\,\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

```
[Out] (2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[Sin[d + e*x]])/(e*S
qrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3243

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\,dx = \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\right.}{\sqrt{b+c\cos(d+ex)}}$$

$$= \frac{\left(\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\right.}{\sqrt{\frac{b+c}{}}}$$

$$= \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}(d+\right.}{e\sqrt{\frac{b+c\cos(d+e}{b+\sqrt{}}}}$$

Mathematica [F]
time = 7.89, size = 0, normalized size = 0.00

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

```
[Out] Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]], x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.85, size = 12365, normalized size = 104.79

method	result	size
risch	Expression too large to display	1851
default	Expression too large to display	12365

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(c*cot(x*e + d) + b*csc(x*e + d) + a)*sqrt(sin(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.38, size = 1378, normalized size = 11.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(a*b + I*b*c)*sqrt(I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4
*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5
+ 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^
2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a
^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqr
t(2)*(a*b - I*b*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^
2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c
^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a
^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)
*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) - 3*sqrt(2)*(
I*a^2 + I*c^2)*sqrt(I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2
*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4)
, -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4
*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 +
 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 +
```

```
 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
+ c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2
*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/
(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*
cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) - 3*sqrt(2)*(-
I*a^2 - I*c^2)*sqrt(-I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^
2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4
), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4
*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 +
 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 +
 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
+ c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*
b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*co
s(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))))*e^(-1)/(a^2 +
c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,\sqrt{\sin(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)*sin(e*x+d)**(1/2),x)
[Out] Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin(d+e\,x)}\,\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
[Out] int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

3.469 $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

Optimal. Leaf size=118

$$\frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

```
[Out] 2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2
)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(
1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3243, 3206, 2740}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),x]
```

```
[Out] (2*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
+ c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3243

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,\sqrt{\sin(d+ex)}}\,dx = \frac{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}\int \frac{}{\sqrt{b+c}}}{\sqrt{a+c\cot(d+ex)+b\csc(d+e}}$$

$$= \frac{\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}\int \frac{}{\sqrt{\overline{b+}}}}{\sqrt{a+c\cot(d+ex)+}}$$

$$= \frac{2F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.07, size = 519, normalized size = 4.40

$$\frac{4\left(-ia-b+c+i\sqrt{a^2-b^2+c^2}\right)F\left(\text{ArcSin}\left(\sqrt{\frac{\left(-a-ib+ic+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{-a+ib-ic+\sqrt{a^2-b^2+c^2}}}\right)|\frac{ib+\sqrt{a^2-b^2+c^2}}{ib-\sqrt{a^2-b^2+c^2}}\right)\sqrt{\frac{\left(-a-ib+ic+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{-a+ib-ic+\sqrt{a^2-b^2+c^2}}}(\cos(d+ex)+i\sin(d+ex))\sqrt{-\frac{i\left(a+\sqrt{a^2-b^2+c^2}+(b-c)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(a-ib+ic+\sqrt{a^2-b^2+c^2}\right)\left(-i+\tan\left(\frac{1}{2}(d+ex)\right)\right)}}\sqrt{-\frac{i\left(-a+\sqrt{a^2-b^2+c^2}+(-b+c)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{\left(-a+ib-ic+\sqrt{a^2-b^2+c^2}\right)\left(-i+\tan\left(\frac{1}{2}(d+ex)\right)\right)}}}{\left(a+ib-ic-\sqrt{a^2-b^2+c^2}\right)e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),
x]
```

```
[Out] (4*((-I)*a - b + c + I*Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[((-a -
I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a +
I*b - I*c + Sqrt[a^2 - b^2 + c^2])]], (I*b + Sqrt[a^2 - b^2 + c^2])/(I*b -
Sqrt[a^2 - b^2 + c^2])]*Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Co
s[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])]*(Co
s[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(a + Sqrt[a^2 - b^2 + c^2] + (b - c
)*Tan[(d + e*x)/2]))/((a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d
+ e*x)/2]))]*Sqrt[((-I)*(-a + Sqrt[a^2 - b^2 + c^2] + (-b + c)*Tan[(d + e*x
)/2]))/((-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*x)/2]))])
```

```
/((a + I*b - I*c - Sqrt[a^2 - b^2 + c^2])*e*Sqrt[a + c*Cot[d + e*x] + b*Csc
[d + e*x]]*Sqrt[Sin[d + e*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.86, size = 695, normalized size = 5.89

method	result
default	$\frac{4i\sqrt{\frac{b+c\cos(ex+d)+a\sin(ex+d)}{\sin(ex+d)}}\sqrt{\frac{i\left(\cos(ex+d)\sqrt{a^2-b^2+c^2}-a\cos(ex+d)-b\sin(ex+d)+c\sin(ex+d)+\sqrt{a^2-b^2+c^2}\right.}{(i\cos(ex+d)+\sin(ex+d)+i)\left(-ib+ic+\sqrt{a^2-b^2+c^2}-a\right)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 4*I/e*((b+c*cos(e*x+d)+a*sin(e*x+d))/sin(e*x+d))^(1/2)*(I*(cos(e*x+d)*(a^2-
b^2+c^2)^(1/2)-a*cos(e*x+d)-b*sin(e*x+d)+c*sin(e*x+d)+(a^2-b^2+c^2)^(1/2)-a
)/(I*cos(e*x+d)+sin(e*x+d)+I)/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)-a))^(1/2)*(I*(c
os(e*x+d)*(a^2-b^2+c^2)^(1/2)+a*cos(e*x+d)+b*sin(e*x+d)-c*sin(e*x+d)+(a^2-b
^2+c^2)^(1/2)+a)/(I*cos(e*x+d)+sin(e*x+d)+I)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a
))^(1/2)*(-(I*sin(e*x+d)+cos(e*x+d))*(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)/(I*b-
I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(cos(e*x+d)+1)^2*EllipticF((-(I*sin(e*x+d
)+cos(e*x+d))*(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)
+a))^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)-a)
/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)+a)/(-I*b+I*c+(a^2-b^2+c^2)^(1/2)-a))^(1/2))*
(cos(e*x+d)-1)^2*(I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*a*sin(e*x+d)-I*b*cos(e
*x+d)+I*c*cos(e*x+d)-cos(e*x+d)*(a^2-b^2+c^2)^(1/2)-b*sin(e*x+d)+c*sin(e*x+
d)-a*cos(e*x+d))/sin(e*x+d)^(7/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))/(I*b-I*c-(a
^2-b^2+c^2)^(1/2)-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/(sqrt(c*cot(x*e + d) + b*csc(x*e + d) + a)*sqrt(sin(x*e + d))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 509, normalized size = 4.31

$$\frac{\left(\sqrt{2}\,(a+i\,c)\sqrt{i\,a+c}\ \mathrm{weierstrassPInverse}\left(\frac{4\left(3\,a^4-4\,a^2b^2+4\,b^2c^2+6i\,ac^3-3\,c^4+2i\left(3\,a^3-4\,ab^2\right)c\right)}{3\left(a^4+2\,a^2c^2+c^4\right)},-\frac{8\left(-9i\,a^5b+8i\,a^3b^3+27i\,abc^4-9\,bc^5+2\left(9\,a^2b+4\,b^3\right)c^3+6i\left(3\,a^3b-4\,ab^3\right)c^2+3\left(9\,a^4b-8\,a^2b^3\right)c\right)}{27\left(a^6+3\,a^4c^2+3\,a^2c^4+c^6\right)},\frac{-2i\,ab+2\,bc+3\left(a^2+c^2\right)\cos(xe+d)-3\left(i\,a^2+i\,c^2\right)\sin(xe+d)}{3\left(a^2+c^2\right)}\right)+\sqrt{2}\,(a-i\,c)\sqrt{-i\,a+c}\ \mathrm{weierstrassPInverse}\left(\frac{4\left(3\,a^4-4\,a^2b^2+4\,b^2c^2-6i\,ac^3-3\,c^4-2i\left(3\,a^3-4\,ab^2\right)c\right)}{3\left(a^4+2\,a^2c^2+c^4\right)},-\frac{8\left(9i\,a^5b-8i\,a^3b^3-27i\,abc^4-9\,bc^5+2\left(9\,a^2b+4\,b^3\right)c^3-6i\left(3\,a^3b-4\,ab^3\right)c^2+3\left(9\,a^4b-8\,a^2b^3\right)c\right)}{27\left(a^6+3\,a^4c^2+3\,a^2c^4+c^6\right)},\frac{2i\,ab+2\,bc+3\left(a^2+c^2\right)\cos(xe+d)-3\left(-i\,a^2-i\,c^2\right)\sin(xe+d)}{3\left(a^2+c^2\right)}\right)\right)e^{(-1)}}{a^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="fricas")
```

```
[Out] (sqrt(2)*(a + I*c)*sqrt(I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2
 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^
2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a
^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2
)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2)) + sqrt(2)*(a
- I*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^
2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -
8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3
)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a
^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d
) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)))*e^(-1)/(a^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\csc(d + ex) + c\cot(d + ex)}\,\sqrt{\sin(d + ex)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/sin(e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(d + e\,x)}\,\sqrt{a + c\cot(d + e\,x) + \frac{b}{\sin(d + e\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)),x)
[Out] int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)), x)
```

3.470 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

Optimal. Leaf size=240

$$-\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}-\frac{2}{}$$

```
[Out] -2*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2)/
e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)-2*(cos(1/2*d+1/2*e*x
-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)
))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b
*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+
c^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3243, 3207, 3198, 2732}

$$-\frac{2(a\sin(d+ex)+b+c\cos(d+ex))^2E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}-\frac{2(a\cos(d+ex)-c\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))}{e(a^2-b^2+c^2)\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)),x]
```

```
[Out] (-2*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)/((a^2 - b^2 + c^2)*e*(a
+ c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)*Sqrt[(b + c*Cos
[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e
*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
```

```
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3207

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Dist[1/(a^2 - b^
2 - c^2), Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3243

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}\int \frac{}{(b+c\cos(d+}}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+e}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+e}$$

$$= -\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2}}{b+\sqrt{a^2}}\right.}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+e}$$

Mathematica [F]
time = 16.76, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)
), x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.77, size = 12464, normalized size = 51.93

method	result	size
default	Expression too large to display	12464

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(x*e + d) + b*csc(x*e + d) + a)^(3/2)*sin(x*e + d)^(3/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.51, size = 1773, normalized size = 7.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="fricas")
```

```
[Out] -1/3*((sqrt(2)*(a*b*c + I*b*c^2)*cos(x*e + d) + sqrt(2)*(a^2*b + I*a*b*c)*s
in(x*e + d) + sqrt(2)*(a*b^2 + I*b^2*c))*sqrt(I*a + c)*weierstrassPInverse(
4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b
^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*
c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(
```

```
9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b
+ 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2
 + c^2)) + (sqrt(2)*(a*b*c - I*b*c^2)*cos(x*e + d) + sqrt(2)*(a^2*b - I*a*b
*c)*sin(x*e + d) + sqrt(2)*(a*b^2 - I*b^2*c))*sqrt(-I*a + c)*weierstrassPIn
verse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 -
 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I
*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2
+ 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*
a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))
/(a^2 + c^2)) + 3*(sqrt(2)*(-I*a^2*c - I*c^3)*cos(x*e + d) + sqrt(2)*(-I*a^
3 - I*a*c^2)*sin(x*e + d) + sqrt(2)*(-I*a^2*b - I*b*c^2))*sqrt(I*a + c)*wei
erstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(
3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^
3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b
^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), w
eierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 +
 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*
a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b -
 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c
^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)
*sin(x*e + d))/(a^2 + c^2))) + 3*(sqrt(2)*(I*a^2*c + I*c^3)*cos(x*e + d) +
sqrt(2)*(I*a^3 + I*a*c^2)*sin(x*e + d) + sqrt(2)*(I*a^2*b + I*b*c^2))*sqrt(
-I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 -
3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b
- 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a
^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c
^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c
^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a
^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I
*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*
a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(-I*a
^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 6*((a^3 + a*c^2)*cos(x*e + d) - (
a^2*c + c^3)*sin(x*e + d))*sqrt((c*cos(x*e + d) + a*sin(x*e + d) + b)/sin(x
*e + d))*sqrt(sin(x*e + d)))/((c^5 + (2*a^2 - b^2)*c^3 + (a^4 - a^2*b^2)*c)
*cos(x*e + d)*e + (a^5 - a^3*b^2 + a*c^4 + (2*a^3 - a*b^2)*c^2)*e*sin(x*e +
 d) + (a^4*b - a^2*b^3 + b*c^4 + (2*a^2*b - b^3)*c^2)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/sin(e*x+d)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(d+e\,x\right)^{3/2}\,{\left(a+c\,\mathrm{cot}\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)),x)
```

```
[Out] int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)), x)
```

3.471 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

Optimal. Leaf size=492

$$\frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^3}{3(a^2-b^2+c^2)^2e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}+\text{–}$$

```
[Out] -2/3*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2
)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*(b+c*cos(e*x+d
)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-b^2+c^2)^2/e/(a+c*co
t(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-1/2*
arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1
/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+c*cot(e*x+d)+b*cs
c(e*x+d))^(5/2)/sin(e*x+d)^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2
)^(1/2)))^(1/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/
2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d
))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a^2-b^2+c^2
)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)
```

Rubi [A]
time = 0.35, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3243, 3208, 3235, 3228, 3198, 2732, 3206, 2740}

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a\sin(d+ex)+b+c\cos(d+ex))^2F\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8b(a\sin(d+ex)+b+c\cos(d+ex))^3E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3e(a^2-b^2+c^2)^2\sin^{\frac{5}{2}}(d+ex)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}+\frac{8(ab\cos(d+ex)-bc\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))^2}{3e(a^2-b^2+c^2)^2\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}-\frac{2(a\cos(d+ex)-c\sin(d+ex))(a\sin(d+ex)+b+c\cos(d+ex))}{3e(a^2-b^2+c^2)\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)),x]
```

```
[Out] (8*b*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^
2 + c^2])]*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3)/(3*(a^2 - b^2 + c^2)^2*
e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)*Sqrt[(b +
c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*EllipticF[(d
+ e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*(b + c*
Cos[d + e*x] + a*Sin[d + e*x])^2*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])
/(b + Sqrt[a^2 + c^2])])/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc
[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) - (2*(b + c*Cos[d + e*x] + a*Sin[d + e
*x])*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(a + c*Cot[d
 + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)) + (8*(b + c*Cos[d + e*x
```

```
] + a*Sin[d + e*x])^2*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/(3*(a^2 - b^2
+ c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3208

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rule 3243

```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Dist[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
, Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d,
 e}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \frac{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}\int \frac{}{(b+c\cos(d}}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\,\mathrm{s}}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d}$$

$$= -\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d}$$

$$= \frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a}}{b+\sqrt{}}\right.}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+}$$

Mathematica [F]
time = 31.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
),x]
```

```
[Out] Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)
), x]
```

Maple [C] Result contains complex when optimal does not.
time = 1.31, size = 64185, normalized size = 130.46

method	result	size
default	Expression too large to display	64185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((c*cot(x*e + d) + b*csc(x*e + d) + a)^(5/2)*sin(x*e + d)^(5/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.10, size = 2839, normalized size = 5.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="fricas")
```

```
[Out] 1/9*((sqrt(2)*(3*a^5 + a^3*b^2 - a*b^2*c^2 - I*b^2*c^3 - 3*a*c^4 - 3*I*c^5
+ I*(3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 - 2*sqrt(2)*(3*a*b*c^3 + 3*I*b*c^4
+ I*(3*a^2*b + b^3)*c^2 + (3*a^3*b + a*b^3)*c)*cos(x*e + d) - 2*(sqrt(2)*(3
*a^2*c^3 + 3*I*a*c^4 + I*(3*a^3 + a*b^2)*c^2 + (3*a^4 + a^2*b^2)*c)*cos(x*e
 + d) + sqrt(2)*(3*a^4*b + a^2*b^3 + 3*a^2*b*c^2 + 3*I*a*b*c^3 + I*(3*a^3*b
 + a*b^3)*c))*sin(x*e + d) - sqrt(2)*(3*a^5 + 4*a^3*b^2 + a*b^4 + 3*I*(a^2
+ b^2)*c^3 + 3*(a^3 + a*b^2)*c^2 + I*(3*a^4 + 4*a^2*b^2 + b^4)*c))*sqrt(I*a
 + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 -
3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b
 + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*
a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*
c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(x*e + d) - 3*(I*a^2 +
 I*c^2)*sin(x*e + d))/(a^2 + c^2)) + (sqrt(2)*(3*a^5 + a^3*b^2 - a*b^2*c^2
+ I*b^2*c^3 - 3*a*c^4 + 3*I*c^5 - I*(3*a^4 + a^2*b^2)*c)*cos(x*e + d)^2 - 2
```

sqrt(2)(3*a*b*c^3 - 3*I*b*c^4 - I*(3*a^2*b + b^3)*c^2 + (3*a^3*b + a*b^3)
*c)*cos(x*e + d) - 2*(sqrt(2)*(3*a^2*c^3 - 3*I*a*c^4 - I*(3*a^3 + a*b^2)*c^
2 + (3*a^4 + a^2*b^2)*c)*cos(x*e + d) + sqrt(2)*(3*a^4*b + a^2*b^3 + 3*a^2*
b*c^2 - 3*I*a*b*c^3 - I*(3*a^3*b + a*b^3)*c))*sin(x*e + d) - sqrt(2)*(3*a^5
 + 4*a^3*b^2 + a*b^4 - 3*I*(a^2 + b^2)*c^3 + 3*(a^3 + a*b^2)*c^2 - I*(3*a^4
 + 4*a^2*b^2 + b^4)*c))*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2
*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b
^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 +
 c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2)) + 12*(sq
rt(2)*(-I*a^4*b + I*b*c^4)*cos(x*e + d)^2 + 2*sqrt(2)*(I*a^2*b^2*c + I*b^2*
c^3)*cos(x*e + d) + 2*(sqrt(2)*(I*a^3*b*c + I*a*b*c^3)*cos(x*e + d) + sqrt(
2)*(I*a^3*b^2 + I*a*b^2*c^2))*sin(x*e + d) + sqrt(2)*(I*a^4*b + I*a^2*b^3 +
 I*(a^2*b + b^3)*c^2))*sqrt(I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2
 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^
2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a
^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 +
2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*
b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2
 + c^2)*cos(x*e + d) - 3*(I*a^2 + I*c^2)*sin(x*e + d))/(a^2 + c^2))) + 12*(
sqrt(2)*(I*a^4*b - I*b*c^4)*cos(x*e + d)^2 + 2*sqrt(2)*(-I*a^2*b^2*c - I*b^
2*c^3)*cos(x*e + d) + 2*(sqrt(2)*(-I*a^3*b*c - I*a*b*c^3)*cos(x*e + d) + sq
rt(2)*(-I*a^3*b^2 - I*a*b^2*c^2))*sin(x*e + d) + sqrt(2)*(-I*a^4*b - I*a^2*
b^3 - I*(a^2*b + b^3)*c^2))*sqrt(-I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2
*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b
^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4
 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^
5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*
a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(
a^2 + c^2)*cos(x*e + d) - 3*(-I*a^2 - I*c^2)*sin(x*e + d))/(a^2 + c^2))) +
6*(4*a^3*b*c + 4*a*b*c^3 - 8*(a^3*b*c + a*b*c^3)*cos(x*e + d)^2 + (a^5 - 5*
a^3*b^2 + a*c^4 + (2*a^3 - 5*a*b^2)*c^2)*cos(x*e + d) - (c^5 + (2*a^2 - 5*b
^2)*c^3 + (a^4 - 5*a^2*b^2)*c + 4*(a^4*b - b*c^4)*cos(x*e + d))*sin(x*e + d
))*sqrt((c*cos(x*e + d) + a*sin(x*e + d) + b)/sin(x*e + d))*sqrt(sin(x*e +
d)))/((a^8 - 2*a^6*b^2 + a^4*b^4 - c^8 - 2*(a^2 - b^2)*c^6 + (2*a^2*b^2 - b
^4)*c^4 + 2*(a^6 - a^4*b^2)*c^2)*cos(x*e + d)^2*e - 2*(b*c^7 + (3*a^2*b - 2
*b^3)*c^5 + (3*a^4*b - 4*a^2*b^3 + b^5)*c^3 + (a^6*b - 2*a^4*b^3 + a^2*b^5)
*c)*cos(x*e + d)*e - (a^8 - a^6*b^2 - a^4*b^4 + a^2*b^6 + (a^2 + b^2)*c^6 +

```
(3*a^4 + a^2*b^2 - 2*b^4)*c^4 + (3*a^6 - a^4*b^2 - 3*a^2*b^4 + b^6)*c^2)*e
 - 2*((a*c^7 + (3*a^3 - 2*a*b^2)*c^5 + (3*a^5 - 4*a^3*b^2 + a*b^4)*c^3 + (a
^7 - 2*a^5*b^2 + a^3*b^4)*c)*cos(x*e + d)*e + (a^7*b - 2*a^5*b^3 + a^3*b^5
+ a*b*c^6 + (3*a^3*b - 2*a*b^3)*c^4 + (3*a^5*b - 4*a^3*b^3 + a*b^5)*c^2)*e)
*sin(x*e + d))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2)/sin(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(d+e\,x\right)^{5/2}\,{\left(a+c\,\mathrm{cot}\left(d+e\,x\right)+\frac{b}{\sin\left(d+e\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)),x)
```

```
[Out] int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)), x)
```

3.472 $\int \frac{1}{\cos^2(x)+\sin^2(x)} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4465, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cos^2(x)+\sin^2(x)} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [A]
time = 0.06, size = 2, normalized size = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\left(\tan^4\left(\frac{x}{2}\right)\right)+2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] x
```

Maxima [A]
time = 0.47, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="maxima")
[Out] x
```

Fricas [A]
time = 2.06, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="fricas")
[Out] x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.
time = 0.16, size = 10, normalized size = 10.00

$$\frac{x}{\sin^2(x) + \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2),x)
```

```
[Out] x/(sin(x)**2 + cos(x)**2)
```

Giac [A]
time = 0.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 2.64, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2),x)
```

```
[Out] x
```

3.473 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4465, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + Sin[x]^2)^(-2),x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx &= \int 1\,dx\\ &= x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-2),x]

[Out] x

Maple [A]
time = 0.06, size = 2, normalized size = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\left(\tan^{8}\left(\frac{x}{2}\right)\right)+4x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+6x\left(\tan^{4}\left(\frac{x}{2}\right)\right)+4x\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{4}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2+cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

[Out] x

Maxima [A]
time = 0.47, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="maxima")
```

[Out] x

Fricas [A]
time = 2.33, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="fricas")
```

[Out] x

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(0) = 0$.
time = 0.31, size = 22, normalized size = 22.00

$$\frac{x}{\sin^{4}(x) + 2\sin^{2}(x)\cos^{2}(x) + \cos^{4}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**2,x)
```

```
[Out] x/(sin(x)**4 + 2*sin(x)**2*cos(x)**2 + cos(x)**4)
```

Giac [A]
time = 0.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 2.63, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2)^2,x)
```

```
[Out] x
```

3.474 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3} dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4465, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]^2 + Sin[x]^2)^(-3),x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x]^2 + Sin[x]^2)^(-3),x]

[Out] x

Maple [A]
time = 0.07, size = 2, normalized size = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\left(\tan^{12}\left(\frac{x}{2}\right)\right)+6x\left(\tan^{2}\left(\frac{x}{2}\right)\right)+15x\left(\tan^{4}\left(\frac{x}{2}\right)\right)+20x\left(\tan^{6}\left(\frac{x}{2}\right)\right)+15x\left(\tan^{8}\left(\frac{x}{2}\right)\right)+6x\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{6}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2+cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.47, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 2.84, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(0) = 0$.
time = 0.60, size = 34, normalized size = 34.00

$$\frac{x}{\sin^{6}(x) + 3\sin^{4}(x)\cos^{2}(x) + 3\sin^{2}(x)\cos^{4}(x) + \cos^{6}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+sin(x)**2)**3,x)
```

```
[Out] x/(sin(x)**6 + 3*sin(x)**4*cos(x)**2 + 3*sin(x)**2*cos(x)**4 + cos(x)**6)
```

Giac [A]
time = 0.42, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 2.59, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + sin(x)^2)^3,x)
```

```
[Out] x
```

3.475 $\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))$$

```
[Out] 1/2*arctanh(2*cos(x)*sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {212}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.01, size = 23, normalized size = 2.09

$$-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2

Maple [A]
time = 0.08, size = 4, normalized size = 0.36

method	result	size
default	$\operatorname{arctanh}(\tan(x))$	4
risch	$\frac{\ln(e^{2ix}+i)}{2}-\frac{\ln(e^{2ix}-i)}{2}$	24
norman	$\frac{\ln(\tan^2(\frac{x}{2})-2\tan(\frac{x}{2})-1)}{2}-\frac{\ln(\tan^2(\frac{x}{2})+2\tan(\frac{x}{2})-1)}{2}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(x)^2-sin(x)^2),x,method=_RETURNVERBOSE)

[Out] arctanh(tan(x))

Maxima [A]
time = 0.26, size = 15, normalized size = 1.36

$$\frac{1}{2}\log(\tan(x)+1)-\frac{1}{2}\log(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="maxima")

[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 2.20, size = 23, normalized size = 2.09

$$\frac{1}{4}\log(2\cos(x)\sin(x)+1)-\frac{1}{4}\log(-2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="fricas")

[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.
time = 0.15, size = 36, normalized size = 3.27

$$\frac{\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cos(x)**2-sin(x)**2),x)

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 - log(tan(x/2)**2 + 2*tan(x/2) - 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(9) = 18$.
time = 0.43, size = 33, normalized size = 3.00

$$\frac{1}{8}\log\left(\left|\frac{1}{\sin(2x)} + \sin(2x) + 2\right|\right) - \frac{1}{8}\log\left(\left|\frac{1}{\sin(2x)} + \sin(2x) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(2*x) + sin(2*x) + 2)) - 1/8*log(abs(1/sin(2*x) + sin(2*x)
 - 2))
```

Mupad [B]
time = 2.90, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2),x)
```

```
[Out] atanh(tan(x))
```

3.476 $\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2} \, dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{1-\tan^2(x)}$$

```
[Out] tan(x)/(1-tan(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {391}

$$\frac{\tan(x)}{1-\tan^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[x]/(1 - Tan[x]^2)
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2} \, dx &= \text{Subst}\left(\int \frac{1+x^2}{\left(1-x^2\right)^2} \, dx, x, \tan(x)\right) \\ &= \frac{\tan(x)}{1-\tan^2(x)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 0.62

$$\frac{1}{2} \tan(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

```
[Out] Tan[2*x]/2
```

Maple [A]
time = 0.09, size = 18, normalized size = 1.38

method	result	size
risch	$\frac{i}{e^{4ix}+1}$	13
default	$-\frac{1}{2(\tan(x)-1)}-\frac{1}{2(1+\tan(x))}$	18
norman	$\frac{-2\left(\tan^3\left(\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\left(\tan^2\left(\frac{x}{2}\right)\right)+1}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/(tan(x)-1)-1/2/(1+tan(x))
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.92

$$-\frac{\tan(x)}{\tan(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="maxima")
[Out] -tan(x)/(tan(x)^2 - 1)
```

Fricas [A]
time = 2.84, size = 15, normalized size = 1.15

$$\frac{\cos(x)\sin(x)}{2\cos(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="fricas")
[Out] cos(x)*sin(x)/(2*cos(x)^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(8) = 16$.
time = 0.61, size = 48, normalized size = 3.69

$$-\frac{2\tan^3\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**2,x)
```

```
[Out] -2*tan(x/2)**3/(tan(x/2)**4 - 6*tan(x/2)**2 + 1) + 2*tan(x/2)/(tan(x/2)**4
- 6*tan(x/2)**2 + 1)
```

Giac [**A**]
time = 0.43, size = 6, normalized size = 0.46

$$\frac{1}{2}\tan\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*tan(2*x)
```

Mupad [**B**]
time = 2.63, size = 6, normalized size = 0.46

$$\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2)^2,x)
```

```
[Out] tan(2*x)/2
```

3.477 $\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} \, dx$

Optimal. Leaf size=32

$$\frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x)) + \frac{\sec^2(x)\tan(x)}{2(1-\tan^2(x))^2}$$

```
[Out] 1/4*arctanh(2*cos(x)*sin(x))+1/2*sec(x)^2*tan(x)/(1-tan(x)^2)^2
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {424, 21, 212}

$$\frac{\tan(x)\sec^2(x)}{2(1-\tan^2(x))^2} + \frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/4 + (Sec[x]^2*Tan[x])/(2*(1 - Tan[x]^2)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x^2)^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\sec^2(x)\tan(x)}{2(1-\tan^2(x))^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-2+2x^2}{(1-x^2)^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\sec^2(x)\tan(x)}{2(1-\tan^2(x))^2} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x)) + \frac{\sec^2(x)\tan(x)}{2(1-\tan^2(x))^2}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.69

$$\frac{1}{4}\tanh^{-1}(\sin(2x)) + \frac{1}{4}\sec(2x)\tan(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

```
[Out] ArcTanh[Sin[2*x]]/4 + (Sec[2*x]*Tan[2*x])/4
```

Maple [A]
time = 0.12, size = 48, normalized size = 1.50

method	result	size
default	$-\frac{1}{4(1+\tan(x))^2}+\frac{1}{4+4\tan(x)}+\frac{\ln(1+\tan(x))}{4}+\frac{1}{4(\tan(x)-1)^2}+\frac{1}{4\tan(x)-4}-\frac{\ln(\tan(x)-1)}{4}$	48
risch	$-\frac{i(e^{6ix}-e^{2ix})}{2(e^{4ix}+1)^2}-\frac{\ln(e^{2ix}-i)}{4}+\frac{\ln(e^{2ix}+i)}{4}$	49
norman	$\frac{\tan^3(\frac{x}{2})-(\tan^5(\frac{x}{2}))-(\tan^7(\frac{x}{2}))+\tan(\frac{x}{2})}{(\tan^4(\frac{x}{2})-6(\tan^2(\frac{x}{2}))+1)^2}+\frac{\ln(\tan^2(\frac{x}{2})-2\tan(\frac{x}{2})-1)}{4}-\frac{\ln(\tan^2(\frac{x}{2})+2\tan(\frac{x}{2})-1)}{4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2-sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(1+tan(x))^2+1/4/(1+tan(x))+1/4*ln(1+tan(x))+1/4/(tan(x)-1)^2+1/4/(tan
(x)-1)-1/4*ln(tan(x)-1)
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.19

$$\frac{\tan(x)^3+\tan(x)}{2\left(\tan(x)^4-2\tan(x)^2+1\right)} + \frac{1}{4}\log(\tan(x)+1) - \frac{1}{4}\log(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(tan(x)^3 + tan(x))/(tan(x)^4 - 2*tan(x)^2 + 1) + 1/4*log(tan(x) + 1) -
 1/4*log(tan(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.
time = 3.07, size = 74, normalized size = 2.31

$$\frac{(4\cos(x)^4 - 4\cos(x)^2 + 1)\log(2\cos(x)\sin(x) + 1) - (4\cos(x)^4 - 4\cos(x)^2 + 1)\log(-2\cos(x)\sin(x) + 1) + 4\cos(x)\sin(x)}{8(4\cos(x)^4 - 4\cos(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((4*cos(x)^4 - 4*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1) - (4*cos(x)^4 -
 4*cos(x)^2 + 1)*log(-2*cos(x)*sin(x) + 1) + 4*cos(x)*sin(x))/(4*cos(x)^4 -
 4*cos(x)^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(29) = 58$.
time = 1.56, size = 765, normalized size = 23.91

$$\frac{\log(\tan^2(\frac{x}{2}) - 2\tan(\frac{x}{2}) - 1)\tan^8(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{12\log(\tan^2(\frac{x}{2}) - 2\tan(\frac{x}{2}) - 1)\tan^6(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{38\log(\tan^2(\frac{x}{2}) - 2\tan(\frac{x}{2}) - 1)\tan^4(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{12\log(\tan^2(\frac{x}{2}) - 2\tan(\frac{x}{2}) - 1)\tan^2(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{\log(\tan^2(\frac{x}{2}) - 2\tan(\frac{x}{2}) - 1)}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{\log(\tan^2(\frac{x}{2}) + 2\tan(\frac{x}{2}) - 1)\tan^8(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{12\log(\tan^2(\frac{x}{2}) + 2\tan(\frac{x}{2}) - 1)\tan^6(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{38\log(\tan^2(\frac{x}{2}) + 2\tan(\frac{x}{2}) - 1)\tan^4(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{12\log(\tan^2(\frac{x}{2}) + 2\tan(\frac{x}{2}) - 1)\tan^2(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{\log(\tan^2(\frac{x}{2}) + 2\tan(\frac{x}{2}) - 1)}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{4\tan^7(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} - \frac{4\tan^5(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{4\tan^3(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4} + \frac{4\tan(\frac{x}{2})}{4\tan^8(\frac{x}{2}) - 48\tan^6(\frac{x}{2}) + 152\tan^4(\frac{x}{2}) - 48\tan^2(\frac{x}{2}) + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2-sin(x)**2)**3,x)
```

```
[Out] log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)*
*6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(tan(x/2)**2 - 2*tan(x/2
) - 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*t
an(x/2)**2 + 4) + 38*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x
/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(ta
n(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 1
52*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + log(tan(x/2)**2 - 2*tan(x/2) - 1)/(4
*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log
(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)**6
+ 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x/2)**2 + 2*tan(x/2) -
 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(
x/2)**2 + 4) - 38*log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**4/(4*tan(x/2)
**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x
/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*
tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log(tan(x/2)**2 + 2*tan(x/2) - 1)/(4*ta
n(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 4*tan(
x/2)**7/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2
```

```
+ 4) - 4*tan(x/2)**5/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48
*tan(x/2)**2 + 4) + 4*tan(x/2)**3/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan
(x/2)**4 - 48*tan(x/2)**2 + 4) + 4*tan(x/2)/(4*tan(x/2)**8 - 48*tan(x/2)**6
 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4)
```

Giac [A]
time = 0.45, size = 37, normalized size = 1.16

$$-\frac{\sin(2x)}{4\left(\sin(2x)^2-1\right)}+\frac{1}{8}\log(\sin(2x)+1)-\frac{1}{8}\log(-\sin(2x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*x)/(sin(2*x)^2 - 1) + 1/8*log(sin(2*x) + 1) - 1/8*log(-sin(2*x)
+ 1)
```

Mupad [B]
time = 2.65, size = 32, normalized size = 1.00

$$\frac{\operatorname{atanh}(\tan(x))}{2}+\frac{\frac{\tan(x)^3}{2}+\frac{\tan(x)}{2}}{\tan(x)^4-2\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 - sin(x)^2)^3,x)
```

```
[Out] atanh(tan(x))/2 + (tan(x)/2 + tan(x)^3/2)/(tan(x)^4 - 2*tan(x)^2 + 1)
```

3.478 $\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\operatorname{ArcTan}(a\tan(x))}{a}$$

```
[Out] arctan(a*tan(x))/a
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {209}

$$\frac{\operatorname{ArcTan}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{1+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}(a\tan(x))}{a}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 1.00

$$\frac{\operatorname{ArcTan}(a\tan(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[a*Tan[x]]/a
```

Maple [A]
time = 0.12, size = 10, normalized size = 1.11

method	result	size
default	$\frac{\arctan(a\tan(x))}{a}$	10
risch	$\frac{i\ln\left(e^{2ix}-\frac{a+1}{a-1}\right)}{2a}-\frac{i\ln\left(e^{2ix}-\frac{a-1}{a+1}\right)}{2a}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(a*tan(x))/a
```

Maxima [A]
time = 0.48, size = 9, normalized size = 1.00

$$\frac{\arctan\left(a\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(9) = 18.
time = 2.94, size = 35, normalized size = 3.89

$$-\frac{\arctan\left(\frac{(a^2+1)\cos(x)^2-a^2}{2\,a\cos(x)\sin(x)}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + 1)*cos(x)^2 - a^2)/(a*cos(x)*sin(x)))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12007 vs. 2(7) = 14.
time = 10.29, size = 12007, normalized size = 1334.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2
 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**
```

```
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)
*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) +
 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqr
t(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*s
qrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a*
*2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2
 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1)) - 64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(s
qrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2
*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqr
t(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(
a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqr
t(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a
**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a
**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*
a**2 + 2*a*sqrt(a**2 - 1) + 1)) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a
*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2)
)/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a
**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt
(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2
- 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a
*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2
- 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*s
qrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 64*a**6*sqrt(a
**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(sqrt(-2*a**2 + 2*a*sqrt
(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)
*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a*
*2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a*
*5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqr
t(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)) - 112*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a*
*2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a
**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 -
1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1)
 + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*s
qrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 -
```

```
1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2 - 2*a
*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2*sqrt(
a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a*
*2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1)) + 112*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*lo
g(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2
- 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*
sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sq
rt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*
a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*
sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 1
2*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 +
 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(
-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)) + 16*a**5*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
 1) + 1)*log(-sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*s
qrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1
) - 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a
**2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.45, size = 20, normalized size = 2.22

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(a\tan\left(x\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)))/a
```

Mupad [B]
time = 2.83, size = 9, normalized size = 1.00

$$\frac{\operatorname{atan}(a\tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2 + a^2*sin(x)^2),x)
```

```
[Out] atan(a*tan(x))/a
```

3.479 $\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{b}\right)}{b}$$

`[Out] arctan(tan(x)/b)/b`

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {209}

$$\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

`[In] Int[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]`

`[Out] ArcTan[Tan[x]/b]/b`

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx &= \operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\tan(x)}{b}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{\operatorname{ArcTan}\left(\frac{\tan(x)}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[Tan[x]/b]/b
```

Maple [A]
time = 0.12, size = 12, normalized size = 1.09

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$	12
risch	$\frac{i\ln\left(e^{2ix}+\frac{b+1}{b-1}\right)}{2b}-\frac{i\ln\left(e^{2ix}+\frac{b-1}{b+1}\right)}{2b}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x)/b)/b
```

Maxima [A]
time = 0.47, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(tan(x)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.
time = 2.22, size = 31, normalized size = 2.82

$$-\frac{\arctan\left(\frac{(b^2+1)\cos(x)^2-1}{2b\cos(x)\sin(x)}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((b^2 + 1)*cos(x)^2 - 1)/(b*cos(x)*sin(x)))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 30189 vs. $2(7) = 14$.
time = 12.76, size = 30189, normalized size = 2744.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+sin(x)**2),x)
```

```
[Out] Piecewise((tan(x/2)/2 - 1/(2*tan(x/2)), Eq(b, 0)), (x/(sin(x)**2 + cos(x)**
2), Eq(b, -1) | Eq(b, 1)), (-b**10*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)
/b**2 - 2/b**2)*log(-sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(
2*b**12*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/
b**2 - 2/b**2) + 18*b**10*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2
/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 82*b**10*sqrt(1 - 2*sqrt(
1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 240*b**
8*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1
 - b**2)/b**2 - 2/b**2) + 560*b**8*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)
*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 864*b**6*sqrt(1 - b**2)*sqrt(1
- 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)
- 1376*b**6*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b*
*2)/b**2 - 2/b**2) - 1152*b**4*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**
2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 1408*b**4*sqrt(1 - 2
*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 5
12*b**2*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*
sqrt(1 - b**2)/b**2 - 2/b**2) - 512*b**2*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2
/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)) + b**10*sqrt(1 - b**2)*sqr
t(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 + 2*sqrt(1 - b**2)/b**2 -
2/b**2) + tan(x/2))/(2*b**12*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(
1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 18*b**10*sqrt(1 - b**2)*sqrt(1 - 2*sq
rt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 82*b
**10*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**
2 - 2/b**2) - 240*b**8*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b*
*2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 560*b**8*sqrt(1 - 2*sqrt(1 -
 b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 864*b**6*s
qrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 -
b**2)/b**2 - 2/b**2) - 1376*b**6*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*s
qrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 1152*b**4*sqrt(1 - b**2)*sqrt(1 -
 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) +
 1408*b**4*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**
2)/b**2 - 2/b**2) + 512*b**2*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2
- 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 512*b**2*sqrt(1 - 2*sq
rt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)) + b**
10*sqrt(1 - b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(-sqrt(1 - 2*
sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**12*sqrt(1 - 2*sqrt(1 - b**2
)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 18*b**10*sqrt(1
 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)
/b**2 - 2/b**2) - 82*b**10*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1
+ 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 240*b**8*sqrt(1 - b**2)*sqrt(1 - 2*sqrt
(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 560*b*
```

```
*8*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2
- 2/b**2) + 864*b**6*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2
)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 1376*b**6*sqrt(1 - 2*sqrt(1 -
b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 1152*b**4*s
qrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 -
b**2)/b**2 - 2/b**2) + 1408*b**4*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*s
qrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 512*b**2*sqrt(1 - b**2)*sqrt(1 -
2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) -
512*b**2*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)
/b**2 - 2/b**2)) - b**10*sqrt(1 - b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/
b**2)*log(sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**12*sqr
t(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b*
*2) + 18*b**10*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt
(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 82*b**10*sqrt(1 - 2*sqrt(1 - b**2)/b
**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 240*b**8*sqrt(1 -
b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b*
*2 - 2/b**2) + 560*b**8*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2
*sqrt(1 - b**2)/b**2 - 2/b**2) + 864*b**6*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1
- b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 1376*b**6
*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 -
2/b**2) - 1152*b**4*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)
*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 1408*b**4*sqrt(1 - 2*sqrt(1 - b
**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 512*b**2*sqr
t(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b*
*2)/b**2 - 2/b**2) - 512*b**2*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt
(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)) + 10*b**1...
```

Giac [A]
time = 0.43, size = 22, normalized size = 2.00

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(tan(x)/b))/b
```

Mupad [B]
time = 2.83, size = 11, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{\tan(x)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2 + b^2*cos(x)^2),x)
[Out] atan(tan(x)/b)/b
```

3.480 $\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)}\, dx$

Optimal. Leaf size=15

$$\frac{\text{ArcTan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] arctan(a*tan(x)/b)/a/b
```

Rubi [A]

time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 15, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A]
time = 0.13, size = 16, normalized size = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
risch	$-\frac{i\ln\left(e^{2ix}-\frac{a-b}{a+b}\right)}{2ab}+\frac{i\ln\left(e^{2ix}-\frac{a+b}{a-b}\right)}{2ab}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A]
time = 0.48, size = 15, normalized size = 1.00

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(15) = 30$.
time = 2.44, size = 43, normalized size = 2.87

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. $2(10) = 20$.
time = 18.70, size = 71839, normalized size = 4789.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)

[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2)
/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1))
, Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2), Eq(a, b) | Eq(a, -b)), (
8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2
*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2
- b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b*
*2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a
**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6
*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*
a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968
*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**1
0*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2
*a*sqrt(a**2 - b**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*
a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a
**4*b**12*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*s
qrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*
sqrt(a**2 - b**2)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/
b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 -
b**2)/b**2 + 1) - 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2
+ 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**15*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sq
rt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 45568*a**11*b**6*sqrt(-2*a**2/b*
*2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b*
*2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
```

```
+ 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 19968*a**8*b**8*sqrt(
a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a*
*2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 12992*a**7*b**10*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 5824*a**6*b**10*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 2408*a**5*b**12*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 728*a**4*b**12*sqrt(a
**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 170*a**3*b**14*sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1) - 26*a**2*b**14*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
- 2*a*b**16*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)) - 8192*a**14*sqrt(a**2 - b**2)*sq
rt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 +
2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b**2*sqrt(-2*a**2/b
**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b
**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*
sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2
+ 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 26624*a**12*b**4*sqr
t(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**...
```

Giac [A]
time = 0.45, size = 26, normalized size = 1.73

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

Mupad [B]
time = 2.85, size = 15, normalized size = 1.00

$$\frac{\mathrm{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
[Out] atan((a*tan(x))/b)/(a*b)
```

3.481 $\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}}$$

```
[Out] 1/6*x*3^(1/2)-1/12*arctan(cos(1+2*x)*sin(1+2*x)/(3+cos(1+2*x)^2+2*3^(1/2)))
*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {209}

$$\frac{x}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sin(2x+1)\cos(2x+1)}{\cos^2(2x+1)+2\sqrt{3}+3}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) - ArcTan[(Cos[1 + 2*x]*Sin[1 + 2*x])/(3 + 2*Sqrt[3] + Cos[1 +
 2*x]^2)]/(4*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+3x^2}\,dx, x, \tan(1+2x)\right)\\ &= \frac{x}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.47

$$\frac{\text{ArcTan}\left(\frac{1}{2}\sqrt{3}\,\tan(1+2x)\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])
```

Maple [A]
time = 0.21, size = 18, normalized size = 0.34

method	result	size
derivativedivides	$\frac{\sqrt{3}\,\arctan\left(\frac{\tan(1+2x)\sqrt{3}}{2}\right)}{12}$	18
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\tan(1+2x)\sqrt{3}}{2}\right)}{12}$	18
risch	$\frac{i\sqrt{3}\,\ln\left(e^{2i(1+2x)}+4\sqrt{3}+7\right)}{24}-\frac{i\sqrt{3}\,\ln\left(e^{2i(1+2x)}-4\sqrt{3}+7\right)}{24}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*3^(1/2)*arctan(1/2*tan(1+2*x)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 17, normalized size = 0.32

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan\left(2x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/2*sqrt(3)*tan(2*x + 1))
```

Fricas [A]
time = 2.89, size = 43, normalized size = 0.81

$$-\frac{1}{24}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos\left(2x+1\right)^2-3\sqrt{3}}{12\cos\left(2x+1\right)\sin\left(2x+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(2*x + 1)^2 - 3*sqrt(3))/(cos(2*x +
 1)*sin(2*x + 1)))
```

Sympy [A]
time = 0.30, size = 87, normalized size = 1.64

$$\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}+\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)**2+3*sin(1+2*x)**2),x)
```

```
[Out] sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 - sqrt(3)/3) + pi*floor((x - pi/2 +
1/2)/pi))/12 + sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 + sqrt(3)/3) + pi*flo
or((x - pi/2 + 1/2)/pi))/12
```

Giac [A]
time = 0.42, size = 61, normalized size = 1.15

$$\frac{1}{12}\sqrt{3}\left(2x+\arctan\left(-\frac{2\sqrt{3}\sin\left(4x+2\right)-3\sin\left(4x+2\right)}{2\sqrt{3}\cos\left(4x+2\right)+2\sqrt{3}-3\cos\left(4x+2\right)+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*(2*x + arctan(-(2*sqrt(3)*sin(4*x + 2) - 3*sin(4*x + 2))/(2*sq
rt(3)*cos(4*x + 2) + 2*sqrt(3) - 3*cos(4*x + 2) + 3)) + 1)
```

Mupad [B]
time = 2.76, size = 36, normalized size = 0.68

$$\frac{\sqrt{3}\left(2x-\operatorname{atan}(\tan(2x+1))\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\tan(2x+1)}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*sin(2*x + 1)^2 + 4*cos(2*x + 1)^2),x)
```

```
[Out] (3^(1/2)*(2*x - atan(tan(2*x + 1))))/12 + (3^(1/2)*atan((3^(1/2)*tan(2*x +
1))/2))/12
```

3.482 $\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{x}{a-b}+\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}}$$

```
[Out] -x/(a-b)+arctan(b^(1/2)*tan(x)/a^(1/2))*a^(1/2)/(a-b)/b^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {492, 209, 211}

$$\frac{\sqrt{a}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}\,(a-b)}-\frac{x}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

```
[Out] -(x/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/((a - b)*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{x^2}{(1+x^2)(a+bx^2)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{a-b} + \frac{a\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(x)\right)}{a-b}$$
$$= -\frac{x}{a-b} + \frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}}$$

Mathematica [A]
time = 0.07, size = 36, normalized size = 0.84

$$\frac{x-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{-a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
[Out] (x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[b])/(-a + b)
```

Maple [A]
time = 0.17, size = 38, normalized size = 0.88

method	result	size
default	$-\frac{\arctan(\tan(x))}{a-b}+\frac{a\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}$	38
risch	$-\frac{x}{a-b}-\frac{\sqrt{-ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b(a-b)}+\frac{\sqrt{-ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2b(a-b)}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/(a-b)*arctan(tan(x))+a/(a-b)/(a*b)^(1/2)*arctan(b*tan(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.46, size = 35, normalized size = 0.81

$$\frac{a\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}-\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")

[Out] a*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - x/(a - b)

Fricas [A]
time = 2.51, size = 182, normalized size = 4.23

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2+4\left((ab+b^2)\cos(x)^3-b^2\cos(x)\right)\sqrt{-\frac{a}{b}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right)+4x}{4(a-b)}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{((a+b)\cos(x)^2-b)\sqrt{\frac{a}{b}}}{2a\cos(x)\sin(x)}\right)+2x}{2(a-b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")

[Out] [-1/4*(sqrt(-a/b)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 + 4*((a*b + b^2)*cos(x)^3 - b^2*cos(x))*sqrt(-a/b)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) + 4*x)/(a - b), -1
/2*(sqrt(a/b)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(a/b)/(a*cos(x)*sin(x))
) + 2*x)/(a - b)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(32) = 64$.
time = 0.61, size = 201, normalized size = 4.67

$$\begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)} + \frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)} - \frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a = b \\ \frac{-x+\frac{\sin(x)}{\cos(x)}}{a} & \text{for } b = 0 \\ -\frac{2x\sqrt{-\frac{b}{a}}}{2a\sqrt{-\frac{b}{a}}-2b\sqrt{-\frac{b}{a}}} - \frac{\log\left(-\sqrt{-\frac{b}{a}}\sin(x)+\cos(x)\right)}{2a\sqrt{-\frac{b}{a}}-2b\sqrt{-\frac{b}{a}}} + \frac{\log\left(\sqrt{-\frac{b}{a}}\sin(x)+\cos(x)\right)}{2a\sqrt{-\frac{b}{a}}-2b\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2/(a*cos(x)**2+b*sin(x)**2),x)

[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*sin(x)**2/(2*b*
sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) -
sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), ((-x + sin(x)/cos
(x))/a, Eq(b, 0)), (-2*x*sqrt(-b/a)/(2*a*sqrt(-b/a) - 2*b*sqrt(-b/a)) - log

```
(-sqrt(-b/a)*sin(x) + cos(x))/(2*a*sqrt(-b/a) - 2*b*sqrt(-b/a)) + log(sqrt(
-b/a)*sin(x) + cos(x))/(2*a*sqrt(-b/a) - 2*b*sqrt(-b/a)), True))
```

Giac [A]
time = 0.41, size = 48, normalized size = 1.12

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)a}{\sqrt{ab}\,(a-b)}-\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*a/(sqrt(a*b)*(a
- b)) - x/(a - b)
```

Mupad [B]
time = 2.73, size = 51, normalized size = 1.19

$$\begin{cases}\frac{2\,x-\sin(2\,x)}{4\,b} & \text{if}\ \ a=b\\ -\frac{x-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a-b} & \text{if}\ \ a\neq b\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

```
[Out] piecewise(a == b, (2*x - sin(2*x))/(4*b), a ~= b, -(x - (a^(1/2)*atan((b^(1
/2)*tan(x))/a^(1/2)))/b^(1/2))/(a - b))
```

3.483 $\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)}$$

[Out] x/(a-b)-arctan(b^(1/2)*tan(x)/a^(1/2))*b^(1/2)/(a-b)/a^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {400, 209, 211}

$$\frac{x}{a-b}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]

[Out] x/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a - b))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\,(a+bx^2)}\,dx, x, \tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)}{a-b} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \tan(x)\right)}{a-b}\\
&= \frac{x}{a-b} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.84

$$
\frac{x-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
[Out] (x - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[a])/(a - b)
```

Maple [A]
time = 0.15, size = 38, normalized size = 0.88

method	result	size
default	$\frac{\arctan(\tan(x))}{a-b}-\frac{b\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}$	38
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(e^{2ix}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)}-\frac{\sqrt{-ab}\,\ln\left(e^{2ix}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/(a-b)*arctan(tan(x))-b/(a-b)/(a*b)^(1/2)*arctan(b*tan(x)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 35, normalized size = 0.81

$$
-\frac{b\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a-b)}+\frac{x}{a-b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) + x/(a - b)
```

Fricas [A]
time = 2.60, size = 181, normalized size = 4.21

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6ab+b^2)\cos(x)^4-2(3ab+b^2)\cos(x)^2-4\left((a^2+ab)\cos(x)^3-ab\cos(x)\right)\sqrt{-\frac{b}{a}}\sin(x)+b^2}{(a^2-2ab+b^2)\cos(x)^4+2(ab-b^2)\cos(x)^2+b^2}\right)-4x}{4(a-b)}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{\left((a+b)\cos(x)^2-b\right)\sqrt{\frac{b}{a}}}{2b\cos(x)\sin(x)}\right)+2x}{2(a-b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(x
)^2 - 4*((a^2 + a*b)*cos(x)^3 - a*b*cos(x))*sqrt(-b/a)*sin(x) + b^2)/((a^2
- 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) - 4*x)/(a - b), 1/
2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(b/a)/(b*cos(x)*sin(x)))
 + 2*x)/(a - b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(32) = 64$.
time = 0.63, size = 226, normalized size = 5.26

$$\begin{cases} \tilde{\infty}\left(-x-\frac{\cos(x)}{\sin(x)}\right) & \text{for } a=0 \wedge b=0 \\ \frac{-x-\frac{\cos(x)}{\sin(x)}}{b} & \text{for } a=0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a=b \\ \frac{x}{a} & \text{for } b=0 \\ \frac{2ax\sqrt{-\frac{b}{a}}}{2a^2\sqrt{-\frac{b}{a}}-2ab\sqrt{-\frac{b}{a}}}+\frac{b\log\left(-\sqrt{-\frac{b}{a}}\sin(x)+\cos(x)\right)}{2a^2\sqrt{-\frac{b}{a}}-2ab\sqrt{-\frac{b}{a}}}-\frac{b\log\left(\sqrt{-\frac{b}{a}}\sin(x)+\cos(x)\right)}{2a^2\sqrt{-\frac{b}{a}}-2ab\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*(-x - cos(x)/sin(x)), Eq(a, 0) & Eq(b, 0)), ((-x - cos(x)/si
n(x))/b, Eq(a, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)
**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*co
s(x)**2), Eq(a, b)), (x/a, Eq(b, 0)), (2*a*x*sqrt(-b/a)/(2*a**2*sqrt(-b/a)
```

```
- 2*a*b*sqrt(-b/a)) + b*log(-sqrt(-b/a)*sin(x) + cos(x))/(2*a**2*sqrt(-b/a)
 - 2*a*b*sqrt(-b/a)) - b*log(sqrt(-b/a)*sin(x) + cos(x))/(2*a**2*sqrt(-b/a)
 - 2*a*b*sqrt(-b/a)), True))
```

Giac [A]
time = 0.44, size = 48, normalized size = 1.12

$$-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}\,(a-b)}+\frac{x}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -(pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*b/(sqrt(a*b)*(a
 - b)) + x/(a - b)
```

Mupad [B]
time = 2.67, size = 48, normalized size = 1.12

$$\begin{cases}\frac{2\,x+\sin(2\,x)}{4\,b} & \text{if}\ \ a=b\\ \frac{x-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b} & \text{if}\ \ a\neq b\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

```
[Out] piecewise(a == b, (2*x + sin(2*x))/(4*b), a ~= b, (x - (b^(1/2)*atan((b^(1/
2)*tan(x))/a^(1/2)))/a^(1/2))/(a - b))
```

3.484 $\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=36

$$-x+\sqrt{2}\,x+\sqrt{2}\,\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

[Out] -x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1107, 209}

$$\sqrt{2}\,\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)+\sqrt{2}\,x-x$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]^2 + Tan[x]^2)^(-1),x]

[Out] -x + Sqrt[2]*x + Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+3x^2+2x^4}\,dx,x,\tan(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)-2\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx,x,\tan(x)\right)\\ &= -x+\sqrt{2}\,x+\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 0.53

$$-x+\sqrt{2}\,\text{ArcTan}\left(\sqrt{2}\,\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-1),x]
[Out] -x + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

Maple [A]
time = 0.12, size = 18, normalized size = 0.50

method	result	size
default	$\sqrt{2}\,\arctan\left(\tan\left(x\right)\sqrt{2}\right)-\arctan\left(\tan\left(x\right)\right)$	18
risch	$-x+\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*arctan(tan(x)*2^(1/2))-arctan(tan(x))
```

Maxima [A]
time = 0.47, size = 15, normalized size = 0.42

$$\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

Fricas [A]
time = 2.54, size = 35, normalized size = 0.97

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(x\right)^{2}-2\,\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) -
 x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\tan^2(x) + \sec^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2),x)
```

```
[Out] Integral(1/(tan(x)**2 + sec(x)**2), x)
```

Giac [A]
time = 0.41, size = 15, normalized size = 0.42

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

Mupad [B]
time = 2.65, size = 15, normalized size = 0.42

$$\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*tan(x)) - x
```

3.485 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx$

Optimal. Leaf size=49

$$x-\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}+\frac{\tan(x)}{1+2\tan^2(x)}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+tan(
x)/(1+2*tan(x)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.364$, Rules used = {425, 12, 492, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}-\frac{x}{\sqrt{2}}+x+\frac{\tan(x)}{2\tan^2(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2] +
Tan[x]/(1 + 2*Tan[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\,(1+2x^2)^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{1+2\tan^2(x)}-\frac{1}{2}\text{Subst}\left(\int -\frac{2x^2}{(1+x^2)\,(1+2x^2)}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{1+2\tan^2(x)}+\text{Subst}\left(\int \frac{x^2}{(1+x^2)\,(1+2x^2)}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{1+2\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan\right.\\ &= x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}+\frac{\tan(x)}{1+2\tan^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 42, normalized size = 0.86

$$-\frac{\text{ArcTan}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}+\frac{-3x+x\cos(2x)-\sin(2x)}{-3+\cos(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

```
[Out] -(ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]) + (-3*x + x*Cos[2*x] - Sin[2*x])/(-3 + Co
s[2*x])
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.59

method	result	size

default	$\arctan(\tan(x)) + \frac{\tan(x)}{2(\tan^2(x))+1} - \frac{\sqrt{2}\,\arctan\left(\tan(x)\sqrt{2}\right)}{2}$	29
risch	$x + \frac{2i(3\,e^{2ix}-1)}{e^{4ix}-6\,e^{2ix}+1} + \frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}-3\right)}{4} - \frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}-3\right)}{4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))+1/2*tan(x)/(tan(x)^2+1/2)-1/2*2^(1/2)*arctan(tan(x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 27, normalized size = 0.55

$$-\frac{1}{2}\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right) + x + \frac{\tan(x)}{2\,\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

Fricas [A]
time = 3.00, size = 68, normalized size = 1.39

$$\frac{4\,x\cos(x)^2 + \left(\sqrt{2}\,\cos(x)^2 - 2\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\,\cos(x)^2-2\sqrt{2}}{4\,\cos(x)\sin(x)}\right) - 4\,\cos(x)\sin(x) - 8\,x}{4\left(\cos(x)^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 - 2*sqrt(2))*arctan(1/4*(3*sqrt(2)*co
s(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) - 8*x)/(cos(x)^2 - 2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\tan^2(x) + \sec^2(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**2,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-2), x)
```

Giac [A]
time = 0.40, size = 27, normalized size = 0.55

$$-\frac{1}{2}\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)+x+\frac{\tan\left(x\right)}{2\,\tan\left(x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

Mupad [B]
time = 2.66, size = 27, normalized size = 0.55

$$x+\frac{\tan(x)}{2\,\left(\tan\left(x\right)^{2}+\frac{1}{2}\right)}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2)^2,x)
[Out] x + tan(x)/(2*(tan(x)^2 + 1/2)) - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

3.486 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3} dx$

Optimal. Leaf size=74

$$-x+\frac{7x}{4\sqrt{2}}+\frac{7\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}$$

```
[Out] -x+7/8*x*2^(1/2)+7/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/2
*tan(x)/(1+2*tan(x)^2)^2-1/4*tan(x)/(1+2*tan(x)^2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {425, 541, 536, 209}

$$\frac{7\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}+\frac{7x}{4\sqrt{2}}-x-\frac{\tan(x)}{4\left(2\tan^2(x)+1\right)}+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -x + (7*x)/(4*Sqrt[2]) + (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)
])/(4*Sqrt[2]) + Tan[x]/(2*(1 + 2*Tan[x]^2)^2) - Tan[x]/(4*(1 + 2*Tan[x]^2)
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)\,(1+2x^2)^3}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{1}{4}\text{Subst}\left(\int \frac{-2-6x^2}{(1+x^2)\,(1+2x^2)^2}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{1}{8}\text{Subst}\left(\int \frac{6-2x^2}{(1+x^2)\,(1+2x^2)}\,dx, x, \text{t}\right.\\
&= \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(x)\right)-\\
&= -x+\frac{7x}{4\sqrt{2}}+\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}}+\frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x\right.}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 79, normalized size = 1.07

$$-\frac{(-3+\cos(2x))\sec^6(x)\left(-76x+7\sqrt{2}\,\text{ArcTan}\left(\sqrt{2}\,\tan(x)\right)(-3+\cos(2x))^2+48x\cos(2x)-4x\cos(4x)-2\sin(2x)+3\sin(4x)\right)}{64\left(\sec^2(x)+\tan^2(x)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

```
[Out] -1/64*((-3 + Cos[2*x])*Sec[x]^6*(-76*x + 7*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]*(
-3 + Cos[2*x])^2 + 48*x*Cos[2*x] - 4*x*Cos[4*x] - 2*Sin[2*x] + 3*Sin[4*x]))
/(Sec[x]^2 + Tan[x]^2)^3
```

Maple [A]
time = 0.14, size = 42, normalized size = 0.57

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\frac{-\frac{\left(\tan^{3}\left(x\right)\right)}{2}+\frac{\tan\left(x\right)}{4}}{\left(2\left(\tan^{2}\left(x\right)\right)+1\right)^{2}}+\frac{7\sqrt{2}\,\arctan\left(\tan\left(x\right)\sqrt{2}\right)}{8}$	42
risch	$-x-\frac{i\left(17\,{\mathrm e}^{6 i x}-57\,{\mathrm e}^{4 i x}+19\,{\mathrm e}^{2 i x}-3\right)}{2\left({\mathrm e}^{4 i x}-6\,{\mathrm e}^{2 i x}+1\right)^{2}}+\frac{7 i \sqrt{2}\,\ln\left({\mathrm e}^{2 i x}-2\sqrt{2}-3\right)}{16}-\frac{7 i \sqrt{2}\,\ln\left({\mathrm e}^{2 i x}+2\sqrt{2}-3\right)}{16}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2+tan(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(tan(x))+8*(-1/16*tan(x)^3+1/32*tan(x))/(1+2*tan(x)^2)^2+7/8*2^(1/2)
*arctan(tan(x)*2^(1/2))
```

Maxima [A]
time = 0.47, size = 45, normalized size = 0.61

$$\frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)-x-\frac{2\tan\left(x\right)^{3}-\tan\left(x\right)}{4\left(4\tan\left(x\right)^{4}+4\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(4*tan(x
)^4 + 4*tan(x)^2 + 1)
```

Fricas [A]
time = 2.46, size = 100, normalized size = 1.35

$$-\frac{16\,x\cos\left(x\right)^{4}-64\,x\cos\left(x\right)^{2}+7\left(\sqrt{2}\cos\left(x\right)^{4}-4\sqrt{2}\cos\left(x\right)^{2}+4\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-2\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)-4\left(3\cos\left(x\right)^{3}-2\cos\left(x\right)\right)\sin\left(x\right)+64\,x}{16\left(\cos\left(x\right)^{4}-4\cos\left(x\right)^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(16*x*cos(x)^4 - 64*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos(
x)^2 + 4*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x
))) - 4*(3*cos(x)^3 - 2*cos(x))*sin(x) + 64*x)/(cos(x)^4 - 4*cos(x)^2 + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\tan^{2}\left(x\right)+\sec^{2}\left(x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2+tan(x)**2)**3,x)
```

```
[Out] Integral((tan(x)**2 + sec(x)**2)**(-3), x)
```

Giac [A]
time = 0.42, size = 39, normalized size = 0.53

$$\frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)-x-\frac{2\tan\left(x\right)^{3}-\tan\left(x\right)}{4\left(2\tan\left(x\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(2*tan(x
)^2 + 1)^2
```

Mupad [B]
time = 2.70, size = 40, normalized size = 0.54

$$\frac{\frac{\tan(x)}{16}-\frac{\tan(x)^{3}}{8}}{\tan\left(x\right)^{4}+\tan\left(x\right)^{2}+\frac{1}{4}}-x+\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 + tan(x)^2)^3,x)
```

```
[Out] (tan(x)/16 - tan(x)^3/8)/(tan(x)^2 + tan(x)^4 + 1/4) - x + (7*2^(1/2)*atan(
2^(1/2)*tan(x)))/8
```

3.487 $\int \frac{1}{\sec^2(x)-\tan^2(x)} dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4466, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx &= \int 1\,dx\\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

```
[Out] x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.08, size = 4, normalized size = 4.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))
```

Maxima [A]
time = 0.47, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 1.90, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\tan(x)+\sec(x)\right)\left(\tan(x)+\sec(x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2),x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))*(tan(x) + sec(x))), x)
```

Giac [**A**]
time = 0.45, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="giac")
```

```
[Out] x
```

Mupad [**B**]
time = 2.75, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2),x)
```

```
[Out] x
```

3.488 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2} dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4466, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2} dx = \int 1 \, dx$$
$$= x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

```
[Out] x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.06, size = 4, normalized size = 4.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))
```

Maxima [A]
time = 0.49, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 2.86, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\tan(x)+\sec(x)\right)^2\left(\tan(x)+\sec(x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**2,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**2*(tan(x) + sec(x))**2), x)
```

Giac [**A**]
time = 0.42, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

Mupad [**B**]
time = 2.58, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2)^2,x)
```

```
[Out] x
```

3.489 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3}\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4466, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4466

```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx &= \int 1 \, dx\\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

```
[Out] x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.06, size = 4, normalized size = 4.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2-tan(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))
```

Maxima [A]
time = 0.47, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 2.38, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-\tan(x) + \sec(x))^3 (\tan(x) + \sec(x))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2-tan(x)**2)**3,x)
```

```
[Out] Integral(1/((-tan(x) + sec(x))**3*(tan(x) + sec(x))**3), x)
```

Giac [A]
time = 0.43, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="giac")
[Out] x
```

Mupad [B]
time = 2.57, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2 - tan(x)^2)^3,x)
[Out] x
```

3.490 $\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$

Optimal. Leaf size=37

$$-x+\sqrt{2}\,x-\sqrt{2}\,\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

[Out] -x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1144, 209}

$$-\sqrt{2}\,\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)+\sqrt{2}\,x-x$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]^2 + Csc[x]^2)^(-1),x]

[Out] -x + Sqrt[2]*x - Sqrt[2]*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{2+3x^2+x^4}\,dx,x,\tan(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= -x+\sqrt{2}\,x-\sqrt{2}\,\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 0.51

$$-x+\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-1),x]
[Out] -x + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]
```

Maple [A]
time = 0.15, size = 19, normalized size = 0.51

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)$	19
risch	$-x+\frac{i\sqrt{2}\,\ln\left(e^{2ix}+2\sqrt{2}+3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(e^{2ix}-2\sqrt{2}+3\right)}{2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2),x,method=_RETURNVERBOSE)
[Out] -arctan(tan(x))+2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Maxima [A]
time = 0.49, size = 16, normalized size = 0.43

$$\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\tan\left(x\right)\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="maxima")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

Fricas [A]
time = 3.06, size = 35, normalized size = 0.95

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos\left(x\right)^{2}-\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) - x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cot^2(x) + \csc^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2),x)
```

```
[Out] Integral(1/(cot(x)**2 + csc(x)**2), x)
```

Giac [A]
time = 0.42, size = 49, normalized size = 1.32

$$\sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - \sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - \cos(2x) + 1}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqrt
(2) - cos(2*x) + 1))) - x
```

Mupad [B]
time = 2.71, size = 16, normalized size = 0.43

$$\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2),x)
```

```
[Out] 2^(1/2)*atan((2^(1/2)*tan(x))/2) - x
```

3.491 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2} \, dx$

Optimal. Leaf size=47

$$x-\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}-\frac{\tan(x)}{2+\tan^2(x)}$$

```
[Out] x-1/2*x*2^(1/2)+1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-tan(
x)/(2+tan(x)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {481, 12, 400, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}-\frac{x}{\sqrt{2}}+x-\frac{\tan(x)}{\tan^2(x)+2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

```
[Out] x - x/Sqrt[2] + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)]/Sqrt[2] -
Tan[x]/(2 + Tan[x]^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
```

```
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\frac{1}{2}\text{Subst}\left(\int \frac{2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(2+x^2\right)}\,dx, x, \tan(x)\right)\\
&= -\frac{\tan(x)}{2+\tan^2(x)}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \tan(
\\
&= x-\frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}-\frac{\tan(x)}{2+\tan^2(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 1.36

$$
\frac{(3+\cos(2x))\csc^4(x)\left(6x+2x\cos(2x)-\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)(3+\cos(2x))-2\sin(2x)\right)}{8\left(\cot^2(x)+\csc^2(x)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-2),x]
[Out] ((3 + Cos[2*x])*Csc[x]^4*(6*x + 2*x*Cos[2*x] - Sqrt[2]*ArcTan[Tan[x]/Sqrt[2
]]*(3 + Cos[2*x]) - 2*Sin[2*x]))/(8*(Cot[x]^2 + Csc[x]^2)^2)
```

Maple [A]
time = 0.16, size = 30, normalized size = 0.64

method	result	size
default	$\arctan(\tan(x))-\frac{\tan(x)}{2+\tan^2(x)}-\frac{\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{2}$	30

risch	$x-\frac{2i(3\,\mathrm{e}^{2ix}+1)}{\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{4}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^2,x,method=_RETURNVERBOSE)
[Out] arctan(tan(x))-tan(x)/(2+tan(x)^2)-1/2*2^(1/2)*arctan(1/2*tan(x)*2^(1/2))
```

Maxima [A]
time = 0.46, size = 27, normalized size = 0.57

$$-\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\tan\left(x\right)\right)+x-\frac{\tan\left(x\right)}{\tan\left(x\right)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) + x - tan(x)/(tan(x)^2 + 2)
```

Fricas [A]
time = 2.66, size = 66, normalized size = 1.40

$$\frac{4\,x\cos\left(x\right)^{2}+\left(\sqrt{2}\,\cos\left(x\right)^{2}+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\,\cos(x)^{2}-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\,\cos\left(x\right)\sin\left(x\right)+4\,x}{4\left(\cos\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="fricas")
[Out] 1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(
x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) + 4*x)/(cos(x)^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\cot^{2}\left(x\right)+\csc^{2}\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**2,x)
[Out] Integral((cot(x)**2 + csc(x)**2)**(-2), x)
```

Giac [A]
time = 0.42, size = 60, normalized size = 1.28

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\,\sin\left(2\,x\right)-\sin\left(2\,x\right)}{\sqrt{2}\,\cos\left(2\,x\right)+\sqrt{2}-\cos\left(2\,x\right)+1}\right)\right)+x-\frac{\tan\left(x\right)}{\tan\left(x\right)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) + x - tan(x)/(tan(x)^2 + 2)
```

Mupad [B]
time = 2.69, size = 27, normalized size = 0.57

$$x-\frac{\tan(x)}{\tan\left(x\right)^2+2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2)^2,x)
```

```
[Out] x - tan(x)/(tan(x)^2 + 2) - (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

3.492 $\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx$

Optimal. Leaf size=72

$$-x+\frac{7x}{4\sqrt{2}}-\frac{7\mathrm{ArcTan}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}$$

```
[Out] -x+7/8*x*2^(1/2)-7/8*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-1/2
*tan(x)^3/(2+tan(x)^2)^2+1/4*tan(x)/(2+tan(x)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {481, 592, 536, 209}

$$-\frac{7\mathrm{ArcTan}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}+\frac{7x}{4\sqrt{2}}-x+\frac{\tan(x)}{4\left(\tan^2(x)+2\right)}-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

```
[Out] -x + (7*x)/(4*Sqrt[2]) - (7*ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Cos[x]^2)
])/(4*Sqrt[2]) - Tan[x]^3/(2*(2 + Tan[x]^2)^2) + Tan[x]/(4*(2 + Tan[x]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 592

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[g^(n - 1)*(b*e - a*f)*(
g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*
(p + 1))), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}\,dx &= \text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)\left(2+x^2\right)^3}\,dx,x,\tan(x)\right)\\ &= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{1}{4}\text{Subst}\left(\int \frac{x^2\left(6+2x^2\right)}{\left(1+x^2\right)\left(2+x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{2-6x^2}{\left(1+x^2\right)\left(2+x^2\right)}\,dx,x,\tan\right.\\ &= -\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}+\frac{7}{4}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\tan(x)\right)-\text{Sul}\\ &= -x+\frac{7x}{4\sqrt{2}}-\frac{7\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}}-\frac{\tan^3(x)}{2\left(2+\tan^2(x)\right)^2}+\frac{\tan(x)}{4\left(2+\tan^2(x)\right)}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 0.92

$$\frac{-76x-48x\cos(2x)+7\sqrt{2}\,\text{ArcTan}\left(\frac{\tan(x)}{\sqrt{2}}\right)(3+\cos(2x))^2-4x\cos(4x)+2\sin(2x)+3\sin(4x)}{8(3+\cos(2x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

```
[Out] (-76*x - 48*x*Cos[2*x] + 7*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2
- 4*x*Cos[4*x] + 2*Sin[2*x] + 3*Sin[4*x])/(8*(3 + Cos[2*x])^2)
```

Maple [A]
time = 0.19, size = 41, normalized size = 0.57

method	result	size
default	$-\arctan(\tan(x))+\frac{-\frac{(\tan^3(x))}{4}+\frac{\tan(x)}{2}}{(2+\tan^2(x))^2}+\frac{7\sqrt{2}\,\arctan\left(\frac{\tan(x)\sqrt{2}}{2}\right)}{8}$	41
risch	$-x+\frac{i(17\,\mathrm{e}^{6ix}+57\,\mathrm{e}^{4ix}+19\,\mathrm{e}^{2ix}+3)}{2(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1)^2}+\frac{7i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{16}-\frac{7i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{16}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2+csc(x)^2)^3,x,method=_RETURNVERBOSE)
[Out] -arctan(tan(x))+2*(-1/8*tan(x)^3+1/4*tan(x))/(2+tan(x)^2)^2+7/8*2^(1/2)*arc
tan(1/2*tan(x)*2^(1/2))
```

Maxima [A]
time = 0.48, size = 42, normalized size = 0.58

$$\frac{7}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\tan(x)\right)-x-\frac{\tan(x)^3-2\,\tan(x)}{4\left(\tan(x)^4+4\,\tan(x)^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="maxima")
[Out] 7/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan
(x)^4 + 4*tan(x)^2 + 4)
```

Fricas [A]
time = 2.33, size = 98, normalized size = 1.36

$$-\frac{16\,x\cos(x)^4+32\,x\cos(x)^2+7\left(\sqrt{2}\,\cos(x)^4+2\sqrt{2}\,\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\,\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\left(3\,\cos(x)^3-\cos(x)\right)\sin(x)+16\,x}{16\left(\cos(x)^4+2\,\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="fricas")
[Out] -1/16*(16*x*cos(x)^4 + 32*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos(
x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x)))
- 4*(3*cos(x)^3 - cos(x))*sin(x) + 16*x)/(cos(x)^4 + 2*cos(x)^2 + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2+csc(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 69, normalized size = 0.96

$$\frac{7}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-\cos\left(2x\right)+1}\right)\right)-x-\frac{\tan\left(x\right)^{3}-2\tan\left(x\right)}{4\left(\tan\left(x\right)^{2}+2\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="giac")
```

```
[Out] 7/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
sqrt(2) - cos(2*x) + 1))) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan(x)^2 + 2)^2
```

Mupad [B]
time = 2.68, size = 43, normalized size = 0.60

$$\frac{\frac{\tan(x)}{2}-\frac{\tan(x)^{3}}{4}}{\tan\left(x\right)^{4}+4\tan\left(x\right)^{2}+4}-x+\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 + 1/sin(x)^2)^3,x)
```

```
[Out] (tan(x)/2 - tan(x)^3/4)/(4*tan(x)^2 + tan(x)^4 + 4) - x + (7*2^(1/2)*atan((
2^(1/2)*tan(x))/2))/8
```

3.493 $\int \frac{1}{\cot^2(x)-\csc^2(x)}\,dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4467, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

```
[Out] -x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\cot^2(x)-\csc^2(x)}\,dx &= -\int 1\,dx \\ &= -x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

```
[Out] -x
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.08, size = 6, normalized size = 2.00

method	result	size
risch	$-x$	4
default	$-\arctan(\tan(x))$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2),x,method=_RETURNVERBOSE)
[Out] -arctan(tan(x))
```

Maxima [A]
time = 0.49, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="maxima")
[Out] -x
```

Fricas [A]
time = 2.46, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="fricas")
[Out] -x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x) - \csc(x))(\cot(x) + \csc(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2),x)
[Out] Integral(1/((cot(x) - csc(x))*(cot(x) + csc(x))), x)
```

Giac [A]
time = 0.41, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="giac")
[Out] -x
```

Mupad [B]
time = 2.73, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2),x)
[Out] -x
```

3.494 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4467, 8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} \, dx &= \int 1 \, dx \\ &= x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

[Out] x

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.06, size = 4, normalized size = 4.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2-csc(x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctan(tan(x))
```

Maxima [A]
time = 0.46, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 2.40, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\cot(x)-\csc(x))^2(\cot(x)+\csc(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)**2-csc(x)**2)**2,x)
```

```
[Out] Integral(1/((cot(x) - csc(x))**2*(cot(x) + csc(x))**2), x)
```

Giac [A]
time = 0.43, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 2.65, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2)^2,x)
```

```
[Out] x
```

3.495 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3} \, dx$

Optimal. Leaf size=3

$$-x$$

```
[Out] -x
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4467, 8}

$$-x$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

```
[Out] -x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4467

```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3} \, dx &= -\int 1 \, dx \\ &= -x \end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

[Out] -x

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 1.
time = 0.07, size = 6, normalized size = 2.00

method	result	size
risch	$-x$	4
default	$-\arctan\left(\tan\left(x\right)\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cot(x)^2-csc(x)^2)^3,x,method=_RETURNVERBOSE)

[Out] -arctan(tan(x))

Maxima [A]
time = 0.47, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="maxima")

[Out] -x

Fricas [A]
time = 2.52, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="fricas")

[Out] -x

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(cot(x)**2-csc(x)**2)**3,x)

[Out] Timed out

Giac [A]
time = 0.42, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="giac")
```

```
[Out] -x
```

Mupad [B]
time = 2.62, size = 3, normalized size = 1.00

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cot(x)^2 - 1/sin(x)^2)^3,x)
```

```
[Out] -x
```

3.496 $\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

```
[Out] arctan((a+c)^(1/2)*tan(x)/(a+b)^(1/2))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+b+(a+c)x^2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 1.00

$$\frac{\text{ArcTan}\left(\frac{\sqrt{a+c}\,\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

Maple [A]
time = 0.15, size = 27, normalized size = 0.82

method	result
default	$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia^2+2iab+2iac+2ibc+2a\sqrt{-a^2-ab-ac-cb}+b\sqrt{-a^2-ab-ac-cb}+c\sqrt{-a^2-ab-ac-cb}}{\sqrt{-a^2-ab-ac-cb}\,(b-c)}\right.}{2\sqrt{-a^2-ab-ac-cb}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((a+b)*(a+c))^(1/2)*arctan((a+c)*tan(x)/((a+b)*(a+c))^(1/2))
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.79

$$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan((a + c)*tan(x)/sqrt((a + b)*(a + c)))/sqrt((a + b)*(a + c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.
time = 3.52, size = 259, normalized size = 7.85

$$\left[-\frac{\sqrt{-a^2-ab-(a+b)c}\,\log\left(\frac{(8a^2+8ab+b^2+2(4a+3b)c+c^2)\cos(x)^4-2(4a^2+3ab+(5a+3b)c+c^2)\cos(x)^2+4\left((2a+b+c)\cos(x)^3-(a+c)\cos(x)\right)\sqrt{-a^2-ab-(a+b)c}\,\sin(x)+a^2+2ac+c^2}{(b^2-2bc+c^2)\cos(x)^4+2(ab-(a-b)c-c^2)\cos(x)^2+a^2+2ac+c^2}\right)}{4(a^2+ab+(a+b)c)}, -\frac{\arctan\left(\frac{(2a+b+c)\cos(x)^2-a-c}{2\sqrt{a^2+ab+(a+b)c}\,\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab+(a+b)c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b - (a + b)*c)*log(((8*a^2 + 8*a*b + b^2 + 2*(4*a + 3*b
)*c + c^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b + (5*a + 3*b)*c + c^2)*cos(x)^2 + 4*
```

```
((2*a + b + c)*cos(x)^3 - (a + c)*cos(x))*sqrt(-a^2 - a*b - (a + b)*c)*sin(
x) + a^2 + 2*a*c + c^2)/((b^2 - 2*b*c + c^2)*cos(x)^4 + 2*(a*b - (a - b)*c
- c^2)*cos(x)^2 + a^2 + 2*a*c + c^2))/(a^2 + a*b + (a + b)*c), -1/2*arctan(
1/2*((2*a + b + c)*cos(x)^2 - a - c)/(sqrt(a^2 + a*b + (a + b)*c)*cos(x)*si
n(x)))/sqrt(a^2 + a*b + (a + b)*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\cos^2(x) + c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(1/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.
time = 0.42, size = 61, normalized size = 1.85

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,c)+\arctan\left(\frac{a\tan(x)+c\tan(x)}{\sqrt{a^2+ab+ac+bc}}\right)}{\sqrt{a^2+ab+ac+bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*c) + arctan((a*tan(x) + c*tan(x))/sqrt(a^
2 + a*b + a*c + b*c)))/sqrt(a^2 + a*b + a*c + b*c)
```

Mupad [B]
time = 2.85, size = 43, normalized size = 1.30

$$\frac{\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,c)}{2\,\sqrt{a\,b+a\,c+b\,c+a^2}}\right)}{\sqrt{a\,b+a\,c+b\,c+a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

```
[Out] atan((tan(x)*(2*a + 2*c))/(2*(a*b + a*c + b*c + a^2)^(1/2)))/(a*b + a*c + b
*c + a^2)^(1/2)
```

3.497 $\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=239

$$-\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}+\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}-\frac{\text{PolyLog}\left(2,-\frac{(b}{2a+b+c-2\sqrt{}}\right.}{4\sqrt{a+b}\,\sqrt{a}}$$

```
[Out] -1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(
1/2)/(a+c)^(1/2)+1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)
^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/4*polylog(2,-(b-c)*exp(2*I*x)/(2*a+b+c-2
*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/4*polylog(2,-(b-c)*exp
(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4683, 3402, 2296, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{4\sqrt{a+b}\,\sqrt{a+c}}+\frac{\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{4\sqrt{a+b}\,\sqrt{a+c}}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\,\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}+\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\,\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-1/2*I)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt
[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x*Log[1 + ((b - c)*E^((2*I)*x
))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) -
PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]
))]/(4*Sqrt[a + b]*Sqrt[a + c]) + PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a +
 b + c + 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*Sqrt[a + b]*Sqrt[a + c])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

```
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4683

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rubi steps

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x}{2a+b+c+(b-c)\cos(2x)}\,dx$$
$$= 4\int \frac{e^{2ix}x}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$
$$= \frac{(2(b-c))\int \frac{e^{2ix}x}{-4\sqrt{a+b}\,\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\,\sqrt{a+c}} - \frac{(2(b-c))\int \frac{}{4\sqrt{a}}}{}$$
$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$

$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$
$$= -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 507 vs. $2(239) = 478$.
time = 2.13, size = 507, normalized size = 2.12

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)\left(2x+\frac{i\left(\log\left(\frac{\sqrt{a+c}(1+i\tan(x))}{\sqrt{a+b}+\sqrt{a+c}}\right)\log\left(1-\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)-\log\left(\frac{i\sqrt{a+c}(i+\tan(x))}{\sqrt{a+b}-\sqrt{a+c}}\right)\log\left(1-\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)+\log\left(\frac{\sqrt{a+c}(1-i\tan(x))}{\sqrt{a+b}+\sqrt{a+c}}\right)\log\left(1+\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)-\log\left(\frac{\sqrt{a+c}(1+i\tan(x))}{-\sqrt{a+b}+\sqrt{a+c}}\right)\log\left(1+\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}-\sqrt{a+c}}\right)+\operatorname{PolyLog}\left(2,\frac{\sqrt{a+b}-i\sqrt{a+c}\tan(x)}{\sqrt{a+b}+\sqrt{a+c}}\right)-\operatorname{PolyLog}\left(2,\frac{\sqrt{a+b}+i\sqrt{a+c}\tan(x)}{\sqrt{a+b}-\sqrt{a+c}}\right)+\operatorname{PolyLog}\left(2,\frac{\sqrt{a+b}+i\sqrt{a+c}\tan(x)}{\sqrt{a+b}+\sqrt{a+c}}\right)\right)}{\log\left(1-\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)-\log\left(1+\frac{i\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]*(2*x + (I*(Log[(Sqrt[a + c]*(1 +
I*Tan[x]))/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt
[a + b]] - Log[(I*Sqrt[a + c]*(I + Tan[x]))/(Sqrt[a + b] - Sqrt[a + c])]*Lo
g[1 - (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] + Log[(Sqrt[a + c]*(1 - I*Tan[x])
)/(Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]]
- Log[(Sqrt[a + c]*(1 + I*Tan[x]))/(-Sqrt[a + b] + Sqrt[a + c])]*Log[1 + (I
*Sqrt[a + c]*Tan[x])/Sqrt[a + b]] - PolyLog[2, (Sqrt[a + b] - I*Sqrt[a + c]
*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] - I*Sqrt[a
+ c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])] - PolyLog[2, (Sqrt[a + b] + I*Sqr
t[a + c]*Tan[x])/(Sqrt[a + b] - Sqrt[a + c])] + PolyLog[2, (Sqrt[a + b] + I
*Sqrt[a + c]*Tan[x])/(Sqrt[a + b] + Sqrt[a + c])]))/(Log[1 - (I*Sqrt[a + c]
*Tan[x])/Sqrt[a + b]] - Log[1 + (I*Sqrt[a + c]*Tan[x])/Sqrt[a + b]])))/(2*S
qrt[a + b]*Sqrt[a + c])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(189) = 378$.
time = 0.21, size = 820, normalized size = 3.43

method	result
risch	$-\frac{i \ln\left(1-\frac{(b-c)\,\mathrm{e}^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)x}{-2\sqrt{(a+b)(a+c)}-2a-b-c}-\frac{i \ln\left(1-\frac{(b-c)\,\mathrm{e}^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)ax}{\sqrt{(a+b)(a+c)}\left(-2\sqrt{(a+b)(a+c)}-2a-b-c\right)}-\frac{\cdots}{2\sqrt{(a \cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))
^(1/2)-2*a-b-c))*x-I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*l
n(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a*x-1/2*I/((a+b)*(a+
c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c))*b*x-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/
2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c*x-1/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^2-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))
^(1/2)-2*a-b-c)*a*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b
-c)*b*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x^2-1/
2/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a
+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a-1/4/((a+b
)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/
(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*b-1/4/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))*c-1/2*I/((a+b)*(a+c))^(1/2)*x*ln(1-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1
/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x^2-1/4/((a+b)*(a+c))^(1/2)*polylog(2
,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2869 vs. $2(189) = 378$.
time = 5.28, size = 2869, normalized size = 12.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")

[Out] -1/4*(-I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b - c)) - I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + b - c)/(b - c)) - I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - b + c)/(b - c)) - I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*c)*sin(x) + 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) + 2*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - b + c)/(b - c)) - (b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-(((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b - c) + 1) - (b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*s

```
qrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^
2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/
(b - c) + 1) - (b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*di
log(-(((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(
x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
)*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a
+ b + c)/(b - c)) + b - c)/(b - c) + 1) - (b - c)*sqrt((a^2 + a*b + (a + b)
*c)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c
)*sin(x) - 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(
b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c) + 1) + (b - c)
*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-(((2*a + b + c)*c
os(x) + (2*I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x
))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + b -
c)/(b - c) + 1) + (b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))
*dilog((((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos
(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a
- b - c)/(b - c)) - b + c)/(b - c) + 1) + (b - c)*sqrt((a^2 + a*b + (a + b)
*c)/(b^2 - 2*b*c + c^2))*dilog(-(((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*
c)*sin(x) + 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a +
b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b
^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) + b - c)/(b - c) + 1) + (b - c)*
sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b + c)*cos
(x) - (-2*I*a - I*b - I*c)*sin(x) + 2*((b - c)*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b\cos^2(x) + c\sin^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(x/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{b\cos(x)^2 + c\sin(x)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + c*sin(x)^2 + b*cos(x)^2),x)
[Out] int(x/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

3.498 $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=365

$$-\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}+\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}-\frac{x\text{PolyLog}\left(2,-\frac{\quad}{2a+b+c-2}\right.}{2\sqrt{a+b}\,\sqrt{}}$$

```
[Out] -1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)
^(1/2)/(a+c)^(1/2)+1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(
a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/2*x*polylog(2,-(b-c)*exp(2*I*x)/(2*a
+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/2*x*polylog(2,-(
b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2
)-1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a
+b)^(1/2)/(a+c)^(1/2)+1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1
/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4683, 3402, 2296, 2221, 2611, 2320, 6724}

$$-\frac{x\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}+\frac{x\text{Li}_2\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}-\frac{i\text{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\,\sqrt{a+c}}\right)}{4\sqrt{a+b}\,\sqrt{a+c}}+\frac{i\text{Li}_3\left(-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\,\sqrt{a+c}}\right)}{4\sqrt{a+b}\,\sqrt{a+c}}-\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\,\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}+\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{2\sqrt{a+b}\,\sqrt{a+c}+2a+b+c}\right)}{2\sqrt{a+b}\,\sqrt{a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-1/2*I)*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sq
rt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/2)*x^2*Log[1 + ((b - c)*E^((2*
I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(Sqrt[a + b]*Sqrt[a + c]
) - (x*PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt
[a + c]))])/(2*Sqrt[a + b]*Sqrt[a + c]) + (x*PolyLog[2, -(((b - c)*E^((2*I)
*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(2*Sqrt[a + b]*Sqrt[a + c
]) - ((I/4)*PolyLog[3, -(((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]
*Sqrt[a + c]))])/(Sqrt[a + b]*Sqrt[a + c]) + ((I/4)*PolyLog[3, -(((b - c)*E
^((2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))])/(Sqrt[a + b]*Sqrt[
a + c])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3402

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[2, Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e + f
*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(e
 + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4683

```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(2*a
+ b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g},
 x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx = 2\int \frac{x^2}{2a+b+c+(b-c)\cos(2x)}\,dx$$

$$= 4\int \frac{e^{2ix}x^2}{b-c+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}}\,dx$$

$$= \frac{(2(b-c))\int \frac{e^{2ix}x^2}{-4\sqrt{a+b}\sqrt{a+c}+2(2a+b+c)+2(b-c)e^{2ix}}\,dx}{\sqrt{a+b}\sqrt{a+c}} - \frac{(2(b-c))\int \frac{}{4\sqrt{a+}}}{\sqrt{}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{}}\right.}{2\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{}}\right.}{2\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{}}\right.}{2\sqrt{a+b}\sqrt{a+c}}$$

$$= -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{}}\right.}{2\sqrt{a+b}\sqrt{a+c}}$$

Mathematica [**A**]
time = 2.73, size = 258, normalized size = 0.71

$$-\frac{i\left(2x^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right)-2x^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right)-2ix\text{PolyLog}\left(2,\frac{(-b+c)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right)+2ix\text{PolyLog}\left(2,\frac{(-b+c)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right)+\text{PolyLog}\left(3,\frac{(-b+c)e^{2ix}}{2a+b+c-2\sqrt{(a+b)(a+c)}}\right)-\text{PolyLog}\left(3,\frac{(-b+c)e^{2ix}}{2a+b+c+2\sqrt{(a+b)(a+c)}}\right)\right)}{4\sqrt{(a+b)(a+c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

```
[Out] ((-1/4*I)*(2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[(a + b
)*(a + c)])] - 2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[(a
 + b)*(a + c)])] - (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a + b + c -
 2*Sqrt[(a + b)*(a + c)])] + (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a
 + b + c + 2*Sqrt[(a + b)*(a + c)])] + PolyLog[3, ((-b + c)*E^((2*I)*x))/(2
*a + b + c - 2*Sqrt[(a + b)*(a + c)])] - PolyLog[3, ((-b + c)*E^((2*I)*x))/
(2*a + b + c + 2*Sqrt[(a + b)*(a + c)])]))/Sqrt[(a + b)*(a + c)]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1160 vs. 2(289) = 578.
time = 0.18, size = 1161, normalized size = 3.18

method	result	size
risch	Expression too large to display	1161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c))-1/2*I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(3,(b
-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-2/3/((a+b)*(a+c))^(1/2)/(-
2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^3-I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c)
)^(1/2)-2*a-b-c)*a*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))-1/3/((a+b)*(a+c))^(1/2)*x^3-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))
^(1/2)-2*a-b-c)*c*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))-1/2/((a+b)*(a+c))^(1/2)*x*polylog(2,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(
1/2)-2*a-b-c))-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b
*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/((a+b)*(a+c
))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x*polylog(2,(b-c)*exp(2*I*x)/(-
2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2)*x^2*ln(1-(b-c)*ex
p(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)/(-2*((a+b
)*(a+c))^(1/2)-2*a-b-c)*b*x^3-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(
1/2)-2*a-b-c)*c*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))
-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x*polylog(2,(b-
c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2)/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c))-1/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)
*c*x^3-1/4*I/((a+b)*(a+c))^(1/2)*polylog(3,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c)
)^(1/2)-2*a-b-c))-2/3/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^3-I/(-2*((a+b)*(a+
c))^(1/2)-2*a-b-c)*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x*polylog(2,
(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2
)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*
(a+c))^(1/2)-2*a-b-c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4289 vs. 2(288) = 576.

time = 5.58, size = 4289, normalized size = 11.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(-I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(
(((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) + (
I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(
-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c
)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(
b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x
) - 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b
^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x^2*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (
-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 +
a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b -
 c)) - I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(
-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) -
 (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sq
rt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b
+ c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c
)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin
(x) + 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)) - 2*a - b - c)/(b - c)) + b - c)/(b - c)) - I*(b - c)*x^2*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x)
- (2*I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqr
t((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 +
 a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - b + c)/(b
- c)) - I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log
((((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*c)*sin(x) + 2*((b - c)*cos(x) -
 (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqr
t((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b -
c)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin
(x) + 2*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2)) - 2*a - b - c)/(b - c)) - b + c)/(b - c)) - 2*(b - c)*x*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(-(((2*a + b + c)*cos(x) +
 (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) + (I*b - I*c)*sin(x))*sqrt(
(a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 +
```

```
a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b -
 c) + 1) - 2*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*di
log((((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x)
 - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*s
qrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b
 + c)/(b - c)) - b + c)/(b - c) + 1) - 2*(b - c)*x*sqrt((a^2 + a*b + (a + b
)*c)/(b^2 - 2*b*c + c^2))*dilog(-(((2*a + b + c)*cos(x) + (-2*I*a - I*b - I
*c)*sin(x) - 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a
+ b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b - c)/(b - c) + 1) - 2*(b
- c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b +
 c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) - (-I*b + I*c)
*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c
)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))
 - b + c)/(b - c) + 1) + 2*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*
b*c + c^2))*dilog(-(((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) + 2*
((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 -
2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
c^2)) - 2*a - b - c)/(b - c)) + b - c)/(b - c) + 1) + 2*(b - c)*x*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b + c)*cos(x) - (2*
I*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*sqrt((a^
2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt((2*(b - c)*sqrt((a^2 + a*b
+ (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b - c)/(b - c)) - b + c)/(b - c)
+ 1) + 2*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*dilog(
-(((2*a + b + c)*cos(x) + (-2*I*a - I*b - I*c)*sin(x) + 2*((b - c)*cos(x) -
 (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqr
t((2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) - 2*a - b -
c)/(b - c)) + b - c)/(b - c) + 1) + 2*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c
)/(b^2 - 2*b*c + c^2))*dilog((((2*a + b + c)*co...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)**2+c*sin(x)**2),x)
```

```
[Out] Integral(x**2/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{b\cos(x)^2 + c\sin(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

```
[Out] int(x^2/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

3.499 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+$

Optimal. Leaf size=195

$$\frac{3}{8}a\left(a^4+12a^2b^2+8b^4\right)x-\frac{b(32a^4+69a^2b^2+4b^4)\cos(d+ex)}{10e}-\frac{a(15a^4+82a^2b^2+8b^4)\cos(d+ex)\sin(d+$$

```
[Out] 3/8*a*(a^4+12*a^2*b^2+8*b^4)*x-1/10*b*(32*a^4+69*a^2*b^2+4*b^4)*cos(e*x+d)/
e-1/40*a*(15*a^4+82*a^2*b^2+8*b^4)*cos(e*x+d)*sin(e*x+d)/e-1/20*b*(17*a^2+4
*b^2)*cos(e*x+d)*(b+a*sin(e*x+d))^2/e-1/20*(5*a^2+4*b^2)*cos(e*x+d)*(b+a*si
n(e*x+d))^3/e-1/5*b*cos(e*x+d)*(b+a*sin(e*x+d))^4/e
```

Rubi [A]
time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3369, 2832, 2813}

$$-\frac{(5a^2+4b^2)\cos(d+ex)(a\sin(d+ex)+b)^3}{20e}-\frac{b(17a^2+4b^2)\cos(d+ex)(a\sin(d+ex)+b)^2}{20e}-\frac{b(32a^4+69a^2b^2+4b^4)\cos(d+ex)}{10e}-\frac{a(15a^4+82a^2b^2+8b^4)\sin(d+ex)\cos(d+ex)}{40e}+\frac{3}{8}ax\left(a^4+12a^2b^2+8b^4\right)-\frac{b\cos(d+ex)(a\sin(d+ex)+b)^4}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```

```
[Out] (3*a*(a^4 + 12*a^2*b^2 + 8*b^4)*x)/8 - (b*(32*a^4 + 69*a^2*b^2 + 4*b^4)*Cos
[d + e*x])/(10*e) - (a*(15*a^4 + 82*a^2*b^2 + 8*b^4)*Cos[d + e*x]*Sin[d + e
*x])/(40*e) - (b*(17*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^2)/(20*
e) - ((5*a^2 + 4*b^2)*Cos[d + e*x]*(b + a*Sin[d + e*x])^3)/(20*e) - (b*Cos[
d + e*x]*(b + a*Sin[d + e*x])^4)/(5*e)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3369

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx=\frac{\int \left(2ab+2a^2\sin(d+ex)\right)^4 (a+b\sin(d+}{16a^4}$$

$$=-\frac{b\cos(d+ex)(b+a\sin(d+ex))^4}{5e}+\int$$

$$=-\frac{(5a^2+4b^2)\cos(d+ex)(b+a\sin(d+}{20e}$$

$$=-\frac{b(17a^2+4b^2)\cos(d+ex)(b+a\sin(d}{20e}$$

$$=\frac{3}{8}a\left(a^4+12a^2b^2+8b^4\right)x-\frac{b(32a^4+69}{}$$

Mathematica [A]
time = 0.64, size = 149, normalized size = 0.76

$$\frac{-20b(29a^4+68a^2b^2+8b^4)\cos(d+ex)+a(60(a^4+12a^2b^2+8b^4)(d+ex)+10(7a^3b+8ab^3)\cos(3(d+ex))-2a^3b\cos(5(d+ex))-40(a^4+10a^2b^2+4b^4)\sin(2(d+ex))+5(a^4+4a^2b^2)\sin(4(d+ex)))}{160e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] (-20*b*(29*a^4 + 68*a^2*b^2 + 8*b^4)*Cos[d + e*x] + a*(60*(a^4 + 12*a^2*b^2
 + 8*b^4)*(d + e*x) + 10*(7*a^3*b + 8*a*b^3)*Cos[3*(d + e*x)] - 2*a^3*b*Cos
[5*(d + e*x)] - 40*(a^4 + 10*a^2*b^2 + 4*b^4)*Sin[2*(d + e*x)] + 5*(a^4 + 4
*a^2*b^2)*Sin[4*(d + e*x)]))/(160*e)
```

Maple [A]
time = 0.19, size = 255, normalized size = 1.31

method	result
risch	$\frac{3a^5x}{8}+\frac{9a^3b^2x}{2}+3ab^4x-\frac{29b\cos(ex+d)a^4}{8e}-\frac{17b^3\cos(ex+d)a^2}{2e}-\frac{b^5\cos(ex+d)}{e}-\frac{a^4b\cos(5ex+5d)}{80e}+\sin($
derivativedivides	$\frac{ab^4(ex+d)-4b^3a^2\cos(ex+d)+6a^3b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{4ba^4\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3}+a^5\left(-\frac{\left(\sin^3(ex+d)\right.}{}\right.}{}$

default	$\frac{a\,b^4(ex+d)-4b^3a^2\cos(ex+d)+6a^3b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{4b\,a^4\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3}+a^5\left(-\frac{\left(\sin^3(ex+\cdots\right.}{}\right.}{}$
norman	$\frac{(3a\,b^4+\frac{3}{8}a^5+\frac{9}{2}a^3b^2)x+(3a\,b^4+\frac{3}{8}a^5+\frac{9}{2}a^3b^2)x\left(\tan^{10}\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(15a\,b^4+\frac{15}{8}a^5+\frac{45}{2}a^3b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(15a\,b^4+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(a*b^4*(e*x+d)-4*b^3*a^2*cos(e*x+d)+6*a^3*b^2*(-1/2*cos(e*x+d)*sin(e*x+
d)+1/2*e*x+1/2*d)-4/3*b*a^4*(2+sin(e*x+d)^2)*cos(e*x+d)+a^5*(-1/4*(sin(e*x+
d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-b^5*cos(e*x+d)+4*a*b^4*(-1/2
*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-2*b^3*a^2*(2+sin(e*x+d)^2)*cos(e*x+d)
+4*a^3*b^2*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*x+d)+3/8*e*x+3/8*d)-1/
5*b*a^4*(8/3+sin(e*x+d)^4+4/3*sin(e*x+d)^2)*cos(e*x+d))
```

Maxima [A]
time = 0.27, size = 265, normalized size = 1.36

$$\frac{1}{480}\left(15\left(12\,xe+12\,d+\sin\left(4\,xe+4\,d\right)-8\,\sin\left(2\,xe+2\,d\right)\right)a^5-32\left(3\,\cos\left(xe+d\right)^5-10\,\cos\left(xe+d\right)^3+15\,\cos\left(xe+d\right)\right)a^4b+640\left(\cos\left(xe+d\right)^3-3\,\cos\left(xe+d\right)\right)a^4b+60\left(12\,xe+12\,d+\sin\left(4\,xe+4\,d\right)-8\,\sin\left(2\,xe+2\,d\right)\right)a^3b^2+720\left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)a^3b^2+960\left(\cos\left(xe+d\right)^3-3\,\cos\left(xe+d\right)\right)a^2b^3+480\left(2\,xe+2\,d-\sin\left(2\,xe+2\,d\right)\right)ab^4+480\left(xe+d\right)ab^4-1920\,a^2b^3\cos\left(xe+d\right)-480\,b^5\cos\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/480*(15*(12*x*e + 12*d + sin(4*x*e + 4*d) - 8*sin(2*x*e + 2*d))*a^5 - 32*
(3*cos(x*e + d)^5 - 10*cos(x*e + d)^3 + 15*cos(x*e + d))*a^4*b + 640*(cos(x
*e + d)^3 - 3*cos(x*e + d))*a^4*b + 60*(12*x*e + 12*d + sin(4*x*e + 4*d) -
8*sin(2*x*e + 2*d))*a^3*b^2 + 720*(2*x*e + 2*d - sin(2*x*e + 2*d))*a^3*b^2
+ 960*(cos(x*e + d)^3 - 3*cos(x*e + d))*a^2*b^3 + 480*(2*x*e + 2*d - sin(2*
x*e + 2*d))*a*b^4 + 480*(x*e + d)*a*b^4 - 1920*a^2*b^3*cos(x*e + d) - 480*b
^5*cos(x*e + d))*e^(-1)
```

Fricas [A]
time = 2.63, size = 156, normalized size = 0.80

$$-\frac{1}{40}\left(8\,a^4b\cos\left(xe+d\right)^5-80\left(a^4b+a^2b^3\right)\cos\left(xe+d\right)^3-15\left(a^5+12\,a^3b^2+8\,ab^4\right)xe+40\left(5\,a^4b+10\,a^2b^3+b^5\right)\cos\left(xe+d\right)-5\left(2\left(a^5+4\,a^3b^2\right)\cos\left(xe+d\right)^3-\left(5\,a^5+44\,a^3b^2+16\,ab^4\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/40*(8*a^4*b*cos(x*e + d)^5 - 80*(a^4*b + a^2*b^3)*cos(x*e + d)^3 - 15*(a
^5 + 12*a^3*b^2 + 8*a*b^4)*x*e + 40*(5*a^4*b + 10*a^2*b^3 + b^5)*cos(x*e +
d) - 5*(2*(a^5 + 4*a^3*b^2)*cos(x*e + d)^3 - (5*a^5 + 44*a^3*b^2 + 16*a*b^4
)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(178) = 356$.
time = 0.39, size = 566, normalized size = 2.90

$$\begin{cases} \frac{3a^5x\sin^4(d+ex)}{8} + \frac{3a^5x\sin^2(d+ex)\cos^2(d+ex)}{4} + \frac{3a^5x\cos^4(d+ex)}{8} - \frac{5a^5\sin^3(d+ex)\cos(d+ex)}{8e} - \frac{3a^5\sin(d+ex)\cos^3(d+ex)}{8e} - \frac{a^4b\sin^4(d+ex)\cos(d+ex)}{e} - \frac{4a^4b\sin^2(d+ex)\cos^3(d+ex)}{3e} - \frac{4a^4b\sin^2(d+ex)\cos(d+ex)}{e} - \frac{8a^4b\cos^5(d+ex)}{15e} - \frac{8a^4b\cos^3(d+ex)}{3e} + \frac{3a^3b^2x\sin^4(d+ex)}{2} + 3a^3b^2x\sin^2(d+ex)\cos^2(d+ex) + 3a^3b^2x\sin^2(d+ex) + \frac{3a^3b^2x\cos^4(d+ex)}{2} + 3a^3b^2x\cos^2(d+ex) - \frac{5a^3b^2\sin^3(d+ex)\cos(d+ex)}{2e} - \frac{3a^3b^2\sin(d+ex)\cos^3(d+ex)}{2e} - \frac{3a^3b^2\sin(d+ex)\cos(d+ex)}{e} - \frac{6a^2b^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{4a^2b^3\cos^3(d+ex)}{e} - \frac{4a^2b^3\cos(d+ex)}{e} + 2ab^4x\sin^2(d+ex) + 2ab^4x\cos^2(d+ex) + ab^4x - \frac{2ab^4\sin(d+ex)\cos(d+ex)}{e} - \frac{b^5\cos(d+ex)}{e} & \text{for}\: e \neq 0 \\ x(a+b\sin(d))(a^2\sin^2(d)+2ab\sin(d)+b^2)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((3*a**5*x*sin(d + e*x)**4/8 + 3*a**5*x*sin(d + e*x)**2*cos(d + e*
x)**2/4 + 3*a**5*x*cos(d + e*x)**4/8 - 5*a**5*sin(d + e*x)**3*cos(d + e*x)/
(8*e) - 3*a**5*sin(d + e*x)*cos(d + e*x)**3/(8*e) - a**4*b*sin(d + e*x)**4*
cos(d + e*x)/e - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)**3/(3*e) - 4*a**4*b*
sin(d + e*x)**2*cos(d + e*x)/e - 8*a**4*b*cos(d + e*x)**5/(15*e) - 8*a**4*b
*cos(d + e*x)**3/(3*e) + 3*a**3*b**2*x*sin(d + e*x)**4/2 + 3*a**3*b**2*x*si
n(d + e*x)**2*cos(d + e*x)**2 + 3*a**3*b**2*x*sin(d + e*x)**2 + 3*a**3*b**2
*x*cos(d + e*x)**4/2 + 3*a**3*b**2*x*cos(d + e*x)**2 - 5*a**3*b**2*sin(d +
e*x)**3*cos(d + e*x)/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)**3/(2*e)
 - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b**3*sin(d + e*x)**2*co
s(d + e*x)/e - 4*a**2*b**3*cos(d + e*x)**3/e - 4*a**2*b**3*cos(d + e*x)/e +
 2*a*b**4*x*sin(d + e*x)**2 + 2*a*b**4*x*cos(d + e*x)**2 + a*b**4*x - 2*a*b
**4*sin(d + e*x)*cos(d + e*x)/e - b**5*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b
*sin(d))*(a**2*sin(d)**2 + 2*a*b*sin(d) + b**2)**2, True))
```

Giac [A]
time = 0.44, size = 158, normalized size = 0.81

$$-\frac{a^4b\cos(5ex+5d)}{80e} + \frac{3}{8}(a^5+12a^3b^2+8ab^4)x + \frac{(7a^4b+8a^2b^3)\cos(3ex+3d)}{16e} - \frac{(29a^4b+68a^2b^3+8b^5)\cos(ex+d)}{8e} + \frac{(a^5+4a^3b^2)\sin(4ex+4d)}{32e} - \frac{(a^5+10a^3b^2+4ab^4)\sin(2ex+2d)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -1/80*a^4*b*cos(5*e*x + 5*d)/e + 3/8*(a^5 + 12*a^3*b^2 + 8*a*b^4)*x + 1/16*
(7*a^4*b + 8*a^2*b^3)*cos(3*e*x + 3*d)/e - 1/8*(29*a^4*b + 68*a^2*b^3 + 8*b
^5)*cos(e*x + d)/e + 1/32*(a^5 + 4*a^3*b^2)*sin(4*e*x + 4*d)/e - 1/4*(a^5 +
 10*a^3*b^2 + 4*a*b^4)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 4.49, size = 456, normalized size = 2.34

$$\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+6\,a\,b^4\right)}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+4\,a\,b^4\right)-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^9\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+4\,a\,b^4\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(\frac{7\,a^5}{2}+26\,a^3\,b^2+8\,a\,b^4\right)-{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^7\,\left(\frac{7\,a^5}{2}+26\,a^3\,b^2+8\,a\,b^4\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6\,\left(16\,a^4\,b+56\,a^2\,b^3+8\,b^5\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(32\,a^4\,b+72\,a^2\,b^3+8\,b^5\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(48\,a^4\,b+104\,a^2\,b^3+12\,b^5\right)+\frac{32\,a^4\,b}{5}+2\,b^5+16\,a^2\,b^3+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^8\,\left(8\,a^2\,b^3+2\,b^5\right)}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^8+10\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+10\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+5\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2+1\right)}-\frac{3\,a\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2,
x)
```

```
[Out] (3*a*atan((3*a*tan(d/2 + (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*(6*a*b^4 +
 (3*a^5)/4 + 9*a^3*b^2)))*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e) - (tan(d/2 + (e
*x)/2)*(4*a*b^4 + (3*a^5)/4 + 9*a^3*b^2) - tan(d/2 + (e*x)/2)^9*(4*a*b^4 +
(3*a^5)/4 + 9*a^3*b^2) + tan(d/2 + (e*x)/2)^3*(8*a*b^4 + (7*a^5)/2 + 26*a^3
*b^2) - tan(d/2 + (e*x)/2)^7*(8*a*b^4 + (7*a^5)/2 + 26*a^3*b^2) + tan(d/2 +
 (e*x)/2)^6*(16*a^4*b + 8*b^5 + 56*a^2*b^3) + tan(d/2 + (e*x)/2)^2*(32*a^4*
b + 8*b^5 + 72*a^2*b^3) + tan(d/2 + (e*x)/2)^4*(48*a^4*b + 12*b^5 + 104*a^2
*b^3) + (32*a^4*b)/5 + 2*b^5 + 16*a^2*b^3 + tan(d/2 + (e*x)/2)^8*(2*b^5 + 8
*a^2*b^3))/(e*(5*tan(d/2 + (e*x)/2)^2 + 10*tan(d/2 + (e*x)/2)^4 + 10*tan(d/
2 + (e*x)/2)^6 + 5*tan(d/2 + (e*x)/2)^8 + tan(d/2 + (e*x)/2)^10 + 1)) - (3*
a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e)
```

3.500 $\int (a+b\sin(d+ex)) \left(b^2 + 2ab\sin(d+ex) + a^2\sin^2(d+\right.$

Optimal. Leaf size=109

$$\frac{1}{2}a\left(a^2+4b^2\right)x+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+ex)}{3be}+\frac{a\left(a^2-6b^2\right)\cos(d+ex)\sin(d+ex)}{6e}-\frac{a^2\cos(d+ex)(a+b\,\mathrm{s}}{3be}$$

```
[Out] 1/2*a*(a^2+4*b^2)*x+1/3*(a^4-8*a^2*b^2-3*b^4)*cos(e*x+d)/b/e+1/6*a*(a^2-6*b
^2)*cos(e*x+d)*sin(e*x+d)/e-1/3*a^2*cos(e*x+d)*(a+b*sin(e*x+d))^2/b/e
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {3102, 2813}

$$\frac{a\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{6e}+\frac{1}{2}ax\left(a^2+4b^2\right)-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+ex)}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] (a*(a^2 + 4*b^2)*x)/2 + ((a^4 - 8*a^2*b^2 - 3*b^4)*Cos[d + e*x])/(3*b*e) +
(a*(a^2 - 6*b^2)*Cos[d + e*x]*Sin[d + e*x])/(6*e) - (a^2*Cos[d + e*x]*(a +
b*Sin[d + e*x])^2)/(3*b*e)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx=-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}+\int$$

$$=\frac{1}{2}a\left(a^2+4b^2\right)x+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(}{3be}$$

Mathematica [A]
time = 0.21, size = 77, normalized size = 0.71

$$\frac{-3b(11a^2+4b^2)\cos(d+ex)+a(6(a^2+4b^2)(d+ex)+ab\cos(3(d+ex))-3(a^2+2b^2)\sin(2(d+ex)))}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] (-3*b*(11*a^2 + 4*b^2)*Cos[d + e*x] + a*(6*(a^2 + 4*b^2)*(d + e*x) + a*b*Co
s[3*(d + e*x)] - 3*(a^2 + 2*b^2)*Sin[2*(d + e*x)]))/(12*e)
```

Maple [A]
time = 0.12, size = 115, normalized size = 1.06

method	result
risch	$\frac{a^3x}{2}+2a\,b^2x-\frac{11b\cos(ex+d)a^2}{4e}-\frac{b^3\cos(ex+d)}{e}+\frac{a^2b\cos(3ex+3d)}{12e}-\frac{a^3\sin(2ex+2d)}{4e}-\frac{a\sin(2ex+2d)b^2}{2e}$
derivativedivides	$\frac{-\frac{a^2b\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3}+a^3\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2a\,b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-2\cos(ex+d)}{e}$
default	$\frac{-\frac{a^2b\left(2+\sin^2(ex+d)\right)\cos(ex+d)}{3}+a^3\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2a\,b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-2\cos(ex+d)}{e}$
norman	$\frac{(2a\,b^2+\frac{1}{2}a^3)x+(\frac{3}{2}a^3+6a\,b^2)x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(\frac{3}{2}a^3+6a\,b^2)x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+(2a\,b^2+\frac{1}{2}a^3)x\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{a(a^2}{}}{\left(1+\tan^2\left(\frac{d}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/e*(-1/3*a^2*b*(2+sin(e*x+d)^2)*cos(e*x+d)+a^3*(-1/2*cos(e*x+d)*sin(e*x+d)
+1/2*e*x+1/2*d)+2*a*b^2*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-2*cos(e*
x+d)*a^2*b-b^3*cos(e*x+d)+a*b^2*(e*x+d))
```

Maxima [A]
time = 0.29, size = 120, normalized size = 1.10

$$\frac{1}{12}\left(3\left(2xe+2d-\sin\left(2xe+2d\right)\right)a^3+4\left(\cos\left(xe+d\right)^3-3\cos\left(xe+d\right)\right)a^2b+6\left(2xe+2d-\sin\left(2xe+2d\right)\right)ab^2+12\left(xe+d\right)ab^2-24a^2b\cos\left(xe+d\right)-12b^3\cos\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*x*e + 2*d - sin(2*x*e + 2*d))*a^3 + 4*(cos(x*e + d)^3 - 3*cos(x*
e + d))*a^2*b + 6*(2*x*e + 2*d - sin(2*x*e + 2*d))*a*b^2 + 12*(x*e + d)*a*b
^2 - 24*a^2*b*cos(x*e + d) - 12*b^3*cos(x*e + d))*e^(-1)
```

Fricas [A]
time = 2.79, size = 80, normalized size = 0.73

$$\frac{1}{6}\left(2\,a^{2}b\cos\left(xe+d\right)^{3}+3\,\left(a^{3}+4\,ab^{2}\right)xe-3\,\left(a^{3}+2\,ab^{2}\right)\cos\left(xe+d\right)\sin\left(xe+d\right)-6\,\left(3\,a^{2}b+b^{3}\right)\cos\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(2*a^2*b*cos(x*e + d)^3 + 3*(a^3 + 4*a*b^2)*x*e - 3*(a^3 + 2*a*b^2)*cos
(x*e + d)*sin(x*e + d) - 6*(3*a^2*b + b^3)*cos(x*e + d))*e^(-1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(95) = 190.
time = 0.14, size = 204, normalized size = 1.87

$$\begin{cases} \frac{a^{3}x\sin^{2}\left(d+ex\right)}{2}+\frac{a^{3}x\cos^{2}\left(d+ex\right)}{2}-\frac{a^{3}\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}-\frac{a^{2}b\sin^{2}\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{2a^{2}b\cos^{3}\left(d+ex\right)}{3e}-\frac{2a^{2}b\cos\left(d+ex\right)}{e}+ab^{2}x\sin^{2}\left(d+ex\right)+ab^{2}x\cos^{2}\left(d+ex\right)+ab^{2}x-\frac{ab^{2}\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{b^{3}\cos\left(d+ex\right)}{e} & \text{for}\: e\neq 0 \\ x\left(a+b\sin\left(d\right)\right)\left(a^{2}\sin^{2}\left(d\right)+2ab\sin\left(d\right)+b^{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Piecewise((a**3*x*sin(d + e*x)**2/2 + a**3*x*cos(d + e*x)**2/2 - a**3*sin(d
 + e*x)*cos(d + e*x)/(2*e) - a**2*b*sin(d + e*x)**2*cos(d + e*x)/e - 2*a**2
*b*cos(d + e*x)**3/(3*e) - 2*a**2*b*cos(d + e*x)/e + a*b**2*x*sin(d + e*x)*
*2 + a*b**2*x*cos(d + e*x)**2 + a*b**2*x - a*b**2*sin(d + e*x)*cos(d + e*x)
/e - b**3*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*
a*b*sin(d) + b**2), True))
```

Giac [A]
time = 0.40, size = 79, normalized size = 0.72

$$\frac{a^{2}b\cos\left(3\,ex+3\,d\right)}{12\,e}+\frac{1}{2}\left(a^{3}+4\,ab^{2}\right)x-\frac{\left(11\,a^{2}b+4\,b^{3}\right)\cos\left(ex+d\right)}{4\,e}-\frac{\left(a^{3}+2\,ab^{2}\right)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/12*a^2*b*cos(3*e*x + 3*d)/e + 1/2*(a^3 + 4*a*b^2)*x - 1/4*(11*a^2*b + 4*b
^3)*cos(e*x + d)/e - 1/4*(a^3 + 2*a*b^2)*sin(2*e*x + 2*d)/e
```

Mupad [B]
time = 2.90, size = 88, normalized size = 0.81

$$-\frac{6\,b^{3}\,\cos\left(d+e\,x\right)+\frac{3\,a^{3}\,\sin\left(2\,d+2\,e\,x\right)}{2}-\frac{a^{2}\,b\,\cos\left(3\,d+3\,e\,x\right)}{2}+3\,a\,b^{2}\,\sin\left(2\,d+2\,e\,x\right)+\frac{33\,a^{2}\,b\,\cos\left(d+e\,x\right)}{2}-3\,a^{3}\,e\,x-12\,a\,b^{2}\,e\,x}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
```

```
[Out] -(6*b^3*cos(d + e*x) + (3*a^3*sin(2*d + 2*e*x))/2 - (a^2*b*cos(3*d + 3*e*x)
)/2 + 3*a*b^2*sin(2*d + 2*e*x) + (33*a^2*b*cos(d + e*x))/2 - 3*a^3*e*x - 12
*a*b^2*e*x)/(6*e)
```

3.501 $\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

```
[Out] -cos(e*x+d)/e/(b+a*sin(e*x+d))
```

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3369, 2833, 8}

$$-\frac{\cos(d+ex)}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3369

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = (4a^2)\int \frac{a+b\sin(d+ex)}{(2ab+2a^2\sin(d+ex))^2}\,dx$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= -\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 1.00

$$-\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2),x]
```

```
[Out] -(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(23) = 46$.
time = 0.32, size = 54, normalized size = 2.35

method	result	size
derivativedivides	$\frac{-\frac{a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{b}-1}{e\left(\frac{b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{b}{2}\right)}$	54
default	$\frac{-\frac{a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{b}-1}{e\left(\frac{b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2}+a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{b}{2}\right)}$	54
risch	$-\frac{2\left(ia+b\,e^{i(ex+d)}\right)}{ae\left(a\,e^{2i(ex+d)}+2ib\,e^{i(ex+d)}-a\right)}$	55
norman	$\frac{\frac{2\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{2\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}+\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{be}+\frac{2a\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{be}}{\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)\left(b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+b\right)}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/e*(-1/2*a/b*tan(1/2*d+1/2*e*x)-1/2)/(1/2*b*tan(1/2*d+1/2*e*x)^2+a*tan(1/2
*d+1/2*e*x)+1/2*b)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 2.29, size = 27, normalized size = 1.17

$$-\frac{\cos(xe+d)}{ae\sin(xe+d)+be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] -cos(x*e + d)/(a*e*sin(x*e + d) + b*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.
time = 0.43, size = 50, normalized size = 2.17

$$-\frac{2\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b\right)}{\left(b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b\right)be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -2*(a*tan(1/2*e*x + 1/2*d) + b)/((b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan(1/2*e*
x + 1/2*d) + b)*b*e)
```

Mupad [B]
time = 2.84, size = 39, normalized size = 1.70

$$-\frac{a\,\sin\left(d+e\,x\right)+b\,\left(\cos\left(d+e\,x\right)+1\right)}{b\,e\,\left(b+a\,\sin\left(d+e\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x)
[Out] -(a*sin(d + e*x) + b*(cos(d + e*x) + 1))/(b*e*(b + a*sin(d + e*x)))
```

3.502 $\int \frac{a+b\sin(d+ex)}{(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex))^2}\,dx$

Optimal. Leaf size=157

$$\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}e}-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}+\frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2}-\frac{(2a^2+b^2)\cos(d+ex)}{3(a^2-b^2)^2e(b+a\sin(d+ex))}$$

```
[Out] 2*a*b*arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/e-1
/3*cos(e*x+d)/e/(b+a*sin(e*x+d))^3+1/3*b*cos(e*x+d)/(a^2-b^2)/e/(b+a*sin(e*
x+d))^2-1/3*(2*a^2+b^2)*cos(e*x+d)/(a^2-b^2)^2/e/(b+a*sin(e*x+d))
```

Rubi [A]

time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3369, 2833, 12, 2739, 632, 212}

$$-\frac{(2a^2+b^2)\cos(d+ex)}{3e(a^2-b^2)^2(a\sin(d+ex)+b)}+\frac{b\cos(d+ex)}{3e(a^2-b^2)(a\sin(d+ex)+b)^2}+\frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{e(a^2-b^2)^{5/2}}-\frac{\cos(d+ex)}{3e(a\sin(d+ex)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2,
x]
```

```
[Out] (2*a*b*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*e) - Cos[d + e*x]/(3*e*(b + a*Sin[d + e*x])^3) + (b*Cos[d + e*x])/(3*(a^2
 - b^2)*e*(b + a*Sin[d + e*x])^2) - ((2*a^2 + b^2)*Cos[d + e*x])/(3*(a^2 -
b^2)^2*e*(b + a*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3369

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx = (16a^4)\int \frac{a+b\sin(d+ex)}{\left(2ab+2a^2\sin(d+ex)\right)^4}\,dx$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{(4a^2)\int \frac{4a(a^2-b^2)\sin(d+ex)}{(2ab+2a^2\sin(d+ex))^3}\,dx}{3(a^2-b^2)}$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{1}{3}(16a^3)\int \frac{\sin(d+ex)}{(2ab+2a^2\sin(d+e$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2$$

$$= -\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{b\cos(d+ex)}{3(a^2-b^2)e(b+a\sin(d+ex))^2$$

$$= \frac{2ab\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}e} - \frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3} + \frac{}{3$$

Mathematica [A]
time = 0.67, size = 140, normalized size = 0.89

$$-\frac{\frac{6ab\,\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{\cos(d+ex)\left(a^4-a^2b^2+3b^4+3ab(a^2+b^2)\sin(d+ex)+a^2(2a^2+b^2)\sin^2(d+ex)\right)}{(a-b)^2(a+b)^2(b+a\sin(d+ex))^3}}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^2,x]
```

```
[Out] -1/3*((6*a*b*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2
)^(5/2) + (Cos[d + e*x]*(a^4 - a^2*b^2 + 3*b^4 + 3*a*b*(a^2 + b^2)*Sin[d +
e*x] + a^2*(2*a^2 + b^2)*Sin[d + e*x]^2))/((a - b)^2*(a + b)^2*(b + a*Sin[d
 + e*x])^3))/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(146) = 292$.

time = 0.68, size = 398, normalized size = 2.54

method	result
risch	$-\frac{2(2ia^5+9a^4b\,e^{i(ex+d)}+ia^3b^2-6ia^5e^{2i(ex+d)}-12ia^3b^2e^{2i(ex+d)}+15ia^3b^2e^{4i(ex+d)}-12a^4b\,e^{3i(ex+d)}-14a^2b^3e^{3i(ex+d)}}{3(a\,e^{2i(ex+d)}+2ib\,e^{i(ex+d)}-a)^3(-a^2+b^2)^2ea}$
derivativedivides	$\frac{\frac{-\frac{2a(a^4-2a^2b^2+2b^4)(\tan^5(\frac{d}{2}+\frac{ex}{2}))}{b(a^4-2a^2b^2+b^4)}-\frac{2(2a^6-3a^4b^2+5a^2b^4+b^6)(\tan^4(\frac{d}{2}+\frac{ex}{2}))}{b^2(a^4-2a^2b^2+b^4)}-\frac{4a(2a^6+a^4b^2+3a^2b^4+9b^6)(\tan^3(\frac{d}{2}+\frac{ex}{2}))}{3b^3(a^4-2a^2b^2+b^4)}}{(b(\tan^2(\frac{d}{2}+\frac{ex}{2}))+2a\tan(\frac{d}{2}+\frac{ex}{2})+b)^3}}{e}$
default	$\frac{\frac{-\frac{2a(a^4-2a^2b^2+2b^4)(\tan^5(\frac{d}{2}+\frac{ex}{2}))}{b(a^4-2a^2b^2+b^4)}-\frac{2(2a^6-3a^4b^2+5a^2b^4+b^6)(\tan^4(\frac{d}{2}+\frac{ex}{2}))}{b^2(a^4-2a^2b^2+b^4)}-\frac{4a(2a^6+a^4b^2+3a^2b^4+9b^6)(\tan^3(\frac{d}{2}+\frac{ex}{2}))}{3b^3(a^4-2a^2b^2+b^4)}}{(b(\tan^2(\frac{d}{2}+\frac{ex}{2}))+2a\tan(\frac{d}{2}+\frac{ex}{2})+b)^3}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(16*(-1/8*a*(a^4-2*a^2*b^2+2*b^4)/b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e
*x)^5-1/8*(2*a^6-3*a^4*b^2+5*a^2*b^4+b^6)/b^2/(a^4-2*a^2*b^2+b^4)*tan(1/2*d
+1/2*e*x)^4-1/12*a/b^3*(2*a^6+a^4*b^2+3*a^2*b^4+9*b^6)/(a^4-2*a^2*b^2+b^4)*
tan(1/2*d+1/2*e*x)^3-1/4/b^2*(a^6+3*a^2*b^4+b^6)/(a^4-2*a^2*b^2+b^4)*tan(1/
2*d+1/2*e*x)^2-1/8*a*(a^4+4*b^4)/b/(a^4-2*a^2*b^2+b^4)*tan(1/2*d+1/2*e*x)-1
/24*(a^4-a^2*b^2+3*b^4)/(a^4-2*a^2*b^2+b^4))/(b*tan(1/2*d+1/2*e*x)^2+2*a*ta
n(1/2*d+1/2*e*x)+b)^3-16*a*b/(8*a^4-16*a^2*b^2+8*b^4)/(-a^2+b^2)^(1/2)*arct
an(1/2*(2*b*tan(1/2*d+1/2*e*x)+2*a)/(-a^2+b^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(149) = 298$.

time = 2.77, size = 832, normalized size = 5.30

$$\left[-\frac{2\left(2a^6-a^4b^2-a^2b^4\right)\cos\left(xe+d\right)^3-6\left(a^5b-ab^5\right)\cos\left(xe+d\right)\sin\left(xe+d\right)-3\left(3a^3b^2\cos\left(xe+d\right)^2-3a^3b^2-ab^4+\left(a^4b\cos\left(xe+d\right)^2-a^4b-3a^2b^3\right)\sin\left(xe+d\right)\right)\sqrt{a^2-b^2}\log\left(\frac{\left(a^2-2b^2\right)\cos(xe+d)^2+2ab\sin(xe+d)+a^2+b^2+2\left(b\cos(xe+d)\sin(xe+d)+a\cos(xe+d)\right)\sqrt{a^2-b^2}}{a^2\cos(xe+d)^2-2ab\sin(xe+d)-a^2-b^2}\right)-6\left(a^6-a^4b^2+a^2b^4-b^6\right)\cos\left(xe+d\right)}{6\left(3\left(a^8b-3a^6b^3+3a^4b^5-a^2b^7\right)\cos\left(xe+d\right)^2e-\left(3a^8b-8a^6b^3+6a^4b^5-b^9\right)e+\left(\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\cos\left(xe+d\right)^2e-\left(a^9-6a^5b^4+8a^3b^6-3ab^8\right)e\right)\sin\left(xe+d\right)\right)},-\frac{\left(2a^6-a^4b^2-a^2b^4\right)\cos\left(xe+d\right)^3-3\left(a^5b-ab^5\right)\cos\left(xe+d\right)\sin\left(xe+d\right)-3\left(3a^3b^2\cos\left(xe+d\right)^2-3a^3b^2-ab^4+\left(a^4b\cos\left(xe+d\right)^2-a^4b-3a^2b^3\right)\sin\left(xe+d\right)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\sin(xe+d)+a\right)}{\left(a^2-b^2\right)\cos(xe+d)}\right)-3\left(a^6-a^4b^2+a^2b^4-b^6\right)\cos\left(xe+d\right)}{3\left(3\left(a^8b-3a^6b^3+3a^4b^5-a^2b^7\right)\cos\left(xe+d\right)^2e-\left(3a^8b-8a^6b^3+6a^4b^5-b^9\right)e+\left(\left(a^9-3a^7b^2+3a^5b^4-a^3b^6\right)\cos\left(xe+d\right)^2e-\left(a^9-6a^5b^4+8a^3b^6-3ab^8\right)e\right)\sin\left(xe+d\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [-1/6*(2*(2*a^6 - a^4*b^2 - a^2*b^4)*cos(x*e + d)^3 - 6*(a^5*b - a*b^5)*cos
(x*e + d)*sin(x*e + d) - 3*(3*a^3*b^2*cos(x*e + d)^2 - 3*a^3*b^2 - a*b^4 +
(a^4*b*cos(x*e + d)^2 - a^4*b - 3*a^2*b^3)*sin(x*e + d))*sqrt(a^2 - b^2)*lo
g(((a^2 - 2*b^2)*cos(x*e + d)^2 + 2*a*b*sin(x*e + d) + a^2 + b^2 + 2*(b*cos
(x*e + d)*sin(x*e + d) + a*cos(x*e + d))*sqrt(a^2 - b^2))/(a^2*cos(x*e + d)
^2 - 2*a*b*sin(x*e + d) - a^2 - b^2)) - 6*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*c
os(x*e + d))/(3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(x*e + d)^2*e
- (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*cos(x*e + d)^2*e - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*s
in(x*e + d)), -1/3*((2*a^6 - a^4*b^2 - a^2*b^4)*cos(x*e + d)^3 - 3*(a^5*b -
 a*b^5)*cos(x*e + d)*sin(x*e + d) - 3*(3*a^3*b^2*cos(x*e + d)^2 - 3*a^3*b^2
 - a*b^4 + (a^4*b*cos(x*e + d)^2 - a^4*b - 3*a^2*b^3)*sin(x*e + d))*sqrt(-a
^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x*e + d) + a)/((a^2 - b^2)*cos(x*
e + d))) - 3*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*cos(x*e + d))/(3*(a^8*b - 3*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(x*e + d)^2*e - (3*a^8*b - 8*a^6*b^3 + 6*a^
4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(x*e + d)^2*e
- (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*sin(x*e + d))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(146) = 292.
time = 0.45, size = 435, normalized size = 2.77

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(b\right)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{3a^5b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-6a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+6ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+6a^6b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^4-9a^4b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^4+15a^2b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^4+3b^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^4+4a^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+2a^5b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+6a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+18ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+6a^6b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+18a^2b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+6b^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+3a^5b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+12ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a^4b^3-a^2b^5+3b^7}{\left(a^4b^3-2a^2b^5+b^7\right)\left(b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b\right)^3}\right)}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*e*x +
1/2*d) + a)/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2
)) + (3*a^5*b^2*tan(1/2*e*x + 1/2*d)^5 - 6*a^3*b^4*tan(1/2*e*x + 1/2*d)^5 +
 6*a*b^6*tan(1/2*e*x + 1/2*d)^5 + 6*a^6*b*tan(1/2*e*x + 1/2*d)^4 - 9*a^4*b^
3*tan(1/2*e*x + 1/2*d)^4 + 15*a^2*b^5*tan(1/2*e*x + 1/2*d)^4 + 3*b^7*tan(1/
2*e*x + 1/2*d)^4 + 4*a^7*tan(1/2*e*x + 1/2*d)^3 + 2*a^5*b^2*tan(1/2*e*x + 1
/2*d)^3 + 6*a^3*b^4*tan(1/2*e*x + 1/2*d)^3 + 18*a*b^6*tan(1/2*e*x + 1/2*d)^
3 + 6*a^6*b*tan(1/2*e*x + 1/2*d)^2 + 18*a^2*b^5*tan(1/2*e*x + 1/2*d)^2 + 6*
b^7*tan(1/2*e*x + 1/2*d)^2 + 3*a^5*b^2*tan(1/2*e*x + 1/2*d) + 12*a*b^6*tan(
1/2*e*x + 1/2*d) + a^4*b^3 - a^2*b^5 + 3*b^7)/((a^4*b^3 - 2*a^2*b^5 + b^7)*
(b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan(1/2*e*x + 1/2*d) + b)^3))/e
```

Mupad [B]
time = 6.06, size = 497, normalized size = 3.17

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{\left(2\,a+2\,b\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}\right)}{e\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}-\frac{\frac{2\,\left(a^4-a^2\,b^2+3\,b^4\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(a^6+3\,a^2\,b^4+b^6\right)}{b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(2\,a^6-3\,a^4\,b^2+5\,a^2\,b^4+b^6\right)}{b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(a^4+4\,b^4\right)}{b\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\,\left(a^4-2\,a^2\,b^2+2\,b^4\right)}{b\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,a\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(2\,a^2+3\,b^2\right)\,\left(a^4-a^2\,b^2+3\,b^4\right)}{3\,b^3\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{e\,\left(b^3\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^6+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(8\,a^3+12\,a\,b^2\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(12\,a^2\,b+3\,b^3\right)+{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4\,\left(12\,a^2\,b+3\,b^3\right)+b^3+6\,a\,b^2\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+6\,a\,b^2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2,
x)
```

```
[Out] (2*a*b*atanh(((2*a + 2*b*tan(d/2 + (e*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(2*(a
 + b)^(5/2)*(a - b)^(5/2))))/(e*(a + b)^(5/2)*(a - b)^(5/2)) - ((2*(a^4 + 3
*b^4 - a^2*b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(d/2 + (e*x)/2)^2*(a^6
 + b^6 + 3*a^2*b^4))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(d/2 + (e*x)/2)^
4*(2*a^6 + b^6 + 5*a^2*b^4 - 3*a^4*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2
*a*tan(d/2 + (e*x)/2)*(a^4 + 4*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*tan
(d/2 + (e*x)/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) +
(4*a*tan(d/2 + (e*x)/2)^3*(2*a^2 + 3*b^2)*(a^4 + 3*b^4 - a^2*b^2))/(3*b^3*(
a^4 + b^4 - 2*a^2*b^2)))/(e*(b^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^
3*(12*a*b^2 + 8*a^3) + tan(d/2 + (e*x)/2)^2*(12*a^2*b + 3*b^3) + tan(d/2 +
(e*x)/2)^4*(12*a^2*b + 3*b^3) + b^3 + 6*a*b^2*tan(d/2 + (e*x)/2) + 6*a*b^2*
tan(d/2 + (e*x)/2)^5))
```

3.503 $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\cdots\right.}{\sqrt{b^2-2c(a-\cdots}}$$

```
[Out] arctan(1/2*(2*c+(b-(-4*a*c+b^2)^(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)-b
*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b^
2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2)+arctan(1/2*(2*c+(b+(-4*a*c+b^2)^(1/
2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1
/2)
```

Rubi [A]
time = 0.61, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3373, 2739, 632, 210}

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+2c}{\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+2c}{\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] (Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b - Sqrt[b^2
- 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]])
])/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2*c + (b + Sqrt[b^2 - 4*a*c])*Tan[x/2])/(Sqr
t[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])])/Sqrt[b^2 - 2*c*(a + c
) + b*Sqrt[b^2 - 4*a*c]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3373

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Sin[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Sin[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\sin(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}+4cx+\left(b+\sqrt{b^2-4a\right.}}\right.$$

$$= -\left(\left(4\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{4\left(4c^2-\left(b+\sqrt{b^2-4ac}\right)^2\right)-x^2}\right.\right.$$

$$= \frac{\sqrt{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}+$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 286, normalized size = 1.18

$$\frac{\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\text{ArcTan}\left(\frac{2c+\left(b-i\sqrt{-b^2+4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\text{ArcTan}\left(\frac{2c+\left(b+i\sqrt{-b^2+4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}}}{\sqrt{-\frac{b^2}{2}+2ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

```
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b - I*Sqrt[-b^
2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a
*c]])])/Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]] + (((2*I)*c*d + ((
-I)*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])*Tan
[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])])/Sqrt[b^
2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])/Sqrt[-1/2*b^2 + 2*a*c]
```

Maple [A]
time = 4.99, size = 263, normalized size = 1.09

method	result
default	$2a\left(\frac{\left(\sqrt{-4ac+b^2}\,d+2ae-db\right)\sqrt{-4ac+b^2}\,\arctan\left(\frac{-2a\tan\left(\frac{x}{2}\right)+\sqrt{-4ac+b^2}-b}{\sqrt{4ac-2b^2+2b\sqrt{-4ac+b^2}+4a^2}}\right)}{(4ac-b^2)a\sqrt{4ac-2b^2+2b\sqrt{-4ac+b^2}+4a^2}}-\frac{\sqrt{-4a}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*(((-4*a*c+b^2)^(1/2)*d+2*a*e-d*b)*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a/(4*a
*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*
c+b^2)^(1/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))-(-4*a*c+b
^2)^(1/2)*((-4*a*c+b^2)^(1/2)*d-2*a*e+d*b)/(4*a*c-b^2)/a/(4*a*c-2*b^2-2*b*(
-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-4*a*c+b^2)^(1/2))
/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6647 vs. 2(214) = 428.
time = 20.50, size = 6647, normalized size = 27.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 4*a*b*c*e^4 + 12*(a*b*c + b*c^2)*d^2*e^2 - 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 + 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 + (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + a*b^2*e^4 + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*sin(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(4*b*c^2*d^4 - 4*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e +

```
4*a*b*c*e^4 + 12*(a*b*c + b*c^2)*d^2*e^2 - 4*(2*a*c^2 + (2*a^2 + b^2)*c)*d
*e^3 - 2*((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2
 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^
3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2
- 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*
a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^
2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3
 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin
(x) + sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3
- 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)
*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a
*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 - 4*(a*b + b*c)
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*
d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^
3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*
a*b^4)*c))*cos(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*
b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*
c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*cos(x))*sqrt(-((b
^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((
b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b
^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + 2*(b^2*c*d^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d
^3*e + a*b^2*e^4 + 3*(a*b^2 + b^2*c)*d^2*e^2 - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)**2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 17.11, size = 2500, normalized size = 10.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*sin(x))/(a + c*sin(x)^2 + b*sin(x)),x)
```

```
[Out] atan(((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d
^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^
3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 +
 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*
c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^
2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) +
2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e
 - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/
(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^
2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^
4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 -
2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*
e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*
b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16
*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*
c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 + 512*a^2*c^3 + 768
*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 128*a^2*b*c^2 - 32
*a*b^3*c - 128*a^3*b*c) + tan(x/2)*(64*a^2*b^2*e - 256*a^2*c^2*e - 64*a*b^3
*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) - 32*a^2*b^2*d + 128*a^2*c
^2*d + 32*a*b^3*e + 128*a^3*c*d - 32*a*b^2*c*d - 128*a^2*b*c*e) - tan(x/2)*
(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2 - 64*a^2*c*d^2 +
128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e^2 + 32*a*b*c*d^2
 - 128*a^2*c*d*e)*1i + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*
e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e
*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^
4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*
a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*
a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*
d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*
e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e
```

```
+ 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 +
 b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)
*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a
^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 -
 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a
*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4
+ 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^
2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*b^2 +
512*a^2*c^3 + 768*a^3*c^2 - 128*a*b^2*c^2 - 576*a^2*b^2*c) + 32*a^2*b^3 + 1
28*a^2*b*c^2 - 32*a*b^3*c - 128*a^3*b*c) - tan(x/2)*(64*a^2*b^2*e - 256*a^2
*c^2*e - 64*a*b^3*d - 256*a^3*c*e + 256*a^2*b*c*d + 64*a*b^2*c*e) + 32*a^2*
b^2*d - 128*a^2*c^2*d - 32*a*b^3*e - 128*a^3*c*d + 32*a*b^2*c*d + 128*a^2*b
*c*e) - tan(x/2)*(64*a^3*e^2 + 32*a*b^2*d^2 - 64*a*b^2*e^2 - 128*a*c^2*d^2
- 64*a^2*c*d^2 + 128*a^2*c*e^2 - 64*a^2*b*d*e + 128*a*b*c*d*e) + 32*a^2*b*e
^2 + 32*a*b*c*d^2 - 128*a^2*c*d*e)*1i)/(2*tan(x/2)*(64*a^2*e^3 - 64*a*b*d*e
^2 + 64*a*c*d^2*e) + (-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^
2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(
-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
- b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^
3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*
c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b
^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^
2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*
(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e +
8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b
^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(
(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4...
```

3.504 $\int (a+b\sin(d+ex)) \left(b^2 + 2ab\sin(d+ex) + a^2\sin^2(d+$

Optimal. Leaf size=331

$$-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{4e}-\frac{\left(4a^4+28a^2b^2+3b^4\right)\cos(d+ex)\left(b^2+2ab\sin(\right.}{6e(b+a\sin(d+ex))}$$

```
[Out] -1/4*b*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e-1/6*(4*a^
4+28*a^2*b^2+3*b^4)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2
)/e/(b+a*sin(e*x+d))^3-1/12*(4*a^2+3*b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+
a^2*sin(e*x+d)^2)^(3/2)/e/(b+a*sin(e*x+d))+5/8*a^4*b*(3*a^2+4*b^2)*x*(b^2+2
*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/(a*b+a^2*sin(e*x+d))^3-1/24*a^4*b*(
29*a^2+6*b^2)*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)
^(3/2)/e/(a*b+a^2*sin(e*x+d))^3
```

Rubi [A]
time = 0.22, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3371, 2832, 2813}

$$-\frac{b\cos(d+ex)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{4e}-\frac{\left(4a^2+3b^2\right)\cos(d+ex)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{12e(a\sin(d+ex)+b)}+\frac{5a^4bx\left(3a^2+4b^2\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{8\left(a^2\sin(d+ex)+ab\right)^3}-\frac{a^4b\left(29a^2+6b^2\right)\sin(d+ex)\cos(d+ex)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{24e\left(a^2\sin(d+ex)+ab\right)^3}-\frac{\left(4a^4+28a^2b^2+3b^4\right)\cos(d+ex)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}{6e(a\sin(d+ex)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```

```
[Out] -1/4*(b*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))
/e - ((4*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] +
 a^2*Sin[d + e*x]^2)^(3/2))/(6*e*(b + a*Sin[d + e*x])^3) - ((4*a^2 + 3*b^2)
*Cos[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2))/(12*e*
(b + a*Sin[d + e*x])) + (5*a^4*b*(3*a^2 + 4*b^2)*x*(b^2 + 2*a*b*Sin[d + e*x
] + a^2*Sin[d + e*x]^2)^(3/2))/(8*(a*b + a^2*Sin[d + e*x])^3) - (a^4*b*(29*
a^2 + 6*b^2)*Cos[d + e*x]*Sin[d + e*x]*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[
d + e*x]^2)^(3/2))/(24*e*(a*b + a^2*Sin[d + e*x])^3)
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 3371

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex}{(2ab}$$

$$=-\frac{b\cos(d+ex)\,(b^2+2ab\sin(d+ex)+}{4e}$$

$$=-\frac{b\cos(d+ex)\,(b^2+2ab\sin(d+ex)+}{4e}$$

$$=-\frac{b\cos(d+ex)\,(b^2+2ab\sin(d+ex)+}{4e}$$

Mathematica [A]
time = 0.60, size = 140, normalized size = 0.42

$$\frac{\sqrt{(b+a\sin(d+ex))^2}\,\left(-24(3a^4+21a^2b^2+4b^4)\cos(d+ex)+8a(a^3+3ab^2)\cos(3(d+ex))+3ab(20(3a^2+4b^2)(d+ex)-8(4a^2+3b^2)\sin(2(d+ex))+a^2\sin(4(d+ex)))\right)}{96e(b+a\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sin[d + e*x])^2]*(-24*(3*a^4 + 21*a^2*b^2 + 4*b^4)*Cos[d + e*x
] + 8*a*(a^3 + 3*a*b^2)*Cos[3*(d + e*x)] + 3*a*b*(20*(3*a^2 + 4*b^2)*(d + e
*x) - 8*(4*a^2 + 3*b^2)*Sin[2*(d + e*x)] + a^2*Sin[4*(d + e*x)])))/(96*e*(b
 + a*Sin[d + e*x]))
```

Maple [A]
time = 1.06, size = 269, normalized size = 0.81

method	result
default	$-\frac{\left(-a^2\left(\cos^2(ex+d)\right)+2ab\sin(ex+d)+a^2+b^2\right)^{\frac{3}{2}}\left(6\left(\cos^3(ex+d)\right)\sin(ex+d)a^3b+8a^4\left(\cos^3(ex+d)\right)+24a^2b^2\left(\cos^3(ex+d)\right)-51\sin(ex-\right.}{24e\left(\left(\cos^2(ex+d)\right)\sin(ex+d)a^3+3\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/24/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)*(6*cos(e*x+d)^3*
sin(e*x+d)*a^3*b+8*a^4*cos(e*x+d)^3+24*a^2*b^2*cos(e*x+d)^3-51*sin(e*x+d)*c
os(e*x+d)*a^3*b-36*cos(e*x+d)*sin(e*x+d)*a*b^3-24*a^4*cos(e*x+d)-144*a^2*b^
2*cos(e*x+d)-24*cos(e*x+d)*b^4+45*(e*x+d)*a^3*b+60*(e*x+d)*a*b^3-16*a^4-120
*a^2*b^2-24*b^4)/(cos(e*x+d)^2*sin(e*x+d)*a^3+3*cos(e*x+d)^2*a^2*b-a^3*sin(
e*x+d)-3*sin(e*x+d)*a*b^2-3*a^2*b-b^3)
```

Maxima [A]
time = 0.52, size = 595, normalized size = 1.80

$$\frac{1}{12}\left(4\left(3\left(3a^2b+2b^3\right)\arctan\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)-\frac{4a^3+18ab^2+\frac{9a^2b\sin(xe+d)}{\cos(xe+d)+1}+\frac{18ab^2\sin(xe+d)^4}{(\cos(xe+d)+1)^4}-\frac{9a^2b\sin(xe+d)^5}{(\cos(xe+d)+1)^5}+\frac{12(a^3+3ab^2)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}}{\frac{3\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{3\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+\frac{\sin(xe+d)^6}{(\cos(xe+d)+1)^6}+1}\right)a+3\left(3\left(a^3+4ab^2\right)\arctan\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)-\frac{16a^2b+8b^3+\frac{8b^3\sin(xe+d)^6}{(\cos(xe+d)+1)^6}+\frac{3(a^3+4ab^2)\sin(xe+d)}{\cos(xe+d)+1}+\frac{8(8a^2b+3b^3)\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{(11a^3+12ab^2)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{24(2a^2b+b^3)\sin(xe+d)^4}{(\cos(xe+d)+1)^4}-\frac{(11a^3+12ab^2)\sin(xe+d)^5}{(\cos(xe+d)+1)^5}-\frac{3(a^3+4ab^2)\sin(xe+d)^7}{(\cos(xe+d)+1)^7}}{\frac{4\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{6\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+\frac{4\sin(xe+d)^6}{(\cos(xe+d)+1)^6}+\frac{\sin(xe+d)^8}{(\cos(xe+d)+1)^8}+1}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/12*(4*(3*(3*a^2*b + 2*b^3)*arctan(sin(x*e + d)/(cos(x*e + d) + 1)) - (4*a
^3 + 18*a*b^2 + 9*a^2*b*sin(x*e + d)/(cos(x*e + d) + 1) + 18*a*b^2*sin(x*e
+ d)^4/(cos(x*e + d) + 1)^4 - 9*a^2*b*sin(x*e + d)^5/(cos(x*e + d) + 1)^5 +
 12*(a^3 + 3*a*b^2)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2)/(3*sin(x*e + d)^2/
(cos(x*e + d) + 1)^2 + 3*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + sin(x*e + d)
^6/(cos(x*e + d) + 1)^6 + 1))*a + 3*(3*(a^3 + 4*a*b^2)*arctan(sin(x*e + d)/
(cos(x*e + d) + 1)) - (16*a^2*b + 8*b^3 + 8*b^3*sin(x*e + d)^6/(cos(x*e + d
) + 1)^6 + 3*(a^3 + 4*a*b^2)*sin(x*e + d)/(cos(x*e + d) + 1) + 8*(8*a^2*b +
 3*b^3)*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 + (11*a^3 + 12*a*b^2)*sin(x*e +
 d)^3/(cos(x*e + d) + 1)^3 + 24*(2*a^2*b + b^3)*sin(x*e + d)^4/(cos(x*e + d
) + 1)^4 - (11*a^3 + 12*a*b^2)*sin(x*e + d)^5/(cos(x*e + d) + 1)^5 - 3*(a^3
 + 4*a*b^2)*sin(x*e + d)^7/(cos(x*e + d) + 1)^7)/(4*sin(x*e + d)^2/(cos(x*e
 + d) + 1)^2 + 6*sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + 4*sin(x*e + d)^6/(co
s(x*e + d) + 1)^6 + sin(x*e + d)^8/(cos(x*e + d) + 1)^8 + 1))*b)*e^(-1)
```

Fricas [A]
time = 2.24, size = 117, normalized size = 0.35

$$\frac{1}{24}\left(8\left(a^4+3a^2b^2\right)\cos(xe+d)^3+15\left(3a^3b+4ab^3\right)xe-24\left(a^4+6a^2b^2+b^4\right)\cos(xe+d)+3\left(2a^3b\cos(xe+d)^3-\left(17a^3b+12ab^3\right)\cos(xe+d)\right)\sin(xe+d)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/24*(8*(a^4 + 3*a^2*b^2)*cos(x*e + d)^3 + 15*(3*a^3*b + 4*a*b^3)*x*e - 24*
(a^4 + 6*a^2*b^2 + b^4)*cos(x*e + d) + 3*(2*a^3*b*cos(x*e + d)^3 - (17*a^3*
b + 12*a*b^3)*cos(x*e + d))*sin(x*e + d))*e^(-1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 229, normalized size = 0.69

$$\frac{a^3b\mathrm{sgn}(a\sin(ex+d)+b)\sin(4ex+4d)}{32e}+\frac{5}{8}\left(3a^3b\mathrm{sgn}(a\sin(ex+d)+b)+4ab^3\mathrm{sgn}(a\sin(ex+d)+b)\right)x+\frac{\left(a^4\mathrm{sgn}(a\sin(ex+d)+b)+3a^2b^2\mathrm{sgn}(a\sin(ex+d)+b)\right)\cos(3ex+3d)}{12e}-\frac{\left(3a^4\mathrm{sgn}(a\sin(ex+d)+b)+21a^2b^2\mathrm{sgn}(a\sin(ex+d)+b)+4b^4\mathrm{sgn}(a\sin(ex+d)+b)\right)\cos(ex+d)}{4e}-\frac{\left(4a^3b\mathrm{sgn}(a\sin(ex+d)+b)+3ab^3\mathrm{sgn}(a\sin(ex+d)+b)\right)\sin(2ex+2d)}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/32*a^3*b*sgn(a*sin(e*x + d) + b)*sin(4*e*x + 4*d)/e + 5/8*(3*a^3*b*sgn(a*
sin(e*x + d) + b) + 4*a*b^3*sgn(a*sin(e*x + d) + b))*x + 1/12*(a^4*sgn(a*si
n(e*x + d) + b) + 3*a^2*b^2*sgn(a*sin(e*x + d) + b))*cos(3*e*x + 3*d)/e - 1
/4*(3*a^4*sgn(a*sin(e*x + d) + b) + 21*a^2*b^2*sgn(a*sin(e*x + d) + b) + 4*
b^4*sgn(a*sin(e*x + d) + b))*cos(e*x + d)/e - 1/4*(4*a^3*b*sgn(a*sin(e*x +
d) + b) + 3*a*b^3*sgn(a*sin(e*x + d) + b))*sin(2*e*x + 2*d)/e
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\sin(d+e\,x))\,\left(a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2), x)
```

3.505 $\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d}$

Optimal. Leaf size=185

$$-\frac{(a^2+b^2)\cos(d+ex)\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}{e(b+a\sin(d+ex))}+\frac{3a^2bx\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d-}}{2\,(ab+a^2\sin(d+ex))}$$

```
[Out] -(a^2+b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/e/(b+a*
sin(e*x+d))+3/2*a^2*b*x*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/(a*b+
a^2*sin(e*x+d))-1/2*a^2*b*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*s
in(e*x+d)^2)^(1/2)/e/(a*b+a^2*sin(e*x+d))
```

Rubi [A]

time = 0.07, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {3371, 2813}

$$\frac{3a^2bx\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2\,(a^2\sin(d+ex)+ab)}-\frac{a^2b\sin(d+ex)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{2e\,(a^2\sin(d+ex)+ab)}-\frac{(a^2+b^2)\cos(d+ex)\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}{e(a\sin(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```

```
[Out] -(((a^2 + b^2)*Cos[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2])/(e*(b + a*Sin[d + e*x]))) + (3*a^2*b*x*Sqrt[b^2 + 2*a*b*Sin[d + e*x]
+ a^2*Sin[d + e*x]^2])/(2*(a*b + a^2*Sin[d + e*x])) - (a^2*b*Cos[d + e*x]*S
in[d + e*x]*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])/(2*e*(a*b
+ a^2*Sin[d + e*x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3371

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}{2ab+}$$

$$= -\frac{(a^2+b^2)\cos(d+ex)\sqrt{b^2+2ab\sin(d-}}{e(b+a\sin(d+e}$$

Mathematica [A]
time = 0.13, size = 70, normalized size = 0.38

$$-\frac{\sqrt{(b+a\sin(d+ex))^2}\,(4(a^2+b^2)\cos(d+ex)+ab(-6(d+ex)+\sin(2(d+ex))))}{4e(b+a\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] -1/4*(Sqrt[(b + a*Sin[d + e*x])^2]*(4*(a^2 + b^2)*Cos[d + e*x] + a*b*(-6*(d
 + e*x) + Sin[2*(d + e*x)])))/(e*(b + a*Sin[d + e*x]))
```

Maple [A]
time = 0.42, size = 107, normalized size = 0.58

method	result
default	$-\frac{\sqrt{-a^2(\cos^2(ex+d))+2ab\sin(ex+d)+a^2+b^2}\,(\cos(ex+d)\sin(ex+d)ab+2a^2\cos(ex+d)+2\cos(ex+d)b^2-3a}{2e(b+a\sin(ex+d))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/e*(-a^2*cos(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(1/2)*(cos(e*x+d)*sin(e
*x+d)*a*b+2*a^2*cos(e*x+d)+2*cos(e*x+d)*b^2-3*a*b*(e*x+d)+2*a^2+2*b^2)/(b+a
*sin(e*x+d))
```

Maxima [A]
time = 0.50, size = 202, normalized size = 1.09

$$\left(2\left(b\arctan\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)-\frac{a}{\frac{\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+1}\right)a+\left(a\arctan\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)-\frac{2b+\frac{a\sin(xe+d)}{\cos(xe+d)+1}+\frac{2b\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{a\sin(xe+d)^3}{(\cos(xe+d)+1)^3}}{\frac{2\sin(xe+d)^2}{(\cos(xe+d)+1)^2}+\frac{\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+1}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] (2*(b*arctan(sin(x*e + d)/(cos(x*e + d) + 1)) - a/(sin(x*e + d)^2/(cos(x*e
+ d) + 1)^2 + 1))*a + (a*arctan(sin(x*e + d)/(cos(x*e + d) + 1)) - (2*b + a
*sin(x*e + d)/(cos(x*e + d) + 1) + 2*b*sin(x*e + d)^2/(cos(x*e + d) + 1)^2
- a*sin(x*e + d)^3/(cos(x*e + d) + 1)^3)/(2*sin(x*e + d)^2/(cos(x*e + d) +
1)^2 + sin(x*e + d)^4/(cos(x*e + d) + 1)^4 + 1))*b)*e^(-1)
```

Fricas [A]
time = 2.22, size = 46, normalized size = 0.25

$$\frac{1}{2}\left(3\,abxe - ab\cos\left(xe+d\right)\sin\left(xe+d\right) - 2\left(a^2+b^2\right)\cos\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(3*a*b*x*e - a*b*cos(x*e + d)*sin(x*e + d) - 2*(a^2 + b^2)*cos(x*e + d)
)*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(d+ex\right)\right)\sqrt{\left(a\sin\left(d+ex\right)+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sin(d + e*x))*sqrt((a*sin(d + e*x) + b)**2), x)
```

Giac [A]
time = 0.45, size = 94, normalized size = 0.51

$$\frac{3}{2}\,abx\mathrm{sgn}(a\sin\left(ex+d\right)+b) - \frac{a^2\cos\left(ex+d\right)\mathrm{sgn}(a\sin\left(ex+d\right)+b)}{e} - \frac{b^2\cos\left(ex+d\right)\mathrm{sgn}(a\sin\left(ex+d\right)+b)}{e} - \frac{ab\mathrm{sgn}(a\sin\left(ex+d\right)+b)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] 3/2*a*b*x*sgn(a*sin(e*x + d) + b) - a^2*cos(e*x + d)*sgn(a*sin(e*x + d) + b
)/e - b^2*cos(e*x + d)*sgn(a*sin(e*x + d) + b)/e - 1/4*a*b*sgn(a*sin(e*x +
d) + b)*sin(2*e*x + 2*d)/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\sin(d+ex))\sqrt{a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2), x)
```

3.506 $\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$

Optimal. Leaf size=137

$$\frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}-\frac{2\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)(b+a\sin(d+ex))}{ae\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}$$

```
[Out] b*x*(b+a*sin(e*x+d))/a/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)-2*arct
anh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(b+a*sin(e*x+d))*(a^2-b^2)^(1
/2)/a/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3371, 2814, 2739, 632, 212}

$$\frac{bx(a\sin(d+ex)+b)}{a\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}-\frac{2\sqrt{a^2-b^2}(a\sin(d+ex)+b)\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{ae\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2
],x]
```

```
[Out] (b*x*(b + a*Sin[d + e*x]))/(a*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e
*x]^2]) - (2*Sqrt[a^2 - b^2]*ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^
2]]*(b + a*Sin[d + e*x]))/(a*e*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3371

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \frac{(2ab+2a^2\sin(d+ex))\int \frac{a+b\sin(d+ex)}{2ab+2a^2\sin(d+ex)}\,dx}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{((-2a^3+2a}{2a^2\sqrt{}}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{((-2a^3+2a}{}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} + \frac{(2(-2a^3+2}{}$$

$$= \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{2\sqrt{a^2-b^2}\,\mathrm{t}}{ae\sqrt{b}}$$

Mathematica [A]

time = 0.13, size = 85, normalized size = 0.62

$$\frac{\left(b(d+ex)-2\sqrt{-a^2+b^2}\,\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)\right)(b+a\sin(d+ex))}{ae\sqrt{(b+a\sin(d+ex))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
e*x]^2],x]
```

```
[Out] ((b*(d + e*x) - 2*Sqrt[-a^2 + b^2]*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^
2 + b^2]])*(b + a*Sin[d + e*x]))/(a*e*Sqrt[(b + a*Sin[d + e*x])^2])
```

Maple [A]
time = 0.34, size = 170, normalized size = 1.24

method	result
default	$\frac{\left(2\arctan\left(\frac{a\sin(ex+d)-b\cos(ex+d)+b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)a^2-2\arctan\left(\frac{a\sin(ex+d)-b\cos(ex+d)+b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)b^2+b(ex+d)\sqrt{-a^2+b^2}\right)(b+a\sin(ex+d))}{e\sqrt{-a^2\left(\cos^2(ex+d)\right)+2ab\sin(ex+d)+a^2+b^2}\,a\sqrt{-a^2+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/e*(2*arctan((a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^
2-2*arctan((a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*b^2+b
*(e*x+d)*(-a^2+b^2)^(1/2))*(b+a*sin(e*x+d))/(-a^2*cos(e*x+d)^2+2*a*b*sin(e*
x+d)+a^2+b^2)^(1/2)/a/(-a^2+b^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.01, size = 213, normalized size = 1.55

$$\left[\frac{\left(2\,bxe+\sqrt{a^2-b^2}\,\log\left(-\frac{(a^2-2\,b^2)\cos(xe+d)^2+2\,ab\sin(xe+d)+a^2+b^2-2\,(b\cos(xe+d)\sin(xe+d)+a\cos(xe+d))\sqrt{a^2-b^2}}{a^2\cos(xe+d)^2-2\,ab\sin(xe+d)-a^2-b^2}\right)\right)e^{(-1)}}{2\,a},\ \frac{\left(bxe-\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\sin(xe+d)+a)}{(a^2-b^2)\cos(xe+d)}\right)\right)e^{(-1)}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*b*x*e + sqrt(a^2 - b^2)*log(-((a^2 - 2*b^2)*cos(x*e + d)^2 + 2*a*b*
sin(x*e + d) + a^2 + b^2 - 2*(b*cos(x*e + d)*sin(x*e + d) + a*cos(x*e + d))
*sqrt(a^2 - b^2))/(a^2*cos(x*e + d)^2 - 2*a*b*sin(x*e + d) - a^2 - b^2)))*e
^(-1)/a, (b*x*e - sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x*e + d)
 + a)/((a^2 - b^2)*cos(x*e + d))))*e^(-1)/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sin(d+ex)}{\sqrt{(a\sin(d+ex)+b)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sin(d + e*x))/sqrt((a*sin(d + e*x) + b)**2), x)
```

Giac [A]
time = 0.44, size = 110, normalized size = 0.80

$$\frac{\frac{(ex+d)b}{a\operatorname{sgn}(a\sin(ex+d)+b)}+\frac{2\left(\pi\left\lfloor\frac{ex+d}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)(a^2-b^2)}{\sqrt{-a^2+b^2}\,a\operatorname{sgn}(a\sin(ex+d)+b)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((e*x + d)*b/(a*sgn(a*sin(e*x + d) + b)) + 2*(pi*floor(1/2*(e*x + d)/pi + 1
/2)*sgn(b) + arctan((b*tan(1/2*e*x + 1/2*d) + a)/sqrt(-a^2 + b^2)))*(a^2 -
b^2)/(sqrt(-a^2 + b^2)*a*sgn(a*sin(e*x + d) + b)))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\sin\left(d+e\,x\right)}{\sqrt{a^2\,{\sin\left(d+e\,x\right)}^2+2\,a\,b\,\sin\left(d+e\,x\right)+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2),x)
[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(1
/2), x)
```

3.507 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$-\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)\left(ab+a^2\sin(d+ex)\right)^3}{a^2\left(a^2-b^2\right)^{3/2}e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}+$$

```
[Out] -1/2*cos(e*x+d)*(b+a*sin(e*x+d))/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^
(3/2)-arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(a*b+a^2*sin(e*x+d)
)^3/a^2/(a^2-b^2)^(3/2)/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)+1/2
*b*cos(e*x+d)*(a*b+a^2*sin(e*x+d))^3/(a^2-b^2)/e/(a^3*b+a^4*sin(e*x+d))/(b^
2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)
```

Rubi [A]
time = 0.17, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {3371, 2833, 12, 2739, 632, 212}

$$-\frac{\cos(d+ex)(a\sin(d+ex)+b)}{2e\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}-\frac{\left(a^2\sin(d+ex)+ab\right)^3\tanh^{-1}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{a^2e\left(a^2-b^2\right)^{3/2}\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}+\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^3}{2e\left(a^2-b^2\right)\left(a^4\sin(d+ex)+a^3b\right)\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3
/2),x]
```

```
[Out] -1/2*(Cos[d + e*x]*(b + a*Sin[d + e*x]))/(e*(b^2 + 2*a*b*Sin[d + e*x] + a^2
*Sin[d + e*x]^2)^(3/2)) - (ArcTanh[(a + b*Tan[(d + e*x)/2])/Sqrt[a^2 - b^2]
]*(a*b + a^2*Sin[d + e*x])^3)/(a^2*(a^2 - b^2)^(3/2)*e*(b^2 + 2*a*b*Sin[d +
 e*x] + a^2*Sin[d + e*x]^2)^(3/2)) + (b*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x
])^3)/(2*(a^2 - b^2)*e*(a^3*b + a^4*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x]
 + a^2*Sin[d + e*x]^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3371

```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n), Int[(A + B*Si
n[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\sin(d+ex)\right)^3\int\frac{a+b\sin(d+ex)}{(2ab+2a^2\sin(d+ex))^3}\,dx}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{(2}{8a^2\,(a^2}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{(2ab+}{4a\,(b^2}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\,(a^2-}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\,(a^2-}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\,(a^2-}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} + \frac{}{2\,(a^2-}$$

$$= -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}} - \frac{\mathrm{ta}}{a^2\,(a^2}$$

Mathematica [A]
time = 0.25, size = 144, normalized size = 0.60

$$\frac{-2a\,\mathrm{ArcTan}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)(b+a\sin(d+ex))^2+\sqrt{-a^2+b^2}\cos(d+ex)\left(a^2-2b^2-ab\sin(d+ex)\right)}{2(-a+b)(a+b)\sqrt{-a^2+b^2}\,e(b+a\sin(d+ex))\sqrt{(b+a\sin(d+ex))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]
^2)^(3/2),x]
```

```
[Out] (-2*a*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(b + a*Sin[d + e*x]
)^2 + Sqrt[-a^2 + b^2]*Cos[d + e*x]*(a^2 - 2*b^2 - a*b*Sin[d + e*x]))/(2*(-
a + b)*(a + b)*Sqrt[-a^2 + b^2]*e*(b + a*Sin[d + e*x])*Sqrt[(b + a*Sin[d +
e*x])^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(224) = 448.
time = 0.34, size = 726, normalized size = 3.04

method	result
default	$-\frac{2\sin(ex+d)\left(\cos^2(ex+d)\right)\arctan\left(\frac{a\sin(ex+d)-b\cos(ex+d)+b}{\sin(ex+d)\sqrt{-a^2+b^2}}\right)a^4b^2-\sin(ex+d)\left(\cos^2(ex+d)\right)\sqrt{-a^2+b^2}\,a^5+2\sin(ex+d)(\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/e*(2*sin(e*x+d)*cos(e*x+d)^2*arctan((a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(
e*x+d)/(-a^2+b^2)^(1/2))*a^4*b^2-sin(e*x+d)*cos(e*x+d)^2*(-a^2+b^2)^(1/2)*a
^5+2*sin(e*x+d)*cos(e*x+d)^2*(-a^2+b^2)^(1/2)*a^3*b^2+cos(e*x+d)^3*(-a^2+b^
2)^(1/2)*a^2*b^3+6*cos(e*x+d)^2*arctan((a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*
x+d)/(-a^2+b^2)^(1/2))*a^3*b^3+sin(e*x+d)*cos(e*x+d)*(-a^2+b^2)^(1/2)*a^3*b
^2-3*sin(e*x+d)*cos(e*x+d)*(-a^2+b^2)^(1/2)*a*b^4-2*sin(e*x+d)*arctan((a*si
n(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^4*b^2-6*sin(e*x+d)*
arctan((a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^2*b^4-3
*cos(e*x+d)^2*(-a^2+b^2)^(1/2)*a^4*b+6*cos(e*x+d)^2*(-a^2+b^2)^(1/2)*a^2*b^
3+sin(e*x+d)*(-a^2+b^2)^(1/2)*a^5+sin(e*x+d)*(-a^2+b^2)^(1/2)*a^3*b^2-6*sin
(e*x+d)*(-a^2+b^2)^(1/2)*a*b^4-2*cos(e*x+d)*(-a^2+b^2)^(1/2)*b^5-6*arctan((
a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a^3*b^3-2*arctan(
(a*sin(e*x+d)-b*cos(e*x+d)+b)/sin(e*x+d)/(-a^2+b^2)^(1/2))*a*b^5+3*(-a^2+b^
2)^(1/2)*a^4*b-5*(-a^2+b^2)^(1/2)*a^2*b^3-2*(-a^2+b^2)^(1/2)*b^5)/(-a^2*cos
(e*x+d)^2+2*a*b*sin(e*x+d)+a^2+b^2)^(3/2)/(-a^2+b^2)^(1/2)/(a^2-b^2)/b^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.45, size = 556, normalized size = 2.33

$$\left[-\frac{2\left(a^3b-ab^3\right)\cos\left(xe+d\right)\sin\left(xe+d\right)+\left(a^3\cos\left(xe+d\right)^2-2\,a^2b\sin\left(xe+d\right)-a^3-ab^2\right)\sqrt{a^2-b^2}\log\left(\frac{\left(a^2-2\,b^2\right)\cos(xe+d)^2+2\,ab\sin(xe+d)+a^2+b^2+2\left(b\cos(xe+d)\sin(xe+d)+a\cos(xe+d)\right)\sqrt{a^2-b^2}}{a^2\cos(xe+d)^2-2\,ab\sin(xe+d)-a^2-b^2}\right)-2\left(a^4-3\,a^2b^2+2\,b^4\right)\cos\left(xe+d\right)}{4\left(\left(a^6-2\,a^4b^2+a^2b^4\right)\cos\left(xe+d\right)^2e-2\left(a^5b-2\,a^3b^3+ab^5\right)e\sin\left(xe+d\right)-\left(a^6-a^4b^2-a^2b^4+b^6\right)e\right)},-\frac{\left(a^3b-ab^3\right)\cos\left(xe+d\right)\sin\left(xe+d\right)+\left(a^3\cos\left(xe+d\right)^2-2\,a^2b\sin\left(xe+d\right)-a^3-ab^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\left(b\sin(xe+d)+a\right)}{\left(a^2-b^2\right)\cos(xe+d)}\right)-\left(a^4-3\,a^2b^2+2\,b^4\right)\cos\left(xe+d\right)}{2\left(\left(a^6-2\,a^4b^2+a^2b^4\right)\cos\left(xe+d\right)^2e-2\left(a^5b-2\,a^3b^3+ab^5\right)e\sin\left(xe+d\right)-\left(a^6-a^4b^2-a^2b^4+b^6\right)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^3*b - a*b^3)*cos(x*e + d)*sin(x*e + d) + (a^3*cos(x*e + d)^2 -
2*a^2*b*sin(x*e + d) - a^3 - a*b^2)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(
x*e + d)^2 + 2*a*b*sin(x*e + d) + a^2 + b^2 + 2*(b*cos(x*e + d)*sin(x*e + d
) + a*cos(x*e + d))*sqrt(a^2 - b^2))/(a^2*cos(x*e + d)^2 - 2*a*b*sin(x*e +
d) - a^2 - b^2)) - 2*(a^4 - 3*a^2*b^2 + 2*b^4)*cos(x*e + d))/((a^6 - 2*a^4*
b^2 + a^2*b^4)*cos(x*e + d)^2*e - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(x*e +
 d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e), -1/2*((a^3*b - a*b^3)*cos(x*e + d
)*sin(x*e + d) + (a^3*cos(x*e + d)^2 - 2*a^2*b*sin(x*e + d) - a^3 - a*b^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x*e + d) + a)/((a^2 - b^2)
*cos(x*e + d))) - (a^4 - 3*a^2*b^2 + 2*b^4)*cos(x*e + d))/((a^6 - 2*a^4*b^2
 + a^2*b^4)*cos(x*e + d)^2*e - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin(x*e + d)
 - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/2
),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 292, normalized size = 1.22

$$\frac{\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a}{\left(a^2\operatorname{sgn}(a\sin(ex+d)+b)-b^2\operatorname{sgn}(a\sin(ex+d)+b)\right)\sqrt{-a^2+b^2}}-\frac{2a^3b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-3ab^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+2a^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-3a^2b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-2b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2a^3b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-5ab^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a^2b^2-2b^4}{\left(a^2b^2\operatorname{sgn}(a\sin(ex+d)+b)-b^4\operatorname{sgn}(a\sin(ex+d)+b)\right)\left(b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b\right)^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] ((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*e*x + 1/2*d)
+ a)/sqrt(-a^2 + b^2)))*a/((a^2*sgn(a*sin(e*x + d) + b) - b^2*sgn(a*sin(e*x
 + d) + b))*sqrt(-a^2 + b^2)) - (2*a^3*b*tan(1/2*e*x + 1/2*d)^3 - 3*a*b^3*t
an(1/2*e*x + 1/2*d)^3 + 2*a^4*tan(1/2*e*x + 1/2*d)^2 - 3*a^2*b^2*tan(1/2*e*
x + 1/2*d)^2 - 2*b^4*tan(1/2*e*x + 1/2*d)^2 + 2*a^3*b*tan(1/2*e*x + 1/2*d)
- 5*a*b^3*tan(1/2*e*x + 1/2*d) + a^2*b^2 - 2*b^4)/((a^2*b^2*sgn(a*sin(e*x +
 d) + b) - b^4*sgn(a*sin(e*x + d) + b))*(b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan
(1/2*e*x + 1/2*d) + b)^2))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\sin(d+e\,x)}{\left(a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(3
/2), x)
```

3.508 $\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\sin(x)}{b+a\cos(x)}$$

```
[Out] sin(x)/(b+a*cos(x))
```

Rubi [A]
time = 0.05, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3370, 2833, 8}

$$\frac{\sin(x)}{a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

```
[Out] Sin[x]/(b + a*Cos[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3370

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx = (4a^2)\int \frac{a+b\cos(x)}{(2ab+2a^2\cos(x))^2}\,dx$$
$$= \frac{\sin(x)}{b+a\cos(x)} + \frac{\int 0\,dx}{a^2-b^2}$$
$$= \frac{\sin(x)}{b+a\cos(x)}$$

Mathematica [A]
time = 0.04, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{b+a\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
[Out] Sin[x]/(b + a*Cos[x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(11) = 22.
time = 0.11, size = 33, normalized size = 3.00

method	result	size
default	$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b}$	33
risch	$\frac{2i\left(b\,e^{ix}+a\right)}{a\left(a\,e^{2ix}+2b\,e^{ix}+a\right)}$	35
norman	$\frac{-2\left(\tan^3\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)-a-b\right)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2-b*tan(1/2*x)^2-a-b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.13, size = 11, normalized size = 1.00

$$\frac{\sin(x)}{a\cos(x)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="fricas
")
```

```
[Out] sin(x)/(a*cos(x) + b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b**2+2*a*b*cos(x)+a**2*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(11) = 22$.
time = 0.45, size = 32, normalized size = 2.91

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2-a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)
```

Mupad [B]
time = 2.94, size = 24, normalized size = 2.18

$$\frac{2\tan\left(\frac{x}{2}\right)}{(b-a)\tan\left(\frac{x}{2}\right)^2+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x))/(a^2*cos(x)^2 + b^2 + 2*a*b*cos(x)),x)
```

```
[Out] (2*tan(x/2))/(a + b - tan(x/2)^2*(a - b))
```

3.509 $\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$

Optimal. Leaf size=246

$$\frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{ArcTan}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{ArcTan}\left(\frac{\sqrt{b-2c+}}{\sqrt{b+2c}}\right.}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+}}$$

```
[Out] 2*arctan((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tan(1/2*x)/(b+2*c-(-4*a*c+b^2)^(1
/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2))^
(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctan((b-2*c+(-4*a*c+b^2)^(1/2))^
(1/2)*tan(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b
^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2
)
```

Rubi [A]
time = 0.56, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3374, 2738, 211}

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\operatorname{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{ArcTan}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 -
4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqrt[
b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3374

```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Dist[B + (b*B - 2*A*c)/q, Int[1/(b + q + 2*c*Cos[d + e*x]), x
], x] + Dist[B - (b*B - 2*A*c)/q, Int[1/(b - q + 2*c*Cos[d + e*x]), x], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+\sqrt{b^2-4ac}+2c\cos(x)}\,dx + \left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)$$

$$= \left(2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+2c+\sqrt{b^2-4ac}+\left(b-2c+\sqrt{b^2-\right.}}\right.$$

$$= \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2c}{\sqrt{b^2}}\right.}{\sqrt{b}}$$

Mathematica [A]
time = 0.38, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(-\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

```
[Out] (Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((b - 2*c + Sqrt[
b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]]]
)/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt[b^
2 - 4*a*c])*e)*ArcTanh[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^
2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b*Sqrt
[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A]
time = 4.93, size = 254, normalized size = 1.03

method	result
default	$2(a-b+c)\left(\frac{\left(\sqrt{-4ac+b^2}\,d-e\sqrt{-4ac+b^2}+2ae-db-be+2cd\right)\operatorname{arctanh}\left(\frac{(-a+b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a}}\right.}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}-a+c\right)(a-b+c)}}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a-b+c)*(1/2*((-4*a*c+b^2)^(1/2)*d-e*(-4*a*c+b^2)^(1/2)+2*a*e-d*b-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arc
tanh((-a+b-c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2))+1/2*((-4
*a*c+b^2)^(1/2)*d-e*(-4*a*c+b^2)^(1/2)-2*a*e+d*b+b*e-2*c*d)/(-4*a*c+b^2)^(1
/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*arctan((a-b+c)*tan(1/2
*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x)*e + d)/(c*cos(x)^2 + b*cos(x) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6649 vs. $2(211) = 422$.
time = 23.19, size = 6649, normalized size = 27.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 -
 b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3
 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c
 + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b
^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*
```

a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 2*a*b*c*e^4 + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) + 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3 - 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3*e + a*b^2*e^4 + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2*a^2*b + b^3 + 2*a*b*c)*d*e^3)*cos(x)) - 1/4*sqrt(2)*sqrt(-((b^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c))*log(2*b*c^2*d^4 - 2*(b^2*c + 2*a*c^2 + 2*c^3)*d^3*e + 2*a*b*c*e^4 + 6*(a*b*c + b*c^2)*d^2*e^2 - 2*(2*a*c^2 + (2*a^2 + b^2)*c)*d*e^3 - ((4*a*c^4 + (8*a^2 - b^2)*c^3 + 2*(2*a^3 - 3*a*b^2)*c^2 - (a^2*b^2 - b^4)*c)*d^2 + (a^2*b^3 - b^5 - 4*a*b*c^3 - (8*a^2*b - b^3)*c^2 - 2*(2*a^3*b - 3*a*b^3)*c)*d*e - (a^3*b^2 - a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 - 2*(2*a^4 - 3*a^2*b^2)*c)*e^2)*sqrt((b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c))*cos(x) - 1/2*sqrt(2)*(((a^2*b^4 - b^6 + 8*a*c^5 + 2*(12*a^2 - b^2)*c^4 + 6*(4*a^3

```
- 3*a*b^2)*c^3 + (8*a^4 - 22*a^2*b^2 + 3*b^4)*c^2 - 2*(3*a^3*b^2 - 4*a*b^4)
*c)*d - (a^3*b^3 - a*b^5 + 4*a*b*c^4 + (4*a^2*b - b^3)*c^3 - (4*a^3*b + 5*a
*b^3)*c^2 - (4*a^4*b - 5*a^2*b^3 - b^5)*c)*e)*sqrt((b^2*d^4 - 4*(a*b + b*c)
*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b^2*e^4 - 4*(a*b + b*c)*
d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^2 - b^2)*c^4 - 12*(2*a^
3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 - 4*(a^5 - 3*a^3*b^2 + 2*
a*b^4)*c))*sin(x) + ((b^4 - 4*a*b^2*c)*d^3 - 3*(a*b^3 - 4*a*b*c^2 - (4*a^2*
b - b^3)*c)*d^2*e + (2*a^2*b^2 + b^4 - 8*a^3*c - 8*a*c^3 - 2*(8*a^2 - b^2)*
c^2)*d*e^2 - (a*b^3 - 4*a*b*c^2 - (4*a^2*b - b^3)*c)*e^3)*sin(x))*sqrt(-((b
^2 - 2*a*c - 2*c^2)*d^2 - 2*(a*b - b*c)*d*e + (2*a^2 - b^2 + 2*a*c)*e^2 - (
a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)*c^2 - 2*(2*a^3 - 3*a*b^2)*c)*sqrt((
b^2*d^4 - 4*(a*b + b*c)*d^3*e + 2*(2*a^2 + b^2 + 4*a*c + 2*c^2)*d^2*e^2 + b
^2*e^4 - 4*(a*b + b*c)*d*e^3)/(a^4*b^2 - 2*a^2*b^4 + b^6 - 4*a*c^5 - (16*a^
2 - b^2)*c^4 - 12*(2*a^3 - a*b^2)*c^3 - 2*(8*a^4 - 11*a^2*b^2 + b^4)*c^2 -
4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c)))/(a^2*b^2 - b^4 - 4*a*c^3 - (8*a^2 - b^2)
*c^2 - 2*(2*a^3 - 3*a*b^2)*c)) + (b^2*c*d^4 - (b^3 + 2*a*b*c + 2*b*c^2)*d^3
*e + a*b^2*e^4 + 3*(a*b^2 + b^2*c)*d^2*e^2 - (2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)**2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5301 vs. $2(206) = 412$.
time = 1.99, size = 5301, normalized size = 21.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

```
[Out] -((2*a^2*b^3 - 8*a*b^4 + 6*b^5 - 8*a^3*b*c + 52*a^2*b^2*c - 44*a*b^3*c - 4*
b^4*c - 80*a^3*c^2 + 80*a^2*b*c^2 + 40*a*b^2*c^2 - 6*b^3*c^2 - 96*a^2*c^3 +
 24*a*b*c^3 + 4*b^2*c^3 - 16*a*c^4 - 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^
2 - 4*a*c)*(a - b + c))*a^2*b^2 + 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 -
 4*a*c)*(a - b + c))*a*b^3 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*
c)*(a - b + c))*b^4 + 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a
- b + c))*a^3*c - 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b +
 c))*a^2*b*c - 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c
))*a*b^2*c - 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*
```

b^3*c + 56*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*
c^2 + 24*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b*c^
2 + 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^2*c^2 -
 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*c^3 + 3*s
qrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c
)*a^2*b - 2*(b^2 - 4*a*c)*a^2*b - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 -
 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 + 8*(b^2 - 4*a*c)*a*b^2 - 5*sq
rt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)
b^3 - 6(b^2 - 4*a*c)*b^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*
c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*c - 20*(b^2 - 4*a*c)*a^2*c + 10*sqrt(
a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*
b*c + 20*(b^2 - 4*a*c)*a*b*c - 4*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*
a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^2*c + 4*(b^2 - 4*a*c)*b^2*c + 28*sqrt
(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a
c^2 - 24(b^2 - 4*a*c)*a*c^2 + 7*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4
*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b*c^2 + 6*(b^2 - 4*a*c)*b*c^2 - 10*sqr
t(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*
c^3 - 4*(b^2 - 4*a*c)*c^3)*d*abs(a - b + c) - (4*a^3*b^2 - 6*a^2*b^3 + 4*a*
b^4 - 2*b^5 - 16*a^4*c + 24*a^3*b*c - 24*a^2*b^2*c + 20*a*b^3*c + 32*a^3*c^
2 - 48*a^2*b*c^2 - 12*a*b^2*c^2 + 2*b^3*c^2 + 48*a^2*c^3 - 8*a*b*c^3 + 3*sq
rt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b^2 - 2*sqrt(
a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^3 - 5*sqrt(a^2 -
 a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^4 - 12*sqrt(a^2 - a*b +
 b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^3*c + 8*sqrt(a^2 - a*b + b*c
 - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b*c + 34*sqrt(a^2 - a*b + b*c -
c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^2*c + 6*sqrt(a^2 - a*b + b*c - c^2
 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^3*c - 56*sqrt(a^2 - a*b + b*c - c^2 + s
qrt(b^2 - 4*a*c)*(a - b + c))*a^2*c^2 - 24*sqrt(a^2 - a*b + b*c - c^2 + sqr
t(b^2 - 4*a*c)*(a - b + c))*a*b*c^2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b
^2 - 4*a*c)*(a - b + c))*b^2*c^2 + 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2
 - 4*a*c)*(a - b + c))*a*c^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*
a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^3 - 4*(b^2 - 4*a*c)*a^3 - sqrt(a^2 -
a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a^2*b +
6*(b^2 - 4*a*c)*a^2*b - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(
a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 - 4*(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 -
a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 + 2*
(b^2 - 4*a*c)*b^3 + 28*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a -
b + c))*sqrt(b^2 - 4*a*c)*a^2*c + 8*(b^2 - 4*a*c)*a^2*c + 26*sqrt(a^2 - a*b
 + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c - 12*
(b^2 - 4*a*c)*a*b*c + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a -
 b + c))*sqrt(b^2 - 4*a*c)*b^2*c - 10*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2
 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*c^2 + 12*(b^2 - 4*a*c)*a*c^2 - 5
*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a
*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*e*abs(a - b + c))*(pi*floor(1/2*x/pi + 1
/2) + arctan(2*sqrt(1/2)*tan(1/2*x)/sqrt((2*a - 2*c + sqrt(-4*(a + b + c)*(

```
a - b + c) + 4*(a - c)^2))/(a - b + c))))/(3*a^5*b^2 - 5*a^4*b^3 - 6*a^3*b^
4 + 10*a^2*b^5 + 3*a*b^6 - 5*b^7 - 12*a^6*c + 20*a^5*b*c + 47*a^4*b^2*c - 6
0*a^3*b^3*c - 46*a^2*b^4*c + 40*a*b^5*c + 11*b^6*c - 92*a^5*c^2 + 80*a^4*b*
c^2 + 182*a^3*b^2*c^2 - 94*a^2*b^3*c^2 - 78*a*b^4*c^2 - 6*b^5*c^2 - 184*a^4
*c^3 + 56*a^3*b*c^3 + 166*a^2*b^2*c^3 + 36*a*b^3*c^3 - 6*b^4*c^3 - 120*a^3*
c^4 - 48*a^2*b*c^4 + 23*a*b^2*c^4 + 11*b^3*c^4 + 4*a^2*c^5 - 44*a*b*c^5 - 5
*b^2*c^5 + 20*a*c^6) - ((2*a^2*b^3 - 2*b^5 - 8*a^3*b*c - 12*a^2*b^2*c + 20*
a*b^3*c + 4*b^4*c + 48*a^3*c^2 - 48*a^2*b*c^2 - 24*a*b^2*c^2 - 6*b^3*c^2 +
32*a^2*c^3 + 24*a*b*c^3 + 4*b^2*c^3 - 16*a*c^4 - 3*sqrt(a^2 - a*b + b*c - c
^2 - sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*b^2 + 2*sqrt(a^2 - a*b + b*c - c^2
- sqrt(b^2 - 4*a*c)*(a - b + c))*a*b^3 + 5*sqrt(a^2 - a*b + b*c - c^2 - sqr
t(b^2 - 4*a*c)*(a - b + c))*b^4 + 12*sqrt(a^2 - a*b + b*c - c^2 - sqrt(b^2
- 4*a*c)*(a - b + c))*a^3*c - 8*sqrt(a^2 - a*b ...
```

Mupad [B]
time = 16.77, size = 2500, normalized size = 10.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*cos(x))/(a + b*cos(x) + c*cos(x)^2),x)
```

```
[Out] - atan((((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^
2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^
2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a
^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32
*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d
 - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e +
tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^
2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^
2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a
^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32
*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b
^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 +
128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) +
64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3
*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c
*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) + tan(x/2)*(64*a^3*e^2 -
 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a
^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*
b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 6
```

4*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i - ((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*c^2*e - tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 256*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3 + 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) - tan(x/2)*(64*a^3*e^2 - 32*b^3*d^2 - 32*b^3*e^2 + 64*c^3*d^2 + 32*a*b^2*d^2 + 96*a*b^2*e^2 - 128*a^2*b*e^2 - 64*a^2*c*d^2 - 64*a*c^2*e^2 - 128*b*c^2*d^2 + 96*b^2*c*d^2 + 32*b^2*c*e^2 + 64*a*b^2*d*e - 64*a^2*b*d*e + 256*a*c^2*d*e + 256*a^2*c*d*e - 64*b*c^2*d*e + 64*b^2*c*d*e - 384*a*b*c*d*e))*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*1i)/(((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e ...

3.510 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d\right.$

Optimal. Leaf size=144

$$a\left(a^2-3b^2\right)\left(a^2+b^2\right)x+\frac{b(3a^2-b^2)\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-\frac{a(a^4-b^4)\tan(d+ex)}{e}+\frac{b(a^2+b^2)\left(b+a\,\mathrm{ta}\right.}{2e}$$

```
[Out] a*(a^2-3*b^2)*(a^2+b^2)*x+b*(3*a^2-b^2)*(a^2+b^2)*ln(cos(e*x+d))/e-a*(a^4-b
^4)*tan(e*x+d)/e+1/2*b*(a^2+b^2)*(b+a*tan(e*x+d))^2/e+1/3*(a^2+b^2)*(b+a*ta
n(e*x+d))^3/e+1/4*b*(b+a*tan(e*x+d))^4/e
```

Rubi [A]

time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3789, 3609, 12, 3606, 3556}

$$-\frac{a(a^4-b^4)\tan(d+ex)}{e}+\frac{(a^2+b^2)(a\tan(d+ex)+b)^3}{3e}+\frac{b(a^2+b^2)(a\tan(d+ex)+b)^2}{2e}+\frac{b(3a^2-b^2)(a^2+b^2)\log(\cos(d+ex))}{e}+ax(a^2-3b^2)(a^2+b^2)+\frac{b(a\tan(d+ex)+b)^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```

```
[Out] a*(a^2 - 3*b^2)*(a^2 + b^2)*x + (b*(3*a^2 - b^2)*(a^2 + b^2)*Log[Cos[d + e*
x]])/e - (a*(a^4 - b^4)*Tan[d + e*x])/e + (b*(a^2 + b^2)*(b + a*Tan[d + e*x
])^2)/(2*e) + ((a^2 + b^2)*(b + a*Tan[d + e*x])^3)/(3*e) + (b*(b + a*Tan[d
+ e*x])^4)/(4*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
```

```
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3789

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx = \frac{\int \left(2ab+2a^2\tan(d+ex)\right)^4 (a+b\tan(\ldots}{16a^4}$$

$$= \frac{b(b+a\tan(d+ex))^4}{4e} + \frac{\int 2a(a^2+b^2)\ldots}{\ldots}$$

$$= \frac{b(b+a\tan(d+ex))^4}{4e} + \frac{(a^2+b^2)\int \tan\ldots}{\ldots}$$

$$= \frac{(a^2+b^2)(b+a\tan(d+ex))^3}{3e} + \frac{b(b+\ldots}{\ldots}$$

$$= \frac{b(a^2+b^2)(b+a\tan(d+ex))^2}{2e} + \frac{(a^2\ldots}{\ldots}$$

$$= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x - \frac{a(a^4-b^4)\tan\ldots}{e}$$

$$= a\left(a^2-3b^2\right)\left(a^2+b^2\right)x + \frac{b(3a^2-b^2)(\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 153, normalized size = 1.06

$$\frac{6(a^2+b^2)\left(-i(a-ib)^3\log(i-\tan(d+ex))+i(a+ib)^3\log(i+\tan(d+ex))\right)-12a(a^4-2a^2b^2-4b^4)\tan(d+ex)+18a^2b(a^2+2b^2)\tan^2(d+ex)+4a^3(a^2+4b^2)\tan^3(d+ex)+3a^4b\tan^4(d+ex)}{12e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (6*(a^2 + b^2)*((-I)*(a - I*b)^3*Log[I - Tan[d + e*x]] + I*(a + I*b)^3*Log[
I + Tan[d + e*x]]) - 12*a*(a^4 - 2*a^2*b^2 - 4*b^4)*Tan[d + e*x] + 18*a^2*b
*(a^2 + 2*b^2)*Tan[d + e*x]^2 + 4*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3 + 3*a^4*
b*Tan[d + e*x]^4)/(12*e)
```

Maple [A]
time = 0.05, size = 173, normalized size = 1.20

method	result
norman	$\left(a^{5}-2a^{3}b^{2}-3a\,b^{4}\right)x-\frac{a\left(a^{4}-2a^{2}b^{2}-4b^{4}\right)\tan\left(ex+d\right)}{e}+\frac{a^{3}\left(a^{2}+4b^{2}\right)\left(\tan^{3}\left(ex+d\right)\right)}{3e}+\frac{b\,a^{4}\left(\tan^{4}\left(ex+d\right)\right)}{4e}$
derivativedivides	$\frac{\frac{b\,a^{4}\left(\tan^{4}\left(ex+d\right)\right)}{4}+\frac{a^{5}\left(\tan^{3}\left(ex+d\right)\right)}{3}+\frac{4a^{3}b^{2}\left(\tan^{3}\left(ex+d\right)\right)}{3}+\frac{3a^{4}b\left(\tan^{2}\left(ex+d\right)\right)}{2}+3a^{2}b^{3}\left(\tan^{2}\left(ex+d\right)\right)-a^{5}\tan\left(ex+d\right)+2a^{3}b^{2}}{e}$
default	$\frac{\frac{b\,a^{4}\left(\tan^{4}\left(ex+d\right)\right)}{4}+\frac{a^{5}\left(\tan^{3}\left(ex+d\right)\right)}{3}+\frac{4a^{3}b^{2}\left(\tan^{3}\left(ex+d\right)\right)}{3}+\frac{3a^{4}b\left(\tan^{2}\left(ex+d\right)\right)}{2}+3a^{2}b^{3}\left(\tan^{2}\left(ex+d\right)\right)-a^{5}\tan\left(ex+d\right)+2a^{3}b^{2}}{e}$
risch	$-3ia^{4}bx-2ia^{2}b^{3}x+\frac{2ib^{5}d}{e}+a^{5}x-2a^{3}b^{2}x-3a\,b^{4}x-\frac{4ia\left(3a^{4}e^{6i(ex+d)}+3a^{2}b^{2}e^{6i(ex+d)}-6b^{4}e^{6i(e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(1/4*b*a^4*tan(e*x+d)^4+1/3*a^5*tan(e*x+d)^3+4/3*a^3*b^2*tan(e*x+d)^3+3
/2*a^4*b*tan(e*x+d)^2+3*a^2*b^3*tan(e*x+d)^2-a^5*tan(e*x+d)+2*a^3*b^2*tan(e
*x+d)+4*a*b^4*tan(e*x+d)+1/2*(-3*a^4*b-2*a^2*b^3+b^5)*ln(1+tan(e*x+d)^2)+(a
^5-2*a^3*b^2-3*a*b^4)*arctan(tan(e*x+d)))
```

Maxima [A]
time = 0.49, size = 155, normalized size = 1.08

$$\frac{1}{12}\left(3\,a^{4}b\tan\left(xe+d\right)^{4}+4\left(a^{5}+4\,a^{3}b^{2}\right)\tan\left(xe+d\right)^{3}+18\left(a^{4}b+2\,a^{2}b^{3}\right)\tan\left(xe+d\right)^{2}+12\left(a^{5}-2\,a^{3}b^{2}-3\,ab^{4}\right)\left(xe+d\right)-6\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\log\left(\tan\left(xe+d\right)^{2}+1\right)-12\left(a^{5}-2\,a^{3}b^{2}-4\,ab^{4}\right)\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/12*(3*a^4*b*tan(x*e + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(x*e + d)^3 + 18*(a^4
*b + 2*a^2*b^3)*tan(x*e + d)^2 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*(x*e + d) -
 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(tan(x*e + d)^2 + 1) - 12*(a^5 - 2*a^3*b^
2 - 4*a*b^4)*tan(x*e + d))*e^(-1)
```

Fricas [A]
time = 3.30, size = 154, normalized size = 1.07

$$\frac{1}{12}\left(3\,a^{4}b\tan\left(xe+d\right)^{4}+4\left(a^{5}+4\,a^{3}b^{2}\right)\tan\left(xe+d\right)^{3}+12\left(a^{5}-2\,a^{3}b^{2}-3\,ab^{4}\right)xe+18\left(a^{4}b+2\,a^{2}b^{3}\right)\tan\left(xe+d\right)^{2}+6\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\log\left(\frac{1}{\tan\left(xe+d\right)^{2}+1}\right)-12\left(a^{5}-2\,a^{3}b^{2}-4\,ab^{4}\right)\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*a^4*b*tan(x*e + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(x*e + d)^3 + 12*(a^5
 - 2*a^3*b^2 - 3*a*b^4)*x*e + 18*(a^4*b + 2*a^2*b^3)*tan(x*e + d)^2 + 6*(3*
a^4*b + 2*a^2*b^3 - b^5)*log(1/(tan(x*e + d)^2 + 1)) - 12*(a^5 - 2*a^3*b^2
- 4*a*b^4)*tan(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.17, size = 248, normalized size = 1.72

$$\begin{cases} a^5x + \frac{a^5\tan^3(d+ex)}{3e} - \frac{a^5\tan(d+ex)}{e} - \frac{3a^4b\log(\tan^2(d+ex)+1)}{2e} + \frac{a^4b\tan^4(d+ex)}{4e} + \frac{3a^4b\tan^2(d+ex)}{2e} - 2a^3b^2x + \frac{4a^3b^2\tan^3(d+ex)}{3e} + \frac{2a^3b^2\tan(d+ex)}{e} - \frac{a^2b^3\log(\tan^2(d+ex)+1)}{e} + \frac{3a^2b^3\tan^2(d+ex)}{e} - 3ab^4x + \frac{4ab^4\tan(d+ex)}{e} + \frac{b^5\log(\tan^2(d+ex)+1)}{2e} & \text{for } e \neq 0 \\ x(a+b\tan(d))(a^2\tan^2(d)+2ab\tan(d)+b^2)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Piecewise((a**5*x + a**5*tan(d + e*x)**3/(3*e) - a**5*tan(d + e*x)/e - 3*a*
*4*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**4*b*tan(d + e*x)**4/(4*e) + 3*a**4
*b*tan(d + e*x)**2/(2*e) - 2*a**3*b**2*x + 4*a**3*b**2*tan(d + e*x)**3/(3*e
) + 2*a**3*b**2*tan(d + e*x)/e - a**2*b**3*log(tan(d + e*x)**2 + 1)/e + 3*a
**2*b**3*tan(d + e*x)**2/e - 3*a*b**4*x + 4*a*b**4*tan(d + e*x)/e + b**5*lo
g(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(d)**2
+ 2*a*b*tan(d) + b**2)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. $2(138) = 276$.
time = 2.35, size = 2157, normalized size = 14.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/12*(12*a^5*e*x*tan(e*x)^4*tan(d)^4 - 24*a^3*b^2*e*x*tan(e*x)^4*tan(d)^4 -
 36*a*b^4*e*x*tan(e*x)^4*tan(d)^4 + 18*a^4*b*log(4*(tan(e*x)^4*tan(d)^2 - 2
*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) +
 1)/(tan(d)^2 + 1))*tan(e*x)^4*tan(d)^4 + 12*a^2*b^3*log(4*(tan(e*x)^4*tan(
d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*
tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^4*tan(d)^4 - 6*b^5*log(4*(tan(e*x)^4*t
an(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*
x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^4*tan(d)^4 - 48*a^5*e*x*tan(e*x)^3*
tan(d)^3 + 96*a^3*b^2*e*x*tan(e*x)^3*tan(d)^3 + 144*a*b^4*e*x*tan(e*x)^3*ta
n(d)^3 + 15*a^4*b*tan(e*x)^4*tan(d)^4 + 36*a^2*b^3*tan(e*x)^4*tan(d)^4 - 72
*a^4*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)
^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^3*tan(d)^
3 - 48*a^2*b^3*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^
2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^3
```

```
*tan(d)^3 + 24*b^5*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e
*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*
x)^3*tan(d)^3 + 12*a^5*tan(e*x)^4*tan(d)^3 - 24*a^3*b^2*tan(e*x)^4*tan(d)^3
 - 48*a*b^4*tan(e*x)^4*tan(d)^3 + 12*a^5*tan(e*x)^3*tan(d)^4 - 24*a^3*b^2*t
an(e*x)^3*tan(d)^4 - 48*a*b^4*tan(e*x)^3*tan(d)^4 + 72*a^5*e*x*tan(e*x)^2*t
an(d)^2 - 144*a^3*b^2*e*x*tan(e*x)^2*tan(d)^2 - 216*a*b^4*e*x*tan(e*x)^2*ta
n(d)^2 + 18*a^4*b*tan(e*x)^4*tan(d)^2 + 36*a^2*b^3*tan(e*x)^4*tan(d)^2 - 24
*a^4*b*tan(e*x)^3*tan(d)^3 - 72*a^2*b^3*tan(e*x)^3*tan(d)^3 + 18*a^4*b*tan(
e*x)^2*tan(d)^4 + 36*a^2*b^3*tan(e*x)^2*tan(d)^4 - 4*a^5*tan(e*x)^4*tan(d)
- 16*a^3*b^2*tan(e*x)^4*tan(d) + 108*a^4*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*t
an(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1
)/(tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 + 72*a^2*b^3*log(4*(tan(e*x)^4*tan(d)
^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*ta
n(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 - 36*b^5*log(4*(tan(e*x)^4*ta
n(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x
)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 - 48*a^5*tan(e*x)^3*tan(d
)^2 + 24*a^3*b^2*tan(e*x)^3*tan(d)^2 + 144*a*b^4*tan(e*x)^3*tan(d)^2 - 48*a
^5*tan(e*x)^2*tan(d)^3 + 24*a^3*b^2*tan(e*x)^2*tan(d)^3 + 144*a*b^4*tan(e*x
)^2*tan(d)^3 - 4*a^5*tan(e*x)*tan(d)^4 - 16*a^3*b^2*tan(e*x)*tan(d)^4 + 3*a
^4*b*tan(e*x)^4 - 48*a^5*e*x*tan(e*x)*tan(d) + 96*a^3*b^2*e*x*tan(e*x)*tan(
d) + 144*a*b^4*e*x*tan(e*x)*tan(d) - 24*a^4*b*tan(e*x)^3*tan(d) - 72*a^2*b^
3*tan(e*x)^3*tan(d) + 36*a^4*b*tan(e*x)^2*tan(d)^2 + 72*a^2*b^3*tan(e*x)^2*
tan(d)^2 - 24*a^4*b*tan(e*x)*tan(d)^3 - 72*a^2*b^3*tan(e*x)*tan(d)^3 + 3*a^
4*b*tan(d)^4 + 4*a^5*tan(e*x)^3 + 16*a^3*b^2*tan(e*x)^3 - 72*a^4*b*log(4*(t
an(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2
 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)*tan(d) - 48*a^2*b^3*log(
4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*
x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)*tan(d) + 24*b^5*log(
4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*
x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)*tan(d) + 48*a^5*tan(
e*x)^2*tan(d) - 24*a^3*b^2*tan(e*x)^2*tan(d) - 144*a*b^4*tan(e*x)^2*tan(d)
+ 48*a^5*tan(e*x)*tan(d)^2 - 24*a^3*b^2*tan(e*x)*tan(d)^2 - 144*a*b^4*tan(e
*x)*tan(d)^2 + 4*a^5*tan(d)^3 + 16*a^3*b^2*tan(d)^3 + 12*a^5*e*x - 24*a^3*b
^2*e*x - 36*a*b^4*e*x + 18*a^4*b*tan(e*x)^2 + 36*a^2*b^3*tan(e*x)^2 - 24*a^
4*b*tan(e*x)*tan(d) - 72*a^2*b^3*tan(e*x)*tan(d) + 18*a^4*b*tan(d)^2 + 36*a
^2*b^3*tan(d)^2 + 18*a^4*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d)
 + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1)
) + 12*a^2*b^3*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^
2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1)) - 6*b^5*lo
g(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(
e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1)) - 12*a^5*tan(e*x) + 24*a^3*
b^2*tan(e*x) + 48*a*b^4*tan(e*x) - 12*a^5*tan(d) + 24*a^3*b^2*tan(d) + 48*a
*b^4*tan(d) + 15*a^4*b + 36*a^2*b^3)/(e*tan(e*x)^4*tan(d)^4 - 4*e*tan(e*x)^
3*tan(d)^3 + 6*e*tan(e*x)^2*tan(d)^2 - 4*e*tan(e*x)*tan(d) + e)
```

Mupad [B]

time = 2.85, size = 205, normalized size = 1.42

$$\frac{\tan(d+e\,x)^3\,\left(\frac{a^5}{3}+\frac{4\,a^3\,b^2}{3}\right)}{e}+\frac{\tan(d+e\,x)\,\left(-a^5+2\,a^3\,b^2+4\,a\,b^4\right)}{e}-\frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{3\,a^4\,b}{2}+a^2\,b^3-\frac{b^5}{2}\right)}{e}+\frac{\tan(d+e\,x)^2\,\left(\frac{3\,a^4\,b}{2}+3\,a^2\,b^3\right)}{e}+\frac{a^4\,b\,\tan(d+e\,x)^4}{4\,e}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(d+e\,x)\,(a^2-3\,b^2)\,(a^2+b^2)}{-a^5+2\,a^3\,b^2+3\,a\,b^4}\right)\,\left(a^2-3\,b^2\right)\,\left(a^2+b^2\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2,
x)
```

```
[Out] (tan(d + e*x)^3*(a^5/3 + (4*a^3*b^2)/3))/e + (tan(d + e*x)*(4*a*b^4 - a^5 +
 2*a^3*b^2))/e - (log(tan(d + e*x)^2 + 1)*((3*a^4*b)/2 - b^5/2 + a^2*b^3))/
e + (tan(d + e*x)^2*((3*a^4*b)/2 + 3*a^2*b^3))/e + (a^4*b*tan(d + e*x)^4)/(
4*e) - (a*atan((a*tan(d + e*x)*(a^2 - 3*b^2)*(a^2 + b^2))/(3*a*b^4 - a^5 +
2*a^3*b^2))*(a^2 - 3*b^2)*(a^2 + b^2))/e
```

3.511 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d\right.$

Optimal. Leaf size=72

$$-a\left(a^2+b^2\right)x-\frac{b(a^2+b^2)\log(\cos(d+ex))}{e}+\frac{2ab^2\tan(d+ex)}{e}+\frac{a^2(a+b\tan(d+ex))^2}{2be}$$

```
[Out] -a*(a^2+b^2)*x-b*(a^2+b^2)*ln(cos(e*x+d))/e+2*a*b^2*tan(e*x+d)/e+1/2*a^2*(a
+b*tan(e*x+d))^2/b/e
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {3711, 3606, 3556}

$$-\frac{b(a^2+b^2)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*Log[Cos[d + e*x]])/e + (2*a*b^2*Tan[d +
 e*x])/e + (a^2*(a + b*Tan[d + e*x])^2)/(2*b*e)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3711

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx = \frac{a^2(a+b\tan(d+ex))^2}{2be} + \int (a+b\tan($$

$$= -a\left(a^2+b^2\right)x + \frac{2ab^2\tan(d+ex)}{e} + \frac{a^2}{}$$

$$= -a\left(a^2+b^2\right)x - \frac{b(a^2+b^2)\log(\cos(d+}{e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 88, normalized size = 1.22

$$\frac{(a^2+b^2)\left((ia+b)\log(i-\tan(d+ex))+(-ia+b)\log(i+\tan(d+ex))\right)+2a(a^2+2b^2)\tan(d+ex)+a^2b\tan^2(d+ex)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((a^2 + b^2)*((I*a + b)*Log[I - Tan[d + e*x]] + ((-I)*a + b)*Log[I + Tan[d
+ e*x]]) + 2*a*(a^2 + 2*b^2)*Tan[d + e*x] + a^2*b*Tan[d + e*x]^2)/(2*e)
```

Maple [A]
time = 0.03, size = 84, normalized size = 1.17

method	result
norman	$\left(-a^3-a\,b^2\right)x+\frac{a\left(a^2+2b^2\right)\tan(ex+d)}{e}+\frac{a^2b\left(\tan^2(ex+d)\right)}{2e}+\frac{b\left(a^2+b^2\right)\ln\left(1+\tan^2(ex+d)\right)}{2e}$
derivativedivides	$\frac{\frac{a^2b\left(\tan^2(ex+d)\right)}{2}+a^3\tan(ex+d)+2a\,b^2\tan(ex+d)+\frac{\left(a^2b+b^3\right)\ln\left(1+\tan^2(ex+d)\right)}{2}+\left(-a^3-a\,b^2\right)\arctan(\tan(ex+d))}{e}$
default	$\frac{\frac{a^2b\left(\tan^2(ex+d)\right)}{2}+a^3\tan(ex+d)+2a\,b^2\tan(ex+d)+\frac{\left(a^2b+b^3\right)\ln\left(1+\tan^2(ex+d)\right)}{2}+\left(-a^3-a\,b^2\right)\arctan(\tan(ex+d))}{e}$
risch	$ia^2bx+ib^3x-a^3x-a\,b^2x+\frac{2ib\,a^2d}{e}+\frac{2ib^3d}{e}+\frac{2ia\left(e^{2i(ex+d)}a^2+2\,e^{2i(ex+d)}b^2-iab\,e^{2i(ex+d)}+a^2+2b^2\right)}{e\left(1+e^{2i(ex+d)}\right)^2}$ –

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/e*(1/2*a^2*b*tan(e*x+d)^2+a^3*tan(e*x+d)+2*a*b^2*tan(e*x+d)+1/2*(a^2*b+b^
3)*ln(1+tan(e*x+d)^2)+(-a^3-a*b^2)*arctan(tan(e*x+d)))
```

Maxima [A]
time = 0.48, size = 77, normalized size = 1.07

$$\frac{1}{2}\left(a^2b\tan\left(xe+d\right)^2-2\left(a^3+ab^2\right)(xe+d)+\left(a^2b+b^3\right)\log\left(\tan\left(xe+d\right)^2+1\right)+2\left(a^3+2\,ab^2\right)\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(a^2*b*tan(x*e + d)^2 - 2*(a^3 + a*b^2)*(x*e + d) + (a^2*b + b^3)*log(t
an(x*e + d)^2 + 1) + 2*(a^3 + 2*a*b^2)*tan(x*e + d))*e^(-1)
```

Fricas [A]
time = 3.64, size = 77, normalized size = 1.07

$$\frac{1}{2}\left(a^{2}b\tan\left(xe+d\right)^{2}-2\left(a^{3}+ab^{2}\right)xe-\left(a^{2}b+b^{3}\right)\log\left(\frac{1}{\tan\left(xe+d\right)^{2}+1}\right)+2\left(a^{3}+2\,ab^{2}\right)\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(a^2*b*tan(x*e + d)^2 - 2*(a^3 + a*b^2)*x*e - (a^2*b + b^3)*log(1/(tan(
x*e + d)^2 + 1)) + 2*(a^3 + 2*a*b^2)*tan(x*e + d))*e^(-1)
```

Sympy [A]
time = 0.09, size = 122, normalized size = 1.69

$$\begin{cases} -a^{3}x+\frac{a^{3}\tan\left(d+ex\right)}{e}+\frac{a^{2}b\log\left(\tan^{2}\left(d+ex\right)+1\right)}{2e}+\frac{a^{2}b\tan^{2}\left(d+ex\right)}{2e}-ab^{2}x+\frac{2ab^{2}\tan\left(d+ex\right)}{e}+\frac{b^{3}\log\left(\tan^{2}\left(d+ex\right)+1\right)}{2e} & \text{for}\ e\neq 0\\ x\left(a+b\tan\left(d\right)\right)\left(a^{2}\tan^{2}\left(d\right)+2ab\tan\left(d\right)+b^{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

```
[Out] Piecewise((-a**3*x + a**3*tan(d + e*x)/e + a**2*b*log(tan(d + e*x)**2 + 1)/
(2*e) + a**2*b*tan(d + e*x)**2/(2*e) - a*b**2*x + 2*a*b**2*tan(d + e*x)/e +
 b**3*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*ta
n(d)**2 + 2*a*b*tan(d) + b**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(70) = 140.
time = 0.78, size = 652, normalized size = 9.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*a^3*e*x*tan(e*x)^2*tan(d)^2 + 2*a*b^2*e*x*tan(e*x)^2*tan(d)^2 + a^2
*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 +
```

```
 tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 +
b^3*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2
+ tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 -
 4*a^3*e*x*tan(e*x)*tan(d) - 4*a*b^2*e*x*tan(e*x)*tan(d) - a^2*b*tan(e*x)^2
*tan(d)^2 - 2*a^2*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(
e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e
*x)*tan(d) - 2*b^3*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e
*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*tan(e*
x)*tan(d) + 2*a^3*tan(e*x)^2*tan(d) + 4*a*b^2*tan(e*x)^2*tan(d) + 2*a^3*tan
(e*x)*tan(d)^2 + 4*a*b^2*tan(e*x)*tan(d)^2 + 2*a^3*e*x + 2*a*b^2*e*x - a^2*
b*tan(e*x)^2 - a^2*b*tan(d)^2 + a^2*b*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*
x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(ta
n(d)^2 + 1)) + b^3*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e
*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1)) - 2*a^
3*tan(e*x) - 4*a*b^2*tan(e*x) - 2*a^3*tan(d) - 4*a*b^2*tan(d) - a^2*b)/(e*t
an(e*x)^2*tan(d)^2 - 2*e*tan(e*x)*tan(d) + e)
```

Mupad [B]
time = 2.77, size = 105, normalized size = 1.46

$$\frac{\tan(d+e\,x)\,\left(a^3+2\,a\,b^2\right)}{e}+\frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{a^2\,b}{2}+\frac{b^3}{2}\right)}{e}+\frac{a^2\,b\tan(d+e\,x)^2}{2\,e}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(d+e\,x)\,\left(a^2+b^2\right)}{a^3+a\,b^2}\right)\,\left(a^2+b^2\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x)
```

```
[Out] (tan(d + e*x)*(2*a*b^2 + a^3))/e + (log(tan(d + e*x)^2 + 1)*((a^2*b)/2 + b^
3/2))/e + (a^2*b*tan(d + e*x)^2)/(2*e) - (a*atan((a*tan(d + e*x)*(a^2 + b^2
))/(a*b^2 + a^3))*(a^2 + b^2))/e
```

3.512 $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

Optimal. Leaf size=101

$$-\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^2}+\frac{b(3a^2-b^2)\log(b\cos(d+ex)+a\sin(d+ex))}{(a^2+b^2)^2\,e}-\frac{a^2-b^2}{(a^2+b^2)\,e(b+a\tan(d+ex))}$$

```
[Out] -a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))/(a
^2+b^2)^2/e+(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d))
```

Rubi [A]
time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3789, 3610, 3612, 3611}

$$-\frac{a^2-b^2}{e\,(a^2+b^2)\,(a\tan(d+ex)+b)}+\frac{b(3a^2-b^2)\log(a\sin(d+ex)+b\cos(d+ex))}{e\,(a^2+b^2)^2}-\frac{ax(a^2-3b^2)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x]
```

```
[Out] -((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^2) + (b*(3*a^2 - b^2)*Log[b*Cos[d + e*x]
+ a*Sin[d + e*x]])/((a^2 + b^2)^2*e) - (a^2 - b^2)/((a^2 + b^2)*e*(b + a*Ta
n[d + e*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
```

```
Q[a*c + b*d, 0]
```

Rule 3789

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = (4a^2)\int \frac{a+b\tan(d+ex)}{(2ab+2a^2\tan(d+ex))^2}\,dx$$

$$= -\frac{a^2-b^2}{(a^2+b^2)\,e(b+a\tan(d+ex))} + \frac{\int \frac{4a^2b-2a(a^2-b^2)\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{a^2+b^2}$$

$$= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^2} - \frac{a^2-b^2}{(a^2+b^2)\,e(b+a\tan(d+ex))} + \frac{(b(3a^2-$$

$$= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^2} + \frac{b(3a^2-b^2)\log(b\cos(d+ex)+a\sin(d+e}{(a^2+b^2)^2\,e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.68, size = 187, normalized size = 1.85

$$\frac{\frac{b(-((a+ib)\log(i-\tan(d+ex)))-(a-ib)\log(i+\tan(d+ex))+2a\log(b+a\tan(d+ex)))}{a^2+b^2}+(a-b)(a+b)\left(\frac{i\log(i-\tan(d+ex))}{(a-ib)^2}-\frac{i\log(i+\tan(d+ex))}{(a+ib)^2}+\frac{2a\left(2b\log(b+a\tan(d+ex))-\frac{a^2+b^2}{b+a\tan(d+ex)}\right)}{(a^2+b^2)^2}\right)}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2),x]
```

```
[Out] ((b*(-((a + I*b)*Log[I - Tan[d + e*x]]) - (a - I*b)*Log[I + Tan[d + e*x]] +
 2*a*Log[b + a*Tan[d + e*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I - Ta
n[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*
b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^
2))/(2*a*e)
```

Maple [A]
time = 0.19, size = 125, normalized size = 1.24

method	result

derivativedivides	$\frac{\frac{\frac{(-3a^2b+b^3)\ln(1+\tan^2(ex+d))}{2}+(-a^3+3a\,b^2)\arctan(\tan(ex+d))}{(a^2+b^2)^2}-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(ex+d))}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{(a^2+b^2)^2}}{e}$
default	$\frac{\frac{\frac{(-3a^2b+b^3)\ln(1+\tan^2(ex+d))}{2}+(-a^3+3a\,b^2)\arctan(\tan(ex+d))}{(a^2+b^2)^2}-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(ex+d))}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{(a^2+b^2)^2}}{e}$
norman	$\frac{\frac{(a^2-b^2)a\tan(ex+d)}{be(a^2+b^2)}-\frac{a^2(a^2-3b^2)x\tan(ex+d)}{(a^2+b^2)^2}-\frac{(a^2-3b^2)abx}{(a^2+b^2)^2}}{b+a\tan(ex+d)}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{e(a^4+2a^2b^2+b^4)}-\frac{b(3a^2-b^2)\ln(1+\tan}{2e(a^4+2a^2b^2-}$
risch	$\frac{ixb}{2iab+a^2-b^2}-\frac{xa}{2iab+a^2-b^2}-\frac{6ib\,a^2x}{a^4+2a^2b^2+b^4}+\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{6ib\,a^2d}{e(a^4+2a^2b^2+b^4)}+\frac{2ib^3d}{e(a^4+2a^2b^2+b^4)}-\frac{}{(-ib}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/e*(1/(a^2+b^2)^2*(1/2*(-3*a^2*b+b^3)*ln(1+tan(e*x+d)^2)+(-a^3+3*a*b^2)*ar
ctan(tan(e*x+d)))-(a^2-b^2)/(a^2+b^2)/(b+a*tan(e*x+d))+b*(3*a^2-b^2)/(a^2+b
^2)^2*ln(b+a*tan(e*x+d)))
```

Maxima [A]
time = 0.47, size = 164, normalized size = 1.62

$$-\frac{1}{2}\left(\frac{2\,(a^3-3\,ab^2)(xe+d)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(3\,a^2b-b^3)\log\left(a\tan\left(xe+d\right)+b\right)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b-b^3)\log\left(\tan\left(xe+d\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(a^2-b^2)}{a^2b+b^3+(a^3+ab^2)\tan\left(xe+d\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(x*e + d)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b^
3)*log(a*tan(x*e + d) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*log(ta
n(x*e + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 + (a
^3 + a*b^2)*tan(x*e + d)))*e^(-1)
```

Fricas [A]
time = 2.72, size = 201, normalized size = 1.99

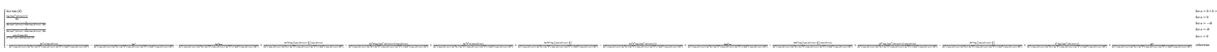
$$-\frac{2\,a^4-2\,a^2b^2+2\,(a^3b-3\,ab^3)xe-(3\,a^2b^2-b^4+(3\,a^3b-ab^3)\tan\left(xe+d\right))\log\left(\frac{a^2\tan(xe+d)^2+2\,ab\tan(xe+d)+b^2}{\tan(xe+d)^2+1}\right)-2\,(a^3b-ab^3-(a^4-3\,a^2b^2)xe)\tan\left(xe+d\right)}{2\,((a^5+2\,a^3b^2+ab^4)e\tan\left(xe+d\right)+(a^4b+2\,a^2b^3+b^5)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="fricas")
```

[Out] -1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*x*e - (3*a^2*b^2 - b^4 + (3*a
^3*b - a*b^3)*tan(x*e + d))*log((a^2*tan(x*e + d)^2 + 2*a*b*tan(x*e + d) +
b^2)/(tan(x*e + d)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*x*e)*tan(
x*e + d))/((a^5 + 2*a^3*b^2 + a*b^4)*e*tan(x*e + d) + (a^4*b + 2*a^2*b^3 +
b^5)*e)

Sympy [C] Result contains complex when optimal does not.
time = 0.76, size = 1358, normalized size = 13.45



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)

[Out] Piecewise((zoo*x*tan(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d + e*x)
2 + 1)/(2*b*e), Eq(a, 0)), (1/(2*b*e*tan(d + e*x)2 + 4*I*b*e*tan(d + e*
x) - 2*b*e), Eq(a, -I*b)), (1/(2*b*e*tan(d + e*x)**2 - 4*I*b*e*tan(d + e*x)
 - 2*b*e), Eq(a, I*b)), (x*(a + b*tan(d))/(a**2*tan(d)**2 + 2*a*b*tan(d) +
b**2), Eq(e, 0)), (-2*a**4*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4
*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x)
 + 2*b**5*e) - 2*a**4/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*t
an(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*a**3*
b*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*
a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 6*a**3*b*log(tan(d + e*
x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*
tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 3*a**3
*b*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*
e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) +
2*b**5*e) + 6*a**2*b**2*log(tan(d + e*x) + b/a)/(2*a**5*e*tan(d + e*x) + 2*
a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d +
e*x) + 2*b**5*e) - 3*a**2*b**2*log(tan(d + e*x)**2 + 1)/(2*a**5*e*tan(d + e
*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*
tan(d + e*x) + 2*b**5*e) + 6*a*b**3*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e
 + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2
*b**5*e) - 2*a*b**3*log(tan(d + e*x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d +
e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e
*tan(d + e*x) + 2*b**5*e) + a*b**3*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2
*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**
3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 2*b**4*log(tan(d + e*x) + b/a)/
(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b
3*e + 2*a*b4*e*tan(d + e*x) + 2*b**5*e) + b**4*log(tan(d + e*x)**2 + 1)
/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*
b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 2*b**4/(2*a**5*e*tan(d + e*x

```
) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*ta
n(d + e*x) + 2*b**5*e), True))
```

Giac [A]
time = 0.70, size = 199, normalized size = 1.97

$$-\frac{\frac{2\,(a^3-3\,ab^2)(ex+d)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b-b^3)\log\left(\tan(ex+d)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(3\,a^3b-ab^3)\log(|a\tan(ex+d)+b|)}{a^5+2\,a^3b^2+ab^4}+\frac{2\,(3\,a^3b\tan(ex+d)-ab^3\tan(ex+d)+a^4+3\,a^2b^2-2\,b^4)}{(a^4+2\,a^2b^2+b^4)(a\tan(ex+d)+b)}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)
*log(tan(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*log(
abs(a*tan(e*x + d) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*tan(e*x + d
) - a*b^3*tan(e*x + d) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b^4)*
(a*tan(e*x + d) + b)))/e
```

Mupad [B]
time = 3.09, size = 152, normalized size = 1.50

$$\frac{b\,\ln\left(b+a\tan(d+e\,x)\right)\,\left(3\,a^2-b^2\right)}{e\,{\left(a^2+b^2\right)}^2}-\frac{\ln\left(\tan(d+e\,x)+1{}\mathrm{i}\right)\,\left(a-b\,1{}\mathrm{i}\right)}{2\,e\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)}-\frac{a^2-b^2}{e\,\left(a^2+b^2\right)\,\left(b+a\tan\left(d+e\,x\right)\right)}-\frac{\ln\left(\tan(d+e\,x)-\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)}{2\,e\,\left(a^2\,1{}\mathrm{i}+2\,a\,b-b^2\,1{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x)
```

```
[Out] (b*log(b + a*tan(d + e*x))*(3*a^2 - b^2))/(e*(a^2 + b^2)^2) - (log(tan(d +
e*x) + 1i)*(a - b*1i))/(2*e*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(e*(a^
2 + b^2)*(b + a*tan(d + e*x))) - (log(tan(d + e*x) - 1i)*(a + b*1i))/(2*e*(
2*a*b + a^2*1i - b^2*1i))
```

3.513 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=197

$$\frac{a(a^4-10a^2b^2+5b^4)\,x}{(a^2+b^2)^4}-\frac{b(5a^4-10a^2b^2+b^4)\log(b\cos(d+ex)+a\sin(d+ex))}{(a^2+b^2)^4\,e}-\frac{a^2-b^2}{3\,(a^2+b^2)\,e(b+a\tan(d+$$

```
[Out] a*(a^4-10*a^2*b^2+5*b^4)*x/(a^2+b^2)^4-b*(5*a^4-10*a^2*b^2+b^4)*ln(b*cos(e*
x+d)+a*sin(e*x+d))/(a^2+b^2)^4/e+1/3*(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d)
)^3-1/2*b*(3*a^2-b^2)/(a^2+b^2)^2/e/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^4)/
(a^2+b^2)^3/e/(b+a*tan(e*x+d))
```

Rubi [A]

time = 0.36, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3789, 3610, 3612, 3611}

$$-\frac{a^2-b^2}{3e\,(a^2+b^2)\,(a\tan(d+ex)+b)^3}-\frac{b(3a^2-b^2)}{2e\,(a^2+b^2)^2\,(a\tan(d+ex)+b)^2}+\frac{a^4-6a^2b^2+b^4}{e\,(a^2+b^2)^3\,(a\tan(d+ex)+b)}-\frac{b(5a^4-10a^2b^2+b^4)\log(a\sin(d+ex)+b\cos(d+ex))}{e\,(a^2+b^2)^4}+\frac{ax(a^4-10a^2b^2+5b^4)}{(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2,
x]
```

```
[Out] (a*(a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)^4 - (b*(5*a^4 - 10*a^2*b^2 + b
^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)^4*e) - (a^2 - b^2)/(
3*(a^2 + b^2)*e*(b + a*Tan[d + e*x])^3) - (b*(3*a^2 - b^2))/(2*(a^2 + b^2)^
2*e*(b + a*Tan[d + e*x])^2) + (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)^3*e*(b +
 a*Tan[d + e*x]))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3789

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx = \left(16a^4\right)\int \frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^4}\,dx$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} + \frac{\left(4a^2\right)\int \frac{4a^2b-2a\left(a^2-b\right.}{\left(2ab+2a^2\tan\right.}}{a^2+b^2}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b(3a^2-}{2\left(a^2+b^2\right)^2e(b+a}$$

$$= -\frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex))^3} - \frac{b(3a^2-}{2\left(a^2+b^2\right)^2e(b+a}$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{a^2-b^2}{3\left(a^2+b^2\right)e(b+a\tan(d+ex)}$$

$$= \frac{a\left(a^4-10a^2b^2+5b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{b\left(5a^4-10a^2b^2+b^4\right)\log(b\cos(}{\left(a^2+b^2\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.77, size = 308, normalized size = 1.56

$$\frac{-\left((a-b)(a+b)\left(\frac{3i\log(i-\tan(d+ex))}{(a-ib)^4}-\frac{3i\log(i+\tan(d+ex))}{(a+ib)^4}+\frac{24a(a-b)b(a+b)\log(b+a\tan(d+ex))}{\left(a^2+b^2\right)^4}+\frac{2a}{\left(a^2+b^2\right)(b+a\tan(d+ex))^3}+\frac{6ab}{\left(a^2+b^2\right)^2(b+a\tan(d+ex))^2}-\frac{6a\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3(b+a\tan(d+ex))}\right)\right)+3b\left(\frac{\log(i-\tan(d+ex))}{(a-ib)^3}+\frac{\log(i+\tan(d+ex))}{(a+ib)^3}+\frac{a\left(-2\left(a^2-3b^2\right)\log(b+a\tan(d+ex))-\frac{\left(a^2+b^2\right)\left(a^2+5b^2+4ab\tan(d+ex)\right)}{(b+a\tan(d+ex))^2}\right)}{\left(a^2+b^2\right)^3}\right)}{6ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^2,x]
```

```
[Out] (-((a - b)*(a + b)*(((3*I)*Log[I - Tan[d + e*x]])/(a - I*b)^4 - ((3*I)*Log[
I + Tan[d + e*x]])/(a + I*b)^4 + (24*a*(a - b)*b*(a + b)*Log[b + a*Tan[d +
```

```
e*x]])/(a^2 + b^2)^4 + (2*a)/((a^2 + b^2)*(b + a*Tan[d + e*x])^3) + (6*a*b)
/((a^2 + b^2)^2*(b + a*Tan[d + e*x])^2) - (6*a*(a^2 - 3*b^2))/((a^2 + b^2)^
3*(b + a*Tan[d + e*x])))) + 3*b*(Log[I - Tan[d + e*x]]/(a - I*b)^3 + Log[I
+ Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d + e*x]]
- ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*x])^2))
/(a^2 + b^2)^3))/(6*a*e)
```

Maple [A]
time = 0.25, size = 218, normalized size = 1.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(1/(a^2+b^2)^4*(1/2*(5*a^4*b-10*a^2*b^3+b^5)*ln(1+tan(e*x+d)^2)+(a^5-10
*a^3*b^2+5*a*b^4)*arctan(tan(e*x+d)))-1/3*(a^2-b^2)/(a^2+b^2)/(b+a*tan(e*x+
d))^3-1/2*b*(3*a^2-b^2)/(a^2+b^2)^2/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^4)/
(a^2+b^2)^3/(b+a*tan(e*x+d))-b*(5*a^4-10*a^2*b^2+b^4)/(a^2+b^2)^4*ln(b+a*ta
n(e*x+d)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(194) = 388.
time = 0.49, size = 426, normalized size = 2.16

$$\frac{1}{6}\left(\frac{6\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)(xe+d)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{6\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\log\left(a\tan\left(xe+d\right)+b\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}+\frac{3\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\log\left(\tan\left(xe+d\right)^{2}+1\right)}{a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}}-\frac{2\,a^{6}+5\,a^{4}b^{2}+40\,a^{2}b^{4}-11\,b^{6}-6\left(a^{6}-6\,a^{4}b^{2}+a^{2}b^{4}\right)\tan\left(xe+d\right)^{2}-3\left(a^{5}b-26\,a^{3}b^{3}+5\,ab^{5}\right)\tan\left(xe+d\right)}{a^{6}b^{3}+3\,a^{4}b^{5}+3\,a^{2}b^{7}+b^{9}+\left(a^{9}+3\,a^{7}b^{2}+3\,a^{5}b^{4}+a^{3}b^{6}\right)\tan\left(xe+d\right)^{3}+3\left(a^{8}b+3\,a^{6}b^{3}+3\,a^{4}b^{5}+a^{2}b^{7}\right)\tan\left(xe+d\right)^{2}+3\left(a^{7}b^{2}+3\,a^{5}b^{4}+3\,a^{3}b^{6}+ab^{8}\right)\tan\left(xe+d\right)}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(x*e + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) - 6*(5*a^4*b - 10*a^2*b^3 + b^5)*log(a*tan(x*e + d) + b)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(x*e + d)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8) - (2*a^6 + 5*a^4*b^2 + 40*a^2*b^4 - 11*b^6 - 6*(a^6 - 6*a^4*b^2 + a^2*b
^4)*tan(x*e + d)^2 - 3*(a^5*b - 26*a^3*b^3 + 5*a*b^5)*tan(x*e + d))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6)*t
an(x*e + d)^3 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(x*e + d)^2
+ 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(x*e + d)))*e^(-1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(194) = 388.
time = 2.57, size = 600, normalized size = 3.05

$$-\frac{2\,a^{8}+7\,a^{6}b^{2}+66\,a^{4}b^{4}-27\,a^{2}b^{6}+\left(21\,a^{7}b-56\,a^{5}b^{3}+11\,a^{3}b^{5}-6\left(a^{8}-10\,a^{6}b^{2}+5\,a^{4}b^{4}\right)xe\right)\tan\left(xe+d\right)^{3}-6\left(a^{5}b^{3}-10\,a^{3}b^{5}+5\,ab^{7}\right)xe-3\left(2\,a^{8}-31\,a^{6}b^{2}+46\,a^{4}b^{4}-9\,a^{2}b^{6}+6\left(a^{7}b-10\,a^{5}b^{3}+5\,a^{3}b^{5}\right)xe\right)\tan\left(xe+d\right)^{2}+3\left(5\,a^{4}b^{4}-10\,a^{2}b^{6}+b^{8}+\left(5\,a^{7}b-10\,a^{5}b^{3}+a^{3}b^{5}\right)\tan\left(xe+d\right)^{3}+3\left(5\,a^{6}b^{2}-10\,a^{4}b^{4}+a^{2}b^{6}\right)\tan\left(xe+d\right)^{2}+3\left(5\,a^{5}b^{3}-10\,a^{3}b^{5}+ab^{7}\right)\tan\left(xe+d\right)\right)\log\left(\frac{a^{2}\tan\left(xe+d\right)^{2}+2\,ab\tan\left(xe+d\right)+b^{2}}{\tan\left(xe+d\right)^{2}+1}\right)-3\left(a^{7}b-46\,a^{5}b^{3}+35\,a^{3}b^{5}-6\,ab^{7}+6\left(a^{6}b^{2}-10\,a^{4}b^{4}+5\,a^{2}b^{6}\right)xe\right)\tan\left(xe+d\right)}{6\left(\left(a^{11}+4\,a^{9}b^{2}+6\,a^{7}b^{4}+4\,a^{5}b^{6}+a^{3}b^{8}\right)e\tan\left(xe+d\right)^{3}+3\left(a^{10}b+4\,a^{8}b^{3}+6\,a^{6}b^{5}+4\,a^{4}b^{7}+a^{2}b^{9}\right)e\tan\left(xe+d\right)^{2}+3\left(a^{9}b^{2}+4\,a^{7}b^{4}+6\,a^{5}b^{6}+4\,a^{3}b^{8}+ab^{10}\right)e\tan\left(xe+d\right)+\left(a^{8}b^{3}+4\,a^{6}b^{5}+6\,a^{4}b^{7}+4\,a^{2}b^{9}+b^{11}\right)e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] -1/6*(2*a^8 + 7*a^6*b^2 + 66*a^4*b^4 - 27*a^2*b^6 + (21*a^7*b - 56*a^5*b^3
+ 11*a^3*b^5 - 6*(a^8 - 10*a^6*b^2 + 5*a^4*b^4)*x*e)*tan(x*e + d)^3 - 6*(a^
5*b^3 - 10*a^3*b^5 + 5*a*b^7)*x*e - 3*(2*a^8 - 31*a^6*b^2 + 46*a^4*b^4 - 9*
a^2*b^6 + 6*(a^7*b - 10*a^5*b^3 + 5*a^3*b^5)*x*e)*tan(x*e + d)^2 + 3*(5*a^4
*b^4 - 10*a^2*b^6 + b^8 + (5*a^7*b - 10*a^5*b^3 + a^3*b^5)*tan(x*e + d)^3 +
 3*(5*a^6*b^2 - 10*a^4*b^4 + a^2*b^6)*tan(x*e + d)^2 + 3*(5*a^5*b^3 - 10*a^
3*b^5 + a*b^7)*tan(x*e + d))*log((a^2*tan(x*e + d)^2 + 2*a*b*tan(x*e + d) +
 b^2)/(tan(x*e + d)^2 + 1)) - 3*(a^7*b - 46*a^5*b^3 + 35*a^3*b^5 - 6*a*b^7
+ 6*(a^6*b^2 - 10*a^4*b^4 + 5*a^2*b^6)*x*e)*tan(x*e + d))/((a^11 + 4*a^9*b^
2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*e*tan(x*e + d)^3 + 3*(a^10*b + 4*a^8*b
^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*e*tan(x*e + d)^2 + 3*(a^9*b^2 + 4*a^7
*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*e*tan(x*e + d) + (a^8*b^3 + 4*a^6*b^
5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*e)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(193) = 386.
time = 0.95, size = 437, normalized size = 2.22

$$\frac{\frac{6\,(a^5-10\,a^3b^2+5\,ab^4)(ex+d)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(5\,a^4b-10\,a^2b^3+b^5)\log\left(\tan(ex+d)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(5\,a^5b-10\,a^3b^3+ab^5)\log(|a\tan(ex+d)+b|)}{a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8}+\frac{55\,a^7b\tan(ex+d)^3-110\,a^5b^3\tan(ex+d)^3+11\,a^3b^5\tan(ex+d)^3+6\,a^8\tan(ex+d)^2+135\,a^6b^2\tan(ex+d)^2-360\,a^4b^4\tan(ex+d)^2+39\,a^2b^6\tan(ex+d)^2+3\,a^7b\tan(ex+d)+90\,a^5b^3\tan(ex+d)-393\,a^3b^5\tan(ex+d)+48\,ab^7\tan(ex+d)-2\,a^8-7\,a^6b^2+10\,a^4b^4-139\,a^2b^6+22\,b^8}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(a\tan(ex+d)+b)^3}}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(e*x + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*log(tan(e*x + d)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(5*a^5*b - 10*a^3*b^3
+ a*b^5)*log(abs(a*tan(e*x + d) + b))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8) + (55*a^7*b*tan(e*x + d)^3 - 110*a^5*b^3*tan(e*x + d)^3 + 11*a
^3*b^5*tan(e*x + d)^3 + 6*a^8*tan(e*x + d)^2 + 135*a^6*b^2*tan(e*x + d)^2 -
 360*a^4*b^4*tan(e*x + d)^2 + 39*a^2*b^6*tan(e*x + d)^2 + 3*a^7*b*tan(e*x +
 d) + 90*a^5*b^3*tan(e*x + d) - 393*a^3*b^5*tan(e*x + d) + 48*a*b^7*tan(e*x
```

```
 + d) - 2*a^8 - 7*a^6*b^2 + 10*a^4*b^4 - 139*a^2*b^6 + 22*b^8)/((a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(a*tan(e*x + d) + b)^3))/e
```

Mupad [B]
time = 3.26, size = 388, normalized size = 1.97

$$\frac{\frac{\tan(d+e\,x)^2\,\left(a^6-6\,a^4\,b^2+a^2\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\,a^6+5\,a^4\,b^2+40\,a^2\,b^4-11\,b^6}{6\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(d+e\,x)\,\left(a^5\,b-26\,a^3\,b^3+5\,a\,b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{e\,\left(a^3\tan\left(d+e\,x\right)^3+3\,a^2\,b\tan\left(d+e\,x\right)^2+3\,a\,b^2\tan\left(d+e\,x\right)+b^3\right)}-\frac{\ln\left(b+a\tan(d+e\,x)\right)\,\left(\frac{5\,b}{{\left(a^2+b^2\right)}^2}-\frac{20\,b^3}{{\left(a^2+b^2\right)}^3}+\frac{16\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{e}+\frac{\ln\left(\tan(d+e\,x)-\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,e\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}-\frac{\ln\left(\tan(d+e\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)}{2\,e\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2,
x)
```

```
[Out] ((tan(d + e*x)^2*(a^6 + a^2*b^4 - 6*a^4*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) - (2*a^6 - 11*b^6 + 40*a^2*b^4 + 5*a^4*b^2)/(6*(a^6 + b^6 + 3*a^2*b^
4 + 3*a^4*b^2)) + (tan(d + e*x)*(5*a*b^5 + a^5*b - 26*a^3*b^3))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(e*(b^3 + a^3*tan(d + e*x)^3 + 3*a^2*b*tan(d
+ e*x)^2 + 3*a*b^2*tan(d + e*x))) - (log(b + a*tan(d + e*x))*((5*b)/(a^2 +
b^2)^2 - (20*b^3)/(a^2 + b^2)^3 + (16*b^5)/(a^2 + b^2)^4))/e + (log(tan(d +
 e*x) - 1i)*(a + b*1i))/(2*e*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2
*6i)) - (log(tan(d + e*x) + 1i)*(a - b*1i))/(2*e*(4*a*b^3 - 4*a^3*b + a^4*1
i + b^4*1i - a^2*b^2*6i))
```

3.514 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d$

Optimal. Leaf size=284

$$\frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}{3e}+\frac{(a^4-b^4)\log(\cos(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d\right.}{e(b+a\tan(d+ex))^3}$$

```
[Out] 1/3*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e+(a^4-b^4)*ln(cos(e*x+
d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d))^3+1/2*
(a^2+b^2)*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d))-
2*a^4*b*(a^2+b^2)*x*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/(a*b+a^2*
tan(e*x+d))^3+a^4*b*(a^2+b^2)*tan(e*x+d)*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+
d)^2)^(3/2)/e/(a*b+a^2*tan(e*x+d))^3
```

Rubi [A]

time = 0.15, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3791, 3609, 12, 3606, 3556}

$$\frac{b(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}{3e}+\frac{(a^2+b^2)(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}{2e(a\tan(d+ex)+b)}-\frac{2a^4bx(a^2+b^2)(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}{(a^2\tan(d+ex)+ab)^3}+\frac{a^4b(a^2+b^2)\tan(d+ex)(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}{e(a^2\tan(d+ex)+ab)^3}+\frac{(a^4-b^4)\log(\cos(d+ex))(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}{e(a\tan(d+ex)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] (b*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(3*e) + ((a^4 - b
^4)*Log[Cos[d + e*x]]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)
)/(e*(b + a*Tan[d + e*x])^3) + ((a^2 + b^2)*(b^2 + 2*a*b*Tan[d + e*x] + a^2
*Tan[d + e*x]^2)^(3/2))/(2*e*(b + a*Tan[d + e*x])) - (2*a^4*b*(a^2 + b^2)*x
*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2))/(a*b + a^2*Tan[d +
e*x])^3 + (a^4*b*(a^2 + b^2)*Tan[d + e*x]*(b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2)^(3/2))/(e*(a*b + a^2*Tan[d + e*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3791

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\,dx = \frac{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{(2}$$

$$= \frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{3e}$$

$$= \frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{3e}$$

$$= \frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{3e}$$

$$= \frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{3e}$$

$$= \frac{b(b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{3e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.37, size = 147, normalized size = 0.52

$$\frac{\sqrt{(b+a\tan(d+ex))^2}\left(-3(a^2+b^2)\left((a-ib)^2\log(i-\tan(d+ex))+(a+ib)^2\log(i+\tan(d+ex))\right)+6ab(2a^2+3b^2)\tan(d+ex)+3a^2(a^2+3b^2)\tan^2(d+ex)+2a^3b\tan^3(d+ex)\right)}{6e(b+a\tan(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-3*(a^2 + b^2)*((a - I*b)^2*Log[I - Tan[d +
e*x]] + (a + I*b)^2*Log[I + Tan[d + e*x]]) + 6*a*b*(2*a^2 + 3*b^2)*Tan[d +
e*x] + 3*a^2*(a^2 + 3*b^2)*Tan[d + e*x]^2 + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(
b + a*Tan[d + e*x]))
```

Maple [A]
time = 0.38, size = 158, normalized size = 0.56

method	result
derivativedivides	$-\frac{\left(b^2+2ab\tan(ex+d)+a^2\left(\tan^2(ex+d)\right)\right)^{\frac{3}{2}}\left(-2a^3b\left(\tan^3(ex+d)\right)-3a^4\left(\tan^2(ex+d)\right)-9a^2b^2\left(\tan^2(ex+d)\right)+3\ln\left(1+\tan\right.\right.}{6e(b}$
default	$-\frac{\left(b^2+2ab\tan(ex+d)+a^2\left(\tan^2(ex+d)\right)\right)^{\frac{3}{2}}\left(-2a^3b\left(\tan^3(ex+d)\right)-3a^4\left(\tan^2(ex+d)\right)-9a^2b^2\left(\tan^2(ex+d)\right)+3\ln\left(1+\tan\right.\right.}{6e(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/6/e*((b+a*tan(e*x+d))^2)^(3/2)*(-2*a^3*b*tan(e*x+d)^3-3*a^4*tan(e*x+d)^2
-9*a^2*b^2*tan(e*x+d)^2+3*ln(1+tan(e*x+d)^2)*a^4-3*ln(1+tan(e*x+d)^2)*b^4+1
2*arctan(tan(e*x+d))*a^3*b+12*arctan(tan(e*x+d))*a*b^3-12*a^3*b*tan(e*x+d)-
18*a*b^3*tan(e*x+d))/(b+a*tan(e*x+d))^3
```

Maxima [A]
time = 0.48, size = 174, normalized size = 0.61

$$\frac{1}{6}\left(3\left(a^3\tan\left(xe+d\right)^2+6a^2b\tan\left(xe+d\right)-2\left(3a^2b-b^3\right)\left(xe+d\right)-\left(a^3-3ab^2\right)\log\left(\tan\left(xe+d\right)^2+1\right)\right)a+\left(2a^3\tan\left(xe+d\right)^3+9a^2b\tan\left(xe+d\right)^2+6\left(a^3-3ab^2\right)\left(xe+d\right)-3\left(3a^2b-b^3\right)\log\left(\tan\left(xe+d\right)^2+1\right)-6\left(a^3-3ab^2\right)\tan\left(xe+d\right)\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(a^3*tan(x*e + d)^2 + 6*a^2*b*tan(x*e + d) - 2*(3*a^2*b - b^3)*(x*e
+ d) - (a^3 - 3*a*b^2)*log(tan(x*e + d)^2 + 1))*a + (2*a^3*tan(x*e + d)^3 +
 9*a^2*b*tan(x*e + d)^2 + 6*(a^3 - 3*a*b^2)*(x*e + d) - 3*(3*a^2*b - b^3)*l
og(tan(x*e + d)^2 + 1) - 6*(a^3 - 3*a*b^2)*tan(x*e + d))*b)*e^(-1)
```

Fricas [A]
time = 2.72, size = 106, normalized size = 0.37

$$\frac{1}{6}\left(2a^3b\tan\left(xe+d\right)^3-12\left(a^3b+ab^3\right)xe+3\left(a^4+3a^2b^2\right)\tan\left(xe+d\right)^2+3\left(a^4-b^4\right)\log\left(\frac{1}{\tan\left(xe+d\right)^2+1}\right)+6\left(2a^3b+3ab^3\right)\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*b*tan(x*e + d)^3 - 12*(a^3*b + a*b^3)*x*e + 3*(a^4 + 3*a^2*b^2)*
tan(x*e + d)^2 + 3*(a^4 - b^4)*log(1/(tan(x*e + d)^2 + 1)) + 6*(2*a^3*b + 3
*a*b^3)*tan(x*e + d))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\tan(d + ex)) \left((a\tan(d + ex) + b)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*((a*tan(d + e*x) + b)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1615 vs. 2(270) = 540.
time = 1.35, size = 1615, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/6*(12*a^3*b*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 + 12*a*b^3*e
*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 - 3*a^4*log(4*(tan(e*x)^4*ta
n(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x
)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 +
 3*b^4*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)
^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d)
 + b)*tan(e*x)^3*tan(d)^3 - 36*a^3*b*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^2
*tan(d)^2 - 36*a*b^3*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 - 3*a^
4*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 - 9*a^2*b^2*sgn(a*tan(e*x + d
) + b)*tan(e*x)^3*tan(d)^3 + 9*a^4*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^
3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d
)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 - 9*b^4*log(4*(tan(e*
x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*
tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan
(d)^2 + 12*a^3*b*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^2 + 18*a*b^3*sgn
(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^2 + 12*a^3*b*sgn(a*tan(e*x + d) + b)
```

```
*tan(e*x)^2*tan(d)^3 + 18*a*b^3*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^3
 + 36*a^3*b*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) + 36*a*b^3*e*x*sgn(
a*tan(e*x + d) + b)*tan(e*x)*tan(d) - 3*a^4*sgn(a*tan(e*x + d) + b)*tan(e*x
)^3*tan(d) - 9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d) + 3*a^4*sg
n(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(e*x + d) +
b)*tan(e*x)^2*tan(d)^2 - 3*a^4*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d)^3 -
9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d)^3 + 2*a^3*b*sgn(a*tan(e*x
 + d) + b)*tan(e*x)^3 - 9*a^4*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan
(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 +
 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) + 9*b^4*log(4*(tan(e*x)^4*tan(
d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*
tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) - 18*a^
3*b*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d) - 36*a*b^3*sgn(a*tan(e*x + d)
 + b)*tan(e*x)^2*tan(d) - 18*a^3*b*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d)^
2 - 36*a*b^3*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d)^2 + 2*a^3*b*sgn(a*tan(
e*x + d) + b)*tan(d)^3 - 12*a^3*b*e*x*sgn(a*tan(e*x + d) + b) - 12*a*b^3*e*
x*sgn(a*tan(e*x + d) + b) + 3*a^4*sgn(a*tan(e*x + d) + b)*tan(e*x)^2 + 9*a^
2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)^2 - 3*a^4*sgn(a*tan(e*x + d) + b)*ta
n(e*x)*tan(d) - 9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) + 3*a^4*s
gn(a*tan(e*x + d) + b)*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(d)^
2 + 3*a^4*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan
(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x +
 d) + b) - 3*b^4*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x
)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*ta
n(e*x + d) + b) + 12*a^3*b*sgn(a*tan(e*x + d) + b)*tan(e*x) + 18*a*b^3*sgn(
a*tan(e*x + d) + b)*tan(e*x) + 12*a^3*b*sgn(a*tan(e*x + d) + b)*tan(d) + 18
*a*b^3*sgn(a*tan(e*x + d) + b)*tan(d) + 3*a^4*sgn(a*tan(e*x + d) + b) + 9*a
^2*b^2*sgn(a*tan(e*x + d) + b))/(e*tan(e*x)^3*tan(d)^3 - 3*e*tan(e*x)^2*tan
(d)^2 + 3*e*tan(e*x)*tan(d) - e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\tan(d+e\,x))\,\left(a^2\tan(d+e\,x)^2+2\,a\,b\tan(d+e\,x)+b^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2), x)
```

3.515 $\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

Optimal. Leaf size=122

$$-\frac{(a^2+b^2)\log(\cos(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}{e(b+a\tan(d+ex))}+\frac{a^2b\tan(d+ex)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}{e\left(ab+a^2\tan(d+ex)\right)}$$

```
[Out] -(a^2+b^2)*ln(cos(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)/e/(
b+a*tan(e*x+d))+a^2*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*tan(e*x
+d)/e/(a*b+a^2*tan(e*x+d))
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3791, 3606, 3556}

$$\frac{a^2b\tan(d+ex)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e\left(a^2\tan(d+ex)+ab\right)}-\frac{(a^2+b^2)\log(\cos(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}{e(a\tan(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[d + e*x]]*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d
+ e*x]^2])/(e*(b + a*Tan[d + e*x]))) + (a^2*b*Tan[d + e*x]*Sqrt[b^2 + 2*a*b
*Tan[d + e*x] + a^2*Tan[d + e*x]^2])/(e*(a*b + a^2*Tan[d + e*x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3791

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+}{2a}$$

$$= \frac{a^2b\tan(d+ex)\sqrt{b^2+2ab\tan(d+ex}{e\left(ab+a^2\tan(d+e\right.}$$

$$= -\frac{(a^2+b^2)\log(\cos(d+ex))\sqrt{b^2+2a}{e(b+a\tan(}$$

Mathematica [A]
time = 0.20, size = 58, normalized size = 0.48

$$\frac{\sqrt{(b+a\tan(d+ex))^2}\,(-((a^2+b^2)\log(\cos(d+ex)))+ab\tan(d+ex))}{e(b+a\tan(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] (Sqrt[(b + a*Tan[d + e*x])^2]*(-((a^2 + b^2)*Log[Cos[d + e*x]]) + a*b*Tan[d
 + e*x]))/(e*(b + a*Tan[d + e*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 75, normalized size = 0.61

method	result	size
derivativedivides	$\frac{\operatorname{csgn}(b+a\tan(ex+d))\left(\ln\left(a^2\left(\tan^2(ex+d)\right)+a^2\right)a^2+\ln\left(a^2\left(\tan^2(ex+d)\right)+a^2\right)b^2+2ab\tan(ex+d)+2b^2\right)}{2e}$	75
default	$\frac{\operatorname{csgn}(b+a\tan(ex+d))\left(\ln\left(a^2\left(\tan^2(ex+d)\right)+a^2\right)a^2+\ln\left(a^2\left(\tan^2(ex+d)\right)+a^2\right)b^2+2ab\tan(ex+d)+2b^2\right)}{2e}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/e*csgn(b+a*tan(e*x+d))*(ln(a^2*tan(e*x+d)^2+a^2)*a^2+ln(a^2*tan(e*x+d)^
2+a^2)*b^2+2*a*b*tan(e*x+d)+2*b^2)
```

Maxima [A]
time = 0.47, size = 69, normalized size = 0.57

$$\frac{1}{2}\left(\left(2\,(xe+d)b+a\log\left(\tan\left(xe+d\right)^2+1\right)\right)a-\left(2\,(xe+d)a-b\log\left(\tan\left(xe+d\right)^2+1\right)-2\,a\tan\left(xe+d\right)\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="maxima")
```

```
[Out] 1/2*((2*(x*e + d)*b + a*log(tan(x*e + d)^2 + 1))*a - (2*(x*e + d)*a - b*log
(tan(x*e + d)^2 + 1) - 2*a*tan(x*e + d))*b)*e^(-1)
```

Fricas [A]
time = 2.39, size = 39, normalized size = 0.32

$$\frac{1}{2}\left(2\,ab\tan\left(xe+d\right)-\left(a^2+b^2\right)\log\left(\frac{1}{\tan\left(xe+d\right)^2+1}\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*tan(x*e + d) - (a^2 + b^2)*log(1/(tan(x*e + d)^2 + 1)))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tan\left(d+ex\right)\right)\sqrt{\left(a\tan\left(d+ex\right)+b\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2)*(a+b*tan(e*x+d)
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))*sqrt((a*tan(d + e*x) + b)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(118) = 236.
time = 0.55, size = 363, normalized size = 2.98

$$-\frac{a^2\log\left(\frac{4\left(\tan(ex)^4\tan(d)^2-2\tan(ex)^3\tan(d)+\tan(ex)^2\tan(d)^2+\tan(ex)^2-2\tan(ex)\tan(d)+1\right)}{\tan(d)^2+1}\right)\mathrm{sgn}(a\tan\left(ex+d\right)+b)\tan\left(ex\right)\tan\left(d\right)+b^2\log\left(\frac{4\left(\tan(ex)^4\tan(d)^2-2\tan(ex)^3\tan(d)+\tan(ex)^2\tan(d)^2+\tan(ex)^2-2\tan(ex)\tan(d)+1\right)}{\tan(d)^2+1}\right)\mathrm{sgn}(a\tan\left(ex+d\right)+b)\tan\left(ex\right)\tan\left(d\right)-a^2\log\left(\frac{4\left(\tan(ex)^4\tan(d)^2-2\tan(ex)^3\tan(d)+\tan(ex)^2\tan(d)^2+\tan(ex)^2-2\tan(ex)\tan(d)+1\right)}{\tan(d)^2+1}\right)\mathrm{sgn}(a\tan\left(ex+d\right)+b)-b^2\log\left(\frac{4\left(\tan(ex)^4\tan(d)^2-2\tan(ex)^3\tan(d)+\tan(ex)^2\tan(d)^2+\tan(ex)^2-2\tan(ex)\tan(d)+1\right)}{\tan(d)^2+1}\right)\mathrm{sgn}(a\tan\left(ex+d\right)+b)+2\,ab\mathrm{sgn}(a\tan\left(ex+d\right)+b)\tan\left(ex\right)+2\,ab\mathrm{sgn}(a\tan\left(ex+d\right)+b)\tan\left(d\right)}{2\left(e\tan\left(ex\right)\tan\left(d\right)-e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,
 algorithm="giac")
```

```
[Out] -1/2*(a^2*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan
(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x +
 d) + b)*tan(e*x)*tan(d) + b^2*log(4*(tan(e*x)^4*tan(d)^2 - 2*tan(e*x)^3*ta
n(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(d)^2
+ 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) - a^2*log(4*(tan(e*x)^4*tan(d
```

```
)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e*x)*t
an(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b) - b^2*log(4*(tan(e*x)^4*
tan(d)^2 - 2*tan(e*x)^3*tan(d) + tan(e*x)^2*tan(d)^2 + tan(e*x)^2 - 2*tan(e
*x)*tan(d) + 1)/(tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b) + 2*a*b*sgn(a*tan(e
*x + d) + b)*tan(e*x) + 2*a*b*sgn(a*tan(e*x + d) + b)*tan(d))/(e*tan(e*x)*t
an(d) - e)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\tan(d+e\,x))\,\sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2), x)
```

3.516 $\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$

Optimal. Leaf size=138

$$\frac{(a^2-b^2)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d+ex))}{(a^2+b^2)\,e\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}+\frac{2bx(ab+a^2\tan(d+ex))}{(a^2+b^2)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

```
[Out] (a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2
*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)+2*b*x*(a*b+a^2*tan(e*x+d))/(a^2+b^2
)/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {3791, 3612, 3611}

$$\frac{2bx(a^2\tan(d+ex)+ab)}{(a^2+b^2)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}+\frac{(a^2-b^2)(a\tan(d+ex)+b)\log(a\sin(d+ex)+b\cos(d+ex))}{e(a^2+b^2)\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2
],x]
```

```
[Out] ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x]))/((a
^2 + b^2)*e*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2]) + (2*b*x*(
a*b + a^2*Tan[d + e*x]))/((a^2 + b^2)*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*T
an[d + e*x]^2])
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3791

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{(2ab+2a^2\tan(d+ex))\int\frac{a+b\tan(d+ex)}{2ab+2a^2\tan(d+ex)}\,dx}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

$$= \frac{2bx(ab+a^2\tan(d+ex))}{(a^2+b^2)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}} + \frac{(}{}$$

$$= \frac{(a^2-b^2)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d}{(a^2+b^2)\,e\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex}$$

Mathematica [A]
time = 0.47, size = 88, normalized size = 0.64

$$\frac{(4ab\text{ArcTan}(\tan(d+ex))-(a^2-b^2)(\log(\sec^2(d+ex))-2\log(b+a\tan(d+ex))))(b+a\tan(d+ex))}{2(a^2+b^2)\,e\sqrt{(b+a\tan(d+ex))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
e*x]^2],x]
```

```
[Out] ((4*a*b*ArcTan[Tan[d + e*x]] - (a^2 - b^2)*(Log[Sec[d + e*x]^2] - 2*Log[b +
 a*Tan[d + e*x]]))*(b + a*Tan[d + e*x]))/(2*(a^2 + b^2)*e*Sqrt[(b + a*Tan[d
 + e*x])^2])
```

Maple [A]
time = 0.20, size = 114, normalized size = 0.83

method	result
derivativedivides	$-\frac{(b+a\tan(ex+d))(\ln(1+\tan^2(ex+d))a^2-\ln(1+\tan^2(ex+d))b^2-4ab\arctan(\tan(ex+d))-2\ln(b+a\tan(ex+d))a^2+2\ln}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2(\tan^2(ex+d))}\,(a^2+b^2)}$
default	$-\frac{(b+a\tan(ex+d))(\ln(1+\tan^2(ex+d))a^2-\ln(1+\tan^2(ex+d))b^2-4ab\arctan(\tan(ex+d))-2\ln(b+a\tan(ex+d))a^2+2\ln}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2(\tan^2(ex+d))}\,(a^2+b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] -1/2/e*(b+a*tan(e*x+d))*(ln(1+tan(e*x+d)^2)*a^2-ln(1+tan(e*x+d)^2)*b^2-4*a*
b*arctan(tan(e*x+d))-2*ln(b+a*tan(e*x+d))*a^2+2*ln(b+a*tan(e*x+d))*b^2)/((b
+a*tan(e*x+d))^2)^(1/2)/(a^2+b^2)
```

Maxima [A]
time = 0.48, size = 142, normalized size = 1.03

$$\frac{1}{2}\left(a\left(\frac{2(xe+d)b}{a^2+b^2}+\frac{2a\log(a\tan(xe+d)+b)}{a^2+b^2}-\frac{a\log(\tan(xe+d)^2+1)}{a^2+b^2}\right)+\left(\frac{2(xe+d)a}{a^2+b^2}-\frac{2b\log(a\tan(xe+d)+b)}{a^2+b^2}+\frac{b\log(\tan(xe+d)^2+1)}{a^2+b^2}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] 1/2*(a*(2*(x*e + d)*b/(a^2 + b^2) + 2*a*log(a*tan(x*e + d) + b)/(a^2 + b^2)
 - a*log(tan(x*e + d)^2 + 1)/(a^2 + b^2)) + (2*(x*e + d)*a/(a^2 + b^2) - 2*
b*log(a*tan(x*e + d) + b)/(a^2 + b^2) + b*log(tan(x*e + d)^2 + 1)/(a^2 + b^
2))*b)*e^(-1)
```

Fricas [A]
time = 2.54, size = 74, normalized size = 0.54

$$\frac{\left(4\,abxe+(a^2-b^2)\log\left(\frac{a^2\tan(xe+d)^2+2\,ab\tan(xe+d)+b^2}{\tan(xe+d)^2+1}\right)\right)e^{(-1)}}{2\,(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x*e + (a^2 - b^2)*log((a^2*tan(x*e + d)^2 + 2*a*b*tan(x*e + d) +
 b^2)/(tan(x*e + d)^2 + 1)))*e^(-1)/(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\tan(d+ex)}{\sqrt{(a\tan(d+ex)+b)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/sqrt((a*tan(d + e*x) + b)**2), x)
```

Giac [A]
time = 0.53, size = 161, normalized size = 1.17

$$\frac{\frac{4 (ex+d)ab}{a^2 \mathrm{sgn}(a\tan(ex+d)+b)+b^2 \mathrm{sgn}(a\tan(ex+d)+b)} - \frac{(a^2-b^2)\log\left(\tan(ex+d)^2+1\right)}{a^2 \mathrm{sgn}(a\tan(ex+d)+b)+b^2 \mathrm{sgn}(a\tan(ex+d)+b)} + \frac{2 (a^3-ab^2)\log(|a\tan(ex+d)+b|)}{a^3 \mathrm{sgn}(a\tan(ex+d)+b)+ab^2 \mathrm{sgn}(a\tan(ex+d)+b)}}{2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] 1/2*(4*(e*x + d)*a*b/(a^2*sgn(a*tan(e*x + d) + b) + b^2*sgn(a*tan(e*x + d)
+ b)) - (a^2 - b^2)*log(tan(e*x + d)^2 + 1)/(a^2*sgn(a*tan(e*x + d) + b) +
b^2*sgn(a*tan(e*x + d) + b)) + 2*(a^3 - a*b^2)*log(abs(a*tan(e*x + d) + b))
/(a^3*sgn(a*tan(e*x + d) + b) + a*b^2*sgn(a*tan(e*x + d) + b)))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\tan(d+e\,x)}{\sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(1
/2), x)
```

3.517 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$-\frac{(a^2-b^2)\,(b+a\tan(d+ex))}{2\,(a^2+b^2)\,e\,\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}-\frac{(a^4-6a^2b^2+b^4)\log(b\cos(d+ex)+a\sin(d+ex}{(a^2+b^2)^3\,e\,(b^2+2ab\tan(d+ex)+a^2\tan}$$

```
[Out] -1/2*(a^2-b^2)*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e
*x+d)^2)^(3/2)-(a^4-6*a^2*b^2+b^4)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e
*x+d))^3/(a^2+b^2)^3/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)-4*b*(a
^2-b^2)*x*(a*b+a^2*tan(e*x+d))^3/a^2/(a^2+b^2)^3/(b^2+2*a*b*tan(e*x+d)+a^2*
tan(e*x+d)^2)^(3/2)-b*(3*a^2-b^2)*(a*b+a^2*tan(e*x+d))^3/(a^2+b^2)^2/e/(a^3
*b+a^4*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3791, 3610, 3612, 3611}

$$-\frac{(a^2-b^2)\,(a\tan(d+ex)+b)}{2e\,(a^2+b^2)\,(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}-\frac{4bx(a^2-b^2)\,(a^2\tan(d+ex)+ab)^3}{a^2\,(a^2+b^2)^3\,(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}-\frac{(a^4-6a^2b^2+b^4)\,(a\tan(d+ex)+b)^3\log(a\sin(d+ex)+b\cos(d+ex))}{e\,(a^2+b^2)^3\,(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}-\frac{b(3a^2-b^2)\,(a^2\tan(d+ex)+ab)^3}{e\,(a^2+b^2)^2\,(a^4\tan(d+ex)+a^3b)\,(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3
/2),x]
```

```
[Out] -1/2*((a^2 - b^2)*(b + a*Tan[d + e*x]))/((a^2 + b^2)*e*(b^2 + 2*a*b*Tan[d +
 e*x] + a^2*Tan[d + e*x]^2)^(3/2)) - ((a^4 - 6*a^2*b^2 + b^4)*Log[b*Cos[d +
 e*x] + a*Sin[d + e*x]]*(b + a*Tan[d + e*x])^3)/((a^2 + b^2)^3*e*(b^2 + 2*a
*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)) - (4*b*(a^2 - b^2)*x*(a*b + a^
2*Tan[d + e*x])^3)/(a^2*(a^2 + b^2)^3*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d
 + e*x]^2)^(3/2)) - (b*(3*a^2 - b^2)*(a*b + a^2*Tan[d + e*x])^3)/((a^2 + b^
2)^2*e*(a^3*b + a^4*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e
*x]^2)^(3/2))
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3791

```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n), Int[(A + B*Ta
n[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\tan(d+ex)\right)^3\int\frac{a+b\tan(d+ex)}{\left(2ab+2a^2\tan(d+ex)\right)^3}\,dx}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(a^2-b^2\right)\left(b+a\tan(d+ex)\right)}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)}$$

$$= -\frac{\left(a^2-b^2\right)\left(b+a\tan(d+ex)\right)}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)}$$

$$= -\frac{\left(a^2-b^2\right)\left(b+a\tan(d+ex)\right)}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)}$$

$$= -\frac{\left(a^2-b^2\right)\left(b+a\tan(d+ex)\right)}{2\left(a^2+b^2\right)e\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.25, size = 268, normalized size = 0.85

$$\frac{(b+a\tan(d+ex))^3\left(b\left(\frac{i\log(i-\tan(d+ex))}{(a-ib)^2}-\frac{i\log(i+\tan(d+ex))}{(a+ib)^2}+\frac{2a\left(2b\log(b+a\tan(d+ex))-\frac{a^2+b^2}{b+a\tan(d+ex)}\right)}{(a^2+b^2)^2}\right)+(a-b)(a+b)\left(\frac{\log(i-\tan(d+ex))}{(a-ib)^3}+\frac{\log(i+\tan(d+ex))}{(a+ib)^3}+\frac{a\left(-2\left(a^2-3b^2\right)\log(b+a\tan(d+ex))-\frac{(a^2+b^2)(a^2+5b^2+4ab\tan(d+ex))}{(b+a\tan(d+ex))^2}\right)}{(a^2+b^2)^3}\right)\right)}{2ae\left((b+a\tan(d+ex))^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((b + a*Tan[d + e*x])^3*(b*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log[
I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2 +
 b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^2) + (a - b)*(a + b)*(Log[I - Tan[
d + e*x]]/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3
*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e
*x]))/(b + a*Tan[d + e*x])^2))/(a^2 + b^2)^3)))/(2*a*e*((b + a*Tan[d + e*x]
)^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(306) = 612$.
time = 0.18, size = 620, normalized size = 1.96

method	result
derivativedivides	$\frac{(-a^6-3a^2b^4-7a^4b^2+3b^6+\ln(1+\tan^2(ex+d))b^6-16\arctan(\tan(ex+d))a^4b^2\tan(ex+d)+16\arctan(\tan(ex+d))a^2b^4\tan(}{}$
default	$\frac{(-a^6-3a^2b^4-7a^4b^2+3b^6+\ln(1+\tan^2(ex+d))b^6-16\arctan(\tan(ex+d))a^4b^2\tan(ex+d)+16\arctan(\tan(ex+d))a^2b^4\tan(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(ln(1+tan(e*x+d)^2)*a^6*tan(e*x+d)^2-a^6-3*a^2*b^4-7*a^4*b^2+3*b^6+8*
arctan(tan(e*x+d))*a^3*b^3*tan(e*x+d)^2+12*ln(b+a*tan(e*x+d))*a^4*b^2*tan(e
*x+d)^2-2*ln(b+a*tan(e*x+d))*a^2*b^4*tan(e*x+d)^2+2*ln(1+tan(e*x+d)^2)*a^5*
b*tan(e*x+d)-12*ln(1+tan(e*x+d)^2)*a^3*b^3*tan(e*x+d)+2*ln(1+tan(e*x+d)^2)*
a*b^5*tan(e*x+d)-16*arctan(tan(e*x+d))*a^4*b^2*tan(e*x+d)+16*arctan(tan(e*x
+d))*a^2*b^4*tan(e*x+d)-4*ln(b+a*tan(e*x+d))*a^5*b*tan(e*x+d)+24*ln(b+a*tan
(e*x+d))*a^3*b^3*tan(e*x+d)-4*ln(b+a*tan(e*x+d))*a*b^5*tan(e*x+d)-6*a^5*b*t
an(e*x+d)-4*a^3*b^3*tan(e*x+d)+2*a*b^5*tan(e*x+d)-2*ln(b+a*tan(e*x+d))*a^6*
tan(e*x+d)^2+ln(1+tan(e*x+d)^2)*a^4*b^2-6*ln(1+tan(e*x+d)^2)*a^2*b^4-8*arct
an(tan(e*x+d))*a^3*b^3+8*arctan(tan(e*x+d))*a*b^5-2*ln(b+a*tan(e*x+d))*a^4*
b^2+12*ln(b+a*tan(e*x+d))*a^2*b^4-6*ln(1+tan(e*x+d)^2)*a^4*b^2*tan(e*x+d)^2
+ln(1+tan(e*x+d)^2)*a^2*b^4*tan(e*x+d)^2-8*arctan(tan(e*x+d))*a^5*b*tan(e*x
+d)^2+ln(1+tan(e*x+d)^2)*b^6-2*ln(b+a*tan(e*x+d))*b^6)*(b+a*tan(e*x+d))/(a^
2+b^2)^3/((b+a*tan(e*x+d))^2)^(3/2)
```

Maxima [A]
time = 0.50, size = 509, normalized size = 1.61

$$-\frac{1}{2}\left(\left(\frac{2\,(3\,a^2b-b^3)(xe+d)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(a^3-3\,ab^2)\log\left(a\tan\left(xe+d\right)+b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(a^3-3\,ab^2)\log\left(\tan\left(xe+d\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{4\,a^2b\tan\left(xe+d\right)+a^3+5\,ab^2}{a^4b^2+2\,a^2b^4+b^6+(a^6+2\,a^4b^2+a^2b^4)\tan\left(xe+d\right)^2+2\,(a^5b+2\,a^3b^3+ab^5)\tan\left(xe+d\right)}\right)a+\left(\frac{2\,(a^3-3\,ab^2)(xe+d)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(3\,a^2b-b^3)\log\left(a\tan\left(xe+d\right)+b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^2b-b^3)\log\left(\tan\left(xe+d\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^2b-3\,b^3+2\,(a^3-ab^2)\tan\left(xe+d\right)}{a^4b^2+2\,a^2b^4+b^6+(a^6+2\,a^4b^2+a^2b^4)\tan\left(xe+d\right)^2+2\,(a^5b+2\,a^3b^3+ab^5)\tan\left(xe+d\right)}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] -1/2*((2*(3*a^2*b - b^3)*(x*e + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*
(a^3 - 3*a*b^2)*log(a*tan(x*e + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 - (a^3 - 3*a*b^2)*log(tan(x*e + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (4*a^2*b*tan(x*e + d) + a^3 + 5*a*b^2)/(a^4*b^2 + 2*a^2*b^4 + b^6 + (
a^6 + 2*a^4*b^2 + a^2*b^4)*tan(x*e + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*t
an(x*e + d)))*a + (2*(a^3 - 3*a*b^2)*(x*e + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) - 2*(3*a^2*b - b^3)*log(a*tan(x*e + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6) + (3*a^2*b - b^3)*log(tan(x*e + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6) + (a^2*b - 3*b^3 + 2*(a^3 - a*b^2)*tan(x*e + d))/(a^4*b^2 +
 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(x*e + d)^2 + 2*(a^5*b +
2*a^3*b^3 + a*b^5)*tan(x*e + d)))*b)*e^(-1)
```

Fricas [**A**]
time = 2.18, size = 370, normalized size = 1.17

$$-\frac{a^6+8\,a^4b^2-5\,a^2b^4+8\,(a^3b^3-ab^5)xe+(a^6-8\,a^4b^2+3\,a^2b^4+8\,(a^5b-a^3b^3)xe)\tan(xe+d)^2+(a^4b^2-6\,a^2b^4+b^6+(a^6-6\,a^4b^2+a^2b^4)\tan(xe+d)^2+2\,(a^5b-6\,a^3b^3+ab^5)\tan(xe+d))\log\left(\frac{a^2\tan(xe+d)^2+2\,ab\tan(xe+d)+b^2}{\tan(xe+d)^2+1}\right)+4\,(2\,a^5b-3\,a^3b^3+ab^5+4\,(a^4b^2-a^2b^4)xe)\tan(xe+d)}{2\,((a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)e\tan(xe+d)^2+2\,(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7)e\tan(xe+d)+(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(a^6 + 8*a^4*b^2 - 5*a^2*b^4 + 8*(a^3*b^3 - a*b^5)*x*e + (a^6 - 8*a^4*
b^2 + 3*a^2*b^4 + 8*(a^5*b - a^3*b^3)*x*e)*tan(x*e + d)^2 + (a^4*b^2 - 6*a^
2*b^4 + b^6 + (a^6 - 6*a^4*b^2 + a^2*b^4)*tan(x*e + d)^2 + 2*(a^5*b - 6*a^3
*b^3 + a*b^5)*tan(x*e + d))*log((a^2*tan(x*e + d)^2 + 2*a*b*tan(x*e + d) +
b^2)/(tan(x*e + d)^2 + 1)) + 4*(2*a^5*b - 3*a^3*b^3 + a*b^5 + 4*(a^4*b^2 -
a^2*b^4)*x*e)*tan(x*e + d))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*e*tan(
x*e + d)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*e*tan(x*e + d) + (a^
6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\tan(d+ex)}{\left((a\tan(d+ex)+b)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*tan(d + e*x))/((a*tan(d + e*x) + b)**2)**(3/2), x)
```

Giac [**A**]
time = 0.73, size = 481, normalized size = 1.52

$$-\frac{\frac{8\,(a^3b-ab^3)(ex+d)}{a^6\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^4b^2\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^2b^4\mathrm{sgn}(a\tan(ex+d)+b)+b^6\mathrm{sgn}(a\tan(ex+d)+b)}-\frac{(a^4-6\,a^2b^2+b^4)\log\left(\tan(ex+d)^2+1\right)}{a^6\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^4b^2\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^2b^4\mathrm{sgn}(a\tan(ex+d)+b)+b^6\mathrm{sgn}(a\tan(ex+d)+b)}+\frac{2\,(a^5-6\,a^3b^2+ab^4)\log(|a\tan(ex+d)+b|)}{a^7\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^5b^2\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^3b^4\mathrm{sgn}(a\tan(ex+d)+b)+ab^6\mathrm{sgn}(a\tan(ex+d)+b)}-\frac{3\,a^6\tan(ex+d)^2-18\,a^4b^2\tan(ex+d)^2+3\,a^2b^4\tan(ex+d)^2-40\,a^3b^3\tan(ex+d)+8\,ab^5\tan(ex+d)-a^6-4\,a^4b^2-21\,a^2b^4+6\,b^6}{(a^6\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^4b^2\mathrm{sgn}(a\tan(ex+d)+b)+3\,a^2b^4\mathrm{sgn}(a\tan(ex+d)+b)+b^6\mathrm{sgn}(a\tan(ex+d)+b))(a\tan(ex+d)+b)^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] -1/2*(8*(a^3*b - a*b^3)*(e*x + d)/(a^6*sgn(a*tan(e*x + d) + b) + 3*a^4*b^2*
sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn(a*tan(e*x + d) + b) + b^6*sgn(a*tan
(e*x + d) + b)) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(e*x + d)^2 + 1)/(a^6*sgn(
a*tan(e*x + d) + b) + 3*a^4*b^2*sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn(a*t
an(e*x + d) + b) + b^6*sgn(a*tan(e*x + d) + b)) + 2*(a^5 - 6*a^3*b^2 + a*b^
4)*log(abs(a*tan(e*x + d) + b))/(a^7*sgn(a*tan(e*x + d) + b) + 3*a^5*b^2*sg
n(a*tan(e*x + d) + b) + 3*a^3*b^4*sgn(a*tan(e*x + d) + b) + a*b^6*sgn(a*tan
(e*x + d) + b)) - (3*a^6*tan(e*x + d)^2 - 18*a^4*b^2*tan(e*x + d)^2 + 3*a^2
*b^4*tan(e*x + d)^2 - 40*a^3*b^3*tan(e*x + d) + 8*a*b^5*tan(e*x + d) - a^6
- 4*a^4*b^2 - 21*a^2*b^4 + 6*b^6)/((a^6*sgn(a*tan(e*x + d) + b) + 3*a^4*b^2
*sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn(a*tan(e*x + d) + b) + b^6*sgn(a*ta
n(e*x + d) + b))*(a*tan(e*x + d) + b)^2))/e
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\tan(d+e\,x)}{\left(a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2),x)
```

```
[Out] int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(3
/2), x)
```

3.518 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d-$

Optimal. Leaf size=184

$$ab^4x+\frac{b(19a^4+56a^2b^2+8b^4)\tanh^{-1}(\sin(d+ex))}{8e}+\frac{a(4a^4+50a^2b^2+19b^4)\tan(d+ex)}{6e}+\frac{a^2b(41a^2+26b^2)}{}$$

```
[Out] a*b^4*x+1/8*b*(19*a^4+56*a^2*b^2+8*b^4)*arctanh(sin(e*x+d))/e+1/6*a*(4*a^4+
50*a^2*b^2+19*b^4)*tan(e*x+d)/e+1/24*a^2*b*(41*a^2+26*b^2)*sec(e*x+d)*tan(e
*x+d)/e+1/12*(4*a^2+7*b^2)*(a*b+a^2*sec(e*x+d))^2*tan(e*x+d)/a/e+1/4*b*(a*b
+a^2*sec(e*x+d))^3*tan(e*x+d)/a^2/e
```

Rubi [A]
time = 0.28, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {4257, 4003, 4141, 4133, 3855, 3852, 8}

$$\frac{a^2b(41a^2+26b^2)\tan(d+ex)\sec(d+ex)}{24e}+\frac{(4a^2+7b^2)\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^2}{12ae}+\frac{b\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^3}{4a^2e}+\frac{a(4a^4+50a^2b^2+19b^4)\tan(d+ex)}{6e}+\frac{b(19a^4+56a^2b^2+8b^4)\tanh^{-1}(\sin(d+ex))}{8e}+ab^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] a*b^4*x + (b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/(8*e) + (
a*(4*a^4 + 50*a^2*b^2 + 19*b^4)*Tan[d + e*x])/(6*e) + (a^2*b*(41*a^2 + 26*b
^2)*Sec[d + e*x]*Tan[d + e*x])/(24*e) + ((4*a^2 + 7*b^2)*(a*b + a^2*Sec[d +
 e*x])^2*Tan[d + e*x])/(12*a*e) + (b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x
])/(4*a^2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4257

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx = \frac{\int (2ab+2a^2\sec(d+ex))^4\,(a+b\sec(\cdots}{16a^4}$$

$$= \frac{b(ab+a^2\sec(d+ex))^3\tan(d+ex)}{4a^2e} + \cdots$$

$$= \frac{(4a^2+7b^2)\,(ab+a^2\sec(d+ex))^2\tan(\cdots}{12ae}$$

$$= \frac{a^2b(41a^2+26b^2)\sec(d+ex)\tan(d+\cdots}{24e}$$

$$= ab^4x + \frac{a^2b(41a^2+26b^2)\sec(d+ex)\,\mathrm{ta}\cdots}{24e}$$

$$= ab^4x + \frac{b(19a^4+56a^2b^2+8b^4)\tanh^{-1}\cdots}{8e}$$

$$= ab^4x + \frac{b(19a^4+56a^2b^2+8b^4)\tanh^{-1}\cdots}{8e}$$

Mathematica [A]

time = 0.55, size = 130, normalized size = 0.71

$$\frac{24ab^4ex+3b(19a^4+56a^2b^2+8b^4)\tanh^{-1}(\sin(d+ex))+3a(8(a^4+10a^2b^2+4b^4)+ab(19a^2+24b^2)\sec(d+ex)+2a^3b\sec^3(d+ex))\tan(d+ex)+8a^3(a^2+4b^2)\tan^3(d+ex)}{24e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] (24*a*b^4*e*x + 3*b*(19*a^4 + 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]] + 3
*a*(8*(a^4 + 10*a^2*b^2 + 4*b^4) + a*b*(19*a^2 + 24*b^2)*Sec[d + e*x] + 2*a
^3*b*Sec[d + e*x]^3)*Tan[d + e*x] + 8*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3)/(24
*e)
```

Maple [A]

time = 0.16, size = 256, normalized size = 1.39

method	result
derivativedivides	$\frac{a\,b^4(ex+d)+4b^3a^2\ln(\sec(ex+d)+\tan(ex+d))+6a^3b^2\tan(ex+d)+4b\,a^4\left(\frac{\sec(ex+d)\tan(ex+d)}{2}+\frac{\ln(\sec(ex+d)+\tan(ex+d))}{2}\right)\cdots}{\cdots}$
default	$\frac{a\,b^4(ex+d)+4b^3a^2\ln(\sec(ex+d)+\tan(ex+d))+6a^3b^2\tan(ex+d)+4b\,a^4\left(\frac{\sec(ex+d)\tan(ex+d)}{2}+\frac{\ln(\sec(ex+d)+\tan(ex+d))}{2}\right)\cdots}{\cdots}$

norman	$\frac{a\,b^4x+a\,b^4x\left(\tan^8\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-4a\,b^4x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+6a\,b^4x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-4a\,b^4x\left(\tan^6\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{a\left(8a^4-21a^3b+80\right.}{}}{}$
risch	$a\,b^4x+\frac{ia\left(-57a^3b\,\mathrm{e}^{7i(ex+d)}-72a\,b^3\mathrm{e}^{7i(ex+d)}+144a^2b^2\mathrm{e}^{6i(ex+d)}+96b^4\mathrm{e}^{6i(ex+d)}-81a^3b\,\mathrm{e}^{5i(ex+d)}-72a\,b^3\mathrm{e}^{5i(ex+d)}+4\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(a*b^4*(e*x+d)+4*b^3*a^2*ln(sec(e*x+d)+tan(e*x+d))+6*a^3*b^2*tan(e*x+d)
+4*b*a^4*(1/2*sec(e*x+d)*tan(e*x+d)+1/2*ln(sec(e*x+d)+tan(e*x+d)))-a^5*(-2/
3-1/3*sec(e*x+d)^2)*tan(e*x+d)+b^5*ln(sec(e*x+d)+tan(e*x+d))+4*a*b^4*tan(e*
x+d)+6*b^3*a^2*(1/2*sec(e*x+d)*tan(e*x+d)+1/2*ln(sec(e*x+d)+tan(e*x+d)))-4*
a^3*b^2*(-2/3-1/3*sec(e*x+d)^2)*tan(e*x+d)+b*a^4*(-(-1/4*sec(e*x+d)^3-3/8*s
ec(e*x+d))*tan(e*x+d)+3/8*ln(sec(e*x+d)+tan(e*x+d))))
```

<u>Maxima</u> [**A**]
time = 0.28, size = 323, normalized size = 1.76

$$\frac{1}{48}\left(16\left(\tan\left(xe+d\right)^{3}+3\,\tan\left(xe+d\right)\right)a^{5}+64\left(\tan\left(xe+d\right)^{3}+3\,\tan\left(xe+d\right)\right)a^{3}b^{2}+48\left(xe+d\right)ab^{4}-3\,a^{4}b\left(\frac{2\left(3\,\sin\left(xe+d\right)^{3}-5\,\sin\left(xe+d\right)\right)}{\sin\left(xe+d\right)^{4}-2\,\sin\left(xe+d\right)^{2}+1}-3\,\log\left(\sin\left(xe+d\right)+1\right)+3\,\log\left(\sin\left(xe+d\right)-1\right)\right)-48\,a^{4}b\left(\frac{2\,\sin\left(xe+d\right)}{\sin\left(xe+d\right)^{2}-1}-\log\left(\sin\left(xe+d\right)+1\right)+\log\left(\sin\left(xe+d\right)-1\right)\right)-72\,a^{2}b^{3}\left(\frac{2\,\sin\left(xe+d\right)}{\sin\left(xe+d\right)^{2}-1}-\log\left(\sin\left(xe+d\right)+1\right)+\log\left(\sin\left(xe+d\right)-1\right)\right)+192\,a^{2}b^{3}\log\left(\sec\left(xe+d\right)+\tan\left(xe+d\right)\right)+48\,b^{5}\log\left(\sec\left(xe+d\right)+\tan\left(xe+d\right)\right)+288\,a^{3}b^{2}\tan\left(xe+d\right)+192\,ab^{4}\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] 1/48*(16*(tan(x*e + d)^3 + 3*tan(x*e + d))*a^5 + 64*(tan(x*e + d)^3 + 3*tan
(x*e + d))*a^3*b^2 + 48*(x*e + d)*a*b^4 - 3*a^4*b*(2*(3*sin(x*e + d)^3 - 5*
sin(x*e + d))/(sin(x*e + d)^4 - 2*sin(x*e + d)^2 + 1) - 3*log(sin(x*e + d)
+ 1) + 3*log(sin(x*e + d) - 1)) - 48*a^4*b*(2*sin(x*e + d)/(sin(x*e + d)^2
- 1) - log(sin(x*e + d) + 1) + log(sin(x*e + d) - 1)) - 72*a^2*b^3*(2*sin(x
*e + d)/(sin(x*e + d)^2 - 1) - log(sin(x*e + d) + 1) + log(sin(x*e + d) - 1
)) + 192*a^2*b^3*log(sec(x*e + d) + tan(x*e + d)) + 48*b^5*log(sec(x*e + d)
 + tan(x*e + d)) + 288*a^3*b^2*tan(x*e + d) + 192*a*b^4*tan(x*e + d))*e^(-1
)
```

<u>Fricas</u> [**A**]
time = 3.03, size = 208, normalized size = 1.13

$$\frac{\left(48\,ab^{4}x\cos\left(xe+d\right)^{4}e+3\left(19\,a^{4}b+56\,a^{2}b^{3}+8\,b^{5}\right)\cos\left(xe+d\right)^{4}\log\left(\sin\left(xe+d\right)+1\right)-3\left(19\,a^{4}b+56\,a^{2}b^{3}+8\,b^{5}\right)\cos\left(xe+d\right)^{4}\log\left(-\sin\left(xe+d\right)+1\right)+2\left(6\,a^{4}b+16\left(a^{5}+13\,a^{3}b^{2}+6\,ab^{4}\right)\cos\left(xe+d\right)^{3}+3\left(19\,a^{4}b+24\,a^{2}b^{3}\right)\cos\left(xe+d\right)^{2}+8\left(a^{5}+4\,a^{3}b^{2}\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}}{48\,\cos\left(xe+d\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] 1/48*(48*a*b^4*x*cos(x*e + d)^4*e + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(x
*e + d)^4*log(sin(x*e + d) + 1) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(x*e
 + d)^4*log(-sin(x*e + d) + 1) + 2*(6*a^4*b + 16*(a^5 + 13*a^3*b^2 + 6*a*b^
4)*cos(x*e + d)^3 + 3*(19*a^4*b + 24*a^2*b^3)*cos(x*e + d)^2 + 8*(a^5 + 4*a
^3*b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)/cos(x*e + d)^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(d+ex))\left(a\sec(d+ex)+b\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(174) = 348.
time = 0.53, size = 447, normalized size = 2.43

$$\frac{24\,(ex+d)ab^4 + 3\,(19\,a^4b + 56\,a^2b^3 + 8\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) + 1\right|\right) - 3\,(19\,a^4b + 56\,a^2b^3 + 8\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) - 1\right|\right) - \frac{2\left(24\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^7 - 63\,a^4b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^7 + 240\,a^3b^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^7 - 72\,a^2b^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^7 + 96\,ab^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^7 - 40\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^5 + 39\,a^4b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^5 - 592\,a^3b^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^5 + 72\,a^2b^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^5 - 288\,ab^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^5 + 40\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3 + 39\,a^4b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3 + 592\,a^3b^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3 + 72\,a^2b^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3 + 288\,ab^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3 - 24\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right) - 63\,a^4b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right) - 240\,a^3b^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right) - 72\,a^2b^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right) - 96\,ab^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right)}{\left(\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2 - 1\right)^4}}{24\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
```

```
[Out] 1/24*(24*(e*x + d)*a*b^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/
2*e*x + 1/2*d) + 1)) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*e*
x + 1/2*d) - 1)) - 2*(24*a^5*tan(1/2*e*x + 1/2*d)^7 - 63*a^4*b*tan(1/2*e*x
+ 1/2*d)^7 + 240*a^3*b^2*tan(1/2*e*x + 1/2*d)^7 - 72*a^2*b^3*tan(1/2*e*x +
1/2*d)^7 + 96*a*b^4*tan(1/2*e*x + 1/2*d)^7 - 40*a^5*tan(1/2*e*x + 1/2*d)^5
+ 39*a^4*b*tan(1/2*e*x + 1/2*d)^5 - 592*a^3*b^2*tan(1/2*e*x + 1/2*d)^5 + 72
*a^2*b^3*tan(1/2*e*x + 1/2*d)^5 - 288*a*b^4*tan(1/2*e*x + 1/2*d)^5 + 40*a^5
*tan(1/2*e*x + 1/2*d)^3 + 39*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 592*a^3*b^2*tan
(1/2*e*x + 1/2*d)^3 + 72*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + 288*a*b^4*tan(1/2
*e*x + 1/2*d)^3 - 24*a^5*tan(1/2*e*x + 1/2*d) - 63*a^4*b*tan(1/2*e*x + 1/2*
d) - 240*a^3*b^2*tan(1/2*e*x + 1/2*d) - 72*a^2*b^3*tan(1/2*e*x + 1/2*d) - 9
6*a*b^4*tan(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^4)/e
```

Mupad [B]
time = 3.30, size = 323, normalized size = 1.76

$$\frac{2\,a^5\,\sin\left(d+e\,x\right)}{3\,e\,\cos\left(d+e\,x\right)}+\frac{a^5\,\sin\left(d+e\,x\right)}{3\,e\,{\cos\left(d+e\,x\right)}^3}+\frac{2\,a\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{4\,a\,b^4\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{19\,a^4\,b\,\sin\left(d+e\,x\right)}{8\,e\,{\cos\left(d+e\,x\right)}^2}+\frac{a^4\,b\,\sin\left(d+e\,x\right)}{4\,e\,{\cos\left(d+e\,x\right)}^4}+\frac{26\,a^3\,b^2\,\sin\left(d+e\,x\right)}{3\,e\,\cos\left(d+e\,x\right)}+\frac{3\,a^2\,b^3\,\sin\left(d+e\,x\right)}{e\,{\cos\left(d+e\,x\right)}^2}+\frac{4\,a^3\,b^2\,\sin\left(d+e\,x\right)}{3\,e\,{\cos\left(d+e\,x\right)}^3}-\frac{b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,2{}\mathrm{i}}{e}-\frac{a^2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,14{}\mathrm{i}}{e}-\frac{a^4\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,19{}\mathrm{i}}{4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
2,x)
```

```
[Out] (2*a^5*sin(d + e*x))/(3*e*cos(d + e*x)) - (b^5*atan((sin(d/2 + (e*x)/2)*1i)
/cos(d/2 + (e*x)/2))*2i)/e + (a^5*sin(d + e*x))/(3*e*cos(d + e*x)^3) - (a^2
*b^3*atan((sin(d/2 + (e*x)/2)*1i)/cos(d/2 + (e*x)/2))*14i)/e + (2*a*b^4*ata
n(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e - (a^4*b*atan((sin(d/2 + (e*x)/
2)*1i)/cos(d/2 + (e*x)/2))*19i)/(4*e) + (4*a*b^4*sin(d + e*x))/(e*cos(d + e
*x)) + (19*a^4*b*sin(d + e*x))/(8*e*cos(d + e*x)^2) + (a^4*b*sin(d + e*x))/
(4*e*cos(d + e*x)^4) + (26*a^3*b^2*sin(d + e*x))/(3*e*cos(d + e*x)) + (3*a^
2*b^3*sin(d + e*x))/(e*cos(d + e*x)^2) + (4*a^3*b^2*sin(d + e*x))/(3*e*cos(
d + e*x)^3)
```

3.519 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d-\right.$

Optimal. Leaf size=76

$$ab^2x+\frac{b(5a^2+2b^2)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a(a^2+2b^2)\tan(d+ex)}{e}+\frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}$$

```
[Out] a*b^2*x+1/2*b*(5*a^2+2*b^2)*arctanh(sin(e*x+d))/e+a*(a^2+2*b^2)*tan(e*x+d)/
e+1/2*a^2*b*sec(e*x+d)*tan(e*x+d)/e
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {4133, 3855, 3852, 8}

$$\frac{a(a^2+2b^2)\tan(d+ex)}{e}+\frac{b(5a^2+2b^2)\tanh^{-1}(\sin(d+ex))}{2e}+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}+ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] a*b^2*x + (b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]])/(2*e) + (a*(a^2 + 2*b^2
)*Tan[d + e*x])/e + (a^2*b*Sec[d + e*x]*Tan[d + e*x])/(2*e)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)\,dx = \frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+\frac{1}{2}\int \left(2ab\right.$$
$$= ab^2x+\frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}+(a$$
$$= ab^2x+\frac{b(5a^2+2b^2)\tanh^{-1}(\sin(d+ex))}{2e}$$
$$= ab^2x+\frac{b(5a^2+2b^2)\tanh^{-1}(\sin(d+ex))}{2e}$$

Mathematica [A]
time = 0.19, size = 64, normalized size = 0.84

$$\frac{2ab^2ex+b(5a^2+2b^2)\tanh^{-1}(\sin(d+ex))+a(2a^2+4b^2+ab\sec(d+ex))\tan(d+ex)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*a*b^2*e*x + b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]] + a*(2*a^2 + 4*b^2 +
 a*b*Sec[d + e*x])*Tan[d + e*x])/(2*e)
```

Maple [A]
time = 0.13, size = 112, normalized size = 1.47

method	result
derivativedivides	$\frac{a\,b^2(ex+d)+2a^2b\ln(\sec(ex+d)+\tan(ex+d))+a^3\tan(ex+d)+b^3\ln(\sec(ex+d)+\tan(ex+d))+2a\,b^2\tan(ex+d)+a^2b\left(\frac{\sec(ex}{}\right.}{e}$
default	$\frac{a\,b^2(ex+d)+2a^2b\ln(\sec(ex+d)+\tan(ex+d))+a^3\tan(ex+d)+b^3\ln(\sec(ex+d)+\tan(ex+d))+2a\,b^2\tan(ex+d)+a^2b\left(\frac{\sec(ex}{}\right.}{e}$
risch	$a\,b^2x-\frac{ia\left(ab\,e^{3i(ex+d)}-2\,e^{2i(ex+d)}a^2-4\,e^{2i(ex+d)}b^2-ab\,e^{i(ex+d)}-2a^2-4b^2\right)}{e\left(1+e^{2i(ex+d)}\right)^2}-\frac{5b\ln\left(e^{i(ex+d)}-i\right)a^2}{2e}-\frac{b^3\ln(e^{i(ex+}}{e}$
norman	$\frac{a\,b^2x+a\,b^2x\left(\tan^4\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+\frac{a\left(2a^2+ab+4b^2\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e}-2a\,b^2x\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{a\left(2a^2-ab+4b^2\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e}}{\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)^2}-\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/e*(a*b^2*(e*x+d)+2*a^2*b*ln(sec(e*x+d)+tan(e*x+d))+a^3*tan(e*x+d)+b^3*ln(
sec(e*x+d)+tan(e*x+d))+2*a*b^2*tan(e*x+d)+a^2*b*(1/2*sec(e*x+d)*tan(e*x+d)+
1/2*ln(sec(e*x+d)+tan(e*x+d))))
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.79

$$\frac{1}{4}\left(4\,(xe+d)ab^2-a^2b\left(\frac{2\,\sin\left(xe+d\right)}{\sin\left(xe+d\right)^2-1}-\log\left(\sin\left(xe+d\right)+1\right)+\log\left(\sin\left(xe+d\right)-1\right)\right)+8\,a^2b\log\left(\sec\left(xe+d\right)+\tan\left(xe+d\right)\right)+4\,b^3\log\left(\sec\left(xe+d\right)+\tan\left(xe+d\right)\right)+4\,a^3\tan\left(xe+d\right)+8\,ab^2\tan\left(xe+d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*(x*e + d)*a*b^2 - a^2*b*(2*sin(x*e + d)/(sin(x*e + d)^2 - 1) - log(s
in(x*e + d) + 1) + log(sin(x*e + d) - 1)) + 8*a^2*b*log(sec(x*e + d) + tan(
x*e + d)) + 4*b^3*log(sec(x*e + d) + tan(x*e + d)) + 4*a^3*tan(x*e + d) + 8
*a*b^2*tan(x*e + d))*e^(-1)
```

Fricas [A]
time = 2.90, size = 133, normalized size = 1.75

$$\frac{\left(4\,ab^2x\cos\left(xe+d\right)^2e+\left(5\,a^2b+2\,b^3\right)\cos\left(xe+d\right)^2\log\left(\sin\left(xe+d\right)+1\right)-\left(5\,a^2b+2\,b^3\right)\cos\left(xe+d\right)^2\log\left(-\sin\left(xe+d\right)+1\right)+2\left(a^2b+2\left(a^3+2\,ab^2\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}}{4\,\cos\left(xe+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(4*a*b^2*x*cos(x*e + d)^2*e + (5*a^2*b + 2*b^3)*cos(x*e + d)^2*log(sin(
x*e + d) + 1) - (5*a^2*b + 2*b^3)*cos(x*e + d)^2*log(-sin(x*e + d) + 1) + 2
*(a^2*b + 2*(a^3 + 2*a*b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)/cos(x*e + d)
^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(d+ex\right)\right)\left(a\sec\left(d+ex\right)+b\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(72) = 144.
time = 0.46, size = 182, normalized size = 2.39

$$\frac{2\,(ex+d)ab^2+\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right|\right)-\left(5\,a^2b+2\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-1\right|\right)-\frac{2\left(2\,a^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-a^2b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+4\,ab^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-2\,a^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-a^2b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-4\,ab^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right)}{\left(\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2-1\right)^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] 1/2*(2*(e*x + d)*a*b^2 + (5*a^2*b + 2*b^3)*log(abs(tan(1/2*e*x + 1/2*d) + 1
)) - (5*a^2*b + 2*b^3)*log(abs(tan(1/2*e*x + 1/2*d) - 1)) - 2*(2*a^3*tan(1/
2*e*x + 1/2*d)^3 - a^2*b*tan(1/2*e*x + 1/2*d)^3 + 4*a*b^2*tan(1/2*e*x + 1/2
*d)^3 - 2*a^3*tan(1/2*e*x + 1/2*d) - a^2*b*tan(1/2*e*x + 1/2*d) - 4*a*b^2*t
an(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^2)/e
```

Mupad [B]
time = 2.91, size = 160, normalized size = 2.11

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{a^3\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{2\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{5\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{2\,a\,b^2\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{a^2\,b\,\sin\left(d+e\,x\right)}{2\,e\,{\cos\left(d+e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)),
x)
```

```
[Out] (2*b^3*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (a^3*sin(d + e*x))
/(e*cos(d + e*x)) + (2*a*b^2*atan(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e
 + (5*a^2*b*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (2*a*b^2*sin(
d + e*x))/(e*cos(d + e*x)) + (a^2*b*sin(d + e*x))/(2*e*cos(d + e*x)^2)
```

3.520 $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

Optimal. Leaf size=92

$$\frac{ax}{b^2}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}-\frac{a^2\tan(d+ex)}{be\,(ab+a^2\sec(d+ex))}$$

```
[Out] a*x/b^2-2*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(a-b)^(1/2)*(a
+b)^(1/2)/b^2/e-a^2*tan(e*x+d)/b/e/(a*b+a^2*sec(e*x+d))
```

Rubi [A]
time = 0.21, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4257, 4008, 4004, 3916, 2738, 211}

$$-\frac{a^2\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}+\frac{ax}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2]
)/Sqrt[a + b]])/(b^2*e) - (a^2*Tan[d + e*x])/(b*e*(a*b + a^2*Sec[d + e*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4257

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \left(4a^2\right)\int \frac{a+b\sec(d+ex)}{(2ab+2a^2\sec(d+ex))^2}\,dx$$

$$= -\frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} + \frac{\int \frac{4a^3(a^2-b^2)+4a^2b(a^2-b^2)\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{2ab\left(a^2-b^2\right)}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{(2a(a^2-b^2))\int \frac{\sec(d+ex)}{2ab+2a^2\sec(d+e}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{\left(a^2-b^2\right)\int \frac{1}{1+\frac{b\cos(d+ex)}{a}}\,dx}{ab^2}$$

$$= \frac{ax}{b^2} - \frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)} - \frac{(2(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{b}{a}+(}\right.}{a}$$

$$= \frac{ax}{b^2} - \frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e} - \frac{}{be\,(a}$$

Mathematica [A]

time = 0.27, size = 97, normalized size = 1.05

$$\frac{2\sqrt{-a^2+b^2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)+\frac{a(ad+aex+b(d+ex)\cos(d+ex)-b\sin(d+ex))}{a+b\cos(d+ex)}}{b^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2),x]
```

```
[Out] (2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]] +
 (a*(a*d + a*e*x + b*(d + e*x)*Cos[d + e*x] - b*Sin[d + e*x]))/(a + b*Cos[d
 + e*x]))/(b^2*e)
```

Maple [A]
time = 0.30, size = 120, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{2a\arctan\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b^2}-\frac{2\left(\frac{ab\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}}{e}$
default	$\frac{\frac{2a\arctan\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{b^2}-\frac{2\left(\frac{ab\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-b\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}}{e}$
risch	$\frac{ax}{b^2}-\frac{2ia\left(a\,e^{i(ex+d)}+b\right)}{b^2e\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(ex+d)}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{e\,b^2}-\frac{\sqrt{-a^2+b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/e*(2*a/b^2*arctan(tan(1/2*d+1/2*e*x))-2/b^2*(a*b*tan(1/2*d+1/2*e*x)/(a*ta
n(1/2*d+1/2*e*x)^2-b*tan(1/2*d+1/2*e*x)^2+a+b)+(a^2-b^2)/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tan(1/2*d+1/2*e*x)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.02, size = 303, normalized size = 3.29

$$\left[\frac{2\,abx\cos\left(xe+d\right)e+2\,a^2xe-2\,ab\sin\left(xe+d\right)+\sqrt{-a^2+b^2}\left(b\cos\left(xe+d\right)+a\right)\log\left(\frac{2\,ab\cos(xe+d)+(2\,a^2-b^2)\cos(xe+d)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(xe+d)+b)\sin(xe+d)-a^2+2\,b^2}{b^2\cos(xe+d)^2+2\,ab\cos(xe+d)+a^2}\right)}{2\,\left(b^3\cos\left(xe+d\right)e+ab^2e\right)}, \frac{abx\cos\left(xe+d\right)e+a^2xe-ab\sin\left(xe+d\right)-\sqrt{a^2-b^2}\left(b\cos\left(xe+d\right)+a\right)\arctan\left(-\frac{a\cos(xe+d)+b}{\sqrt{a^2-b^2}\,\sin(xe+d)}\right)}{b^3\cos\left(xe+d\right)e+ab^2e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(2*a*b*x*cos(x*e + d)*e + 2*a^2*x*e - 2*a*b*sin(x*e + d) + sqrt(-a^2 +
 b^2)*(b*cos(x*e + d) + a)*log((2*a*b*cos(x*e + d) + (2*a^2 - b^2)*cos(x*e
+ d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x*e + d) + b)*sin(x*e + d) - a^2 + 2*b^2
)/(b^2*cos(x*e + d)^2 + 2*a*b*cos(x*e + d) + a^2)))/(b^3*cos(x*e + d)*e + a
*b^2*e), (a*b*x*cos(x*e + d)*e + a^2*x*e - a*b*sin(x*e + d) - sqrt(a^2 - b^
2)*(b*cos(x*e + d) + a)*arctan(-(a*cos(x*e + d) + b)/(sqrt(a^2 - b^2)*sin(x
*e + d))))/(b^3*cos(x*e + d)*e + a*b^2*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec\left(d + ex\right)}{\left(a\sec\left(d + ex\right) + b\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**2, x)
```

Giac [A]
time = 0.47, size = 139, normalized size = 1.51

$$\frac{\frac{(ex+d)a}{b^2}-\frac{2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+a+b\right)b}-\frac{2\left(\pi\left\lfloor\frac{ex+d}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algor
ithm="giac")
```

```
[Out] ((e*x + d)*a/b^2 - 2*a*tan(1/2*e*x + 1/2*d)/((a*tan(1/2*e*x + 1/2*d)^2 - b*
tan(1/2*e*x + 1/2*d)^2 + a + b)*b) - 2*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sg
n(2*a - 2*b) + arctan((a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d))/sqr
t(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)/e
```

Mupad [B]

time = 3.02, size = 444, normalized size = 4.83

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^3\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sqrt{b^2-a^2}}{64\,a^4-128\,a^3\,b+128\,a\,b^3-64\,b^4}-\frac{192\,a^2\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sqrt{b^2-a^2}}{128\,a\,b^2-128\,a^3-64\,b^3+\frac{64\,a^4}{b}}+\frac{192\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}-\frac{64\,b\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}\right)\,\sqrt{b^2-a^2}}{b^2\,e}-\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}+\frac{64\,a^3\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b^2-64\,a^2\,b-64\,a^3+\frac{64\,a^4}{b}}-\frac{64\,a^4\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a^4-64\,a^3\,b-64\,a^2\,b^2+64\,a\,b^3}-\frac{64\,a\,b\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}\right)}{b^2\,e}-\frac{2\,a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{b\,e\,\left(\left(a-b\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x)),
x)
```

```
[Out] (2*atanh((64*a^3*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^3 - 128*a^3
*b + 64*a^4 - 64*b^4) - (192*a^2*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128
*a*b^2 - 128*a^3 - 64*b^3 + (64*a^4)/b) + (192*a*tan(d/2 + (e*x)/2)*(b^2 -
a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b^2) - (64*b*tan(d/2
 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b
^2))*(b^2 - a^2)^(1/2))/(b^2*e) - (2*a*atan((64*a^2*tan(d/2 + (e*x)/2))/(64
*a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2) + (64*a^3*tan(d/2 + (e*x)/2))/(6
4*a*b^2 - 64*a^2*b - 64*a^3 + (64*a^4)/b) - (64*a^4*tan(d/2 + (e*x)/2))/(64
*a*b^3 - 64*a^3*b + 64*a^4 - 64*a^2*b^2) - (64*a*b*tan(d/2 + (e*x)/2))/(64*
a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2)))/(b^2*e) - (2*a*tan(d/2 + (e*x)/
2))/(b*e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b)))
```

3.521 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

Optimal. Leaf size=230

$$\frac{ax}{b^4}-\frac{(a^2-2b^2)\left(2a^4-a^2b^2+b^4\right)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}e}-\frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2}-\frac{a(6a^4\ \cdots}{6b^3\,(a\ \cdots}$$

```
[Out] a*x/b^4-(a^2-2*b^2)*(2*a^4-a^2*b^2+b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*
d)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/e-1/6*a*(3*a^2-5*b^2)*tan(e*x+d
)/b^2/(a^2-b^2)/e/(b+a*sec(e*x+d))^2-1/6*a*(6*a^4-11*a^2*b^2+11*b^4)*tan(e*
x+d)/b^3/(a^2-b^2)^2/e/(b+a*sec(e*x+d))-1/3*a^4*tan(e*x+d)/b/e/(a*b+a^2*sec
(e*x+d))^3
```

Rubi [A]
time = 0.57, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {4257, 4008, 4145, 4004, 3916, 2738, 211}

$$-\frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2e\left(a^2-b^2\right)(a\sec(d+ex)+b)^2}-\frac{(a^2-2b^2)\left(2a^4-a^2b^2+b^4\right)\operatorname{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^4e(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{6b^3e\left(a^2-b^2\right)^2(a\sec(d+ex)+b)}-\frac{a^4\tan(d+ex)}{3be\left(a^2\sec(d+ex)+ab\right)^3}+\frac{ax}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2,
x]
```

```
[Out] (a*x)/b^4 - ((a^2 - 2*b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan[(Sqrt[a - b]*Tan[
(d + e*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*e) - (a*(3*a^2
 - 5*b^2)*Tan[d + e*x])/(6*b^2*(a^2 - b^2)*e*(b + a*Sec[d + e*x])^2) - (a*(
6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])/(6*b^3*(a^2 - b^2)^2*e*(b + a*Se
c[d + e*x])) - (a^4*Tan[d + e*x])/(3*b*e*(a*b + a^2*Sec[d + e*x])^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4257

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[1/(4^n*c^n
), Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{
a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^2}\,dx = (16a^4)\int \frac{a+b\sec(d+ex)}{(2ab+2a^2\sec(d+ex))^4}\,dx$$

$$= -\frac{a^4\tan(d+ex)}{3be\,(ab+a^2\sec(d+ex))^3} + \frac{(2a)\int \frac{12a^3(a^2-b^2)+12a^2b(a^2-b^2 \ldots}{(2ab+2 \ldots}}{3b\,(a \ldots}$$

$$= -\frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\,(a^2-b^2)\,e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\,(ab+a^2\sec(d+ \ldots}$$

$$= -\frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\,(a^2-b^2)\,e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ex)}{3be\,(ab+a^2\sec(d+ \ldots}$$

$$= \frac{ax}{b^4} - \frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\,(a^2-b^2)\,e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ \ldots}{3be\,(ab+a^2\sec \ldots}$$

$$= \frac{ax}{b^4} - \frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\,(a^2-b^2)\,e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ \ldots}{3be\,(ab+a^2\sec \ldots}$$

$$= \frac{ax}{b^4} - \frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\,(a^2-b^2)\,e(b+a\sec(d+ex))^2} - \frac{a^4\tan(d+ \ldots}{3be\,(ab+a^2\sec \ldots}$$

$$= \frac{ax}{b^4} - \frac{(a^2-2b^2)\,(2a^4-a^2b^2+b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+e \ldots}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^4(a+b)^{5/2}e}$$

Mathematica [A]
time = 1.03, size = 276, normalized size = 1.20

$$\frac{(a+b\cos(d+ex))\sec^3(d+ex)(a+b\sec(d+ex))\left(6a(d+ex)(a+b\cos(d+ex))^3+\frac{6(-2a^6+5a^4b^2-3a^2b^4+2b^6)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\cos(d+ex))^3}{(-a^2+b^2)^{5/2}}-2a^3b\sin(d+ex)+\frac{a^2b(7a^2-9b^2)(a+b\cos(d+ex))\sin(d+ex)}{(a-b)(a+b)}-\frac{ab(11a^4-23a^2b^2+18b^4)(a+b\cos(d+ex))^2\sin(d+ex)}{(a-b)^2(a+b)^2}\right)}{6b^4e(b+a\cos(d+ex))(b+a\sec(d+ex))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^2,x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^3*(a + b*Sec[d + e*x])*(6*a*(d + e*x)*(a
 + b*Cos[d + e*x])^3 + (6*(-2*a^6 + 5*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*ArcTanh[
((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^3)/(-a^2
 + b^2)^(5/2) - 2*a^3*b*Sin[d + e*x] + (a^2*b*(7*a^2 - 9*b^2)*(a + b*Cos[d
+ e*x])*Sin[d + e*x])/((a - b)*(a + b)) - (a*b*(11*a^4 - 23*a^2*b^2 + 18*b^
4)*(a + b*Cos[d + e*x])^2*Sin[d + e*x])/((a - b)^2*(a + b)^2)))/(6*b^4*e*(b
 + a*Cos[d + e*x])*(b + a*Sec[d + e*x])^4)
```

Maple [A]
time = 0.57, size = 307, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/e*(2*a/b^4*arctan(tan(1/2*d+1/2*e*x))-2/b^4*((1/2*(2*a^4-a^3*b-4*a^2*b^2+
3*a*b^3+6*b^4)*a*b/(a^2+2*a*b+b^2)*tan(1/2*d+1/2*e*x)^5+2/3*(3*a^4-8*a^2*b^
2+9*b^4)*a*b/(a+b)/(a-b)*tan(1/2*d+1/2*e*x)^3+1/2*(2*a^4+a^3*b-4*a^2*b^2-3*
a*b^3+6*b^4)*a*b/(a^2-2*a*b+b^2)*tan(1/2*d+1/2*e*x))/(a*tan(1/2*d+1/2*e*x)^
2-b*tan(1/2*d+1/2*e*x)^2+a+b)^3+1/2*(2*a^6-5*a^4*b^2+3*a^2*b^4-2*b^6)/(a^4-
2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d+1/2*e*x)/((a+b)*(
a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(218) = 436$.
time = 3.73, size = 1383, normalized size = 6.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="fricas")
```

```
[Out] [1/12*(12*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*x*cos(x*e + d)^3*e + 36
*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*x*cos(x*e + d)^2*e + 36*(a^9*b
 - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*x*cos(x*e + d)*e + 12*(a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*x*e + 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6
+ (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(x*e + d)^3 + 3*(2*a^7*b^2
 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(x*e + d)^2 + 3*(2*a^8*b - 5*a^6*b^3
 + 3*a^4*b^5 - 2*a^2*b^7)*cos(x*e + d))*sqrt(-a^2 + b^2)*log((2*a*b*cos(x*e
```

```
 + d) + (2*a^2 - b^2)*cos(x*e + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x*e + d) +
 b)*sin(x*e + d) - a^2 + 2*b^2)/(b^2*cos(x*e + d)^2 + 2*a*b*cos(x*e + d) +
a^2)) - 2*(6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a^7*b^3 - 3
4*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(x*e + d)^2 + 3*(5*a^8*b^2 - 15*a^6*b
^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(x*e + d))*sin(x*e + d))/((a^6*b^7 - 3*a^4*
b^9 + 3*a^2*b^11 - b^13)*cos(x*e + d)^3*e + 3*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*
b^10 - a*b^12)*cos(x*e + d)^2*e + 3*(a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*
b^11)*cos(x*e + d)*e + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^3*b^10)*e), 1/6
*(6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*x*cos(x*e + d)^3*e + 18*(a^8*
b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*x*cos(x*e + d)^2*e + 18*(a^9*b - 3*a
^7*b^3 + 3*a^5*b^5 - a^3*b^7)*x*cos(x*e + d)*e + 6*(a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*x*e - 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*b^6 + (2*a^
6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(x*e + d)^3 + 3*(2*a^7*b^2 - 5*a^
5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(x*e + d)^2 + 3*(2*a^8*b - 5*a^6*b^3 + 3*a^
4*b^5 - 2*a^2*b^7)*cos(x*e + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(x*e + d) +
b)/(sqrt(a^2 - b^2)*sin(x*e + d))) - (6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 1
1*a^3*b^7 + (11*a^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(x*e + d)^
2 + 3*(5*a^8*b^2 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(x*e + d))*sin(x
*e + d))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*cos(x*e + d)^3*e + 3*(a
^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*cos(x*e + d)^2*e + 3*(a^8*b^5 - 3
*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*cos(x*e + d)*e + (a^9*b^4 - 3*a^7*b^6 + 3*
a^5*b^8 - a^3*b^10)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec{\left(d + ex \right)}}{\left(a \sec{\left(d + ex \right)} + b\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x)
[Out] Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(215) = 430.
time = 0.54, size = 469, normalized size = 2.04

$$\frac{\frac{3\left(2a^6-5a^4b^2+3a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3\left(ex+d\right)a}{b^4}-\frac{6a^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-15a^6b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+30a^4b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-12a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-27a^2b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+18ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+12a^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-44a^5b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+68a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-36ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+6a^7\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+15a^6b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-30a^4b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-12a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+27a^2b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+18ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{\left(a^4b^3-2a^2b^5+b^7\right)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+a+b\right)^3}}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, alg
orithm="giac")
[Out] 1/3*(3*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(e*x + d)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1
```

```
/2*d))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*
(e*x + d)*a/b^4 - (6*a^7*tan(1/2*e*x + 1/2*d)^5 - 15*a^6*b*tan(1/2*e*x + 1/
2*d)^5 + 30*a^4*b^3*tan(1/2*e*x + 1/2*d)^5 - 12*a^3*b^4*tan(1/2*e*x + 1/2*d
)^5 - 27*a^2*b^5*tan(1/2*e*x + 1/2*d)^5 + 18*a*b^6*tan(1/2*e*x + 1/2*d)^5 +
 12*a^7*tan(1/2*e*x + 1/2*d)^3 - 44*a^5*b^2*tan(1/2*e*x + 1/2*d)^3 + 68*a^3
*b^4*tan(1/2*e*x + 1/2*d)^3 - 36*a*b^6*tan(1/2*e*x + 1/2*d)^3 + 6*a^7*tan(1
/2*e*x + 1/2*d) + 15*a^6*b*tan(1/2*e*x + 1/2*d) - 30*a^4*b^3*tan(1/2*e*x +
1/2*d) - 12*a^3*b^4*tan(1/2*e*x + 1/2*d) + 27*a^2*b^5*tan(1/2*e*x + 1/2*d)
+ 18*a*b^6*tan(1/2*e*x + 1/2*d))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*e*
x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + a + b)^3))/e
```

Mupad [B]
time = 11.47, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
2,x)
```

```
[Out] - ((tan(d/2 + (e*x)/2)*(6*a*b^4 + a^4*b + 2*a^5 - 3*a^2*b^3 - 4*a^3*b^2))/(
b^5 - 2*a*b^4 + a^2*b^3) + (tan(d/2 + (e*x)/2)^5*(6*a*b^4 - a^4*b + 2*a^5 +
 3*a^2*b^3 - 4*a^3*b^2))/(b^3*(a + b)^2) + (4*tan(d/2 + (e*x)/2)^3*(9*a*b^4
 + 3*a^5 - 8*a^3*b^2))/(3*(a*b^3 - b^4)*(a + b)))/(e*(3*a*b^2 - tan(d/2 + (
e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(d/2 + (e*x)/2)^2*(3*a*b
^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(d/2 + (e*x)/2)^6*
(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (2*a*atan(((a*((a*((8*(4*b^18 - 14*a^2*
b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 +
 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3
*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(d/2 + (e*x)/
2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^
6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^
12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7
*b^6)))*1i)/b^4 - (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2
*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*
a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^
10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))/b^4 - (a*((a*((8*(4*b^18
- 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a
^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^1
4 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(d/2
 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^1
3 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(
b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*
b^7 - a^7*b^6)))*1i)/b^4 + (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^1
2 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*
```

b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))/b^4)/((a*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 - (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))*1i)/b^4 - (16*(4*a*b^11 - 2*a^11*b + 4*a^12 - 4*a^2*b^10 - 8*a^3*b^9 + 14*a^4*b^8 + 15*a^5*b^7 - 26*a^6*b^6 - 12*a^7*b^5 + 30*a^8*b^4 + 7*a^9*b^3 - 18*a^10*b^2))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*tan(d/2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*1i)/b^4 + (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))*1i)/b^4)))/(b^4*e) - (atan((((a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 - 4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 48*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (4*tan(d/2 + (e*x)/2)*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^4 + b^4 - a^2*b^2))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(2*a^4 + b^4 - a^2*b^2)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + ((a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^...

3.522 $\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d-\right.$

Optimal. Leaf size=359

$$\frac{(a^4+9a^2b^2+2b^4)\tanh^{-1}(\sin(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}{2e(b+a\sec(d+ex))^3}+\frac{a^4b^3x(b^2+2ab\sec(d+}{(ab+a^2\text{se}}$$

```
[Out] 1/2*(a^4+9*a^2*b^2+2*b^4)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec
(e*x+d)^2)^(3/2)/e/(b+a*sec(e*x+d))^3+a^4*b^3*x*(b^2+2*a*b*sec(e*x+d)+a^2*s
ec(e*x+d)^2)^(3/2)/(a*b+a^2*sec(e*x+d))^3+1/3*a^4*b*(11*a^2+8*b^2)*(b^2+2*a
*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^3+1
/6*a^5*(3*a^2+5*b^2)*sec(e*x+d)*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/
2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^3+1/3*b*(a^2*b+a^3*sec(e*x+d))^2*(b^2+
2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^
3
```

Rubi [A]
time = 0.19, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {4259, 4003, 4133, 3855, 3852, 8}

$$\frac{a^5(3a^2+5b^2)\tan(d+ex)\sec(d+ex)\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{6e\left(a^2\sec(d+ex)+ab\right)^3}+\frac{a^4b(11a^2+8b^2)\tan(d+ex)\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{3e\left(a^2\sec(d+ex)+ab\right)^3}+\frac{(a^4+9a^2b^2+2b^4)\tanh^{-1}(\sin(d+ex))\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{2e(a\sec(d+ex)+b)^3}+\frac{a^4b^3x(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}{\left(a^2\sec(d+ex)+ab\right)^3}+\frac{b\tan(d+ex)\left(a^3\sec(d+ex)+a^2b\right)^2\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}{3e\left(a^2\sec(d+ex)+ab\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```

```
[Out] ((a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh[Sin[d + e*x]]*(b^2 + 2*a*b*Sec[d + e*x]
+ a^2*Sec[d + e*x]^2)^(3/2))/(2*e*(b + a*Sec[d + e*x])^3) + (a^4*b^3*x*(b^2
 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))/(a*b + a^2*Sec[d + e*x])
^3 + (a^4*b*(11*a^2 + 8*b^2)*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
)^(3/2)*Tan[d + e*x])/(3*e*(a*b + a^2*Sec[d + e*x])^3) + (a^5*(3*a^2 + 5*b^
2)*Sec[d + e*x]*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d
 + e*x])/(6*e*(a*b + a^2*Sec[d + e*x])^3) + (b*(a^2*b + a^3*Sec[d + e*x])^2
*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*Tan[d + e*x])/(3*e*(
a*b + a^2*Sec[d + e*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4259

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx = \frac{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+e}{(2a}$$

$$= \frac{b(a^2b+a^3\sec(d+ex))^2\left(b^2+2ab\operatorname{se}\right.}{3e\,(ab+}$$

$$= \frac{a^5(3a^2+5b^2)\sec(d+ex)\left(b^2+2ab\,\mathrm{s}\right.}{6e\,(ab+}$$

$$= \frac{a^4b^3x(b^2+2ab\sec(d+ex)+a^2\sec^2}{(ab+a^2\sec(d+ex))^3}$$

$$= \frac{(a^4+9a^2b^2+2b^4)\tanh^{-1}(\sin(d+e}{2e(b+}$$

$$= \frac{(a^4+9a^2b^2+2b^4)\tanh^{-1}(\sin(d+e}{2e(b+}$$

Mathematica [A]
time = 0.57, size = 128, normalized size = 0.36

$$\frac{\cos(d+ex)\sqrt{(b+a\sec(d+ex))^2}\,(6ab^3ex+3(a^4+9a^2b^2+2b^4)\tanh^{-1}(\sin(d+ex))+3a(8a^2b+6b^3+a(a^2+3b^2)\sec(d+ex))\tan(d+ex)+2a^3b\tan^3(d+ex))}{6e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*(6*a*b^3*e*x + 3*(a^4 + 9*a^2*b^
2 + 2*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*a^2*b + 6*b^3 + a*(a^2 + 3*b^2)*S
ec[d + e*x])*Tan[d + e*x] + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(a + b*Cos[d + e*
x]))
```

Maple [A]
time = 1.29, size = 390, normalized size = 1.09

method	result
default	$\frac{\left(3\ln\left(-\frac{\cos(ex+d)-1-\sin(ex+d)}{\sin(ex+d)}\right)(\cos^3(ex+d))a^4+27\ln\left(-\frac{\cos(ex+d)-1-\sin(ex+d)}{\sin(ex+d)}\right)(\cos^3(ex+d))a^2b^2+6\ln\left(-\frac{\cos(ex+d)-1-\sin(ex+d)}{\sin(ex+d)}\right.\right.}{}$
risch	$\frac{(1+e^{2i(ex+d)})\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}\,a\,b^3x}{b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b}-\frac{i\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}\,a(3a^3e^{5i(ex+d)}+9a\,b^2e^{5i(ex+d)}-18a}{3(b\,e^{2i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/6/e*(3*ln(-(cos(e*x+d)-1-sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^4+27*ln(-
(cos(e*x+d)-1-sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*a^2*b^2+6*ln(-(cos(e*x+d
)-1-sin(e*x+d))/sin(e*x+d))*cos(e*x+d)^3*b^4-3*ln(-(cos(e*x+d)-1+sin(e*x+d)
)/sin(e*x+d))*cos(e*x+d)^3*a^4-27*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))
*cos(e*x+d)^3*a^2*b^2-6*ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*cos(e*x+d
)^3*b^4+6*cos(e*x+d)^3*a*b^3*(e*x+d)+22*cos(e*x+d)^2*sin(e*x+d)*a^3*b+18*co
s(e*x+d)^2*sin(e*x+d)*a*b^3+3*cos(e*x+d)*sin(e*x+d)*a^4+9*cos(e*x+d)*sin(e*
x+d)*a^2*b^2+2*a^3*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+d)^2)^(3/2)/(b
*cos(e*x+d)+a)^3
```

Maxima [A]
time = 0.50, size = 469, normalized size = 1.31

$$\frac{1}{6}\left(3\left(4\,b^3\arctan\left(\frac{\sin\left(xe+d\right)}{\cos\left(xe+d\right)+1}\right)+\left(a^3+6\,ab^2\right)\log\left(\frac{\sin\left(xe+d\right)}{\cos\left(xe+d\right)+1}+1\right)-\left(a^3+6\,ab^2\right)\log\left(\frac{\sin\left(xe+d\right)}{\cos\left(xe+d\right)+1}-1\right)-\frac{2\left(\frac{\left(a^3+6\,a^2b\right)\sin(xe+d)}{\cos(xe+d)+1}+\frac{\left(a^3-6\,a^2b\right)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}\right)}{\frac{2\,\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{\sin(xe+d)^4}{(\cos(xe+d)+1)^4}-1}\right)a+\left(3\left(3\,a^2b+2\,b^3\right)\log\left(\frac{\sin\left(xe+d\right)}{\cos\left(xe+d\right)+1}+1\right)-3\left(3\,a^2b+2\,b^3\right)\log\left(\frac{\sin\left(xe+d\right)}{\cos\left(xe+d\right)+1}-1\right)-\frac{2\left(\frac{3\left(2\,a^3+3\,a^2b+6\,ab^2\right)\sin(xe+d)}{\cos(xe+d)+1}-\frac{4\left(a^3+9\,ab^2\right)\sin(xe+d)^3}{(\cos(xe+d)+1)^3}+\frac{3\left(2\,a^3-3\,a^2b+6\,ab^2\right)\sin(xe+d)^5}{(\cos(xe+d)+1)^5}\right)}{\frac{3\,\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-\frac{3\,\sin(xe+d)^4}{(\cos(xe+d)+1)^4}+\frac{\sin(xe+d)^6}{(\cos(xe+d)+1)^6}-1}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] 1/6*(3*(4*b^3*arctan(sin(x*e + d)/(cos(x*e + d) + 1)) + (a^3 + 6*a*b^2)*log
(sin(x*e + d)/(cos(x*e + d) + 1) + 1) - (a^3 + 6*a*b^2)*log(sin(x*e + d)/(c
os(x*e + d) + 1) - 1) - 2*((a^3 + 6*a^2*b)*sin(x*e + d)/(cos(x*e + d) + 1)
+ (a^3 - 6*a^2*b)*sin(x*e + d)^3/(cos(x*e + d) + 1)^3)/(2*sin(x*e + d)^2/(c
os(x*e + d) + 1)^2 - sin(x*e + d)^4/(cos(x*e + d) + 1)^4 - 1))*a + (3*(3*a^
2*b + 2*b^3)*log(sin(x*e + d)/(cos(x*e + d) + 1) + 1) - 3*(3*a^2*b + 2*b^3)
*log(sin(x*e + d)/(cos(x*e + d) + 1) - 1) - 2*(3*(2*a^3 + 3*a^2*b + 6*a*b^2
)*sin(x*e + d)/(cos(x*e + d) + 1) - 4*(a^3 + 9*a*b^2)*sin(x*e + d)^3/(cos(x
*e + d) + 1)^3 + 3*(2*a^3 - 3*a^2*b + 6*a*b^2)*sin(x*e + d)^5/(cos(x*e + d)
 + 1)^5)/(3*sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - 3*sin(x*e + d)^4/(cos(x*e
 + d) + 1)^4 + sin(x*e + d)^6/(cos(x*e + d) + 1)^6 - 1))*b)*e^(-1)
```

Fricas [A]
time = 3.83, size = 171, normalized size = 0.48

$$\frac{\left(12\,ab^3x\cos\left(xe+d\right)^3e+3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos\left(xe+d\right)^3\log\left(\sin\left(xe+d\right)+1\right)-3\left(a^4+9\,a^2b^2+2\,b^4\right)\cos\left(xe+d\right)^3\log\left(-\sin\left(xe+d\right)+1\right)+2\left(2\,a^3b+2\left(11\,a^3b+9\,ab^3\right)\cos\left(xe+d\right)^2+3\left(a^4+3\,a^2b^2\right)\cos\left(xe+d\right)\right)\sin\left(xe+d\right)\right)e^{(-1)}}{12\,\cos\left(xe+d\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(12*a*b^3*x*cos(x*e + d)^3*e + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(x*e + d
)^3*log(sin(x*e + d) + 1) - 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(x*e + d)^3*log(
-sin(x*e + d) + 1) + 2*(2*a^3*b + 2*(11*a^3*b + 9*a*b^3)*cos(x*e + d)^2 + 3
*(a^4 + 3*a^2*b^2)*cos(x*e + d))*sin(x*e + d))*e^(-1)/cos(x*e + d)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec{(d + ex)}) \left((a \sec{(d + ex)} + b)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*((a*sec(d + e*x) + b)**2)**(3/2), x)
```

Giac [A]
time = 0.57, size = 605, normalized size = 1.69

$$\frac{6\,(ex+d)ab^3\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)+3\left(a^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)+9a^2b^2\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)+2b^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\right)\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)-3\left(a^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)+9a^2b^2\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)+2b^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\right)\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-1\right|\right)+\frac{2\left(3a^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-24a^3b\,\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+9a^2b^2\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-18ab^3\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+40a^3b\,\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+36ab^3\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-3a^4\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-24a^3b\,\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-9a^2b^2\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-18ab^3\mathrm{sgn}\left(b\cos\left(ex+d\right)^2+a\cos\left(ex+d\right)\right)\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)\right)}{\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-1\right)^3}}{6\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] 1/6*(6*(e*x + d)*a*b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 3*(a^4*sgn(
b*cos(e*x + d)^2 + a*cos(e*x + d)) + 9*a^2*b^2*sgn(b*cos(e*x + d)^2 + a*cos
(e*x + d)) + 2*b^4*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)))*log(abs(tan(1/2*
e*x + 1/2*d) + 1)) - 3*(a^4*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 9*a^2*
b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 2*b^4*sgn(b*cos(e*x + d)^2 + a
*cos(e*x + d)))*log(abs(tan(1/2*e*x + 1/2*d) - 1)) + 2*(3*a^4*sgn(b*cos(e*x
 + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 - 24*a^3*b*sgn(b*cos(e*x +
 d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 + 9*a^2*b^2*sgn(b*cos(e*x +
d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 - 18*a*b^3*sgn(b*cos(e*x + d)
^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 + 40*a^3*b*sgn(b*cos(e*x + d)^2
 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^3 + 36*a*b^3*sgn(b*cos(e*x + d)^2 +
 a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^3 - 3*a^4*sgn(b*cos(e*x + d)^2 + a*co
s(e*x + d))*tan(1/2*e*x + 1/2*d) - 24*a^3*b*sgn(b*cos(e*x + d)^2 + a*cos(e*
x + d))*tan(1/2*e*x + 1/2*d) - 9*a^2*b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x +
 d))*tan(1/2*e*x + 1/2*d) - 18*a*b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))
*tan(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^3)/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(d+e\,x\right)}\right)\,{\left(b^2+\frac{a^2}{{\cos\left(d+e\,x\right)}^2}+\frac{2\,a\,b}{\cos\left(d+e\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2),x)
```

```
[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2), x)
```

3.523 $\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d$

Optimal. Leaf size=173

$$\frac{(a^2+b^2)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}{e(b+a\sec(d+ex))}+\frac{a^2bx\sqrt{b^2+2ab\sec(d+ex)+a^2\mathrm{se}}}{ab+a^2\sec(d+ex)}$$

```
[Out] (a^2+b^2)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
/e/(b+a*sec(e*x+d))+a^2*b*x*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)/(
a*b+a^2*sec(e*x+d))+a^2*b*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)*tan
(e*x+d)/e/(a*b+a^2*sec(e*x+d))
```

Rubi [A]
time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {4259, 3999, 3852, 8, 3855}

$$\frac{a^2bx\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{a^2\sec(d+ex)+ab}+\frac{a^2b\tan(d+ex)\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e\left(a^2\sec(d+ex)+ab\right)}+\frac{(a^2+b^2)\tanh^{-1}(\sin(d+ex))\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}{e(a\sec(d+ex)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```

```
[Out] ((a^2 + b^2)*ArcTanh[Sin[d + e*x]]*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[
d + e*x]^2])/(e*(b + a*Sec[d + e*x])) + (a^2*b*x*Sqrt[b^2 + 2*a*b*Sec[d + e
*x] + a^2*Sec[d + e*x]^2])/(a*b + a^2*Sec[d + e*x]) + (a^2*b*Sqrt[b^2 + 2*a
*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]*Tan[d + e*x])/(e*(a*b + a^2*Sec[d + e
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4259

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex}}{2ab+}$$

$$= \frac{a^2bx\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d}}{ab+a^2\sec(d+ex)}$$

$$= \frac{(a^2+b^2)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab}}{e(b+a\sec(d+}$$

$$= \frac{(a^2+b^2)\tanh^{-1}(\sin(d+ex))\sqrt{b^2+2ab}}{e(b+a\sec(d+}$$

Mathematica [A]
time = 0.19, size = 67, normalized size = 0.39

$$\frac{\cos(d+ex)\sqrt{(b+a\sec(d+ex))^2}\,\left((a^2+b^2)\tanh^{-1}(\sin(d+ex))+ab(ex+\tan(d+ex))\right)}{e(a+b\cos(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] (Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*((a^2 + b^2)*ArcTanh[Sin[d + e*x
]] + a*b*(e*x + Tan[d + e*x])))/(e*(a + b*Cos[d + e*x]))
```

Maple [A]
time = 0.42, size = 210, normalized size = 1.21

method	result
default	$\frac{\left(\ln\left(-\frac{\cos(ex+d)-1-\sin(ex+d)}{\sin(ex+d)}\right)\cos(ex+d)a^2+\ln\left(-\frac{\cos(ex+d)-1-\sin(ex+d)}{\sin(ex+d)}\right)\cos(ex+d)b^2-\ln\left(-\frac{\cos(ex+d)-1+\sin(ex+d)}{\sin(ex+d)}\right)\cos(ex+d)a}{e(b\cos(ex+d)+a)}$
risch	$\frac{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\left(1+e^{2i(ex+d)}\right)abx}{b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b}+\frac{2i\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\,ab}{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)e}+\frac{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+\right.}{\left(1+e^{2i(ex+d)}\right)^2}}}{\left(b\,e^{2i(ex\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/e*(ln(-(cos(e*x+d)-1-sin(e*x+d))/sin(e*x+d))*cos(e*x+d)*a^2+ln(-(cos(e*x+
d)-1-sin(e*x+d))/sin(e*x+d))*cos(e*x+d)*b^2-ln(-(cos(e*x+d)-1+sin(e*x+d))/s
in(e*x+d))*cos(e*x+d)*a^2-ln(-(cos(e*x+d)-1+sin(e*x+d))/sin(e*x+d))*cos(e*x
+d)*b^2+cos(e*x+d)*a*b*(e*x+d)+a*b*sin(e*x+d))*((b*cos(e*x+d)+a)^2/cos(e*x+
d)^2)^(1/2)/(b*cos(e*x+d)+a)
```

Maxima [A]
time = 0.50, size = 177, normalized size = 1.02

$$\left(\left(2\,b\arctan\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}\right)+a\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)-a\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}-1\right)\right)a+\left(b\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}+1\right)-b\log\left(\frac{\sin(xe+d)}{\cos(xe+d)+1}-1\right)-\frac{2\,a\sin(xe+d)}{\left(\frac{\sin(xe+d)^2}{(\cos(xe+d)+1)^2}-1\right)(\cos(xe+d)+1)}\right)b\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] ((2*b*arctan(sin(x*e + d)/(cos(x*e + d) + 1)) + a*log(sin(x*e + d)/(cos(x*e
 + d) + 1) + 1) - a*log(sin(x*e + d)/(cos(x*e + d) + 1) - 1))*a + (b*log(si
n(x*e + d)/(cos(x*e + d) + 1) + 1) - b*log(sin(x*e + d)/(cos(x*e + d) + 1)
- 1) - 2*a*sin(x*e + d)/((sin(x*e + d)^2/(cos(x*e + d) + 1)^2 - 1)*(cos(x*e
 + d) + 1)))*b)*e^(-1)
```

Fricas [A]
time = 3.76, size = 92, normalized size = 0.53

$$\frac{(2\,abx\cos(xe+d)\,e+(a^2+b^2)\cos(xe+d)\log(\sin(xe+d)+1)-(a^2+b^2)\cos(xe+d)\log(-\sin(xe+d)+1)+2\,ab\sin(xe+d))e^{(-1)}}{2\,\cos(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x*cos(x*e + d)*e + (a^2 + b^2)*cos(x*e + d)*log(sin(x*e + d) + 1
) - (a^2 + b^2)*cos(x*e + d)*log(-sin(x*e + d) + 1) + 2*a*b*sin(x*e + d))*e
^(-1)/cos(x*e + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(d + ex))\sqrt{(a\sec(d + ex) + b)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))*sqrt((a*sec(d + e*x) + b)**2), x)
```

Giac [A]
time = 0.47, size = 208, normalized size = 1.20

$$\frac{(ex+d)ab\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right) - \frac{2\,ab\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right)\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right)}{\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right)^2 - 1} + \left(a^2\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right) + b^2\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right)\right)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) + 1\right|\right) - \left(a^2\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right) + b^2\mathrm{sgn}\left(b\cos(ex+d)^2 + a\cos(ex+d)\right)\right)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) - 1\right|\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] ((e*x + d)*a*b*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - 2*a*b*sgn(b*cos(e*x
 + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)/(tan(1/2*e*x + 1/2*d)^2 - 1)
 + (a^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos(e*x + d)^2 +
 a*cos(e*x + d)))*log(abs(tan(1/2*e*x + 1/2*d) + 1)) - (a^2*sgn(b*cos(e*x +
 d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)))*log(a
bs(tan(1/2*e*x + 1/2*d) - 1)))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{\cos(d + e\,x)}\right)\sqrt{b^2 + \frac{a^2}{\cos(d + e\,x)^2} + \frac{2\,a\,b}{\cos(d + e\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2),x)
```

```
[Out] int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2), x)
```

3.524 $\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))}{be\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}+\frac{x(ab+a^2\sec(d+ex))}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d}}$$

```
[Out] x*(a*b+a^2*sec(e*x+d))/b/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)-2*ar
ctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(b+a*sec(e*x+d))*(a-b)^(1/
2)*(a+b)^(1/2)/b/e/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {4259, 4004, 3916, 2738, 211}

$$\frac{x(a^2\sec(d+ex)+ab)}{b\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,(a\sec(d+ex)+b)\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2
],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a +
b]]*(b + a*Sec[d + e*x]))/(b*e*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2]) + (x*(a*b + a^2*Sec[d + e*x]))/(b*Sqrt[b^2 + 2*a*b*Sec[d + e*x] +
a^2*Sec[d + e*x]^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4259

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx = \frac{(2ab+2a^2\sec(d+ex))\int \frac{a+b\sec(d+ex)}{2ab+2a^2\sec(d+ex)}\,dx}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

$$= \frac{x(ab+a^2\sec(d+ex))}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{((2a^3-2ab}{2ab\sqrt{}}$$

$$= \frac{x(ab+a^2\sec(d+ex))}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{((2a^3-2ab}{4a^3b\sqrt{b}}$$

$$= \frac{x(ab+a^2\sec(d+ex))}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} - \frac{((2a^3-2ab}{}$$

$$= -\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\,\mathrm{se}}{be\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

Mathematica [A]
time = 0.26, size = 92, normalized size = 0.65

$$\frac{\left(a(d+ex)+2\sqrt{-a^2+b^2}\,\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)\right)(a+b\cos(d+ex))\sec(d+ex)}{be\sqrt{(b+a\sec(d+ex))^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
e*x]^2],x]
```

```
[Out] ((a*(d + e*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt
[-a^2 + b^2]])*(a + b*Cos[d + e*x])*Sec[d + e*x])/(b*e*Sqrt[(b + a*Sec[d +
e*x])^2])
```

Maple [A]
time = 0.48, size = 157, normalized size = 1.11

method	result
default	$\frac{(b\cos(ex+d)+a)\left(a(ex+d)\sqrt{(a+b)(a-b)}+2\arctan\left(\frac{(a-b)(\cos(ex+d)-1)}{\sin(ex+d)\sqrt{(a+b)(a-b)}}\right)a^2-2\arctan\left(\frac{(a-b)(\cos(\cdots}{\sin(ex+d)\sqrt{(a+\cdots}}\right.\right.}{e\cos(ex+d)\sqrt{\frac{(b\cos(ex+d)+a)^2}{\cos(ex+d)^2}}\,b\sqrt{(a+b)(a-b)}}$
risch	$\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)ax}{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\left(1+e^{2i(ex+d)}\right)b}+\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)\sqrt{-a^2+b^2}\,\ln\left(e^{i(ex+d)}+\frac{i\sqrt{-a^2+b^2}}{b}\cdots\right.}{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\left(1+e^{2i(ex+d)}\right)eb}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/e*(b*cos(e*x+d)+a)*(a*(e*x+d)*((a+b)*(a-b))^(1/2)+2*arctan((a-b)*(cos(e*x
+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^2-2*arctan((a-b)*(cos(e*x+d)-1)/si
n(e*x+d)/((a+b)*(a-b))^(1/2))*b^2)/cos(e*x+d)/((b*cos(e*x+d)+a)^2/cos(e*x+d
)^2)^(1/2)/b/((a+b)*(a-b))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.66, size = 192, normalized size = 1.35

$$\left[\frac{\left(2\,axe+\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(xe+d)+(2\,a^2-b^2)\cos(xe+d)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(xe+d)+b)\sin(xe+d)-a^2+2\,b^2}{b^2\cos(xe+d)^2+2\,ab\cos(xe+d)+a^2}\right)\right)e^{(-1)}}{2\,b},\ \frac{\left(axe-\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\cos(xe+d)+b}{\sqrt{a^2-b^2}\,\sin(xe+d)}\right)\right)e^{(-1)}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*e + sqrt(-a^2 + b^2)*log((2*a*b*cos(x*e + d) + (2*a^2 - b^2)*co
s(x*e + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x*e + d) + b)*sin(x*e + d) - a^2 +
 2*b^2)/(b^2*cos(x*e + d)^2 + 2*a*b*cos(x*e + d) + a^2)))*e^(-1)/b, (a*x*e
- sqrt(a^2 - b^2)*arctan(-(a*cos(x*e + d) + b)/(sqrt(a^2 - b^2)*sin(x*e + d
))))*e^(-1)/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\sec(d+ex)}{\sqrt{(a\sec(d+ex)+b)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/sqrt((a*sec(d + e*x) + b)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(129) = 258.
time = 0.83, size = 383, normalized size = 2.70

$$-\frac{\frac{\left(a|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|-b|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|-2a^2\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)+ab\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)+b^2\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right)(ex+d)}{a|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)-b^2}+\frac{2\left(\sqrt{a^2-b^2}\,|a-b||b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|+\sqrt{a^2-b^2}\,(2a+b)|a-b|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right)\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\sqrt{a^2-b^2}\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a+b}\right)\right)}{(a^2-ab)|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)+(a-b)b^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] -1/2*((a*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))) - b*abs(b)*abs(
sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))) - 2*a^2*sgn(b*cos(e*x + d)^2 + a*co
s(e*x + d)) + a*b*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos(e*
x + d)^2 + a*cos(e*x + d)))*(e*x + d)/(a*abs(b)*abs(sgn(b*cos(e*x + d)^2 +
a*cos(e*x + d)))*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^2) + 2*(sqrt(a^
2 - b^2)*abs(a - b)*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))) + sq
rt(a^2 - b^2)*(2*a + b)*abs(a - b)*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)))*
(pi*floor(1/2*(e*x + d)/pi + 1/2) + arctan(sqrt(a^2 - b^2)*tan(1/2*e*x + 1/
2*d)/(a + b)))/((a^2 - a*b)*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d
)))*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + (a - b)*b^2))/e
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{\cos(d+e\,x)}}{\sqrt{b^2+\frac{a^2}{\cos\left(d+e\,x\right)^2}+\frac{2\,a\,b}{\cos\left(d+e\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2),x)
```

```
[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(1/2), x)
```

3.525 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{(2a^4-3a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))^3}{(a-b)^{3/2}b^3(a+b)^{3/2}e\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}+\frac{x(ab+a^2\sec(d+ex)}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec\right.}$$

```
[Out] -(2*a^4-3*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))
*(b+a*sec(e*x+d))^3/(a-b)^(3/2)/b^3/(a+b)^(3/2)/e/(b^2+2*a*b*sec(e*x+d)+a^2
*sec(e*x+d)^2)^(3/2)+x*(a*b+a^2*sec(e*x+d))^3/a^2/b^3/(b^2+2*a*b*sec(e*x+d)
+a^2*sec(e*x+d)^2)^(3/2)-1/2*(a*b+a^2*sec(e*x+d))*tan(e*x+d)/b/e/(b^2+2*a*b
*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)-1/2*(2*a^2-3*b^2)*(a*b+a^2*sec(e*x+d))^
3*tan(e*x+d)/b^2/(a^2-b^2)/e/(a^2*b+a^3*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a
^2*sec(e*x+d)^2)^(3/2)
```

Rubi [A]

time = 0.37, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4259, 4008, 4145, 4004, 3916, 2738, 211}

$$-\frac{\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)}{2be\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}+\frac{x\left(a^2\sec(d+ex)+ab\right)^3}{a^2b^3\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{(2a^4-3a^2b^2+2b^4)(a\sec(d+ex)+b)^3\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^3e(a-b)^{3/2}(a+b)^{3/2}\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}-\frac{(2a^2-3b^2)\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^3}{2b^2e\left(a^2-b^2\right)\left(a^3\sec(d+ex)+a^2b\right)\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3
/2),x]
```

```
[Out] -(((2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a
 + b]]*(b + a*Sec[d + e*x])^3)/((a - b)^(3/2)*b^3*(a + b)^(3/2)*e*(b^2 + 2*
a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2))) + (x*(a*b + a^2*Sec[d + e*x]
)^3)/(a^2*b^3*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)) - ((a*
b + a^2*Sec[d + e*x])*Tan[d + e*x])/(2*b*e*(b^2 + 2*a*b*Sec[d + e*x] + a^2*
Sec[d + e*x]^2)^(3/2)) - ((2*a^2 - 3*b^2)*(a*b + a^2*Sec[d + e*x])^3*Tan[d
+ e*x])/(2*b^2*(a^2 - b^2)*e*(a^2*b + a^3*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d
+ e*x] + a^2*Sec[d + e*x]^2)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4259

```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Dist[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n), Int[(A + B*Se
c[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e, A
, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx = \frac{\left(2ab+2a^2\sec(d+ex)\right)^3\int \frac{a+b\sec(d+ex)}{(2ab+2a^2\sec(d+ex))^3}\,dx}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} + \frac{(2ab \cdots}{\cdots}$$

$$= -\frac{\left(ab+a^2\sec(d+ex)\right)\tan(d+ex)}{2be\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\cdots}{2ab^2 \cdots}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\cdots}{2be\,(b^2 \cdots}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\cdots}{2be\,(b^2 \cdots}$$

$$= \frac{x\left(ab+a^2\sec(d+ex)\right)^3}{a^2b^3\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}} - \frac{\cdots}{2be\,(b^2 \cdots}$$

$$= -\frac{\left(2a^4-3a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a \cdots}{(a-b)^{3/2}b^3(a+b)^{3/2}e\left(b^2+2ab\sec(d+ex)+a^2\sec^2 \cdots\right.}$$

Mathematica [A]
time = 0.73, size = 216, normalized size = 0.65

$$\frac{(a+b\cos(d+ex))\sec^2(d+ex)(a+b\sec(d+ex))\left(2a(d+ex)(a+b\cos(d+ex))^2+\frac{2(2a^4-3a^2b^2+2b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\cos(d+ex))^2}{(-a^2+b^2)^{3/2}}+a^2b\sin(d+ex)+\frac{ab(3a^2-4b^2)(a+b\cos(d+ex))\sin(d+ex)}{(-a+b)(a+b)}\right)}{2b^3e(b+a\cos(d+ex))\left((b+a\sec(d+ex))^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]
^2)^(3/2),x]
```

```
[Out] ((a + b*Cos[d + e*x])*Sec[d + e*x]^2*(a + b*Sec[d + e*x])*(2*a*(d + e*x)*(a
 + b*Cos[d + e*x])^2 + (2*(2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan
[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^2)/(-a^2 + b^2)^(3/2)
 + a^2*b*Sin[d + e*x] + (a*b*(3*a^2 - 4*b^2)*(a + b*Cos[d + e*x])*Sin[d + e
*x])/((-a + b)*(a + b))))/(2*b^3*e*(b + a*Cos[d + e*x])*((b + a*Sec[d + e*x
])^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(309) = 618.
time = 0.50, size = 756, normalized size = 2.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(b*cos(e*x+d)+a)*(4*cos(e*x+d)^2*arctan((a-b)*(cos(e*x+d)-1)/sin(e*x+
d)/((a+b)*(a-b))^(1/2))*a^4*b^2-6*cos(e*x+d)^2*arctan((a-b)*(cos(e*x+d)-1)/
sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^2*b^4+4*cos(e*x+d)^2*arctan((a-b)*(cos(e*
x+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*b^6+2*((a+b)*(a-b))^(1/2)*cos(e*x+d
)^2*a^3*b^2*(e*x+d)-2*((a+b)*(a-b))^(1/2)*cos(e*x+d)^2*a*b^4*(e*x+d)+8*cos(
e*x+d)*arctan((a-b)*(cos(e*x+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^5*b-12
*cos(e*x+d)*arctan((a-b)*(cos(e*x+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^3
*b^3+8*cos(e*x+d)*arctan((a-b)*(cos(e*x+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2
))*a*b^5-3*((a+b)*(a-b))^(1/2)*sin(e*x+d)*cos(e*x+d)*a^3*b^2+4*((a+b)*(a-b)
)^(1/2)*sin(e*x+d)*cos(e*x+d)*a*b^4+4*((a+b)*(a-b))^(1/2)*cos(e*x+d)*a^4*b*
(e*x+d)-4*((a+b)*(a-b))^(1/2)*cos(e*x+d)*a^2*b^3*(e*x+d)+4*arctan((a-b)*(co
s(e*x+d)-1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^6-6*arctan((a-b)*(cos(e*x+d)-
1)/sin(e*x+d)/((a+b)*(a-b))^(1/2))*a^4*b^2+4*arctan((a-b)*(cos(e*x+d)-1)/si
n(e*x+d)/((a+b)*(a-b))^(1/2))*a^2*b^4-2*((a+b)*(a-b))^(1/2)*a^4*b*sin(e*x+d
)+3*((a+b)*(a-b))^(1/2)*a^2*b^3*sin(e*x+d)+2*(e*x+d)*((a+b)*(a-b))^(1/2)*a^
5-2*((a+b)*(a-b))^(1/2)*a^3*b^2*(e*x+d))/cos(e*x+d)^3/((b*cos(e*x+d)+a)^2/c
os(e*x+d)^2)^(3/2)/((a+b)*(a-b))^(1/2)/(a^2-b^2)/b^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.43, size = 834, normalized size = 2.53

$$\left[\frac{4\,(a^5b^2-2\,a^3b^4+ab^6)x\cos(xe+d)^2e+8\,(a^6b-2\,a^4b^3+a^2b^5)x\cos(xe+d)\,e+4\,(a^7-2\,a^5b^2+a^3b^4)xe+\left(2\,a^6-3\,a^4b^2+2\,a^2b^4+(2\,a^4b^2-3\,a^2b^4+2\,b^6)\cos(xe+d)^2+2\,(2\,a^5b-3\,a^3b^3+2\,ab^5)\cos(xe+d)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(xe+d)+(2\,a^2-b^2)\cos(xe+d)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(xe+d)+b)\sin(xe+d)-a^2+2\,b^2}{b^2\cos(xe+d)^2+2\,ab\cos(xe+d)+a^2}\right)-2\,\left(2\,a^6b-5\,a^4b^3+3\,a^2b^5+(3\,a^5b^2-7\,a^3b^4+4\,ab^6)\cos(xe+d)\right)\sin(xe+d)}{4\,\left((a^4b^5-2\,a^2b^7+b^9)\cos(xe+d)^2e+2\,(a^5b^4-2\,a^3b^6+ab^8)\cos(xe+d)\,e+(a^6b^3-2\,a^4b^5+a^2b^7)e\right)},\ \frac{2\,(a^5b^2-2\,a^3b^4+ab^6)x\cos(xe+d)^2e+4\,(a^6b-2\,a^4b^3+a^2b^5)x\cos(xe+d)\,e+2\,(a^7-2\,a^5b^2+a^3b^4)xe-\left(2\,a^6-3\,a^4b^2+2\,a^2b^4+(2\,a^4b^2-3\,a^2b^4+2\,b^6)\cos(xe+d)^2+2\,(2\,a^5b-3\,a^3b^3+2\,ab^5)\cos(xe+d)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(xe+d)+b}{\sqrt{a^2-b^2}\sin(xe+d)}\right)-\left(2\,a^6b-5\,a^4b^3+3\,a^2b^5+(3\,a^5b^2-7\,a^3b^4+4\,ab^6)\cos(xe+d)\right)\sin(xe+d)}{2\,\left((a^4b^5-2\,a^2b^7+b^9)\cos(xe+d)^2e+2\,(a^5b^4-2\,a^3b^6+ab^8)\cos(xe+d)\,e+(a^6b^3-2\,a^4b^5+a^2b^7)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x*e + d)^2*e + 8*(a^6*b - 2*a^4
*b^3 + a^2*b^5)*x*cos(x*e + d)*e + 4*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*e + (2*a
^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(x*e + d)^2
 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x*e + d))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(x*e + d) + (2*a^2 - b^2)*cos(x*e + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(x*e + d) + b)*sin(x*e + d) - a^2 + 2*b^2)/(b^2*cos(x*e + d)^2 + 2*a*b*cos(
x*e + d) + a^2)) - 2*(2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*
b^4 + 4*a*b^6)*cos(x*e + d))*sin(x*e + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*cos
(x*e + d)^2*e + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*cos(x*e + d)*e + (a^6*b^3 -
 2*a^4*b^5 + a^2*b^7)*e), 1/2*(2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x*e +
d)^2*e + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5)*x*cos(x*e + d)*e + 2*(a^7 - 2*a^5*
b^2 + a^3*b^4)*x*e - (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^
4 + 2*b^6)*cos(x*e + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(x*e + d))
*sqrt(a^2 - b^2)*arctan(-(a*cos(x*e + d) + b)/(sqrt(a^2 - b^2)*sin(x*e + d)
)) - (2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b^4 + 4*a*b^6)*c
os(x*e + d))*sin(x*e + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*cos(x*e + d)^2*e +
2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*cos(x*e + d)*e + (a^6*b^3 - 2*a^4*b^5 + a^2
*b^7)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \sec{\left(d + ex \right)}}{\left(\left(a \sec{\left(d + ex \right)} + b\right)^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/2
),x)
```

```
[Out] Integral((a + b*sec(d + e*x))/((a*sec(d + e*x) + b)**2)**(3/2), x)
```

Giac [A]
time = 0.54, size = 405, normalized size = 1.23

$$-\frac{\frac{\left(2\,a^{4}-3\,a^{2}b^{2}+2\,b^{4}\right)\left(\pi\left\lfloor\frac{ex+d}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{3}\mathrm{sgn}\left(b\cos\left(ex+d\right)^{2}+a\cos\left(ex+d\right)\right)-b^{5}\mathrm{sgn}\left(b\cos\left(ex+d\right)^{2}+a\cos\left(ex+d\right)\right)\right)\sqrt{a^{2}-b^{2}}}+\frac{2\,a^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-3\,a^{3}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}-3\,a^{2}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+4\,ab^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{3}+2\,a^{4}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+3\,a^{3}b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-3\,a^{2}b^{2}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-4\,ab^{3}\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\left(a^{2}b^{2}\mathrm{sgn}\left(b\cos\left(ex+d\right)^{2}+a\cos\left(ex+d\right)\right)-b^{4}\mathrm{sgn}\left(b\cos\left(ex+d\right)^{2}+a\cos\left(ex+d\right)\right)\right)\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}-b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^{2}+a+b\right)^{2}}-\frac{\left(ex+d\right)a}{b^{3}\mathrm{sgn}\left(b\cos\left(ex+d\right)^{2}+a\cos\left(ex+d\right)\right)}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,
 algorithm="giac")
```

```
[Out] -((2*a^4 - 3*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(2*a - 2
*b) + arctan((a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d))/sqrt(a^2 - b
^2)))/((a^2*b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^5*sgn(b*cos(e*x
+ d)^2 + a*cos(e*x + d)))*sqrt(a^2 - b^2)) + (2*a^4*tan(1/2*e*x + 1/2*d)^3
- 3*a^3*b*tan(1/2*e*x + 1/2*d)^3 - 3*a^2*b^2*tan(1/2*e*x + 1/2*d)^3 + 4*a*b
```

```
^3*tan(1/2*e*x + 1/2*d)^3 + 2*a^4*tan(1/2*e*x + 1/2*d) + 3*a^3*b*tan(1/2*e*
x + 1/2*d) - 3*a^2*b^2*tan(1/2*e*x + 1/2*d) - 4*a*b^3*tan(1/2*e*x + 1/2*d))
/((a^2*b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^4*sgn(b*cos(e*x + d)^
2 + a*cos(e*x + d)))*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 +
 a + b)^2) - (e*x + d)*a/(b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))))/e
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac {a+\frac {b}{\cos (d+e\,x)}}{{\left (b^2+\frac {a^2}{{\cos (d+e\,x)}^2}+\frac {2\,a\,b}{\cos (d+e\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2),x)
```

```
[Out] int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))^
(3/2), x)
```

3.526 $\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

```
[Out] 1/2*I*(cos(x)-I*sin(x))^2
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4470}

$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*(Cos[x] - I*Sin[x])^2
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx = \frac{1}{2}i(\cos(x)-i\sin(x))^2$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.12

$$\frac{1}{2}i\cos(2x)+\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

```
[Out] (I/2)*Cos[2*x] + Sin[2*x]/2
```

Maple [A]
time = 0.19, size = 8, normalized size = 0.47

method	result	size
default	$\frac{1}{\tan(x)-i}$	8
risch	$\frac{ie^{-2ix}}{2}$	9
norman	$\frac{-4i(\tan^2(\frac{x}{2}))-2(\tan^3(\frac{x}{2}))+2\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(tan(x)-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.75, size = 6, normalized size = 0.35

$$\frac{1}{2}i\,e^{(-2i\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.47

$$\frac{ie^{-2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
```

```
[Out] I*exp(-2*I*x)/2
```

Giac [A]
time = 0.42, size = 14, normalized size = 0.82

$$-\frac{2 \tan\left(\frac{1}{2} x\right)}{\left(\tan\left(\frac{1}{2} x\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x) - I)^2
```

Mupad [B]
time = 2.76, size = 16, normalized size = 0.94

$$-\frac{\cos\left(x\right)}{-\sin\left(x\right)+\cos\left(x\right)\,1{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - sin(x)*1i)/(cos(x) + sin(x)*1i),x)
```

```
[Out] -cos(x)/(cos(x)*1i - sin(x))
```

3.527 $\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx$

Optimal. Leaf size=17

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

```
[Out] -1/2*I/(cos(x)-I*sin(x))^2
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {4470}

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
[Out] (-1/2*I)/(Cos[x] - I*Sin[x])^2
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx=-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.12

$$-\frac{1}{2}i\cos(2x)+\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
[Out] (-1/2*I)*Cos[2*x] + Sin[2*x]/2
```

Maple [A]
time = 0.18, size = 8, normalized size = 0.47

method	result	size
default	$\frac{1}{\tan(x)+i}$	8
risch	$-\frac{i e^{2ix}}{2}$	9
norman	$\frac{4i(\tan^2(\frac{x}{2}))-2(\tan^3(\frac{x}{2}))+2\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/(tan(x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.86, size = 6, normalized size = 0.35

$$-\frac{1}{2} i e^{(2 i x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="fricas")
[Out] -1/2*I*e^(2*I*x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.59

$$-\frac{ie^{2ix}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
[Out] -I*exp(2*I*x)/2
```

Giac [A]
time = 0.42, size = 14, normalized size = 0.82

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)+i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="giac")
[Out] -2*tan(1/2*x)/(tan(1/2*x) + I)^2
```

Mupad [B]
time = 2.74, size = 13, normalized size = 0.76

$$\frac{\sin(x)}{\cos(x)-\sin(x)\ 1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x)*1i)/(cos(x) - sin(x)*1i),x)
[Out] sin(x)/(cos(x) - sin(x)*1i)
```

3.528 $\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=6

$$\log(\cos(x)+\sin(x))$$

```
[Out] ln(cos(x)+sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3212}

$$\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
[Out] Log[Cos[x] + Sin[x]]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx = \log(\cos(x)+\sin(x))$$

Mathematica [A]
time = 0.02, size = 6, normalized size = 1.00

$$\log(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
[Out] Log[Cos[x] + Sin[x]]
```

Maple [A]
time = 0.07, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\ln(\cos(x)+\sin(x))$	7
default	$\ln(\cos(x)+\sin(x))$	7
risch	$-ix+\ln(\mathrm{e}^{2ix}+i)$	15
norman	$-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)+\ln\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))/(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(cos(x)+sin(x))
```

Maxima [A]
time = 0.26, size = 6, normalized size = 1.00

$$\log(\cos(x)+\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
[Out] log(cos(x) + sin(x))
```

Fricas [A]
time = 3.11, size = 11, normalized size = 1.83

$$\frac{1}{2}\log(2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
[Out] 1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 1.17

$$\log(\sin(x)+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x)
[Out] log(sin(x) + cos(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.45, size = 16, normalized size = 2.67

$$-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + log(abs(tan(x) + 1))
```

Mupad [B]
time = 2.92, size = 32, normalized size = 5.33

$$2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{x}{2}\right)+128}{16\tan\left(\frac{x}{2}\right)^2+32\tan\left(\frac{x}{2}\right)+48}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) - sin(x))/(cos(x) + sin(x)),x)
```

```
[Out] 2*atanh((128*tan(x/2) + 128)/(32*tan(x/2) + 16*tan(x/2)^2 + 48) - 3)
```

3.529 $\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3212}

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 +
 c^2)
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Mathematica [A]
time = 0.10, size = 39, normalized size = 0.83

$$\frac{(bB+cC)x+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
[Out] ((b*B + c*C)*x + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [A]
time = 0.15, size = 66, normalized size = 1.40

method	result	size
default	$\frac{\frac{(-Bc+bC)\ln\left(\tan^2(x)+1\right)}{2}+(bB+Cc)\arctan(\tan(x))}{b^2+c^2}+\frac{(Bc-bC)\ln(c\tan(x)+b)}{b^2+c^2}$	66
norman	$\frac{\frac{(bB+Cc)x}{b^2+c^2}+\frac{(bB+Cc)x\left(\tan^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{(Bc-bC)\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2}-\frac{(Bc-bC)\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b^2+c^2}$	122
risch	$\frac{iCx}{ic-b}-\frac{Bx}{ic-b}-\frac{2ixBc}{b^2+c^2}+\frac{2ixbC}{b^2+c^2}+\frac{\ln\left(e^{2ix}-\frac{ic+b}{ic-b}\right)Bc}{b^2+c^2}-\frac{\ln\left(e^{2ix}-\frac{ic+b}{ic-b}\right)bC}{b^2+c^2}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(tan(x)^2+1)+(B*b+C*c)*arctan(tan(x)))+(B*c-C
*b)/(b^2+c^2)*ln(c*tan(x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(48) = 96.
time = 0.48, size = 181, normalized size = 3.85

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)+C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\frac{b\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) -
 b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2
) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2))
```

Fricas [A]
time = 2.92, size = 59, normalized size = 1.26

$$\frac{2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\cos\left(x\right)\sin\left(x\right)+\left(b^2-c^2\right)\cos\left(x\right)^2+c^2\right)}{2\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2))/(b^2 + c^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 360, normalized size = 7.66

$$\begin{cases} \tilde{\infty}\left(B\log\left(\sin\left(x\right)\right)+Cx\right) & \text{for}\ b=0\wedge c=0 \\ \frac{B\log\left(\sin\left(x\right)\right)+Cx}{c} & \text{for}\ b=0 \\ -\frac{Bx\sin\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)}+\frac{iBx\cos\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)}-\frac{B\cos\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)}+\frac{iCx\sin\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)}+\frac{Cx\cos\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)}-\frac{iC\cos\left(x\right)}{2ic\sin\left(x\right)+2c\cos\left(x\right)} & \text{for}\ b=-ic \\ -\frac{Bx\sin\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)}-\frac{iBx\cos\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)}-\frac{B\cos\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)}-\frac{iCx\sin\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)}+\frac{Cx\cos\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)}+\frac{iC\cos\left(x\right)}{-2ic\sin\left(x\right)+2c\cos\left(x\right)} & \text{for}\ b=ic \\ \frac{Bbx}{b^2+c^2}+\frac{Bc\log\left(\cos\left(x\right)+\frac{c\sin\left(x\right)}{b}\right)}{b^2+c^2}-\frac{Cb\log\left(\cos\left(x\right)+\frac{c\sin\left(x\right)}{b}\right)}{b^2+c^2}+\frac{Ccx}{b^2+c^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(B*log(sin(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*log(sin(x))
 + C*x)/c, Eq(b, 0)), (-B*x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*x*cos(
x)/(2*I*c*sin(x) + 2*c*cos(x)) - B*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*C
*x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(
x)) - I*C*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-B*x*sin(x)/(-
2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - B*
cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos
(x)) + C*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) + I*C*cos(x)/(-2*I*c*sin(x)
+ 2*c*cos(x)), Eq(b, I*c)), (B*b*x/(b**2 + c**2) + B*c*log(cos(x) + c*sin(x
)/b)/(b**2 + c**2) - C*b*log(cos(x) + c*sin(x)/b)/(b**2 + c**2) + C*c*x/(b*
*2 + c**2), True))
```

Giac [A]
time = 0.46, size = 77, normalized size = 1.64

$$\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(x\right)^2+1\right)}{2\left(b^2+c^2\right)}-\frac{(Cbc-Bc^2)\log\left(|c\tan\left(x\right)+b|\right)}{b^2c+c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) + 1/2*(C*b - B*c)*log(tan(x)^2 + 1)/(b^2 + c^2) -
 (C*b*c - B*c^2)*log(abs(c*tan(x) + b))/(b^2*c + c^3)
```

Mupad [B]
time = 12.82, size = 1976, normalized size = 42.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(b + 2*c*tan(x/2) - b*tan(x/2)^2)*(B*c - C*b))/(b^2 + c^2) - (log(1/(co
s(x) + 1))*(2*B*c - 2*C*b))/(2*(b^2 + c^2)) + (2*atan(((((32*B*C^2*b^2 - ((
2*B*c - 2*C*b)*(((2*B*c - 2*C*b)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64
*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2))))/(2*(
b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c + 64*B*C*b^3 + 64*B*C*b*c^2))/(2*
(b^2 + c^2)) + ((B*b + C*c)*(((B*b + C*c)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b
*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2
))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2
*(b^2 + c^2)^2)))/(b^2 + c^2) - 32*B^2*C*b*c + ((B*b + C*c)^2*(2*B*c - 2*C*
b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2)^3))*(12*B^2*b*c^3 - 6*B^2*b^3*c
- 6*C^2*b*c^3 + 12*C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^2*c^2))/((b
^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*b*c)^2) - ta
n(x/2)*(((32*B^3*b*c - 32*B^2*C*b^2 - 64*C^3*b^2 + ((2*B*c - 2*C*b)*(32*B^2
*b^3 - 96*B^2*b*c^2 + 64*C^2*b*c^2 - ((2*B*c - 2*C*b)*(32*C*b^2*c^2 - 64*C*
b^4 + 32*B*b*c^3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/
(2*(b^2 + c^2))))/(2*(b^2 + c^2)) + 192*B*C*b^2*c))/(2*(b^2 + c^2)) + ((B*b
 + C*c)*(((B*b + C*c)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 + 128*B*b^3*c -
 ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/(b^2 + c^2) -
((B*b + C*c)*(2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2)^2)))/(
b^2 + c^2) + 64*B*C^2*b*c - ((B*b + C*c)^2*(2*B*c - 2*C*b)*(96*b*c^4 + 96*b
^3*c^2))/(2*(b^2 + c^2)^3))*(12*B^2*b*c^3 - 6*B^2*b^3*c - 6*C^2*b*c^3 + 12*
C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^2*c^2))/((b^2 + c^2)^2*(B^2*b^
2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*b*c)^2) + ((((B*b + C*c)^3*(96*
b*c^4 + 96*b^3*c^2))/(b^2 + c^2)^3 - ((B*b + C*c)*(32*B^2*b^3 - 96*B^2*b*c^
2 + 64*C^2*b*c^2 - ((2*B*c - 2*C*b)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 +
 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/
(2*(b^2 + c^2)) + 192*B*C*b^2*c))/(b^2 + c^2) + ((2*B*c - 2*C*b)*(((B*b + C
*c)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*
(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2))))/(b^2 + c^2) - ((B*b + C*c)*(2*B*
c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2)^2)))/(2*(b^2 + c^2)))*(B
^2*b^4 + 4*B^2*c^4 - 4*C^2*b^4 - C^2*c^4 - 13*B^2*b^2*c^2 + 13*C^2*b^2*c^2
- 18*B*C*b*c^3 + 18*B*C*b^3*c))/((b^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2
*b^2 + C^2*c^2 - 6*B*C*b*c)^2)) + ((((B*b + C*c)*(((2*B*c - 2*C*b)*(64*B*b^
2*c^2 - 32*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 9
6*b^2*c^3))/(2*(b^2 + c^2))))/(2*(b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c
 + 64*B*C*b^3 + 64*B*C*b*c^2))/(b^2 + c^2) - ((B*b + C*c)^3*(96*b^4*c + 96*
b^2*c^3))/(b^2 + c^2)^3 + ((2*B*c - 2*C*b)*(((B*b + C*c)*(64*B*b^2*c^2 - 32
*B*b^4 + 32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3)
)/(2*(b^2 + c^2))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c +
96*b^2*c^3))/(2*(b^2 + c^2)^2)))/(2*(b^2 + c^2)))*(B^2*b^4 + 4*B^2*c^4 - 4*
C^2*b^4 - C^2*c^4 - 13*B^2*b^2*c^2 + 13*C^2*b^2*c^2 - 18*B*C*b*c^3 + 18*B*C
*b^3*c))/((b^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*
b*c)^2))*(b^4 + c^4 + 2*b^2*c^2))/(32*B*b^2 + 32*C*b*c))*(B*b + C*c))/(b^2
```

```
+ c^2)
```

3.530 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

```
[Out] -(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B
*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3232, 3153, 212}

$$-\frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
```

```
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx &= -\frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= -\frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\text{Subst}\left(\int \frac{1}{b^2+c^2-x^2}\,dx,x,c\cos(x)\right.}{b^2+c^2}\\ &= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 75, normalized size = 1.01

$$\frac{2(bB+cC)\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{-Bc+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (-(B*c) + b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.26, size = 113, normalized size = 1.53

method	result
default	$-\frac{2\left(-\frac{c(Bc-bC)\tan\left(\frac{x}{2}\right)}{b\left(b^2+c^2\right)}-\frac{Bc-bC}{b^2+c^2}\right)}{b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2(bB+Cc)\,\text{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}$
risch	$-\frac{2\,e^{ix}(Bc-bC)}{(ic+b)(-ic+b)(-ic\,e^{2ix}+b\,e^{2ix}+ic+b)}+\frac{bB\ln\left(e^{ix}+\frac{ib^3+ib\,c^2-b^2c-c^3}{(b^2+c^2)^{\frac{3}{2}}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{cC\ln\left(e^{ix}+\frac{ib^3+ib\,c^2-b^2c-c^3}{(b^2+c^2)^{\frac{3}{2}}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{bB\ln\left(e^{ix}-i\right.}{(b^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*(-c*(B*c-C*b)/b/(b^2+c^2)*tan(1/2*x)-(B*c-C*b)/(b^2+c^2))/(b*tan(1/2*x)^
2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)
-2*c)/(b^2+c^2)^(1/2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(69) = 138$.
time = 0.48, size = 271, normalized size = 3.66

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1}-\frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt
(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/
2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(c
os(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(69) = 138$.
time = 3.03, size = 194, normalized size = 2.62

$$\frac{2\,Cb^3-2\,Bb^2c+2\,Cbc^2-2\,Bc^3+\sqrt{b^2+c^2}\left((Bb^2+Cbc)\cos(x)+(Bbc+Cc^2)\sin(x)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)}{2\left((b^5+2\,b^3c^2+bc^4)\cos(x)+(b^4c+2\,b^2c^3+c^5)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/((b^5 + 2*b^3*c^2 + b*c^4)
*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.45, size = 132, normalized size = 1.78

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\left(b^2+c^2\right)^{\frac{3}{2}}}-\frac{2\left(Cbc\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{\left(b^3+bc^2\right)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(C*b*c*tan(1/2*x)
- B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*ta
n(1/2*x) - b))
```

Mupad [B]
time = 3.01, size = 129, normalized size = 1.74

$$-\frac{\frac{2\,(B\,c-C\,b)}{b^2+c^2}+\frac{2\,c\tan\left(\frac{x}{2}\right)\,(B\,c-C\,b)}{b\,(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b}+\frac{\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,(b^2+c^2)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,(B\,b+C\,c)\,2\mathrm{i}}{(b^2+c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(B
*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) + (2*c*tan(x
/2)*(B*c - C*b))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

3.531 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=66

$$-\frac{Bc-bC}{2\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)^2}+\frac{\left(bB+cC\right)\sin(x)}{b\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)}$$

```
[Out] 1/2*(-B*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(B*b+C*c)*sin(x)/b/(b^2+c^2)
/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3235, 12, 3154}

$$\frac{\sin(x)(bB+cC)}{b\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)}-\frac{Bc-bC}{2\left(b^2+c^2\right)\left(b\cos(x)+c\sin(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -1/2*(B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[x
])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx &= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2(bB+cC)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}\\ &= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\int \frac{1}{(b\cos(x)+c\sin(x))^2}\,dx}{b^2+c^2}\\ &= -\frac{Bc-bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{(bB+cC)\sin(x)}{b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 64, normalized size = 0.97

$$\frac{\left(b^2+c^2\right)C-c(bB+cC)\cos(2x)+b(bB+cC)\sin(2x)}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((b^2 + c^2)*C - c*(b*B + c*C)*Cos[2*x] + b*(b*B + c*C)*Sin[2*x])/(2*b*(b^2
 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A]
time = 0.25, size = 37, normalized size = 0.56

method	result	size
default	$-\frac{Bc-bC}{2c^2(c\tan(x)+b)^2}-\frac{C}{c^2(c\tan(x)+b)}$	37
risch	$\frac{2i\left(Bb\,e^{2ix}-Cc\,e^{2ix}-iBc\,e^{2ix}-iCb\,e^{2ix}+bB+Cc\right)}{(-ic+b)^2\left(-ic\,e^{2ix}+b\,e^{2ix}+ic+b\right)^2}$	80
norman	$\frac{\frac{(2Bc+2bC)\left(\tan^2\left(\frac{x}{2}\right)\right)}{b^2}+\frac{(2Bc+2bC)\left(\tan^4\left(\frac{x}{2}\right)\right)}{b^2}+\frac{2B\tan\left(\frac{x}{2}\right)}{b}-\frac{2B\left(\tan^5\left(\frac{x}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(B*c-C*b)/c^2/(c*tan(x)+b)^2-C/c^2/(c*tan(x)+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(64) = 128.
time = 0.28, size = 199, normalized size = 3.02

$$\frac{2\,B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^4+\frac{4\,b^3c\sin(x)}{\cos(x)+1}-\frac{4\,b^3c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^4\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\,(b^4-2\,b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}+\frac{2\,C\sin(x)^2}{\left(b^3+\frac{4\,b^2c\sin(x)}{\cos(x)+1}-\frac{4\,b^2c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\,(b^3-2\,bc^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
 + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) + 1)
^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x) + 1
)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(x)^3/
(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x)^2/(
cos(x) + 1)^2)*(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.
time = 2.70, size = 152, normalized size = 2.30

$$\frac{Cb^3 + Bb^2c + 3\,Cbc^2 - Bc^3 - 4\,(Bb^2c + Cbc^2)\cos(x)^2 + 2\,(Bb^3 + Cb^2c - Bbc^2 - Cc^3)\cos(x)\sin(x)}{2\,(b^4c^2 + 2\,b^2c^4 + c^6 + (b^6 + b^4c^2 - b^2c^4 - c^6)\cos(x)^2 + 2\,(b^5c + 2\,b^3c^3 + bc^5)\cos(x)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*b^3 + B*b^2*c + 3*C*b*c^2 - B*c^3 - 4*(B*b^2*c + C*b*c^2)*cos(x)^2 +
 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x)*sin(x))/(b^4*c^2 + 2*b^2*c^4
+ c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b
*c^5)*cos(x)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 26, normalized size = 0.39

$$-\frac{2\,Cc\tan(x) + Cb + Bc}{2\,(c\tan(x) + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*c*tan(x) + C*b + B*c)/((c*tan(x) + b)^2*c^2)
```

Mupad [B]
time = 2.82, size = 95, normalized size = 1.44

$$\frac{\frac{2\tan\left(\frac{x}{2}\right)^2\,\left(B\,c+C\,b\right)}{b^2}-\frac{2\,B\,\tan\left(\frac{x}{2}\right)^3}{b}+\frac{2\,B\,\tan\left(\frac{x}{2}\right)}{b}}{b^2-\tan\left(\frac{x}{2}\right)^2\,\left(2\,b^2-4\,c^2\right)+b^2\tan\left(\frac{x}{2}\right)^4+4\,b\,c\tan\left(\frac{x}{2}\right)-4\,b\,c\tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((2*tan(x/2)^2*(B*c + C*b))/b^2 - (2*B*tan(x/2)^3)/b + (2*B*tan(x/2))/b)/(b
^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*t
an(x/2)^3)
```

3.532 $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{(bB+cC)x}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((
c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3215, 3153, 212}

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c
^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
```

```
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}+A\int \frac{1}{b\cos(x)+c\sin(}$$
$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}-A\text{Subst}\left(\int \frac{1}{b^2+c^2-}\right.$$
$$= \frac{(bB+cC)x}{b^2+c^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{(Bc-bC)\log(b\cos(x)+c\,\text{si}}{b^2+c^2}$$

Mathematica [A]
time = 0.18, size = 78, normalized size = 0.93

$$\frac{(bB+cC)x+2A\sqrt{b^2+c^2}\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
[Out] ((b*B + c*C)*x + 2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c
^2]] + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(80) = 160$.
time = 0.30, size = 167, normalized size = 1.99

method	result
default	$\frac{\frac{(bBc-b^2C)\ln\left(b\left(\tan^2\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)}{b}-\frac{2\left(-Ab^2-Ac^2-Bc^2+Cbc+\frac{(bBc-b^2C)c}{b}\right)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}}{b^2+c^2}+\frac{(-Bc+bC)\ln(1+}{}$
risch	$\frac{iCx}{ic-b}-\frac{Bx}{ic-b}-\frac{2ixB\,b^2c}{b^4+2b^2c^2+c^4}-\frac{2ixB\,c^3}{b^4+2b^2c^2+c^4}+\frac{2iCx\,b^3}{b^4+2b^2c^2+c^4}+\frac{2iCx\,c^2b}{b^4+2b^2c^2+c^4}+\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A(b^2+c^2)}\right)Bc}{b^2+c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/(b^2+c^2)*(1/2*(B*b*c-C*b^2)/b*ln(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)-(-A*b^
2-A*c^2-B*c^2+C*b*c+(B*b*c-C*b^2)*c/b)/(b^2+c^2)^(1/2)*arctanh(1/2*(2*b*tan
```

```
(1/2*x)-2*c)/(b^2+c^2)^(1/2)))+2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^
2)+(B*b+C*c)*arctan(tan(1/2*x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(81) = 162$.

time = 0.48, size = 243, normalized size = 2.89

$$B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)+C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\frac{b\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)-\frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(cos
(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x)
+ 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) -
 b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2
) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(
cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)
))/sqrt(b^2 + c^2)
```

Fricas [A]

time = 2.67, size = 155, normalized size = 1.85

$$\frac{\sqrt{b^2+c^2}\,A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)+2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2\right)}{2\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2
*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x)
+ (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*co
s(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)
```

Sympy [C] Result contains complex when optimal does not.

time = 21.55, size = 1030, normalized size = 12.26

$$\begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx\right) & \text{for}\ b=0\wedge c=0 \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)-B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)+B\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx}{c} & \text{for}\ b=0 \\ -\frac{2A}{2ic\sin(x)+2c\cos(x)}-\frac{Bx\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iBx\cos(x)}{2ic\sin(x)+2c\cos(x)}-\frac{B\cos(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iCx\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{Cx\cos(x)}{2ic\sin(x)+2c\cos(x)}-\frac{iC\cos(x)}{2ic\sin(x)+2c\cos(x)} & \text{for}\ b=-ic \\ -\frac{2A}{-2ic\sin(x)+2c\cos(x)}-\frac{Bx\sin(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iBx\cos(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{B\cos(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iCx\sin(x)}{-2ic\sin(x)+2c\cos(x)}+\frac{Cx\cos(x)}{-2ic\sin(x)+2c\cos(x)}+\frac{iC\cos(x)}{-2ic\sin(x)+2c\cos(x)} & \text{for}\ b=ic \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bbx\sqrt{b^2+c^2}}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Bc\sqrt{b^2+c^2}\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bc\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Bc\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Cb\sqrt{b^2+c^2}\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Cb\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}-\frac{Cb\sqrt{b^2+c^2}\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}}+\frac{Ccx\sqrt{b^2+c^2}}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2))
+ C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) +
B*log(tan(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) - B*
```

```
x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) + 2*c*cos
(x)) - B*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*sin(x) +
2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) - I*C*cos(x)/(2*I*c*si
n(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x) + 2*c*cos(x)) - B*x*
sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos
(x)) - B*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x)
+ 2*c*cos(x)) + C*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) + I*C*cos(x)/(-2*I*
c*sin(x) + 2*c*cos(x)), Eq(b, I*c)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**
2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log
(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(
b**2 + c**2)) - A*c**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt
(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(
b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*b*x*s
qrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - B*c*sq
rt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b*
*2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/
b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2
)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*
sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(
b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2)
 - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c*
*2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**
2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c*x*sqrt(b**2 + c**2)/(b*
*2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), True))
```

Giac [A]
time = 0.48, size = 148, normalized size = 1.76

$$-\frac{A\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}+\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{b^2+c^2}-\frac{(Cb-Bc)\log\left(\left|b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right|\right)}{b^2+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] -A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2
*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2) + (B*b + C*c)*x/(b^2 + c^2) + (C*b
 - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - (C*b - B*c)*log(abs(b*tan(1/2*x
)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2)
```

Mupad [B]
time = 9.41, size = 1099, normalized size = 13.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
[Out] log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/
2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b +
 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2
)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A
*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) - 32*B^2*b^2*c - 32*C^2*b^2*c - 64*A^2*b^2*
c - 64*A*C*b^3 + 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B
*b^2*c - C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C
*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c
+ 4*B*b^2*c + C*b*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2)
+ B*c^3 - C*b^3 + B*b^2*c - C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*
b^2*c + 64*B*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*((B*c -
 C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B
*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x/2)*(B^3*c -
2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c)
- ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*c^2)*(64*A^2*b^
2*c + 32*B^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*
b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) + 64*A*C*b
^3 - 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b
*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 - 32*C*b*c^3 + 64*
C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 4*B*b^2*c
+ C*b*c^2) - (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*
b^3 - B*b^2*c + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2 - 64*A*B*b^2*c - 64*B
*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b*c)*((B*c - C*b)/(b^2 +
 c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) - (log(tan(x/2) + 1i)*(B -
 C*1i))/(b*1i + c) + (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c)
```

3.533 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

```
[Out] -(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B
*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3232, 3153, 212}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^
(3/2)) - (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*S
in[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
```

```
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx &= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}+\frac{(bB+cC)\int\frac{1}{b\cos(x)+c\sin(x)}\,dx}{b^2+c^2}\\ &= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\mathrm{Subst}\left(\int\frac{1}{b^2+c^2-x^2}\,dx,x,c\,c\right.}{b^2+c^2}\\ &= -\frac{(bB+cC)\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 92, normalized size = 1.08

$$\frac{2(bB+cC)\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{b(-Bc+bC)+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2
) + (b*(-(B*c) + b*C) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*
Sin[x]))
```

Maple [A]
time = 0.24, size = 124, normalized size = 1.46

method	result
default	$\frac{-\frac{2\left(A\,b^{2}+A\,c^{2}-B\,c^{2}+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^{2}+c^{2}\right)}+\frac{2(Bc-bC)}{b^{2}+c^{2}}}{b\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2(bB+Cc)\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{2i\left(Ac-iAb+Bc\,\mathrm{e}^{ix}-bC\,\mathrm{e}^{ix}\right)}{(-ib+c)(ib+c)\left(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib\right)}+\frac{bB\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}+\frac{cC\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{bB\ln\left(\mathrm{e}^{ix}-\frac{ib^{3}+\cdots}{\left(\cdots\right.}\right.}{\left(b^{2}+c\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(A*b^2+A*c^2-B*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)+(B*c-C*b)/(b^2+c^2))/(
b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2
*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(81) = 162$.
time = 0.51, size = 286, normalized size = 3.36

$$-B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)-C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1}-\frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)-\frac{A}{c^2\tan(x)+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] -B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(
x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(x)
 + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b^2*
c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt
(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/
2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(c
os(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(81) = 162$.
time = 2.00, size = 226, normalized size = 2.66

$$\frac{2\,Cb^3-2\,Bb^2c+2\,Cbc^2-2\,Bc^3+\sqrt{b^2+c^2}\left((Bb^2+Cbc)\cos(x)+(Bbc+Cc^2)\sin(x)\right)\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)-2\,(Ab^2c+Ac^3)\cos(x)+2\,(Ab^3+Abc^2)\sin(x)}{2\left((b^5+2\,b^3c^2+bc^4)\cos(x)+(b^4c+2\,b^2c^3+c^5)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2 -
c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b
*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x
) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c
+ 2*b^2*c^3 + c^5)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.45, size = 150, normalized size = 1.76

$$-\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(
1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x)
+ C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/(
(b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))
```

Mupad [B]
time = 3.05, size = 141, normalized size = 1.66

$$-\frac{\frac{2\,(B\,c-C\,b)}{b^2+c^2}-\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+A\,c^2-B\,c^2+C\,b\,c\right)}{b\,(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b}+\frac{\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,(B\,b+C\,c)\,2\mathrm{i}}{(b^2+c^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(B
*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) - (2*tan(x/2
)*(A*b^2 + A*c^2 - B*c^2 + C*b*c))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*t
an(x/2)^2)
```

3.534 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

```
[Out] -1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-B
*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c*(B*b+C*c)
*cos(x)+b*(B*b+C*c)*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3235, 3232, 3153, 212}

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{c\cos(x)(bB+cC)-b\sin(x)(bB+cC)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -1/2*(A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) -
 (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])
^2) - (c*(b*B + c*C)*Cos[x] - b*(b*B + c*C)*Sin[x])/((b^2 + c^2)^2*(b*Cos[x
] + c*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
```

```
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx &= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}+\frac{\int \frac{2(bB+cC)+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx}{2\left(b^2+c^2\right)}\\ &= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}\\ &= -\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+cC)\cos(x)-b(bB+cC)\sin}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}\\ &= -\frac{A\tanh^{-1}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}-\frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}-\frac{c(bB+}{(b}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 122, normalized size = 0.95

$$\frac{A\tanh^{-1}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}+\frac{b^2C+c^2C-Abc\cos(x)-c(bB+cC)\cos(2x)+Ab^2\sin(x)+b^2B\sin(2x)+bcC\sin(2x)}{2b\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2*C +
 c^2*C - A*b*c*Cos[x] - c*(b*B + c*C)*Cos[2*x] + A*b^2*Sin[x] + b^2*B*Sin[2
*x] + b*c*C*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

Maple [A]
time = 0.30, size = 218, normalized size = 1.69

method	result
default	$-\frac{2\left(-\frac{\left(A\,b^{2}+2A\,c^{2}-2B\,b^{2}-2B\,c^{2}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b}-\frac{\left(A\,b^{2}c-2A\,c^{3}+2B\,b^{2}c+2B\,c^{3}+2C\,b^{3}+2Cb\,c^{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2\left(b^{2}+c^{2}\right)b^{2}}-\frac{\left(A\,b^{2}-2A\,c^{2}+2B\,b^{2}+2B\,c^{2}\right)\tan(}{2\left(b^{2}+c^{2}\right)b}}{\left(b\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}$
risch	$-\frac{i\left(2iB\,b^{2}\mathrm{e}^{2ix}+2C\,b^{2}\mathrm{e}^{2ix}+2iB\,c^{2}\mathrm{e}^{2ix}+iA\,b^{2}\mathrm{e}^{ix}+iA\,c^{2}\mathrm{e}^{ix}+2iB\,b^{2}+2C\,c^{2}\mathrm{e}^{2ix}-2C\,c^{2}-2Abc\,\mathrm{e}^{3ix}-iA\,b^{2}\mathrm{e}^{3ix}+iA\,c^{2}\mathrm{e}^{3ix}-2bBc+2iC\right.}{\left(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib\right)^{2}\left(-ib+c\right)\left(ib+c\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^
2*c-2*A*c^3+2*B*b^2*c+2*B*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2
-1/2*(A*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*A*c/(b^2+c^
2))/(b*tan(1/2*x)^2-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*
tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(121) = 242.
time = 0.51, size = 451, normalized size = 3.50

$$-\frac{1}{2}A\left(\frac{2\left(b^{2}c-\frac{(b^{3}-2bc^{2})\sin(x)}{\cos(x)+1}-\frac{(b^{2}c-2c^{3})\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{(b^{3}+2bc^{2})\sin(x)^{3}}{(\cos(x)+1)^{3}}\right)}{b^{6}+b^{4}c^{2}+\frac{4(b^{5}c+b^{3}c^{3})\sin(x)}{\cos(x)+1}-\frac{2(b^{6}-b^{4}c^{2}-2b^{2}c^{4})\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{4(b^{5}c+b^{3}c^{3})\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{(b^{6}+b^{4}c^{2})\sin(x)^{4}}{(\cos(x)+1)^{4}}}+\frac{\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^{2}+c^{2}}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^{2}+c^{2}}}\right)}{(b^{2}+c^{2})^{\frac{3}{2}}}\right)+\frac{2B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{b\sin(x)^{3}}{(\cos(x)+1)^{3}}\right)}{b^{4}+\frac{4b^{3}c\sin(x)}{\cos(x)+1}-\frac{4b^{3}c\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{b^{4}\sin(x)^{4}}{(\cos(x)+1)^{4}}-\frac{2(b^{4}-2b^{2}c^{2})\sin(x)^{2}}{(\cos(x)+1)^{2}}}+\frac{2C\sin(x)^{2}}{\left(b^{3}+\frac{4b^{2}c\sin(x)}{\cos(x)+1}-\frac{4b^{2}c\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{b^{3}\sin(x)^{4}}{(\cos(x)+1)^{4}}-\frac{2(b^{3}-2bc^{2})\sin(x)^{2}}{(\cos(x)+1)^{2}}\right)(\cos(x)+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] -1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*si
n(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b^4
*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^2*c
^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)^3 +
 (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x) + 1)
+ sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^
2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(
x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/
(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2
/(cos(x) + 1)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2
*c*sin(x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2
)*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(121) = 242.

time = 2.78, size = 311, normalized size = 2.41

$$\frac{2\,Cb^{3}+2\,Bb^{2}c+6\,Cbc^{2}-2\,Bc^{3}-8\,\left(Bb^{2}c+Cbc^{2}\right)\cos\left(x\right)^{2}+\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\cos\left(x\right)^{2}\right)\sqrt{b^{2}+c^{2}}\,\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^{2}-c^{2})\cos(x)^{2}-2\,b^{2}-c^{2}+2\,\sqrt{b^{2}+c^{2}}\,(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^{2}-c^{2})\cos(x)^{2}+c^{2}}\right)-2\,\left(Ab^{2}c+Ac^{3}\right)\cos\left(x\right)+2\,\left(Ab^{3}+Abc^{2}+2\,\left(Bb^{3}+Cb^{2}c-Bbc^{2}-Cc^{3}\right)\cos\left(x\right)\right)\sin\left(x\right)}{4\,\left(b^{4}c^{2}+2\,b^{2}c^{4}+c^{6}+\left(b^{6}+b^{4}c^{2}-b^{2}c^{4}-c^{6}\right)\cos\left(x\right)^{2}+2\,\left(b^{5}c+2\,b^{3}c^{3}+bc^{5}\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*C*b^3 + 2*B*b^2*c + 6*C*b*c^2 - 2*B*c^3 - 8*(B*b^2*c + C*b*c^2)*cos(
x)^2 + (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2
+ c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*s
qrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*co
s(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2 + 2*(B*b^3
 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 + c^6 +
(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*c^5)*co
s(x)*sin(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(121) = 242.
time = 0.50, size = 270, normalized size = 2.09

$$\frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^{2}+c^{2}}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^{2}+c^{2}}\right|}\right)}{2\,\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}+\frac{Ab^{3}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,Bb^{3}\tan\left(\frac{1}{2}\,x\right)^{3}+2\,Abc^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,Bbc^{2}\tan\left(\frac{1}{2}\,x\right)^{3}+2\,Cb^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+Ab^{2}c\tan\left(\frac{1}{2}\,x\right)^{2}+2\,Bb^{2}c\tan\left(\frac{1}{2}\,x\right)^{2}+2\,Cbc^{2}\tan\left(\frac{1}{2}\,x\right)^{2}-2\,Ac^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+2\,Bc^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+Ab^{3}\tan\left(\frac{1}{2}\,x\right)+2\,Bb^{3}\tan\left(\frac{1}{2}\,x\right)-2\,Abc^{2}\tan\left(\frac{1}{2}\,x\right)+2\,Bbc^{2}\tan\left(\frac{1}{2}\,x\right)-Ab^{2}c}{\left(b^{4}+b^{2}c^{2}\right)\left(b\tan\left(\frac{1}{2}\,x\right)^{2}-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*x
) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2*B
*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + 2*C*b
^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 + 2*C*b*c^2
*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2
*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x) + 2*B*b*c^2*tan(1/2*x) - A*
b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

Mupad [B]
time = 3.44, size = 264, normalized size = 2.05

$$\frac{\frac{\tan\left(\frac{x}{2}\right)\,\left(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,B\,c^3-2\,A\,c^3+2\,C\,b^3+A\,b^2\,c+2\,B\,b^2\,c+2\,C\,b\,c^2\right)}{b^2\,\left(b^2+c^2\right)}+\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(A\,b^2+2\,A\,c^2-2\,B\,b^2-2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{b^2-{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,b^2-4\,c^2\right)+b^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,b\,c\,\tan\left(\frac{x}{2}\right)-4\,b\,c\,{\tan\left(\frac{x}{2}\right)}^3}+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\,\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,1\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/(
b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + 2*C*b^3 + A*b^2*c + 2*B*b^2*c
 + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 -
 2*B*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2
)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*t
an(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

3.535 $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=115

$$\frac{bBx}{b^2+c^2}-\frac{2(abB-A(b^2+c^2))\,\mathrm{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] b*B*x/(b^2+c^2)+B*c*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*(a*b*B-A*(b^2+c^2))
*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3217, 3203, 632, 210}

$$-\frac{2(abB-A(b^2+c^2))\,\mathrm{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (b*B*x)/(b^2 + c^2) - (2*(a*b*B - A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (B*c*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3217

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] + Simp[c*B*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)} \\ &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int \right. \\ &= \frac{bBx}{b^2+c^2}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{abB}{b^2+c^2}\right)\right)\text{Subst}\left(\int \right. \\ &= \frac{bBx}{b^2+c^2}+\frac{2\left(A-\frac{abB}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c}{b^2+c^2}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 95, normalized size = 0.83

$$\frac{-\frac{2(-abB+A(b^2+c^2))\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+B(bx+c\log(a+b\cos(x)+c\sin(x)))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(-(a*b*B) + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 +
b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + B*(b*x + c*Log[a + b*Cos[x] + c*Sin[x
]]))/(b^2 + c^2)
```

Maple [A]
time = 0.44, size = 185, normalized size = 1.61

method	result
default	$\frac{\frac{2(aBc-bBc)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(A\,b^2+A\,c^2-abB+B\,c^2-\frac{(aBc-bBc)c}{a-b}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(1/2*(B*a*c-B*b*c)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*t
an(1/2*x)+a+b)+(A*b^2+A*c^2-a*b*B+B*c^2-(B*a*c-B*b*c)*c/(a-b))/(a^2-b^2-c^2
)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))+2*B/(b^2+
c^2)*(-1/2*c*ln(1+tan(1/2*x)^2)+b*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see 'assume?' for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(109) = 218$.
time = 2.90, size = 625, normalized size = 5.43

$$\left[-\frac{(Bab-Ab^2-Ac^2)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^3)c)\cos(x))\sin(x)-2(2abc\cos(x)^2-abc+(b^2c+c^3)\cos(x)-(b^3+bc^2+(ab^2-ac^2)\cos(x))\sin(x))\sqrt{-a^2+b^2+c^2}}{2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)}\right)-2(Ba^2b-Bb^3-Bbc^2)x+(Bc^3-(Ba^2-Bb^2)c)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)},-\frac{2(Bab-Ab^2-Ac^2)\sqrt{a^2-b^2-c^2}\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)-2(Ba^2b-Bb^3-Bbc^2)x+(Bc^3-(Ba^2-Bb^2)c)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*b - A*b^2 - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4
- c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*
c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b
^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b
*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
- 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*c
```

```
os(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a
^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 - A*c^2)*sq
rt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2
- b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x)))
 - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(
a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 178, normalized size = 1.55

$$\frac{Bbx}{b^2+c^2}+\frac{Bc\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}-\frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{2\left(Bab-Ab^2-Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] B*b*x/(b^2 + c^2) + B*c*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*b
- A*b^2 - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

Mupad [**B**]
time = 25.56, size = 1709, normalized size = 14.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 - 32*A^2*B*b^2 - 32*B^3*a*b +
 32*A^2*B*a*b - ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + A*c^2*(
b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b
^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a
^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b - 2*A*B*a*b) + 6
```

4*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c + ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^(1/2))*(32*A*a^2*c^2 - 32*B*b^4 - 32*A*a^2*b^2 - 32*A*b^4 - 32*B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64*B*b^2*c^2 + 64*A*a*b^3 + 64*B*a*b^3 - 96*B*a*b*c^2 + 32*c*tan(x/2)*(a - b)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2 - 2*A*a*b - 4*B*a*b) + (32*(a - b)*(B*c^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + 32*B*c*tan(x/2)*(A - B)^2*(a - b))*(B*c^3 + b^2*(A*(b^2 - a^2 + c^2)^(1/2) + B*c) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) - B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) - (B*log(tan(x/2) + 1i))/(b*1i + c) - (B*log(tan(x/2) - 1i)*1i)/(b + c*1i) - (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 - 32*A^2*B*b^2 - 32*B^3*a*b + 32*A^2*B*a*b - ((B*c^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c + B*a*b*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b - 2*A*B*a*b) + 64*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c + ((B*c^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c + B*a*b*(b^2 - a^2 + c^2)^(1/2))*(32*A*a^2*c^2 - 32*B*b^4 - 32*A*a^2*b^2 - 32*A*b^4 - 32*B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64*B*b^2*c^2 + 64*A*a*b^3 + 64*B*a*b^3 - 96*B*a*b*c^2 + 32*c*tan(x/2)*(a - b)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2 - 2*A*a*b - 4*B*a*b) + (32*(a - b)*(B*c^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c + B*a*b*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + 32*B*c*tan(x/2)*(A - B)^2*(a - b))*(b^2*(A*(b^2 - a^2 + c^2)^(1/2) - B*c) - B*c^3 + B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) - B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))

3.536 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=113

$$\frac{2(aA-bB)\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}$$

```
[Out] 2*(A*a-B*b)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)+(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3234, 3203, 632, 210}

$$\frac{2(aA-bB)\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}+\frac{(aA-bB)\int\frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}+\frac{(2(aA-bB))\text{Subst}\left(\int\frac{1}{a+b+2cx+(\ldots}\right.}{a^2-b^2-c^2}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}-\frac{(4(aA-bB))\text{Subst}\left(\int\frac{1}{-4(a^2-b^2-\ldots}\right.}{a^2-\ldots}$$

$$= \frac{2(aA-bB)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{Bc+Ac\cos(x)-(Ab-aB)\sin\ldots}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin\ldots}$$

Mathematica [A]
time = 0.24, size = 118, normalized size = 1.04

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}}+\frac{(aA-bB)c+(-abB+A(b^2+c^2))\sin(x)}{b\,(-a^2+b^2+c^2)\,(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + ((a*A - b*B)*c + (-(a*b*B) + A*(b^2 + c^2))*Sin[x])/
(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.34, size = 206, normalized size = 1.82

method	result

default	$-\frac{\frac{2\left(aAb-A\,b^{2}-A\,c^{2}-a^{2}B+abB+B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{2(aA-bB)c}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(aA-bB)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2i\left(-iAac\,e^{ix}+iBbc\,e^{ix}+Aab\,e^{ix}-B\,a^{2}e^{ix}+B\,c^{2}e^{ix}+A\,b^{2}+A\,c^{2}-abB\right)}{\left(-a^{2}+b^{2}+c^{2}\right)\left(-ic+b\right)\left(-ic\,e^{2ix}+b\,e^{2ix}+ic+2a\,e^{ix}+b\right)}-\frac{\ln\left(e^{ix}+\frac{iac\sqrt{-a^{2}+b^{2}+c^{2}}+ia^{2}b-ib^{3}-ib\,c^{2}+ab\sqrt{\cdots}}{\left(b^{2}+c^{2}\right)\sqrt{-a^{2}+b\cdots}}\right.}{\sqrt{-a^{2}+b^{2}+c^{2}}\,(a^{2}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)
*tan(1/2*x)+(A*a-B*b)*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^2-
b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2*(A*a-B*b)/(a^2-b^2-c^2)^(3/2)*arctan(1
/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(106) = 212$.
time = 2.82, size = 1277, normalized size = 11.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*c^5 - 4*(B*a^2 - B*b^2)*c^3 + (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B*
a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*c^
3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b
^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2
 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^
3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) -
 (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*
```

```
a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)
)) + 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(A*c^5 - (A*a^2 + B*a*b - 2*A*b^
2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) - 2*(B*a^3*b^2 -
 A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2)
*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^
5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*
b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 -
3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)
*sin(x)), -(B*c^5 - 2*(B*a^2 - B*b^2)*c^3 - (A*a^2*b^2 - B*a*b^3 + (A*a^2 -
 B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)
*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(
x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*c
os(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c +
(A*c^5 - (A*a^2 + B*a*b - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A
*b^4)*c)*cos(x) - (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a
^2*b + B*a*b^2 - 2*A*b^3)*c^2)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6
 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a
^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*
c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 209, normalized size = 1.85

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)+Ab^2\tan\left(\frac{1}{2}x\right)+Ac^2\tan\left(\frac{1}{2}x\right)-Bc^2\tan\left(\frac{1}{2}x\right)+Aac-Bbc\right)}{\left(a^3-a^2b-ab^2+b^3-ac^2+bc^2\right)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b)/(a^2 - b^2 - c^2)^(3/2) +
 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*tan(1/2*
x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + A*a*c - B*b*c)/((a^3 - a^2*b - a
*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*
x) + a + b))
```

Mupad [B]
time = 3.25, size = 205, normalized size = 1.81

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(A\,a-B\,b)}{\left(-a^2+b^2+c^2\right)^{3/2}}-\frac{\frac{2\,(A\,a\,c-B\,b\,c)}{(a-b)\,\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+B\,a^2+A\,c^2-B\,c^2-A\,a\,b-B\,a\,b\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2
))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - B*b))/(b^2 - a^2 + c^2)^(3/2) - ((2*
(A*a*c - B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 + B*a^2 +
 A*c^2 - B*c^2 - A*a*b - B*a*b))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*
tan(x/2) + tan(x/2)^2*(a - b))
```

3.537 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{\left(2a^2A-3abB+A(b^2+c^2)\right)\operatorname{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}+\frac{aBc+(\ldots}{}$$

```
[Out] (2*a^2*A-3*a*b*B+A*(b^2+c^2))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)
/(a+b*cos(x)+c*sin(x))^2+1/2*(a*B*c+(3*A*a-2*B*b)*c*cos(x)-(3*A*a*b-B*a^2-2
*B*b^2)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.19, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3237, 3232, 3203, 632, 210}

$$\frac{\left(2a^2A-3abB+A(b^2+c^2)\right)\operatorname{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{2\left(a^2-b^2-c^2\right)^2\left(a+b\cos(x)+c\sin(x)\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*B*c + (3*a*A
- 2*b*B)*c*Cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
```

```
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3237

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2
- b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x
], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 -
 b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(x)+Ac\,s}{(a+b\cos(x)+c\sin(x))^2}}{2\left(a^2-b^2-c^2\right)}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)}{2\left(a^2-b^2-c^2\right)^2(a}$$

$$= \frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc+Ac\cos}{2\left(a^2-b^2-c^2\right)}$$

Mathematica [A]

time = 0.62, size = 326, normalized size = 1.63

$$-\frac{(2a^2A-3abB+A(b^2+c^2))\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}+\frac{-6a^3Ac-3aAb^2c+9a^2bBc-3aAc^3-2bc(2a^2A-3abB+A(b^2+c^2))\cos(x)+c(-a^2bB+3aA(b^2+c^2)-2bB(b^2+c^2))\cos(2x)-8a^2Ab^2\sin(x)+2Ab^4\sin(x)+4a^3bB\sin(x)+2ab^3B\sin(x)-12a^2Ac^2\sin(x)+2Ab^2c^2\sin(x)+8abBc^2\sin(x)-3aAb^3\sin(2x)+a^2b^2B\sin(2x)+2b^4B\sin(2x)-3aAbc^2\sin(2x)+2b^2Bc^2\sin(2x)}{4b(-a^2+b^2+c^2)^2(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c +
 9*a^2*b*B*c - 3*a*A*c^3 - 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*Cos[x]
 + c*(-(a^2*b*B) + 3*a*A*(b^2 + c^2) - 2*b*B*(b^2 + c^2))*Cos[2*x] - 8*a^2*
A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^
2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] - 3*a*A*b^3*Sin[2*
x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] + 2*b^2*B
*c^2*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(190) = 380$.

time = 0.79, size = 853, normalized size = 4.26

method	result
default	$-\frac{\frac{\left(4A\,a^3b-7A\,a^2b^2-5A\,a^2c^2+2Aa\,b^3+2Aab\,c^2+A\,b^4+3A\,b^2c^2+2A\,c^4-2B\,a^4+3B\,a^3b-2B\,a^2b^2+4B\,a^2c^2+3Ba\,b^3-2B\,b^4-4B\,b^2c^2-2B\,c^4\right)\left(\tan^3\left(\right.\right.}{(a-b)\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan
(1/2*x)^3+1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2+7*A*a^2*c^2-6*A*a*b^3-6*A*
a*b*c^2+A*b^4-A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2-4*B*a^2*c^2-
9*B*a*b^3+2*B*b^4+4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^
2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c
^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^
2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4)/(a^4-2*a
^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*c*(4*A*a
^4-3*A*a^2*b^2-A*a^2*c^2-A*b^4-A*b^2*c^2-5*B*a^3*b+5*B*a*b^3+2*B*a*b*c^2)/(
a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2
-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+(2*A*a^2+A*b^2+A*c^2-3*B*a*b)/(a^4-2*
a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b
)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. 2(187) = 374.
time = 4.44, size = 3402, normalized size = 17.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*c^7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(3*B*a^4 - 3*A*a^3*b
- 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 4*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*
a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B
*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 +
A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*
b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6
 + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a
^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A
*a*b^3)*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 +
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2
 + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt
(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2
*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*
b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos
(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x
))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a
^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 3*A*a^3
*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(A*c^7 - (5*A*a^2 - B*a*b - 3
*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3
- (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*
cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 -
 A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (4*A*a^4*b + B*a^3
*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4*b^3 - 3*A*a^3*b^4 +
```

B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (3*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*((3*A*a*b - 2*B*b^2)*c^5 - (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 + (2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + (A*c^7 - (5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 - A*b^6)*c)*cos(x) + (2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 - B*a*b^6 - A*b^7 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^4 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c^2 + (B*a^4*b^3 - 3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 - (3*A*a - 2*B*b)*c^6 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(187) = 374.
time = 0.47, size = 1162, normalized size = 5.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
+ (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(1/2*x)^3 +
 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b^3*tan(1/2
*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b^4*tan(1/2
*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^
3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*t
an(1/2*x)^3 - A*a*b^2*c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b
^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2
*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 + 4
*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2
- 9*B*a^3*b*c*tan(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*t
an(1/2*x)^2 - 6*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c
*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 - 4*B*a^2
*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^2 + 4*B
*b^2*c^3*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a
^5*tan(1/2*x) - 4*A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*t
an(1/2*x) + B*a^3*b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1
/2*x) - 5*A*a*b^4*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*
B*b^5*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 3*A*a
^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1/2*x) +
 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) -
 2*A*a*c^4*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b
*c^4*tan(1/2*x) + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c + 5*B*a*b^3*c - A
*b^4*c - A*a^2*c^3 + 2*B*a*b*c^3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4
```

```
*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2
+ 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)
^2 + 2*c*tan(1/2*x) + a + b)^2)
```

Mupad [B]
time = 6.84, size = 946, normalized size = 4.73

$$-\frac{\frac{-4Aa^4c+5Ba^3bc+3Aa^2b^2c+Aa^2c^3-5Bab^3c-2Babc^3+Ab^4c+Ab^2c^3}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}-\frac{\tan\left(\frac{x}{2}\right)(Ab^5+2Ba^5+2Bb^5+3Aa^2b^3+5Aa^3b^2+11Aa^3c^2+Ba^2b^3+Ba^3b^2-Ab^3c^2-4Ba^3c^2+4Bb^3c^2-5Aab^4-4Aa^4b-2Aac^4-3Bab^4-3Ba^4b-2Abc^4+2Bac^4+2Bbc^4-7Aab^2c^2-3Aa^2bc^2+8Bab^2c^2-8Ba^2bc^2)}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}-\frac{\tan\left(\frac{x}{2}\right)^2(2Bc^5-2Ac^5+7Aa^2c^3-Ab^2c^3-4Ba^2c^3+4Bb^2c^3+4Aa^4c+Ab^4c+2Ba^4c+2Bb^4c-6Aabc^3-6Aab^3c-12Aa^3bc-9Bab^3c-9Ba^3bc+13Aa^2b^2c+14Ba^2b^2c)}{(a-b)^2(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}+\frac{\tan\left(\frac{x}{2}\right)^3(Ab^4-2Ba^4+2Ac^4-2Bb^4-2Bc^4-7Aa^2b^2-5Aa^2c^2-2Ba^2b^2+3Ab^2c^2+4Ba^2c^2-4Bb^2c^2+2Aab^3+4Aa^3b+3Bab^3+3Ba^3b+2Aabc^2)}{(a-b)(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}}{\tan\left(\frac{x}{2}\right)^4(a^2-2ab+b^2)+2ab+\tan\left(\frac{x}{2}\right)(4ac+4bc)+\tan\left(\frac{x}{2}\right)^3(4ac-4bc)+a^2+b^2+\tan\left(\frac{x}{2}\right)^2(2a^2-2b^2+4c^2)}-\frac{\mathrm{atanh}\left(\frac{2a^4c-4a^2b^2c-4a^2c^3+2b^4c+4b^2c^3+2c^5}{2(-a^2+b^2+c^2)^{5/2}}+\frac{\tan\left(\frac{x}{2}\right)(2a-2b)(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4)}{2(-a^2+b^2+c^2)^{5/2}}\right)(2Aa^2-3Bab+Ab^2+Ac^2)}{(-a^2+b^2+c^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((A*a^2*c^3 + A*b^2*c^3 - 4*A*a^4*c + A*b^4*c - 2*B*a*b*c^3 - 5*B*a*b^3*c
 + 5*B*a^3*b*c + 3*A*a^2*b^2*c)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5 + 3*A*a^2*b^3
 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3*c^2 - 4*B*a^3
*c^2 + 4*B*b^3*c^2 - 5*A*a*b^4 - 4*A*a^4*b - 2*A*a*c^4 - 3*B*a*b^4 - 3*B*a^
4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 7*A*a*b^2*c^2 - 3*A*a^2*b*c^2 + 8
*B*a*b^2*c^2 - 8*B*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*
a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*A*c^5 + 7*A*a^2*c^3 - A*b^
2*c^3 - 4*B*a^2*c^3 + 4*B*b^2*c^3 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*c + 2*B*b
^4*c - 6*A*a*b*c^3 - 6*A*a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c
 + 13*A*a^2*b^2*c + 14*B*a^2*b^2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^
2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*
b^4 - 2*B*c^4 - 7*A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B
*a^2*c^2 - 4*B*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 2*
A*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))
)/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(
x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) -
(atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2
*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*
b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*A*a^2 + A*b^2
 + A*c^2 - 3*B*a*b))/(b^2 - a^2 + c^2)^(5/2)
```

3.538 $\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{(2aA-bB)x}{2a^2}+\frac{iB\cos(x)}{2a}+\frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{B\sin(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2+1/2*I*B*cos(x)/a+1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*c
os(x)+I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3211}

$$\frac{i(a^2(-B)+2aAb-b^2B)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*Cos[x])/a + ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3211

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}+\frac{iB\cos(x)}{2a}+\frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)+ib\sin$$

Mathematica [A]
time = 0.16, size = 147, normalized size = 1.75

$$\frac{2aAbx+a^2Bx-b^2Bx+2(-2aAb+a^2B+b^2B)\,\text{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iabB\cos(x)+2iaAb\log\left(a^2+b^2+2ab\cos(x)\right)-ia^2B\log\left(a^2+b^2+2ab\cos(x)\right)-ib^2B\log\left(a^2+b^2+2ab\cos(x)\right)+2abB\sin(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] + (2*I)*a*b*B*Cos[x] + (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] - I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] - I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

Maple [A]
time = 0.30, size = 104, normalized size = 1.24

method	result	siz
risch	$\frac{iB\,e^{-ix}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}+\frac{i\ln\left(e^{ix}+\frac{a}{b}\right)A}{a}-\frac{i\ln\left(e^{ix}+\frac{a}{b}\right)B}{2b}-\frac{ib\ln\left(e^{ix}+\frac{a}{b}\right)B}{2a^2}$	86
default	$\frac{i(2aAb-a^2B-B\,b^2)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^2b}-\frac{i(2aA-bB)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{B}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(2*A*a*b-B*a^2-B*b^2)/a^2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))-1/2
*I*(2*A*a-B*b)/a^2*ln(-I+tan(1/2*x))+B/a/(-I+tan(1/2*x))+1/2*I*B/b*ln(tan(1
/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.00, size = 72, normalized size = 0.86

$$\frac{\left(i\,Bab+(2\,Aab-Bb^2)xe^{(i\,x)}+(-i\,Ba^2+2i\,Aab-i\,Bb^2)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*B*a*b + (2*A*a*b - B*b^2)*x*e^(I*x) + (-I*B*a^2 + 2*I*A*a*b - I*B*b^
2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A]
time = 0.32, size = 94, normalized size = 1.12

$$\begin{cases} \frac{iBe^{-ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{2Aa-Bb}{2a^2} + \frac{2Aa+Ba-Bb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-Bb)}{2a^2} - \frac{i(-2Aab+Ba^2+Bb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((I*B*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - B*b)/(2*a**2) + (2
*A*a + B*a - B*b)/(2*a**2)), True)) + x*(2*A*a - B*b)/(2*a**2) - I*(-2*A*a*
b + B*a**2 + B*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.
time = 0.41, size = 168, normalized size = 2.00

$$-\frac{(-2i\,Aa+i\,Bb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+b\tan\left(\frac{1}{2}\,x\right)^2-2i\,a\tan\left(\frac{1}{2}\,x\right)+a+b\right)}{4\,a^2}-\frac{(2i\,Aa-i\,Bb)\log\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}{2\,a^2}+\frac{(2\,Ba^2-2\,Aab+Bb^2)\left(x+2\,\arctan\left(\frac{-i\,a\cos(x)-a\sin(x)-i\,a}{a\cos(x)-i\,a\sin(x)-a+2\,b}\right)\right)}{4\,a^2b}-\frac{-2i\,Aa\tan\left(\frac{1}{2}\,x\right)+i\,Bb\tan\left(\frac{1}{2}\,x\right)-2\,Aa-2\,Ba+Bb}{2\,a^2\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a + I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/
2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*
B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*co
s(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*t
an(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) - I))
```

Mupad [B]
time = 4.33, size = 99, normalized size = 1.18

$$\frac{B}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{B\,\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(A\,a\,1\mathrm{i}-\frac{B\,b\,1\mathrm{i}}{2}\right)}{a^2}-\frac{\ln\left(a+b-a\,\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\,\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)\,1\mathrm{i}}{2\,a^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] B/(a*(tan(x/2) - 1i)) + (B*log(tan(x/2) + 1i)*1i)/(2*b) - (log(tan(x/2) - 1
i)*(A*a*1i - (B*b*1i)/2))/a^2 - (log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)
*(B*a^2 + B*b^2 - 2*A*a*b)*1i)/(2*a^2*b)
```

3.539 $\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=84

$$\frac{(2aA-bB)x}{2a^2}-\frac{iB\cos(x)}{2a}-\frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}+\frac{B\sin(x)}{2a}$$

```
[Out] 1/2*(2*A*a-B*b)*x/a^2-1/2*I*B*cos(x)/a-1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*c
os(x)-I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3211}

$$-\frac{i(a^2(-B)+2aAb-b^2B)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B -
b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

Rule 3211

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB)x}{2a^2}-\frac{iB\cos(x)}{2a}-\frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)-ib\sin($$

Mathematica [A]
time = 0.13, size = 147, normalized size = 1.75

$$\frac{2aAbx+a^2Bx-b^2Bx+2(-2aAb+a^2B+b^2B)\,\text{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iabB\cos(x)-2iaAb\log\left(a^2+b^2+2ab\cos(x)\right)+ia^2B\log\left(a^2+b^2+2ab\cos(x)\right)+ib^2B\log\left(a^2+b^2+2ab\cos(x)\right)+2abB\sin(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
b)*Cot[x/2])/(a - b)] - (2*I)*a*b*B*Cos[x] - (2*I)*a*A*b*Log[a^2 + b^2 + 2*
a*b*Cos[x]] + I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*b^2*B*Log[a^2 + b^2
 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

Maple [A]
time = 0.30, size = 116, normalized size = 1.38

method	result
risch	$-\frac{iB\,\mathrm{e}^{ix}}{2a}+\frac{Bx}{2b}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)A}{a}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$-\frac{iB\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}+\frac{i\left(2aAb-a^2B-B\,b^2\right)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}+\frac{i(2aA-bB)\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2a^2}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)+i\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*B/b*ln(-I+tan(1/2*x))+1/2*I*(2*A*a*b-B*a^2-B*b^2)*(a-b)/a^2/b/(-a+b)
*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))+1/2*I*(2*A*a-B*b)/a^2*ln(tan(1/2*x)+
I)+B/a/(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.68, size = 56, normalized size = 0.67

$$\frac{Ba^2x-i\,Babe^{(i\,x)}+\left(i\,Ba^2-2i\,Aab+i\,Bb^2\right)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*x - I*B*a*b*e^(I*x) + (I*B*a^2 - 2*I*A*a*b + I*B*b^2)*log((a*e^(
I*x) + b)/a))/(a^2*b)
```

Sympy [A]
time = 0.32, size = 73, normalized size = 0.87

$$\frac{Bx}{2b} + \begin{cases} -\frac{iBe^{ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{B}{2b} + \frac{Ba+Bb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{i(-2Aab + Ba^2 + Bb^2)\log\left(e^{ix} + \frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] B*x/(2*b) + Piecewise((-I*B*exp(I*x)/(2*a), Ne(a, 0)), (x*(-B/(2*b) + (B*a
+ B*b)/(2*a*b)), True)) + I*(-2*A*a*b + B*a**2 + B*b**2)*log(exp(I*x) + b/a
)/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.
time = 0.41, size = 168, normalized size = 2.00

$$-\frac{(2iAa - iBb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 + 2ia\tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2} - \frac{(-2iAa + iBb)\log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2} + \frac{(2Ba^2 - 2Aab + Bb^2)\left(x + 2\arctan\left(\frac{ia\cos(x) - a\sin(x) + ia}{a\cos(x) + ia\sin(x) - a + 2b}\right)\right)}{4a^2b} - \frac{2iAa\tan\left(\frac{1}{2}x\right) - iBb\tan\left(\frac{1}{2}x\right) - 2Aa - 2Ba + Bb}{2a^2\left(\tan\left(\frac{1}{2}x\right) + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*A*a - I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2
*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*
B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*cos
(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan
(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) + I))
```

Mupad [B]
time = 8.39, size = 584, normalized size = 6.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] symsum(log(tan(x/2)*(a - b)^2*(B*a - 2*A*a + B*b)^2 - root(a^4*b^2*d^3*64i
- A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i
 + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*
a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*(4*A*a^3*(a - b)^2 - 8*root(a^4*b^2
*d^3*64i - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*
b^2*d*64i + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2
 - 8*B^3*a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*a^2*(a - b)^2*(a^2*tan(x/2
) + b^2*tan(x/2) - a^2*1i + b^2*1i - a*b*tan(x/2)) + 4*a*tan(x/2)*(a - b)^2
```

```
*(A*a^2*1i + B*a^2*1i + B*b^2*1i - A*a*b*2i - B*a*b*1i)) - (a - b)^2*(4*A^2
*a^2 - B^2*a^2 + B^2*b^2 - 4*A*B*a*b)*1i)*root(a^4*b^2*d^3*64i - A*B*a*b^3*
d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i + B^2*b^4*d
*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*a^2*b + 16*A
*B^2*a^3 - 8*B^3*b^3, d, k), k, 1, 3) + B/(a*(tan(x/2) + 1i))
```

3.540 $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=116

$$\frac{cCx}{b^2+c^2}+\frac{2(A(b^2+c^2)-acC)\operatorname{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] c*C*x/(b^2+c^2)-b*C*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2+c^2)-a*c*C)
*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3216, 3203, 632, 210}

$$\frac{2(A(b^2+c^2)-acC)\operatorname{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] (c*C*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*c*C)*ArcTan[(c + (a - b)*Tan[x/
2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*C*Log[
a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3216

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[b*C*(Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(A-\frac{acC}{b^2+c^2}\right)\int\frac{1}{a+b\cos(x)\ \cdots}\\ &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\left(2\left(A-\frac{acC}{b^2+c^2}\right)\right)\mathrm{Subst}\left(\int \cdots\right.\\ &= \frac{cCx}{b^2+c^2}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\left(4\left(A-\frac{acC}{b^2+c^2}\right)\right)\mathrm{Subst}\left(\int \cdots\right.\\ &= \frac{cCx}{b^2+c^2}+\frac{2\left(A-\frac{acC}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\ \cdots}{b^2+c^2}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 96, normalized size = 0.83

$$\frac{-\frac{2\left(A\left(b^2+c^2\right)-acC\right)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+C(cx-b\log(a+b\cos(x)+c\sin(x)))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((-2*(A*(b^2 + c^2) - a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2
 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + C*(c*x - b*Log[a + b*Cos[x] + c*Sin[x]])
)/(b^2 + c^2)
```

Maple [A]
time = 0.44, size = 187, normalized size = 1.61

method	result
default	$\frac{\frac{2\left(-abC+b^{2}C\right)\ln\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(A\,b^{2}+A\,c^{2}-acC-Cbc-\frac{\left(-abC+b^{2}C\right)c}{a-b}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\sqrt{a^{2}-b^{2}-c^{2}}}}{b^{2}+c^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(1/2*(-C*a*b+C*b^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*
tan(1/2*x)+a+b)+(A*b^2+A*c^2-a*c*C-C*b*c-(-C*a*b+C*b^2)*c/(a-b))/(a^2-b^2-c
^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))+2*C/(b^
2+c^2)*(1/2*b*ln(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(110) = 220.
time = 4.48, size = 625, normalized size = 5.39

$$\left[\frac{(Ab^2-Cac+Ac^2)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^3)c)\cos(x))\sin(x)-2\left(2abc\cos(x)^2-abc+(b^2c+c^3)\cos(x)-(b^3+bc^2+(ab^2-ac^2)\cos(x))\sin(x)\right)\sqrt{-a^2+b^2+c^2}}{2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)}\right)-2(Cc^3-(Ca^2-Cb^2)c)x-(Ca^2b-Cb^3-Cbc^2)\log(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x))}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)}, \frac{2(Ab^2-Cac+Ac^2)\sqrt{a^2-b^2-c^2}\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)-2(Cc^3-(Ca^2-Cb^2)c)x-(Ca^2b-Cb^3-Cbc^2)\log(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x))}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b^2 - C*a*c + A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 -
 c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2
*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c
)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^
3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*
cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
```

```
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), 1/2*(2*(A*b^2 - C*a*c + A*c^2)*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^
2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 177, normalized size = 1.53

$$\frac{Ccx}{b^2+c^2}-\frac{Cb\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}-\frac{2\left(Ab^2-Cac+Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] C*c*x/(b^2 + c^2) - C*b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2*
x) - a - b)/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - 2*(A*b^2
- C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*
(b^2 + c^2))
```

Mupad [**B**]
time = 24.86, size = 1741, normalized size = 15.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (C*log(tan(x/2) + 1i))/(b - c*1i) + (C*log(tan(x/2) - 1i)*1i)/(b*1i - c) -
(log(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b
*c + 32*C*tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b^3
+ A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) +
 C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c + 32*C^2*a^2*c + 32
```

*C^2*b^2*c + 64*A*C*b^3 + 32*tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*C^2*a^2 + 2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*A*b^4 + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*A*b^2*c^2 - 32*tan(x/2)*(a - b)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c) - 64*A*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c - 128*C*a*b^2*c + 64*C*a^2*b*c + (32*(a - b)*(C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - b*(C*a^2 - C*c^2) + A*b^2*(b^2 - a^2 + c^2)^(1/2) + A*c^2*(b^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (log(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*C*tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 64*A*C*b^3 + 32*tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*C^2*a^2 + 2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*A*b^4 + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*A*b^2*c^2 - 32*tan(x/2)*(a - b)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c) - 64*A*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c - 128*C*a*b^2*c + 64*C*a^2*b*c + (32*(a - b)*(C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*(C*a^2 - C*c^2) - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) + A*c^2*(b^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))

3.541 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=114

$$\frac{2(aA-cC)\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}-\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

```
[Out] 2*(A*a-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)^
(3/2)+(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x)
)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3233, 3203, 632, 210}

$$\frac{2(aA-cC)\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2 -
 b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))} + \frac{(aA-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))} + \frac{(2(aA-cC))\text{Subst}\left(\int \frac{1}{a+b+2cx-}\right.}{a^2-b^2-}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))} - \frac{(4(aA-cC))\text{Subst}\left(\int \frac{1}{-4(a^2-b^2}\right.}{a^2}$$

$$= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}} - \frac{bC-(Ac-aC)\cos(x)+Ab\sin}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin}$$

Mathematica [A]
time = 0.28, size = 123, normalized size = 1.08

$$\frac{2(aA-cC)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} + \frac{aAc-a^2C+b^2C+(A(b^2+c^2)-acC)\sin(x)}{b(-a^2+b^2+c^2)(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b^2*C + (A*(b^2 + c^2) - a*c*C)*Sin
[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.34, size = 207, normalized size = 1.82

method	result
default	$\frac{-\frac{2\left(aAb-A\,b^{2}-A\,c^{2}+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{2\left(aAc-a^{2}C+b^{2}C\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}}{a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(aA-Cc)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2i\left(-iA\,b^{2}-iA\,c^{2}+iacC-iAab\,\mathrm{e}^{ix}+iCbc\,\mathrm{e}^{ix}-Aac\,\mathrm{e}^{ix}+C\,a^{2}\mathrm{e}^{ix}-C\,b^{2}\mathrm{e}^{ix}\right)}{\left(a^{2}-b^{2}-c^{2}\right)(ib+c)\left(-ic\,\mathrm{e}^{2ix}+b\,\mathrm{e}^{2ix}+ic+2a\,\mathrm{e}^{ix}+b\right)}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{iac\sqrt{-a^{2}+b^{2}+c^{2}}+ia^{2}b-ib^{3}-ib\,c^{2}+ab}{\left(b^{2}+c^{2}\right)\sqrt{-a^{2}+}}\right.}{\sqrt{-a^{2}+b^{2}+c^{2}}\,(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)

[Out] 2*(-(A*a*b-A*b^2-A*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1
/2*x)+(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)^
2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2*(A*a-C*c)/(a^2-b^2-c^2)^(3/2)*arctan
(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(107) = 214$.
time = 4.02, size = 1301, normalized size = 11.41

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")

[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 4*(C*a^2*b - C*b^3)*c
^2 - (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c + A*
a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x
))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 -
 (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x)
- 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(2*a*

```
b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2
)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x
)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(C*a*c^4 - A*c^5 + (A*a
^2 - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) -
 2*(A*a^2*b^3 - A*b^5 + C*a*b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*
a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 -
 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^
7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x)
 + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 -
2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - 2*(
C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*
b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*
c^3 - C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x)
 + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b
- b^3 - b*c^2)*sin(x))) + (C*a*c^4 - A*c^5 + (A*a^2 - 2*A*b^2)*c^3 - (C*a^3
 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c)*cos(x) - (A*a^2*b^3 - A*b^5 + C*a*
b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3)*c^2 - (C*a^3*b - C*a*b^3)*c)*sin(x))/
(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3
*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3
)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^
5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 206, normalized size = 1.81

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Cc)}{(a^2-b^2-c^2)^{\frac{3}{2}}}-\frac{2\left(Aab\tan\left(\frac{1}{2}x\right)-Ab^2\tan\left(\frac{1}{2}x\right)+Cac\tan\left(\frac{1}{2}x\right)-Cbc\tan\left(\frac{1}{2}x\right)-Ac^2\tan\left(\frac{1}{2}x\right)+Ca^2-Cb^2-Aac\right)}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - C*c)/(a^2 - b^2 - c^2)^(3/2) -
 2*(A*a*b*tan(1/2*x) - A*b^2*tan(1/2*x) + C*a*c*tan(1/2*x) - C*b*c*tan(1/2*
x) - A*c^2*tan(1/2*x) + C*a^2 - C*b^2 - A*a*c)/((a^3 - a^2*b - a*b^2 + b^3
```

```
- a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)
)
```

Mupad [B]
time = 3.19, size = 204, normalized size = 1.79

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(A\,a-C\,c)}{\left(-a^2+b^2+c^2\right)^{3/2}}-\frac{\frac{2\,\left(-C\,a^2+A\,c\,a+C\,b^2\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+C\,b\,c-A\,a\,b+A\,c^2-C\,a\,c\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2
))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - C*c))/(b^2 - a^2 + c^2)^(3/2) - ((2*
(C*b^2 - C*a^2 + A*a*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 +
 A*c^2 - A*a*b - C*a*c + C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*
tan(x/2) + tan(x/2)^2*(a - b))
```

3.542 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=200

$$\frac{(2a^2A+A(b^2+c^2)-3acC)\,\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}-\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\,(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))^2}-\frac{abC-(}{}$$

```
[Out] (2*a^2*A+A*(b^2+c^2)-3*a*c*C)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/
2))/(a^2-b^2-c^2)^(5/2)+1/2*(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2
)/(a+b*cos(x)+c*sin(x))^2+1/2*(-a*b*C+(3*A*a*c-C*a^2-2*C*c^2)*cos(x)-b*(3*A
*a-2*C*c)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.18, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3236, 3232, 3203, 632, 210}

$$\frac{(2a^2A-3acC+A(b^2+c^2))\,\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2
 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*S
in[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^2) - (a*b*C - (3*a*A*
c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(2*(a^2 - b^2 - c^2
)^2*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
```

```
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3236

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b
^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*
(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^
2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{\int \frac{-2(aA-cC)+Ab\cos(x)+(Ac-aC}{(a+b\cos(x)+c\sin(x))^2}}{2\left(a^2-b^2-c^2\right)}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{abC-\left(3aAc-a^2C-2c^2\right.}{2\left(a^2-b^2-c^2\right)^2(}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{abC-\left(3aAc-a^2C-2c^2\right.}{2\left(a^2-b^2-c^2\right)^2(}$$

$$= -\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{abC-\left(3aAc-a^2C-2c^2\right.}{2\left(a^2-b^2-c^2\right)^2(}$$

$$= \frac{\left(2a^2A+A\left(b^2+c^2\right)-3acC\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}-\frac{bC-(Ac-}{2\left(a^2-b^2-c^2\right)}$$

Mathematica [A]

time = 0.65, size = 361, normalized size = 1.80

$$-\frac{(2a^2A+A(b^2+c^2)-3acC)\tanh^{-1}\left(\frac{c+(a-b)\tan(\frac{x}{2})}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}+\frac{-6a^3Ac-3aAb^2c-3aAc^3+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C-2bc(2a^2A+A(b^2+c^2)-3acC)\cos(x)-c(-3aA(b^2+c^2)+a^2cC+2c(b^2+c^2)C)\cos(2x)-8a^2Ab^2\sin(x)+2Ab^4\sin(x)-12a^2Ac^2\sin(x)+2Ab^2c^2\sin(x)+4a^3cC\sin(x)+2ab^2cC\sin(x)+8ac^3C\sin(x)-3aAb^3\sin(2x)-3aAbc^2\sin(2x)+a^2bcC\sin(2x)+2b^3cC\sin(2x)+2bc^3C\sin(2x)}{4b(-a^2+b^2+c^2)^2(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[
-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c -
 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C +
2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*Cos[x] - c*(-3*a*A*(b^2
 + c^2) + a^2*c*C + 2*c*(b^2 + c^2)*C)*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*
b^4*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 4*a^3*c*C*Sin[x] +
2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] - 3*a*A*b*c^2*Si
n[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x]])/(4*b
*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(192) = 384$.

time = 0.79, size = 832, normalized size = 4.16

method	result
default	$-\frac{\left(4Aa^3b-7Aa^2b^2-5Aa^2c^2+2Aab^3+2Aabc^2+Ab^4+3Ab^2c^2+2Ac^4+3Ca^3c-6Ca^2bc+3Cab^2c\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)(a-b)}+\left(4Aa^4c-12Aa^3bc+13Aa^2b^2c+7\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2
+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*a^2
*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5-2*C*a^
5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13
*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*
c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*
b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^
4-A*b^5+A*b^3*c^2+2*A*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3
+5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-
2*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*
c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a
^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2
*c*tan(1/2*x)+a+b)^2+(2*A*a^2+A*b^2+A*c^2-3*C*a*c)/(a^4-2*a^2*b^2-2*a^2*c^2
```

```
+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)
/(a^2-b^2-c^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. $2(187) = 374$.
time = 4.34, size = 3513, normalized size = 17.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 + 6*A*a*b*c^5 - 6*C*b
*c^6 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 - 6*(A*a^3*b - 2*A*a*b^3)*c^3 - 2*(2*C*a
^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 4*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b -
 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^
5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 - (2*A*a^4*b^2 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a
*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*
C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2
 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2
*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c
^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 +
 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)
*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
+ 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*
b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*si
n(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
(a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 6*(A*a^3*b^
3 - A*a*b^5)*c + 2*(C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2)*c^5 + (C*a^3 + 2*
C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^
2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)*c)*cos(x) - 2*(4*A*a
```

^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*A*a^2*b - 3*A*b^3)*
c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*b^3 + 3*A*b^5)*c^2
- (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3*A*a*b^6 - 3*A*a*b^
4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*(A*a^3 - A*a*b^2)*c
^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*s
in(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*
c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4
 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6
 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4
)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*
cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b
- 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 +
 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a
^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3
 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)
c^7 + 3(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 -
 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1
/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 + 3*A*a*b*c^5 - 3*C*b*c^6 +
 (4*C*a^2*b - 7*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 - (2*C*a^4*b - 7*C
*a^2*b^3 + 5*C*b^5)*c^2 - 2*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*b - 4*C*b^3)*
c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - 3
*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 + (2*A*a^4*b^2 + A*a^2*b^4 - 3*C*a^3*b^2
*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3
+ (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b^6 - 3*C*a*b^4*c
+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*
a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + 2*(A*a^3*b
+ A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - 2*(A*
a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c
^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*
sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)
sin(x))) - 3(A*a^3*b^3 - A*a*b^5)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b
^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^3
- (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 + (4*A*a^4*b^2 - 5*A*a^2*b^4 + A*b^6)
*c)*cos(x) - (4*A*a^4*b^3 - 5*A*a^2*b^5 + A*b^7 + C*a*b*c^5 + A*b*c^6 - (5*
A*a^2*b - 3*A*b^3)*c^4 + (C*a^3*b + 2*C*a*b^3)*c^3 + (4*A*a^4*b - 10*A*a^2*
b^3 + 3*A*b^5)*c^2 - (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (3*A*a^3*b^4 - 3
*A*a*b^6 - 3*A*a*b^4*c^2 + 3*A*a*c^6 - 2*C*c^7 + (C*a^2 - 2*C*b^2)*c^5 - 3*
(A*a^3 - A*a*b^2)*c^4 + (C*a^4 + 2*C*b^4)*c^3 - (C*a^4*b^2 + C*a^2*b^4 - 2*
C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10
 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a
^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (
a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4
- 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a
^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 -

```
a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(187) = 374$.
time = 0.48, size = 1054, normalized size = 5.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(2*A*a^2 + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a
^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2))
- (4*A*a^4*b*tan(1/2*x)^3 - 11*A*a^3*b^2*tan(1/2*x)^3 + 9*A*a^2*b^3*tan(1/2
*x)^3 - A*a*b^4*tan(1/2*x)^3 - A*b^5*tan(1/2*x)^3 + 3*C*a^4*c*tan(1/2*x)^3
- 9*C*a^3*b*c*tan(1/2*x)^3 + 9*C*a^2*b^2*c*tan(1/2*x)^3 - 3*C*a*b^3*c*tan(1
/2*x)^3 - 5*A*a^3*c^2*tan(1/2*x)^3 + 7*A*a^2*b*c^2*tan(1/2*x)^3 + A*a*b^2*c
^2*tan(1/2*x)^3 - 3*A*b^3*c^2*tan(1/2*x)^3 + 2*A*a*c^4*tan(1/2*x)^3 - 2*A*b
*c^4*tan(1/2*x)^3 + 2*C*a^5*tan(1/2*x)^2 - 2*C*a^4*b*tan(1/2*x)^2 - 4*C*a^3
*b^2*tan(1/2*x)^2 + 4*C*a^2*b^3*tan(1/2*x)^2 + 2*C*a*b^4*tan(1/2*x)^2 - 2*C
*b^5*tan(1/2*x)^2 - 4*A*a^4*c*tan(1/2*x)^2 + 12*A*a^3*b*c*tan(1/2*x)^2 - 13
*A*a^2*b^2*c*tan(1/2*x)^2 + 6*A*a*b^3*c*tan(1/2*x)^2 - A*b^4*c*tan(1/2*x)^2
 + 5*C*a^3*c^2*tan(1/2*x)^2 - 14*C*a^2*b*c^2*tan(1/2*x)^2 + 13*C*a*b^2*c^2*
tan(1/2*x)^2 - 4*C*b^3*c^2*tan(1/2*x)^2 - 7*A*a^2*c^3*tan(1/2*x)^2 + 6*A*a*
b*c^3*tan(1/2*x)^2 + A*b^2*c^3*tan(1/2*x)^2 + 2*C*a*c^4*tan(1/2*x)^2 - 2*C*
b*c^4*tan(1/2*x)^2 + 2*A*c^5*tan(1/2*x)^2 + 4*A*a^4*b*tan(1/2*x) - 5*A*a^3*
b^2*tan(1/2*x) - 3*A*a^2*b^3*tan(1/2*x) + 5*A*a*b^4*tan(1/2*x) - A*b^5*tan(
1/2*x) + 5*C*a^4*c*tan(1/2*x) - 5*C*a^3*b*c*tan(1/2*x) - 5*C*a^2*b^2*c*tan(
1/2*x) + 5*C*a*b^3*c*tan(1/2*x) - 11*A*a^3*c^2*tan(1/2*x) + 3*A*a^2*b*c^2*t
an(1/2*x) + 7*A*a*b^2*c^2*tan(1/2*x) + A*b^3*c^2*tan(1/2*x) + 4*C*a^2*c^3*t
an(1/2*x) - 4*C*a*b*c^3*tan(1/2*x) + 2*A*a*c^4*tan(1/2*x) + 2*A*b*c^4*tan(1
/2*x) + 2*C*a^5 - 4*C*a^3*b^2 + 2*C*a*b^4 - 4*A*a^4*c + 3*A*a^2*b^2*c + A*b
^4*c + C*a^3*c^2 - C*a*b^2*c^2 + A*a^2*c^3 + A*b^2*c^3)/((a^6 - 2*a^5*b - a
^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*
```

```
a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*
tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)
```

Mupad [B]
time = 6.73, size = 912, normalized size = 4.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c +
 A*b^4*c + 2*C*a*b^4 + 3*A*a^2*b^2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 +
 c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)*(A*b^3*c^2 - 3*A*a^2
*b^3 - 5*A*a^3*b^2 - 11*A*a^3*c^2 - A*b^5 + 4*C*a^2*c^3 + 5*A*a*b^4 + 4*A*a
^4*b + 2*A*a*c^4 + 2*A*b*c^4 + 5*C*a^4*c - 4*C*a*b*c^3 + 5*C*a*b^3*c - 5*C*
a^3*b*c + 7*A*a*b^2*c^2 + 3*A*a^2*b*c^2 - 5*C*a^2*b^2*c))/((a - b)^2*(a^4 +
 b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^2*(2*A*c^5 + 2
*C*a^5 - 2*C*b^5 - 7*A*a^2*c^3 + A*b^2*c^3 + 4*C*a^2*b^3 - 4*C*a^3*b^2 + 5*
C*a^3*c^2 - 4*C*b^3*c^2 - 4*A*a^4*c - A*b^4*c + 2*C*a*b^4 - 2*C*a^4*b + 2*C
*a*c^4 - 2*C*b*c^4 + 6*A*a*b*c^3 + 6*A*a*b^3*c + 12*A*a^3*b*c - 13*A*a^2*b^
2*c + 13*C*a*b^2*c^2 - 14*C*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2
*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 + 2*A*c^4 - 7*A*a^2*b^2
 - 5*A*a^2*c^2 + 3*A*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*C*a^3*c + 2*A*a*b*
c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*
(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^
2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2
*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/2)*(2*a - 2*b)*(a^
4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5
/2)))*(2*A*a^2 + A*b^2 + A*c^2 - 3*C*a*c))/(b^2 - a^2 + c^2)^(5/2)
```

3.543 $\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(2aA-ibC)x}{2a^2}-\frac{C\cos(x)}{2a}+\frac{(2iaAb-a^2C+b^2C)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{iC\sin(x)}{2a}$$

```
[Out] 1/2*(2*a*A-I*b*C)*x/a^2-1/2*C*cos(x)/a+1/2*(2*I*a*A*b-a^2*C+b^2*C)*ln(a+b*c
os(x)+I*b*sin(x))/a^2/b+1/2*I*C*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3210}

$$\frac{(a^2(-C)+2iaAb+b^2C)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b^
2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a
```

Rule 3210

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-ibC)x}{2a^2}-\frac{C\cos(x)}{2a}+\frac{(2iaAb-a^2C+b^2C)\log(a+b\cos(x)+ib\sin$$

Mathematica [A]
time = 0.20, size = 152, normalized size = 1.79

$$\frac{2aAbx-ia^2Cx-ib^2Cx+(-4aAb-2ia^2C+2ib^2C)\,\mathrm{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2abC\cos(x)+2iaAb\log\left(a^2+b^2+2ab\cos(x)\right)-a^2C\log\left(a^2+b^2+2ab\cos(x)\right)+b^2C\log\left(a^2+b^2+2ab\cos(x)\right)+2iabC\sin(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x - I*a^2*C*x - I*b^2*C*x + (-4*a*A*b - (2*I)*a^2*C + (2*I)*b^2*C)
*ArcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] + (2*I)*a*A*b*Log[a^2
+ b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2
 + b^2 + 2*a*b*Cos[x]] + (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

Maple [A]
time = 0.27, size = 107, normalized size = 1.26

method	result	s
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^{2}}+\frac{xA}{a}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2a^{2}}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)A}{a}$	8
default	$\frac{i\left(iC\,a^{2}-iC\,b^{2}+2aAb\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^{2}b}+\frac{iC}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{\left(-2iAa-bC\right)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^{2}}+\frac{C\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}$	-

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(I*C*a^2-I*C*b^2+2*a*A*b)/a^2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))
+I*C/a/(-I+tan(1/2*x))+1/2/a^2*(-2*I*A*a-b*C)*ln(-I+tan(1/2*x))+1/2*C/b*ln(
tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.99, size = 71, normalized size = 0.84

$$-\frac{\left(Cab-\left(2\,Aab-i\,Cb^{2}\right)xe^{(i\,x)}+\left(Ca^{2}-2i\,Aab-Cb^{2}\right)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(C*a*b - (2*A*a*b - I*C*b^2)*x*e^(I*x) + (C*a^2 - 2*I*A*a*b - C*b^2)*e
^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A]
time = 0.42, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{Ce^{-ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{2Aa-iCb}{2a^2} + \frac{2Aa+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-iCb)}{2a^2} - \frac{(-2iAab+Ca^2-Cb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise((-C*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - I*C*b)/(2*a**2) + (
2*A*a + I*C*a - I*C*b)/(2*a**2)), True)) + x*(2*A*a - I*C*b)/(2*a**2) - (-2
*I*A*a*b + C*a**2 - C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.
time = 0.43, size = 169, normalized size = 1.99

$$-\frac{(-2i\,Aa - Cb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2 + b\tan\left(\frac{1}{2}\,x\right)^2 - 2i\,a\tan\left(\frac{1}{2}\,x\right) + a + b\right)}{4\,a^2} - \frac{(2i\,Aa + Cb)\log\left(\tan\left(\frac{1}{2}\,x\right) - i\right)}{2\,a^2} - \frac{(2i\,Ca^2 + 2\,Aab - i\,Cb^2)\left(x + 2\,\arctan\left(\frac{-i\,a\cos(x) - a\sin(x) - i\,a}{a\cos(x) - i\,a\sin(x) - a + 2\,b}\right)\right)}{4\,a^2b} - \frac{-2i\,Aa\tan\left(\frac{1}{2}\,x\right) - Cb\tan\left(\frac{1}{2}\,x\right) - 2\,Aa - 2i\,Ca + i\,Cb}{2\,a^2\left(\tan\left(\frac{1}{2}\,x\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*
x) + a + b)/a^2 - 1/2*(2*I*A*a + C*b)*log(tan(1/2*x) - I)/a^2 - 1/4*(2*I*C*
a^2 + 2*A*a*b - I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a*co
s(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) - C*b*tan
(1/2*x) - 2*A*a - 2*I*C*a + I*C*b)/(a^2*(tan(1/2*x) - I))
```

Mupad [B]
time = 4.35, size = 96, normalized size = 1.13

$$\ln\left(a + b - a\tan\left(\frac{x}{2}\right)\,1\mathrm{i} + b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\frac{C\,b}{2\,a^2} - \frac{C}{2\,b} + \frac{A\,1\mathrm{i}}{a}\right) + \frac{C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)} + \frac{C\,\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)}{2\,b} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)\,\left(\frac{C\,b}{2} + A\,a\,1\mathrm{i}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)*((A*1i)/a - C/(2*b) + (C*b)/(2*a
^2)) + (C*1i)/(a*(tan(x/2) - 1i)) + (C*log(tan(x/2) + 1i))/(2*b) - (log(tan
(x/2) - 1i)*(A*a*1i + (C*b)/2))/a^2
```

3.544 $\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=85

$$\frac{(2aA+ibC)x}{2a^2}-\frac{C\cos(x)}{2a}-\frac{(2iaAb+a^2C-b^2C)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{iC\sin(x)}{2a}$$

```
[Out] 1/2*(2*a*A+I*b*C)*x/a^2-1/2*C*cos(x)/a-1/2*(2*I*a*A*b+a^2*C-b^2*C)*ln(a+b*c
os(x)-I*b*sin(x))/a^2/b-1/2*I*C*sin(x)/a
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3210}

$$-\frac{(a^2C+2iaAb-b^2C)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A + I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b^
2*C)*Log[a + b*Cos[x] - I*b*Sin[x]])/(2*a^2*b) - ((I/2)*C*Sin[x])/a
```

Rule 3210

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA+ibC)x}{2a^2}-\frac{C\cos(x)}{2a}-\frac{(2iaAb+a^2C-b^2C)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{iC\sin(x)}{2a}$$

Mathematica [A]
time = 0.18, size = 152, normalized size = 1.79

$$\frac{2aAbx+ia^2Cx+ib^2Cx+2i(2iaAb+a^2C-b^2C)\,\text{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2abC\cos(x)-2iaAb\log\left(a^2+b^2+2ab\cos(x)\right)-a^2C\log\left(a^2+b^2+2ab\cos(x)\right)+b^2C\log\left(a^2+b^2+2ab\cos(x)\right)-2iabC\sin(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] (2*a*A*b*x + I*a^2*C*x + I*b^2*C*x + (2*I)*((2*I)*a*A*b + a^2*C - b^2*C)*Ar
cTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] - (2*I)*a*A*b*Log[a^2 + b
^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2 +
b^2 + 2*a*b*Cos[x]] - (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

Maple [A]
time = 0.29, size = 119, normalized size = 1.40

method	result
risch	$-\frac{C\,\mathrm{e}^{ix}}{2a}+\frac{ixC}{2b}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2a^2}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)A}{a}$
default	$\frac{C\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}+\frac{i\left(-iC\,a^2+iC\,b^2+2aAb\right)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}-\frac{iC}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}+\frac{(2iAa-bC)\ln\left(\tan\left(\frac{x}{2}\right)+\right.}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*C/b*ln(-I+tan(1/2*x))+1/2*I*(-I*C*a^2+I*C*b^2+2*a*A*b)*(a-b)/a^2/b/(-a+
b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))-I*C/a/(tan(1/2*x)+I)+1/2*(2*I*A*a-
b*C)/a^2*ln(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.78, size = 57, normalized size = 0.67

$$\frac{i\,Ca^2x - Cabe^{(i\,x)} - (Ca^2 + 2i\,Aab - Cb^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(I*C*a^2*x - C*a*b*e^(I*x) - (C*a^2 + 2*I*A*a*b - C*b^2)*log((a*e^(I*x)
 + b)/a))/(a^2*b)
```

Sympy [A]
time = 0.40, size = 78, normalized size = 0.92

$$\frac{iCx}{2b}+\begin{cases}-\frac{Ce^{ix}}{2a} & \text{for } a\neq 0\\ x\left(-\frac{iC}{2b}+\frac{iCa-iCb}{2ab}\right) & \text{otherwise}\end{cases}-\frac{(2iAab+Ca^2-Cb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] I*C*x/(2*b) + Piecewise((-C*exp(I*x)/(2*a), Ne(a, 0)), (x*(-I*C/(2*b) + (I*
C*a - I*C*b)/(2*a*b)), True)) - (2*I*A*a*b + C*a**2 - C*b**2)*log(exp(I*x)
+ b/a)/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.
time = 0.43, size = 169, normalized size = 1.99

$$-\frac{(2iAa-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2}-\frac{(-2iAa+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2}-\frac{(-2iCa^2+2Aab+iCb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b}-\frac{2iAa\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa+2iCa-iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x
) + a + b)/a^2 - 1/2*(-2*I*A*a + C*b)*log(tan(1/2*x) + I)/a^2 - 1/4*(-2*I*C
*a^2 + 2*A*a*b + I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*co
s(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - C*b*tan(
1/2*x) - 2*A*a + 2*I*C*a - I*C*b)/(a^2*(tan(1/2*x) + I))
```

Mupad [B]
time = 4.34, size = 96, normalized size = 1.13

$$-\ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\frac{C}{2\,b}-\frac{C\,b}{2\,a^2}+\frac{A\,1\mathrm{i}}{a}\right)-\frac{C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}+\frac{C\,\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(-\frac{C\,b}{2}+A\,a\,1\mathrm{i}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] (C*log(tan(x/2) - 1i))/(2*b) - (C*1i)/(a*(tan(x/2) + 1i)) - log(a + b + a*t
an(x/2)*1i - b*tan(x/2)*1i)*((A*1i)/a + C/(2*b) - (C*b)/(2*a^2)) + (log(tan
(x/2) + 1i)*(A*a*1i - (C*b)/2))/a^2
```

3.545 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=119

$$\frac{(bB+cC)x}{b^2+c^2}-\frac{2a(bB+cC)\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*(B*b+
C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^
2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3215, 3203, 632, 210}

$$-\frac{2a(bB+cC)\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2]
)/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + ((B*c - b*C
)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(a(bB+cC))\int \frac{}{a+b\,c}}{b^2+c^2}$$

$$=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\frac{(2a(bB+cC))\text{Subst}}{}$$

$$=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{(4a(bB+cC))\text{Subst}}{}$$

$$=\frac{(bB+cC)x}{b^2+c^2}-\frac{2a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{(Bc-bC)\log(a+b}{b^2+}$$

Mathematica [A]
time = 0.25, size = 98, normalized size = 0.82

$$\frac{(bB+cC)x+\frac{2a(bB+cC)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2
+ b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*Si
n[x]])/(b^2 + c^2)
```

Maple [A]
time = 0.51, size = 217, normalized size = 1.82

method	result
default	$\frac{\frac{2\left(aBc-bBc-abC+b^{2}C\right)\ln\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-b\left(\tan^{2}\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(-abB+B\,c^{2}-acC-Cbc-\frac{\left(aBc-bBc-abC+b^{2}C\right)c}{a-b}\right)\arctan\left(\frac{2}{2\sqrt{}}\right.}{\sqrt{a^{2}-b^{2}-c^{2}}}}{b^{2}+c^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)

[Out] 2/(b^2+c^2)*(1/2*(B*a*c-B*b*c-C*a*b+C*b^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+(-a*b*B+B*c^2-a*c*C-C*b*c-(B*a*c-B*b*c-C*a*b+C*b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^2)+(B*b+C*c)*arctan(tan(1/2*x)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(114) = 228.
time = 3.39, size = 687, normalized size = 5.77

$$\left[-\frac{(Bab+Cac)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(2a^2b-b^3)c)\cos(x))\sin(x)-2\left(2abc\cos(x)^2-abc+(b^2c+c^3)\cos(x)-(b^3+bc^2+(ab^2-ac^2)\cos(x))\sin(x)\right)\sqrt{-a^2+b^2+c^2}}{2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)}\right)-2(Ba^2b-Bb^3-Bbc^2-Cc^3+(Ca^2-Cb^2)c)x+(Ca^2b-Cb^3-Cbc^2+Bc^3-(Ba^2-Bb^2)c)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)},-\frac{2(Bab+Cac)\sqrt{a^2-b^2-c^2}\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)-2(Ba^2b-Bb^3-Bbc^2-Cc^3+(Ca^2-Cb^2)c)x+(Ca^2b-Cb^3-Cbc^2+Bc^3-(Ba^2-Bb^2)c)\log\left(2ab\cos(x)+(b^2-c^2)\cos(x)^2+a^2+c^2+2(bc\cos(x)+ac)\sin(x)\right)}{2(a^2b^2-b^4-c^4+(a^2-2b^2)c^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")

[Out] [-1/2*((B*a*b + C*a*c)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a

```
^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 -
C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^
2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 -
 2*b^2)*c^2), -1/2*(2*(B*a*b + C*a*c)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*co
s(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)
*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C
*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 -
 B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos
(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 187, normalized size = 1.57

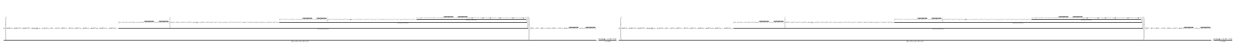
$$\frac{(Bb+Cc)x}{b^2+c^2}-\frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{2(Bab+Cac)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a
^2 - b^2 - c^2)*(b^2 + c^2))
```

Mupad [B]
time = 28.56, size = 1864, normalized size = 15.66



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i)
)/(b*1i + c) - (log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*
(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C*b + 2*B*C^2*c) - 32*
B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c + ((C*b^3 - B*c^3 + B*
```

```
a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c
*(b^2 - a^2 + c^2)^(1/2))*(32*B^2*b^2*c - 32*B^2*a^2*c + 32*C^2*a^2*c + 32*
C^2*b^2*c + 32*tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^
2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*B*C*b*c) - 128*B*C*a^
3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*C^2*a*b*c
 + ((C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a
^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B*b^4 + 32*B*a^2*b^2 -
 32*B*a^2*c^2 - 64*B*b^2*c^2 - 32*tan(x/2)*(a - b)*(B*c^3 - 2*C*b^3 + 2*C*a
*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) - 64*B*a*b^3 + 32*C*a*c
^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c
+ (32*(a - b)*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*
b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2)
+ a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b
^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/
2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((
b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c
 + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))
)/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C
^2*b^2 + 32*tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C
*b + 2*B*C^2*c) - 32*B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c -
 ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2
 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B^2*b^2*c - 32*B^2*a^2*c
 + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 -
 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*
B*C*b*c) - 128*B*C*a^3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C
*b*c^2 - 64*C^2*a*b*c + ((B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b^2*c - C*b
*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B
*a^2*c^2 - 32*B*a^2*b^2 - 32*B*b^4 + 64*B*b^2*c^2 + 32*tan(x/2)*(a - b)*(B*
c^3 - 2*C*b^3 + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) +
64*B*a*b^3 - 32*C*a*c^3 + 32*C*b*c^3 - 64*C*b^3*c - 96*B*a*b*c^2 + 128*C*a*
b^2*c - 64*C*a^2*b*c + (32*(a - b)*(B*c^3 - C*b^3 - B*a^2*c + C*a^2*b + B*b
^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1
/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a
^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b
*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b
^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(B*c^3 - C*b^3 - B*a^2*
c + C*a^2*b + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^
2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

3.546 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{2(bB+cC)\operatorname{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}$$

```
[Out] -2*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)
^(3/2)+(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3232, 3203, 632, 210}

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}-\frac{2(bB+cC)\operatorname{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^2
- b^2 - c^2)^(3/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/((a^2 - b^2 - c^
2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} - \frac{(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}\,dx}{a^2-b^2-c^2}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} - \frac{(2(bB+cC))\text{Subst}\left(\int \frac{1}{a+b+2cx+}\right.}{a^2-b^2-}$$

$$= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} + \frac{(4(bB+cC))\text{Subst}\left(\int \frac{1}{-4(a^2-b^2}\right.}{a^2-}$$

$$= -\frac{2(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}} + \frac{Bc-bC-aC\cos(x)+aB}{(a^2-b^2-c^2)\,(a+b\cos(x)+}$$

Mathematica [A]
time = 0.31, size = 116, normalized size = 1.05

$$-\frac{2(bB+cC)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} - \frac{bBc+a^2C-b^2C+a(bB+cC)\sin(x)}{b\,(-a^2+b^2+c^2)\,(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (-2*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a
^2 + b^2 + c^2)^(3/2) - (b*B*c + a^2*C - b^2*C + a*(b*B + c*C)*Sin[x])/(b*(
-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Maple [A]
time = 0.36, size = 206, normalized size = 1.87

method	result

default	$-\frac{2\left(-\frac{(a^2B-abB-Bc^2-acC+Cbc)\tan(\frac{x}{2})}{a^3-a^2b-ab^2-ac^2+b^3+c^2b}+\frac{bBc+a^2C-b^2C}{a^3-a^2b-ab^2-ac^2+b^3+c^2b}\right)}{a(\tan^2(\frac{x}{2}))-b(\tan^2(\frac{x}{2}))+2c\tan(\frac{x}{2})+a+b}-\frac{2(bB+Cc)\arctan\left(\frac{2(a-b)\tan(\frac{x}{2})+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{\frac{3}{2}}}$
risch	$\frac{-2iBab-2iacC-2iBa^2e^{ix}+2iBc^2e^{ix}-2iCbce^{ix}-2Bbce^{ix}-2Ca^2e^{ix}+2Cb^2e^{ix}}{(-a^2+b^2+c^2)(-ic+b)(-ice^{2ix}+be^{2ix}+ic+2ae^{ix}+b)}-\frac{\ln\left(e^{ix}+\frac{iac\sqrt{-a^2+b^2+c^2}-ia^2b+ib^3+i}{(b^2+c^2)\sqrt{-}}\right.}{\sqrt{-a^2+b^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-(B*a^2-B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(
1/2*x)+(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(a*tan(1/2*x)
^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)-2*(B*b+C*c)/(a^2-b^2-c^2)^(3/2)*arcta
n(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(105) = 210$.
time = 2.88, size = 1316, normalized size = 11.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 + (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C
*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*
c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4
 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
```

```
 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)
*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (
b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*
b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))
 - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*
a*b^2)*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + 2*(B*a^3*b^2 - B*a*b^4 - B*a*b
^2*c^2 - C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 +
a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (
a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*
b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 +
3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3
 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b - C*b^3)*c^
2 - (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c
^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt
(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 -
b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
 (B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (B*a*b*c^3 + C*a*c^4 - (C*a^3 - C*a*b^2)
*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + (B*a^3*b^2 - B*a*b^4 - B*a*b^2*c^2 -
 C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 +
a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3
- 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*
b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c
^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 205, normalized size = 1.86

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Bb+Cc)}{(a^2-b^2-c^2)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)-Cac\tan\left(\frac{1}{2}x\right)+Cbc\tan\left(\frac{1}{2}x\right)-Bc^2\tan\left(\frac{1}{2}x\right)-Ca^2+Cb^2-Bbc\right)}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(B*b + C*c)/(a^2 - b^2 - c^2)^(3/2) +
2*(B*a^2*tan(1/2*x) - B*a*b*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x
```

```
) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 -
 a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

Mupad [B]
time = 3.24, size = 202, normalized size = 1.84

$$\frac{\frac{2\,\left(C\,a^2-C\,b^2+B\,c\,b\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(-B\,a^2+C\,a\,c+B\,b\,a+B\,c^2-C\,b\,c\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(B\,b+C\,c)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*(C*a^2 - C*b^2 + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(B*c
^2 - B*a^2 + B*a*b + C*a*c - C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b +
2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b
^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2)))*(B*b +
 C*c))/(b^2 - a^2 + c^2)^(3/2)
```

3.547 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=197

$$-\frac{3a(bB+cC)\mathrm{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}+\frac{a(Bc-bC)-(2bBc \ldots}{2\,(a \ldots}$$

```
[Out] -3*a*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^
2)^(5/2)+1/2*(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*si
n(x))^2+1/2*(a*(B*c-C*b)-(2*b*B*c+(a^2+2*c^2)*C)*cos(x)+(a^2*B+2*b*(B*b+C*c
))*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.16, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3235, 3232, 3203, 632, 210}

$$-\frac{3a(bB+cC)\mathrm{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{-\cos(x)\left(C\left(a^2+2c^2\right)+2bBc\right)+\sin(x)\left(a^2B+2b(bB+cC)\right)+a(Bc-bC)}{2\left(a^2-b^2-c^2\right)^2(a+b\cos(x)+c\sin(x))}+\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (-3*a*(b*B + c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(a^
2 - b^2 - c^2)^(5/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/(2*(a^2 - b^2
- c^2)*(a + b*Cos[x] + c*Sin[x])^2) + (a*(B*c - b*C) - (2*b*B*c + (a^2 + 2*
c^2)*C)*Cos[x] + (a^2*B + 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(
a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
```

```
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx &= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{\int \frac{2(bB+cC)-aB\cos(x)-aC\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx}{2\left(a^2-b^2-c^2\right)} \\
&= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}+\frac{a(Bc-bC)-(2bBc+(a^2+}{2\left(a^2-b^2-c\right.} \\
&= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}+\frac{a(Bc-bC)-(2bBc+(a^2+}{2\left(a^2-b^2-c\right.} \\
&= \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}+\frac{a(Bc-bC)-(2bBc+(a^2+}{2\left(a^2-b^2-c\right.} \\
&= -\frac{3a(bB+cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}}+\frac{Bc-bC-aC\cos(x)+aB}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+}
\end{aligned}
$$

Mathematica [A]

time = 0.64, size = 311, normalized size = 1.58

$$\frac{3a(bB+cC)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}+\frac{9a^2bBc+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C+6abc(bB+cC)\cos(x)-c(a^2+2(b^2+c^2))(bB+cC)\cos(2x)+4a^3bB\sin(x)+2ab^3B\sin(x)+8abBc^2\sin(x)+4a^3cC\sin(x)+2ab^2cC\sin(x)+8ac^3C\sin(x)+a^2b^2B\sin(2x)+2b^4B\sin(2x)+2b^2Bc^2\sin(2x)+a^2bcC\sin(2x)+2b^3cC\sin(2x)+2bc^3C\sin(2x)}{4b(-a^2+b^2+c^2)^2(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] (3*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-
a^2 + b^2 + c^2)^(5/2) + (9*a^2*b*B*c + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5
*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C + 6*a*b*c*(b*B + c*C)*Cos[x] - c*(a^2 +
2*(b^2 + c^2))*(b*B + c*C)*Cos[2*x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] +
 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin
[x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*
c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x]])/(4*b*(-a^2 + b^2 +
c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(187) = 374$.
time = 0.73, size = 795, normalized size = 4.04

method	result
default	$-\frac{2\left(-\frac{\left(2B\,a^4-3B\,a^3b+2B\,a^2b^2-4B\,a^2c^2-3Ba\,b^3+2B\,b^4+4B\,b^2c^2+2B\,c^4-3C\,a^3c+6C\,a^2bc-3Ca\,b^2c\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)(a-b)}-\frac{\left(2B\,a^4c-9B\,a^3bc+14B\,a^2\right.}{}\right.}{}$
risch	$\frac{i\left(9iC\,a^2bc\,e^{2ix}+3iCa\,b^2c\,e^{3ix}+5iCa\,b^2c\,e^{ix}+2iB\,b^4+4iB\,b^2c^2e^{2ix}+9B\,a^2bc\,e^{2ix}+6Ba\,b^2c\,e^{3ix}+3iBa\,b^3e^{3ix}+5iB\,a^2b^2e^{2ix}-4iB\,a^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2-4*B*a^2*c^2-3*B*a*b^3+2*B*b^4+4*B*b
^2*c^2+2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3-1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b
^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*
b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c
^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(2*B*a^5-3*B*a^4*b+B*a^3*b^2-
4*B*a^3*c^2+B*a^2*b^3-8*B*a^2*b*c^2-3*B*a*b^4+8*B*a*b^2*c^2+2*B*a*c^4+2*B*b
^5+4*B*b^3*c^2+2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c-4*C*a^2*c^3-5*
C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a
*b+b^2)*tan(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c-2*B*b*c^3+2*C*a^4-4*C*a^2*b
^2+C*a^2*c^2+2*C*b^4-C*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)
/(a^2-2*a*b+b^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2-3*a*
```

```
(B*b+C*c)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*a
rctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(187) = 374$.
time = 3.31, size = 3264, normalized size = 16.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B*a^4 - 5*B
*a^2*b^2 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*B*
b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 -
 (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)
*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a
*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*
a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)
*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b
^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(-a^2 + b
^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4
 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*
c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*
b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))
*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 +
 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 2*B*a^2*b^4 + B*b
^6)*c + 2*(B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 + (B*a^3*b + 2*B*a
*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^5*b - B*a^3*b^3 -
B*a*b^5)*c)*cos(x) + 2*(2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^6 - B*a*b^2*c^4 - C
```

*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2*B*a*b^4)*c^2 + (2*C*a
^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^5 - 2*B*b^7 + 2*B*b*c^
6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*b^3)*c^4 - (C*a^4 + 2*
C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C*a^2*b^4 - 2*C*b^6)*c)
*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8 - c^10 + 2*(a^2
 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*a^4*b^2 - 3*a^2*b^4 +
4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 - b^8)*c^2 + (a^6*b^4 -
 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*c^8 + (3*a^4 - 6*a^2*b
^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(a^4*b^4 - 2*a^2*b^6 +
b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9 - a*b*c^8 +
 (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c^4 + (a^7*b - 6
*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^9 - (3*a^3 - 4*a*b^2)*
c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^5*b^2 + 9*a^3*b^4 - 4*
a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*c + (b*c^9 - (3*a^2*
b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 - (a^6*b - 6*a^4*b^3 + 9
*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c)*cos(x))*
sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5 - C*b^7 - 3*C*b*c^6 + B*c
^7 - (3*B*a^2 - B*b^2)*c^5 + (4*C*a^2*b - 7*C*b^3)*c^4 + (3*B*a^4 - 5*B*a^2
*b^2 - B*b^4)*c^3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*B*b^2*c^
5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c^3 - (C*a^
4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^6)*c)*cos(x
)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3 + C*a*b^2)*
c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c^4 - C*a*c^5)
cos(x)^2 + 2(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^2*b*c^3)*cos(x
) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4 + (B*a*b^4*c +
 C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^
2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^
3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a
^4*b^2 + 2*B*a^2*b^4 + B*b^6)*c + (B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2
)*c^4 + (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (
2*B*a^5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + (2*B*a^5*b^2 - B*a^3*b^4 - B*a
*b^6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + 2
*B*a*b^4)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 + B*a^2*b^
5 - 2*B*b^7 + 2*B*b*c^6 + 2*C*c^7 - (C*a^2 - 2*C*b^2)*c^5 - (B*a^2*b - 2*B*
b^3)*c^4 - (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b + 2*B*b^5)*c^2 + (C*a^4*b^2 + C
*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a
^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 - 3*
a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^6 -
 b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^2)*
c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 - 3*(
a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b
^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 + 2*a
*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a*c^
9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6*a^
5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8

```
)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5 -
(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a^2*
b^7 - b^9)*c)*cos(x))*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(187) = 374.
time = 0.51, size = 1034, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

```
[Out] 3*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a^4 - 2*a^2*b^2 + b^
4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (2*B*a^5*tan(1/2*
x)^3 - 5*B*a^4*b*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(
1/2*x)^3 + 5*B*a*b^4*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan(1/
2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b^3*
c*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 + 4*
B*a*b^2*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^
3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*tan(1/2*x)^2
+ 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*b^4*tan(1/2*x
)^2 + 2*C*b^5*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x
)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + 2*B*b^4*c*ta
n(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*
a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x
)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + B*
a^3*b^2*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + 2*B*b^5*
tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*
tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x) - 8*B*a^2*b*c^
2*tan(1/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1/2*x) - 4*C*a^2*
c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) + 2*B*a*c^4*tan(1/2*x) + 2*B*b*c^4*
tan(1/2*x) - 2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 - 5*B*a^3*b*c + 5*B*a*b^3*c
- C*a^3*c^2 + C*a*b^2*c^2 + 2*B*a*b*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*
```

```
b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b
^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 +
2*c*tan(1/2*x) + a + b)^2)
```

Mupad [B]
time = 6.36, size = 923, normalized size = 4.69

$$\frac{\frac{\tan\left(\frac{x}{2}\right)^3\,\left(2\,B\,a^4-3\,B\,a^3\,b-3\,C\,a^3\,c+2\,B\,a^2\,b^2+6\,C\,a^2\,b\,c-4\,B\,a^2\,c^2-3\,B\,a\,b^3-3\,C\,a\,b^2\,c+2\,B\,b^4+4\,B\,b^2\,c^2+2\,B\,c^4\right)}{(a-b)\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}-\frac{2\,C\,a^5-4\,C\,a^3\,b^2+5\,B\,a^3\,b\,c+C\,a^3\,c^2+2\,C\,a\,b^4-5\,B\,a\,b^3\,c-C\,a\,b^2\,c^2-2\,B\,a\,b\,c^3}{(a-b)^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{x}{2}\right)^2\,\left(-2\,C\,a^5+2\,C\,a^4\,b+2\,B\,a^4\,c+4\,C\,a^3\,b^2-9\,B\,a^3\,b\,c-5\,C\,a^3\,c^2-4\,C\,a^2\,b^3+14\,B\,a^2\,b^2\,c+14\,C\,a^2\,b\,c^2-4\,B\,a^2\,c^3-2\,C\,a\,b^4-9\,B\,a\,b^3\,c-13\,C\,a\,b^2\,c^2-2\,C\,a\,c^4+2\,C\,b^5+2\,B\,b^4\,c+4\,C\,b^3\,c^2+4\,B\,b^2\,c^3+2\,C\,b\,c^4+2\,B\,c^5\right)}{(a-b)^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{x}{2}\right)\,\left(2\,B\,a^5-3\,B\,a^4\,b-5\,C\,a^4\,c+B\,a^3\,b^2+5\,C\,a^3\,b\,c-4\,B\,a^3\,c^2+B\,a^2\,b^3+5\,C\,a^2\,b^2\,c-8\,B\,a^2\,b\,c^2-4\,C\,a^2\,c^3-3\,B\,a\,b^4-5\,C\,a\,b^3\,c+8\,B\,a\,b^2\,c^2+4\,C\,a\,b\,c^3+2\,B\,a\,c^4+2\,B\,b^5+4\,B\,b^3\,c^2+2\,B\,b\,c^4\right)}{(a-b)^2\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}}{\tan\left(\frac{x}{2}\right)^4\,\left(a^2-2\,a\,b+b^2\right)+2\,a\,b+\tan\left(\frac{x}{2}\right)\,\left(4\,a\,c+4\,b\,c\right)+\tan\left(\frac{x}{2}\right)^3\,\left(4\,a\,c-4\,b\,c\right)+a^2+b^2+\tan\left(\frac{x}{2}\right)^2\,\left(2\,a^2-2\,b^2+4\,c^2\right)}+\frac{3\,a\,\mathrm{atanh}\left(\frac{3\,a\,\left(B\,b+C\,c\right)\,\left(\tan\left(\frac{x}{2}\right)\,\left(2\,a-2\,b\right)+\frac{2\,a^4\,c-4\,a^2\,b^2\,c-4\,a^2\,c^3+2\,b^4\,c+4\,b^2\,c^3+2\,c^5}{a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4}\right)\,\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}{2\,\left(3\,B\,a\,b+3\,C\,a\,c\right)\,{\left(-a^2+b^2+c^2\right)}^{5/2}}\right)\,\left(B\,b+C\,c\right)}{{\left(-a^2+b^2+c^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(2*B*a^4 + 2*B*b^4 + 2*B*c^4 + 2*B*a^2*b^2 - 4*B*a^2*c^2 + 4*B
*b^2*c^2 - 3*B*a*b^3 - 3*B*a^3*b - 3*C*a^3*c - 3*C*a*b^2*c + 6*C*a^2*b*c))/
((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (2*C*a^5
- 4*C*a^3*b^2 + C*a^3*c^2 + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3
*b*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2
*b^2*c^2)) + (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 + 2*C*b^5 - 4*B*a^2*c^3 - 4*C*a
^2*b^3 + 4*C*a^3*b^2 + 4*B*b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 2*B*a^4*c
- 2*C*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 9*B*a*b^3*c -
 9*B*a^3*b*c + 14*B*a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/((a - b)^
2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)*(2*B*a
^5 + 2*B*b^5 + B*a^2*b^3 + B*a^3*b^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 4*C*a^2*
c^3 - 3*B*a*b^4 - 3*B*a^4*b + 2*B*a*c^4 + 2*B*b*c^4 - 5*C*a^4*c + 4*C*a*b*c
^3 - 5*C*a*b^3*c + 5*C*a^3*b*c + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2 + 5*C*a^2*b^
2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(t
an(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(x/2)
^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) + (3*a
*atanh((3*a*(B*b + C*c)*(tan(x/2)*(2*a - 2*b) + (2*a^4*c + 2*b^4*c + 2*c^5
- 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2
*c^2 + 2*b^2*c^2))*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(
2*(3*B*a*b + 3*C*a*c)*(b^2 - a^2 + c^2)^(5/2)))*(B*b + C*c))/(b^2 - a^2 + c
^2)^(5/2)
```

3.548 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{b(B+iC)x}{2a^2}-\frac{(ib^2(B+iC)+a^2(iB+C))\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{(iB-C)(\cos(x)-i\sin(x))}{2a}$$

```
[Out] -1/2*b*(B+I*C)*x/a^2-1/2*(I*b^2*(B+I*C)+a^2*(I*B+C))*ln(a+b*cos(x)+I*b*sin(
x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3209}

$$-\frac{\left(\frac{ib^2(B+iC)}{a^2}+iB+C\right)\log(a+ib\sin(x)+b\cos(x))}{2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] -1/2*(b*(B + I*C)*x)/a^2 - ((I*B + (I*b^2*(B + I*C))/a^2 + C)*Log[a + b*Cos
[x] + I*b*Sin[x]])/(2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)
```

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=-\frac{b(B+iC)x}{2a^2}-\frac{\left(iB+\frac{ib^2(B+iC)}{a^2}+C\right)\log(a+b\cos(x)+ib\sin(x))}{2b}+\frac{(iB-$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 195 vs. 2(92) = 184.
time = 0.23, size = 195, normalized size = 2.12

$$\frac{(a^2B-b^2B-ia^2C-ib^2C)\,x}{4a^2b}-\frac{(a^2B+b^2B-ia^2C+ib^2C)\,\text{ArcTan}\left(\frac{(a+b)\cos\left(\frac{x}{2}\right)}{-a\sin\left(\frac{x}{2}\right)+b\sin\left(\frac{x}{2}\right)}\right)}{2a^2b}+\frac{i(B+iC)\cos(x)}{2a}-\frac{i(a^2B+b^2B-ia^2C+ib^2C)\log\left(a^2+b^2+2ab\cos(x)\right)}{4a^2b}+\frac{(B+iC)\sin(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B - I*a^2*C - I*b^2*C)*x)/(4*a^2*b) - ((a^2*B + b^2*B - I*a^2
*C + I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(-(a*Sin[x/2]) + b*Sin[x/2])])/(2*a
^2*b) + ((I/2)*(B + I*C)*Cos[x])/a - ((I/4)*(a^2*B + b^2*B - I*a^2*C + I*b^
2*C)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B + I*C)*Sin[x])/(2*a)
```

Maple [A]
time = 0.30, size = 123, normalized size = 1.34

method	result
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}+\frac{iB\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^2}-\frac{bxB}{2a^2}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2a^2}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2b}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2a^2}$
default	$-\frac{i\left(-iC\,a^2+iC\,b^2+a^2B+B\,b^2\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^2b}-\frac{-iC-B}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{b(iB-C)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{i(-iC+B)\ln}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(-I*C*a^2+I*C*b^2+a^2*B+B*b^2)/a^2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1
/2*x))-(-I*C-B)/a/(-I+tan(1/2*x))+1/2*b*(I*B-C)/a^2*ln(-I+tan(1/2*x))+1/2*I
*(B-I*C)/b*ln(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.39, size = 78, normalized size = 0.85

$$-\frac{\left((B+i\,C)b^2xe^{(i\,x)}-(i\,B-C)ab-((-i\,B-C)a^2+(-i\,B+C)b^2)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] -1/2*((B + I*C)*b^2*x*e^(I*x) - (I*B - C)*a*b - ((-I*B - C)*a^2 + (-I*B + C
)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

Sympy [A]
time = 0.46, size = 110, normalized size = 1.20

$$\begin{cases} \frac{(iB-C)e^{-ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{-Bb-iCb}{2a^2}+\frac{Ba-Bb+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-Bb-iCb)}{2a^2} - \frac{i(Ba^2+Bb^2-iCa^2+iCb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise(((I*B - C)*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(-B*b - I*C*b)/(2*a**
2) + (B*a - B*b + I*C*a - I*C*b)/(2*a**2)), True)) + x*(-B*b - I*C*b)/(2*a*
*2) - I*(B*a**2 + B*b**2 - I*C*a**2 + I*C*b**2)*log(a/b + exp(I*x))/(2*a**2
*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(70) = 140$.
time = 0.42, size = 178, normalized size = 1.93

$$-\frac{(iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2}-\frac{(-iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)-i\right)}{2a^2}+\frac{(2Ba^2-2iCa^2+Bb^2+iCb^2)\left(x+2\arctan\left(\frac{-ia\cos(x)-a\sin(x)-ia}{a\cos(x)-ia\sin(x)-a+2b}\right)\right)}{4a^2b}-\frac{iBb\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Ba-2iCa+Bb+iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*x)
+ a + b)/a^2 - 1/2*(-I*B*b + C*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*B*a^2 -
2*I*C*a^2 + B*b^2 + I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(
a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(I*B*b*tan(1/2*x) - C*b*ta
n(1/2*x) - 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

Mupad [B]
time = 5.32, size = 118, normalized size = 1.28

$$-\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{-\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B+C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(-C\,b+B\,b\,1\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] (B + C*1i)/(a*(tan(x/2) - 1i)) - log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)
*(((B*1i)/2 + C/2)/b + ((B*b^2*1i)/2 - (C*b^2)/2)/(a^2*b)) + (log(tan(x/2)
+ 1i)*(B*1i + C))/(2*b) + (log(tan(x/2) - 1i)*(B*b*1i - C*b))/(2*a^2)
```

3.549 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=90

$$-\frac{b(B-iC)x}{2a^2}+\frac{(ia^2(B+iC)+b^2(iB+C))\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{(iB+C)(\cos(x)+i\sin(x))}{2a}$$

```
[Out] -1/2*b*(B-I*C)*x/a^2+1/2*(I*a^2*(B+I*C)+b^2*(I*B+C))*ln(a+b*cos(x)-I*b*sin(
x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {3209}

$$-\frac{bx(B-iC)}{2a^2}+\frac{1}{2}\left(\frac{b(C+iB)}{a^2}+\frac{i(B+iC)}{b}\right)\log(a-ib\sin(x)+b\cos(x))-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] -1/2*(b*(B - I*C)*x)/a^2 + (((I*(B + I*C))/b + (b*(I*B + C))/a^2)*Log[a + b
*Cos[x] - I*b*Sin[x]])/2 - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)
```

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=-\frac{b(B-iC)x}{2a^2}+\frac{1}{2}\left(\frac{i(B+iC)}{b}+\frac{b(iB+C)}{a^2}\right)\log(a+b\cos(x)-ib\sin(x))$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 195 vs. 2(90) = 180.
time = 0.22, size = 195, normalized size = 2.17

$$\frac{(a^2B-b^2B+ia^2C+ib^2C)\,x}{4a^2b}+\frac{(a^2B+b^2B+ia^2C-ib^2C)\,\text{ArcTan}\left(\frac{(a+b)\cos\left(\frac{x}{2}\right)}{a\sin\left(\frac{x}{2}\right)-b\sin\left(\frac{x}{2}\right)}\right)}{2a^2b}-\frac{i(B-iC)\cos(x)}{2a}+\frac{i(a^2B+b^2B+ia^2C-ib^2C)\log\left(a^2+b^2+2ab\cos(x)\right)}{4a^2b}+\frac{(B-iC)\sin(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((a^2*B - b^2*B + I*a^2*C + I*b^2*C)*x)/(4*a^2*b) + ((a^2*B + b^2*B + I*a^2
*C - I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(a*Sin[x/2] - b*Sin[x/2])])/(2*a^2*
b) - ((I/2)*(B - I*C)*Cos[x])/a + ((I/4)*(a^2*B + b^2*B + I*a^2*C - I*b^2*C
)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B - I*C)*Sin[x])/(2*a)
```

Maple [A]
time = 0.29, size = 133, normalized size = 1.48

method	result
risch	$-\frac{C\,e^{ix}}{2a}-\frac{iB\,e^{ix}}{2a}+\frac{ixC}{2b}+\frac{Bx}{2b}-\frac{\ln\left(e^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(e^{ix}+\frac{b}{a}\right)C}{2a^2}+\frac{i\ln\left(e^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(e^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$-\frac{i(iC+B)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}-\frac{i(iC\,a^2-iC\,b^2+a^2B+B\,b^2)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}-\frac{iC-B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}-\frac{b(iB+C)\ln(\ldots}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(B+I*C)/b*ln(-I+tan(1/2*x))-1/2*I*(I*C*a^2-I*C*b^2+a^2*B+B*b^2)*(a-b
)/a^2/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))-(I*C-B)/a/(tan(1/2*x)+
I)-1/2*b*(I*B+C)/a^2*ln(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.27, size = 68, normalized size = 0.76

$$\frac{(B+i\,C)a^2x+(-i\,B-C)abe^{(i\,x)}+((i\,B-C)a^2+(i\,B+C)b^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas"
)
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 + (I*B + C)*
b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

Sympy [A]
time = 0.43, size = 99, normalized size = 1.10

$$\begin{cases} \frac{(-iB-C)e^{ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{B+iC}{2b} + \frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{x(B+iC)}{2b} + \frac{i(Ba^2+Bb^2+iCa^2-iCb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] Piecewise(((-I*B - C)*exp(I*x)/(2*a), Ne(a, 0)), (x*(-(B + I*C)/(2*b) + (B*
a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) + x*(B + I*C)/(2*b) + I*(B*a**2 +
 B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(73) = 146.
time = 0.44, size = 178, normalized size = 1.98

$$-\frac{(-iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2} + \frac{(2Ba^2+2iCa^2+Bb^2-iCb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{-iBb\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Ba+2iCa+Bb-iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x)
 + a + b)/a^2 - 1/2*(I*B*b + C*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*B*a^2 +
2*I*C*a^2 + B*b^2 - I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a
*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-I*B*b*tan(1/2*x) - C*b*ta
n(1/2*x) - 2*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

Mupad [B]
time = 4.51, size = 118, normalized size = 1.31

$$\ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B-C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C\,b+B\,b\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1i
)/2 + (C*b^2)/2)/(a^2*b)) + (B - C*1i)/(a*(tan(x/2) + 1i)) - (log(tan(x/2)
+ 1i)*(B*b*1i + C*b))/(2*a^2) - (log(tan(x/2) - 1i)*(B*1i - C))/(2*b)
```

3.550 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

Optimal. Leaf size=131

$$\frac{(bB+cC)x}{b^2+c^2}+\frac{2(A(b^2+c^2)-a(bB+cC))\operatorname{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

```
[Out] (B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2
+c^2)-a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^
2)/(a^2-b^2-c^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3215, 3203, 632, 210}

$$\frac{2(A(b^2+c^2)-a(bB+cC))\operatorname{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x)/(b^2 + c^2) + (2*(A*(b^2 + c^2) - a*(b*B + c*C))*ArcTan[(c
+ (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c
^2)) + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3215

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a
*(b*B + c*C))/(b^2 + c^2), Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*x]), x],
 x] + Simp[(c*B - b*C)*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 +
c^2))), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && N
eQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\Bigg(A-\frac{a(bB+cC}{b^2+c^2}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\Bigg(2\Bigg(A-\frac{a(bB+}{b^2+}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}-\Bigg(4\Bigg(A-\frac{a(bB+}{b^2+}$$

$$= \frac{(bB+cC)x}{b^2+c^2}+\frac{2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\,\text{lo}}{}$$

Mathematica [A]
time = 0.26, size = 110, normalized size = 0.84

$$\frac{(bB+cC)x+\frac{2(-A(b^2+c^2)+a(bB+cC))\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

```
[Out] ((b*B + c*C)*x + (2*(-(A*(b^2 + c^2)) + a*(b*B + c*C))*ArcTanh[(c + (a - b)
*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Lo
g[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

Maple [A]
time = 0.81, size = 227, normalized size = 1.73

method	result

default	$\frac{\frac{2\left(aBc-bBc-abC+b^2C\right)\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(A\,b^2+A\,c^2-abB+B\,c^2-acC-Cbc-\frac{\left(aBc-bBc-abC+b^2C\right)c}{a-b}\right)\operatorname{arct}}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b^2+c^2)*(1/2*(B*a*c-B*b*c-C*a*b+C*b^2)/(a-b)*ln(a*tan(1/2*x)^2-b*tan(1/
2*x)^2+2*c*tan(1/2*x)+a+b)+(A*b^2+A*c^2-a*b*B+B*c^2-a*c*C-C*b*c-(B*a*c-B*b*
c-C*a*b+C*b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+
2*c)/(a^2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^2)+(
B*b+C*c)*arctan(tan(1/2*x)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(126) = 252$.
time = 2.90, size = 711, normalized size = 5.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas"
)
```

```
[Out] [-1/2*((B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2
- 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos
(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b
 - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos
(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)
```

```
)/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*s
in(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*
a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^
2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^
4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*
b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 +
(a^2 - 2*b^2)*c^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 199, normalized size = 1.52

$$\frac{(Bb+Cc)x}{b^2+c^2}-\frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}+\frac{2\left(Bab-Ab^2+Cac-Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

```
[Out] (B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^
2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 + 1)
/(b^2 + c^2) + 2*(B*a*b - A*b^2 + C*a*c - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*
sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2
- c^2)))/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2))
```

Mupad [**B**]
time = 55.11, size = 2711, normalized size = 20.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

```
[Out] (log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i)
)/(b*1i + c) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2
```

- 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*A*b^4 + 32*B*b^4 - 32*tan(x/2)*(a - b)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c - 4*B*a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c^2 - 32*B*a^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*(a - b)*(B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(c*(B*b^2 - B*a^2 + C*a*(b^2 - a^2 + c^2)^(1/2)) + B*c^3 - C*b^3 - c^2*(A*(b^2 - a^2 + c^2)^(1/2) + C*b) - A*b^2*(b^2 - a^2 + c^2)^(1/2) + C*a^2*b + B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a^2 - 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*B^2*C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A^2*B*a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*B^2*C*a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c - 4*A*C*b*c - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*b^3 + 64*A*B*a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/

```
2))*(32*A*b^4 + 32*B*b^4 - 32*tan(x/2)*(a - b)*(2*A*c^3 + B*c^3 - 2*C*b^3 +
 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 2*A*a*b*c - 4*B*
a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*B*a^2*b^2 + 32*A*b^2*c^2 - 32*B*a
^2*c^2 - 64*B*b^2*c^2 - 64*A*a*b^3 - 64*B*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 +
 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c + 64*C*a^2*b*c - (32*(a - b)*(B*
c^3 - C*b^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b + A*c^2*(b^
2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 - B*a*b*(b^2 - a^2 + c^2)^(1/2) -
C*a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c
+ 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*ta
n(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2
 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)
))*(B*c^3 - c*(B*a^2 - B*b^2 + C*a*(b^2 - a^2 + c^2)^(1/2)) - C*b^3 + c^2*(
A*(b^2 - a^2 + c^2)^(1/2) - C*b) + A*b^2*(b^2 - a^2 + c^2)^(1/2) + C*a^2*b
- B*a*b*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))
```

3.551 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=127

$$\frac{2(aA-bB-cC)\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

```
[Out] 2*(A*a-B*b-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c
^2)^(3/2)+(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*co
s(x)+c*sin(x))
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3232, 3203, 632, 210}

$$\frac{2(aA-bB-cC)\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTan[(c + (a - b)*Tan[x/2])/Sqrt[a^2 - b^2 - c^2]])/
(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin
[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
```

```
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} + \frac{(aA-bB-cC)\int \cdots}{a^2-b^2\cdots}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} + \frac{(2(aA-bB-cC))\cdots}{\cdots}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))} - \frac{(4(aA-bB-cC))\cdots}{\cdots}$$

$$= \frac{2(aA-bB-cC)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}} + \frac{Bc-bC+(Ac-aC)\cos\cdots}{(a^2-b^2-c^2)\,(a+b\cdots}$$

Mathematica [A]
time = 0.36, size = 137, normalized size = 1.08

$$\frac{2(aA-bB-cC)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} + \frac{aAc-a^2C+b(-Bc+bC)+(A(b^2+c^2)-a(bB+cC))\sin(x)}{b\,(-a^2+b^2+c^2)\,(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] (2*(a*A - b*B - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]]
)/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b*(-(B*c) + b*C) + (A*(b^2 +
c^2) - a*(b*B + c*C))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x
]))
```

Maple [A]
time = 0.38, size = 231, normalized size = 1.82

method	result
default	$\frac{-\frac{2\left(aAb-A\,b^2-A\,c^2-a^2B+abB+B\,c^2+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}+\frac{2\left(aAc-bBc-a^2C+b^2C\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(aA-bB-Cc)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}$
risch	$\frac{2iA\,b^2+2iA\,c^2-2iBab-2iacC+2iAab\,e^{ix}-2iB\,a^2e^{ix}+2iB\,c^2e^{ix}-2iCbc\,e^{ix}+2Aac\,e^{ix}-2Bbc\,e^{ix}-2C\,a^2e^{ix}+2C\,b^2e^{ix}}{(-a^2+b^2+c^2)(-ic+b)(-ic\,e^{2ix}+b\,e^{2ix}+ic+2a\,e^{ix}+b)}-\frac{\ln\left(e^{ix}+\frac{ia\cdots}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^
2+b^3+b*c^2)*tan(1/2*x)+(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^
3+b*c^2))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)+2*(A*a-B*b-C*c
)/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/
2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(122) = 244$.
time = 2.66, size = 1556, normalized size = 12.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="frica
s")
```

```
[Out] [1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C
*a*c^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*
```

```
b - B*b^2)*c^2)*cos(x) - (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 -
B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2
 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a
*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*s
in(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 +
 (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b
^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^4 -
 2*B*a^2*b^2 + B*b^4)*c + 2*(C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b^2)*c^
3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(
x) + 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 - (A*
a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 -
2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*
b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^
4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 -
4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b -
 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*a^2*b
 - C*b^3)*c^2 + (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C*a*c^3 + (A*a^2 - B*a*b
)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A*a*b - B*b^2)*c^2)*cos(x)
- (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(
a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) -
(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + (C*a*c^4 - A*c^5 + (A*a^2 + B*a*b - 2*A*b
^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c
)*cos(x) + (B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 + A*b*c^4 -
 (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^
2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 +
3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 +
 (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^
4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 241, normalized size = 1.90

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb-Cc)}{(a^2-b^2-c^2)^{\frac{3}{2}}}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)+Ab^2\tan\left(\frac{1}{2}x\right)-Cac\tan\left(\frac{1}{2}x\right)+Cbc\tan\left(\frac{1}{2}x\right)+Ac^2\tan\left(\frac{1}{2}x\right)-Bc^2\tan\left(\frac{1}{2}x\right)-Ca^2+Cb^2+Aac-Bbc\right)}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac"
)
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b - C*c)/(a^2 - b^2 - c^2)^(
3/2) + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*ta
n(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*t
an(1/2*x) - C*a^2 + C*b^2 + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*
c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b))
```

Mupad [B]
time = 3.49, size = 227, normalized size = 1.79

$$\frac{\frac{2\,\left(C\,a^2-A\,c\,a-C\,b^2+B\,c\,b\right)}{\left(a-b\right)\,\left(-a^2+b^2+c^2\right)}-\frac{2\,\mathrm{tan}\left(\frac{x}{2}\right)\,\left(A\,b^2+B\,a^2+A\,c^2-B\,c^2-A\,a\,b-B\,a\,b-C\,a\,c+C\,b\,c\right)}{\left(a-b\right)\,\left(-a^2+b^2+c^2\right)}}{\left(a-b\right)\,{\mathrm{tan}\left(\frac{x}{2}\right)}^2+2\,c\,\mathrm{tan}\left(\frac{x}{2}\right)+a+b}-\frac{2\,\mathrm{atanh}\left(\frac{\mathrm{tan}\left(\frac{x}{2}\right)\,\left(2\,a-2\,b\right)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,\left(B\,b-A\,a+C\,c\right)}{{\left(-a^2+b^2+c^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] ((2*(C*a^2 - C*b^2 - A*a*c + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) - (2*tan(x
/2)*(A*b^2 + B*a^2 + A*c^2 - B*c^2 - A*a*b - B*a*b - C*a*c + C*b*c))/((a -
b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*ata
nh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*
(b^2 - a^2 + c^2)^(1/2)))*(B*b - A*a + C*c))/(b^2 - a^2 + c^2)^(3/2)
```

3.552 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

Optimal. Leaf size=237

$$\frac{(2a^2A+A(b^2+c^2)-3a(bB+cC))\,\text{ArcTan}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}+\frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB}{2\,(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x}$$

```
[Out] (2*a^2*A+A*(b^2+c^2)-3*a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^
2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x
))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*(B*c-C*b)+(3*a*A*c-a^2*C-2*
c*(B*b+C*c))*cos(x)-(3*a*A*b-a^2*B-2*b*(B*b+C*c))*sin(x))/(a^2-b^2-c^2)^2/(
a+b*cos(x)+c*sin(x))
```

Rubi [A]
time = 0.21, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3235, 3232, 3203, 632, 210}

$$\frac{(2a^2A-3a(bB+cC)+A(b^2+c^2))\,\text{ArcTan}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)+c}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2c(bB+cC)\right)+a(Bc-bC)}{2\,(a^2-b^2-c^2)^2\,(a+b\cos(x)+c\sin(x))}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\,(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] ((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTan[(c + (a - b)*Tan[x/2])/
Sqrt[a^2 - b^2 - c^2]])/(a^2 - b^2 - c^2)^(5/2) + (B*c - b*C + (A*c - a*C)*
Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])
^2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*Cos[x] - (3*a*A*
b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/(2*(a^2 - b^2 - c^2)^2*(a + b*Cos[x] +
 c*Sin[x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3203

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[2*(f
/e), Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3232

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Dist[(a*A - b*B - c*C)/(a^2 - b^2 - c^2), Int[1/(a + b*Cos[d + e*x] + c*Si
n[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
- c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} - \frac{\int \frac{-2(aA-bB-cC)+(Ab-a}{(a+b\cos(x)}}{2\left(a^2-\right.}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+(3aAc}{}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+(3aAc}{}$$

$$= \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+(3aAc}{}$$

$$= \frac{\left(2a^2A+A(b^2+c^2)-3a(bB+cC)\right)\tan^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{5/2}} + \frac{Bc-bC}{2\left(a^2\right.}$$

Mathematica [A]
time = 0.87, size = 452, normalized size = 1.91

$$-\frac{\left(2a^2A+A\left(b^2+c^2\right)-3a(bB+cC)\right)\tanh^{-1}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{5/2}}+\frac{-6a^3Ac-3aAb^2c+9a^2bBc-3aAc^3+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C-2bc\left(2a^2A+A\left(b^2+c^2\right)-3a(bB+cC)\right)\cos(x)-c\left(-3aA\left(b^2+c^2\right)+a^2(bB+cC)+2\left(b^2+c^2\right)(bB+cC)\right)\cos(2x)-8a^2Ab^2\sin(x)+2Ab^4\sin(x)+4a^3bB\sin(x)+2ab^3B\sin(x)-12a^2Ac^2\sin(x)+2Ab^2c^2\sin(x)+8abBc^2\sin(x)+4a^3cC\sin(x)+2ab^2cC\sin(x)+8ac^3C\sin(x)-3aAb^3\sin(2x)+a^2b^2B\sin(2x)+2b^4B\sin(2x)-3aAbc^2\sin(2x)+2b^2Bc^2\sin(2x)+a^2bcC\sin(2x)+2b^3cC\sin(2x)+2bc^3C\sin(2x)}{4b\left(-a^2+b^2+c^2\right)^2(a+b\cos(x)+c\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

```
[Out] -(((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/2
])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A
*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*
c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B +
 c*C))*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*(b*B + c*C) + 2*(b^2 + c^2)*(b*
B + c*C))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x]
 + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^
2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A
*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x
] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^
3*C*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(227) = 454$.
time = 0.92, size = 1080, normalized size = 4.56

method	result
default	$-\frac{\left(4A\,a^3b-7A\,a^2b^2-5A\,a^2c^2+2Aa\,b^3+2Aab\,c^2+A\,b^4+3A\,b^2c^2+2A\,c^4-2B\,a^4+3B\,a^3b-2B\,a^2b^2+4B\,a^2c^2+3Ba\,b^3-2B\,b^4-4B\,b^2c^2-2B\,c^4+3C\,a}{(a-b)\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A*
b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b^4
-4*B*b^2*c^2-2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a-b)/(a^4-2*a^2*b^
2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*
A*a^2*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5+2
*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b
^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a
^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/
(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/
2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7
*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^
2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B
```

```
*b^5-4*B*b^3*c^2-2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*C*a^2*c^3+
5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2
*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b^4*c-A*b^2*c
^3-5*B*a^3*b*c+5*B*a*b^3*c+2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*
b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2
))/(a*tan(1/2*x)^2-b*tan(1/2*x)^2+2*c*tan(1/2*x)+a+b)^2+(2*A*a^2+A*b^2+A*c^
2-3*B*a*b-3*C*a*c)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2
)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2038 vs. 2(225) = 450.
time = 2.88, size = 4240, normalized size = 17.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="frica
s")
```

```
[Out] [1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^7
 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B
*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*
C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*a^2*
b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 - (C*a
^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^4 + 3*
A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 - 3
*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 - 3*(C
*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^
4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a
*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*A*a^3*b^3 -
 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*c^4 + (2*A*a
^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*
```

c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*a^3*b^2 - 3*B*
a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - (2*A*a^2*b
- 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c
^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos
(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2*(
2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a
*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*c
os(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^
2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(C*a*c^6 + A*c^7 -
(5*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a
^3*b - 10*A*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a
*b^4)*c^2 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4 - B*a*b^5 -
A*b^6)*c)*cos(x) + 2*(2*B*a^5*b^2 - 4*A*a^4*b^3 - B*a^3*b^4 + 5*A*a^2*b^5 -
B*a*b^6 - A*b^7 - C*a*b*c^5 - A*b*c^6 + (5*A*a^2*b - B*a*b^2 - 3*A*b^3)*c^
4 - (C*a^3*b + 2*C*a*b^3)*c^3 - (4*A*a^4*b + B*a^3*b^2 - 10*A*a^2*b^3 + 2*B
*a*b^4 + 3*A*b^5)*c^2 + (2*C*a^5*b - C*a^3*b^3 - C*a*b^5)*c + (B*a^4*b^3 -
3*A*a^3*b^4 + B*a^2*b^5 + 3*A*a*b^6 - 2*B*b^7 + 2*C*c^7 - (3*A*a - 2*B*b)*c
^6 - (C*a^2 - 2*C*b^2)*c^5 + (3*A*a^3 - B*a^2*b - 3*A*a*b^2 + 2*B*b^3)*c^4
- (C*a^4 + 2*C*b^4)*c^3 - (B*a^4*b - 3*A*a*b^4 + 2*B*b^5)*c^2 + (C*a^4*b^2
+ C*a^2*b^4 - 2*C*b^6)*c)*cos(x))*sin(x))/(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6
- a^2*b^8 - c^10 + 2*(a^2 - 2*b^2)*c^8 + (5*a^2*b^2 - 6*b^4)*c^6 - (2*a^6 -
3*a^4*b^2 - 3*a^2*b^4 + 4*b^6)*c^4 + (a^8 - 5*a^6*b^2 + 6*a^4*b^4 - a^2*b^
6 - b^8)*c^2 + (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10 + c^10 - 3*(a^2 - b^
2)*c^8 + (3*a^4 - 6*a^2*b^2 + 2*b^4)*c^6 - (a^6 - 3*a^4*b^2 + 2*b^6)*c^4 -
3*(a^4*b^4 - 2*a^2*b^6 + b^8)*c^2)*cos(x)^2 + 2*(a^7*b^3 - 3*a^5*b^5 + 3*a^
3*b^7 - a*b^9 - a*b*c^8 + (3*a^3*b - 4*a*b^3)*c^6 - 3*(a^5*b - 3*a^3*b^3 +
2*a*b^5)*c^4 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*c^2)*cos(x) - 2*(a
*c^9 - (3*a^3 - 4*a*b^2)*c^7 + 3*(a^5 - 3*a^3*b^2 + 2*a*b^4)*c^5 - (a^7 - 6
*a^5*b^2 + 9*a^3*b^4 - 4*a*b^6)*c^3 - (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*
b^8)*c + (b*c^9 - (3*a^2*b - 4*b^3)*c^7 + 3*(a^4*b - 3*a^2*b^3 + 2*b^5)*c^5
- (a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*c^3 - (a^6*b^3 - 3*a^4*b^5 + 3*a
^2*b^7 - b^9)*c)*cos(x))*sin(x)), 1/2*(C*a^6*b - 3*C*a^4*b^3 + 3*C*a^2*b^5
- C*b^7 - 3*C*b*c^6 + B*c^7 - (3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + (4*C*a^2*b
- 7*C*b^3)*c^4 + (3*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^
3 - (2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 2*(2*C*b*c^6 - (3*A*a*b - 2*B
*b^2)*c^5 - (C*a^2*b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 +
4*B*b^4)*c^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b
^3 + B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 + (2*A*a^4*b^2 - 3*B*a^3*
b^3 + A*a^2*b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b +
2*A*b^2)*c^4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2
- 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c
+ A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2
+ 2*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 +
A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2

```
*b^2*c^2 + 3*C*a^2*c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 -
(2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*
b*c^5 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 +
A*b^5)*c)*cos(x))*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*s
in(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a
^2*b - b^3 - b*c^2)*sin(x))) - (B*a^6 - 4*B*a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2
*b^4 - 3*A*a*b^5 + B*b^6)*c + (C*a*c^6 + A*c^7 - (5*A*a^2 - B*a*b - 3*A*b^2
)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a
*b^3 + 3*A*b^4)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(225) = 450$.
time = 0.50, size = 1506, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac"
)
```

```
[Out] -(2*A*a^2 - 3*B*a*b + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 -
c^2)))/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^
2 - c^2)) + (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(
1/2*x)^3 + 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b
^3*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b
^4*tan(1/2*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*tan
(1/2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*a*b
^3*c*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7
*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 - A*a*b^2*c^2*tan(1/
2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^
2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^
4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*
tan(1/2*x)^2 + 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*
b^4*tan(1/2*x)^2 + 2*C*b^5*tan(1/2*x)^2 + 4*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*
c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x)^2 + 13*
```

```
A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 6*A*a*b^3*c*tan(1/
2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/
2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2*x)^2 - 13*C*a*b^
2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2*x)^2 -
4*B*a^2*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*tan(1/2*x)^
2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*b*c^4*tan(1/2*x
)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 4*
A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*x) + 5*A*a^3*b^2*tan(1/2*x) + B*a^3*
b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 5*A*a*b^4*
tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) + A*b^5*tan(1/2*x) + 2*B*b^5*tan(1/2*x) -
 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x) + 5*C*a^2*b^2*c*tan(1/2*x) -
 5*C*a*b^3*c*tan(1/2*x) + 11*A*a^3*c^2*tan(1/2*x) - 4*B*a^3*c^2*tan(1/2*x)
- 3*A*a^2*b*c^2*tan(1/2*x) - 8*B*a^2*b*c^2*tan(1/2*x) - 7*A*a*b^2*c^2*tan(1
/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) - A*b^3*c^2*tan(1/2*x) + 4*B*b^3*c^2*tan(1
/2*x) - 4*C*a^2*c^3*tan(1/2*x) + 4*C*a*b*c^3*tan(1/2*x) - 2*A*a*c^4*tan(1/2
*x) + 2*B*a*c^4*tan(1/2*x) - 2*A*b*c^4*tan(1/2*x) + 2*B*b*c^4*tan(1/2*x) -
2*C*a^5 + 4*C*a^3*b^2 - 2*C*a*b^4 + 4*A*a^4*c - 5*B*a^3*b*c - 3*A*a^2*b^2*c
 + 5*B*a*b^3*c - A*b^4*c - C*a^3*c^2 + C*a*b^2*c^2 - A*a^2*c^3 + 2*B*a*b*c^
3 - A*b^2*c^3)/((a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 +
b^6 - 2*a^4*c^2 + 4*a^3*b*c^2 - 4*a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c
^4 + b^2*c^4)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)^2)
```

Mupad [B]
time = 8.21, size = 1160, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

```
[Out] - ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c +
 A*b^4*c + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3*b*c + 3*A*a^2*b^
2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*
b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*b^4 - 2*B*c^4 - 7*
A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B*a^2*c^2 - 4*B*b^2
*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 3*C*a^3*c + 2*A*a*b*
c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2
*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 - 2*A*c^5 + 2*C*b^5
 + 7*A*a^2*c^3 - A*b^2*c^3 - 4*B*a^2*c^3 - 4*C*a^2*b^3 + 4*C*a^3*b^2 + 4*B*
b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*c - 2*C
*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 6*A*a*b*c^3 - 6*A*
a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c + 13*A*a^2*b^2*c + 14*B*
a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 -
 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5
 + 3*A*a^2*b^3 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3
```

```
*c^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 4*C*a^2*c^3 - 5*A*a*b^4 - 4*A*a^4*b - 2*
A*a*c^4 - 3*B*a*b^4 - 3*B*a^4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 5*C*a
^4*c + 4*C*a*b*c^3 - 5*C*a*b^3*c + 5*C*a^3*b*c - 7*A*a*b^2*c^2 - 3*A*a^2*b*
c^2 + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2 + 5*C*a^2*b^2*c))/((a - b)^2*(a^4 + b^4
 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^
2) + 2*a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 +
b^2 + tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c
^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (ta
n(x/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(
2*(b^2 - a^2 + c^2)^(5/2)))*(2*A*a^2 + A*b^2 + A*c^2 - 3*B*a*b - 3*C*a*c))/
(b^2 - a^2 + c^2)^(5/2)
```

3.553 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

Optimal. Leaf size=105

$$\frac{(2aA-b(B+iC))x}{2a^2}+\frac{i(2aAb-a^2(B-iC)-b^2(B+iC))\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{(iB-C)(\cos(x)}{2a}$$

```
[Out] 1/2*(2*a*A-b*(B+I*C))*x/a^2+1/2*I*(2*a*A*b-a^2*(B-I*C)-b^2*(B+I*C))*ln(a+b*
cos(x)+I*b*sin(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3209}

$$\frac{i(-(a^2(B-iC))+2aAb-b^2(B+iC))\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*(
B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] - I*
Sin[x]))/(2*a)
```

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(2aA-b(B+iC))x}{2a^2}+\frac{i(2aAb-a^2(B-iC)-b^2(B+iC))\log(a+b\cos}{2a^2b}$$

Mathematica [A]
time = 0.30, size = 165, normalized size = 1.57

$$\frac{(2aAb+a^2(B-iC)-b^2(B+iC))x+2(-2aAb+a^2(B-iC)+b^2(B+iC))\,\mathrm{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iab(B+iC)\cos(x)+(2iaAb+a^2(-iB-C)+b^2(-iB+C))\log\left(a^2+b^2+2ab\cos(x)\right)+2ab(B+iC)\sin(x)}{4a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

```
[Out] ((2*a*A*b + a^2*(B - I*C) - b^2*(B + I*C))*x + 2*(-2*a*A*b + a^2*(B - I*C)
+ b^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)] + (2*I)*a*b*(B + I*C)*C
os[x] + ((2*I)*a*A*b + a^2*((-I)*B - C) + b^2*((-I)*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]] + 2*a*b*(B + I*C)*Sin[x])/(4*a^2*b)
```

Maple [A]
time = 0.30, size = 136, normalized size = 1.30

method	result
default	$\frac{i\left(iC\,a^{2}-iC\,b^{2}+2aAb-a^{2}B-B\,b^{2}\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^{2}b}+\frac{\left(-2iAa+iBb-bC\right)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^{2}}-\frac{-iC-B}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{i(-$
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}+\frac{iB\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^{2}}+\frac{xA}{a}-\frac{bxB}{2a^{2}}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2a^{2}}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)A}{a}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2b}-\frac{ib\ln(\mathrm{e}^{i}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(I*C*a^2-I*C*b^2+2*a*A*b-a^2*B-B*b^2)/a^2/b*ln(I*a+I*b+a*tan(1/2*x)-b
*tan(1/2*x))+1/2/a^2*(-2*I*A*a+I*B*b-b*C)*ln(-I+tan(1/2*x))-(-I*C-B)/a/(-I+
tan(1/2*x))+1/2*I*(B-I*C)/b*ln(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.83, size = 89, normalized size = 0.85

$$\frac{\left((i\,B-C)ab+(2\,Aab-(B+i\,C)b^{2})xe^{(i\,x)}+((-i\,B-C)a^{2}+2i\,Aab+(-i\,B+C)b^{2})e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*B - C)*a*b + (2*A*a*b - (B + I*C)*b^2)*x*e^(I*x) + ((-I*B - C)*a^2
+ 2*I*A*a*b + (-I*B + C)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2
*b)
```

Sympy [A]
time = 0.74, size = 129, normalized size = 1.23

$$\begin{cases} \frac{(iB-C)e^{-ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{2Aa-Bb-iCb}{2a^2} + \frac{2Aa+Ba-Bb+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-Bb-iCb)}{2a^2} - \frac{i(-2Aab+Ba^2+Bb^2-iCa^2+iCb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

```
[Out] Piecewise(((I*B - C)*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - B*b - I*C*b)
/(2*a**2) + (2*A*a + B*a - B*b + I*C*a - I*C*b)/(2*a**2)), True)) + x*(2*A*
a - B*b - I*C*b)/(2*a**2) - I*(-2*A*a*b + B*a**2 + B*b**2 - I*C*a**2 + I*C*
b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(83) = 166.
time = 0.42, size = 203, normalized size = 1.93

$$-\frac{(-2i\,Aa+i\,Bb-Cb)\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+b\tan\left(\frac{1}{2}\,x\right)^2-2i\,a\tan\left(\frac{1}{2}\,x\right)+a+b\right)}{4\,a^2} - \frac{(2i\,Aa-i\,Bb+Cb)\log\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}{2\,a^2} + \frac{(2\,Ba^2-2i\,Ca^2-2\,Aab+Bb^2+i\,Cb^2)\left(x+2\,\arctan\left(\frac{-i\,a\cos(x)-a\sin(x)-i\,a}{a\cos(x)-i\,a\sin(x)-a+2\,b}\right)\right)}{4\,a^2b} - \frac{-2i\,Aa\tan\left(\frac{1}{2}\,x\right)+i\,Bb\tan\left(\frac{1}{2}\,x\right)-Cb\tan\left(\frac{1}{2}\,x\right)-2\,Aa-2\,Ba-2i\,Ca+Bb+i\,Cb}{2\,a^2\left(\tan\left(\frac{1}{2}\,x\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -1/4*(-2*I*A*a + I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*
tan(1/2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b + C*b)*log(tan(1/2*x) - I)/a
^2 + 1/4*(2*B*a^2 - 2*I*C*a^2 - 2*A*a*b + B*b^2 + I*C*b^2)*(x + 2*arctan((-
I*a*cos(x) - a*sin(x) - I*a)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) -
1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*
a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

Mupad [B]
time = 6.93, size = 132, normalized size = 1.26

$$-\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}-\frac{\frac{C\,b^2}{2}-\frac{B\,b^2\,1\mathrm{i}}{2}+A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C+B\,1\mathrm{i}\right)}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(B\,b-2\,A\,a+C\,b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a^2}+\frac{5\,B+C\,5\mathrm{i}}{5\,a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

```
[Out] (log(tan(x/2) + 1i)*(B*1i + C))/(2*b) - log(a + b - a*tan(x/2)*1i + b*tan(x
/2)*1i)*(((B*1i)/2 + C/2)/b - ((C*b^2)/2 - (B*b^2*1i)/2 + A*a*b*1i)/(a^2*b)
) + (log(tan(x/2) - 1i)*(B*b - 2*A*a + C*b*1i)*1i)/(2*a^2) + (5*B + C*5i)/(
5*a*(tan(x/2) - 1i))
```

3.554 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

Optimal. Leaf size=103

$$\frac{(2aA-bB+ibC)x}{2a^2}-\frac{i(2aAb-b^2(B-iC)-a^2(B+iC))\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{(iB+C)(\cos(x)+}{2a}$$

[Out] 1/2*(2*a*A-b*B+I*b*C)*x/a^2-1/2*I*(2*a*A*b-b^2*(B-I*C)-a^2*(B+I*C))*ln(a+b*cos(x)-I*b*sin(x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3209}

$$-\frac{i(-(a^2(B+iC))+2aAb-b^2(B-iC))\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]

[Out] ((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*(B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)

Rule 3209

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA-bB+ibC)x}{2a^2}-\frac{i(2aAb-b^2(B-iC)-a^2(B+iC))\log(a+b\cos(x}{2a^2b}$$

Mathematica [A]
time = 0.32, size = 167, normalized size = 1.62

$$\frac{\left(2aA-b(B-iC)+\frac{a^2(B+iC)}{b}\right)x+\frac{2(-2aAb+b^2(B-iC)+a^2(B+iC))\text{ArcTan}\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)}{b}-2ia(B-iC)\cos(x)+\frac{(-2iaAb+ia^2(B+iC)+b^2(iB+C))\log(a^2+b^2+2ab\cos(x))}{b}+2a(B-iC)\sin(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

```
[Out] ((2*a*A - b*(B - I*C) + (a^2*(B + I*C))/b)*x + (2*(-2*a*A*b + b^2*(B - I*C)
 + a^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)])/b - (2*I)*a*(B - I*C)
*Cos[x] + (((-2*I)*a*A*b + I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a^2 + b^2 +
 2*a*b*Cos[x]])/b + 2*a*(B - I*C)*Sin[x])/(4*a^2)
```

Maple [A]
time = 0.30, size = 148, normalized size = 1.44

method	result
risch	$-\frac{C\,e^{ix}}{2a}-\frac{iB\,e^{ix}}{2a}+\frac{ixC}{2b}+\frac{Bx}{2b}-\frac{\ln\left(e^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(e^{ix}+\frac{b}{a}\right)C}{2a^2}-\frac{i\ln\left(e^{ix}+\frac{b}{a}\right)A}{a}+\frac{i\ln\left(e^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(e^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$-\frac{i(iC+B)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}+\frac{i(-iC\,a^2+iC\,b^2+2aAb-a^2B-B\,b^2)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}-\frac{iC-B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}+\frac{(2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(B+I*C)/b*ln(-I+tan(1/2*x))+1/2*I*(-I*C*a^2+I*C*b^2+2*a*A*b-a^2*B-B*
b^2)*(a-b)/a^2/b/(-a+b)*ln(I*a+I*b-a*tan(1/2*x)+b*tan(1/2*x))-(I*C-B)/a/(ta
n(1/2*x)+I)+1/2*(-b*C+2*I*A*a-I*B*b)/a^2*ln(tan(1/2*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.23, size = 73, normalized size = 0.71

$$\frac{(B+i\,C)a^2x+(-i\,B-C)abe^{(i\,x)}+((i\,B-C)a^2-2i\,Aab+(i\,B+C)b^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="frica
s")
```

```
[Out] 1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 - 2*I*A*a*b
+ (I*B + C)*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

Sympy [A]
time = 0.69, size = 105, normalized size = 1.02

$$\begin{cases} \frac{(-iB-C)e^{ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{B+iC}{2b} + \frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{x(B+iC)}{2b} + \frac{i(-2Aab+Ba^2+Bb^2+iCa^2-iCb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

```
[Out] Piecewise(((-I*B - C)*exp(I*x)/(2*a), Ne(a, 0)), (x*(-(B + I*C)/(2*b) + (B*
a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) + x*(B + I*C)/(2*b) + I*(-2*A*a*b
 + B*a**2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(85) = 170.
time = 0.43, size = 203, normalized size = 1.97

$$-\frac{(2iAa-iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(-2iAa+iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2} + \frac{(2Ba^2+2iCa^2-2Aab+Bb^2-iCb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{2iAa\tan\left(\frac{1}{2}x\right)-iBb\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa-2Ba+2iCa+Bb-iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac"
)
```

```
[Out] -1/4*(2*I*A*a - I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*t
an(1/2*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b + C*b)*log(tan(1/2*x) + I)/a
^2 + 1/4*(2*B*a^2 + 2*I*C*a^2 - 2*A*a*b + B*b^2 - I*C*b^2)*(x + 2*arctan((I
*a*cos(x) - a*sin(x) + I*a)/(a*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1
/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2*B*a
+ 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

Mupad [B]
time = 6.86, size = 133, normalized size = 1.29

$$\ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\,\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{B\,b^2\,1\mathrm{i}}{2}+\frac{C\,b^2}{2}-A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(2\,A\,a-B\,b+C\,b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a^2}+\frac{5\,B-C\,5\mathrm{i}}{5\,a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

```
[Out] log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1i
)/2 + (C*b^2)/2 - A*a*b*1i)/(a^2*b)) + (log(tan(x/2) + 1i)*(2*A*a - B*b + C
*b*1i)*1i)/(2*a^2) + (5*B - C*5i)/(5*a*(tan(x/2) + 1i)) - (log(tan(x/2) - 1
i)*(B*1i - C))/(2*b)
```

3.555 $\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{c\cos(x)-b\sin(x)}{a+b\cos(x)+c\sin(x)}$$

```
[Out] (-c*cos(x)+b*sin(x))/(a+b*cos(x)+c*sin(x))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 2.83, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3229}

$$-\frac{c\cos(x)\left(a^2-b^2-c^2\right)-b\sin(x)\left(a^2-b^2-c^2\right)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

```
[Out] -((c*(a^2 - b^2 - c^2)*Cos[x] - b*(a^2 - b^2 - c^2)*Sin[x])/((a^2 - b^2 - c
^2)*(a + b*Cos[x] + c*Sin[x])))
```

Rule 3229

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

Rubi steps

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=-\frac{c(a^2-b^2-c^2)\cos(x)-b(a^2-b^2-c^2)\sin(x)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 1.33

$$\frac{ac+b^2\sin(x)+c^2\sin(x)}{b(a+b\cos(x)+c\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2
,x]
```

```
[Out] (a*c + b^2*Sin[x] + c^2*Sin[x])/(b*(a + b*Cos[x] + c*Sin[x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.
time = 0.23, size = 70, normalized size = 2.92

method	result	size
risch	$-\frac{2i\left(-ib+c-ia\,e^{ix}\right)}{c\,e^{2ix}+ib\,e^{2ix}-c+2ia\,e^{ix}+ib}$	54
default	$-\frac{2\left(-\frac{\left(ab-b^2-c^2\right)\tan\left(\frac{x}{2}\right)}{a-b}+\frac{ac}{a-b}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b}$	70
norman	$\frac{-\frac{2ab+2b^2+2c^2}{2c}-\frac{\left(2ab-2b^2-2c^2\right)\left(\tan^4\left(\frac{x}{2}\right)\right)}{2c}-\frac{2ab\left(\tan^2\left(\frac{x}{2}\right)\right)}{c}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-b\left(\tan^2\left(\frac{x}{2}\right)\right)+2c\tan\left(\frac{x}{2}\right)+a+b\right)}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETUR
NVERBOSE)
```

```
[Out] -2*(-(a*b-b^2-c^2)/(a-b)*tan(1/2*x)+a*c/(a-b))/(a*tan(1/2*x)^2-b*tan(1/2*x)
^2+2*c*tan(1/2*x)+a+b)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.31, size = 24, normalized size = 1.00

$$-\frac{c\cos(x)-b\sin(x)}{b\cos(x)+c\sin(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="fricas")
```

```
[Out] -(c*cos(x) - b*sin(x))/(b*cos(x) + c*sin(x) + a)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2+c**2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.
time = 0.43, size = 68, normalized size = 2.83

$$\frac{2\left(ab\tan\left(\frac{1}{2}x\right)-b^2\tan\left(\frac{1}{2}x\right)-c^2\tan\left(\frac{1}{2}x\right)-ac\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algori
thm="giac")
```

```
[Out] 2*(a*b*tan(1/2*x) - b^2*tan(1/2*x) - c^2*tan(1/2*x) - a*c)/((a*tan(1/2*x)^2
 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)*(a - b))
```

Mupad [**B**]
time = 3.03, size = 62, normalized size = 2.58

$$-\frac{\frac{2ac}{a-b}+\frac{2\tan\left(\frac{x}{2}\right)\left(b^2-ab+c^2\right)}{a-b}}{(a-b)\tan\left(\frac{x}{2}\right)^2+2c\tan\left(\frac{x}{2}\right)+a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2 + c^2 + a*c*sin(x) + a*b*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

```
[Out] -((2*a*c)/(a - b) + (2*tan(x/2)*(b^2 - a*b + c^2))/(a - b))/(a + b + 2*c*ta
n(x/2) + tan(x/2)^2*(a - b))
```

3.556 $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=390

$$\frac{2(161a^2d+63(b^2+c^2)\,d+15a^3e+145a(b^2+c^2)\,e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\cdots}}{105\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

```
[Out] -2/7*(a+b*cos(x)+c*sin(x))^(5/2)*(c*e*cos(x)-b*e*sin(x))-2/35*(a+b*cos(x)+c
*sin(x))^(3/2)*(c*(5*a*e+7*d)*cos(x)-b*(5*a*e+7*d)*sin(x))-2/105*(c*(56*a*d
+15*a^2*e+25*(b^2+c^2)*e)*cos(x)-b*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*sin(x))
*(a+b*cos(x)+c*sin(x))^(1/2)+2/105*(161*a^2*d+63*(b^2+c^2)*d+15*a^3*e+145*a
*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c
))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^
2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^
2+c^2)^(1/2)))^(1/2)-2/105*(a^2-b^2-c^2)*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*(
cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(si
n(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^
(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3225, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2(a^2-b^2-c^2)\,(15a^2e+56ad+25e(b^2+c^2))\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(15a^3e+161a^2d+145ae(b^2+c^2)+63d(b^2+c^2))\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{105\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2}{105}\sqrt{a+b\cos(x)+c\sin(x)}\,(c\cos(x)\,(15a^2e+56ad+25e(b^2+c^2))-b\sin(x)\,(15a^2e+56ad+25e(b^2+c^2)))-\frac{2}{35}(a+b\cos(x)+c\sin(x))^{3/2}(c\cos(x)(5ae+7d)-b\sin(x)(5ae+7d))-\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b^2 + c^2)*e)*Elliptic
E[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a +
 b*Cos[x] + c*Sin[x]])/(105*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 +
c^2])]) - (2*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Ellip
ticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[
(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(105*Sqrt[a + b*Cos[x] +
c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(5/2)*(c*e*Cos[x] - b*e*Sin[x]))/
7 - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*(7*d + 5*a*e)*Cos[x] - b*(7*d + 5
*a*e)*Sin[x]))/35 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(56*a*d + 15*a^2*e
```

```
+ 25*(b^2 + c^2)*e)*Cos[x] - b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e)*Sin[x
]))/105
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
```

```
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin$$

$$= -\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin$$

$$= \frac{2(161a^2d+63(b^2+c^2)\,d+15a^3e+145a(b^2+\ldots}{105\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.68, size = 7823, normalized size = 20.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3451 vs. $2(414) = 828$.
time = 1.82, size = 3452, normalized size = 8.85

method	result	size
default	Expression too large to display	3452

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURNV
ERBOSE)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)*((b^6*e+3*b^4*c^2*e+3*b^
2*c^4*e+c^6*e)*(-2/7/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))^2*(cos(x-arctan(-b
,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+12/35/(b^2+c^2)*a*sin
(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)+a))^(1/2)-2/3*(5/7+24/35/(b^2+c^2)*a^2)/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2*(-4/35/(b^2+c^2)*a^2
+5/21)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+
(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1
/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))
^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2
)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(-48*a^3-44*
a*b^2-44*a*c^2)/(105*(b^2+c^2)^(1/2)*b^2+105*(b^2+c^2)^(1/2)*c^2)*(1/(b^2+c
^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2))
)^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2
)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos
(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2
+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-Elli
pticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),(
(-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+(3*(b^2+c^2)^(1/2)*a*b^
4*e+6*(b^2+c^2)^(1/2)*a*b^2*c^2*e+3*(b^2+c^2)^(1/2)*a*c^4*e+(b^2+c^2)^(1/2)
*b^4*d+2*(b^2+c^2)^(1/2)*b^2*c^2*d+(b^2+c^2)^(1/2)*c^4*d)*(-2/5/(b^2+c^2)^(
1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arct
an(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a
+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((s
in(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-
b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1
/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^(
1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/
2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-
sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-ar
```

```
ctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)
^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)
^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF
((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(
b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+(3*a^2*b^4*e+6*a^2*b^2*c^2*e
+3*a^2*c^4*e+3*a*b^4*d+6*a*b^2*c^2*d+3*a*c^4*d)*(-2/3/(b^2+c^2)^(1/2)*(cos(
x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^
2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*Ellipt
icF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-
a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b
^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1
/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^
(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/
(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/
(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+
(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-
EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/
2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+2*((b^2+c^2)^(1/2)*
a^3*b^2*e+(b^2+c^2)^(1/2)*a^3*c^2*e+(b^2+c^2)^(3/2)*a^2*d+2*a^2*b^2*d*(b^2+
c^2)^(1/2)+2*a^2*c^2*d*(b^2+c^2)^(1/2))*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)
^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,
c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1
)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE
((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(
b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*si
n(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(
b^2+c^2)^(1/2)))^(1/2)))+2*a^3*b^2*d*(1/(b^2+c^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.39, size = 2027, normalized size = 5.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-7*I*(a^3*b - 33*a*b^3 - 33*a*b*c^2)*d + 7*(33*a*c^3 - (a^3
 - 33*a*b^2)*c)*d - 5*I*(6*a^4*b - 23*a^2*b^3 - 15*b^5 - 15*b*c^4 - (23*a^2
*b + 30*b^3)*c^2)*e + 5*(15*c^5 + (23*a^2 + 30*b^2)*c^3 - (6*a^4 - 23*a^2*b
^2 - 15*b^4)*c)*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4
 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^
2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3
+ 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(
b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*co
s(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(7*I*(a^3*b - 33*a*
b^3 - 33*a*b*c^2)*d + 7*(33*a*c^3 - (a^3 - 33*a*b^2)*c)*d + 5*I*(6*a^4*b -
23*a^2*b^3 - 15*b^5 - 15*b*c^4 - (23*a^2*b + 30*b^3)*c^2)*e + 5*(15*c^5 + (
23*a^2 + 30*b^2)*c^3 - (6*a^4 - 23*a^2*b^2 - 15*b^4)*c)*e)*sqrt(b - I*c)*we
ierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 +
2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^
5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b
^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6),
 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(
b^2 + c^2)) - 3*sqrt(2)*(7*I*(23*a^2*b^2 + 9*b^4 + 9*c^4 + (23*a^2 + 18*b^2
)*c^2)*d + 5*I*(3*a^3*b^2 + 29*a*b^4 + 29*a*c^4 + (3*a^3 + 58*a*b^2)*c^2)*e
)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*
c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^
3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4
*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b
^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I
*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8
*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6
*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 +
3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 +
I*c^2)*sin(x))/(b^2 + c^2))) - 3*sqrt(2)*(-7*I*(23*a^2*b^2 + 9*b^4 + 9*c^4
+ (23*a^2 + 18*b^2)*c^2)*d - 5*I*(3*a^3*b^2 + 29*a*b^4 + 29*a*c^4 + (3*a^3
+ 58*a*b^2)*c^2)*e)*sqrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 -
4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 +
 c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9
*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6
 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4
 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^
2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3
+ 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(
b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*co
s(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))) - 6*(15*(3*b^4*c + 2*b^2*c^
```

```
3 - c^5)*cos(x)^3*e + 6*(7*(b^3*c + b*c^3)*d + 15*(a*b^3*c + a*b*c^3)*e)*co
s(x)^2 - 21*(b^3*c + b*c^3)*d + (77*(a*b^2*c + a*c^3)*d + 5*(8*c^5 + (9*a^2
 + 7*b^2)*c^3 + (9*a^2*b^2 - b^4)*c)*e)*cos(x) - 45*(a*b^3*c + a*b*c^3)*e -
 (15*(b^5 - 2*b^3*c^2 - 3*b*c^4)*cos(x)^2*e + 77*(a*b^3 + a*b*c^2)*d + 3*(7
*(b^4 - c^4)*d + 15*(a*b^4 - a*c^4)*e)*cos(x) + 5*(9*a^2*b^3 + 5*b^5 + 8*b*
c^4 + (9*a^2*b + 13*b^3)*c^2)*e)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a))/(b^
2 + c^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\cos(x) + c\sin(x))^{5/2} (d + b\,e\cos(x) + c\,e\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

```
[Out] int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.557 $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=294

$$\frac{2(20ad+3a^2e+9(b^2+c^2)\,e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{15\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2(a^2-b^2-$$

```
[Out] -2/5*(a+b*cos(x)+c*sin(x))^(3/2)*(c*e*cos(x)-b*e*sin(x))-2/15*(c*(3*a*e+5*d
)*cos(x)-b*(3*a*e+5*d)*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/15*(20*a*d+3*a
^2*e+9*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arct
an(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(
b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/
(a+(b^2+c^2)^(1/2)))^(1/2)-2/15*(a^2-b^2-c^2)*(3*a*e+5*d)*(cos(1/2*x-1/2*ar
ctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arct
an(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+
c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3225, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2(a^2-b^2-c^2)\,(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(3a^2e+20ad+9e(b^2+c^2))\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{15\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2}{15}\sqrt{a+b\cos(x)+c\sin(x)}\,(c\cos(x)(3ae+5d)-b\sin(x)(3ae+5d))-\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*
Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(15*
Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^
2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + S
qrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(15
*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*e*C
os[x] - b*e*Sin[x]))/5 - (2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*(5*d + 3*a*e)*
Cos[x] - b*(5*d + 3*a*e)*Sin[x]))/15
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int (a + b\cos(x) + c\sin(x))^{3/2}(d + be\cos(x) + ce\sin(x))\,dx = -\frac{2}{5}(a + b\cos(x) + c\sin(x))^{3/2}(ce\cos(x) - be\,\mathrm{s}$$

$$= -\frac{2}{5}(a + b\cos(x) + c\sin(x))^{3/2}(ce\cos(x) - be\,\mathrm{s}$$

$$= -\frac{2}{5}(a + b\cos(x) + c\sin(x))^{3/2}(ce\cos(x) - be\,\mathrm{s}$$

$$= -\frac{2}{5}(a + b\cos(x) + c\sin(x))^{3/2}(ce\cos(x) - be\,\mathrm{s}$$

$$= \frac{2(20ad + 3a^2e + 9(b^2 + c^2)\,e)\,E\left(\frac{1}{2}(x - \tan^{-1}\right.}{15\sqrt{\frac{a + b\cos}{a +}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.40, size = 5218, normalized size = 17.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2204 vs. $2(322) = 644$.
time = 1.01, size = 2205, normalized size = 7.50

method	result	size
default	Expression too large to display	2205

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURNV
ERBOSE)

[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*((b^4*e+2*b^2*c^2*
e+c^4*e)*(-2/5/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+8/15/(b^2+c^2)*a*(cos(x-arcta
n(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+4/15/(b^2+c^2)^(1
/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^
(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)
))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(b^
2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a
)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(
b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^
2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c)
)+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arc
tan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2
)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+(2
*(b^2+c^2)^(1/2)*a*b^2*e+2*(b^2+c^2)^(1/2)*a*c^2*e+(b^2+c^2)^(1/2)*b^2*d+(b
^2+c^2)^(1/2)*c^2*d)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*((b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-
b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))
+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x
-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2
+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(
-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c)
)+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*Ellipt
icE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-
a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)
*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-
a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a^2*b^2*e+a^2*c^2*e+2*a*b^2*d+2*a*c^2*d)*(1
/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)
^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)
))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/
2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((
-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/
(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2
```

```
))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^
(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))+2*a^2*d*(b^2+c^2)
^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+
(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1
/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)
^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2
))/(-a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,
c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.74, size = 1703, normalized size = 5.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] 1/45*(sqrt(2)*(5*I*(a^2*b + 3*b^3 + 3*b*c^2)*d + 5*(3*c^3 + (a^2 + 3*b^2)*c
)*d - 6*I*(a^3*b - 3*a*b^3 - 3*a*b*c^2)*e + 6*(3*a*c^3 - (a^3 - 3*a*b^2)*c)
*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 +
6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27
*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3
- 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2
 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2
 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(-5*I*(a^2*b + 3*b^3 + 3*b*c^2)*d
+ 5*(3*c^3 + (a^2 + 3*b^2)*c)*d + 6*I*(a^3*b - 3*a*b^3 - 3*a*b*c^2)*e + 6*(
3*a*c^3 - (a^3 - 3*a*b^2)*c)*e)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^
2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b
^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5
- 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 -
9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(
b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2)) - 3*sqrt(2)*(20
```

```
*I*(a*b^2 + a*c^2)*d + 3*I*(a^2*b^2 + 3*b^4 + 3*c^4 + (a^2 + 6*b^2)*c^2)*e)
*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c
^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3
*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*
a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^
2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*
b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*
a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*
(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3
*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I
*c^2)*sin(x))/(b^2 + c^2))) - 3*sqrt(2)*(-20*I*(a*b^2 + a*c^2)*d - 3*I*(a^2
*b^2 + 3*b^4 + 3*c^4 + (a^2 + 6*b^2)*c^2)*e)*sqrt(b - I*c)*weierstrassZeta(
4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b
^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9
*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a
^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInver
se(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b -
3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
+ 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(
8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2
*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))) -
6*(6*(b^3*c + b*c^3)*cos(x)^2*e + (5*(b^2*c + c^3)*d + 6*(a*b^2*c + a*c^3)*
e)*cos(x) - 3*(b^3*c + b*c^3)*e - (3*(b^4 - c^4)*cos(x)*e + 5*(b^3 + b*c^2)
*d + 6*(a*b^3 + a*b*c^2)*e)*sin(x))*sqrt(b*cos(x) + c*sin(x) + a))/(b^2 + c
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\cos(x) + c\sin(x))^{\frac{3}{2}} (be\cos(x) + ce\sin(x) + d)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
[Out] Integral((a + b*cos(x) + c*sin(x))**(3/2)*(b*e*cos(x) + c*e*sin(x) + d), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos (x) + c \sin (x))^{3/2} (d + b e \cos (x) + c e \sin (x)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
[Out] int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.558 $\int \sqrt{a+b\cos(x)+c\sin(x)}\,(d+be\cos(x)+ce\sin(x))\,dx$

Optimal. Leaf size=229

$$\frac{2(3d+ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2\left(a^2-b^2-c^2\right)eF\left(\frac{1}{2}(x-\tan^{-1}\right.}{3\sqrt{a\,}}$$

```
[Out] -2/3*(c*e*cos(x)-b*e*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(a*e+3*d)*(cos
(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin(1
/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*
(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/
2)-2/3*(a^2-b^2-c^2)*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*a
rctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(
a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2
)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3225, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] (2*(3*d + a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqr
t[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*Sqrt[(a + b*Cos[x] + c*Sin
[x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*e*EllipticF[(x - ArcTan
[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] +
c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*Sqrt[a + b*Cos[x] + c*Sin[x]]) - (2*Sq
rt[a + b*Cos[x] + c*Sin[x]]*(c*e*Cos[x] - b*e*Sin[x]))/3
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3225

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Dist[1/(a*(n + 1
)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
+ a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
+ (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /; F
reeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(x)+c\sin(x)}\,(d+be\cos(x)+ce\sin(x))\,dx = -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin($$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin($$

$$= -\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}\,(ce\cos(x)-be\sin($$

$$= \frac{2(3d+ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\sqrt{\frac{a+b\cos(x)+c\sin(}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.25, size = 3006, normalized size = 13.13

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*b*(3*d + a*e))/(3*c) - (2*c*e*Cos[x])/3 +
 (2*b*e*Sin[x])/3) + (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]
*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^
2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[
(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*S
qrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]
)/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c) + (2*b^2*e*AppellF
1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*
c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c
))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)
/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[
```

(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sq
rt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/
(3*Sqrt[1 + b^2/c^2]*c) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1
 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2
/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b
^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sq
rt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*
Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c
]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[
b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]) + (b^2*d*(-((
c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan
[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt
[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1
+ c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^
2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(
b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sq
rt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])
/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x -
 ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]
))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/c + c*d*(-((c*Appel
lF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])
/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^
2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b
^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2
)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c
^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*S
qrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a +
b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan
[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt
[a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]) + (a*b^2*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(3*c) + (a*c*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b

```
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(261) = 522$.
time = 0.68, size = 1438, normalized size = 6.28

method	result	size
default	Expression too large to display	1438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURNV
ERBOSE)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(((b^2+c^2)^(1/2)*
b^2*e+(b^2+c^2)^(1/2)*c^2*e)*(-2/3/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a+1)*(
((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arcta
n(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))
^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/
(-a+(b^2+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*
(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x
-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arct
an(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1
)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(
1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(
1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))+2*(a*b^2*e+a*c^2*e+b^2*d+c^2*d)*(1/(b^
2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(
b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-E
llipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2
),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))+2*a*d*(b^2+c^2)^(1/2)
*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c
^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1
```

```
/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^
(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2)
)*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*
sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a
+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,c))+c^
2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.00, size = 1506, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="fricas")
```

```
[Out] 1/9*(sqrt(2)*(3*I*a*b*d + 3*a*c*d - I*(2*a^2*b - 3*b^3 - 3*b*c^2)*e + (3*c^
3 - (2*a^2 - 3*b^2)*c)*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2
- 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2
*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*
(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^
4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 +
c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(-3*I*a*b*d
+ 3*a*c*d + I*(2*a^2*b - 3*b^3 - 3*b*c^2)*e + (3*c^3 - (2*a^2 - 3*b^2)*c)*e
)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*
I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(
8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 -
6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 +
 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2
- I*c^2)*sin(x))/(b^2 + c^2)) - 3*sqrt(2)*(3*I*(b^2 + c^2)*d + I*(a*b^2 + a
*c^2)*e)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 +
 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/2
7*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3
 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^
2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^
```

```
2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -
8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*
c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4
*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I
*b^2 + I*c^2)*sin(x))/(b^2 + c^2))) - 3*sqrt(2)*(-3*I*(b^2 + c^2)*d - I*(a*
b^2 + a*c^2)*e)*sqrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^
2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4
), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b
^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3
*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4
*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 +
c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*
a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6
+ 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x)
 - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))) - 6*((b^2*c + c^3)*cos(x)*e - (
b^3 + b*c^2)*e*sin(x))*sqrt(b*cos(x) + c*sin(x) + a))/(b^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\cos(x) + c\sin(x)}\,(be\cos(x) + ce\sin(x) + d)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))**(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
[Out] Integral(sqrt(a + b*cos(x) + c*sin(x))*(b*e*cos(x) + c*e*sin(x) + d), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorith
m="giac")
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\,\cos(x) + c\,\sin(x)}\,\left(d + b\,e\,\cos(x) + c\,e\,\sin(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
[Out] int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.559 $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

Optimal. Leaf size=180

$$\frac{2eE\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2(d-ae)F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\cdots}{\sqrt{a+b\cos(x)+\cdots}}$$

```
[Out] 2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*Ellipti
cE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))
^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/
2)))^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*ar
ctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a
+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)
/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3228, 3198, 2732, 3206, 2740}

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^
2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqr
t[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 +
 c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2
+ c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx = e\int \sqrt{a+b\cos(x)+c\sin(x)}\,dx + (d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

$$= \frac{\left(e\sqrt{a+b\cos(x)+c\sin(x)}\right)\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}} + \frac{\sqrt{b^2+c^2}\cos(x-\mathrm{t}}{a+\sqrt{b^2+c}}}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

$$= \frac{2eE\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2(d}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 5.56, size = 570, normalized size = 3.17

$$\frac{2dF_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{a+b\cos(x)+c\sin(x)}{a-\sqrt{1+\frac{b^2}{c^2}}\,c},\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{1+\frac{b^2}{c^2}}\,c}\right)\sec\left(x+\mathrm{ArcTan}\left(\frac{b}{c}\right)\right)\sqrt{\frac{\sqrt{1+\frac{b^2}{c^2}}\,c-b\cos(x)-c\sin(x)}{a+\sqrt{1+\frac{b^2}{c^2}}\,c}}\sqrt{a+b\cos(x)+c\sin(x)}\sqrt{\frac{b\cos(x)+c\left(\sqrt{1+\frac{b^2}{c^2}}+\sin(x)\right)}{-a+\sqrt{1+\frac{b^2}{c^2}}\,c}}}{\sqrt{1+\frac{b^2}{c^2}}\,c} - \frac{(b^2+c^2)\,eF_1\left(-\frac{1}{2};-\frac{1}{2},-\frac{1}{2};\frac{1}{2};\frac{a+b\cos(x)+c\sin(x)}{a-b\sqrt{1+\frac{c^2}{b^2}}},\frac{a+b\cos(x)+c\sin(x)}{a+b\sqrt{1+\frac{c^2}{b^2}}}\right)\sin\left(x-\mathrm{ArcTan}\left(\frac{c}{b}\right)\right)}{b\sqrt{1+\frac{c^2}{b^2}}\sqrt{\frac{b\sqrt{1+\frac{c^2}{b^2}}-b\cos(x)-c\sin(x)}{a+b\sqrt{1+\frac{c^2}{b^2}}}}\sqrt{a+b\cos(x)+c\sin(x)}\sqrt{\frac{b\sqrt{1+\frac{c^2}{b^2}}+b\cos(x)+c\sin(x)}{-a+b\sqrt{1+\frac{c^2}{b^2}}}}} + \frac{e\left(2b^3\sqrt{1+\frac{c^2}{b^2}}\cos(x)-2b(b^2+c^2)\cos\left(x-\mathrm{ArcTan}\left(\frac{c}{b}\right)\right)+2b^2c\sqrt{1+\frac{c^2}{b^2}}\sin(x)+b^2c\sin\left(x-\mathrm{ArcTan}\left(\frac{c}{b}\right)\right)+c^3\sin\left(x-\mathrm{ArcTan}\left(\frac{c}{b}\right)\right)\right)}{bc\sqrt{1+\frac{c^2}{b^2}}\sqrt{a+b\cos(x)+c\sin(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

```
[Out] (2*d*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cos[x] + c*Sin[x])/(a - Sqrt[1 + b
^2/c^2]*c), (a + b*Cos[x] + c*Sin[x])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[x + Ar
cTan[b/c]]*Sqrt[(Sqrt[1 + b^2/c^2]*c - b*Cos[x] - c*Sin[x])/(a + Sqrt[1 + b
^2/c^2]*c)]*Sqrt[a + b*Cos[x] + c*Sin[x]]*Sqrt[(b*Cos[x] + c*(Sqrt[1 + b^2/
c^2] + Sin[x]))/(-a + Sqrt[1 + b^2/c^2]*c)])/(Sqrt[1 + b^2/c^2]*c) - ((b^2
+ c^2)*e*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cos[x] + c*Sin[x])/(a - b*S
qrt[1 + c^2/b^2]), (a + b*Cos[x] + c*Sin[x])/(a + b*Sqrt[1 + c^2/b^2])]*Sin
[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[1 + c^2/b^2] - b*Cos[x
] - c*Sin[x])/(a + b*Sqrt[1 + c^2/b^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]]*Sqrt
[(b*Sqrt[1 + c^2/b^2] + b*Cos[x] + c*Sin[x])/(-a + b*Sqrt[1 + c^2/b^2])]) +
 (e*(2*b^3*Sqrt[1 + c^2/b^2]*Cos[x] - 2*b*(b^2 + c^2)*Cos[x - ArcTan[c/b]]
+ 2*b^2*c*Sqrt[1 + c^2/b^2]*Sin[x] + b^2*c*Sin[x - ArcTan[c/b]] + c^3*Sin[x
 - ArcTan[c/b]]))/(b*c*Sqrt[1 + c^2/b^2]*Sqrt[a + b*Cos[x] + c*Sin[x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(220) = 440$.
time = 1.10, size = 766, normalized size = 4.26

method	result
default	$\sqrt{-\frac{\left(-b^2\sin(x-\arctan(-b,c))-c^2\sin(x-\arctan(-b,c))-a\sqrt{b^2+c^2}\right)(\cos^2(x-\arctan(-b,c)))}{\sqrt{b^2+c^2}}}\left(\frac{2\left(b^2e+c^2e\right)\left(\frac{a}{\sqrt{b^2+c^2}}+1\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2*e+c^2*e)*(
1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2
)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2
)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1
/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*(
(-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/
2))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))
^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))+2*(b^2+c^2)^(1/2
)*d*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^
2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)
^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)
))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1
/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/
(-a+(b^2+c^2)^(1/2)))^(1/2)))/cos(x-arctan(-b,c))/((b^2*sin(x-arctan(-b,c))
+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.83, size = 1358, normalized size = 7.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="fricas")
[Out] 1/3*(-3*I*sqrt(2)*(b^2 + c^2)*sqrt(b + I*c)*e*weierstrassZeta(4/3*(4*a^2*b^
2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 +
 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*
I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*
b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2
*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^
4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9
*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b
^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2))) + 3*I*sqrt(2)*(b
^2 + c^2)*sqrt(b - I*c)*e*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^
2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -
8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*
c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4
*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2
*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4)
, -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^
2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*
b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3
*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))) + sqrt(2)*(-2*I*a*b*e - 2*a*c*e + 3
*I*b*d + 3*c*d)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 -
4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 +
 c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9
*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6
 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x
) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(2*I*a*b*e - 2*a*c*e -
 3*I*b*d + 3*c*d)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4
- 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2
 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 +
 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b
^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos
(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2)))/(b^2 + c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{be\cos(x) + ce\sin(x) + d}{\sqrt{a + b\cos(x) + c\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(1/2),x)
```

```
[Out] Integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(a + b*cos(x) + c*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)}{\sqrt{a+b\,\cos\left(x\right)+c\,\sin\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2),x)
```

```
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2), x)
```

3.560 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=250

$$\frac{2(d-ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2eF\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2-}}{a+\sqrt{b^2}}\right.}{\sqrt{a+b\cos(}}$$

```
[Out] 2*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan
(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^
2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)/((a+b*cos(x
)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)+2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(
1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2
)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b
^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3235, 3228, 3198, 2732, 3206, 2740}

$$\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2e\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] (2*(d - a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[
b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/((a^2 - b^2 - c^2)*Sqrt[(a + b*
Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) + (2*e*EllipticF[(x - ArcTan[b,
c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Si
n[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]] + (2*(c*(d - a*
e)*Cos[x] - b*(d - a*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a + b*Cos[x] + c*S
in[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}} - \frac{2\int \frac{\frac{1}{2}(-ad+(b^2+c^2)e)-\frac{1}{2}b(d-ae)\,c\cdots}{\sqrt{a+b\cos(x)+\cdots}}}{a^2-b^2-\cdots}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}} + e\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(\cdots}}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}} + \frac{\left((d-ae)\sqrt{a+b\cos(x)+\cdots}\right.}{(\cdots}$$

$$= \frac{2(d-ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x\cdots}}{(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.40, size = 3176, normalized size = 12.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

```
[Out] Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*(b^2 + c^2)*(-d + a*e))/(b*c*(-a^2 + b^2
+ c^2)) - (2*(-(a*c*d) + a^2*c*e - b^2*d*Sin[x] - c^2*d*Sin[x] + a*b^2*e*Si
n[x] + a*c^2*e*Sin[x]))/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))) -
 (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + Arc
Tan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqr
t[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 +
 b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*S
qrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c
^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2
+ c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*b^2*e*AppellF1
[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt
[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c
*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c)
```

)]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/
c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqr
t[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(
Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3/
2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 -
 a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/
c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[
b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTa
n[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Si
n[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]
*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*
(-a^2 + b^2 + c^2)) - (b^2*d*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqr
t[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqr
t[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Co
s[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c
^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2
 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b
*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - Ar
cTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) - (c*d*(-((c*AppellF1[-1/2, -1/2, -1/2
, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2
]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTa
n[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[x - Arc
Tan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2
 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a +
b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2]
 + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b
^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]))/(b^2 + c^2)
 - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/
b^2]*Cos[x - ArcTan[c/b]]]))/(-a^2 + b^2 + c^2) + (a*b^2*e*(-((c*AppellF1[-
1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*S
qrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b*Sqrt[1 + c^2/b^2
]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))
))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2
] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b
^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(
b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(-a + b*Sqr
t[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]
]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a +
b*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]]]))/(c*(-a^2 + b^2 + c^2)) + (a*c*e
*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a + b

```
*Sqrt[1 + c^2/b^2]*Cos[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 - a/(b*Sq
rt[1 + c^2/b^2]))))]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqr
t[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]])/(a + b*S
qrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c/b]
]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[x - ArcTan[c
/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Co
s[x - ArcTan[c/b]]))/(b^2 + c^2) - (c*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2
/b^2]))/Sqrt[a + b*Sqrt[1 + c^2/b^2]*Cos[x - Ar...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2911 vs. $2(288) = 576$.
time = 5.62, size = 2912, normalized size = 11.65

method	result	size
default	Expression too large to display	2912

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] (-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2+c^2)^(1/2)
*e*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^
(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2))
)^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/
2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(
-a+(b^2+c^2)^(1/2)))^(1/2))-(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-a
rctan(-b,c))+a)*(b^2+c^2))^(1/2)*(b^2*sin(x-arctan(-b,c))^2+c^2*sin(x-arcta
n(-b,c))^2-a^2)*(b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2
)^(1/2))/((cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b
^2+c^2))^(1/2)*sin(x-arctan(-b,c))*b^2+(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1
/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(x-arctan(-b,c))*c^2-(cos(x-
arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*a*b^2-(cos(x
-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*a*c^2)/(b^2
*sin(x-arctan(-b,c))^2+c^2*sin(x-arctan(-b,c))^2+2*sin(x-arctan(-b,c))*a*(b
^2+c^2)^(1/2)+a^2)*((b^2+c^2)*cos(x-arctan(-b,c))^2*(a*e-d)/(a^2-b^2-c^2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+a*(b^2
+c^2)^(1/2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)
*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)
*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1
/2)*sin(x-arctan(-b,c))+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(
-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1
```

```
/2)))^(1/2))+(a*e-d)*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arcta
n(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,
c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*Elli
pticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),(
(-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/
2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/
(-a+(b^2+c^2)^(1/2)))^(1/2)))-(1/2*a*e-1/2*d)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arct
an(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b
,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*(b^2+c^2)^(1/2)/a*EllipticPi(
(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-
1/(b^2+c^2)^(1/2)*a-1)*(b^2+c^2)^(1/2)/a,((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2
)^(1/2)))^(1/2))-(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(x-arctan(-b,c))^2/(a^2-b^2-
c^2)*(a*e-d)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
*(b^2+c^2))^(1/2)+a*(b^2+c^2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a+1)
*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(
x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arc
tan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c
))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*EllipticF((((
b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+
c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(-1/2*(b^2+c^2)^(3/2)*(a*e-d)/(a
^2-b^2-c^2)+1/2*(b^2+c^2)^(1/2)*(2*b^2+2*c^2)/(a^2-b^2-c^2)*(a*e-d))*(1/(b^
2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/
2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(
cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1
/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c
))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2))
)^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1
/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))+1/2*(a*b^2*
e+a*c^2*e-b^2*d-c^2*d)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arct
an(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^
(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-a
rctan(-b,c))+a)*(b^2+c^2))^(1/2)/a*EllipticPi((((b^2+c^2)^(1/2)*sin(x-arcta
n(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a-1)*(b^2+c
^2)^(1/2)/a,((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))/cos(x-arct
an(-b,c))/((b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/
2))/(b^2+c^2)^(1/2))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.05, size = 2106, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="fricas")
```

```
[Out] -1/3*((sqrt(2)*(-I*a*b^2*d - a*b*c*d - I*(2*a^2*b^2 - 3*b^4 - 3*b^2*c^2)*e
+ (3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e)*cos(x) + sqrt(2)*(-I*a*b*c*d - a*c^2*d
 + I*(3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e + (3*c^4 - (2*a^2 - 3*b^2)*c^2)*e)*s
in(x) + sqrt(2)*(-I*a^2*b*d - a^2*c*d - I*(2*a^3*b - 3*a*b^3 - 3*a*b*c^2)*e
 + (3*a*c^3 - (2*a^3 - 3*a*b^2)*c)*e))*sqrt(b + I*c)*weierstrassPInverse(4/
3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3
)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I
*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3
*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*
c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + (sqrt(2
)*(I*a*b^2*d - a*b*c*d + I*(2*a^2*b^2 - 3*b^4 - 3*b^2*c^2)*e + (3*b*c^3 - (
2*a^2*b - 3*b^3)*c)*e)*cos(x) + sqrt(2)*(I*a*b*c*d - a*c^2*d - I*(3*b*c^3 -
 (2*a^2*b - 3*b^3)*c)*e + (3*c^4 - (2*a^2 - 3*b^2)*c^2)*e)*sin(x) + sqrt(2)
*(I*a^2*b*d - a^2*c*d + I*(2*a^3*b - 3*a*b^3 - 3*a*b*c^2)*e + (3*a*c^3 - (2
*a^3 - 3*a*b^2)*c)*e))*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3
*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^
2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*
a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*
c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2
)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2)) - 3*(sqrt(2)*(-I*(b^3 +
b*c^2)*d + I*(a*b^3 + a*b*c^2)*e)*cos(x) + sqrt(2)*(-I*(b^2*c + c^3)*d + I*
(a*b^2*c + a*c^3)*e)*sin(x) + sqrt(2)*(-I*(a*b^2 + a*c^2)*d + I*(a^2*b^2 +
a^2*c^2)*e))*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c
^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4),
-8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)
*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^
```

```
4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^
2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4
), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b
^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3
*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) -
3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2))) - 3*(sqrt(2)*(I*(b^3 + b*c^2)*d - I
*(a*b^3 + a*b*c^2)*e)*cos(x) + sqrt(2)*(I*(b^2*c + c^3)*d - I*(a*b^2*c + a*
c^3)*e)*sin(x) + sqrt(2)*(I*(a*b^2 + a*c^2)*d - I*(a^2*b^2 + a^2*c^2)*e))*s
qrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3
 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b
^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^
3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*
c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*
c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^
3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4
*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b
^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*
c^2)*sin(x))/(b^2 + c^2))) - 6*(((b^2*c + c^3)*d - (a*b^2*c + a*c^3)*e)*cos
(x) - ((b^3 + b*c^2)*d - (a*b^3 + a*b*c^2)*e)*sin(x))*sqrt(b*cos(x) + c*sin
(x) + a))/(a^3*b^2 - a*b^4 - a*c^4 + (a^3 - 2*a*b^2)*c^2 + (a^2*b^3 - b^5 -
 b*c^4 + (a^2*b - 2*b^3)*c^2)*cos(x) - (c^5 - (a^2 - 2*b^2)*c^3 - (a^2*b^2
- b^4)*c)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorith
m="giac")
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)}{{\left(a+b\,\cos\left(x\right)+c\,\sin\left(x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2),x)
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2), x)
```

3.561 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

Optimal. Leaf size=378

$$\frac{2(4ad-a^2e-3(b^2+c^2)\,e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{3\,(a^2-b^2-c^2)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2(d-ae)F\Big(\frac{1}{2}(\cdots}{}$$

```
[Out] 2/3*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x
))^(3/2)+2/3*(c*(4*a*d-a^2*e-3*(b^2+c^2)*e)*cos(x)-b*(4*a*d-a^2*e-3*(b^2+c^
2)*e)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(4*a*d-a^2*e-
3*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,
c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)^2/((a+b*cos(x)
+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,
c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)
),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x
))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3235, 3228, 3198, 2732, 3206, 2740}

$$-\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,F\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\,(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(a^2(-e)+4ad-3e(b^2+c^2))\sqrt{a+b\cos(x)+c\sin(x)}\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{3\,(a^2-b^2-c^2)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\,(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))^{3/2}}+\frac{2(c\cos(x)\,(a^2(-e)+4ad-3e(b^2+c^2))-b\sin(x)\,(a^2(-e)+4ad-3e(b^2+c^2)))}{3\,(a^2-b^2-c^2)^2\sqrt{a+b\cos(x)+c\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

```
[Out] (2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqr
t[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*(a^2
 - b^2 - c^2)^2*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])]) - (2
*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^
2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/(3*(a^2 -
 b^2 - c^2)*Sqrt[a + b*Cos[x] + c*Sin[x]]) + (2*(c*(d - a*e)*Cos[x] - b*(d
- a*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])^(3/2)) + (2*
(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Cos[x] - b*(4*a*d - a^2*e - 3*(b^2 + c
^2)*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)^2*Sqrt[a + b*Cos[x] + c*Sin[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3198

```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Dist[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])], Int[Sqrt[a/(a + Sq
rt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - ArcT
an[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
&& NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3206

```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], Int[1/Sqrt[a/(a
+ Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

Rule 3228

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],
x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[A*
b - a*B, 0]
```

Rule 3235

```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a + b*Cos
[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n + 2
)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
```

```
0] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx = \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(ad-\left(b^2+c^2\right)e\right)+\frac{1}{2}b(d-ae)}{(a+b\cos(x)+c\,s}}{3\left(a^2-b^2\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2(c(4ad-a^2e-3(b^2+c^2)}{3\left(a^2-b^2-\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2(c(4ad-a^2e-3(b^2+c^2)}{3\left(a^2-b^2-\right.}$$

$$= \frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}} + \frac{2(c(4ad-a^2e-3(b^2+c^2)}{3\left(a^2-b^2-\right.}$$

$$= \frac{2\left(4ad-a^2e-3\left(b^2+c^2\right)e\right)E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b}}{3\left(a^2-b^2-c^2\right)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 6.64, size = 5554, normalized size = 14.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4019 vs. 2(406) = 812.
time = 9.22, size = 4020, normalized size = 10.63

method	result	size
default	Expression too large to display	4020

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,
c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*(b^4*sin(x-a
rctan(-b,c))^4+2*b^2*c^2*sin(x-arctan(-b,c))^4+c^4*sin(x-arctan(-b,c))^4-2*
a^2*b^2*sin(x-arctan(-b,c))^2-2*a^2*c^2*sin(x-arctan(-b,c))^2+a^4)*(sin(x-a
rctan(-b,c))^2*b^2*e+sin(x-arctan(-b,c))^2*c^2*e+e*a*sin(x-arctan(-b,c))*(b
^2+c^2)^(1/2)+d*sin(x-arctan(-b,c))*(b^2+c^2)^(1/2)+a*d)/(2*(cos(x-arctan(-
b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*sin(x-arctan(-b,c))^
2*a*b^2*e+2*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))
^(1/2)*sin(x-arctan(-b,c))^2*a*c^2*e-(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(x-arctan(-b,c))^3*b^2*e-(cos(
x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*
sin(x-arctan(-b,c))^3*c^2*e-(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-a
rctan(-b,c))+a))^(1/2)*sin(x-arctan(-b,c))^2*b^2*d-(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)*sin(x-arctan(-b,c))^2*c^2*d-e
*a^2*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arct
an(-b,c))+a)*(b^2+c^2))^(1/2)-d*a^2*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)
*sin(x-arctan(-b,c))+a))^(1/2)+2*d*a*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c
))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2))/(b^4*sin(x-a
rctan(-b,c))^3+2*b^2*c^2*sin(x-arctan(-b,c))^3+c^4*sin(x-arctan(-b,c))^3+3*
b^2*sin(x-arctan(-b,c))^2*a*(b^2+c^2)^(1/2)+3*c^2*sin(x-arctan(-b,c))^2*a*(
b^2+c^2)^(1/2)+3*a^2*b^2*sin(x-arctan(-b,c))+3*a^2*c^2*sin(x-arctan(-b,c))+
(b^2+c^2)^(1/2)*a^3)*(1/4/a/(a^2-b^2-c^2)*(a*e-d)*(b^2+c^2)^(3/2)*(cos(x-ar
ctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)/(b^2*sin(x-arc
tan(-b,c))+c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))-1/3/(a^2-b^2-c^2)*(a*
e-d)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)/
(sin(x-arctan(-b,c))+1/(b^2+c^2)^(1/2)*a)^2-1/3*(b^2+c^2)*cos(x-arctan(-b,c
))^2/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(cos(x-arctan(-b,c))^2*(
(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^(1/2)+2*(1/24*(a*e-d)*(b^2+c^2)^(1/
2)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)^(1/2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(a^2-b
^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b
^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2
)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))
+a))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^
(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(1/8*(a*
```

```
b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-b^2-c^2)-1/6*(b^2+c^2)*(a^2*e+3*b^2*e+3*c
^2*e-4*a*d)/(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(
x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2
+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)
^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*s
in(x-arctan(-b,c))+a))^(1/2)*((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1
/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(
-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1
/2)))^(1/2)))+1/8*(a^3*b^2*e+a^3*c^2*e+3*a*b^4*e+6*a*b^2*c^2*e+3*a*c^4*e-5*
a^2*b^2*d-5*a^2*c^2*d+b^4*d+2*b^2*c^2*d+c^4*d)/a^2/(a^2-b^2-c^2)/(b^2+c^2)^
(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^
2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/
2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a))^
(1/2)*EllipticPi((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2
)))^(1/2),-1/2*(-1/(b^2+c^2)^(1/2)*a-1)*(b^2+c^2)^(1/2)/a,((-a-(b^2+c^2)^(1
/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-1/4*(a*b^2*e+a*c^2*e-b^2*d-c^2*d)/a/(a^2-
b^2-c^2)*(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^
2+c^2))^(1/2)/(b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^
(1/2))-1/3/(a^2-b^2-c^2)*(a*e-d)/(b^2+c^2)^(1/2)*(cos(x-arctan(-b,c))^2*((b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)/(sin(x-arctan(-b,c))+
1/(b^2+c^2)^(1/2)*a)^2+1/3*(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(x-arctan(-b,c))^2
/(a^2-b^2-c^2)^2*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/(cos(x-arctan(-b,c))^2*((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)+2*(7/24*(a*b^2*e+a*c^2*
e-b^2*d-c^2*d)/(a^2-b^2-c^2)-1/6*a*(b^2+c^2)*(a^2*e+3*b^2*e+3*c^2*e-4*a*d)/
(a^2-b^2-c^2)^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,
c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/
(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b
^2+c^2)^(1/2)))^(1/2)/(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(
-b,c))+a)*(b^2+c^2))^(1/2)*EllipticF((((b^2+c^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="maxima")
```

```
[Out] integrate((b*cos(x)*e + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.63, size = 4092, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="fricas")
```

```
[Out] 1/9*((sqrt(2)*(I*(a^2*b^3 + 3*b^5 - a^2*b*c^2 - 3*b*c^4)*d - (a^2*c^3 + 3*c
^5 - (a^2*b^2 + 3*b^4)*c)*d + 2*I*(a^3*b^3 - 3*a*b^5 - a^3*b*c^2 + 3*a*b*c^
4)*e - 2*(a^3*c^3 - 3*a*c^5 - (a^3*b^2 - 3*a*b^4)*c)*e)*cos(x)^2 - 2*sqrt(2
)*(-I*(a^3*b^2 + 3*a*b^4 + 3*a*b^2*c^2)*d - (3*a*b*c^3 + (a^3*b + 3*a*b^3)*
c)*d - 2*I*(a^4*b^2 - 3*a^2*b^4 - 3*a^2*b^2*c^2)*e + 2*(3*a^2*b*c^3 - (a^4*
b - 3*a^2*b^3)*c)*e)*cos(x) - 2*(sqrt(2)*(-I*(3*b^2*c^3 + (a^2*b^2 + 3*b^4)
*c)*d - (3*b*c^4 + (a^2*b + 3*b^3)*c^2)*d + 2*I*(3*a*b^2*c^3 - (a^3*b^2 - 3
*a*b^4)*c)*e + 2*(3*a*b*c^4 - (a^3*b - 3*a*b^3)*c^2)*e)*cos(x) + sqrt(2)*(-
I*(3*a*b*c^3 + (a^3*b + 3*a*b^3)*c)*d - (3*a*c^4 + (a^3 + 3*a*b^2)*c^2)*d +
 2*I*(3*a^2*b*c^3 - (a^4*b - 3*a^2*b^3)*c)*e + 2*(3*a^2*c^4 - (a^4 - 3*a^2*
b^2)*c^2)*e))*sin(x) + sqrt(2)*(I*(a^4*b + 3*a^2*b^3 + 3*b*c^4 + (4*a^2*b +
 3*b^3)*c^2)*d + (3*c^5 + (4*a^2 + 3*b^2)*c^3 + (a^4 + 3*a^2*b^2)*c)*d + 2*
I*(a^5*b - 3*a^3*b^3 - 3*a*b*c^4 - (2*a^3*b + 3*a*b^3)*c^2)*e - 2*(3*a*c^5
+ (2*a^3 + 3*a*b^2)*c^3 - (a^5 - 3*a^3*b^2)*c)*e))*sqrt(b + I*c)*weierstras
sPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a
^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a
*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2
- 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*
a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2
)) + (sqrt(2)*(-I*(a^2*b^3 + 3*b^5 - a^2*b*c^2 - 3*b*c^4)*d - (a^2*c^3 + 3*
c^5 - (a^2*b^2 + 3*b^4)*c)*d - 2*I*(a^3*b^3 - 3*a*b^5 - a^3*b*c^2 + 3*a*b*c
^4)*e - 2*(a^3*c^3 - 3*a*c^5 - (a^3*b^2 - 3*a*b^4)*c)*e)*cos(x)^2 - 2*sqrt(
2)*(I*(a^3*b^2 + 3*a*b^4 + 3*a*b^2*c^2)*d - (3*a*b*c^3 + (a^3*b + 3*a*b^3)*
c)*d + 2*I*(a^4*b^2 - 3*a^2*b^4 - 3*a^2*b^2*c^2)*e + 2*(3*a^2*b*c^3 - (a^4*
b - 3*a^2*b^3)*c)*e)*cos(x) - 2*(sqrt(2)*(I*(3*b^2*c^3 + (a^2*b^2 + 3*b^4)*
c)*d - (3*b*c^4 + (a^2*b + 3*b^3)*c^2)*d - 2*I*(3*a*b^2*c^3 - (a^3*b^2 - 3*
a*b^4)*c)*e + 2*(3*a*b*c^4 - (a^3*b - 3*a*b^3)*c^2)*e)*cos(x) + sqrt(2)*(I*
(3*a*b*c^3 + (a^3*b + 3*a*b^3)*c)*d - (3*a*c^4 + (a^3 + 3*a*b^2)*c^2)*d - 2
*I*(3*a^2*b*c^3 - (a^4*b - 3*a^2*b^3)*c)*e + 2*(3*a^2*c^4 - (a^4 - 3*a^2*b^
2)*c^2)*e))*sin(x) + sqrt(2)*(-I*(a^4*b + 3*a^2*b^3 + 3*b*c^4 + (4*a^2*b +
3*b^3)*c^2)*d + (3*c^5 + (4*a^2 + 3*b^2)*c^3 + (a^4 + 3*a^2*b^2)*c)*d - 2*I
*(a^5*b - 3*a^3*b^3 - 3*a*b*c^4 - (2*a^3*b + 3*a*b^3)*c^2)*e - 2*(3*a*c^5 +
 (2*a^3 + 3*a*b^2)*c^3 - (a^5 - 3*a^3*b^2)*c)*e))*sqrt(b - I*c)*weierstrass
PInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^
2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*
b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 +
 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a
*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2
)) - 3*(sqrt(2)*(4*I*(a*b^4 - a*c^4)*d - I*(a^2*b^4 + 3*b^6 + 3*b^4*c^2 - 3
*c^6 - (a^2 + 3*b^2)*c^4)*e)*cos(x)^2 + 2*sqrt(2)*(4*I*(a^2*b^3 + a^2*b*c^2
```

```
)*d - I*(a^3*b^3 + 3*a*b^5 + 3*a*b*c^4 + (a^3*b + 6*a*b^3)*c^2)*e)*cos(x) +
 2*(sqrt(2)*(4*I*(a*b^3*c + a*b*c^3)*d - I*(3*b*c^5 + (a^2*b + 6*b^3)*c^3 +
 (a^2*b^3 + 3*b^5)*c)*e)*cos(x) + sqrt(2)*(4*I*(a^2*b^2*c + a^2*c^3)*d - I*
(3*a*c^5 + (a^3 + 6*a*b^2)*c^3 + (a^3*b^2 + 3*a*b^4)*c)*e))*sin(x) + sqrt(2
)*(4*I*(a^3*b^2 + a*c^4 + (a^3 + a*b^2)*c^2)*d - I*(a^4*b^2 + 3*a^2*b^4 + 3
*c^6 + 2*(2*a^2 + 3*b^2)*c^4 + (a^4 + 7*a^2*b^2 + 3*b^4)*c^2)*e))*sqrt(b +
I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4
 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a
*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*
a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^
6), weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*
c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 -
9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b -
 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 +
 c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(
x))/(b^2 + c^2))) - 3*(sqrt(2)*(-4*I*(a*b^4 - a*c^4)*d + I*(a^2*b^4 + 3*b^6
 + 3*b^4*c^2 - 3*c^6 - (a^2 + 3*b^2)*c^4)*e)*cos(x)^2 + 2*sqrt(2)*(-4*I*(a^
2*b^3 + a^2*b*c^2)*d + I*(a^3*b^3 + 3*a*b^5 + 3*a*b*c^4 + (a^3*b + 6*a*b^3)
*c^2)*e)*cos(x) + 2*(sqrt(2)*(-4*I*(a*b^3*c + a*b*c^3)*d + I*(3*b*c^5 + (a^
2*b + 6*b^3)*c^3 + (a^2*b^3 + 3*b^5)*c)*e)*cos(x) + sqrt(2)*(-4*I*(a^2*b^2*
c + a^2*c^3)*d + I*(3*a*c^5 + (a^3 + 6*a*b^2)*c^3 + (a^3*b^2 + 3*a*b^4)*c)*
e))*sin(x) + sqrt(2)*(-4*I*(a^3*b^2 + a*c^4 + (a^3 + a*b^2)*c^2)*d + I*(a^4
*b^2 + 3*a^2*b^4 + 3*c^6 + 2*(2*a^2 + 3*b^2)*c^4 + (a^4 + 7*a^2*b^2 + 3*b^4
)*c^2)*e))*sqrt(b - I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2
 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8
/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c
^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorith
m="giac")
```

```
[Out] integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)}{{\left(a+b\,\cos\left(x\right)+c\,\sin\left(x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2),x)
[Out] int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2), x)
```

3.562 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

Optimal. Leaf size=84

$$\frac{Cx}{c}+\frac{2(Ac-aC)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}\,e}+\frac{B\log(a+c\sin(d+ex))}{ce}$$

```
[Out] C*x/c+B*ln(a+c*sin(e*x+d))/c/e+2*(A*c-C*a)*arctan((c+a*tan(1/2*e*x+1/2*d))/
(a^2-c^2)^(1/2))/c/e/(a^2-c^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4461, 2814, 2739, 632, 210, 2747, 31}

$$\frac{2(Ac-aC)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

```
[Out] (C*x)/c + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/
(c*Sqrt[a^2 - c^2]*e) + (B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = B\int \frac{\cos(d+ex)}{a+c\sin(d+ex)}\,dx+\int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$
$$= \frac{Cx}{c}-\frac{(-Ac+aC)\int \frac{1}{a+c\sin(d+ex)}\,dx}{c}+\frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,c\,\text{si}\right.}{ce}$$
$$= \frac{Cx}{c}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{(2(Ac-aC))\text{Subst}\left(\int \frac{1}{a+2cx+}\right.}{ce}$$
$$= \frac{Cx}{c}+\frac{B\log(a+c\sin(d+ex))}{ce}-\frac{(4(Ac-aC))\text{Subst}\left(\int \frac{}{-4(a^2-}\right.}{}$$
$$= \frac{Cx}{c}+\frac{2(Ac-aC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}\,e}+\frac{B\log(a+c\sin(}{ce}$$

Mathematica [A]

time = 0.35, size = 80, normalized size = 0.95

$$\frac{C(d+ex)+\frac{2(Ac-aC)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\sqrt{a^2-c^2}}+B\log(a+c\sin(d+ex))}{ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^
2]])/Sqrt[a^2 - c^2] + B*Log[a + c*Sin[d + e*x]])/(c*e)
```

Maple [A]
time = 0.39, size = 128, normalized size = 1.52

method	result
derivativedivides	$\frac{\frac{-B\ln\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2C\arctan\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}+\frac{B\ln\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a\right)+\frac{2(Ac-Ca)\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-c^2}}}{c}}{e}$
default	$\frac{\frac{-B\ln\left(1+\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2C\arctan\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{c}+\frac{B\ln\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a\right)+\frac{2(Ac-Ca)\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-c^2}}}{c}}{e}$
risch	$\frac{ixB}{c}+\frac{Cx}{c}-\frac{2iB\,a^2c\,e^2x}{a^2c^2e^2-c^4e^2}+\frac{2iB\,c^3e^2x}{a^2c^2e^2-c^4e^2}-\frac{2iB\,a^2cde}{a^2c^2e^2-c^4e^2}+\frac{2iB\,c^3de}{a^2c^2e^2-c^4e^2}+\frac{\ln\left(e^{i(ex+d)}+\frac{iaAc-iC\,a^2+\sqrt{-}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x,method=_RETURNVERBOSE)
[Out] 1/e*(2/c*(-1/2*B*ln(1+tan(1/2*d+1/2*e*x)^2)+C*arctan(tan(1/2*d+1/2*e*x)))+2
/c*(1/2*B*ln(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)+(A*c-C*a)/(a^
2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.08, size = 359, normalized size = 4.27

$$\left[\frac{\left(2\,(Ca^2-Cc^2)xe+(Ca-Ac)\sqrt{-a^2+c^2}\log\left(\frac{(2a^2-c^2)\cos(xe+d)^2-2ac\sin(xe+d)-a^2-c^2+2\,(a\cos(xe+d)\sin(xe+d)+c\cos(xe+d))\sqrt{-a^2+c^2}}{c^2\cos(xe+d)^2-2ac\sin(xe+d)-a^2-c^2}\right)+(Ba^2-Bc^2)\log\left(-c^2\cos(xe+d)^2+2ac\sin(xe+d)+a^2+c^2\right)\right)e^{(-1)}}{2\,(a^2c-c^3)},\ \frac{\left(2\,(Ca^2-Cc^2)xe+2\,(Ca-Ac)\sqrt{a^2-c^2}\arctan\left(-\frac{a\sin(xe+d)+c}{\sqrt{a^2-c^2}\cos(xe+d)}\right)+(Ba^2-Bc^2)\log\left(-c^2\cos(xe+d)^2+2ac\sin(xe+d)+a^2+c^2\right)\right)e^{(-1)}}{2\,(a^2c-c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(C*a^2 - C*c^2)*x*e + (C*a - A*c)*sqrt(-a^2 + c^2)*log(((2*a^2 - c^
2)*cos(x*e + d)^2 - 2*a*c*sin(x*e + d) - a^2 - c^2 + 2*(a*cos(x*e + d)*sin(
x*e + d) + c*cos(x*e + d))*sqrt(-a^2 + c^2))/(c^2*cos(x*e + d)^2 - 2*a*c*si
n(x*e + d) - a^2 - c^2)) + (B*a^2 - B*c^2)*log(-c^2*cos(x*e + d)^2 + 2*a*c*
sin(x*e + d) + a^2 + c^2))*e^(-1)/(a^2*c - c^3), 1/2*(2*(C*a^2 - C*c^2)*x*e
 + 2*(C*a - A*c)*sqrt(a^2 - c^2)*arctan(-(a*sin(x*e + d) + c)/(sqrt(a^2 - c
^2)*cos(x*e + d))) + (B*a^2 - B*c^2)*log(-c^2*cos(x*e + d)^2 + 2*a*c*sin(x*
e + d) + a^2 + c^2))*e^(-1)/(a^2*c - c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. $2(70) = 140$.
time = 27.03, size = 1110, normalized size = 13.21

$$\begin{cases} \frac{\tilde{\infty} x \left(A + B\cos(d) + C\sin(d)\right)}{\sin(d)} & \text{for}\: a = 0 \wedge c = 0 \wedge e = 0 \\ \frac{\frac{A\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e} - \frac{B\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{e} + \frac{B\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{e} + Cx}{c} & \text{for}\: a = 0 \\ \frac{2A}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} + \frac{2B\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} - \frac{2B\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} - \frac{B\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} + \frac{B\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} + \frac{Cex\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} - \frac{Cex}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} + \frac{2C}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ce} & \text{for}\: a = -c \\ -\frac{2A}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} + \frac{2B\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} + \frac{2B\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} - \frac{B\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} - \frac{B\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} + \frac{Cex\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} + \frac{Cex}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} + \frac{2C}{ce\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+ce} & \text{for}\: a = c \\ \frac{Ax + \frac{B\sin(d+ex)}{e} - \frac{C\cos(d+ex)}{e}}{a} & \text{for}\: c = 0 \\ \frac{x\left(A + B\cos(d) + C\sin(d)\right)}{a + c\sin(d)} & \text{for}\: e = 0 \\ -\frac{Ac\sqrt{-a^2+c^2}\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}-\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} + \frac{Ac\sqrt{-a^2+c^2}\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}+\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} - \frac{Ba^2\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{a^2ce-c^3e} + \frac{Ba^2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}-\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} + \frac{Ba^2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}+\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} + \frac{Bc^2\log\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{a^2ce-c^3e} - \frac{Bc^2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}-\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} - \frac{Bc^2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}+\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} + \frac{Ca^2ex}{a^2ce-c^3e} + \frac{Ca\sqrt{-a^2+c^2}\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}-\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} - \frac{Ca\sqrt{-a^2+c^2}\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+\frac{c}{a}+\frac{\sqrt{-a^2+c^2}}{a}\right)}{a^2ce-c^3e} - \frac{Cc^2ex}{a^2ce-c^3e} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(d) + C*sin(d))/sin(d), Eq(a, 0) & Eq(c, 0) & Eq
(e, 0)), ((A*log(tan(d/2 + e*x/2))/e - B*log(tan(d/2 + e*x/2)**2 + 1)/e + B
*log(tan(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*A/(c*e*tan(d/2 + e*x/2) -
c*e) + 2*B*log(tan(d/2 + e*x/2) - 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2)
 - c*e) - 2*B*log(tan(d/2 + e*x/2) - 1)/(c*e*tan(d/2 + e*x/2) - c*e) - B*lo
g(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) +
B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) - c*e) + C*e*x*tan(d/2
 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - C*e*x/(c*e*tan(d/2 + e*x/2) - c*e)
 + 2*C/(c*e*tan(d/2 + e*x/2) - c*e), Eq(a, -c)), (-2*A/(c*e*tan(d/2 + e*x/2
) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*
x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)/(c*e*tan(d/2 + e*x/2) + c*e) -
B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e
```

```
) - B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x*tan
(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x/(c*e*tan(d/2 + e*x/2) +
c*e) + 2*C/(c*e*tan(d/2 + e*x/2) + c*e), Eq(a, c)), ((A*x + B*sin(d + e*x)/
e - C*cos(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cos(d) + C*sin(d))/(a + c*sin
(d)), Eq(e, 0)), (-A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt
(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + A*c*sqrt(-a**2 + c**2)*log(tan(d/2
+ e*x/2) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*a**2*log(tan
(d/2 + e*x/2)**2 + 1)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2) + c
/a - sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*a**2*log(tan(d/2 + e*x/2
) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + B*c**2*log(tan(d/2 +
e*x/2)**2 + 1)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a - sq
rt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - B*c**2*log(tan(d/2 + e*x/2) + c/a
 + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + C*a**2*e*x/(a**2*c*e - c**3*
e) + C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a - sqrt(-a**2 + c**2)
/a)/(a**2*c*e - c**3*e) - C*a*sqrt(-a**2 + c**2)*log(tan(d/2 + e*x/2) + c/a
 + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) - C*c**2*e*x/(a**2*c*e - c**3*
e), True))
```

Giac [A]
time = 0.43, size = 136, normalized size = 1.62

$$\frac{\frac{(ex+d)C}{c} + \frac{B\log\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+2\,c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right)}{c} - \frac{B\log\left(\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+1\right)}{c} - \frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c}{\sqrt{a^2-c^2}}\right)\right)(Ca-Ac)}{\sqrt{a^2-c^2}\,c}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="giac
")
```

```
[Out] ((e*x + d)*C/c + B*log(a*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e*x + 1/2*d)
+ a)/c - B*log(tan(1/2*e*x + 1/2*d)^2 + 1)/c - 2*(pi*floor(1/2*(e*x + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))*(C*a
 - A*c)/(sqrt(a^2 - c^2)*c))/e
```

Mupad [B]
time = 9.63, size = 1143, normalized size = 13.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x)),x)
```

```
[Out] (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^2*a^2 + 32*a*tan(d/2
 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*B^2*C*a - 2*A*C^2*c
+ 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c - ((B*a^2 - B*c^2 +
A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2))*(32*C^2*a^2*c - 32*B^2*a^2*c
```

```
 - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*a^2 -
 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*a^2*c
+ 64*B*C*a*c^2 + ((B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^
(1/2))*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a*c^2*tan(d/2 + (e*x)
/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*tan(d/2 + (e*x)/2) + 3*c^2
*tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^
2)^(1/2)))/(a^2 - c^2)))/(c*(a^2 - c^2))))/(c*(a^2 - c^2)))*(B*a^2 - B*c^2
+ A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2)))/(c*e*(a^2 - c^2)) - (log(
tan(d/2 + (e*x)/2) + 1i)*(B - C*1i))/(c*e) - (log(tan(d/2 + (e*x)/2) - 1i)*
(B + C*1i))/(c*e) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^
2*a^2 + 32*a*tan(d/2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*
B^2*C*a - 2*A*C^2*c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c
- ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2) + C*a*(c^2 - a^2)^(1/2))*(32*C^2*
a^2*c - 32*B^2*a^2*c - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A
^2*c^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*
a*c) + 64*A*B*a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2
) + C*a*(c^2 - a^2)^(1/2))*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a
*c^2*tan(d/2 + (e*x)/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*tan(d/
2 + (e*x)/2) + 3*c^2*tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(
1/2) + C*a*(c^2 - a^2)^(1/2)))/(a^2 - c^2)))/(c*(a^2 - c^2))))/(c*(a^2 - c^
2)))*(B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2) + C*a*(c^2 - a^2)^(1/2)))/(c*e*
(a^2 - c^2))
```

3.563 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

Optimal. Leaf size=118

$$\frac{2(aA-cC)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{3/2}\,e}-\frac{B}{ce(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))}$$

```
[Out] 2*(A*a-C*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a^2-c^2)^(3/2
)/e-B/c/e/(a+c*sin(e*x+d))+(A*c-C*a)*cos(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d)
)
```

Rubi [A]
time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$\frac{2(aA-cC)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\,(a^2-c^2)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e\,(a^2-c^2)\,(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

```
[Out] (2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/((a^2 - c^
2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d + e*x])/((a
^2 - c^2)*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^2}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))} + \frac{\int \frac{-aA+cC}{a+c\sin(d+ex)}\,dx}{-a^2+c^2} + \frac{B\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))} + \frac{(aA-\cdots}{\cdots}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))} + \frac{(2(aA\cdots}{\cdots}$$

$$= -\frac{B}{ce(a+c\sin(d+ex))} + \frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))} - \frac{(4(aA\cdots}{\cdots}$$

$$= \frac{2(aA-cC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{3/2}\,e} - \frac{B}{ce(a+c\sin(d+ex))} + \cdots$$

Mathematica [A]

time = 0.35, size = 114, normalized size = 0.97

$$\frac{\frac{2(aA-cC)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{3/2}} + \frac{B(a^2-c^2)-c(Ac-aC)\cos(d+ex)}{c(-a+c)(a+c)(a+c\sin(d+ex))}}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

```
[Out] ((2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c^
2)^(3/2) + (B*(a^2 - c^2) - c*(A*c - a*C)*Cos[d + e*x])/(c*(-a + c)*(a + c)
*(a + c*Sin[d + e*x])))/e
```

Maple [A]

time = 0.46, size = 155, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{\frac{2\left(A\,c^2+a^2B-B\,c^2-acC\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{a\left(a^2-c^2\right)}+\frac{2(Ac-Ca)}{a^2-c^2}}{a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a}+\frac{2(aA-Cc)\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{\frac{3}{2}}}}{e}$

default	$\frac{\frac{\frac{2\left(A\,c^2+a^2B-B\,c^2-acC\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{a\left(a^2-c^2\right)}+\frac{2(Ac-Ca)}{a^2-c^2}}{a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)\right)+2c\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+a}+\frac{2(aA-Cc)\arctan\left(\frac{2a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+2c}{2\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{\frac{3}{2}}}}{e}$
risch	$-\frac{2i\left(-iB\,a^2\mathrm{e}^{i(ex+d)}+iB\,c^2\mathrm{e}^{i(ex+d)}+iA\,c^2+Aac\,\mathrm{e}^{i(ex+d)}-iacC-C\,a^2\mathrm{e}^{i(ex+d)}\right)}{ce\left(a^2-c^2\right)\left(-ic\,\mathrm{e}^{2i(ex+d)}+ic+2a\,\mathrm{e}^{i(ex+d)}\right)}-\frac{\ln\left(\mathrm{e}^{i(ex+d)}+\frac{i\sqrt{-a^2+c^2}\,a-}{\sqrt{-a^2+c^2}}\right.}{\sqrt{-a^2+c^2}\,(a+c)(a-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/e*(2*((A*c^2+B*a^2-B*c^2-C*a*c)/a/(a^2-c^2)*tan(1/2*d+1/2*e*x)+(A*c-C*a)/
(a^2-c^2))/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+a)+2*(A*a-C*c)/(a
^2-c^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.78, size = 477, normalized size = 4.04

$$\left[-\frac{2\,Ba^4-4\,Ba^2c^2+2\,Bc^4+(Aa^2c-Cac^2+(Aac^2-Cc^3)\sin(xe+d))\sqrt{-a^2+c^2}\log\left(\frac{(2\,a^2-c^2)\cos(xe+d)^2-2\,ac\sin(xe+d)-a^2-c^2+2\,(a\cos(xe+d)\sin(xe+d)+c\cos(xe+d))\sqrt{-a^2+c^2}}{c^2\cos(xe+d)^2-2\,ac\sin(xe+d)-a^2-c^2}\right)+2\,(Ca^3c-Aa^2c^2-Cac^3+Ac^4)\cos(xe+d)}{2\,((a^4c^2-2\,a^2c^4+c^6)e\sin(xe+d)+(a^5c-2\,a^3c^3+ac^5)e)},-\frac{Ba^4-2\,Ba^2c^2+Bc^4+(Aa^2c-Cac^2+(Aac^2-Cc^3)\sin(xe+d))\sqrt{a^2-c^2}\arctan\left(-\frac{a\sin(xe+d)+c}{\sqrt{a^2-c^2}\cos(xe+d)}\right)+(Ca^3c-Aa^2c^2-Cac^3+Ac^4)\cos(xe+d)}{(a^4c^2-2\,a^2c^4+c^6)e\sin(xe+d)+(a^5c-2\,a^3c^3+ac^5)e}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*B*a^4 - 4*B*a^2*c^2 + 2*B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*
c^3)*sin(x*e + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(x*e + d)^2 - 2*a
*c*sin(x*e + d) - a^2 - c^2 + 2*(a*cos(x*e + d)*sin(x*e + d) + c*cos(x*e +
d))*sqrt(-a^2 + c^2))/(c^2*cos(x*e + d)^2 - 2*a*c*sin(x*e + d) - a^2 - c^2)
) + 2*(C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(x*e + d))/((a^4*c^2 - 2*a
^2*c^4 + c^6)*e*sin(x*e + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e), -(B*a^4 - 2*
```

```
B*a^2*c^2 + B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*sin(x*e + d))*sq
rt(a^2 - c^2)*arctan(-(a*sin(x*e + d) + c)/(sqrt(a^2 - c^2)*cos(x*e + d)))
+ (C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(x*e + d))/((a^4*c^2 - 2*a^2*c
^4 + c^6)*e*sin(x*e + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 180, normalized size = 1.53

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c}{\sqrt{a^2-c^2}}\right)\right)(Aa-Cc)}{(a^2-c^2)^{\frac{3}{2}}}+\frac{Ba^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-Cac\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+Ac^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-Bc^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-Ca^2+Aac}{(a^3-ac^2)\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+2\,c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right)}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="gi
ac")
```

```
[Out] 2*((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*d
) + c)/sqrt(a^2 - c^2)))*(A*a - C*c)/(a^2 - c^2)^(3/2) + (B*a^2*tan(1/2*e*x
 + 1/2*d) - C*a*c*tan(1/2*e*x + 1/2*d) + A*c^2*tan(1/2*e*x + 1/2*d) - B*c^2
*tan(1/2*e*x + 1/2*d) - C*a^2 + A*a*c)/((a^3 - a*c^2)*(a*tan(1/2*e*x + 1/2*
d)^2 + 2*c*tan(1/2*e*x + 1/2*d) + a)))/e
```

Mupad [B]
time = 3.33, size = 227, normalized size = 1.92

$$\frac{\frac{2\,(A\,c-C\,a)}{a^2-c^2}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(B\,a^2+A\,c^2-B\,c^2-C\,a\,c)}{a\,(a^2-c^2)}}{e\,\left(a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a\right)}+\frac{2\,\mathrm{atan}\left(\frac{(a^2-c^2)\,\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(A\,a-C\,c)}{(a+c)^{3/2}\,(a-c)^{3/2}}+\frac{2\,\left(a^2\,c-c^3\right)\,(A\,a-C\,c)}{(a+c)^{3/2}\,\left(a^2-c^2\right)\,(a-c)^{3/2}}\right)}{2\,(A\,a-C\,c)}\right)\,(A\,a-C\,c)}{e\,(a+c)^{3/2}\,(a-c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^2,x)
```

```
[Out] ((2*(A*c - C*a))/(a^2 - c^2) + (2*tan(d/2 + (e*x)/2)*(B*a^2 + A*c^2 - B*c^2
 - C*a*c))/(a*(a^2 - c^2)))/(e*(a + 2*c*tan(d/2 + (e*x)/2) + a*tan(d/2 + (e
*x)/2)^2)) + (2*atan(((a^2 - c^2)*((2*a*tan(d/2 + (e*x)/2)*(A*a - C*c))/((a
 + c)^(3/2)*(a - c)^(3/2)) + (2*(a^2*c - c^3)*(A*a - C*c))/((a + c)^(3/2)*(
a^2 - c^2)*(a - c)^(3/2))))/(2*(A*a - C*c)))*(A*a - C*c))/(e*(a + c)^(3/2)*
(a - c)^(3/2))
```

3.564 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

Optimal. Leaf size=185

$$\frac{(2a^2A+Ac^2-3acC)\,\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{5/2}\,e}-\frac{B}{2ce(a+c\sin(d+ex))^2}+\frac{(Ac-aC)\cos(d+ex)}{2\,(a^2-c^2)\,e(a+c\sin(d+ex))^2}$$

```
[Out] (2*A*a^2+A*c^2-3*C*a*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a
^2-c^2)^(5/2)/e-1/2*B/c/e/(a+c*sin(e*x+d))^2+1/2*(A*c-C*a)*cos(e*x+d)/(a^2-
c^2)/e/(a+c*sin(e*x+d))^2+1/2*(3*A*a*c-C*a^2-2*C*c^2)*cos(e*x+d)/(a^2-c^2)^
2/e/(a+c*sin(e*x+d))
```

Rubi [A]

time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$\frac{(2a^2A-3acC+Ac^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\,(a^2-c^2)^{5/2}}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{2e\,(a^2-c^2)^2\,(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex)}{2e\,(a^2-c^2)\,(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

```
[Out] ((2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2
]])/((a^2 - c^2)^(5/2)*e) - B/(2*c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)
*Cos[d + e*x])/(2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((3*a*A*c - a^2*C
 - 2*c^2*C)*Cos[d + e*x])/(2*(a^2 - c^2)^2*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^3}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} - \frac{\int \frac{-2(aA-cC)+(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}}{2(a^2-c^2)}$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} +$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} +$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} +$$

$$= -\frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} +$$

$$= \frac{(2a^2A+Ac^2-3acC)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{5/2}e} - \frac{B}{2ce(a+c\sin($$

Mathematica [A]
time = 0.62, size = 174, normalized size = 0.94

$$\frac{\frac{2(2a^2A+Ac^2-3acC)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{5/2}} + \frac{B(-a^2+c^2)+c(Ac-aC)\cos(d+ex)}{(a-c)c(a+c)(a+c\sin(d+ex))^2} - \frac{(-3aAc+a^2C+2c^2C)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
[Out] ((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 -
c^2]])/(a^2 - c^2)^(5/2) + (B*(-a^2 + c^2) + c*(A*c - a*C)*Cos[d + e*x])/((
a - c)*c*(a + c)*(a + c*Sin[d + e*x])^2) - ((-3*a*A*c + a^2*C + 2*c^2*C)*Co
s[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])))/(2*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(174) = 348$.
time = 0.64, size = 419, normalized size = 2.26

method	result

derivativedivides	$\frac{\frac{\left(5A\,a^2c^2-2A\,c^4+2B\,a^4-4B\,a^2c^2+2B\,c^4-3C\,a^3c\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{\left(a^4-2a^2c^2+c^4\right)a}+\frac{\left(4A\,a^4c+7A\,a^2c^3-2A\,c^5+2B\,a^4c-4B\,a^2c^3+2B\,c^5-2C\,a^5-5C}{\left(a^4-2a^2c^2+c^4\right)a^2}}{\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right.\right.\right.}$
default	$\frac{\frac{\left(5A\,a^2c^2-2A\,c^4+2B\,a^4-4B\,a^2c^2+2B\,c^4-3C\,a^3c\right)\left(\tan^3\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{\left(a^4-2a^2c^2+c^4\right)a}+\frac{\left(4A\,a^4c+7A\,a^2c^3-2A\,c^5+2B\,a^4c-4B\,a^2c^3+2B\,c^5-2C\,a^5-5C}{\left(a^4-2a^2c^2+c^4\right)a^2}}{\left(a\left(\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right.\right.\right.}$
risch	$-\frac{-3iAa\,c^3-2iC\,a^4e^{2i(ex+d)}+2c^2A\,a^2e^{3i(ex+d)}+c^4A\,e^{3i(ex+d)}+3ic^3Aa\,e^{2i(ex+d)}+6icA\,a^3e^{2i(ex+d)}+2iC\,c^4-3c^3Ca\,e^{3i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/e*(2*(1/2*(5*A*a^2*c^2-2*A*c^4+2*B*a^4-4*B*a^2*c^2+2*B*c^4-3*C*a^3*c)/(a^
4-2*a^2*c^2+c^4)/a*tan(1/2*d+1/2*e*x)^3+1/2*(4*A*a^4*c+7*A*a^2*c^3-2*A*c^5+
2*B*a^4*c-4*B*a^2*c^3+2*B*c^5-2*C*a^5-5*C*a^3*c^2-2*C*a*c^4)/(a^4-2*a^2*c^2
+c^4)/a^2*tan(1/2*d+1/2*e*x)^2+1/2*(11*A*a^2*c^2-2*A*c^4+2*B*a^4-4*B*a^2*c^
2+2*B*c^4-5*C*a^3*c-4*C*a*c^3)/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*d+1/2*e*x)+1/2
*(4*A*a^2*c-A*c^3-2*C*a^3-C*a*c^2)/(a^4-2*a^2*c^2+c^4))/(a*tan(1/2*d+1/2*e*
x)^2+2*c*tan(1/2*d+1/2*e*x)+a)^2+(2*A*a^2+A*c^2-3*C*a*c)/(a^4-2*a^2*c^2+c^4
)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(175) = 350$.

time = 4.57, size = 909, normalized size = 4.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*B*a^6 - 6*B*a^4*c^2 + 6*B*a^2*c^4 - 2*B*c^6 + 2*(C*a^4*c^2 - 3*A*a^
3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(x*e + d)*sin(x*e + d) + (2*A*a
^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*C*a
*c^4 + A*c^5)*cos(x*e + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*sin(
x*e + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(x*e + d)^2 - 2*a*c*sin(x*
e + d) - a^2 - c^2 + 2*(a*cos(x*e + d)*sin(x*e + d) + c*cos(x*e + d))*sqrt(
-a^2 + c^2))/(c^2*cos(x*e + d)^2 - 2*a*c*sin(x*e + d) - a^2 - c^2)) + 2*(2*
C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(x*e
+ d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*cos(x*e + d)^2*e - 2*(a^7*c^
2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(x*e + d) - (a^8*c - 2*a^6*c^3 + 2*
a^2*c^7 - c^9)*e), 1/2*(B*a^6 - 3*B*a^4*c^2 + 3*B*a^2*c^4 - B*c^6 + (C*a^4*
c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(x*e + d)*sin(x*e +
 d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2
*c^3 - 3*C*a*c^4 + A*c^5)*cos(x*e + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A
*a*c^4)*sin(x*e + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(x*e + d) + c)/(sqrt(a^
2 - c^2)*cos(x*e + d))) + (2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^
4 - C*a*c^5 - A*c^6)*cos(x*e + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)
*cos(x*e + d)^2*e - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(x*e +
 d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(173) = 346.
time = 0.45, size = 571, normalized size = 3.09

$$\frac{\frac{(2Aa^2-3Cac+Ac^2)\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-c^2}}\right)\right)}{(a^4-2a^2c^2+c^4)\sqrt{a^2-c^2}}+\frac{2Ba^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-3Ca^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+5Aa^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-4Ba^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-2Aac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+2Bac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-2Ca^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+4Aa^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2Ba^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-5Ca^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+7Aa^2c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-4Ba^2c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-2Cac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-2Ac^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2Bc^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2Ba^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-5Ca^4c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+11Aa^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-4Ba^3c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-4Ca^2c^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2Aac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+2Bac^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2Ca^5+4Aa^4c-Ca^3c^2-Aa^2c^3}{(a^6-2a^4c^2+a^2c^4)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a\right)^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="gi
ac")
```

```
[Out] ((2*A*a^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(a) + arc
tan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^4 - 2*a^2*c^2 + c^4)
```

```
*sqrt(a^2 - c^2)) + (2*B*a^5*tan(1/2*e*x + 1/2*d)^3 - 3*C*a^4*c*tan(1/2*e*x
 + 1/2*d)^3 + 5*A*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 - 4*B*a^3*c^2*tan(1/2*e*x
+ 1/2*d)^3 - 2*A*a*c^4*tan(1/2*e*x + 1/2*d)^3 + 2*B*a*c^4*tan(1/2*e*x + 1/2
*d)^3 - 2*C*a^5*tan(1/2*e*x + 1/2*d)^2 + 4*A*a^4*c*tan(1/2*e*x + 1/2*d)^2 +
 2*B*a^4*c*tan(1/2*e*x + 1/2*d)^2 - 5*C*a^3*c^2*tan(1/2*e*x + 1/2*d)^2 + 7*
A*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 - 4*B*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 - 2*C
*a*c^4*tan(1/2*e*x + 1/2*d)^2 - 2*A*c^5*tan(1/2*e*x + 1/2*d)^2 + 2*B*c^5*ta
n(1/2*e*x + 1/2*d)^2 + 2*B*a^5*tan(1/2*e*x + 1/2*d) - 5*C*a^4*c*tan(1/2*e*x
 + 1/2*d) + 11*A*a^3*c^2*tan(1/2*e*x + 1/2*d) - 4*B*a^3*c^2*tan(1/2*e*x + 1
/2*d) - 4*C*a^2*c^3*tan(1/2*e*x + 1/2*d) - 2*A*a*c^4*tan(1/2*e*x + 1/2*d) +
 2*B*a*c^4*tan(1/2*e*x + 1/2*d) - 2*C*a^5 + 4*A*a^4*c - C*a^3*c^2 - A*a^2*c
^3)/((a^6 - 2*a^4*c^2 + a^2*c^4)*(a*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e*
x + 1/2*d) + a)^2))/e
```

Mupad [B]
time = 5.27, size = 557, normalized size = 3.01

$$\frac{\operatorname{atan}\left(\frac{\left(\frac{\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)\,\left(2\,a^4\,c-4\,a^2\,c^3+2\,c^5\right)}{2\,{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}\,\left(a^4-2\,a^2\,c^2+c^4\right)}+\frac{a\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)}{{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}}\right)\,\left(a^4-2\,a^2\,c^2+c^4\right)}{2\,A\,a^2-3\,C\,a\,c+A\,c^2}\right)\,\left(2\,A\,a^2-3\,C\,a\,c+A\,c^2\right)}{e\,{\left(a+c\right)}^{5/2}\,{\left(a-c\right)}^{5/2}}-\frac{\frac{2\,C\,a^3-4\,A\,a^2\,c+C\,a\,c^2+A\,c^3}{a^4-2\,a^2\,c^2+c^4}-\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3\,\left(2\,B\,a^4-2\,A\,c^4+2\,B\,c^4+5\,A\,a^2\,c^2-4\,B\,a^2\,c^2-3\,C\,a^3\,c\right)}{a\,\left(a^4-2\,a^2\,c^2+c^4\right)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(2\,A\,c^4-2\,B\,a^4-2\,B\,c^4-11\,A\,a^2\,c^2+4\,B\,a^2\,c^2+4\,C\,a\,c^3+5\,C\,a^3\,c\right)}{a\,\left(a^4-2\,a^2\,c^2+c^4\right)}+\frac{{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(2\,A\,c^5+2\,C\,a^5-2\,B\,c^5-7\,A\,a^2\,c^3+4\,B\,a^2\,c^3+5\,C\,a^3\,c^2-4\,A\,a^4\,c-2\,B\,a^4\,c+2\,C\,a\,c^4\right)}{a^2\,\left(a^4-2\,a^2\,c^2+c^4\right)}}{e\,\left({\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^2\,\left(2\,a^2+4\,c^2\right)+a^2\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^4+a^2+4\,a\,c\,{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}^3+4\,a\,c\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^3,x)
```

```
[Out] (atan(((((2*A*a^2 + A*c^2 - 3*C*a*c)*(2*a^4*c + 2*c^5 - 4*a^2*c^3))/(2*(a +
 c)^(5/2)*(a - c)^(5/2)*(a^4 + c^4 - 2*a^2*c^2)) + (a*tan(d/2 + (e*x)/2)*(2
*A*a^2 + A*c^2 - 3*C*a*c))/((a + c)^(5/2)*(a - c)^(5/2)))*(a^4 + c^4 - 2*a^
2*c^2))/(2*A*a^2 + A*c^2 - 3*C*a*c))*(2*A*a^2 + A*c^2 - 3*C*a*c))/(e*(a + c
)^(5/2)*(a - c)^(5/2)) - ((A*c^3 + 2*C*a^3 - 4*A*a^2*c + C*a*c^2)/(a^4 + c^
4 - 2*a^2*c^2) - (tan(d/2 + (e*x)/2)^3*(2*B*a^4 - 2*A*c^4 + 2*B*c^4 + 5*A*a
^2*c^2 - 4*B*a^2*c^2 - 3*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 +
 (e*x)/2)*(2*A*c^4 - 2*B*a^4 - 2*B*c^4 - 11*A*a^2*c^2 + 4*B*a^2*c^2 + 4*C*a
*c^3 + 5*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*A
*c^5 + 2*C*a^5 - 2*B*c^5 - 7*A*a^2*c^3 + 4*B*a^2*c^3 + 5*C*a^3*c^2 - 4*A*a^
4*c - 2*B*a^4*c + 2*C*a*c^4))/(a^2*(a^4 + c^4 - 2*a^2*c^2))))/(e*(tan(d/2 +
(e*x)/2)^2*(2*a^2 + 4*c^2) + a^2*tan(d/2 + (e*x)/2)^4 + a^2 + 4*a*c*tan(d/2
 + (e*x)/2)^3 + 4*a*c*tan(d/2 + (e*x)/2)))
```

3.565 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

Optimal. Leaf size=258

$$\frac{(2a^3A+3aAc^2-4a^2cC-c^3C)\,\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{7/2}\,e}-\frac{B}{3ce(a+c\sin(d+ex))^3}+\frac{(Ac-aC)\cos(d\ldots}{3\,(a^2-c^2)\,e(a+c\sin\ldots}$$

```
[Out] (2*A*a^3+3*A*a*c^2-4*C*a^2*c-C*c^3)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^
2)^(1/2))/(a^2-c^2)^(7/2)/e-1/3*B/c/e/(a+c*sin(e*x+d))^3+1/3*(A*c-C*a)*cos(
e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d))^3+1/6*(5*A*a*c-2*C*a^2-3*C*c^2)*cos(e*x
+d)/(a^2-c^2)^2/e/(a+c*sin(e*x+d))^2+1/6*(11*A*a^2*c+4*A*c^3-2*C*a^3-13*C*a
*c^2)*cos(e*x+d)/(a^2-c^2)^3/e/(a+c*sin(e*x+d))
```

Rubi [A]
time = 0.28, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{6e\,(a^2-c^2)^2\,(a+c\sin(d+ex))^2}+\frac{(Ac-aC)\cos(d+ex)}{3e\,(a^2-c^2)\,(a+c\sin(d+ex))^3}+\frac{(2a^3A-4a^2cC+3aAc^2-c^3C)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(d+ex)\right)+c}{\sqrt{a^2-c^2}}\right)}{e\,(a^2-c^2)^{7/2}}+\frac{(-2a^3C+11a^2Ac-13ac^2C+4Ac^3)\cos(d+ex)}{6e\,(a^2-c^2)^3\,(a+c\sin(d+ex))}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/
Sqrt[a^2 - c^2]])/((a^2 - c^2)^(7/2)*e) - B/(3*c*e*(a + c*Sin[d + e*x])^3)
+ ((A*c - a*C)*Cos[d + e*x])/(3*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^3) + ((5
*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/(6*(a^2 - c^2)^2*e*(a + c*Sin[d +
 e*x])^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/(6
*(a^2 - c^2)^3*e*(a + c*Sin[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx = B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^4}\,dx + \int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$

$$= \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} - \frac{\int \frac{-3(aA-cC)+2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}}{3\left(a^2-c^2\right)}$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} +$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} +$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} +$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} +$$

$$= -\frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\left(a^2-c^2\right)e(a+c\sin(d+ex))^3} +$$

$$= \frac{\left(2a^3A+3aAc^2-4a^2cC-c^3C\right)\tan^{-1}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}e} - \frac{}{3ce}$$

Mathematica [A]
time = 1.72, size = 244, normalized size = 0.95

$$\frac{\frac{6\left(2a^3A+3aAc^2-4a^2cC-c^3C\right)\text{ArcTan}\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\left(a^2-c^2\right)^{7/2}} + \frac{2B\left(-a^2+c^2\right)+2c(Ac-aC)\cos(d+ex)}{(a-c)c(a+c)(a+c\sin(d+ex))^3} + \frac{\left(5aAc-2a^2C-3c^2C\right)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))^2} + \frac{\left(11a^2Ac+4Ac^3-2a^3C-13ac^2C\right)\cos(d+ex)}{(a-c)^3(a+c)^3(a+c\sin(d+ex))}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/2
])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(7/2) + (2*B*(-a^2 + c^2) + 2*c*(A*c - a*C
)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^3) + ((5*a*A*c - 2*
a^2*C - 3*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])^2)
 + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/((a - c)^3*
(a + c)^3*(a + c*Sin[d + e*x])))/(6*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(245) = 490$.
time = 0.91, size = 860, normalized size = 3.33

method	result
derivativedivides	$\frac{\frac{\left(9A\,a^4c^2-6A\,a^2c^4+2A\,c^6+2B\,a^6-6B\,a^4c^2+6B\,a^2c^4-2B\,c^6-4C\,a^5c-C\,a^3c^3\right)\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{a\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}+\frac{\left(6A\,a^6c+27A\,a^4c^3-12A\,a^2c^5+4A\right.}{}}{}$
default	$\frac{\frac{\left(9A\,a^4c^2-6A\,a^2c^4+2A\,c^6+2B\,a^6-6B\,a^4c^2+6B\,a^2c^4-2B\,c^6-4C\,a^5c-C\,a^3c^3\right)\left(\tan^5\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{a\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}+\frac{\left(6A\,a^6c+27A\,a^4c^3-12A\,a^2c^5+4A\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/e*(2*(1/2*(9*A*a^4*c^2-6*A*a^2*c^4+2*A*c^6+2*B*a^6-6*B*a^4*c^2+6*B*a^2*c^
4-2*B*c^6-4*C*a^5*c-C*a^3*c^3)/a/(a^6-3*a^4*c^2+3*a^2*c^4-c^6)*tan(1/2*d+1/
2*e*x)^5+1/2*(6*A*a^6*c+27*A*a^4*c^3-12*A*a^2*c^5+4*A*c^7+4*B*a^6*c-12*B*a^
4*c^3+12*B*a^2*c^5-4*B*c^7-2*C*a^7-14*C*a^5*c^2-11*C*a^3*c^4+2*C*a*c^6)/(a^
6-3*a^4*c^2+3*a^2*c^4-c^6)/a^2*tan(1/2*d+1/2*e*x)^4+1/3/a^3*(54*A*a^6*c^2+2
1*A*a^4*c^4-4*A*a^2*c^6+4*A*c^8+6*B*a^8-14*B*a^6*c^2+6*B*a^4*c^4+6*B*a^2*c^
6-4*B*c^8-18*C*a^7*c-42*C*a^5*c^3-17*C*a^3*c^5+2*C*a*c^7)/(a^6-3*a^4*c^2+3*
a^2*c^4-c^6)*tan(1/2*d+1/2*e*x)^3+(6*A*a^6*c+20*A*a^4*c^3-3*A*a^2*c^5+2*A*c
^7+2*B*a^6*c-6*B*a^4*c^3+6*B*a^2*c^5-2*B*c^7-2*C*a^7-10*C*a^5*c^2-14*C*a^3*
c^4+C*a*c^6)/(a^6-3*a^4*c^2+3*a^2*c^4-c^6)/a^2*tan(1/2*d+1/2*e*x)^2+1/2*(27
*A*a^4*c^2-4*A*a^2*c^4+2*A*c^6+2*B*a^6-6*B*a^4*c^2+6*B*a^2*c^4-2*B*c^6-8*C*
a^5*c-19*C*a^3*c^3+2*C*a*c^5)/a/(a^6-3*a^4*c^2+3*a^2*c^4-c^6)*tan(1/2*d+1/2
*e*x)+1/6*(18*A*a^4*c-5*A*a^2*c^3+2*A*c^5-6*C*a^5-10*C*a^3*c^2+C*a*c^4)/(a^
6-3*a^4*c^2+3*a^2*c^4-c^6))/(a*tan(1/2*d+1/2*e*x)^2+2*c*tan(1/2*d+1/2*e*x)+
a)^3+(2*A*a^3+3*A*a*c^2-4*C*a^2*c-C*c^3)/(a^6-3*a^4*c^2+3*a^2*c^4-c^6)/(a^2
-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d+1/2*e*x)+2*c)/(a^2-c^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(248) = 496$.
time = 3.44, size = 1448, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(4*B*a^8 - 16*B*a^6*c^2 + 24*B*a^4*c^4 - 16*B*a^2*c^6 + 4*B*c^8 - 2*(
2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 + 4*A*
c^8)*cos(x*e + d)^3 + 6*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*a^3*
c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(x*e + d)*sin(x*e + d) + 3*(2*A*a^
6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^6 -
3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(x*e + d)^2 + (6*A
*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^7 -
(2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(x*e + d)^2)*sin(x*e + d
))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(x*e + d)^2 - 2*a*c*sin(x*e + d)
- a^2 - c^2 + 2*(a*cos(x*e + d)*sin(x*e + d) + c*cos(x*e + d))*sqrt(-a^2 +
c^2))/(c^2*cos(x*e + d)^2 - 2*a*c*sin(x*e + d) - a^2 - c^2)) + 12*(C*a^7*c
- 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 + A*c^8)*cos(x*e + d))/
(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c^11)*cos(x*e + d)^2*e
- (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3*c^9 + 3*a*c^11)*e + (
(a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)*cos(x*e + d)^2*e - (3
*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2*c^10 + c^12)*e)*sin(x
*e + d)), 1/6*(2*B*a^8 - 8*B*a^6*c^2 + 12*B*a^4*c^4 - 8*B*a^2*c^6 + 2*B*c^8
 - (2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 +
4*A*c^8)*cos(x*e + d)^3 + 3*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*
a^3*c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(x*e + d)*sin(x*e + d) + 3*(2*
A*a^6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c^
6 - 3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(x*e + d)^2 +
(6*A*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C*c^
7 - (2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(x*e + d)^2)*sin(x*e
 + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(x*e + d) + c)/(sqrt(a^2 - c^2)*cos(x*
e + d))) + 6*(C*a^7*c - 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 +
 A*c^8)*cos(x*e + d))/(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c
^11)*cos(x*e + d)^2*e - (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3
*c^9 + 3*a*c^11)*e + ((a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)
*cos(x*e + d)^2*e - (3*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2
*c^10 + c^12)*e)*sin(x*e + d))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(245) = 490$.
time = 0.46, size = 1281, normalized size = 4.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*A*a^3 - 4*C*a^2*c + 3*A*a*c^2 - C*c^3)*(pi*floor(1/2*(e*x + d)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^
6 - 3*a^4*c^2 + 3*a^2*c^4 - c^6)*sqrt(a^2 - c^2)) + (6*B*a^8*tan(1/2*e*x +
1/2*d)^5 - 12*C*a^7*c*tan(1/2*e*x + 1/2*d)^5 + 27*A*a^6*c^2*tan(1/2*e*x + 1
/2*d)^5 - 18*B*a^6*c^2*tan(1/2*e*x + 1/2*d)^5 - 3*C*a^5*c^3*tan(1/2*e*x + 1
/2*d)^5 - 18*A*a^4*c^4*tan(1/2*e*x + 1/2*d)^5 + 18*B*a^4*c^4*tan(1/2*e*x +
1/2*d)^5 + 6*A*a^2*c^6*tan(1/2*e*x + 1/2*d)^5 - 6*B*a^2*c^6*tan(1/2*e*x + 1
/2*d)^5 - 6*C*a^8*tan(1/2*e*x + 1/2*d)^4 + 18*A*a^7*c*tan(1/2*e*x + 1/2*d)^
4 + 12*B*a^7*c*tan(1/2*e*x + 1/2*d)^4 - 42*C*a^6*c^2*tan(1/2*e*x + 1/2*d)^4
 + 81*A*a^5*c^3*tan(1/2*e*x + 1/2*d)^4 - 36*B*a^5*c^3*tan(1/2*e*x + 1/2*d)^
4 - 33*C*a^4*c^4*tan(1/2*e*x + 1/2*d)^4 - 36*A*a^3*c^5*tan(1/2*e*x + 1/2*d)
^4 + 36*B*a^3*c^5*tan(1/2*e*x + 1/2*d)^4 + 6*C*a^2*c^6*tan(1/2*e*x + 1/2*d)
^4 + 12*A*a*c^7*tan(1/2*e*x + 1/2*d)^4 - 12*B*a*c^7*tan(1/2*e*x + 1/2*d)^4
+ 12*B*a^8*tan(1/2*e*x + 1/2*d)^3 - 36*C*a^7*c*tan(1/2*e*x + 1/2*d)^3 + 108
*A*a^6*c^2*tan(1/2*e*x + 1/2*d)^3 - 28*B*a^6*c^2*tan(1/2*e*x + 1/2*d)^3 - 8
4*C*a^5*c^3*tan(1/2*e*x + 1/2*d)^3 + 42*A*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 +
12*B*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 - 34*C*a^3*c^5*tan(1/2*e*x + 1/2*d)^3 -
 8*A*a^2*c^6*tan(1/2*e*x + 1/2*d)^3 + 12*B*a^2*c^6*tan(1/2*e*x + 1/2*d)^3 +
 4*C*a*c^7*tan(1/2*e*x + 1/2*d)^3 + 8*A*c^8*tan(1/2*e*x + 1/2*d)^3 - 8*B*c^
8*tan(1/2*e*x + 1/2*d)^3 - 12*C*a^8*tan(1/2*e*x + 1/2*d)^2 + 36*A*a^7*c*tan
(1/2*e*x + 1/2*d)^2 + 12*B*a^7*c*tan(1/2*e*x + 1/2*d)^2 - 60*C*a^6*c^2*tan(
1/2*e*x + 1/2*d)^2 + 120*A*a^5*c^3*tan(1/2*e*x + 1/2*d)^2 - 36*B*a^5*c^3*ta
n(1/2*e*x + 1/2*d)^2 - 84*C*a^4*c^4*tan(1/2*e*x + 1/2*d)^2 - 18*A*a^3*c^5*t
an(1/2*e*x + 1/2*d)^2 + 36*B*a^3*c^5*tan(1/2*e*x + 1/2*d)^2 + 6*C*a^2*c^6*t
an(1/2*e*x + 1/2*d)^2 + 12*A*a*c^7*tan(1/2*e*x + 1/2*d)^2 - 12*B*a*c^7*tan(
1/2*e*x + 1/2*d)^2 + 6*B*a^8*tan(1/2*e*x + 1/2*d) - 24*C*a^7*c*tan(1/2*e*x
+ 1/2*d) + 81*A*a^6*c^2*tan(1/2*e*x + 1/2*d) - 18*B*a^6*c^2*tan(1/2*e*x + 1
/2*d) - 57*C*a^5*c^3*tan(1/2*e*x + 1/2*d) - 12*A*a^4*c^4*tan(1/2*e*x + 1/2*
d) + 18*B*a^4*c^4*tan(1/2*e*x + 1/2*d) + 6*C*a^3*c^5*tan(1/2*e*x + 1/2*d) +
 6*A*a^2*c^6*tan(1/2*e*x + 1/2*d) - 6*B*a^2*c^6*tan(1/2*e*x + 1/2*d) - 6*C*
```

```
a^8 + 18*A*a^7*c - 10*C*a^6*c^2 - 5*A*a^5*c^3 + C*a^4*c^4 + 2*A*a^3*c^5)/((
a^9 - 3*a^7*c^2 + 3*a^5*c^4 - a^3*c^6)*(a*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(
1/2*e*x + 1/2*d) + a)^3))/e
```

Mupad [B]
time = 6.22, size = 1085, normalized size = 4.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^4,x)
```

```
[Out] ((2*A*c^5 - 6*C*a^5 - 5*A*a^2*c^3 - 10*C*a^3*c^2 + 18*A*a^4*c + C*a*c^4)/(3
*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (tan(d/2 + (e*x)/2)*(2*B*a^6 + 2*A*
c^6 - 2*B*c^6 - 4*A*a^2*c^4 + 27*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 - 19
*C*a^3*c^3 + 2*C*a*c^5 - 8*C*a^5*c))/(a*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)
) + (2*tan(d/2 + (e*x)/2)^2*(2*A*c^7 - 2*C*a^7 - 2*B*c^7 - 3*A*a^2*c^5 + 20
*A*a^4*c^3 + 6*B*a^2*c^5 - 6*B*a^4*c^3 - 14*C*a^3*c^4 - 10*C*a^5*c^2 + 6*A*
a^6*c + 2*B*a^6*c + C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (
tan(d/2 + (e*x)/2)^4*(4*A*c^7 - 2*C*a^7 - 4*B*c^7 - 12*A*a^2*c^5 + 27*A*a^4
*c^3 + 12*B*a^2*c^5 - 12*B*a^4*c^3 - 11*C*a^3*c^4 - 14*C*a^5*c^2 + 6*A*a^6*
c + 4*B*a^6*c + 2*C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) - (ta
n(d/2 + (e*x)/2)^5*(2*B*c^6 - 2*A*c^6 - 2*B*a^6 + 6*A*a^2*c^4 - 9*A*a^4*c^2
 - 6*B*a^2*c^4 + 6*B*a^4*c^2 + C*a^3*c^3 + 4*C*a^5*c))/(a*(a^6 - c^6 + 3*a^
2*c^4 - 3*a^4*c^2)) + (2*tan(d/2 + (e*x)/2)^3*(3*a^2 + 2*c^2)*(2*B*a^6 + 2*
A*c^6 - 2*B*c^6 - 5*A*a^2*c^4 + 18*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 -
10*C*a^3*c^3 + C*a*c^5 - 6*C*a^5*c))/(3*a^3*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*
c^2)))/(e*(a^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^2*(12*a*c^2 + 3*a^
3) + tan(d/2 + (e*x)/2)^4*(12*a*c^2 + 3*a^3) + tan(d/2 + (e*x)/2)^3*(12*a^2
*c + 8*c^3) + a^3 + 6*a^2*c*tan(d/2 + (e*x)/2) + 6*a^2*c*tan(d/2 + (e*x)/2)
^5)) + (atan(((((2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4*C*a^2*c)*(2*a^6*c - 2*c^7
+ 6*a^2*c^5 - 6*a^4*c^3))/(2*(a + c)^(7/2)*(a - c)^(7/2)*(a^6 - c^6 + 3*a^2
*c^4 - 3*a^4*c^2)) + (a*tan(d/2 + (e*x)/2)*(2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4
*C*a^2*c))/((a + c)^(7/2)*(a - c)^(7/2)))*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^
2))/(2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4*C*a^2*c))*(2*A*a^3 - C*c^3 + 3*A*a*c^2
 - 4*C*a^2*c))/(e*(a + c)^(7/2)*(a - c)^(7/2))
```

3.566 $\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx$

Optimal. Leaf size=131

$$-\frac{F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx}{2a+b}\right.}{\sqrt{2}\,d\sqrt{1+\sin(2c+2dx)}}$$

```
[Out] -1/2*AppellF1(1/2,-m,1/2,3/2,b*(1-sin(2*d*x+2*c))/(2*a+b),1/2-1/2*sin(2*d*x
+2*c))*cos(2*d*x+2*c)*(a+1/2*b*sin(2*d*x+2*c))^m/d/(((2*a+b*sin(2*d*x+2*c))
/(2*a+b))^m)*2^(1/2)/(1+sin(2*d*x+2*c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2745, 2744, 144, 143}

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\sin(2c+2dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] -((AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[2*c + 2*d*x])/2, (b*(1 - Sin[2*c +
2*d*x]))/(2*a + b)]*Cos[2*c + 2*d*x]*(a + (b*Sin[2*c + 2*d*x])/2)^m)/(Sqrt[
2]*d*Sqrt[1 + Sin[2*c + 2*d*x]]*((2*a + b*Sin[2*c + 2*d*x])/(2*a + b))^m))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx=\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\,dx$$

$$=\frac{\cos(2c+2dx)\text{Subst}\left(\int \frac{\left(a+\frac{bx}{2}\right)^m}{\sqrt{1-x}\,\sqrt{1+x}}\,dx,x,\sin(2c+2dx)\right)}{2d\sqrt{1-\sin(2c+2dx)}\,\sqrt{1+\sin(2c+2dx)}}$$

$$=\frac{\left(\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(-\frac{a+\frac{1}{2}b\sin(2c+2dx)}{-a-\frac{b}{2}}\right)^{-m}\right)\text{Su}}{2d\sqrt{1-\sin(2c+2dx)}\,\sqrt{1+\text{s}}}$$

$$=-\frac{F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)\cos(2c+2c}{\sqrt{2}\,d\sqrt{1+\sin(2c+2}}$$

Mathematica [A]
time = 0.43, size = 145, normalized size = 1.11

$$\frac{F_1\left(1+m;\frac{1}{2},\frac{1}{2};2+m;\frac{2a+b\sin(2(c+dx))}{2a-b},\frac{2a+b\sin(2(c+dx))}{2a+b}\right)\sec(2(c+dx))\sqrt{-\frac{b(-1+\sin(2(c+dx)))}{2a+b}}\sqrt{\frac{b(1+\sin(2(c+dx)))}{-2a+b}}\left(a+\frac{1}{2}b\sin(2(c+dx))\right)^{1+m}}{bd(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

```
[Out] (AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sin[2*(c + d*x)])/(2*a - b), (2*
a + b*Sin[2*(c + d*x)])/(2*a + b)]*Sec[2*(c + d*x)]*Sqrt[-((b*(-1 + Sin[2*(
c + d*x)]))/(2*a + b))]*Sqrt[(b*(1 + Sin[2*(c + d*x)]))/(-2*a + b)]*(a + (b
*Sin[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a+b\cos \left(dx+c\right)\sin \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
[Out] int((a+b*cos(d*x+c)*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^m,x)
```

```
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^m, x)
```

3.567 $\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$

Optimal. Leaf size=107

$$\frac{1}{8}a\left(8a^2+3b^2\right)x-\frac{b(16a^2+b^2)\cos(2c+2dx)}{24d}-\frac{5ab^2\cos(2c+2dx)\sin(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin(}{48d}$$

[Out] 1/8*a*(8*a^2+3*b^2)*x-1/24*b*(16*a^2+b^2)*cos(2*d*x+2*c)/d-5/48*a*b^2*cos(2*d*x+2*c)*sin(2*d*x+2*c)/d-1/48*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^2/d

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2745, 2735, 2813}

$$-\frac{b(16a^2+b^2)\cos(2c+2dx)}{24d}+\frac{1}{8}ax\left(8a^2+3b^2\right)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]

[Out] (a*(8*a^2 + 3*b^2)*x)/8 - (b*(16*a^2 + b^2)*Cos[2*c + 2*d*x])/(24*d) - (5*a*b^2*Cos[2*c + 2*d*x]*Sin[2*c + 2*d*x])/(48*d) - (b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^2)/(48*d)

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3 dx$$
$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d} + \frac{1}{3}\int \left(a+\frac{1}{2}b\sin(2c+2\right.$$
$$= \frac{1}{8}a\left(8a^2+3b^2\right)x - \frac{b(16a^2+b^2)\cos(2c+2dx)}{24d} - \frac{5ab^2\cos(2c+2dx)}{48d}$$

Mathematica [A]
time = 0.20, size = 75, normalized size = 0.70

$$\frac{-9(16a^2b+b^3)\cos(2(c+dx))+b^3\cos(6(c+dx))+6a(4(8a^2+3b^2)(c+dx)-3b^2\sin(4(c+dx)))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

```
[Out] (-9*(16*a^2*b + b^3)*Cos[2*(c + d*x)] + b^3*Cos[6*(c + d*x)] + 6*a*(4*(8*a^
2 + 3*b^2)*(c + d*x) - 3*b^2*Sin[4*(c + d*x)]))/(192*d)
```

Maple [A]
time = 0.31, size = 106, normalized size = 0.99

method	result
risch	$a^3x+\frac{3ab^2x}{8}+\frac{b^3\cos(6dx+6c)}{192d}-\frac{3ab^2\sin(4dx+4c)}{32d}-\frac{3b\cos(2dx+2c)a^2}{4d}-\frac{3b^3\cos(2dx+2c)}{64d}$
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-3a^2b(}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-3a^2b(}{d}$
norman	$(a^3+\frac{3}{8}ab^2)x+(a^3+\frac{3}{8}ab^2)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^3+\frac{9}{4}ab^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^3+\frac{9}{4}ab^2)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(15a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/4*s
in(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/2*a^2*b*c
os(d*x+c)^2+a^3*(d*x+c))
```

Maxima [A]
time = 0.30, size = 80, normalized size = 0.75

$$a^3x - \frac{3\,a^2b\cos\left(dx+c\right)^2}{2\,d} + \frac{3\,(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right))ab^2}{32\,d} - \frac{\left(2\,\sin\left(dx+c\right)^6 - 3\,\sin\left(dx+c\right)^4\right)b^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 3/2*a^2*b*cos(d*x + c)^2/d + 3/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*
a*b^2/d - 1/12*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*b^3/d
```

Fricas [A]
time = 2.87, size = 97, normalized size = 0.91

$$\frac{4\,b^3\cos\left(dx+c\right)^6 - 6\,b^3\cos\left(dx+c\right)^4 - 36\,a^2b\cos\left(dx+c\right)^2 + 3\,(8\,a^3+3\,ab^2)dx - 9\left(2\,ab^2\cos\left(dx+c\right)^3 - ab^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/24*(4*b^3*cos(d*x + c)^6 - 6*b^3*cos(d*x + c)^4 - 36*a^2*b*cos(d*x + c)^2
 + 3*(8*a^3 + 3*a*b^2)*d*x - 9*(2*a*b^2*cos(d*x + c)^3 - a*b^2*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [A]
time = 0.44, size = 190, normalized size = 1.78

$$\begin{cases} a^3x - \frac{3a^2b\cos^2\left(c+dx\right)}{2d} + \frac{3ab^2x\sin^4\left(c+dx\right)}{8} + \frac{3ab^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4} + \frac{3ab^2x\cos^4\left(c+dx\right)}{8} + \frac{3ab^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d} - \frac{3ab^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d} + \frac{b^3\sin^6\left(c+dx\right)}{12d} + \frac{b^3\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{4d} & \text{for}\: d \neq 0 \\ x\left(a+b\sin\left(c\right)\cos\left(c\right)\right)^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x - 3*a**2*b*cos(c + d*x)**2/(2*d) + 3*a*b**2*x*sin(c + d*x
)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c +
d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c +
d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d*x)**6/(12*d) + b**3*sin(c + d*x
)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**3, True))
```

Giac [A]
time = 0.43, size = 75, normalized size = 0.70

$$\frac{b^3\cos\left(6\,dx+6\,c\right)}{192\,d} - \frac{3\,ab^2\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{1}{8}\left(8\,a^3+3\,ab^2\right)x - \frac{3\,(16\,a^2b+b^3)\cos\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

[Out] 1/192*b^3*cos(6*d*x + 6*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*a^3 + 3*a*b^2)*x - 3/64*(16*a^2*b + b^3)*cos(2*d*x + 2*c)/d

Mupad [B]
time = 3.44, size = 125, normalized size = 1.17

$$a^3\,x-\frac{\tan(c+d\,x)^2\,(72\,a^2\,b+6\,b^3)+36\,a^2\,b+2\,b^3+36\,a^2\,b\tan(c+d\,x)^4-9\,a\,b^2\tan(c+d\,x)^5+9\,a\,b^2\tan(c+d\,x)}{d\,\left(24\tan\left(c+d\,x\right)^6+72\tan\left(c+d\,x\right)^4+72\tan\left(c+d\,x\right)^2+24\right)}+\frac{3\,a\,b^2\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*cos(c + d*x)*sin(c + d*x))^3,x)

[Out] a^3*x - (tan(c + d*x)^2*(72*a^2*b + 6*b^3) + 36*a^2*b + 2*b^3 + 36*a^2*b*tan(c + d*x)^4 - 9*a*b^2*tan(c + d*x)^5 + 9*a*b^2*tan(c + d*x))/(d*(72*tan(c + d*x)^2 + 72*tan(c + d*x)^4 + 24*tan(c + d*x)^6 + 24)) + (3*a*b^2*x)/8

3.568 $\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$

Optimal. Leaf size=61

$$\frac{1}{8}\left(8a^2+b^2\right)x-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\cos(2c+2dx)\sin(2c+2dx)}{16d}$$

```
[Out] 1/8*(8*a^2+b^2)*x-1/2*a*b*cos(2*d*x+2*c)/d-1/16*b^2*cos(2*d*x+2*c)*sin(2*d*
x+2*c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2745, 2723}

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]*
Sin[2*c + 2*d*x])/(16*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx &= \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2\,dx\\ &= \frac{1}{8}\left(8a^2+b^2\right)x-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\cos(2c+2dx)\sin(2c+2dx)}{16d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.79

$$-\frac{-4(8a^2+b^2)(c+dx)+16ab\cos(2(c+dx))+b^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

```
[Out] -1/32*(-4*(8*a^2 + b^2)*(c + d*x) + 16*a*b*Cos[2*(c + d*x)] + b^2*Sin[4*(c
+ d*x)])/d
```

Maple [A]
time = 0.24, size = 69, normalized size = 1.13

method	result
risch	$a^2x+\frac{b^2x}{8}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{2d}$
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-ab\left(\cos^2(dx+c)\right)+a^2(dx+c)}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-ab\left(\cos^2(dx+c)\right)+a^2(dx+c)}{d}$
norman	$\left(a^2+\frac{b^2}{8}\right)x+\left(a^2+\frac{b^2}{8}\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^2+\frac{b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^2+\frac{b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(6a^2+\frac{3b^2}{4}\right)x($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-a*b*cos(d*x+c)^2+a^2*(d*x+c))
```

Maxima [A]
time = 0.26, size = 48, normalized size = 0.79

$$a^2x-\frac{ab\cos(dx+c)^2}{d}+\frac{(4\,dx+4\,c-\sin(4\,dx+4\,c))b^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x - a*b*cos(d*x + c)^2/d + 1/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2/d
```

Fricas [A]
time = 3.01, size = 63, normalized size = 1.03

$$-\frac{8\,ab\cos(dx+c)^2-(8\,a^2+b^2)dx+\left(2\,b^2\cos(dx+c)^3-b^2\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b*cos(d*x + c)^2 - (8*a^2 + b^2)*d*x + (2*b^2*cos(d*x + c)^3 - b^
2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.
time = 0.19, size = 129, normalized size = 2.11

$$\begin{cases} a^2x - \frac{ab\cos^2(c+dx)}{d} + \frac{b^2x\sin^4(c+dx)}{8} + \frac{b^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{b^2x\cos^4(c+dx)}{8} + \frac{b^2\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{b^2\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x(a + b\sin(c)\cos(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**2
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin(
c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), N
e(d, 0)), (x*(a + b*sin(c)*cos(c))**2, True))
```

Giac [A]
time = 0.42, size = 46, normalized size = 0.75

$$\frac{1}{8}\left(8a^2 + b^2\right)x - \frac{ab\cos\left(2dx + 2c\right)}{2d} - \frac{b^2\sin\left(4dx + 4c\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d - 1/32*b^2*sin(4*d*x + 4*c
)/d
```

Mupad [B]
time = 3.03, size = 78, normalized size = 1.28

$$x\left(a^2 + \frac{b^2}{8}\right) - \frac{-\frac{b^2\tan(c+dx)^3}{8} + \frac{b^2\tan(c+dx)}{8} + a\,b\tan(c+d\,x)^2 + a\,b}{d\,\left(\tan\left(c+d\,x\right)^4 + 2\tan\left(c+d\,x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

```
[Out] x*(a^2 + b^2/8) - (a*b + (b^2*tan(c + d*x))/8 - (b^2*tan(c + d*x)^3)/8 + a*
b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.569 $\int (a+b\cos(c+dx)\sin(c+dx))\,dx$

Optimal. Leaf size=20

$$ax+\frac{b\sin^2(c+dx)}{2d}$$

[Out] a*x+1/2*b*sin(d*x+c)^2/d

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2644, 30}

$$ax+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Cos[c + d*x]*Sin[c + d*x],x]

[Out] a*x + (b*Sin[c + d*x]^2)/(2*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx)\sin(c+dx))\,dx &= ax+b\int \cos(c+dx)\sin(c+dx)\,dx\\ &= ax+\frac{b\text{Subst}(\int x\,dx,x,\sin(c+dx))}{d}\\ &= ax+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.90

$$ax-\frac{b\cos(2c)\cos(2dx)}{4d}+\frac{b\sin(2c)\sin(2dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[c + d*x]*Sin[c + d*x],x]
[Out] a*x - (b*Cos[2*c]*Cos[2*d*x])/(4*d) + (b*Sin[2*c]*Sin[2*d*x])/(4*d)
```

Maple [A]
time = 0.11, size = 19, normalized size = 0.95

method	result	size
default	$ax+\frac{b\left(\sin^{2}\left(dx+c\right)\right)}{2d}$	19
risch	$ax-\frac{b\cos\left(2dx+2c\right)}{4d}$	20
derivativedivides	$\frac{\left(dx+c\right)a+\frac{b\left(\sin^{2}\left(dx+c\right)\right)}{2}}{d}$	24
norman	$\frac{ax+ax\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(d*x+c)*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*sin(d*x+c)^2/d
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.90

$$ax-\frac{b\cos\left(dx+c\right)^{2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
[Out] a*x - 1/2*b*cos(d*x + c)^2/d
```

Fricas [A]
time = 3.38, size = 22, normalized size = 1.10

$$\frac{2\,adx-b\cos\left(dx+c\right)^{2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
[Out] 1/2*(2*a*d*x - b*cos(d*x + c)^2)/d
```

Sympy [A]
time = 0.06, size = 26, normalized size = 1.30

$$ax + b\left(\begin{cases} -\frac{\cos^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\sin(c)\cos(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((-cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*sin(c)*cos(c), Tru
e))
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.90

$$ax + \frac{b\sin(dx+c)^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*sin(d*x + c)^2/d
```

Mupad [B]
time = 2.94, size = 22, normalized size = 1.10

$$-\frac{\frac{b\cos(c+dx)^2}{2} - a\,d\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*cos(c + d*x)*sin(c + d*x),x)
```

```
[Out] -((b*cos(c + d*x)^2)/2 - a*d*x)/d
```

3.570 $\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{2\text{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}\,d}$$

```
[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2745, 2739, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
```

`x]`

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx &= \int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}\\
&= -\frac{2\operatorname{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx,x,b+2a\tan\left(\frac{1}{2}(2c+2dx)\right)\right)}{d}\\
&= \frac{2\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}\,d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 1.00

$$\frac{2\operatorname{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
[Out] (2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

Maple [A]
time = 0.29, size = 45, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$	45
default	$\frac{2\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$	45
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\sqrt{-4a^2+b^2}-4a^2+b^2}{b\sqrt{-4a^2+b^2}}\right)}{\sqrt{-4a^2+b^2}\,d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\sqrt{-4a^2+b^2}+4a^2-b^2}{b\sqrt{-4a^2+b^2}}\right)}{\sqrt{-4a^2+b^2}\,d}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 2.48, size = 290, normalized size = 6.04

$$\left[-\frac{\sqrt{-4\,a^2+b^2}\,\log\left(-\frac{2\,(8\,a^2-b^2)\cos(dx+c)^4-4\,ab\cos(dx+c)\sin(dx+c)-2\,(8\,a^2-b^2)\cos(dx+c)^2+2\,a^2-b^2+\left(2\,b\cos(dx+c)^2+4\,\left(2\,a\cos(dx+c)^3-a\cos(dx+c)\right)\sin(dx+c)-b\right)\sqrt{-4\,a^2+b^2}}{b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-a^2}\right)}{2\,(4\,a^2-b^2)d}, -\frac{\arctan\left(-\frac{(4\,a\cos(dx+c)\sin(dx+c)+b)\sqrt{4\,a^2-b^2}}{2\,(4\,a^2-b^2)\cos(dx+c)^2-4\,a^2+b^2}\right)}{\sqrt{4\,a^2-b^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d
*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*
cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*
sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*
x + c)*sin(d*x + c) - a^2))/((4*a^2 - b^2)*d), -arctan(-(4*a*cos(d*x + c)*s
in(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2
+ b^2))/(sqrt(4*a^2 - b^2)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 61, normalized size = 1.27

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)}{\sqrt{4\,a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqrt
(4*a^2 - b^2)))/(sqrt(4*a^2 - b^2)*d)
```

Mupad [B]
time = 3.11, size = 44, normalized size = 0.92

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,a\,\tan\left(c+d\,x\right)}{\sqrt{4\,a^2-b^2}}\right)}{d\,\sqrt{4\,a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x)),x)
```

```
[Out] (2*atan((b + 2*a*tan(c + d*x))/(4*a^2 - b^2)^(1/2)))/(d*(4*a^2 - b^2)^(1/2)
)
```

3.571 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$

Optimal. Leaf size=95

$$\frac{8a\text{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}\,d}+\frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))}$$

[Out] 8*a*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d+2*b*cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2745, 2743, 12, 2739, 632, 210}

$$\frac{8a\text{ArcTan}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\,(4a^2-b^2)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\,(4a^2-b^2)\,(2a+b\sin(2c+2dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]

[Out] (8*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2 - b^2)^(3/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx &= \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}\,dx \\
&= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))} + \frac{4\int \frac{a}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2} \\
&= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))} + \frac{(4a)\int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}\,dx}{4a^2-b^2} \\
&= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))} + \frac{(4a)\text{Subst}\left(\int \frac{1}{a+bx+ax^2}\,dx, x, \text{t}\right.}{(4a^2-b^2)\,d} \\
&= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))} - \frac{(8a)\text{Subst}\left(\int \frac{1}{-4a^2+b^2-x^2}\,dx, x\right.}{(4a^2-} \\
&= \frac{8a\tan^{-1}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}\,d} + \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 94, normalized size = 0.99

$$
\frac{2\left(\frac{4a\text{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}}+\frac{b\cos(2(c+dx))}{(2a-b)(2a+b)(2a+b\sin(2(c+dx)))}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

```
[Out] (2*((4*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a^2 - b^2)^(3
/2) + (b*Cos[2*(c + d*x)])/((2*a - b)*(2*a + b)*(2*a + b*Sin[2*(c + d*x)]))
))/d
```

Maple [A]
time = 0.46, size = 114, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\frac{b^2\tan(dx+c)}{a(4a^2-b^2)}+\frac{2b}{4a^2-b^2}}{a\left(\tan^2(dx+c)\right)+b\tan(dx+c)+a}+\frac{8a\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{\frac{\frac{b^2\tan(dx+c)}{a(4a^2-b^2)}+\frac{2b}{4a^2-b^2}}{a\left(\tan^2(dx+c)\right)+b\tan(dx+c)+a}+\frac{8a\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{\frac{3}{2}}}}{d}$
risch	$\frac{8a\,e^{2i(dx+c)}+4ib}{(4a^2-b^2)d\left(b\,e^{4i(dx+c)}+4ia\,e^{2i(dx+c)}-b\right)}-\frac{4a\ln\left(e^{2i(dx+c)}+\frac{2ia\sqrt{-4a^2+b^2}-4a^2+b^2}{b\sqrt{-4a^2+b^2}}\right)}{\sqrt{-4a^2+b^2}\,(2a+b)(2a-b)d}+\frac{4a\ln\left(e^{2i(dx+c)}+\frac{2ia}{}\right.}{\sqrt{-4a^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((b^2/a/(4*a^2-b^2)*tan(d*x+c)+2*b/(4*a^2-b^2))/(a*tan(d*x+c)^2+b*tan(d
*x+c)+a)+8*a/(4*a^2-b^2)^(3/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(91) = 182.
time = 2.84, size = 493, normalized size = 5.19

$$\left[-\frac{4a^2b-b^3-2(4a^2b-b^3)\cos(dx+c)^2-2(ab\cos(dx+c)\sin(dx+c)+a^2)\sqrt{-4a^2+b^2}\log\left(\frac{2(8a^2-b^2)\cos(dx+c)^4-4ab\cos(dx+c)\sin(dx+c)-2(8a^2-b^2)\cos(dx+c)^2+2a^2-b^2-\left(2b\cos(dx+c)^2+4\left(2a\cos(dx+c)^3-a\cos(dx+c)\right)\sin(dx+c)-b\right)\sqrt{-4a^2+b^2}}{b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2ab\cos(dx+c)\sin(dx+c)-a^2}\right)}{(16a^4b-8a^2b^3+b^5)d\cos(dx+c)\sin(dx+c)+(16a^5-8a^3b^2+ab^4)d},\ -\frac{4a^2b-b^3-2(4a^2b-b^3)\cos(dx+c)^2+4(ab\cos(dx+c)\sin(dx+c)+a^2)\sqrt{4a^2-b^2}\arctan\left(-\frac{(4a\cos(dx+c)\sin(dx+c)+b)\sqrt{4a^2-b^2}}{2(4a^2-b^2)\cos(dx+c)^2-4a^2+b^2}\right)}{(16a^4b-8a^2b^3+b^5)d\cos(dx+c)\sin(dx+c)+(16a^5-8a^3b^2+ab^4)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")

[Out] [-(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 - 2*(a*b*cos(d*x + c)*sin(d*x + c) + a^2)*sqrt(-4*a^2 + b^2)*log((2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 - (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2)))/((16*a^4*b - 8*a^2*b^3 + b^5)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d), -(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b*cos(d*x + c)*sin(d*x + c) + a^2)*sqrt(4*a^2 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)))/((16*a^4*b - 8*a^2*b^3 + b^5)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**2,x)

[Out] Timed out

Giac [**A**]
time = 0.43, size = 116, normalized size = 1.22

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)a}{(4\,a^2-b^2)^{\frac{3}{2}}}+\frac{b^2\tan(dx+c)+2\,ab}{(4\,a^3-ab^2)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")

[Out] (8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqrt(4*a^2 - b^2)))*a/(4*a^2 - b^2)^(3/2) + (b^2*tan(d*x + c) + 2*a*b)/((4*a^3 - a*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d

Mupad [**B**]
time = 3.05, size = 181, normalized size = 1.91

$$\frac{\frac{2\,b}{4\,a^2-b^2}+\frac{b^2\tan(c+d\,x)}{a\,(4\,a^2-b^2)}}{d\,\left(a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+a\right)}+\frac{8\,a\,\mathrm{atan}\left(\frac{(4\,a^2-b^2)\left(\frac{8\,a^2\tan(c+d\,x)}{(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}+\frac{4\,a\,\left(4\,a^2\,b-b^3\right)}{(2\,a+b)^{3/2}\,\left(4\,a^2-b^2\right)\,(2\,a-b)^{3/2}}\right)}{4\,a}\right)}{d\,(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

```
[Out] ((2*b)/(4*a^2 - b^2) + (b^2*tan(c + d*x))/(a*(4*a^2 - b^2)))/(d*(a + b*tan(
c + d*x) + a*tan(c + d*x)^2)) + (8*a*atan(((4*a^2 - b^2)*((8*a^2*tan(c + d*
x))/((2*a + b)^(3/2)*(2*a - b)^(3/2)) + (4*a*(4*a^2*b - b^3))/((2*a + b)^(3
/2)*(4*a^2 - b^2)*(2*a - b)^(3/2))))/(4*a)))/(d*(2*a + b)^(3/2)*(2*a - b)^(
3/2))
```

3.572 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{4(8a^2+b^2)\operatorname{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}d}+\frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2}+\frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+2dx))}$$

```
[Out] 4*(8*a^2+b^2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(5/2
)/d+2*b*cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^2+12*a*b*cos(2*
d*x+2*c)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))
```

Rubi [A]

time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2745, 2743, 2833, 12, 2739, 632, 210}

$$\frac{4(8a^2+b^2)\operatorname{ArcTan}\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\,(4a^2-b^2)^{5/2}}+\frac{12ab\cos(2c+2dx)}{d\,(4a^2-b^2)^2\,(2a+b\sin(2c+2dx))}+\frac{2b\cos(2c+2dx)}{d\,(4a^2-b^2)\,(2a+b\sin(2c+2dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (4*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/((4*a^2
- b^2)^(5/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c
+ 2*d*x])^2) + (12*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)^2*d*(2*a + b*Sin[2*
c + 2*d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3}\,dx$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} - \frac{2\int \frac{-2a+\frac{1}{2}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}\,dx}{4a^2-b^2}$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+$$

$$= \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2\,d(2a+b\sin(2c+$$

$$= \frac{4(8a^2+b^2)\tan^{-1}\left(\frac{\frac{b+2a\tan(c+dx)}{1}}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}\,d} + \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2$$

Mathematica [A]
time = 0.65, size = 120, normalized size = 0.81

$$\frac{2\left(\frac{2(8a^2+b^2)\text{ArcTan}\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}} + \frac{b\cos(2(c+dx))(16a^2-b^2+6ab\sin(2(c+dx)))}{(-4a^2+b^2)^2(2a+b\sin(2(c+dx)))^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

```
[Out] (2*((2*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a
^2 - b^2)^(5/2) + (b*Cos[2*(c + d*x)]*(16*a^2 - b^2 + 6*a*b*Sin[2*(c + d*x)
]))/((-4*a^2 + b^2)^2*(2*a + b*Sin[2*(c + d*x)])^2)))/d
```

Maple [A]
time = 0.66, size = 271, normalized size = 1.82

method	result

derivativedivides	$\frac{\frac{\frac{b^{2}\left(10a^{2}-b^{2}\right)\left(\tan^{3}(dx+c)\right)}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(32a^{4}+14a^{2}b^{2}-b^{4}\right)\left(\tan^{2}(dx+c)\right)}{2\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(22a^{2}-b^{2}\right)\tan(dx+c)}{a\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)}+\frac{b\left(16a^{2}-b^{2}\right)}{16a^{4}-8a^{2}b^{2}+b^{4}}}{\left(a\left(\tan^{2}(dx+c)\right)+b\tan(dx+c)+a\right)^{2}}+\frac{4\left(8a^{2}+b^{2}\right)\arctan\left(\right.}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right.}}{d}$
default	$\frac{\frac{\frac{b^{2}\left(10a^{2}-b^{2}\right)\left(\tan^{3}(dx+c)\right)}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(32a^{4}+14a^{2}b^{2}-b^{4}\right)\left(\tan^{2}(dx+c)\right)}{2\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(22a^{2}-b^{2}\right)\tan(dx+c)}{a\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)}+\frac{b\left(16a^{2}-b^{2}\right)}{16a^{4}-8a^{2}b^{2}+b^{4}}}{\left(a\left(\tan^{2}(dx+c)\right)+b\tan(dx+c)+a\right)^{2}}+\frac{4\left(8a^{2}+b^{2}\right)\arctan\left(\right.}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right.}}{d}$
risch	$-\frac{4i\left(-8ia^{2}b\,e^{6i(dx+c)}-ib^{3}e^{6i(dx+c)}+48a^{3}e^{4i(dx+c)}+6a\,b^{2}e^{4i(dx+c)}+40ia^{2}b\,e^{2i(dx+c)}-ib^{3}e^{2i(dx+c)}-6a\,b^{2}\right)}{\left(ib-ib\,e^{4i(dx+c)}+4a\,e^{2i(dx+c)}\right)^{2}\left(4a^{2}-b^{2}\right)^{2}d}-\frac{16\ln\left(e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((b^2*(10*a^2-b^2)/(16*a^4-8*a^2*b^2+b^4)/a*tan(d*x+c)^3+1/2*b*(32*a^4+
14*a^2*b^2-b^4)/(16*a^4-8*a^2*b^2+b^4)/a^2*tan(d*x+c)^2+b^2*(22*a^2-b^2)/a/
(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)+b*(16*a^2-b^2)/(16*a^4-8*a^2*b^2+b^4))/(a
*tan(d*x+c)^2+b*tan(d*x+c)+a)^2+4*(8*a^2+b^2)/(16*a^4-8*a^2*b^2+b^4)/(4*a^2
-b^2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(145) = 290$.
time = 2.71, size = 969, normalized size = 6.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x
 + c)^2 - 2*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^
```

```
2 + b^4)*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sq
rt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)*
sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*cos(d*x
+ c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sqrt(-4*
a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*s
in(d*x + c) - a^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(d*x + c)^3 - (4*a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^
6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*
cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*d*cos(d*x +
 c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)*d), 1/2*(64
*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*x + c)^2
- 4*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*b^2 + b^4)
*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(4*a^2
 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4
*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(
d*x + c)^3 - (4*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 -
 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4
 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b
^5 - a*b^7)*d*cos(d*x + c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4
 - a^2*b^6)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 252, normalized size = 1.69

$$\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2a\tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)(8a^2+b^2)}{(16a^4-8a^2b^2+b^4)\sqrt{4a^2-b^2}}+\frac{20a^3b^2\tan(dx+c)^3-2ab^4\tan(dx+c)^3+32a^4b\tan(dx+c)^2+14a^2b^3\tan(dx+c)^2-b^5\tan(dx+c)^2+44a^3b^2\tan(dx+c)-2ab^4\tan(dx+c)+32a^4b-2a^2b^3}{(16a^6-8a^4b^2+a^2b^4)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)
/sqrt(4*a^2 - b^2)))*(8*a^2 + b^2)/((16*a^4 - 8*a^2*b^2 + b^4)*sqrt(4*a^2 -
 b^2)) + (20*a^3*b^2*tan(d*x + c)^3 - 2*a*b^4*tan(d*x + c)^3 + 32*a^4*b*tan
(d*x + c)^2 + 14*a^2*b^3*tan(d*x + c)^2 - b^5*tan(d*x + c)^2 + 44*a^3*b^2*t
an(d*x + c) - 2*a*b^4*tan(d*x + c) + 32*a^4*b - 2*a^2*b^3)/((16*a^6 - 8*a^4
*b^2 + a^2*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 3.87, size = 396, normalized size = 2.66

$$\frac{\frac{16a^2b-b^3}{16a^4-8a^2b^2+b^4}+\frac{b\tan(c+dx)\,(22a^2b-b^3)}{a\,(16a^4-8a^2b^2+b^4)}+\frac{\tan(c+dx)^2\,(16a^2b-b^3)\,(2a^2+b^2)}{2a^2\,(16a^4-8a^2b^2+b^4)}+\frac{b\tan(c+dx)^3\,(10a^2b-b^3)}{a\,(16a^4-8a^2b^2+b^4)}}{d\,\left(\tan(c+dx)^2\,(2a^2+b^2)+a^2+a^2\tan(c+dx)^4+2ab\tan(c+dx)+2ab\tan(c+dx)^3\right)}+\frac{4\,\mathrm{atan}\left(\frac{\left(\frac{4a\tan(c+dx)\,(8a^2+b^2)}{(2a+b)^{5/2}\,(2a-b)^{5/2}}+\frac{2\,(8a^2+b^2)\,(16a^4b-8a^2b^3+b^5)}{(2a+b)^{5/2}\,(2a-b)^{5/2}\,(16a^4-8a^2b^2+b^4)}\right)\,(16a^4-8a^2b^2+b^4)}{16a^2+2b^2}\right)\,(8a^2+b^2)}{d\,(2a+b)^{5/2}\,(2a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

```
[Out] ((16*a^2*b - b^3)/(16*a^4 + b^4 - 8*a^2*b^2) + (b*tan(c + d*x)*(22*a^2*b -
b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)) + (tan(c + d*x)^2*(16*a^2*b - b^3)*(2*
a^2 + b^2))/(2*a^2*(16*a^4 + b^4 - 8*a^2*b^2)) + (b*tan(c + d*x)^3*(10*a^2*
b - b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b^2)
+ a^2 + a^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) +
(4*atan((((4*a*tan(c + d*x)*(8*a^2 + b^2))/((2*a + b)^(5/2)*(2*a - b)^(5/2)
) + (2*(8*a^2 + b^2)*(16*a^4*b + b^5 - 8*a^2*b^3))/((2*a + b)^(5/2)*(2*a -
b)^(5/2)*(16*a^4 + b^4 - 8*a^2*b^2)))*(16*a^4 + b^4 - 8*a^2*b^2))/(16*a^2 +
 2*b^2))*(8*a^2 + b^2))/(d*(2*a + b)^(5/2)*(2*a - b)^(5/2))
```

3.573 $\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=265

$$-\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}\,d}+\frac{(92a^2+9b^2)\,E}{60}$$

```
[Out] -1/40*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(3/2)/d*2^(1/2)-2/15*a*b*cos(
2*d*x+2*c)*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d-1/120*(92*a^2+9*b^2)*(sin
(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/
2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a+b*sin(2*
d*x+2*c))/(2*a+b))^(1/2)+2/15*a*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin
(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/
2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2745, 2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\sqrt{2}\,a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{15d\sqrt{2a+b\sin(2c+2dx)}}+\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{60\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}\,d}-\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*a*b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(15*d) - (
b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^(3/2))/(20*Sqrt[2]*d) + ((92*
a^2 + 9*b^2)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*
c + 2*d*x]])/(60*Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - (2
*Sqrt[2]*a*(4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2
*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(15*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}\,dx$$

$$= -\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}\,d} + \frac{2}{5}\int \sqrt{a+\frac{1}{2}b\sin(2c}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx}{}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx}{}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx}{}$$

$$= -\frac{2\sqrt{2}\,ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx}{}$$

Mathematica [**A**]
time = 1.28, size = 202, normalized size = 0.76

$$\frac{2(184a^3+92a^2b+18ab^2+9b^3)\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}} - 32a(4a^2-b^2)\,F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}} - b(88a^2\cos(2(c+dx))+b(28a+3b\sin(2(c+dx)))\sin(4(c+dx)))}{120d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*(184*a^3 + 92*a^2*b + 18*a*b^2 + 9*b^3)*EllipticE[c - Pi/4 + d*x, (2*b)/
(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - 32*a*(4*a^2 - b^2)*
EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/
(2*a + b)] - b*(88*a^2*Cos[2*(c + d*x)] + b*(28*a + 3*b*Sin[2*(c + d*x)])*S
in[4*(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(293) = 586$.
time = 0.55, size = 1138, normalized size = 4.29

method	result
default	$\frac{240a^4\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)+64}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*(240*a^4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b
/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+64*((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-
b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c
))*b/(2*a-b))^(1/2)*a^3*b-24*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(
sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*El
lipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2
-16*a*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b)
)^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-9*((2*a+b*sin(2*d*x+2*c))/
(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2
*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/
(2*a-b))^(1/2)*b^4-368*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2
*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2
*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^4+56*((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))
^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+
sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2*b^2+9*((2*a+b*sin(2*d*x+2*c))/(2*a-b))
^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(
1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))
^(1/2)*b^4+3*b^4*sin(2*d*x+2*c)^4+28*a*b^3*sin(2*d*x+2*c)^3+44*a^2*b^2*sin(
2*d*x+2*c)^2-3*b^4*sin(2*d*x+2*c)^2-28*a*b^3*sin(2*d*x+2*c)-44*a^2*b^2)/b/c
os(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(-(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin
(d*x + c) - a^2)*sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \cos(c + d\,x) \sin(c + d\,x))^{5/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

3.574 $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=212

$$-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d}+\frac{2\sqrt{2}\,aE\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{3d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{(4a^2-b^2)\,F}{}$$

```
[Out] -1/12*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)-2/3*a*(sin(c+
1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2)*
(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d/((2*a+b*sin(2*d*x
+2*c))/(2*a+b))^(1/2)+1/12*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/
4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*((2*a+b*si
n(2*d*x+2*c))/(2*a+b))^(1/2)/d*2^(1/2)/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2745, 2735, 2831, 2742, 2740, 2734, 2732}

$$-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{6\sqrt{2}\,d\sqrt{2a+b\sin(2c+2dx)}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d}+\frac{2\sqrt{2}\,a\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{3d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(Sqrt[2]*d) + (2*S
qrt[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c +
2*d*x]])/(3*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*
EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/
(2*a + b)])/(6*Sqrt[2]*d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2} dx$$
$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{2}{3}\int \frac{\frac{1}{8}(12a^2+b^2)+ab}{\sqrt{a+\frac{1}{2}b\sin(2}}$$
$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{1}{3}(4a)\int \sqrt{a+\frac{1}{2}b\sin(}$$
$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{\left(4a\sqrt{a+\frac{1}{2}b\sin(2c+}\right.}{3\sqrt{}}$$
$$= -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}\,d} + \frac{2\sqrt{2}\,aE\left(c-\frac{\pi}{4}+dx\middle|\frac{}{2a}\right.}{3d\sqrt{\frac{2a+}{}}}$$

Mathematica [**A**]
time = 1.05, size = 167, normalized size = 0.79

$$\frac{-b\cos(2(c+dx))(2a+b\sin(2(c+dx)))+8a(2a+b)E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}-(4a^2-b^2)\,F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}{6d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]*(2*a + b*Sin[2*(c + d*x)])) + 8*a*(2*a + b)*EllipticE
[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)
] - (4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*
Sin[2*(c + d*x)])/(2*a + b)])/(6*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(246) = 492$.
time = 0.42, size = 844, normalized size = 3.98

method	result

default	$\frac{24a^3\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)+4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(24*a^3*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(
2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+4*((2*a+b*sin(2*d*x+2*c)
)/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/
(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*
b/(2*a-b))^(1/2)*a^2*b-6*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*
x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(
((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2*a-((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(
1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b
))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-32*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(
1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/
2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(
1/2)*a^3+8*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a*b^2+b^3*sin(2*d*x+2*c)^
3+2*a*b^2*sin(2*d*x+2*c)^2-b^3*sin(2*d*x+2*c)-2*a*b^2)/b/cos(2*d*x+2*c)/(4*
a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin (c + dx)\cos (c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\cos \left(c+d\,x\right)\,\sin \left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

3.575 $\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] -1/2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d
*x),2^(1/2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a
+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2745, 2734, 2732}

$$\frac{\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
/(Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx &= \int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\,dx\\ &= \frac{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\int \sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}\,dx}{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}}\\ &= \frac{E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{\sqrt{2}\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 0.99

$$\frac{(2a+b)E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}{d\sqrt{4a+2b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] ((2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(
c + d*x)])/(2*a + b)])/(d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(99) = 198$.
time = 0.34, size = 312, normalized size = 4.11

method	result
default	$-\frac{\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,(2a-b)\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right.\right.}{b\cos(2dx+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/
2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)/b*(2*a-b)*(2*EllipticE(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a+EllipticE(((2*a+b*si
```

```
n(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b-2*a*EllipticF(((2*a
+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-EllipticF(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b)/cos(2*d*x+2*c)
/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin (c + dx) \cos (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \cos(c + dx) \sin(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
[Out] int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

3.576 $\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{2}\,F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{d\sqrt{2a+b\sin(2c+2dx)}}$$

```
[Out] -(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),
2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d
/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2745, 2742, 2740}

$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c +
 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}\,dx$$

$$= \frac{\sqrt{\frac{a+\frac{1}{2}b\sin(2c+2dx)}{a+\frac{b}{2}}}\int \frac{1}{\sqrt{\frac{a}{a+\frac{b}{2}}+\frac{b\sin(2c+2dx)}{2\left(a+\frac{b}{2}\right)}}}\,dx}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}$$

$$= \frac{\sqrt{2}\,F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{d\sqrt{2a+b\sin(2c+2dx)}}$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 0.92

$$\frac{F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}{d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
[Out] (EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])
/(2*a + b)])/(d*Sqrt[a + (b*Sin[2*(c + d*x)])/2])
```

Maple [A]
time = 0.28, size = 165, normalized size = 2.17

method	result
default	$\frac{2(2a-b)\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)}{b\cos(2dx+2c)\sqrt{4a+2b\sin(2dx+2c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(2*a-b)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*
a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d
```

```
*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))/b/cos(2*d*x+2*c)/(4*a+2*b*
sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.49, size = 302, normalized size = 3.97

$$-\frac{\left(b\sqrt{-\frac{4a^2-b^2}{b^2}}-2ia\right)\sqrt{4ib}\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}+2ia}{b}}\,\mathrm{ellipticF}\left(\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}+2ia}{b}}\left(\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right),\frac{4iab\sqrt{-\frac{4a^2-b^2}{b^2}}+8a^2-b^2}{b^2}\right)+\left(\sqrt{-4ib}\,b\sqrt{-\frac{4a^2-b^2}{b^2}}+2ia\sqrt{-4ib}\right)\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}-2ia}{b}}\,\mathrm{ellipticF}\left(\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}-2ia}{b}}\left(\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right),\frac{-4iab\sqrt{-\frac{4a^2-b^2}{b^2}}+8a^2-b^2}{b^2}\right)}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)*sqrt(4*I*b)*sqrt((b*sqrt(-(4*a^2
 - b^2)/b^2) + 2*I*a)/b)*ellipticF(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a
)/b)*(cos(d*x + c) + I*sin(d*x + c)), (4*I*a*b*sqrt(-(4*a^2 - b^2)/b^2) + 8
*a^2 - b^2)/b^2) + (sqrt(-4*I*b)*b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a*sqrt(-4
*I*b))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*ellipticF(sqrt((b*sqrt(
-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*(cos(d*x + c) - I*sin(d*x + c)), (-4*I*a*b*
sqrt(-(4*a^2 - b^2)/b^2) + 8*a^2 - b^2)/b^2))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin\left(c + dx\right)\cos\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

3.577 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2\sqrt{2}\,b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{(4a^2-b^2)\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

```
[Out] 2*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(1/2)-2*(si
n(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1
/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4*a^2-b^2)/d/(
(2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2745, 2743, 21, 2734, 2732}

$$\frac{2\sqrt{2}\,b\cos(2c+2dx)}{d\,(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}\middle|\frac{2b}{2a+b}\right)}{d\,(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[2]*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a + b*Sin[2*c + 2*d*
x]]) + (2*Sqrt[2]*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*S
in[2*c + 2*d*x]])/((4*a^2 - b^2)*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b
)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,dx$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}} - \frac{8\int \frac{-\frac{a}{2}-\frac{1}{4}b\sin(2c+2dx)}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}} + \frac{4\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}{4a^2-b^2}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}} + \frac{\left(4\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}\right.}{(4a^2-b^2)\sqrt{}}$$

$$= \frac{2\sqrt{2}\,b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}} + \frac{2\sqrt{2}\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)}{(4a^2-b^2)\,d\sqrt{\frac{2a+b}{}}}$$

Mathematica [A]

time = 0.31, size = 101, normalized size = 0.71

$$\frac{2\left(b\cos(2(c+dx))+(2a+b)E\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}\right)}{(4a^2-b^2)\,d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

```
[Out] (2*(b*Cos[2*(c + d*x)] + (2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b
)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)]))/((4*a^2 - b^2)*d*Sqrt[a + (
b*Sin[2*(c + d*x)])/2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(161) = 322$.
time = 0.38, size = 570, normalized size = 3.99

method	result
default	$\frac{16a^2\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\text{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)-4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*(4*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2
*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*
d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-((2*a+b*sin(2*d*x+2*c))/(
2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*
a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(
2*a-b))^(1/2)*b^2-4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)
-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2+((2*a+b*sin(
2*d*x+2*c))/(2*a-b))^(1/2)*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)
,((2*a-b)/(2*a+b))^(1/2))*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*
d*x+2*c))*b/(2*a-b))^(1/2)*b^2-b^2*sin(2*d*x+2*c)^2+b^2)/(4*a^2-b^2)/cos(2*
d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin\left(c + dx\right)\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

3.578 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=295

$$\frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\left(4a^2-b^2\right)d(2a+b\sin(2c+2dx))^{3/2}}+\frac{32\sqrt{2}\,ab\cos(2c+2dx)}{3\left(4a^2-b^2\right)^2d\sqrt{2a+b\sin(2c+2dx)}}+\frac{32\sqrt{2}\,aE\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right.}{3\left(4a^2-b^2\right)^2d\sqrt{\frac{2a}{}}}$$

```
[Out] 4/3*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(3/2)+32/
3*a*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))^(1/2)-3
2/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+
d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4*a^2
-b^2)^2/d/((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)+4/3*(sin(c+1/4*Pi+d*x)^2)^
(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/
2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin
(2*d*x+2*c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2745, 2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\frac{32\sqrt{2}\,ab\cos(2c+2dx)}{3d\left(4a^2-b^2\right)^2\sqrt{2a+b\sin(2c+2dx)}}+\frac{4\sqrt{2}\,b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}-\frac{4\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,F\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{3d\left(4a^2-b^2\right)\sqrt{2a+b\sin(2c+2dx)}}+\frac{32\sqrt{2}\,a\sqrt{2a+b\sin(2c+2dx)}\,E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{3d\left(4a^2-b^2\right)^2\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (4*Sqrt[2]*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x]
)^(3/2)) + (32*Sqrt[2]*a*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)^2*d*Sqrt[2*a
+ b*Sin[2*c + 2*d*x]]) + (32*Sqrt[2]*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a
 + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(3*(4*a^2 - b^2)^2*d*Sqrt[(2*a + b*S
in[2*c + 2*d*x])/(2*a + b)]) - (4*Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(
2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(3*(4*a^2 - b^2)*d*Sq
rt[2*a + b*Sin[2*c + 2*d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2745

```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}}\,dx$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\,(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} - \frac{8\int \frac{-\frac{3a}{2}+\frac{1}{4}b\sin(2c+2dx)}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}\,d}{3\,(4a^2-b^2)}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\,(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c}{3\,(4a^2-b^2)^2\,d\sqrt{2a+b}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\,(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c}{3\,(4a^2-b^2)^2\,d\sqrt{2a+b}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\,(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c}{3\,(4a^2-b^2)^2\,d\sqrt{2a+b}}$$

$$= \frac{4\sqrt{2}\,b\cos(2c+2dx)}{3\,(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} + \frac{32\sqrt{2}\,ab\cos(2c}{3\,(4a^2-b^2)^2\,d\sqrt{2a+b}}$$

Mathematica [A]
time = 1.06, size = 201, normalized size = 0.68

$$-\frac{4\sqrt{2}\left(-\frac{8aE\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)(2a+b\sin(2(c+dx)))^2}{\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}+(2a-b)(2a+b)^2F\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\left(\frac{2a+b\sin(2(c+dx))}{2a+b}\right)^{3/2}+b\cos(2(c+dx))\left(-20a^2+b^2-8ab\sin(2(c+dx))\right)\right)}{3\left(-4a^2+b^2\right)^2 d(2a+b\sin(2(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

```
[Out] (-4*Sqrt[2]*((-8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*(2*a + b*Sin[
2*(c + d*x)])^2)/Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] + (2*a - b)*(2*
a + b)^2*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*((2*a + b*Sin[2*(c + d*
x)])/(2*a + b))^(3/2) + b*Cos[2*(c + d*x)]*(-20*a^2 + b^2 - 8*a*b*Sin[2*(c
+ d*x)])))/(3*(-4*a^2 + b^2)^2*d*(2*a + b*Sin[2*(c + d*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1552 vs. $2(323) = 646$.
time = 0.56, size = 1553, normalized size = 5.26

method	result	size
default	Expression too large to display	1553

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3*(-8*a*b^3*cos(2*d*x+2*c)^2*sin(2*d*x+2*c)+(-20*a^2*b^2+b^4)*cos(2*d*x+
2*c)^2-(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2
*c)-b/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*b*(24*Elli
pticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))
*a^3+4*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a
+b))^(1/2))*a^2*b-6*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),
((2*a-b)/(2*a+b))^(1/2))*a*b^2-EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a
-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-32*EllipticE((b/(2*a-b)*sin(2*d*x+2
*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^3+8*EllipticE((b/(2*a-b)*
sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a*b^2)*sin(2*d*x
+2*c)+64*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2
*a+b))^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2
*d*x+2*c)-b/(2*a-b))^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*a^4
-16*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b)
)^(1/2))*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+
2*c)-b/(2*a-b))^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*a^2*b^2-
48*(-b/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b
/(2*a-b))^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(
2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^4-8*(-b
/(2*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-
b))^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)
*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^3*b+12*(-b/(2
*a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))
^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*si
n(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b^2+2*(-b/(2*a
+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(
1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*EllipticF((b/(2*a-b)*sin(
2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a*b^3)/(2*a+b*sin(2*
d*x+2*c))/(4*a^2-b^2)^2/b/cos(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\sin\left(c + dx\right)\cos\left(c + dx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)*cos(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
```

```
[Out] int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

3.579 $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=461

$$-\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{3x^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{PolyL}}{}$$

```
[Out] -I*x^3*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^3
*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*x^2*pol
ylog(2,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2*x^2*po
lylog(2,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*I*x*p
olylog(3,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2*I*x*
polylog(3,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/4*pol
ylog(4,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/4*polylo
g(4,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4680, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{3ix\text{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\text{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3\text{Li}_4\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}-\frac{3\text{Li}_4\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}-\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
 - b^2] + (I*x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2]
)])/(2*Sqrt[4*a^2 - b^2]) + (3*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt
[4*a^2 - b^2])])/(2*Sqrt[4*a^2 - b^2]) - (((3*I)/2)*x*PolyLog[3, (I*b*E^((2
*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (((3*I)/2)*x*PolyLo
g[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2] + (3*P
olyLog[4, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*a^2 - b^2
]) - (3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(4*Sqrt[4*
a^2 - b^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4680

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx &= \int \frac{x^3}{a+\frac{1}{2}b\sin(2x)}\,dx\\
&= 2\int \frac{e^{2ix}x^3}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx\\
&= -\frac{(2ib)\int \frac{e^{2ix}x^3}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}+\frac{(2ib)\int \frac{e^{2ix}x^3}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{(3i)\int x^2\log\left(\right.}{}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{3x^2\mathrm{Li}_2\left(\frac{}{2a-\surd}\right.}{2\sqrt{4a^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{3x^2\mathrm{Li}_2\left(\frac{}{2a-\surd}\right.}{2\sqrt{4a^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{3x^2\mathrm{Li}_2\left(\frac{}{2a-\surd}\right.}{2\sqrt{4a^2}}\\
&= -\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{3x^2\mathrm{Li}_2\left(\frac{}{2a-\surd}\right.}{2\sqrt{4a^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.53, size = 340, normalized size = 0.74

$$
\frac{-4ix^3\log\left(1+\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+4ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-6x^2\mathrm{PolyLog}\left(2,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+6x^2\mathrm{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-6ix\mathrm{PolyLog}\left(3,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+6ix\mathrm{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)+3\mathrm{PolyLog}\left(4,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)-3\mathrm{PolyLog}\left(4,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-4*I)*x^3*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (4*I)*x
^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - 6*x^2*PolyLog[2,
```

```
((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + 6*x^2*PolyLog[2, (I*b*E^
((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (6*I)*x*PolyLog[3, ((-I)*b*E^((2*I)
*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (6*I)*x*PolyLog[3, (I*b*E^((2*I)*x))/(2*
a + Sqrt[4*a^2 - b^2])] + 3*PolyLog[4, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*
a^2 - b^2])] - 3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/(
4*Sqrt[4*a^2 - b^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2281 vs. 2(389) = 778.
time = 0.21, size = 2282, normalized size = 4.95

method	result	size
risch	Expression too large to display	2282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a
+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+
b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^3-6/(
8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I
*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-6*I/(8*a^2-2*b
^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a
-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x-4*I/(8*a^2-2*b^2)/(-2*I*a
-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/
2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^3+4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*
a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)
*(2*a+b))^(1/2)*a*x^3+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*p
olylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))
^(1/2)*a*x+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I
*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*
b^2*x^3-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b)
)^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,
b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-3/(8*a^2-2*b^2)/(-2*I
*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+
b))^(1/2)))*b^2*x+3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog
(4,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)
*a+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^4-3/2*I/(8*a^2-2
*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2)))*b^2+6/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)
)*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+
b))^(1/2)*a*x^2+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)
*(2*a+b))^(1/2)*a*x^4+12/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*po
lylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-3/(8*a^2-2*b^
```

```
2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-
b)*(2*a+b))^(1/2)))*b^2*x-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))
*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b
))^(1/2)*a+I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^4-4*I/(8
*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2
*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*
a+b))^(1/2)))*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*pol
ylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2-4*I/(8*a^2-2*b^2
)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*
a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2
)))*a^2-12*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*ex
p(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x^2+3*I/(8*a^2-2*b^2)/(-2*I
*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+
b))^(1/2)))*b^2*x^2+3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*pol
ylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2-12*I/(8*a^2-
2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(
2*a-b)*(2*a+b))^(1/2)))*a^2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3308 vs. 2(367) = 734.
time = 3.72, size = 3308, normalized size = 7.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b*
cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/
b^2) + 2*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos
(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((
b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^3*sqrt(-(4*a^2 - b^2)/
b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^
2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^
3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) +
I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2
```

```
*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(
4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(
-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)
/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) - b*x^3*sqrt(-(
4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x)
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) -
 b)/b) - b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) - 2*a*sin(x) -
 (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
b^2)/b^2) - 2*I*a)/b) - b)/b) + 3*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((
2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2
))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + 3*I*b*x^2*sqr
t(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*
sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)
/b) - b)/b + 1) + 3*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) -
 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sq
rt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + 3*I*b*x^2*sqrt(-(4*a^2 - b
^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b +
 1) - 3*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x)
+ (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 -
b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 3*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilo
g(((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2
)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 3*I*b*x^
2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) -
I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2
*I*a)/b) - b)/b + 1) - 3*I*b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*co
s(x) - 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(
-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) + 6*b*x*sqrt(-(4*a^2 -
 b^2)/b^2)*polylog(3, -(2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))
*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b)
+ 6*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) - 2*a*sin(x) +
(b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^
2)/b^2) + 2*I*a)/b)/b) - 6*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*
cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqr
t(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 6*b*x*sqrt(-(4*a^2 - b^2)/b
^2)*polylog(3, -(-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 6*b
*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) + (b*cos
(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2
) - 2*I*a)/b)/b) + 6*b*x*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x
) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*
sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 6*b*x*sqrt(-(4*a^2 - b^2)/b^2)*po
lylog(3, -(2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 6*b*x*sqrt
```

```
(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) -
I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2
*I*a)/b)/b) - 6*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(2*I*a*cos(x) + 2*
a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-
(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 6*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(
4, -(-2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) - 6*I*b*sqrt(-(4*a
^2 - b^2)/b^2)*polylog(4, -(2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin
(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b
)/b) - 6*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(-2*I*a*cos(x) + 2*a*sin(
x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^
2 - b^2)/b^2) + 2*I*a)/b)/b) + 6*I*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(4, -(
2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*cos(x)*sin(x)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a+b\,\cos\left(x\right)\,\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*cos(x)*sin(x)),x)
[Out] int(x^3/(a + b*cos(x)*sin(x)), x)
```

3.580 $\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=340

$$-\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{x\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{PolyLog}}{}$$

```
[Out] -I*x^2*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^2
*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-x*polylog(2
,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+x*polylog(2,I*b*
exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*I*polylog(3,I*b*e
xp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*I*polylog(3,I*b*ex
p(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4680, 3404, 2296, 2221, 2611, 2320, 6724}

$$-\frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{i\text{Li}_3\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\text{Li}_3\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2
- b^2] + (I*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[
4*a^2 - b^2] - (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/
Sqrt[4*a^2 - b^2] + (x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2
])])/Sqrt[4*a^2 - b^2] - ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*
a^2 - b^2])])/Sqrt[4*a^2 - b^2] + ((I/2)*PolyLog[3, (I*b*E^((2*I)*x))/(2*a
+ Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 - b^2]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4680

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{a+\frac{1}{2}b\sin(2x)}\,dx$$

$$= 2\int \frac{e^{2ix}x^2}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx$$

$$= -\frac{(2ib)\int \frac{e^{2ix}x^2}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} + \frac{(2ib)\int \frac{e^{2ix}x^2}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{(2i)\int x\log(}{}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\mathrm{Li}_2\left(\frac{i}{2a-\sqrt{4}}\right.}{\sqrt{4a^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\mathrm{Li}_2\left(\frac{i}{2a-\sqrt{4}}\right.}{\sqrt{4a^2}}$$

$$= -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\mathrm{Li}_2\left(\frac{i}{2a-\sqrt{4}}\right.}{\sqrt{4a^2}}$$

Mathematica [A]
time = 0.44, size = 256, normalized size = 0.75

$$-\frac{i\left(2x^2\log\left(1+\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)-2x^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-2ix\mathrm{PolyLog}\left(2,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+2ix\mathrm{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)+\mathrm{PolyLog}\left(3,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)-\mathrm{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)\right)}{2\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-1/2*I)*(2*x^2*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] - 2*
x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (2*I)*x*PolyLog[
2, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (2*I)*x*PolyLog[2, (I
*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] + PolyLog[3, ((-I)*b*E^((2*I)*x)
)/(-2*a + Sqrt[4*a^2 - b^2])] - PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*
a^2 - b^2])]))/Sqrt[4*a^2 - b^2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1781 vs. 2(290) = 580.
time = 0.15, size = 1782, normalized size = 5.24

method	result	size
risch	Expression too large to display	1782

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/
2)*a*x^3-2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a
+b))^(1/2)*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a+4/3*
I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^3-16/3*I/(8*a^2-2*b
^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^3+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*
a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a
^2*x^2-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/
(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2-4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b
)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*polylog(2,b*exp(2*I*x)/(-2*I*a-(
-(2*a-b)*(2*a+b))^(1/2)))*a*x-8*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^
(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x+4*I/
(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(
-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2+2*I/(8*a^2-2*b^2)/
(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*
(2*a+b))^(1/2)))*b^2*x+4/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*po
lylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2-1/(8*a^2-2*b^2)
/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)
*(2*a+b))^(1/2)))*b^2+8/3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(
-(2*a-b)*(2*a+b))^(1/2)*a*x^3+4/3*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b)
)^(1/2))*b^2*x^3+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylo
g(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2
)*a+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I
*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a
-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^
2*x^2-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(
-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)
*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*
(-(2*a-b)*(2*a+b))^(1/2)*a*x-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(
1/2))*(-(2*a-b)*(2*a+b))^(1/2)*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))
^(1/2)))*a*x^2-16/3*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x
^3-8*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*
x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x+4/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-
b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))
)*a^2-1/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I
*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2492 vs. 2(272) = 544.
time = 4.84, size = 2492, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b*
cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/
b^2) + 2*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos
(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((
b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^2*sqrt(-(4*a^2 - b^2)/
b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^
2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^
2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) +
I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2
*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(
4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(
-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)
/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) - b*x^2*sqrt(-(
4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x)
)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) -
 b)/b) - b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) - 2*a*sin(x) -
 (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 -
b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*
I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))
*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + 2*I*b*x*sqrt(-(
4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)
- b)/b + 1) + 2*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*s
in(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4
*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + 2*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)
*dilog(((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2
```

```
- b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - 2*
I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x
) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2)
- 2*I*a)/b) - b)/b + 1) - 2*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*c
os(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt
((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - 2*I*b*x*sqrt(-(4*a^2
 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sq
rt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/
b + 1) - 2*I*b*x*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) - 2*a*sin(x
) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2
 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) + 2*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog
(3, -(2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 2*b*sqrt(-(4*a^2
 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/
b) - 2*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) -
(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b
^2)/b^2) + 2*I*a)/b)/b) - 2*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*
cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqr
t(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)/b) + 2*b*sqrt(-(4*a^2 - b^2)/b^2
)*polylog(3, -(2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4
*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) + 2*b*sqr
t(-(4*a^2 - b^2)/b^2)*polylog(3, -(-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) +
 I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2
*I*a)/b)/b) - 2*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3, -(2*I*a*cos(x) + 2*a*
sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(
4*a^2 - b^2)/b^2) - 2*I*a)/b)/b) - 2*b*sqrt(-(4*a^2 - b^2)/b^2)*polylog(3,
-(-2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)/b))/(4*a^2 - b^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\sin{(x)}\cos{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x**2/(a + b*sin(x)*cos(x)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="giac")
[Out] integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{a + b\,\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*cos(x)*sin(x)),x)
[Out] int(x^2/(a + b*cos(x)*sin(x)), x)
```

3.581 $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

Optimal. Leaf size=225

$$-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}$$

```
[Out] -I*x*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x*ln(
1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*polylog(2,I
*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*polylog(2,I*b*
exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4680, 3404, 2296, 2221, 2317, 2438}

$$-\frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\text{Li}_2\left(\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}-\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{\sqrt{4a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((-I)*x*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/Sqrt[4*a^2 -
b^2] + (I*x*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])])/Sqrt[4*a^
2 - b^2] - PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/(2*Sqrt[
4*a^2 - b^2]) + PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]/(2*
Sqrt[4*a^2 - b^2])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4680

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{a+b\cos(x)\sin(x)}\,dx &= \int \frac{x}{a+\frac{1}{2}b\sin(2x)}\,dx \\
&= 2\int \frac{e^{2ix}x}{\frac{ib}{2}+2ae^{2ix}-\frac{1}{2}ibe^{4ix}}\,dx \\
&= -\frac{(2ib)\int \frac{e^{2ix}x}{2a-\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}}+\frac{(2ib)\int \frac{e^{2ix}x}{2a+\sqrt{4a^2-b^2}-ibe^{2ix}}\,dx}{\sqrt{4a^2-b^2}} \\
&= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\ldots \\
&= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\ldots \\
&= -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}-\ldots
\end{aligned}
$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 788 vs. $2(225) = 450$.
time = 0.99, size = 788, normalized size = 3.50

$$\frac{1}{2}\left(\frac{\pi \text{ArcTan}\left(\frac{b+2a\tan(x)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{2\text{ArcCos}\left(-\frac{2a}{b}\right)\tanh^{-1}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)+(\pi-4x)\tanh^{-1}\left(\frac{(2a+b)\tan\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)-\left(\text{ArcCos}\left(-\frac{2a}{b}\right)+2i\tanh^{-1}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)\right)\log\left(\frac{(2a+b)\left(-2a+b-i\sqrt{-4a^2+b^2}\right)\left(1+i\cot\left(\frac{\pi}{4}+x\right)\right)}{b\left(2a+b+\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}\right)-\left(\text{ArcCos}\left(-\frac{2a}{b}\right)-2i\tanh^{-1}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)\right)\log\left(\frac{(2a+b)\left(2ia-ib+\sqrt{-4a^2+b^2}\right)\left(i+\cot\left(\frac{\pi}{4}+x\right)\right)}{b\left(2a+b+\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}\right)+\left(\text{ArcCos}\left(-\frac{2a}{b}\right)+2i\left(\tanh^{-1}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)+\tanh^{-1}\left(\frac{(2a+b)\tan\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)\right)\right)\log\left(\frac{\sqrt[4]{-1}\sqrt{-4a^2+b^2}e^{-ix}}{2\sqrt{b}\sqrt{a+b\cos(x)\sin(x)}}\right)+\left(\text{ArcCos}\left(-\frac{2a}{b}\right)-2i\tanh^{-1}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)-2i\tanh^{-1}\left(\frac{(2a+b)\tan\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)\right)\log\left(-\frac{(-1)^{3/4}\sqrt{-4a^2+b^2}e^{ix}}{2\sqrt{b}\sqrt{a+b\cos(x)\sin(x)}}\right)+i\left(\text{PolyLog}\left(2,\frac{\left(2a-i\sqrt{-4a^2+b^2}\right)\left(2a+b-\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}{b\left(2a+b+\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}\right)-\text{PolyLog}\left(2,\frac{\left(2a+i\sqrt{-4a^2+b^2}\right)\left(2a+b-\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}{b\left(2a+b+\sqrt{-4a^2+b^2}\cot\left(\frac{\pi}{4}+x\right)\right)}\right)\right)}{\sqrt{-4a^2+b^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Cos[x]*Sin[x]),x]
```

```
[Out] ((Pi*ArcTan[(b + 2*a*Tan[x])/Sqrt[4*a^2 - b^2]])/Sqrt[4*a^2 - b^2] + (2*Arc
Cos[(-2*a)/b]*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + (Pi -
 4*x)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (ArcCos[(-2*a
)/b] + (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2
*a + b)*(-2*a + b - I*Sqrt[-4*a^2 + b^2])*(1 + I*Cot[Pi/4 + x]))/(b*(2*a +
b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - (ArcCos[(-2*a)/b] - (2*I)*ArcTanh
[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2*a + b)*((2*I)*a - I
*b + Sqrt[-4*a^2 + b^2])*(I + Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b
^2]*Cot[Pi/4 + x]))] + (ArcCos[(-2*a)/b] + (2*I)*(ArcTanh[((2*a - b)*Cot[Pi
/4 + x])/Sqrt[-4*a^2 + b^2]] + ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^
2 + b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 + b^2])/(2*Sqrt[b]*E^(I*x)*Sqrt[a +
 b*Cos[x]*Sin[x]])] + (ArcCos[(-2*a)/b] - (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4
 + x])/Sqrt[-4*a^2 + b^2]] - (2*I)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-
4*a^2 + b^2]])*Log[-1/2*((-1)^(3/4)*Sqrt[-4*a^2 + b^2]*E^(I*x))/(Sqrt[b]*Sq
rt[a + b*Cos[x]*Sin[x]])] + I*(PolyLog[2, ((2*a - I*Sqrt[-4*a^2 + b^2])*(2*
a + b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]
*Cot[Pi/4 + x]))] - PolyLog[2, ((2*a + I*Sqrt[-4*a^2 + b^2])*(2*a + b - Sqr
t[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 +
 x]))]))/Sqrt[-4*a^2 + b^2])/2
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(191) = 382$.
time = 0.15, size = 1284, normalized size = 5.71

method	result	size
risch	Expression too large to display	1284

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-
2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a
+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x+2/(8
*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*
a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a+4/(8*a^2-2*b^2)/(-2
*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-4/(8*a^2-2*b^
```

```
2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-4*I/(8*
a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x+I/(8*a^2-2*b^2)/(-2*I*a
-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b)
)^(1/2)))*b^2+2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*x^2*b^2-8
*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^2-4*I/(8*a^2-2*b^2
)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b
)*(2*a+b))^(1/2)))*a^2-2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln
(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-2/(8*a^2-2*b^2)/(-
2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2
*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)
*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x
-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*
a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a
+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2+4
*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*
a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x-8*I/(8*a^2-2*b^2)
/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^2+2*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a
-b)*(2*a+b))^(1/2))*x^2*b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1676 vs. $2(179) = 358$.
time = 4.75, size = 1676, normalized size = 7.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
[Out] -1/2*(b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b*co
s(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^
2) + 2*I*a)/b) - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x)
- 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sq
rt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x*sqrt(-(4*a^2 - b^2)/b^2)*l
og(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^
2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x*sqrt(-
```

```
(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) + I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)
 - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) +
(b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^
2)/b^2) - 2*I*a)/b) - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*co
s(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(
(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) - b*x*sqrt(-(4*a^2 - b^2)/b
^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2
 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) - b*x*s
qrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) - I*b
*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*
a)/b) - b)/b) + I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin
(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^
2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(
((-2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) + I*b*sqrt(-
(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(
x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)
 - b)/b + 1) + I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin
(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a
^2 - b^2)/b^2) + 2*I*a)/b) - b)/b + 1) - I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog
(((2*I*a*cos(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - I*b*sqrt(-
(4*a^2 - b^2)/b^2)*dilog(((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin
(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)
 - b)/b + 1) - I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(
x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^
2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1) - I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(
((-2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/
b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b + 1))/(4*a^2 - b
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \sin (x) \cos (x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x)
```

```
[Out] Integral(x/(a + b*sin(x)*cos(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*cos(x)*sin(x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a + b\cos(x)\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*cos(x)*sin(x)),x)
```

```
[Out] int(x/(a + b*cos(x)*sin(x)), x)
```

3.582 $\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$

Optimal. Leaf size=20

$$\text{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+1/2*b*sin(2*x)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*Cos[x]*Sin[x])),x]
[Out] Defer[Int][1/(x*(a + (b*Sin[2*x])/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}\,dx$$

Mathematica [A]
time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*Cos[x]*Sin[x])),x]
[Out] Integrate[1/(x*(a + b*Cos[x]*Sin[x])), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(a+b\cos(x)\sin(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*cos(x)*sin(x)),x)
[Out] int(1/x/(a+b*cos(x)*sin(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
[Out] integral(1/(b*x*cos(x)*sin(x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\sin\left(x\right)\cos\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x)
[Out] Integral(1/(x*(a + b*sin(x)*cos(x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
[Out] integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\left(a+b\,\cos\left(x\right)\,\sin\left(x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*cos(x)*sin(x))),x)
[Out] int(1/(x*(a + b*cos(x)*sin(x))), x)
```

3.583 $\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$

Optimal. Leaf size=79

$$\frac{b(bx)^{1-n}\sin^{-1+n}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}+\frac{b^2(1-n)\mathrm{Int}\left((bx)^{-n}\sin^{-2+n}(ax),x\right)}{a^2c^2}$$

```
[Out] b*(b*x)^(1-n)*sin(a*x)^(-1+n)/a^2/(a*c^2*x*cos(a*x)-c^2*sin(a*x))+b^2*(1-n)
*Unintegrable(sin(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] (b*(b*x)^(1 - n)*Sin[a*x]^(-1 + n))/(a^2*(a*c^2*x*Cos[a*x] - c^2*Sin[a*x]))
 + (b^2*(1 - n)*Defer[Int][Sin[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx=\frac{b(bx)^{1-n}\sin^{-1+n}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int (bx)^{-n}\sin^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A]
time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2, x]
```

Maple [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\left(\sin^n(ax)\right)}{\left(acx\cos(ax)-c\sin(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(b*x)^(-n + 2)*sin(a*x)^n/(2*a*c^2*x*cos(a*x)*sin(a*x) - (a^2*c^2
*x^2 - c^2)*cos(a*x)^2 - c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*sin(a*x)**n/(a*c*x*cos(a*x)-c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a\,x)^n\,(b\,x)^{2-n}}{\left(c\,\sin(a\,x)-a\,c\,x\,\cos(a\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2,x)
[Out] int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2, x)
```

3.584 $\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{b(bx)^{1-n}\cos^{-1+n}(ax)}{a^2\left(c^2\cos(ax)+ac^2x\sin(ax)\right)}+\frac{b^2(1-n)\text{Int}\left((bx)^{-n}\cos^{-2+n}(ax),x\right)}{a^2c^2}$$

```
[Out] -b*(b*x)^(1-n)*cos(a*x)^(-1+n)/a^2/(c^2*cos(a*x)+a*c^2*x*sin(a*x))+b^2*(1-n
)*Unintegrable(cos(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] -((b*(b*x)^(1 - n)*Cos[a*x]^(-1 + n))/(a^2*(c^2*Cos[a*x] + a*c^2*x*Sin[a*x]
))) + (b^2*(1 - n)*Defer[Int][Cos[a*x]^(-2 + n)/(b*x)^n, x])/(a^2*c^2)
```

Rubi steps

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx=-\frac{b(bx)^{1-n}\cos^{-1+n}(ax)}{a^2\left(c^2\cos(ax)+ac^2x\sin(ax)\right)}+\frac{\left(b^2(1-n)\right)\int (bx)^{-n}\cos^{-2+n}(ax)\,dx}{a^2c^2}$$

Mathematica [A]
time = 3.15, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

```
[Out] Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2, x]
```

Maple [A]
time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{(bx)^{2-n}\left(\cos^n(ax)\right)}{\left(c\cos(ax)+acx\sin(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

```
[Out] int((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="maxima")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="fricas")
```

```
[Out] integral((b*x)^(-n + 2)*cos(a*x)^n/(a^2*c^2*x^2 + 2*a*c^2*x*cos(a*x)*sin(a*
x) - (a^2*c^2*x^2 - c^2)*cos(a*x)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**(2-n)*cos(a*x)**n/(c*cos(a*x)+a*c*x*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorithm
="giac")
```

```
[Out] integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a\,x\right)^n\,{\left(b\,x\right)}^{2-n}}{{\left(c\,\cos\left(a\,x\right)+a\,c\,x\,\sin\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2,x)
[Out] int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2, x)
```

3.585 $\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=175

$$\frac{a^2}{x}+\frac{a\cos(ax)\sin(ax)}{x^2}+\frac{\sin^2(ax)}{x^3}-\frac{10a^2\sin^2(ax)}{x}+\frac{\cos(ax)\sin^3(ax)}{ax^4}-\frac{8a\cos(ax)\sin^3(ax)}{3x^2}+\frac{\sin^4(ax)}{a^2x^5}-\frac{4\sin^4(}{3x^3}$$

```
[Out] a^2/x-2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)+a*cos(a*x)*sin(a*x)/x^2+sin(a*x)
^2/x^3-10*a^2*sin(a*x)^2/x+cos(a*x)*sin(a*x)^3/a/x^4-8/3*a*cos(a*x)*sin(a*x
)^3/x^2+sin(a*x)^4/a^2/x^5-4/3*sin(a*x)^4/x^3+32/3*a^2*sin(a*x)^4/x+sin(a*x
)^5/a^2/x^5/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]
time = 0.20, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4694, 3395, 30, 3394, 12, 3380}

$$-\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\sin^4(ax)}{a^2x^5}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}+\frac{a^2}{x}+\frac{32a^2\sin^4(ax)}{3x}-\frac{10a^2\sin^2(ax)}{x}+\frac{\sin^3(ax)\cos(ax)}{ax^4}-\frac{4\sin^4(ax)}{3x^3}+\frac{\sin^2(ax)}{x^3}-\frac{8a\sin^3(ax)\cos(ax)}{3x^2}+\frac{a\sin(ax)\cos(ax)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] a^2/x + (a*Cos[a*x]*Sin[a*x])/x^2 + Sin[a*x]^2/x^3 - (10*a^2*Sin[a*x]^2)/x
+ (Cos[a*x]*Sin[a*x]^3)/(a*x^4) - (8*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) + Sin[a
*x]^4/(a^2*x^5) - (4*Sin[a*x]^4)/(3*x^3) + (32*a^2*Sin[a*x]^4)/(3*x) + Sin[
a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
```

```
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4694

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - \frac{5\int \frac{\sin^4(ax)}{x^6}\,dx}{a^2}\\
&= \frac{\cos(ax)\sin^3(ax)}{ax^4} + \frac{\sin^4(ax)}{a^2x^5} + \frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))} - 3\int \frac{\sin^2(ax}{x^4}\\
&= \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} + \frac{\cos(ax)\sin^3(ax)}{ax^4} - \frac{8a\cos(ax)\sin^3(ax)}{3x^2} +\\
&= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} -\\
&= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} -\\
&= \frac{a^2}{x} + \frac{a\cos(ax)\sin(ax)}{x^2} + \frac{\sin^2(ax)}{x^3} - \frac{10a^2\sin^2(ax)}{x} + \frac{\cos(ax)\sin^3(ax)}{ax^4} -\end{aligned}$$

Mathematica [A]
time = 0.69, size = 198, normalized size = 1.13

$$\frac{8ax\cos(ax)-8a^3x^3\cos(ax)-12ax\cos(3ax)+24a^3x^3\cos(3ax)+4ax\cos(5ax)+32a^3x^3\cos(5ax)+10\sin(ax)-12a^2x^2\sin(ax)-5\sin(3ax)+44a^2x^2\sin(3ax)+\sin(5ax)-24a^2x^2\sin(5ax)-32a^3x^3(ax\cos(ax)-\sin(ax))\text{Si}(2ax)+256a^3x^3(ax\cos(ax)-\sin(ax))\text{Si}(4ax)}{48x^3(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (8*a*x*Cos[a*x] - 8*a^3*x^3*Cos[a*x] - 12*a*x*Cos[3*a*x] + 24*a^3*x^3*Cos[3
*a*x] + 4*a*x*Cos[5*a*x] + 32*a^3*x^3*Cos[5*a*x] + 10*Sin[a*x] - 12*a^2*x^2
*Sin[a*x] - 5*Sin[3*a*x] + 44*a^2*x^2*Sin[3*a*x] + Sin[5*a*x] - 24*a^2*x^2*
Sin[5*a*x] - 32*a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x] + 256*
a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(a*x*Cos[a*x]
 - Sin[a*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4 (ax\cos(ax) - \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.72, size = 186, normalized size = 1.06

$$\frac{4\,(8\,a^3x^3 + ax)\cos(ax)^5 - 2\,(17\,a^3x^3 + 4\,ax)\cos(ax)^3 + (16\,a^4x^4\,\mathrm{Si}(4\,ax) - 2\,a^4x^4\,\mathrm{Si}(2\,ax) + 5\,a^3x^3 + 4\,ax)\cos(ax) - (16\,a^3x^3\,\mathrm{Si}(4\,ax) - 2\,a^3x^3\,\mathrm{Si}(2\,ax) + (24\,a^2x^2 - 1)\cos(ax)^4 + 5\,a^2x^2 - (29\,a^2x^2 - 2)\cos(ax)^2 - 1)\sin(ax)}{3\,(ax^4\cos(ax) - x^3\sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*(8*a^3*x^3 + a*x)*cos(a*x)^5 - 2*(17*a^3*x^3 + 4*a*x)*cos(a*x)^3 + (
16*a^4*x^4*sin_integral(4*a*x) - 2*a^4*x^4*sin_integral(2*a*x) + 5*a^3*x^3
+ 4*a*x)*cos(a*x) - (16*a^3*x^3*sin_integral(4*a*x) - 2*a^3*x^3*sin_integra
l(2*a*x) + (24*a^2*x^2 - 1)*cos(a*x)^4 + 5*a^2*x^2 - (29*a^2*x^2 - 2)*cos(a
*x)^2 - 1)*sin(a*x))/(a*x^4*cos(a*x) - x^3*sin(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**6/x**4/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**6/(x**4*(a*x*cos(a*x) - sin(a*x))**2), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 7347, normalized size = 41.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x
)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan
(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)
^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*ta
n(2*a*x)^2*tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)
^2*tan(a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(
a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2
 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^8*x^8*sin_i
ntegral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(
4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*
x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))
*tan(2*a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*t
an(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*ta
n(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 +
32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^8
*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*i
mag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_
part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integ
ral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(a*
x)^2*tan(1/2*a*x)^2 + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^
2*tan(a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*
a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_integral(-2*a*x))*
tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integral(-4
```

```
*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*
x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*ta
n(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^
8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2 - 4*a^8*x^8*imag_part(cos
_integral(-2*a*x))*tan(2*a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)
)*tan(2*a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^8*x^8*si
n_integral(2*a*x)*tan(2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x))*
tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^8*x^
8*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_int
egral(-4*a*x))*tan(a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(a*x)^2 + 8*a
^8*x^8*sin_integral(2*a*x)*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(4
*a*x))*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(1/2*a*
x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 32*a^8*x^
8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(
4*a*x)*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 64*a
^6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^
2 - 8*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/
2*a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^
2*tan(1/2*a*x)^2 - 64*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x)^2 + 128*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*ta
n(a*x)^2*tan(1/2*a*x)^2 - 16*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x)^2 + 12*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*imag
_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^7*x^7*imag_part(
cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^7*x^7*imag_part(cos_in
tegral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*imag_part(cos_integr
al(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_integral(4*a*x)*tan
(2*a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/
2*a*x) + 64*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x)
- 8*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^7*
x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^7*x^7*im
ag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^7*x^7*sin_int
egral(4*a*x)*tan(a*x)^2*tan(1/2*a*x) - 16*a^7*x^7*sin_integral(2*a*x)*tan(a
*x)^2*tan(1/2*a*x) - 20*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x)^2 + 20*a^7*x^7*ta
n(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a*x)) + 4*a^8
*x^8*imag_part(cos_integral(2*a*x)) - 4*a^8*x^8*imag_part(cos_integral(-2*a
*x)) + 32*a^8*x^8*imag_part(cos_integral(-4*a*x)) - 64*a^8*x^8*sin_integral
(4*a*x) + 8*a^8*x^8*sin_integral(2*a*x) - 64*a^6*x^6*imag_part(cos_integral
(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 8*a^6*x^6*imag_part(cos_integral(2*a*x))
*tan(2*a*x)^2*tan(a*x)^2 - 8*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*
a*x)^2*tan(a*x)^2 + 64*a^6*x^6*imag_part(cos_in...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^6}{x^4 (\sin(ax) - ax \cos(ax))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.586 $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=131

$$\frac{a\cos(ax)}{x}+\frac{\sin(ax)}{x^2}+\frac{\cos(ax)\sin^2(ax)}{ax^3}-\frac{9a\cos(ax)\sin^2(ax)}{2x}+\frac{\sin^3(ax)}{a^2x^4}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}$$

```
[Out] a*cos(a*x)/x-1/8*a^2*Si(a*x)+27/8*a^2*Si(3*a*x)+sin(a*x)/x^2+cos(a*x)*sin(a
*x)^2/a/x^3-9/2*a*cos(a*x)*sin(a*x)^2/x+sin(a*x)^3/a^2/x^4-3/2*sin(a*x)^3/x
^2+sin(a*x)^4/a^2/x^4/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]
time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4694, 3395, 3378, 3380, 3393}

$$-\frac{1}{8}a^2\text{Si}(ax)+\frac{27}{8}a^2\text{Si}(3ax)+\frac{\sin^3(ax)}{a^2x^4}+\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}+\frac{\sin^2(ax)\cos(ax)}{ax^3}-\frac{3\sin^3(ax)}{2x^2}+\frac{\sin(ax)}{x^2}+\frac{a\cos(ax)}{x}-\frac{9a\sin^2(ax)\cos(ax)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (a*Cos[a*x])/x + Sin[a*x]/x^2 + (Cos[a*x]*Sin[a*x]^2)/(a*x^3) - (9*a*Cos[a*
x]*Sin[a*x]^2)/(2*x) + Sin[a*x]^3/(a^2*x^4) - (3*Sin[a*x]^3)/(2*x^2) + Sin[
a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (a^2*SinIntegral[a*x])/8 + (27
*a^2*SinIntegral[3*a*x])/8
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4694

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\int \frac{\sin^3(ax)}{x^5}\,dx}{a^2} \\
&= \frac{\cos(ax)\sin^2(ax)}{ax^3} + \frac{\sin^3(ax)}{a^2x^4} + \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - 2\int \frac{\sin(ax)}{x^3} \\
&= \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax)}{a^2x^4} - \frac{3\sin^3(ax}{2x^2} \\
&= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax}{a^2x^4} \\
&= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax}{a^2x^4} \\
&= \frac{a\cos(ax)}{x} + \frac{\sin(ax)}{x^2} + \frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x} + \frac{\sin^3(ax}{a^2x^4}
\end{aligned}$$

Mathematica [A]
time = 0.50, size = 142, normalized size = 1.08

$$\frac{3-a^2x^2-4\cos(2ax)+8a^2x^2\cos(2ax)+\cos(4ax)+9a^2x^2\cos(4ax)+12ax\sin(2ax)-6ax\sin(4ax)-2a^2x^2(ax\cos(ax)-\sin(ax))\text{Si}(ax)+54a^2x^2(ax\cos(ax)-\sin(ax))\text{Si}(3ax)}{16x^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (3 - a^2*x^2 - 4*Cos[2*a*x] + 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^2
*Cos[4*a*x] + 12*a*x*Sin[2*a*x] - 6*a*x*Sin[4*a*x] - 2*a^2*x^2*(a*x*Cos[a*x
```

```
] - Sin[a*x])*SinIntegral[a*x] + 54*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*SinIn
tegral[3*a*x])/(16*x^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^5(ax)}{x^3 (ax\cos(ax) - \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 3.20, size = 142, normalized size = 1.08

$$\frac{4\,(9\,a^2x^2+1)\cos(ax)^4 - 4\,(7\,a^2x^2+2)\cos(ax)^2 + (27\,a^3x^3\,\mathrm{Si}(3\,ax) - a^3x^3\,\mathrm{Si}(ax))\cos(ax) - (24\,ax\cos(ax)^3 + 27\,a^2x^2\,\mathrm{Si}(3\,ax) - a^2x^2\,\mathrm{Si}(ax) - 24\,ax\cos(ax))\sin(ax) + 4}{8\,(ax^3\cos(ax) - x^2\sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 4*(7*a^2*x^2 + 2)*cos(a*x)^2 + (27*a^3*
x^3*sin_integral(3*a*x) - a^3*x^3*sin_integral(a*x))*cos(a*x) - (24*a*x*cos
(a*x)^3 + 27*a^2*x^2*sin_integral(3*a*x) - a^2*x^2*sin_integral(a*x) - 24*a
*x*cos(a*x))*sin(a*x) + 4)/(a*x^3*cos(a*x) - x^2*sin(a*x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**5/x**3/(a*x*cos(a*x)-sin(a*x))**2,x)
```

[Out] Timed out

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 4175, normalized size = 31.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + a
^7*x^7*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^7
*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7
*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_inte
gral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^7*x^7*imag_part(cos_integral
(3*a*x))*tan(1/2*a*x)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(1/2*a*x)
^4 + a^7*x^7*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^7*x^7*imag
_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*sin_integral(3*a*x)
*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_integral(a*x)*tan(1/2*a*x)^4 + 54*a^6*x^6*i
mag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*a^6*x^6*ima
g_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_pa
rt(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^6*x^6*imag_part
(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^6*x^6*sin_inte
gral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan
(3/2*a*x)^2*tan(1/2*a*x)^3 - 16*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*
a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^7*x^7*imag_part(c
os_integral(a*x))*tan(3/2*a*x)^2 - a^7*x^7*imag_part(cos_integral(-a*x))*ta
n(3/2*a*x)^2 + 27*a^7*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 -
54*a^7*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^7*x^7*sin_integral(a*x)
*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*
tan(1/2*a*x)^4 - 2*a^5*x^5*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2
*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 4
*a^5*x^5*sin_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^6*x^6*imag_
part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 2*a^6*x^6*imag_part
(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^6*x^6*imag_part(cos_i
ntegral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integ
ral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*
tan(3/2*a*x)^2*tan(1/2*a*x) - 4*a^6*x^6*sin_integral(a*x)*tan(3/2*a*x)^2*ta
n(1/2*a*x) - 4*a^6*x^6*tan(3/2*a*x)^2*tan(1/2*a*x)^2 + 54*a^6*x^6*imag_part
(cos_integral(3*a*x))*tan(1/2*a*x)^3 - 2*a^6*x^6*imag_part(cos_integral(a*x
))*tan(1/2*a*x)^3 + 2*a^6*x^6*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3
- 54*a^6*x^6*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 108*a^6*x^6*s
```

```
in_integral(3*a*x)*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x
)^3 + 20*a^6*x^6*tan(1/2*a*x)^4 - 27*a^7*x^7*imag_part(cos_integral(3*a*x))
 + a^7*x^7*imag_part(cos_integral(a*x)) - a^7*x^7*imag_part(cos_integral(-a
*x)) + 27*a^7*x^7*imag_part(cos_integral(-3*a*x)) - 54*a^7*x^7*sin_integral
(3*a*x) + 2*a^7*x^7*sin_integral(a*x) - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a
*x)^3 + 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - 2*a^5*x^
5*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 + 2*a^5*x^5*imag_part(cos_int
egral(-a*x))*tan(1/2*a*x)^4 - 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*ta
n(1/2*a*x)^4 + 108*a^5*x^5*sin_integral(3*a*x)*tan(1/2*a*x)^4 - 4*a^5*x^5*s
in_integral(a*x)*tan(1/2*a*x)^4 + 12*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 +
20*a^6*x^6*tan(3/2*a*x)^2 + 54*a^6*x^6*imag_part(cos_integral(3*a*x))*tan(1
/2*a*x) - 2*a^6*x^6*imag_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^6*x^6*i
mag_part(cos_integral(-a*x))*tan(1/2*a*x) - 54*a^6*x^6*imag_part(cos_integr
al(-3*a*x))*tan(1/2*a*x) + 108*a^6*x^6*sin_integral(3*a*x)*tan(1/2*a*x) - 4
*a^6*x^6*sin_integral(a*x)*tan(1/2*a*x) - 4*a^6*x^6*tan(1/2*a*x)^2 + 108*a^
4*x^4*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*
x^4*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^4*x^4*
imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 108*a^4*x^4*i
mag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 216*a^4*x^4*
sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 8*a^4*x^4*sin_integral(
a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 32*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - 54*a^5*x^5*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a^5*x^5*i
mag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - 2*a^5*x^5*imag_part(cos_integr
al(-a*x))*tan(3/2*a*x)^2 + 54*a^5*x^5*imag_part(cos_integral(-3*a*x))*tan(3
/2*a*x)^2 - 108*a^5*x^5*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 4*a^5*x^5*sin_
integral(a*x)*tan(3/2*a*x)^2 - 36*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 36*
a^5*x^5*tan(1/2*a*x)^3 + 27*a^3*x^3*imag_part(cos_integral(3*a*x))*tan(3/2*
a*x)^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2
*tan(1/2*a*x)^4 + a^3*x^3*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 - 27*a^3*x^3*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 + 54*a^3*x^3*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
 2*a^3*x^3*sin_integral(a*x)*tan(3/2*a*x)^2*tan...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a\,x)^5}{x^3\,{\left(\sin (a\,x)-a\,x\,\cos (a\,x)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2),x)
[Out] int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.587 $\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{1}{x}+\frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}-\frac{2\sin^2(ax)}{x}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\mathrm{Si}(2ax)$$

```
[Out] 1/x+2*a*Si(2*a*x)+cos(a*x)*sin(a*x)/a/x^2+sin(a*x)^2/a^2/x^3-2*sin(a*x)^2/x
+sin(a*x)^3/a^2/x^3/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4694, 3395, 30, 3394, 12, 3380}

$$\frac{\sin^2(ax)}{a^2x^3}+\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\mathrm{Si}(2ax)+\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\sin^2(ax)}{x}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] x^(-1) + (Cos[a*x]*Sin[a*x])/(a*x^2) + Sin[a*x]^2/(a^2*x^3) - (2*Sin[a*x]^2
)/x + Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) + 2*a*SinIntegral[2*a*
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4694

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} - \frac{3\int \frac{\sin^2(ax)}{x^4}\,dx}{a^2}\\ &= \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))} + 2\int \frac{\sin^2(a}{x^2}\\ &= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(a}\\ &= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(a}\\ &= \frac{1}{x} + \frac{\cos(ax)\sin(ax)}{ax^2} + \frac{\sin^2(ax)}{a^2x^3} - \frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(a}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 77, normalized size = 0.96

$$\frac{2ax\cos(ax)+2ax\cos(3ax)+3\sin(ax)-\sin(3ax)+8ax(ax\cos(ax)-\sin(ax))\text{Si}(2ax)}{4x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (2*a*x*Cos[a*x] + 2*a*x*Cos[3*a*x] + 3*Sin[a*x] - Sin[3*a*x] + 8*a*x*(a*x*C
os[a*x] - Sin[a*x])*SinIntegral[2*a*x])/(4*x*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.39, size = 77, normalized size = 0.96

$$\frac{2\,ax\cos(ax)^3 + (2\,a^2x^2\,\mathrm{Si}(2\,ax) - ax)\cos(ax) - \left(2\,ax\,\mathrm{Si}(2\,ax) + \cos(ax)^2 - 1\right)\sin(ax)}{ax^2\cos(ax) - x\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*x*cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) - a*x)*cos(a*x) - (2*a*x
*sin_integral(2*a*x) + cos(a*x)^2 - 1)*sin(a*x))/(a*x^2*cos(a*x) - x*sin(a*
x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(ax)}{x^2 (ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**4/x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

[Out] Integral(sin(a*x)**4/(x**2*(a*x*cos(a*x) - sin(a*x))**2), x)

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 0.58, size = 1033, normalized size = 12.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")

[Out] (a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(2*a*x)) + a^4*x^4*imag_part(cos_integral(-2*a*x)) - 2*a^4*x^4*sin_integral(2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*tan(a*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x) + a^3*x^3*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2 + a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 - 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2 - 2*a^2*x^2*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(1/2*a*x)^2 + a^2*x^2*tan(a*x)*tan(1/2*a*x)^2 - a^3*x^3 + 2*a*x*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a*x*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^2*x^2*imag_part(cos_integral(2*a*x)) + a^2*x^2*imag_part(cos_integral(-2*a*x)) - 2*a^2*x^2*sin_integral(2*a*x) - a^2*x^2*tan(a*x) + 2*a^2*x^2*tan(1/2*a*x) + 2*a*x*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a*x*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a*x*sin_integral(2*a*x)*tan(1/2*a*x) + 2*a*x*tan(a*x)*tan(1/2*a*x) + a*x*tan(1/2*a*x)^2 - 2*tan(a*x)^2*tan(1/2*a*x) - a*x)/(a^3*x^4*tan(a*x)^2*tan(1/2*a*x)^2 - a^3*x^4*tan(a*x)^2 + a^3*x^4*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^4 + a*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^2*x^3*tan(1/2*a*x) - a*x^2*tan(a*x)^2 + a*x^2*tan(1/2*a*x)^2 + 2*x*tan(a*x)^2*tan(1/2*a*x) - a*x^2 + 2*x*tan(1/2*a*x))

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(ax)^4}{x^2(\sin(ax) - ax\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2),x)
[Out] int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.588 $\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=56

$$\frac{\cos(ax)}{ax}+\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)$$

```
[Out] cos(a*x)/a/x+Si(a*x)+sin(a*x)/a^2/x^2+sin(a*x)^2/a^2/x^2/(a*x*cos(a*x)-sin(
a*x))
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {4694, 3378, 3380}

$$\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)+\frac{\cos(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a*x) + Sin[a*x]/(a^2*x^2) + Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - S
in[a*x])) + SinIntegral[a*x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4694

```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[(b*
x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[
a*c + d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{2\int \frac{\sin(ax)}{x^3}\,dx}{a^2}$$
$$= \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} - \frac{\int \frac{\cos(ax)}{x^2}\,dx}{a}$$
$$= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \int \frac{\sin(ax)}{x}\,dx$$
$$= \frac{\cos(ax)}{ax} + \frac{\sin(ax)}{a^2x^2} + \frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))} + \text{Si}(ax)$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.75, size = 242, normalized size = 4.32

$$\frac{1+\cos(2ax)+iaex\cos(ax)\text{Ei}(-1-iax)-iaex\cos(ax)\text{Ei}(-1+iax)-ie\text{CosIntegral}(i-ax)(ax\cos(ax)-\sin(ax))+ie\text{CosIntegral}(i+ax)(ax\cos(ax)-\sin(ax))-ie\text{Ei}(-1-iax)\sin(ax)+ie\text{Ei}(-1+iax)\sin(ax)+2ax\cos(ax)\text{Si}(ax)-2\sin(ax)\text{Si}(ax)+aex\cos(ax)\text{Si}(i-ax)-e\sin(ax)\text{Si}(i-ax)-aex\cos(ax)\text{Si}(i+ax)+e\sin(ax)\text{Si}(i+ax)}{2ax\cos(ax)-2\sin(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

```
[Out] (1 + Cos[2*a*x] + I*a*E*x*Cos[a*x]*ExpIntegralEi[-1 - I*a*x] - I*a*E*x*Cos[
a*x]*ExpIntegralEi[-1 + I*a*x] - I*E*CosIntegral[I - a*x]*(a*x*Cos[a*x] - S
in[a*x]) + I*E*CosIntegral[I + a*x]*(a*x*Cos[a*x] - Sin[a*x]) - I*E*ExpInte
gralEi[-1 - I*a*x]*Sin[a*x] + I*E*ExpIntegralEi[-1 + I*a*x]*Sin[a*x] + 2*a*
x*Cos[a*x]*SinIntegral[a*x] - 2*Sin[a*x]*SinIntegral[a*x] + a*E*x*Cos[a*x]*
SinIntegral[I - a*x] - E*Sin[a*x]*SinIntegral[I - a*x] - a*E*x*Cos[a*x]*Sin
Integral[I + a*x] + E*Sin[a*x]*SinIntegral[I + a*x])/(2*a*x*Cos[a*x] - 2*Si
n[a*x])
```

Maple [C] Result contains complex when optimal does not.
time = 2.98, size = 108, normalized size = 1.93

method	result	size
risch	$\frac{i e^{iax}}{2iax-2}+\frac{i\,\text{expIntegral}(1,-iax)}{2}+\frac{i e^{-iax}}{2iax+2}-\frac{i\,\text{expIntegral}(1,iax)}{2}+\frac{2\,e^{iax}}{(ax+i)(ax-i)(ax\,e^{2iax}+i e^{2iax}+ax-i)}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*exp(I*a*x)/(-1+I*a*x)+1/2*I*Ei(1,-I*a*x)+1/2*I*exp(-I*a*x)/(I*a*x+1)-
1/2*I*Ei(1,I*a*x)+2*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)+I*exp(2*I*
a*x)+a*x-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 3.03, size = 45, normalized size = 0.80

$$\frac{ax\cos(ax)\,\mathrm{Si}(ax)+\cos(ax)^2-\sin(ax)\,\mathrm{Si}(ax)}{ax\cos(ax)-\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a*x*cos(a*x)*sin_integral(a*x) + cos(a*x)^2 - sin(a*x)*sin_integral(a*x))/
(a*x*cos(a*x) - sin(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^3(ax)}{x\,(ax\cos(ax)-\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**3/x/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(sin(a*x)**3/(x*(a*x*cos(a*x) - sin(a*x))**2), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 496, normalized size = 8.86

$$\frac{a^3x^3\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)^4-a^3x^3\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)^4+2a^3x^3\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)^4+2a^2x^2\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)^3-2a^2x^2\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)^3+4a^2x^2\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)^3-2a^2x^2\tan\left(\tfrac12ax\right)^4-a^3x^3\Im(\mathrm{Ci}(ax))+a^3x^3\Im(\mathrm{Ci}(-ax))-2a^3x^3\,\mathrm{Si}(ax)+ax\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)^4-ax\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)^4+2ax\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)^4+2a^2x^2\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)-2a^2x^2\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)+4a^2x^2\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)+4a^2x^2\tan\left(\tfrac12ax\right)^2-2a^2x^2+2\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)^3-2\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)^3+4\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)^3-4\tan\left(\tfrac12ax\right)^4-ax\Im(\mathrm{Ci}(ax))+ax\Im(\mathrm{Ci}(-ax))-2ax\,\mathrm{Si}(ax)+2\Im(\mathrm{Ci}(ax))\tan\left(\tfrac12ax\right)-2\Im(\mathrm{Ci}(-ax))\tan\left(\tfrac12ax\right)+4\,\mathrm{Si}(ax)\tan\left(\tfrac12ax\right)-4}{2\left(a^3x^3\tan\left(\tfrac12ax\right)^4+2a^2x^2\tan\left(\tfrac12ax\right)^3-a^3x^3+ax\tan\left(\tfrac12ax\right)^4+2a^2x^2\tan\left(\tfrac12ax\right)+2\tan\left(\tfrac12ax\right)^3-ax+2\tan\left(\tfrac12ax\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*x^3*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_par
t(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a^3*x^3*sin_integral(a*x)*tan(1/2*
a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*a^2*x^2*
imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*a^2*x^2*sin_integral(a*x)*
tan(1/2*a*x)^3 - 2*a^2*x^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_integral(
a*x)) + a^3*x^3*imag_part(cos_integral(-a*x)) - 2*a^3*x^3*sin_integral(a*x)
 + a*x*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a*x*imag_part(cos_inte
gral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*sin_integral(a*x)*tan(1/2*a*x)^4 + 2*a^2
*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_part(cos_in
```

```
tegral(-a*x))*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*a*x) + 4*a
^2*x^2*tan(1/2*a*x)^2 - 2*a^2*x^2 + 2*imag_part(cos_integral(a*x))*tan(1/2*
a*x)^3 - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*sin_integral(a*
x)*tan(1/2*a*x)^3 - 4*tan(1/2*a*x)^4 - a*x*imag_part(cos_integral(a*x)) + a
*x*imag_part(cos_integral(-a*x)) - 2*a*x*sin_integral(a*x) + 2*imag_part(co
s_integral(a*x))*tan(1/2*a*x) - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x
) + 4*sin_integral(a*x)*tan(1/2*a*x) - 4)/(a^3*x^3*tan(1/2*a*x)^4 + 2*a^2*x
^2*tan(1/2*a*x)^3 - a^3*x^3 + a*x*tan(1/2*a*x)^4 + 2*a^2*x^2*tan(1/2*a*x) +
 2*tan(1/2*a*x)^3 - a*x + 2*tan(1/2*a*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a\,x\right)^3}{x\,{\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2),x)
[Out] int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.589 $\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

[Out] 1/a^2/x+sin(a*x)/a^2/x/(a*x*cos(a*x)-sin(a*x))

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.043$, Rules used = {4692}

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

[In] Int[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] 1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))

Rule 4692

```
Int[Sin[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] + Simp[Sin[a*x]/(a*d*x*(d*x*Cos[a*x] + c
*Sin[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

Mathematica [A]
time = 0.22, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]

[Out] Cos[a*x]/(a^2*x*Cos[a*x] - a*Sin[a*x])

Maple [C] Result contains complex when optimal does not.
time = 1.16, size = 54, normalized size = 1.54

method	result	size
risch	$\frac{1}{a(ax+i)}+\frac{2i}{(ax+i)(ax\,\mathrm{e}^{2iax}+i\mathrm{e}^{2iax}+ax-i)a}$	54
norman	$\frac{\frac{\tan^4(\frac{ax}{2})}{a}+\frac{\tan^6(\frac{ax}{2})}{a}-\frac{1}{a}-\frac{\tan^2(\frac{ax}{2})}{a}}{(1+\tan^2(\frac{ax}{2}))^2(ax(\tan^2(\frac{ax}{2}))-ax+2\tan(\frac{ax}{2}))}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(a*x+I)+2*I/(a*x+I)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(35) = 70$.
time = 0.26, size = 114, normalized size = 3.26

$$\frac{ax\cos(2\,ax)^2+ax\sin(2\,ax)^2+2\,ax\cos(2\,ax)+ax-2\,\sin(2\,ax)}{(a^2x^2+(a^2x^2+1)\cos(2\,ax)^2-4\,ax\sin(2\,ax)+(a^2x^2+1)\sin(2\,ax)^2+2\,(a^2x^2-1)\cos(2\,ax)+1)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 + 2*a*x*cos(2*a*x) + a*x - 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

Fricas [A]
time = 2.59, size = 24, normalized size = 0.69

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] cos(a*x)/(a^2*x*cos(a*x) - a*sin(a*x))
```

Sympy [A]
time = 1.81, size = 20, normalized size = 0.57

$$\frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] cos(a*x)/(a**2*x*cos(a*x) - a*sin(a*x))
```

Giac [A]
time = 0.44, size = 39, normalized size = 1.11

$$\frac{\tan\left(\frac{1}{2}\,ax\right)^2-1}{a^2x\tan\left(\frac{1}{2}\,ax\right)^2-a^2x+2\,a\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (tan(1/2*a*x)^2 - 1)/(a^2*x*tan(1/2*a*x)^2 - a^2*x + 2*a*tan(1/2*a*x))
```

Mupad [B]
time = 3.03, size = 24, normalized size = 0.69

$$-\frac{\cos\left(a\,x\right)}{a\,\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

```
[Out] -cos(a*x)/(a*(sin(a*x) - a*x*cos(a*x)))
```

3.590 $\int \frac{x \sin(ax)}{(ax \cos(ax) - \sin(ax))^2} \, dx$

Optimal. Leaf size=20

$$\frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

```
[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6818}

$$\frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x \sin(ax)}{(ax \cos(ax) - \sin(ax))^2} \, dx = \frac{1}{a^2(ax \cos(ax) - \sin(ax))}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{a^2(-ax \cos(ax) + \sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] -(1/(a^2*(-(a*x*Cos[a*x]) + Sin[a*x])))
```

Maple [A]
time = 0.25, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$	21
default	$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$	21
risch	$\frac{2\,e^{iax}}{a^2(ax\,e^{2iax}+ie^{2iax}+ax-i)}$	38
norman	$\frac{-\frac{1}{a^2}-\frac{2\left(\tan^2\left(\frac{ax}{2}\right)\right)}{a^2}-\frac{\tan^4\left(\frac{ax}{2}\right)}{a^2}}{\left(1+\tan^2\left(\frac{ax}{2}\right)\right)\left(ax\left(\tan^2\left(\frac{ax}{2}\right)\right)-ax+2\tan\left(\frac{ax}{2}\right)\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(a*x*cos(a*x)-sin(a*x))
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.00

$$\frac{1}{(ax\cos(ax)-\sin(ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] 1/((a*x*cos(a*x) - sin(a*x))*a^2)
```

Fricas [A]
time = 2.28, size = 21, normalized size = 1.05

$$\frac{1}{a^3x\cos(ax)-a^2\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/(a^3*x*cos(a*x) - a^2*sin(a*x))
```

Sympy [A]
time = 1.78, size = 19, normalized size = 0.95

$$\frac{1}{a^3x\cos(ax)-a^2\sin(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] 1/(a**3*x*cos(a*x) - a**2*sin(a*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.
time = 0.42, size = 42, normalized size = 2.10

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2+1\right)}{a^3x\tan\left(\frac{1}{2}\,ax\right)^2-a^3x+2\,a^2\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
[Out] -2*(tan(1/2*a*x)^2 + 1)/(a^3*x*tan(1/2*a*x)^2 - a^3*x + 2*a^2*tan(1/2*a*x))
```

Mupad [B]
time = 0.12, size = 23, normalized size = 1.15

$$-\frac{1}{a^2\,\sin\left(a\,x\right)-a^3\,x\,\cos\left(a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a*x))/(sin(a*x) - a*x*cos(a*x))^2,x)
[Out] -1/(a^2*sin(a*x) - a^3*x*cos(a*x))
```

3.591 $\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\cot(ax)}{a^3}+\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] -cot(a*x)/a^3+x*csc(a*x)/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]

time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4690, 3852, 8}

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
[Out] -(Cot[a*x]/a^3) + (x*Csc[a*x])/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4690

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{\int \csc^2(ax)\,dx}{a^2}\\ &= \frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\text{Subst}\left(\int 1\,dx,x,\cot(ax)\right)}{a^3}\\ &= -\frac{\cot(ax)}{a^3}+\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 32, normalized size = 0.91

$$\frac{\cos(ax) + ax\sin(ax)}{a^3(ax\cos(ax) - \sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (Cos[a*x] + a*x*Sin[a*x])/(a^3*(a*x*Cos[a*x] - Sin[a*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.88, size = 39, normalized size = 1.11

method	result	size
risch	$\frac{2i(ax-i)}{a^3(ax\,\mathrm{e}^{2iax}+i\mathrm{e}^{2iax}+ax-i)}$	39
norman	$\frac{\frac{\tan^2\left(\frac{ax}{2}\right)}{a^3}-\frac{1}{a^3}-\frac{2x\tan\left(\frac{ax}{2}\right)}{a^2}}{ax\left(\tan^2\left(\frac{ax}{2}\right)\right)-ax+2\tan\left(\frac{ax}{2}\right)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*(a*x-I)/a^3/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(35) = 70$.
time = 0.27, size = 100, normalized size = 2.86

$$\frac{2\left(2\,ax\cos\left(2\,ax\right) + \left(a^2x^2 - 1\right)\sin\left(2\,ax\right)\right)}{\left(a^2x^2 + \left(a^2x^2 + 1\right)\cos\left(2\,ax\right)^2 - 4\,ax\sin\left(2\,ax\right) + \left(a^2x^2 + 1\right)\sin\left(2\,ax\right)^2 + 2\left(a^2x^2 - 1\right)\cos\left(2\,ax\right) + 1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*c
os(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2 -
1)*cos(2*a*x) + 1)*a^3)
```

Fricas [A]
time = 3.65, size = 34, normalized size = 0.97

$$\frac{ax\sin\left(ax\right) + \cos\left(ax\right)}{a^4x\cos\left(ax\right) - a^3\sin\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a*x*sin(a*x) + cos(a*x))/(a^4*x*cos(a*x) - a^3*sin(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax\cos{(ax)} - \sin{(ax)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**2/(a*x*cos(a*x) - sin(a*x))**2, x)
```

Giac [A]
time = 0.45, size = 53, normalized size = 1.51

$$-\frac{2\,ax\tan\left(\frac{1}{2}\,ax\right) - \tan\left(\frac{1}{2}\,ax\right)^2 + 1}{a^4x\tan\left(\frac{1}{2}\,ax\right)^2 - a^4x + 2\,a^3\tan\left(\frac{1}{2}\,ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*tan(1/2*a*x) - tan(1/2*a*x)^2 + 1)/(a^4*x*tan(1/2*a*x)^2 - a^4*x +
2*a^3*tan(1/2*a*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(\sin{(a\,x)} - a\,x\,\cos{(a\,x)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

```
[Out] int(x^2/(sin(a*x) - a*x*cos(a*x))^2, x)
```

3.592 $\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{\csc(ax)}{a^4}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{i\text{PolyLog}\left(2,-e^{iax}\right)}{a^4}-\frac{i\text{PolyLog}\left(2,e^{iax}\right)}{a^4}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

```
[Out] -2*x*arctanh(exp(I*a*x))/a^3-csc(a*x)/a^4-x*cot(a*x)*csc(a*x)/a^3+I*polylog
(2,-exp(I*a*x))/a^4-I*polylog(2,exp(I*a*x))/a^4+x^2*csc(a*x)^2/a^2/(a*x*cos
(a*x)-sin(a*x))
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4696, 4270, 4268, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{iax}\right)}{a^4}-\frac{i\text{Li}_2\left(e^{iax}\right)}{a^4}-\frac{\csc(ax)}{a^4}-\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3}-\frac{x\cot(ax)\csc(ax)}{a^3}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (-2*x*ArcTanh[E^(I*a*x)])/a^3 - Csc[a*x]/a^4 - (x*Cot[a*x]*Csc[a*x])/a^3 +
(I*PolyLog[2, -E^(I*a*x)])/a^4 - (I*PolyLog[2, E^(I*a*x)])/a^4 + (x^2*Csc[a
*x]^2)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4696

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Csc[a*x]^(n
+ 1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] + Dist[b^2*((n + 1)/d^2), Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{2\int x\csc^3(ax)\,dx}{a^2}$$
$$= -\frac{\csc(ax)}{a^4} - \frac{x\cot(ax)\csc(ax)}{a^3} + \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{\int x\csc(ax)\,d}{a^2}$$
$$= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x\cot(ax)\csc(ax)}{a^3} + \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(a}$$
$$= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x\cot(ax)\csc(ax)}{a^3} + \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(a}$$
$$= -\frac{2x\tanh^{-1}\left(e^{iax}\right)}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x\cot(ax)\csc(ax)}{a^3} + \frac{i\text{Li}_2(-e^{iax})}{a^4} - \frac{i\text{Li}_2(e^{i}}{a^4}$$

Mathematica [A]
time = 0.50, size = 157, normalized size = 1.51

$$\frac{\csc(ax)+a^2x^2\csc(ax)-ax\log\left(1-e^{iax}\right)+a^2x^2\cot(ax)\log\left(1-e^{iax}\right)+ax\log\left(1+e^{iax}\right)-a^2x^2\cot(ax)\log\left(1+e^{iax}\right)+i(-1+ax\cot(ax))\text{PolyLog}(2,-e^{iax})-i(-1+ax\cot(ax))\text{PolyLog}(2,e^{iax})}{a^4(-1+ax\cot(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (Csc[a*x] + a^2*x^2*Csc[a*x] - a*x*Log[1 - E^(I*a*x)] + a^2*x^2*Cot[a*x]*Lo
g[1 - E^(I*a*x)] + a*x*Log[1 + E^(I*a*x)] - a^2*x^2*Cot[a*x]*Log[1 + E^(I*a
*x)] + I*(-1 + a*x*Cot[a*x])*PolyLog[2, -E^(I*a*x)] - I*(-1 + a*x*Cot[a*x])
*PolyLog[2, E^(I*a*x)])/(a^4*(-1 + a*x*Cot[a*x]))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

```
[Out] int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(89) = 178.
time = 2.76, size = 295, normalized size = 2.84

$$\frac{2 a^2x^2 - (i ax\cos(ax) - i\sin(ax))\mathrm{Li}_2(\cos(ax) + i\sin(ax)) - (-i ax\cos(ax) + i\sin(ax))\mathrm{Li}_2(\cos(ax) - i\sin(ax)) - (i ax\cos(ax) - i\sin(ax))\mathrm{Li}_2(-\cos(ax) + i\sin(ax)) - (-i ax\cos(ax) + i\sin(ax))\mathrm{Li}_2(-\cos(ax) - i\sin(ax)) - (a^2x^2\cos(ax) - ax\sin(ax))\log(\cos(ax) + i\sin(ax) + 1) - (a^2x^2\cos(ax) - ax\sin(ax))\log(\cos(ax) - i\sin(ax) + 1) + (a^2x^2\cos(ax) - ax\sin(ax))\log(-\cos(ax) + i\sin(ax) + 1) + (a^2x^2\cos(ax) - ax\sin(ax))\log(-\cos(ax) - i\sin(ax) + 1) + 2}{2(a^5x\cos(ax) - a^4\sin(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2 - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(cos(a*x) + I*sin(a*x))
 - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(cos(a*x) - I*sin(a*x)) - (I*a*x*cos
(a*x) - I*sin(a*x))*dilog(-cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I*si
n(a*x))*dilog(-cos(a*x) - I*sin(a*x)) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*l
og(cos(a*x) + I*sin(a*x) + 1) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(cos(a
*x) - I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) + I
*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) - I*sin(a*
x) + 1) + 2)/(a^5*x*cos(a*x) - a^4*sin(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc(ax)}{(ax \cos(ax) - \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sin\left(a\,x\right)\,{\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.593 $\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{2i\text{PolyLog}(2,e^{2iax})}{a^5}+\frac{}{a^2}$$

```
[Out] -2*I*x^2/a^3-cot(a*x)/a^5-2*x^2*cot(a*x)/a^3-x*csc(a*x)^2/a^4-x^2*cot(a*x)*
csc(a*x)^2/a^3+4*x*ln(1-exp(2*I*a*x))/a^4-2*I*polylog(2,exp(2*I*a*x))/a^5+x
^3*csc(a*x)^3/a^2/(a*x*cos(a*x)-sin(a*x))
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {4696, 4271, 3852, 8, 4269, 3798, 2221, 2317, 2438}

$$-\frac{2i\text{Li}_2(e^{2iax})}{a^5}-\frac{\cot(ax)}{a^5}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4}-\frac{x\csc^2(ax)}{a^4}-\frac{2ix^2}{a^3}-\frac{2x^2\cot(ax)}{a^3}-\frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x^2)/a^3 - Cot[a*x]/a^5 - (2*x^2*Cot[a*x])/a^3 - (x*Csc[a*x]^2)/a^4
 - (x^2*Cot[a*x]*Csc[a*x]^2)/a^3 + (4*x*Log[1 - E^((2*I)*a*x)])/a^4 - ((2*I
)*PolyLog[2, E^((2*I)*a*x)])/a^5 + (x^3*Csc[a*x]^3)/(a^2*(a*x*Cos[a*x] - Si
n[a*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4696

```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Csc[a*x]^(n
+ 1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] + Dist[b^2*((n + 1)/d^2), Int[(
b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && Eq
Q[a*c + d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{3\int x^2\csc^4(ax)\,dx}{a^2}$$
$$= -\frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))} + \frac{\int \csc^2(ax}{a^4}$$
$$= -\frac{2x^2\cot(ax)}{a^3} - \frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax}$$
$$= -\frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2\cot(ax)}{a^3} - \frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{}{a^2(a}$$
$$= -\frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2\cot(ax)}{a^3} - \frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{4x\,\mathrm{lo}}{}$$
$$= -\frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2\cot(ax)}{a^3} - \frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{4x\,\mathrm{lo}}{}$$
$$= -\frac{2ix^2}{a^3} - \frac{\cot(ax)}{a^5} - \frac{2x^2\cot(ax)}{a^3} - \frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3} + \frac{4x\,\mathrm{lo}}{}$$

Mathematica [A]
time = 0.61, size = 102, normalized size = 0.80

$$\frac{\frac{1}{x} + a^2x - a^3x^2\cot(ax) + 4a^2x\log\left(1-e^{2iax}\right) - 2ia\left(a^2x^2 + \mathrm{PolyLog}\left(2,e^{2iax}\right)\right) + \frac{\left(1+a^2x^2\right)^2\sin(ax)}{x(ax\cos(ax)-\sin(ax))}}{a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

```
[Out] (x^(-1) + a^2*x - a^3*x^2*Cot[a*x] + 4*a^2*x*Log[1 - E^((2*I)*a*x)] - (2*I)
*a*(a^2*x^2 + PolyLog[2, E^((2*I)*a*x)]) + ((1 + a^2*x^2)^2*Sin[a*x])/(x*(a
*x*Cos[a*x] - Sin[a*x])))/a^6
```

Maple [A]
time = 0.77, size = 172, normalized size = 1.35

method	result
risch	$-\frac{2i\left(2ia^2x^2e^{2iax}+2a^3x^3-2ia^2x^2-ax\,e^{2iax}+ie^{2iax}+ax-i\right)}{\left(e^{2iax}-1\right)\left(ax\,e^{2iax}+ie^{2iax}+ax-i\right)a^5} - \frac{4ix^2}{a^3} + \frac{4x\ln\left(e^{iax}+1\right)}{a^4} - \frac{4i\,\mathrm{polylog}\left(2,-e^{iax}\right)}{a^5} + \frac{4x\ln\left(1-e^{iax}\right)}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(2*I*a^2*x^2*exp(2*I*a*x)+2*a^3*x^3-2*I*a^2*x^2-a*x*exp(2*I*a*x)+I*exp
(2*I*a*x)+a*x-I)/(exp(2*I*a*x)-1)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)/a
```

```
^5-4*I/a^3*x^2+4/a^4*x*ln(exp(I*a*x)+1)-4*I/a^5*polylog(2,-exp(I*a*x))+4/a^
4*x*ln(1-exp(I*a*x))-4*I/a^5*polylog(2,exp(I*a*x))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(116) = 232$.
time = 0.31, size = 594, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*x + 2*(a^2*x^2 + 2*I*a*x*cos(2*a*x) - 2*a*x*sin(2*a*x) - I*a*x - (a^2
*x^2 + I*a*x)*cos(4*a*x) + (-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(sin(a*x),
 cos(a*x) + 1) - 2*(a^2*x^2 + 2*I*a*x*cos(2*a*x) - 2*a*x*sin(2*a*x) - I*a*x
 - (a^2*x^2 + I*a*x)*cos(4*a*x) - (I*a^2*x^2 - a*x)*sin(4*a*x))*arctan2(sin
(a*x), -cos(a*x) + 1) + 2*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) + (-2*I*a^2*x^2
- a*x + I)*cos(2*a*x) - 2*(a*x - (a*x + I)*cos(4*a*x) - (I*a*x - 1)*sin(4*a
*x) + 2*I*cos(2*a*x) - 2*sin(2*a*x) - I)*dilog(-e^(I*a*x)) - 2*(a*x - (a*x
+ I)*cos(4*a*x) - (I*a*x - 1)*sin(4*a*x) + 2*I*cos(2*a*x) - 2*sin(2*a*x) -
I)*dilog(e^(I*a*x)) + (-I*a^2*x^2 + 2*a*x*cos(2*a*x) + 2*I*a*x*sin(2*a*x) -
 a*x + (I*a^2*x^2 - a*x)*cos(4*a*x) - (a^2*x^2 + I*a*x)*sin(4*a*x))*log(cos
(a*x)^2 + sin(a*x)^2 + 2*cos(a*x) + 1) + (-I*a^2*x^2 + 2*a*x*cos(2*a*x) + 2
*I*a*x*sin(2*a*x) - a*x + (I*a^2*x^2 - a*x)*cos(4*a*x) - (a^2*x^2 + I*a*x)*
sin(4*a*x))*log(cos(a*x)^2 + sin(a*x)^2 - 2*cos(a*x) + 1) + 2*(I*a^3*x^3 -
a^2*x^2)*sin(4*a*x) + (2*a^2*x^2 - I*a*x - 1)*sin(2*a*x) - I)/((I*a*x + (-I
*a*x + 1)*cos(4*a*x) + (a*x + I)*sin(4*a*x) - 2*cos(2*a*x) - 2*I*sin(2*a*x)
 + 1)*a^5)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(116) = 232$.
time = 2.97, size = 410, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 + a*x - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2 - I)*dilog(cos(a*x) + I*
sin(a*x)) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2 + I)*dilog(cos(a*x)
- I*sin(a*x)) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2 + I)*dilog(-cos(
a*x) + I*sin(a*x)) - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2 - I)*dilog(-
cos(a*x) - I*sin(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*
x)*log(cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(
a*x)^2 - a*x)*log(cos(a*x) - I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x)
```

```
 + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a
*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) - I*sin(a*x) + 1))/(a^6*
x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2 - a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*csc(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

```
[Out] Integral(x**4*csc(a*x)**2/(a*x*cos(a*x) - sin(a*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*csc(a*x)^2/(a*x*cos(a*x) - sin(a*x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sin(a\,x)^2\,(\sin(a\,x) - a\,x\,\cos(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

```
[Out] int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.594 $\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=176

$$\frac{a^2}{x}+\frac{\cos^2(ax)}{x^3}-\frac{10a^2\cos^2(ax)}{x}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}+\frac{32a^2\cos^4(ax)}{3x}-\frac{a\cos(ax)\sin(ax)}{x^2}-\frac{\cos^3(ax)\sin(ax)}{ax^4}+$$

```
[Out] a^2/x+cos(a*x)^2/x^3-10*a^2*cos(a*x)^2/x+cos(a*x)^4/a^2/x^5-4/3*cos(a*x)^4/
x^3+32/3*a^2*cos(a*x)^4/x+2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)-a*cos(a*x)*s
in(a*x)/x^2-cos(a*x)^3*sin(a*x)/a/x^4+8/3*a*cos(a*x)^3*sin(a*x)/x^2-cos(a*x
)^5/a^2/x^5/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A]
time = 0.21, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4695, 3395, 30, 3394, 12, 3380}

$$\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)+\frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}+\frac{a^2}{x}+\frac{32a^2\cos^4(ax)}{3x}-\frac{10a^2\cos^2(ax)}{x}-\frac{\sin(ax)\cos^3(ax)}{ax^4}-\frac{4\cos^4(ax)}{3x^3}+\frac{\cos^2(ax)}{x^3}+\frac{8a\sin(ax)\cos^3(ax)}{3x^2}-\frac{a\sin(ax)\cos(ax)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] a^2/x + Cos[a*x]^2/x^3 - (10*a^2*Cos[a*x]^2)/x + Cos[a*x]^4/(a^2*x^5) - (4*
Cos[a*x]^4)/(3*x^3) + (32*a^2*Cos[a*x]^4)/(3*x) - (a*Cos[a*x]*Sin[a*x])/x^2
 - (Cos[a*x]^3*Sin[a*x])/(a*x^4) + (8*a*Cos[a*x]^3*Sin[a*x])/(3*x^2) - Cos[
a*x]^5/(a^2*x^5*(Cos[a*x] + a*x*Sin[a*x])) + (2*a^3*SinIntegral[2*a*x])/3 +
 (16*a^3*SinIntegral[4*a*x])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
```

```
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4695

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[
(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^5(ax)}{a^2x^5(\cos(ax)+ax\sin(ax))}-\frac{5\int \frac{\cos^4(ax)}{x^6}\,dx}{a^2}\\
&= \frac{\cos^4(ax)}{a^2x^5}-\frac{\cos^3(ax)\sin(ax)}{ax^4}-\frac{\cos^5(ax)}{a^2x^5(\cos(ax)+ax\sin(ax))}-3\int \frac{\cos^2(a
}{x^4}\\
&= \frac{\cos^2(ax)}{x^3}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}-\frac{a\cos(ax)\sin(ax)}{x^2}-\frac{\cos^3(ax)\sin(ax
}{ax^4}\\
&= \frac{a^2}{x}+\frac{\cos^2(ax)}{x^3}-\frac{10a^2\cos^2(ax)}{x}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}+\frac{32a^2\cos^4(ax)}{3x}\\
&= \frac{a^2}{x}+\frac{\cos^2(ax)}{x^3}-\frac{10a^2\cos^2(ax)}{x}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}+\frac{32a^2\cos^4(ax)}{3x}\\
&= \frac{a^2}{x}+\frac{\cos^2(ax)}{x^3}-\frac{10a^2\cos^2(ax)}{x}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}+\frac{32a^2\cos^4(ax)}{3x}
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 194, normalized size = 1.10

$$\frac{-10\cos(ax)+12a^2x^2\cos(ax)-5\cos(3ax)+44a^2x^2\cos(3ax)-\cos(5ax)+24a^2x^2\cos(5ax)+8ax\sin(ax)-8a^3x^3\sin(ax)+12ax\sin(3ax)-24a^3x^3\sin(3ax)+4ax\sin(5ax)+32a^3x^3\sin(5ax)+32a^3x^3(\cos(ax)+ax\sin(ax))\text{Si}(2ax)+256a^3x^3(\cos(ax)+ax\sin(ax))\text{Si}(4ax)}{48x^3(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]

[Out] (-10*Cos[a*x] + 12*a^2*x^2*Cos[a*x] - 5*Cos[3*a*x] + 44*a^2*x^2*Cos[3*a*x] - Cos[5*a*x] + 24*a^2*x^2*Cos[5*a*x] + 8*a*x*Sin[a*x] - 8*a^3*x^3*Sin[a*x] + 12*a*x*Sin[3*a*x] - 24*a^3*x^3*Sin[3*a*x] + 4*a*x*Sin[5*a*x] + 32*a^3*x^3 *Sin[5*a*x] + 32*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x] + 256 *a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(Cos[a*x] + a*x*Sin[a*x]))

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4 (\cos(ax) + ax\sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)

[Out] int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [A]
time = 3.35, size = 162, normalized size = 0.92

$$-\frac{19\,a^2x^2\cos(ax)^3 - (24\,a^2x^2 - 1)\cos(ax)^5 - 2\,(8\,a^3x^3\,\mathrm{Si}(4\,ax) + a^3x^3\,\mathrm{Si}(2\,ax))\cos(ax) - (16\,a^4x^4\,\mathrm{Si}(4\,ax) + 2\,a^4x^4\,\mathrm{Si}(2\,ax) - 30\,a^3x^3\cos(ax)^2 + 3\,a^3x^3 + 4\,(8\,a^3x^3 + ax)\cos(ax)^4)\sin(ax)}{3\,(ax^4\sin(ax) + x^3\cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")

[Out] -1/3*(19*a^2*x^2*cos(a*x)^3 - (24*a^2*x^2 - 1)*cos(a*x)^5 - 2*(8*a^3*x^3*sin_integral(4*a*x) + a^3*x^3*sin_integral(2*a*x))*cos(a*x) - (16*a^4*x^4*sin_integral(4*a*x) + 2*a^4*x^4*sin_integral(2*a*x) - 30*a^3*x^3*cos(a*x)^2 + 3*a^3*x^3 + 4*(8*a^3*x^3 + a*x)*cos(a*x)^4)*sin(a*x))/(a*x^4*sin(a*x) + x^3 *cos(a*x))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(ax)}{x^4(ax\sin(ax)+\cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**6/x**4/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**6/(x**4*(a*x*sin(a*x) + cos(a*x))**2), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 7279, normalized size = 41.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/12*(64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan
(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^
2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan
(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2
*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integr
al(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integra
l(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*
a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_inte
gral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2
*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x)
)*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*ta
n(2*a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(
1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(1/2*a*x) + 64*a^
8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_in
tegral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*t
an(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*
a*x) + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2 +
4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 4*a^7*x^
7*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2 - 32*a^7*x^7*imag
```

```
_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2 + 64*a^7*x^7*sin_integr
al(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*a*x
)^2*tan(a*x)^2 - 40*a^7*x^7*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) - 32*a^7*x
^7*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_
part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_pa
rt(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integ
ral(4*a*x)*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin_integral(2*a*x)*tan(
2*a*x)^2*tan(1/2*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x
)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(a*x)^2*ta
n(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2
*a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^2*tan(1/2*a*x
)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*
sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_part(cos_in
tegral(4*a*x))*tan(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(
1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) - 64*a^8*
x^8*imag_part(cos_integral(-4*a*x))*tan(1/2*a*x) + 128*a^8*x^8*sin_integral
(4*a*x)*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(1/2*a*x) + 128*a^
6*x^6*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x) +
 16*a^6*x^6*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*
a*x) - 16*a^6*x^6*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*t
an(1/2*a*x) - 128*a^6*x^6*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*tan(
a*x)^2*tan(1/2*a*x) + 256*a^6*x^6*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)
^2*tan(1/2*a*x) + 32*a^6*x^6*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x) + 20*a^6*x^6*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7
*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2 + 4*a^7*x^7*imag_part(cos_inte
gral(2*a*x))*tan(2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(-2*a*x))*tan(2
*a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2 + 64*a^7*
x^7*sin_integral(4*a*x)*tan(2*a*x)^2 + 8*a^7*x^7*sin_integral(2*a*x)*tan(2*
a*x)^2 + 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(a*x)^2 + 4*a^7*x^7*i
mag_part(cos_integral(2*a*x))*tan(a*x)^2 - 4*a^7*x^7*imag_part(cos_integral
(-2*a*x))*tan(a*x)^2 - 32*a^7*x^7*imag_part(cos_integral(-4*a*x))*tan(a*x)^
2 + 64*a^7*x^7*sin_integral(4*a*x)*tan(a*x)^2 + 8*a^7*x^7*sin_integral(2*a*
x)*tan(a*x)^2 - 24*a^7*x^7*tan(2*a*x)^2*tan(1/2*a*x) + 24*a^7*x^7*tan(a*x)^
2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(1/2*a*x)^2 -
 4*a^7*x^7*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_p
art(cos_integral(-2*a*x))*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(cos_integra
l(-4*a*x))*tan(1/2*a*x)^2 - 64*a^7*x^7*sin_integral(4*a*x)*tan(1/2*a*x)^2 -
 8*a^7*x^7*sin_integral(2*a*x)*tan(1/2*a*x)^2 - 64*a^5*x^5*imag_part(cos_in
tegral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^5*x^5*imag_part
(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x)^2*t...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(a x\right)^{6}}{x^{4} \left(\cos \left(a x\right)+a x \sin \left(a x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.595 $\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=132

$$\frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin(ax)}{ax^3}$$

```
[Out] -1/8*a^2*Ci(a*x)-27/8*a^2*Ci(3*a*x)+cos(a*x)/x^2+cos(a*x)^3/a^2/x^4-3/2*cos
(a*x)^3/x^2-a*sin(a*x)/x-cos(a*x)^2*sin(a*x)/a/x^3+9/2*a*cos(a*x)^2*sin(a*x
)/x-cos(a*x)^4/a^2/x^4/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A]
time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4695, 3395, 3378, 3383, 3393}

$$-\frac{1}{8}a^2\text{CosIntegral}(ax)-\frac{27}{8}a^2\text{CosIntegral}(3ax)+\frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}-\frac{\sin(ax)\cos^2(ax)}{ax^3}-\frac{3\cos^3(ax)}{2x^2}+\frac{\cos(ax)}{x^2}-\frac{a\sin(ax)}{x}+\frac{9a\sin(ax)\cos^2(ax)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/x^2 + Cos[a*x]^3/(a^2*x^4) - (3*Cos[a*x]^3)/(2*x^2) - (a^2*CosInte
gral[a*x])/8 - (27*a^2*CosIntegral[3*a*x])/8 - (a*Sin[a*x])/x - (Cos[a*x]^2
*Sin[a*x])/(a*x^3) + (9*a*Cos[a*x]^2*Sin[a*x])/(2*x) - Cos[a*x]^4/(a^2*x^4*
(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4695

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[
(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))}-\frac{4\int\frac{\cos^3(ax)}{x^5}\,dx}{a^2}\\ &= \frac{\cos^3(ax)}{a^2x^4}-\frac{\cos^2(ax)\sin(ax)}{ax^3}-\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))}-2\int\frac{\cos(ax}{x^3}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{\cos^2(ax)\sin(ax)}{ax^3}+\frac{9a\cos^2(ax)\sin(a}{2x}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}+9a^2\mathrm{Ci}(ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin}{ax^3}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}+10a^2\mathrm{Ci}(ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin}{ax^3}\\ &= \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{1}{8}a^2\mathrm{Ci}(ax)-\frac{27}{8}a^2\mathrm{Ci}(3ax)-\frac{a\sin(a}{x}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 136, normalized size = 1.03

$$-\frac{3-a^2x^2+4\cos(2ax)-8a^2x^2\cos(2ax)+\cos(4ax)+9a^2x^2\cos(4ax)+2a^2x^2\mathrm{CosIntegral}(ax)(\cos(ax)+ax\sin(ax))+54a^2x^2\mathrm{CosIntegral}(3ax)(\cos(ax)+ax\sin(ax))-12ax\sin(2ax)-6ax\sin(4ax)}{16x^2(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -1/16*(3 - a^2*x^2 + 4*Cos[2*a*x] - 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a
^2*x^2*Cos[4*a*x] + 2*a^2*x^2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x]) +
```

```
54*a^2*x^2*CosIntegral[3*a*x]*(Cos[a*x] + a*x*Sin[a*x]) - 12*a*x*Sin[2*a*x]
 - 6*a*x*Sin[4*a*x])/(x^2*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 2.75, size = 185, normalized size = 1.40

$$\frac{88\,a^2x^2\cos(ax)^2 - 8\,(9\,a^2x^2+1)\cos(ax)^4 - 16\,a^2x^2 - (27\,a^2x^2\,\mathrm{Ci}(3\,ax) + a^2x^2\,\mathrm{Ci}(ax) + a^2x^2\,\mathrm{Ci}(-ax) + 27\,a^2x^2\,\mathrm{Ci}(-3\,ax))\cos(ax) - \left(27\,a^3x^3\,\mathrm{Ci}(3\,ax) + a^3x^3\,\mathrm{Ci}(ax) + a^3x^3\,\mathrm{Ci}(-ax) + 27\,a^3x^3\,\mathrm{Ci}(-3\,ax) - 48\,ax\cos(ax)^3\right)\sin(ax)}{16\,(ax^3\sin(ax) + x^2\cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(88*a^2*x^2*cos(a*x)^2 - 8*(9*a^2*x^2 + 1)*cos(a*x)^4 - 16*a^2*x^2 - (
27*a^2*x^2*cos_integral(3*a*x) + a^2*x^2*cos_integral(a*x) + a^2*x^2*cos_in
tegral(-a*x) + 27*a^2*x^2*cos_integral(-3*a*x))*cos(a*x) - (27*a^3*x^3*cos_
integral(3*a*x) + a^3*x^3*cos_integral(a*x) + a^3*x^3*cos_integral(-a*x) +
27*a^3*x^3*cos_integral(-3*a*x) - 48*a*x*cos(a*x)^3)*sin(a*x))/(a*x^3*sin(a
*x) + x^2*cos(a*x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)**5/x**3/(cos(a*x)+a*x*sin(a*x))**2,x)

[Out] Timed out

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 3130, normalized size = 23.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")

```
[Out] -1/16*(54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x
)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3
+ 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 5
4*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2
7*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^
6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^6*x^6*
real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^6*x^6*re
al_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^7*x^7*re
al_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_p
art(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*real_part(co
s_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_in
tegral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_part(cos_inte
gral(3*a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(1/
2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 54*a^7*
x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 - 27*a^6*x^6*real_part(c
os_integral(3*a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(a*x))*t
an(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a
^6*x^6*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*real_par
t(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(a*x)
)*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 54*
a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x) - 8*a^6*x^6*tan(3/2*a*
x)^2*tan(1/2*a*x)^2 - 72*a^6*x^6*tan(3/2*a*x)*tan(1/2*a*x)^3 + 108*a^5*x^5*
real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*re
al_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 4*a^5*x^5*real_p
art(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^5*x^5*real_pa
rt(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 27*a^6*x^6*real_pa
rt(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(a*x
))*tan(3/2*a*x)^2 + a^6*x^6*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2 +
27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2 - 12*a^5*x^5*tan(
3/2*a*x)^2*tan(1/2*a*x)^3 + 36*a^5*x^5*tan(3/2*a*x)*tan(1/2*a*x)^4 - 54*a^4
*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x
^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^4*x^4*r
eal_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 54*a^4*x^4*rea
l_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 72*a^6*x^6*tan
```

```
(3/2*a*x)*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(3*a*x))*tan(3/2
*a*x)^2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(3/2*a*x)^
2*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan
(1/2*a*x) + 108*a^5*x^5*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(
1/2*a*x) - 8*a^6*x^6*tan(1/2*a*x)^2 + 108*a^5*x^5*real_part(cos_integral(3*
a*x))*tan(1/2*a*x)^3 + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x)^
3 + 4*a^5*x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 108*a^5*x^5*re
al_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 + 8*a^4*x^4*tan(3/2*a*x)^2*tan
(1/2*a*x)^4 + 27*a^6*x^6*real_part(cos_integral(3*a*x)) + a^6*x^6*real_part
(cos_integral(a*x)) + a^6*x^6*real_part(cos_integral(-a*x)) + 27*a^6*x^6*re
al_part(cos_integral(-3*a*x)) - 12*a^5*x^5*tan(3/2*a*x)^2*tan(1/2*a*x) + 12
*a^5*x^5*tan(1/2*a*x)^3 - 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(1/2
*a*x)^4 - 2*a^4*x^4*real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - 2*a^4*x^4
*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 54*a^4*x^4*real_part(cos_in
tegral(-3*a*x))*tan(1/2*a*x)^4 + 108*a^5*x^5*real_part(cos_integral(3*a*x))
*tan(1/2*a*x) + 4*a^5*x^5*real_part(cos_integral(a*x))*tan(1/2*a*x) + 4*a^5
*x^5*real_part(cos_integral(-a*x))*tan(1/2*a*x) + 108*a^5*x^5*real_part(cos
_integral(-3*a*x))*tan(1/2*a*x) - 4*a^4*x^4*tan(3/2*a*x)^2*tan(1/2*a*x)^2 -
 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integra
l(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(
a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^3*x^3*real_part(cos_integral(-a*x
))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 54*a^3*x^3*real_part(cos_integral(-3*a*x
))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^4*x^4*tan(1/2*a*x)^4 - 36*a^5*x^5*ta
n(3/2*a*x) + 54*a^4*x^4*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + 2*a
^4*x^4*real_part(cos_integral(a*x))*tan(3/2*a*x)^2 + 2*a^4*x^4*real_part(co
s_integral(-a*x))*tan(3/2*a*x)^2 + 54*a^4*x^4*real_part(cos_integral(-3*a*x
))*tan(3/2*a*x)^2 + 12*a^5*x^5*tan(1/2*a*x) + 64*a^3*x^3*tan(3/2*a*x)*tan(1
/2*a*x)^4 - 27*a^2*x^2*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/
2*a*x)^4 - a^2*x^2*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)
^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
27*a^2*x^2*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 -
4*a^4*x^4*tan(3/2*a*x)^2 - 128*a^4*x^4*tan(3/2*a*x)*tan(1/2*a*x) + 54*a^3*x
^3*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a\,x)^5}{x^3\,{\left(\cos (a\,x)+a\,x\,\sin (a\,x)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.596 $\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=80

$$\frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-2a\mathrm{Si}(2ax)$$

```
[Out] 1/x+cos(a*x)^2/a^2/x^3-2*cos(a*x)^2/x-2*a*Si(2*a*x)-cos(a*x)*sin(a*x)/a/x^2
-cos(a*x)^3/a^2/x^3/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A]
time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4695, 3395, 30, 3394, 12, 3380}

$$\frac{\cos^2(ax)}{a^2x^3}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}-2a\mathrm{Si}(2ax)-\frac{\sin(ax)\cos(ax)}{ax^2}-\frac{2\cos^2(ax)}{x}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] x^(-1) + Cos[a*x]^2/(a^2*x^3) - (2*Cos[a*x]^2)/x - (Cos[a*x]*Sin[a*x])/(a*x
^2) - Cos[a*x]^3/(a^2*x^3*(Cos[a*x] + a*x*Sin[a*x])) - 2*a*SinIntegral[2*a*
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4695

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[
(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-\frac{3\int\frac{\cos^2(ax)}{x^4}\,dx}{a^2}\\
&= \frac{\cos^2(ax)}{a^2x^3}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}+2\int\frac{\cos^2(a}{x^2}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(}\\
&= \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 71, normalized size = 0.89

$$-\frac{3\cos(ax)+\cos(3ax)-2ax\sin(ax)+2ax\sin(3ax)+8ax(\cos(ax)+ax\sin(ax))\mathrm{Si}(2ax)}{4x(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] -1/4*(3*Cos[a*x] + Cos[3*a*x] - 2*a*x*Sin[a*x] + 2*a*x*Sin[3*a*x] + 8*a*x*(
Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x])/(x*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax) + ax\sin(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.80, size = 73, normalized size = 0.91

$$-\frac{2\,ax\cos(ax)\operatorname{Si}(2\,ax) + \cos(ax)^3 + \left(2\,a^2x^2\operatorname{Si}(2\,ax) + 2\,ax\cos(ax)^2 - ax\right)\sin(ax)}{ax^2\sin(ax) + x\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*x*cos(a*x)*sin_integral(2*a*x) + cos(a*x)^3 + (2*a^2*x^2*sin_integral
(2*a*x) + 2*a*x*cos(a*x)^2 - a*x)*sin(a*x))/(a*x^2*sin(a*x) + x*cos(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(ax)}{x^2(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**4/x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**4/(x**2*(a*x*sin(a*x) + cos(a*x))**2), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 997, normalized size = 12.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^4*
x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*sin
_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_integral(2
*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*
tan(a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 + 2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*a^4*x^
4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^4*x^4*sin_integral(2*a
*x)*tan(1/2*a*x) + a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a^3*
x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a^3*x^3*sin_integral(2*a
*x)*tan(a*x)^2 - 2*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_
integral(2*a*x))*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*x))*t
an(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^2*x^2*im
ag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^2*x^2*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(2*a*
x)*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*im
ag_part(cos_integral(2*a*x)) - a^3*x^3*imag_part(cos_integral(-2*a*x)) + 2*
a^3*x^3*sin_integral(2*a*x) + 2*a^3*x^3*tan(1/2*a*x) - a*x*imag_part(cos_in
tegral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*
x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a*x*sin_integral(2*a*x)*tan(a*x)^2*tan(1/
2*a*x)^2 - a^2*x^2*tan(a*x)^2 + 2*a^2*x^2*imag_part(cos_integral(2*a*x))*ta
n(1/2*a*x) - 2*a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^2
*x^2*sin_integral(2*a*x)*tan(1/2*a*x) + 2*a^2*x^2*tan(a*x)*tan(1/2*a*x) - a
^2*x^2*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(2*a*x))*tan(a*x)^2 - a*x
*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a*x*sin_integral(2*a*x)*tan
(a*x)^2 - 2*a*x*tan(a*x)^2*tan(1/2*a*x) - a*x*imag_part(cos_integral(2*a*x)
)*tan(1/2*a*x)^2 + a*x*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x)^2 - 2*a
*x*sin_integral(2*a*x)*tan(1/2*a*x)^2 - a*x*tan(a*x)*tan(1/2*a*x)^2 + a^2*x
^2 + a*x*imag_part(cos_integral(2*a*x)) - a*x*imag_part(cos_integral(-2*a*x
)) + 2*a*x*sin_integral(2*a*x) + a*x*tan(a*x) - tan(1/2*a*x)^2 + 1)/(2*a^3*
x^4*tan(a*x)^2*tan(1/2*a*x) - a^2*x^3*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^3*x^4
*tan(1/2*a*x) + a^2*x^3*tan(a*x)^2 - a^2*x^3*tan(1/2*a*x)^2 + 2*a*x^2*tan(a
*x)^2*tan(1/2*a*x) + a^2*x^3 - x*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a*x^2*tan(1/
2*a*x) + x*tan(a*x)^2 - x*tan(1/2*a*x)^2 + x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a x)^{4}}{x^{2} (\cos (a x) + a x \sin (a x))^{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.597 $\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2} dx$

Optimal. Leaf size=56

$$\frac{\cos(ax)}{a^2x^2} + \text{CosIntegral}(ax) - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax) + ax\sin(ax))}$$

```
[Out] Ci(a*x)+cos(a*x)/a^2/x^2-sin(a*x)/a/x-cos(a*x)^2/a^2/x^2/(cos(a*x)+a*x*sin(
a*x))
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4695, 3378, 3383}

$$\frac{\cos(ax)}{a^2x^2} - \frac{\cos^2(ax)}{a^2x^2(ax\sin(ax) + \cos(ax))} + \text{CosIntegral}(ax) - \frac{\sin(ax)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] Cos[a*x]/(a^2*x^2) + CosIntegral[a*x] - Sin[a*x]/(a*x) - Cos[a*x]^2/(a^2*x^
2*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4695

```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Dist[b^2*((n - 1)/d^2), Int[
(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c - d, 0] && EqQ[m, 2 - n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}-\frac{2\int \frac{\cos(ax)}{x^3}\,dx}{a^2}\\ &= \frac{\cos(ax)}{a^2x^2}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}+\frac{\int \frac{\sin(ax)}{x^2}\,dx}{a}\\ &= \frac{\cos(ax)}{a^2x^2}-\frac{\sin(ax)}{ax}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}+\int \frac{\cos(ax)}{x}\,dx\\ &= \frac{\cos(ax)}{a^2x^2}+\operatorname{Ci}(ax)-\frac{\sin(ax)}{ax}-\frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.69, size = 237, normalized size = 4.23

$$\frac{-1+\cos(2ax)-e\cos(ax)\text{CosIntegral}(i+ax)+e\cos(ax)\text{Ei}(-1-iax)+e\cos(ax)\text{Ei}(-1+iax)-aex\text{CosIntegral}(i+ax)\sin(ax)+aex\text{Ei}(-1-iax)\sin(ax)+aex\text{Ei}(-1+iax)\sin(ax)+2\text{CosIntegral}(ax)(\cos(ax)+ax\sin(ax))-e\text{CosIntegral}(i-ax)(\cos(ax)+ax\sin(ax))-ie\cos(ax)\text{Si}(i-ax)-iaex\sin(ax)\text{Si}(i-ax)-ie\cos(ax)\text{Si}(i+ax)-iaex\sin(ax)\text{Si}(i+ax)}{2(\cos(ax)+ax\sin(ax))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

```
[Out] (-1 + Cos[2*a*x] - E*Cos[a*x]*CosIntegral[I + a*x] + E*Cos[a*x]*ExpIntegral
Ei[-1 - I*a*x] + E*Cos[a*x]*ExpIntegralEi[-1 + I*a*x] - a*E*x*CosIntegral[I
 + a*x]*Sin[a*x] + a*E*x*ExpIntegralEi[-1 - I*a*x]*Sin[a*x] + a*E*x*ExpInte
gralEi[-1 + I*a*x]*Sin[a*x] + 2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x])
- E*CosIntegral[I - a*x]*(Cos[a*x] + a*x*Sin[a*x]) - I*E*Cos[a*x]*SinIntegr
al[I - a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I - a*x] - I*E*Cos[a*x]*SinInteg
ral[I + a*x] - I*a*E*x*Sin[a*x]*SinIntegral[I + a*x])/(2*(Cos[a*x] + a*x*Si
n[a*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 3.09, size = 106, normalized size = 1.89

method	result	size
risch	$-\frac{e^{iax}}{2(iax-1)}-\frac{\text{expIntegral}(1,-iax)}{2}+\frac{e^{-iax}}{2iax+2}-\frac{\text{expIntegral}(1,iax)}{2}-\frac{2ie^{iax}}{(ax+i)(ax-i)(ax\,e^{2iax}-ax+ie^{2iax}+i)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(I*a*x)/(-1+I*a*x)-1/2*Ei(1,-I*a*x)+1/2*exp(-I*a*x)/(I*a*x+1)-1/2*E
i(1,I*a*x)-2*I*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a
*x)+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.15, size = 62, normalized size = 1.11

$$\frac{(\mathrm{Ci}\,(ax)+\mathrm{Ci}\,(-ax))\cos\,(ax)+2\,\cos\,(ax)^2+(ax\,\mathrm{Ci}\,(ax)+ax\,\mathrm{Ci}\,(-ax))\sin\,(ax)-2}{2\,(ax\sin\,(ax)+\cos\,(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(a*x) + cos_integral(-a*x))*cos(a*x) + 2*cos(a*x)^2 + (a*
x*cos_integral(a*x) + a*x*cos_integral(-a*x))*sin(a*x) - 2)/(a*x*sin(a*x) +
 cos(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^3\,(ax)}{x\,(ax\sin\,(ax)+\cos\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**3/x/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(cos(a*x)**3/(x*(a*x*sin(a*x) + cos(a*x))**2), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 366, normalized size = 6.54

$$\frac{2\,a^3x^3\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)^3+2\,a^3x^3\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)^3-a^2x^2\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)^4-a^2x^2\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)^4+2\,a^3x^3\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)+2\,a^3x^3\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)-8\,a^2x^2\tan\left(\tfrac{1}{2}\,ax\right)^2+2\,ax\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)^3+2\,ax\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)^3+a^2x^2\Re(\mathrm{Ci}\,(ax))+a^2x^2\Re(\mathrm{Ci}\,(-ax))-\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)^4-\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)^4+2\,ax\Re(\mathrm{Ci}\,(ax))\tan\left(\tfrac{1}{2}\,ax\right)+2\,ax\Re(\mathrm{Ci}\,(-ax))\tan\left(\tfrac{1}{2}\,ax\right)-2\,\tan\left(\tfrac{1}{2}\,ax\right)^4-12\,\tan\left(\tfrac{1}{2}\,ax\right)^2+\Re(\mathrm{Ci}\,(ax))+\Re(\mathrm{Ci}\,(-ax))-2}{2\left(2\,a^3x^3\tan\left(\tfrac{1}{2}\,ax\right)^3-a^2x^2\tan\left(\tfrac{1}{2}\,ax\right)^4+2\,a^3x^3\tan\left(\tfrac{1}{2}\,ax\right)+2\,ax\tan\left(\tfrac{1}{2}\,ax\right)^3+a^2x^2-\tan\left(\tfrac{1}{2}\,ax\right)^4+2\,ax\tan\left(\tfrac{1}{2}\,ax\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*real
_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - a^2*x^2*real_part(cos_integral(a
*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1/2*a*x)^4
+ 2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_part
(cos_integral(-a*x))*tan(1/2*a*x) - 8*a^2*x^2*tan(1/2*a*x)^2 + 2*a*x*real_p
art(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a*x*real_part(cos_integral(-a*x))
*tan(1/2*a*x)^3 + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*real_part(
cos_integral(-a*x)) - real_part(cos_integral(a*x))*tan(1/2*a*x)^4 - real_pa
```

```
rt(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*real_part(cos_integral(a*x))*
tan(1/2*a*x) + 2*a*x*real_part(cos_integral(-a*x))*tan(1/2*a*x) - 2*tan(1/2
*a*x)^4 - 12*tan(1/2*a*x)^2 + real_part(cos_integral(a*x)) + real_part(cos_
integral(-a*x)) - 2)/(2*a^3*x^3*tan(1/2*a*x)^3 - a^2*x^2*tan(1/2*a*x)^4 + 2
*a^3*x^3*tan(1/2*a*x) + 2*a*x*tan(1/2*a*x)^3 + a^2*x^2 - tan(1/2*a*x)^4 + 2
*a*x*tan(1/2*a*x) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a\,x\right)^3}{x\,{\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.598 $\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(\cos(ax)+ax\sin(ax))}$$

```
[Out] 1/a^2/x-cos(a*x)/a^2/x/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {4693}

$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] 1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))
```

Rule 4693

```
Int[Cos[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] - Simp[Cos[a*x]/(a*d*x*(d*x*Sin[a*x] + c
*Cos[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rubi steps

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(\cos(ax)+ax\sin(ax))}$$

Mathematica [A]
time = 0.16, size = 22, normalized size = 0.65

$$\frac{\sin(ax)}{a(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] Sin[a*x]/(a*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 1.17, size = 55, normalized size = 1.62

method	result	size
risch	$\frac{1}{a(ax+i)}-\frac{2i}{(ax+i)(ax\,\mathrm{e}^{2iax}-ax+i\mathrm{e}^{2iax}+i)a}$	55
norman	$\frac{\frac{2\tan\left(\frac{ax}{2}\right)}{a}+\frac{4\left(\tan^3\left(\frac{ax}{2}\right)\right)}{a}+\frac{2\left(\tan^5\left(\frac{ax}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{ax}{2}\right)\right)^2\left(2ax\tan\left(\frac{ax}{2}\right)-\left(\tan^2\left(\frac{ax}{2}\right)\right)+1\right)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(a*x+I)-2*I/(a*x+I)/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(34) = 68$.
time = 0.28, size = 114, normalized size = 3.35

$$\frac{ax\cos\left(2\,ax\right)^2+ax\sin\left(2\,ax\right)^2-2\,ax\cos\left(2\,ax\right)+ax+2\,\sin\left(2\,ax\right)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos\left(2\,ax\right)^2+4\,ax\sin\left(2\,ax\right)+\left(a^2x^2+1\right)\sin\left(2\,ax\right)^2-2\left(a^2x^2-1\right)\cos\left(2\,ax\right)+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] (a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 - 2*a*x*cos(2*a*x) + a*x + 2*sin(2*a*x
))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1
)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

Fricas [A]
time = 2.06, size = 23, normalized size = 0.68

$$\frac{\sin\left(ax\right)}{a^2x\sin\left(ax\right)+a\cos\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] sin(a*x)/(a^2*x*sin(a*x) + a*cos(a*x))
```

Sympy [A]
time = 1.69, size = 20, normalized size = 0.59

$$\frac{\sin\left(ax\right)}{a^2x\sin\left(ax\right)+a\cos\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] sin(a*x)/(a**2*x*sin(a*x) + a*cos(a*x))
```

Giac [A]
time = 0.44, size = 32, normalized size = 0.94

$$\frac{2\,\tan\left(\frac{1}{2}\,ax\right)}{2\,a^{2}x\tan\left(\frac{1}{2}\,ax\right)-a\tan\left(\frac{1}{2}\,ax\right)^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] 2*tan(1/2*a*x)/(2*a^2*x*tan(1/2*a*x) - a*tan(1/2*a*x)^2 + a)
```

Mupad [B]
time = 0.16, size = 22, normalized size = 0.65

$$\frac{\sin\left(a\,x\right)}{a\,\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a*x)^2/(cos(a*x) + a*x*sin(a*x))^2,x)
```

```
[Out] sin(a*x)/(a*(cos(a*x) + a*x*sin(a*x)))
```

3.599 $\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

```
[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))
```

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{6818\}$

$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
[Out] -(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

Maple [A]
time = 0.26, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$	20
default	$-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$	20
risch	$-\frac{2ie^{iax}}{a^2(ax\,e^{2iax}-ax+ie^{2iax}+i)}$	40
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{ax}{2}\right)\right)}{a^2}-\frac{2}{a^2}-\frac{2x\tan\left(\frac{ax}{2}\right)}{a}-\frac{2x\left(\tan^3\left(\frac{ax}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{ax}{2}\right)\right)\left(2ax\tan\left(\frac{ax}{2}\right)-\left(\tan^2\left(\frac{ax}{2}\right)\right)+1\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/(cos(a*x)+a*x*sin(a*x))
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.00

$$-\frac{1}{(ax\sin(ax)+\cos(ax))a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x*sin(a*x) + cos(a*x))*a^2)
```

Fricas [A]
time = 2.60, size = 22, normalized size = 1.16

$$-\frac{1}{a^3x\sin(ax)+a^2\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/(a^3*x*sin(a*x) + a^2*cos(a*x))
```

Sympy [A]
time = 1.71, size = 20, normalized size = 1.05

$$-\frac{1}{a^3x\sin(ax)+a^2\cos(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] -1/(a**3*x*sin(a*x) + a**2*cos(a*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 0.45, size = 40, normalized size = 2.11

$$-\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2+1\right)}{2\,a^3x\tan\left(\frac{1}{2}\,ax\right)-a^2\tan\left(\frac{1}{2}\,ax\right)^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*a*x)^2 + 1)/(2*a^3*x*tan(1/2*a*x) - a^2*tan(1/2*a*x)^2 + a^2)
```

Mupad [B]
time = 0.09, size = 22, normalized size = 1.16

$$-\frac{1}{a^2\,\cos\left(a\,x\right)+a^3\,x\,\sin\left(a\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a*x))/(cos(a*x) + a*x*sin(a*x))^2,x)
```

```
[Out] -1/(a^2*cos(a*x) + a^3*x*sin(a*x))
```

3.600 $\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=33

$$-\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^3}$$

```
[Out] -x*sec(a*x)/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^3
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4691, 3852, 8}

$$\frac{\tan(ax)}{a^3}-\frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
[Out] -((x*Sec[a*x])/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + Tan[a*x]/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4691

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[-x/(a*d*Cos[a*x]*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Dist[1/d^2, I
nt[1/Cos[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx &= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\int \sec^2(ax)\,dx}{a^2}\\ &= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))}-\frac{\text{Subst}(\int 1\,dx,x,-\tan(ax))}{a^3}\\ &= -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 31, normalized size = 0.94

$$\frac{-ax\cos(ax)+\sin(ax)}{a^3(\cos(ax)+ax\sin(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] (-(a*x*Cos[a*x]) + Sin[a*x])/(a^3*(Cos[a*x] + a*x*Sin[a*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.84, size = 40, normalized size = 1.21

method	result	size
risch	$-\frac{2i(ax-i)}{a^3(ax\,e^{2iax}-ax+ie^{2iax}+i)}$	40
norman	$\frac{\frac{x\left(\tan^2\left(\frac{ax}{2}\right)\right)}{a^2}+\frac{2\tan\left(\frac{ax}{2}\right)}{a^3}-\frac{x}{a^2}}{2ax\tan\left(\frac{ax}{2}\right)-\left(\tan^2\left(\frac{ax}{2}\right)\right)+1}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(a*x-I)/a^3/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(33) = 66.
time = 0.28, size = 100, normalized size = 3.03

$$-\frac{2\left(2\,ax\cos\left(2\,ax\right)+\left(a^2x^2-1\right)\sin\left(2\,ax\right)\right)}{\left(a^2x^2+\left(a^2x^2+1\right)\cos\left(2\,ax\right)^2+4\,ax\sin\left(2\,ax\right)+\left(a^2x^2+1\right)\sin\left(2\,ax\right)^2-2\left(a^2x^2-1\right)\cos\left(2\,ax\right)+1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*
cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2 -
 1)*cos(2*a*x) + 1)*a^3)
```

Fricas [A]
time = 2.37, size = 36, normalized size = 1.09

$$-\frac{ax\cos\left(ax\right)-\sin\left(ax\right)}{a^4x\sin\left(ax\right)+a^3\cos\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -(a*x*cos(a*x) - sin(a*x))/(a^4*x*sin(a*x) + a^3*cos(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(ax\sin(ax) + \cos(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

Giac [A]
time = 0.41, size = 52, normalized size = 1.58

$$\frac{ax\tan\left(\frac{1}{2}\,ax\right)^2 - ax + 2\,\tan\left(\frac{1}{2}\,ax\right)}{2\,a^4x\tan\left(\frac{1}{2}\,ax\right) - a^3\tan\left(\frac{1}{2}\,ax\right)^2 + a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] (a*x*tan(1/2*a*x)^2 - a*x + 2*tan(1/2*a*x))/(2*a^4*x*tan(1/2*a*x) - a^3*tan
(1/2*a*x)^2 + a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(\cos(a\,x) + a\,x\,\sin(a\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a*x) + a*x*sin(a*x))^2,x)
```

```
[Out] int(x^2/(cos(a*x) + a*x*sin(a*x))^2, x)
```

3.601 $\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{2ix\text{ArcTan}(e^{iax})}{a^3}+\frac{i\text{PolyLog}(2,-ie^{iax})}{a^4}-\frac{i\text{PolyLog}(2,ie^{iax})}{a^4}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\ldots}{a\ldots}$$

```
[Out] -2*I*x*arctan(exp(I*a*x))/a^3+I*polylog(2,-I*exp(I*a*x))/a^4-I*polylog(2,I*
exp(I*a*x))/a^4-sec(a*x)/a^4-x^2*sec(a*x)^2/a^2/(cos(a*x)+a*x*sin(a*x))+x*s
ec(a*x)*tan(a*x)/a^3
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4697, 4270, 4266, 2317, 2438}

$$\frac{i\text{Li}_2(-ie^{iax})}{a^4}-\frac{i\text{Li}_2(ie^{iax})}{a^4}-\frac{\sec(ax)}{a^4}-\frac{2ix\text{ArcTan}(e^{iax})}{a^3}+\frac{x\tan(ax)\sec(ax)}{a^3}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x*ArcTan[E^(I*a*x)])/a^3 + (I*PolyLog[2, (-I)*E^(I*a*x)])/a^4 - (I*
PolyLog[2, I*E^(I*a*x)])/a^4 - Sec[a*x]/a^4 - (x^2*Sec[a*x]^2)/(a^2*(Cos[a*
x] + a*x*Sin[a*x])) + (x*Sec[a*x]*Tan[a*x])/a^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4697

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Sec[a*x]^
(n + 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] + Dist[b^2*((n + 1)/d^2), In
t[(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
 EqQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^3\sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{2\int x\sec^3(ax)\,dx}{a^2}$$

$$= -\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(ax)}{a^3}+\frac{\int x\sec(ax)\,d}{a^2}$$

$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(}{a^3}$$

$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{x\sec(ax)\tan(}{a^3}$$

$$= -\frac{2ix\tan^{-1}\left(e^{iax}\right)}{a^3}+\frac{i\text{Li}_2\left(-ie^{iax}\right)}{a^4}-\frac{i\text{Li}_2\left(ie^{iax}\right)}{a^4}-\frac{\sec(ax)}{a^4}-\frac{x^2\sec^2}{a^2(\cos(ax)+}$$

Mathematica [A]
time = 0.51, size = 176, normalized size = 1.60

$$-\frac{-ax\log\left(1-ie^{iax}\right)+ax\log\left(1+ie^{iax}\right)+\sec(ax)+a^2x^2\sec(ax)-a^2x^2\log\left(1-ie^{iax}\right)\tan(ax)+a^2x^2\log\left(1+ie^{iax}\right)\tan(ax)-i\text{PolyLog}(2,-ie^{iax})\,(1+ax\tan(ax))+i\text{PolyLog}(2,ie^{iax})\,(1+ax\tan(ax))}{a^4(1+ax\tan(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] -((-(a*x*Log[1 - I*E^(I*a*x)]) + a*x*Log[1 + I*E^(I*a*x)] + Sec[a*x] + a^2*
x^2*Sec[a*x] - a^2*x^2*Log[1 - I*E^(I*a*x)]*Tan[a*x] + a^2*x^2*Log[1 + I*E^
(I*a*x)]*Tan[a*x] - I*PolyLog[2, (-I)*E^(I*a*x)]*(1 + a*x*Tan[a*x]) + I*Pol
yLog[2, I*E^(I*a*x)]*(1 + a*x*Tan[a*x]))/(a^4*(1 + a*x*Tan[a*x])))
```

Maple [F]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax \sin(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

```
[Out] int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(89) = 178.
time = 2.52, size = 290, normalized size = 2.64

$$-\frac{2\,a^2x^2 - (-i\,ax\sin(ax) - i\cos(ax))\mathrm{Li}_2(i\cos(ax) + \sin(ax)) - (-i\,ax\sin(ax) - i\cos(ax))\mathrm{Li}_2(i\cos(ax) - \sin(ax)) - (i\,ax\sin(ax) + i\cos(ax))\mathrm{Li}_2(-i\cos(ax) + \sin(ax)) - (i\,ax\sin(ax) + i\cos(ax))\mathrm{Li}_2(-i\cos(ax) - \sin(ax)) - (a^2x^2\sin(ax) + ax\cos(ax))\log(i\cos(ax) + \sin(ax) + 1) + (a^2x^2\sin(ax) + ax\cos(ax))\log(i\cos(ax) - \sin(ax) + 1) - (a^2x^2\sin(ax) + ax\cos(ax))\log(-i\cos(ax) + \sin(ax) + 1) + (a^2x^2\sin(ax) + ax\cos(ax))\log(-i\cos(ax) - \sin(ax) + 1) + 2}{2\,(a^5x\sin(ax) + a^4\cos(ax))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*x^2 - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) + sin(a*x
)) - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) - sin(a*x)) - (I*a*x*s
in(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) + sin(a*x)) - (I*a*x*sin(a*x) + I*c
os(a*x))*dilog(-I*cos(a*x) - sin(a*x)) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*
log(I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(I*co
s(a*x) - sin(a*x) + 1) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x)
+ sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x) - sin(a
*x) + 1) + 2)/(a^5*x*sin(a*x) + a^4*cos(a*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sec(ax)}{(ax \sin(ax) + \cos(ax))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
[Out] Integral(x**3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
[Out] integrate(x^3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos\left(a\,x\right)\,{\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2),x)
[Out] int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.602 $\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{2ix^2}{a^3}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{2i\mathrm{PolyLog}\left(2,-e^{2iax}\right)}{a^5}-\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^5}+\frac{2x^2\tan(}{a^3}$$

```
[Out] -2*I*x^2/a^3+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5-x*
sec(a*x)^2/a^4-x^3*sec(a*x)^3/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^5+2*x^
2*tan(a*x)/a^3+x^2*sec(a*x)^2*tan(a*x)/a^3
```

Rubi [A]
time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4697, 4271, 3852, 8, 4269, 3800, 2221, 2317, 2438}

$$-\frac{2i\mathrm{Li}_2\left(-e^{2iax}\right)}{a^5}+\frac{\tan(ax)}{a^5}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{x\sec^2(ax)}{a^4}-\frac{2ix^2}{a^3}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\tan(ax)\sec^2(ax)}{a^3}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] ((-2*I)*x^2)/a^3 + (4*x*Log[1 + E^((2*I)*a*x)])/a^4 - ((2*I)*PolyLog[2, -E^
((2*I)*a*x)])/a^5 - (x*Sec[a*x]^2)/a^4 - (x^3*Sec[a*x]^3)/(a^2*(Cos[a*x] +
a*x*Sin[a*x])) + Tan[a*x]/a^5 + (2*x^2*Tan[a*x])/a^3 + (x^2*Sec[a*x]^2*Tan[
a*x])/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4697

```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Sec[a*x]^
(n + 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] + Dist[b^2*((n + 1)/d^2), In
t[(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
 EqQ[a*c - d, 0] && EqQ[m, n + 2]
```

Rubi steps

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = -\frac{x^3 \sec^3(ax)}{a^2(\cos(ax) + ax\sin(ax))} + \frac{3\int x^2 \sec^4(ax)\,dx}{a^2}$$

$$= -\frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{x^2\sec^2(ax)\tan(ax)}{a^3} + \frac{\int \sec^2(ax}{a^4}$$

$$= -\frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{2x^2\tan(ax)}{a^3} + \frac{x^2\sec^2(ax)\tan(a}{a^3}$$

$$= -\frac{2ix^2}{a^3} - \frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\tan(ax)}{a^5} + \frac{2x^2\tan(ax)}{a^3}$$

$$= -\frac{2ix^2}{a^3} + \frac{4x\log(1+e^{2iax})}{a^4} - \frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\tan(}{a}$$

$$= -\frac{2ix^2}{a^3} + \frac{4x\log(1+e^{2iax})}{a^4} - \frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{\tan(}{a}$$

$$= -\frac{2ix^2}{a^3} + \frac{4x\log(1+e^{2iax})}{a^4} - \frac{2i\operatorname{Li}_2(-e^{2iax})}{a^5} - \frac{x\sec^2(ax)}{a^4} - \frac{x^3\sec^3(a}{a^2(\cos(ax)+ax}$$

Mathematica [A]
time = 0.48, size = 130, normalized size = 1.05

$$\frac{-ax(1+2iax+a^2x^2-4\log(1+e^{2iax})) + (1+2a^2x^2-2ia^3x^3+4a^2x^2\log(1+e^{2iax}))\tan(ax) + a^3x^3\tan^2(ax) - 2i\text{PolyLog}(2,-e^{2iax})(1+ax\tan(ax))}{a^5(1+ax\tan(ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

```
[Out] (-(a*x*(1 + (2*I)*a*x + a^2*x^2 - 4*Log[1 + E^((2*I)*a*x)])) + (1 + 2*a^2*x
^2 - (2*I)*a^3*x^3 + 4*a^2*x^2*Log[1 + E^((2*I)*a*x)])*Tan[a*x] + a^3*x^3*T
an[a*x]^2 - (2*I)*PolyLog[2, -E^((2*I)*a*x)]*(1 + a*x*Tan[a*x]))/(a^5*(1 +
a*x*Tan[a*x]))
```

Maple [A]
time = 0.87, size = 141, normalized size = 1.14

method	result	size
risch	$-\frac{2i(-2ia^2x^2e^{2iax}+2a^3x^3-2ia^2x^2+ax\,e^{2iax}-ie^{2iax}+ax-i)}{(1+e^{2iax})(ax\,e^{2iax}-ax+ie^{2iax}+i)a^5} - \frac{4ix^2}{a^3} + \frac{4x\ln(1+e^{2iax})}{a^4} - \frac{2i\,\text{polylog}(2,-e^{2iax})}{a^5}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(-2*I*a^2*x^2*exp(2*I*a*x)+2*a^3*x^3-2*I*a^2*x^2+a*x*exp(2*I*a*x)-I*ex
p(2*I*a*x)+a*x-I)/(1+exp(2*I*a*x))/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)/
a^5-4*I/a^3*x^2+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(113) = 226$.
time = 0.51, size = 372, normalized size = 3.00

$$-\frac{2\left(ax + 2\left(a^{2}x^{2} - 2i\,ax\cos\left(2\,ax\right) + 2\,ax\sin\left(2\,ax\right) - i\,ax - \left(a^{2}x^{2} + i\,ax\right)\cos\left(4\,ax\right) + \left(-i\,a^{2}x^{2} + ax\right)\sin\left(4\,ax\right)\right)\arctan\left(\sin\left(2\,ax\right), \cos\left(2\,ax\right) + 1\right) + 2\left(a^{3}x^{3} + i\,a^{2}x^{2}\right)\cos\left(4\,ax\right) + \left(2i\,a^{2}x^{2} + ax - i\right)\cos\left(2\,ax\right) - \left(ax - \left(ax + i\right)\cos\left(4\,ax\right) - \left(i\,ax - 1\right)\sin\left(4\,ax\right) - 2i\,\cos\left(2\,ax\right) + 2\,\sin\left(2\,ax\right) - i\right){\rm Li}_2\left(-e^{\left(2i\,ax\right)}\right) + \left(-i\,a^{2}x^{2} - 2\,ax\cos\left(2\,ax\right) - 2i\,ax\sin\left(2\,ax\right) - ax + \left(i\,a^{2}x^{2} - ax\right)\cos\left(4\,ax\right) - \left(a^{2}x^{2} + i\,ax\right)\sin\left(4\,ax\right)\right)\log\left(\cos\left(2\,ax\right)^{2} + \sin\left(2\,ax\right)^{2} + 2\,\cos\left(2\,ax\right) + 1\right) + 2\left(i\,a^{3}x^{3} - a^{2}x^{2}\right)\sin\left(4\,ax\right) - \left(2\,a^{2}x^{2} - i\,ax - 1\right)\sin\left(2\,ax\right) - i\right)}{\left(i\,ax + \left(-i\,ax + 1\right)\cos\left(4\,ax\right) + \left(ax + i\right)\sin\left(4\,ax\right) + 2\,\cos\left(2\,ax\right) + 2i\,\sin\left(2\,ax\right) + 1\right)a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*x + 2*(a^2*x^2 - 2*I*a*x*cos(2*a*x) + 2*a*x*sin(2*a*x) - I*a*x - (a^2
*x^2 + I*a*x)*cos(4*a*x) + (-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(sin(2*a*x
), cos(2*a*x) + 1) + 2*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) + (2*I*a^2*x^2 + a*
x - I)*cos(2*a*x) - (a*x - (a*x + I)*cos(4*a*x) - (I*a*x - 1)*sin(4*a*x) -
2*I*cos(2*a*x) + 2*sin(2*a*x) - I)*dilog(-e^(2*I*a*x)) + (-I*a^2*x^2 - 2*a*
x*cos(2*a*x) - 2*I*a*x*sin(2*a*x) - a*x + (I*a^2*x^2 - a*x)*cos(4*a*x) - (a
^2*x^2 + I*a*x)*sin(4*a*x))*log(cos(2*a*x)^2 + sin(2*a*x)^2 + 2*cos(2*a*x)
+ 1) + 2*(I*a^3*x^3 - a^2*x^2)*sin(4*a*x) - (2*a^2*x^2 - I*a*x - 1)*sin(2*a
*x) - I)/((I*a*x + (-I*a*x + 1)*cos(4*a*x) + (a*x + I)*sin(4*a*x) + 2*cos(2
*a*x) + 2*I*sin(2*a*x) + 1)*a^5)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(113) = 226$.
time = 3.18, size = 382, normalized size = 3.08

$$\frac{a^{3}x^{3} - \left(2\,a^{3}x^{3} + ax\right)\cos\left(ax\right)^{2} + \left(2\,a^{2}x^{2} + 1\right)\cos\left(ax\right)\sin\left(ax\right) - 2\left(-i\,ax\cos\left(ax\right)\sin\left(ax\right) - i\,\cos\left(ax\right)^{2}\right){\rm Li}_2\left(i\,\cos\left(ax\right) + \sin\left(ax\right)\right) - 2\left(i\,ax\cos\left(ax\right)\sin\left(ax\right) + i\,\cos\left(ax\right)^{2}\right){\rm Li}_2\left(i\,\cos\left(ax\right) - \sin\left(ax\right)\right) - 2\left(i\,ax\cos\left(ax\right)\sin\left(ax\right) + i\,\cos\left(ax\right)^{2}\right){\rm Li}_2\left(-i\,\cos\left(ax\right) + \sin\left(ax\right)\right) - 2\left(-i\,ax\cos\left(ax\right)\sin\left(ax\right) - i\,\cos\left(ax\right)^{2}\right){\rm Li}_2\left(-i\,\cos\left(ax\right) - \sin\left(ax\right)\right) + 2\left(a^{2}x^{2}\cos\left(ax\right)\sin\left(ax\right) + ax\cos\left(ax\right)^{2}\right)\log\left(i\,\cos\left(ax\right) + \sin\left(ax\right) + 1\right) + 2\left(a^{2}x^{2}\cos\left(ax\right)\sin\left(ax\right) + ax\cos\left(ax\right)^{2}\right)\log\left(i\,\cos\left(ax\right) - \sin\left(ax\right) + 1\right) + 2\left(a^{2}x^{2}\cos\left(ax\right)\sin\left(ax\right) + ax\cos\left(ax\right)^{2}\right)\log\left(-i\,\cos\left(ax\right) + \sin\left(ax\right) + 1\right) + 2\left(a^{2}x^{2}\cos\left(ax\right)\sin\left(ax\right) + ax\cos\left(ax\right)^{2}\right)\log\left(-i\,\cos\left(ax\right) - \sin\left(ax\right) + 1\right)}{a^{6}x\cos\left(ax\right)\sin\left(ax\right) + a^{5}\cos\left(ax\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] (a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x)
 - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2)*dilog(I*cos(a*x) + sin(a*x))
 - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2)*dilog(I*cos(a*x) - sin(a*x))
- 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2)*dilog(-I*cos(a*x) + sin(a*x))
- 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2)*dilog(-I*cos(a*x) - sin(a*x))
 + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x) + sin(a*x)
 + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x) - sin
(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(-I*cos(a*x)
 + sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(-I*co
s(a*x) - sin(a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\sec^{2}\left(ax\right)}{\left(ax\sin\left(ax\right) + \cos\left(ax\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*sec(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

```
[Out] Integral(x**4*sec(a*x)**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*sec(a*x)^2/(a*x*sin(a*x) + cos(a*x))^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\cos\left(a\,x\right)^2\,{\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2),x)
```

```
[Out] int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.603 $\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=157

$$-\frac{2c\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}}-\frac{4\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{35b}-6$$

```
[Out] -6/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c-2/5*c*tan(2*b*x+2*a)/b
/(-c+c*sec(2*b*x+2*a))^(1/2)+1/7*c*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)/b/(-c+c*
sec(2*b*x+2*a))^(1/2)-4/35*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A]
time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4482, 3888, 3885, 4086, 3877}

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35bc}-\frac{4\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{35b}-\frac{2c\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-2*c*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sec[2*a +
2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (4*Sqrt[-c
 + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(35*b) - (6*(-c + c*Sec[2*a + 2*b*
x])^(3/2)*Tan[2*a + 2*b*x])/(35*b*c)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
```

```
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^4(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6}{7}\int \sec^3(2a+2bx$$
$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{6(-c+c\sec(2a+}{}$$
$$= \frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{4\sqrt{-c+c\sec(2a}}{}$$
$$= -\frac{2c\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sec^3(2a+2bx)\tan(}{7b\sqrt{-c+c\sec(2a}}$$

Mathematica [A]
time = 0.17, size = 64, normalized size = 0.41

$$-\frac{(7\cos(3(a+bx))+2\cos(7(a+bx)))\csc(a+bx)\sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/35*((7*Cos[3*(a + b*x)] + 2*Cos[7*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*
x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

Maple [A]
time = 2.68, size = 98, normalized size = 0.62

method	result	size
default	$-\frac{\sqrt{2}\,\cos(bx+a)\sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}}\,(128(\cos^6(bx+a))-224(\cos^4(bx+a))+140(\cos^2(bx+a))-35)\sqrt{4}}{70b\sin(bx+a)(2(\cos^2(bx+a))-1)^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/70*2^(1/2)/b*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(128*c
os(b*x+a)^6-224*cos(b*x+a)^4+140*cos(b*x+a)^2-35)/sin(b*x+a)/(2*cos(b*x+a)^
2-1)^3*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -8/35*(70*(b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2 + 2*b*cos(4*b*x + 4*
a) + b)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(3/4)*sqrt(c)*integrate(-(((cos(20*b*x + 20*a)*cos(4*b*x + 4*a) + 4*cos(16*
b*x + 16*a)*cos(4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 4*co
s(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*b*x + 20*a)*s
in(4*b*x + 4*a) + 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*sin(12*b*x + 12
*a)*sin(4*b*x + 4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*
a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*
x + 4*a)*sin(20*b*x + 20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos
(4*b*x + 4*a)*sin(12*b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - co
s(20*b*x + 20*a)*sin(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) -
 6*cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a
))*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(5/2*arcta
n2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(20*b*x + 2
0*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b*x + 4*a)*sin(12*b*
x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(20*b*x + 20*a)*sin(4*
b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6*cos(12*b*x + 12*a)*s
in(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(
4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(20*b*x + 20*a)*cos(4*b*x + 4*a
) + 4*cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x
+ 4*a) + 4*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*
b*x + 20*a)*sin(4*b*x + 4*a) + 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*si
```

```
n(12*b*x + 12*a)*sin(4*b*x + 4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + s
in(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
))*sin(5/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/(((2*(4*cos(16*b*x
 + 16*a) + 6*cos(12*b*x + 12*a) + 4*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*co
s(20*b*x + 20*a) + cos(20*b*x + 20*a)^2 + 8*(6*cos(12*b*x + 12*a) + 4*cos(8
*b*x + 8*a) + cos(4*b*x + 4*a))*cos(16*b*x + 16*a) + 16*cos(16*b*x + 16*a)^
2 + 12*(4*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 36*cos(
12*b*x + 12*a)^2 + 16*cos(8*b*x + 8*a)^2 + 8*cos(8*b*x + 8*a)*cos(4*b*x + 4
*a) + cos(4*b*x + 4*a)^2 + 2*(4*sin(16*b*x + 16*a) + 6*sin(12*b*x + 12*a) +
 4*sin(8*b*x + 8*a) + sin(4*b*x + 4*a))*sin(20*b*x + 20*a) + sin(20*b*x + 2
0*a)^2 + 8*(6*sin(12*b*x + 12*a) + 4*sin(8*b*x + 8*a) + sin(4*b*x + 4*a))*s
in(16*b*x + 16*a) + 16*sin(16*b*x + 16*a)^2 + 12*(4*sin(8*b*x + 8*a) + sin(
4*b*x + 4*a))*sin(12*b*x + 12*a) + 36*sin(12*b*x + 12*a)^2 + 16*sin(8*b*x +
 8*a)^2 + 8*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2
*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (2*(4*cos(16*b*x + 1
6*a) + 6*cos(12*b*x + 12*a) + 4*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(20
*b*x + 20*a) + cos(20*b*x + 20*a)^2 + 8*(6*cos(12*b*x + 12*a) + 4*cos(8*b*x
 + 8*a) + cos(4*b*x + 4*a))*cos(16*b*x + 16*a) + 16*cos(16*b*x + 16*a)^2 +
12*(4*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 36*cos(12*b
*x + 12*a)^2 + 16*cos(8*b*x + 8*a)^2 + 8*cos(8*b*x + 8*a)*cos(4*b*x + 4*a)
+ cos(4*b*x + 4*a)^2 + 2*(4*sin(16*b*x + 16*a) + 6*sin(12*b*x + 12*a) + 4*s
in(8*b*x + 8*a) + sin(4*b*x + 4*a))*sin(20*b*x + 20*a) + sin(20*b*x + 20*a)
^2 + 8*(6*sin(12*b*x + 12*a) + 4*sin(8*b*x + 8*a) + sin(4*b*x + 4*a))*sin(1
6*b*x + 16*a) + 16*sin(16*b*x + 16*a)^2 + 12*(4*sin(8*b*x + 8*a) + sin(4*b*
x + 4*a))*sin(12*b*x + 12*a) + 36*sin(12*b*x + 12*a)^2 + 16*sin(8*b*x + 8*a
)^2 + 8*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arc
tan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*x + 4*a)^2 + sin
(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) + (7*cos(7/2*arctan2(s
in(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*a) + (7*cos(4*b*x +
4*a) + 2)*sin(7/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sqrt(c
))/((b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2 + 2*b*cos(4*b*x + 4*a) + b
)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4))
```

Fricas [A]
time = 2.79, size = 106, normalized size = 0.68

$$-\frac{\sqrt{2}\left(35\tan\left(bx+a\right)^{6}-35\tan\left(bx+a\right)^{4}+49\tan\left(bx+a\right)^{2}-9\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{35\left(b\tan\left(bx+a\right)^{7}-3\,b\tan\left(bx+a\right)^{5}+3\,b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/35*sqrt(2)*(35*tan(b*x + a)^6 - 35*tan(b*x + a)^4 + 49*tan(b*x + a)^2 -
```

```
9)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^7 - 3*b*tan
(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 8.88, size = 463, normalized size = 2.95

$$-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}\,16\mathrm{i}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{\left(\frac{8\mathrm{i}}{7\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,8\mathrm{i}}{7\,b}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^3}-\frac{\left(\frac{8\mathrm{i}}{5\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,64\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}\,8\mathrm{i}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^4,x)
```

```
[Out] ((8i/(7*b) - (exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*1i - 1i
)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4
i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^3) - (e
xp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i -
 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*16i)/(35*b
*(exp(a*2i + b*x*2i) - 1)) - ((8i/(5*b) - (exp(a*2i + b*x*2i)*64i)/(35*b))*
((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i +
 b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(
exp(a*4i + b*x*4i) + 1)^2) - (exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i
 - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i +
b*x*4i) + 1)))^(1/2)*8i)/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i)
 + 1))
```

3.604 $\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=110

$$\frac{7c\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}}+\frac{2\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{15b}+\frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5bc}$$

```
[Out] 1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c+7/15*c*tan(2*b*x+2*a)/b/
(-c+c*sec(2*b*x+2*a))^(1/2)+2/15*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A]
time = 0.20, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3885, 4086, 3877}

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}+\frac{2\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15b}+\frac{7c\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*c*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*Sqrt[-c +
c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b*c)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
```

```
e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5bc} + \frac{2\int \sec(2$$
$$= \frac{2\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{15b} + \frac{(-c+c$$
$$= \frac{7c\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{2\sqrt{-c+c\sec(2a+}{15$$

Mathematica [A]
time = 0.13, size = 62, normalized size = 0.56

$$\frac{(5\cos(a+bx)+2\cos(5(a+bx)))\csc(a+bx)\sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
[Out] ((5*Cos[a + b*x] + 2*Cos[5*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*x)]^2*Sqrt
[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(15*b)
```

Maple [A]
time = 0.78, size = 88, normalized size = 0.80

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}}\,\cos(bx+a)(32(\cos^4(bx+a))-40(\cos^2(bx+a))+15)\sqrt{4}}{30b\sin(bx+a)(2(\cos^2(bx+a))-1)^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

[Out] 1/30*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+15)/sin(b*x+a)/(2*cos(b*x+a)^2-1)^2*4^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] 4/15*(30*(b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2 + 2*b*cos(4*b*x + 4*a
) + b)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)*sqrt(c)*integrate(-((((cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*cos(12*b
*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b
*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x + 12*a)*si
n(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*
cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*
a)*sin(16*b*x + 16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*b*x
 + 4*a)*sin(8*b*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(12*b
*x + 12*a)*sin(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*
arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*
x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 3*co
s(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - c
os(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(12*b*x + 12*a)*sin(4*b*x + 4*a)
- 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)) - (cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*cos(12*b*x
 + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x
 + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x + 12*a)*sin(
4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*si
n(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(si
n(4*b*x + 4*a), cos(4*b*x + 4*a))))/(((2*(3*cos(12*b*x + 12*a) + 3*cos(8*b*
x + 8*a) + cos(4*b*x + 4*a))*cos(16*b*x + 16*a) + cos(16*b*x + 16*a)^2 + 6*
(3*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 9*cos(12*b*x +
 12*a)^2 + 9*cos(8*b*x + 8*a)^2 + 6*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + 2*(3*sin(12*b*x + 12*a) + 3*sin(8*b*x + 8*a) + sin(4*b*x
+ 4*a))*sin(16*b*x + 16*a) + sin(16*b*x + 16*a)^2 + 6*(3*sin(8*b*x + 8*a) +
 sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + 9*sin(12*b*x + 12*a)^2 + 9*sin(8*b*
x + 8*a)^2 + 6*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(
1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (2*(3*cos(12*b*x
+ 12*a) + 3*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(16*b*x + 16*a) + cos(1
6*b*x + 16*a)^2 + 6*(3*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x + 12
*a) + 9*cos(12*b*x + 12*a)^2 + 9*cos(8*b*x + 8*a)^2 + 6*cos(8*b*x + 8*a)*co
s(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(3*sin(12*b*x + 12*a) + 3*sin(8*b*x
```

```
 + 8*a) + sin(4*b*x + 4*a))*sin(16*b*x + 16*a) + sin(16*b*x + 16*a)^2 + 6*(
3*sin(8*b*x + 8*a) + sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + 9*sin(12*b*x +
12*a)^2 + 9*sin(8*b*x + 8*a)^2 + 6*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(
4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4))
, x) - (5*cos(5/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b
*x + 4*a) + (5*cos(4*b*x + 4*a) + 2)*sin(5/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1)))*sqrt(c))/((b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2
 + 2*b*cos(4*b*x + 4*a) + b)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*c
os(4*b*x + 4*a) + 1)^(1/4))
```

Fricas [**A**]
time = 2.71, size = 84, normalized size = 0.76

$$\frac{\sqrt{2}\,\left(15\,\tan\left(bx+a\right)^{4}-10\,\tan\left(bx+a\right)^{2}+7\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{15\,\left(b\tan\left(bx+a\right)^{5}-2\,b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/15*sqrt(2)*(15*tan(b*x + a)^4 - 10*tan(b*x + a)^2 + 7)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*tan(b
*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 11.03, size = 148, normalized size = 1.35

$$\frac{4\left(e^{a\,4i+b\,x\,4i}\,5i+e^{a\,6i+b\,x\,6i}\,5i+e^{a\,10i+b\,x\,10i}\,2i+2i\right)\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{15\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^3,x)
```

```
[Out] (4*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)*2i
 + 2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp
(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(15*b*(exp(a*2i + b*
x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

3.605 $\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=72

$$-\frac{c\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}}+\frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3b}$$

[Out] -1/3*c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b

Rubi [A]
time = 0.14, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4482, 3883, 3877}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

[In] Int[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] -1/3*(c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sec^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3b} - \frac{1}{3}\int \sec(2a$$
$$= -\frac{c\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2b}}{3b}$$

Mathematica [A]
time = 0.13, size = 44, normalized size = 0.61

$$\frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,(-\cot(a+bx)+\tan(2(a+bx)))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]*(-Cot[a + b*x] + Tan[2*(a + b*x)]))/
(3*b)
```

Maple [A]
time = 0.94, size = 78, normalized size = 1.08

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}}\,\cos(bx+a)\left(4\left(\cos^2(bx+a)\right)-3\right)\sqrt{4}}{6b\sin(bx+a)\left(2\left(\cos^2(bx+a)\right)-1\right)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)*(4*cos(
b*x+a)^2-3)/sin(b*x+a)/(2*cos(b*x+a)^2-1)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -2/3*(6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(3/4)*b*sqrt(c)*integrate(-(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8
*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(
4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)
*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12
*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2
(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a
), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x
 + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*
x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*
sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), c
os(4*b*x + 4*a))))/(((2*(2*cos(8*b*x + 8*a) + cos(4*b*x + 4*a))*cos(12*b*x
+ 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^2 + 4*cos(8*b*x + 8*a)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b*x + 8*a) + sin(4*b*x +
 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4*sin(8*b*x + 8*a)^2 + 4
*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arctan2(si
n(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (2*(2*cos(8*b*x + 8*a) + cos(4*
b*x + 4*a))*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^
2 + 4*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b
*x + 8*a) + sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4
*sin(8*b*x + 8*a)^2 + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a
)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*
x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) + sqrt
(c)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4)*b)
```

Fricas [A]
time = 3.46, size = 64, normalized size = 0.89

$$-\frac{\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(3\,\tan\left(bx+a\right)^{2}-1\right)}{3\,\left(b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/3*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(3*tan(b*x + a)^2
 - 1)/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 413066 vs. $2(64) = 128$.
time = 120.44, size = 413066, normalized size = 5737.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/3*sqrt(2)*(((((((c^5*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 88
0*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^1
0 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(
1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a
)^2 + 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)^2*tan
(1/2*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 + tan(1
/2*b*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*x + 2*
a)^2*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2*a)^4
- 6*tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(1/2*a)
^2 + 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)*tan(1
/2*a)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2*a)*tan(
1/2*a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15*tan(1/2*
b*x + 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*
a))*tan(1/2*a)^174 - 147*c^5*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*ta
n(1/2*b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 -
6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^
8 - 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/
2*a)^10 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 6
```

```
6*tan(1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x
+ 2*a)^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1
/2*b*x + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan
(1/2*a)^2 + 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)
^2*tan(1/2*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 +
 tan(1/2*b*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2
*a)^4 - 6*tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(
1/2*a)^2 + 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)
*tan(1/2*a)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2*a
)*tan(1/2*a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*ta
n(1/2*a))*tan(1/2*a)^172 + 441*c^5*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 -
 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)
^11 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1
/2*a)^8 - 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*
tan(1/2*a)^10 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*t
an(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 -
 2970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)
^9 - 66*tan(1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/
2*b*x + 2*a)^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168
*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)
^4*tan(1/2*a)^2 + 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*
a)^6 + tan(1/2*b*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*t
an(1/2*a)^4 - 6*tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 6
6*tan(1/2*a)^2 + 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x
+ 2*a)*tan(1/2*a)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) +
 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a)
+ 3*tan(1/2*a))*tan(1/2*a)^170 + 185709*c^5*sgn(tan(1/2*b*x + 2*a)^4*tan(1/
2*a)^12 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^11 - 6*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^8 - 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^10 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^1
2 - 924*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1
/2*a)^7 - 2970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*t
an(1/2*a)^9 - 66*tan(1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 31
68*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^6 - 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^7 + 495...
```

Mupad [B]

time = 7.34, size = 129, normalized size = 1.79

$$-\frac{2\left(\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^2,x)
```

```
[Out] -(2*(exp(a*6i + b*x*6i)*1i + 1i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i
 + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(
1/2))/(3*b*(exp(a*2i + b*x*2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) -
1))
```

3.606 $\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=33

$$\frac{c\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4482, 3877}

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx &= \int \sec(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx\\ &= \frac{c\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 0.91

$$\frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cot[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

Maple [A]
time = 0.81, size = 52, normalized size = 1.58

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\frac{c\left(\sin^{2}\left(bx+a\right)\right)}{2\left(\cos^{2}\left(bx+a\right)\right)-1}}\,\cos\left(bx+a\right)\sqrt{4}}{2b\sin\left(bx+a\right)}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2*2^(1/2)/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*cos(b*x+a)/sin(b*x+
a)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] (2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)
*b*sqrt(c)*integrate(-(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*
a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(8*b*x
 + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x + 4*
a), -cos(4*b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x +
4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x +
 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(8*b
*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*
x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/(((cos
(8*b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ sin(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*
a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(8*
b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + s
in(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^
```

```
2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)), x) - sqrt(c
)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))/((cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*b)
```

Fricas [**A**]
time = 3.37, size = 40, normalized size = 1.21

$$\frac{\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 80097 vs. $2(31) = 62$.
time = 30.73, size = 80097, normalized size = 2427.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] -sqrt(2)*(((c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 880*tan(1/2
*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 48*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x + 2*a)^4*t
an(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/2*b*x + 2
*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(1/2*a)^10
```

+ 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)^3*tan(1/
2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x + 2*a)*t
an(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 + 880
*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^4
+ 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 + tan(1/2*b*x +
2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*x + 2*a)^2*tan(
1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2*a)^4 - 6*tan(1
/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(1/2*a)^2 + 1)*s
gn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)*tan(1/2*a)^6 +
10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^4
+ 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15*tan(1/2*b*x + 2*a
)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))*tan(1
/2*a)^78 + 21*c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*b*x + 2
*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 880*tan(1
/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 48*
tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/2*b*x +
2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(1/2*a)^
10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)^3*tan(
1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x + 2*a)
*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2 + 8
80*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)^2*tan(1/2*a)
^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 + tan(1/2*b*x
+ 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*x + 2*a)^2*ta
n(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2*a)^4 - 6*tan
(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(1/2*a)^2 + 1)
*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)*tan(1/2*a)^6
+ 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^
4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15*tan(1/2*b*x + 2
*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))*tan
(1/2*a)^76 + 105*c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*b*x
+ 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(1/2*
b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 880*ta
n(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 -
48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/2*b*
x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(1/2*
a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)^3*t
an(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x + 2
*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^2
+ 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)^2*tan(1/2
*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 + tan(1/2*b
*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*x + 2*a)^2
*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2*a)^4 - 6*

```
tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(1/2*a)^2 +
 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a)*tan(1/2*a
)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2*a)*tan(1/2*
a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15*tan(1/2*b*x
+ 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) + 3*tan(1/2*a))*
tan(1/2*a)^74 - 1771*c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(
1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 88
0*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^1
0 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x +
 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/
2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(
1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)
^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*ta...
```

Mupad [B]
time = 3.68, size = 87, normalized size = 2.64

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{c\,\left(\cos\left(2\,a+2\,b\,x\right)-\cos\left(6\,a+6\,b\,x\right)\right)}{3\,\cos\left(2\,a+2\,b\,x\right)+2\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+2}}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(sin(2*a + 2*b*x)*((c*(cos(2*a + 2*b*x) - cos(6*a + 6*b*x)))/(3*cos(2*a +
2*b*x) + 2*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 2))^(1/2))/(b*(cos(2*a + 2
*b*x) - 1))
```

3.607 $\int \sqrt{c \tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

[Out] -arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4482, 3859, 213}

$$-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]

[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/b)

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$
$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 1.62

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a + b
*x)]]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(Sqrt[2]*b))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(39) = 78$.
time = 0.89, size = 136, normalized size = 3.02

method	result	size
default	$-\frac{\sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a))-1}}\sin(bx+a)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{4}}{2b(-1+\cos(bx+a))}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+
cos(b*x+a))*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(39) = 78$.

time = 0.52, size = 430, normalized size = 9.56

$$\frac{\sqrt{c}\left(\log\left(4\sqrt{\cos\left(4bx+4a\right)^{2}+\sin\left(4bx+4a\right)^{2}+2\cos\left(4bx+4a\right)+1}\cos\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)^{2}+4\sqrt{\cos\left(4bx+4a\right)^{2}+\sin\left(4bx+4a\right)^{2}+2\cos\left(4bx+4a\right)+1}\sin\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)^{2}+8\left(\cos\left(4bx+4a\right)^{2}+\sin\left(4bx+4a\right)^{2}+2\cos\left(4bx+4a\right)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)+4\right)-\log\left(\cos\left(2bx+2a\right)^{2}+\sin\left(2bx+2a\right)^{2}+\sqrt{\cos\left(4bx+4a\right)^{2}+\sin\left(4bx+4a\right)^{2}+2\cos\left(4bx+4a\right)+1}\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)^{2}+\sin\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)^{2}\right)+2\left(\cos\left(4bx+4a\right)^{2}+\sin\left(4bx+4a\right)^{2}+2\cos\left(4bx+4a\right)+1\right)^{\frac{1}{4}}\left(\cos\left(2bx+2a\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)+\sin\left(2bx+2a\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(4bx+4a\right),\cos\left(4bx+4a\right)+1\right)\right)\right)\right)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c)*(log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 +
 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*s
in(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*(cos(4*b*x +
4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2) + 2*(cos(4*b
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(2*b*x
 + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + sin(2*b*
x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)))))/b
```

Fricas [A]
time = 3.65, size = 201, normalized size = 4.47

$$\left[\frac{\sqrt{c}\log\left(-\frac{c\tan\left(bx+a\right)^{5}-14\,c\tan\left(bx+a\right)^{3}-4\sqrt{2}\left(\tan\left(bx+a\right)^{4}-4\tan\left(bx+a\right)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\sqrt{c}+17\,c\tan\left(bx+a\right)}{\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)}\right)}{4b},\ \frac{\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)\sqrt{-c}}{c\tan\left(bx+a\right)^{3}-3\,c\tan\left(bx+a\right)}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(
b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 -
 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(
b*x + a)))/b, 1/2*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
 + a)))/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.608 $\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] 1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b-1
/2*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4482, 3890, 3859, 213}

$$\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}\int \sqrt{-c+c\sec(2a$$
$$= -\frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx,\right.}{}$$
$$= \frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} - \frac{c\,\mathrm{si}}{2b\sqrt{-c}}$$

Mathematica [A]
time = 0.17, size = 92, normalized size = 1.10

$$-\frac{\left(\cos(a+bx)-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}+\cos(3(a+bx))\right)\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/4*((Cos[a + b*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a
+ b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Ta
n[a + b*x]*Tan[2*(a + b*x)]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(72) = 144$.
time = 1.11, size = 391, normalized size = 4.65

method	result
default	$\frac{\sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a))-1}}\,\sin(bx+a)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\mathrm{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{4}}{2b(-1+\cos(bx+a))} - \frac{\sqrt{2}\,\sin(bx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+c
os(b*x+a))*4^(1/2)-1/8*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+
a)^2-1))^(1/2)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*x+a)*4^(1/2)*(-
1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-4*cos(b*x+a)^3+2*c
os(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(72) = 144$.
time = 0.59, size = 1049, normalized size = 12.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/16*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*x
+ 2*a) + (cos(2*b*x + 2*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)))*sqrt(c) - sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(s
qrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1
/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1)) + 1) + log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
 + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*s
in(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1))) + 1) - log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*co
```

```
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))) + 1)))/b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(72) = 144$.
time = 3.21, size = 351, normalized size = 4.18

$$\left[\frac{\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^{5}-14\,c\tan(bx+a)^{3}+4\sqrt{2}\left(\tan(bx+a)^{4}-4\tan(bx+a)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^{5}+2\tan(bx+a)^{3}+\tan(bx+a)}\right)+4\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)}{8\left(b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}, -\frac{\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{c\tan(bx+a)^{3}-3\,c\tan(bx+a)}\right)-2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)}{4\left(b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/8*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c
*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c
*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x
 + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4*sqrt(2)*sqrt(-c*tan(b*x + a
)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b
*x + a)), -1/4*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(
c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*tan(b*x + a
))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)\,\sqrt{c\tan(a+bx)\,\tan(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.609 $\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=129

$$-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b}+\frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}}-\frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] -3/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b+
3/8*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/4*c*cos(2*b*x+2*a)*sin
(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3890, 3859, 213}

$$\frac{3c\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (-3*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(8*b) + (3*c*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c
*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^2(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$
$$= -\frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{3}{4}\int \cos(2a+2bx$$
$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a}{4b\sqrt{-c+c\sec(2a+}$$
$$= \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a}{4b\sqrt{-c+c\sec(2a+}$$
$$= -\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} + \frac{3}{8b\sqrt{-}}$$

Mathematica [A]
time = 0.18, size = 105, normalized size = 0.81

$$\frac{\left(-3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)+2(\cot(a+bx)-\sin(2(a+bx))+\sin(4(a+bx)))\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Co
s[2*(a + b*x)]]*Csc[a + b*x] + 2*(Cot[a + b*x] - Sin[2*(a + b*x)] + Sin[4*(
a + b*x)]))*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(113) = 226$.
time = 0.82, size = 657, normalized size = 5.09

method	result

default	$-\frac{\sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a))-1}}\,\sin(bx+a)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{4}}{2b(-1+\cos(bx+a))}+\frac{\sqrt{2}\,\sin(bx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x
+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x
+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+
cos(b*x+a))*4^(1/2)+1/4*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x
+a)^2-1))^(1/2)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^
2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/
(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*x+a)*4^(1/2)*(
-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(
1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-4*cos(b*x+a)^3+2*
cos(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)+1/32*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*
x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(16*cos(b*x+a)^5-3*arctanh(1/2*cos(b*x+a)
*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)
^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1
/2)+4*cos(b*x+a)^3-3*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x
+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2
-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-6*cos(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. $2(113) = 226$.
time = 0.62, size = 1421, normalized size = 11.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/64*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*
x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) - co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*a) -
(cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) - ((cos(4*b*x + 4*a) - 2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)) + sin(4*b*x + 4*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
```

```
b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))*sqr
t(c) - 3*sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x
 + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arct
an2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) + 1) + log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*
arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x
 + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))
^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
+ 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))
)) + 1) - log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arc
tan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x +
4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)
*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(
cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))
+ 1)))/b
```

Fricas [A]

time = 3.04, size = 419, normalized size = 3.25

$$\left[\frac{3\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17c\tan(bx+a)}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right)-4\sqrt{2}\left(5\tan(bx+a)^4-4\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{32\left(b\tan(bx+a)^5+2b\tan(bx+a)^3+b\tan(bx+a)\right)}, \frac{3\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3c\tan(bx+a)}\right)-2\sqrt{2}\left(5\tan(bx+a)^4-4\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{16\left(b\tan(bx+a)^5+2b\tan(bx+a)^3+b\tan(bx+a)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c
*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b
```

```
*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*
tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(
2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*
x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a)), 1
/16*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(5*tan(b*x + a)
^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b
*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(2\,a+2\,b\,x\right)^2\,\sqrt{c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.610 $\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

Optimal. Leaf size=176

$$\frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b}-\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}}+\frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}}-c\,c$$

```
[Out] 5/16*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b-
5/16*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+5/24*c*cos(2*b*x+2*a)*s
in(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/6*c*cos(2*b*x+2*a)^2*sin(2*b*
x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3890, 3859, 213}

$$-\frac{5c\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}+\frac{5c\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}+\frac{5\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (5*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(16*b) - (5*c*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
5*c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
 - (c*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \cos^3(2a+2bx)\sqrt{-c+c\sec(2a+2bx)}\,dx$$

$$= -\frac{c\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{5}{6}\int \cos^2(2a+$$

$$= \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos^2(2a+2bx)\,\mathrm{s}}{6b\sqrt{-c+c\sec(}}$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\,\mathrm{si}}{24b\sqrt{-c+c\sec(}}$$

$$= -\frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\,\mathrm{si}}{24b\sqrt{-c+c\sec(}}$$

$$= \frac{5\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} - \frac{5}{16b\sqrt{-}}$$

Mathematica [A]
time = 0.20, size = 116, normalized size = 0.66

$$\frac{\left(-26\cot(a+bx)+15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)+30\sin(2(a+bx))-2\sin(4(a+bx))+4\sin(6(a+bx))\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((-26*Cot[a + b*x] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] + 30*Sin[2*(a + b*x)] - 2*S
in[4*(a + b*x)] + 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]
)/(96*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(156) = 312$.
time = 0.79, size = 933, normalized size = 5.30

method	result	size
default	Expression too large to display	933

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/b*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+
a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))/(-1+c
os(b*x+a))*4^(1/2)-3/8*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+
a)^2-1))^(1/2)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*x+a)*4^(1/2)*(-
1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-4*cos(b*x+a)^3+2*c
os(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)-3/32*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x
+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(16*cos(b*x+a)^5-3*arctanh(1/2*cos(b*x+a)*
4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/
2)+4*cos(b*x+a)^3-3*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+
a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-6*cos(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)-1
/192*2^(1/2)/b*sin(b*x+a)*(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(-1
28*cos(b*x+a)^7-16*cos(b*x+a)^5+15*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b
*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+15*arctanh(1/2
*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2)-20*cos(b*x+a)^3+30*cos(b*x+a))/(cos(b*x+a)^2-1)*4^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2333 vs. $2(156) = 312$.
time = 0.89, size = 2333, normalized size = 13.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/384*(8*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(s
```

```
in(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos
(6*b*x + 6*a))) - 1))*sin(6*b*x + 6*a) + (cos(6*b*x + 6*a) + 1)*sin(3/2*arc
tan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 12*(cos(2/3*arctan2(
sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
)) + 1)^(1/4)*((sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 5*si
n(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))))*cos(1/2*arctan2(sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) - 1)) + (cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*
b*x + 6*a))) - 3*cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 4)*
sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(
2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 15*sqrt(c
)*(log(sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*
b*x + 6*a), cos(6*b*x + 6*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
 + 6*a))) - 1))^2 + sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)
))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*a
rctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*
a), cos(6*b*x + 6*a))) - 1))^2 + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)) + 1) - log(sqrt(cos(2/3*arcta
n2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a
), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
*a))) + 1)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*
a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + sqrt(
cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a),
cos(6*b*x + 6*a))) + 1)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), c
os(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) -
1))^2 - 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b
*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a))) - 1)) + 1) + log(((cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos
(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3
*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), co
s(6*b*x + 6*a)))^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6
*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^
```

```
2)*sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*ar
ctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a))) + 1) + 2*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(
6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 +
 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arc
tan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))*sin(1/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) + cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6
*a)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))),
 -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))) + 1) - log(((
cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(1/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*cos(1/2*arctan2(sin(2/3*arctan2(sin(6
*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a))) - 1))^2 + (cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))
^2 + sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2)*sin(1/2*arctan
2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(si
n(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1))^2)*sqrt(cos(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
+ 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a)...
```

Fricas [A]
time = 2.90, size = 481, normalized size = 2.73

$$\left[\frac{15\left(\tan\left(bx+a\right)^{7}+3\tan\left(bx+a\right)^{5}+3\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^{5}-14c\tan(bx+a)^{3}+4\sqrt{2}\left(\tan(bx+a)^{4}-4\tan(bx+a)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\sqrt{c}+17c\tan(bx+a)}{\tan(bx+a)^{5}+2\tan(bx+a)^{3}+\tan(bx+a)}\right)+4\sqrt{2}\left(33\tan\left(bx+a\right)^{6}-19\tan\left(bx+a\right)^{4}-\tan\left(bx+a\right)^{2}-13\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{192\left(b\tan\left(bx+a\right)^{7}+3b\tan\left(bx+a\right)^{5}+3b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)},-\frac{15\left(\tan\left(bx+a\right)^{7}+3\tan\left(bx+a\right)^{5}+3\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{c\tan(bx+a)^{3}-3c\tan(bx+a)}\right)-2\sqrt{2}\left(33\tan\left(bx+a\right)^{6}-19\tan\left(bx+a\right)^{4}-\tan\left(bx+a\right)^{2}-13\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{96\left(b\tan\left(bx+a\right)^{7}+3b\tan\left(bx+a\right)^{5}+3b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(15*(tan(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x
+ a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a))) + 4*sqrt(2)*(33*tan(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x +
a)^2 - 13)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7
+ 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a)), -1/96*(15*(tan
(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*
arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)
^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(33*tan
(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x + a)^2 - 13)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(
b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(2\,a+2\,b\,x\right)^3\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.611 $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{34c^2\tan(2a+2bx)}{45b\sqrt{-c+c\sec(2a+2bx)}}-\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}}+\frac{68c\sqrt{-c}}{}$$

```
[Out] 34/105*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+34/45*c^2*tan(2*b*x+2*a
)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-17/63*c^2*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)/b
/(-c+c*sec(2*b*x+2*a))^(1/2)+1/9*c^2*sec(2*b*x+2*a)^4*tan(2*b*x+2*a)/b/(-c+
c*sec(2*b*x+2*a))^(1/2)+68/315*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A]
time = 0.34, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4482, 3899, 21, 3888, 3885, 4086, 3877}

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17c^2\tan(2a+2bx)\sec^3(2a+2bx)}{63b\sqrt{c\sec(2a+2bx)-c}}+\frac{34c^2\tan(2a+2bx)}{45b\sqrt{c\sec(2a+2bx)-c}}+\frac{34\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{105b}+\frac{68c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{315b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (34*c^2*Tan[2*a + 2*b*x])/(45*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (17*c^2*Se
c[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(63*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Sec[2*a + 2*b*x]^4*Tan[2*a + 2*b*x])/(9*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) + (68*c*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(315*b) + (34*(-c
 + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(105*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^4(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{9}(2c)\int \frac{\sec^4(2a}{}$$
$$= \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{9}(17c)\int \sec^4(2a$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+}{9b\sqrt{-c+}}$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+}{9b\sqrt{-c+}}$$
$$= -\frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+}{9b\sqrt{-c+}}$$
$$= \frac{34c^2\tan(2a+2bx)}{45b\sqrt{-c+c\sec(2a+2bx)}} - \frac{17c^2\sec^3(2a+2bx)\,\mathrm{t}}{63b\sqrt{-c+c\sec(2}}$$

Mathematica [A]
time = 0.24, size = 85, normalized size = 0.41

$$\frac{\cot(a+bx)\left(-84+188\cot(a+bx)\cot(2(a+bx))+52\sec(2(a+bx))-50\sec^2(2(a+bx))+35\sec^3(2(a+bx))\right)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Cot[a + b*x]*(-84 + 188*Cot[a + b*x]*Cot[2*(a + b*x)] + 52*Sec[2*(a + b*x)
] - 50*Sec[2*(a + b*x)]^2 + 35*Sec[2*(a + b*x)]^3)*(c*Tan[a + b*x]*Tan[2*(a
 + b*x)])^(3/2))/(315*b)
```

Maple [A]
time = 1.16, size = 105, normalized size = 0.50

method	result	siz
default	$\frac{2\sqrt{2}\,\left(2176\left(\cos^8(bx+a)\right)-4896\left(\cos^6(bx+a)\right)+4284\left(\cos^4(bx+a)\right)-1785\left(\cos^2(bx+a)\right)+315\right)\cos(bx+a)\left(\frac{c\left(\sin^2(bx+a)\right)}{2\left(\cos^2(bx+a)\right)-1}\right)^{\frac{3}{2}}}{315b(2(\cos^2(bx+a))-1)^3\sin(bx+a)^3}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/315*2^(1/2)/b*(2176*cos(b*x+a)^8-4896*cos(b*x+a)^6+4284*cos(b*x+a)^4-1785
*cos(b*x+a)^2+315)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*
cos(b*x+a)^2-1)^3/sin(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] -8/315*(630*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) +
 1)^(1/4)*((b*c*cos(4*b*x + 4*a)^4 + b*c*sin(4*b*x + 4*a)^4 + 4*b*c*cos(4*b
*x + 4*a)^3 + 6*b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(4*b*x + 4*a) + 2*(b*c*co
s(4*b*x + 4*a)^2 + 2*b*c*cos(4*b*x + 4*a) + b*c)*sin(4*b*x + 4*a)^2 + b*c)*
integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) +
1)^(1/4)*(((cos(20*b*x + 20*a)*cos(4*b*x + 4*a) + 4*cos(16*b*x + 16*a)*cos(
4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 4*cos(8*b*x + 8*a)*c
os(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*b*x + 20*a)*sin(4*b*x + 4*a)
+ 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*sin(12*b*x + 12*a)*sin(4*b*x +
4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(3/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(20*
b*x + 20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b*x + 4*a)*si
n(12*b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(20*b*x + 20*a)
*sin(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6*cos(12*b*x +
12*a)*sin(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3/2*arcta
n2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(7/2*arctan2(sin(4*b*x + 4
*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(20*b*x + 20*a) + 4*cos(4*b
*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 4*co
s(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(20*b*x + 20*a)*sin(4*b*x + 4*a) - 4*c
os(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6*cos(12*b*x + 12*a)*sin(4*b*x + 4*a)
- 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)) - (cos(20*b*x + 20*a)*cos(4*b*x + 4*a) + 4*cos(16*b*x
 + 16*a)*cos(4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 4*cos(8
*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*b*x + 20*a)*sin(
4*b*x + 4*a) + 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*sin(12*b*x + 12*a)
*sin(4*b*x + 4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^
2)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(7/2*arcta
n2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x +
 4*a)^4 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1
)*cos(20*b*x + 20*a)^2 + 16*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*co
s(4*b*x + 4*a) + 1)*cos(16*b*x + 16*a)^2 + 36*(cos(4*b*x + 4*a)^2 + sin(4*b
*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(12*b*x + 12*a)^2 + 16*(cos(4*b*x
```

```
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a)^2
+ 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)*sin(20*b*x + 20*a)^2 + 16*(cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(16*b*x + 16*a)^2 + 36*(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a)^2 +
 16*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(
8*b*x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x
+ 4*a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 4*
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(16*b
*x + 16*a) + 6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)*cos(12*b*x + 12*a) + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*
cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x +
 4*a))*cos(20*b*x + 20*a) + 8*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*
b*x + 4*a)^2 + 6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)*cos(12*b*x + 12*a) + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x
 + 4*a))*cos(16*b*x + 16*a) + 12*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin
(4*b*x + 4*a)^2 + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos
(12*b*x + 12*a) + 8*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)
^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(4*b*x
+ 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)
^2 + 2*cos(4*b*x + 4*a) + 1)*sin(16*b*x + 16*a) + 6*(cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a) + 4*(cos(4*b
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)
 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(20*b
*x + 20*a) + 8*(sin(4*b*x + 4*a)^3 + 6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*
a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a) + 4*(cos(4*b*x + 4*a)^2 +
 sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a) + (cos(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(16*b*x + 16*a) +
12*(sin(4*b*x + 4*a)^3 + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos
(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a) + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)*sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + 8*(sin(4*b*x + 4*a)^3 + (co
s(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*
a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*...
```

Fricas [A]
time = 2.44, size = 132, normalized size = 0.63

$$\frac{2\sqrt{2}\left(315\,c\tan\left(bx+a\right)^{8}-525\,c\tan\left(bx+a\right)^{6}+819\,c\tan\left(bx+a\right)^{4}-423\,c\tan\left(bx+a\right)^{2}+94\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{315\left(b\tan\left(bx+a\right)^{9}-4\,b\tan\left(bx+a\right)^{7}+6\,b\tan\left(bx+a\right)^{5}-4\,b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/315*sqrt(2)*(315*c*tan(b*x + a)^8 - 525*c*tan(b*x + a)^6 + 819*c*tan(b*x
+ a)^4 - 423*c*tan(b*x + a)^2 + 94*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))/(b*tan(b*x + a)^9 - 4*b*tan(b*x + a)^7 + 6*b*tan(b*x + a)^5 - 4*b*t
an(b*x + a)^3 + b*tan(b*x + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
[Out] Timed out
```

Mupad [B]
time = 10.18, size = 594, normalized size = 2.86

$$\frac{\left(\frac{c\,16{}\mathrm{i}}{9\,b}+\frac{c\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,16{}\mathrm{i}}{9\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}^4}-\frac{\left(\frac{c\,40{}\mathrm{i}}{7\,b}+\frac{c\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,88{}\mathrm{i}}{63\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}^3}+\frac{\left(\frac{c\,24{}\mathrm{i}}{5\,b}-\frac{c\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,176{}\mathrm{i}}{105\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}^2}+\frac{c\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}\,272{}\mathrm{i}}{315\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}+\frac{c\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}}\,136{}\mathrm{i}}{315\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^4,x)
[Out] (((c*16i)/(9*b) + (c*exp(a*2i + b*x*2i)*16i)/(9*b))*((c*(exp(a*2i + b*x*2i)
*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i
 + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)
^4) - (((c*40i)/(7*b) + (c*exp(a*2i + b*x*2i)*88i)/(63*b))*((c*(exp(a*2i +
b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(e
xp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4
i) + 1)^3) + (((c*24i)/(5*b) - (c*exp(a*2i + b*x*2i)*176i)/(105*b))*((c*(ex
p(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i
) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4
i + b*x*4i) + 1)^2) + (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i
)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4
i) + 1)))^(1/2)*272i)/(315*b*(exp(a*2i + b*x*2i) - 1)) + (c*exp(a*2i + b*x*
2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*
2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*136i)/(315*b*(exp(a*2i +
 b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1))
```

3.612 $\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=148

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{-c+c\sec(2a+2bx)}}+\frac{19c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{105b}+\frac{2(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35b}$$

```
[Out] 2/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+1/7*(-c+c*sec(2*b*x+2*a))
^(5/2)*tan(2*b*x+2*a)/b/c-76/105*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))
^(1/2)+19/105*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A]
time = 0.24, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4482, 3885, 4086, 3878, 3877}

$$-\frac{76c^2\tan(2a+2bx)}{105b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}+\frac{2\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{35b}+\frac{19c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{105b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-76*c^2*Tan[2*a + 2*b*x])/(105*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (19*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(105*b) + (2*(-c + c*Sec[2*a
+ 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(35*b) + ((-c + c*Sec[2*a + 2*b*x])^(5/2)
*Tan[2*a + 2*b*x])/(7*b*c)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
```

```
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{(-c+c\sec(2a+2bx))^{5/2}\tan(2a+2bx)}{7bc} + \frac{2\int \sec$$

$$= \frac{2(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35b} + \frac{(-c+$$

$$= \frac{19c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{105b} + \frac{2(-c$$

$$= -\frac{76c^2\tan(2a+2bx)}{105b\sqrt{-c+c\sec(2a+2bx)}} + \frac{19c\sqrt{-c+c\sec$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.49

$$-\frac{\cot(a+bx)\left(-28+76\cot(a+bx)\cot(2(a+bx))+24\sec(2(a+bx))-15\sec^2(2(a+bx))\right)(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/105*(Cot[a + b*x]*(-28 + 76*Cot[a + b*x]*Cot[2*(a + b*x)] + 24*Sec[2*(a
+ b*x)] - 15*Sec[2*(a + b*x)]^2)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/b
```

Maple [A]
time = 0.65, size = 95, normalized size = 0.64

method	result	size
default	$-\frac{2\sqrt{2}\,\left(416\left(\cos^{6}\left(bx+a\right)\right)-728\left(\cos^{4}\left(bx+a\right)\right)+455\left(\cos^{2}\left(bx+a\right)\right)-105\right)\cos\left(bx+a\right)\left(\frac{c\left(\sin^{2}\left(bx+a\right)\right)}{2\left(\cos^{2}\left(bx+a\right)\right)-1}\right)^{\frac{3}{2}}}{105b\left(2\left(\cos^{2}\left(bx+a\right)\right)-1\right)^{2}\sin\left(bx+a\right)^{3}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*2^(1/2)/b*(416*cos(b*x+a)^6-728*cos(b*x+a)^4+455*cos(b*x+a)^2-105)*c
os(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)^2/si
n(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] 4/105*(210*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) +
1)^(3/4)*(3*(b*c*cos(4*b*x + 4*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 2*b*c*cos(4*
b*x + 4*a) + b*c)*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*c
os(4*b*x + 4*a) + 1)^(1/4)*(((cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*cos(1
2*b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x + 12*a)
*sin(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^
2)*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x +
 4*a)*sin(16*b*x + 16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*
b*x + 4*a)*sin(8*b*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(1
2*b*x + 12*a)*sin(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3
/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(5/2*arctan2(sin(4
*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 3
*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*b*x + 4*a)*sin(8*b*x + 8*a)
- cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(12*b*x + 12*a)*sin(4*b*x + 4*
a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1)) - (cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*cos(12*
b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*
b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x + 12*a)*s
in(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)
*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(5/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x + 4
```

```
*a)^4 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*
cos(16*b*x + 16*a)^2 + 9*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)*cos(12*b*x + 12*a)^2 + 9*(cos(4*b*x + 4*a)^2 + sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3
 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1
6*b*x + 16*a)^2 + 9*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)*sin(12*b*x + 12*a)^2 + 9*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)
^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2
*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b
*x + 4*a)*sin(4*b*x + 4*a)^2 + 3*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
 2*cos(4*b*x + 4*a) + 1)*cos(12*b*x + 12*a) + 3*(cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*
a)^2 + cos(4*b*x + 4*a))*cos(16*b*x + 16*a) + 6*(cos(4*b*x + 4*a)^3 + cos(4
*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 3*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2
 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*
b*x + 4*a))*cos(12*b*x + 12*a) + 6*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*
a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + 3*(cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a) + 3*(cos(4*b
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)
 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(16*b
*x + 16*a) + 6*(sin(4*b*x + 4*a)^3 + 3*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*
a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a) + (cos(4*b*x + 4*a)^2 + 2*c
os(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + 6*(sin(4*b*x +
4*a)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*si
n(8*b*x + 8*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2
 + (cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a)^2 + sin(4*b
*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(16*b*x + 16*a)^2 + 9*(cos(4*b*x +
 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(12*b*x + 12*a)^2
 + 9*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos
(8*b*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x +
4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(16*b*x + 16*a)^2 + 9*(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a)^2 + 9*
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*
x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*
a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 3*(cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(12*b*x +
 12*a) + 3*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) +
1)*cos(8*b*x + 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(16*b*x +
 16*a) + 6*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 3*(c
os(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x
+ 8*a) + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 6*(c
os(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*
x + 4*a)^3 + 3*(cos(4*b*x + 4*a)^2 + sin(4*b*x ...
```

Fricas [A]
time = 2.49, size = 111, normalized size = 0.75

$$-\frac{2\sqrt{2}\left(105\,c\tan\left(bx+a\right)^{6}-140\,c\tan\left(bx+a\right)^{4}+133\,c\tan\left(bx+a\right)^{2}-38\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{105\left(b\tan\left(bx+a\right)^{7}-3\,b\tan\left(bx+a\right)^{5}+3\,b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/105*sqrt(2)*(105*c*tan(b*x + a)^6 - 140*c*tan(b*x + a)^4 + 133*c*tan(b*x
 + a)^2 - 38*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a
)^7 - 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 9.26, size = 479, normalized size = 3.24

$$\frac{\left(\frac{c\,8\mathrm{i}}{7\,b}-\frac{c\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,8\mathrm{i}}{7\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^3}-\frac{\left(\frac{c\,4\mathrm{i}}{5\,b}-\frac{c\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,92\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}-\frac{\left(\frac{c\,4\mathrm{i}}{3\,b}+\frac{c\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,52\mathrm{i}}{105\,b}\right)\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}-\frac{c\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{\frac{c\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}\,104\mathrm{i}}{105\,b\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^3,x)
```

```
[Out] (((c*8i)/(7*b) - (c*exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*1
i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i +
```

```
 b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^3
) - (((c*4i)/(5*b) - (c*exp(a*2i + b*x*2i)*92i)/(35*b))*((c*(exp(a*2i + b*x
*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(
a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i)
 + 1)^2) - (((c*4i)/(3*b) + (c*exp(a*2i + b*x*2i)*52i)/(105*b))*((c*(exp(a*2
i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1
)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b
*x*4i) + 1)) - (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(
a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)
))^(1/2)*104i)/(105*b*(exp(a*2i + b*x*2i) - 1))
```

3.613 $\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}}-\frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{5b}+\frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2b}{5b}$$

```
[Out] 1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+4/5*c^2*tan(2*b*x+2*a)/b/(
-c+c*sec(2*b*x+2*a))^(1/2)-1/5*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)
/b
```

Rubi [A]
time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3883, 3878, 3877}

$$\frac{4c^2\tan(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{5b}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (4*c^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) - (c*Sqrt[-c +
 c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(5*b) + ((-c + c*Sec[2*a + 2*b*x])^(
3/2)*Tan[2*a + 2*b*x])/(5*b)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5b} - \frac{3}{5}\int \text{se}$$
$$= -\frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{5b} + \frac{(-c}{}$$
$$= \frac{4c^2\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\sqrt{-c+c\sec(2a+}}{5}$$

Mathematica [A]
time = 0.15, size = 59, normalized size = 0.54

$$\frac{\cot(a+bx)(-2+4\cot(a+bx)\cot(2(a+bx))+\sec(2(a+bx)))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
[Out] (Cot[a + b*x]*(-2 + 4*Cot[a + b*x]*Cot[2*(a + b*x)] + Sec[2*(a + b*x)])*(c*
Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/(5*b)
```

Maple [A]
time = 0.60, size = 85, normalized size = 0.77

method	result	size
default	$\frac{2\sqrt{2}\,(12(\cos^4(bx+a))-15(\cos^2(bx+a))+5)\cos(bx+a)\left(\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}\right)^{\frac{3}{2}}}{5b(2(\cos^2(bx+a))-1)\sin(bx+a)^3}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
[Out] 2/5*2^(1/2)/b*(12*cos(b*x+a)^4-15*cos(b*x+a)^2+5)*cos(b*x+a)*(c*sin(b*x+a)^
2/(2*cos(b*x+a)^2-1))^(3/2)/(2*cos(b*x+a)^2-1)/sin(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] -2/5*(10*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*((b*c*cos(4*b*x + 4*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 2*b*c*cos(4*b*x
+ 4*a) + b*c)*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x
 + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*
x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(3/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin
(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*
sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(5/2*arctan2(sin(4*b*x + 4*a), c
os(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4
*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x +
8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4*b*x
 + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*sin(
8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(3/2*arctan2(sin(4*b
*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(5/2*arctan2(sin(4*b*x + 4*a), cos(4
*b*x + 4*a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a
)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(12*b*x + 12*a)^2 + 4
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b
*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)
^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a)^2 + 4*(cos(4*b*x + 4*a)^2 +
 sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)^2 + (2*cos(4
*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x +
 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x + 4*a)^2 + sin
(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) + 2*cos(4*b*x +
4*a)^2 + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 4*(cos(4*b*x + 4*a)^3 + cos
(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))
*cos(8*b*x + 8*a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + 2*(cos(4*b
*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)
 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(12*b
*x + 12*a) + 4*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*a))*cos(3/2*arctan2(sin(4*b*x + 4*a
), -cos(4*b*x + 4*a) - 1))^2 + (cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (
cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(12*b*
x + 12*a)^2 + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
```

```
a) + 1)*cos(8*b*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(12*b*x + 12*a)^2 + 4*(cos(4
*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*
a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a)^2 +
 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x
 + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a) +
 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(12*b*x + 12*a) + 4*(cos(4*b*x
 + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + co
s(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)
^3 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*s
in(8*b*x + 8*a) + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x +
 4*a))*sin(12*b*x + 12*a) + 4*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + 2
*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*a))*sin(3/2*arctan2(
sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2), x) + 3*(b*c*cos(4*b*x + 4*a)^
2 + b*c*sin(4*b*x + 4*a)^2 + 2*b*c*cos(4*b*x + 4*a) + b*c)*integrate(-(cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(((cos(
12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*
sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(3/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x
+ 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x
 + 8*a)*sin(4*b*x + 4*a))*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*
a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*
x + 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*
b*x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2
*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(12*b*x + 12*a)*co
s(4*b*x + 4*a) + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 +
 sin(12*b*x + 12*a)*sin(4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a)
+ sin(4*b*x + 4*a)^2)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/((cos(4*b*x + 4
*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a)^...
```

Fricas [A]
time = 2.52, size = 88, normalized size = 0.80

$$\frac{2\sqrt{2}\left(5c\tan\left(bx+a\right)^{4}-5c\tan\left(bx+a\right)^{2}+2c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{5\left(b\tan\left(bx+a\right)^{5}-2b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/5*sqrt(2)*(5*c*tan(b*x + a)^4 - 5*c*tan(b*x + a)^2 + 2*c)*sqrt(-c*tan(b*x
 + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*ta
n(b*x + a))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 11.04, size = 149, normalized size = 1.35

$$\frac{2\,c\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,10\mathrm{i}+b\,x\,10\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{5\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^2,x)
```

```
[Out] (2*c*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)*
3i + 3i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((e
xp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(5*b*(exp(a*2i + b
*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

3.614 $\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=75

$$-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3b}$$

```
[Out] -4/3*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*c*(-c+c*sec(2*b*x
+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

Rubi [A]
time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4482, 3878, 3877}

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-4*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c
+ c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \sec(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3b} - \frac{1}{3}(4c)\int \mathrm{s}$$

$$= -\frac{4c^2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c\sqrt{-c+c\sec(2a+2}}{3b}$$

Mathematica [A]
time = 0.11, size = 51, normalized size = 0.68

$$-\frac{\cot(a+bx)(-1+4\cot(a+bx)\cot(2(a+bx)))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/3*(Cot[a + b*x]*(-1 + 4*Cot[a + b*x]*Cot[2*(a + b*x)])*(c*Tan[a + b*x]*T
an[2*(a + b*x)])^(3/2))/b
```

Maple [A]
time = 0.55, size = 61, normalized size = 0.81

method	result	size
default	$-\frac{2\sqrt{2}\,(5(\cos^2(bx+a))-3)\cos(bx+a)\left(\frac{c(\sin^2(bx+a))}{2(\cos^2(bx+a))-1}\right)^{\frac{3}{2}}}{3b\sin(bx+a)^3}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*2^(1/2)/b*(5*cos(b*x+a)^2-3)*cos(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/sin(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] 1/3*(6*(3*b*c*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)^(1/4)*(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4
*a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(3/2*arc
tan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(8*b*
x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3/2*arctan2(sin(4*b*x + 4
*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x +
 4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x
+ 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(8*
b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b
*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*
x + 4*a) - 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))))/((cos
(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*
a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 +
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*
x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*
a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(4*b*x + 4*a)^2 +
2*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4
*b*x + 4*a))*sin(8*b*x + 8*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
 + 4*a) - 1))^2 + (cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a)^2 + 2
*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*
x + 4*a) + 1)*sin(8*b*x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4
*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*a) +
 cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4
*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*a))*sin(3/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2), x) + b*c*integrate(-(cos(4*b*x + 4*
a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(((cos(8*b*x + 8*
a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a
) + sin(4*b*x + 4*a)^2)*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
 - 1)) + (cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x +
4*a))*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(1/2*ar
ctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x +
 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1)) - (cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x
+ 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(3/2*
arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*
x + 4*a), cos(4*b*x + 4*a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (
cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x
 + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2
 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*c
os(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x
 + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8
*b*x + 8*a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a
```

```
)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*a))*cos(3/2*a
rctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(4*b*x + 4*a)^4 +
sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
 + 4*a) + 1)*cos(8*b*x + 8*a)^2 + 2*cos(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^
2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(8*b*x + 8*a)^2 + (2*co
s(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*
x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + c
os(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a
)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8
*b*x + 8*a))*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2),
x))*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4
)*sqrt(c) - (3*c*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*
sin(4*b*x + 4*a) + (3*c*cos(4*b*x + 4*a) + 5*c)*sin(3/2*arctan2(sin(4*b*x +
 4*a), -cos(4*b*x + 4*a) - 1)))*sqrt(c))/((cos(4*b*x + 4*a)^2 + sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(3/4)*b)
```

Fricas [A]
time = 2.70, size = 67, normalized size = 0.89

$$-\frac{2\,\sqrt{2}\,\left(3\,c\tan\left(bx+a\right)^{2}-2\,c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{3\,\left(b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] -2/3*sqrt(2)*(3*c*tan(b*x + a)^2 - 2*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a
)^2 - 1))/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 7.87, size = 158, normalized size = 2.11

$$-\frac{c\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,5\mathrm{i}+5\mathrm{i}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x),x)
```

```
[Out] -(c*(exp(a*2i + b*x*2i)*3i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*5i
+ 5i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(
a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(3*b*(exp(a*2i + b*x*
2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) - 1))
```

3.615 $\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b+c^2*t
an(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4482, 3860, 21, 3859, 213}

$$\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])
/b + (c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx &= \int (-c+c\sec(2a+2bx))^{3/2}\,dx\\ &= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{-\frac{c}{2}+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\ &= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - c\int \sqrt{-c+c\sec(2a+2bx)}\,dx\\ &= \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}\\ &= \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 86, normalized size = 1.08

$$\frac{c\left(2\cot(a+bx)+\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(2*Cot[a + b*x] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
+ b*x)]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(72) = 144.
time = 3.35, size = 253, normalized size = 3.16

method	result
default	$\frac{\sqrt{2}\,(2(\cos^2(bx+a))-1)\left(\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\cos(bx+a)\sqrt{2}+\operatorname{arctanh}\left(\frac{\cos(bx\cdots}{2\sin(b\cdots}\right.\right.}{b\sin(bx+a)^3\left(2+\sqrt{2}\right)\left(\sqrt{2}-2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a)
)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*x
+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-2*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2)
)/(2^(1/2)-2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(72) = 144$.
time = 0.61, size = 1317, normalized size = 16.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1
/4)*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(c
os(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 +
 sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b
*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*log(sqrt(cos(4*b*x + 4*a)^2 + s
in(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a
), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2
 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4
*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1
) + c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + s
in(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2
(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x
+ 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqr
```

```
t(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(co
s(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(
1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x
+ 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1)
 - c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + si
n(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*cos(1/2*arctan2(
sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x +
 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2)*sqrt
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) - 2*(cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/2*arctan2(sin(4*b*x +
 4*a), -cos(4*b*x + 4*a) - 1)) + cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b
*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))) + 1))
*sqrt(c) + 8*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + c*cos(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(1/2*arctan2(sin(4*b*x + 4*a), co
s(4*b*x + 4*a))) + c*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1)))*sqrt(c))/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)^(1/4)*b)
```

Fricas [A]
time = 3.00, size = 296, normalized size = 3.70

$$\left[\frac{c^{\frac{3}{2}}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\,\sqrt{2}\left(\tan(bx+a)^4-4\,\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\,\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}\right)\tan\left(bx+a\right)+4\,\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\,c}{4\,b\tan\left(bx+a\right)},-\frac{\sqrt{-c}\,c\arctan\left(\frac{2\,\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3\,c\tan(bx+a)}\right)\tan\left(bx+a\right)-2\,\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\,c}{2\,b\tan\left(bx+a\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c^(3/2)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan
(b*x + a)))*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*c)/(b*tan(b*x + a)), -1/2*(sqrt(-c)*c*arctan(2*sqrt(2)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x +
a)^3 - 3*c*tan(b*x + a)))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.616 $\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=86

$$-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b}+\frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] -3/2*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b+
1/2*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4482, 3899, 21, 3890, 3859, 213}

$$\frac{c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/(2*b) + (c^2*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3899

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[b/(m + n - 1), Int[(a +
b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
- 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2
, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (2c)\int \frac{\cos(2a+2bx)}{\sqrt{-c}}$$
$$= -\frac{c^2\sin(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}} - (3c)\int \cos(2a+2bx)$$
$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}(3c)\int \sqrt{-c+c\sec(}$$
$$= \frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{-c+x^2}\,d\right.}{}$$
$$= -\frac{3c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} + \frac{c}{2b\sqrt{-}}$$

Mathematica [A]
time = 0.17, size = 93, normalized size = 1.08

$$\frac{c\left(\cos(a+bx)-3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}+\cos(3(a+bx))\right)\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(Cos[a + b*x] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a +
 b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*Tan
[a + b*x]*Tan[2*(a + b*x)]])/(4*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(74) = 148$.
time = 1.05, size = 518, normalized size = 6.02

method	result
default	$-\frac{\sqrt{2}\,(2(\cos^2(bx+a))-1)\left(\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\cos(bx+a)\sqrt{2}+\operatorname{arctanh}\left(\cdots\right.\right.}{b\sin(bx+a)^3\left(2+\sqrt{2}\right)\left(\sqrt{2}\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a
))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*
x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-2*c
os(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2
))/(2^(1/2)-2)-2*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/
2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)*2^(1/2)+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(3/2)/sin(b*x+a)^3/(2^(1/2)-2)^3/(2+2^(1/2))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(74) = 148$.
time = 0.60, size = 1058, normalized size = 12.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] -1/16*(4*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2
*b*x + 2*a) + (c*cos(2*b*x + 2*a) + c)*sin(1/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)))*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b
*x + 4*a) + 1)^(1/4)*sqrt(c) - 3*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos
(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^
(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*lo
g(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin
(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*
x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a)
, -cos(4*b*x + 4*a) - 1)) + 1) + c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*
x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1))*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1))) + 1) - c*log(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*
b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1) - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
+ 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)
)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1))) + 1))*sqrt(c))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(74) = 148$.
time = 2.73, size = 369, normalized size = 4.29

$$\left[\frac{3\,\left(c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{c}\,\log\left(-\frac{c\tan(bx+a)^{5}-14\,c\tan(bx+a)^{3}-4\,\sqrt{2}\,\left(\tan(bx+a)^{4}-4\,\tan(bx+a)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^{5}+2\,\tan(bx+a)^{3}+\tan(bx+a)}\right)-4\,\sqrt{2}\,\left(c\tan\left(bx+a\right)^{2}-c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{8\,\left(b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)},\ \frac{3\,\left(c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{-c}\,\arctan\left(\frac{2\,\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{c\tan(bx+a)^{3}-3\,c\tan(bx+a)}\right)-2\,\sqrt{2}\,\left(c\tan\left(bx+a\right)^{2}-c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{4\,\left(b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5
- 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(t
an(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(c*tan(b*x +
```

```
a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3 +
 b*tan(b*x + a)), 1/4*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)*arcta
n(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(c*tan(b*x +
 a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^3
+ b*tan(b*x + a))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)\ (c\tan(a+bx)\ \tan(2a+2bx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.617 $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b}-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] 7/8*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b-7
/8*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*c^2*cos(2*b*x+2*a)*
sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4482, 3898, 21, 3890, 3859, 213}

$$\frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b}-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (7*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]
])/(8*b) - (7*c^2*Sin[2*a + 2*b*x])/(8*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (
c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^2(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$
$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{2}c\int \frac{\cos(2a+2}{\sqrt{-}}$$
$$= \frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{4}(7c)\int \cos(2a -$$
$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\,\mathrm{si}}{4b\sqrt{-c+c\sec(2}}$$
$$= -\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos(2a+2bx)\,\mathrm{si}}{4b\sqrt{-c+c\sec(2}}$$
$$= \frac{7c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} - \frac{7}{8b\sqrt{-}}$$

Mathematica [A]

time = 0.20, size = 105, normalized size = 0.79

$$\frac{c\left(-5\cos(a+bx)+7\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}-6\cos(3(a+bx))+\cos(5(a+bx))\right)\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(-5*Cos[a + b*x] + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(
a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] - 6*Cos[3*(a + b*x)] + Cos[5*(a + b*x)])
*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(117) = 234$.
time = 0.70, size = 792, normalized size = 5.95

method	result	size
default	Expression too large to display	792

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a)
)/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*x
+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-2*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2)
)/(2^(1/2)-2)+4*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/2
)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*
2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arc
tanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)*2^(1/2)+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(3/2)/sin(b*x+a)^3/(2^(1/2)-2)^3/(2+2^(1/2))^3-2*2^(1/2)/b*(2*cos(b*x+a)^
2-1)*(16*cos(b*x+a)^5+9*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)-12*cos(b*x+a)^3+9*arctanh
(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*
2^(1/2)+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)
^3/(2^(1/2)-2)^5/(2+2^(1/2))^5
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.52, size = 437, normalized size = 3.29

$$\left[\frac{7\left(c\tan\left(bx+a\right)^{5}+2c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{c}\log\left(-\frac{c\tan\left(bx+a\right)^{5}-14c\tan\left(bx+a\right)^{3}+4\sqrt{2}\left(\tan\left(bx+a\right)^{4}-4\tan\left(bx+a\right)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\sqrt{c}+17c\tan\left(bx+a\right)}{\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)}\right)+4\sqrt{2}\left(9c\tan\left(bx+a\right)^{4}-4c\tan\left(bx+a\right)^{2}-5c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{32\left(b\tan\left(bx+a\right)^{5}+2b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)},\ -\frac{7\left(c\tan\left(bx+a\right)^{5}+2c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)\sqrt{-c}}{c\tan\left(bx+a\right)^{3}-3c\tan\left(bx+a\right)}\right)-2\sqrt{2}\left(9c\tan\left(bx+a\right)^{4}-4c\tan\left(bx+a\right)^{2}-5c\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{16\left(b\tan\left(bx+a\right)^{5}+2b\tan\left(bx+a\right)^{3}+b\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/32*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*l
og(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4
*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) +
 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4
*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(
b*x + a)), -1/16*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a)
)*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(t
an(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt
(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x +
 a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(2a+2bx)^2 (c\tan(a+bx)\tan(2a+2bx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.618 $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

Optimal. Leaf size=182

$$-\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\ \tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b}+\frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}}-\frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] -11/16*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b+11/16*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-11/24*c^2*cos(2*b*
x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/6*c^2*cos(2*b*x+2*a)^
2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4482, 3898, 21, 3890, 3859, 213}

$$-\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\ \tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{16b}+\frac{11c^2\sin(2a+2bx)}{16b\sqrt{c\sec(2a+2bx)-c}}+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11c^2\sin(2a+2bx)\cos(2a+2bx)}{24b\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-11*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x
]]])/(16*b) + (11*c^2*Sin[2*a + 2*b*x])/(16*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) - (11*c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(24*b*Sqrt[-c + c*Sec[2*a +
2*b*x]]) + (c^2*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2
*a + 2*b*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3898

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos^3(2a+2bx)(-c+c\sec(2a+2bx))^{3/2}\,dx$$

$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} - \frac{1}{3}c\int \frac{\cos^2(2a+2}{\sqrt{}}$$

$$= \frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{6}(11c)\int \cos^2(2a$$

$$= -\frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\cos^2(2a+2}{6b\sqrt{-c+c}}$$

$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\,\mathrm{si}}{24b\sqrt{-c+c\sec(2}}$$

$$= \frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} - \frac{11c^2\cos(2a+2bx)\,\mathrm{si}}{24b\sqrt{-c+c\sec(2}}$$

$$= -\frac{11c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} + \frac{}{16b}$$

Mathematica [A]
time = 0.17, size = 117, normalized size = 0.64

$$\frac{c\left(38\cot(a+bx)-33\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\,\csc(a+bx)-42\sin(2(a+bx))+14\sin(4(a+bx))-4\sin(6(a+bx))\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (c*(38*Cot[a + b*x] - 33*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*
(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] - 42*Sin[2*(a + b*x)] + 14
*Sin[4*(a + b*x)] - 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)
]])/(96*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. $2(162) = 324$.
time = 0.74, size = 1078, normalized size = 5.92

method	result	size
default	Expression too large to display	1078

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a
))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+arctanh(1/2*cos(b*
x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)-2*c
os(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/(2+2^(1/2
))/(2^(1/2)-2)-6*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(arctanh(1/2*cos(b*x+a)*4^(1/
2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)+ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)*2^(1/2)+4*cos(b*x+a)^3+2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(3/2)/sin(b*x+a)^3/(2^(1/2)-2)^3/(2+2^(1/2))^3+6*2^(1/2)/b*(2*cos(b*x+a)
^2-1)*(16*cos(b*x+a)^5+9*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin
(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)-12*cos(b*x+a)^3+9*arctan
h(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*2^(1/2)+18*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a
)^3/(2^(1/2)-2)^5/(2+2^(1/2))^5-4/3*2^(1/2)/b*(2*cos(b*x+a)^2-1)*(128*cos(b
```

```
*x+a)^7-80*cos(b*x+a)^5+75*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/s
in(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)*2^(1/2)-100*cos(b*x+a)^3+75*ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)*2^(1/2)+150*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(
b*x+a)^3/(2^(1/2)-2)^7/(2+2^(1/2))^7
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.79, size = 503, normalized size = 2.76

$$\left[\frac{33\left(c\tan(bx+a)^7+3c\tan(bx+a)^5+3c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17c\tan(bx+a)}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right)-4\sqrt{2}\left(63c\tan(bx+a)^6-13c\tan(bx+a)^4-31c\tan(bx+a)^2-19c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{192\left(b\tan(bx+a)^7+3b\tan(bx+a)^5+3b\tan(bx+a)^3+b\tan(bx+a)\right)}, \frac{33\left(c\tan(bx+a)^7+3c\tan(bx+a)^5+3c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3c\tan(bx+a)}\right)-2\sqrt{2}\left(63c\tan(bx+a)^6-13c\tan(bx+a)^4-31c\tan(bx+a)^2-19c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{96\left(b\tan(bx+a)^7+3b\tan(bx+a)^5+3b\tan(bx+a)^3+b\tan(bx+a)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c*
tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt
(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)
^3 + tan(b*x + a))) - 4*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4
- 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))
/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x +
a)), 1/96*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 +
 c*tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x
+ a))) - 2*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 - 31*c*tan(b*
x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x +
 a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(2\,a+2\,b\,x\right)^3\,{\left(c\,\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.619 $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=175

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}}+\frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}}+\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}}+\sqrt{-$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+14/15*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/5*
sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/15*(-c+c*se
c(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

Rubi [A]

time = 0.42, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4482, 3907, 4095, 4086, 3880, 213}

$$\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{15bc}+\frac{14\tan(2a+2bx)}{15b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (14*Tan[2*a + 2*b*x])/(15*b*Sqrt[-c + c*Sec[2*a +
 2*b*x]]) + (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(15*b*c)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
```

```
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[d^2/(b*(2*n - 3)), Int[(d
*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e + f
*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^4(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\int \frac{\sec^2(2a+2bx)(4c+c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{5c}$$

$$= \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2}{15bc}$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-}}{}$$

$$= \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+}}$$

Mathematica [A]
time = 0.49, size = 112, normalized size = 0.64

$$\frac{\cos(a+bx)\sec^3(2(a+bx))\sin(a+bx)\left(38+4\cos(2(a+bx))+26\cos(4(a+bx))+30\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\cos^2(2(a+bx))\sqrt{-1+\tan^2(a+bx)}\right)}{30b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]*Sec[2*(a + b*x)]^3*Sin[a + b*x]*(38 + 4*Cos[2*(a + b*x)] + 26
*Cos[4*(a + b*x)] + 30*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[2*(a + b*x)]^2
*Sqrt[-1 + Tan[a + b*x]^2]))/(30*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(154) = 308$.
time = 1.72, size = 984, normalized size = 5.62

method	result	size
default	Expression too large to display	984

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/120*2^(1/2)/b*(-1+cos(b*x+a))*(208*cos(b*x+a)^6*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)+120*cos(b*x+a)^6*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+
1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+120*cos(b*x+a)^6*arctanh(1/2*4^(1/2)*(
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/
sin(b*x+a)^2)+208*cos(b*x+a)^5*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-
200*cos(b*x+a)^4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-180*cos(b*x+a)
^4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*
x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)
^2)-180*cos(b*x+a)^4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-200*cos(b*x+a)^3*((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+60*((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)*cos(b*x+a)^2+90*cos(b*x+a)^2*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+90*cos(b*x+a)^2*arctanh(1/2*4^(
1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)/sin(b*x+a)^2)+60*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)-15*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos
(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x
+a)^2)-15*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a
)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(2*cos(b*x+a)^2-1)^3/sin(b*x+
a)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*(1-cos(b*x+a)^2)/(2*cos(b
*x+a)^2-1))^(1/2)*4^(1/2)/(3+2*2^(1/2))^3/(-3+2*2^(1/2))^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^4/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A]
time = 2.81, size = 380, normalized size = 2.17

$$\left[\frac{4\sqrt{2}\left(15\tan\left(bx+a\right)^{4}-20\tan\left(bx+a\right)^{2}+17\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}+\frac{15\sqrt{2}\left(c\tan(bx+a)^{5}-2c\tan(bx+a)^{3}+c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^{3}-\frac{2\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^{3}}\right)}{\sqrt{c}}}{60\left(bc\tan\left(bx+a\right)^{5}-2bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)}, -\frac{15\sqrt{2}\left(c\tan\left(bx+a\right)^{5}-2c\tan\left(bx+a\right)^{3}+c\tan\left(bx+a\right)\right)\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)-2\sqrt{2}\left(15\tan\left(bx+a\right)^{4}-20\tan\left(bx+a\right)^{2}+17\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{30\left(bc\tan\left(bx+a\right)^{5}-2bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/60*(4*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1)) + 15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*
x + a)^3 + c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x
 + a)^3)/sqrt(c))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x
+ a)), -1/30*(15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*tan(b*x
 + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) - 2*sqrt(2)*(15*tan(b*x + a)^4 - 2
0*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*t
an(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2\,a+2\,b\,x)^4\,\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.620 $\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=129

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}}+\frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+2/3*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*(-
c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

Rubi [A]
time = 0.24, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4482, 3885, 4086, 3880, 213}

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}+\frac{2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + (2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]]) + (Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b*c)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
```

```
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec^3(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$= \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx)}{3bc} + \frac{2\int \frac{\sec(2a+2bx)\left(\frac{c}{2}+c\sec(2a+2b}{\sqrt{-c+c\sec(2a+2b}}}{3c}$$
$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx}{3bc}$$
$$= \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\,\tan(2a+2bx}{3bc}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}} + \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2}}$$

Mathematica [A]
time = 0.28, size = 89, normalized size = 0.69

$$\frac{\cos^2(a+bx)\csc(2(a+bx))\left(2+2\sec(2(a+bx))+3\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (Cos[a + b*x]^2*Csc[2*(a + b*x)]*(2 + 2*Sec[2*(a + b*x)] + 3*ArcTan[Sqrt[-1
 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a
 + b*x)]])/(3*b*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(112) = 224$.
time = 0.81, size = 677, normalized size = 5.25

method	result
default	$-\frac{\sqrt{2}\,(-1+\cos(bx+a))\left(8\left(\cos^4(bx+a)\right)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}+12\left(\cos^4(bx+a)\right)\ln\left(-\frac{2\left(\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\left(\cos^2(bx+a)\right)-2(\right.}{\mathrm{s}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/24*2^(1/2)/b*(-1+cos(b*x+a))*(8*cos(b*x+a)^4*((2*cos(b*x+a)^2-1)/(cos(b*
x+a)+1)^2)^(1/2)+12*cos(b*x+a)^4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^
(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+12*cos(b*x+a)^4*arctanh(1/2*4^(1/2)*(2*co
s(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(
b*x+a)^2)+8*cos(b*x+a)^3*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-12*cos
(b*x+a)^2*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2
*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin
(b*x+a)^2)-12*cos(b*x+a)^2*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)
+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+3*ln(-2*(((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+3*arctanh(
1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)/sin(b*x+a)^2))/(2*cos(b*x+a)^2-1)^2/(c*(1-cos(b*x+a)^2)/(2*cos
(b*x+a)^2-1))^(1/2)/sin(b*x+a)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*
4^(1/2)/(-3+2*2^(1/2))^2/(3+2*2^(1/2))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

[Out] integrate(sec(2*b*x + 2*a)^3/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)

Fricas [A]
time = 2.50, size = 294, normalized size = 2.28

$$\left[\frac{\frac{3\sqrt{2}\left(c\tan(bx+a)^3-c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{\sqrt{c}}-8\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{12\left(bc\tan(bx+a)^3-bc\tan(bx+a)\right)}, -\frac{3\sqrt{2}\left(c\tan(bx+a)^3-c\tan(bx+a)\right)\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)+4\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{6\left(bc\tan(bx+a)^3-bc\tan(bx+a)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm ="fricas")

[Out] [1/12*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c) - 8*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a)), -1/6*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a))]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)

[Out] Timed out

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm ="giac")

[Out] Timed out

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^3\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.621 $\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=88

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3883, 3880, 213}

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3883

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Dist[a*(m/(b*(m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /
```

```
; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx &= \int \frac{\sec^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\ &= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}+\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx\\ &= \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}-\frac{\text{Subst}\left(\int \frac{1}{-2c+x^2}\,dx,x,-\frac{c\tan(2a}{\sqrt{-c+c\sec}}\right.}{b}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 67, normalized size = 0.76

$$\frac{\left(2+\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ((2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a
 + b*x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(77) = 154.
time = 1.01, size = 478, normalized size = 5.43

method	result

default	$\frac{\sqrt{2}\left(\ln\left(-\frac{2\left(\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\left(\cos^2(bx+a)\right)-2\left(\cos^2(bx+a)\right)-\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}+\cos(bx+a)+1\right)}{\sin(bx+a)^2}\right)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\ c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*2^(1/2)/b*(ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a
)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1
)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)+((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2
-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*
cos(b*x+a)+ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-
2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/si
n(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+arctanh(1/2*4^(1/2)
*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+a))*s
in(b*x+a)/(2*cos(b*x+a)^2-1)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A]
time = 1.91, size = 245, normalized size = 2.78

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)+4\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{4\,bc\tan(bx+a)},\ -\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)\tan(bx+a)-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{2\,bc\tan(bx+a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b
*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)
^3)*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/
(b*c*tan(b*x + a)), -1/2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))*tan(b
*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(
b*x + a))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^2\,\sqrt{c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.622 $\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] -1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4482, 3880, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)]/(Sqrt[2]*b*Sqrt[c]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Mathematica [**A**]
time = 0.12, size = 64, normalized size = 1.16

$$\frac{\operatorname{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\,\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]*Tan[2*(a + b*x
)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(46) = 92$.
time = 0.71, size = 236, normalized size = 4.29

method	result
default	$\frac{\sqrt{2}\,(\cos(bx+a)+1)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a))-1}}\left(\ln\left(-\frac{2\left(\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\left(\cos^2(bx+a)\right)-2\left(\cos^2(bx+a)\right)-\right.}{\sin(bx+a)^2}\right.\right.}{8b\sin(bx+a)c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/8*2^(1/2)/b*(cos(b*x+a)+1)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*(c
*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)*(ln(-2*(((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+arctanh(1/2*4^(1/2)*(2*cos(b*
```

```
x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+
a)^2))/sin(b*x+a)/c*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A]
time = 2.05, size = 146, normalized size = 2.65

$$\left[\frac{\sqrt{2}\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{4b\sqrt{c}}, -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/(b*sq
rt(c)), -1/2*sqrt(2)*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))/b]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
[Out] integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.623 $\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

```
[Out] arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1/2*a
rctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b*2^
(1/2)/c^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4482, 3861, 3859, 213, 3880}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[c
]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])]/(Sqrt[2]*b*Sqrt[c])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{1}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$= -\frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c} + \int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+}{\sqrt{2}\sqrt{-c+c\sec(}}\right.}{\sqrt{2}\,b\sqrt{c}}$$

Mathematica [A]
time = 0.41, size = 96, normalized size = 0.96

$$-\frac{\left(\sqrt{2}\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)-2\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\right)\tan(a+bx)}{b\sqrt{2-2\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -(((Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - 2*ArcTanh[Sqrt[1 - Tan[a +
b*x]^2]/Sqrt[2]])*Tan[a + b*x])/(b*Sqrt[2 - 2*Tan[a + b*x]^2]*Sqrt[c*Tan[a
+ b*x]*Tan[2*(a + b*x)]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(85) = 170.
time = 0.70, size = 298, normalized size = 2.98

method	result
default	$\frac{\sqrt{2}\,(-1+\cos(bx+a))\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)-\ln\left(-\frac{2\left(\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,\left(\cos^2(bx+a)\right)-2\left(\cos\right.\right.}{\sin(b}\right.\right.}{8b\sqrt{\frac{c(1-(\cos^2(bx+a)))}{2(\cos^2(bx+a))-1}}\,\sin(bx+a)\sqrt{\frac{2}{(}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)/b*(-1+cos(b*x+a))*(2*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1
+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2))-ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b
*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a
)^2)-arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^
2-1))^(1/2)/sin(b*x+a)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*4^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 1.71, size = 309, normalized size = 3.09

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+2\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^3+\tan(bx+a)}\right)}{4bc},\ -\frac{\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)}{2bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
+ a)^3) + 2*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(t
```

```
an(b*x + a)^3 + tan(b*x + a))))/(b*c), -1/2*(sqrt(2)*sqrt(-c)*arctan(sqrt(-
c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan
(b*x + a))) - 2*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))))/(b*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.624 $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=138

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] 1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1
/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b*2^(1/2)/c^(1/2)+1/2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4482, 3908, 3989, 3972, 492, 212}

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(2*b*Sqrt
[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*
b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(2*b*Sqrt[-c + c*Sec[2*a + 2
*b*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{-c-c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c}$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} + \frac{1}{2}c\int \frac{\tan^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\text{Subst}\left(\int \frac{x^2}{(1-cx^2)(2-cx^2)}\,dx, x, -\frac{}{\sqrt{-c}}\right.}{2b}$$

$$= \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, -\frac{\tan(2a+2}{\sqrt{-c+c\sec(2}}\right.}{2b}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2b}{\sqrt{2}\sqrt{-c+c\sec(2}}\right.}{\sqrt{2}b\sqrt{c}}$$

Mathematica [A]
time = 1.80, size = 163, normalized size = 1.18

$$-\frac{\tan(a+bx)\left(\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)-\sqrt{2}\left(\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)+\cos^2(a+bx)\sqrt{\frac{1}{1+\sec(2(a+bx))}}\left(2+\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sqrt{-1+\tan^2(a+bx)}\right)\right)\right)}{2b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/2*(Tan[a + b*x]*(ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - Sqrt[2]*(ArcTanh[Sq
rt[1 - Tan[a + b*x]^2]/Sqrt[2]] + Cos[a + b*x]^2*Sqrt[(1 + Sec[2*(a + b*x)]
)^(-1)]*(2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sqrt[-1 + T
an[a + b*x]^2]))))/(b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a
 + b*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(117) = 234$.
time = 1.14, size = 1023, normalized size = 7.41

method	result	size
default	Expression too large to display	1023

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/16*2^(1/2)/b*(-1+cos(b*x+a))^2*(4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(3/2)*cos(b*x+a)^3+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/
2)*cos(b*x+a)^2+5*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b
*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*arctanh(1/2*c
os(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*
x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+
a)^2)*cos(b*x+a)+4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b
*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+
a)+1)/sin(b*x+a)^2)*cos(b*x+a)-6*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1
+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2))+6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*arctanh(1/2*
4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)/sin(b*x+a)^2)+4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+co
s(b*x+a)+1)/sin(b*x+a)^2)+4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))/((
2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(1/2)/sin(b*x+a)^3*4^(1/2)+1/8*2^(1/2)/b*(-1+cos(b*x+a))*(-2*2^(1/2)*arc
tanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*co
s(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+ln(-2
*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-(
(2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2))/(c*
(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)/((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A]
time = 2.35, size = 481, normalized size = 3.49

$$\left[\frac{\sqrt{2}\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}-2\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^{3}}\right)+\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}+2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^{3}+\tan(bx+a)}\right)-2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)}{4\left(bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)},\ -\frac{\sqrt{2}\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-\left(\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)+\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan\left(bx+a\right)^{2}-1\right)}{2\left(bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3
 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt
(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + (tan(b*x + a)^3 + tan(b*x + a))*s
qrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3
 + tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1))/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/2*(sqrt(2
)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - (tan(
b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) +
sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/
(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(2\,a+2\,b\,x\right)}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.625 $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

Optimal. Leaf size=182

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}\,b\sqrt{c}}+\frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] 7/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1
/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/
b*2^(1/2)/c^(1/2)+1/8*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*cos(
2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4482, 3908, 4107, 4005, 3859, 213, 3880}

$$\frac{\sin(2a+2bx)}{8b\sqrt{c\sec(2a+2bx)-c}}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8b\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}\,b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*b*
Sqrt[c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])]/(Sqrt[2]*b*Sqrt[c]) + Sin[2*a + 2*b*x]/(8*b*Sqrt[-c + c*Sec[2*a
 + 2*b*x]]) + (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3908

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Dist[1/(2*b*d*n), Int[(d*Csc[e + f*x])^(n + 1)*((a
+ b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos^2(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx$$

$$= \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{\cos(2a+2bx)(-c-3c\sec(2a+2bx))}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \frac{}{\sqrt{}}}{}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} - \frac{7\int \sqrt{}}{}$$

$$= \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\mathrm{Sub}}{}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a}{\sqrt{2}\sqrt{-c+c\sec}}\right.}{\sqrt{2}b\sqrt{c}}$$

Mathematica [A]
time = 2.65, size = 186, normalized size = 1.02

$$-\frac{\tan(a+bx)\left(7\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)-\sqrt{2}\left(7\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)+\cos^2(a+bx)\sec(2(a+bx))\sqrt{\frac{1}{1+\sec(2(a+bx))}}\left(2(1+\cos(2(a+bx))+\cos(4(a+bx)))+\mathrm{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\right)\right)\right)}{8b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

```
[Out] -1/8*(Tan[a + b*x]*(7*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - Sqrt[2]*(7*ArcTan
h[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]] + Cos[a + b*x]^2*Sec[2*(a + b*x)]*Sqrt[
(1 + Sec[2*(a + b*x)])^(-1)]*(2*(1 + Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) +
 ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]))))/(b*Sqrt[1
- Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(157) = 314$.
time = 0.75, size = 1828, normalized size = 10.04

method	result	size
default	Expression too large to display	1828

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVE RBOSE)

[Out] -1/8*2^(1/2)/b*(-1+cos(b*x+a))*(-2*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2))/(c*(1-cos(b*x+a)^2)/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*4^(1/2)-1/8*2^(1/2)/b*(-1+cos(b*x+a))^2*(4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^3+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^2+5*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)+4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)-6*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)^3*4^(1/2)+1/64*2^(1/2)/b*(-1+cos(b*x+a))^2*(4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^5+16*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^4+33*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^3+52*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)^2+49*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*cos(b*x+a)+18*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-46*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)+32*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)+54*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+32*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)-46*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+32*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+36*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+32*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+c

```
os(b*x+a)+1)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/(c*
sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/sin(b*x+a)^3*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

Fricas [A]
time = 2.31, size = 569, normalized size = 3.13

$$\left[\frac{4\sqrt{2}\left(\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}-2\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^{3}}\right)+7\left(\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}+2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^{3}+\tan(bx+a)}\right)+2\sqrt{2}\left(\tan\left(bx+a\right)^{4}-4\tan\left(bx+a\right)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{16\left(bc\tan\left(bx+a\right)^{5}+2bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)},-\frac{4\sqrt{2}\left(\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-7\left(\tan\left(bx+a\right)^{5}+2\tan\left(bx+a\right)^{3}+\tan\left(bx+a\right)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)-\sqrt{2}\left(\tan\left(bx+a\right)^{4}-4\tan\left(bx+a\right)^{2}+3\right)\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{8\left(bc\tan\left(bx+a\right)^{5}+2bc\tan\left(bx+a\right)^{3}+bc\tan\left(bx+a\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)
*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(ta
n(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 7*(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3 + 2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*
sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(t
an(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1
/8*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a))) - 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x +
a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
)*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(tan(b*x + a)^4
 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c
*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(2\,a+2\,b\,x\right)^2}{\sqrt{c\tan \left(a+b\,x\right)\,\tan \left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

```
[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.626 $\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}+\frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}}+$$

```
[Out] -11/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b/c^(3/2)*2^(1/2)-1/4*sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+
2*a))^(3/2)+13/6*tan(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)+7/12*(-c+c*
sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c^2
```

Rubi [A]
time = 0.34, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4482, 3901, 4095, 4086, 3880, 213}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{12bc^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}+\frac{13\tan(2a+2bx)}{6bc\sqrt{c\sec(2a+2bx)-c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(4*b*(
-c + c*Sec[2*a + 2*b*x])^(3/2)) + (13*Tan[2*a + 2*b*x])/(6*b*c*Sqrt[-c + c*
Sec[2*a + 2*b*x]]) + (7*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(12
*b*c^2)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
```

```
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^4(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec^2(2a+2bx)\left(2c+\frac{7}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{7\sqrt{-c+c\sec(2a+2bx)}\,\tan(}{12bc^2}$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} +$$

$$= -\frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} +$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\sec^2(2a+2bx)\,\text{ta}}{4b(-c+c\sec(2a}$$

Mathematica [A]
time = 0.91, size = 100, normalized size = 0.56

$$-\frac{\cot(a+bx)\left(\csc^2(a+bx)(-24+(11+19\cos(4(a+bx)))\sec(2(a+bx)))-66\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{48bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/48*(Cot[a + b*x]*(Csc[a + b*x]^2*(-24 + (11 + 19*Cos[4*(a + b*x)])*Sec[2
*(a + b*x)]) - 66*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^
2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. 2(157) = 314.
time = 1.96, size = 1211, normalized size = 6.73

method	result	size
default	Expression too large to display	1211

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

[Out] 1/96*2^(1/2)/b*(-1+cos(b*x+a))^2*(152*cos(b*x+a)^5*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+132*cos(b*x+a)^5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+132*cos(b*x+a)^5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-132*cos(b*x+a)^4*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)-132*cos(b*x+a)^4*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)-200*cos(b*x+a)^3*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-132*cos(b*x+a)^3*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)-132*cos(b*x+a)^3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+132*cos(b*x+a)^2*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+132*cos(b*x+a)^2*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)+54*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+33*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)+33*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)-33*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)-33*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(2*cos(b*x+a)^2-1)^2/sin(b*x+a)^3/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)/(-3+2*2^(1/2))^3/(3+2*2^(1/2))^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(2*b*x + 2*a)^4/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)

Fricas [A]
time = 1.82, size = 350, normalized size = 1.94

$$\left[\frac{33 \sqrt{2} \left(\tan\left(bx+a\right)^{5}-\tan\left(bx+a\right)^{3}\right) \sqrt{c} \log\left(\frac{c \tan(bx+a)^{3}-2 \sqrt{-\frac{c \tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}} \left(\tan(bx+a)^{2}-1\right) \sqrt{c}-2 c \tan(bx+a)}{\tan(bx+a)^{3}}\right)+2 \sqrt{2} \left(27 \tan\left(bx+a\right)^{4}-46 \tan\left(bx+a\right)^{2}+3\right) \sqrt{-\frac{c \tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{48 \left(b c^{2} \tan\left(bx+a\right)^{5}-b c^{2} \tan\left(bx+a\right)^{3}\right)}, -\frac{33 \sqrt{2} \left(\tan\left(bx+a\right)^{5}-\tan\left(bx+a\right)^{3}\right) \sqrt{-c} \arctan\left(\frac{\sqrt{-\frac{c \tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}} \left(\tan(bx+a)^{2}-1\right) \sqrt{-c}}{c \tan(bx+a)}\right)-\sqrt{2} \left(27 \tan\left(bx+a\right)^{4}-46 \tan\left(bx+a\right)^{2}+3\right) \sqrt{-\frac{c \tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}}{24 \left(b c^{2} \tan\left(bx+a\right)^{5}-b c^{2} \tan\left(bx+a\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(33*sqrt(2)*(tan(b*x + a)^5 - tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x
+ a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 2*sqrt(2)*(27*tan(b*x + a)^
4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b
*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3), -1/24*(33*sqrt(2)*(tan(b*x + a
)^5 - tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(27*tan(b
*x + a)^4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 -
 1)))/(b*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^4\,{\left(c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.627 $\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}}$$

```
[Out] -7/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+tan(2*
b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4482, 3884, 4086, 3880, 213}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}+\frac{\tan(2a+2bx)}{bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]]])]/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2)) + Tan[2*a + 2*b*x]/(b*c*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^3(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \frac{\sec(2a+2bx)\left(\frac{3c}{2}+2c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\int}{}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{7S}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2b}{4b(-c+c\sec(2a+}$$

Mathematica [A]
time = 0.42, size = 94, normalized size = 0.73

$$\frac{\left(-5+4\sec(2(a+bx))+7\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sin^2(a+bx)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ((-5 + 4*Sec[2*(a + b*x)] + 7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a +
b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*T
an[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(111) = 222$.
time = 0.75, size = 930, normalized size = 7.27

method	result	size
default	Expression too large to display	930

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/16*2^(1/2)/b*(7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b
*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+
a)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^
3+7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*cos(
b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*
x+a)^2)*cos(b*x+a)^3+7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*c
os(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(
b*x+a)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x
+a)^2+7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(2*
cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/si
n(b*x+a)^2)*cos(b*x+a)^2-7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+
cos(b*x+a)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos
(b*x+a)-7*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*arctanh(1/2*4^(1/2)*(
2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/
sin(b*x+a)^2)*cos(b*x+a)+20*cos(b*x+a)^3-7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)-7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(
b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)-18*cos(b*x+a))*sin(b*x+a)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/(2*cos(b*x+a)^2-1)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^3/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A]
time = 1.74, size = 276, normalized size = 2.16

$$\left[\frac{7\,\sqrt{2}\,\sqrt{c}\,\log\left(\frac{c\tan\left(bx+a\right)^{3}-2\,\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(\tan\left(bx+a\right)^{2}-1\right)\sqrt{c}-2\,c\tan\left(bx+a\right)}{\tan\left(bx+a\right)^{3}}\right)\tan\left(bx+a\right)^{3}+2\,\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(9\,\tan\left(bx+a\right)^{2}-1\right)}{16\,bc^{2}\tan\left(bx+a\right)^{3}},\ -\frac{7\,\sqrt{2}\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(\tan\left(bx+a\right)^{2}-1\right)\sqrt{-c}}{c\tan\left(bx+a\right)}\right)\tan\left(bx+a\right)^{3}-\sqrt{2}\,\sqrt{-\frac{c\tan\left(bx+a\right)^{2}}{\tan\left(bx+a\right)^{2}-1}}\,\left(9\,\tan\left(bx+a\right)^{2}-1\right)}{8\,bc^{2}\tan\left(bx+a\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(7*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(7*sqrt(2)*sqr
t(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a
)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(2a+2bx)^3 \left(c\tan(a+bx)\,\tan(2a+2bx)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
[Out] int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.628 $\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

```
[Out] -3/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4482, 3882, 3880, 213}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x
]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x
])^(3/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3882

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{3\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\mathrm{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c}{\sqrt{-c+}}\right.}{4bc}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2bx}{4b(-c+c\sec(2a+}$$

Mathematica [A]
time = 0.39, size = 84, normalized size = 0.90

$$\frac{\left(-1+3\mathrm{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sin^2(a+bx)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ((-1 + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2*
Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(a
+ b*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(78) = 156.
time = 0.71, size = 433, normalized size = 4.66

method	result

default	$-\frac{\sqrt{2}\,(-1+\cos(bx+a))^2\left(3\ln\left(-\frac{2\left(\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\,(\cos^2(bx+a))-2(\cos^2(bx+a))-\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}+\cos(bx+a)+1\right)}{\sin(bx+a)^2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(3*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)+3*arctanh(1/2*4^(1/2)*(2*c
os(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin
(b*x+a)^2)*cos(b*x+a)+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
/2)-3*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos
(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x
+a)^2)-3*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)
^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^
2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A]
time = 2.66, size = 272, normalized size = 2.92

$$\left[\frac{3\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{16\,bc^2\tan(bx+a)^3}, -\frac{3\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)\tan(bx+a)^3-\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{8\,bc^2\tan(bx+a)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(3*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(3*sqrt(2)*sqrt(
-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^2\,{\left(c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.629 $\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

```
[Out] 1/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))
/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4482, 3881, 3880, 213}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]
/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3
/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{4c}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{1}{-2c+x^2}\,dx, x, -\frac{c\,\mathrm{ta}}{\sqrt{-c+c}}\right.}{4bc}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx}$$

Mathematica [A]
time = 0.42, size = 83, normalized size = 0.89

$$-\frac{\left(1+\operatorname{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sin^2(a+bx)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] -1/4*((1 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^
2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(b*(c*Tan[a + b*x]*Tan[2*(a
+ b*x)])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(78) = 156$.
time = 0.70, size = 599, normalized size = 6.44

method	result

default	$\frac{\sqrt{2}\,(-1+\cos(bx+a))^3\left(2\sqrt{4}\,\left(\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}\right)^{\frac{3}{2}}(\cos^2(bx+a))+4\sqrt{4}\,\left(\frac{2\left(\cos^2(bx+a)\right)-1}{(\cos(bx+a)+1)^2}\right)^{\frac{3}{2}}\cos(bx+a)+2\sqrt{4}\,\left(\frac{2\left(\cos^2(bx\right.}{(\cos(bx+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/32*2^(1/2)/b*(-1+cos(b*x+a))^3*(2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(3/2)*cos(b*x+a)^2+4*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
3/2)*cos(b*x+a)+2*4^(1/2)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)-6*((2
*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-cos(b*x+a)^2*arctanh(
1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)/sin(b*x+a)^2)-cos(b*x+a)^2*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(
cos(b*x+a)+1)^2)^(1/2)+4*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+arctan
h(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+
a)+1)^2)^(1/2)/sin(b*x+a)^2)+ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2
)+cos(b*x+a)+1)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/si
n(b*x+a)^5/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A]
time = 3.37, size = 269, normalized size = 2.89

$$\left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan\left(bx+a\right)^3+2\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\left(\tan\left(bx+a\right)^2-1\right)}{16\,bc^2\tan\left(bx+a\right)^3}, \frac{\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)\tan\left(bx+a\right)^3+\sqrt{2}\sqrt{-\frac{c\tan\left(bx+a\right)^2}{\tan\left(bx+a\right)^2-1}}\left(\tan\left(bx+a\right)^2-1\right)}{8\,bc^2\tan\left(bx+a\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x
 + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), 1/8*(sqrt(2)*sqrt(-c)*a
rctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)\,{\left(c\,\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.630 $\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx)}$$

```
[Out] -arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)+5/8*
arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c
^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4482, 3862, 4005, 3859, 213, 3880}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{bc^{3/2}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*c^(3
/2))) + (5*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a
+ 2*b*x])^(3/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{2c+\frac{1}{2}c\sec(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\int \sqrt{-c+c\sec(2a+2bx)}\,dx}{c^2} - 5$$

$$= -\frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, -\frac{c\tan}{\sqrt{-c+c}}\right.}{bc}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc^{3/2}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(}{\sqrt{2}\,\sqrt{-c+c\,s}}\right.}{4\sqrt{2}\,bc^{3/2}}$$

Mathematica [A]

time = 2.58, size = 198, normalized size = 1.43

$$-\frac{\cot(a+bx)\left(-4\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)\cos(2(a+bx))\sec^2(a+bx)+4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\cos(2(a+bx))\sec^2(a+bx)+\cot^2(a+bx)\left(\cos(2(a+bx))\sec^2(a+bx)\right)^{3/2}+\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-\left(-1+\tan^2(a+bx)\right)^2}\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{8bc^2\sqrt{1-\tan^2(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

```
[Out] -1/8*(Cot[a + b*x]*(-4*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a + b*x)]*S
ec[a + b*x]^2 + 4*Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]]*Cos[2*(
a + b*x)]*Sec[a + b*x]^2 + Cot[a + b*x]^2*(Cos[2*(a + b*x)]*Sec[a + b*x]^2)
^(3/2) + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-(-1 + Tan[a + b*x]^2)^2])*
Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2*Sqrt[1 - Tan[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(117) = 234$.
time = 0.65, size = 561, normalized size = 4.07

method	result
default	$-\frac{\sqrt{2}\,(-1+\cos(bx+a))^2\left(8\,\mathrm{arctanh}\left(\frac{\cos(bx+a)\sqrt{4}\,(-1+\cos(bx+a))\sqrt{2}}{2\sin(bx+a)^2\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}}\right)\cos(bx+a)\sqrt{2}+2\cos(bx+a)\sqrt{\frac{2(\cos^2(bx+a))-1}{(\cos(bx+a)+1)^2}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos
(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*
cos(b*x+a)*2^(1/2)+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
-5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*
x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)
^2)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*co
s(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)-8*2^(1/2)*ar
ctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-
1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a
)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1
)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^
2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)
)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")

[Out] integrate((c*tan(2*b*x + 2*a)*tan(b*x + a))^(-3/2), x)

Fricas [A]
time = 2.74, size = 438, normalized size = 3.17

$$\left[\frac{5\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)^3+8\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^3+\tan(bx+a)}\right)\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{16\,bc^2\tan(bx+a)^3}, \frac{5\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)\tan(bx+a)^3-8\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)\tan(bx+a)^3+\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{8\,bc^2\tan(bx+a)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")

[Out] [1/16*(5*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3)*tan(b*x + a)^3 + 8*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a)))*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), 1/8*(5*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - 8*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)

[Out] Timed out

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.631 $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=178

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx)}$$

```
[Out] -3/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)-
1/4*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+9/8*arctanh(1/2*c^(1/2)*ta
n(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)*2^(1/2)-3/4*sin
(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4482, 3902, 4107, 4005, 3859, 213, 3880}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2bc^{3/2}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{c}\,\tan(2a+2bx)}{\sqrt{2}\,\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(2*b
*c^(3/2)) + (9*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[
2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - Sin[2*a + 2*b*x]/(4*b*(-c + c*Sec[
2*a + 2*b*x])^(3/2)) - (3*Sin[2*a + 2*b*x])/(4*b*c*Sqrt[-c + c*Sec[2*a + 2*
b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos(2a+2bx)\left(3c+\frac{3}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{3\int \cdots}{\cdots}$$

$$= -\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{3S\cdots}{\cdots}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2bc^{3/2}} + \frac{9\tanh^{-1}\left(\frac{\sqrt{c}\,\mathrm{ta}\cdots}{\sqrt{2}\sqrt{-c+c\cdots}}\right)}{4\sqrt{2}\,bc^{3\cdots}}$$

Mathematica [A]
time = 4.97, size = 217, normalized size = 1.22

$$\frac{\csc(a+bx)\sec(a+bx)\left(6\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)\cos(2(a+bx))-6\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\cos(2(a+bx))+(1-3\cos(2(a+bx))+\cos(4(a+bx)))\cot^2(a+bx)\sqrt{\cos(2(a+bx))\sec^2(a+bx)}-3\text{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\cos^2(a+bx)\sqrt{-(-1+\tan^2(a+bx))^2}\right)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{8bc^2\sqrt{1-\tan^2(a+bx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*Sec[a + b*x]*(6*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a +
b*x)] - 6*Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]]*Cos[2*(a + b*x)
] + (1 - 3*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*Cot[a + b*x]^2*Sqrt[Cos[2*(
a + b*x)]*Sec[a + b*x]^2] - 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[a + b*x
]^2*Sqrt[-(-1 + Tan[a + b*x]^2)^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/
(8*b*c^2*Sqrt[1 - Tan[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. $2(151) = 302$.
time = 1.08, size = 1157, normalized size = 6.50

method	result	size
default	Expression too large to display	1157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(
b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*c
os(b*x+a)*2^(1/2)+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-
5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x
+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^
2)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)-8*2^(1/2)*arc
tanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2
-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))
/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)+1/16*2^(1/2)/b*(-1+cos(b*x+a))^2*(4*cos(b*
x+a)^3*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-10*arctanh(1/2*cos(b*x+a
)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2
)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)-6*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)+7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(
b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x
+a)+1)/sin(b*x+a)^2)*cos(b*x+a)+7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos
(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*
x+a)+10*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2
/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-7*ln(-2*(((2*cos(b*x+
a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^
2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)-7*arctanh(1/2*4^(1
/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(
1/2)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/
((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A]
time = 2.64, size = 528, normalized size = 2.97

$$\left[\frac{9\sqrt{2}\left(\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+12\left(\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^3+\tan(bx+a)}\right)+2\sqrt{2}\left(5\tan(bx+a)^4-4\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{16\left(bc^2\tan(bx+a)^5+bc^2\tan(bx+a)^3\right)}, \frac{9\sqrt{2}\left(\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-12\left(\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)+\sqrt{2}\left(5\tan(bx+a)^4-4\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{8\left(bc^2\tan(bx+a)^5+bc^2\tan(bx+a)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/16*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x +
 a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)
*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 12*(tan(b*x + a)^5 + tan(b*x
 + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(
b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^
2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^5
+ b*c^2*tan(b*x + a)^3), 1/8*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*s
qrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 12*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqr
t(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(
b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(5*tan(b*x + a)^4 - 4*
tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*ta
n(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (2\,a+2\,b\,x)}{{\left(c\tan (a+b\,x)\,\tan (2\,a+2\,b\,x)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.632 $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{\cos(2a+2bx)\sin(2a}{4b(-c+c\sec(2a+2}$$

```
[Out] -19/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)
-1/4*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+13/8*arcta
nh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2
)*2^(1/2)-7/8*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)-1/2*cos(2*b*x+
2*a)*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4482, 3902, 4107, 4005, 3859, 213, 3880}

$$-\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{8bc^{3/2}}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}\,bc^{3/2}}-\frac{7\sin(2a+2bx)}{8bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{2bc\sqrt{c\sec(2a+2bx)-c}}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (-19*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(8*
b*c^(3/2)) + (13*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Se
c[2*a + 2*b*x]])])/(4*Sqrt[2]*b*c^(3/2)) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x])/(4*b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) - (7*Sin[2*a + 2*b*x])/(8*b*c*Sqr
t[-c + c*Sec[2*a + 2*b*x]]) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(2*b*c*Sq
rt[-c + c*Sec[2*a + 2*b*x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
 f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos^2(2a+2bx)}{(-c+c\sec(2a+2bx))^{3/2}}\,dx$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\int \frac{\cos^2(2a+2bx)\left(4c+\frac{5}{2}c\sec(2a+2bx)\right)}{\sqrt{-c+c\sec(2a+2bx)}}\,dx}{2c^2}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{2bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\int}{}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\mathrm{co}}{2b}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\mathrm{co}}{2b}$$

$$= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\mathrm{co}}{2b}$$

$$= -\frac{19\tanh^{-1}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8bc^{3/2}} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{2}\sqrt{-c}}\right)}{4\sqrt{2}\,b}$$

Mathematica [A]

time = 6.16, size = 357, normalized size = 1.53

$$\frac{\tan^{\frac{3}{2}}(a+bx)\left(-\frac{7\mathrm{ArcTan}\left(\sqrt{-1+\tan^2(a+bx)}\right)\csc^2(a+bx)\sec^2(a+bx)\tan^{\frac{3}{2}}(a+bx)\sqrt{-1+\tan^2(a+bx)}\sqrt{\tan(2(a+bx))}}{(1+\tan^2(a+bx))^2}+\frac{19\left(\sqrt{2}\tanh^{-1}\left(\sqrt{1-\tan^2(a+bx)}\right)-2\tanh^{-1}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\right)\cos(2(a+bx))\csc^2(a+bx)\sec^2(a+bx)\tan^{\frac{3}{2}}(a+bx)\sqrt{\tan(2(a+bx))}}{\sqrt{2}\sqrt{1-\tan^2(a+bx)}\,(1+\tan^2(a+bx))}\right)\tan^{\frac{3}{2}}(2(a+bx))}{16b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}} + \frac{\left(-\frac{5}{8}\cot(a+bx)-\frac{1}{8}\cot(a+bx)\csc^2(a+bx)+\frac{7}{8}\sin(2(a+bx))+\frac{1}{8}\sin(4(a+bx))\right)\tan^2(a+bx)\tan^2(2(a+bx))}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

```
[Out] (Tan[a + b*x]^(3/2)*((-7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*S
ec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a +
b*x)]])/(1 + Tan[a + b*x]^2)^2 + (19*(Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]
^2]] - 2*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]])*Cos[2*(a + b*x)]*Csc[a
+ b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sqrt[2]
*Sqrt[1 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/2))/(1
6*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + (((-5*Cot[a + b*x])/8 - (Cot
[a + b*x]*Csc[a + b*x]^2)/8 + (7*Sin[2*(a + b*x)])/8 + Sin[4*(a + b*x)]/8)*
Tan[a + b*x]^2*Tan[2*(a + b*x)]^2)/(b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1786 vs. 2(203) = 406.
time = 0.70, size = 1787, normalized size = 7.64

method	result	size
default	Expression too large to display	1787

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*cos(b*x+a)^5*((2*cos(b*x+a)^2-1)/(cos(b
*x+a)+1)^2)^(1/2)+14*cos(b*x+a)^3*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/
2)-51*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)-30*cos(b*x+a
)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+36*ln(-2*(((2*cos(b*x+a)^2-1)
/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)+36*arctanh(1/2
*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)
^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)+51*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2
)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*
2^(1/2))-36*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2
-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/s
in(b*x+a)^2)-36*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2))/((2*cos(b*x+a)^2-1)/(co
s(b*x+a)+1)^2)^(3/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3
*4^(1/2)-1/32*2^(1/2)/b*(-1+cos(b*x+a))^2*(8*arctanh(1/2*cos(b*x+a)*4^(1/2)
*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2
^(1/2))*cos(b*x+a)*2^(1/2)+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^
2)^(1/2)-5*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-
2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/si
n(b*x+a)^2)*cos(b*x+a)-5*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1
)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2)*cos(b*x+a)-8*2^
(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b
*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))+5*ln(-2*(((2*cos(b*x+a)^2-1)/(c
os(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(
b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)+5*arctanh(1/2*4^(1/2)*(2*cos
(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b
*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b*x+a)^3/((2*cos(b*
x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)-1/8*2^(1/2)/b*(-1+cos(b*x+a))^2*(
4*cos(b*x+a)^3*((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)-10*arctanh(1/2*c
os(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x
+a)+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)*2^(1/2)-6*cos(b*x+a)*((2*cos(b*x+a)^2-1
)/(cos(b*x+a)+1)^2)^(1/2)+7*ln(-2*(((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1
```

```
/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)
+cos(b*x+a)+1)/sin(b*x+a)^2)*cos(b*x+a)+7*arctanh(1/2*4^(1/2)*(2*cos(b*x+a)
^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)/sin(b*x+a)^2
)*cos(b*x+a)+10*2^(1/2)*arctanh(1/2*cos(b*x+a)*4^(1/2)*(-1+cos(b*x+a))/sin(
b*x+a)^2/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2))-7*ln(-2*(((2*
cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^2-2*cos(b*x+a)^2-((2*cos
(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)+1)/sin(b*x+a)^2)-7*arctanh(
1/2*4^(1/2)*(2*cos(b*x+a)^2-3*cos(b*x+a)+1)/((2*cos(b*x+a)^2-1)/(cos(b*x+a)
+1)^2)^(1/2)/sin(b*x+a)^2))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(3/2)/sin(b
*x+a)^3/((2*cos(b*x+a)^2-1)/(cos(b*x+a)+1)^2)^(3/2)*4^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

Fricas [A]
time = 2.60, size = 616, normalized size = 2.63

$$\left[\frac{13\sqrt{2}\left(\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right)+19\left(\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-3c\tan(bx+a)}{\tan(bx+a)^3+\tan(bx+a)}\right)+2\sqrt{2}\left(4\tan(bx+a)^6+5\tan(bx+a)^4-8\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{16\left(bc^2\tan(bx+a)^7+2bc^2\tan(bx+a)^5+bc^2\tan(bx+a)^3\right)}, \frac{13\sqrt{2}\left(\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-19\left(\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{2c\tan(bx+a)}\right)+\sqrt{2}\left(4\tan(bx+a)^6+5\tan(bx+a)^4-8\tan(bx+a)^2-1\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{8\left(bc^2\tan(bx+a)^7+2bc^2\tan(bx+a)^5+bc^2\tan(bx+a)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/16*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt
(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*
(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 19*(tan(
b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)
^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt
(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x + a)^2 - 1)*sqrt(-c*ta
n(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^7 + 2*b*c^2*tan(b*x
 + a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*
x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 19*(tan(b*x
+ a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(1/2*sqrt(2)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*
tan(b*x + a))) + sqrt(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x +
```

```
a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a
)^7 + 2*b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm
="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(2\,a+2\,b\,x\right)^2}{{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
[Out] int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.633 $\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

```
[Out] -2/3*cos(x)*cot(x)/sin(2*x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4475, 30}

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

```
[Out] (-2*Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx &= \frac{\sin(x)\int \frac{\csc^2(x)}{\sqrt{\tan(x)}}\,dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= \frac{\sin(x)\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \tan(x)\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}\\
&= -\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{3}\cot(x)\csc(x)\sqrt{\sin(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

```
[Out] -1/3*(Cot[x]*Csc[x]*Sqrt[Sin[2*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.20, size = 119, normalized size = 7.44

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\left(4\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1}\right.\right.}{6\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(4*(ta
n(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((ta
n(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)+tan(1/2*x)^4-1)/(tan(1/2*x)*(tan(
1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 2.55, size = 29, normalized size = 1.81

$$\frac{\sqrt{2}\,\sqrt{\cos\left(x\right)\sin\left(x\right)}\,\cos\left(x\right)+\cos\left(x\right)^{2}-1}{3\,\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + cos(x)^2 - 1)/(cos(x)^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

Mupad [B]
time = 3.10, size = 14, normalized size = 0.88

$$-\frac{\sqrt{\sin(2\,x)}\,\cos(x)}{3\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(2*x)^(1/2)*sin(x)),x)
```

```
[Out] -(sin(2*x)^(1/2)*cos(x))/(3*sin(x)^2)
```

3.634 $\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}\,(-2+\tan(x))}\,dx$

Optimal. Leaf size=69

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin(x)}{2\sqrt{2}\,\sqrt{\sin(2x)}\,\sqrt{\tan(x)}}$$

```
[Out] 1/2*cos(x)/sin(2*x)^(1/2)+1/3*cos(x)*cot(x)/sin(2*x)^(1/2)-5/4*arctanh(1/2*
tan(x)^(1/2)*2^(1/2))*sin(x)*2^(1/2)/sin(2*x)^(1/2)/tan(x)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4475, 912, 1276, 213}

$$\frac{\cos(x)}{2\sqrt{\sin(2x)}}-\frac{5\sin(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\,\sqrt{\sin(2x)}\,\sqrt{\tan(x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

```
[Out] Cos[x]/(2*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]]) - (5*ArcTanh
[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x])/(2*Sqrt[2]*Sqrt[Sin[2*x]]*Sqrt[Tan[x]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}\,(-2+\tan(x))}\,dx &= \frac{\sin(x)\int \frac{\csc^3(x)\sec(x)\sqrt{\tan(x)}}{-2+\tan(x)}\,dx}{\sqrt{\sin(2x)}\,\sqrt{\tan(x)}} \\
&= \frac{\sin(x)\text{Subst}\left(\int \frac{1+x^2}{(-2+x)x^{5/2}}\,dx, x, \tan(x)\right)}{\sqrt{\sin(2x)}\,\sqrt{\tan(x)}} \\
&= \frac{(2\sin(x))\text{Subst}\left(\int \frac{1+x^4}{x^4(-2+x^2)}\,dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\,\sqrt{\tan(x)}} \\
&= \frac{(2\sin(x))\text{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4(-2+x^2)}\right)dx, x, \sqrt{\tan(x)}\right)}{\sqrt{\sin(2x)}\,\sqrt{\tan(x)}} \\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}+\frac{(5\sin(x))\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{2\sqrt{\sin(2x)}\,\sqrt{\tan(x)}} \\
&= \frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin(x)}{2\sqrt{2}\,\sqrt{\sin(2x)}\,\sqrt{\tan(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.82, size = 80, normalized size = 1.16

$$\frac{1}{4}\sqrt{\sin(2x)}\left(\left(1+\frac{2\cot(x)}{3}\right)\csc(x)-\frac{5\text{ArcTan}\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{\cos(x)}{2+2\cos(x)}}\,\sec(x)}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

```
[Out] (Sqrt[Sin[2*x]]*((1 + (2*Cot[x])/3)*Csc[x] - (5*ArcTan[Sqrt[Tan[x/2]]/Sqrt[
-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x]))]*Sec[x])/Sqrt[Tan[x/2]]))/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 396, normalized size = 5.74

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(240\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\mathrm{EllipticE}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(240*(tan(1/2*x)+1
)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((tan(1/2*x)+1
)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)-140*(ta
n(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((ta
n(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*
x)+(tan(1/2*x)^3-tan(1/2*x))^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11
)*(_alpha^3+2*_alpha-3)*(tan(1/2*x)+1)^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2
*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((tan(1/2*x)+1)^(1
/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^
2-_Z+1))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*2^(1/2)*tan(1/2*x)+40*(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan(1/2*x)^3-tan
(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-40*(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(50) = 100$.
time = 2.81, size = 120, normalized size = 1.74

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right)+15\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)^2+\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\sin(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}\right)+4}{48\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/48*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 - 1
5*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x))
 + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos(
x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
4)/(cos(x)^2 - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*sec(x)/sin(2*x)**(1/2)/(-2+tan(x)),x)
```

```
[Out] Integral(csc(x)**2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sin(2\,x)}\,\cos(x)\,\sin(x)^2\,\left(\tan(x)-2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)),x)
```

```
[Out] int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)), x)
```

3.635 $\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=79

$$\frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] 1/3*cos(x)^4*sin(x)/sin(2*x)^(5/2)+1/2*cos(x)^3*sin(x)^2/sin(2*x)^(5/2)-5/4
*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin(2*x)^(5/2)*2^(1/2)/tan(x)^(
5/2)
```

Rubi [A]
time = 0.40, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4475, 912, 1276, 213}

$$\frac{\sin(x)\cos^4(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\sin^2(x)\cos^3(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Cos[x]^4*Sin[x])/(3*Sin[2*x]^(5/2)) + (Cos[x]^3*Sin[x]^2)/(2*Sin[2*x]^(5/2
)) - (5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x]^5)/(2*Sqrt[2]*Sin[2*x]^(5/2)*T
an[x]^(5/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx &= \frac{\sin^5(x)\int \frac{\csc^2(x)\sqrt{\tan(x)}}{\sin^2(x)-\sin(2x)}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\sin^5(x)\text{Subst}\left(\int \frac{-1-x^2}{(2-x)x^{5/2}}\,dx,x,\tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \frac{-1-x^4}{x^4(2-x^2)}\,dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\left(2\sin^5(x)\right)\text{Subst}\left(\int \left(-\frac{1}{2x^4}-\frac{1}{4x^2}+\frac{5}{4(-2+x^2)}\right)dx,x,\sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}+\frac{\left(5\sin^5(x)\right)\text{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\sqrt{\text{t}\right.}{2\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}\\
&= \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)}-\frac{5\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 122, normalized size = 1.54

$$\frac{\csc^2\left(\frac{x}{2}\right)\sqrt{\sin(2x)}\left(-15\text{ArcTan}\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)(-1+\cos(x))+\sqrt{2}\sqrt{-\frac{\cos(x)}{1+\cos(x)}}\,(2\cos(x)+3\sin(x))\sqrt{\tan\left(\frac{x}{2}\right)}\right)}{96(1+\cos(x))\sqrt{\tan\left(\frac{x}{2}\right)}\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Csc[x/2]^2*Sqrt[Sin[2*x]]*(-15*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]
]*(-1 + Cos[x]) + Sqrt[2]*Sqrt[-(Cos[x]/(1 + Cos[x]))]*(2*Cos[x] + 3*Sin[x]
)*Sqrt[Tan[x/2]]))/(96*(1 + Cos[x])*Sqrt[Tan[x/2]]*Sqrt[-1 + Tan[x/2]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 396, normalized size = 5.01

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\left(240\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\mathrm{EllipticE}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/1920*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(240*(tan(1/2*x)+
1)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((tan(1/2*x)+
1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)-140*(t
an(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((t
an(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2
*x)+(tan(1/2*x)^3-tan(1/2*x))^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-1
1)*(_alpha^3+2*_alpha-3)*(tan(1/2*x)+1)^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/
2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((tan(1/2*x)+1)^(
1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z
^2-_Z+1))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*2^(1/2)*tan(1/2*x)+40*(tan(1/
2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan(1/2*x)^3-ta
n(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-40*(tan(1/2*
x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.00, size = 120, normalized size = 1.52

$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right)+15\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)^2+\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\,\sin(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}\right)+4}{192\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/192*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 -
15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x)
) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*cos
(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2) +
 4)/(cos(x)^2 - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate(cos(x)^2*sin(x)/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\cos(x)^2\,\sin(x)}{\sin(2\,x)^{5/2}\,\left(\sin(2\,x)-\sin(x)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

```
[Out] -int((cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

3.636 $\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=95

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

```
[Out] 1/5*cos(x)^5/sin(2*x)^(5/2)+1/6*cos(x)^4*sin(x)/sin(2*x)^(5/2)-3/4*cos(x)^3
*sin(x)^2/sin(2*x)^(5/2)+3/8*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin
(2*x)^(5/2)*2^(1/2)/tan(x)^(5/2)
```

Rubi [A]
time = 0.41, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4475, 912, 1276, 213}

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) + (Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (3*Cos
[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Sin[x
]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 4475

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Dist[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m)), Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{\sin^5(x)\int \frac{\cos(2x)\csc^2(x)}{(\sin^2(x)-\sin(2x))\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\sin^5(x)\text{Subst}\left(\int \frac{-1+x^2}{(2-x)x^{7/2}}\,dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{(2\sin^5(x))\,\text{Subst}\left(\int \frac{-1+x^4}{x^6(2-x^2)}\,dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{(2\sin^5(x))\,\text{Subst}\left(\int \left(-\frac{1}{2x^6}-\frac{1}{4x^4}+\frac{3}{8x^2}-\frac{3}{8(-2+x^2)}\right)dx, x, \sqrt{\tan(x)}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}-\frac{(3\sin^5(x))\,\text{Subst}\left(\int \right.}{4\sin^{\frac{5}{2}}(2}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right.}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\,\text{ta}}$$

Mathematica [A]

time = 3.26, size = 92, normalized size = 0.97

$$\frac{\sec(x)\sqrt{\sin(2x)}\left(4\cot(x)\csc^2(x)(-33+57\cos(2x)+10\sin(2x))+\frac{180\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{1}{1+\sec(x)}}}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

```
[Out] (Sec[x]*Sqrt[Sin[2*x]]*(4*Cot[x]*Csc[x]^2*(-33 + 57*Cos[2*x] + 10*Sin[2*x])
 + (180*Sqrt[2]*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(1 + Sec
[x])^(-1)])/Sqrt[Tan[x/2]]))/3840
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 761, normalized size = 8.01

method	result	size
default	Expression too large to display	761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(1772*(tan(1/2*x)+
1)^(1/2)*(-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/
2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan
(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-4464*(tan(1/2*x)+1)^(1/2)*(-ta
n(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(-2*tan(1/
2*x)+2)^(1/2)*EllipticE((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(
1/2*x)^2-1))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1
))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+3*sum((6*_alpha^3
+7*_alpha^2+6*_alpha+1)*(_alpha^3+2*_alpha-3)*(tan(1/2*x)+1)^(1/2)*(1-tan(1
/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((tan(1/2*x)+1)^(1/2),-1/4*_alph
a^3-1/2*_alpha+3/4,1/2*2^(1/2))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2),_alpha=
RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*
(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*2^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^
(1/2)*tan(1/2*x)^2-40*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan
(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^5-1272*tan(1/2*x)^4*(tan(1/2*x)*
(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)-1920*t
an(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(
1/2)-24*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(
```

```
1/2*x)*(tan(1/2*x)^2-1))^(1/2)+1272*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/
2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)^2-24*(tan(1/2*x)*(tan(
1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2
*x)^2+40*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x)*(tan(1
/2*x)*(tan(1/2*x)^2-1))^(1/2)+24*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))
^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)*(tan(1/2*x)-1)*(tan
(1/2*x)+1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.01, size = 136, normalized size = 1.43

$$-\frac{45\,\left(\cos\left(x\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\sqrt{2}\,\sqrt{\cos\left(x\right)\sin\left(x\right)}\,\left(4\,\cos\left(x\right)+3\,\sin\left(x\right)\right)+\frac{1}{2}\,\cos\left(x\right)^{2}+\frac{7}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)-45\,\left(\cos\left(x\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(x\right)^{2}+\frac{1}{2}\,\sqrt{2}\,\sqrt{\cos\left(x\right)\sin\left(x\right)}\,\sin\left(x\right)-\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)+4\,\sqrt{2}\,\left(57\,\cos\left(x\right)^{2}+10\,\cos\left(x\right)\sin\left(x\right)-45\right)\sqrt{\cos\left(x\right)\sin\left(x\right)}+268\,\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}{1920\,\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/1920*(45*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) +
 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 45*(cos(x)^2
- 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)
*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(57*cos(x)^2 + 10*cos(x)*sin(x) - 45)*sqr
t(cos(x)*sin(x)) + 268*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*cos(2*x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate(cos(2*x)*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\cos\left(2\,x\right)\,\cos\left(x\right)^3}{\sin\left(2\,x\right)^{5/2}\,\left(\sin\left(2\,x\right)-\sin\left(x\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

```
[Out] int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

3.637 $\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{(b\sec(c+dx)+a\sin(c+dx))^{1+n}}{d(1+n)}$$

```
[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4470}

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^{1+n}}{d(1+n)}$$

Mathematica [A]
time = 0.86, size = 51, normalized size = 1.70

$$\frac{\sec(c+dx)(b\sec(c+dx)+a\sin(c+dx))^n(2b+a\sin(2(c+dx)))}{2d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(2*b + a*Sin[2*(c + d*x)]
))/(2*d*(1 + n))
```

Maple [A]
time = 0.59, size = 31, normalized size = 1.03

method	result	size
derivativedivides	$\frac{(b\sec(dx+c)+a\sin(dx+c))^{1+n}}{d(1+n)}$	31
default	$\frac{(b\sec(dx+c)+a\sin(dx+c))^{1+n}}{d(1+n)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x,
method=_RETURNVERBOSE)
```

```
[Out] (b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)
```

Maxima [A]
time = 0.29, size = 30, normalized size = 1.00

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] (b*sec(d*x + c) + a*sin(d*x + c))^(n + 1)/(d*(n + 1))
```

Fricas [A]
time = 2.66, size = 59, normalized size = 1.97

$$\frac{(a\cos(dx+c)\sin(dx+c)+b)\left(\frac{a\cos(dx+c)\sin(dx+c)+b}{\cos(dx+c)}\right)^n}{(dn+d)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c)*sin(d*x + c) + b)*((a*cos(d*x + c)*sin(d*x + c) + b)/cos(d*
x + c))^n/((d*n + d)*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(24) = 48$.

time = 19.49, size = 138, normalized size = 4.60

$$\begin{cases} \frac{x(a\cos{(c)}+b\tan{(c)}\sec{(c)})}{a\sin{(c)}+b\sec{(c)}} & \text{for } d=0 \wedge n=-1 \\ x(a\sin{(c)}+b\sec{(c)})^n\,(a\cos{(c)}+b\tan{(c)}\sec{(c)}) & \text{for } d=0 \\ \frac{\log\left(\frac{a\sin{(c+dx)}}{b}+\sec{(c+dx)}\right)}{d} & \text{for } n=-1 \\ \frac{a(a\sin{(c+dx)}+b\sec{(c+dx)})^n\sin{(c+dx)}}{dn+d}+\frac{b(a\sin{(c+dx)}+b\sec{(c+dx)})^n\sec{(c+dx)}}{dn+d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0) &
 Eq(n, -1)), (x*(a*sin(c) + b*sec(c))**n*(a*cos(c) + b*tan(c)*sec(c)), Eq(d
, 0)), (log(a*sin(c + d*x)/b + sec(c + d*x))/d, Eq(n, -1)), (a*(a*sin(c + d
*x) + b*sec(c + d*x))**n*sin(c + d*x)/(d*n + d) + b*(a*sin(c + d*x) + b*sec
(c + d*x))**n*sec(c + d*x)/(d*n + d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(30) = 60$.
time = 1.11, size = 85, normalized size = 2.83

$$\frac{\left(-\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}\right)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] (-(b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x
+ 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)/(tan(1/2*d*x + 1/2*c)^4 - 1))^(n
 + 1)/(d*(n + 1))
```

Mupad [B]
time = 5.54, size = 63, normalized size = 2.10

$$\begin{cases} \frac{\ln\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)}{d} & \text{if } n=-1 \\ \frac{\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)^{n+1}}{d\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^n*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] piecewise(n == -1, log(a*sin(c + d*x) + b/cos(c + d*x))/d, n ~= -1, (a*sin(
c + d*x) + b/cos(c + d*x))^(n + 1)/(d*(n + 1)))
```

3.638 $\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(b\sec(c+dx)+a\sin(c+dx))^4}{4d}$$

```
[Out] 1/4*(b*sec(d*x+c)+a*sin(d*x+c))^4/d
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{4470\}$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+ }{4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 938 vs. 2(26) = 52.
time = 6.39, size = 938, normalized size = 36.08

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

[Out] (8*b^4*Cos[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (a^4*Cos[4*c]*Cos[4*d*x]*Cos[c + d*x]^5*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (16*a*b^2*Cos[c + d*x]^3*Sec[c]*(3*a*Cos[c] + 2*b*Sin[c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (4*a^3*Cos[2*d*x]*Cos[c + d*x]^5*(a*Cos[2*c] + 4*b*Sin[2*c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a*b^3*Cos[c + d*x]^2*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (32*a^3*b*Cos[c + d*x]^4*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (4*a^3*Cos[c + d*x]^5*(-4*b*Cos[2*c] + a*Sin[2*c])*Sin[2*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (a^4*Cos[c + d*x]^5*Sin[4*c]*Sin[4*d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(24) = 48$.
time = 0.56, size = 187, normalized size = 7.19

method	result
derivativedivides	$\frac{\frac{a^4\left(\sin^4(dx+c)\right)}{4}+a^3b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
default	$\frac{\frac{a^4\left(\sin^4(dx+c)\right)}{4}+a^3b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
risch	$\frac{a^4 e^{4i(dx+c)}}{64d}+\frac{ia^3 e^{2i(dx+c)}b}{4d}-\frac{a^4 e^{2i(dx+c)}}{16d}-\frac{ia^3 e^{-2i(dx+c)}b}{4d}-\frac{a^4 e^{-2i(dx+c)}}{16d}+\frac{a^4 e^{-4i(dx+c)}}{64d}+\frac{2b\left(ia^3 e^{6i(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/4*a^4*sin(d*x+c)^4+a^3*b*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+3*a^3*b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))-3*a^2*b^2*ln(cos(

```
d*x+c))+a*b^3*sin(d*x+c)^3/cos(d*x+c)^3+a*b^3*tan(d*x+c)+1/4*b^4/cos(d*x+c)
^4)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.92

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="maxima")
```

```
[Out] 1/4*(b*sec(d*x + c) + a*sin(d*x + c))^4/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(24) = 48$.
time = 2.80, size = 122, normalized size = 4.69

$$\frac{8\,a^4\cos(dx+c)^8-16\,a^4\cos(dx+c)^6+5\,a^4\cos(dx+c)^4+48\,a^2b^2\cos(dx+c)^2+8\,b^4-32\left(a^3b\cos(dx+c)^5-a^3b\cos(dx+c)^3-ab^3\cos(dx+c)\right)\sin(dx+c)}{32\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^4*cos(d*x + c)^8 - 16*a^4*cos(d*x + c)^6 + 5*a^4*cos(d*x + c)^4 +
 48*a^2*b^2*cos(d*x + c)^2 + 8*b^4 - 32*(a^3*b*cos(d*x + c)^5 - a^3*b*cos(d
*x + c)^3 - a*b^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(20) = 40$.
time = 3.53, size = 129, normalized size = 4.96

$$\begin{cases} \frac{a^4\sin^4(c+dx)}{4d}+\frac{a^3b\sin^3(c+dx)\sec(c+dx)}{d}+\frac{3a^2b^2\sin^2(c+dx)\sec^2(c+dx)}{2d}+\frac{ab^3\sin(c+dx)\sec^3(c+dx)}{d}+\frac{b^4\sec^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x(a\sin(c)+b\sec(c))^3\,(a\cos(c)+b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**4/(4*d) + a**3*b*sin(c + d*x)**3*sec(c + d*x)
/d + 3*a**2*b**2*sin(c + d*x)**2*sec(c + d*x)**2/(2*d) + a*b**3*sin(c + d*x
)*sec(c + d*x)**3/d + b**4*sec(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) +
 b*sec(c))**3*(a*cos(c) + b*tan(c)*sec(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(24) = 48$.

time = 0.92, size = 142, normalized size = 5.46

$$\frac{b^4\tan(dx+c)^4 + 4\,ab^3\tan(dx+c)^3 + 6\,a^2b^2\tan(dx+c)^2 + 2\,b^4\tan(dx+c)^2 + 4\,a^3b\tan(dx+c) + 4\,ab^3\tan(dx+c) - \frac{4\,a^3b\tan(dx+c)^3 + 2\,a^4\tan(dx+c)^2 + 4\,a^3b\tan(dx+c) + a^4}{\left(\tan(dx+c)^2+1\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] 1/4*(b^4*tan(d*x + c)^4 + 4*a*b^3*tan(d*x + c)^3 + 6*a^2*b^2*tan(d*x + c)^2
 + 2*b^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) - (4*
a^3*b*tan(d*x + c)^3 + 2*a^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + a^4)/(
tan(d*x + c)^2 + 1)^2)/d
```

Mupad [B]

time = 3.57, size = 185, normalized size = 7.12

$$\frac{a^4\cos(2\,c+2\,d\,x)^4 - 2\,a^4\cos(2\,c+2\,d\,x)^2 + a^4 - 8\,\sin(2\,c+2\,d\,x)\,a^3\,b\cos(2\,c+2\,d\,x)^2 + 8\,\sin(2\,c+2\,d\,x)\,a^3\,b - 24\,a^2\,b^2\cos(2\,c+2\,d\,x)^2 + 24\,a^2\,b^2 + 32\,\sin(2\,c+2\,d\,x)\,a\,b^3 - 4\,b^4\cos(2\,c+2\,d\,x)^2 - 8\,b^4\,\cos(2\,c+2\,d\,x) + 12\,b^4}{d\,\left(16\cos(2\,c+2\,d\,x)^2 + 32\,\cos(2\,c+2\,d\,x) + 16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^3*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] (a^4*cos(2*c + 2*d*x)^4 - 2*a^4*cos(2*c + 2*d*x)^2 - 4*b^4*cos(2*c + 2*d*x)
^2 + a^4 + 12*b^4 + 24*a^2*b^2 - 8*b^4*cos(2*c + 2*d*x) - 24*a^2*b^2*cos(2*
c + 2*d*x)^2 + 32*a*b^3*sin(2*c + 2*d*x) + 8*a^3*b*sin(2*c + 2*d*x) - 8*a^3
*b*cos(2*c + 2*d*x)^2*sin(2*c + 2*d*x))/(d*(32*cos(2*c + 2*d*x) + 16*cos(2*
c + 2*d*x)^2 + 16))
```

3.639 $\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(b\sec(c+dx)+a\sin(c+dx))^3}{3d}$$

```
[Out] 1/3*(b*sec(d*x+c)+a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{4470\}$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*Ta
n[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+$$

Mathematica [A]
time = 0.84, size = 31, normalized size = 1.19

$$\frac{\sec^3(c+dx)(2b+a\sin(2(c+dx)))^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d
*x]*Tan[c + d*x]),x]
```

[Out] (Sec[c + d*x]^3*(2*b + a*Sin[2*(c + d*x)])^3)/(24*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(24) = 48$.
time = 0.50, size = 152, normalized size = 5.85

method	result
derivativedivides	$\frac{\frac{a^3\left(\sin^3(dx+c)\right)}{3}+a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)-2\cos(dx+c)a^2b+2ab^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(}{}\right.}{d}$
default	$\frac{\frac{a^3\left(\sin^3(dx+c)\right)}{3}+a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)-2\cos(dx+c)a^2b+2ab^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(}{}\right.}{d}$
risch	$\frac{ia^3e^{3i(dx+c)}}{24d}-\frac{e^{i(dx+c)}a^2b}{2d}-\frac{ie^{i(dx+c)}a^3}{8d}-\frac{e^{-i(dx+c)}a^2b}{2d}+\frac{ie^{-i(dx+c)}a^3}{8d}-\frac{ia^3e^{-3i(dx+c)}}{24d}+\frac{2b(-3iab\,e^{5i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, method=_RETURNVERBOSE)

[Out] 1/d*(1/3*a^3*sin(d*x+c)^3+a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))-2*cos(d*x+c)*a^2*b+2*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/3*b^3/cos(d*x+c)^3)

Maxima [A]
time = 0.29, size = 24, normalized size = 0.92

$$\frac{\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/3*(b*sec(d*x + c) + a*sin(d*x + c))^3/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.
time = 2.42, size = 92, normalized size = 3.54

$$-\frac{3\,a^2b\cos\left(dx+c\right)^4-3\,a^2b\cos\left(dx+c\right)^2-b^3+\left(a^3\cos\left(dx+c\right)^5-a^3\cos\left(dx+c\right)^3-3\,ab^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="fricas")

```
[Out] -1/3*(3*a^2*b*cos(d*x + c)^4 - 3*a^2*b*cos(d*x + c)^2 - b^3 + (a^3*cos(d*x
+ c)^5 - a^3*cos(d*x + c)^3 - 3*a*b^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.
time = 1.41, size = 100, normalized size = 3.85

$$\begin{cases} \frac{a^3\sin^3(c+dx)}{3d} + \frac{a^2b\sin^2(c+dx)\sec(c+dx)}{d} + \frac{ab^2\sin(c+dx)\sec^2(c+dx)}{d} + \frac{b^3\sec^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\sin(c) + b\sec(c))^2(a\cos(c) + b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))**2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**3/(3*d) + a**2*b*sin(c + d*x)**2*sec(c + d*x)
/d + a*b**2*sin(c + d*x)*sec(c + d*x)**2/d + b**3*sec(c + d*x)**3/(3*d), Ne
(d, 0)), (x*(a*sin(c) + b*sec(c))**2*(a*cos(c) + b*tan(c)*sec(c)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+
c)),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.22, size = 100, normalized size = 3.85

$$\frac{a^3\sin(c+dx)}{3d} + \frac{a^2b\cos(c+dx)^2 + \sin(c+dx)\,a\,b^2\cos(c+dx) + \frac{b^3}{3}}{d\cos(c+dx)^3} - \frac{a^3\cos(c+dx)^2\sin(c+dx)}{3d} - \frac{a^2b\cos(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))^2*(a*cos(c + d*x) + (b*tan(c + d*x))/
cos(c + d*x)),x)
```

```
[Out] (a^3*sin(c + d*x))/(3*d) + (b^3/3 + a^2*b*cos(c + d*x)^2 + a*b^2*cos(c + d*
x)*sin(c + d*x))/(d*cos(c + d*x)^3) - (a^3*cos(c + d*x)^2*sin(c + d*x))/(3*
d) - (a^2*b*cos(c + d*x))/d
```

3.640 $\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{(b\sec(c+dx)+a\sin(c+dx))^2}{2d}$$

```
[Out] 1/2*(b*sec(d*x+c)+a*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = $\{4470\}$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[
c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^2}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.
time = 0.03, size = 67, normalized size = 2.58

$$abx-\frac{ab\text{ArcTan}(\tan(c+dx))}{d}-\frac{a^2\cos^2(c+dx)}{2d}+\frac{b^2\sec^2(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x
]*Tan[c + d*x]),x]
```

```
[Out] a*b*x - (a*b*ArcTan[Tan[c + d*x]])/d - (a^2*Cos[c + d*x]^2)/(2*d) + (b^2*Se
c[c + d*x]^2)/(2*d) + (a*b*Tan[c + d*x])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.
time = 0.46, size = 57, normalized size = 2.19

method	result	size
derivativedivides	$\frac{-\frac{(\cos^2(dx+c))a^2}{2}+ab(\tan(dx+c)-dx-c)+ab(dx+c)+\frac{b^2}{2\cos(dx+c)^2}}{d}$	57
default	$\frac{-\frac{(\cos^2(dx+c))a^2}{2}+ab(\tan(dx+c)-dx-c)+ab(dx+c)+\frac{b^2}{2\cos(dx+c)^2}}{d}$	57
risch	$-\frac{a^2 e^{2i(dx+c)}}{8d}-\frac{a^2 e^{-2i(dx+c)}}{8d}+\frac{2b(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+ia)}{d(e^{2i(dx+c)}+1)^2}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*cos(d*x+c)^2*a^2+a*b*(tan(d*x+c)-d*x-c)+a*b*(d*x+c)+1/2*b^2/cos(d
*x+c)^2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.92

$$\frac{(b\sec(dx+c)+a\sin(dx+c))^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="maxima")
```

```
[Out] 1/2*(b*sec(d*x + c) + a*sin(d*x + c))^2/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.
time = 2.16, size = 61, normalized size = 2.35

$$-\frac{2\,a^2\cos(dx+c)^4-a^2\cos(dx+c)^2-4\,ab\cos(dx+c)\sin(dx+c)-2\,b^2}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*
x + c) - 2*b^2)/(d*cos(d*x + c)^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(20) = 40$.
time = 0.56, size = 73, normalized size = 2.81

$$\begin{cases} -\frac{a^2\cos^2(c+dx)}{2d} + \frac{ab\sin(c+dx)\sec(c+dx)}{d} + \frac{b^2\sec^2(c+dx)}{2d} & \text{for } d\neq 0 \\ x(a\sin(c) + b\sec(c))(a\cos(c) + b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**2/(2*d) + a*b*sin(c + d*x)*sec(c + d*x)/d +
b**2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))*(a*cos(c) +
 b*tan(c)*sec(c)), True))
```

Giac [A]
time = 0.57, size = 45, normalized size = 1.73

$$\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) - \frac{a^2}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)
),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) - a^2/(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 3.18, size = 61, normalized size = 2.35

$$-\frac{\frac{a^2\left(2\sin(2\,c+2\,d\,x)^2-1\right)}{16} + \frac{a^2}{16} + \frac{b^2}{2} + \frac{a\,b\,\sin(2\,c+2\,d\,x)}{2}}{d\,\left(\sin(c+d\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b/cos(c + d*x))*(a*cos(c + d*x) + (b*tan(c + d*x))/co
s(c + d*x)),x)
```

```
[Out] -((a^2*(2*sin(2*c + 2*d*x)^2 - 1))/16 + a^2/16 + b^2/2 + (a*b*sin(2*c + 2*d
*x))/2)/(d*(sin(c + d*x)^2 - 1))
```

3.641 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(b\sec(c+dx)+a\sin(c+dx))}{d}$$

[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{4468\}$

$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x]),x]

[Out] Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d

Rule 4468

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], ActivateTrig[u], x]}, Simp[q*Log[RemoveContent[ActivateTrig[y], x]], x] /;  !FalseQ[q]] /;  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \frac{\log(b\sec(c+dx)+a\sin(c+dx))}{d}$$

Mathematica [A]
time = 0.33, size = 29, normalized size = 1.32

$$\frac{-\log(\cos(c+dx))+\log(2b+a\sin(2(c+dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[c + d*x]),x]

[Out] (-Log[Cos[c + d*x]] + Log[2*b + a*Sin[2*(c + d*x)]])/d

Maple [A]
time = 0.61, size = 23, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\ln(b\sec(dx+c)+a\sin(dx+c))}{d}$	23
default	$\frac{\ln(b\sec(dx+c)+a\sin(dx+c))}{d}$	23
risch	$-ix-\frac{2ic}{d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}+\frac{\ln\left(e^{4i(dx+c)}+\frac{4ib\,e^{2i(dx+c)}}{a}-1\right)}{d}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x,me
thod=_RETURNVERBOSE)
```

```
[Out] ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

Maxima [A]
time = 0.27, size = 22, normalized size = 1.00

$$\frac{\log\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="maxima")
```

```
[Out] log(b*sec(d*x + c) + a*sin(d*x + c))/d
```

Fricas [A]
time = 2.34, size = 33, normalized size = 1.50

$$\frac{\log\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\right)-\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="fricas")
```

```
[Out] (log(a*cos(d*x + c)*sin(d*x + c) + b) - log(-cos(d*x + c)))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

time = 3.76, size = 61, normalized size = 2.77

$$\begin{cases} \frac{x\cos(c)}{\sin(c)} & \text{for } b = 0 \wedge d = 0 \\ \frac{\log(\sin(c+dx))}{d} & \text{for } b = 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{a\sin(c)+b\sec(c)} & \text{for } d = 0 \\ \frac{\log\left(\frac{a\sin(c+dx)}{b}+\sec(c+dx)\right)}{d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x)
```

```
[Out] Piecewise((x*cos(c)/sin(c), Eq(b, 0) & Eq(d, 0)), (log(sin(c + d*x))/d, Eq(
b, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)), (
log(a*sin(c + d*x)/b + sec(c + d*x))/d, True))
```

Giac [A]
time = 111.90, size = 42, normalized size = 1.91

$$\frac{2\log\left(b\tan(dx+c)^2+a\tan(dx+c)+b\right)-\log\left(\tan(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
),x, algorithm="giac")
```

```
[Out] 1/2*(2*log(b*tan(d*x + c)^2 + a*tan(d*x + c) + b) - log(tan(d*x + c)^2 + 1)
)/d
```

Mupad [B]
time = 4.86, size = 133, normalized size = 6.05

$$\frac{\operatorname{atan}\left(\frac{-\cos(c+d\,x)\,a^6+8\,\cos(c+d\,x)\,a^4\,b^2-16\,\cos(c+d\,x)\,a^2\,b^4+\frac{\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6}{1\mathrm{i}\,\cos(c+d\,x)\,a^6-8\mathrm{i}\,\cos(c+d\,x)\,a^4\,b^2+16\mathrm{i}\,\cos(c+d\,x)\,a^2\,b^4+\frac{1\mathrm{i}\,\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6\,1\mathrm{i}}\right)\,2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x)),x)
```

```
[Out] (atan((b^6 - a^6*cos(c + d*x) - 16*a^2*b^4*cos(c + d*x) + 8*a^4*b^2*cos(c +
 d*x) + (a*b^5*sin(2*c + 2*d*x))/2)/(a^6*cos(c + d*x)*1i + b^6*1i + a^2*b^4
*cos(c + d*x)*16i - a^4*b^2*cos(c + d*x)*8i + (a*b^5*sin(2*c + 2*d*x)*1i)/2
))*2i)/d
```

3.642 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{d(b\sec(c+dx)+a\sin(c+dx))}$$

```
[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4470}

$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^2,x]
```

```
[Out] -(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx = -\frac{1}{d(b\sec(c+dx)+a\sin(c+dx))}$$

Mathematica [A]
time = 0.21, size = 27, normalized size = 1.12

$$-\frac{2\cos(c+dx)}{d(2b+a\sin(2(c+dx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Cos[c + d*x])/(d*(2*b + a*Sin[2*(c + d*x)]))
```

Maple [A]
time = 0.66, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$-\frac{1}{d(b\sec(dx+c)+a\sin(dx+c))}$	25
default	$-\frac{1}{d(b\sec(dx+c)+a\sin(dx+c))}$	25
risch	$-\frac{2i\left(e^{3i(dx+c)}+e^{i(dx+c)}\right)}{d\left(a\,e^{4i(dx+c)}+4ib\,e^{2i(dx+c)}-a\right)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^2,x,
method=_RETURNVERBOSE)
```

```
[Out] -1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

Maxima [A]
time = 0.26, size = 24, normalized size = 1.00

$$-\frac{1}{(b\sec(dx+c)+a\sin(dx+c))d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sec(d*x + c) + a*sin(d*x + c))*d)
```

Fricas [A]
time = 2.66, size = 29, normalized size = 1.21

$$-\frac{\cos(dx+c)}{ad\cos(dx+c)\sin(dx+c)+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="fricas")
```

```
[Out] -cos(d*x + c)/(a*d*cos(d*x + c)*sin(d*x + c) + b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.
time = 11.29, size = 49, normalized size = 2.04

$$\begin{cases} -\frac{1}{ad\sin(c+dx)+bd\sec(c+dx)} & \text{for } d\neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**2,x)
```

```
[Out] Piecewise((-1/(a*d*sin(c + d*x) + b*d*sec(c + d*x)), Ne(d, 0)), (x*(a*cos(c
) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(24) = 48.
time = 0.70, size = 108, normalized size = 4.50

$$\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\right)}{\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^2,x, algorithm="giac")
```

```
[Out] 2*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/
2*c) - b)/((b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan
(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)*b*d)
```

Mupad [B]
time = 3.24, size = 47, normalized size = 1.96

$$-\frac{b\left(\cos\left(c+d\,x\right)+1\right)+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{2}}{b\,d\,\left(b+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x))^2,x)
```

```
[Out] -(b*(cos(c + d*x) + 1) + (a*sin(2*c + 2*d*x))/2)/(b*d*(b + (a*sin(2*c + 2*d
*x))/2))
```

3.643 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2d(b\sec(c+dx)+a\sin(c+dx))^2}$$

```
[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {4470}

$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin[
c + d*x])^3,x]
```

```
[Out] -1/2*1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)
```

Rule 4470

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = -\frac{1}{2d(b\sec(c+dx)+a\sin(c+dx))^2}$$

Mathematica [A]
time = 0.51, size = 29, normalized size = 1.12

$$-\frac{2\cos^2(c+dx)}{d(2b+a\sin(2(c+dx)))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*Cos[c + d*x]^2)/(d*(2*b + a*Sin[2*(c + d*x)])^2)
```

Maple [A]
time = 0.74, size = 25, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{1}{2d(b\sec(dx+c)+a\sin(dx+c))^2}$	25
default	$-\frac{1}{2d(b\sec(dx+c)+a\sin(dx+c))^2}$	25
risch	$\frac{2\,e^{6i(dx+c)}+4\,e^{4i(dx+c)}+2\,e^{2i(dx+c)}}{\left(a\,e^{4i(dx+c)}+4ib\,e^{2i(dx+c)}-a\right)^2 d}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x,
method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.92

$$-\frac{1}{2\left(b\sec\left(dx+c\right)+a\sin\left(dx+c\right)\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*sec(d*x + c) + a*sin(d*x + c))^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.
time = 2.27, size = 63, normalized size = 2.42

$$\frac{\cos\left(dx+c\right)^2}{2\left(a^2d\cos\left(dx+c\right)^4-a^2d\cos\left(dx+c\right)^2-2\,abd\cos\left(dx+c\right)\sin\left(dx+c\right)-b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos(d*x + c)^2/(a^2*d*cos(d*x + c)^4 - a^2*d*cos(d*x + c)^2 - 2*a*b*d*c
os(d*x + c)*sin(d*x + c) - b^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(24) = 48.

time = 21.73, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{1}{2a^2d\sin^2{(c+dx)}+4abd\sin{(c+dx)}\sec{(c+dx)}+2b^2d\sec^2{(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(a\cos{(c)}+b\tan{(c)}\sec{(c)})}{(a\sin{(c)}+b\sec{(c)})^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)**3,x)
```

```
[Out] Piecewise((-1/(2*a**2*d*sin(c + d*x)**2 + 4*a*b*d*sin(c + d*x)*sec(c + d*x)
 + 2*b**2*d*sec(c + d*x)**2), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a
*sin(c) + b*sec(c))**3, True))
```

Giac [A]
time = 121.28, size = 37, normalized size = 1.42

$$-\frac{\tan{(dx+c)}^2+1}{2\left(b\tan{(dx+c)}^2+a\tan{(dx+c)}+b\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)
)^3,x, algorithm="giac")
```

```
[Out] -1/2*(tan(d*x + c)^2 + 1)/((b*tan(d*x + c)^2 + a*tan(d*x + c) + b)^2*d)
```

Mupad [B]
time = 6.31, size = 291, normalized size = 11.19

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+b^2)}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(a^2+b^2)}{b^2}+\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-b^2)}{b^2}+\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{b}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{b}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{b}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4a^2+4b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4a^2+4b^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8a^2-6b^2)+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+b^2+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/co
s(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^2 + (2*tan(c/2 + (d*x)/2)^6*(a^2 +
b^2))/b^2 + (2*a*tan(c/2 + (d*x)/2))/b - (4*tan(c/2 + (d*x)/2)^4*(a^2 - b^2
))/b^2 + (2*a*tan(c/2 + (d*x)/2)^3)/b - (2*a*tan(c/2 + (d*x)/2)^5)/b - (2*a
*tan(c/2 + (d*x)/2)^7)/b)/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/
2 + (d*x)/2)^6*(4*a^2 + 4*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^2 - 6*b^2) + b^2
*tan(c/2 + (d*x)/2)^8 + b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 +
(d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.644 $\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$

Optimal. Leaf size=21

$$\text{Int}(F(c,d,\cos(a+bx),r,s)\sin(a+bx),x)$$

```
[Out] CannotIntegrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cos[a + b*x]]/b)
```

Rubi steps

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = -\frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx, x, \cos(a+bx)\right)}{b}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
[Out] Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int F(c,d,\cos(bx+a),r,s)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
[Out] int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="fricas")
[Out] integral(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\cos{(a+bx)},r,s)\sin{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
[Out] Integral(F(c, d, cos(a + b*x), r, s)*sin(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \sin\left(a+b\,x\right)\,F(c,d,\cos\left(a+b\,x\right),r,s)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s),x)
[Out] int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s), x)
```

3.645 $\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$

Optimal. Leaf size=21

$$\text{Int}(\cos(a+bx)F(c,d,\sin(a+bx),r,s),x)$$

```
[Out] CannotIntegrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Sin[a + b*x]]/b
```

Rubi steps

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx,x,\sin(a+bx)\right)}{b}$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
[Out] Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)F(c,d,\sin\left(bx+a\right),r,s)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

```
[Out] int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="maxima")
```

```
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="fricas")
```

```
[Out] integral(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F(c,d,\sin{(a+bx)},r,s)\cos{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

```
[Out] Integral(F(c, d, sin(a + b*x), r, s)*cos(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="giac")
```

```
[Out] integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \cos\left(a+b\,x\right)\,F\left(c,d,\sin\left(a+b\,x\right),r,s\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s),x)
[Out] int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s), x)
```

3.646 $\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(F(c,d,\tan(a+bx),r,s)\sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
[Out] Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Tan[a + b*x]]/b
```

Rubi steps

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int F(c,d,x,r,s)\,dx, x, \tan(a+bx)\right)}{b}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
[Out] Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int F(c,d,\tan(bx+a),r,s)\left(\sec^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
[Out] int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \tan{(a + bx)}, r, s) \sec^{2}{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)**2,x)
[Out] Integral(F(c, d, tan(a + b*x), r, s)*sec(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c, d, \tan(a + b\,x)\,, r, s)}{\cos\left(a + b\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2,x)
[Out] int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2, x)
```

3.647 $\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\csc^2(a+bx)F(c,d,\cot(a+bx),r,s),x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
[Out] -(Defer[Subst][Defer[Int][F[c, d, x, r, s], x], x, Cot[a + b*x]]/b)
```

Rubi steps

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = -\frac{\text{Subst}(\int F(c,d,x,r,s)\,dx,x,\cot(a+bx))}{b}$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
[Out] Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\csc^2(bx+a)\right) F(c,d,\cot(bx+a),r,s)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
[Out] int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="maxima")
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="fricas")
[Out] integral(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F(c, d, \cot{(a + bx)}, r, s) \csc^2{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*F(c,d,cot(b*x+a),r,s),x)
[Out] Integral(F(c, d, cot(a + b*x), r, s)*csc(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="giac")
[Out] integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{F(c, d, \cot(a + b\,x)\,, r, s)}{\sin\left(a + b\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2,x)
[Out] int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2, x)
```

3.648 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cos(x))}{b}$$

```
[Out] -ln(a+b*cos(x))/b
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 31}

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a + b*Cos[x]),x]
[Out] -(Log[a + b*Cos[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a+b\cos(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{\log(a+b\cos(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(a+b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a + b*Cos[x]),x]
[Out] -(Log[a + b*Cos[x]]/b)
```

Maple [A]
time = 0.05, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\cos(x))}{b}$	13
default	$-\frac{\ln(a+b\cos(x))}{b}$	13
risch	$\frac{ix}{b}-\frac{\ln\left(e^{2ix}+1+\frac{2a\,e^{ix}}{b}\right)}{b}$	33
norman	$\frac{\ln(1+\tan^2(\frac{x}{2}))}{b}-\frac{\ln(a(\tan^2(\frac{x}{2}))-b(\tan^2(\frac{x}{2}))+a+b)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*cos(x))/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
[Out] -log(b*cos(x) + a)/b
```

Fricas [A]
time = 3.63, size = 15, normalized size = 1.25

$$-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
[Out] -log(-b*cos(x) - a)/b
```

Sympy [A]
time = 0.15, size = 17, normalized size = 1.42

$$\begin{cases} -\frac{\log\left(\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for}\ b\neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x)
```

```
[Out] Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Giac [A]
time = 0.42, size = 13, normalized size = 1.08

$$-\frac{\log\left(|b\cos\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -log(abs(b*cos(x) + a))/b
```

Mupad [B]
time = 0.06, size = 12, normalized size = 1.00

$$-\frac{\ln\left(a+b\,\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a + b*cos(x)),x)
```

```
[Out] -log(a + b*cos(x))/b
```

3.649 $\int (a+b\cos(x))^n \sin(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$$

```
[Out] -(a+b*cos(x))^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 32}

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[x])^n*Sin[x],x]
```

```
[Out] -((a + b*Cos[x])^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(x))^n \sin(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\cos(x)\right)}{b}\\ &= -\frac{(a+b\cos(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 0.95

$$-\frac{(a+b\cos(x))^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[x])^n*Sin[x],x]
[Out] -((a + b*Cos[x])^(1 + n)/(b + b*n))
```

Maple [A]
time = 1.05, size = 21, normalized size = 1.05

method	result
derivativedivides	$-\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$
default	$-\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$
norman	$\frac{-\frac{(a+b)\,\mathrm{e}^{n\ln\left(a+\frac{b\left(1-\left(\tan^2\left(\frac{x}{2}\right)\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}}{(1+n)b}-\frac{(a-b)\left(\tan^2\left(\frac{x}{2}\right)\right)\mathrm{e}^{n\ln\left(a+\frac{b\left(1-\left(\tan^2\left(\frac{x}{2}\right)\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}}{(1+n)b}}{1+\tan^2\left(\frac{x}{2}\right)}$
risch	$-\frac{\left(a\,\mathrm{e}^{ix}+\frac{b\,\mathrm{e}^{2ix}}{2}+\frac{b}{2}\right)^n\left(\mathrm{e}^{i\Re(x)}\right)^{-n}\mathrm{e}^{-ix}\,\mathrm{e}^{-\frac{in\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}\left(ia\,\mathrm{e}^{ix}+\frac{ib\,\mathrm{e}^{2ix}}{2}+\frac{ib}{2}\right)\mathrm{csgn}(ia+ib\cos(x))}{2}}\,\mathrm{e}^{\frac{in\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}(ia\cdots}{2}}\cdots}{2(1+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))^n*sin(x),x,method=_RETURNVERBOSE)
[Out] -(a+b*cos(x))^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.00

$$-\frac{(b\cos(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="maxima")
[Out] -(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 4.29, size = 23, normalized size = 1.15

$$-\frac{(b\cos(x)+a)(b\cos(x)+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="fricas")
[Out] -(b*cos(x) + a)*(b*cos(x) + a)^n/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.
time = 0.50, size = 63, normalized size = 3.15

$$\begin{cases} -\frac{\cos(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ -a^n \cos(x) & \text{for } b = 0 \\ -\frac{\log\left(\frac{a}{b} + \cos(x)\right)}{b} & \text{for } n = -1 \\ -\frac{a(a+b\cos(x))^n}{bn+b} - \frac{b(a+b\cos(x))^n \cos(x)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))**n*sin(x),x)
```

```
[Out] Piecewise((-cos(x)/a, Eq(b, 0) & Eq(n, -1)), (-a**n*cos(x), Eq(b, 0)), (-lo
g(a/b + cos(x))/b, Eq(n, -1)), (-a*(a + b*cos(x))**n/(b*n + b) - b*(a + b*c
os(x))**n*cos(x)/(b*n + b), True))
```

Giac [A]
time = 0.41, size = 20, normalized size = 1.00

$$-\frac{(b\cos(x) + a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^n*sin(x),x, algorithm="giac")
```

```
[Out] -(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 3.15, size = 20, normalized size = 1.00

$$-\frac{(a + b\,\cos(x))^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a + b*cos(x))^n,x)
```

```
[Out] -(a + b*cos(x))^(n + 1)/(b*(n + 1))
```

3.650 $\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=5

$$-\sinh^{-1}(\cos(x))$$

```
[Out] -arcsinh(cos(x))
```

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3269, 221}

$$-\sinh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] -ArcSinh[Cos[x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= -\sinh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.
time = 0.02, size = 16, normalized size = 3.20

$$-\tanh^{-1}\left(\frac{\cos(x)}{\sqrt{1+\cos^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] -ArcTanh[Cos[x]/Sqrt[1 + Cos[x]^2]]
```

Maple [A]
time = 0.06, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$-\operatorname{arcsinh}\left(\cos \left(x\right)\right)$	6
default	$-\operatorname{arcsinh}\left(\cos \left(x\right)\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(cos(x))
```

Maxima [A]
time = 0.48, size = 5, normalized size = 1.00

$$-\operatorname{arsinh}\left(\cos \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsinh(cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(5) = 10.
time = 3.80, size = 36, normalized size = 7.20

$$\frac{1}{4} \, \log \left(8 \, \cos \left(x\right)^{4} + 8 \, \cos \left(x\right)^{2} - 4 \, {\left(2 \, \cos \left(x\right)^{3} + \cos \left(x\right)\right)} \sqrt{\cos \left(x\right)^{2} + 1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 + 8*cos(x)^2 - 4*(2*cos(x)^3 + cos(x))*sqrt(cos(x)^2 + 1
) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(x \right)}}{\sqrt{\cos^{2}{\left(x \right)} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)**2)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(cos(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.
time = 0.42, size = 14, normalized size = 2.80

$$\log\left(\sqrt{\cos\left(x\right)^2 + 1} - \cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(sqrt(cos(x)^2 + 1) - cos(x))
```

Mupad [B]
time = 3.08, size = 5, normalized size = 1.00

$$-\operatorname{asinh}(\cos\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^2 + 1)^(1/2),x)
```

```
[Out] -asinh(cos(x))
```

3.651 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -sin(cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4420, 2717}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(\cos(x))\sin(x)\,dx &= -\text{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right)\\ &= -\sin(\cos(x))\end{aligned}$$

Mathematica [A]
time = 1.72, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
[Out] -Sin[Cos[x]]
```

Maple [A]
time = 0.03, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$-\sin(\cos(x))$	6
default	$-\sin(\cos(x))$	6
risch	$-\sin(\cos(x))$	6
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right)\tan\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2+2\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)-2\tan\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2+2\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)}{\left(1+\tan^2\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(1+\tan^2\left(\frac{x}{2}\right)\right)}\right)\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x,method=_RETURNVERBOSE)
[Out] -sin(cos(x))
```

Maxima [A]
time = 0.28, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
[Out] -sin(cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(5) = 10$.
time = 3.18, size = 20, normalized size = 4.00

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A]
time = 0.13, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Giac [A]
time = 0.40, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -sin(cos(x))
```

Mupad [B]
time = 0.09, size = 5, normalized size = 1.00

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

3.652 $\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx$

Optimal. Leaf size=28

$$\frac{\cos(x)}{4}-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))-\frac{1}{2}\cos(x)\sin^2(\cos(x))$$

```
[Out] 1/4*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/2*cos(x)*sin(cos(x))^2
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4420, 3524, 2715, 8}

$$\frac{\cos(x)}{4}-\frac{1}{2}\cos(x)\sin^2(\cos(x))-\frac{1}{4}\cos(\cos(x))\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

```
[Out] Cos[x]/4 - (Cos[Cos[x]]*Sin[Cos[x]])/4 - (Cos[x]*Sin[Cos[x]]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3524

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx &= -\mathrm{Subst}(\int x\cos(x)\sin(x)\,dx, x, \cos(x))\\ &= -\frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{2}\mathrm{Subst}\left(\int \sin^2(x)\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x)) + \frac{1}{4}\mathrm{Subst}(\\ &= \frac{\cos(x)}{4} - \frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.95, size = 21, normalized size = 0.75

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
[Out] (Cos[x]*Cos[2*Cos[x]])/4 - Sin[2*Cos[x]]/8
```

Maple [A]
time = 0.06, size = 23, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\cos(x)\left(\cos^2(\cos(x))\right)}{2} - \frac{\cos(\cos(x))\sin(\cos(x))}{4} - \frac{\cos(x)}{4}$	23
default	$\frac{\cos(x)\left(\cos^2(\cos(x))\right)}{2} - \frac{\cos(\cos(x))\sin(\cos(x))}{4} - \frac{\cos(x)}{4}$	23
risch	$\frac{\cos(-2\cos(x)+x)}{8} + \frac{\cos(2\cos(x)+x)}{8} - \frac{\sin(2\cos(x))}{8}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*cos(x)*cos(cos(x))^2-1/4*cos(cos(x))*sin(cos(x))-1/4*cos(x)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.61

$$\frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="maxima")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(22) = 44$.
time = 3.28, size = 73, normalized size = 2.61

$$\frac{1}{2}\cos\left(x\right)\cos\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)^{2}+\frac{1}{4}\cos\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\sin\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-\frac{1}{4}\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 + 1/4*cos((tan(1/2*
x)^2 - 1)/(tan(1/2*x)^2 + 1))*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1)) -
1/4*cos(x)
```

Sympy [A]
time = 0.81, size = 34, normalized size = 1.21

$$-\frac{\sin^{2}\left(\cos\left(x\right)\right)\cos\left(x\right)}{4}-\frac{\sin\left(\cos\left(x\right)\right)\cos\left(\cos\left(x\right)\right)}{4}+\frac{\cos\left(x\right)\cos^{2}\left(\cos\left(x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

```
[Out] -sin(cos(x))**2*cos(x)/4 - sin(cos(x))*cos(cos(x))/4 + cos(x)*cos(cos(x))**
2/4
```

Giac [A]
time = 0.41, size = 17, normalized size = 0.61

$$\frac{1}{4}\cos\left(x\right)\cos\left(2\,\cos\left(x\right)\right)-\frac{1}{8}\sin\left(2\,\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="giac")
```

```
[Out] 1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

Mupad [B]
time = 3.05, size = 22, normalized size = 0.79

$$\frac{\cos\left(x\right)\,\cos\left(\cos\left(x\right)\right)^{2}}{2}-\frac{\sin\left(\cos\left(x\right)\right)\,\cos\left(\cos\left(x\right)\right)}{4}-\frac{\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(cos(x))*cos(x)*sin(x),x)
```

```
[Out] (cos(cos(x))^2*cos(x))/2 - cos(x)/4 - (cos(cos(x))*sin(cos(x)))/4
```

3.653 $\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx$

Optimal. Leaf size=26

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

```
[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4420, 4439, 2717}

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

```
[Out] -1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 4439

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\text{Subst}\left(\int \cos(x)\sin^2(6x)\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{\cos(x)}{2} - \frac{1}{4}\cos(11x) - \frac{1}{4}\cos(13x)\right)\,dx, x, \cos(x)\right)$$
$$= \frac{1}{4}\text{Subst}(\int \cos(11x)\,dx, x, \cos(x)) + \frac{1}{4}\text{Subst}(\int \cos(13x)\,dx, x, \cos(x$$
$$= -\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Mathematica [**A**]
time = 3.03, size = 26, normalized size = 1.00

$$-\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
[Out] -1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

Maple [**A**]
time = 0.14, size = 21, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$	21
default	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$	21
risch	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

Maxima [**A**]
time = 0.28, size = 20, normalized size = 0.77

$$\frac{1}{52}\sin\left(13\cos\left(x\right)\right) + \frac{1}{44}\sin\left(11\cos\left(x\right)\right) - \frac{1}{2}\sin\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(20) = 40.
time = 2.93, size = 168, normalized size = 6.46

$$-\frac{4}{143}\left(2816\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{12}-6912\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{10}+6048\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{8}-2240\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{6}+315\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{4}-9\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{2}-18\right)\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="fricas")
[Out] -4/143*(2816*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^12 - 6912*cos((tan(
1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^10 + 6048*cos((tan(1/2*x)^2 - 1)/(tan(1/2
*x)^2 + 1))^8 - 2240*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^6 + 315*cos
((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^4 - 9*cos((tan(1/2*x)^2 - 1)/(tan(1
/2*x)^2 + 1))^2 - 18)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(22) = 44.
time = 3.99, size = 54, normalized size = 2.08

$$-\frac{71\sin(\cos(x))\sin^2(6\cos(x))}{143}-\frac{72\sin(\cos(x))\cos^2(6\cos(x))}{143}+\frac{12\sin(6\cos(x))\cos(\cos(x))\cos(6\cos(x))}{143}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))**2,x)
[Out] -71*sin(cos(x))*sin(6*cos(x))**2/143 - 72*sin(cos(x))*cos(6*cos(x))**2/143
+ 12*sin(6*cos(x))*cos(cos(x))*cos(6*cos(x))/143
```

Giac [A]
time = 0.42, size = 20, normalized size = 0.77

$$\frac{1}{52}\sin(13\cos(x))+\frac{1}{44}\sin(11\cos(x))-\frac{1}{2}\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="giac")
[Out] 1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

Mupad [B]
time = 3.13, size = 20, normalized size = 0.77

$$\frac{\sin(11\cos(x))}{44}-\frac{\sin(\cos(x))}{2}+\frac{\sin(13\cos(x))}{52}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(6*cos(x))^2*sin(x),x)
[Out] sin(11*cos(x))/44 - sin(cos(x))/2 + sin(13*cos(x))/52
```

3.654 $\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx$

Optimal. Leaf size=36

$$\frac{a(a+b\cos^2(x))^4}{8b^2}-\frac{(a+b\cos^2(x))^5}{10b^2}$$

```
[Out] 1/8*a*(a+b*cos(x)^2)^4/b^2-1/10*(a+b*cos(x)^2)^5/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4420, 272, 45}

$$\frac{a(a+b\cos^2(x))^4}{8b^2}-\frac{(a+b\cos^2(x))^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (a*(a + b*Cos[x]^2)^4)/(8*b^2) - (a + b*Cos[x]^2)^5/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx &= -\text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)\,dx,x,\cos^2(x)\right)\right)\\
&= \frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(36) = 72.
time = 0.23, size = 137, normalized size = 3.81

$$\frac{1}{32}\left(-12a^2b\cos^4(x)-8ab^2\cos^6(x)-2b^3\cos^8(x)-4a^3\cos(2x)-4a^2b\cos^3(x)\cos(3x)-a^3\cos(4x)-\frac{1}{32}ab^2(48\cos(2x)+36\cos(4x)+16\cos(6x)+3\cos(8x))-\frac{1}{320}b^3(140\cos(2x)+100\cos(4x)+50\cos(6x)+15\cos(8x)+2\cos(10x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

```
[Out] (-12*a^2*b*Cos[x]^4 - 8*a*b^2*Cos[x]^6 - 2*b^3*Cos[x]^8 - 4*a^3*Cos[2*x] -
4*a^2*b*Cos[x]^3*Cos[3*x] - a^3*Cos[4*x] - (a*b^2*(48*Cos[2*x] + 36*Cos[4*x
] + 16*Cos[6*x] + 3*Cos[8*x]))/32 - (b^3*(140*Cos[2*x] + 100*Cos[4*x] + 50*
Cos[6*x] + 15*Cos[8*x] + 2*Cos[10*x]))/320)/32
```

Maple [A]
time = 0.08, size = 40, normalized size = 1.11

method	result
derivativedivides	$-\frac{b^3(\cos^{10}(x))}{10}-\frac{3a\,b^2(\cos^8(x))}{8}-\frac{a^2b(\cos^6(x))}{2}-\frac{a^3(\cos^4(x))}{4}$
default	$-\frac{b^3(\cos^{10}(x))}{10}-\frac{3a\,b^2(\cos^8(x))}{8}-\frac{a^2b(\cos^6(x))}{2}-\frac{a^3(\cos^4(x))}{4}$
risch	$-\frac{b^3\cos(10x)}{5120}-\frac{3\cos(8x)a\,b^2}{1024}-\frac{\cos(8x)b^3}{512}-\frac{\cos(6x)a^2b}{64}-\frac{3\cos(6x)a\,b^2}{128}-\frac{9\cos(6x)b^3}{1024}-\frac{\cos(4x)a^3}{32}-$ 3 cos

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*b^3*cos(x)^10-3/8*a*b^2*cos(x)^8-1/2*a^2*b*cos(x)^6-1/4*a^3*cos(x)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(32) = 64.
time = 0.28, size = 103, normalized size = 2.86

$$\frac{1}{10}b^3\sin(x)^{10}-\frac{1}{8}\left(3ab^2+4b^3\right)\sin(x)^8+\frac{1}{2}\left(a^2b+3ab^2+2b^3\right)\sin(x)^6-\frac{1}{4}\left(a^3+6a^2b+9ab^2+4b^3\right)\sin(x)^4+\frac{1}{2}\left(a^3+3a^2b+3ab^2+b^3\right)\sin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 - 1/8*(3*a*b^2 + 4*b^3)*sin(x)^8 + 1/2*(a^2*b + 3*a*b^2
+ 2*b^3)*sin(x)^6 - 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*sin(x)^4 + 1/2*(a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(x)^2
```

Fricas [A]
time = 2.50, size = 39, normalized size = 1.08

$$-\frac{1}{10}\,b^{3}\cos\left(x\right)^{10}-\frac{3}{8}\,ab^{2}\cos\left(x\right)^{8}-\frac{1}{2}\,a^{2}b\cos\left(x\right)^{6}-\frac{1}{4}\,a^{3}\cos\left(x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

Sympy [A]
time = 1.36, size = 46, normalized size = 1.28

$$-\frac{a^{3}\cos^{4}\left(x\right)}{4}-\frac{a^{2}b\cos^{6}\left(x\right)}{2}-\frac{3ab^{2}\cos^{8}\left(x\right)}{8}-\frac{b^{3}\cos^{10}\left(x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(a+b*cos(x)**2)**3*sin(x),x)
```

```
[Out] -a**3*cos(x)**4/4 - a**2*b*cos(x)**6/2 - 3*a*b**2*cos(x)**8/8 - b**3*cos(x)
**10/10
```

Giac [A]
time = 0.42, size = 39, normalized size = 1.08

$$-\frac{1}{10}\,b^{3}\cos\left(x\right)^{10}-\frac{3}{8}\,ab^{2}\cos\left(x\right)^{8}-\frac{1}{2}\,a^{2}b\cos\left(x\right)^{6}-\frac{1}{4}\,a^{3}\cos\left(x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*cos
(x)^4
```

Mupad [B]
time = 0.09, size = 39, normalized size = 1.08

$$-\frac{a^{3}\cos\left(x\right)^{4}}{4}-\frac{a^{2}\,b\cos\left(x\right)^{6}}{2}-\frac{3\,a\,b^{2}\cos\left(x\right)^{8}}{8}-\frac{b^{3}\cos\left(x\right)^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)*(a + b*cos(x)^2)^3,x)
```

```
[Out] - (a^3*cos(x)^4)/4 - (b^3*cos(x)^10)/10 - (a^2*b*cos(x)^6)/2 - (3*a*b^2*cos
(x)^8)/8
```

3.655 $\int \sin(3x)\sin(\cos(3x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{3}\cos(\cos(3x))$$

[Out] 1/3*cos(cos(3*x))

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4420, 2718}

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

[In] Int[Sin[3*x]*Sin[Cos[3*x]],x]

[Out] Cos[Cos[3*x]]/3

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sin(3x)\sin(\cos(3x))\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \sin(x)\,dx, x, \cos(3x)\right)\right)\\ &= \frac{1}{3}\cos(\cos(3x))\end{aligned}$$

Mathematica [A]
time = 1.76, size = 9, normalized size = 1.00

$$\frac{1}{3}\cos(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]*Sin[Cos[3*x]],x]
```

```
[Out] Cos[Cos[3*x]]/3
```

Maple [A]
time = 0.03, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\cos(\cos(3x))}{3}$	8
default	$\frac{\cos(\cos(3x))}{3}$	8
risch	$\frac{\cos(\cos(3x))}{3}$	8
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{1-\left(\tan^2\left(\frac{3x}{2}\right)\right)}{2\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}\right)\right)}{3}-\frac{2\left(\tan^2\left(\frac{3x}{2}\right)\right)\left(\tan^2\left(\frac{1-\left(\tan^2\left(\frac{3x}{2}\right)\right)}{2\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}\right)\right)}{3}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2\left(\frac{1-\left(\tan^2\left(\frac{3x}{2}\right)\right)}{2\left(1+\tan^2\left(\frac{3x}{2}\right)\right)}\right)\right)}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(cos(3*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*cos(cos(3*x))
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.78

$$\frac{1}{3}\cos\left(\cos\left(3\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="maxima")
```

```
[Out] 1/3*cos(cos(3*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(7) = 14.
time = 2.96, size = 22, normalized size = 2.44

$$\frac{1}{3}\cos\left(\frac{\tan\left(\frac{3}{2}\,x\right)^2-1}{\tan\left(\frac{3}{2}\,x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="fricas")
```

```
[Out] 1/3*cos((tan(3/2*x)^2 - 1)/(tan(3/2*x)^2 + 1))
```

Sympy [A]
time = 0.13, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] cos(cos(3*x))/3
```

Giac [A]
time = 0.45, size = 7, normalized size = 0.78

$$\frac{1}{3}\cos(\cos(3x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="giac")
```

```
[Out] 1/3*cos(cos(3*x))
```

Mupad [B]
time = 3.01, size = 7, normalized size = 0.78

$$\frac{\cos(\cos(3x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(cos(3*x)),x)
```

```
[Out] cos(cos(3*x))/3
```

3.656 $\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx$

Optimal. Leaf size=31

$$\frac{1}{3}e^{\cos(1+3x)}-\frac{1}{3}e^{\cos(1+3x)}\cos(1+3x)$$

```
[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4420, 2207, 2225}

$$\frac{1}{3}e^{\cos(3x+1)}-\frac{1}{3}e^{\cos(3x+1)}\cos(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

```
[Out] E^Cos[1 + 3*x]/3 - (E^Cos[1 + 3*x]*Cos[1 + 3*x])/3
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\left(\frac{1}{3}\text{Subst}\left(\int e^x x\,dx, x, \cos(1+3x)\right)\right)$$
$$= -\frac{1}{3}e^{\cos(1+3x)}\cos(1+3x) + \frac{1}{3}\text{Subst}\left(\int e^x\,dx, x, \cos(1+3x)\right)$$
$$= \frac{1}{3}e^{\cos(1+3x)} - \frac{1}{3}e^{\cos(1+3x)}\cos(1+3x)$$

Mathematica [A]
time = 0.08, size = 24, normalized size = 0.77

$$\frac{2}{3}e^{\cos(1+3x)}\sin^2\left(\frac{1}{2}(1+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
[Out] (2*E^Cos[1 + 3*x]*Sin[(1 + 3*x)/2]^2)/3
```

Maple [A]
time = 0.12, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$\frac{e^{\cos(1+3x)}}{3}-\frac{e^{\cos(1+3x)}\cos(1+3x)}{3}$	26
default	$\frac{e^{\cos(1+3x)}}{3}-\frac{e^{\cos(1+3x)}\cos(1+3x)}{3}$	26
norman	$\frac{\frac{2\left(\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)\right)e^{\frac{1-\left(\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)\right)}{1+\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)}}}{3}+\frac{2\left(\tan^4\left(\frac{1}{2}+\frac{3x}{2}\right)\right)e^{\frac{1-\left(\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)\right)}{1+\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)}}}{3}}{\left(1+\tan^2\left(\frac{1}{2}+\frac{3x}{2}\right)\right)^2}$	87
risch	$-\frac{e^{3ix}e^{\frac{e^{i(1+3x)}}{2}}e^{\frac{e^{-i(1+3x)}}{2}}e^{i}}{6}+\frac{e^{\frac{e^{i(1+3x)}}{2}}e^{\frac{e^{-i(1+3x)}}{2}}}{3}-\frac{e^{-3ix}e^{\frac{e^{i(1+3x)}}{2}}e^{\frac{e^{-i(1+3x)}}{2}}e^{-i}}{6}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x,method=_RETURNVERBOSE)
[Out] 1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="maxima")
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

Fricas [A]
time = 2.27, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="fricas")
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

Sympy [A]
time = 0.16, size = 26, normalized size = 0.84

$$-\frac{e^{\cos\left(3x+1\right)}\cos\left(3x+1\right)}{3}+\frac{e^{\cos\left(3x+1\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
[Out] -exp(cos(3*x + 1))*cos(3*x + 1)/3 + exp(cos(3*x + 1))/3
```

Giac [A]
time = 0.42, size = 17, normalized size = 0.55

$$-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="giac")
[Out] -1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

Mupad [B]
time = 0.12, size = 17, normalized size = 0.55

$$-\frac{\mathrm{e}^{\cos\left(3\,x+1\right)}\,\left(\cos\left(3\,x+1\right)-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(3*x + 1))*cos(3*x + 1)*sin(3*x + 1),x)
[Out] -(exp(cos(3*x + 1))*(cos(3*x + 1) - 1))/3
```

3.657 $\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx$

Optimal. Leaf size=9

$$-\frac{1}{3}\text{ArcSin}\left(\cos^3(x)\right)$$

```
[Out] -1/3*arcsin(cos(x)^3)
```

Rubi [A]
time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4420, 281, 222}

$$-\frac{1}{3}\text{ArcSin}\left(\cos^3(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] -1/3*ArcSin[Cos[x]^3]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = -\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^6}}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cos^3(x)\right)\right)$$
$$= -\frac{1}{3}\sin^{-1}\left(\cos^3(x)\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 1.46, size = 162, normalized size = 18.00

$$-\frac{i\cos^2(x)\Pi\left(\frac{3}{2}+\frac{i\sqrt{3}}{2};i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\tan(x)\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)\sin(x)\sqrt{1-\frac{2i\tan^2(x)}{-3i+\sqrt{3}}}\sqrt{1+\frac{2i\tan^2(x)}{3i+\sqrt{3}}}}{\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{3}}}\sqrt{1-\cos^6(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

```
[Out] ((-I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I
+ Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3])]*Sin[x]*Sqrt[1 - ((2*I
)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])])/(
Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[3])]*Sqrt[1 - Cos[x]^6])
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(x)\right)\sin(x)}{\sqrt{1-\left(\cos^6(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

```
[Out] int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.
time = 0.49, size = 18, normalized size = 2.00

$$\frac{1}{3}\arctan\left(\frac{\sqrt{-\cos(x)^6+1}}{\cos(x)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(sqrt(-cos(x)^6 + 1)/cos(x)^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(7) = 14$.
time = 1.71, size = 29, normalized size = 3.22

$$\frac{1}{6}\arctan\left(\frac{2\sqrt{-\cos\left(x\right)^{6}+1}\,\cos\left(x\right)^{3}}{2\,\cos\left(x\right)^{6}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(2*sqrt(-cos(x)^6 + 1)*cos(x)^3/(2*cos(x)^6 - 1))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(1-cos(x)**6)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 7, normalized size = 0.78

$$-\frac{1}{3}\arcsin\left(\cos\left(x\right)^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*arcsin(cos(x)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{\cos\left(x\right)^{2}\,\sin\left(x\right)}{\sqrt{1-\cos\left(x\right)^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2),x)
```

```
[Out] int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2), x)
```

3.658 $\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx$

Optimal. Leaf size=71

$$\frac{1152\sqrt{1-5\cos(x)}}{3125}+\frac{64(1-5\cos(x))^{3/2}}{3125}-\frac{88(1-5\cos(x))^{5/2}}{15625}-\frac{8(1-5\cos(x))^{7/2}}{21875}+\frac{2(1-5\cos(x))^{9/2}}{28125}$$

```
[Out] 64/3125*(1-5*cos(x))^(3/2)-88/15625*(1-5*cos(x))^(5/2)-8/21875*(1-5*cos(x))
^(7/2)+2/28125*(1-5*cos(x))^(9/2)+1152/3125*(1-5*cos(x))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2747, 711}

$$\frac{2(1-5\cos(x))^{9/2}}{28125}-\frac{8(1-5\cos(x))^{7/2}}{21875}-\frac{88(1-5\cos(x))^{5/2}}{15625}+\frac{64(1-5\cos(x))^{3/2}}{3125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (1152*Sqrt[1 - 5*Cos[x]])/3125 + (64*(1 - 5*Cos[x])^(3/2))/3125 - (88*(1 -
5*Cos[x])^(5/2))/15625 - (8*(1 - 5*Cos[x])^(7/2))/21875 + (2*(1 - 5*Cos[x])
^(9/2))/28125
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{\text{Subst}\left(\int \frac{(25-x^2)^2}{\sqrt{1+x}}\,dx, x, -5\cos(x)\right)}{3125}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{576}{\sqrt{1+x}} + 96\sqrt{1+x} - 44(1+x)^{3/2} - 4(1+x)^{5/2} + (1+x)^{7/2}\right)\,dx, x$$
$$= \frac{1152\sqrt{1-5\cos(x)}}{3125} + \frac{64(1-5\cos(x))^{3/2}}{3125} - \frac{88(1-5\cos(x))^{5/2}}{15625} - \frac{8(1-5\cos(x))^{7}}{21875}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.83

$$\frac{180607\left(-1+\sqrt{1-5\cos(x)}\right)}{562500} + \sqrt{1-5\cos(x)}\left(-\frac{6772\cos(x)}{196875} - \frac{2227\cos(2x)}{39375} + \frac{4\cos(3x)}{1575} + \frac{1}{180}\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

```
[Out] (180607*(-1 + Sqrt[1 - 5*Cos[x]]))/562500 + Sqrt[1 - 5*Cos[x]]*((-6772*Cos[
x])/196875 - (2227*Cos[2*x])/39375 + (4*Cos[3*x])/1575 + Cos[4*x]/180)
```

Maple [A]
time = 0.25, size = 49, normalized size = 0.69

method	result	size
default	$\frac{32\sqrt{10\left(\sin^2\left(\frac{x}{2}\right)\right)-4}\,\left(21875\left(\sin^8\left(\frac{x}{2}\right)\right)-46250\left(\sin^6\left(\frac{x}{2}\right)\right)+17175\left(\sin^4\left(\frac{x}{2}\right)\right)+9160\left(\sin^2\left(\frac{x}{2}\right)\right)+7328\right)}{984375}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5/(1-5*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 32/984375*(10*sin(1/2*x)^2-4)^(1/2)*(21875*sin(1/2*x)^8-46250*sin(1/2*x)^6+
17175*sin(1/2*x)^4+9160*sin(1/2*x)^2+7328)
```

Maxima [A]
time = 0.26, size = 51, normalized size = 0.72

$$\frac{2}{28125}\left(-5\cos(x)+1\right)^{\frac{9}{2}} - \frac{8}{21875}\left(-5\cos(x)+1\right)^{\frac{7}{2}} - \frac{88}{15625}\left(-5\cos(x)+1\right)^{\frac{5}{2}} + \frac{64}{3125}\left(-5\cos(x)+1\right)^{\frac{3}{2}} + \frac{1152}{3125}\sqrt{-5\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="maxima")
```

[Out] 2/28125*(-5*cos(x) + 1)^(9/2) - 8/21875*(-5*cos(x) + 1)^(7/2) - 88/15625*(-5*cos(x) + 1)^(5/2) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)

Fricas [A]
time = 2.30, size = 34, normalized size = 0.48

$$\frac{2}{984375}\left(21875\cos\left(x\right)^{4}+5000\cos\left(x\right)^{3}-77550\cos\left(x\right)^{2}-20680\cos\left(x\right)+188603\right)\sqrt{-5\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="fricas")

[Out] 2/984375*(21875*cos(x)^4 + 5000*cos(x)^3 - 77550*cos(x)^2 - 20680*cos(x) + 188603)*sqrt(-5*cos(x) + 1)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**5/(1-5*cos(x))**(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep

Giac [A]
time = 0.40, size = 75, normalized size = 1.06

$$\frac{2}{28125}\left(5\cos\left(x\right)-1\right)^{4}\sqrt{-5\cos\left(x\right)+1}+\frac{8}{21875}\left(5\cos\left(x\right)-1\right)^{3}\sqrt{-5\cos\left(x\right)+1}-\frac{88}{15625}\left(5\cos\left(x\right)-1\right)^{2}\sqrt{-5\cos\left(x\right)+1}+\frac{64}{3125}\left(-5\cos\left(x\right)+1\right)^{\frac{3}{2}}+\frac{1152}{3125}\sqrt{-5\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="giac")

[Out] 2/28125*(5*cos(x) - 1)^4*sqrt(-5*cos(x) + 1) + 8/21875*(5*cos(x) - 1)^3*sqrt(-5*cos(x) + 1) - 88/15625*(5*cos(x) - 1)^2*sqrt(-5*cos(x) + 1) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(x\right)^5}{\sqrt{1-5\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^5/(1 - 5*cos(x))^(1/2),x)

[Out] int(sin(x)^5/(1 - 5*cos(x))^(1/2), x)

3.659 $\int e^{n\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=18

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

```
[Out] -exp(n*cos(b*x+a))/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4420, 2225}

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{e^{n\cos(a+bx)}}{bn}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

```
[Out] -(E^(n*Cos[a + b*x])/(b*n))
```

Maple [A]
time = 0.04, size = 18, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{e^{n\cos(bx+a)}}{bn}$	18
default	$-\frac{e^{n\cos(bx+a)}}{bn}$	18
risch	$-\frac{e^{n\cos(bx+a)}}{bn}$	18
norman	$\frac{-\frac{e^{\frac{n\left(1-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}}}{nb}-\frac{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)e^{\frac{n\left(1-\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}}}{nb}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(n*cos(b*x+a))/b/n
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

Fricas [A]
time = 2.95, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

time = 0.12, size = 39, normalized size = 2.17

$$\begin{cases} x\sin(a) & \text{for } b = 0 \wedge n = 0 \\ xe^{n\cos(a)}\sin(a) & \text{for } b = 0 \\ -\frac{\cos(a+bx)}{b} & \text{for } n = 0 \\ -\frac{e^{n\cos(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cos(a))*sin(a), Eq(b, 0
)), (-cos(a + b*x)/b, Eq(n, 0)), (-exp(n*cos(a + b*x))/(b*n), True))
```

Giac [A]

time = 0.42, size = 17, normalized size = 0.94

$$-\frac{e^{(n\cos(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*x + a))/(b*n)
```

Mupad [B]

time = 0.10, size = 17, normalized size = 0.94

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(a + b*x),x)
```

```
[Out] -exp(n*cos(a + b*x))/(b*n)
```

3.660 $\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx$

Optimal. Leaf size=23

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

[Out] -exp(n*cos(c*(b*x+a)))/b/c/n

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4420, 2225}

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]

[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx,x,\cos(c(a+bx))\right)}{bc}\\ &= -\frac{e^{n\cos(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 23, normalized size = 1.00

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

```
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Maple [A]
time = 0.09, size = 24, normalized size = 1.04

method	result	size
risch	$-\frac{e^{n\cos(c(bx+a))}}{bcn}$	23
default	$-\frac{e^{n\cos(bcx+ac)}}{bcn}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Fricas [A]
time = 1.77, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

time = 5.55, size = 53, normalized size = 2.30

$$\begin{cases} xe^{n\cos(ac)}\sin(ac) & \text{for } b = 0 \\ 0 & \text{for } c = 0 \\ \begin{cases} x\sin(ac) & \text{for } b = 0 \\ 0 & \text{for } c = 0 \\ -\frac{\cos(c(a+bx))}{bc} & \text{otherwise} \end{cases} & \text{for } n = 0 \\ -\frac{e^{n\cos(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewise
((x*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (-cos(c*(a + b*x))/(b*c), True)), E
q(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))
```

Giac [A]
time = 0.44, size = 23, normalized size = 1.00

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Mupad [B]
time = 3.16, size = 23, normalized size = 1.00

$$-\frac{e^{n\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
```

```
[Out] -exp(n*cos(a*c + b*c*x))/(b*c*n)
```

3.661 $\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx$

Optimal. Leaf size=24

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

```
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4420, 2225}

$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

```
[Out] -(E^(n*Cos[a*c + b*c*x])/(b*c*n))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx &= -\frac{\text{Subst}\left(\int e^{nx}\,dx, x, \cos(ac+bcx)\right)}{bc}\\ &= -\frac{e^{n\cos(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 23, normalized size = 0.96

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
[Out] -(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

Maple [A]
time = 0.05, size = 24, normalized size = 1.00

method	result	size
risch	$-\frac{e^{n\cos(c(bx+a))}}{bcn}$	23
default	$-\frac{e^{n\cos(bcx+ac)}}{bcn}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] -exp(n*cos(b*c*x+a*c))/b/c/n
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="maxima")
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Fricas [A]
time = 1.63, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="fricas")
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(19) = 38$.
time = 0.31, size = 51, normalized size = 2.12

$$\begin{cases} 0 & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \cos{\left(a c \right)}} \sin{\left(a c \right)} & \text{for}\: b = 0 \\ - \frac{\cos{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ 0 & \text{for}\: c = 0 \\ - \frac{e^{n \cos{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cos(a*c))*sin(a*c),
 Eq(b, 0)), (-cos(a*c + b*c*x)/(b*c), Eq(n, 0)), (0, Eq(c, 0)), (-exp(n*cos
(a*c + b*c*x))/(b*c*n), True))
```

Giac [A]
time = 0.40, size = 23, normalized size = 0.96

$$-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -e^(n*cos(b*c*x + a*c))/(b*c*n)
```

Mupad [B]
time = 3.02, size = 23, normalized size = 0.96

$$-\frac{\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(a + b*x)))*sin(a*c + b*c*x),x)
```

```
[Out] -exp(n*cos(a*c + b*c*x))/(b*c*n)
```

3.662 $\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

```
[Out] -Ei(n*cos(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4424, 2209}

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(a+bx)}\tan(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\text{Ei}(n\cos(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 14, normalized size = 1.00

$$-\frac{\text{Ei}(n\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

Maple [A]
time = 0.44, size = 16, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\mathrm{expIntegral}(1,-n\cos(bx+a))}{b}$	16
default	$\frac{\mathrm{expIntegral}(1,-n\cos(bx+a))}{b}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*tan(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*Ei(1,-n*cos(b*x+a))
```

Maxima [A]
time = 0.32, size = 14, normalized size = 1.00

$$-\frac{\mathrm{Ei}(n\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*x + a))/b
```

Fricas [A]
time = 2.31, size = 14, normalized size = 1.00

$$-\frac{\mathrm{Ei}(n\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x)
[Out] Integral(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*x + a))*tan(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int e^{n\,\cos(a+b\,x)}\,\tan(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*tan(a + b*x),x)
[Out] int(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

3.663 $\int e^{n \cos(ac+bcx)} \tan(c(a+bx))\, dx$

Optimal. Leaf size=19

$$-\frac{\text{Ei}(n \cos(c(a+bx)))}{bc}$$

```
[Out] -Ei(n*cos(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4424, 2209}

$$-\frac{\text{Ei}(n \cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned} \int e^{n \cos(ac+bcx)} \tan(c(a+bx))\, dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\, dx, x, \cos(c(a+bx))\right)}{bc} \\ &= -\frac{\text{Ei}(n \cos(c(a+bx)))}{bc} \end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 1.00

$$-\frac{\text{Ei}(n \cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Maple [A]
time = 0.57, size = 22, normalized size = 1.16

method	result	size
default	$\frac{\mathrm{expIntegral}(1,-n\cos(bcx+ac))}{cb}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

Maxima [A]
time = 0.31, size = 20, normalized size = 1.05

$$-\frac{\mathrm{Ei}(n\cos(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Fricas [A]
time = 2.67, size = 20, normalized size = 1.05

$$-\frac{\mathrm{Ei}(n\cos(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos(ac+bcx)}\tan(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

```
[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*c*x + a*c))*tan((b*x + a)*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(c\,(a+b\,x))\,\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
[Out] int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)), x)
```

3.664 $\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$

Optimal. Leaf size=20

$$-\frac{\text{Ei}(n\cos(ac+bcx))}{bc}$$

```
[Out] -Ei(n*cos(b*c*x+a*c))/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4424, 2209}

$$-\frac{\text{Ei}(n\cos(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[a*c + b*c*x]]/(b*c))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx &= -\frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx,x,\cos(ac+bcx)\right)}{bc}\\ &= -\frac{\text{Ei}(n\cos(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.95

$$-\frac{\text{Ei}(n\cos(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

```
[Out] -(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

Maple [A]
time = 0.05, size = 22, normalized size = 1.10

method	result	size
default	$\frac{\mathrm{expIntegral}(1,-n\cos(bcx+ac))}{cb}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b*Ei(1,-n*cos(b*c*x+a*c))
```

Maxima [A]
time = 0.33, size = 20, normalized size = 1.00

$$-\frac{\mathrm{Ei}(n\cos{(bcx+ac)})}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Fricas [A]
time = 3.18, size = 20, normalized size = 1.00

$$-\frac{\mathrm{Ei}(n\cos{(bcx+ac)})}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] -Ei(n*cos(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos{(ac+bcx)}}\tan{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

```
[Out] Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos((b*x + a)*c))*tan(b*c*x + a*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x),x)
```

```
[Out] int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x), x)
```

3.665 $\int \frac{\cos(x)}{a+b\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sin(x))}{b}$$

```
[Out] ln(a+b*sin(x))/b
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2747, 31}

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]),x]
```

```
[Out] Log[a + b*Sin[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{\log(a+b\sin(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\log(a+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]),x]
[Out] Log[a + b*Sin[x]]/b
```

Maple [A]
time = 0.05, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(x))}{b}$	12
default	$\frac{\ln(a+b\sin(x))}{b}$	12
risch	$-\frac{ix}{b}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{b}$	33
norman	$\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{b}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sin(x))/b
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sin\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="maxima")
[Out] log(b*sin(x) + a)/b
```

Fricas [A]
time = 2.24, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sin\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="fricas")
[Out] log(b*sin(x) + a)/b
```

Sympy [A]
time = 0.15, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\sin\left(x\right)\right)}{b} & \text{for}\ b\neq 0\\ \frac{\sin\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x)
```

```
[Out] Piecewise((log(a/b + sin(x))/b, Ne(b, 0)), (sin(x)/a, True))
```

Giac [A]
time = 0.41, size = 12, normalized size = 1.09

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*sin(x) + a))/b
```

Mupad [B]
time = 0.03, size = 11, normalized size = 1.00

$$\frac{\ln\left(a+b\,\sin\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)),x)
```

```
[Out] log(a + b*sin(x))/b
```

3.666 $\int \cos(x)(a+b\sin(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\sin(x))^{1+n}}{b(1+n)}$$

[Out] (a+b*sin(x))^(1+n)/b/(1+n)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2747, 32}

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(a + b*Sin[x])^n,x]
```

```
[Out] (a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)(a+b\sin(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\sin(x)\right)}{b}\\ &= \frac{(a+b\sin(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.95

$$\frac{(a+b\sin(x))^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(a + b*Sin[x])^n,x]
[Out] (a + b*Sin[x])^(1 + n)/(b + b*n)
```

Maple [A]
time = 0.13, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(a+b\sin(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b\sin(x))^{1+n}}{b(1+n)}$	20
norman	$\frac{\frac{a\,{\mathrm e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)}}{b(1+n)}+\frac{a\left(\tan^{2}\left(\frac{x}{2}\right)\right){\mathrm e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)}}{b(1+n)}+\frac{2\tan\left(\frac{x}{2}\right){\mathrm e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)}}{1+n}}{1+\tan^{2}\left(\frac{x}{2}\right)}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(a+b*sin(x))^n,x,method=_RETURNVERBOSE)
[Out] (a+b*sin(x))^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 1.00

$$\frac{(b\sin(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="maxima")
[Out] (b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 2.80, size = 22, normalized size = 1.16

$$\frac{(b\sin(x)+a)(b\sin(x)+a)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="fricas")
[Out] (b*sin(x) + a)*(b*sin(x) + a)^n/(b*n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(14) = 28$.
time = 0.53, size = 56, normalized size = 2.95

$$\begin{cases} \frac{\sin(x)}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n \sin(x) & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + \sin(x)\right)}{b} & \text{for } n = -1 \\ \frac{a(a + b\sin(x))^n}{bn + b} + \frac{b(a + b\sin(x))^n \sin(x)}{bn + b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))**n,x)
```

```
[Out] Piecewise((sin(x)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sin(x), Eq(b, 0)), (log(a
/b + sin(x))/b, Eq(n, -1)), (a*(a + b*sin(x))**n/(b*n + b) + b*(a + b*sin(x
))**n*sin(x)/(b*n + b), True))
```

Giac [A]
time = 0.42, size = 19, normalized size = 1.00

$$\frac{(b\sin(x) + a)^{n+1}}{b(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="giac")
```

```
[Out] (b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 3.13, size = 19, normalized size = 1.00

$$\frac{(a + b\,\sin(x))^{n+1}}{b\,(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(a + b*sin(x))^n,x)
```

```
[Out] (a + b*sin(x))^(n + 1)/(b*(n + 1))
```

3.667 $\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} \, dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\sin(x))$$

```
[Out] arcsinh(sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3269, 221}

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sin(x)\right) \\ &= \sinh^{-1}(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\sinh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] ArcSinh[Sin[x]]
```

Maple [A]
time = 0.07, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\operatorname{arcsinh}(\sin(x))$	4
default	$\operatorname{arcsinh}(\sin(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(sin(x))
```

Maxima [A]
time = 0.49, size = 3, normalized size = 1.00

$$\operatorname{arsinh}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(3) = 6.
time = 2.55, size = 39, normalized size = 13.00

$$\frac{1}{4}\log\left(8\cos(x)^4 - 4\left(2\cos(x)^2 - 3\right)\sqrt{-\cos(x)^2 + 2}\sin(x) - 24\cos(x)^2 + 17\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 3)*sqrt(-cos(x)^2 + 2)*sin(x) - 24*cos
(x)^2 + 17)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{\sin^2(x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(sin(x)**2 + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.
time = 0.40, size = 16, normalized size = 5.33

$$-\log\left(\sqrt{\sin\left(x\right)^{2}+1}-\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(sin(x)^2 + 1) - sin(x))
```

Mupad [B]
time = 0.02, size = 9, normalized size = 3.00

$$-\mathrm{asin}(\sin\left(x\right)\ 1\mathrm{i})\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] -asin(sin(x)*1i)*1i
```

3.668 $\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx$

Optimal. Leaf size=7

$$\text{ArcSin}\left(\frac{\sin(x)}{2}\right)$$

```
[Out] arcsin(1/2*sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3269, 222}

$$\text{ArcSin}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)\\ &= \sin^{-1}\left(\frac{\sin(x)}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$\text{ArcSin}\left(\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

```
[Out] ArcSin[Sin[x]/2]
```

Maple [A]
time = 0.08, size = 6, normalized size = 0.86

method	result	size
derivativedivides	$\arcsin\left(\frac{\sin(x)}{2}\right)$	6
default	$\arcsin\left(\frac{\sin(x)}{2}\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(1/2*sin(x))
```

Maxima [A]
time = 0.49, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/2*sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(5) = 10$.
time = 2.46, size = 53, normalized size = 7.57

$$\frac{1}{2}\arctan\left(\frac{\sqrt{\cos\left(x\right)^{2}+3}\left(\cos\left(x\right)^{2}+1\right)\sin\left(x\right)-4\cos\left(x\right)\sin\left(x\right)}{\cos\left(x\right)^{4}+6\cos\left(x\right)^{2}-3}\right)+\frac{1}{2}\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(co
s(x)^4 + 6*cos(x)^2 - 3)) + 1/2*arctan(sin(x)/cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(x\right)}{\sqrt{-\left(\sin\left(x\right)-2\right)\left(\sin\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(-(sin(x) - 2)*(sin(x) + 2)), x)
```

Giac [A]
time = 0.43, size = 5, normalized size = 0.71

$$\arcsin\left(\frac{1}{2}\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sin(x))
```

Mupad [B]
time = 2.98, size = 5, normalized size = 0.71

$$\mathrm{asin}\left(\frac{\sin\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(4 - sin(x)^2)^(1/2),x)
```

```
[Out] asin(sin(x)/2)
```

3.669 $\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(3x)\right)$$

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Rubi [A]

time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3269, 222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
[Out] ArcSin[Sin[3*x]/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(3x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{1}{2}\sin(3x)\right)\end{aligned}$$

Mathematica [A]

time = 0.02, size = 13, normalized size = 1.00

$$\frac{1}{3}\text{ArcSin}\left(\frac{1}{2}\sin(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

```
[Out] ArcSin[Sin[3*x]/2]/3
```

Maple [A]
time = 0.10, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$	10
default	$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(4-sin(3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Maxima [A]
time = 0.49, size = 9, normalized size = 0.69

$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin\left(3x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(1/2*sin(3*x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(9) = 18.
time = 3.44, size = 71, normalized size = 5.46

$$\frac{1}{6}\arctan\left(\frac{\sqrt{\cos\left(3x\right)^{2}+3}\,\left(\cos\left(3x\right)^{2}+1\right)\sin\left(3x\right)-4\,\cos\left(3x\right)\sin\left(3x\right)}{\cos\left(3x\right)^{4}+6\,\cos\left(3x\right)^{2}-3}\right)+\frac{1}{6}\arctan\left(\frac{\sin\left(3x\right)}{\cos\left(3x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan((sqrt(cos(3*x)^2 + 3)*(cos(3*x)^2 + 1)*sin(3*x) - 4*cos(3*x)*sin
(3*x))/(cos(3*x)^4 + 6*cos(3*x)^2 - 3)) + 1/6*arctan(sin(3*x)/cos(3*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(3x\right)}{\sqrt{-\left(\sin\left(3x\right)-2\right)\left(\sin\left(3x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)**2)**(1/2),x)
```

```
[Out] Integral(cos(3*x)/sqrt(-(sin(3*x) - 2)*(sin(3*x) + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(3*x)/sqrt(-sin(3*x)^2 + 4), x)
```

Mupad [B]
time = 2.98, size = 9, normalized size = 0.69

$$\frac{\operatorname{asin}\left(\frac{\sin(3\,x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(4 - sin(3*x)^2)^(1/2),x)
```

```
[Out] asin(sin(3*x)/2)/3
```

3.670 $\int \cos(x)\sqrt{1+\csc(x)}\,dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\sqrt{1+\csc(x)}\right)+\sqrt{1+\csc(x)}\,\sin(x)$$

```
[Out] arctanh((1+csc(x))^(1/2))+sin(x)*(1+csc(x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3958, 43, 65, 213}

$$\sin(x)\sqrt{\csc(x)+1}+\tanh^{-1}\left(\sqrt{\csc(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
```

```
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sqrt{1+\csc(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\,\sin(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \csc(x)\right)\\ &= \sqrt{1+\csc(x)}\,\sin(x) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\csc(x)}\right)\\ &= \tanh^{-1}\left(\sqrt{1+\csc(x)}\right) + \sqrt{1+\csc(x)}\,\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\tanh^{-1}\left(\sqrt{1+\csc(x)}\right) + \sqrt{1+\csc(x)}\,\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[1 + Csc[x]],x]
[Out] ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(17) = 34.
time = 0.12, size = 48, normalized size = 2.29

method	result
derivativedivides	$\frac{1}{2\sqrt{1+\csc(x)}-2} - \frac{\ln\left(\sqrt{1+\csc(x)}-1\right)}{2} + \frac{1}{2\sqrt{1+\csc(x)}+2} + \frac{\ln\left(\sqrt{1+\csc(x)}+1\right)}{2}$
default	$\frac{1}{2\sqrt{1+\csc(x)}-2} - \frac{\ln\left(\sqrt{1+\csc(x)}-1\right)}{2} + \frac{1}{2\sqrt{1+\csc(x)}+2} + \frac{\ln\left(\sqrt{1+\csc(x)}+1\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+csc(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((1+csc(x))^(1/2)-1)-1/2*ln((1+csc(x))^(1/2)-1)+1/2/((1+csc(x))^(1/2)+1
)+1/2*ln((1+csc(x))^(1/2)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.27, size = 38, normalized size = 1.81

$$\sqrt{\frac{1}{\sin(x)}+1}\,\sin(x)+\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(1/sin(x) + 1)*sin(x) + 1/2*log(sqrt(1/sin(x) + 1) + 1) - 1/2*log(sqrt(
1/sin(x) + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(17) = 34$.
time = 2.61, size = 79, normalized size = 3.76

$$\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)+\sin(x)+1\right)}{\cos(x)+\sin(x)+1}\right)-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\,\sin(x)-\sin(x)-1\right)}{\cos(x)+\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt((sin(x) + 1)/sin(x))*sin(x) + 1/2*log(2*(sqrt((sin(x) + 1)/sin(x))*sin
(x) + sin(x) + 1)/(cos(x) + sin(x) + 1)) - 1/2*log(-2*(sqrt((sin(x) + 1)/si
n(x))*sin(x) - sin(x) - 1)/(cos(x) + sin(x) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\csc(x)+1}\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(csc(x) + 1)*cos(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.41, size = 38, normalized size = 1.81

$$\frac{1}{2}\left(2\sqrt{\sin(x)^2+\sin(x)}-\log\left(\left|2\sqrt{\sin(x)^2+\sin(x)}-2\sin(x)-1\right|\right)\right)\operatorname{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sin(x)^2 + sin(x)) - log(abs(2*sqrt(sin(x)^2 + sin(x)) - 2*sin(
x) - 1)))*sgn(sin(x))
```

Mupad [B]
time = 3.10, size = 47, normalized size = 2.24

$$\sin(x)\sqrt{\frac{1}{\sin(x)}+1}+\frac{\ln\left(\sin(x)+\sqrt{\sin(x)^2+\sin(x)}+\frac{1}{2}\right)\sin(x)\sqrt{\frac{1}{\sin(x)}+1}}{2\sqrt{\sin(x)^2+\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1/sin(x) + 1)^(1/2),x)
```

```
[Out] sin(x)*(1/sin(x) + 1)^(1/2) + (log(sin(x) + (sin(x) + sin(x)^2)^(1/2) + 1/2
)*sin(x)*(1/sin(x) + 1)^(1/2))/(2*(sin(x) + sin(x)^2)^(1/2))
```

3.671 $\int \cos(x)\sqrt{4-\sin^2(x)}\,dx$

Optimal. Leaf size=28

$$2\text{ArcSin}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {3269, 201, 222}

$$2\text{ArcSin}\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sqrt[4 - Sin[x]^2],x]

[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \text{Subst}\left(\int \sqrt{4-x^2}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)} + 2\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\sin^{-1}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$2\text{ArcSin}\left(\frac{\sin(x)}{2}\right) + \frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[4 - Sin[x]^2],x]
```

```
[Out] 2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

Maple [A]
time = 0.06, size = 23, normalized size = 0.82

method	result	size
derivativedivides	$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\sqrt{4-\left(\sin^2(x)\right)}}{2}$	23
default	$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\sqrt{4-\left(\sin^2(x)\right)}}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(4-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 22, normalized size = 0.79

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\,\sin(x) + 2\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 2.57, size = 61, normalized size = 2.18

$$\frac{1}{2}\sqrt{\cos(x)^2+3}\,\sin(x)+\arctan\left(\frac{\sqrt{\cos(x)^2+3}\,(\cos(x)^2+1)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right)+\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(cos(x)^2 + 3)*sin(x) + arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*s
in(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + arctan(sin(x)/cos(x
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(\sin(x)-2)(\sin(x)+2)}\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)**2)**(1/2),x)
[Out] Integral(sqrt(-(sin(x) - 2)*(sin(x) + 2))*cos(x), x)
```

Giac [A]
time = 0.41, size = 22, normalized size = 0.79

$$\frac{1}{2}\sqrt{-\sin(x)^2+4}\,\sin(x)+2\arcsin\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

Mupad [B]
time = 2.97, size = 20, normalized size = 0.71

$$2\,\mathrm{asin}\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\,\sqrt{\cos(x)^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(4 - sin(x)^2)^(1/2),x)
[Out] 2*asin(sin(x)/2) + (sin(x)*(cos(x)^2 + 3)^(1/2))/2
```

3.672 $\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}$$

```
[Out] 1/3*(1+sin(x)^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3277, 267}

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3277

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx &= \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(x)\right)\\ &= \frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] (1 + Sin[x]^2)^(3/2)/3
```

Maple [A]
time = 0.02, size = 11, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\left(1+\sin^2(x)\right)^{\frac{3}{2}}}{3}$	11
default	$\frac{\left(1+\sin^2(x)\right)^{\frac{3}{2}}}{3}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+sin(x)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.71

$$\frac{1}{3}\left(\sin\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

Fricas [A]
time = 4.03, size = 12, normalized size = 0.86

$$\frac{1}{3}\left(-\cos\left(x\right)^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-cos(x)^2 + 2)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.16, size = 27, normalized size = 1.93

$$\frac{\sqrt{\sin^{2}\left(x\right)+1}\,\sin^{2}\left(x\right)}{3}+\frac{\sqrt{\sin^{2}\left(x\right)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)**2)**(1/2),x)
```

```
[Out] sqrt(sin(x)**2 + 1)*sin(x)**2/3 + sqrt(sin(x)**2 + 1)/3
```

Giac [A]
time = 0.39, size = 10, normalized size = 0.71

$$\frac{1}{3}\left(\sin\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sin(x)^2 + 1)^(3/2)
```

Mupad [B]
time = 0.10, size = 10, normalized size = 0.71

$$\frac{\left(\sin\left(x\right)^{2}+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)*(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] (sin(x)^2 + 1)^(3/2)/3
```

3.673 $\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx$

Optimal. Leaf size=19

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$

[Out] 2*arctanh(sin(x)/(2*sin(x)+sin(x)^2)^(1/2))

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3339, 634, 212}

$$2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]

[Out] 2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 3339

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{2x+x^2}}\,dx, x, \sin(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$
$$= 2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.
time = 0.01, size = 40, normalized size = 2.11

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{\sin(x)}}{\sqrt{2}}\right)\sqrt{\sin(x)}\sqrt{2+\sin(x)}}{\sqrt{\sin(x)(2+\sin(x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
[Out] (2*ArcSinh[Sqrt[Sin[x]]/Sqrt[2]]*Sqrt[Sin[x]]*Sqrt[2 + Sin[x]])/Sqrt[Sin[x]
*(2 + Sin[x])]
```

Maple [**A**]
time = 0.19, size = 17, normalized size = 0.89

method	result	size
derivativedivides	$\ln\left(1+\sin(x)+\sqrt{2\sin(x)+\sin^2(x)}\right)$	17
default	$\ln\left(1+\sin(x)+\sqrt{2\sin(x)+\sin^2(x)}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(1+sin(x)+(2*sin(x)+sin(x)^2)^(1/2))
```

Maxima [**A**]
time = 0.27, size = 20, normalized size = 1.05

$$\log\left(2\sqrt{\sin(x)^2+2\sin(x)}+2\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(sin(x)^2 + 2*sin(x)) + 2*sin(x) + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.
time = 5.25, size = 35, normalized size = 1.84

$$\frac{1}{2}\log\left(-2\cos(x)^2 + 2\sqrt{-\cos(x)^2 + 2\sin(x) + 1}\,(\sin(x) + 1) + 4\sin(x) + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-2*cos(x)^2 + 2*sqrt(-cos(x)^2 + 2*sin(x) + 1)*(sin(x) + 1) + 4*sin
(x) + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\sqrt{(\sin(x) + 2)\sin(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt((sin(x) + 2)*sin(x)), x)
```

Giac [A]
time = 0.42, size = 20, normalized size = 1.05

$$-\log\left(-\sqrt{\sin(x)^2 + 2\sin(x)} + \sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(sin(x)^2 + 2*sin(x)) + sin(x) + 1)
```

Mupad [B]
time = 3.17, size = 14, normalized size = 0.74

$$\ln\left(\sin(x) + \sqrt{\sin(x)\,(\sin(x) + 2)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(2*sin(x) + sin(x)^2)^(1/2),x)
```

```
[Out] log(sin(x) + (sin(x)*(sin(x) + 2))^(1/2) + 1)
```

3.674 $\int \cos(x)\cos(\sin(x))\,dx$

Optimal. Leaf size=3

$$\sin(\sin(x))$$

```
[Out] sin(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4419, 2717}

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]],x]
```

```
[Out] Sin[Sin[x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(\sin(x))\,dx &= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(x)\right)\\ &= \sin(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.92, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]],x]
[Out] Sin[Sin[x]]
```

Maple [A]
time = 0.03, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\sin(\sin(x))$	4
default	$\sin(\sin(x))$	4
risch	$\sin(\sin(x))$	4
norman	$\frac{2(\tan^2(\frac{x}{2}))\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)+2\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)}{(1+\tan^2(\frac{x}{2}))\left(1+\tan^2\left(\frac{\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x)),x,method=_RETURNVERBOSE)
[Out] sin(sin(x))
```

Maxima [A]
time = 0.31, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="maxima")
[Out] sin(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.
time = 2.96, size = 17, normalized size = 5.67

$$\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="fricas")
[Out] sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

Sympy [A]
time = 0.13, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x)
```

```
[Out] sin(sin(x))
```

Giac [A]
time = 0.43, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x)),x, algorithm="giac")
```

```
[Out] sin(sin(x))
```

Mupad [B]
time = 2.95, size = 3, normalized size = 1.00

$$\sin(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*cos(x),x)
```

```
[Out] sin(sin(x))
```

3.675 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

Optimal. Leaf size=4

$$\sin(\sin(\sin(x)))$$

```
[Out] sin(sin(sin(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4419, 2717}

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

```
[Out] Sin[Sin[Sin[x]]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx &= \text{Subst}\left(\int \cos(x)\cos(\sin(x))\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \cos(x)\,dx, x, \sin(\sin(x))\right)\\
&= \sin(\sin(\sin(x)))
\end{aligned}$$

Mathematica [A]
time = 6.53, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
[Out] Sin[Sin[Sin[x]]]
```

Maple [A]
time = 0.04, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\sin(\sin(\sin(x)))$	5
default	$\sin(\sin(\sin(x)))$	5
risch	$\sin(\sin(\sin(x)))$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x,method=_RETURNVERBOSE)
[Out] sin(sin(sin(x)))
```

Maxima [A]
time = 0.31, size = 4, normalized size = 1.00

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
[Out] sin(sin(sin(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(4) = 8$.
time = 2.50, size = 41, normalized size = 10.25

$$\sin\left(\frac{2\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
[Out] sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1)
)^2 + 1))
```

Sympy [A]
time = 1.08, size = 5, normalized size = 1.25

$$\sin(\sin(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

```
[Out] sin(sin(sin(x)))
```

Giac [A]
time = 0.40, size = 4, normalized size = 1.00

$$\sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

```
[Out] sin(sin(sin(x)))
```

Mupad [B]
time = 3.00, size = 4, normalized size = 1.00

$$\sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)
```

```
[Out] sin(sin(sin(x)))
```

3.676 $\int \cos(x)\sec(\sin(x))\,dx$

Optimal. Leaf size=4

$$\tanh^{-1}(\sin(\sin(x)))$$

```
[Out] arctanh(sin(sin(x)))
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4419, 3855}

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sec[Sin[x]],x]
```

```
[Out] ArcTanh[Sin[Sin[x]]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sec(\sin(x))\,dx &= \text{Subst}\left(\int \sec(x)\,dx, x, \sin(x)\right)\\ &= \tanh^{-1}(\sin(\sin(x)))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$\tanh^{-1}(\sin(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sec[Sin[x]],x]
[Out] ArcTanh[Sin[Sin[x]]]
```

Maple [A]
time = 0.03, size = 9, normalized size = 2.25

method	result	size
derivativedivides	$\ln(\sec(\sin(x)) + \tan(\sin(x)))$	9
default	$\ln(\sec(\sin(x)) + \tan(\sin(x)))$	9
risch	$-\ln\left(e^{i\sin(x)} - i\right) + \ln\left(e^{i\sin(x)} + i\right)$	24
norman	$-\ln\left(\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right) - 1\right) + \ln\left(\tan\left(\frac{\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right) + 1\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sec(sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(sec(sin(x))+tan(sin(x)))
```

Maxima [A]
time = 0.33, size = 8, normalized size = 2.00

$$\log(\sec(\sin(x)) + \tan(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="maxima")
[Out] log(sec(sin(x)) + tan(sin(x)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(4) = 8.
time = 2.78, size = 47, normalized size = 11.75

$$\frac{1}{2}\log\left(\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) + 1\right) - \frac{1}{2}\log\left(-\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="fricas")
[Out] 1/2*log(sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)) + 1) - 1/2*log(-sin(2*tan(1/2*
x)/(tan(1/2*x)^2 + 1)) + 1)
```

Sympy [A]
time = 0.74, size = 10, normalized size = 2.50

$$\log(\tan(\sin(x)) + \sec(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x)
```

```
[Out] log(tan(sin(x)) + sec(sin(x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.
time = 0.42, size = 29, normalized size = 7.25

$$\frac{1}{4}\log\left(\left|\frac{1}{\sin(\sin(x))}+\sin(\sin(x))+2\right|\right)-\frac{1}{4}\log\left(\left|\frac{1}{\sin(\sin(x))}+\sin(\sin(x))-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sec(sin(x)),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(sin(x)) + sin(sin(x)) + 2)) - 1/4*log(abs(1/sin(sin(x)) +
 sin(sin(x)) - 2))
```

Mupad [B]
time = 3.24, size = 21, normalized size = 5.25

$$-\mathrm{atan}\left(\mathrm{e}^{-\frac{\mathrm{e}^{-x\,1\mathrm{i}}}{2}}\,\mathrm{e}^{\frac{\mathrm{e}^{x\,1\mathrm{i}}}{2}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/cos(sin(x)),x)
```

```
[Out] -atan(exp(-exp(-x*1i)/2)*exp(exp(x*1i)/2))*2i
```

3.677 $\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=36

$$-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}$$

[Out] -1/8*a*(a+b*sin(x)^2)^4/b^2+1/10*(a+b*sin(x)^2)^5/b^2

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {3277, 272, 45}

$$\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]

[Out] -1/8*(a*(a + b*Sin[x]^2)^4)/b^2 + (a + b*Sin[x]^2)^5/(10*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3277

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx = \text{Subst}\left(\int x^3\left(a+bx^2\right)^3\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^3\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx, x, \sin^2(x)\right)$$
$$= -\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(36) = 72.
time = 0.29, size = 128, normalized size = 3.56

$$\frac{-20(64a^3+24ab^2+7b^3)\cos(2x)+20(16a^3+18ab^2+5b^3)\cos(4x)+b(-10b(16a+5b)\cos(6x)+15b(2a+b)\cos(8x)-2b^2\cos(10x)+3840a^2\sin^4(x)+2560ab\sin^6(x)+640b^2\sin^8(x)-1280a^2\sin^3(x)\sin(3x))}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

```
[Out] (-20*(64*a^3 + 24*a*b^2 + 7*b^3)*Cos[2*x] + 20*(16*a^3 + 18*a*b^2 + 5*b^3)*
Cos[4*x] + b*(-10*b*(16*a + 5*b)*Cos[6*x] + 15*b*(2*a + b)*Cos[8*x] - 2*b^2
*Cos[10*x] + 3840*a^2*Sin[x]^4 + 2560*a*b*Sin[x]^6 + 640*b^2*Sin[x]^8 - 128
0*a^2*Sin[x]^3*Sin[3*x]))/10240
```

Maple [A]
time = 0.06, size = 40, normalized size = 1.11

method	result
derivativedivides	$\frac{b^3(\sin^{10}(x))}{10}+\frac{3a\,b^2(\sin^8(x))}{8}+\frac{a^2b(\sin^6(x))}{2}+\frac{a^3(\sin^4(x))}{4}$
default	$\frac{b^3(\sin^{10}(x))}{10}+\frac{3a\,b^2(\sin^8(x))}{8}+\frac{a^2b(\sin^6(x))}{2}+\frac{a^3(\sin^4(x))}{4}$
risch	$-\frac{b^3\cos(10x)}{5120}+\frac{3\cos(8x)a\,b^2}{1024}+\frac{\cos(8x)b^3}{512}-\frac{\cos(6x)a^2b}{64}-\frac{3\cos(6x)a\,b^2}{128}-\frac{9\cos(6x)b^3}{1024}+\frac{\cos(4x)a^3}{32}+\frac{3\cos(4}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^3*sin(x)^10+3/8*a*b^2*sin(x)^8+1/2*a^2*b*sin(x)^6+1/4*a^3*sin(x)^4
```

Maxima [A]
time = 0.34, size = 39, normalized size = 1.08

$$\frac{1}{10}\,b^3\sin\left(x\right)^{10}+\frac{3}{8}\,ab^2\sin\left(x\right)^8+\frac{1}{2}\,a^2b\sin\left(x\right)^6+\frac{1}{4}\,a^3\sin\left(x\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(32) = 64$.
time = 2.84, size = 103, normalized size = 2.86

$$-\frac{1}{10}\,b^{3}\cos\left(x\right)^{10}+\frac{1}{8}\left(3\,ab^{2}+4\,b^{3}\right)\cos\left(x\right)^{8}-\frac{1}{2}\left(a^{2}b+3\,ab^{2}+2\,b^{3}\right)\cos\left(x\right)^{6}+\frac{1}{4}\left(a^{3}+6\,a^{2}b+9\,ab^{2}+4\,b^{3}\right)\cos\left(x\right)^{4}-\frac{1}{2}\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*b^3*cos(x)^10 + 1/8*(3*a*b^2 + 4*b^3)*cos(x)^8 - 1/2*(a^2*b + 3*a*b^2
 + 2*b^3)*cos(x)^6 + 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(x)^4 - 1/2*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2
```

Sympy [A]
time = 1.32, size = 44, normalized size = 1.22

$$\frac{a^{3}\sin^{4}\left(x\right)}{4}+\frac{a^{2}b\sin^{6}\left(x\right)}{2}+\frac{3ab^{2}\sin^{8}\left(x\right)}{8}+\frac{b^{3}\sin^{10}\left(x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3*(a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*sin(x)**4/4 + a**2*b*sin(x)**6/2 + 3*a*b**2*sin(x)**8/8 + b**3*sin(x)*
*10/10
```

Giac [A]
time = 0.41, size = 39, normalized size = 1.08

$$\frac{1}{10}\,b^{3}\sin\left(x\right)^{10}+\frac{3}{8}\,ab^{2}\sin\left(x\right)^{8}+\frac{1}{2}\,a^{2}b\sin\left(x\right)^{6}+\frac{1}{4}\,a^{3}\sin\left(x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin(
x)^4
```

Mupad [B]
time = 0.07, size = 73, normalized size = 2.03

$$\frac{b^{2}\cos\left(x\right)^{8}\left(3\,a+4\,b\right)}{8}-\frac{b^{3}\cos\left(x\right)^{10}}{10}-\frac{\cos\left(x\right)^{2}\left(a+b\right)^{3}}{2}-\frac{b\cos\left(x\right)^{6}\left(a^{2}+3\,a\,b+2\,b^{2}\right)}{2}+\frac{\cos\left(x\right)^{4}\left(a+b\right)^{2}\left(a+4\,b\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3*(a + b*sin(x)^2)^3,x)
```

```
[Out] (b^2*cos(x)^8*(3*a + 4*b))/8 - (b^3*cos(x)^10)/10 - (cos(x)^2*(a + b)^3)/2
- (b*cos(x)^6*(3*a*b + a^2 + 2*b^2))/2 + (cos(x)^4*(a + b)^2*(a + 4*b))/4
```

3.678 $\int e^{\sin(x)} \cos(x) \sin(x) \, dx$

Optimal. Leaf size=14

$$-e^{\sin(x)} + e^{\sin(x)} \sin(x)$$

[Out] -exp(sin(x))+exp(sin(x))*sin(x)

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4419, 2207, 2225}

$$e^{\sin(x)} \sin(x) - e^{\sin(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Sin[x]*Cos[x]*Sin[x],x]

[Out] -E^Sin[x] + E^Sin[x]*Sin[x]

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int e^{\sin(x)} \cos(x) \sin(x) \, dx &= \text{Subst}\left(\int e^x x \, dx, x, \sin(x)\right) \\
&= e^{\sin(x)} \sin(x) - \text{Subst}\left(\int e^x \, dx, x, \sin(x)\right) \\
&= -e^{\sin(x)} + e^{\sin(x)} \sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 0.64

$$e^{\sin(x)}(-1+\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[E^Sin[x]*Cos[x]*Sin[x],x]

[Out] E^Sin[x]*(-1 + Sin[x])

Maple [A]
time = 0.04, size = 13, normalized size = 0.93

method	result	siz
derivativedivides	$-\mathrm{e}^{\sin(x)}+\mathrm{e}^{\sin(x)}\sin(x)$	13
default	$-\mathrm{e}^{\sin(x)}+\mathrm{e}^{\sin(x)}\sin(x)$	13
risch	$-\mathrm{e}^{\sin(x)}+\mathrm{e}^{\sin(x)}\sin(x)$	13
norman	$\frac{2\tan\left(\frac{x}{2}\right)\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}}-2\left(\tan^{2}\left(\frac{x}{2}\right)\right)\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}}+2\left(\tan^{3}\left(\frac{x}{2}\right)\right)\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}}-\left(\tan^{4}\left(\frac{x}{2}\right)\right)\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}}-\mathrm{e}^{\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	13

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(sin(x))*cos(x)*sin(x),x,method=_RETURNVERBOSE)

[Out] -exp(sin(x))+exp(sin(x))*sin(x)

Maxima [A]
time = 0.31, size = 8, normalized size = 0.57

$$(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="maxima")

[Out] (sin(x) - 1)*e^sin(x)

Fricas [A]
time = 3.00, size = 8, normalized size = 0.57

$$(\sin(x)-1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="fricas")

```
[Out] (sin(x) - 1)*e^sin(x)
```

Sympy [A]
time = 0.15, size = 12, normalized size = 0.86

$$e^{\sin(x)}\sin(x) - e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x)
[Out] exp(sin(x))*sin(x) - exp(sin(x))
```

Giac [A]
time = 0.41, size = 8, normalized size = 0.57

$$(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="giac")
[Out] (sin(x) - 1)*e^sin(x)
```

Mupad [B]
time = 2.91, size = 8, normalized size = 0.57

$$e^{\sin(x)}(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*cos(x)*sin(x),x)
[Out] exp(sin(x))*(sin(x) - 1)
```

3.679 $\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}} dx$

Optimal. Leaf size=25

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3}\sqrt{\sin^3(x)}$$

[Out] -2*sin(x)/(sin(x)^3)^(1/2)-2/3*(sin(x)^3)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3286, 2644, 14}

$$-\frac{2\sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3}\sqrt{\sin^3(x)}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/Sqrt[Sin[x]^3],x]

[Out] (-2*Sin[x])/Sqrt[Sin[x]^3] - (2*Sqrt[Sin[x]^3])/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{\sin^3(x)}}$$

$$= \frac{\sin^{\frac{3}{2}}(x)\mathrm{Subst}\left(\int \frac{1-x^2}{x^{3/2}}\,dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}}$$

$$= \frac{\sin^{\frac{3}{2}}(x)\mathrm{Subst}\left(\int \left(\frac{1}{x^{3/2}} - \sqrt{x}\right)\,dx, x, \sin(x)\right)}{\sqrt{\sin^3(x)}}$$

$$= -\frac{2\sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3}\sqrt{\sin^3(x)}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.80

$$\frac{(-7+\cos(2x))\sin(x)}{3\sqrt{\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]^3],x]
[Out] ((-7 + Cos[2*x])*Sin[x])/(3*Sqrt[Sin[x]^3])
```

Maple [A]
time = 0.23, size = 14, normalized size = 0.56

method	result	size
default	$-\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3} - \frac{2}{\sqrt{\sin(x)}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*sin(x)^(3/2)-2/sin(x)^(1/2)
```

Maxima [A]
time = 0.32, size = 19, normalized size = 0.76

$$-\frac{2}{3}\sqrt{\sin(x)^3} - \frac{2\sin(x)}{\sqrt{\sin(x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="maxima")
[Out] -2/3*sqrt(sin(x)^3) - 2*sin(x)/sqrt(sin(x)^3)
```

Fricas [A]
time = 2.64, size = 28, normalized size = 1.12

$$-\frac{2\left(\cos\left(x\right)^{2}-4\right)\sqrt{-\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}}{3\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="fricas")
[Out] -2/3*(cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)*sin(x))/(cos(x)^2 - 1)
```

Sympy [A]
time = 0.54, size = 36, normalized size = 1.44

$$-\frac{8\sin^{3}\left(x\right)}{3\sqrt{\sin^{3}\left(x\right)}}-\frac{2\sin\left(x\right)\cos^{2}\left(x\right)}{\sqrt{\sin^{3}\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(sin(x)**3)**(1/2),x)
[Out] -8*sin(x)**3/(3*sqrt(sin(x)**3)) - 2*sin(x)*cos(x)**2/sqrt(sin(x)**3)
```

Giac [A]
time = 0.44, size = 13, normalized size = 0.52

$$-\frac{2}{3}\sin\left(x\right)^{\frac{3}{2}}-\frac{2}{\sqrt{\sin\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="giac")
[Out] -2/3*sin(x)^(3/2) - 2/sqrt(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(x\right)^{3}}{\sqrt{\sin\left(x\right)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(sin(x)^3)^(1/2),x)
[Out] int(cos(x)^3/(sin(x)^3)^(1/2), x)
```

3.680 $\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} dx$

Optimal. Leaf size=10

$$2e^{\sqrt{\sin(x)}}$$

[Out] 2*exp(sin(x)^(1/2))

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4419, 2240}

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

[In] Int[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]

[Out] 2*E^Sqrt[Sin[x]]

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}} dx &= \text{Subst}\left(\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx, x, \sin(x)\right)\\ &= 2e^{\sqrt{\sin(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$2e^{\sqrt{\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
```

```
[Out] 2*E^Sqrt[Sin[x]]
```

Maple [A]
time = 0.03, size = 8, normalized size = 0.80

method	result	size
derivativedivides	$2\,e^{\sqrt{\sin}\,(x)}$	8
default	$2\,e^{\sqrt{\sin}\,(x)}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(sin(x)^(1/2))
```

Maxima [A]
time = 0.31, size = 7, normalized size = 0.70

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*e^sqrt(sin(x))
```

Fricas [A]
time = 3.46, size = 7, normalized size = 0.70

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*e^sqrt(sin(x))
```

Sympy [A]
time = 0.14, size = 8, normalized size = 0.80

$$2e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)**(1/2))*cos(x)/sin(x)**(1/2),x)
```

```
[Out] 2*exp(sqrt(sin(x)))
```

Giac [A]
time = 0.42, size = 7, normalized size = 0.70

$$2\,e^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*e^sqrt(sin(x))
```

Mupad [B]
time = 3.00, size = 7, normalized size = 0.70

$$2\,\mathrm{e}^{\sqrt{\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(sin(x)^(1/2))*cos(x))/sin(x)^(1/2),x)
```

```
[Out] 2*exp(sin(x)^(1/2))
```

3.681 $\int e^{4+\sin(x)}\cos(x)\,dx$

Optimal. Leaf size=6

$$e^{4+\sin(x)}$$

```
[Out] exp(4+sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4419, 2225}

$$e^{\sin(x)+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4 + Sin[x])*Cos[x],x]
```

```
[Out] E^(4 + Sin[x])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{4+\sin(x)}\cos(x)\,dx &= \text{Subst}\left(\int e^{4+x}\,dx, x, \sin(x)\right)\\ &= e^{4+\sin(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$e^{4+\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4 + Sin[x])*Cos[x],x]
[Out] E^(4 + Sin[x])
```

Maple [A]
time = 0.03, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$e^{4+\sin(x)}$	6
default	$e^{4+\sin(x)}$	6
risch	$e^{4+\sin(x)}$	6
norman	$\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)e^{4+\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}+e^{4+\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}}{1+\tan^2\left(\frac{x}{2}\right)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4+sin(x))*cos(x),x,method=_RETURNVERBOSE)
[Out] exp(4+sin(x))
```

Maxima [A]
time = 0.30, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="maxima")
[Out] e^(sin(x) + 4)
```

Fricas [A]
time = 2.32, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="fricas")
[Out] e^(sin(x) + 4)
```

Sympy [A]
time = 0.13, size = 7, normalized size = 1.17

$$e^{4}e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x)
```

```
[Out] exp(4)*exp(sin(x))
```

Giac [**A**]
time = 0.42, size = 5, normalized size = 0.83

$$e^{(\sin(x)+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4+sin(x))*cos(x),x, algorithm="giac")
```

```
[Out] e^(sin(x) + 4)
```

Mupad [**B**]
time = 2.91, size = 6, normalized size = 1.00

$$e^{\sin(x)}\,e^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x) + 4)*cos(x),x)
```

```
[Out] exp(sin(x))*exp(4)
```

3.682 $\int e^{\cos(x)\sin(x)} \cos(2x) \, dx$

Optimal. Leaf size=10

$$e^{\frac{1}{2}\sin(2x)}$$

[Out] exp(1/2*sin(2*x))

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4441, 2225}

$$e^{\frac{1}{2}\sin(2x)}$$

Antiderivative was successfully verified.

[In] Int[E^(Cos[x]*Sin[x])*Cos[2*x],x]

[Out] E^(Sin[2*x]/2)

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned} \int e^{\cos(x)\sin(x)} \cos(2x) \, dx &= \frac{1}{2}\text{Subst}\left(\int e^{x/2} \, dx, x, \sin(2x)\right) \\ &= e^{\frac{1}{2}\sin(2x)} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 7, normalized size = 0.70

$$e^{\cos(x)\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]*Sin[x])*Cos[2*x],x]
[Out] E^(Cos[x]*Sin[x])
```

Maple [A]
time = 0.08, size = 7, normalized size = 0.70

method	result	size
derivativedivides	$e^{\cos(x)\sin(x)}$	7
default	$e^{\cos(x)\sin(x)}$	7
risch	$e^{\cos(x)\sin(x)}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)*sin(x))*cos(2*x),x,method=_RETURNVERBOSE)
[Out] exp(cos(x)*sin(x))
```

Maxima [A]
time = 0.89, size = 7, normalized size = 0.70

$$e^{\left(\frac{1}{2}\sin(2x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="maxima")
[Out] e^(1/2*sin(2*x))
```

Fricas [A]
time = 4.16, size = 6, normalized size = 0.60

$$e^{(\cos(x)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="fricas")
[Out] e^(cos(x)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x)
[Out] Timed out
```

Giac [A]
time = 0.42, size = 12, normalized size = 1.20

$$e^{\left(\frac{\tan(x)}{\tan(x)^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="giac")
[Out] e^(tan(x)/(tan(x)^2 + 1))
```

Mupad [B]
time = 2.99, size = 7, normalized size = 0.70

$$e^{\frac{\sin(2x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*exp(cos(x)*sin(x)),x)
[Out] exp(sin(2*x)/2)
```

3.683 $\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx$

Optimal. Leaf size=10

$$2e^{\frac{\sin(x)}{2}}$$

```
[Out] 2*exp(1/2*sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4441, 2225}

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

```
[Out] 2*E^(Sin[x]/2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx &= \text{Subst}\left(\int e^{x/2}\,dx, x, \sin(x)\right)\\ &= 2e^{\frac{\sin(x)}{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$2e^{\frac{\sin(x)}{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

```
[Out] 2*E^(Sin[x]/2)
```

Maple [A]
time = 0.09, size = 13, normalized size = 1.30

method	result	size
derivativedivides	$2\,\mathrm{e}^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}$	13
default	$2\,\mathrm{e}^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}$	13
risch	$2\,\mathrm{e}^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*exp(cos(1/2*x)*sin(1/2*x))
```

Maxima [A]
time = 0.33, size = 7, normalized size = 0.70

$$2\,e^{\left(\frac{1}{2}\,\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="maxima")
```

```
[Out] 2*e^(1/2*sin(x))
```

Fricas [A]
time = 3.76, size = 12, normalized size = 1.20

$$2\,e^{\left(\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="fricas")
```

```
[Out] 2*e^(cos(1/2*x)*sin(1/2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\cos\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

[Out] Integral(exp(sin(x/2)*cos(x/2))*cos(x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.
time = 0.41, size = 18, normalized size = 1.80

$$2 e^{\left(\frac{\tan\left(\frac{1}{2} x\right)}{\tan\left(\frac{1}{2} x\right)^{2}+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="giac")
```

```
[Out] 2*e^(tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

Mupad [B]
time = 2.95, size = 7, normalized size = 0.70

$$2 \mathrm{e}^{\frac{\sin(x)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x/2)*sin(x/2))*cos(x),x)
```

```
[Out] 2*exp(sin(x)/2)
```

3.684 $\int e^{n\sin(a+bx)}\cos(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{e^{n\sin(a+bx)}}{bn}$$

[Out] exp(n*sin(b*x+a))/b/n

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4419, 2225}

$$\frac{e^{n\sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

[In] Int[E^(n*Sin[a + b*x])*Cos[a + b*x],x]

[Out] E^(n*Sin[a + b*x])/(b*n)

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(a+bx)}\cos(a+bx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{e^{n\sin(a+bx)}}{bn}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{e^{n\sin(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

```
[Out] E^(n*Sin[a + b*x])/(b*n)
```

Maple [A]
time = 0.04, size = 17, normalized size = 1.00

method	result	size
derivativedivides	$\frac{{\mathrm e}^{n \sin \left(b x +a \right)}}{b n}$	17
default	$\frac{{\mathrm e}^{n \sin \left(b x +a \right)}}{b n}$	17
risch	$\frac{{\mathrm e}^{n \sin \left(b x +a \right)}}{b n}$	17
norman	$\frac{\frac{{\mathrm e}^{\frac{2 n \tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{1+\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)}}}{n b}+\frac{\left(\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right) {\mathrm e}^{\frac{2 n \tan \left(\frac{b x}{2}+\frac{a}{2}\right)}{1+\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)}}}{n b}}{1+\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] exp(n*sin(b*x+a))/b/n
```

Maxima [A]
time = 0.31, size = 16, normalized size = 0.94

$$\frac{e^{(n \sin (bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

Fricas [A]
time = 4.58, size = 16, normalized size = 0.94

$$\frac{e^{(n \sin (bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.
time = 0.12, size = 36, normalized size = 2.12

$$\begin{cases} x\cos{(a)} & \text{for}\: b = 0 \wedge n = 0 \\ xe^{n\sin{(a)}}\cos{(a)} & \text{for}\: b = 0 \\ \frac{\sin{(a+bx)}}{b} & \text{for}\: n = 0 \\ \frac{e^{n\sin{(a+bx)}}}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sin(a))*cos(a), Eq(b, 0
)), (sin(a + b*x)/b, Eq(n, 0)), (exp(n*sin(a + b*x))/(b*n), True))
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.94

$$\frac{e^{(n\sin(bx+a))}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*x + a))/(b*n)
```

Mupad [B]
time = 0.11, size = 16, normalized size = 0.94

$$\frac{e^{n\sin(a+b\,x)}}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*exp(n*sin(a + b*x)),x)
```

```
[Out] exp(n*sin(a + b*x))/(b*n)
```

3.685 $\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx$

Optimal. Leaf size=22

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

```
[Out] exp(n*sin(c*(b*x+a)))/b/c/n
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4419, 2225}

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

```
[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{e^{n\sin(c(a+bx))}}{bcn}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 22, normalized size = 1.00

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

```
[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

Maple [A]
time = 0.07, size = 23, normalized size = 1.05

method	result	size
risch	$\frac{e^{n \sin(c(bx+a))}}{bcn}$	22
default	$\frac{e^{n \sin(bcx+ac)}}{bcn}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Maxima [A]
time = 0.31, size = 22, normalized size = 1.00

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Fricas [A]
time = 2.54, size = 22, normalized size = 1.00

$$\frac{e^{(n \sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

time = 5.26, size = 49, normalized size = 2.23

$$\begin{cases} xe^{n\sin(ac)}\cos(ac) & \text{for } b = 0 \\ x & \text{for } c = 0 \\ \begin{cases} x\cos(ac) & \text{for } b = 0 \\ x & \text{for } c = 0 \\ \frac{\sin(c(a+bx))}{bc} & \text{otherwise} \end{cases} & \text{for } n = 0 \\ \frac{e^{n\sin(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

```
[Out] Piecewise((x*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewise
((x*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sin(c*(a + b*x))/(b*c), True)), Eq
(n, 0)), (exp(n*sin(a*c + b*c*x))/(b*c*n), True))
```

Giac [A]
time = 0.42, size = 22, normalized size = 1.00

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Mupad [B]
time = 3.08, size = 22, normalized size = 1.00

$$\frac{e^{n\sin(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)
```

```
[Out] exp(n*sin(a*c + b*c*x))/(b*c*n)
```

3.686 $\int e^{n \sin(c(a+bx))} \cos(ac+bcx)\,dx$

Optimal. Leaf size=23

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4419, 2225}

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx &= \frac{\text{Subst}\left(\int e^{nx}\,dx, x, \sin(ac+bcx)\right)}{bc}\\ &= \frac{e^{n\sin(ac+bcx)}}{bcn}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.96

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

```
[Out] E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

Maple [A]
time = 0.05, size = 23, normalized size = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{n\sin(c(bx+a))}}{bcn}$	22
default	$\frac{\mathrm{e}^{n\sin(bcx+ac)}}{bcn}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] exp(n*sin(b*c*x+a*c))/b/c/n
```

Maxima [A]
time = 0.30, size = 22, normalized size = 0.96

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Fricas [A]
time = 4.14, size = 22, normalized size = 0.96

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.
time = 0.29, size = 48, normalized size = 2.09

$$\begin{cases} x & \text{for}\: b = 0 \wedge c = 0 \wedge n = 0 \\ x e^{n \sin{\left(a c \right)}} \cos{\left(a c \right)} & \text{for}\: b = 0 \\ \frac{\sin{\left(a c + b c x \right)}}{b c} & \text{for}\: n = 0 \\ x & \text{for}\: c = 0 \\ \frac{e^{n \sin{\left(a c + b c x \right)}}}{b c n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

```
[Out] Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sin(a*c))*cos(a*c),
 Eq(b, 0)), (sin(a*c + b*c*x)/(b*c), Eq(n, 0)), (x, Eq(c, 0)), (exp(n*sin(a
*c + b*c*x))/(b*c*n), True))
```

Giac [**A**]
time = 0.44, size = 22, normalized size = 0.96

$$\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] e^(n*sin(b*c*x + a*c))/(b*c*n)
```

Mupad [**B**]
time = 2.99, size = 22, normalized size = 0.96

$$\frac{e^{n\sin(a\,c+b\,c\,x)}}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(a + b*x)))*cos(a*c + b*c*x),x)
```

```
[Out] exp(n*sin(a*c + b*c*x))/(b*c*n)
```

3.687 $\int e^{n \sin(a+bx)} \cot(a+bx) \, dx$

Optimal. Leaf size=13

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

```
[Out] Ei(n*sin(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4423, 2209}

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \int e^{n \sin(a+bx)} \cot(a+bx) \, dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x} \, dx, x, \sin(a+bx)\right)}{b} \\ &= \frac{\text{Ei}(n \sin(a+bx))}{b} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 1.00

$$\frac{\text{Ei}(n \sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
[Out] ExpIntegralEi[n*Sin[a + b*x]]/b
```

Maple [A]
time = 0.44, size = 17, normalized size = 1.31

method	result	size
derivativedivides	$-\frac{\mathrm{expIntegral}(1,-n\sin(bx+a))}{b}$	17
default	$-\frac{\mathrm{expIntegral}(1,-n\sin(bx+a))}{b}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*cot(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b*Ei(1,-n*sin(b*x+a))
```

Maxima [A]
time = 0.36, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}(n\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="maxima")
[Out] Ei(n*sin(b*x + a))/b
```

Fricas [A]
time = 3.55, size = 13, normalized size = 1.00

$$\frac{\mathrm{Ei}(n\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="fricas")
[Out] Ei(n*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

```
[Out] Integral(exp(n*sin(a + b*x))*cot(a + b*x), x)
```

Giac [**A**]
time = 0.41, size = 13, normalized size = 1.00

$$\frac{\operatorname{Ei}(n\sin{(bx+a)})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="giac")
```

```
[Out] Ei(n*sin(b*x + a))/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \cot(a+b\,x)\ \mathrm{e}^{n\ \sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)*exp(n*sin(a + b*x)),x)
```

```
[Out] int(cot(a + b*x)*exp(n*sin(a + b*x)), x)
```

3.688 $\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx$

Optimal. Leaf size=18

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

```
[Out] Ei(n*sin(c*(b*x+a)))/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4423, 2209}

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx &= \frac{\text{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(c(a+bx))\right)}{bc}\\ &= \frac{\text{Ei}(n\sin(c(a+bx)))}{bc}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 18, normalized size = 1.00

$$\frac{\text{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Maple [A]
time = 0.53, size = 23, normalized size = 1.28

method	result	size
default	$-\frac{\mathrm{expIntegral}(1,-n\sin(bcx+ac))}{cb}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

Maxima [A]
time = 0.36, size = 19, normalized size = 1.06

$$\frac{\mathrm{Ei}(n\sin{(bcx+ac)})}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Fricas [A]
time = 4.44, size = 19, normalized size = 1.06

$$\frac{\mathrm{Ei}(n\sin{(bcx+ac)})}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin{(ac+bcx)}}\cot{(ac+bcx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="giac")

[Out] integrate(cot((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \cot(c\,(a+b\,x))\,\mathrm{e}^{n\,\sin(a\,c+b\,c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)

[Out] int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)), x)

3.689 $\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx$

Optimal. Leaf size=19

$$\frac{\operatorname{Ei}(n\sin(ac+bcx))}{bc}$$

```
[Out] Ei(n*sin(b*c*x+a*c))/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4423, 2209}

$$\frac{\operatorname{Ei}(n\sin(ac+bxc))}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{e^{nx}}{x}\,dx, x, \sin(ac+bcx)\right)}{bc}\\ &= \frac{\operatorname{Ei}(n\sin(ac+bcx))}{bc}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 0.95

$$\frac{\operatorname{Ei}(n\sin(c(a+bx)))}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

```
[Out] ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

Maple [A]
time = 0.05, size = 23, normalized size = 1.21

method	result	size
default	$-\frac{\operatorname{expIntegral}(1,-n\sin(bcx+ac))}{cb}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/b*Ei(1,-n*sin(b*c*x+a*c))
```

Maxima [A]
time = 0.36, size = 19, normalized size = 1.00

$$\frac{\operatorname{Ei}(n\sin(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Fricas [A]
time = 1.95, size = 19, normalized size = 1.00

$$\frac{\operatorname{Ei}(n\sin(bcx+ac))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] Ei(n*sin(b*c*x + a*c))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin(ac+bcx)}\cot(ac+bcx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

```
[Out] Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="giac")
[Out] integrate(cot(b*c*x + a*c)*e^(n*sin((b*x + a)*c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \mathrm{e}^{n\,\sin\left(c\,\left(a+b\,x\right)\right)}\,\mathrm{cot}\left(a\,c+b\,c\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x),x)
[Out] int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x), x)
```

3.690 $\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\tan(x))}{b}$$

[Out] ln(a+b*tan(x))/b

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3587, 31}

$$\frac{\log(a+b\tan(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(a + b*Tan[x]),x]

[Out] Log[a + b*Tan[x]]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\tan(x)\right)}{b}\\ &= \frac{\log(a+b\tan(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 20, normalized size = 1.82

$$\frac{-\log(\cos(x))+\log(a\cos(x)+b\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Tan[x]),x]
```

```
[Out] (-Log[Cos[x]] + Log[a*Cos[x] + b*Sin[x]])/b
```

Maple [A]
time = 0.11, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(x))}{b}$	12
default	$\frac{\ln(a+b\tan(x))}{b}$	12
risch	$\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{b}-\frac{\ln(e^{2ix}+1)}{b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*tan(x))/b
```

Maxima [A]
time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log\left(b\tan\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="maxima")
```

```
[Out] log(b*tan(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(11) = 22$.
time = 3.51, size = 40, normalized size = 3.64

$$\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\log\left(\cos\left(x\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(cos(x)^2))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(x\right)}{a+b\tan\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(a + b*tan(x)), x)
```

Giac [A]
time = 0.40, size = 12, normalized size = 1.09

$$\frac{\log\left(|b\tan\left(x\right) + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(x) + a))/b
```

Mupad [B]
time = 3.03, size = 11, normalized size = 1.00

$$\frac{\ln\left(a + b\tan(x)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*tan(x))),x)
```

```
[Out] log(a + b*tan(x))/b
```

3.691 $\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))$$

```
[Out] 1/2*arctanh(2*cos(x)*sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3756, 212}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.01, size = 23, normalized size = 2.09

$$-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 - Tan[x]^2),x]
[Out] -1/2*Log[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x]]/2
```

Maple [A]
time = 0.10, size = 4, normalized size = 0.36

method	result	size
default	$\operatorname{arctanh}(\tan(x))$	4
risch	$\frac{\ln(e^{2ix}+i)}{2}-\frac{\ln(e^{2ix}-i)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-tan(x)^2),x,method=_RETURNVERBOSE)
[Out] arctanh(tan(x))
```

Maxima [A]
time = 0.30, size = 15, normalized size = 1.36

$$\frac{1}{2}\log(\tan(x)+1)-\frac{1}{2}\log(\tan(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="maxima")
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.
time = 3.44, size = 23, normalized size = 2.09

$$\frac{1}{4}\log(2\cos(x)\sin(x)+1)-\frac{1}{4}\log(-2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="fricas")
[Out] 1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec^2(x)}{\tan^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-tan(x)**2),x)
```

```
[Out] -Integral(sec(x)**2/(tan(x)**2 - 1), x)
```

Giac [A]
time = 0.41, size = 17, normalized size = 1.55

$$\frac{1}{2}\log\left(|\tan(x)+1|\right)-\frac{1}{2}\log\left(|\tan(x)-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

Mupad [B]
time = 3.08, size = 3, normalized size = 0.27

$$\operatorname{atanh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(tan(x)^2 - 1)),x)
```

```
[Out] atanh(tan(x))
```

3.692 $\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{3}-\frac{1}{3}\text{ArcTan}\left(\frac{2\cos(x)\sin(x)}{1+2\cos^2(x)}\right)$$

[Out] 1/3*x-1/3*arctan(2*cos(x)*sin(x)/(1+2*cos(x)^2))

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3756, 209}

$$\frac{x}{3}-\frac{1}{3}\text{ArcTan}\left(\frac{2\sin(x)\cos(x)}{2\cos^2(x)+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(9 + Tan[x]^2),x]

[Out] x/3 - ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Cos[x]^2)]/3

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{x}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\cos(x)\sin(x)}{1+2\cos^2(x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 0.33

$$-\frac{1}{3}\text{ArcTan}(3\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(9 + Tan[x]^2),x]
[Out] -1/3*ArcTan[3*Cot[x]]
```

Maple [A]
time = 0.09, size = 8, normalized size = 0.30

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{3}\right)}{3}$	8
risch	$\frac{i\ln\left(e^{2ix}+2\right)}{6}-\frac{i\ln\left(e^{2ix}+\frac{1}{2}\right)}{6}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(9+tan(x)^2),x,method=_RETURNVERBOSE)
[Out] 1/3*arctan(1/3*tan(x))
```

Maxima [A]
time = 0.54, size = 7, normalized size = 0.26

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="maxima")
[Out] 1/3*arctan(1/3*tan(x))
```

Fricas [A]
time = 3.46, size = 21, normalized size = 0.78

$$-\frac{1}{6}\arctan\left(\frac{10\cos\left(x\right)^{2}-1}{6\cos\left(x\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="fricas")
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\tan^2(x)+9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(9+tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(tan(x)**2 + 9), x)
```

Giac [A]
time = 0.41, size = 7, normalized size = 0.26

$$\frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/3*arctan(1/3*tan(x))
```

Mupad [B]
time = 2.88, size = 7, normalized size = 0.26

$$\frac{\operatorname{atan}\left(\frac{\tan(x)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 + 9)),x)
```

```
[Out] atan(tan(x)/3)/3
```

3.693 $\int \sec^2(x)(a+b\tan(x))^n\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$$

[Out] (a+b*tan(x))^(1+n)/b/(1+n)

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3587, 32}

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*(a + b*Tan[x])^n,x]

[Out] (a + b*Tan[x])^(1 + n)/(b*(1 + n))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)(a+b\tan(x))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(x)\right)}{b}\\ &= \frac{(a+b\tan(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 18, normalized size = 0.95

$$\frac{(a+b\tan(x))^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(a + b*Tan[x])^n,x]
```

```
[Out] (a + b*Tan[x])^(1 + n)/(b + b*n)
```

Maple [A]
time = 0.15, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$	20
default	$\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^n,x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*tan(x))^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 1.00

$$\frac{(b\tan{(x)}+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="maxima")
```

```
[Out] (b*tan(x) + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 3.94, size = 37, normalized size = 1.95

$$\frac{(a\cos{(x)}+b\sin{(x)})\left(\frac{a\cos(x)+b\sin(x)}{\cos(x)}\right)^{n}}{(bn+b)\cos{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="fricas")
```

```
[Out] (a*cos(x) + b*sin(x))*((a*cos(x) + b*sin(x))/cos(x))^n/((b*n + b)*cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\tan{(x)})^{n}\sec^{2}{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**n,x)
```

```
[Out] Integral((a + b*tan(x))**n*sec(x)**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [**B**]
time = 3.56, size = 37, normalized size = 1.95

$$\begin{cases} \frac{\ln(a+b\tan(x))}{b} & \text{if}\ \ n=-1 \\ \frac{(a+b\tan(x))^{n+1}}{b\,(n+1)} & \text{if}\ \ n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^n/cos(x)^2,x)
```

```
[Out] piecewise(n == -1, log(a + b*tan(x))/b, n ~= -1, (a + b*tan(x))^(n + 1)/(b*
(n + 1)))
```

3.694 $\int \sec^2(x) \left(1+\frac{1}{1+\tan^2(x)}\right) dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

[Out] x+tan(x)

Rubi [A]
time = 0.03, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{209\}$

$$x+\tan(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]

[Out] x + Tan[x]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right) dx &= \text{Subst}\left(\int \left(1+\frac{1}{1+x^2}\right) dx, x, \tan(x)\right)\\ &= \tan(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \tan(x)\right)\\ &= x+\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]

[Out] x + Tan[x]

Maple [A]
time = 0.09, size = 5, normalized size = 1.25

method	result	size
default	$x+\tan(x)$	5
risch	$x+\frac{2i}{e^{2ix}+1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+1/(tan(x)^2+1)),x,method=_RETURNVERBOSE)
```

```
[Out] x+tan(x)
```

Maxima [A]
time = 0.54, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="maxima")
```

```
[Out] x + tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.
time = 4.08, size = 12, normalized size = 3.00

$$\frac{x\cos(x)+\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) + sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(3) = 6.
time = 0.29, size = 27, normalized size = 6.75

$$\frac{x\sec^2(x)}{\tan^2(x)+1}+\frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+1/(1+tan(x)**2)),x)
```

```
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

Giac [**A**]
time = 0.42, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="giac")
```

```
[Out] x + tan(x)
```

Mupad [**B**]
time = 2.94, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(tan(x)^2 + 1) + 1)/cos(x)^2,x)
```

```
[Out] x + tan(x)
```

3.695 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^2(x)} \, dx$

Optimal. Leaf size=4

$$x + \tan(x)$$

```
[Out] x+tan(x)
```

Rubi [A]
time = 0.05, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3738, 3554, 8}

$$x + \tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

```
[Out] x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^2(x)} \, dx &= \int \left(2+\tan^2(x)\right) \, dx \\
&= 2x + \int \tan^2(x) \, dx \\
&= 2x + \tan(x) - \int 1 \, dx \\
&= x + \tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
[Out] x + Tan[x]
```

Maple [A]
time = 0.10, size = 5, normalized size = 1.25

method	result	size
default	$x+\tan\left(x\right)$	5
risch	$x+\frac{2i}{e^{2ix}+1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(tan(x)^2+1),x,method=_RETURNVERBOSE)
[Out] x+tan(x)
```

Maxima [A]
time = 0.53, size = 4, normalized size = 1.00

$$x+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="maxima")
[Out] x + tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.
time = 2.44, size = 12, normalized size = 3.00

$$\frac{x\cos\left(x\right)+\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="fricas")
[Out] (x*cos(x) + sin(x))/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(3) = 6$.
time = 0.29, size = 27, normalized size = 6.75

$$\frac{x\sec^{2}\left(x\right)}{\tan^{2}\left(x\right)+1}+\frac{\tan\left(x\right)\sec^{2}\left(x\right)}{\tan^{2}\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**2),x)
[Out] x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

Giac [A]
time = 0.43, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="giac")
[Out] x + tan(x)
```

Mupad [B]
time = 3.00, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^2 + 1)),x)
[Out] x + tan(x)
```

3.696 $\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx$

Optimal. Leaf size=33

$$x-\operatorname{ArcTan}\left(\frac{1-2\cos^2(x)+\cos(x)\sin(x)}{2+\cos^2(x)+2\cos(x)\sin(x)}\right)$$

```
[Out] x-arctan((1-2*cos(x)^2+cos(x)*sin(x))/(2+cos(x)^2+2*cos(x)*sin(x)))
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4427, 631, 210}

$$x-\operatorname{ArcTan}\left(\frac{-2\cos^2(x)+\sin(x)\cos(x)+1}{\cos^2(x)+2\sin(x)\cos(x)+2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

```
[Out] x - ArcTan[(1 - 2*Cos[x]^2 + Cos[x]*Sin[x])/(2 + Cos[x]^2 + 2*Cos[x]*Sin[x]
)]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{2+2x+x^2}\,dx, x, \tan(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\tan(x)\right)$$
$$= x-\tan^{-1}\left(\frac{1-2\cos^2(x)+\cos(x)\sin(x)}{2+\cos^2(x)+2\cos(x)\sin(x)}\right)$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.94

$$2\left(-\frac{1}{4}\text{ArcTan}\left(\frac{\cos(x)}{\cos(x)+\sin(x)}\right)+\frac{1}{4}\text{ArcTan}(\sec(x)(\cos(x)+\sin(x)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
[Out] 2*(-1/4*ArcTan[Cos[x]/(Cos[x] + Sin[x])] + ArcTan[Sec[x]*(Cos[x] + Sin[x])]
/4)
```

Maple [A]
time = 0.14, size = 6, normalized size = 0.18

method	result	size
default	$\arctan(1+\tan(x))$	6
risch	$\frac{i\ln(\mathrm{e}^{2ix}+1+2i)}{2}-\frac{i\ln(\mathrm{e}^{2ix}+\frac{1}{5}+\frac{2i}{5})}{2}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(2+2*tan(x)+tan(x)^2),x,method=_RETURNVERBOSE)
[Out] arctan(1+tan(x))
```

Maxima [A]
time = 0.52, size = 5, normalized size = 0.15

$$\arctan(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="maxima")
[Out] arctan(tan(x) + 1)
```

Fricas [A]
time = 2.80, size = 35, normalized size = 1.06

$$-\frac{1}{2}\arctan\left(-\frac{3\cos\left(x\right)^{2}+6\cos\left(x\right)\sin\left(x\right)+1}{2\left(2\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="fricas")
[Out] -1/2*arctan(-1/2*(3*cos(x)^2 + 6*cos(x)*sin(x) + 1)/(2*cos(x)^2 - cos(x)*si
n(x) - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(x\right)}{\tan^{2}\left(x\right)+2\tan\left(x\right)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(2+2*tan(x)+tan(x)**2),x)
[Out] Integral(sec(x)**2/(tan(x)**2 + 2*tan(x) + 2), x)
```

Giac [A]
time = 0.41, size = 5, normalized size = 0.15

$$\arctan\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="giac")
[Out] arctan(tan(x) + 1)
```

Mupad [B]
time = 3.12, size = 5, normalized size = 0.15

$$\mathrm{atan}(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(2*tan(x) + tan(x)^2 + 2)),x)
[Out] atan(tan(x) + 1)
```

3.697 $\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx$

Optimal. Leaf size=10

$$-\cot(x)+\log(1+\cot(x))$$

[Out] -cot(x)+ln(1+cot(x))

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{4427, 46\}$

$$-\cot(x)-\log(\tan(x))+\log(\tan(x)+1)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]

[Out] -Cot[x] - Log[Tan[x]] + Log[1 + Tan[x]]

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx,x,\tan(x)\right)\\ &= -\cot(x)-\log(\tan(x))+\log(1+\tan(x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.60

$$-\cot(x) - \log(\sin(x)) + \log(\cos(x) + \sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
[Out] -Cot[x] - Log[Sin[x]] + Log[Cos[x] + Sin[x]]
```

Maple [A]
time = 0.11, size = 18, normalized size = 1.80

method	result	size
default	$-\frac{1}{\tan(x)} - \ln(\tan(x)) + \ln(1 + \tan(x))$	18
risch	$-\frac{2i}{e^{2ix}-1} + \ln(e^{2ix} + i) - \ln(e^{2ix} - 1)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(tan(x)^2+tan(x)^3),x,method=_RETURNVERBOSE)
[Out] -1/tan(x)-ln(tan(x))+ln(1+tan(x))
```

Maxima [A]
time = 0.36, size = 17, normalized size = 1.70

$$-\frac{1}{\tan(x)} + \log(\tan(x) + 1) - \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="maxima")
[Out] -1/tan(x) + log(tan(x) + 1) - log(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.
time = 1.80, size = 36, normalized size = 3.60

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - \log(2\cos(x)\sin(x) + 1)\sin(x) + 2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="fricas")
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(2*cos(x)*sin(x) + 1)*sin(x) + 2
*cos(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x)+1)\tan^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(tan(x)**2+tan(x)**3),x)
[Out] Integral(sec(x)**2/((tan(x) + 1)*tan(x)**2), x)
```

Giac [A]
time = 0.43, size = 19, normalized size = 1.90

$$-\frac{1}{\tan(x)} + \log(|\tan(x)+1|) - \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="giac")
[Out] -1/tan(x) + log(abs(tan(x) + 1)) - log(abs(tan(x)))
```

Mupad [B]
time = 3.10, size = 16, normalized size = 1.60

$$2\,\mathrm{atanh}(2\tan(x)+1) - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 + tan(x)^3)),x)
[Out] 2*atanh(2*tan(x) + 1) - 1/tan(x)
```

3.698 $\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx$

Optimal. Leaf size=10

$$\cot(x)+\log(1-\cot(x))$$

```
[Out] cot(x)+ln(1-cot(x))
```

Rubi [A]

time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4427, 46}

$$\cot(x)+\log(1-\tan(x))-\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

```
[Out] Cot[x] + Log[1 - Tan[x]] - Log[Tan[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(-1+x)x^2}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx,x,\tan(x)\right)\\ &= \cot(x)+\log(1-\tan(x))-\log(\tan(x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.60

$$\cot(x) + \log(\cos(x) - \sin(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
[Out] Cot[x] + Log[Cos[x] - Sin[x]] - Log[Sin[x]]
```

Maple [A]
time = 0.11, size = 16, normalized size = 1.60

method	result	size
default	$\ln(\tan(x) - 1) + \frac{1}{\tan(x)} - \ln(\tan(x))$	16
risch	$\frac{2i}{e^{2ix}-1} + \ln(e^{2ix} - i) - \ln(e^{2ix} - 1)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-tan(x)^2+tan(x)^3),x,method=_RETURNVERBOSE)
[Out] ln(tan(x)-1)+1/tan(x)-ln(tan(x))
```

Maxima [A]
time = 0.29, size = 15, normalized size = 1.50

$$\frac{1}{\tan(x)} + \log(\tan(x) - 1) - \log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="maxima")
[Out] 1/tan(x) + log(tan(x) - 1) - log(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.
time = 2.57, size = 36, normalized size = 3.60

$$-\frac{\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - \log(-2\cos(x)\sin(x) + 1)\sin(x) - 2\cos(x)}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="fricas")
[Out] -1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(-2*cos(x)*sin(x) + 1)*sin(x) -
2*cos(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{(\tan(x) - 1)\tan^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-tan(x)**2+tan(x)**3),x)
```

```
[Out] Integral(sec(x)**2/((tan(x) - 1)*tan(x)**2), x)
```

Giac [A]
time = 0.41, size = 17, normalized size = 1.70

$$\frac{1}{\tan(x)} + \log(|\tan(x) - 1|) - \log(|\tan(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/tan(x) + log(abs(tan(x) - 1)) - log(abs(tan(x)))
```

Mupad [B]
time = 2.98, size = 14, normalized size = 1.40

$$\frac{1}{\tan(x)} - 2\,\mathrm{atanh}(2\tan(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(tan(x)^2 - tan(x)^3)),x)
```

```
[Out] 1/tan(x) - 2*atanh(2*tan(x) - 1)
```

3.699 $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

Optimal. Leaf size=176

$$\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}-\frac{\text{ArcTan}\left(\frac{6^{2/3}-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\cos(x)\sin(x)}{3\ 2^{2/3}\sqrt[6]{3}+4\sqrt[3]{6}+\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\cos(x)\sin(x)}\right)}{3\ 2^{2/3}\sqrt[6]{3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}+\frac{\log\left(3^{2/3}\right.}{}$$

```
[Out] 1/18*x*2^(1/3)*3^(5/6)-1/18*arctan((6^(2/3)-2*6^(2/3)*cos(x)^2+2*(3-2*6^(1/
3))*cos(x)*sin(x))/(3*2^(2/3)*3^(1/6)+4*6^(1/3)+(6-4*6^(1/3))*cos(x)^2+2*6^
(2/3)*cos(x)*sin(x)))*2^(1/3)*3^(5/6)-1/18*ln(3^(1/3)-2^(2/3)*tan(x))*6^(1/
3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tan(x)+2*2^(1/3)*tan(x)^2)*6^(1/3)
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3756, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\sin(x)\cos(x)+6^{2/3}}{\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\sin(x)\cos(x)+4\sqrt[3]{6}+3\ 2^{2/3}\sqrt[6]{3}}\right)}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{x}{3\ 2^{2/3}\sqrt[6]{3}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{6\ 6^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

```
[Out] x/(3*2^(2/3)*3^(1/6)) - ArcTan[(6^(2/3) - 2*6^(2/3)*Cos[x]^2 + 2*(3 - 2*6^(
1/3))*Cos[x]*Sin[x])/(3*2^(2/3)*3^(1/6) + 4*6^(1/3) + (6 - 4*6^(1/3))*Cos[x
]^2 + 2*6^(2/3)*Cos[x]*Sin[x])]/(3*2^(2/3)*3^(1/6)) - Log[3^(1/3) - 2^(2/3)
*Tan[x]]/(3*6^(2/3)) + Log[3^(2/3) + 2^(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan
[x]^2]/(6*6^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \text{Subst}\left(\int \frac{1}{3-4x^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{3}-2^{2/3}x}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{3}+2^{2/3}x}{3^{2/3}+2^{2/3}\sqrt[3]{3}\,x+2\sqrt[3]{2}\,x^2}\,dx, x, \tan(x)\right)}{3\ 3^{2/3}}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{3^{2/3}+2^{2/3}\sqrt[3]{3}\,x+2\sqrt[3]{2}\,x^2}\,dx, x, \tan(x)\right)}{2\sqrt[3]{3}} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tan(x)+2\sqrt[3]{2}\tan^2(x)\right)}{6\ 6^{2/3}} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{x}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\tan^{-1}\left(\frac{6^{2/3}-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\cos(x)\sin(x)}{3\ 2^{2/3}\sqrt[6]{3}+4\sqrt[3]{6}+2\left(3-2\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\cos(x)\sin(x)}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}\right. -}{3}$$

Mathematica [A]
time = 0.08, size = 74, normalized size = 0.42

$$\frac{2\sqrt{3}\,\text{ArcTan}\left(\frac{3+2\ 6^{2/3}\tan(x)}{3\sqrt{3}}\right)-2\log\left(3-6^{2/3}\tan(x)\right)+\log\left(3+6^{2/3}\tan(x)+2\sqrt[3]{6}\tan^2(x)\right)}{6\ 6^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(3 - 4*Tan[x]^3),x]
[Out] (2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tan[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*T
an[x]] + Log[3 + 6^(2/3)*Tan[x] + 2*6^(1/3)*Tan[x]^2])/(6*6^(2/3))
```

Maple [A]
time = 0.12, size = 80, normalized size = 0.45

method	result	size
risch	$4\left(\sum_{\_R=\text{RootOf}(62208\_Z^3+1)}\_R\ln\left(e^{2ix}+\left(\frac{41472}{25}-\frac{31104i}{25}\right)\_R^2+\left(\frac{864}{25}+\frac{1152i}{25}\right)\_R-\frac{7}{25}+\frac{24i}{25}\right)\right)$	38
default	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan^2(x)+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tan(x)}{3}+1\right)}{3}\right)}{36}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2/(3-4*tan(x)^3),x,method=_RETURNVERBOSE)

[Out] -1/36*3^(1/3)*4^(2/3)*ln(tan(x)-1/4*3^(1/3)*4^(2/3))+1/72*3^(1/3)*4^(2/3)*ln(tan(x)^2+1/4*3^(1/3)*4^(2/3)*tan(x)+1/4*3^(2/3)*4^(1/3))+1/36*3^(5/6)*4^(2/3)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*4^(1/3)*tan(x)+1))

Maxima [A]
time = 0.50, size = 89, normalized size = 0.51

$$\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{5}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 4^{\frac{2}{3}}\tan\left(x\right)+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)+\frac{1}{72}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\tan\left(x\right)^{2}+4^{\frac{1}{3}}3^{\frac{1}{3}}\tan\left(x\right)+3^{\frac{2}{3}}\right)-\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(\frac{1}{4}\cdot 4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\tan\left(x\right)-3^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="maxima")

[Out] 1/36*4^(2/3)*3^(5/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*4^(2/3)*tan(x) + 4^(1/3)*3^(1/3))) + 1/72*4^(2/3)*3^(1/3)*log(4^(2/3)*tan(x)^2 + 4^(1/3)*3^(1/3)*tan(x) + 3^(2/3)) - 1/36*4^(2/3)*3^(1/3)*log(1/4*4^(2/3)*(4^(1/3)*tan(x) - 3^(1/3)))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(131) = 262.
time = 4.17, size = 441, normalized size = 2.51

$$-\frac{1}{36}\cdot 36^{\frac{1}{6}}\sqrt{3}\left(-1\right)^{\frac{1}{3}}\arctan\left(-\frac{36^{\frac{1}{6}}\left(28\left(36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}-9\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)^{6}-4\left(14\cdot 36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}+36\cdot 36^{\frac{1}{3}}\sqrt{3}-63\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)^{4}+\left(37\cdot 36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}+144\cdot 36^{\frac{1}{3}}\sqrt{3}+144\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)^{2}-6\left(16\left(36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}-9\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)^{5}-\left(24\cdot 36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}-7\cdot 36^{\frac{1}{3}}\sqrt{3}-72\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)^{3}+4\left(36^{\frac{2}{3}}\sqrt{3}\left(-1\right)^{\frac{2}{3}}-4\cdot 36^{\frac{1}{3}}\sqrt{3}+9\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)\cos\left(x\right)\right)\sin\left(x\right)-18\cdot 36^{\frac{1}{3}}\sqrt{3}-144\sqrt{3}\left(-1\right)^{\frac{1}{3}}\right)}{108\left(48\cos\left(x\right)^{6}-72\cos\left(x\right)^{4}+18\cos\left(x\right)^{2}+14\left(\cos\left(x\right)^{5}-\cos\left(x\right)^{3}\right)\sin\left(x\right)+3\right)}\right)-\frac{1}{432}\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(-3\left(2\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}-8\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+25\right)\cos\left(x\right)^{4}+3\left(3\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}-4\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+32\right)\cos\left(x\right)^{2}-2\left(\left(4\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+9\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}\right)\cos\left(x\right)^{3}-4\left(36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}-9\right)\cos\left(x\right)\right)\sin\left(x\right)-12\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}-48\right)+\frac{1}{216}\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}\log\left(3\left(2\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}+8\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}-7\right)\cos\left(x\right)^{2}+2\left(4\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}-9\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+36\right)\cos\left(x\right)\sin\left(x\right)-3\cdot 36^{\frac{2}{3}}\left(-1\right)^{\frac{1}{3}}-12\cdot 36^{\frac{1}{3}}\left(-1\right)^{\frac{2}{3}}+48\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="fricas")

[Out] -1/36*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/108*36^(1/6)*(28*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^6 - 4*(14*36^(2/3)*sqrt(3)*(-1)^(2/3) + 36*36^(1/3)*sqrt(3) - 63*sqrt(3)*(-1)^(1/3))*cos(x)^4 + (37*36^(2/3)*sqrt(3)*(-1)^(2/3) + 144*36^(1/3)*sqrt(3) + 144*sqrt(3)*(-1)^(1/3))*cos(x)^2 - 6*(16*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^5 - (24*36^(2/3)*sqrt(3)*(-1)^(2/3) - 7*36^(1/3)*sqrt(3) - 72*sqrt(3)*(-1)^(1/3))*cos(x)^3 + 4*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 4*36^(1/3)*sqrt(3) + 9*sqrt(3)*(-1)^(1/3))*cos(x))*sin(x) - 18*36^(1/3)*sqrt(3) - 144*sqrt(3)*(-1)^(1/3))/(48*cos(x)^6 - 72*cos(x)^4 + 18*cos(x)^2 + 14*(cos(x)^5 - cos(x)^3)*sin(x) + 3)) - 1/432*36^(2/3)*(-1)^(1/3)*log(-3*(2*36^(2/3)*(-1)^(1/3) - 8*36^(1/3)*(-1)^(2/3) + 25)*cos(x)^4 + 3*(3*36^(2/3)*(-1)^(1/3) - 4*36^(1/3)*(-1)^(2/3) + 32)*cos(x)^2 - 2*((4*36^(2/3)*(-1)^(1/3) + 9*36^(1/3)*(-1)^(2/3))*cos(x)^3 - 4*(36^(2/3)*(-1)^(1/3) - 9)*cos(x))*sin(x) - 12*36^(1/3)*(-1)^(2/3) - 48) + 1/216*36^(2/3)*(-1)^(1/3)*log(3*(2*36^(2/3)*(-1)^(1/3) + 8*36^(1/3)*(-1)^(2/3) - 7)*cos(x)^2 + 2*(4*36^(2/3)*(-1)^(1/3) - 9*36^(1/3)*(-1)^(2/3) + 36)*cos(x)*sin(x) - 3*36^(2/3)*(-1)^(1/3) - 12*36^(1/3)*(-1)^(2/3) + 48)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sec^2(x)}{4\tan^3(x)-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(3-4*tan(x)**3),x)
```

```
[Out] -Integral(sec(x)**2/(4*tan(x)**3 - 3), x)
```

Giac [A]
time = 0.40, size = 61, normalized size = 0.35

$$\frac{1}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{1}{3}}\arctan\left(\frac{4}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\tan(x)\right)\right)+\frac{1}{36}\cdot 6^{\frac{1}{3}}\log\left(\tan(x)^2+\left(\frac{3}{4}\right)^{\frac{1}{3}}\tan(x)+\left(\frac{3}{4}\right)^{\frac{2}{3}}\right)-\frac{1}{9}\left(\frac{3}{4}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{3}{4}\right)^{\frac{1}{3}}+\tan(x)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(3/4)^(1/3)*arctan(4/9*sqrt(3)*(3/4)^(2/3)*((3/4)^(1/3) + 2*tan
(x))) + 1/36*6^(1/3)*log(tan(x)^2 + (3/4)^(1/3)*tan(x) + (3/4)^(2/3)) - 1/9
*(3/4)^(1/3)*log(abs(-(3/4)^(1/3) + tan(x)))
```

Mupad [B]
time = 3.31, size = 75, normalized size = 0.43

$$-\frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}}{2}\right)}{18}-\frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{36}+\frac{6^{1/3}\ln\left(\tan(x)+\frac{6^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)^2*(4*tan(x)^3 - 3)),x)
```

```
[Out] (6^(1/3)*log(tan(x) + (6^(1/3)*(3^(1/2)*1i + 1))/4)*(3^(1/2)*1i + 1))/36 -
(6^(1/3)*log(tan(x) - (6^(1/3)*(3^(1/2)*1i - 1))/4)*(3^(1/2)*1i - 1))/36 -
(6^(1/3)*log(tan(x) - 6^(1/3)/2))/18
```

3.700 $\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx$

Optimal. Leaf size=53

$$\frac{2x}{\sqrt{195}}-\frac{2\mathrm{ArcTan}\left(\frac{-5+10\cos^2(x)+12\cos(x)\sin(x)}{10+\sqrt{195}+12\cos^2(x)-10\cos(x)\sin(x)}\right)}{\sqrt{195}}$$

```
[Out] 2/195*x*195^(1/2)-2/195*arctan((-5+10*cos(x)^2+12*cos(x)*sin(x))/(10+12*cos
(x)^2-10*cos(x)*sin(x)+195^(1/2)))*195^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4427, 632, 210}

$$\frac{2x}{\sqrt{195}}-\frac{2\mathrm{ArcTan}\left(\frac{10\cos^2(x)+12\sin(x)\cos(x)-5}{12\cos^2(x)-10\sin(x)\cos(x)+\sqrt{195}+10}\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

```
[Out] (2*x)/Sqrt[195] - (2*ArcTan[(-5 + 10*Cos[x]^2 + 12*Cos[x]*Sin[x])/(10 + Sqr
t[195] + 12*Cos[x]^2 - 10*Cos[x]*Sin[x])])/Sqrt[195]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{11-5x+5x^2}\,dx, x, \tan(x)\right)$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-195-x^2}\,dx, x, -5+10\tan(x)\right)\right)$$
$$= \frac{2x}{\sqrt{195}} + \frac{2\tan^{-1}\left(\frac{5-10\cos^2(x)-12\cos(x)\sin(x)}{10+\sqrt{195}+12\cos^2(x)-10\cos(x)\sin(x)}\right)}{\sqrt{195}}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 0.42

$$-\frac{2\text{ArcTan}\left(\sqrt{\frac{5}{39}}\,(1-2\tan(x))\right)}{\sqrt{195}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
[Out] (-2*ArcTan[Sqrt[5/39]*(1 - 2*Tan[x])])/Sqrt[195]
```

Maple [A]
time = 0.16, size = 18, normalized size = 0.34

method	result	size
default	$\frac{2\sqrt{195}\,\arctan\left(\frac{(10\tan(x)-5)\sqrt{195}}{195}\right)}{195}$	18
risch	$\frac{i\sqrt{195}\,\ln\left(e^{2ix}+\frac{6\sqrt{195}}{61}-\frac{5i\sqrt{195}}{61}+\frac{96}{61}-\frac{80i}{61}\right)}{195}-\frac{i\sqrt{195}\,\ln\left(e^{2ix}-\frac{6\sqrt{195}}{61}+\frac{5i\sqrt{195}}{61}+\frac{96}{61}-\frac{80i}{61}\right)}{195}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x,method=_RETURNVERBOSE)
[Out] 2/195*195^(1/2)*arctan(1/195*(10*tan(x)-5)*195^(1/2))
```

Maxima [A]
time = 0.54, size = 17, normalized size = 0.32

$$\frac{2}{195}\sqrt{195}\,\arctan\left(\frac{1}{39}\sqrt{195}\,(2\tan(x)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="maxima")
```

```
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

Fricas [A]
time = 4.23, size = 48, normalized size = 0.91

$$\frac{1}{195}\sqrt{195}\arctan\left(-\frac{192\sqrt{195}\cos(x)^2-160\sqrt{195}\cos(x)\sin(x)-35\sqrt{195}}{195\left(10\cos(x)^2+12\cos(x)\sin(x)-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/195*sqrt(195)*arctan(-1/195*(192*sqrt(195)*cos(x)^2 - 160*sqrt(195)*cos(x
)*sin(x) - 35*sqrt(195))/(10*cos(x)^2 + 12*cos(x)*sin(x) - 5))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(x)}{5\tan^2(x)-5\tan(x)+11}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(11-5*tan(x)+5*tan(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(5*tan(x)**2 - 5*tan(x) + 11), x)
```

Giac [A]
time = 0.39, size = 17, normalized size = 0.32

$$\frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}\,(2\tan(x)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="giac")
```

```
[Out] 2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

Mupad [B]
time = 3.11, size = 17, normalized size = 0.32

$$\frac{2\sqrt{195}\,\mathrm{atan}\left(\frac{\sqrt{195}\,(2\tan(x)-1)}{39}\right)}{195}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(5*tan(x)^2 - 5*tan(x) + 11)),x)
```

```
[Out] (2*195^(1/2)*atan((195^(1/2)*(2*tan(x) - 1))/39))/195
```

3.701 $\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}+\frac{b\tan(x)}{d}$$

```
[Out] -(-a*d+b*c)*ln(c+d*tan(x))/d^2+b*tan(x)/d
```

Rubi [A]
time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {4427, 45}

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
[Out] -(((b*c - a*d)*Log[c + d*Tan[x]])/d^2) + (b*Tan[x])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx &= \text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx,x,\tan(x)\right)\\ &= -\frac{(bc-ad)\log(c+d\tan(x))}{d^2}+\frac{b\tan(x)}{d}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 54, normalized size = 1.93

$$\frac{\cos(x)(a+b\tan(x))((bc-ad)(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))+bd\tan(x))}{d^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
[Out] (Cos[x]*(a + b*Tan[x])*((b*c - a*d)*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]]
) + b*d*Tan[x]))/(d^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.16, size = 28, normalized size = 1.00

method	result	size
default	$\frac{b\tan(x)}{d}+\frac{(ad-cb)\ln(c+d\tan(x))}{d^2}$	28
risch	$\frac{2ib}{d(e^{2ix}+1)}-\frac{\ln(e^{2ix}+1)a}{d}+\frac{\ln(e^{2ix}+1)cb}{d^2}+\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right)a}{d}-\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right)cb}{d^2}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x,method=_RETURNVERBOSE)
[Out] b*tan(x)/d+(a*d-b*c)/d^2*ln(c+d*tan(x))
```

Maxima [A]
time = 0.32, size = 28, normalized size = 1.00

$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(d\tan(x)+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="maxima")
[Out] b*tan(x)/d - (b*c - a*d)*log(d*tan(x) + c)/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.
time = 2.98, size = 71, normalized size = 2.54

$$-\frac{(bc-ad)\cos(x)\log\left(2\,cd\cos(x)\sin(x)+(c^2-d^2)\cos(x)^2+d^2\right)-(bc-ad)\cos(x)\log\left(\cos(x)^2\right)-2\,bd\sin(x)}{2\,d^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*c - a*d)*cos(x)*log(2*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d
^2) - (b*c - a*d)*cos(x)*log(cos(x)^2) - 2*b*d*sin(x))/(d^2*cos(x))
```

Sympy [A]
time = 2.27, size = 29, normalized size = 1.04

$$\frac{b\tan{(x)}}{d}+\frac{(ad-bc)\left(\begin{cases}\frac{\tan{(x)}}{c} & \text{for}\ d=0\\ \frac{\log{(c+d\tan{(x)})}}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))/(c+d*tan(x)),x)
[Out] b*tan(x)/d + (a*d - b*c)*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))
/d, True))/d
```

Giac [A]
time = 0.41, size = 29, normalized size = 1.04

$$\frac{b\tan{(x)}}{d}-\frac{(bc-ad)\log{(|d\tan{(x)}+c|)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="giac")
[Out] b*tan(x)/d - (b*c - a*d)*log(abs(d*tan(x) + c))/d^2
```

Mupad [B]
time = 3.09, size = 27, normalized size = 0.96

$$\frac{b\tan(x)}{d}+\frac{\ln{(c+d\tan(x))}\ (a\,d-b\,c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))/(cos(x)^2*(c + d*tan(x))),x)
[Out] (b*tan(x))/d + (log(c + d*tan(x))*(a*d - b*c))/d^2
```

3.702 $\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx$

Optimal. Leaf size=53

$$\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b(bc-ad)\tan(x)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

```
[Out] (-a*d+b*c)^2*ln(c+d*tan(x))/d^3-b*(-a*d+b*c)*tan(x)/d^2+1/2*(a+b*tan(x))^2/
d
```

Rubi [A]
time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {4427, 45}

$$\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] ((b*c - a*d)^2*Log[c + d*Tan[x]])/d^3 - (b*(b*c - a*d)*Tan[x])/d^2 + (a + b
*Tan[x])^2/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx &= \text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx,x,\tan(x)\right)\\ &= \frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b(bc-ad)\tan(x)}{d^2}+\frac{(a+b\tan(x))^2}{2d}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 62, normalized size = 1.17

$$\frac{b^2d^2\sec^2(x)-2\left((bc-ad)^2(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))+bd(bc-2ad)\tan(x)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

```
[Out] (b^2*d^2*Sec[x]^2 - 2*((b*c - a*d)^2*(Log[Cos[x]] - Log[c*Cos[x] + d*Sin[x]
]) + b*d*(b*c - 2*a*d)*Tan[x]))/(2*d^3)
```

Maple [A]
time = 0.19, size = 60, normalized size = 1.13

method	result
default	$\frac{b\left(\frac{b\left(\tan^2(x)\right)d}{2}+2ad\tan(x)-cb\tan(x)\right)}{d^2}+\frac{\left(d^2a^2-2cdab+b^2c^2\right)\ln(c+d\tan(x))}{d^3}$
risch	$\frac{2ib\left(2ad\,e^{2ix}-bc\,e^{2ix}-ibd\,e^{2ix}+2ad-cb\right)}{\left(e^{2ix}+1\right)^2d^2}-\frac{\ln\left(e^{2ix}+1\right)a^2}{d}+\frac{2\ln\left(e^{2ix}+1\right)cab}{d^2}-\frac{\ln\left(e^{2ix}+1\right)b^2c^2}{d^3}+\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right)a^2}{d}-\frac{2\ln\left(e^{2\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/2*b*tan(x)^2*d+2*a*d*tan(x)-c*b*tan(x))+(a^2*d^2-2*a*b*c*d+b^2*c^2
)/d^3*ln(c+d*tan(x))
```

Maxima [A]
time = 0.32, size = 63, normalized size = 1.19

$$\frac{b^2d\tan(x)^2-2\left(b^2c-2\,abd\right)\tan(x)}{2\,d^2}+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(d\tan(x)+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*(b^2*c - 2*a*b*d)*tan(x))/d^2 + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(d*tan(x) + c)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(51) = 102$.
time = 2.77, size = 122, normalized size = 2.30

$$\frac{b^2d^2+\left(b^2c^2-2\,abcd+a^2d^2\right)\cos(x)^2\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-\left(b^2c^2-2\,abcd+a^2d^2\right)\cos(x)^2\log\left(\cos(x)^2\right)-2\left(b^2cd-2\,abd^2\right)\cos(x)\sin(x)}{2\,d^3\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2*log(2*c*d*cos(x)*si
n(x) + (c^2 - d^2)*cos(x)^2 + d^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)
^2*log(cos(x)^2) - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x))/(d^3*cos(x)^2)
```

Sympy [A]
time = 3.12, size = 56, normalized size = 1.06

$$\frac{b^2\tan^2(x)}{2d}+\frac{(ad-bc)^2\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2}+\frac{(2abd-b^2c)\tan(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**2/(c+d*tan(x)),x)
```

```
[Out] b**2*tan(x)**2/(2*d) + (a*d - b*c)**2*Piecewise((tan(x)/c, Eq(d, 0)), (log(
c + d*tan(x))/d, True))/d**2 + (2*a*b*d - b**2*c)*tan(x)/d**2
```

Giac [A]
time = 0.43, size = 64, normalized size = 1.21

$$\frac{b^2d\tan(x)^2-2\,b^2c\tan(x)+4\,abd\tan(x)}{2\,d^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|d\tan(x)+c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*tan(x)^2 - 2*b^2*c*tan(x) + 4*a*b*d*tan(x))/d^2 + (b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(abs(d*tan(x) + c))/d^3
```

Mupad [B]
time = 2.98, size = 65, normalized size = 1.23

$$\frac{\ln(c+d\tan(x))\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{d^3}-\tan(x)\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{b^2\tan(x)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^2/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] (log(c + d*tan(x))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - tan(x)*((b^2*c)/d
^2 - (2*a*b)/d) + (b^2*tan(x)^2)/(2*d)
```

3.703 $\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b(bc-ad)^2\tan(x)}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}+\frac{(a+b\tan(x))^3}{3d}$$

```
[Out] -(-a*d+b*c)^3*ln(c+d*tan(x))/d^4+b*(-a*d+b*c)^2*tan(x)/d^3-1/2*(-a*d+b*c)*(
a+b*tan(x))^2/d^2+1/3*(a+b*tan(x))^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4427, 45}

$$-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}+\frac{(a+b\tan(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] -(((b*c - a*d)^3*Log[c + d*Tan[x]])/d^4) + (b*(b*c - a*d)^2*Tan[x])/d^3 - (
(b*c - a*d)*(a + b*Tan[x])^2)/(2*d^2) + (a + b*Tan[x])^3/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx &= \text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)\\ &= -\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b(bc-ad)^2\tan(x)}{d^3}-\frac{(bc-ad)(a+b\tan(x)}{2d^2}\end{aligned}$$

Mathematica [A]
time = 0.68, size = 133, normalized size = 1.71

$$\frac{(c\cos(x)+d\sin(x))(a+b\tan(x))^3\left(6(bc-ad)^3\cos^2(x)(\log(\cos(x))-\log(c\cos(x)+d\sin(x)))-b^3d(-3c^2+d^2)\sin(2x)+bd^2(9a(-bc+ad)\sin(2x)+b(-3bc+9ad+2bd\tan(x)))\right)}{6d^4(a\cos(x)+b\sin(x))^3(c+d\tan(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

```
[Out] ((c*Cos[x] + d*Sin[x])*(a + b*Tan[x])^3*(6*(b*c - a*d)^3*Cos[x]^2*(Log[Cos[
x]] - Log[c*Cos[x] + d*Sin[x]]) - b^3*d*(-3*c^2 + d^2)*Sin[2*x] + b*d^2*(9*
a*(-(b*c) + a*d)*Sin[2*x] + b*(-3*b*c + 9*a*d + 2*b*d*Tan[x]))))/(6*d^4*(a*
Cos[x] + b*Sin[x])^3*(c + d*Tan[x]))
```

Maple [A]
time = 0.16, size = 116, normalized size = 1.49

method	result
default	$\frac{b\left(\frac{b^2\left(\tan^3(x)\right)d^2}{3}+\frac{3ab\,d^2\left(\tan^2(x)\right)}{2}-\frac{b^2cd\left(\tan^2(x)\right)}{2}+3d^2a^2\tan(x)-3cdab\tan(x)+b^2c^2\tan(x)\right)}{d^3}+\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2-b^3c^3)}{d^4}$
risch	$\frac{2ib\left(9a^2d^2\mathrm{e}^{4ix}-9abcd\,\mathrm{e}^{4ix}+3b^2c^2\mathrm{e}^{4ix}-3b^2d^2\mathrm{e}^{4ix}-9iab\,d^2\mathrm{e}^{4ix}+3ib^2cd\,\mathrm{e}^{4ix}+18a^2d^2\mathrm{e}^{2ix}-18abcd\,\mathrm{e}^{2ix}+6b^2c^2\mathrm{e}^{2ix}-9iab\,d^2\mathrm{e}^{2ix}+3ib^2\right.}{3d^3\left(\mathrm{e}^{2ix}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(1/3*b^2*tan(x)^3*d^2+3/2*a*b*d^2*tan(x)^2-1/2*b^2*c*d*tan(x)^2+3*d^2
*a^2*tan(x)-3*c*d*a*b*tan(x)+b^2*c^2*tan(x))+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2
*c^2*d-b^3*c^3)/d^4*ln(c+d*tan(x))
```

Maxima [A]
time = 0.31, size = 118, normalized size = 1.51

$$\frac{2\,b^3d^2\tan(x)^3-3\,(b^3cd-3\,ab^2d^2)\tan(x)^2+6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)\tan(x)}{6\,d^3}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(d\tan(x)+c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*tan(x)^2 + 6*(b^3*c^2 -
 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*log(d*tan(x) + c)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(74) = 148$.
time = 2.44, size = 201, normalized size = 2.58

$$-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\cos(x)^3\log\left(2\,cd\cos(x)\sin(x)+(c^2-d^2)\cos(x)^2+d^2\right)-3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\cos(x)^3\log\left(\cos(x)^2\right)+3\,(b^3cd^2-3\,ab^2d^3)\cos(x)-2\,\left(b^3d^3+(3\,b^3c^2d-9\,ab^2cd^2+(9\,a^2b-b^3)d^3)\cos(x)^2\right)\sin(x)}{6\,d^4\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(2*
c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d^2) - 3*(b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(cos(x)^2) + 3*(b^3*c*d^2 - 3*a*b^
2*d^3)*cos(x) - 2*(b^3*d^3 + (3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)
*d^3)*cos(x)^2)*sin(x))/(d^4*cos(x)^3)
```

Sympy [A]
time = 4.61, size = 95, normalized size = 1.22

$$\frac{b^3\tan^3(x)}{3d}+\frac{(3ab^2d-b^3c)\tan^2(x)}{2d^2}+\frac{(ad-bc)^3\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{(3a^2bd^2-3ab^2cd+b^3c^2)\tan(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(a+b*tan(x))**3/(c+d*tan(x)),x)
```

```
[Out] b**3*tan(x)**3/(3*d) + (3*a*b**2*d - b**3*c)*tan(x)**2/(2*d**2) + (a*d - b*
c)**3*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**3 + (
3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*tan(x)/d**3
```

Giac [A]
time = 0.42, size = 123, normalized size = 1.58

$$\frac{2b^3d^2\tan(x)^3-3b^3cd\tan(x)^2+9ab^2d^2\tan(x)^2+6b^3c^2\tan(x)-18ab^2cd\tan(x)+18a^2bd^2\tan(x)}{6d^3}-\frac{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\log(|d\tan(x)+c|)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*tan(x)^3 - 3*b^3*c*d*tan(x)^2 + 9*a*b^2*d^2*tan(x)^2 + 6*b^3
*c^2*tan(x) - 18*a*b^2*c*d*tan(x) + 18*a^2*b*d^2*tan(x))/d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(d*tan(x) + c))/d^4
```

Mupad [B]
time = 2.96, size = 122, normalized size = 1.56

$$\tan(x)\left(\frac{3a^2b}{d}-\frac{c\left(\frac{3ab^2}{d}-\frac{b^3c}{d^2}\right)}{d}\right)+\tan(x)^2\left(\frac{3ab^2}{2d}-\frac{b^3c}{2d^2}\right)+\frac{b^3\tan(x)^3}{3d}+\frac{\ln(c+d\tan(x))\,(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tan(x))^3/(cos(x)^2*(c + d*tan(x))),x)
```

```
[Out] tan(x)*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3*c)/d^2))/d) + tan(x)^2*((3*a*b
^2)/(2*d) - (b^3*c)/(2*d^2)) + (b^3*tan(x)^3)/(3*d) + (log(c + d*tan(x))*(a
^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/d^4
```

3.704 $\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\left(2+\tan^3(x)\right)}$$

```
[Out] -1/3/(2+tan(x)^3)
```

Rubi [A]
time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4427, 267}

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/3*1/(2 + Tan[x]^3)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\tan^2(x)}{\left(2+\tan^3(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{x^2}{\left(2+x^3\right)^2}\,dx, x, \tan(x)\right)\\ &= -\frac{1}{3\left(2+\tan^3(x)\right)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{3\left(2+\tan^3(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

```
[Out] -1/3*1/(2 + Tan[x]^3)
```

Maple [A]
time = 0.12, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{3(2+\tan^3(x))}$	11
default	$-\frac{1}{3(2+\tan^3(x))}$	11
risch	$\frac{\left(-\frac{8}{15}-\frac{2i}{5}\right)\left(3\,e^{4ix}+1\right)}{5\,e^{6ix}+9\,e^{4ix}-12ie^{4ix}+15\,e^{2ix}+3-4i}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(2+tan(x)^3)
```

Maxima [A]
time = 0.31, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tan\left(x\right)^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.
time = 2.46, size = 36, normalized size = 3.00

$$-\frac{\cos\left(x\right)^3+2\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)}{15\left(2\cos\left(x\right)^3-\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(x)^3 + 2*(cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - (cos(x)^2 - 1)*sin
(x))
```

Sympy [A]
time = 170.87, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tan^{3}\left(x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**2/(2+tan(x)**3)**2,x)
```

```
[Out] -1/(3*(tan(x)**3 + 2))
```

Giac [A]
time = 0.42, size = 10, normalized size = 0.83

$$-\frac{1}{3\left(\tan\left(x\right)^{3}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="giac")
```

```
[Out] -1/3/(tan(x)^3 + 2)
```

Mupad [B]
time = 2.93, size = 12, normalized size = 1.00

$$-\frac{1}{3\,\left({\mathrm{tan}\left(x\right)}^3+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(cos(x)^2*(tan(x)^3 + 2)^2),x)
```

```
[Out] -1/(3*(tan(x)^3 + 2))
```

3.705 $\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx$

Optimal. Leaf size=33

$$\frac{\tan^7(x)}{7}+\frac{\tan^9(x)}{3}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{13}(x)}{13}$$

```
[Out] 1/7*tan(x)^7+1/3*tan(x)^9+3/11*tan(x)^11+1/13*tan(x)^13
```

Rubi [A]
time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3738, 2687, 276}

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx &= \int \sec^8(x)\tan^6(x)\,dx\\ &= \text{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx, x, \tan(x)\right)\\ &= \frac{\tan^7(x)}{7}+\frac{\tan^9(x)}{3}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{13}(x)}{13}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.
time = 0.02, size = 67, normalized size = 2.03

$$-\frac{16\tan(x)}{3003}-\frac{8\sec^2(x)\tan(x)}{3003}-\frac{2\sec^4(x)\tan(x)}{1001}-\frac{5\sec^6(x)\tan(x)}{3003}+\frac{53}{429}\sec^8(x)\tan(x)-\frac{27}{143}\sec^{10}(x)\tan(x)+\frac{1}{13}\sec^{12}(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

```
[Out] (-16*Tan[x])/3003 - (8*Sec[x]^2*Tan[x])/3003 - (2*Sec[x]^4*Tan[x])/1001 - (
5*Sec[x]^6*Tan[x])/3003 + (53*Sec[x]^8*Tan[x])/429 - (27*Sec[x]^10*Tan[x])/
143 + (Sec[x]^12*Tan[x])/13
```

Maple [A]
time = 0.07, size = 42, normalized size = 1.27

method	result	size
default	$\frac{\sin^{13}(x)}{13\cos(x)^{13}}+\frac{3\left(\sin^{11}(x)\right)}{11\cos(x)^{11}}+\frac{\sin^9(x)}{3\cos(x)^9}+\frac{\sin^7(x)}{7\cos(x)^7}$	42
risch	$-\frac{32i\left(3003\,e^{18ix}-9009\,e^{16ix}+18018\,e^{14ix}-16302\,e^{12ix}+10296\,e^{10ix}-2288\,e^{8ix}+286\,e^{6ix}+78\,e^{4ix}+13\,e^{2ix}+1\right)}{3003\left(e^{2ix}+1\right)^{13}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^6*(tan(x)^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*sin(x)^13/cos(x)^13+3/11*sin(x)^11/cos(x)^11+1/3*sin(x)^9/cos(x)^9+1/7
*sin(x)^7/cos(x)^7
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.76

$$\frac{1}{13}\tan\left(x\right)^{13}+\frac{3}{11}\tan\left(x\right)^{11}+\frac{1}{3}\tan\left(x\right)^{9}+\frac{1}{7}\tan\left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

Fricas [A]
time = 2.78, size = 46, normalized size = 1.39

$$-\frac{\left(16 \cos \left(x\right)^{12}+8 \cos \left(x\right)^{10}+6 \cos \left(x\right)^{8}+5 \cos \left(x\right)^{6}-371 \cos \left(x\right)^{4}+567 \cos \left(x\right)^{2}-231\right) \sin \left(x\right)}{3003 \cos \left(x\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/3003*(16*cos(x)^12 + 8*cos(x)^10 + 6*cos(x)^8 + 5*cos(x)^6 - 371*cos(x)^
4 + 567*cos(x)^2 - 231)*sin(x)/cos(x)^13
```

Sympy [A]
time = 13.77, size = 27, normalized size = 0.82

$$\frac{\tan^{13}\left(x\right)}{13}+\frac{3\tan^{11}\left(x\right)}{11}+\frac{\tan^{9}\left(x\right)}{3}+\frac{\tan^{7}\left(x\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**6*(1+tan(x)**2)**3,x)
```

```
[Out] tan(x)**13/13 + 3*tan(x)**11/11 + tan(x)**9/3 + tan(x)**7/7
```

Giac [A]
time = 0.43, size = 25, normalized size = 0.76

$$\frac{1}{13} \tan \left(x\right)^{13}+\frac{3}{11} \tan \left(x\right)^{11}+\frac{1}{3} \tan \left(x\right)^{9}+\frac{1}{7} \tan \left(x\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

Mupad [B]
time = 2.92, size = 25, normalized size = 0.76

$$\frac{{\tan\left(x\right)}^{13}}{13}+\frac{3\,{\tan\left(x\right)}^{11}}{11}+\frac{{\tan\left(x\right)}^{9}}{3}+\frac{{\tan\left(x\right)}^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^6*(tan(x)^2 + 1)^3)/cos(x)^2,x)
```

```
[Out] tan(x)^7/7 + tan(x)^9/3 + (3*tan(x)^11)/11 + tan(x)^13/13
```

3.706 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)} dx$

Optimal. Leaf size=46

$$\frac{2x}{\sqrt{3}} + \frac{2\text{ArcTan}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{\sqrt{3}} + \log(1+\tan(x))$$

```
[Out] ln(1+tan(x))+2/3*x*3^(1/2)+2/3*arctan((1-2*cos(x)^2)/(2-2*cos(x)*sin(x)+3^(
1/2)))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {4427, 1877, 31, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{1-2\cos^2(x)}{-2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}} + \frac{2x}{\sqrt{3}} + \log(\tan(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

```
[Out] (2*x)/Sqrt[3] + (2*ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] - 2*Cos[x]*Sin[x])]
)/Sqrt[3] + Log[1 + Tan[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1877

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Dist[C
```

```
/b, Int[1/(q + x), x], x] + Dist[(B + C*q)/b, Int[1/(q^2 - q*x + x^2), x],
x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a, b}
, x] && PolyQ[P2, x, 2]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx &= \text{Subst}\left(\int \frac{2+x^2}{1+x^3}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan(x)\right)+\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tan(x)\right)\\ &= \log(1+\tan(x))-2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tan(x)\right)\\ &= \frac{2x}{\sqrt{3}}+\frac{2\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{\sqrt{3}}+\log(1+\tan(x))\end{aligned}$$

Mathematica [A]
time = 0.16, size = 32, normalized size = 0.70

$$-\frac{2\text{ArcTan}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\log(\cos(x))+\log(\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

```
[Out] (-2*ArcTan[(1 - 2*Tan[x])/Sqrt[3]])/Sqrt[3] - Log[Cos[x]] + Log[Cos[x] + Si
n[x]]
```

Maple [A]
time = 0.17, size = 24, normalized size = 0.52

method	result	size

default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{3}+\ln\left(1+\tan\left(x\right)\right)$	24
risch	$-\ln\left(\mathrm{e}^{2ix}+1\right)+\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}-2i-i\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}-2i+i\sqrt{3}\right)}{3}+\ln\left(\mathrm{e}^{2ix}+i\right)$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))+ln(1+tan(x))
```

Maxima [A]
time = 0.51, size = 23, normalized size = 0.50

$$\frac{2}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,\tan\left(x\right)-1)\right)+\log\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(tan(x) + 1)
```

Fricas [A]
time = 3.56, size = 52, normalized size = 1.13

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{4\,\sqrt{3}\,\cos\left(x\right)\sin\left(x\right)-\sqrt{3}}{3\,\left(2\,\cos\left(x\right)^{2}-1\right)}\right)-\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}\right)+\frac{1}{2}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 - 1)
) - 1/2*log(cos(x)^2) + 1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A]
time = 4.55, size = 41, normalized size = 0.89

$$\frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\,\left(\tan\left(x\right)-\frac{1}{2}\right)}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}+\log\left(\tan\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**3),x)
```

```
[Out] 2*sqrt(3)*(atan(2*sqrt(3)*(tan(x) - 1/2)/3) + pi*floor((x - pi/2)/pi))/3 +
log(tan(x) + 1)
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.52

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(x\right)-1\right)\right)+\log\left(\left|\tan\left(x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(abs(tan(x) + 1))
```

Mupad [B]
time = 2.97, size = 30, normalized size = 0.65

$$\ln\left(\tan(x)+1\right)-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3}\,\tan(x)}{\tan(x)+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^3 + 1)),x)
```

```
[Out] log(tan(x) + 1) - (2*3^(1/2)*atan((3^(1/2) - 3^(1/2)*tan(x))/(tan(x) + 1)))
/3
```

3.707 $\int (1+\cos^2(x)) \sec^2(x) \, dx$

Optimal. Leaf size=4

$$x+\tan(x)$$

```
[Out] x+tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3091, 8}

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)*Sec[x]^2,x]
```

```
[Out] x + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (1+\cos^2(x)) \sec^2(x) \, dx &= \tan(x)+\int 1 \, dx \\ &= x+\tan(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)*Sec[x]^2,x]
```

[Out] x + Tan[x]

Maple [A]
time = 0.06, size = 5, normalized size = 1.25

method	result	size
default	$x+\tan(x)$	5
risch	$x+\frac{2i}{e^{2ix}+1}$	15
norman	$\frac{x\left(\tan^4\left(\frac{x}{2}\right)\right)+x\left(\tan^6\left(\frac{x}{2}\right)\right)-x-4\left(\tan^3\left(\frac{x}{2}\right)\right)-2\left(\tan^5\left(\frac{x}{2}\right)\right)-x\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+cos(x)^2)*sec(x)^2,x,method=_RETURNVERBOSE)

[Out] x+tan(x)

Maxima [A]
time = 0.51, size = 4, normalized size = 1.00

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="maxima")

[Out] x + tan(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.
time = 4.41, size = 12, normalized size = 3.00

$$\frac{x\cos(x)+\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="fricas")

[Out] (x*cos(x) + sin(x))/cos(x)

Sympy [A]
time = 3.91, size = 3, normalized size = 0.75

$$x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+cos(x)**2)*sec(x)**2,x)

[Out] x + tan(x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(4) = 8$.
time = 0.42, size = 15, normalized size = 3.75

$$-\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="giac")
```

```
[Out] -pi*floor(x/pi + 1/2) + x + tan(x)
```

Mupad [B]
time = 2.88, size = 4, normalized size = 1.00

$$x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)/cos(x)^2,x)
```

```
[Out] x + tan(x)
```

3.708 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$-\log(\cos(x)-\sin(x))+\log(2\cos(x)-\sin(x))$$

```
[Out] -ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

Rubi [A]
time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {630, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,\tan(x)\right)\\
&= -\log(1-\tan(x))+\log(2-\tan(x))
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.38

$$2\left(-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(2\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

Maple [A]
time = 0.14, size = 14, normalized size = 0.67

method	result	size
default	$-\ln\left(\tan\left(x\right)-1\right)+\ln\left(-2+\tan\left(x\right)\right)$	14
risch	$\ln\left({e}^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)-\ln\left({e}^{2ix}-i\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(tan(x)-1)+ln(-2+tan(x))
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.62

$$-\log\left(\tan\left(x\right)-1\right)+\log\left(\tan\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A]
time = 3.31, size = 29, normalized size = 1.38

$$\frac{1}{2}\,\log\left(\frac{3}{4}\,\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\frac{1}{2}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(x\right)}{-3\tan\left(x\right)+\sec^{2}\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.71

$$-\log\left(|\tan\left(x\right)-1|\right)+\log\left(|\tan\left(x\right)-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

Mupad [B]
time = 3.51, size = 9, normalized size = 0.43

$$-2\,\mathrm{atanh}(2\tan(x)-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
[Out] -2*atanh(2*tan(x) - 3)
```

3.709 $\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx$

Optimal. Leaf size=9

$$\text{ArcSin}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

[Out] arcsin(1/3*tan(x)*3^(1/2))

Rubi [A]
time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4231, 222}

$$\text{ArcSin}\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]

[Out] ArcSin[Tan[x]/Sqrt[3]]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4231

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, S
ubst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2), x
]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[
m/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{3-x^2}}\,dx, x, \tan(x)\right)\\ &= \sin^{-1}\left(\frac{\tan(x)}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(9) = 18.
time = 0.03, size = 43, normalized size = 4.78

$$\frac{\operatorname{ArcTan}\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)\sqrt{1+2\cos(2x)}\sec(x)}{\sqrt{4-\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
[Out] (ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]]*Sqrt[1 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 -
Sec[x]^2]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 103, normalized size = 11.44

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{2\cos(x)-1}{1+\cos(x)}}\sqrt{6}\sqrt{\frac{1+2\cos(x)}{1+\cos(x)}}\left(\operatorname{EllipticF}\left(\frac{(\cos(x)-1)\sqrt{3}}{\sin(x)},\frac{1}{3}\right)-2\operatorname{EllipticPi}\left(\frac{(\cos(x)-1)\sqrt{3}}{\sin(x)},\frac{1}{3},\frac{1}{3}\right)\right)(\sin^2(x))\sqrt{3}}{9\sqrt{\frac{4(\cos^2(x))-1}{\cos(x)^2}}\cos(x)(\cos(x)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(4-sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/9*2^(1/2)*((2*cos(x)-1)/(1+cos(x)))^(1/2)*6^(1/2)*((1+2*cos(x))/(1+cos(x
)))^(1/2)*(EllipticF((cos(x)-1)*3^(1/2)/sin(x),1/3)-2*EllipticPi((cos(x)-1)
*3^(1/2)/sin(x),1/3,1/3))*sin(x)^2/((4*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)/(
cos(x)-1)*3^(1/2)
```

Maxima [A]
time = 0.50, size = 8, normalized size = 0.89

$$\arcsin\left(\frac{1}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(1/3*sqrt(3)*tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(8) = 16.
time = 3.05, size = 25, normalized size = 2.78

$$-\arctan\left(\frac{\sqrt{\frac{4\cos(x)^2-1}{\cos(x)^2}}\cos(x)}{\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt((4*cos(x)^2 - 1)/cos(x)^2)*cos(x)/sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(x)}{\sqrt{-(\sec(x)-2)(\sec(x)+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(4-sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(sec(x) - 2)*(sec(x) + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(x)^2/sqrt(-sec(x)^2 + 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\cos(x)^2\sqrt{4-\frac{1}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)), x)
```

3.710 $\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\text{ArcSin}(2\tan(x))$$

[Out] 1/2*arcsin(2*tan(x))

Rubi [A]
time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3756, 222}

$$\frac{1}{2}\text{ArcSin}(2\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]

[Out] ArcSin[2*Tan[x]]/2

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-4x^2}}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\sin^{-1}(2\tan(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(9) = 18.
time = 0.04, size = 47, normalized size = 5.22

$$\frac{\operatorname{ArcTan}\left(\frac{2\sin(x)}{\sqrt{1-5\sin^2(x)}}\right)\sqrt{-3+5\cos(2x)}\sec(x)}{2\sqrt{2-8\tan^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

```
[Out] (ArcTan[(2*Sin[x])/Sqrt[1 - 5*Sin[x]^2]]*Sqrt[-3 + 5*Cos[2*x]]*Sec[x])/(2*S
qrt[2 - 8*Tan[x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.62, size = 171, normalized size = 19.00

method	result
default	$-\frac{\sqrt{2}\sqrt{\frac{2\cos(x)\sqrt{5}-2\sqrt{5}+5\cos(x)-4}{1+\cos(x)}}\sqrt{-\frac{2\left(2\cos(x)\sqrt{5}-2\sqrt{5}-5\cos(x)+4\right)}{1+\cos(x)}}\left(\operatorname{EllipticF}\left(\frac{(\cos(x)-1)\left(\sqrt{5}+2\right)}{\sin(x)},\right.\right.}{\sqrt{\frac{5(\cos^2(x))-4}{\cos(x)^2}}\cos(x)(\cos(x)-1)\sqrt{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1-4*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)+5*cos(x)-4)/(1+cos(x)))^(1/2)*(-2*(2*
cos(x)*5^(1/2)-2*5^(1/2)-5*cos(x)+4)/(1+cos(x)))^(1/2)*(EllipticF((cos(x)-1
)*(5^(1/2)+2)/sin(x),9-4*5^(1/2))-2*EllipticPi((9+4*5^(1/2))^(1/2)*(cos(x)-
1)/sin(x),1/(9+4*5^(1/2)),(9-4*5^(1/2))^(1/2)/(9+4*5^(1/2))^(1/2)))*sin(x)^
2/((5*cos(x)^2-4)/cos(x)^2)^(1/2)/cos(x)/(cos(x)-1)/(9+4*5^(1/2))^(1/2)
```

Maxima [A]
time = 0.51, size = 7, normalized size = 0.78

$$\frac{1}{2}\arcsin\left(2\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(7) = 14.
time = 2.96, size = 45, normalized size = 5.00

$$-\frac{1}{4}\arctan\left(\frac{(9\cos(x)^3-8\cos(x))\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}}{4\,(5\cos(x)^2-4)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(1/4*(9*cos(x)^3 - 8*cos(x))*sqrt((5*cos(x)^2 - 4)/cos(x)^2)/((5
*cos(x)^2 - 4)*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(x)}{\sqrt{-(2\tan(x)-1)(2\tan(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1-4*tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt(-(2*tan(x) - 1)*(2*tan(x) + 1)), x)
```

Giac [A]
time = 0.43, size = 7, normalized size = 0.78

$$\frac{1}{2}\arcsin(2\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(2*tan(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.11

$$\int\frac{1}{\cos(x)^2\sqrt{1-4\tan(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)), x)
```

3.711 $\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

```
[Out] arctanh(tan(x)/(-4+tan(x)^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3756, 223, 212}

$$\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

```
[Out] ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-4+x^2}}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.
time = 0.04, size = 46, normalized size = 3.29

$$\frac{\text{ArcTan}\left(\frac{\sin(x)}{\sqrt{4-5\sin^2(x)}}\right)\sqrt{3+5\cos(2x)}\,\sec(x)}{\sqrt{2}\sqrt{-4+\tan^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
[Out] (ArcTan[Sin[x]/Sqrt[4 - 5*Sin[x]^2]]*Sqrt[3 + 5*Cos[2*x]]*Sec[x])/(Sqrt[2]*
Sqrt[-4 + Tan[x]^2])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.72, size = 171, normalized size = 12.21

method	result
default	$-\frac{\sqrt{-\frac{2\left(\cos(x)\sqrt{5}-\sqrt{5}-5\cos(x)+1\right)}{1+\cos(x)}}\sqrt{2}\sqrt{\frac{\cos(x)\sqrt{5}+5\cos(x)-\sqrt{5}-1}{1+\cos(x)}}\left(\text{EllipticF}\left(\frac{(\cos(x)-1)\left(\sqrt{5}-1\right)}{2\sin(x)},\frac{3}{2}+\frac{\sqrt{5}}{2}\right.\right.}{4\sqrt{-\frac{5(\cos^2(x))-1}{\cos(x)^2}}\cos(x)(\cos(x)-1)\sqrt{\frac{3}{2}-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(-4+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(-2*(cos(x)*5^(1/2)-5^(1/2)-5*cos(x)+1)/(1+cos(x)))^(1/2)*2^(1/2)*((co
s(x)*5^(1/2)+5*cos(x)-5^(1/2)-1)/(1+cos(x)))^(1/2)*(EllipticF(1/2*(cos(x)-1
```

```
)*(5^(1/2)-1)/sin(x),3/2+1/2*5^(1/2))-2*EllipticPi((3/2-1/2*5^(1/2))^(1/2)*
(cos(x)-1)/sin(x),-2/(5^(1/2)-3),(3/2+1/2*5^(1/2))^(1/2)/(3/2-1/2*5^(1/2))^
(1/2)))*sin(x)^2/(-(5*cos(x)^2-1)/cos(x)^2)^(1/2)/cos(x)/(cos(x)-1)/(3/2-1/
2*5^(1/2))^(1/2)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.14

$$\log\left(2\sqrt{\tan\left(x\right)^{2}-4}+2\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(tan(x)^2 - 4) + 2*tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(12) = 24$.
time = 2.84, size = 67, normalized size = 4.79

$$\frac{1}{4}\log\left(\frac{1}{2}\sqrt{-\frac{5\cos\left(x\right)^{2}-1}{\cos\left(x\right)^{2}}}\cos\left(x\right)\sin\left(x\right)-\frac{3}{2}\cos\left(x\right)^{2}+\frac{1}{2}\right)-\frac{1}{4}\log\left(-\frac{1}{2}\sqrt{-\frac{5\cos\left(x\right)^{2}-1}{\cos\left(x\right)^{2}}}\cos\left(x\right)\sin\left(x\right)-\frac{3}{2}\cos\left(x\right)^{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2 +
 1/2) - 1/4*log(-1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*c
os(x)^2 + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(x\right)}{\sqrt{\left(\tan\left(x\right)-2\right)\left(\tan\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(-4+tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sec(x)**2/sqrt((tan(x) - 2)*(tan(x) + 2)), x)
```

Giac [A]
time = 0.42, size = 17, normalized size = 1.21

$$-\log\left(\left|\sqrt{\tan\left(x\right)^{2}-4}-\tan\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(sqrt(tan(x)^2 - 4) - tan(x)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\cos(x)^2 \sqrt{\tan(x)^2 - 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)),x)
```

```
[Out] int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)), x)
```

3.712 $\int \sqrt{1-\cot^2(x)} \sec^2(x) \, dx$

Optimal. Leaf size=19

$$\text{ArcSin}(\cot(x)) + \sqrt{1-\cot^2(x)} \tan(x)$$

```
[Out] arcsin(cot(x))+(1-cot(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3744, 283, 222}

$$\text{ArcSin}(\cot(x)) + \tan(x)\sqrt{1-\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

```
[Out] ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 3744

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[c*(ff^(m + 1)/f), Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/2
 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
 && IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = -\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2}\,dx, x, \cot(x)\right)$$
$$= \sqrt{1-\cot^2(x)}\tan(x) + \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \cot(x)\right)$$
$$= \sin^{-1}(\cot(x)) + \sqrt{1-\cot^2(x)}\tan(x)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.
time = 0.34, size = 52, normalized size = 2.74

$$\left(-\text{ArcTan}\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\cos(x)\sqrt{-\cos(2x)}+\cos(2x)\right)\sqrt{1-\cot^2(x)}\sec(2x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

```
[Out] (-(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Cos[x]*Sqrt[-Cos[2*x]]) + Cos[2*x])*Sqrt[
1 - Cot[x]^2]*Sec[2*x]*Tan[x]
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.58, size = 220, normalized size = 11.58

method	result
default	$-\frac{(\cos(x)-1)\left(4i\cos(x)\ln\left(-\frac{4(\cos(x)-1)\left(\cos(x)\sqrt{-\frac{2(\cos^2(x))-1}{(1+\cos(x))^2}}-2i\cos(x)+\sqrt{-\frac{2(\cos^2(x))-1}{(1+\cos(x))^2}}-i\right)}{\sin(x)^2}\right)+2\cos(x)\sqrt{-\frac{2(\cos^2(x)}{(1+\cos(}}}{2\cos(x)\sin(x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(cos(x)-1)*(4*I*cos(x)*ln(-4*(cos(x)-1)*(cos(x)*(-(2*cos(x)^2-1)/(1+co
s(x))^2)^(1/2)-2*I*cos(x)+(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-I)/sin(x)^2)
+2*cos(x)*(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2)-3*cos(x)*arcsin(1/2*(1+2*cos
(x))/(1+cos(x))*2^(1/2))-cos(x)*arctan((2*cos(x)^2-3*cos(x)+1)/(-(2*cos(x)^
2-1)/(1+cos(x))^2)^(1/2)/sin(x)^2)+2*(-(2*cos(x)^2-1)/(1+cos(x))^2)^(1/2))*
((2*cos(x)^2-1)/(cos(x)^2-1))^(1/2)/cos(x)/sin(x)/(-(2*cos(x)^2-1)/(1+cos(x
))^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 30, normalized size = 1.58

$$\sqrt{-\frac{1}{\tan(x)^2}+1}\,\tan(x)-\arctan\left(\sqrt{-\frac{1}{\tan(x)^2}+1}\,\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(-1/tan(x)^2 + 1)*tan(x) - arctan(sqrt(-1/tan(x)^2 + 1)*tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.
time = 2.47, size = 78, normalized size = 4.11

$$-\frac{\arctan\left(\frac{\left(3\cos(x)^2-1\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\left(2\cos(x)^3-\cos(x)\right)}\right)\cos(x)-2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*(arctan(1/2*(3*cos(x)^2 - 1)*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))*sin
(x)/(2*cos(x)^3 - cos(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))
*sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(\cot(x)-1\right)\left(\cot(x)+1\right)}\,\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-cot(x)**2)**(1/2),x)
[Out] Integral(sqrt(-(cot(x) - 1)*(cot(x) + 1))*sec(x)**2, x)
```

Giac [C] Result contains complex when optimal does not.
time = 0.44, size = 142, normalized size = 7.47

$$-\frac{1}{2}\left(\pi+2\arctan(-i)+2i\right)\mathrm{sgn}(\sin(x))+\frac{1}{4}\left(2\pi\mathrm{sgn}(\cos(x))+\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}}{\cos(x)}-\frac{4\cos(x)}{\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}}\right)+4\arctan\left(-\frac{\sqrt{2}\left(\frac{\left(\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}\right)^2}{\cos(x)^2}-4\right)\cos(x)}{4\left(\sqrt{2}\sqrt{-2\cos(x)^2+1}-\sqrt{2}\right)}\right)\right)\mathrm{sgn}(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(pi + 2*arctan(-I) + 2*I)*sgn(sin(x)) + 1/4*(2*pi*sgn(cos(x)) + sqrt(2
)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*sqr
t(-2*cos(x)^2 + 1) - sqrt(2))) + 4*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2*co
s(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2 + 1
) - sqrt(2))))*sgn(sin(x))
```

Mupad [B]
time = 3.06, size = 19, normalized size = 1.00

$$\operatorname{asin}(\cot(x)) + \frac{\sqrt{1-\cot(x)^2}}{\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - cot(x)^2)^(1/2)/cos(x)^2,x)
```

```
[Out] asin(cot(x)) + (1 - cot(x)^2)^(1/2)/cot(x)
```

3.713 $\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}\text{ArcSin}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}$$

```
[Out] 1/2*arcsin(tan(x))+1/2*(1-tan(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3756, 201, 222}

$$\frac{1}{2}\text{ArcSin}(\tan(x))+\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\sin^{-1}(\tan(x)) + \frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.
time = 0.08, size = 63, normalized size = 2.42

$$\frac{\cos(2x)\tan(x) + \text{ArcSin}\left(\frac{\sin(x)}{\sqrt{\cos^2(x)}}\right)\cos(x)\sqrt{\cos^2(x)}\sqrt{1-\tan^2(x)}}{2\sqrt{\cos^2(x)}\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

```
[Out] (Cos[2*x]*Tan[x] + ArcSin[Sin[x]/Sqrt[Cos[x]^2]]*Cos[x]*Sqrt[Cos[x]^2]*Sqrt
[1 - Tan[x]^2])/(2*Sqrt[Cos[x]^2]*Sqrt[Cos[2*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 492, normalized size = 18.92

method	result
default	$\sin(x)\left(-(\cos^2(x))\sin(x)\sqrt{2}\sqrt{\frac{\cos(x)\sqrt{2}-\sqrt{2}+2\cos(x)-1}{1+\cos(x)}}\sqrt{-\frac{2\left(\cos(x)\sqrt{2}-\sqrt{2}-2\cos(x)+1\right)}{1+\cos(x)}}\ \text{EllipticF}\left(\frac{\left(1+\sqrt{2}\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x)*(-cos(x)^2*sin(x)*2^(1/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(
1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*
EllipticF((1+2^(1/2))*(cos(x)-1)/sin(x),3-2*2^(1/2))+2*cos(x)^2*sin(x)*2^(1
/2)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1
/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticPi((3+2*2^(1/2))^(1/2)*(c
os(x)-1)/sin(x),1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-2*
cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*
```

```
(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+cos(x)))^(1/2)*EllipticF((1+2^(1/2))
*(cos(x)-1)/sin(x),3-2*2^(1/2))+4*cos(x)^2*sin(x)*((cos(x)*2^(1/2)-2^(1/2)+
2*cos(x)-1)/(1+cos(x)))^(1/2)*(-2*(cos(x)*2^(1/2)-2^(1/2)-2*cos(x)+1)/(1+co
s(x)))^(1/2)*EllipticPi((3+2*2^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(3+2*2^(1/2
)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))+4*cos(x)^3*2^(1/2)+6*cos(x)^3-4
*cos(x)^2*2^(1/2)-6*cos(x)^2-2*cos(x)*2^(1/2)-3*cos(x)+2*2^(1/2)+3)*((2*cos
(x)^2-1)/cos(x)^2)^(1/2)/(cos(x)-1)/(2*cos(x)^2-1)/cos(x)/(3+2*2^(1/2))^(1/
2)/(1+2^(1/2))
```

Maxima [A]
time = 0.50, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan\left(x\right)^{2}+1}\,\tan\left(x\right)+\frac{1}{2}\arcsin\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(20) = 40.
time = 2.49, size = 72, normalized size = 2.77

$$-\frac{\arctan\left(\frac{\left(3\cos\left(x\right)^{3}-2\cos\left(x\right)\right)\sqrt{\frac{2\cos\left(x\right)^{2}-1}{\cos\left(x\right)^{2}}}}{2\left(2\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}\right)\cos\left(x\right)-2\sqrt{\frac{2\cos\left(x\right)^{2}-1}{\cos\left(x\right)^{2}}}\,\sin\left(x\right)}{4\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(arctan(1/2*(3*cos(x)^3 - 2*cos(x))*sqrt((2*cos(x)^2 - 1)/cos(x)^2)/((
2*cos(x)^2 - 1)*sin(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/cos(x)^2)*sin(x))
/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(\tan\left(x\right)-1\right)\left(\tan\left(x\right)+1\right)}\,\sec^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1-tan(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(tan(x) - 1)*(tan(x) + 1))*sec(x)**2, x)
```

Giac [A]
time = 0.40, size = 20, normalized size = 0.77

$$\frac{1}{2}\sqrt{-\tan\left(x\right)^{2}+1}\,\tan\left(x\right)+\frac{1}{2}\,\arcsin\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-\tan\left(x\right)^{2}}}{\cos\left(x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - tan(x)^2)^(1/2)/cos(x)^2,x)
```

```
[Out] int((1 - tan(x)^2)^(1/2)/cos(x)^2, x)
```

3.714 $\int e^{\tan(x)} \sec^2(x) \, dx$

Optimal. Leaf size=4

$$e^{\tan(x)}$$

[Out] exp(tan(x))

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4427, 2225}

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Tan[x]*Sec[x]^2,x]

[Out] E^Tan[x]

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4427

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rubi steps

$$\begin{aligned} \int e^{\tan(x)} \sec^2(x) \, dx &= \text{Subst}\left(\int e^x \, dx, x, \tan(x)\right) \\ &= e^{\tan(x)} \end{aligned}$$

Mathematica [A]
time = 0.05, size = 4, normalized size = 1.00

$$e^{\tan(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Tan[x]*Sec[x]^2,x]
```

```
[Out] E^Tan[x]
```

Maple [A]
time = 0.04, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$e^{\tan(x)}$	4
default	$e^{\tan(x)}$	4
risch	$e^{\frac{\sin(x)}{\cos(x)}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(tan(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(tan(x))
```

Maxima [A]
time = 0.29, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] e^tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.
time = 2.76, size = 8, normalized size = 2.00

$$e^{\frac{\sin(x)}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] e^(sin(x)/cos(x))
```

Sympy [A]
time = 0.55, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)**2,x)
```

```
[Out] exp(tan(x))
```

Giac [A]
time = 0.43, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(tan(x))*sec(x)^2,x, algorithm="giac")
[Out] e^tan(x)
```

Mupad [B]
time = 3.10, size = 3, normalized size = 0.75

$$e^{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(tan(x))/cos(x)^2,x)
[Out] exp(tan(x))
```

3.715 $\int \sec^4(x) \left(-1+\sec^2(x)\right)^2 \tan(x) \, dx$

Optimal. Leaf size=17

$$\frac{\tan^6(x)}{6}+\frac{\tan^8(x)}{8}$$

```
[Out] 1/6*tan(x)^6+1/8*tan(x)^8
```

Rubi [A]
time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 2687, 14}

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Tan[x]^6/6 + Tan[x]^8/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx = \int \sec^4(x)\tan^5(x)\,dx$$
$$= \text{Subst}\left(\int x^5\left(1+x^2\right)\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, \tan(x)\right)$$
$$= \frac{\tan^6(x)}{6}+\frac{\tan^8(x)}{8}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.47

$$\frac{\sec^4(x)}{4}-\frac{\sec^6(x)}{3}+\frac{\sec^8(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

```
[Out] Sec[x]^4/4 - Sec[x]^6/3 + Sec[x]^8/8
```

Maple [A]
time = 0.06, size = 20, normalized size = 1.18

method	result	size
derivativedivides	$\frac{\left(\sec^8(x)\right)}{8}-\frac{\left(\sec^6(x)\right)}{3}+\frac{\left(\sec^4(x)\right)}{4}$	20
default	$\frac{\left(\sec^8(x)\right)}{8}-\frac{\left(\sec^6(x)\right)}{3}+\frac{\left(\sec^4(x)\right)}{4}$	20
risch	$\frac{4\,e^{12ix}-\frac{16\,e^{10ix}}{3}+\frac{40\,e^{8ix}}{3}-\frac{16\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}+1\right)^8}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sec(x)^8-1/3*sec(x)^6+1/4*sec(x)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.
time = 0.29, size = 42, normalized size = 2.47

$$\frac{6\,\sin(x)^4-4\,\sin(x)^2+1}{24\,\left(\sin(x)^8-4\,\sin(x)^6+6\,\sin(x)^4-4\,\sin(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="maxima")
[Out] 1/24*(6*sin(x)^4 - 4*sin(x)^2 + 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*
sin(x)^2 + 1)
```

Fricas [A]
time = 2.62, size = 20, normalized size = 1.18

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="fricas")
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

Sympy [A]
time = 1.08, size = 19, normalized size = 1.12

$$\frac{\sec^8(x)}{8} - \frac{\sec^6(x)}{3} + \frac{\sec^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*(-1+sec(x)**2)**2*tan(x),x)
[Out] sec(x)**8/8 - sec(x)**6/3 + sec(x)**4/4
```

Giac [A]
time = 0.43, size = 20, normalized size = 1.18

$$\frac{6 \cos(x)^4 - 8 \cos(x)^2 + 3}{24 \cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="giac")
[Out] 1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

Mupad [B]
time = 2.92, size = 14, normalized size = 0.82

$$\frac{\tan(x)^6 \left(3 \tan(x)^2 + 4\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(1/cos(x)^2 - 1)^2)/cos(x)^4,x)
[Out] (tan(x)^6*(3*tan(x)^2 + 4))/24
```

3.716 $\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\cot(x))}{b}$$

```
[Out] -ln(a+b*cot(x))/b
```

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3587, 31}

$$-\frac{\log(a+b\cot(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a + b*Cot[x]),x]
```

```
[Out] -(Log[a + b*Cot[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\cot(x)\right)}{b}\\ &= -\frac{\log(a+b\cot(x))}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))-\log(b\cos(x)+a\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a + b*Cot[x]),x]
[Out] (Log[Sin[x]] - Log[b*Cos[x] + a*Sin[x]])/b
```

Maple [A]
time = 0.09, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\cot(x))}{b}$	13
default	$-\frac{\ln(a+b\cot(x))}{b}$	13
risch	$\frac{\ln(\mathrm{e}^{2ix}-1)}{b}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)}{b}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a+b*cot(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*cot(x))/b
```

Maxima [A]
time = 0.30, size = 12, normalized size = 1.00

$$-\frac{\log\left(b\cot\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="maxima")
[Out] -log(b*cot(x) + a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.
time = 2.50, size = 45, normalized size = 3.75

$$-\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^2-b^2\right)\cos\left(x\right)^2+a^2\right)-\log\left(-\frac{1}{4}\cos\left(x\right)^2+\frac{1}{4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="fricas")
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2) - log(-1/4*cos(
x)^2 + 1/4))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(x\right)}{a+b\cot\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a+b*cot(x)),x)
```

```
[Out] Integral(csc(x)**2/(a + b*cot(x)), x)
```

Giac [A]
time = 0.43, size = 22, normalized size = 1.83

$$-\frac{\log\left(|a\tan\left(x\right)+b|\right)}{b}+\frac{\log\left(|\tan\left(x\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="giac")
```

```
[Out] -log(abs(a*tan(x) + b))/b + log(abs(tan(x)))/b
```

Mupad [B]
time = 3.01, size = 16, normalized size = 1.33

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,a\,\tan(x)}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a + b*cot(x))),x)
```

```
[Out] -(2*atanh((2*a*tan(x))/b + 1))/b
```

3.717 $\int (a+b\cot(x))^n \csc^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$$

```
[Out] -(a+b*cot(x))^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3587, 32}

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cot[x])^n*Csc[x]^2,x]
[Out] -((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int (a+b\cot(x))^n \csc^2(x)\,dx &= -\frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\cot(x)\right)}{b}\\ &= -\frac{(a+b\cot(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 19, normalized size = 0.95

$$-\frac{(a+b\cot(x))^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cot[x])^n*Csc[x]^2,x]
[Out] -((a + b*Cot[x])^(1 + n)/(b + b*n))
```

Maple [A]
time = 0.12, size = 21, normalized size = 1.05

method	result	size
derivativedivides	$-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$	21
default	$-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^n*csc(x)^2,x,method=_RETURNVERBOSE)
[Out] -(a+b*cot(x))^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.00

$$-\frac{(b\cot(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="maxima")
[Out] -(b*cot(x) + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 2.12, size = 38, normalized size = 1.90

$$-\frac{(b\cos(x)+a\sin(x))\left(\frac{b\cos(x)+a\sin(x)}{\sin(x)}\right)^n}{(bn+b)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="fricas")
[Out] -(b*cos(x) + a*sin(x))*((b*cos(x) + a*sin(x))/sin(x))^n/((b*n + b)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\cot(x))^n\csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**n*csc(x)**2,x)
```

```
[Out] Integral((a + b*cot(x))**n*csc(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.41, size = 41, normalized size = 2.05

$$-\frac{\left(-\frac{b\tan\left(\frac{1}{2}x\right)^2-2a\tan\left(\frac{1}{2}x\right)-b}{2\tan\left(\frac{1}{2}x\right)}\right)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")
```

```
[Out] -(-1/2*(b*tan(1/2*x)^2 - 2*a*tan(1/2*x) - b)/tan(1/2*x))^(n + 1)/(b*(n + 1)
)
```

Mupad [B]
time = 3.19, size = 43, normalized size = 2.15

$$\begin{cases} -\frac{\ln\left(a+\frac{b}{\tan(x)}\right)}{b} & \text{if } n=-1 \\ -\frac{\left(a+\frac{b}{\tan(x)}\right)^{n+1}}{b\,(n+1)} & \text{if } n\neq-1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))^n/sin(x)^2,x)
```

```
[Out] piecewise(n == -1, -log(a + b/tan(x))/b, n ~= -1, -(a + b/tan(x))^(n + 1)/(
b*(n + 1)))
```

3.718 $\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx$

Optimal. Leaf size=6

$$x - \cot(x)$$

```
[Out] x-cot(x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{3091, 8\}$

$$x - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*(1 + Sin[x]^2),x]
```

```
[Out] x - Cot[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \csc^2(x) \left(1+\sin^2(x)\right) \, dx &= -\cot(x)+\int 1 \, dx \\ &= x-\cot(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*(1 + Sin[x]^2),x]
```

```
[Out] x - Cot[x]
```

Maple [A]
time = 0.05, size = 7, normalized size = 1.17

method	result	size
default	$x-\cot\left(x\right)$	7
risch	$x-\frac{2i}{{\mathrm e}^{2ix}-1}$	15
norman	$\frac{-\frac{1}{2}+x\tan\left(\frac{x}{2}\right)+x\left(\tan^{5}\left(\frac{x}{2}\right)\right)-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}+\frac{\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2}+\frac{\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{2}+2x\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{\tan\left(\frac{x}{2}\right)\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{2}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(1+sin(x)^2),x,method=_RETURNVERBOSE)
[Out] x-cot(x)
```

Maxima [A]
time = 0.51, size = 8, normalized size = 1.33

$$x-\frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="maxima")
[Out] x - 1/tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 2.82, size = 14, normalized size = 2.33

$$\frac{x\sin\left(x\right)-\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="fricas")
[Out] (x*sin(x) - cos(x))/sin(x)
```

Sympy [A]
time = 2.56, size = 3, normalized size = 0.50

$$x-\cot\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(1+sin(x)**2),x)
```

```
[Out] x - cot(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.42, size = 16, normalized size = 2.67

$$x - \frac{1}{2\,\tan\left(\frac{1}{2}\,x\right)} + \frac{1}{2}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="giac")
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

Mupad [B]
time = 2.93, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2 + 1)/sin(x)^2,x)
[Out] x - cot(x)
```

3.719 $\int \left(1+\frac{1}{1+\cot^2(x)}\right) \csc^2(x)\, dx$

Optimal. Leaf size=6

$$x-\cot(x)$$

[Out] x-cot(x)

Rubi [A]
time = 0.03, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {14, 209}

$$x-\cot(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]

[Out] x - Cot[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(1+\frac{1}{1+\cot^2(x)}\right) \csc^2(x)\, dx &= \text{Subst}\left(\int \frac{1+\frac{1}{1+\frac{1}{x^2}}}{x^2}\, dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{1+x^2}\right) dx, x, \tan(x)\right)\\
&= -\cot(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \tan(x)\right)\\
&= x-\cot(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
[Out] x - Cot[x]
```

Maple [A]
time = 0.07, size = 7, normalized size = 1.17

method	result	size
default	$x - \cot(x)$	7
risch	$x - \frac{2i}{e^{2ix}-1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/(1+cot(x)^2))*csc(x)^2,x,method=_RETURNVERBOSE)
[Out] x-cot(x)
```

Maxima [A]
time = 0.51, size = 8, normalized size = 1.33

$$x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="maxima")
[Out] x - 1/tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 3.44, size = 14, normalized size = 2.33

$$\frac{x\sin(x) - \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="fricas")
[Out] (x*sin(x) - cos(x))/sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(3) = 6$.
time = 0.25, size = 27, normalized size = 4.50

$$\frac{x\csc^2(x)}{\cot^2(x)+1} - \frac{\cot(x)\csc^2(x)}{\cot^2(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)**2))*csc(x)**2,x)
```

```
[Out] x*csc(x)**2/(cot(x)**2 + 1) - cot(x)*csc(x)**2/(cot(x)**2 + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.43, size = 16, normalized size = 2.67

$$x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="giac")
```

```
[Out] x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

Mupad [B]
time = 2.94, size = 6, normalized size = 1.00

$$x - \cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(cot(x)^2 + 1) + 1)/sin(x)^2,x)
```

```
[Out] x - cot(x)
```

3.720 $\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\cot(x)}{d}+\frac{(bc-ad)\log(c+d\cot(x))}{d^2}$$

```
[Out] -b*cot(x)/d+(-a*d+b*c)*ln(c+d*cot(x))/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4429, 45}

$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*Cot[x])/d) + ((b*c - a*d)*Log[c + d*Cot[x]])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx &= -\text{Subst}\left(\int \frac{a+bx}{c+dx}\,dx,x,\cot(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)\,dx,x,\cot(x)\right)\\
&= -\frac{b\cot(x)}{d}+\frac{(bc-ad)\log(c+d\cot(x))}{d^2}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 56, normalized size = 2.00

$$\frac{(a+b\cot(x))(-bd\cot(x)-(bc-ad)(\log(\sin(x))-\log(d\cos(x)+c\sin(x))))\sin(x)}{d^2(b\cos(x)+a\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])*(-(b*d*Cot[x]) - (b*c - a*d)*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]]))*Sin[x])/(d^2*(b*Cos[x] + a*Sin[x]))
```

Maple [A]
time = 0.13, size = 47, normalized size = 1.68

method	result	size
default	$-\frac{b}{d\tan(x)}+\frac{(ad-cb)\ln(\tan(x))}{d^2}-\frac{(ad-cb)\ln(c\tan(x)+d)}{d^2}$	47
risch	$-\frac{2ib}{d(e^{2ix}-1)}+\frac{\ln(e^{2ix}-1)a}{d}-\frac{\ln(e^{2ix}-1)cb}{d^2}-\frac{\ln\left(e^{2ix}+\frac{id-c}{id+c}\right)a}{d}+\frac{\ln\left(e^{2ix}+\frac{id-c}{id+c}\right)cb}{d^2}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/d/tan(x)+(a*d-b*c)/d^2*ln(tan(x))-(a*d-b*c)/d^2*ln(c*tan(x)+d)
```

Maxima [A]
time = 0.30, size = 46, normalized size = 1.64

$$\frac{(bc-ad)\log(c\tan(x)+d)}{d^2}-\frac{(bc-ad)\log(\tan(x))}{d^2}-\frac{b}{d\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*log(c*tan(x) + d)/d^2 - (b*c - a*d)*log(tan(x))/d^2 - b/(d*tan(
x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.
time = 2.51, size = 76, normalized size = 2.71

$$-\frac{2\,bd\cos(x)-(bc-ad)\log\left(2\,cd\cos(x)\sin(x)-(c^2-d^2)\cos(x)^2+c^2\right)\sin(x)+(bc-ad)\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\sin(x)}{2\,d^2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*cos(x) - (b*c - a*d)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos(
x)^2 + c^2)*sin(x) + (b*c - a*d)*log(-1/4*cos(x)^2 + 1/4)*sin(x))/(d^2*sin(
x))
```

Sympy [A]
time = 3.55, size = 31, normalized size = 1.11

$$-\frac{b\cot(x)}{d}-\frac{(ad-bc)\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b*cot(x)/d - (a*d - b*c)*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x)
)/d, True))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.
time = 0.42, size = 68, normalized size = 2.43

$$-\frac{(bc-ad)\log(|\tan(x)|)}{d^2}+\frac{(bc^2-acd)\log(|c\tan(x)+d|)}{cd^2}+\frac{bc\tan(x)-ad\tan(x)-bd}{d^2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b*c - a*d)*log(abs(tan(x)))/d^2 + (b*c^2 - a*c*d)*log(abs(c*tan(x) + d))/
(c*d^2) + (b*c*tan(x) - a*d*tan(x) - b*d)/(d^2*tan(x))
```

Mupad [B]
time = 3.07, size = 35, normalized size = 1.25

$$-\frac{b}{d\tan(x)}-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\tan(x)}{d}+1\right)\,(a\,d-b\,c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))/(sin(x)^2*(c + d*cot(x))),x)
```

```
[Out] - b/(d*tan(x)) - (2*atanh((2*c*tan(x))/d + 1)*(a*d - b*c))/d^2
```

3.721 $\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=53

$$\frac{b(bc-ad)\cot(x)}{d^2}-\frac{(a+b\cot(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}$$

```
[Out] b*(-a*d+b*c)*cot(x)/d^2-1/2*(a+b*cot(x))^2/d-(-a*d+b*c)^2*ln(c+d*cot(x))/d^
3
```

Rubi [A]
time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {4429, 45}

$$-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}+\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(a+b\cot(x))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (b*(b*c - a*d)*Cot[x])/d^2 - (a + b*Cot[x])^2/(2*d) - ((b*c - a*d)^2*Log[c
+ d*Cot[x]])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^2}{c+dx}\,dx,x,\cot(x)\right)\\ &= -\text{Subst}\left(\int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx,x,\cot(x)\right)\\ &= \frac{b(bc-ad)\cot(x)}{d^2}-\frac{(a+b\cot(x))^2}{2d}-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 62, normalized size = 1.17

$$\frac{2bd(bc-2ad)\cot(x)-b^2d^2\csc^2(x)+2(bc-ad)^2(\log(\sin(x))-\log(d\cos(x)+c\sin(x)))}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] (2*b*d*(b*c - 2*a*d)*Cot[x] - b^2*d^2*Csc[x]^2 + 2*(b*c - a*d)^2*(Log[Sin[x
]] - Log[d*Cos[x] + c*Sin[x]]))/(2*d^3)
```

Maple [A]
time = 0.13, size = 94, normalized size = 1.77

method	result
default	$-\frac{b^2}{2d\tan(x)^2}+\frac{(d^2a^2-2cdab+b^2c^2)\ln(\tan(x))}{d^3}-\frac{b(2ad-cb)}{d^2\tan(x)}-\frac{(d^2a^2-2cdab+b^2c^2)\ln(c\tan(x)+d)}{d^3}$
risch	$\frac{2ib(-2ad\,e^{2ix}+bc\,e^{2ix}-ibd\,e^{2ix}+2ad-cb)}{(e^{2ix}-1)^2d^2}+\frac{\ln(e^{2ix}-1)a^2}{d}-\frac{2\ln(e^{2ix}-1)cab}{d^2}+\frac{\ln(e^{2ix}-1)b^2c^2}{d^3}-\frac{\ln\left(e^{2ix}+\frac{id-c}{id+c}\right)a^2}{d}+$ 2 ln

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^2/d/tan(x)^2+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(tan(x))-b*(2*a*d-b*c
)/d^2/tan(x)-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(c*tan(x)+d)
```

Maxima [A]
time = 0.31, size = 92, normalized size = 1.74

$$-\frac{(b^2c^2-2\,abcd+a^2d^2)\log(c\tan(x)+d)}{d^3}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(\tan(x))}{d^3}-\frac{b^2d-2\,(b^2c-2\,abd)\tan(x)}{2\,d^2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] -(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(c*tan(x) + d)/d^3 + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(tan(x))/d^3 - 1/2*(b^2*d - 2*(b^2*c - 2*a*b*d)*tan(x))/(d
^2*tan(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(51) = 102$.
time = 3.43, size = 182, normalized size = 3.43

$$\frac{b^2d^2-2\,(b^2cd-2\,abd^2)\cos(x)\sin(x)+(b^2c^2-2\,abcd+a^2d^2-(b^2c^2-2\,abcd+a^2d^2)\cos(x)^2)\log(2\,cd\cos(x)\sin(x)-(c^2-d^2)\cos(x)^2+c^2)-(b^2c^2-2\,abcd+a^2d^2-(b^2c^2-2\,abcd+a^2d^2)\cos(x)^2)\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)}{2\,(d^3\cos(x)^2-d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2 - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(2*c*d*cos(x)*sin
(x) - (c^2 - d^2)*cos(x)^2 + c^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(-1/4*cos(x)^2 + 1/4))/(d^3*cos(x)^2
 - d^3)
```

Sympy [A]
time = 7.72, size = 58, normalized size = 1.09

$$-\frac{b^2\cot^2(x)}{2d}-\frac{(ad-bc)^2\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{(2abd-b^2c)\cot(x)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**2*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**2*cot(x)**2/(2*d) - (a*d - b*c)**2*Piecewise((cot(x)/c, Eq(d, 0)), (log
(c + d*cot(x))/d, True))/d**2 - (2*a*b*d - b**2*c)*cot(x)/d**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(51) = 102.
time = 0.44, size = 139, normalized size = 2.62

$$\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|\tan(x)|)}{d^3}-\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\log(|c\tan(x)+d|)}{cd^3}-\frac{3\,b^2c^2\tan(x)^2-6\,abcd\tan(x)^2+3\,a^2d^2\tan(x)^2-2\,b^2cd\tan(x)+4\,abd^2\tan(x)+b^2d^2}{2\,d^3\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(tan(x)))/d^3 - (b^2*c^3 - 2*a*b*c^2
*d + a^2*c*d^2)*log(abs(c*tan(x) + d))/(c*d^3) - 1/2*(3*b^2*c^2*tan(x)^2 -
6*a*b*c*d*tan(x)^2 + 3*a^2*d^2*tan(x)^2 - 2*b^2*c*d*tan(x) + 4*a*b*d^2*tan(
x) + b^2*d^2)/(d^3*tan(x)^2)
```

Mupad [B]
time = 3.07, size = 92, normalized size = 1.74

$$-\frac{\frac{b^2}{2\,d}+\frac{b\tan(x)\,(2\,a\,d-b\,c)}{d^2}}{\tan(x)^2}-\frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^2}{d\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))^2/(sin(x)^2*(c + d*cot(x))),x)
```

```
[Out] - (b^2/(2*d) + (b*tan(x)*(2*a*d - b*c))/d^2)/tan(x)^2 - (2*atanh(((d + 2*c*
tan(x))*(a*d - b*c)^2)/(d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/
d^3
```

3.722 $\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{b(bc-ad)^2\cot(x)}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}-\frac{(a+b\cot(x))^3}{3d}+\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}$$

```
[Out] -b*(-a*d+b*c)^2*cot(x)/d^3+1/2*(-a*d+b*c)*(a+b*cot(x))^2/d^2-1/3*(a+b*cot(x
))^3/d+(-a*d+b*c)^3*ln(c+d*cot(x))/d^4
```

Rubi [A]

time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4429, 45}

$$\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}-\frac{(a+b\cot(x))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] -((b*(b*c - a*d)^2*Cot[x])/d^3) + ((b*c - a*d)*(a + b*Cot[x])^2)/(2*d^2) -
(a + b*Cot[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Cot[x]])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^3}{c+dx}\,dx, x, \cot(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)}{d^3(c+dx)}\right.\right.\\ &= -\frac{b(bc-ad)^2\cot(x)}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}-\frac{(a+b\cot(x))^3}{3d}+\frac{(bc-a}{}\end{aligned}$$

Mathematica [A]
time = 0.91, size = 135, normalized size = 1.73

$$\frac{(a+b\cot(x))^3(d\cos(x)+c\sin(x))\left(-2b^3d^3\cot(x)-6(bc-ad)^3(\log(\sin(x))-\log(d\cos(x)+c\sin(x)))\sin^2(x)+bd(3bd(bc-3ad)+(9abcd-9a^2d^2+b^2(-3c^2+d^2))\sin(2x))\right)}{6d^4(c+d\cot(x))(b\cos(x)+a\sin(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

```
[Out] ((a + b*Cot[x])^3*(d*Cos[x] + c*Sin[x])*(-2*b^3*d^3*Cot[x] - 6*(b*c - a*d)^
3*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]])*Sin[x]^2 + b*d*(3*b*d*(b*c - 3*a
*d) + (9*a*b*c*d - 9*a^2*d^2 + b^2*(-3*c^2 + d^2))*Sin[2*x])))/(6*d^4*(c +
d*Cot[x])*(b*Cos[x] + a*Sin[x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(74) = 148.
time = 0.16, size = 158, normalized size = 2.03

method	result
default	$-\frac{b^3}{3d\tan(x)^3}+\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2-b^3c^3)\ln(\tan(x))}{d^4}-\frac{b(3d^2a^2-3cdab+b^2c^2)}{d^3\tan(x)}-\frac{b^2(3ad-cb)}{2d^2\tan(x)^2}-\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\dots}{}$
risch	$-\frac{2ib(9a^2d^2e^{4ix}-9abcd\,e^{4ix}+3b^2c^2e^{4ix}-3b^2d^2e^{4ix}+9iab\,d^2e^{4ix}-3ib^2cd\,e^{4ix}-18a^2d^2e^{2ix}+18abcd\,e^{2ix}-6b^2c^2e^{2ix}-9iab\,d^2e^{2ix}+3ib^2\dots}{3d^3(e^{2ix}-1)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^3/d/tan(x)^3+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(ta
n(x))-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3/tan(x)-1/2*b^2*(3*a*d-b*c)/d^2/ta
n(x)^2-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(c*tan(x)+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(74) = 148.
time = 0.30, size = 161, normalized size = 2.06

$$\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(c\tan(x)+d)}{d^4}-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(\tan(x))}{d^4}-\frac{2\,b^3d^2+6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)\tan(x)^2-3\,(b^3cd-3\,ab^2d^2)\tan(x)}{6\,d^3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(c*tan(x) + d)/d^4 -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(tan(x))/d^4 - 1/6*
(2*b^3*d^2 + 6*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x)^2 - 3*(b^3*c*d
- 3*a*b^2*d^2)*tan(x))/(d^3*tan(x)^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(74) = 148$.
time = 3.15, size = 320, normalized size = 4.10

$$-\frac{2\,(3\,b^3c^2d - 9\,ab^2cd^2 + (9\,a^2b - b^3)d^3)\cos(x)^3 + 3\,\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3 - (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\cos(x)^2\right)\log\left(2\,cd\cos(x)\sin(x) - (c^2 - d^2)\cos(x)^2 + c^2\right)\sin(x) - 3\,\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3 - (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\cos(x)^2\right)\log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right)\sin(x) - 6\,(b^3c^2d - 3\,ab^2cd^2 + 3\,a^2bd^3)\cos(x) + 3\,(b^3cd^2 - 3\,ab^2d^3)\sin(x)}{6\,\left(d^4\cos(x)^2 - d^4\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*cos(x)^3 + 3*(b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)
*cos(x)^2 + c^2)*sin(x) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(-1/
4*cos(x)^2 + 1/4)*sin(x) - 6*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*cos(
x) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*sin(x))/((d^4*cos(x)^2 - d^4)*sin(x))
```

Sympy [A]
time = 13.60, size = 97, normalized size = 1.24

$$-\frac{b^3\cot^3(x)}{3d} - \frac{(3ab^2d - b^3c)\cot^2(x)}{2d^2} - \frac{(ad-bc)^3\left(\begin{cases}\frac{\cot(x)}{c} & \text{for}\ d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{(3a^2bd^2 - 3ab^2cd + b^3c^2)\cot(x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))**3*csc(x)**2/(c+d*cot(x)),x)
```

```
[Out] -b**3*cot(x)**3/(3*d) - (3*a*b**2*d - b**3*c)*cot(x)**2/(2*d**2) - (a*d - b
*c)**3*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d**3 -
(3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*cot(x)/d**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(74) = 148$.
time = 0.45, size = 232, normalized size = 2.97

$$-\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log(|\tan(x)|)}{d^4} + \frac{(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3)\log(|c\tan(x) + d|)}{cd^4} + \frac{11\,b^3c^3\tan(x)^3 - 33\,ab^2c^2d\tan(x)^3 + 33\,a^2bcd^2\tan(x)^3 - 11\,a^3d^3\tan(x)^3 - 6\,b^3c^2d\tan(x)^2 + 18\,ab^2cd^2\tan(x)^2 - 18\,a^2bd^3\tan(x)^2 + 3\,b^3cd^2\tan(x) - 9\,ab^2d^3\tan(x) - 2\,b^3d^3}{6\,d^4\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(tan(x)))/d^4 +
 (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*log(abs(c*tan(x) +
 d))/(c*d^4) + 1/6*(11*b^3*c^3*tan(x)^3 - 33*a*b^2*c^2*d*tan(x)^3 + 33*a^2*
b*c*d^2*tan(x)^3 - 11*a^3*d^3*tan(x)^3 - 6*b^3*c^2*d*tan(x)^2 + 18*a*b^2*c*
d^2*tan(x)^2 - 18*a^2*b*d^3*tan(x)^2 + 3*b^3*c*d^2*tan(x) - 9*a*b^2*d^3*tan
(x) - 2*b^3*d^3)/(d^4*tan(x)^3)
```

Mupad [B]
time = 3.08, size = 141, normalized size = 1.81

$$-\frac{\frac{b^3}{3\,d}+\frac{b^2\,\tan(x)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{b\,\tan(x)^2\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{d^3}}{\tan\left(x\right)^3}-\frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^3}{d\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,\left(a\,d-b\,c\right)^3}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cot(x))^3/(sin(x)^2*(c + d*cot(x))),x)
```

```
[Out] - (b^3/(3*d) + (b^2*tan(x)*(3*a*d - b*c))/(2*d^2) + (b*tan(x)^2*(3*a^2*d^2
+ b^2*c^2 - 3*a*b*c*d))/d^3)/tan(x)^3 - (2*atanh(((d + 2*c*tan(x))*(a*d - b
*c)^3)/(d*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)
^3)/d^4
```

3.723 $\int e^{-\cot(x)} \csc^2(x) \, dx$

Optimal. Leaf size=6

$$e^{-\cot(x)}$$

[Out] exp(-cot(x))

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4429, 2225}

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^2/E^Cot[x],x]

[Out] E^(-Cot[x])

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4429

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cot[c*(a +
 b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc] |
| EqQ[F, csc])
```

Rubi steps

$$\begin{aligned} \int e^{-\cot(x)} \csc^2(x) \, dx &= -\text{Subst}\left(\int e^{-x} \, dx, x, \cot(x)\right) \\ &= e^{-\cot(x)} \end{aligned}$$

Mathematica [A]
time = 0.06, size = 6, normalized size = 1.00

$$e^{-\cot(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/E^Cot[x],x]
```

```
[Out] E^(-Cot[x])
```

Maple [A]
time = 0.05, size = 6, normalized size = 1.00

method	result	size
derivativedivides	$\mathrm{e}^{-\cot(x)}$	6
default	$\mathrm{e}^{-\cot(x)}$	6
risch	$\mathrm{e}^{-\frac{\cos(x)}{\sin(x)}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/exp(cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/exp(cot(x))
```

Maxima [A]
time = 0.30, size = 5, normalized size = 0.83

$$e^{(-\cot(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="maxima")
```

```
[Out] e^(-cot(x))
```

Fricas [A]
time = 3.50, size = 9, normalized size = 1.50

$$e^{\left(-\frac{\cos(x)}{\sin(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="fricas")
```

```
[Out] e^(-cos(x)/sin(x))
```

Sympy [A]
time = 14.04, size = 5, normalized size = 0.83

$$e^{-\cot(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/exp(cot(x)),x)
```

```
[Out] exp(-cot(x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/exp(cot(x)),x, algorithm="giac")
[Out] integrate(csc(x)^2*e^(-cot(x)), x)
```

Mupad [B]
time = 2.94, size = 7, normalized size = 1.17

$$e^{-\frac{1}{\tan(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-cot(x))/sin(x)^2,x)
[Out] exp(-1/tan(x))
```

3.724 $\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\sec(x))}{b}$$

[Out] ln(a+b*sec(x))/b

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4424, 36, 29, 31}

$$\frac{\log(a\cos(x)+b)}{b} - \frac{\log(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]

[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right)}{b} + \frac{a\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \cos(x)\right)}{b}$$
$$= -\frac{\log(\cos(x))}{b} + \frac{\log(b+a\cos(x))}{b}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.82

$$-\frac{\log(\cos(x))}{b} + \frac{\log(b+a\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
[Out] -(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

Maple [A]
time = 0.07, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sec(x))}{b}$	12
default	$\frac{\ln(a+b\sec(x))}{b}$	12
risch	$-\frac{\ln(e^{2ix}+1)}{b} + \frac{\ln\left(e^{2ix}+\frac{2b\,e^{ix}}{a}+1\right)}{b}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(a+b*sec(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sec(x))/b
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.00

$$\frac{\log\left(b\sec\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="maxima")
[Out] log(b*sec(x) + a)/b
```

Fricas [A]
time = 3.70, size = 19, normalized size = 1.73

$$\frac{\log\left(a\cos\left(x\right)+b\right)-\log\left(-\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="fricas")
[Out] (log(a*cos(x) + b) - log(-cos(x)))/b
```

Sympy [A]
time = 0.20, size = 14, normalized size = 1.27

$$\begin{cases}\frac{\log\left(\frac{a}{b}+\sec\left(x\right)\right)}{b} & \text{for}\ b\neq 0\\ \frac{\sec\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x)
[Out] Piecewise((log(a/b + sec(x))/b, Ne(b, 0)), (sec(x)/a, True))
```

Giac [A]
time = 0.43, size = 22, normalized size = 2.00

$$\frac{\log\left(\left|a\cos\left(x\right)+b\right|\right)}{b}-\frac{\log\left(\left|\cos\left(x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="giac")
[Out] log(abs(a*cos(x) + b))/b - log(abs(cos(x)))/b
```

Mupad [B]
time = 3.25, size = 48, normalized size = 4.36

$$\frac{\mathrm{atan}\left(\frac{b\sin\left(\frac{x}{2}\right)^2}{a\cos\left(\frac{x}{2}\right)^2\,1\mathrm{i}+b\cos\left(\frac{x}{2}\right)^2\,1\mathrm{i}-a\sin\left(\frac{x}{2}\right)^2\,1\mathrm{i}}\right)\,2\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(a + b/cos(x))),x)
[Out] (atan((b*sin(x/2)^2)/(a*cos(x/2)^2*1i + b*cos(x/2)^2*1i - a*sin(x/2)^2*1i))
*2i)/b
```

3.725 $\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx$

Optimal. Leaf size=5

$$-\text{ArcTan}(\cos(x))$$

```
[Out] -arctan(cos(x))
```

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4424, 209}

$$-\text{ArcTan}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
```

```
[Out] -ArcTan[Cos[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cos(x)\right)\\ &= -\tan^{-1}(\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 5, normalized size = 1.00

$$-\text{ArcTan}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
```

```
[Out] -ArcTan[Cos[x]]
```

Maple [A]
time = 0.06, size = 4, normalized size = 0.80

method	result	size
derivativedivides	$\arctan\left(\sec\left(x\right)\right)$	4
default	$\arctan\left(\sec\left(x\right)\right)$	4
risch	$-\frac{i\ln\left(2i\,\mathrm{e}^{ix}+\mathrm{e}^{2ix}+1\right)}{2}+\frac{i\ln\left(-2i\,\mathrm{e}^{ix}+\mathrm{e}^{2ix}+1\right)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+sec(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sec(x))
```

Maxima [A]
time = 0.50, size = 5, normalized size = 1.00

$$-\arctan\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="maxima")
```

```
[Out] -arctan(cos(x))
```

Fricas [A]
time = 3.67, size = 5, normalized size = 1.00

$$-\arctan\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="fricas")
```

```
[Out] -arctan(cos(x))
```

Sympy [A]
time = 0.08, size = 3, normalized size = 0.60

$$\operatorname{atan}\left(\sec\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)**2),x)
```

```
[Out] atan(sec(x))
```

Giac [A]
time = 0.42, size = 5, normalized size = 1.00

$$-\arctan\left(\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(cos(x))
```

Mupad [B]
time = 2.95, size = 7, normalized size = 1.40

$$\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 1)),x)
```

```
[Out] atan(tan(x/2)^2)
```

3.726 $\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}\text{ArcTan}\left(\frac{3\cos(x)}{2}\right)$$

[Out] -1/6*arctan(3/2*cos(x))

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4424, 209}

$$-\frac{1}{6}\text{ArcTan}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]

[Out] -1/6*ArcTan[(3*Cos[x])/2]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{4+9x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{6}\tan^{-1}\left(\frac{3\cos(x)}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{6}\text{ArcTan}\left(\frac{3\cos(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

```
[Out] -1/6*ArcTan[(3*Cos[x])/2]
```

Maple [A]
time = 0.06, size = 8, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$	8
default	$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$	8
risch	$-\frac{i\ln\left(\frac{4i e^{ix}}{3}+e^{2ix}+1\right)}{12}+\frac{i\ln\left(-\frac{4i e^{ix}}{3}+e^{2ix}+1\right)}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(9+4*sec(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(2/3*sec(x))
```

Maxima [A]
time = 0.51, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

Fricas [A]
time = 3.65, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="fricas")
```

[Out] -1/6*arctan(3/2*cos(x))

Sympy [A]
time = 0.09, size = 8, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{2\sec(x)}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)**2),x)
```

```
[Out] atan(2*sec(x)/3)/6
```

Giac [A]
time = 0.42, size = 7, normalized size = 0.64

$$-\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="giac")
```

```
[Out] -1/6*arctan(3/2*cos(x))
```

Mupad [B]
time = 3.04, size = 13, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{13\tan\left(\frac{x}{2}\right)^2}{12}-\frac{5}{12}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(4/cos(x)^2 + 9)),x)
```

```
[Out] atan((13*tan(x/2)^2)/12 - 5/12)/6
```

3.727 $\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx$

Optimal. Leaf size=7

$$-\log(1+\cos(x))$$

```
[Out] -ln(cos(x)+1)
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4424, 31}

$$-\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

```
[Out] -Log[1 + Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\\ &= -\log(1+\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.29

$$-2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

```
[Out] -2*Log[Cos[x/2]]
```

Maple [A]
time = 0.07, size = 12, normalized size = 1.71

method	result	size
derivativedivides	$\ln(\sec(x)) - \ln(1 + \sec(x))$	12
default	$\ln(\sec(x)) - \ln(1 + \sec(x))$	12
risch	$ix - 2\ln(e^{ix} + 1)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(sec(x)+sec(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sec(x))-ln(1+sec(x))
```

Maxima [A]
time = 0.29, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="maxima")
```

```
[Out] -log(cos(x) + 1)
```

Fricas [A]
time = 3.31, size = 9, normalized size = 1.29

$$-\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="fricas")
```

```
[Out] -log(1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 0.09, size = 15, normalized size = 2.14

$$\frac{\log(\tan^2(x) + 1)}{2} - \log(\sec(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)**2),x)
```

```
[Out] log(tan(x)**2 + 1)/2 - log(sec(x) + 1)
```

Giac [A]
time = 0.45, size = 7, normalized size = 1.00

$$-\log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="giac")
```

```
[Out] -log(cos(x) + 1)
```

Mupad [B]
time = 3.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x) + 1/cos(x)^2)),x)
```

```
[Out] log(tan(x/2)^2 + 1)
```

3.728 $\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx$

Optimal. Leaf size=5

$$\operatorname{csch}^{-1}(2\cos(x))$$

[Out] arccsch(2*cos(x))

Rubi [A]
time = 0.03, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4424, 342, 221}

$$\operatorname{csch}^{-1}(2\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]

[Out] ArcCsch[2*Cos[x]]

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{\sqrt{4+\frac{1}{x^2}}\,x^2}\,dx, x, \cos(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sec(x)\right)$$
$$= \sinh^{-1}\left(\frac{\sec(x)}{2}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(5) = 10$.
time = 0.02, size = 38, normalized size = 7.60

$$\frac{\tanh^{-1}\left(\sqrt{1+4\cos^2(x)}\right)\sqrt{3+2\cos(2x)}\,\sec(x)}{\sqrt{4+\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

```
[Out] (ArcTanh[Sqrt[1 + 4*Cos[x]^2]]*Sqrt[3 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 + Sec[x]
^2]
```

Maple [A]
time = 0.08, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$	6
default	$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(1/2*sec(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(5) = 10$.
time = 0.30, size = 33, normalized size = 6.60

$$\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\,\cos(x)+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\,\cos(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) + 1) - 1/2*log(sqrt(1/cos(x)^2 + 4)*cos
(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(5) = 10.
time = 3.70, size = 27, normalized size = 5.40

$$\log\left(-\frac{\sqrt{\frac{4\cos(x)^2+1}{\cos(x)^2}}\cos(x)+1}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(sqrt((4*cos(x)^2 + 1)/cos(x)^2)*cos(x) + 1)/cos(x))
```

Sympy [A]
time = 0.40, size = 5, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sec(x)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)**2)**(1/2),x)
```

```
[Out] asinh(sec(x)/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(5) = 10.
time = 0.41, size = 36, normalized size = 7.20

$$\frac{\log\left(\sqrt{4\cos(x)^2+1}+1\right)-\log\left(\sqrt{4\cos(x)^2+1}-1\right)}{2\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(sqrt(4*cos(x)^2 + 1) + 1) - log(sqrt(4*cos(x)^2 + 1) - 1))/sgn(cos
(x))
```

Mupad [B]
time = 3.10, size = 7, normalized size = 1.40

$$\operatorname{asinh}\left(\frac{1}{2\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(1/cos(x)^2 + 4)^(1/2)),x)
```

```
[Out] asinh(1/(2*cos(x)))
```

3.729 $\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

Optimal. Leaf size=13

$$\sqrt{1+\cos^2(x)}\,\sec(x)$$

```
[Out] sec(x)*(1+cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {270}

$$\sqrt{\cos^2(x)+1}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \cos(x)\right)\\ &= \sqrt{1+\cos^2(x)}\,\sec(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{1+\cos^2(x)}\,\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

```
[Out] Sqrt[1 + Cos[x]^2]*Sec[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.
time = 0.11, size = 25, normalized size = 1.92

method	result	size
derivativedivides	$\frac{1+\sec^2(x)}{\sqrt{\frac{1+\sec^2(x)}{\sec(x)^2}}\,\sec(x)}$	25
default	$\frac{1+\sec^2(x)}{\sqrt{\frac{1+\sec^2(x)}{\sec(x)^2}}\,\sec(x)}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((1+sec(x)^2)/sec(x)^2)^(1/2)/sec(x)*(1+sec(x)^2)
```

Maxima [A]
time = 0.50, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos\left(x\right)^2+1}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(cos(x)^2 + 1)/cos(x)
```

Fricas [A]
time = 4.49, size = 16, normalized size = 1.23

$$\frac{\sqrt{\cos\left(x\right)^2+1}+\cos\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] (sqrt(cos(x)^2 + 1) + cos(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)\sec\left(x\right)}{\sqrt{\cos^2\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)**2)**(1/2),x)
[Out] Integral(tan(x)*sec(x)/sqrt(cos(x)**2 + 1), x)
```

Giac [A]
time = 0.43, size = 21, normalized size = 1.62

$$-\frac{2}{\left(\sqrt{\cos\left(x\right)^{2}+1}-\cos\left(x\right)\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
[Out] -2/((sqrt(cos(x)^2 + 1) - cos(x))^2 - 1)
```

Mupad [B]
time = 0.11, size = 13, normalized size = 1.00

$$\frac{\sqrt{\cos\left(x\right)^{2}+1}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cos(x)*(cos(x)^2 + 1)^(1/2)),x)
[Out] (cos(x)^2 + 1)^(1/2)/cos(x)
```

3.730 $\int e^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=4

$$e^{\sec(x)}$$

[Out] exp(sec(x))

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4424, 2240}

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

[In] Int[E^Sec[x]*Sec[x]*Tan[x],x]

[Out] E^Sec[x]

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int e^{\sec(x)} \sec(x) \tan(x) \, dx &= -\text{Subst}\left(\int \frac{e^{\frac{1}{x}}}{x^2} \, dx, x, \cos(x)\right)\\ &= e^{\sec(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 4, normalized size = 1.00

$$e^{\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sec[x]*Sec[x]*Tan[x],x]
```

```
[Out] E^Sec[x]
```

Maple [A]
time = 0.03, size = 4, normalized size = 1.00

method	result	size
derivativedivides	$e^{\sec(x)}$	4
default	$e^{\sec(x)}$	4
risch	$e^{\frac{1}{\cos(x)}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sec(x))*sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(sec(x))
```

Maxima [A]
time = 0.31, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] e^sec(x)
```

Fricas [A]
time = 3.79, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] e^(1/cos(x))
```

Sympy [A]
time = 0.18, size = 3, normalized size = 0.75

$$e^{\sec(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x)
```

```
[Out] exp(sec(x))
```

Giac [A]
time = 0.42, size = 5, normalized size = 1.25

$$e^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] e^(1/cos(x))
```

Mupad [B]
time = 3.09, size = 5, normalized size = 1.25

$$\mathrm{e}^{\frac{1}{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(1/cos(x))*tan(x))/cos(x),x)
```

```
[Out] exp(1/cos(x))
```

3.731 $\int 2^{\sec(x)} \sec(x) \tan(x) \, dx$

Optimal. Leaf size=9

$$\frac{2^{\sec(x)}}{\log(2)}$$

[Out] 2^sec(x)/ln(2)

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4424, 2240}

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sec[x]*Sec[x]*Tan[x],x]

[Out] 2^Sec[x]/Log[2]

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int 2^{\sec(x)} \sec(x) \tan(x) \, dx &= -\text{Subst}\left(\int \frac{2^{\frac{1}{x}}}{x^2} \, dx, x, \cos(x)\right)\\ &= \frac{2^{\sec(x)}}{\log(2)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sec[x]*Sec[x]*Tan[x],x]
[Out] 2^Sec[x]/Log[2]
```

Maple [A]
time = 0.08, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{2^{\sec(x)}}{\ln(2)}$	10
default	$\frac{2^{\sec(x)}}{\ln(2)}$	10
risch	$\frac{2^{\frac{1}{\cos(x)}}}{\ln(2)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sec(x)*sec(x)*tan(x),x,method=_RETURNVERBOSE)
[Out] 2^sec(x)/ln(2)
```

Maxima [A]
time = 0.30, size = 9, normalized size = 1.00

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="maxima")
[Out] 2^sec(x)/log(2)
```

Fricas [A]
time = 3.19, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 2^(1/cos(x))/log(2)
```

Sympy [A]
time = 0.18, size = 7, normalized size = 0.78

$$\frac{2^{\sec(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sec(x)*sec(x)*tan(x),x)
[Out] 2**sec(x)/log(2)
```

Giac [A]
time = 0.42, size = 11, normalized size = 1.22

$$\frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="giac")
[Out] 2^(1/cos(x))/log(2)
```

Mupad [B]
time = 3.12, size = 11, normalized size = 1.22

$$\frac{2^{\frac{1}{\cos(x)}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/cos(x))*tan(x))/cos(x),x)
[Out] 2^(1/cos(x))/log(2)
```

3.732 $\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{\sqrt{1+\sec(2x)}}$$

```
[Out] -1/(1+sec(2*x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {4424, 267}

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

```
[Out] -(1/Sqrt[1 + Sec[2*x]])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2}x^2}\,dx, x, \cos(2x)\right)\right)\\ &= -\frac{1}{\sqrt{1+\sec(2x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.67

$$-\frac{2\cos^2(x)\sec(2x)}{(1+\sec(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

```
[Out] (-2*Cos[x]^2*Sec[2*x])/(1 + Sec[2*x])^(3/2)
```

Maple [A]
time = 0.09, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{1}{\sqrt{1+\sec\left(2x\right)}}$	11
default	$-\frac{1}{\sqrt{1+\sec\left(2x\right)}}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1+sec(2*x))^(1/2)
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.83

$$-\frac{1}{\sqrt{\sec\left(2\,x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(10) = 20.
time = 3.57, size = 29, normalized size = 2.42

$$-\frac{\sqrt{\frac{\cos\left(2\,x\right)+1}{\cos\left(2\,x\right)}}\cos\left(2\,x\right)}{\cos\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt((cos(2*x) + 1)/cos(2*x))*cos(2*x)/(cos(2*x) + 1)
```

Sympy [A]
time = 0.39, size = 12, normalized size = 1.00

$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))**(3/2),x)
```

```
[Out] -1/sqrt(sec(2*x) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(10) = 20$.
time = 0.43, size = 31, normalized size = 2.58

$$\frac{1}{\left(\sqrt{\cos(2x)^2+\cos(2x)}-\cos(2x)-1\right)\operatorname{sgn}(\cos(2x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/((sqrt(cos(2*x)^2 + cos(2*x)) - cos(2*x) - 1)*sgn(cos(2*x)))
```

Mupad [B]
time = 3.11, size = 18, normalized size = 1.50

$$-\frac{1}{\sqrt{\cos(2x)+1}\,\sqrt{\frac{1}{\cos(2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(2*x)/(cos(2*x)*(1/cos(2*x) + 1)^(3/2)),x)
```

```
[Out] -1/((cos(2*x) + 1)^(1/2)*(1/cos(2*x))^(1/2))
```

3.733 $\int \sqrt{1+5\cos^2(3x)}\ \sec(3x)\tan(3x)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)+\frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)$$

```
[Out] -1/3*arcsinh(cos(3*x)*5^(1/2))*5^(1/2)+1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {283, 221}

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\ \sec(3x)-\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -1/3*(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]]) + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]
)/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+5\cos^2(3x)}\ \sec(3x)\tan(3x)\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{1+5x^2}}{x^2}\,dx,x,\cos(3x)\right)\right)\\
&= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)-\frac{5}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1+5x^2}}\,dx,x,\cos(3x)\right)\\
&= -\frac{1}{3}\sqrt{5}\ \sinh^{-1}\left(\sqrt{5}\ \cos(3x)\right)+\frac{1}{3}\sqrt{1+5\cos^2(3x)}\ \sec(3x)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 1.00

$$-\frac{1}{3}\sqrt{5}\sinh^{-1}\left(\sqrt{5}\cos(3x)\right)+\frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

```
[Out] -1/3*(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]]) + (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]
)/3
```

Maple [A]
time = 0.11, size = 65, normalized size = 1.51

method	result	size
derivativedivides	$\frac{\sqrt{\frac{\sec^2(3x)+5}{\sec(3x)^2}}\,\sec(3x)\left(\sqrt{\sec^2(3x)+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec^2(3x)+5}}\right)\right)}{3\sqrt{\sec^2(3x)+5}}$	65
default	$\frac{\sqrt{\frac{\sec^2(3x)+5}{\sec(3x)^2}}\,\sec(3x)\left(\sqrt{\sec^2(3x)+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec^2(3x)+5}}\right)\right)}{3\sqrt{\sec^2(3x)+5}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)*sec(3*x)/(sec(3*x)^2+5)^(1/2)*((sec(3
*x)^2+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(sec(3*x)^2+5)^(1/2)))
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.81

$$-\frac{1}{3}\sqrt{5}\,\operatorname{arsinh}\left(\sqrt{5}\cos\left(3\,x\right)\right)+\frac{\sqrt{5\,\cos\left(3\,x\right)^2+1}}{3\,\cos\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(5)*arcsinh(sqrt(5)*cos(3*x)) + 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(33) = 66.
time = 3.58, size = 122, normalized size = 2.84

$$\frac{\sqrt{5}\cos\left(3\,x\right)\log\left(80000\,\cos\left(3\,x\right)^8+32000\,\cos\left(3\,x\right)^6+4000\,\cos\left(3\,x\right)^4+160\,\cos\left(3\,x\right)^2-8\left(2000\,\sqrt{5}\cos\left(3\,x\right)^7+600\,\sqrt{5}\cos\left(3\,x\right)^5+50\,\sqrt{5}\cos\left(3\,x\right)^3+\sqrt{5}\cos\left(3\,x\right)\right)\sqrt{5\,\cos\left(3\,x\right)^2+1}+1\right)+8\,\sqrt{5\,\cos\left(3\,x\right)^2+1}}{24\,\cos\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(5)*cos(3*x)*log(80000*cos(3*x)^8 + 32000*cos(3*x)^6 + 4000*cos(3
*x)^4 + 160*cos(3*x)^2 - 8*(2000*sqrt(5)*cos(3*x)^7 + 600*sqrt(5)*cos(3*x)^
5 + 50*sqrt(5)*cos(3*x)^3 + sqrt(5)*cos(3*x))*sqrt(5*cos(3*x)^2 + 1) + 1) +
 8*sqrt(5*cos(3*x)^2 + 1))/cos(3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5\cos^2{(3x)} + 1}\,\tan{(3x)}\sec{(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)**2)**(1/2)*tan(3*x),x)
```

```
[Out] Integral(sqrt(5*cos(3*x)**2 + 1)*tan(3*x)*sec(3*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(5*cos(3*x)^2 + 1)*sec(3*x)*tan(3*x), x)
```

Mupad [B]
time = 3.26, size = 36, normalized size = 0.84

$$\frac{\sqrt{\frac{5\,\cos\left(6\,x\right)}{2}+\frac{7}{2}}}{3\,\cos\left(3\,x\right)}+\frac{\sqrt{5}\,\mathrm{asin}\left(\sqrt{5}\,\cos\left(3\,x\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(3*x)*(5*cos(3*x)^2 + 1)^(1/2))/cos(3*x),x)
```

```
[Out] (5^(1/2)*asin(5^(1/2)*cos(3*x)*1i)*1i)/3 + ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*
cos(3*x))
```

3.734 $\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{3}\sqrt{1+5\cos^2(3x)}\,\sec(3x)$$

```
[Out] 1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{270\}$

$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\,\sec(3x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+5x^2}}\,dx,x,\cos(3x)\right)\right)\\ &= \frac{1}{3}\sqrt{1+5\cos^2(3x)}\,\sec(3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{1}{3}\sqrt{1+5\cos^2(3x)}\,\sec(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

```
[Out] (Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

Maple [A]
time = 0.08, size = 34, normalized size = 1.55

method	result	size
derivativedivides	$\frac{\sec^{2}(3x)+5}{3\sqrt{\frac{\sec^{2}(3x)+5}{\sec(3x)^{2}}}\ \sec(3x)}$	34
default	$\frac{\sec^{2}(3x)+5}{3\sqrt{\frac{\sec^{2}(3x)+5}{\sec(3x)^{2}}}\ \sec(3x)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)/sec(3*x)*(sec(3*x)^2+5)
```

Maxima [A]
time = 0.51, size = 20, normalized size = 0.91

$$\frac{\sqrt{5\cos\left(3\,x\right)^{2}+1}}{3\cos\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

Fricas [A]
time = 2.41, size = 20, normalized size = 0.91

$$\frac{\sqrt{5\cos\left(3\,x\right)^{2}+1}}{3\cos\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(3x\right)\sec\left(3x\right)}{\sqrt{5\cos^{2}\left(3x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)**2)**(1/2),x)
[Out] Integral(tan(3*x)*sec(3*x)/sqrt(5*cos(3*x)**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sec(3*x)*tan(3*x)/sqrt(5*cos(3*x)^2 + 1), x)
```

Mupad [B]
time = 3.03, size = 18, normalized size = 0.82

$$\frac{\sqrt{\frac{5\,\cos\left(6\,x\right)}{2}+\frac{7}{2}}}{3\,\cos\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)/(cos(3*x)*(5*cos(3*x)^2 + 1)^(1/2)),x)
[Out] ((5*cos(6*x))/2 + 7/2)^(1/2)/(3*cos(3*x))
```

3.735 $\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(a+b\csc(x))}{b}$$

[Out] -ln(a+b*csc(x))/b

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {4423, 36, 29, 31}

$$\frac{\log(\sin(x))}{b}-\frac{\log(a\sin(x)+b)}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]

[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx = \text{Subst}\left(\int \frac{1}{x(b+ax)}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{b} - \frac{a\text{Subst}\left(\int \frac{1}{b+ax}\,dx, x, \sin(x)\right)}{b}$$
$$= \frac{\log(\sin(x))}{b} - \frac{\log(b+a\sin(x))}{b}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.67

$$\frac{\log(\sin(x))}{b} - \frac{\log(b+a\sin(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

```
[Out] Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

Maple [A]
time = 0.07, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\csc(x))}{b}$	13
default	$-\frac{\ln(a+b\csc(x))}{b}$	13
risch	$\frac{\ln(e^{2ix}-1)}{b} - \frac{\ln\left(\frac{2ib\,e^{ix}}{a}+e^{2ix}-1\right)}{b}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+b*csc(x))/b
```

Maxima [A]
time = 0.30, size = 12, normalized size = 1.00

$$-\frac{\log(b\csc(x)+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="maxima")
```

```
[Out] -log(b*csc(x) + a)/b
```

Fricas [A]
time = 2.64, size = 20, normalized size = 1.67

$$-\frac{\log\left(a\sin\left(x\right)+b\right)-\log\left(-\frac{1}{2}\sin\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="fricas")
[Out] -(log(a*sin(x) + b) - log(-1/2*sin(x)))/b
```

Sympy [A]
time = 0.20, size = 17, normalized size = 1.42

$$\begin{cases}-\frac{\log\left(\frac{a}{b}+\csc\left(x\right)\right)}{b} & \text{for}\ b\neq 0\\ -\frac{\csc\left(x\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x)
[Out] Piecewise((-log(a/b + csc(x))/b, Ne(b, 0)), (-csc(x)/a, True))
```

Giac [A]
time = 0.41, size = 22, normalized size = 1.83

$$-\frac{\log\left(|a\sin\left(x\right)+b|\right)}{b}+\frac{\log\left(|\sin\left(x\right)|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="giac")
[Out] -log(abs(a*sin(x) + b))/b + log(abs(sin(x)))/b
```

Mupad [B]
time = 3.18, size = 31, normalized size = 2.58

$$-\frac{\ln\left(b\tan\left(\frac{x}{2}\right)^2+2\,a\tan\left(\frac{x}{2}\right)+b\right)-\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(a + b/sin(x))),x)
[Out] -(log(b + 2*a*tan(x/2) + b*tan(x/2)^2) - log(tan(x/2)))/b
```

3.736 $\int 5^{\csc(3x)} \cot(3x) \csc(3x) \, dx$

Optimal. Leaf size=14

$$-\frac{5^{\csc(3x)}}{3 \log(5)}$$

```
[Out] -1/3*5^csc(3*x)/ln(5)
```

Rubi [A]

time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4423, 2240}

$$-\frac{5^{\csc(3x)}}{3 \log(5)}$$

Antiderivative was successfully verified.

```
[In] Int[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

```
[Out] -1/3*5^Csc[3*x]/Log[5]
```

Rule 2240

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned} \int 5^{\csc(3x)} \cot(3x) \csc(3x) \, dx &= \frac{1}{3} \text{Subst}\left(\int \frac{5^{\frac{1}{x}}}{x^2} \, dx, x, \sin(3x)\right) \\ &= -\frac{5^{\csc(3x)}}{3 \log(5)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
[Out] -1/3*5^Csc[3*x]/Log[5]
```

Maple [A]
time = 0.06, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{5^{\csc(3x)}}{3\ln(5)}$	13
default	$-\frac{5^{\csc(3x)}}{3\ln(5)}$	13
risch	$-\frac{5^{\frac{1}{\sin(3x)}}}{3\ln(5)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^csc(3*x)*cot(3*x)*csc(3*x),x,method=_RETURNVERBOSE)
[Out] -1/3*5^csc(3*x)/ln(5)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="maxima")
[Out] -1/3*5^csc(3*x)/log(5)
```

Fricas [A]
time = 3.32, size = 14, normalized size = 1.00

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

Sympy [A]
time = 0.18, size = 12, normalized size = 0.86

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5**csc(3*x)*cot(3*x)*csc(3*x),x)
[Out] -5**csc(3*x)/(3*log(5))
```

Giac [A]
time = 0.46, size = 14, normalized size = 1.00

$$-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\log(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="giac")
[Out] -1/3*5^(1/sin(3*x))/log(5)
```

Mupad [B]
time = 2.97, size = 14, normalized size = 1.00

$$-\frac{5^{\frac{1}{\sin(3x)}}}{3\ln(5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5^(1/sin(3*x))*cot(3*x))/sin(3*x),x)
[Out] -5^(1/sin(3*x))/(3*log(5))
```

3.737 $\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx$

Optimal. Leaf size=3

$$\text{ArcTan}(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4423, 209}

$$\text{ArcTan}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)\\ &= \tan^{-1}(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\text{ArcTan}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A]
time = 0.05, size = 6, normalized size = 2.00

method	result	size
derivativedivides	$-\arctan\left(\csc\left(x\right)\right)$	6
default	$-\arctan\left(\csc\left(x\right)\right)$	6
risch	$\frac{i\ln\left(\mathrm{e}^{2ix}-2\,\mathrm{e}^{ix}-1\right)}{2}-\frac{i\ln\left(\mathrm{e}^{2ix}+2\,\mathrm{e}^{ix}-1\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+csc(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(csc(x))
```

Maxima [A]
time = 0.51, size = 3, normalized size = 1.00

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(sin(x))
```

Fricas [A]
time = 3.64, size = 3, normalized size = 1.00

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [A]
time = 0.09, size = 5, normalized size = 1.67

$$-\operatorname{atan}\left(\csc\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)**2),x)
```

```
[Out] -atan(csc(x))
```

Giac [A]
time = 0.41, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

Mupad [B]
time = 3.22, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(1/sin(x)^2 + 1)),x)
```

```
[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

3.738 $\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}+\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}$$

```
[Out] 1/60*sin(6*x)/(11-5*sin(6*x)^2)-1/3300*arctanh(1/11*sin(6*x)*55^(1/2))*55^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4423, 294, 212}

$$\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

```
[Out] -1/60*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(60*(11 - 5*Sin[6*x]
^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
```

```
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(6x)\csc(6x)}{\left(5-11\csc^2(6x)\right)^2}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{\left(11-5x^2\right)^2}\,dx, x, \sin(6x)\right)\\ &= \frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}-\frac{1}{60}\text{Subst}\left(\int \frac{1}{11-5x^2}\,dx, x, \sin(6x)\right)\\ &= -\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}+\frac{\sin(6x)}{60\left(11-5\sin^2(6x)\right)}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 41, normalized size = 0.95

$$-\frac{\tanh^{-1}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}}+\frac{\sin(6x)}{30(17+5\cos(12x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
[Out] -1/60*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(30*(17 + 5*Cos[12*x
]))
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\csc(6x)}{660(\csc^2(6x))-300}-\frac{\sqrt{55}\,\operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$	35
default	$\frac{\csc(6x)}{660(\csc^2(6x))-300}-\frac{\sqrt{55}\,\operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$	35
risch	$-\frac{i\left(e^{18ix}-e^{6ix}\right)}{30(5\,e^{24ix}+34\,e^{12ix}+5)}-\frac{\sqrt{55}\,\ln\left(e^{12ix}+\frac{2i\sqrt{55}\,e^{6ix}}{5}-1\right)}{6600}+\frac{\sqrt{55}\,\ln\left(e^{12ix}-\frac{2i\sqrt{55}\,e^{6ix}}{5}-1\right)}{6600}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*csc(6*x)/(11*csc(6*x)^2-5)-1/3300*55^(1/2)*arctanh(1/5*csc(6*x)*55^(1/
2))
```

Maxima [A]
time = 0.53, size = 49, normalized size = 1.14

$$\frac{1}{6600}\sqrt{55}\log\left(-\frac{\sqrt{55}-5\sin(6x)}{\sqrt{55}+5\sin(6x)}\right)-\frac{\sin(6x)}{60\left(5\sin(6x)^2-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6600*sqrt(55)*log(-(sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/6
0*sin(6*x)/(5*sin(6*x)^2 - 11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(34) = 68$.
time = 4.74, size = 73, normalized size = 1.70

$$\frac{\left(5\sqrt{55}\cos(6x)^2+6\sqrt{55}\right)\log\left(-\frac{5\cos(6x)^2+2\sqrt{55}\sin(6x)-16}{5\cos(6x)^2+6}\right)+110\sin(6x)}{6600\left(5\cos(6x)^2+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6600*((5*sqrt(55)*cos(6*x)^2 + 6*sqrt(55))*log(-(5*cos(6*x)^2 + 2*sqrt(55
)*sin(6*x) - 16)/(5*cos(6*x)^2 + 6)) + 110*sin(6*x))/(5*cos(6*x)^2 + 6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(34) = 68$.
time = 0.51, size = 151, normalized size = 3.51

$$\frac{11\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}-\frac{5\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000}-\frac{11\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000}+\frac{5\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000}+\frac{110\csc(6x)}{72600\csc^2(6x)-33000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)**2)**2,x)
```

```
[Out] 11*sqrt(55)*log(csc(6*x) - sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 33
000) - 5*sqrt(55)*log(csc(6*x) - sqrt(55)/11)/(72600*csc(6*x)**2 - 33000) -
 11*sqrt(55)*log(csc(6*x) + sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 3
3000) + 5*sqrt(55)*log(csc(6*x) + sqrt(55)/11)/(72600*csc(6*x)**2 - 33000)
+ 110*csc(6*x)/(72600*csc(6*x)**2 - 33000)
```

Giac [A]
time = 0.46, size = 48, normalized size = 1.12

$$\frac{1}{6600}\sqrt{55}\log\left(\frac{\sqrt{55}-5\sin(6x)}{\sqrt{55}+5\sin(6x)}\right)-\frac{\sin(6x)}{60\left(5\sin(6x)^2-11\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6600*sqrt(55)*log((sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/60
*sin(6*x)/(5*sin(6*x)^2 - 11)
```

Mupad [B]
time = 3.15, size = 57, normalized size = 1.33

$$-\frac{55\sin(6x)-11\sqrt{55}\,\mathrm{atanh}\left(\frac{\sqrt{55}\,\sin(6x)}{11}\right)+5\sqrt{55}\,\sin(6x)^2\,\mathrm{atanh}\left(\frac{\sqrt{55}\,\sin(6x)}{11}\right)}{16500\sin(6x)^2-36300}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(6*x)/(sin(6*x)*(11/sin(6*x)^2 - 5)^2),x)
```

```
[Out] -(55*sin(6*x) - 11*55^(1/2)*atanh((55^(1/2)*sin(6*x))/11) + 5*55^(1/2)*sin(
6*x)^2*atanh((55^(1/2)*sin(6*x))/11))/(16500*sin(6*x)^2 - 36300)
```

3.739 $\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx$

Optimal. Leaf size=14

$$-\csc(x)\sqrt{1+\sin^2(x)}$$

```
[Out] -csc(x)*(1+sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {270}

$$\sqrt{\sin^2(x)+1}\,(-\csc(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(x)\right)\\ &= -\csc(x)\sqrt{1+\sin^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\csc(x)\sqrt{1+\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -(Csc[x]*Sqrt[1 + Sin[x]^2])
```

Maple [A]
time = 0.22, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{\sqrt{1+\sin^{2}\left(x\right)}}{\sin\left(x\right)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/sin(x)*(1+sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 14, normalized size = 1.00

$$-\frac{\sqrt{\sin\left(x\right)^{2}+1}}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(sin(x)^2 + 1)/sin(x)
```

Fricas [A]
time = 3.20, size = 21, normalized size = 1.50

$$-\frac{\sqrt{-\cos\left(x\right)^{2}+2}-\sin\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-cos(x)^2 + 2) - sin(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(x\right)\csc\left(x\right)}{\sqrt{\sin^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(cot(x)*csc(x)/sqrt(sin(x)**2 + 1), x)
```

Giac [A]
time = 0.43, size = 21, normalized size = 1.50

$$\frac{2}{\left(\sqrt{\sin\left(x\right)^{2}+1}-\sin\left(x\right)\right)^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(sin(x)^2 + 1) - sin(x))^2 - 1)
```

Mupad [B]
time = 3.11, size = 34, normalized size = 2.43

$$-\frac{\sqrt{\frac{1}{\sin\left(x\right)^{2}}+1}}{\sin\left(x\right)\left(\sqrt{\frac{1}{\sin\left(x\right)^{2}}+1}+1\right)\sqrt{\sin\left(x\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sin(x)*(sin(x)^2 + 1)^(1/2)),x)
```

```
[Out] -(1/sin(x)^2 + 1)^(1/2)/(sin(x)*((1/sin(x)^2 + 1)^(1/2) + 1)*(sin(x)^2 + 1)
^(1/2))
```

3.740 $\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{15}\csc(5x)\sqrt{1+\sin^2(5x)}-\frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}$$

```
[Out] 2/15*csc(5*x)*(1+sin(5*x)^2)^(1/2)-1/15*csc(5*x)^3*(1+sin(5*x)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {277, 270}

$$\frac{2}{15}\sqrt{\sin^2(5x)+1}\,\csc(5x)-\frac{1}{15}\sqrt{\sin^2(5x)+1}\,\csc^3(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

```
[Out] (2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/15 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/15
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)\\
&= -\frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}-\frac{2}{15}\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}}\,dx, x, \sin(5x)\right)\\
&= \frac{2}{15}\csc(5x)\sqrt{1+\sin^2(5x)}-\frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.65

$$-\frac{1}{15}\csc(5x)\left(-2+\csc^2(5x)\right)\sqrt{1+\sin^2(5x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
[Out] -1/15*(Csc[5*x]*(-2 + Csc[5*x]^2)*Sqrt[1 + Sin[5*x]^2])
```

Maple [A]
time = 0.28, size = 38, normalized size = 0.88

method	result	size
default	$-\frac{\sqrt{1+\sin^2(5x)}}{15\sin(5x)^3}+\frac{2\sqrt{1+\sin^2(5x)}}{15\sin(5x)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/15/sin(5*x)^3*(1+sin(5*x)^2)^(1/2)+2/15/sin(5*x)*(1+sin(5*x)^2)^(1/2)
```

Maxima [A]
time = 0.50, size = 37, normalized size = 0.86

$$\frac{2\sqrt{\sin(5x)^2+1}}{15\sin(5x)}-\frac{\sqrt{\sin(5x)^2+1}}{15\sin(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*sqrt(sin(5*x)^2 + 1)/sin(5*x) - 1/15*sqrt(sin(5*x)^2 + 1)/sin(5*x)^3
```

Fricas [A]
time = 3.25, size = 57, normalized size = 1.33

$$-\frac{2\left(\cos(5x)^2-1\right)\sin(5x)-\left(2\cos(5x)^2-1\right)\sqrt{-\cos(5x)^2+2}}{15\left(\cos(5x)^2-1\right)\sin(5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(2*(cos(5*x)^2 - 1)*sin(5*x) - (2*cos(5*x)^2 - 1)*sqrt(-cos(5*x)^2 +
2))/((cos(5*x)^2 - 1)*sin(5*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{\sin^2(5x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)**3/(1+sin(5*x)**2)**(1/2),x)
```

```
[Out] Integral(cot(5*x)*csc(5*x)**3/sqrt(sin(5*x)**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(5*x)*csc(5*x)^3/sqrt(sin(5*x)^2 + 1), x)
```

Mupad [B]
time = 3.14, size = 28, normalized size = 0.65

$$\frac{\sqrt{\sin(5\,x)^2+1}\,\left(2\sin(5\,x)^2-1\right)}{15\sin(5\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(5*x)/(sin(5*x)^3*(sin(5*x)^2 + 1)^(1/2)),x)
```

```
[Out] ((sin(5*x)^2 + 1)^(1/2)*(2*sin(5*x)^2 - 1))/(15*sin(5*x)^3)
```

3.741 $\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$-\frac{2e^{n\sin(a+bx)}}{bn^2}+\frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

```
[Out] -2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {12, 2207, 2225}

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(-1+n\sin(a+bx))}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 104, normalized size = 2.42

method	result	size
risch	$-\frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)} e^{ibx} e^{ia}}{nb} + \frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)} e^{-ibx} e^{-ia}}{nb} - \frac{2\, e^{n(\sin(bx)\cos(a)+\cos(bx)\sin(a))}}{n^2 b}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*e
xp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n
*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

Maxima [A]
time = 0.31, size = 37, normalized size = 0.86

$$\frac{2\left(ne^{(n\sin(bx+a))}\sin(bx+a) - e^{(n\sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

Fricas [A]
time = 3.70, size = 27, normalized size = 0.63

$$\frac{2\left(n\sin\left(bx+a\right)-1\right)e^{\left(n\sin\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

Mupad [B]
time = 3.18, size = 27, normalized size = 0.63

$$\frac{2\,{\mathrm{e}}^{n\,\sin\left(a+b\,x\right)}\,\left(n\,\sin\left(a+b\,x\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

3.742 $\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx$

Optimal. Leaf size=43

$$-\frac{2e^{n\sin(a+bx)}}{bn^2}+\frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

```
[Out] -2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {12, 2207, 2225}

$$\frac{2\sin(a+bx)e^{n\sin(a+bx)}}{bn}-\frac{2e^{n\sin(a+bx)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Sin[a + b*x]))/(b*n^2) + (2*E^(n*Sin[a + b*x])*Sin[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn} - \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \sin(a+bx)\right)}{bn}$$
$$= -\frac{2e^{n\sin(a+bx)}}{bn^2} + \frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.65

$$\frac{2e^{n\sin(a+bx)}(-1+n\sin(a+bx))}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.00, size = 104, normalized size = 2.42

method	result	size
risch	$-\frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)} e^{ibx} e^{ia}}{nb}+\frac{i e^{n\sin(bx)\cos(a)+n\cos(bx)\sin(a)} e^{-ibx} e^{-ia}}{nb}-\frac{2\,e^{n(\sin(bx)\cos(a)+\cos(bx)\sin(a))}}{n^2 b}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -I/n/b*exp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*e
xp(n*sin(b*x)*cos(a)+n*cos(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)-2/n^2/b*exp(n
*(sin(b*x)*cos(a)+cos(b*x)*sin(a)))
```

Maxima [A]
time = 0.32, size = 37, normalized size = 0.86

$$\frac{2\left(ne^{(n\sin(bx+a))}\sin(bx+a) - e^{(n\sin(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

Fricas [A]
time = 4.43, size = 27, normalized size = 0.63

$$\frac{2 (n \sin (bx + a) - 1)e^{(n \sin(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n \sin (a+bx)} \sin (2a + 2bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
[Out] Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

Mupad [B]
time = 0.00, size = 27, normalized size = 0.63

$$\frac{2\, e^{n\, \sin(a+b\, x)}\, (n\, \sin (a + b\, x) - 1)}{b\, n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
[Out] (2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

3.743 $\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*sin(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sin(1/2*a+1/2*b*x))*sin(1/2*a+1/
2*b*x)/b/n
```

Rubi [A]

time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 2207, 2225}

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx &= \frac{2\text{Subst}\left(\int 2e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4\text{Subst}\left(\int e^{nx}x\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} \\
&= \frac{4e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn} - \frac{4\text{Subst}\left(\int e^{nx}\,dx, x, \sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn} \\
&= -\frac{4e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2} + \frac{4e^{n \sin\left(\frac{a}{2}+\frac{bx}{2}\right)} \sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.56

$$
\frac{4e^{n \sin\left(\frac{1}{2}(a+bx)\right)}\left(-1+n \sin\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 122, normalized size = 1.91

method	result
risch	$-\frac{2 i \mathrm{e}^{n \sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right)+n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)} \mathrm{e}^{\frac{i b x}{2}} \mathrm{e}^{\frac{i a}{2}}}{n b}+\frac{2 i \mathrm{e}^{n \sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right)+n \cos\left(\frac{bx}{2}\right) \sin\left(\frac{a}{2}\right)} \mathrm{e}^{-\frac{i b x}{2}} \mathrm{e}^{-\frac{i a}{2}}}{n b}-\frac{4 \mathrm{e}^{n\left(\sin\left(\frac{bx}{2}\right) \cos\left(\frac{a}{2}\right)+\cos\left(\frac{bx}{2}\right.\right.}}{n^2 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(1/2*I
*b*x)*exp(1/2*I*a)+2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin
(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*
a)+cos(1/2*b*x)*sin(1/2*a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A]
time = 3.51, size = 33, normalized size = 0.52

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(50) = 100.
time = 0.46, size = 138, normalized size = 2.16

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

Mupad [B]
time = 3.18, size = 33, normalized size = 0.52

$$\frac{4\,{\mathrm{e}}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.744 $\int e^{n \sin\left(\frac{1}{2}(a+bx)\right)} \sin(a+bx)\, dx$

Optimal. Leaf size=64

$$-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] -4*exp(n*sin(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sin(1/2*a+1/2*b*x))*sin(1/2*a+1/
2*b*x)/b/n
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {12, 2207, 2225}

$$\frac{4\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (-4*E^(n*Sin[a/2 + (b*x)/2]))/(b*n^2) + (4*E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2
 + (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=\frac{2\text{Subst}\left(\int 2e^{nx}x\,dx,x,\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$=\frac{4\text{Subst}\left(\int e^{nx}x\,dx,x,\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$=\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}-\frac{4\text{Subst}\left(\int e^{nx}\,dx,x,\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$=-\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.56

$$\frac{4e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\sin\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
[Out] (4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.00, size = 122, normalized size = 1.91

method	result
risch	$-\frac{2i\mathrm{e}^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{\frac{ibx}{2}}\mathrm{e}^{\frac{ia}{2}}}{nb}+\frac{2i\mathrm{e}^{n\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+n\cos\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\mathrm{e}^{-\frac{ibx}{2}}\mathrm{e}^{-\frac{ia}{2}}}{nb}-\frac{4\,\mathrm{e}^{n\left(\sin\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)+\cos\left(\right.\right.}}{n^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin(1/2*a))*exp(1/2*I
*b*x)*exp(1/2*I*a)+2*I/n/b*exp(n*sin(1/2*b*x)*cos(1/2*a)+n*cos(1/2*b*x)*sin
(1/2*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*b*x)*cos(1/2*
a)+cos(1/2*b*x)*sin(1/2*a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A]
time = 3.16, size = 33, normalized size = 0.52

$$\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] 4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x)
[Out] Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(50) = 100.
time = 0.47, size = 138, normalized size = 2.16

$$\frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*sin(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
[Out] 4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b*
x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 +
1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

Mupad [B]
time = 0.00, size = 33, normalized size = 0.52

$$\frac{4\,\mathrm{e}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
[Out] (4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.745 $\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

```
[Out] 2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {12, 2207, 2225}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx = -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn} + \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \cos(a+bx)\right)}{bn}$$
$$= \frac{2e^{n\cos(a+bx)}}{bn^2} - \frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

Mathematica [A]
time = 0.07, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(-1+n\cos(a+bx))}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 105, normalized size = 2.44

method	result	size
risch	$-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{ibx}e^{ia}}{bn}-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{-ibx}e^{-ia}}{bn}+\frac{2\,e^{n(\cos(bx)\cos(a)-\sin(bx)\sin(a))}}{b\,n^2}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-1/b/n*e
xp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)+2/b/n^2*exp(n
*(cos(b*x)*cos(a)-sin(b*x)*sin(a)))
```

Maxima [A]
time = 0.32, size = 37, normalized size = 0.86

$$-\frac{2\left(n\cos\left(bx+a\right)e^{\left(n\cos\left(bx+a\right)\right)}-e^{\left(n\cos\left(bx+a\right)\right)}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

Fricas [A]
time = 3.39, size = 27, normalized size = 0.63

$$-\frac{2\,(n\cos{(bx+a)}-1)e^{(n\cos(bx+a))}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos{(a+bx)}}\sin{(2a+2bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

```
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

Mupad [B]
time = 3.21, size = 27, normalized size = 0.63

$$-\frac{2\,e^{n\,\cos(a+b\,x)}\,\left(n\,\cos\left(a+b\,x\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
```

```
[Out] -(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

3.746 $\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx$

Optimal. Leaf size=43

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

```
[Out] 2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {12, 2207, 2225}

$$\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2\cos(a+bx)e^{n\cos(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (2*E^(n*Cos[a + b*x]))/(b*n^2) - (2*E^(n*Cos[a + b*x])*Cos[a + b*x])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{\text{Subst}\left(\int 2e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int e^{nx}x\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn} + \frac{2\text{Subst}\left(\int e^{nx}\,dx, x, \cos(a+bx)\right)}{bn}$$
$$= \frac{2e^{n\cos(a+bx)}}{bn^2} - \frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.65

$$-\frac{2e^{n\cos(a+bx)}(-1+n\cos(a+bx))}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

```
[Out] (-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.00, size = 105, normalized size = 2.44

method	result	size
risch	$-\frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{ibx}e^{ia}}{bn} - \frac{e^{n\cos(bx)\cos(a)-n\sin(bx)\sin(a)}e^{-ibx}e^{-ia}}{bn} + \frac{2\,e^{n(\cos(bx)\cos(a)-\sin(bx)\sin(a))}}{b\,n^2}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/n*exp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(I*b*x)*exp(I*a)-1/b/n*e
xp(n*cos(b*x)*cos(a)-n*sin(b*x)*sin(a))*exp(-I*b*x)*exp(-I*a)+2/b/n^2*exp(n
*(cos(b*x)*cos(a)-sin(b*x)*sin(a)))
```

Maxima [A]
time = 0.31, size = 37, normalized size = 0.86

$$-\frac{2\left(n\cos\left(bx+a\right)e^{(n\cos(bx+a))} - e^{(n\cos(bx+a))}\right)}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

Fricas [A]
time = 2.08, size = 27, normalized size = 0.63

$$-\frac{2\left(n\cos\left(bx+a\right)-1\right)e^{\left(n\cos\left(bx+a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] -2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(a+bx\right)}\sin\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
[Out] Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
[Out] integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

Mupad [B]
time = 0.00, size = 27, normalized size = 0.63

$$-\frac{2\,\mathrm{e}^{n\,\cos\left(a+b\,x\right)}\,\left(n\,\cos\left(a+b\,x\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
[Out] -(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

3.747 $\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] 4*exp(n*cos(1/2*a+1/2*b*x))/b/n^2-4*exp(n*cos(1/2*a+1/2*b*x))*cos(1/2*a+1/2
*b*x)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {12, 2207, 2225}

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=-\frac{2\text{Subst}\left(\int 2e^{nx}x\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$=-\frac{4\text{Subst}\left(\int e^{nx}x\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$
$$=-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}+\frac{4\text{Subst}\left(\int e^{nx}\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}$$
$$=\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 0.56

$$-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cos\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 123, normalized size = 1.92

method	result
risch	$-\frac{2\,e^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\,e^{\frac{ibx}{2}}\,e^{\frac{ia}{2}}}{bn}-\frac{2\,e^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}\,e^{-\frac{ibx}{2}}\,e^{-\frac{ia}{2}}}{bn}+\frac{4\,e^{n\left(\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-\sin\left(\frac{bx}{2}\right.\right.}}{b\,n^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2
*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-s
in(1/2*b*x)*sin(1/2*a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A]
time = 2.05, size = 33, normalized size = 0.52

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
```

```
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(50) = 100.
time = 0.46, size = 195, normalized size = 3.05

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

Mupad [B]
time = 3.17, size = 33, normalized size = 0.52

$$-\frac{4\,\mathrm{e}^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.748 $\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$

Optimal. Leaf size=64

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

```
[Out] 4*exp(n*cos(1/2*a+1/2*b*x))/b/n^2-4*exp(n*cos(1/2*a+1/2*b*x))*cos(1/2*a+1/2
*b*x)/b/n
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 2207, 2225}

$$\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

```
[Out] (4*E^(n*Cos[a/2 + (b*x)/2]))/(b*n^2) - (4*E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2
+ (b*x)/2])/(b*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2207

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Dist[d*(m/(f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx &= -\frac{2\text{Subst}\left(\int 2e^{nx}x\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= -\frac{4\text{Subst}\left(\int e^{nx}x\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\\ &= -\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}+\frac{4\text{Subst}\left(\int e^{nx}\,dx,x,\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{bn}\\ &= \frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.56

$$-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cos\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
[Out] (-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.00, size = 123, normalized size = 1.92

method	result
risch	$-\frac{2\,e^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}e^{\frac{ibx}{2}}e^{\frac{ia}{2}}}{bn}-\frac{2\,e^{n\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-n\sin\left(\frac{bx}{2}\right)\sin\left(\frac{a}{2}\right)}e^{-\frac{ibx}{2}}e^{-\frac{ia}{2}}}{bn}+\frac{4\,e^{n\left(\cos\left(\frac{bx}{2}\right)\cos\left(\frac{a}{2}\right)-\sin\left(\frac{bx}{2}\right)\ldots\right)}}{b\,n^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2*a))*exp(1/2*I*b
*x)*exp(1/2*I*a)-2/b/n*exp(n*cos(1/2*b*x)*cos(1/2*a)-n*sin(1/2*b*x)*sin(1/2
*a))*exp(-1/2*I*b*x)*exp(-1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*b*x)*cos(1/2*a)-s
in(1/2*b*x)*sin(1/2*a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

Fricas [A]
time = 2.68, size = 33, normalized size = 0.52

$$-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="fricas")
[Out] -4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x)
[Out] Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(50) = 100.
time = 0.47, size = 195, normalized size = 3.05

$$\frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}+e^{\left(-\frac{n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-n}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*cos(1/2*a+1/2*b*x))*sin(b*x+a),x, algorithm="giac")
[Out] 4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1
/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a
)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(ta
n(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*
x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

Mupad [B]
time = 0.00, size = 33, normalized size = 0.52

$$-\frac{4\,e^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
[Out] -(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.749 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

[Out] 1/2*ln(tan(x))^2

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2700, 29, 6818}

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Maple [A]
time = 0.20, size = 8, normalized size = 0.89

method	result
derivativedivides	$\frac{\ln(\tan(x))^2}{2}$
default	$\frac{\ln(\tan(x))^2}{2}$
risch	$\frac{\ln(e^{2ix}+1)^2}{2} - \ln(e^{2ix}-1)\ln(e^{2ix}+1) + \frac{\ln(e^{2ix}-1)^2}{2} - \frac{i\ln(e^{2ix}-1)\pi\,\mathrm{csgn}\left(\frac{i}{e^{2ix}+1}\right)\mathrm{csgn}\left(i(e^{2ix}-1)\right)\mathrm{csgn}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*ln(tan(x))*sec(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(tan(x))^2
```

Maxima [A]
time = 0.29, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\tan\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x))^2
```

Fricas [A]
time = 3.03, size = 12, normalized size = 1.33

$$\frac{1}{2}\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\tan\left(x\right)\right)\csc\left(x\right)\sec\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*ln(tan(x))*sec(x),x)
[Out] Integral(log(tan(x))*csc(x)*sec(x), x)
```

Giac [A]
time = 0.41, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\tan\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="giac")
[Out] 1/2*log(tan(x))^2
```

Mupad [B]
time = 5.27, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x\,2i}\,1i-i}{e^{x\,2i}+1}\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/(cos(x)*sin(x)),x)
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/2
```

3.750 $\int \csc(2x)\log(\tan(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{4}\log^2(\tan(x))$$

```
[Out] 1/4*ln(tan(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3855, 6818}

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*Log[Tan[x]],x]
```

```
[Out] Log[Tan[x]]^2/4
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log^2(\tan(x))$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{1}{4}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*Log[Tan[x]],x]
```

[Out] Log[Tan[x]]^2/4

Maple [A]
time = 0.12, size = 8, normalized size = 0.89

method	result
derivativedivides	$\frac{\ln(\tan(x))^2}{4}$
default	$\frac{\ln(\tan(x))^2}{4}$
norman	$\frac{\ln(\tan(x))^2}{4}$
risch	$\frac{\ln(\mathrm{e}^{2ix}+1)^2}{4} - \frac{\ln(\mathrm{e}^{2ix}-1)\ln(\mathrm{e}^{2ix}+1)}{2} + \frac{\ln(\mathrm{e}^{2ix}-1)^2}{4} - \frac{i\ln(\mathrm{e}^{2ix}-1)\pi\,\mathrm{csgn}\left(\frac{\mathrm{e}^{2ix}-1}{\mathrm{e}^{2ix}+1}\right)^3}{4} + \frac{i\ln(\mathrm{e}^{2ix}+1)\pi\,\mathrm{csgn}\left(\frac{\mathrm{e}^{2i}}{\mathrm{e}^{2i}}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*ln(tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(tan(x))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(7) = 14$.
time = 0.51, size = 265, normalized size = 29.44

$$\frac{1}{4}\left(\pi - 2\arctan\left(\sin(x), \cos(x)+1\right) - 2\arctan\left(\sin(x), \cos(x)-1\right)\right)\arctan\left(\sin(2x), \cos(2x)+1\right) + \frac{1}{4}\arctan\left(\sin(2x), \cos(2x)+1\right)^2 - \frac{1}{4}\left(\pi - 2\arctan\left(\sin(x), \cos(x)-1\right)\right)\arctan\left(\sin(x), \cos(x)+1\right) + \frac{1}{4}\arctan\left(\sin(x), \cos(x)+1\right)^2 - \frac{1}{4}\pi\arctan\left(\sin(x), \cos(x)-1\right) + \frac{1}{4}\arctan\left(\sin(x), \cos(x)-1\right)^2 + \frac{1}{8}\left(\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\right)\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right) - \frac{1}{16}\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)^2 - \frac{1}{16}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^2 - \frac{1}{8}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right) - \frac{1}{16}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^2 - \frac{1}{2}\log\left(\cot(2x)+\csc(2x)\right)\log\left(\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(pi - 2*arctan2(sin(x), cos(x) + 1) - 2*arctan2(sin(x), cos(x) - 1))*ar
ctan2(sin(2*x), cos(2*x) + 1) + 1/4*arctan2(sin(2*x), cos(2*x) + 1)^2 - 1/4
*(pi - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(x), cos(x) + 1) + 1/4*arc
tan2(sin(x), cos(x) + 1)^2 - 1/4*pi*arctan2(sin(x), cos(x) - 1) + 1/4*arcta
n2(sin(x), cos(x) - 1)^2 + 1/8*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1) - 1/16*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^2 - 1/16*lo
g(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^2 - 1/8*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/16*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1)^2 - 1/2*log(cot(2*x) + csc(2*x))*log(tan(x))
```

Fricas [A]
time = 4.70, size = 7, normalized size = 0.78

$$\frac{1}{4}\log\left(\tan\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x))^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*ln(tan(x)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*log(tan(x)),x, algorithm="giac")
```

```
[Out] integrate(csc(2*x)*log(tan(x)), x)
```

Mupad [**B**]
time = 3.53, size = 27, normalized size = 3.00

$$\frac{\ln\left(-\frac{e^{x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{e^{x\,2\mathrm{i}}+1}\right)^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tan(x))/sin(2*x),x)
```

```
[Out] log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/4
```

3.751 $\int e^{\cos^2(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=3

$$ex$$

```
[Out] exp(1)*x
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 209}

$$ex$$

Antiderivative was successfully verified.

```
[In] Int[E^(Cos[x]^2 + Sin[x]^2),x]
```

```
[Out] E*x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int e^{\cos^2(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{e}{1+x^2}\,dx, x, \tan(x)\right)\\ &= e\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\ &= ex\end{aligned}$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$ex$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(Cos[x]^2 + Sin[x]^2),x]
```

```
[Out] E*x
```

Maple [A]
time = 0.05, size = 5, normalized size = 1.67

method	result	size
default	ex	5
risch	ex	5
norman	$x\,\mathrm{e}^{\frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}+\frac{\left(1-\left(\tan^2\left(\frac{x}{2}\right)\right)\right)^2}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x)^2+cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] exp(1)*x
```

Maxima [A]
time = 0.50, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

```
[Out] x*e
```

Fricas [A]
time = 4.62, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*e
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.
time = 0.05, size = 14, normalized size = 4.67

$$xe^{\sin^2(x)}e^{\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)**2+sin(x)**2),x)
```

```
[Out] x*exp(sin(x)**2)*exp(cos(x)**2)
```

Giac [**A**]
time = 0.41, size = 4, normalized size = 1.33

$$xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

```
[Out] x*e
```

Mupad [**B**]
time = 0.03, size = 4, normalized size = 1.33

$$x\,\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(cos(x)^2 + sin(x)^2),x)
```

```
[Out] x*exp(1)
```

3.752 $\int x \sec^2(x) \, dx$

Optimal. Leaf size=8

$$\log(\cos(x)) + x \tan(x)$$

[Out] ln(cos(x))+x*tan(x)

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4269, 3556}

$$x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]^2,x]

[Out] Log[Cos[x]] + x*Tan[x]

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x \sec^2(x) \, dx &= x \tan(x) - \int \tan(x) \, dx \\ &= \log(\cos(x)) + x \tan(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\log(\cos(x)) + x \tan(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sec[x]^2,x]

```
[Out] Log[Cos[x]] + x*Tan[x]
```

Maple [A]
time = 0.03, size = 9, normalized size = 1.12

method	result	size
default	$\ln(\cos(x)) + x\tan(x)$	9
risch	$-2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix}+1)$	27
norman	$-\frac{2x\tan(\frac{x}{2})}{\tan^2(\frac{x}{2})-1} - \ln\left(1+\tan^2\left(\frac{x}{2}\right)\right) + \ln\left(\tan\left(\frac{x}{2}\right)-1\right) + \ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(cos(x))+x*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(8) = 16.
time = 0.51, size = 74, normalized size = 9.25

$$\frac{\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right) + 4x\sin(2x)}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2
 + 2*cos(2*x) + 1) + 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) +
1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.
time = 3.00, size = 18, normalized size = 2.25

$$\frac{\cos(x)\log(-\cos(x)) + x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)*log(-cos(x)) + x*sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)**2,x)
```

```
[Out] Integral(x*sec(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(8) = 16$.
time = 0.43, size = 103, normalized size = 12.88

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-4x\tan\left(\frac{1}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}x\right)^4-2\tan\left(\frac{1}{2}x\right)^2+1\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)
^2 + 1))*tan(1/2*x)^2 - 4*x*tan(1/2*x) - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)
^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$\ln\left(\cos\left(x\right)\right)+x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x)^2,x)
```

```
[Out] log(cos(x)) + x*tan(x)
```

3.753 $\int x \cos^4(x^2)\, dx$

Optimal. Leaf size=34

$$\frac{3x^2}{16} + \frac{3}{16}\cos(x^2)\sin(x^2) + \frac{1}{8}\cos^3(x^2)\sin(x^2)$$

```
[Out] 3/16*x^2+3/16*cos(x^2)*sin(x^2)+1/8*cos(x^2)^3*sin(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3461, 2715, 8}

$$\frac{3x^2}{16} + \frac{1}{8}\sin(x^2)\cos^3(x^2) + \frac{3}{16}\sin(x^2)\cos(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2]^4,x]
```

```
[Out] (3*x^2)/16 + (3*Cos[x^2]*Sin[x^2])/16 + (Cos[x^2]^3*Sin[x^2])/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int x\cos^4\left(x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cos^4(x)\,dx,x,x^2\right)\\
&= \frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{8}\text{Subst}\left(\int \cos^2(x)\,dx,x,x^2\right)\\
&= \frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)+\frac{3}{16}\text{Subst}\left(\int 1\,dx,x,x^2\right)\\
&= \frac{3x^2}{16}+\frac{3}{16}\cos\left(x^2\right)\sin\left(x^2\right)+\frac{1}{8}\cos^3\left(x^2\right)\sin\left(x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$\frac{3x^2}{16}+\frac{1}{8}\sin\left(2x^2\right)+\frac{1}{64}\sin\left(4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2]^4,x]
[Out] (3*x^2)/16 + Sin[2*x^2]/8 + Sin[4*x^2]/64
```

Maple [A]
time = 0.07, size = 26, normalized size = 0.76

method	result
risch	$\frac{3x^2}{16}+\frac{\sin(4x^2)}{64}+\frac{\sin(2x^2)}{8}$
derivativedivides	$\frac{\left(\cos^3(x^2)+\frac{3\cos(x^2)}{2}\right)\sin(x^2)}{8}+\frac{3x^2}{16}$
default	$\frac{\left(\cos^3(x^2)+\frac{3\cos(x^2)}{2}\right)\sin(x^2)}{8}+\frac{3x^2}{16}$
norman	$\frac{\frac{3x^2}{16}-\frac{3\left(\tan^3\left(\frac{x^2}{2}\right)\right)}{8}+\frac{3\left(\tan^5\left(\frac{x^2}{2}\right)\right)}{8}-\frac{5\left(\tan^7\left(\frac{x^2}{2}\right)\right)}{8}+\frac{3x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{4}+\frac{9x^2\left(\tan^4\left(\frac{x^2}{2}\right)\right)}{8}+\frac{3x^2\left(\tan^6\left(\frac{x^2}{2}\right)\right)}{4}+\frac{3x^2\left(\tan^8\left(\right.\right.}{16}}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)^4,x,method=_RETURNVERBOSE)
[Out] 1/8*(cos(x^2)^3+3/2*cos(x^2))*sin(x^2)+3/16*x^2
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.65

$$\frac{3}{16}\,x^2+\frac{1}{64}\,\sin\left(4\,x^2\right)+\frac{1}{8}\,\sin\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="maxima")
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

Fricas [A]
time = 4.03, size = 27, normalized size = 0.79

$$\frac{3}{16}\,x^{2}+\frac{1}{16}\,\left(2\,\cos\left(x^{2}\right)^{3}+3\,\cos\left(x^{2}\right)\right)\,\sin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="fricas")
[Out] 3/16*x^2 + 1/16*(2*cos(x^2)^3 + 3*cos(x^2))*sin(x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.
time = 0.19, size = 76, normalized size = 2.24

$$\frac{3x^{2}\sin^{4}\left(x^{2}\right)}{16}+\frac{3x^{2}\sin^{2}\left(x^{2}\right)\cos^{2}\left(x^{2}\right)}{8}+\frac{3x^{2}\cos^{4}\left(x^{2}\right)}{16}+\frac{3\sin^{3}\left(x^{2}\right)\cos\left(x^{2}\right)}{16}+\frac{5\sin\left(x^{2}\right)\cos^{3}\left(x^{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)**4,x)
[Out] 3*x**2*sin(x**2)**4/16 + 3*x**2*sin(x**2)**2*cos(x**2)**2/8 + 3*x**2*cos(x*
*2)**4/16 + 3*sin(x**2)**3*cos(x**2)/16 + 5*sin(x**2)*cos(x**2)**3/16
```

Giac [A]
time = 0.41, size = 22, normalized size = 0.65

$$\frac{3}{16}\,x^{2}+\frac{1}{64}\,\sin\left(4\,x^{2}\right)+\frac{1}{8}\,\sin\left(2\,x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)^4,x, algorithm="giac")
[Out] 3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

Mupad [B]
time = 2.97, size = 22, normalized size = 0.65

$$\frac{\sin\left(2\,x^{2}\right)}{8}+\frac{\sin\left(4\,x^{2}\right)}{64}+\frac{3\,x^{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)^4,x)
[Out] sin(2*x^2)/8 + sin(4*x^2)/64 + (3*x^2)/16
```

3.754 $\int \sqrt{\cos(x)} \sin(x) \, dx$

Optimal. Leaf size=10

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

```
[Out] -2/3*cos(x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2645, 30}

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]]*Sin[x],x]
```

```
[Out] (-2*Cos[x]^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(x)} \sin(x) \, dx &= -\text{Subst}\left(\int \sqrt{x} \, dx, x, \cos(x)\right) \\ &= -\frac{2}{3} \cos^{\frac{3}{2}}(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{2}{3} \cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x],x]
[Out] (-2*Cos[x]^(3/2))/3
```

Maple [A]
time = 0.04, size = 7, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right)}{3}$	7
default	$-\frac{2\left(\cos^{\frac{3}{2}}(x)\right)}{3}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*cos(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*cos(x)^(3/2)
```

Maxima [A]
time = 0.29, size = 6, normalized size = 0.60

$$-\frac{2}{3}\cos\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="maxima")
[Out] -2/3*cos(x)^(3/2)
```

Fricas [A]
time = 4.54, size = 6, normalized size = 0.60

$$-\frac{2}{3}\cos\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="fricas")
[Out] -2/3*cos(x)^(3/2)
```

Sympy [A]
time = 0.10, size = 10, normalized size = 1.00

$$-\frac{2\cos^{\frac{3}{2}}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)**(1/2),x)
[Out] -2*cos(x)**(3/2)/3
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.60

$$-\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*cos(x)^(1/2),x, algorithm="giac")
[Out] -2/3*cos(x)^(3/2)
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.60

$$-\frac{2\cos(x)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(1/2)*sin(x),x)
[Out] -(2*cos(x)^(3/2))/3
```

3.755 $\int e^{-2x} \tan\left(e^{-2x}\right) dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

[Out] 1/2*ln(cos(exp(-2*x)))

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2320, 3556}

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

[In] Int[Tan[E^(-2*x)]/E^(2*x),x]

[Out] Log[Cos[E^(-2*x)]]/2

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}\tan\left(e^{-2x}\right)\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \tan(x)\,dx, x, e^{-2x}\right)\right)\\ &= \frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[E^(-2*x)]/E^(2*x),x]
```

```
[Out] Log[Cos[E^(-2*x)]]/2
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.82

method	result	size
default	$\frac{\ln\left(\cos\left(\mathrm{e}^{-2x}\right)\right)}{2}$	9
norman	$-\frac{\ln\left(1+\tan^{2}\left(\mathrm{e}^{-2x}\right)\right)}{4}$	13
risch	$-\frac{i\mathrm{e}^{-2x}}{2}+\frac{\ln\left(\mathrm{e}^{2i\mathrm{e}^{-2x}}+1\right)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(exp(-2*x))/exp(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(cos(exp(-2*x)))
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.73

$$-\frac{1}{2}\log\left(\sec\left(e^{(-2x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(sec(e^(-2*x)))
```

Fricas [A]
time = 3.34, size = 14, normalized size = 1.27

$$\frac{1}{4}\log\left(\frac{1}{\tan\left(e^{(-2x)}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(1/(tan(e^(-2*x))^2 + 1))
```

Sympy [A]
time = 0.16, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan^{2}\left(e^{-2x}\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x)
```

```
[Out] -log(tan(exp(-2*x))**2 + 1)/4
```

Giac [A]
time = 0.40, size = 9, normalized size = 0.82

$$\frac{1}{2} \log \left(\left|\cos \left(e^{(-2\,x)}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(cos(e^(-2*x))))
```

Mupad [B]
time = 3.54, size = 12, normalized size = 1.09

$$-\frac{\ln\left(\tan(\mathrm{e}^{-2\,x})^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*x)*tan(exp(-2*x)),x)
```

```
[Out] -log(tan(exp(-2*x))^2 + 1)/4
```

3.756 $\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx$

Optimal. Leaf size=7

$$-2\log(1+\cos(x))$$

```
[Out] -2*ln(cos(x)+1)
```

Rubi [A]
time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {12, 31}

$$-2\log(\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

```
[Out] -2*Log[1 + Cos[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx &= -\text{Subst}\left(\int \frac{2}{1+x}\,dx, x, \cos(x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right)\right)\\
&= -2\log(1+\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.29

$$-4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
[Out] -4*Log[Cos[x/2]]
```

Maple [A]
time = 0.07, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$-2\ln\left(1+\cos\left(x\right)\right)$	8
default	$-2\ln\left(1+\cos\left(x\right)\right)$	8
risch	$2ix-4\ln\left(\mathrm{e}^{ix}+1\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*sin(2*x)/(1+cos(x)),x,method=_RETURNVERBOSE)
[Out] -2*ln(1+cos(x))
```

Maxima [A]
time = 0.30, size = 7, normalized size = 1.00

$$-2\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="maxima")
[Out] -2*log(cos(x) + 1)
```

Fricas [A]
time = 3.39, size = 9, normalized size = 1.29

$$-2\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="fricas")
[Out] -2*log(1/2*cos(x) + 1/2)
```

Sympy [A]
time = 1.23, size = 8, normalized size = 1.14

$$-2\log\left(\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x)
```

```
[Out] -2*log(cos(x) + 1)
```

Giac [**A**]
time = 0.40, size = 7, normalized size = 1.00

$$-2 \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="giac")
[Out] -2*log(cos(x) + 1)
```

Mupad [**B**]
time = 2.92, size = 7, normalized size = 1.00

$$-2 \ln(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(x)*(cos(x) + 1)),x)
[Out] -2*log(cos(x) + 1)
```

3.757 $\int x \sec^2(3x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{9}\log(\cos(3x)) + \frac{1}{3}x\tan(3x)$$

```
[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4269, 3556}

$$\frac{1}{3}x\tan(3x) + \frac{1}{9}\log(\cos(3x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(3x)\,dx &= \frac{1}{3}x\tan(3x) - \frac{1}{3}\int \tan(3x)\,dx\\ &= \frac{1}{9}\log(\cos(3x)) + \frac{1}{3}x\tan(3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{9}\log(\cos(3x)) + \frac{1}{3}x\tan(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[3*x]^2,x]
```

```
[Out] Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

Maple [A]
time = 0.05, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\ln(\cos(3x))}{9}+\frac{x\tan(3x)}{3}$	16
default	$\frac{\ln(\cos(3x))}{9}+\frac{x\tan(3x)}{3}$	16
risch	$-\frac{2ix}{3}+\frac{2ix}{3(e^{6ix}+1)}+\frac{\ln(e^{6ix}+1)}{9}$	29
norman	$-\frac{2x\tan(\frac{3x}{2})}{3(\tan^2(\frac{3x}{2})-1)}+\frac{\ln(\tan(\frac{3x}{2})-1)}{9}+\frac{\ln(\tan(\frac{3x}{2})+1)}{9}-\frac{\ln(1+\tan^2(\frac{3x}{2}))}{9}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(15) = 30.
time = 0.49, size = 74, normalized size = 3.89

$$\frac{(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1)\log(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1)+12x\sin(6x)}{18(\cos(6x)^2+\sin(6x)^2+2\cos(6x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*((cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1)*log(cos(6*x)^2 + sin(6*x)^
2 + 2*cos(6*x) + 1) + 12*x*sin(6*x))/(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x)
+ 1)
```

Fricas [A]
time = 2.91, size = 28, normalized size = 1.47

$$\frac{\cos(3x)\log(-\cos(3x))+3x\sin(3x)}{9\cos(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(cos(3*x)*log(-cos(3*x)) + 3*x*sin(3*x))/cos(3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec^2(3x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)**2,x)
```

```
[Out] Integral(x*sec(3*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(15) = 30.
time = 0.45, size = 103, normalized size = 5.42

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)\tan\left(\frac{3}{2}x\right)^2-12x\tan\left(\frac{3}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)}{18\left(\tan\left(\frac{3}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/18*(log(4*(tan(3/2*x)^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x
)^2 + 1))*tan(3/2*x)^2 - 12*x*tan(3/2*x) - log(4*(tan(3/2*x)^4 - 2*tan(3/2*
x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1)))/(tan(3/2*x)^2 - 1)
```

Mupad [B]
time = 2.90, size = 15, normalized size = 0.79

$$\frac{\ln\left(\cos\left(3\,x\right)\right)}{9}+\frac{x\tan(3\,x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(3*x)^2,x)
```

```
[Out] log(cos(3*x))/9 + (x*tan(3*x))/3
```

3.758 $\int e^{-2\pi x} \cos(2\pi x)\, dx$

Optimal. Leaf size=37

$$-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}+\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}$$

```
[Out] -1/4*cos(2*Pi*x)/exp(2*Pi*x)/Pi+1/4*sin(2*Pi*x)/exp(2*Pi*x)/Pi
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4518}

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*Pi*x]/E^(2*Pi*x),x]
[Out] -1/4*Cos[2*Pi*x]/(E^(2*Pi*x)*Pi) + Sin[2*Pi*x]/(4*E^(2*Pi*x)*Pi)
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-2\pi x}\cos(2\pi x)\,dx=-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}+\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.70

$$\frac{e^{-2\pi x}(-\cos(2\pi x)+\sin(2\pi x))}{4\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*Pi*x]/E^(2*Pi*x),x]
[Out] (-Cos[2*Pi*x] + Sin[2*Pi*x])/(4*E^(2*Pi*x)*Pi)
```

Maple [A]
time = 0.08, size = 31, normalized size = 0.84

method	result	size
derivativedivides	$\frac{-\frac{e^{-2\pi x}\cos(2\pi x)}{2}+\frac{e^{-2\pi x}\sin(2\pi x)}{2}}{2\pi}$	31
default	$\frac{-\frac{e^{-2\pi x}\cos(2\pi x)}{2}+\frac{e^{-2\pi x}\sin(2\pi x)}{2}}{2\pi}$	31
norman	$\frac{\left(-\frac{1}{4\pi}+\frac{\tan(\pi x)}{2\pi}+\frac{\tan^2(\pi x)}{4\pi}\right)e^{-2\pi x}}{1+\tan^2(\pi x)}$	45
risch	$-\frac{e^{(-2+2i)\pi x}}{8\pi}-\frac{ie^{(-2+2i)\pi x}}{8\pi}-\frac{e^{(-2-2i)\pi x}}{8\pi}+\frac{ie^{(-2-2i)\pi x}}{8\pi}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*Pi*x)/exp(2*Pi*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/Pi*(-1/2*exp(-2*Pi*x)*cos(2*Pi*x)+1/2*exp(-2*Pi*x)*sin(2*Pi*x))
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.70

$$-\frac{(\pi\cos(2\pi x)-\pi\sin(2\pi x))e^{(-2\pi x)}}{4\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="maxima")
```

```
[Out] -1/4*(pi*cos(2*pi*x) - pi*sin(2*pi*x))*e^(-2*pi*x)/pi^2
```

Fricas [A]
time = 4.32, size = 29, normalized size = 0.78

$$-\frac{\cos(2\pi x)e^{(-2\pi x)}-e^{(-2\pi x)}\sin(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(2*pi*x)*e^(-2*pi*x) - e^(-2*pi*x)*sin(2*pi*x))/pi
```

Sympy [A]
time = 0.17, size = 32, normalized size = 0.86

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x)
```

```
[Out] exp(-2*pi*x)*sin(2*pi*x)/(4*pi) - exp(-2*pi*x)*cos(2*pi*x)/(4*pi)
```

Giac [**A**]
time = 0.41, size = 23, normalized size = 0.62

$$-\frac{(\cos(2\,\pi x) - \sin(2\,\pi x))e^{(-2\,\pi x)}}{4\,\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="giac")
```

```
[Out] -1/4*(cos(2*pi*x) - sin(2*pi*x))*e^(-2*pi*x)/pi
```

Mupad [**B**]
time = 2.90, size = 25, normalized size = 0.68

$$-\frac{e^{-2\,\Pi\,x}\,(2\,\cos(2\,\Pi\,x) - 2\,\sin(2\,\Pi\,x))}{8\,\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-2*Pi*x)*cos(2*Pi*x),x)
```

```
[Out] -(exp(-2*Pi*x)*(2*cos(2*Pi*x) - 2*sin(2*Pi*x)))/(8*Pi)
```

3.759 $\int \left(\cos^{12}(x)\sin^{10}(x) - \cos^{10}(x)\sin^{12}(x)\right)\,dx$

Optimal. Leaf size=12

$$\frac{1}{11}\cos^{11}(x)\sin^{11}(x)$$

```
[Out] 1/11*cos(x)^11*sin(x)^11
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.
time = 0.22, antiderivative size = 129, normalized size of antiderivative = 10.75, number of steps used = 25, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2648, 2715, 8}

$$-\frac{1}{22}\sin^9(x)\cos^{13}(x)-\frac{9}{440}\sin^7(x)\cos^{13}(x)-\frac{7}{880}\sin^5(x)\cos^{13}(x)-\frac{7\sin^3(x)\cos^{13}(x)}{2816}-\frac{3\sin(x)\cos^{13}(x)}{5632}+\frac{1}{22}\sin^{11}(x)\cos^{11}(x)+\frac{1}{40}\sin^9(x)\cos^{11}(x)+\frac{1}{80}\sin^7(x)\cos^{11}(x)+\frac{7\sin^5(x)\cos^{11}(x)}{1280}+\frac{1}{512}\sin^3(x)\cos^{11}(x)+\frac{3\sin(x)\cos^{11}(x)}{5632}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (3*Cos[x]^11*Sin[x])/5632 - (3*Cos[x]^13*Sin[x])/5632 + (Cos[x]^11*Sin[x]^3
)/512 - (7*Cos[x]^13*Sin[x]^3)/2816 + (7*Cos[x]^11*Sin[x]^5)/1280 - (7*Cos[
x]^13*Sin[x]^5)/880 + (Cos[x]^11*Sin[x]^7)/80 - (9*Cos[x]^13*Sin[x]^7)/440
+ (Cos[x]^11*Sin[x]^9)/40 - (Cos[x]^13*Sin[x]^9)/22 + (Cos[x]^11*Sin[x]^11)
/22
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \left(\cos^{12}(x)\sin^{10}(x) - \cos^{10}(x)\sin^{12}(x)\right)\,dx = \int \cos^{12}(x)\sin^{10}(x)\,dx - \int \cos^{10}(x)\sin^{12}(x)\,dx$$
$$= -\frac{1}{22}\cos^{13}(x)\sin^{9}(x) + \frac{1}{22}\cos^{11}(x)\sin^{11}(x) + \frac{9}{22}\int \cos^{12}(x)\,\text{s}$$
$$= -\frac{9}{440}\cos^{13}(x)\sin^{7}(x) + \frac{1}{40}\cos^{11}(x)\sin^{9}(x) - \frac{1}{22}\cos^{13}(x)\sin$$
$$= -\frac{7}{880}\cos^{13}(x)\sin^{5}(x) + \frac{1}{80}\cos^{11}(x)\sin^{7}(x) - \frac{9}{440}\cos^{13}(x)\sin$$
$$= -\frac{7\cos^{13}(x)\sin^{3}(x)}{2816} + \frac{7\cos^{11}(x)\sin^{5}(x)}{1280} - \frac{7}{880}\cos^{13}(x)\sin^{5}(x$$
$$= -\frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) - \frac{7\cos^{13}(x)\sin^{3}(x)}{2816}$$
$$= \frac{3\cos^{11}(x)\sin(x)}{5632} - \frac{3\cos^{13}(x)\sin(x)}{5632} + \frac{1}{512}\cos^{11}(x)\sin^{3}(x) -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.
time = 0.03, size = 49, normalized size = 4.08

$$\frac{21\sin(2x)}{1048576} - \frac{15\sin(6x)}{1048576} + \frac{15\sin(10x)}{2097152} - \frac{5\sin(14x)}{2097152} + \frac{\sin(18x)}{2097152} - \frac{\sin(22x)}{23068672}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

```
[Out] (21*Sin[2*x])/1048576 - (15*Sin[6*x])/1048576 + (15*Sin[10*x])/2097152 - (5
*Sin[14*x])/2097152 + Sin[18*x]/2097152 - Sin[22*x]/23068672
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(10) = 20.
time = 0.15, size = 176, normalized size = 14.67

method	result
risch	$-\frac{\sin(22x)}{23068672} + \frac{\sin(18x)}{2097152} - \frac{5\sin(14x)}{2097152} + \frac{15\sin(10x)}{2097152} - \frac{15\sin(6x)}{1048576} + \frac{21\sin(2x)}{1048576}$
default	$-\frac{(\cos^{13}(x))(\sin^{9}(x))}{22} - \frac{9(\sin^{7}(x))(\cos^{13}(x))}{440} - \frac{7(\sin^{5}(x))(\cos^{13}(x))}{880} - \frac{7(\sin^{3}(x))(\cos^{13}(x))}{2816} - \frac{3\sin(x)(\cos^{13}(x))}{5632} + \frac{(\cos^{1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/22*cos(x)^13*sin(x)^9-9/440*sin(x)^7*cos(x)^13-7/880*sin(x)^5*cos(x)^13-
7/2816*sin(x)^3*cos(x)^13-3/5632*sin(x)*cos(x)^13+1/22528*(cos(x)^11+11/10*
cos(x)^9+99/80*cos(x)^7+231/160*cos(x)^5+231/128*cos(x)^3+693/256*cos(x))*s
in(x)+1/22*cos(x)^11*sin(x)^11+1/40*sin(x)^9*cos(x)^11+1/80*sin(x)^7*cos(x)
^11+7/1280*sin(x)^5*cos(x)^11+1/512*sin(x)^3*cos(x)^11+1/2048*sin(x)*cos(x)
^11-1/20480*(cos(x)^9+9/8*cos(x)^7+21/16*cos(x)^5+105/64*cos(x)^3+315/128*c
os(x))*sin(x)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.67

$$\frac{1}{22528} \sin\left(2\,x\right)^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="maxima")
```

```
[Out] 1/22528*sin(2*x)^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.
time = 4.13, size = 39, normalized size = 3.25

$$-\frac{1}{11} \left(\cos\left(x\right)^{21} - 5 \cos\left(x\right)^{19} + 10 \cos\left(x\right)^{17} - 10 \cos\left(x\right)^{15} + 5 \cos\left(x\right)^{13} - \cos\left(x\right)^{11}\right) \sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(cos(x)^21 - 5*cos(x)^19 + 10*cos(x)^17 - 10*cos(x)^15 + 5*cos(x)^13
- cos(x)^11)*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(10) = 20.
time = 0.02, size = 236, normalized size = 19.67

$$-\frac{\sin^{21}\left(x\right)\cos\left(x\right)}{22} + \frac{89\sin^{19}\left(x\right)\cos\left(x\right)}{440} - \frac{301\sin^{17}\left(x\right)\cos\left(x\right)}{880} + \frac{3683\sin^{15}\left(x\right)\cos\left(x\right)}{14080} - \frac{433\sin^{13}\left(x\right)\cos\left(x\right)}{5632} + \frac{\sin^{11}\left(x\right)\cos\left(x\right)}{22528} + \frac{\sin^{9}\left(x\right)\cos\left(x\right)}{20480} + \frac{9\sin^{7}\left(x\right)\cos\left(x\right)}{163840} + \frac{21\sin^{5}\left(x\right)\cos\left(x\right)}{327680} + \frac{21\sin^{3}\left(x\right)\cos\left(x\right)}{262144} - \frac{\sin\left(x\right)\cos^{21}\left(x\right)}{22} + \frac{89\sin\left(x\right)\cos^{19}\left(x\right)}{440} - \frac{301\sin\left(x\right)\cos^{17}\left(x\right)}{880} + \frac{3683\sin\left(x\right)\cos^{15}\left(x\right)}{14080} - \frac{433\sin\left(x\right)\cos^{13}\left(x\right)}{5632} + \frac{\sin\left(x\right)\cos^{11}\left(x\right)}{22528} + \frac{\sin\left(x\right)\cos^{9}\left(x\right)}{20480} + \frac{9\sin\left(x\right)\cos^{7}\left(x\right)}{163840} + \frac{21\sin\left(x\right)\cos^{5}\left(x\right)}{327680} + \frac{21\sin\left(x\right)\cos^{3}\left(x\right)}{262144} + \frac{63\sin\left(x\right)\cos\left(x\right)}{262144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**12*sin(x)**10-cos(x)**10*sin(x)**12,x)
```

```
[Out] -sin(x)**21*cos(x)/22 + 89*sin(x)**19*cos(x)/440 - 301*sin(x)**17*cos(x)/88
0 + 3683*sin(x)**15*cos(x)/14080 - 433*sin(x)**13*cos(x)/5632 + sin(x)**11*
cos(x)/22528 + sin(x)**9*cos(x)/20480 + 9*sin(x)**7*cos(x)/163840 + 21*sin(
x)**5*cos(x)/327680 + 21*sin(x)**3*cos(x)/262144 - sin(x)*cos(x)**21/22 + 8
9*sin(x)*cos(x)**19/440 - 301*sin(x)*cos(x)**17/880 + 3683*sin(x)*cos(x)**1
5/14080 - 433*sin(x)*cos(x)**13/5632 + sin(x)*cos(x)**11/22528 + sin(x)*cos
```

```
(x)**9/20480 + 9*sin(x)*cos(x)**7/163840 + 21*sin(x)*cos(x)**5/327680 + 21*
sin(x)*cos(x)**3/262144 + 63*sin(x)*cos(x)/262144
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.
time = 0.42, size = 37, normalized size = 3.08

$$-\frac{1}{23068672}\sin(22x) + \frac{1}{2097152}\sin(18x) - \frac{5}{2097152}\sin(14x) + \frac{15}{2097152}\sin(10x) - \frac{15}{1048576}\sin(6x) + \frac{21}{1048576}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="giac")
```

```
[Out] -1/23068672*sin(22*x) + 1/2097152*sin(18*x) - 5/2097152*sin(14*x) + 15/2097
152*sin(10*x) - 15/1048576*sin(6*x) + 21/1048576*sin(2*x)
```

Mupad [B]
time = 2.99, size = 49, normalized size = 4.08

$$-\frac{\sin(x)\cos(x)^{21}}{11} + \frac{5\sin(x)\cos(x)^{19}}{11} - \frac{10\sin(x)\cos(x)^{17}}{11} + \frac{10\sin(x)\cos(x)^{15}}{11} - \frac{5\sin(x)\cos(x)^{13}}{11} + \frac{\sin(x)\cos(x)^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^12*sin(x)^10 - cos(x)^10*sin(x)^12,x)
```

```
[Out] (cos(x)^11*sin(x))/11 - (5*cos(x)^13*sin(x))/11 + (10*cos(x)^15*sin(x))/11
- (10*cos(x)^17*sin(x))/11 + (5*cos(x)^19*sin(x))/11 - (cos(x)^21*sin(x))/1
1
```

3.760 $\int x \cot(x^2)\, dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

[Out] 1/2*ln(sin(x^2))

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3833, 3556}

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

Antiderivative was successfully verified.

[In] Int[x*Cot[x^2],x]

[Out] Log[Sin[x^2]]/2

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3833

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int x \cot\left(x^2\right)\, dx &= \frac{1}{2}\text{Subst}\left(\int \cot(x)\, dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(\sin\left(x^2\right)\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.01, size = 19, normalized size = 2.11

$$\frac{1}{2}\log\left(\cos\left(x^2\right)\right)+\frac{1}{2}\log\left(\tan\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cot[x^2],x]
[Out] Log[Cos[x^2]]/2 + Log[Tan[x^2]]/2
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\sin(x^2))}{2}$	8
default	$\frac{\ln(\sin(x^2))}{2}$	8
norman	$\frac{\ln(\tan(x^2))}{2} - \frac{\ln(1+\tan^2(x^2))}{4}$	20
risch	$-\frac{ix^2}{2} + \frac{\ln\left(e^{2ix^2}-1\right)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sin(x^2))
```

Maxima [A]
time = 0.29, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="maxima")
[Out] 1/2*log(sin(x^2))
```

Fricas [A]
time = 4.09, size = 13, normalized size = 1.44

$$\frac{1}{4}\log\left(-\frac{1}{2}\cos\left(2\,x^2\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="fricas")
[Out] 1/4*log(-1/2*cos(2*x^2) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.
time = 0.06, size = 19, normalized size = 2.11

$$-\frac{\log\left(\tan^2\left(x^2\right)+1\right)}{4}+\frac{\log\left(\tan\left(x^2\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x**2),x)
```

```
[Out] -log(tan(x**2)**2 + 1)/4 + log(tan(x**2))/2
```

Giac [A]
time = 0.40, size = 8, normalized size = 0.89

$$\frac{1}{2}\log\left(\left|\sin\left(x^2\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cot(x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(sin(x^2)))
```

Mupad [B]
time = 0.12, size = 19, normalized size = 2.11

$$\frac{\ln\left(\mathrm{e}^{x^2\,2\mathrm{i}}-1\right)}{2}-\frac{x^2\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cot(x^2),x)
```

```
[Out] log(exp(x^2*2i) - 1)/2 - (x^2*1i)/2
```

3.761 $\int x \sec^2(x^2)\,dx$

Optimal. Leaf size=8

$$\frac{\tan(x^2)}{2}$$

[Out] 1/2*tan(x^2)

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4289, 3852, 8}

$$\frac{\tan(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x^2]^2,x]

[Out] Tan[x^2]/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(x^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sec^2(x)\,dx, x, x^2\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int 1\,dx, x, -\tan(x^2)\right)\right)\\ &= \frac{\tan(x^2)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\tan(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]^2,x]
```

```
[Out] Tan[x^2]/2
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\tan(x^2)}{2}$	7
default	$\frac{\tan(x^2)}{2}$	7
risch	$\frac{i}{e^{2ix^2}+1}$	15
norman	$-\frac{\tan\left(\frac{x^2}{2}\right)}{\tan^2\left(\frac{x^2}{2}\right)-1}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tan(x^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(6) = 12$.
time = 0.28, size = 35, normalized size = 4.38

$$\frac{\sin(2x^2)}{\cos(2x^2)^2 + \sin(2x^2)^2 + 2\cos(2x^2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="maxima")
```

```
[Out] sin(2*x^2)/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)
```

Fricas [A]
time = 3.16, size = 12, normalized size = 1.50

$$\frac{\sin(x^2)}{2\cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2)/cos(x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec^2\left(x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2,x)
```

```
[Out] Integral(x*sec(x**2)**2, x)
```

Giac [A]
time = 0.39, size = 6, normalized size = 0.75

$$\frac{1}{2}\tan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*tan(x^2)
```

Mupad [B]
time = 0.10, size = 14, normalized size = 1.75

$$\frac{1\mathrm{i}}{\mathrm{e}^{x^2\,2\mathrm{i}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x^2)^2,x)
```

```
[Out] 1i/(exp(x^2*2i) + 1)
```

3.762 $\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{12}\text{ArcTan}\left(\frac{1}{3}\sin^2(4x)\right)$$

[Out] 1/12*arctan(1/3*sin(4*x)^2)

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {12, 281, 209}

$$\frac{1}{12}\text{ArcTan}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[8*x]/(9 + Sin[4*x]^4),x]

[Out] ArcTan[Sin[4*x]^2/3]/12

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{2x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{9+x^4}\,dx, x, \sin(4x)\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \sin^2(4x)\right)\\ &= \frac{1}{12}\tan^{-1}\left(\frac{1}{3}\sin^2(4x)\right)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{12}\text{ArcTan}\left(\frac{1}{3}\sin^2(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[8*x]/(9 + Sin[4*x]^4),x]
[Out] ArcTan[Sin[4*x]^2/3]/12
```

Maple [**A**]
time = 0.14, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\arctan\left(\frac{\left(\sin^2(4x)\right)}{3}\right)}{12}$	12
risch	$-\frac{i\ln\left(e^{16ix}+(-2+12i)e^{8ix}+1\right)}{24}+\frac{i\ln\left(e^{16ix}+(-2-12i)e^{8ix}+1\right)}{24}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(8*x)/(9+sin(4*x)^4),x,method=_RETURNVERBOSE)
[Out] 1/12*arctan(1/3*sin(4*x)^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="maxima")
```

```
[Out] integrate(sin(8*x)/(sin(4*x)^4 + 9), x)
```

Fricas [**A**]
time = 3.56, size = 13, normalized size = 0.87

$$-\frac{1}{12} \arctan\left(\frac{1}{3} \cos(4x)^2 - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="fricas")
[Out] -1/12*arctan(1/3*cos(4*x)^2 - 1/3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)**4),x)
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 11, normalized size = 0.73

$$\frac{1}{12} \arctan\left(\frac{1}{3} \sin(4x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="giac")
[Out] 1/12*arctan(1/3*sin(4*x)^2)
```

Mupad [**B**]
time = 2.96, size = 13, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{10\tan(4x)^2}{3} + 3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(8*x)/(sin(4*x)^4 + 9),x)
[Out] atan((10*tan(4*x)^2)/3 + 3)/12
```

3.763 $\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\text{ArcTan}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3269, 209}

$$\frac{\text{ArcTan}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

```
[Out] ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{8+x^2}\,dx, x, \sin(2x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 0.87

$$\frac{\text{ArcTan}\left(\frac{\cos(x)\sin(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]/(8 + Sin[2*x]^2),x]
[Out] ArcTan[(Cos[x]*Sin[x])/Sqrt[2]]/(4*Sqrt[2])
```

Maple [A]
time = 0.05, size = 16, normalized size = 0.70

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$	16
default	$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$	16
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4ix}-4\sqrt{2}\,e^{2ix}-1\right)}{16}-\frac{i\sqrt{2}\,\ln\left(e^{4ix}+4\sqrt{2}\,e^{2ix}-1\right)}{16}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/(8+sin(2*x)^2),x,method=_RETURNVERBOSE)
[Out] 1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.65

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="maxima")
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

Fricas [A]
time = 3.43, size = 15, normalized size = 0.65

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin\left(2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

Sympy [A]
time = 0.08, size = 19, normalized size = 0.83

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sin(2x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)**2),x)
[Out] sqrt(2)*atan(sqrt(2)*sin(2*x)/4)/8
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.65

$$\frac{1}{8}\,\sqrt{2}\,\arctan\left(\frac{1}{4}\,\sqrt{2}\,\sin\left(2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="giac")
[Out] 1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

Mupad [B]
time = 0.07, size = 15, normalized size = 0.65

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,\sin(2\,x)}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/(sin(2*x)^2 + 8),x)
[Out] (2^(1/2)*atan((2^(1/2)*sin(2*x))/4))/8
```

3.764 $\int x(\cos^3(x^2) - \sin^3(x^2))\, dx$

Optimal. Leaf size=37

$$\frac{\cos(x^2)}{2} - \frac{1}{6}\cos^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6}\sin^3(x^2)$$

```
[Out] 1/2*cos(x^2)-1/6*cos(x^2)^3+1/2*sin(x^2)-1/6*sin(x^2)^3
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {14, 3461, 2713, 3460}

$$-\frac{1}{6}\sin^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6}\cos^3(x^2) + \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

```
[Out] Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
```

```
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx &= \int\left(x\cos^3\left(x^2\right)-x\sin^3\left(x^2\right)\right)\,dx\\
&= \int x\cos^3\left(x^2\right)\,dx-\int x\sin^3\left(x^2\right)\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int\cos^3(x)\,dx,x,x^2\right)-\frac{1}{2}\text{Subst}\left(\int\sin^3(x)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(x^2\right)\right)-\frac{1}{2}\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin\left(x\right.\right.\\
&= \frac{\cos\left(x^2\right)}{2}-\frac{1}{6}\cos^3\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\sin^3\left(x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.00

$$\frac{3\cos\left(x^2\right)}{8}-\frac{1}{24}\cos\left(3x^2\right)+\frac{\sin\left(x^2\right)}{2}-\frac{1}{6}\sin^3\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
[Out] (3*Cos[x^2])/8 - Cos[3*x^2]/24 + Sin[x^2]/2 - Sin[x^2]^3/6
```

Maple [A]
time = 0.20, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\left(2+\cos^2\left(x^2\right)\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$	30
default	$\frac{\left(2+\cos^2\left(x^2\right)\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$	30
risch	$\frac{3\cos\left(x^2\right)}{8}+\frac{3\sin\left(x^2\right)}{8}-\frac{\cos\left(3x^2\right)}{24}+\frac{\sin\left(3x^2\right)}{24}$	30
norman	$\frac{\tan^5\left(\frac{x^2}{2}\right)-2\left(\tan^4\left(\frac{x^2}{2}\right)\right)-\frac{2\left(\tan^6\left(\frac{x^2}{2}\right)\right)}{3}+\frac{2\left(\tan^3\left(\frac{x^2}{2}\right)\right)}{3}+\tan\left(\frac{x^2}{2}\right)}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^3}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x^2)^3-sin(x^2)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2+cos(x^2)^2)*sin(x^2)+1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A]
time = 0.29, size = 29, normalized size = 0.78

$$-\frac{1}{24}\cos\left(3x^2\right)+\frac{3}{8}\cos\left(x^2\right)+\frac{1}{24}\sin\left(3x^2\right)+\frac{3}{8}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="maxima")
[Out] -1/24*cos(3*x^2) + 3/8*cos(x^2) + 1/24*sin(3*x^2) + 3/8*sin(x^2)
```

Fricas [A]
time = 2.64, size = 29, normalized size = 0.78

$$-\frac{1}{6}\cos\left(x^2\right)^3+\frac{1}{6}\left(\cos\left(x^2\right)^2+2\right)\sin\left(x^2\right)+\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="fricas")
[Out] -1/6*cos(x^2)^3 + 1/6*(cos(x^2)^2 + 2)*sin(x^2) + 1/2*cos(x^2)
```

Sympy [A]
time = 0.15, size = 42, normalized size = 1.14

$$\frac{\sin^3\left(x^2\right)}{3}+\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)\cos^2\left(x^2\right)}{2}+\frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x**2)**3-sin(x**2)**3),x)
[Out] sin(x**2)**3/3 + sin(x**2)**2*cos(x**2)/2 + sin(x**2)*cos(x**2)**2/2 + cos(
x**2)**3/3
```

Giac [A]
time = 0.40, size = 29, normalized size = 0.78

$$-\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{6}\sin\left(x^2\right)^3+\frac{1}{2}\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="giac")
[Out] -1/6*cos(x^2)^3 - 1/6*sin(x^2)^3 + 1/2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]
time = 2.97, size = 33, normalized size = 0.89

$$-\frac{\cos\left(x^2\right)^3}{6}+\frac{\sin\left(x^2\right)\,\cos\left(x^2\right)^2}{6}+\frac{\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x^2)^3 - sin(x^2)^3),x)
```

```
[Out] cos(x^2)/2 + sin(x^2)/3 + (cos(x^2)^2*sin(x^2))/6 - cos(x^2)^3/6
```

3.765 $\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

[Out] cos(x)+ln(1-cos(x))

Rubi [A]

time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2912, 45}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/(1 - Cos[x]),x]

[Out] Cos[x] + Log[1 - Cos[x]]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, -\cos(x)\right)\\ &= \cos(x) + \log(1-\cos(x))\end{aligned}$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.20

$$\cos(x) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

```
[Out] Cos[x] + 2*Log[Sin[x/2]]
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\cos(x)+\ln(\cos(x)-1)$	9
default	$\cos(x)+\ln(\cos(x)-1)$	9
risch	$-ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+2\ln(e^{ix}-1)$	30
norman	$\frac{2\tan(\frac{x}{2})+2(\tan^3(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^2\tan(\frac{x}{2})}+2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)+ln(cos(x)-1)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\cos(x)+\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] cos(x) + log(cos(x) - 1)
```

Fricas [A]
time = 3.65, size = 10, normalized size = 1.00

$$\cos(x)+\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.80

$$\log(\cos(x)-1)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x)
```

```
[Out] log(cos(x) - 1) + cos(x)
```

Giac [A]
time = 0.41, size = 10, normalized size = 1.00

$$\cos(x) + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="giac")
```

```
[Out] cos(x) + log(-cos(x) + 1)
```

Mupad [B]
time = 2.92, size = 8, normalized size = 0.80

$$\ln(\cos(x) - 1) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)*sin(x))/(cos(x) - 1),x)
```

```
[Out] log(cos(x) - 1) + cos(x)
```

3.766 $\int x\cos(x^2)\,dx$

Optimal. Leaf size=8

$$\frac{\sin(x^2)}{2}$$

[Out] 1/2*sin(x^2)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3461, 2717}

$$\frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x^2],x]

[Out] Sin[x^2]/2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos(x^2)\,dx &= \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)\\ &= \frac{\sin(x^2)}{2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2],x]
```

```
[Out] Sin[x^2]/2
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\sin(x^2)}{2}$	7
default	$\frac{\sin(x^2)}{2}$	7
meijerg	$\frac{\sin(x^2)}{2}$	7
risch	$\frac{\sin(x^2)}{2}$	7
norman	$\frac{\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x^2)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 0.75

$$\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2)
```

Fricas [A]
time = 3.65, size = 6, normalized size = 0.75

$$\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2)
```

Sympy [A]
time = 0.06, size = 5, normalized size = 0.62

$$\frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2),x)
```

```
[Out] sin(x**2)/2
```

Giac [A]
time = 0.41, size = 6, normalized size = 0.75

$$\frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2)
```

Mupad [B]
time = 0.06, size = 6, normalized size = 0.75

$$\frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2),x)
```

```
[Out] sin(x^2)/2
```

3.767 $\int x^2 \cos(4x^3)\,dx$

Optimal. Leaf size=10

$$\frac{1}{12}\sin\left(4x^3\right)$$

[Out] 1/12*sin(4*x^3)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3461, 2717}

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

[In] Int[x^2*Cos[4*x^3],x]

[Out] Sin[4*x^3]/12

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2\cos\left(4x^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int\cos(4x)\,dx, x, x^3\right)\\ &= \frac{1}{12}\sin\left(4x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{12}\sin\left(4x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3],x]
```

```
[Out] Sin[4*x^3]/12
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\sin(4x^3)}{12}$	9
default	$\frac{\sin(4x^3)}{12}$	9
meijerg	$\frac{\sin(4x^3)}{12}$	9
risch	$\frac{\sin(4x^3)}{12}$	9
norman	$\frac{\tan(2x^3)}{6+6(\tan^2(2x^3))}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*sin(4*x^3)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{12}\sin\left(4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="maxima")
```

```
[Out] 1/12*sin(4*x^3)
```

Fricas [A]
time = 3.83, size = 8, normalized size = 0.80

$$\frac{1}{12}\sin\left(4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="fricas")
```

```
[Out] 1/12*sin(4*x^3)
```

Sympy [A]
time = 0.07, size = 7, normalized size = 0.70

$$\frac{\sin(4x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3),x)
```

```
[Out] sin(4*x**3)/12
```

Giac [A]
time = 0.40, size = 8, normalized size = 0.80

$$\frac{1}{12}\sin\left(4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3),x, algorithm="giac")
```

```
[Out] 1/12*sin(4*x^3)
```

Mupad [B]
time = 0.06, size = 8, normalized size = 0.80

$$\frac{\sin(4\,x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3),x)
```

```
[Out] sin(4*x^3)/12
```

3.768 $\int x^3 \cos(x^4) \, dx$

Optimal. Leaf size=8

$$\frac{\sin(x^4)}{4}$$

```
[Out] 1/4*sin(x^4)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3461, 2717}

$$\frac{\sin(x^4)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x^4],x]
```

```
[Out] Sin[x^4]/4
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3 \cos(x^4) \, dx &= \frac{1}{4}\text{Subst}\left(\int \cos(x) \, dx, x, x^4\right)\\ &= \frac{\sin(x^4)}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\sin(x^4)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^4],x]
```

```
[Out] Sin[x^4]/4
```

Maple [A]
time = 0.04, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\sin(x^4)}{4}$	7
default	$\frac{\sin(x^4)}{4}$	7
meijerg	$\frac{\sin(x^4)}{4}$	7
risch	$\frac{\sin(x^4)}{4}$	7
norman	$\frac{\tan\left(\frac{x^4}{2}\right)}{2+2\left(\tan^2\left(\frac{x^4}{2}\right)\right)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(x^4)
```

Maxima [A]
time = 0.30, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x^4)
```

Fricas [A]
time = 3.78, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="fricas")
```

```
[Out] 1/4*sin(x^4)
```

Sympy [A]
time = 0.12, size = 5, normalized size = 0.62

$$\frac{\sin(x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**4),x)
```

```
[Out] sin(x**4)/4
```

Giac [A]
time = 0.42, size = 6, normalized size = 0.75

$$\frac{1}{4}\sin\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^4),x, algorithm="giac")
```

```
[Out] 1/4*sin(x^4)
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.75

$$\frac{\sin(x^4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^4),x)
```

```
[Out] sin(x^4)/4
```

3.769 $\int x \sin\left(\frac{x^2}{2}\right) dx$

Optimal. Leaf size=10

$$-\cos\left(\frac{x^2}{2}\right)$$

```
[Out] -cos(1/2*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3460, 2718}

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2/2],x]
[Out] -Cos[x^2/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x \sin\left(\frac{x^2}{2}\right) dx &= \frac{1}{2}\text{Subst}\left(\int \sin\left(\frac{x}{2}\right) dx, x, x^2\right)\\ &= -\cos\left(\frac{x^2}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\cos\left(\frac{x^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2/2],x]
[Out] -Cos[x^2/2]
```

Maple [A]
time = 0.01, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\cos\left(\frac{x^{2}}{2}\right)$	9
default	$-\cos\left(\frac{x^{2}}{2}\right)$	9
risch	$-\cos\left(\frac{x^{2}}{2}\right)$	9
norman	$-\frac{2}{1+\tan^{2}\left(\frac{x^{2}}{4}\right)}$	15
meijerg	$\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{x^{2}}{2}\right)}{\sqrt{\pi}}\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(1/2*x^2),x,method=_RETURNVERBOSE)
[Out] -cos(1/2*x^2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="maxima")
[Out] -cos(1/2*x^2)
```

Fricas [A]
time = 3.75, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="fricas")
[Out] -cos(1/2*x^2)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x**2),x)
```

```
[Out] -cos(x**2/2)
```

Giac [A]
time = 0.40, size = 8, normalized size = 0.80

$$-\cos\left(\frac{1}{2}\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/2*x^2),x, algorithm="giac")
```

```
[Out] -cos(1/2*x^2)
```

Mupad [B]
time = 2.92, size = 8, normalized size = 0.80

$$-\cos\left(\frac{x^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2/2),x)
```

```
[Out] -cos(x^2/2)
```

3.770 $\int x \sec(x^2) \tan(x^2)\, dx$

Optimal. Leaf size=8

$$\frac{\sec(x^2)}{2}$$

```
[Out] 1/2*sec(x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6847, 2686, 8}

$$\frac{\sec(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]*Tan[x^2],x]
```

```
[Out] Sec[x^2]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int x \sec(x^2) \tan(x^2)\, dx &= \frac{1}{2}\text{Subst}\left(\int \sec(x)\tan(x)\, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int 1\, dx, x, \sec(x^2)\right)\\
&= \frac{\sec(x^2)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]*Tan[x^2],x]
```

```
[Out] Sec[x^2]/2
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\sec(x^2)}{2}$	7
default	$\frac{\sec(x^2)}{2}$	7
risch	$\frac{e^{i x^2}}{e^{2 i x^2}+1}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)*tan(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sec(x^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(6) = 12.
time = 0.28, size = 56, normalized size = 7.00

$$\frac{\cos(2\,x^2)\cos(x^2) + \sin(2\,x^2)\sin(x^2) + \cos(x^2)}{\cos(2\,x^2)^2 + \sin(2\,x^2)^2 + 2\,\cos(2\,x^2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="maxima")
```

```
[Out] (cos(2*x^2)*cos(x^2) + sin(2*x^2)*sin(x^2) + cos(x^2))/(cos(2*x^2)^2 + sin(
2*x^2)^2 + 2*cos(2*x^2) + 1)
```

Fricas [A]
time = 3.27, size = 8, normalized size = 1.00

$$\frac{1}{2\,\cos(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="fricas")
[Out] 1/2/cos(x^2)
```

Sympy [A]
time = 0.09, size = 5, normalized size = 0.62

$$\frac{\sec\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)*tan(x**2),x)
[Out] sec(x**2)/2
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.00

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)*tan(x^2),x, algorithm="giac")
[Out] 1/2/cos(x^2)
```

Mupad [B]
time = 0.07, size = 8, normalized size = 1.00

$$\frac{1}{2\cos\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x^2))/cos(x^2),x)
[Out] 1/(2*cos(x^2))
```

3.771 $\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=10

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

```
[Out] 1/x-tan(1/x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3832, 3554, 8}

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x^(-1)]^2/x^2,x]
```

```
[Out] x^(-1) - Tan[x^(-1)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\text{Subst}\left(\int \tan^2(x)\,dx, x, \frac{1}{x}\right)$$
$$= -\tan\left(\frac{1}{x}\right) + \text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

Mathematica [A]
time = 0.02, size = 12, normalized size = 1.20

$$\text{ArcTan}\left(\tan\left(\frac{1}{x}\right)\right) - \tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x^(-1)]^2/x^2,x]
[Out] ArcTan[Tan[x^(-1)]] - Tan[x^(-1)]
```

Maple [A]
time = 0.03, size = 13, normalized size = 1.30

method	result	size
derivativedivides	$-\tan\left(\frac{1}{x}\right)+\arctan\left(\tan\left(\frac{1}{x}\right)\right)$	13
default	$-\tan\left(\frac{1}{x}\right)+\arctan\left(\tan\left(\frac{1}{x}\right)\right)$	13
norman	$\frac{1-\tan\left(\frac{1}{x}\right)x}{x}$	14
risch	$\frac{1}{x}-\frac{2i}{e^{\frac{2i}{x}}+1}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(1/x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -tan(1/x)+arctan(tan(1/x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(10) = 20.
time = 0.29, size = 67, normalized size = 6.70

$$\frac{\cos\left(\frac{2}{x}\right)^2 - 2\,x\sin\left(\frac{2}{x}\right) + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1}{\left(\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] (cos(2/x)^2 - 2*x*sin(2/x) + sin(2/x)^2 + 2*cos(2/x) + 1)/((cos(2/x)^2 + si
n(2/x)^2 + 2*cos(2/x) + 1)*x)
```

Fricas [A]
time = 3.92, size = 13, normalized size = 1.30

$$-\frac{x\tan\left(\frac{1}{x}\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(x*tan(1/x) - 1)/x
```

Sympy [A]
time = 0.08, size = 7, normalized size = 0.70

$$-\tan\left(\frac{1}{x}\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)**2/x**2,x)
```

```
[Out] -tan(1/x) + 1/x
```

Giac [A]
time = 0.40, size = 10, normalized size = 1.00

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(1/x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/x - tan(1/x)
```

Mupad [B]
time = 2.92, size = 10, normalized size = 1.00

$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(1/x)^2/x^2,x)
```

```
[Out] 1/x - tan(1/x)
```

3.772 $\int x \tan(1+x^2)\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\cos\left(1+x^2\right)\right)$$

```
[Out] -1/2*ln(cos(x^2+1))
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3832, 3556}

$$-\frac{1}{2}\log\left(\cos\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[1 + x^2],x]
```

```
[Out] -1/2*Log[Cos[1 + x^2]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x\tan\left(1+x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \tan(1+x)\,dx, x, x^2\right)\\ &= -\frac{1}{2}\log\left(\cos\left(1+x^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{2}\log\left(\cos\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[1 + x^2],x]
[Out] -1/2*Log[Cos[1 + x^2]]
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$-\frac{\ln(\cos(x^2+1))}{2}$	10
default	$-\frac{\ln(\cos(x^2+1))}{2}$	10
norman	$\frac{\ln(1+\tan^2(x^2+1))}{4}$	14
risch	$\frac{i x^2}{2}+i-\frac{\ln\left(e^{2i\left(x^2+1\right)}+1\right)}{2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(cos(x^2+1))
```

Maxima [A]
time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\sec\left(x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="maxima")
[Out] 1/2*log(sec(x^2 + 1))
```

Fricas [A]
time = 2.52, size = 15, normalized size = 1.36

$$-\frac{1}{4}\log\left(\frac{1}{\tan\left(x^2+1\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="fricas")
[Out] -1/4*log(1/(tan(x^2 + 1)^2 + 1))
```

Sympy [A]
time = 0.04, size = 12, normalized size = 1.09

$$\frac{\log\left(\tan^2\left(x^2+1\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x**2+1),x)
```

```
[Out] log(tan(x**2 + 1)**2 + 1)/4
```

Giac [A]
time = 0.45, size = 10, normalized size = 0.91

$$-\frac{1}{2}\,\log\left(\left|\cos\left(x^2+1\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(cos(x^2 + 1)))
```

Mupad [B]
time = 0.28, size = 13, normalized size = 1.18

$$\frac{\ln\left(\tan(x^2+1)^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x^2 + 1),x)
```

```
[Out] log(tan(x^2 + 1)^2 + 1)/4
```

3.773 $\int \sin(\pi(1+2x))\,dx$

Optimal. Leaf size=12

$$\frac{\cos(2\pi x)}{2\pi}$$

[Out] 1/2*cos(2*Pi*x)/Pi

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2718}

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

[In] Int[Sin[Pi*(1 + 2*x)],x]

[Out] Cos[2*Pi*x]/(2*Pi)

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\pi x)}{2\pi}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{\cos(2\pi x)}{2\pi}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[Pi*(1 + 2*x)],x]

[Out] Cos[2*Pi*x]/(2*Pi)

Maple [A]
time = 0.04, size = 11, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\cos(2\pi x)}{2\pi}$	11
default	$\frac{\cos(2\pi x)}{2\pi}$	11
risch	$\frac{\cos(2\pi x)}{2\pi}$	11
meijerg	$-\frac{\frac{1}{\sqrt{\pi}}-\frac{\cos(2\pi x)}{\sqrt{\pi}}}{2\sqrt{\pi}}$	20
norman	$-\frac{1}{\pi\left(1+\tan^2\left(\frac{\pi(1+2x)}{2}\right)\right)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*(1+2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(2*Pi*x)/Pi
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

Fricas [A]
time = 3.41, size = 12, normalized size = 1.00

$$-\frac{\cos(\pi + 2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*cos(pi + 2*pi*x)/pi
```

Sympy [A]
time = 0.46, size = 12, normalized size = 1.00

$$-\frac{\cos(\pi(2x+1))}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x)
```

```
[Out] -cos(pi*(2*x + 1))/(2*pi)
```

Giac [A]
time = 0.40, size = 10, normalized size = 0.83

$$\frac{\cos(2\pi x)}{2\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*(1+2*x)),x, algorithm="giac")
```

```
[Out] 1/2*cos(2*pi*x)/pi
```

Mupad [B]
time = 2.92, size = 13, normalized size = 1.08

$$-\frac{\cos(\Pi(2x+1))}{2\Pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*(2*x + 1)),x)
```

```
[Out] -cos(Pi*(2*x + 1))/(2*Pi)
```

3.774 $\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=21

$$-\cot(x)-\frac{\cot^2(x)}{2}-\frac{\cot^3(x)}{3}$$

```
[Out] -cot(x)-1/2*cot(x)^2-1/3*cot(x)^3
```

Rubi [A]
time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -Cot[x] - Cot[x]^2/2 - Cot[x]^3/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x+x^2}{x^4}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^4}+\frac{1}{x^3}+\frac{1}{x^2}\right)\,dx,x,\tan(x)\right)\\ &= -\cot(x)-\frac{\cot^2(x)}{2}-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.19

$$-\frac{2\cot(x)}{3}-\frac{\csc^2(x)}{2}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
[Out] (-2*Cot[x])/3 - Csc[x]^2/2 - (Cot[x]*Csc[x]^2)/3
```

Maple [A]
time = 0.13, size = 20, normalized size = 0.95

method	result	size
default	$-\frac{1}{2\tan(x)^2}-\frac{1}{\tan(x)}-\frac{1}{3\tan(x)^3}$	20
risch	$\frac{2\,e^{4ix}+4ie^{2ix}-2\,e^{2ix}-\frac{4i}{3}}{(e^{2ix}-1)^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(x)+csc(x)^2)/(1-cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/2/tan(x)^2-1/tan(x)-1/3/tan(x)^3
```

Maxima [A]
time = 0.30, size = 18, normalized size = 0.86

$$-\frac{6\,\tan(x)^2+3\,\tan(x)+2}{6\,\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

Fricas [A]
time = 2.90, size = 29, normalized size = 1.38

$$-\frac{4\,\cos(x)^3-6\,\cos(x)-3\,\sin(x)}{6\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
[Out] -1/6*(4*cos(x)^3 - 6*cos(x) - 3*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\cot(x)}{\cos^2(x)-1}\,dx-\int\frac{\csc^2(x)}{\cos^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)**2)/(1-cos(x)**2),x)
```

```
[Out] -Integral(cot(x)/(cos(x)**2 - 1), x) - Integral(csc(x)**2/(cos(x)**2 - 1),
x)
```

Giac [A]
time = 0.41, size = 18, normalized size = 0.86

$$-\frac{6\,\tan\left(x\right)^{2}+3\,\tan\left(x\right)+2}{6\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

Mupad [B]
time = 3.00, size = 16, normalized size = 0.76

$$-\frac{\cot(x)\,\left(2\cot(x)^{2}+3\cot(x)+6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cot(x) + 1/sin(x)^2)/(cos(x)^2 - 1),x)
```

```
[Out] -(cot(x)*(3*cot(x) + 2*cot(x)^2 + 6))/6
```

3.775 $\int x^2 \cos(4x^3) \cos(5x^3) \, dx$

Optimal. Leaf size=19

$$\frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

```
[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)
```

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4668, 3461, 2717}

$$\frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

```
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 4668

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigRedu
ce[x^m, Cos[v]^p*Cos[w]^q, x], x] /; IGtQ[m, 0] && IGtQ[p, 0] && IGtQ[q, 0]
 && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && I
ndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\int x^2 \cos(4x^3)\cos(5x^3)\,dx = \int \left(\frac{1}{2}x^2\cos(x^3) + \frac{1}{2}x^2\cos(9x^3)\right)\,dx$$
$$= \frac{1}{2}\int x^2\cos(x^3)\,dx + \frac{1}{2}\int x^2\cos(9x^3)\,dx$$
$$= \frac{1}{6}\text{Subst}\left(\int \cos(x)\,dx, x, x^3\right) + \frac{1}{6}\text{Subst}\left(\int \cos(9x)\,dx, x, x^3\right)$$
$$= \frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[4*x^3]*Cos[5*x^3],x]
[Out] Sin[x^3]/6 + Sin[9*x^3]/54
```

Maple [A]
time = 0.17, size = 16, normalized size = 0.84

method	result	size
default	$\frac{\sin(x^3)}{6}+\frac{\sin(9x^3)}{54}$	16
risch	$\frac{\sin(x^3)}{6}+\frac{\sin(9x^3)}{54}$	16
norman	$\frac{\frac{8\tan(2x^3)\left(\tan^2\left(\frac{5x^3}{2}\right)\right)}{27}-\frac{10\left(\tan^2(2x^3)\right)\tan\left(\frac{5x^3}{2}\right)}{27}-\frac{8\tan(2x^3)}{27}+\frac{10\tan\left(\frac{5x^3}{2}\right)}{27}}{(1+\tan^2(2x^3))\left(1+\tan^2\left(\frac{5x^3}{2}\right)\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3)*cos(5*x^3),x,method=_RETURNVERBOSE)
[Out] 1/6*sin(x^3)+1/54*sin(9*x^3)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.79

$$\frac{1}{54}\sin(9x^3) + \frac{1}{6}\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="maxima")
```

```
[Out] 1/54*sin(9*x^3) + 1/6*sin(x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.
time = 3.38, size = 40, normalized size = 2.11

$$\frac{1}{27}\left(128\cos\left(x^3\right)^8 - 224\cos\left(x^3\right)^6 + 120\cos\left(x^3\right)^4 - 20\cos\left(x^3\right)^2 + 5\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="fricas")
```

```
[Out] 1/27*(128*cos(x^3)^8 - 224*cos(x^3)^6 + 120*cos(x^3)^4 - 20*cos(x^3)^2 + 5)
*sin(x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.78, size = 32, normalized size = 1.68

$$-\frac{4\sin\left(4x^3\right)\cos\left(5x^3\right)}{27} + \frac{5\sin\left(5x^3\right)\cos\left(4x^3\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(4*x**3)*cos(5*x**3),x)
```

```
[Out] -4*sin(4*x**3)*cos(5*x**3)/27 + 5*sin(5*x**3)*cos(4*x**3)/27
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.
time = 0.43, size = 39, normalized size = 2.05

$$\frac{128}{27}\sin\left(x^3\right)^9 - \frac{32}{3}\sin\left(x^3\right)^7 + 8\sin\left(x^3\right)^5 - \frac{20}{9}\sin\left(x^3\right)^3 + \frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="giac")
```

```
[Out] 128/27*sin(x^3)^9 - 32/3*sin(x^3)^7 + 8*sin(x^3)^5 - 20/9*sin(x^3)^3 + 1/3*
sin(x^3)
```

Mupad [B]
time = 3.03, size = 15, normalized size = 0.79

$$\frac{\sin\left(x^3\right)}{6} + \frac{\sin\left(9\,x^3\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(4*x^3)*cos(5*x^3),x)
```

```
[Out] sin(x^3)/6 + sin(9*x^3)/54
```

3.776 $\int x^{14} \sin(x^3)\, dx$

Optimal. Leaf size=47

$$-8\cos(x^3) + 4x^6\cos(x^3) - \frac{1}{3}x^{12}\cos(x^3) - 8x^3\sin(x^3) + \frac{4}{3}x^9\sin(x^3)$$

```
[Out] -8*cos(x^3)+4*x^6*cos(x^3)-1/3*x^12*cos(x^3)-8*x^3*sin(x^3)+4/3*x^9*sin(x^3
)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2718}

$$-8x^3\sin(x^3) - 8\cos(x^3) - \frac{1}{3}x^{12}\cos(x^3) + \frac{4}{3}x^9\sin(x^3) + 4x^6\cos(x^3)$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sin[x^3],x]
```

```
[Out] -8*Cos[x^3] + 4*x^6*Cos[x^3] - (x^12*Cos[x^3])/3 - 8*x^3*Sin[x^3] + (4*x^9*
Sin[x^3])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int x^{14}\sin\left(x^3\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int x^4\sin(x)\,dx,x,x^3\right)\\
&= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}\text{Subst}\left(\int x^3\cos(x)\,dx,x,x^3\right)\\
&= -\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-4\text{Subst}\left(\int x^2\sin(x)\,dx,x,x^3\right)\\
&= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)-8\text{Subst}\left(\int x\cos(x)\,dx,x,x^3\right)\\
&= 4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)+8\text{Subst}\left(\int \sin(x)\,dx,x,x^3\right)\\
&= -8\cos\left(x^3\right)+4x^6\cos\left(x^3\right)-\frac{1}{3}x^{12}\cos\left(x^3\right)-8x^3\sin\left(x^3\right)+\frac{4}{3}x^9\sin\left(x^3\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.74

$$-\frac{1}{3}\left(24-12x^6+x^{12}\right)\cos\left(x^3\right)+\frac{4}{3}x^3\left(-6+x^6\right)\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sin[x^3],x]
[Out] -1/3*((24 - 12*x^6 + x^12)*Cos[x^3]) + (4*x^3*(-6 + x^6)*Sin[x^3])/3
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.70

method	result	size
risch	$\left(-\frac{1}{3}x^{12}+4x^6-8\right)\cos\left(x^3\right)+\frac{4x^3\left(x^6-6\right)\sin\left(x^3\right)}{3}$	33
meijerg	$\frac{16\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^{12}-\frac{9}{2}x^6+9\right)\cos\left(x^3\right)}{6\sqrt{\pi}}-\frac{x^3\left(-\frac{3x^6}{2}+9\right)\sin\left(x^3\right)}{6\sqrt{\pi}}\right)}{3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*sin(x^3),x,method=_RETURNVERBOSE)
[Out] (-1/3*x^12+4*x^6-8)*cos(x^3)+4/3*x^3*(x^6-6)*sin(x^3)
```

Maxima [A]
time = 0.30, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^6+24\right)\cos\left(x^3\right)+\frac{4}{3}\left(x^9-6\,x^3\right)\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

Fricas [A]
time = 3.18, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

Sympy [A]
time = 4.88, size = 48, normalized size = 1.02

$$-\frac{x^{12}\cos\left(x^{3}\right)}{3}+\frac{4x^{9}\sin\left(x^{3}\right)}{3}+4x^{6}\cos\left(x^{3}\right)-8x^{3}\sin\left(x^{3}\right)-8\cos\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*sin(x**3),x)
```

```
[Out] -x**12*cos(x**3)/3 + 4*x**9*sin(x**3)/3 + 4*x**6*cos(x**3) - 8*x**3*sin(x**
3) - 8*cos(x**3)
```

Giac [A]
time = 0.45, size = 32, normalized size = 0.68

$$-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*sin(x^3),x, algorithm="giac")
```

```
[Out] -1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

Mupad [B]
time = 3.00, size = 43, normalized size = 0.91

$$4\,x^{6}\,\cos\left(x^{3}\right)-8\,\cos\left(x^{3}\right)-\frac{x^{12}\,\cos\left(x^{3}\right)}{3}-8\,x^{3}\,\sin\left(x^{3}\right)+\frac{4\,x^{9}\,\sin\left(x^{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*sin(x^3),x)
```

```
[Out] 4*x^6*cos(x^3) - 8*cos(x^3) - (x^12*cos(x^3))/3 - 8*x^3*sin(x^3) + (4*x^9*s
in(x^3))/3
```

3.777 $\int e^{-3x^3}x^2\sin(2x^3)\,dx$

Optimal. Leaf size=35

$$-\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)$$

```
[Out] -2/39*cos(2*x^3)/exp(3*x^3)-1/13*sin(2*x^3)/exp(3*x^3)
```

Rubi [A]
time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {6847, 4517}

$$-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)-\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

```
[Out] (-2*Cos[2*x^3])/(39*E^(3*x^3)) - Sin[2*x^3]/(13*E^(3*x^3))
```

Rule 4517

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int e^{-3x^3}x^2\sin\left(2x^3\right)\,dx&=\frac{1}{3}\text{Subst}\left(\int e^{-3x}\sin(2x)\,dx,x,x^3\right)\\&=-\frac{2}{39}e^{-3x^3}\cos\left(2x^3\right)-\frac{1}{13}e^{-3x^3}\sin\left(2x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 0.80

$$-\frac{1}{39}e^{-3x^3}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

```
[Out] -1/39*(2*Cos[2*x^3] + 3*Sin[2*x^3])/E^(3*x^3)
```

Maple [A]
time = 0.03, size = 36, normalized size = 1.03

method	result	size
norman	$\frac{\left(-\frac{2}{39}+\frac{2\left(\tan^2\left(x^3\right)\right)}{39}-\frac{2\tan\left(x^3\right)}{13}\right)e^{-3x^3}}{1+\tan^2\left(x^3\right)}$	36
risch	$-\frac{e^{(-3+2i)x^3}}{39}+\frac{ie^{(-3+2i)x^3}}{26}-\frac{e^{(-3-2i)x^3}}{39}-\frac{ie^{(-3-2i)x^3}}{26}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x^3)/exp(3*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/39+2/39*tan(x^3)^2-2/13*tan(x^3))/(1+tan(x^3)^2)/exp(3*x^3)
```

Maxima [A]
time = 0.30, size = 25, normalized size = 0.71

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="maxima")
```

```
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

Fricas [A]
time = 3.06, size = 29, normalized size = 0.83

$$-\frac{2}{39}\cos\left(2x^3\right)e^{\left(-3x^3\right)}-\frac{1}{13}e^{\left(-3x^3\right)}\sin\left(2x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="fricas")
```

```
[Out] -2/39*cos(2*x^3)*e^(-3*x^3) - 1/13*e^(-3*x^3)*sin(2*x^3)
```

Sympy [A]
time = 0.43, size = 32, normalized size = 0.91

$$-\frac{e^{-3x^3}\sin\left(2x^3\right)}{13}-\frac{2e^{-3x^3}\cos\left(2x^3\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(2*x**3)/exp(3*x**3),x)
[Out] -exp(-3*x**3)*sin(2*x**3)/13 - 2*exp(-3*x**3)*cos(2*x**3)/39
```

Giac [A]
time = 0.42, size = 25, normalized size = 0.71

$$-\frac{1}{39}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)e^{\left(-3x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="giac")
[Out] -1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

Mupad [B]
time = 2.97, size = 25, normalized size = 0.71

$$-\frac{e^{-3x^3}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)}{39}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*exp(-3*x^3)*sin(2*x^3),x)
[Out] -(exp(-3*x^3)*(2*cos(2*x^3) + 3*sin(2*x^3)))/39
```

3.778 $\int 2x \cos(x^2)\, dx$

Optimal. Leaf size=4

$$\sin(x^2)$$

[Out] sin(x^2)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 3461, 2717}

$$\sin(x^2)$$

Antiderivative was successfully verified.

[In] Int[2*x*Cos[x^2],x]

[Out] Sin[x^2]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int 2x\cos(x^2)\,dx &= 2\int x\cos(x^2)\,dx\\ &= \text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)\\ &= \sin(x^2)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Cos[x^2],x]
```

```
[Out] Sin[x^2]
```

Maple [A]
time = 0.02, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\sin\left(x^2\right)$	5
default	$\sin\left(x^2\right)$	5
meijerg	$\sin\left(x^2\right)$	5
risch	$\sin\left(x^2\right)$	5
norman	$\frac{2\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*cos(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x^2)
```

Maxima [A]
time = 0.28, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="maxima")
```

```
[Out] sin(x^2)
```

Fricas [A]
time = 3.58, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="fricas")
```

```
[Out] sin(x^2)
```

Sympy [A]
time = 0.05, size = 3, normalized size = 0.75

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x**2),x)
```

```
[Out] sin(x**2)
```

Giac [A]
time = 0.43, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*cos(x^2),x, algorithm="giac")
```

```
[Out] sin(x^2)
```

Mupad [B]
time = 2.92, size = 4, normalized size = 1.00

$$\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*cos(x^2),x)
```

```
[Out] sin(x^2)
```

3.779 $\int 3x^2 \cos(7+x^3)\,dx$

Optimal. Leaf size=6

$$\sin(7+x^3)$$

```
[Out] sin(x^3+7)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {12, 3461, 2717}

$$\sin(x^3+7)$$

Antiderivative was successfully verified.

```
[In] Int[3*x^2*Cos[7 + x^3],x]
```

```
[Out] Sin[7 + x^3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int 3x^2 \cos(7+x^3)\,dx &= 3\int x^2 \cos(7+x^3)\,dx\\ &= \text{Subst}\left(\int \cos(7+x)\,dx, x, x^3\right)\\ &= \sin(7+x^3)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\sin\left(7+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[7 + x^3],x]
[Out] Sin[7 + x^3]
```

Maple [A]
time = 0.04, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$\sin\left(x^3+7\right)$	7
default	$\sin\left(x^3+7\right)$	7
risch	$\sin\left(x^3+7\right)$	7
norman	$\frac{2\tan\left(\frac{7}{2}+\frac{x^3}{2}\right)}{1+\tan^2\left(\frac{7}{2}+\frac{x^3}{2}\right)}$	25
meijerg	$\cos\left(7\right)\sin\left(x^3\right)-\sin\left(7\right)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^3\right)}{\sqrt{\pi}}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3+7),x,method=_RETURNVERBOSE)
[Out] sin(x^3+7)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="maxima")
[Out] sin(x^3 + 7)
```

Fricas [A]
time = 3.48, size = 6, normalized size = 1.00

$$\sin\left(x^3+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="fricas")
```

```
[Out] sin(x^3 + 7)
```

Sympy [**A**]
time = 0.08, size = 5, normalized size = 0.83

$$\sin\left(x^3 + 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3+7),x)
[Out] sin(x**3 + 7)
```

Giac [**A**]
time = 0.40, size = 6, normalized size = 1.00

$$\sin\left(x^3 + 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3+7),x, algorithm="giac")
[Out] sin(x^3 + 7)
```

Mupad [**B**]
time = 0.06, size = 6, normalized size = 1.00

$$\sin\left(x^3 + 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3 + 7),x)
[Out] sin(x^3 + 7)
```

3.780 $\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx$

Optimal. Leaf size=7

$$\text{ArcTan}(x)-\cos(x)$$

```
[Out] arctan(x)-cos(x)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {209, 2718}

$$\text{ArcTan}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx &= \int \frac{1}{1+x^2}\,dx+\int \sin(x)\,dx\\ &= \tan^{-1}(x)-\cos(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$\text{ArcTan}(x)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-1) + Sin[x],x]
```

```
[Out] ArcTan[x] - Cos[x]
```

Maple [A]
time = 0.08, size = 8, normalized size = 1.14

method	result	size
default	$\arctan(x) - \cos(x)$	8
risch	$\arctan(x) - \cos(x)$	8
meijerg	$\arctan(x) + \sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(x)}{\sqrt{\pi}}\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)+sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)-cos(x)
```

Maxima [A]
time = 0.50, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="maxima")
```

```
[Out] arctan(x) - cos(x)
```

Fricas [A]
time = 3.56, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="fricas")
```

```
[Out] arctan(x) - cos(x)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 0.71

$$-\cos(x) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)+sin(x),x)
```

```
[Out] -cos(x) + atan(x)
```

Giac [A]
time = 0.44, size = 7, normalized size = 1.00

$$\arctan(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)+sin(x),x, algorithm="giac")
```

```
[Out] arctan(x) - cos(x)
```

Mupad [B]
time = 0.05, size = 7, normalized size = 1.00

$$\mathrm{atan}(x) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x) + 1/(x^2 + 1),x)
```

```
[Out] atan(x) - cos(x)
```

3.781 $\int x \sin (1+x^2) \, dx$

Optimal. Leaf size=10

$$-\frac{1}{2} \cos (1+x^2)$$

```
[Out] -1/2*cos(x^2+1)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 2718}

$$-\frac{1}{2} \cos (x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[1 + x^2],x]
```

```
[Out] -1/2*Cos[1 + x^2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int x \sin (1+x^2) \, dx &= \frac{1}{2} \text{Subst}\left(\int \sin(1+x) \, dx, x, x^2\right) \\ &= -\frac{1}{2} \cos (1+x^2) \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.01, size = 21, normalized size = 2.10

$$-\frac{1}{2} \cos(1) \cos (x^2) + \frac{1}{2} \sin(1) \sin (x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[1 + x^2],x]
```

```
[Out] -1/2*(Cos[1]*Cos[x^2]) + (Sin[1]*Sin[x^2])/2
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{\cos(x^2+1)}{2}$	9
default	$-\frac{\cos(x^2+1)}{2}$	9
risch	$-\frac{\cos(x^2+1)}{2}$	9
norman	$-\frac{1}{1+\tan^2\left(\frac{1}{2}+\frac{x^2}{2}\right)}$	17
meijerg	$\frac{\sin(1)\sin(x^2)}{2}+\frac{\cos(1)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x^2)}{\sqrt{\pi}}\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x^2+1)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x^2 + 1)
```

Fricas [A]
time = 3.87, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x^2 + 1)
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2+1),x)
```

```
[Out] -cos(x**2 + 1)/2
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.80

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*cos(x^2 + 1)
```

Mupad [B]
time = 0.04, size = 8, normalized size = 0.80

$$-\frac{\cos\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2 + 1),x)
```

```
[Out] -cos(x^2 + 1)/2
```

3.782 $\int x \cos(1+x^2)\, dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin\left(1+x^2\right)$$

[Out] 1/2*sin(x^2+1)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3461, 2717}

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[1 + x^2],x]

[Out] Sin[1 + x^2]/2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos\left(1+x^2\right)\, dx &= \frac{1}{2}\text{Subst}\left(\int \cos(1+x)\, dx, x, x^2\right)\\ &= \frac{1}{2}\sin\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\sin\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[1 + x^2],x]
```

```
[Out] Sin[1 + x^2]/2
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\sin(x^2+1)}{2}$	9
default	$\frac{\sin(x^2+1)}{2}$	9
risch	$\frac{\sin(x^2+1)}{2}$	9
norman	$\frac{\tan\left(\frac{1}{2}+\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{1}{2}+\frac{x^2}{2}\right)}$	24
meijerg	$\frac{\cos(1)\sin(x^2)}{2}-\frac{\sin(1)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x^2)}{\sqrt{\pi}}\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(x^2+1)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sin(x^2 + 1)
```

Fricas [A]
time = 3.69, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sin(x^2 + 1)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\sin(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2+1),x)
```

```
[Out] sin(x**2 + 1)/2
```

Giac [A]
time = 0.41, size = 8, normalized size = 0.80

$$\frac{1}{2}\sin\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sin(x^2 + 1)
```

Mupad [B]
time = 2.98, size = 8, normalized size = 0.80

$$\frac{\sin(x^2+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2 + 1),x)
```

```
[Out] sin(x^2 + 1)/2
```

3.783 $\int (1+x^2\cos(x^3))\,dx$

Optimal. Leaf size=10

$$x+\frac{\sin(x^3)}{3}$$

[Out] x+1/3*sin(x^3)

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3461, 2717}

$$\frac{\sin(x^3)}{3}+x$$

Antiderivative was successfully verified.

[In] Int[1 + x^2*Cos[x^3],x]

[Out] x + Sin[x^3]/3

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \left(1+x^2\cos\left(x^3\right)\right)\,dx &= x+\int x^2\cos\left(x^3\right)\,dx\\ &= x+\frac{1}{3}\text{Subst}\left(\int \cos(x)\,dx,x,x^3\right)\\ &= x+\frac{\sin\left(x^3\right)}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$x + \frac{\sin(x^3)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^2*Cos[x^3],x]
```

```
[Out] x + Sin[x^3]/3
```

Maple [A]
time = 0.04, size = 9, normalized size = 0.90

method	result	size
default	$x+\frac{\sin(x^3)}{3}$	9
risch	$x+\frac{\sin(x^3)}{3}$	9
norman	$\frac{x+x\left(\tan^2\left(\frac{x^3}{2}\right)\right)+\frac{2\tan\left(\frac{x^3}{2}\right)}{3}}{1+\tan^2\left(\frac{x^3}{2}\right)}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/3*sin(x^3)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$x + \frac{1}{3}\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] x + 1/3*sin(x^3)
```

Fricas [A]
time = 2.82, size = 8, normalized size = 0.80

$$x + \frac{1}{3}\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] x + 1/3*sin(x^3)
```

Sympy [A]
time = 0.08, size = 7, normalized size = 0.70

$$x + \frac{\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x**2*cos(x**3),x)
```

```
[Out] x + sin(x**3)/3
```

Giac [A]
time = 0.43, size = 8, normalized size = 0.80

$$x + \frac{1}{3}\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] x + 1/3*sin(x^3)
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$x + \frac{\sin(x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x^3) + 1,x)
```

```
[Out] x + sin(x^3)/3
```

3.784 $\int x^2 \sin(1+x^3)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos\left(1+x^3\right)$$

```
[Out] -1/3*cos(x^3+1)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3460, 2718}

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[1 + x^3],x]
```

```
[Out] -1/3*Cos[1 + x^3]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^2 \sin\left(1+x^3\right)\, dx &= \frac{1}{3}\text{Subst}\left(\int \sin(1+x)\, dx, x, x^3\right)\\ &= -\frac{1}{3}\cos\left(1+x^3\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.01, size = 21, normalized size = 2.10

$$-\frac{1}{3}\cos(1)\cos\left(x^3\right)+\frac{1}{3}\sin(1)\sin\left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[1 + x^3],x]
[Out] -1/3*(Cos[1]*Cos[x^3]) + (Sin[1]*Sin[x^3])/3
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{\cos\left(x^{3}+1\right)}{3}$	9
default	$-\frac{\cos\left(x^{3}+1\right)}{3}$	9
risch	$-\frac{\cos\left(\left(1+x\right)\left(x^{2}-x+1\right)\right)}{3}$	16
norman	$-\frac{2}{3\left(1+\tan^{2}\left(\frac{1}{2}+\frac{x^{3}}{2}\right)\right)}$	17
meijerg	$\frac{\sin\left(1\right)\sin\left(x^{3}\right)}{3}+\frac{\cos\left(1\right)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^{3}\right)}{\sqrt{\pi}}\right)}{3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*cos(x^3+1)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="maxima")
[Out] -1/3*cos(x^3 + 1)
```

Fricas [A]
time = 2.80, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="fricas")
[Out] -1/3*cos(x^3 + 1)
```

Sympy [A]
time = 0.08, size = 8, normalized size = 0.80

$$-\frac{\cos(x^3+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x**3+1),x)
```

```
[Out] -cos(x**3 + 1)/3
```

Giac [A]
time = 0.43, size = 8, normalized size = 0.80

$$-\frac{1}{3}\cos\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*cos(x^3 + 1)
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$-\frac{\cos(x^3+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x^3 + 1),x)
```

```
[Out] -cos(x^3 + 1)/3
```

3.785 $\int 12x^2 \cos(x^3)\, dx$

Optimal. Leaf size=6

$$4 \sin(x^3)$$

[Out] 4*sin(x^3)

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 3461, 2717}

$$4 \sin(x^3)$$

Antiderivative was successfully verified.

[In] Int[12*x^2*Cos[x^3],x]

[Out] 4*Sin[x^3]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int 12x^2 \cos(x^3)\, dx &= 12 \int x^2 \cos(x^3)\, dx \\
&= 4\text{Subst}\left(\int \cos(x)\, dx, x, x^3\right) \\
&= 4 \sin(x^3)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$4 \sin \left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[12*x^2*Cos[x^3],x]
```

```
[Out] 4*Sin[x^3]
```

Maple [A]
time = 0.04, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$4 \sin \left(x^3\right)$	7
default	$4 \sin \left(x^3\right)$	7
meijerg	$4 \sin \left(x^3\right)$	7
risch	$4 \sin \left(x^3\right)$	7
norman	$\frac{8 \tan \left(\frac{x^3}{2}\right)}{1+\tan^2 \left(\frac{x^3}{2}\right)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(12*x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 4*sin(x^3)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 1.00

$$4 \sin \left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] 4*sin(x^3)
```

Fricas [A]
time = 2.67, size = 6, normalized size = 1.00

$$4 \sin \left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] 4*sin(x^3)
```

Sympy [A]
time = 0.07, size = 5, normalized size = 0.83

$$4\sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x**2*cos(x**3),x)
```

```
[Out] 4*sin(x**3)
```

Giac [A]
time = 0.43, size = 6, normalized size = 1.00

$$4\ \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(12*x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] 4*sin(x^3)
```

Mupad [B]
time = 0.05, size = 6, normalized size = 1.00

$$4\ \sin\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(12*x^2*cos(x^3),x)
```

```
[Out] 4*sin(x^3)
```

3.786 $\int (1+x)\sin(1+x)\,dx$

Optimal. Leaf size=14

$$-((1+x)\cos(1+x))+\sin(1+x)$$

```
[Out] -(1+x)*cos(1+x)+sin(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3377, 2717}

$$\sin(x+1)-(x+1)\cos(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (1+x)\sin(1+x)\,dx &= -(1+x)\cos(1+x)+\int \cos(1+x)\,dx\\ &= -(1+x)\cos(1+x)+\sin(1+x)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 14, normalized size = 1.00

$$-((1+x)\cos(1+x))+\sin(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)*Sin[1 + x],x]
```

```
[Out] -((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

Maple [A]
time = 0.04, size = 15, normalized size = 1.07

method	result
derivativedivides	$-(1+x)\cos(1+x)+\sin(1+x)$
default	$-(1+x)\cos(1+x)+\sin(1+x)$
risch	$(-1-x)\cos(1+x)+\sin(1+x)$
norman	$\frac{x\left(\tan^{2}\left(\frac{1}{2}+\frac{x}{2}\right)\right)-x+2\tan\left(\frac{1}{2}+\frac{x}{2}\right)-2}{1+\tan^{2}\left(\frac{1}{2}+\frac{x}{2}\right)}$
meijerg	$\sin(1)\sin(x)+\cos(1)\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)+2\sin(1)\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x}{2\sqrt{\pi}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)*sin(1+x),x,method=_RETURNVERBOSE)
[Out] -(1+x)*cos(1+x)+sin(1+x)
```

Maxima [A]
time = 0.30, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="maxima")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

Fricas [A]
time = 3.75, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="fricas")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 1.07

$$-x\cos(x+1)+\sin(x+1)-\cos(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x)
[Out] -x*cos(x + 1) + sin(x + 1) - cos(x + 1)
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.00

$$-(x+1)\cos(x+1)+\sin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)*sin(1+x),x, algorithm="giac")
[Out] -(x + 1)*cos(x + 1) + sin(x + 1)
```

Mupad [B]
time = 2.95, size = 14, normalized size = 1.00

$$\sin(x+1)-\cos(x+1)\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x + 1)*(x + 1),x)
[Out] sin(x + 1) - cos(x + 1)*(x + 1)
```

3.787 $\int x^5 \cos(x^3)\, dx$

Optimal. Leaf size=20

$$\frac{\cos(x^3)}{3} + \frac{1}{3}x^3 \sin(x^3)$$

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3461, 3377, 2718}

$$\frac{1}{3}x^3 \sin(x^3) + \frac{\cos(x^3)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^5 \cos \left(x^3\right) \, dx &= \frac{1}{3}\text{Subst}\left(\int x \cos(x) \, dx, x, x^3\right) \\ &= \frac{1}{3}x^3 \sin \left(x^3\right) - \frac{1}{3}\text{Subst}\left(\int \sin(x) \, dx, x, x^3\right) \\ &= \frac{\cos \left(x^3\right)}{3} + \frac{1}{3}x^3 \sin \left(x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\cos \left(x^3\right)}{3} + \frac{1}{3}x^3 \sin \left(x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[x^3],x]
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Maple [A]
time = 0.05, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$	17
default	$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$	17
risch	$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$	17
meijerg	$\frac{2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x^3\right)}{2\sqrt{\pi}}+\frac{x^3\sin\left(x^3\right)}{2\sqrt{\pi}}\right)}{3}$	33
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{x^3}{2}\right)\right)}{3}+\frac{2x^3\tan\left(\frac{x^3}{2}\right)}{3}}{1+\tan^2\left(\frac{x^3}{2}\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x,method=_RETURNVERBOSE)
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$\frac{1}{3} \, x^3 \sin \left(x^3\right) + \frac{1}{3} \, \cos \left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Fricas [A]
time = 3.51, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{1}{3}\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Sympy [A]
time = 0.27, size = 15, normalized size = 0.75

$$\frac{x^3\sin\left(x^3\right)}{3}+\frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

Giac [A]
time = 0.40, size = 16, normalized size = 0.80

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{1}{3}\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Mupad [B]
time = 2.96, size = 16, normalized size = 0.80

$$\frac{\cos\left(x^3\right)}{3}+\frac{x^3\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
```

```
[Out] cos(x^3)/3 + (x^3*sin(x^3))/3
```

3.788 $\int e^{-3x}\cos(x)\,dx$

Optimal. Leaf size=23

$$-\frac{3}{10}e^{-3x}\cos(x)+\frac{1}{10}e^{-3x}\sin(x)$$

```
[Out] -3/10*cos(x)/exp(3*x)+1/10*sin(x)/exp(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{4518\}$

$$\frac{1}{10}e^{-3x}\sin(x)-\frac{3}{10}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/E^(3*x),x]
```

```
[Out] (-3*Cos[x])/(10*E^(3*x)) + Sin[x]/(10*E^(3*x))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(x)\,dx=-\frac{3}{10}e^{-3x}\cos(x)+\frac{1}{10}e^{-3x}\sin(x)$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.70

$$\frac{1}{10}e^{-3x}(-3\cos(x)+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/E^(3*x),x]
```

```
[Out] (-3*Cos[x] + Sin[x])/(10*E^(3*x))
```

Maple [A]
time = 0.04, size = 18, normalized size = 0.78

method	result	size
default	$-\frac{3\,\mathrm{e}^{-3x}\cos(x)}{10}+\frac{\mathrm{e}^{-3x}\sin(x)}{10}$	18
norman	$\frac{\left(-\frac{3}{10}+\frac{3\left(\tan^2\left(\frac{x}{2}\right)\right)}{10}+\frac{\tan\left(\frac{x}{2}\right)}{5}\right)\mathrm{e}^{-3x}}{1+\tan^2\left(\frac{x}{2}\right)}$	34
risch	$-\frac{3\,\mathrm{e}^{(-3+i)x}}{20}-\frac{i\mathrm{e}^{(-3+i)x}}{20}-\frac{3\,\mathrm{e}^{(-3-i)x}}{20}+\frac{i\mathrm{e}^{(-3-i)x}}{20}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/exp(3*x),x,method=_RETURNVERBOSE)
[Out] -3/10*exp(-3*x)*cos(x)+1/10*exp(-3*x)*sin(x)
```

Maxima [A]
time = 0.30, size = 15, normalized size = 0.65

$$-\frac{1}{10}\left(3\cos\left(x\right)-\sin\left(x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="maxima")
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

Fricas [A]
time = 3.09, size = 17, normalized size = 0.74

$$-\frac{3}{10}\cos\left(x\right)e^{\left(-3\,x\right)}+\frac{1}{10}\,e^{\left(-3\,x\right)}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="fricas")
[Out] -3/10*cos(x)*e^(-3*x) + 1/10*e^(-3*x)*sin(x)
```

Sympy [A]
time = 0.17, size = 20, normalized size = 0.87

$$\frac{e^{-3x}\sin\left(x\right)}{10}-\frac{3e^{-3x}\cos\left(x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x)
[Out] exp(-3*x)*sin(x)/10 - 3*exp(-3*x)*cos(x)/10
```

Giac [A]
time = 0.42, size = 15, normalized size = 0.65

$$-\frac{1}{10}\,(3\,\cos\left(x\right)-\sin\left(x\right))e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

Mupad [B]
time = 0.02, size = 15, normalized size = 0.65

$$-\frac{\mathrm{e}^{-3\,x}\,\left(3\,\cos\left(x\right)-\sin\left(x\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-3*x)*cos(x),x)
```

```
[Out] -(exp(-3*x)*(3*cos(x) - sin(x)))/10
```

3.789 $\int x^3 \sin(x^2)\, dx$

Optimal. Leaf size=20

$$-\frac{1}{2}x^2\cos\left(x^2\right)+\frac{\sin\left(x^2\right)}{2}$$

[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3460, 3377, 2717}

$$\frac{\sin\left(x^2\right)}{2}-\frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sin[x^2],x]

[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3 \sin\left(x^2\right) \, dx = \frac{1}{2}\text{Subst}\left(\int x \sin(x) \, dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2 \cos\left(x^2\right) + \frac{1}{2}\text{Subst}\left(\int \cos(x) \, dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2 \cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2}x^2 \cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
[Out] -1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

Maple [A]
time = 0.03, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
default	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
risch	$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$	17
meijerg	$\sqrt{\pi}\,\left(-\frac{x^2 \cos(x^2)}{2\sqrt{\pi}} + \frac{\sin(x^2)}{2\sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^2}{2}+\frac{x^2\left(\tan^2\left(\frac{x^2}{2}\right)\right)}{2}+\tan\left(\frac{x^2}{2}\right)}{1+\tan^2\left(\frac{x^2}{2}\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2 \cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A]
time = 3.35, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A]
time = 0.12, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.80

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Mupad [B]
time = 2.96, size = 16, normalized size = 0.80

$$\frac{\sin\left(x^2\right)}{2}-\frac{x^2\,\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.790 $\int x^3 \cos(x^2)\, dx$

Optimal. Leaf size=20

$$\frac{\cos(x^2)}{2} + \frac{1}{2} x^2 \sin(x^2)$$

```
[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3461, 3377, 2718}

$$\frac{1}{2} x^2 \sin(x^2) + \frac{\cos(x^2)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x^2],x]
```

```
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3\cos\left(x^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int x\cos(x)\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^2\sin\left(x^2\right) - \frac{1}{2}\text{Subst}\left(\int \sin(x)\,dx, x, x^2\right)$$
$$= \frac{\cos\left(x^2\right)}{2} + \frac{1}{2}x^2\sin\left(x^2\right)$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\cos\left(x^2\right)}{2} + \frac{1}{2}x^2\sin\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x^2],x]
[Out] Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

Maple [A]
time = 0.04, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\cos\left(x^2\right)}{2}+\frac{x^2\sin\left(x^2\right)}{2}$	17
default	$\frac{\cos\left(x^2\right)}{2}+\frac{x^2\sin\left(x^2\right)}{2}$	17
risch	$\frac{\cos\left(x^2\right)}{2}+\frac{x^2\sin\left(x^2\right)}{2}$	17
norman	$\frac{x^2\tan\left(\frac{x^2}{2}\right)+1}{1+\tan^2\left(\frac{x^2}{2}\right)}$	26
meijerg	$\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos\left(x^2\right)}{2\sqrt{\pi}}+\frac{x^2\sin\left(x^2\right)}{2\sqrt{\pi}}\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

Fricas [A]
time = 2.97, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

Sympy [A]
time = 0.12, size = 15, normalized size = 0.75

$$\frac{x^2\sin\left(x^2\right)}{2}+\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x**2),x)
```

```
[Out] x**2*sin(x**2)/2 + cos(x**2)/2
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.80

$$\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

Mupad [B]
time = 0.05, size = 16, normalized size = 0.80

$$\frac{\cos\left(x^2\right)}{2}+\frac{x^2\,\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x^2),x)
```

```
[Out] cos(x^2)/2 + (x^2*sin(x^2))/2
```

3.791 $\int \cos(x)\cos(2\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\sin(2\sin(x))$$

[Out] 1/2*sin(2*sin(x))

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4419, 2717}

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Cos[2*Sin[x]],x]

[Out] Sin[2*Sin[x]]/2

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2\sin(x))\,dx &= \text{Subst}\left(\int \cos(2x)\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\sin(2\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.95, size = 9, normalized size = 1.00

$$\frac{1}{2}\sin(2\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*Sin[x]],x]
```

```
[Out] Sin[2*Sin[x]]/2
```

Maple [A]
time = 0.07, size = 8, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\sin(2\sin(x))}{2}$	8
default	$\frac{\sin(2\sin(x))}{2}$	8
risch	$\frac{\sin(2\sin(x))}{2}$	8
norman	$\frac{(\tan^2(\frac{x}{2}))\tan\left(\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)+\tan\left(\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)}{(1+\tan^2(\frac{x}{2}))\left(1+\tan^2\left(\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}\right)\right)}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(2*sin(x))
```

Maxima [A]
time = 0.29, size = 7, normalized size = 0.78

$$\frac{1}{2}\sin\left(2\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.
time = 2.61, size = 19, normalized size = 2.11

$$\frac{1}{2}\sin\left(\frac{4\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*sin(4*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

Sympy [A]
time = 0.13, size = 7, normalized size = 0.78

$$\frac{\sin(2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x)
```

```
[Out] sin(2*sin(x))/2
```

Giac [A]
time = 0.39, size = 7, normalized size = 0.78

$$\frac{1}{2}\sin(2\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*sin(x))
```

Mupad [B]
time = 0.07, size = 7, normalized size = 0.78

$$\frac{\sin(2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*sin(x))*cos(x),x)
```

```
[Out] sin(2*sin(x))/2
```

3.792 $\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(1+\cos^2(x)\right)$$

```
[Out] -1/2*ln(1+cos(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4420, 266}

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

```
[Out] -1/2*Log[1 + Cos[x]^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log\left(1+\cos^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{2}\log(3+\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
[Out] -1/2*Log[3 + Cos[2*x]]
```

Maple [A]
time = 0.04, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$-\frac{\ln\left(1+\cos^{2}\left(x\right)\right)}{2}$	10
default	$-\frac{\ln\left(1+\cos^{2}\left(x\right)\right)}{2}$	10
norman	$-\frac{\ln\left(\tan^{4}\left(\frac{x}{2}\right)+1\right)}{2}+\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)$	22
risch	$ix-\frac{\ln\left(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1\right)}{2}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1+cos(x)^2)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.82

$$-\frac{1}{2}\,\log\left(\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="maxima")
[Out] -1/2*log(cos(x)^2 + 1)
```

Fricas [A]
time = 3.67, size = 11, normalized size = 1.00

$$-\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right)^{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="fricas")
[Out] -1/2*log(1/2*cos(x)^2 + 1/2)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.91

$$-\frac{\log\left(\cos^{2}\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)**2),x)
```

```
[Out] -log(cos(x)**2 + 1)/2
```

Giac [A]
time = 0.44, size = 9, normalized size = 0.82

$$-\frac{1}{2}\log\left(\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x)^2 + 1)
```

Mupad [B]
time = 3.01, size = 17, normalized size = 1.55

$$-\mathrm{atanh}\left(\frac{16}{3\,\left(12\tan\left(x\right)^{2}+16\right)}-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(cos(x)^2 + 1),x)
```

```
[Out] -atanh(16/(3*(12*tan(x)^2 + 16)) - 1/3)
```

3.793 $\int (1+\cos(x))(x+\sin(x))^3\,dx$

Optimal. Leaf size=10

$$\frac{1}{4}(x+\sin(x))^4$$

```
[Out] 1/4*(x+sin(x))^4
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6818}

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{1}{4}(x+\sin(x))^4$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{4}(x+\sin(x))^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*(x + Sin[x])^3,x]
```

```
[Out] (x + Sin[x])^4/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(8) = 16$.
time = 0.09, size = 65, normalized size = 6.50

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+\frac{x(4x^2+3)\sin(x)}{4}+\frac{\cos(4x)}{32}-\frac{x\sin(3x)}{4}+2\left(-\frac{1}{16}-\frac{3x^2}{8}\right)\cos(2x)$
default	$\frac{x^4}{4}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}+x^3\sin(x)-\frac{3x^2(\cos^2(x))}{2}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+x(\sin^3(x))+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*(x+sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+3*x*(-1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2+x^3*sin(x)-3/2*x^2*cos(x)^2+
3*x*(1/2*cos(x)*sin(x)+1/2*x)+x*sin(x)^3+1/4*sin(x)^4
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.80

$$\frac{1}{4}\left(x+\sin\left(x\right)\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(x + sin(x))^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(8) = 16$.
time = 3.79, size = 45, normalized size = 4.50

$$\frac{1}{4}\,x^{4}+\frac{1}{4}\,\cos\left(x\right)^{4}-\frac{1}{2}\,\left(3\,x^{2}+1\right)\cos\left(x\right)^{2}+\frac{3}{2}\,x^{2}+\left(x^{3}-x\cos\left(x\right)^{2}+x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/4*cos(x)^4 - 1/2*(3*x^2 + 1)*cos(x)^2 + 3/2*x^2 + (x^3 - x*cos(
x)^2 + x)*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(7) = 14$.
time = 0.13, size = 36, normalized size = 3.60

$$\frac{x^{4}}{4}+x^{3}\sin\left(x\right)+\frac{3x^{2}\sin^{2}\left(x\right)}{2}+x\sin^{3}\left(x\right)+\frac{\sin^{4}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))**3,x)
```

```
[Out] x**4/4 + x**3*sin(x) + 3*x**2*sin(x)**2/2 + x*sin(x)**3 + sin(x)**4/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(8) = 16$.
time = 0.41, size = 61, normalized size = 6.10

$$\frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{1}{4}(3x^2 - 1)\cos(2x) - \frac{1}{4}x\sin(3x) + \frac{1}{4}(4x^3 - 21x)\sin(x) + 6x\sin(x) + \frac{1}{32}\cos(4x) - \frac{3}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/4*x^2 - 1/4*(3*x^2 - 1)*cos(2*x) - 1/4*x*sin(3*x) + 1/4*(4*x^3
- 21*x)*sin(x) + 6*x*sin(x) + 1/32*cos(4*x) - 3/8*cos(2*x)
```

Mupad [B]
time = 3.15, size = 8, normalized size = 0.80

$$\frac{(x + \sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)*(x + sin(x))^3,x)
```

```
[Out] (x + sin(x))^4/4
```

3.794 $\int (1+\cos(x))\csc^2(x)\,dx$

Optimal. Leaf size=9

$$-\cot(x)-\csc(x)$$

[Out] -cot(x)-csc(x)

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2748, 3852, 8}

$$-\cot(x)-\csc(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])*Csc[x]^2,x]

[Out] -Cot[x] - Csc[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc^2(x)\,dx &= -\csc(x)+\int \csc^2(x)\,dx\\ &= -\csc(x)-\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\cot(x)-\csc(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\cot(x)-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x]^2,x]
```

```
[Out] -Cot[x] - Csc[x]
```

Maple [A]
time = 0.05, size = 12, normalized size = 1.33

method	result	size
default	$-\cot\left(x\right)-\frac{1}{\sin\left(x\right)}$	12
risch	$-\frac{2i}{{\mathrm e}^{i x}-1}$	13
norman	$\frac{-1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -cot(x)-1/sin(x)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 1.44

$$-\frac{1}{\sin\left(x\right)}-\frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/sin(x) - 1/tan(x)
```

Fricas [A]
time = 5.09, size = 10, normalized size = 1.11

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [**A**]
time = 0.97, size = 8, normalized size = 0.89

$$-\cot(x) - \frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)**2,x)
```

```
[Out] -cot(x) - 1/sin(x)
```

Giac [**A**]
time = 0.40, size = 8, normalized size = 0.89

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

Mupad [**B**]
time = 2.93, size = 6, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/sin(x)^2,x)
```

```
[Out] -cot(x/2)
```

3.795 $\int \sin(x)\tan^2(x)\,dx$

Optimal. Leaf size=5

$$\cos(x)+\sec(x)$$

[Out] cos(x)+sec(x)

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2670, 14}

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]*Tan[x]^2,x]

[Out] Cos[x] + Sec[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan^2(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(x)\right)\\ &= -\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(x)\right)\\ &= \cos(x)+\sec(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\cos(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^2,x]
```

```
[Out] Cos[x] + Sec[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.
time = 0.03, size = 20, normalized size = 4.00

method	result	size
default	$\frac{\sin^4(x)}{\cos(x)} + (2 + \sin^2(x))\cos(x)$	20
risch	$\frac{e^{ix}}{2} + \frac{e^{-ix}}{2} + \frac{2\,e^{ix}}{e^{2ix}+1}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

Maxima [A]
time = 0.29, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/cos(x) + cos(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 4.51, size = 11, normalized size = 2.20

$$\frac{\cos(x)^2 + 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] (cos(x)^2 + 1)/cos(x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**2,x)
```

```
[Out] cos(x) + 1/cos(x)
```

Giac [A]
time = 0.43, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/cos(x) + cos(x)
```

Mupad [B]
time = 3.00, size = 7, normalized size = 1.40

$$\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^2,x)
```

```
[Out] cos(x) + 1/cos(x)
```

3.796 $\int e^{\sin(x)} \sec^2(x) (x\cos^3(x) - \sin(x)) \, dx$

Optimal. Leaf size=13

$$e^{\sin(x)}(-1 + x\cos(x))\sec(x)$$

```
[Out] exp(sin(x))*(-1+x*cos(x))*sec(x)
```

Rubi [F]
time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int e^{\sin(x)} \sec^2(x) \left(x\cos^3(x) - \sin(x)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Int[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
[Out] Defer[Int][E^Sin[x]*x*Cos[x], x] - Defer[Int][E^Sin[x]*Sec[x]*Tan[x], x]
```

Rubi steps

$$\begin{aligned} \int e^{\sin(x)} \sec^2(x) \left(x\cos^3(x) - \sin(x)\right) \, dx &= \int \left(e^{\sin(x)}x\cos(x) - e^{\sin(x)}\sec(x)\tan(x)\right) \, dx \\ &= \int e^{\sin(x)}x\cos(x) \, dx - \int e^{\sin(x)}\sec(x)\tan(x) \, dx \end{aligned}$$

Mathematica [A]
time = 0.21, size = 13, normalized size = 1.00

$$e^{\sin(x)}(-1 + x\cos(x))\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
[Out] E^Sin[x]*(-1 + x*Cos[x])*Sec[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 30, normalized size = 2.31

method	result	size
risch	$\frac{(x\,e^{2ix}+x-2\,e^{ix})e^{\sin(x)}}{e^{2ix}+1}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (x*exp(2*I*x)+x-2*exp(I*x))/(exp(2*I*x)+1)*exp(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(12) = 24$.
time = 0.74, size = 88, normalized size = 6.77

$$\frac{x\cos\left(2\,x\right)^{2}e^{\sin\left(x\right)}+xe^{\sin\left(x\right)}\sin\left(2\,x\right)^{2}-2\,e^{\sin\left(x\right)}\sin\left(2\,x\right)\sin\left(x\right)+2\,\left(xe^{\sin\left(x\right)}-\cos\left(x\right)e^{\sin\left(x\right)}\right)\cos\left(2\,x\right)+xe^{\sin\left(x\right)}-2\,\cos\left(x\right)e^{\sin\left(x\right)}}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="maxima")
```

```
[Out] (x*cos(2*x)^2*e^sin(x) + x*e^sin(x)*sin(2*x)^2 - 2*e^sin(x)*sin(2*x)*sin(x)
 + 2*(x*e^sin(x) - cos(x)*e^sin(x))*cos(2*x) + x*e^sin(x) - 2*cos(x)*e^sin(
x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 3.43, size = 14, normalized size = 1.08

$$\frac{(x\cos\left(x\right)-1)e^{\sin\left(x\right)}}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) - 1)*e^sin(x)/cos(x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)**2*(x*cos(x)**3-sin(x)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(12) = 24$.
time = 0.42, size = 794, normalized size = 61.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="giac")
```

```
[Out] (x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 + e^(2*tan
(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 - 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^6 + 12*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7 - x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^8 - 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan
(1/2*x)^6 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7
- e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 30*x*e^(2*tan(1/2*x)/(
tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^4 - 52*x*e^(2*tan(1/2*x)/(tan(1/
2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^5 + 16*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^6 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1
/2*x)^5 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 16*x*e^(2*t
an(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^2 + 52*x*e^(2*tan(1/2
*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^3 - 30*x*e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^4 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3
/2*x)^2*tan(1/2*x)^2 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*ta
n(1/2*x)^3 + x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2 - 12*x*e^(2
*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) + 16*x*e^(2*tan(1/2*x
)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*ta
n(3/2*x)^2 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) -
 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - x*e^(2*tan(1/2*x)/(t
an(1/2*x)^2 + 1)) + e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)))/(tan(3/2*x)^2*tan(
1/2*x)^8 + 2*tan(3/2*x)^2*tan(1/2*x)^6 + tan(1/2*x)^8 + 2*tan(1/2*x)^6 - 2*
tan(3/2*x)^2*tan(1/2*x)^2 - tan(3/2*x)^2 - 2*tan(1/2*x)^2 - 1)
```

Mupad [B]
time = 3.12, size = 14, normalized size = 1.08

$$\frac{e^{\sin(x)}\,(x\,\cos(x)-1)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(exp(sin(x))*(sin(x) - x*cos(x)^3))/cos(x)^2,x)
```

```
[Out] (exp(sin(x))*(x*cos(x) - 1))/cos(x)
```

3.797 $\int x \csc^2(x) \, dx$

Optimal. Leaf size=9

$$-x\cot(x) + \log(\sin(x))$$

```
[Out] -x*cot(x)+ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4269, 3556}

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int x\csc^2(x)\,dx &= -x\cot(x) + \int \cot(x)\,dx \\ &= -x\cot(x) + \log(\sin(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$-x\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]^2,x]
```

[Out] -(x*Cot[x]) + Log[Sin[x]]

Maple [A]
time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$-x\cot\left(x\right)+\ln\left(\sin\left(x\right)\right)$	10
risch	$-2ix-\frac{2ix}{\mathrm{e}^{2ix}-1}+\ln\left(\mathrm{e}^{2ix}-1\right)$	27
norman	$\frac{-\frac{x}{2}+\frac{x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*csc(x)^2,x,method=_RETURNVERBOSE)

[Out] -x*cot(x)+ln(sin(x))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(9) = 18.
time = 0.29, size = 104, normalized size = 11.56

$$\frac{\left(\cos\left(2x\right)^{2}+\sin\left(2x\right)^{2}-2\cos\left(2x\right)+1\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)+\left(\cos\left(2x\right)^{2}+\sin\left(2x\right)^{2}-2\cos\left(2x\right)+1\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)-4x\sin\left(2x\right)}{2\left(\cos\left(2x\right)^{2}+\sin\left(2x\right)^{2}-2\cos\left(2x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="maxima")

[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 3.73, size = 20, normalized size = 2.22

$$-\frac{x\cos\left(x\right)-\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*csc(x)^2,x, algorithm="fricas")

[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(9) = 18$.
time = 0.43, size = 52, normalized size = 5.78

$$\frac{x\tan\left(\frac{1}{2}\,x\right)^2+\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)-x}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
1))*tan(1/2*x) - x)/tan(1/2*x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 1.00

$$\ln\left(\sin\left(x\right)\right)-x\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sin(x)^2,x)
```

```
[Out] log(sin(x)) - x*cot(x)
```

3.798 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal. Leaf size=20

$$\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {4670, 2718}

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx &= \int\left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{2}\int\sin\left(\frac{\pi}{6}+2x\right)\,dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Maple [A]
time = 0.12, size = 15, normalized size = 0.75

method	result	size
default	$\frac{x}{4}-\frac{\cos\left(\frac{\pi}{6}+2x\right)}{4}$	15
risch	$\frac{x}{4}-\frac{\sqrt{3}\,\cos\left(2x\right)}{8}+\frac{\sin\left(2x\right)}{8}$	20
norman	$\frac{x\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\left(\tan^{2}\left(\frac{\pi}{12}+\frac{x}{2}\right)\right)+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)-x\left(\tan^{2}\left(\frac{x}{2}\right)\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)\left(1+\tan^{2}\left(\frac{\pi}{12}+\frac{x}{2}\right)\right)}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(1/6*Pi+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.70

$$\frac{1}{4}\,x-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.
time = 3.38, size = 31, normalized size = 1.55

$$-\frac{1}{4}\,\sqrt{3}\,\cos\left(\frac{1}{6}\,\pi+x\right)^{2}-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+x\right)\sin\left(\frac{1}{6}\,\pi+x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

time = 0.14, size = 37, normalized size = 1.85

$$-\frac{x\sin\left(x\right)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos\left(x\right)}{2}-\frac{\cos\left(x\right)\cos\left(x+\frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 - cos(x)*cos(x + pi/6)
/2
```

Giac [**A**]

time = 0.41, size = 14, normalized size = 0.70

$$\frac{1}{4}\,x-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Mupad [**B**]

time = 0.03, size = 18, normalized size = 0.90

$$\frac{x\,\sin\left(\frac{\Pi}{6}\right)}{2}-\frac{\cos\left(\frac{\Pi}{6}+2\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(Pi/6 + x),x)
```

```
[Out] (x*sin(Pi/6))/2 - cos(Pi/6 + 2*x)/4
```

3.799 $\int x \sin^3\left(x^2\right)\,dx$

Optimal. Leaf size=19

$$-\frac{1}{2}\cos\left(x^2\right)+\frac{1}{6}\cos^3\left(x^2\right)$$

```
[Out] -1/2*cos(x^2)+1/6*cos(x^2)^3
```

Rubi [A]

time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3460, 2713}

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -1/2*Cos[x^2] + Cos[x^2]^3/6
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int x \sin^3\left(x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sin^3(x)\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(x^2\right)\right)\right)\\
&= -\frac{1}{2}\cos\left(x^2\right)+\frac{1}{6}\cos^3\left(x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-\frac{3}{8}\cos\left(x^2\right)+\frac{1}{24}\cos\left(3x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2]^3,x]
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$	15
default	$-\frac{\left(2+\sin^2\left(x^2\right)\right)\cos\left(x^2\right)}{6}$	15
risch	$-\frac{3\cos\left(x^2\right)}{8}+\frac{\cos\left(3x^2\right)}{24}$	16
norman	$\frac{-2\left(\tan^2\left(\frac{x^2}{2}\right)\right)-\frac{2}{3}}{\left(1+\tan^2\left(\frac{x^2}{2}\right)\right)^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.79

$$\frac{1}{24}\cos\left(3\,x^2\right)-\frac{3}{8}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

Fricas [A]
time = 2.36, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Sympy [A]
time = 0.12, size = 22, normalized size = 1.16

$$-\frac{\sin^2(x^2)\cos(x^2)}{2} - \frac{\cos^3(x^2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

Giac [A]
time = 0.40, size = 15, normalized size = 0.79

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Mupad [B]
time = 2.95, size = 14, normalized size = 0.74

$$\frac{\cos(x^2)\left(\cos(x^2)^2 - 3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] (cos(x^2)*(cos(x^2)^2 - 3))/6
```

3.800 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

[Out] 1/2*cos(x)^2-ln(cos(x))

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2670, 14}

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2*Tan[x],x]

[Out] Cos[x]^2/2 - Log[Cos[x]]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{1-x^2}{x}\,dx,x,\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx,x,\cos(x)\right)\\ &= \frac{\cos^2(x)}{2}-\log(\cos(x))\end{aligned}$$

Mathematica [A]

time = 0.00, size = 14, normalized size = 1.00

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A]
time = 0.04, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{\left(\sin^{2}\left(x\right)\right)}{2}-\ln\left(\cos\left(x\right)\right)$	13
risch	$ix+\frac{\mathrm{e}^{2ix}}{8}+\frac{\mathrm{e}^{-2ix}}{8}-\ln\left(\mathrm{e}^{2ix}+1\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A]
time = 0.28, size = 16, normalized size = 1.14

$$-\frac{1}{2}\sin\left(x\right)^{2}-\frac{1}{2}\log\left(\sin\left(x\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A]
time = 2.62, size = 14, normalized size = 1.00

$$\frac{1}{2}\cos\left(x\right)^{2}-\log\left(-\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.71

$$-\log\left(\cos\left(x\right)\right)+\frac{\cos^{2}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*tan(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

Giac [A]
time = 0.41, size = 18, normalized size = 1.29

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(-\sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

Mupad [B]
time = 2.95, size = 16, normalized size = 1.14

$$\frac{\cos(x)^2}{2} + \frac{\ln\left(\tan(x)^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
```

```
[Out] log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

3.801 $\int \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=22

$$-\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}$$

```
[Out] -1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2670, 272, 45}

$$\frac{\sin^2(x)}{2} - \frac{1}{2}\csc^2(x) - 2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -1/2*Csc[x]^2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(x)\cot^3(x)\,dx &= \text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx, x, \sin^2(x)\right)\\
&= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(-\csc^2(x) - 4\log(\sin(x)) + \sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A]
time = 0.05, size = 29, normalized size = 1.32

method	result	size
default	$-\frac{\cos^6(x)}{2\sin(x)^2}-\frac{(\cos^4(x))}{2}-(\cos^2(x))-2\ln(\sin(x))$	29
risch	$2ix-\frac{e^{2ix}}{8}-\frac{e^{-2ix}}{8}+\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}-2\ln(e^{2ix}-1)$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.91

$$\frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.
time = 3.39, size = 37, normalized size = 1.68

$$-\frac{2 \cos(x)^4 - 3 \cos(x)^2 + 8 \left(\cos(x)^2 - 1\right) \log\left(\frac{1}{2} \sin(x)\right) - 1}{4 \left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.91

$$-2 \log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2 \sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*cot(x)**3,x)
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Giac [A]
time = 0.41, size = 28, normalized size = 1.27

$$-\frac{1}{2} \cos(x)^2 + \frac{1}{2 \left(\cos(x)^2 - 1\right)} - \log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

Mupad [B]
time = 2.97, size = 32, normalized size = 1.45

$$\ln\left(\tan(x)^2 + 1\right) - 2 \ln\left(\tan(x)\right) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan(x)^4 + \tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
[Out] log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

3.802 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(1+\sin(x))$$

[Out] ln(1+sin(x))

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2746, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*(1 - Sin[x]),x]

[Out] Log[1 + Sin[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \sec(x)(1-\sin(x))\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, -\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(5) = 10.
time = 0.01, size = 36, normalized size = 7.20

$$\log(\cos(x))-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.04, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\ln(1+\sin(x))$	6
default	$\ln(1+\sin(x))$	6
risch	$-ix+2\ln(e^{ix}+i)$	17
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1-sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+sin(x))
```

Maxima [A]
time = 0.28, size = 5, normalized size = 1.00

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x) + 1)
```

Fricas [A]
time = 3.28, size = 5, normalized size = 1.00

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5)=10$.
time = 1.13, size = 12, normalized size = 2.40

$$\log(\tan(x)+\sec(x))+\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x)
[Out] log(tan(x) + sec(x)) + log(cos(x))
```

Giac [**A**]
time = 0.40, size = 5, normalized size = 1.00

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1-sin(x)),x, algorithm="giac")
[Out] log(sin(x) + 1)
```

Mupad [**B**]
time = 2.94, size = 5, normalized size = 1.00

$$\ln(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(sin(x) - 1)/cos(x),x)
[Out] log(sin(x) + 1)
```

3.803 $\int (1+\cos(x))\csc(x)\,dx$

Optimal. Leaf size=7

$$\log(1-\cos(x))$$

```
[Out] ln(1-cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2746, 31}

$$\log(1-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])*Csc[x],x]
```

```
[Out] Log[1 - Cos[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.
time = 0.01, size = 20, normalized size = 2.86

$$-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A]
time = 0.05, size = 6, normalized size = 0.86

method	result	size
derivativedivides	$\ln(\cos(x)-1)$	6
default	$\ln(\cos(x)-1)$	6
risch	$-ix+2\ln(e^{ix}-1)$	16
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x))*csc(x),x,method=_RETURNVERBOSE)
[Out] ln(cos(x)-1)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
[Out] log(cos(x) - 1)
```

Fricas [A]
time = 2.18, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.
time = 1.03, size = 12, normalized size = 1.71

$$-\log(\cot(x)+\csc(x))+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x)
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Giac [A]
time = 0.45, size = 7, normalized size = 1.00

$$\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
```

```
[Out] log(-cos(x) + 1)
```

Mupad [B]
time = 2.91, size = 5, normalized size = 0.71

$$\ln(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + 1)/sin(x),x)
```

```
[Out] log(cos(x) - 1)
```

3.804 $\int \cos^2(x) \left(1-\tan^2(x)\right) \, dx$

Optimal. Leaf size=5

$$\cos(x)\sin(x)$$

[Out] cos(x)*sin(x)

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3756, 391}

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]

[Out] Cos[x]*Sin[x]

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 3756

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \cos(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.60

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Sin[2*x]/2
```

Maple [A]
time = 0.04, size = 6, normalized size = 1.20

method	result	size
default	$\cos(x)\sin(x)$	6
risch	$\frac{\sin(2x)}{2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)*sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.
time = 0.28, size = 11, normalized size = 2.20

$$\frac{\tan(x)}{\tan(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] tan(x)/(tan(x)^2 + 1)
```

Fricas [A]
time = 1.86, size = 5, normalized size = 1.00

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.
time = 1.85, size = 14, normalized size = 2.80

$$\frac{\sin(x)\cos(x)}{2} + \frac{\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*(1-tan(x)**2),x)
```

```
[Out] sin(x)*cos(x)/2 + sin(2*x)/4
```

Giac [A]
time = 0.39, size = 9, normalized size = 1.80

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/(1/tan(x) + tan(x))
```

Mupad [B]
time = 2.89, size = 6, normalized size = 1.20

$$\frac{\sin(2\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2*(tan(x)^2 - 1),x)
```

```
[Out] sin(2*x)/2
```

3.805 $\int \csc(2x)(\cos(x)+\sin(x))\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -1/2*arctanh(cos(x))+1/2*arctanh(sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4486, 4372, 3855, 4373}

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[x]] + ArcTanh[Sin[x]]/2
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \csc(2x)(\cos(x)+\sin(x))\,dx &= \int (\cos(x)\csc(2x)+\csc(2x)\sin(x))\,dx \\
&= \int \cos(x)\csc(2x)\,dx + \int \csc(2x)\sin(x)\,dx \\
&= \frac{1}{2}\int \csc(x)\,dx + \frac{1}{2}\int \sec(x)\,dx \\
&= -\frac{1}{2}\tanh^{-1}(\cos(x)) + \frac{1}{2}\tanh^{-1}(\sin(x))
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.
time = 0.01, size = 61, normalized size = 4.07

$$
-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -1/2*Log[Cos[x/2]] - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 + Log[Cos
[x/2] + Sin[x/2]]/2
```

Maple [A]
time = 0.21, size = 20, normalized size = 1.33

method	result	size
default	$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(-\cot(x)+\csc(x))}{2}$	20
risch	$\frac{\ln(e^{2ix}+(-1+i)e^{ix}-i)}{2}-\frac{\ln(e^{2ix}+(1-i)e^{ix}-i)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*x)*(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(-cot(x)+csc(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(11) = 22$.
time = 0.50, size = 69, normalized size = 4.60

$$
-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(11) = 22$.
time = 2.79, size = 35, normalized size = 2.33

$$-\frac{1}{4}\log\left(-\frac{1}{2}\left(\cos\left(x\right)+1\right)\sin\left(x\right)+\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\left(\cos\left(x\right)-1\right)\sin\left(x\right)-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 1.12, size = 32, normalized size = 2.13

$$-\frac{\log\left(\sin\left(x\right)-1\right)}{4}+\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 0.41, size = 29, normalized size = 1.93

$$\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))
```

Mupad [B]
time = 3.11, size = 24, normalized size = 1.60

$$\frac{\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+\mathrm{tan}\left(\frac{x}{2}\right)\right)}{2}-\frac{\ln\left(\mathrm{tan}\left(\frac{x}{2}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/sin(2*x),x)
```

```
[Out] log(tan(x/2) + tan(x/2)^2)/2 - log(tan(x/2) - 1)/2
```

3.806 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log\left(2-3\sin(x)+\sin^2(x)\right)$$

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Rubi [A]
time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4419, 642}

$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2}\,dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.
time = 0.07, size = 26, normalized size = 2.36

$$2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log(2-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

Maple [A]
time = 0.06, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\ln\left(2-3\sin\left(x\right)+\sin^{2}\left(x\right)\right)$	12
default	$\ln\left(2-3\sin\left(x\right)+\sin^{2}\left(x\right)\right)$	12
risch	$-2ix+2\ln\left(\mathrm{e}^{ix}-i\right)+\ln\left(-4i\,\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1\right)$	33
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\ln\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)+\ln\left(\tan^{2}\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)+1\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 1.00

$$\log\left(\sin\left(x\right)^{2}-3\,\sin\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

Fricas [A]
time = 3.74, size = 15, normalized size = 1.36

$$\log\left(-\frac{1}{2}\,\sin\left(x\right)+1\right)+\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

Sympy [A]
time = 0.08, size = 12, normalized size = 1.09

$$\log\left(\sin\left(x\right)-2\right)+\log\left(\sin\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

Giac [A]
time = 0.40, size = 15, normalized size = 1.36

$$\log\left(-\sin\left(x\right)+2\right)+\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

Mupad [B]
time = 0.08, size = 11, normalized size = 1.00

$$\ln\left(\sin\left(x\right)^2-3\,\sin\left(x\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

3.807 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\,\text{ArcTan}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4420, 327, 209}

$$\sqrt{5}\,\text{ArcTan}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = -\text{Subst}\left(\int \frac{x^2}{5+x^2}\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + 5\text{Subst}\left(\int \frac{1}{5+x^2}\,dx, x, \cos(x)\right)$$
$$= \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right) - \cos(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. $2(20) = 40$.
time = 0.12, size = 82, normalized size = 4.10

$$\frac{1}{20}\left(-\sqrt{5}\,\text{ArcTan}\left(\frac{\cos(x)}{\sqrt{5}}\right) + 21\sqrt{5}\,\text{ArcTan}\left(\frac{1}{\sqrt{5}} - \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) + 21\sqrt{5}\,\text{ArcTan}\left(\frac{1}{\sqrt{5}} + \sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right) - 20\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

Maple [A]
time = 0.05, size = 18, normalized size = 0.90

method	result	size
derivativedivides	$-\cos(x) + \arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$	18
default	$-\cos(x) + \arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$	18
risch	$-\frac{e^{ix}}{2} - \frac{e^{-ix}}{2} - \frac{i\sqrt{5}\,\ln\left(e^{2ix} - 2i\sqrt{5}\,e^{ix} + 1\right)}{2} + \frac{i\sqrt{5}\,\ln\left(e^{2ix} + 2i\sqrt{5}\,e^{ix} + 1\right)}{2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x,method=_RETURNVERBOSE)
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.85

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Fricas [A]
time = 3.54, size = 17, normalized size = 0.85

$$\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,\cos\left(x\right)\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Sympy [A]
time = 0.13, size = 19, normalized size = 0.95

$$-\cos\left(x\right)+\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\cos\left(x\right)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

Giac [A]
time = 0.42, size = 17, normalized size = 0.85

$$\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,\cos\left(x\right)\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Mupad [B]
time = 2.90, size = 17, normalized size = 0.85

$$\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\cos\left(x\right)}{5}\right)-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)
[Out] 5^(1/2)*atan((5^(1/2)*cos(x))/5) - cos(x)
```

3.808 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(1+\sin(x))$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3339, 629}

$$\log(\sin(x)) - \log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 629

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rule 3339

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x+x^2}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\log(\sin(x)) - \log(1+\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Maple [A]
time = 0.07, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\ln(\sin(x)) - \ln(1 + \sin(x))$	12
default	$\ln(\sin(x)) - \ln(1 + \sin(x))$	12
norman	$-2\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	16
risch	$-2\ln\left(e^{ix} + i\right) + \ln\left(e^{2ix} - 1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.00

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Fricas [A]
time = 2.69, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2}\sin(x)\right) - \log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.91

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Giac [A]
time = 0.41, size = 12, normalized size = 1.09

$$-\log\left(\sin\left(x\right)+1\right)+\log\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

Mupad [B]
time = 2.98, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}(2\,\sin\left(x\right)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^2),x)
```

```
[Out] -2*atanh(2*sin(x) + 1)
```

3.809 $\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)}\,dx$

Optimal. Leaf size=26

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)$$

```
[Out] ln(sin(x))-ln(1+sin(x)^(2^(1/2)-1))*(1+2^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4419, 272, 36, 29, 31}

$$\log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(\sin^{\sqrt{2}-1}(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

```
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
```

```
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx, x, \sin(x)\right)\\ &= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \sin^{-1+\sqrt{2}}(x)\right)\\ &= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin^{-1+\sqrt{2}}(x)\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx\right.\\ &= \log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$\log(\sin(x))-\left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
[Out] Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.38, size = 674, normalized size = 25.92

method	result	size
risch	Expression too large to display	674

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x,method=_RETURNVERBOSE)
[Out] -I*Pi-2*ln(2)+2^(1/2)*ln(exp(2*I*x)-1)-2^(1/2)*ln(2)-2^(1/2)*ln(exp(I*x))-l
n(exp(-1/2*2^(1/2)*(-I*Pi*csgn(sin(x))^3-I*Pi*csgn(sin(x))^2*csgn(I*exp(-I*
x))-I*Pi*csgn(sin(x))^2*csgn(I*(exp(2*I*x)-1))-I*Pi*csgn(sin(x))*csgn(I*exp
(-I*x))*csgn(I*(exp(2*I*x)-1))+I*Pi*csgn(sin(x))*csgn(I*sin(x))^2+I*Pi*csgn
(I*sin(x))^3-I*Pi*csgn(sin(x))*csgn(I*sin(x))-I*Pi*csgn(I*sin(x))^2+I*Pi+2*
ln(2)+2*ln(exp(I*x))-2*ln(exp(2*I*x)-1)))+sin(x))+1/2*I*2^(1/2)*Pi*csgn(sin
(x))*csgn(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1))+2*ln(exp(2*I*x)-1)+1/2*I*2^(1
/2)*Pi*csgn(sin(x))^3-1/2*I*2^(1/2)*Pi*csgn(I*sin(x))^3+1/2*I*2^(1/2)*Pi*cs
```

```
gn(I*sin(x))^2-ln(exp(-1/2*2^(1/2)*(-I*Pi*csgn(sin(x))^3-I*Pi*csgn(sin(x))^
2*csgn(I*exp(-I*x))-I*Pi*csgn(sin(x))^2*csgn(I*(exp(2*I*x)-1))-I*Pi*csgn(si
n(x))*csgn(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1))+I*Pi*csgn(sin(x))*csgn(I*sin
(x))^2+I*Pi*csgn(I*sin(x))^3-I*Pi*csgn(sin(x))*csgn(I*sin(x))-I*Pi*csgn(I*s
in(x))^2+I*Pi+2*ln(2)+2*ln(exp(I*x))-2*ln(exp(2*I*x)-1)))+sin(x))*2^(1/2)-2
*ln(exp(I*x))+I*Pi*csgn(sin(x))^2*csgn(I*exp(-I*x))+I*Pi*csgn(sin(x))^2*csg
n(I*(exp(2*I*x)-1))+I*Pi*csgn(sin(x))*csgn(I*sin(x))+I*Pi*csgn(sin(x))*csgn
(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1))-1/2*I*2^(1/2)*Pi-I*Pi*csgn(I*sin(x))^3
+1/2*I*2^(1/2)*Pi*csgn(sin(x))^2*csgn(I*exp(-I*x))+I*Pi*csgn(sin(x))^3+I*Pi
*csgn(I*sin(x))^2+1/2*I*2^(1/2)*Pi*csgn(sin(x))^2*csgn(I*(exp(2*I*x)-1))-1/
2*I*2^(1/2)*Pi*csgn(sin(x))*csgn(I*sin(x))^2+1/2*I*2^(1/2)*Pi*csgn(sin(x))*
csgn(I*sin(x))-I*Pi*csgn(I*sin(x))^2*csgn(sin(x))
```

Maxima [A]
time = 0.51, size = 34, normalized size = 1.31

$$\frac{\sqrt{2}\,\log\left(\sin\left(x\right)\right)}{\sqrt{2}-1}-\frac{\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)}+\sin\left(x\right)\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="maxima")
[Out] sqrt(2)*log(sin(x))/(sqrt(2) - 1) - log(sin(x)^sqrt(2) + sin(x))/(sqrt(2) -
 1)
```

Fricas [A]
time = 3.06, size = 27, normalized size = 1.04

$$-\left(\sqrt{2}+1\right)\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)}+\sin\left(x\right)\right)+\left(\sqrt{2}+2\right)\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(sin(x)^sqrt(2) + sin(x)) + (sqrt(2) + 2)*log(sin(x))
```

Sympy [A]
time = 0.34, size = 36, normalized size = 1.38

$$-\frac{\log\left(\sin\left(x\right)+\sin^{\sqrt{2}}\left(x\right)\right)}{-1+\sqrt{2}}+\frac{\sqrt{2}\,\log\left(\sin\left(x\right)\right)}{-1+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**(2**(1/2))),x)
```

```
[Out] -log(sin(x) + sin(x)**(sqrt(2)))/(-1 + sqrt(2)) + sqrt(2)*log(sin(x))/(-1 +
 sqrt(2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="giac")
[Out] integrate(cos(x)/(sin(x)^sqrt(2) + sin(x)), x)
```

Mupad [B]
time = 3.08, size = 29, normalized size = 1.12

$$\ln\left(\sin\left(x\right)\right)\,\left(\sqrt{2}+2\right)-\frac{\ln\left(\sin\left(x\right)+{\sin\left(x\right)}^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x) + sin(x)^(2^(1/2))),x)
[Out] log(sin(x))*(2^(1/2) + 2) - log(sin(x) + sin(x)^(2^(1/2)))/(2^(1/2) - 1)
```

3.810 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)$$

```
[Out] 1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 14}

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sin(x)+\sin(2x)}\,dx &= 2\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(1+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

Maple [A]
time = 0.15, size = 24, normalized size = 1.00

method	result	size
default	$\frac{1}{4\cos(x)+4}-\frac{\ln(1+\cos(x))}{8}+\frac{\ln(\cos(x)-1)}{8}$	24
risch	$\frac{e^{ix}}{2(e^{ix}+1)^2}-\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(e^{ix}-1)}{4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)+sin(2*x)),x,method=_RETURNVERBOSE)
[Out] 1/4/(1+cos(x))-1/8*ln(1+cos(x))+1/8*ln(cos(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(16) = 32$.
time = 0.29, size = 220, normalized size = 9.17

$$\frac{4\cos(2x)\cos(x)+8\cos(x)^2-\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(2x)\sin(x)+4\sin(x)^2+4\cos(x)+1\right)\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(2x)\sin(x)+4\sin(x)^2+4\cos(x)+1\right)\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+4\sin(2x)\sin(x)+8\sin(x)^2+4\cos(x)}{8\left(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(2x)\sin(x)+4\sin(x)^2+4\cos(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 2.30, size = 35, normalized size = 1.46

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2\sin(x) + \sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

Giac [A]
time = 0.42, size = 28, normalized size = 1.17

$$-\frac{\cos(x) - 1}{8(\cos(x) + 1)} + \frac{1}{8}\log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B]
time = 3.07, size = 16, normalized size = 0.67

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(2*x) + 2*sin(x)),x)
```

```
[Out] log(tan(x/2))/4 + tan(x/2)^2/8
```

3.811 $\int (-3+4x+x^2)\sin(2x)\,dx$

Optimal. Leaf size=40

$$\frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)$$

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Rubi [A]
time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {6874, 2718, 3377, 2717}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int (-3+4x+x^2)\sin(2x)\,dx &= \int \left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx \\
&= -(3\int \sin(2x)\,dx) + 4\int x\sin(2x)\,dx + \int x^2\sin(2x)\,dx \\
&= \frac{3}{2}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + 2\int \cos(2x)\,dx + \int x\cos(2x)\,dx \\
&= \frac{3}{2}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + \sin(2x) + \frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x) \\
&= \frac{7}{4}\cos(2x) - 2x\cos(2x) - \frac{1}{2}x^2\cos(2x) + \sin(2x) + \frac{1}{2}x\sin(2x)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.72

$$\frac{1}{4}\left((7-8x-2x^2)\cos(2x)+2(2+x)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

Maple [A]
time = 0.05, size = 35, normalized size = 0.88

method	result
risch	$\left(-\frac{1}{2}x^2-2x+\frac{7}{4}\right)\cos(2x)+\frac{(2+x)\sin(2x)}{2}$
derivativedivides	$\frac{7\cos(2x)}{4}-2x\cos(2x)-\frac{x^2\cos(2x)}{2}+\sin(2x)+\frac{x\sin(2x)}{2}$
default	$\frac{7\cos(2x)}{4}-2x\cos(2x)-\frac{x^2\cos(2x)}{2}+\sin(2x)+\frac{x\sin(2x)}{2}$
norman	$\frac{x\tan(x)-2x-\frac{x^2}{2}+2x\left(\tan^2(x)\right)+\frac{x^2\left(\tan^2(x)\right)}{2}+2\tan(x)+\frac{7}{2}}{\tan^2(x)+1}$
meijerg	$\frac{\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{(-2x^2+1)\cos(2x)}{2\sqrt{\pi}}+\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{2}+2\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)-\frac{3\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-3)*sin(2*x),x,method=_RETURNVERBOSE)
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Maxima [A]
time = 0.30, size = 38, normalized size = 0.95

$$-\frac{1}{4}\left(2\,x^{2}-1\right)\cos\left(2\,x\right)-2\,x\cos\left(2\,x\right)+\frac{1}{2}\,x\sin\left(2\,x\right)+\frac{3}{2}\,\cos\left(2\,x\right)+\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

Fricas [A]
time = 3.33, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2\,x^{2}+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.98

$$-\frac{x^{2}\cos\left(2x\right)}{2}+\frac{x\sin\left(2x\right)}{2}-2x\cos\left(2x\right)+\sin\left(2x\right)+\frac{7\cos\left(2x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

Giac [A]
time = 0.40, size = 26, normalized size = 0.65

$$-\frac{1}{4}\left(2\,x^{2}+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Mupad [B]
time = 2.90, size = 34, normalized size = 0.85

$$\frac{7\,\cos\left(2\,x\right)}{4}+\sin\left(2\,x\right)-2\,x\,\cos\left(2\,x\right)+\frac{x\,\sin\left(2\,x\right)}{2}-\frac{x^2\,\cos\left(2\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(4*x + x^2 - 3),x)
```

```
[Out] (7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))/
2
```

3.812 $\int e^{-3x} \cos(4x) \, dx$

Optimal. Leaf size=27

$$-\frac{3}{25}e^{-3x}\cos(4x) + \frac{4}{25}e^{-3x}\sin(4x)$$

```
[Out] -3/25*cos(4*x)/exp(3*x)+4/25*sin(4*x)/exp(3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4518}

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4518

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}e^{-3x}\cos(4x) + \frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(-3\cos(4x) + 4\sin(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

Maple [A]
time = 0.03, size = 22, normalized size = 0.81

method	result	size
default	$-\frac{3\,{\mathrm e}^{-3 x} \cos\left(4 x\right)}{25}+\frac{4\,{\mathrm e}^{-3 x} \sin\left(4 x\right)}{25}$	22
norman	$\frac{\left(-\frac{3}{25}+\frac{3\left(\tan^{2}\left(2 x\right)\right)}{25}+\frac{8 \tan\left(2 x\right)}{25}\right) {\mathrm e}^{-3 x}}{1+\tan^{2}\left(2 x\right)}$	34
risch	$-\frac{3\,{\mathrm e}^{\left(-3+4 i\right) x}}{50}-\frac{2 i {\mathrm e}^{\left(-3+4 i\right) x}}{25}-\frac{3\,{\mathrm e}^{\left(-3-4 i\right) x}}{50}+\frac{2 i {\mathrm e}^{\left(-3-4 i\right) x}}{25}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/exp(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.70

$$-\frac{1}{25} \, {\left(3 \, \cos\left(4 \, x\right) - 4 \, \sin\left(4 \, x\right)\right)} e^{\left(-3 \, x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Fricas [A]
time = 2.57, size = 21, normalized size = 0.78

$$-\frac{3}{25} \, \cos\left(4 \, x\right) e^{\left(-3 \, x\right)} + \frac{4}{25} \, e^{\left(-3 \, x\right)} \sin\left(4 \, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

Sympy [A]
time = 0.17, size = 26, normalized size = 0.96

$$\frac{4 e^{- 3 x} \sin{\left(4 x \right)}}{25} - \frac{3 e^{- 3 x} \cos{\left(4 x \right)}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

Giac [A]
time = 0.43, size = 19, normalized size = 0.70

$$-\frac{1}{25}\left(3 \cos\left(4 x\right) - 4 \sin\left(4 x\right)\right)e^{\left(-3 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.70

$$-\frac{e^{-3 x}\left(3 \cos\left(4 x\right) - 4 \sin\left(4 x\right)\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)*exp(-3*x),x)
[Out] -(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25
```

3.813 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}$$

[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {2912, 45}

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]

[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx,x,\sin(x)\right)\\ &= -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.35

$$\frac{2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2(-2+\sin(x))}{3\sqrt{1+\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$\frac{2(1+\sin(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\sin(x)}$	18
default	$\frac{2(1+\sin(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\sin(x)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+sin(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.74

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

Fricas [A]
time = 2.94, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\sin(x)+1}(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

Sympy [A]
time = 0.09, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin\left(x\right)+1}\,\sin\left(x\right)}{3}-\frac{4\sqrt{\sin\left(x\right)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.
time = 0.40, size = 42, normalized size = 1.83

$$\frac{2\left(2\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)^{3}-3\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*sqrt(2)*cos(-1/4*pi + 1/2*x)^3 - 3*sqrt(2)*cos(-1/4*pi + 1/2*x))/sgn
(cos(-1/4*pi + 1/2*x))
```

Mupad [B]
time = 0.10, size = 12, normalized size = 0.52

$$\frac{2\sqrt{\sin\left(x\right)+1}\,\left(\sin\left(x\right)-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)
```

```
[Out] (2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3
```

3.814 $\int (x+60 \cos^5(x) \sin^4(x)) \, dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+12 \sin^5(x)-\frac{120 \sin^7(x)}{7}+\frac{20 \sin^9(x)}{3}$$

```
[Out] 1/2*x^2+12*sin(x)^5-120/7*sin(x)^7+20/3*sin(x)^9
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2644, 276}

$$\frac{x^2}{2}+\frac{20 \sin^9(x)}{3}-\frac{120 \sin^7(x)}{7}+12 \sin^5(x)$$

Antiderivative was successfully verified.

```
[In] Int[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int (x+60 \cos^5(x) \sin^4(x)) \, dx &= \frac{x^2}{2}+60 \int \cos^5(x) \sin^4(x) \, dx \\
&= \frac{x^2}{2}+60 \text{Subst}\left(\int x^4 (1-x^2)^2 \, dx, x, \sin(x)\right) \\
&= \frac{x^2}{2}+60 \text{Subst}\left(\int (x^4-2 x^6+x^8) \, dx, x, \sin(x)\right) \\
&= \frac{x^2}{2}+12 \sin^5(x)-\frac{120 \sin^7(x)}{7}+\frac{20 \sin^9(x)}{3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.53

$$\frac{x^2}{2}+\frac{45\sin(x)}{32}-\frac{5}{16}\sin(3x)-\frac{3}{16}\sin(5x)+\frac{15}{448}\sin(7x)+\frac{5}{192}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x + 60*Cos[x]^5*Sin[x]^4,x]
```

```
[Out] x^2/2 + (45*Sin[x])/32 - (5*Sin[3*x])/16 - (3*Sin[5*x])/16 + (15*Sin[7*x])/
448 + (5*Sin[9*x])/192
```

Maple [A]
time = 0.08, size = 41, normalized size = 1.37

method	result	size
risch	$\frac{x^2}{2}+\frac{45\sin(x)}{32}+\frac{5\sin(9x)}{192}+\frac{15\sin(7x)}{448}-\frac{3\sin(5x)}{16}-\frac{5\sin(3x)}{16}$	35
default	$\frac{x^2}{2}-\frac{20(\cos^6(x))(\sin^3(x))}{3}-\frac{20\sin(x)(\cos^6(x))}{7}+\frac{4\left(\frac{8}{3}+\cos^4(x)+\frac{4(\cos^2(x))}{3}\right)\sin(x)}{7}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+60*cos(x)^5*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-20/3*cos(x)^6*sin(x)^3-20/7*sin(x)*cos(x)^6+4/7*(8/3+cos(x)^4+4/3*c
os(x)^2)*sin(x)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.80

$$\frac{20}{3}\sin(x)^9-\frac{120}{7}\sin(x)^7+12\sin(x)^5+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

Fricas [A]
time = 1.92, size = 36, normalized size = 1.20

$$\frac{1}{2}x^2+\frac{4}{21}\left(35\cos(x)^8-50\cos(x)^6+3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 4/21*(35*cos(x)^8 - 50*cos(x)^6 + 3*cos(x)^4 + 4*cos(x)^2 + 8)*si
n(x)
```

Sympy [A]
time = 0.01, size = 27, normalized size = 0.90

$$\frac{x^2}{2} + \frac{20\sin^9{(x)}}{3} - \frac{120\sin^7{(x)}}{7} + 12\sin^5{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)**5*sin(x)**4,x)
[Out] x**2/2 + 20*sin(x)**9/3 - 120*sin(x)**7/7 + 12*sin(x)**5
```

Giac [A]
time = 0.42, size = 24, normalized size = 0.80

$$\frac{20}{3}\,\sin\left(x\right)^{9} - \frac{120}{7}\,\sin\left(x\right)^{7} + 12\,\sin\left(x\right)^{5} + \frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="giac")
[Out] 20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

Mupad [B]
time = 3.00, size = 24, normalized size = 0.80

$$\frac{x^2}{2} + \frac{20\sin\left(x\right)^9}{3} - \frac{120\sin\left(x\right)^7}{7} + 12\sin\left(x\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 60*cos(x)^5*sin(x)^4,x)
[Out] 12*sin(x)^5 - (120*sin(x)^7)/7 + (20*sin(x)^9)/3 + x^2/2
```

3.815 $\int \cos(x)(\sec(x) + \tan(x))\,dx$

Optimal. Leaf size=6

$$x - \cos(x)$$

```
[Out] x-cos(x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3240, 2718}

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(Sec[x] + Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3240

```
Int[cos[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos(x)(\sec(x) + \tan(x))\,dx &= \int (1 + \sin(x))\,dx\\ &= x + \int \sin(x)\,dx\\ &= x - \cos(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x] + Tan[x]),x]
[Out] x - Cos[x]
```

Maple [A]
time = 0.07, size = 7, normalized size = 1.17

method	result	size
default	$x-\cos(x)$	7
risch	$x-\cos(x)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] x-cos(x)
```

Maxima [A]
time = 0.29, size = 6, normalized size = 1.00

$$x-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="maxima")
[Out] x - cos(x)
```

Fricas [A]
time = 2.17, size = 6, normalized size = 1.00

$$x-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="fricas")
[Out] x - cos(x)
```

Sympy [A]
time = 0.66, size = 3, normalized size = 0.50

$$x-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x)
[Out] x - cos(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.43, size = 14, normalized size = 2.33

$$x - \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="giac")
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 2.95, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(tan(x) + 1/cos(x)),x)
[Out] x - cos(x)
```

3.816 $\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx$

Optimal. Leaf size=7

$$-\cos(x)+\tan(x)$$

[Out] -cos(x)+tan(x)

Rubi [A]
time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {4486, 3852, 8, 2718}

$$\tan(x)-\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(Sec[x]^3 + Tan[x]),x]

[Out] -Cos[x] + Tan[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx &= \int \left(\sec^2(x)+\sin(x)\right)\,dx\\
&= \int \sec^2(x)\,dx + \int \sin(x)\,dx\\
&= -\cos(x) - \text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\
&= -\cos(x) + \tan(x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\cos(x)+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(Sec[x]^3 + Tan[x]),x]
[Out] -Cos[x] + Tan[x]
```

Maple [A]
time = 0.10, size = 8, normalized size = 1.14

method	result	size
default	$-\cos(x)+\tan(x)$	8
risch	$\frac{2i}{e^{2ix}+1}-\cos(x)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(sec(x)^3+tan(x)),x,method=_RETURNVERBOSE)
[Out] -cos(x)+tan(x)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 1.00

$$-\cos(x)+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="maxima")
[Out] -cos(x) + tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.
time = 2.84, size = 15, normalized size = 2.14

$$-\frac{\cos(x)^2 - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="fricas")
[Out] -(cos(x)^2 - sin(x))/cos(x)
```

Sympy [A]
time = 3.50, size = 8, normalized size = 1.14

$$\frac{\sin(x)}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)**3+tan(x)),x)
[Out] sin(x)/cos(x) - cos(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(7) = 14$.
time = 0.42, size = 30, normalized size = 4.29

$$-\frac{2\left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right) - 1\right)}{\tan\left(\frac{1}{2}x\right)^4 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="giac")
[Out] -2*(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) - 1)/(tan(1/2*x)^4 - 1)
```

Mupad [B]
time = 2.97, size = 12, normalized size = 1.71

$$\frac{\sin(x)}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(tan(x) + 1/cos(x)^3),x)
[Out] sin(x)/cos(x) - cos(x)
```

3.817 $\int \frac{1}{2}(-\cot(x)\csc(x)+\csc^2(x))\,dx$

Optimal. Leaf size=13

$$-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}$$

```
[Out] -1/2*cot(x)+1/2*csc(x)
```

Rubi [A]

time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {12, 2686, 8, 3852}

$$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

```
[Out] -1/2*Cot[x] + Csc[x]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx &= \frac{1}{2}\int \left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx\\
&= -\left(\frac{1}{2}\int \cot(x)\csc(x)\,dx\right)+\frac{1}{2}\int \csc^2(x)\,dx\\
&= -\left(\frac{1}{2}\text{Subst}(\int 1\,dx,x,\cot(x))\right)+\frac{1}{2}\text{Subst}(\int 1\,dx,x,\csc(x))\\
&= -\frac{\cot(x)}{2}+\frac{\csc(x)}{2}
\end{aligned}
$$

Mathematica [**A**]
time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{2}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
[Out] Tan[x/2]/2
```

Maple [**A**]
time = 0.05, size = 10, normalized size = 0.77

method	result	size
default	$-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}$	10
risch	$\frac{i}{e^{ix}+1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*cot(x)+1/2*csc(x)
```

Maxima [**A**]
time = 0.29, size = 13, normalized size = 1.00

$$\frac{1}{2\sin(x)}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="maxima")
[Out] 1/2/sin(x) - 1/2/tan(x)
```

Fricas [A]
time = 3.98, size = 10, normalized size = 0.77

$$\frac{\sin(x)}{2(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="fricas")
[Out] 1/2*sin(x)/(cos(x) + 1)
```

Sympy [A]
time = 0.02, size = 14, normalized size = 1.08

$$-\frac{\cos(x)}{2\sin(x)}+\frac{1}{2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)**2,x)
[Out] -cos(x)/(2*sin(x)) + 1/(2*sin(x))
```

Giac [A]
time = 0.41, size = 13, normalized size = 1.00

$$\frac{1}{2\sin(x)}-\frac{1}{2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="giac")
[Out] 1/2/sin(x) - 1/2/tan(x)
```

Mupad [B]
time = 2.93, size = 6, normalized size = 0.46

$$\frac{\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)^2) - cot(x)/(2*sin(x)),x)
[Out] tan(x/2)/2
```

3.818 $\int (-\csc^2(x)+\sin(2x))\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\cos(2x)+\cot(x)$$

[Out] -1/2*cos(2*x)+cot(x)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3852, 8, 2718}

$$\cot(x)-\frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

[In] Int[-Csc[x]^2 + Sin[2*x],x]

[Out] -1/2*Cos[2*x] + Cot[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\csc^2(x)+\sin(2x))\,dx &= -\int \csc^2(x)\,dx+\int \sin(2x)\,dx\\ &= -\frac{1}{2}\cos(2x)+\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\frac{1}{2}\cos(2x)+\cot(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{1}{2}\cos(2x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Csc[x]^2 + Sin[2*x],x]
```

```
[Out] -1/2*Cos[2*x] + Cot[x]
```

Maple [A]
time = 0.08, size = 10, normalized size = 0.91

method	result	size
default	$-\frac{\cos(2x)}{2}+\cot\left(x\right)$	10
risch	$\frac{2i}{e^{2ix}-1}-\frac{e^{2ix}}{4}-\frac{e^{-2ix}}{4}$	28
norman	$\frac{\frac{1}{2}-\tan\left(\frac{x}{2}\right)+\frac{\left(\tan^{2}(x)\right)}{2}-\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}-\frac{\left(\tan^{2}(x)\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)\left(\tan^{2}(x)+1\right)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-csc(x)^2+sin(2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(2*x)+cot(x)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.00

$$\frac{1}{\tan\left(x\right)}-\frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/tan(x) - 1/2*cos(2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 2.89, size = 22, normalized size = 2.00

$$-\frac{\left(2\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)-2\,\cos\left(x\right)}{2\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*((2*cos(x)^2 - 1)*sin(x) - 2*cos(x))/sin(x)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 1.09

$$-\frac{\cos(2x)}{2}+\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)**2+sin(2*x),x)
[Out] -cos(2*x)/2 + cos(x)/sin(x)
```

Giac [A]
time = 0.42, size = 11, normalized size = 1.00

$$\frac{1}{\tan(x)}-\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-csc(x)^2+sin(2*x),x, algorithm="giac")
[Out] 1/tan(x) - 1/2*cos(2*x)
```

Mupad [B]
time = 2.92, size = 14, normalized size = 1.27

$$\frac{\cos(x)}{\sin(x)}-\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x) - 1/sin(x)^2,x)
[Out] cos(x)/sin(x) - cos(x)^2
```

3.819 $\int (2\cot(2x)-3\sin(3x))\,dx$

Optimal. Leaf size=10

$$\cos(3x)+\log(\sin(2x))$$

```
[Out] cos(3*x)+ln(sin(2*x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {3556, 2718}

$$\cos(3x)+\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[2*Cot[2*x] - 3*Sin[3*x],x]
```

```
[Out] Cos[3*x] + Log[Sin[2*x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2\cot(2x)-3\sin(3x))\,dx &= 2\int \cot(2x)\,dx-3\int \sin(3x)\,dx\\ &= \cos(3x)+\log(\sin(2x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\cos(3x)+\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[2*Cot[2*x] - 3*Sin[3*x],x]
```

```
[Out] Cos[3*x] + Log[Sin[2*x]]
```

Maple [A]
time = 0.07, size = 17, normalized size = 1.70

method	result	size
default	$-\frac{\ln(\cot^2(2x)+1)}{2} + \cos(3x)$	17
risch	$-2ix + \ln(e^{4ix} - 1) + \cos(3x)$	18
norman	$\frac{2}{1+\tan^2(\frac{3x}{2})} - \frac{\ln(1+\tan^2(2x))}{2} + \ln(\tan(2x))$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*cot(2*x)-3*sin(3*x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(cot(2*x)^2+1)+cos(3*x)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\cos(3x) + \log(\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="maxima")
[Out] cos(3*x) + log(sin(2*x))
```

Fricas [A]
time = 2.73, size = 18, normalized size = 1.80

$$4\cos(x)^3 - 3\cos(x) + \log\left(-\frac{1}{2}\cos(x)\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="fricas")
[Out] 4*cos(x)^3 - 3*cos(x) + log(-1/2*cos(x)*sin(x))
```

Sympy [A]
time = 0.01, size = 10, normalized size = 1.00

$$\log(\sin(2x)) + \cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x)
[Out] log(sin(2*x)) + cos(3*x)
```

Giac [A]
time = 0.41, size = 11, normalized size = 1.10

$$\cos(3x) + \log(|\sin(2x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="giac")
```

```
[Out] cos(3*x) + log(abs(sin(2*x)))
```

Mupad [B]
time = 3.08, size = 24, normalized size = 2.40

$$\cos(3x) + \ln\left(\cos\left(\frac{x}{2}\right)\left(\sin\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)^3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*cot(2*x) - 3*sin(3*x),x)
```

```
[Out] cos(3*x) + log(cos(x/2)*(sin(x/2) - 2*sin(x/2)^3))
```

3.820 $\int x \sin(2x^2)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{4}\cos(2x^2)$$

```
[Out] -1/4*cos(2*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3460, 2718}

$$-\frac{1}{4}\cos(2x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[2*x^2],x]
```

```
[Out] -1/4*Cos[2*x^2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x \sin(2x^2)\, dx &= \frac{1}{2}\text{Subst}\left(\int \sin(2x)\, dx, x, x^2\right)\\ &= -\frac{1}{4}\cos(2x^2)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\frac{1}{4}\cos(2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[2*x^2],x]
```

```
[Out] -1/4*Cos[2*x^2]
```

Maple [A]
time = 0.01, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{\cos(2x^2)}{4}$	9
default	$-\frac{\cos(2x^2)}{4}$	9
risch	$-\frac{\cos(2x^2)}{4}$	9
norman	$-\frac{1}{2(1+\tan^2(x^2))}$	13
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x^2)}{\sqrt{\pi}}\right)}{4}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(2*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(2*x^2)
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(2*x^2)
```

Fricas [A]
time = 2.99, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x^2)
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.80

$$-\frac{\cos(2x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x**2),x)
```

```
[Out] -cos(2*x**2)/4
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.80

$$-\frac{1}{4}\cos\left(2\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(2*x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(2*x^2)
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$\frac{\sin(x^2)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(2*x^2),x)
```

```
[Out] sin(x^2)^2/2
```

3.821 $\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx$

Optimal. Leaf size=18

$$\frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}$$

```
[Out] 1/3*(1+sin(-1+x)^2)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3277, 267}

$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 3277

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m
 - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d
, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx &= \text{Subst}\left(\int x\sqrt{1+x^2}\,dx, x, \sin(1-x)\right)\\ &= \frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$\frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

```
[Out] (1 + Sin[1 - x]^2)^(3/2)/3
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.72

method	result	size
derivativedivides	$\frac{\left(1+\sin^{2}\left(-1+x\right)\right)^{\frac{3}{2}}}{3}$	13
default	$\frac{\left(1+\sin^{2}\left(-1+x\right)\right)^{\frac{3}{2}}}{3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+sin(-1+x)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.67

$$\frac{1}{3} \left(\sin \left(x-1\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

Fricas [A]
time = 3.20, size = 14, normalized size = 0.78

$$\frac{1}{3} \left(-\cos \left(x-1\right)^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-cos(x - 1)^2 + 2)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 0.17, size = 32, normalized size = 1.78

$$\frac{\sqrt{\sin^{2}\left(x-1\right)+1}\,\sin^{2}\left(x-1\right)}{3}+\frac{\sqrt{\sin^{2}\left(x-1\right)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)**2)**(1/2),x)
```

```
[Out] sqrt(sin(x - 1)**2 + 1)*sin(x - 1)**2/3 + sqrt(sin(x - 1)**2 + 1)/3
```

Giac [A]
time = 0.39, size = 12, normalized size = 0.67

$$\frac{1}{3}\left(\sin\left(x-1\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sin(x - 1)^2 + 1)^(3/2)
```

Mupad [B]
time = 2.99, size = 12, normalized size = 0.67

$$\frac{\left(\sin\left(x-1\right)^{2}+1\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x - 1)*sin(x - 1)*(sin(x - 1)^2 + 1)^(1/2),x)
```

```
[Out] (sin(x - 1)^2 + 1)^(3/2)/3
```

3.822 $\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

```
[Out] -1/2*sin(1/x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3522}

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

```
[Out] -1/2*Sin[x^(-1)]^2
```

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{2}\cos^2\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

```
[Out] Cos[x^(-1)]^2/2
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{\left(\cos^2\left(\frac{1}{x}\right)\right)}{2}$	9
default	$\frac{\left(\cos^2\left(\frac{1}{x}\right)\right)}{2}$	9
risch	$\frac{\cos\left(\frac{2}{x}\right)}{4}$	9
meijerg	$-\frac{\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{2}{x}\right)}{\sqrt{\pi}}\right)}{4}$	21
norman	$\frac{x+x\left(\tan^4\left(\frac{1}{2x}\right)\right)}{\left(1+\tan^2\left(\frac{1}{2x}\right)\right)^2 x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)*sin(1/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*cos(1/x)^2
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*cos(1/x)^2
```

Fricas [A]
time = 4.68, size = 8, normalized size = 0.80

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(1/x)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.
time = 0.55, size = 31, normalized size = 3.10

$$-\frac{2\tan^2\left(\frac{1}{2x}\right)}{\tan^4\left(\frac{1}{2x}\right)+2\tan^2\left(\frac{1}{2x}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x**2,x)
```

```
[Out] -2*tan(1/(2*x))**2/(tan(1/(2*x))**4 + 2*tan(1/(2*x))**2 + 1)
```

Giac [A]
time = 0.43, size = 8, normalized size = 0.80

$$\frac{1}{2}\cos\left(\frac{1}{x}\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*cos(1/x)^2
```

Mupad [B]
time = 2.92, size = 8, normalized size = 0.80

$$\frac{\cos\left(\frac{1}{x}\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(1/x)*sin(1/x))/x^2,x)
```

```
[Out] cos(1/x)^2/2
```

3.823 $\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{6}\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)$$

```
[Out] 1/6*sin(1/2+3/2*x)^4
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2644, 30}

$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

```
[Out] Sin[1/2 + (3*x)/2]^4/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx &= \frac{2}{3}\text{Subst}\left(\int x^3\,dx, x, \sin\left(\frac{1}{2}+\frac{3x}{2}\right)\right)\\ &= \frac{1}{6}\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.56

$$\frac{1}{2}\left(-\frac{1}{6}\cos(1+3x)+\frac{1}{24}\cos(2+6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
[Out] (-1/6*Cos[1 + 3*x] + Cos[2 + 6*x]/24)/2
```

Maple [A]
time = 0.06, size = 11, normalized size = 0.69

method	result	size
derivativedivides	$\frac{\left(\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)\right)}{6}$	11
default	$\frac{\left(\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)\right)}{6}$	11
risch	$-\frac{\cos(1+3x)}{12}+\frac{\cos(2+6x)}{48}$	18
norman	$\frac{-\frac{16\left(\tan^2\left(\frac{1}{4}+\frac{3x}{4}\right)\right)}{9}-\frac{16\left(\tan^6\left(\frac{1}{4}+\frac{3x}{4}\right)\right)}{9}-\frac{4\left(\tan^8\left(\frac{1}{4}+\frac{3x}{4}\right)\right)}{9}-\frac{4}{9}}{\left(1+\tan^2\left(\frac{1}{4}+\frac{3x}{4}\right)\right)^4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/6*sin(1/2+3/2*x)^4
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.62

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="maxima")
[Out] 1/6*sin(3/2*x + 1/2)^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 2.71, size = 21, normalized size = 1.31

$$\frac{1}{6}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^4-\frac{1}{3}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="fricas")
[Out] 1/6*cos(3/2*x + 1/2)^4 - 1/3*cos(3/2*x + 1/2)^2
```

Sympy [A]
time = 0.13, size = 12, normalized size = 0.75

$$\frac{\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)**3,x)
```

```
[Out] sin(3*x/2 + 1/2)**4/6
```

Giac [A]
time = 0.46, size = 10, normalized size = 0.62

$$\frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="giac")
```

```
[Out] 1/6*sin(3/2*x + 1/2)^4
```

Mupad [B]
time = 0.08, size = 14, normalized size = 0.88

$$\frac{\left(\frac{\cos(3\,x+1)}{2}-\frac{1}{2}\right)^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((3*x)/2 + 1/2)*sin((3*x)/2 + 1/2)^3,x)
```

```
[Out] (cos(3*x + 1)/2 - 1/2)^2/6
```

3.824 $\int 4x \tan(x^2) \, dx$

Optimal. Leaf size=7

$$-2\log(\cos(x^2))$$

[Out] -2*ln(cos(x^2))

Rubi [A]
time = 0.01, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {12, 3832, 3556}

$$-2\log(\cos(x^2))$$

Antiderivative was successfully verified.

[In] Int[4*x*Tan[x^2],x]

[Out] -2*Log[Cos[x^2]]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3832

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int 4x\tan(x^2)\,dx &= 4\int x\tan(x^2)\,dx\\ &= 2\text{Subst}\left(\int \tan(x)\,dx, x, x^2\right)\\ &= -2\log(\cos(x^2))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$-2\log\left(\cos\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Tan[x^2],x]
[Out] -2*Log[Cos[x^2]]
```

Maple [A]
time = 0.02, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$-2\ln\left(\cos\left(x^2\right)\right)$	8
default	$-2\ln\left(\cos\left(x^2\right)\right)$	8
norman	$\ln\left(1+\tan^2\left(x^2\right)\right)$	10
risch	$2ix^2-2\ln\left(e^{2ix^2}+1\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*tan(x^2),x,method=_RETURNVERBOSE)
[Out] -2*ln(cos(x^2))
```

Maxima [A]
time = 0.29, size = 7, normalized size = 1.00

$$2\log\left(\sec\left(x^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="maxima")
[Out] 2*log(sec(x^2))
```

Fricas [A]
time = 3.90, size = 13, normalized size = 1.86

$$-\log\left(\frac{1}{\tan\left(x^2\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="fricas")
[Out] -log(1/(tan(x^2)^2 + 1))
```

Sympy [A]
time = 0.04, size = 8, normalized size = 1.14

$$\log\left(\tan^{2}\left(x^{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x**2),x)
[Out] log(tan(x**2)**2 + 1)
```

Giac [A]
time = 0.42, size = 9, normalized size = 1.29

$$\log\left(\tan\left(x^{2}\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*tan(x^2),x, algorithm="giac")
[Out] log(tan(x^2)^2 + 1)
```

Mupad [B]
time = 0.07, size = 9, normalized size = 1.29

$$\ln\left(\tan\left(x^{2}\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*tan(x^2),x)
[Out] log(tan(x^2)^2 + 1)
```

3.825 $\int x\sec(5-x^2)\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

```
[Out] 1/2*arctanh(sin(x^2-5))
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4289, 3855}

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[5 - x^2],x]
```

```
[Out] -1/2*ArcTanh[Sin[5 - x^2]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x\sec\left(5-x^2\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int \sec(5-x)\,dx,x,x^2\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(5-x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[5 - x^2],x]
[Out] -1/2*ArcTanh[Sin[5 - x^2]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.31

method	result	size
derivativedivides	$\frac{\ln(\sec(x^2-5)+\tan(x^2-5))}{2}$	17
default	$\frac{\ln(\sec(x^2-5)+\tan(x^2-5))}{2}$	17
norman	$-\frac{\ln\left(\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)+1\right)}{2}$	28
risch	$\frac{\ln\left(e^{i\left(x^2-5\right)}+i\right)}{2}-\frac{\ln\left(e^{i\left(x^2-5\right)}-i\right)}{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2-5),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sec(x^2-5)+tan(x^2-5))
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.23

$$\frac{1}{2}\log\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="maxima")
[Out] 1/2*log(sec(x^2 - 5) + tan(x^2 - 5))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(9) = 18.
time = 2.14, size = 25, normalized size = 1.92

$$\frac{1}{4}\log\left(\sin\left(x^2-5\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x^2-5\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="fricas")
[Out] 1/4*log(sin(x^2 - 5) + 1) - 1/4*log(-sin(x^2 - 5) + 1)
```

Sympy [A]
time = 0.57, size = 15, normalized size = 1.15

$$\frac{\log\left(\tan\left(x^2-5\right)+\sec\left(x^2-5\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2-5),x)
```

```
[Out] log(tan(x**2 - 5) + sec(x**2 - 5))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(9) = 18$.
time = 0.40, size = 41, normalized size = 3.15

$$\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)+2\right|\right)-\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2-5),x, algorithm="giac")
```

```
[Out] 1/8*log(abs(1/sin(x^2 - 5) + sin(x^2 - 5) + 2)) - 1/8*log(abs(1/sin(x^2 - 5
) + sin(x^2 - 5) - 2))
```

Mupad [B]
time = 3.53, size = 15, normalized size = 1.15

$$-\mathrm{atan}\left(\mathrm{e}^{-5\mathrm{i}}\,\mathrm{e}^{x^2\,1\mathrm{i}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cos(x^2 - 5),x)
```

```
[Out] -atan(exp(-5i)*exp(x^2*1i))*1i
```

3.826 $\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=5

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

```
[Out] arctanh(cos(1/x))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4290, 3855}

$$\tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x^(-1)]/x^2,x]
```

```
[Out] ArcTanh[Cos[x^(-1)]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \csc(x)\,dx, x, \frac{1}{x}\right)\\ &= \tanh^{-1}\left(\cos\left(\frac{1}{x}\right)\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(5) = 10$.
time = 0.01, size = 21, normalized size = 4.20

$$\log\left(\cos\left(\frac{1}{2x}\right)\right) - \log\left(\sin\left(\frac{1}{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x^(-1)]/x^2,x]
```

```
[Out] Log[Cos[1/(2*x)]] - Log[Sin[1/(2*x)]]
```

Maple [A]
time = 0.02, size = 11, normalized size = 2.20

method	result	size
norman	$-\ln\left(\tan\left(\frac{1}{2x}\right)\right)$	10
derivativedivides	$\ln\left(\csc\left(\frac{1}{x}\right)+\cot\left(\frac{1}{x}\right)\right)$	11
default	$\ln\left(\csc\left(\frac{1}{x}\right)+\cot\left(\frac{1}{x}\right)\right)$	11
risch	$-\ln\left(e^{\frac{i}{x}}-1\right)+\ln\left(e^{\frac{i}{x}}+1\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(1/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(csc(1/x)+cot(1/x))
```

Maxima [A]
time = 0.29, size = 10, normalized size = 2.00

$$\log\left(\cot\left(\frac{1}{x}\right)+\csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] log(cot(1/x) + csc(1/x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(5) = 10$.
time = 3.02, size = 23, normalized size = 4.60

$$\frac{1}{2}\log\left(\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)-\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*log(1/2*cos(1/x) + 1/2) - 1/2*log(-1/2*cos(1/x) + 1/2)
```

Sympy [A]
time = 0.69, size = 10, normalized size = 2.00

$$\log\left(\cot\left(\frac{1}{x}\right)+\csc\left(\frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x**2,x)
```

```
[Out] log(cot(1/x) + csc(1/x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(5) = 10$.
time = 0.41, size = 43, normalized size = 8.60

$$-\frac{1}{2}\log\left(\frac{4\tan\left(\frac{1}{2x}\right)^2}{\tan\left(\frac{1}{2x}\right)^2+1}\right)+\frac{1}{2}\log\left(\frac{4}{\tan\left(\frac{1}{2x}\right)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/x)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*log(4*tan(1/2/x)^2/(tan(1/2/x)^2 + 1)) + 1/2*log(4/(tan(1/2/x)^2 + 1))
```

Mupad [B]
time = 3.68, size = 31, normalized size = 6.20

$$\ln\left(-\mathrm{e}^{1\mathrm{i}/x}\,2\mathrm{i}-2\mathrm{i}\right)-\ln\left(-\mathrm{e}^{1\mathrm{i}/x}\,2\mathrm{i}+2\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*sin(1/x)),x)
```

```
[Out] log(- exp(1i/x)*2i - 2i) - log(2i - exp(1i/x)*2i)
```

3.827 $\int (\csc(x) - \sec(x))(\cos(x) + \sin(x)) \, dx$

Optimal. Leaf size=7

$$\log(\cos(x)) + \log(\sin(x))$$

```
[Out] ln(cos(x))+ln(sin(x))
```

Rubi [A]

time = 0.03, antiderivative size = 9, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {457, 78}

$$\log(\tan(x)) + 2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

```
[Out] 2*Log[Cos[x]] + Log[Tan[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int (\csc(x) - \sec(x))(\cos(x) + \sin(x)) \, dx &= \text{Subst}\left(\int \frac{1 - x^2}{x\,(1 + x^2)} \, dx, x, \tan(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1 - x}{x(1 + x)} \, dx, x, \tan^2(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x} - \frac{2}{1 + x}\right) \, dx, x, \tan^2(x)\right) \\
&= 2\log(\cos(x)) + \log(\tan(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 7, normalized size = 1.00

$$\log(\cos(x)) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

```
[Out] Log[Cos[x]] + Log[Sin[x]]
```

Maple [A]
time = 0.13, size = 8, normalized size = 1.14

method	result	size
default	$\ln(\cos(x)) + \ln(\sin(x))$	8
risch	$-2ix + \ln(e^{4ix} - 1)$	14
norman	$-2\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right) + \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((csc(x)-sec(x))*(cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(cos(x))+ln(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.
time = 0.29, size = 15, normalized size = 2.14

$$\frac{1}{2}\log\left(-\sin(x)^2 + 1\right) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*log(-sin(x)^2 + 1) + log(sin(x))
```

Fricas [A]
time = 1.85, size = 7, normalized size = 1.00

$$\log\left(-\frac{1}{2}\cos(x)\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x)*sin(x))
```

Sympy [**A**]
time = 1.33, size = 8, normalized size = 1.14

$$\log(\sin(x)) + \log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x)
```

```
[Out] log(sin(x)) + log(cos(x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(7) = 14.
time = 0.42, size = 16, normalized size = 2.29

$$\frac{1}{2}\log\left(-\cos(x)^2 + 1\right) + \log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1) + log(abs(cos(x)))
```

Mupad [**B**]
time = 3.25, size = 26, normalized size = 3.71

$$\ln\left(\tan\left(\frac{x}{2}\right)^3 - \tan\left(\frac{x}{2}\right)\right) - 2\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) + sin(x))*(1/cos(x) - 1/sin(x)),x)
```

```
[Out] log(tan(x/2)^3 - tan(x/2)) - 2*log(tan(x/2)^2 + 1)
```

3.828 $\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -cos(x)
```

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4369\}$

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[x]
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx &= -\int \cos(3x)\sin(2x)\,dx+\int \cos(2x)\sin(3x)\,dx\\ &= -\cos(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[x]
```

Maple [A]
time = 0.21, size = 5, normalized size = 1.25

method	result	size
default	$-\cos(x)$	5
risch	$-\cos(x)$	5
norman	$\frac{2(\tan^2(x))+2(\tan^2(\frac{3x}{2}))-4\tan(x)\tan(\frac{3x}{2})}{(1+\tan^2(\frac{3x}{2}))(\tan^2(x)+1)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)
```

Maxima [A]
time = 0.28, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A]
time = 3.25, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(3) = 6$.
time = 0.14, size = 20, normalized size = 5.00

$$-\sin(2x)\sin(3x) - \cos(2x)\cos(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
```

```
[Out] -sin(2*x)*sin(3*x) - cos(2*x)*cos(3*x)
```

Giac [A]
time = 0.42, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

Mupad [B]
time = 3.07, size = 4, normalized size = 1.00

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*sin(3*x) - cos(3*x)*sin(2*x),x)
```

```
[Out] -cos(x)
```

3.829 $\int 4x\sec^2(2x)\,dx$

Optimal. Leaf size=13

$$\log(\cos(2x))+2x\tan(2x)$$

```
[Out] ln(cos(2*x))+2*x*tan(2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 4269, 3556}

$$2x\tan(2x)+\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[4*x*Sec[2*x]^2,x]
```

```
[Out] Log[Cos[2*x]] + 2*x*Tan[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int 4x\sec^2(2x)\,dx &= 4\int x\sec^2(2x)\,dx\\ &= 2x\tan(2x)-2\int \tan(2x)\,dx\\ &= \log(\cos(2x))+2x\tan(2x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.62

$$4\left(\frac{1}{4}\log(\cos(2x))+\frac{1}{2}x\tan(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x*Sec[2*x]^2,x]
```

```
[Out] 4*(Log[Cos[2*x]]/4 + (x*Tan[2*x])/2)
```

Maple [A]
time = 0.04, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$\ln(\cos(2x))+2x\tan(2x)$	14
default	$\ln(\cos(2x))+2x\tan(2x)$	14
risch	$-4ix+\frac{4ix}{e^{4ix}+1}+\ln(e^{4ix}+1)$	27
norman	$-\frac{4x\tan(x)}{\tan^2(x)-1}-\ln(\tan^2(x)+1)+\ln(\tan(x)-1)+\ln(1+\tan(x))$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x*sec(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(cos(2*x))+2*x*tan(2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(13) = 26$.
time = 0.50, size = 74, normalized size = 5.69

$$\frac{(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1)\log(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1)+8x\sin(4x)}{2(\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*log(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1) + 8*x*sin(4*x))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.
time = 3.57, size = 27, normalized size = 2.08

$$\frac{\cos(2x)\log(-\cos(2x))+2x\sin(2x)}{\cos(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="fricas")
```

```
[Out] (cos(2*x)*log(-cos(2*x)) + 2*x*sin(2*x))/cos(2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$4\int x\sec^2(2x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)**2,x)
```

```
[Out] 4*Integral(x*sec(2*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.
time = 0.42, size = 81, normalized size = 6.23

$$\frac{\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)\tan(x)^2-8x\tan(x)-\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)}{2\left(\tan(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x*sec(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)^
2 - 8*x*tan(x) - log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 +
 1)))/(tan(x)^2 - 1)
```

Mupad [B]
time = 2.99, size = 13, normalized size = 1.00

$$\ln(\cos(2x))+2x\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x)/cos(2*x)^2,x)
```

```
[Out] log(cos(2*x)) + 2*x*tan(2*x)
```

3.830 $\int 4\sin^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$-6x+6\tan(x)-2\sin^2(x)\tan(x)$$

```
[Out] -6*x+6*tan(x)-2*sin(x)^2*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {12, 2671, 294, 327, 209}

$$-6x+6\tan(x)-2\sin^2(x)\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Sin[x]^2*Tan[x]^2,x]
```

```
[Out] -6*x + 6*Tan[x] - 2*Sin[x]^2*Tan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int 4\sin^2(x)\tan^2(x)\,dx &= 4\int \sin^2(x)\tan^2(x)\,dx\\
&= 4\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(x)\right)\\
&= -2\sin^2(x)\tan(x) + 6\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 6\tan(x) - 2\sin^2(x)\tan(x) - 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)\\
&= -6x + 6\tan(x) - 2\sin^2(x)\tan(x)
\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 18, normalized size = 1.12

$$4\left(-\frac{3x}{2} + \frac{1}{4}\sin(2x) + \tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Sin[x]^2*Tan[x]^2,x]
[Out] 4*((-3*x)/2 + Sin[2*x]/4 + Tan[x])
```

Maple [**A**]
time = 0.05, size = 28, normalized size = 1.75

method	result	size
default	$\frac{4(\sin^5(x))}{\cos(x)} + 4\left(\sin^3(x) + \frac{3\sin(x)}{2}\right)\cos(x) - 6x$	28
risch	$-6x - \frac{i e^{2ix}}{2} + \frac{i e^{-2ix}}{2} + \frac{8i}{e^{2ix}+1}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*sin(x)^2*tan(x)^2,x,method=_RETURNVERBOSE)
[Out] 4*sin(x)^5/cos(x)+4*(sin(x)^3+3/2*sin(x))*cos(x)-6*x
```

Maxima [A]
time = 0.50, size = 20, normalized size = 1.25

$$-6\,x + \frac{2\,\tan\left(x\right)}{\tan\left(x\right)^{2} + 1} + 4\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="maxima")
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

Fricas [A]
time = 4.30, size = 22, normalized size = 1.38

$$-\frac{2\,\left(3\,x\cos\left(x\right) - \left(\cos\left(x\right)^{2} + 2\right)\sin\left(x\right)\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="fricas")
[Out] -2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.25

$$-6x + \frac{4\sin^{3}\left(x\right)}{\cos\left(x\right)} + 6\sin\left(x\right)\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)**2*tan(x)**2,x)
[Out] -6*x + 4*sin(x)**3/cos(x) + 6*sin(x)*cos(x)
```

Giac [A]
time = 0.41, size = 20, normalized size = 1.25

$$-6\,x + \frac{2\,\tan\left(x\right)}{\tan\left(x\right)^{2} + 1} + 4\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*sin(x)^2*tan(x)^2,x, algorithm="giac")
[Out] -6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

Mupad [B]
time = 2.96, size = 18, normalized size = 1.12

$$2\,\cos\left(x\right)\,\sin\left(x\right) - 6\,x + \frac{4\,\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*sin(x)^2*tan(x)^2,x)
```

```
[Out] 2*cos(x)*sin(x) - 6*x + (4*sin(x))/cos(x)
```

3.831 $\int \cos^4(x)\cot^2(x)\,dx$

Optimal. Leaf size=32

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{5}{8}\cos^2(x)\cot(x)+\frac{1}{4}\cos^4(x)\cot(x)$$

```
[Out] -15/8*x-15/8*cot(x)+5/8*cos(x)^2*cot(x)+1/4*cos(x)^4*cot(x)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2671, 294, 327, 209}

$$-\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{1}{4}\cos^4(x)\cot(x)+\frac{5}{8}\cos^2(x)\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Cot[x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[x])/8 + (5*Cos[x]^2*Cot[x])/8 + (Cos[x]^4*Cot[x])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(x)\cot^2(x)\,dx &= -\mathrm{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx,x,\cot(x)\right)\\
&= \frac{1}{4}\cos^4(x)\cot(x)-\frac{5}{4}\mathrm{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\cot(x)\right)\\
&= \frac{5}{8}\cos^2(x)\cot(x)+\frac{1}{4}\cos^4(x)\cot(x)-\frac{15}{8}\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\cot(x)\right)\\
&= -\frac{15\cot(x)}{8}+\frac{5}{8}\cos^2(x)\cot(x)+\frac{1}{4}\cos^4(x)\cot(x)+\frac{15}{8}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\cot(x)\right)\\
&= -\frac{15x}{8}-\frac{15\cot(x)}{8}+\frac{5}{8}\cos^2(x)\cot(x)+\frac{1}{4}\cos^4(x)\cot(x)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.81

$$-\frac{15x}{8}-\cot(x)-\frac{1}{2}\sin(2x)-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Cot[x]^2,x]
[Out] (-15*x)/8 - Cot[x] - Sin[2*x]/2 - Sin[4*x]/32
```

Maple [A]
time = 0.04, size = 34, normalized size = 1.06

method	result	size
default	$-\frac{\cos^7(x)}{\sin(x)}-\left(\cos^5(x)+\frac{5(\cos^3(x))}{4}+\frac{15\cos(x)}{8}\right)\sin(x)-\frac{15x}{8}$	34
risch	$-\frac{15x}{8}+\frac{i e^{2ix}}{4}-\frac{i e^{-2ix}}{4}-\frac{2i}{e^{2ix}-1}-\frac{\sin(4x)}{32}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cot(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/sin(x)*cos(x)^7-(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-15/8*x
```

Maxima [A]
time = 0.50, size = 35, normalized size = 1.09

$$-\frac{15}{8}x-\frac{15\tan\left(x\right)^{4}+25\tan\left(x\right)^{2}+8}{8\left(\tan\left(x\right)^{5}+2\tan\left(x\right)^{3}+\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="maxima")
[Out] -15/8*x - 1/8*(15*tan(x)^4 + 25*tan(x)^2 + 8)/(tan(x)^5 + 2*tan(x)^3 + tan(
x))
```

Fricas [A]
time = 4.04, size = 28, normalized size = 0.88

$$\frac{2\cos\left(x\right)^{5}+5\cos\left(x\right)^{3}-15\,x\sin\left(x\right)-15\cos\left(x\right)}{8\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="fricas")
[Out] 1/8*(2*cos(x)^5 + 5*cos(x)^3 - 15*x*sin(x) - 15*cos(x))/sin(x)
```

Sympy [A]
time = 0.01, size = 36, normalized size = 1.12

$$-\frac{15x}{8}-\frac{5\sin\left(x\right)\cos^{3}\left(x\right)}{4}-\frac{15\sin\left(x\right)\cos\left(x\right)}{8}-\frac{\cos^{5}\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cot(x)**2,x)
[Out] -15*x/8 - 5*sin(x)*cos(x)**3/4 - 15*sin(x)*cos(x)/8 - cos(x)**5/sin(x)
```

Giac [A]
time = 0.45, size = 31, normalized size = 0.97

$$-\frac{15}{8}x-\frac{7\tan\left(x\right)^{3}+9\tan\left(x\right)}{8\left(\tan\left(x\right)^{2}+1\right)^{2}}-\frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cot(x)^2,x, algorithm="giac")
[Out] -15/8*x - 1/8*(7*tan(x)^3 + 9*tan(x))/(tan(x)^2 + 1)^2 - 1/tan(x)
```

Mupad [B]
time = 3.00, size = 26, normalized size = 0.81

$$\frac{\frac{\cos(x)^5}{4}+\frac{5\cos(x)^3}{8}-\frac{15\,\cos(x)}{8}}{\sin\left(x\right)}-\frac{15\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cot(x)^2,x)
[Out] ((5*cos(x)^3)/8 - (15*cos(x))/8 + cos(x)^5/4)/sin(x) - (15*x)/8
```

3.832 $\int 16 \cos^2(x) \sin^2(x) \, dx$

Optimal. Leaf size=18

$$2x + 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x)$$

```
[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2648, 2715, 8}

$$2x - 4\sin(x)\cos^3(x) + 2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[16*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] 2*x + 2*Cos[x]*Sin[x] - 4*Cos[x]^3*Sin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int 16\cos^2(x)\sin^2(x)\,dx = 16\int \cos^2(x)\sin^2(x)\,dx$$
$$= -4\cos^3(x)\sin(x) + 4\int \cos^2(x)\,dx$$
$$= 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x) + 2\int 1\,dx$$
$$= 2x + 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x)$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.89

$$4\left(\frac{x}{2} - \frac{1}{8}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*Cos[x]^2*Sin[x]^2,x]
[Out] 4*(x/2 - Sin[4*x]/8)
```

Maple [A]
time = 0.02, size = 19, normalized size = 1.06

method	result	size
risch	$2x - \frac{\sin(4x)}{2}$	11
default	$2x + 2\cos(x)\sin(x) - 4(\cos^3(x))\sin(x)$	19
norman	$\frac{2x+28(\tan^3(\frac{x}{2}))-28(\tan^5(\frac{x}{2}))+4(\tan^7(\frac{x}{2}))+8x(\tan^2(\frac{x}{2}))+12x(\tan^4(\frac{x}{2}))+8x(\tan^6(\frac{x}{2}))+2x(\tan^8(\frac{x}{2}))-4\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^4}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*cos(x)^2*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] 2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.56

$$2\,x - \frac{1}{2}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 2*x - 1/2*sin(4*x)
```

Fricas [A]
time = 3.57, size = 19, normalized size = 1.06

$$-2\left(2\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)+2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -2*(2*cos(x)^3 - cos(x))*sin(x) + 2*x
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.67

$$2x-\sin\left(2x\right)\cos\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)**2*sin(x)**2,x)
```

```
[Out] 2*x - sin(2*x)*cos(2*x)
```

Giac [A]
time = 0.44, size = 10, normalized size = 0.56

$$2\,x-\frac{1}{2}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 2*x - 1/2*sin(4*x)
```

Mupad [B]
time = 0.05, size = 18, normalized size = 1.00

$$4\cos\left(x\right)\,\sin\left(x\right)^{3}-2\cos\left(x\right)\,\sin\left(x\right)+2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*cos(x)^2*sin(x)^2,x)
```

```
[Out] 2*x - 2*cos(x)*sin(x) + 4*cos(x)*sin(x)^3
```

3.833 $\int 8\cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)$$

```
[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2648, 2715, 8}

$$\frac{x}{2}-\frac{4}{3}\sin^3(x)\cos^3(x)-\sin(x)\cos^3(x)+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[8*Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin[x] - (4*Cos[x]^3*Sin[x]^3)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\int 8\cos^2(x)\sin^4(x)\,dx &= 8\int \cos^2(x)\sin^4(x)\,dx \\
&= -\frac{4}{3}\cos^3(x)\sin^3(x)+4\int \cos^2(x)\sin^2(x)\,dx \\
&= -\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\int \cos^2(x)\,dx \\
&= \frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)+\frac{\int 1\,dx}{2} \\
&= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$8\left(\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[8*Cos[x]^2*Sin[x]^4,x]
```

```
[Out] 8*(x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192)
```

Maple [A]
time = 0.03, size = 29, normalized size = 0.85

method	result
risch	$\frac{x}{2}+\frac{\sin(6x)}{24}-\frac{\sin(4x)}{8}-\frac{\sin(2x)}{8}$
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}-\left(\cos^3(x)\right)\sin(x)-\frac{4\left(\cos^3(x)\right)\left(\sin^3(x)\right)}{3}$
norman	$\frac{\tan^{11}\left(\frac{x}{2}\right)+\frac{x}{2}-\frac{17\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}+38\left(\tan^5\left(\frac{x}{2}\right)\right)-38\left(\tan^7\left(\frac{x}{2}\right)\right)+\frac{17\left(\tan^9\left(\frac{x}{2}\right)\right)}{3}+3x\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{15x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+10x\left(\tan^6\left(\frac{x}{2}\right)\right)+\frac{15x\left(\tan^8\left(\right.\right.}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*cos(x)^2*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.53

$$-\frac{1}{6}\sin\left(2\,x\right)^3+\frac{1}{2}\,x-\frac{1}{8}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="maxima")
[Out] -1/6*sin(2*x)^3 + 1/2*x - 1/8*sin(4*x)
```

Fricas [A]
time = 4.15, size = 25, normalized size = 0.74

$$\frac{1}{6}\left(8\cos\left(x\right)^{5}-14\cos\left(x\right)^{3}+3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="fricas")
[Out] 1/6*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/2*x
```

Sympy [A]
time = 0.01, size = 32, normalized size = 0.94

$$\frac{x}{2}+\frac{4\sin^{5}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin^{3}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)**2*sin(x)**4,x)
[Out] x/2 + 4*sin(x)**5*cos(x)/3 - sin(x)**3*cos(x)/3 - sin(x)*cos(x)/2
```

Giac [A]
time = 0.40, size = 22, normalized size = 0.65

$$\frac{1}{2}x+\frac{1}{24}\sin\left(6x\right)-\frac{1}{8}\sin\left(4x\right)-\frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*cos(x)^2*sin(x)^4,x, algorithm="giac")
[Out] 1/2*x + 1/24*sin(6*x) - 1/8*sin(4*x) - 1/8*sin(2*x)
```

Mupad [B]
time = 0.05, size = 24, normalized size = 0.71

$$\frac{4\cos\left(x\right)\sin\left(x\right)^{5}}{3}+\frac{x}{2}-\frac{\sin\left(2x\right)}{3}+\frac{\sin\left(4x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*cos(x)^2*sin(x)^4,x)
[Out] x/2 - sin(2*x)/3 + sin(4*x)/24 + (4*cos(x)*sin(x)^5)/3
```

3.834 $\int 35 \cos^3(x) \sin^4(x) \, dx$

Optimal. Leaf size=13

$$7 \sin^5(x) - 5 \sin^7(x)$$

[Out] 7*sin(x)^5-5*sin(x)^7

Rubi [A]

time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {12, 2644, 14}

$$7 \sin^5(x) - 5 \sin^7(x)$$

Antiderivative was successfully verified.

[In] Int[35*Cos[x]^3*Sin[x]^4,x]

[Out] 7*Sin[x]^5 - 5*Sin[x]^7

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int 35 \cos^3(x) \sin^4(x) \, dx &= 35 \int \cos^3(x) \sin^4(x) \, dx \\
&= 35\text{Subst}\left(\int x^4 \left(1-x^2\right) \, dx, x, \sin(x)\right) \\
&= 35\text{Subst}\left(\int \left(x^4-x^6\right) \, dx, x, \sin(x)\right) \\
&= 7 \sin^5(x) - 5 \sin^7(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.
time = 0.01, size = 33, normalized size = 2.54

$$35\left(\frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[35*Cos[x]^3*Sin[x]^4,x]
[Out] 35*((3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.
time = 0.03, size = 29, normalized size = 2.23

method	result	size
risch	$\frac{105\sin(x)}{64}+\frac{5\sin(7x)}{64}-\frac{7\sin(5x)}{64}-\frac{35\sin(3x)}{64}$	24
default	$-5(\cos^4(x))(\sin^3(x))-3\sin(x)(\cos^4(x))+(2+\cos^2(x))\sin(x)$	29
norman	$\frac{224(\tan^5(\frac{x}{2}))-192(\tan^7(\frac{x}{2}))+224(\tan^9(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))^7}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(35*cos(x)^3*sin(x)^4,x,method=_RETURNVERBOSE)
[Out] -5*cos(x)^4*sin(x)^3-3*sin(x)*cos(x)^4+(2+cos(x)^2)*sin(x)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 1.00

$$-5\sin(x)^7+7\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="maxima")
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

Fricas [A]
time = 3.29, size = 21, normalized size = 1.62

$$\left(5\cos(x)^6-8\cos(x)^4+\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="fricas")
```

```
[Out] (5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.92

$$-5\sin^7(x) + 7\sin^5(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)**3*sin(x)**4,x)
[Out] -5*sin(x)**7 + 7*sin(x)**5
```

Giac [A]
time = 0.43, size = 13, normalized size = 1.00

$$-5\sin(x)^7 + 7\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(35*cos(x)^3*sin(x)^4,x, algorithm="giac")
[Out] -5*sin(x)^7 + 7*sin(x)^5
```

Mupad [B]
time = 0.04, size = 13, normalized size = 1.00

$$7\sin(x)^5 - 5\sin(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(35*cos(x)^3*sin(x)^4,x)
[Out] 7*sin(x)^5 - 5*sin(x)^7
```

3.835 $\int 4\cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{32}+\frac{3}{32}\cos(x)\sin(x)+\frac{1}{16}\cos^3(x)\sin(x)-\frac{1}{4}\cos^5(x)\sin(x)-\frac{1}{2}\cos^5(x)\sin^3(x)$$

```
[Out] 3/32*x+3/32*cos(x)*sin(x)+1/16*cos(x)^3*sin(x)-1/4*cos(x)^5*sin(x)-1/2*cos(
x)^5*sin(x)^3
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {12, 2648, 2715, 8}

$$\frac{3x}{32}-\frac{1}{2}\sin^3(x)\cos^5(x)-\frac{1}{4}\sin(x)\cos^5(x)+\frac{1}{16}\sin(x)\cos^3(x)+\frac{3}{32}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[4*Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/32 + (3*Cos[x]*Sin[x])/32 + (Cos[x]^3*Sin[x])/16 - (Cos[x]^5*Sin[x])/
4 - (Cos[x]^5*Sin[x]^3)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\int 4\cos^4(x)\sin^4(x)\,dx &= 4\int \cos^4(x)\sin^4(x)\,dx \\
&= -\frac{1}{2}\cos^5(x)\sin^3(x) + \frac{3}{2}\int \cos^4(x)\sin^2(x)\,dx \\
&= -\frac{1}{4}\cos^5(x)\sin(x) - \frac{1}{2}\cos^5(x)\sin^3(x) + \frac{1}{4}\int \cos^4(x)\,dx \\
&= \frac{1}{16}\cos^3(x)\sin(x) - \frac{1}{4}\cos^5(x)\sin(x) - \frac{1}{2}\cos^5(x)\sin^3(x) + \frac{3}{16}\int \cos^2(x)\,dx \\
&= \frac{3}{32}\cos(x)\sin(x) + \frac{1}{16}\cos^3(x)\sin(x) - \frac{1}{4}\cos^5(x)\sin(x) - \frac{1}{2}\cos^5(x)\sin^3(x) + \frac{3\int 1}{32} \\
&= \frac{3x}{32} + \frac{3}{32}\cos(x)\sin(x) + \frac{1}{16}\cos^3(x)\sin(x) - \frac{1}{4}\cos^5(x)\sin(x) - \frac{1}{2}\cos^5(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.52

$$4\left(\frac{3x}{128} - \frac{1}{128}\sin(4x) + \frac{\sin(8x)}{1024}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[4*Cos[x]^4*Sin[x]^4,x]
```

```
[Out] 4*((3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024)
```

Maple [A]
time = 0.03, size = 36, normalized size = 0.78

method	result
risch	$\frac{3x}{32}+\frac{\sin(8x)}{256}-\frac{\sin(4x)}{32}$
default	$-\frac{(\cos^5(x))(\sin^3(x))}{2}-\frac{(\cos^5(x))\sin(x)}{4}+\frac{\left(\cos^3(x)+\frac{3\cos(x)}{2}\right)\sin(x)}{16}+\frac{3x}{32}$
norman	$\frac{\frac{3x}{32}-\frac{23\left(\tan^3\left(\frac{x}{2}\right)\right)}{16}-\frac{671\left(\tan^7\left(\frac{x}{2}\right)\right)}{16}-\frac{3\tan\left(\frac{x}{2}\right)}{16}+\frac{21x\left(\tan^4\left(\frac{x}{2}\right)\right)}{8}+\frac{333\left(\tan^5\left(\frac{x}{2}\right)\right)}{16}+\frac{23\left(\tan^{13}\left(\frac{x}{2}\right)\right)}{16}+\frac{3\left(\tan^{15}\left(\frac{x}{2}\right)\right)}{16}-\frac{333\left(\tan^{11}\left(\frac{x}{2}\right)\right)}{16}+671(}{(1+\tan^2(\frac{x}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*cos(x)^4*sin(x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x)^5*sin(x)^3-1/4*cos(x)^5*sin(x)+1/16*(cos(x)^3+3/2*cos(x))*sin(x
)+3/32*x
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.35

$$\frac{3}{32}\,x+\frac{1}{256}\,\sin\left(8\,x\right)-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="maxima")
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

Fricas [A]
time = 2.47, size = 31, normalized size = 0.67

$$\frac{1}{32}\,\left(16\,\cos\left(x\right)^{7}-24\,\cos\left(x\right)^{5}+2\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{3}{32}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="fricas")
[Out] 1/32*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/32*x
```

Sympy [A]
time = 0.01, size = 31, normalized size = 0.67

$$\frac{3x}{32}-\frac{\sin^{3}\left(2x\right)\cos\left(2x\right)}{32}-\frac{3\sin\left(2x\right)\cos\left(2x\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)**4*sin(x)**4,x)
[Out] 3*x/32 - sin(2*x)**3*cos(2*x)/32 - 3*sin(2*x)*cos(2*x)/64
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.35

$$\frac{3}{32}\,x+\frac{1}{256}\,\sin\left(8\,x\right)-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*cos(x)^4*sin(x)^4,x, algorithm="giac")
[Out] 3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

Mupad [B]
time = 0.04, size = 33, normalized size = 0.72

$$\frac{3\,x}{32}-\frac{\sin\left(2\,x\right)}{16}+\frac{\sin\left(4\,x\right)}{128}+4\,\sin\left(x\right)^{5}\,\left(\frac{\cos\left(x\right)^{3}}{8}+\frac{\cos\left(x\right)}{16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*cos(x)^4*sin(x)^4,x)
[Out] (3*x)/32 - sin(2*x)/16 + sin(4*x)/128 + 4*sin(x)^5*(cos(x)/16 + cos(x)^3/8)
```

3.836 $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)} \, dx$

Optimal. Leaf size=9

$$\log(\cos(x)) - \log(\sin(x))$$

```
[Out] ln(cos(x))-ln(sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4419, 272, 36, 31, 29}

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

```
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(-1+x^2\right)}\,dx, x, \sin(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, \sin^2(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \sin^2(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) \\
&= \log(\cos(x)) - \log(\sin(x))
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\log(\cos(x)) - \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Sin[x] + Sin[x]^3),x]
[Out] Log[Cos[x]] - Log[Sin[x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.06, size = 21, normalized size = 2.33

method	result	size
risch	$\ln\left(e^{2ix}+1\right)-\ln\left(e^{2ix}-1\right)$	20
derivativedivides	$-\ln\left(\sin\left(x\right)\right)+\frac{\ln(1+\sin(x))}{2}+\frac{\ln(\sin(x)-1)}{2}$	21
default	$-\ln\left(\sin\left(x\right)\right)+\frac{\ln(1+\sin(x))}{2}+\frac{\ln(\sin(x)-1)}{2}$	21
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-sin(x)+sin(x)^3),x,method=_RETURNVERBOSE)
[Out] -ln(sin(x))+1/2*ln(1+sin(x))+1/2*ln(sin(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.29, size = 20, normalized size = 2.22

$$\frac{1}{2}\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\log\left(\sin\left(x\right)-1\right)-\log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - log(sin(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 3.21, size = 19, normalized size = 2.11

$$\frac{1}{2}\log\left(\cos\left(x\right)^{2}\right) - \frac{1}{2}\log\left(-\frac{1}{4}\cos\left(x\right)^{2} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*log(cos(x)^2) - 1/2*log(-1/4*cos(x)^2 + 1/4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.
time = 0.10, size = 20, normalized size = 2.22

$$\frac{\log\left(\sin\left(x\right) - 1\right)}{2} + \frac{\log\left(\sin\left(x\right) + 1\right)}{2} - \log\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)**3),x)
```

```
[Out] log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(sin(x))
```

Giac [A]
time = 0.44, size = 18, normalized size = 2.00

$$\frac{1}{2}\log\left(-\sin\left(x\right)^{2} + 1\right) - \log\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*log(-sin(x)^2 + 1) - log(abs(sin(x)))
```

Mupad [B]
time = 3.01, size = 13, normalized size = 1.44

$$\frac{\ln\left(\cos\left(x\right)^{2}\right)}{2} - \ln\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(sin(x) - sin(x)^3),x)
```

```
[Out] log(cos(x)^2)/2 - log(sin(x))
```

3.837 $\int \left(-1+2 \cos^2(x)+\cos(x) \sin(x)\right) \, dx$

Optimal. Leaf size=14

$$\cos(x)\sin(x)+\frac{\sin^2(x)}{2}$$

```
[Out] cos(x)*sin(x)+1/2*sin(x)^2
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2715, 8, 2644, 30}

$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

```
[Out] Cos[x]*Sin[x] + Sin[x]^2/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx = -x+2\int \cos^2(x)\,dx+\int \cos(x)\sin(x)\,dx$$
$$= -x+\cos(x)\sin(x)+\int 1\,dx+\text{Subst}(\int x\,dx,x,\sin(x))$$
$$= \cos(x)\sin(x)+\frac{\sin^2(x)}{2}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.21

$$-\frac{1}{2}\cos^2(x)+\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

```
[Out] -1/2*Cos[x]^2 + Sin[2*x]/2
```

Maple [A]
time = 0.05, size = 13, normalized size = 0.93

method	result	size
default	$\cos(x)\sin(x)+\frac{(\sin^2(x))}{2}$	13
risch	$-\frac{\cos(2x)}{4}+\frac{\sin(2x)}{2}$	14
meijerg	$\frac{\sin(2x)}{2}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{2(\tan^2(\frac{x}{2}))-2(\tan^3(\frac{x}{2}))+2\tan(\frac{x}{2})}{(1+\tan^2(\frac{x}{2}))^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1+2*cos(x)^2+cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)*sin(x)+1/2*sin(x)^2
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.93

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="maxima")
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

Fricas [A]
time = 2.75, size = 12, normalized size = 0.86

$$-\frac{1}{2}\cos(x)^2+\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="fricas")
[Out] -1/2*cos(x)^2 + cos(x)*sin(x)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.86

$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)**2+cos(x)*sin(x),x)
[Out] sin(x)**2/2 + sin(x)*cos(x)
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.93

$$-\frac{1}{2}\cos(x)^2+\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="giac")
[Out] -1/2*cos(x)^2 + 1/2*sin(2*x)
```

Mupad [B]
time = 2.97, size = 11, normalized size = 0.79

$$-\frac{\cos(x)\,(\cos(x)-2\sin(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x) + 2*cos(x)^2 - 1,x)
[Out] -(cos(x)*(cos(x) - 2*sin(x)))/2
```

3.838 $\int (\cos^2(x)+\sin^2(x))\ dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.01, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2715, 8}

$$x$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2 + Sin[x]^2,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int (\cos^2(x)+\sin^2(x))\ dx &= \int \cos^2(x)\,dx+\int \sin^2(x)\,dx\\ &= 2\frac{\int 1\,dx}{2}\\ &= x\end{aligned}$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] x
```

Maple [A]
time = 0.05, size = 2, normalized size = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\left(\tan^4\left(\frac{x}{2}\right)\right)+2x\left(\tan^2\left(\frac{x}{2}\right)\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2+cos(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 1.94, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2+sin(x)**2,x)
```

```
[Out] x
```

Giac [A]
time = 0.43, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2+sin(x)^2,x, algorithm="giac")
[Out] x
```

Mupad [B]
time = 2.93, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2 + sin(x)^2,x)
[Out] x
```

3.839 $\int \left(-\cos^2(x)+\sin^2(x)\right) \, dx$

Optimal. Leaf size=6

$$-\cos(x)\sin(x)$$

```
[Out] -cos(x)*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2715, 8}

$$\sin(x)(-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[-Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] -(Cos[x]*Sin[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned} \int \left(-\cos^2(x)+\sin^2(x)\right) \, dx &= -\int \cos^2(x) \, dx+\int \sin^2(x) \, dx \\ &= -\cos(x)\sin(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.33

$$-\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Cos[x]^2 + Sin[x]^2,x]
```

```
[Out] -1/2*Sin[2*x]
```

Maple [A]
time = 0.05, size = 7, normalized size = 1.17

method	result	size
default	$-\cos(x)\sin(x)$	7
risch	$-\frac{\sin(2x)}{2}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)^2+sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)*sin(x)
```

Maxima [A]
time = 0.30, size = 6, normalized size = 1.00

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(2*x)
```

Fricas [A]
time = 2.76, size = 6, normalized size = 1.00

$$-\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

```
[Out] -cos(x)*sin(x)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 1.17

$$-\sin(x)\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)**2+sin(x)**2,x)
```

```
[Out] -sin(x)*cos(x)
```

Giac [A]
time = 0.40, size = 6, normalized size = 1.00

$$-\frac{1}{2}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(x)^2+sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(2*x)
```

Mupad [B]
time = 2.94, size = 6, normalized size = 1.00

$$-\frac{\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2 - cos(x)^2,x)
```

```
[Out] -sin(2*x)/2
```

3.840 $\int 2^{\sin(x)} \cos(x) \, dx$

Optimal. Leaf size=9

$$\frac{2^{\sin(x)}}{\log(2)}$$

[Out] 2^sin(x)/ln(2)

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4419, 2225}

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

[In] Int[2^Sin[x]*Cos[x],x]

[Out] 2^Sin[x]/Log[2]

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int 2^{\sin(x)} \cos(x) \, dx &= \text{Subst}\left(\int 2^x \, dx, x, \sin(x)\right)\\ &= \frac{2^{\sin(x)}}{\log(2)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sin[x]*Cos[x],x]
```

```
[Out] 2^Sin[x]/Log[2]
```

Maple [A]
time = 0.06, size = 10, normalized size = 1.11

method	result	size
derivativedivides	$\frac{2^{\sin(x)}}{\ln(2)}$	10
default	$\frac{2^{\sin(x)}}{\ln(2)}$	10
risch	$\frac{2^{\sin(x)}}{\ln(2)}$	10
norman	$\frac{\frac{e^{\frac{2\tan\left(\frac{x}{2}\right)\ln(2)}{1+\tan^2\left(\frac{x}{2}\right)}}}{\ln(2)}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)e^{\frac{2\tan\left(\frac{x}{2}\right)\ln(2)}{1+\tan^2\left(\frac{x}{2}\right)}}}{\ln(2)}}{1+\tan^2\left(\frac{x}{2}\right)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sin(x)*cos(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2^sin(x)/ln(2)
```

Maxima [A]
time = 0.29, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="maxima")
```

```
[Out] 2^sin(x)/log(2)
```

Fricas [A]
time = 2.50, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="fricas")
```

```
[Out] 2^sin(x)/log(2)
```

Sympy [A]
time = 0.10, size = 7, normalized size = 0.78

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**sin(x)*cos(x),x)
```

```
[Out] 2**sin(x)/log(2)
```

Giac [A]
time = 0.44, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^sin(x)*cos(x),x, algorithm="giac")
```

```
[Out] 2^sin(x)/log(2)
```

Mupad [B]
time = 0.07, size = 9, normalized size = 1.00

$$\frac{2^{\sin(x)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^sin(x)*cos(x),x)
```

```
[Out] 2^sin(x)/log(2)
```

3.841 $\int (\tan^3(x)+\tan^5(x))\,dx$

Optimal. Leaf size=8

$$\frac{\tan^4(x)}{4}$$

```
[Out] 1/4*tan(x)^4
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3554, 3556}

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3 + Tan[x]^5,x]
```

```
[Out] Tan[x]^4/4
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (\tan^3(x)+\tan^5(x))\,dx &= \int \tan^3(x)\,dx+\int \tan^5(x)\,dx\\ &= \frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}-\int \tan(x)\,dx-\int \tan^3(x)\,dx\\ &= \log(\cos(x))+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\ &= \frac{\tan^4(x)}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\tan^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3 + Tan[x]^5,x]
[Out] Tan[x]^4/4
```

Maple [A]
time = 0.04, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\tan^4(x))}{4}$	7
default	$\frac{(\tan^4(x))}{4}$	7
norman	$\frac{(\tan^4(x))}{4}$	7
risch	$-\frac{2(e^{6ix}+e^{2ix})}{(e^{2ix}+1)^4}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3+tan(x)^5,x,method=_RETURNVERBOSE)
[Out] 1/4*tan(x)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(6) = 12$.
time = 0.31, size = 35, normalized size = 4.38

$$\frac{4 \sin(x)^2 - 3}{4 \left(\sin(x)^4 - 2 \sin(x)^2 + 1\right)} - \frac{1}{2 \left(\sin(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="maxima")
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2/(sin(x)^2 - 1)
```

Fricas [A]
time = 1.87, size = 6, normalized size = 0.75

$$\frac{1}{4} \tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5) = 10$.
time = 0.05, size = 22, normalized size = 2.75

$$-\frac{4\cos^2(x) - 1}{4\cos^4(x)} + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3+tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) + 1/(2*cos(x)**2)
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.75

$$\frac{1}{4}\tan(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3+tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4
```

Mupad [B]
time = 2.95, size = 6, normalized size = 0.75

$$\frac{\tan(x)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3 + tan(x)^5,x)
```

```
[Out] tan(x)^4/4
```

3.842 $\int x \sec(x)(2 + x \tan(x)) \, dx$

Optimal. Leaf size=6

$$x^2 \sec(x)$$

[Out] x^2*sec(x)

Rubi [A]
time = 0.13, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6874, 4266, 2317, 2438, 3842}

$$x^2 \sec(x)$$

Antiderivative was successfully verified.

[In] Int[x*Sec[x]*(2 + x*Tan[x]),x]

[Out] x^2*Sec[x]

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int x\sec(x)(2+x\tan(x))\,dx &= \int \left(2x\sec(x)+x^2\sec(x)\tan(x)\right)\,dx\\
&= 2\int x\sec(x)\,dx+\int x^2\sec(x)\tan(x)\,dx\\
&= -4ix\tan^{-1}\left(e^{ix}\right)+x^2\sec(x)-2\int \log\left(1-ie^{ix}\right)\,dx+2\int \log\left(1+ie^{ix}\right)\,dx\\
&= x^2\sec(x)+2i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)-2i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx,\right.\\
&= 2i\text{Li}_2\left(-ie^{ix}\right)-2i\text{Li}_2\left(ie^{ix}\right)+x^2\sec(x)-2i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+\\
&= x^2\sec(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 6, normalized size = 1.00

$$x^2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*(2 + x*Tan[x]),x]
[Out] x^2*Sec[x]
```

Maple [A]
time = 0.03, size = 9, normalized size = 1.50

method	result	size
default	$\frac{x^2}{\cos(x)}$	9
risch	$\frac{2x^2e^{ix}}{e^{2ix}+1}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)*(2+x*tan(x)),x,method=_RETURNVERBOSE)
[Out] x^2/cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(6) = 12$.
time = 0.59, size = 51, normalized size = 8.50

$$\frac{2\,(x^2\cos(2\,x)\cos(x) + x^2\sin(2\,x)\sin(x) + x^2\cos(x))}{\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="maxima")
```

```
[Out] 2*(x^2*cos(2*x)*cos(x) + x^2*sin(2*x)*sin(x) + x^2*cos(x))/(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A]
time = 1.94, size = 8, normalized size = 1.33

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="fricas")
```

```
[Out] x^2/cos(x)
```

Sympy [A]
time = 0.14, size = 5, normalized size = 0.83

$$x^2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x)
```

```
[Out] x**2*sec(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 0.41, size = 26, normalized size = 4.33

$$-\frac{x^2\tan\left(\frac{1}{2}\,x\right)^2 + x^2}{\tan\left(\frac{1}{2}\,x\right)^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="giac")
```

```
[Out] -(x^2*tan(1/2*x)^2 + x^2)/(tan(1/2*x)^2 - 1)
```

Mupad [B]
time = 0.08, size = 8, normalized size = 1.33

$$\frac{x^2}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x*tan(x) + 2))/cos(x),x)
```

```
[Out] x^2/cos(x)
```

3.843 $\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-2\csc\left(\sqrt{x}\right)$$

```
[Out] -2*csc(x^(1/2))
```

Rubi [A]

time = 0.15, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 2686, 8}

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -2*Csc[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \cot(x)\csc(x)\,dx, x, \sqrt{x}\right)\\ &= -\left(2\text{Subst}\left(\int 1\,dx, x, \csc\left(\sqrt{x}\right)\right)\right)\\ &= -2\csc\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-2\csc\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
[Out] -2*Csc[Sqrt[x]]
```

Maple [A]
time = 0.07, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-2\csc\left(\sqrt{x}\right)$	7
default	$-2\csc\left(\sqrt{x}\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*csc(x^(1/2))
```

Maxima [A]
time = 0.33, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] -2/sin(sqrt(x))
```

Fricas [A]
time = 1.62, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] -2/sin(sqrt(x))
```

Sympy [A]
time = 0.09, size = 8, normalized size = 1.00

$$-2\csc\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x**(1/2))*csc(x**(1/2))/x**(1/2),x)
[Out] -2*csc(sqrt(x))
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] -2/sin(sqrt(x))
```

Mupad [B]
time = 3.07, size = 8, normalized size = 1.00

$$-\frac{2}{\sin\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x^(1/2))/(x^(1/2)*sin(x^(1/2))),x)
[Out] -2/sin(x^(1/2))
```

3.844 $\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$\sin^2\left(\sqrt{x}\right)$$

```
[Out] sin(x^(1/2))^2
```

Rubi [A]

time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3522}

$$\sin^2\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sin[Sqrt[x]]^2
```

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sin^2\left(\sqrt{x}\right)$$

Mathematica [A]

time = 0.01, size = 12, normalized size = 1.50

$$-\frac{1}{2}\cos\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] -1/2*Cos[2*Sqrt[x]]
```

Maple [A]

time = 0.04, size = 9, normalized size = 1.12

method	result	size
derivativedivides	$-\left(\cos^2\left(\sqrt{x}\right)\right)$	9
default	$-\left(\cos^2\left(\sqrt{x}\right)\right)$	9
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(2\sqrt{x}\right)}{\sqrt{\pi}}\right)}{2}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x^(1/2))^2
```

Maxima [A]
time = 0.33, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] -cos(sqrt(x))^2
```

Fricas [A]
time = 1.60, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -cos(sqrt(x))^2
```

Sympy [A]
time = 0.09, size = 7, normalized size = 0.88

$$\sin^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))*sin(x**(1/2))/x**(1/2),x)
```

```
[Out] sin(sqrt(x))**2
```

Giac [A]
time = 0.41, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] -cos(sqrt(x))^2
```

Mupad [B]
time = 3.05, size = 8, normalized size = 1.00

$$-\cos\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x^(1/2))*sin(x^(1/2)))/x^(1/2),x)
```

```
[Out] -cos(x^(1/2))^2
```

3.845 $\int \frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\sec\left(\sqrt{x}\right)$$

```
[Out] 2*sec(x^(1/2))
```

Rubi [A]
time = 0.13, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6847, 2686, 8}

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*Sec[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \sec(x)\tan(x)\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int 1\,dx, x, \sec\left(\sqrt{x}\right)\right)\\
&= 2\sec\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$2\sec\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*Sec[Sqrt[x]]
```

Maple [A]
time = 0.05, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2\sec\left(\sqrt{x}\right)$	7
default	$2\sec\left(\sqrt{x}\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*sec(x^(1/2))
```

Maxima [A]
time = 0.33, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/cos(sqrt(x))
```

Fricas [A]
time = 1.92, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/cos(sqrt(x))
```

Sympy [**A**]
time = 0.10, size = 7, normalized size = 0.88

$$2\sec\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x**(1/2))*tan(x**(1/2))/x**(1/2),x)
[Out] 2*sec(sqrt(x))
```

Giac [**A**]
time = 0.43, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2/cos(sqrt(x))
```

Mupad [**B**]
time = 3.00, size = 8, normalized size = 1.00

$$\frac{2}{\cos\left(\sqrt{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x^(1/2))/(x^(1/2)*cos(x^(1/2))),x)
[Out] 2/cos(x^(1/2))
```

3.846 $\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\operatorname{ArcTan}\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] -1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1089, 12, 648, 632, 210, 642, 266}

$$\frac{\operatorname{ArcTan}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}-\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[Cos[x]]/(2
*b) - Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1089

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*((-b)*
C*d + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*((-b)*C*d + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{x^2}{(1+x^2)\,(a+2bx+ax^2)}\,dx, x, \tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx, x, \tan(x)\right)}{4b^2} + \frac{\text{Subst}\left(\int -\frac{2abx}{a+2bx+ax^2}\,dx, x, \tan(x)\right)}{4b^2}\\
&= \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right)}{2b} - \frac{a\text{Subst}\left(\int \frac{x}{a+2bx+ax^2}\,dx, x, \tan(x)\right)}{2b}\\
&= -\frac{\log(\cos(x))}{2b} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan(x)\right) - \frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx, x,\right.}{4b}\\
&= -\frac{\log(\cos(x))}{2b} - \frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b} - \text{Subst}\left(\int \frac{1}{-4\,(a^2-b^2)-x^2}\,dx, x,\right.\\
&= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log(\cos(x))}{2b} - \frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 55, normalized size = 1.00

$$\frac{\operatorname{ArcTan}\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Sin[
2*x]]/(4*b)
```

Maple [A]
time = 0.24, size = 80, normalized size = 1.45

method	result
default	$\frac{\ln\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{\ln\left(a+2b\tan(x)+a\left(\tan^2(x)\right)\right)}{2a}-\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\right)}{2b}$
risch	$-\frac{ix}{2b}+\frac{ix\,a^2b}{a^2b^2-b^4}-\frac{ix\,b^3}{a^2b^2-b^4}-\frac{\ln\left(e^{2ix}-\frac{-iab+\sqrt{-a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2-b^2)b}+\frac{b\ln\left(e^{2ix}-\frac{-iab+\sqrt{-a^2b^2+b^4}}{b^2}\right)}{4a^2-4b^2}-\frac{\ln\left(e^{2ix}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*sin(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*ln(tan(x)^2+1)-1/2*a/b*(1/2/a*ln(a+2*b*tan(x)+a*tan(x)^2)-b/a/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(47) = 94.
time = 1.81, size = 320, normalized size = 5.82

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,ab\cos(x)\sin(x)-4\,(2\,a^2-b^2)\cos(x)^2+a^2-2\,b^2+2\,\left(2\,b\cos(x)^2+2\,\left(2\,a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-a^2+b^2}}{4\,b^2\cos(x)^4-4\,b^2\cos(x)^2-4\,ab\cos(x)\sin(x)-a^2}\right)+(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(x)\sin(x)+a^2\right)}{8\,(a^2b-b^3)},-\frac{2\,\sqrt{a^2-b^2}\,b\arctan\left(-\frac{(2\,a\cos(x)\sin(x)+b)\sqrt{a^2-b^2}}{2\,(a^2-b^2)\cos(x)^2-a^2+b^2}\right)+(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(x)\sin(x)+a^2\right)}{8\,(a^2b-b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

Sympy [A]
time = 4.66, size = 162, normalized size = 2.95

$$-\begin{cases}\frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for } b\neq 0\\ \frac{\sin(2x)}{4a} & \text{otherwise}\end{cases}+\begin{cases}\tilde{\infty}\log(\tan(x)) & \text{for } a=0\wedge b=0\\ \frac{\log(\tan(x))}{4b} & \text{for } a=0\\ \frac{\sqrt{b^2}}{2b^2\tan(x)-2b\sqrt{b^2}} & \text{for } a=-\sqrt{b^2}\\ -\frac{\sqrt{b^2}}{2b^2\tan(x)+2b\sqrt{b^2}} & \text{for } a=\sqrt{b^2}\\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}}-\frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] -Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
 Piecewise((zoo*log(tan(x)), Eq(a, 0) & Eq(b, 0)), (log(tan(x))/(4*b), Eq(a
, 0)), (sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (
-sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(
x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)) - log(tan(x) + b/a
+ sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)), True))
```

Giac [A]
time = 0.40, size = 77, normalized size = 1.40

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log\left(a\tan(x)^2+2b\tan(x)+a\right)}{4b}+\frac{\log\left(\tan(x)^2+1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) - 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b + 1/4*log(tan(x)^2
 + 1)/b
```

Mupad [B]
time = 4.18, size = 1108, normalized size = 20.15

$$\frac{\ln\left(\tan(x)^2+1\right)}{4\,b}+\frac{\mathrm{atan}\left(\frac{\frac{2\tan(x)\,{\left(a^2-b^2\right)}^{3/2}\left(\frac{\left(4\,a^2\,b-2\,b^3\right)\left(2\,a\,b-\frac{\frac{8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{8\sqrt{a^2-b^2}\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(4\,a^3-16\,a\,b^2+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}-\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{32\left(a^2-b^2\right)\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{a^3\,{\left(4\,a^2-3\,b^2\right)}^2}-\frac{\left(4\,a^4-5\,a^2\,b^2+2\,b^4\right)\left(\frac{4\,a^3-16\,a\,b^2+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}-\frac{96\,a\,b^4-64\,a^3\,b^2}{64\,{\left(a^2-b^2\right)}^{3/2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(\frac{8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{8\sqrt{a^2-b^2}\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{a^3\sqrt{a^2-b^2}\,{\left(4\,a^2-3\,b^2\right)}^2}\right)}{a}+\frac{2\left(a^2-b^2\right)\left(\frac{6\,a^2\,b-\frac{8\,a^2\,b^3\,{\left(8\,a^2\,b-8\,b^3\right)}^2}{{\left(16\,b^4-16\,a^2\,b^2\right)}^2}}{4\sqrt{a^2-b^2}}+\frac{a^2\,b^3}{2\,{\left(a^2-b^2\right)}^{3/2}}-\frac{4\,a^2\,b^3\,{\left(8\,a^2\,b-8\,b^3\right)}^2}{\sqrt{a^2-b^2}\,{\left(16\,b^4-16\,a^2\,b^2\right)}^2}\right)\left(4\,a^4-5\,a^2\,b^2+2\,b^4\right)}{a^4\,{\left(4\,a^2-3\,b^2\right)}^2}-\frac{2\left(4\,a^2\,b-2\,b^3\right){\left(a^2-b^2\right)}^{3/2}\left(\frac{\left(8\,a^2\,b-8\,b^3\right)\left(6\,a^2\,b-\frac{8\,a^2\,b^3\,{\left(8\,a^2\,b-8\,b^3\right)}^2}{{\left(16\,b^4-16\,a^2\,b^2\right)}^2}\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}-a^2+\frac{3\,a^2\,b^3\left(8\,a^2\,b-8\,b^3\right)}{\left(a^2-b^2\right)\left(16\,b^4-16\,a^2\,b^2\right)}\right)}{a^4\,{\left(4\,a^2-3\,b^2\right)}^2}\right)}{2\sqrt{a^2-b^2}}+\frac{\ln\left(a\tan(x)^2+2\,b\tan(x)+a\right)\left(8\,a^2\,b-8\,b^3\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*sin(2*x)),x)
```

```
[Out] log(tan(x)^2 + 1)/(4*b) + atan((2*tan(x)*(a^2 - b^2)^(3/2)*(((4*a^2*b - 2*b
^3)*(2*a*b - ((8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16
*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 -
 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(
1/2)) + ((8*a^2*b - 8*b^3)*(4*a^3 - 16*a*b^2 + ((8*a^2*b - 8*b^3)*(8*a*b^3
+ ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(
2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(
96*a*b^4 - 64*a^3*b^2))/(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^
2 - 3*b^2)^2) - ((4*a^4 + 2*b^4 - 5*a^2*b^2)*((4*a^3 - 16*a*b^2 + ((8*a^2*b
 - 8*b^3)*(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4
 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) - (96*a*
b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) + ((8*a^2*b - 8*b^3)*((8*a*b^3 + (
(8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a
^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2 - b^
2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(a^3*(a^2 - b
^2)^(1/2)*(4*a^2 - 3*b^2)^2)))/a + (2*(a^2 - b^2)*((6*a^2*b - (8*a^2*b^3*(8
*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2)/(4*(a^2 - b^2)^(1/2)) + (a^2*b^
3)/(2*(a^2 - b^2)^(3/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3)^2)/((a^2 - b^2)^(1/
2)*(16*b^4 - 16*a^2*b^2)^2))*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(a^4*(4*a^2 - 3*b
^2)^2) - (2*(4*a^2*b - 2*b^3)*(a^2 - b^2)^(3/2)*(((8*a^2*b - 8*b^3)*(6*a^2*
b - (8*a^2*b^3*(8*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2))/(2*(16*b^4 -
16*a^2*b^2)) - a^2 + (3*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)*(16*b^4 - 1
6*a^2*b^2))))/(a^4*(4*a^2 - 3*b^2)^2))/(2*(a^2 - b^2)^(1/2)) + (log(a + a*t
an(x)^2 + 2*b*tan(x))*(8*a^2*b - 8*b^3))/(2*(16*b^4 - 16*a^2*b^2))
```

3.847 $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

Optimal. Leaf size=55

$$\frac{\text{ArcTan}\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{4b}$$

```
[Out] 1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {995, 648, 632, 210, 642, 12, 266}

$$\frac{\text{ArcTan}\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{4b}+\frac{\log(\cos(x))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[2*x]),x]
```

```
[Out] ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) + Log[Cos[x]]/(2
*b) + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(4*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 995

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(c^2*
d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Dist[1/q, Int[(c*d
*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c,
d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)(a+2bx+ax^2)}\,dx,x,\tan(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{2bx}{1+x^2}\,dx,x,\tan(x)\right)}{4b^2}+\frac{\text{Subst}\left(\int \frac{4b^2+2abx}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b^2}\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan(x)\right)+\frac{\text{Subst}\left(\int \frac{2b+2ax}{a+2bx+ax^2}\,dx,x,\tan(x)\right)}{4b}-\frac{\text{Subst}(\ldots}{\ldots}\\
&= \frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}-\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx,x,2b\ldots\right.\\
&= \frac{\tan^{-1}\left(\frac{2b+2a\tan(x)}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(\cos(x))}{2b}+\frac{\log\left(a+2b\tan(x)+a\tan^2(x)\right)}{4b}
\end{aligned}
$$

Mathematica [A]

time = 0.05, size = 54, normalized size = 0.98

$$\frac{1}{4}\left(\frac{2\text{ArcTan}\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[2*x]),x]
[Out] ((2*ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + Log[a + b*Sin
[2*x]]/b)/4
```

Maple [A]
time = 0.23, size = 72, normalized size = 1.31

method	result
default	$-\frac{\ln(\tan^2(x)+1)}{4b}+\frac{\frac{\ln\left(a+2b\tan(x)+a\left(\tan^2(x)\right)\right)}{2}+\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2b}$
risch	$\frac{ix}{2b}-\frac{ix\,a^2b}{a^2b^2-b^4}+\frac{ix\,b^3}{a^2b^2-b^4}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{iab+\sqrt{-a^2b^2+b^4}}{b^2}\right)a^2}{4(a^2-b^2)b}-\frac{b\ln\left(\mathrm{e}^{2ix}+\frac{iab+\sqrt{-a^2b^2+b^4}}{b^2}\right)}{4(a^2-b^2)}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{iab+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(2*x)),x,method=_RETURNVERBOSE)
[Out] -1/4/b*ln(tan(x)^2+1)+1/2/b*(1/2*ln(a+2*b*tan(x)+a*tan(x)^2)+b/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(47) = 94$.
time = 2.51, size = 322, normalized size = 5.85

$$\left[-\frac{\sqrt{-a^2+b^2}\,b\log\left(-\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,ab\cos(x)\sin(x)-4\,(2\,a^2-b^2)\cos(x)^2+a^2-2\,b^2+2\,\left(2\,b\cos(x)^2+2\,\left(2\,a\cos(x)^3-a\cos(x)\right)\sin(x)-b\right)\sqrt{-a^2+b^2}}{4\,b^2\cos(x)^4-4\,b^2\cos(x)^2-4\,ab\cos(x)\sin(x)-a^2}\right)-(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(x)\sin(x)+a^2\right)}{8\,(a^2b-b^3)},\ -\frac{2\,\sqrt{a^2-b^2}\,b\arctan\left(-\frac{(2\,a\cos(x)\sin(x)+b)\sqrt{a^2-b^2}}{2\,(a^2-b^2)\cos(x)^2-a^2+b^2}\right)-(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(x)\sin(x)+a^2\right)}{8\,(a^2b-b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*sin
(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*cos
(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*cos
(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b
^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(a^2 -
 b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2)*cos(
x)^2 - a^2 + b^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^2 + 4*a
*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

Sympy [A]
time = 4.68, size = 162, normalized size = 2.95

$$\begin{cases} \frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for}\ b \neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \tilde{\infty}\log(\tan(x)) & \text{for}\ a = 0 \wedge b = 0 \\ \frac{\log(\tan(x))}{4b} & \text{for}\ a = 0 \\ \frac{\sqrt{b^2}}{2b^2\tan(x)-2b\sqrt{b^2}} & \text{for}\ a = -\sqrt{b^2} \\ -\frac{\sqrt{b^2}}{2b^2\tan(x)+2b\sqrt{b^2}} & \text{for}\ a = \sqrt{b^2} \\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} - \frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(2*x)),x)
```

```
[Out] Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
Piecewise((zoo*log(tan(x)), Eq(a, 0) & Eq(b, 0)), (log(tan(x))/(4*b), Eq(a,
 0)), (sqrt(b**2)/(2*b**2*tan(x) - 2*b*sqrt(b**2)), Eq(a, -sqrt(b**2))), (-
sqrt(b**2)/(2*b**2*tan(x) + 2*b*sqrt(b**2)), Eq(a, sqrt(b**2))), (log(tan(x
) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)) - log(tan(x) + b/a +
 sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)), True))
```

Giac [A]
time = 0.42, size = 77, normalized size = 1.40

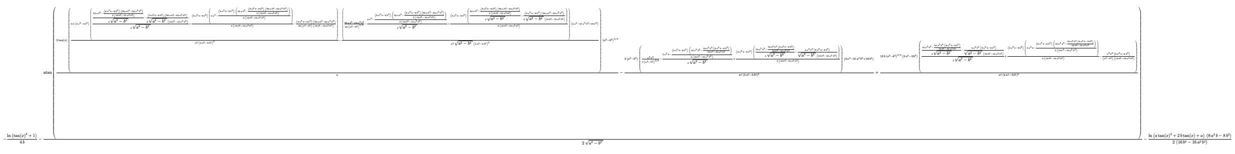
$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log\left(a\tan(x)^2+2\,b\tan(x)+a\right)}{4\,b}-\frac{\log\left(\tan(x)^2+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))/
sqrt(a^2 - b^2) + 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b - 1/4*log(tan(x)^2
 + 1)/b
```

Mupad [B]
time = 3.44, size = 1374, normalized size = 24.98



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(2*x)),x)
```

```
[Out] - log(tan(x)^2 + 1)/(4*b) - atan((2*tan(x)*((6*b*(2*a^2 - 3*b^2)*(((24*a*b^
3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2)))/
(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a^2
 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8
*b^3)*(4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4
- 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(
16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(32*(a^
2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^2 - 3*b^2)^2) - (((96*a*b^4 - 6
4*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) - (4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 -
 ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2
*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*((24*a*
b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))
)/(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(a
^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))*(4*a^4
+ 18*b^4 - 21*a^2*b^2))/(a^3*(a^2 - b^2)^(1/2)*(4*a^2 - 3*b^2)^2))*(a^2 - b
^2)^(3/2))/a - (2*(a^2 - b^2)*((a^2*b^3)/(2*(a^2 - b^2)^(3/2)) - (2*a^2*b -
 ((8*a^2*b - 8*b^3)*(16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 -
16*a^2*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b
- 8*b^3)*((16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^2*b^2
))/(4*(a^2 - b^2)^(1/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)^(1/2)
*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))*(4*a^4 + 18*b^4 - 21*a
^2*b^2))/(a^4*(4*a^2 - 3*b^2)^2) + (12*b*(a^2 - b^2)^(3/2)*(2*a^2 - 3*b^2)*
(((16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^2*b^2))/(4*(a
^2 - b^2)^(1/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3))/((a^2 - b^2)^(1/2)*(16*b^4
 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(2*a^2*b - ((8*
a^2*b - 8*b^3)*(16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(16*b^4 - 16*a^
2*b^2)))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - (a^2*b^3*(
8*a^2*b - 8*b^3))/((a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^4*(4*a^2 - 3*b^2
)^2))/(2*(a^2 - b^2)^(1/2)) - (log(a + a*tan(x)^2 + 2*b*tan(x))*(8*a^2*b -
8*b^3))/(2*(16*b^4 - 16*a^2*b^2))
```

3.848 $\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$-\frac{x}{2b}+\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}$$

```
[Out] -1/2*x/b+1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1144, 211}

$$\frac{\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -1/2*x/b + (Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*Sqrt[a
 - b]*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{x^2}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$

$$= -\left(\frac{1}{2}\left(-1+\frac{a}{b}\right)\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)\right) + \frac{(a+b)\text{Subst}\left(\int \frac{}{a+b+}\right.}{2}$$

$$= -\frac{x}{2b} + \frac{\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}\,b}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.92

$$-\frac{x+\frac{(a+b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] -1/2*(x + ((a + b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/b
```

Maple [A]
time = 0.16, size = 49, normalized size = 0.94

method	result
default	$\frac{(a+b)\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}} - \frac{\arctan(\tan(x))}{2b}$
risch	$-\frac{x}{2b} - \frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}+\frac{i\sqrt{-(a+b)(a-b)}+a}{b}\right)}{4(a-b)b} + \frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}-\frac{i\sqrt{-}}{}\right.}{4(a-b)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a+b*cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a+b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(x)/((a+b)*(a-b))^(1/2))-1/
2/b*arctan(tan(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 3.65, size = 225, normalized size = 4.33

$$\left[\frac{\sqrt{-\frac{a+b}{a-b}}\log\left(\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,(2\,a^2-ab-b^2)\cos(x)^2-4\left(2\,(a^2-ab)\cos(x)^3-(a^2-2\,ab+b^2)\cos(x)\right)\sqrt{-\frac{a+b}{a-b}}\sin(x)+a^2-2\,ab+b^2}{4\,b^2\cos(x)^4+4\,(ab-b^2)\cos(x)^2+a^2-2\,ab+b^2}\right)-4\,x}{8\,b},\ -\frac{\sqrt{\frac{a+b}{a-b}}\arctan\left(\frac{(2\,a\cos(x)^2-a+b)\sqrt{\frac{a+b}{a-b}}}{2\,(a+b)\cos(x)\sin(x)}\right)+2\,x}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a + b)/(a - b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 - 4*(2*(a^2 - a*b)*cos(x)^3 - (a^2 - 2*a*b + b^2)*cos(x))*
sqrt(-(a + b)/(a - b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b
 - b^2)*cos(x)^2 + a^2 - 2*a*b + b^2)) - 4*x)/b, -1/4*(sqrt((a + b)/(a - b)
)*arctan(1/2*(2*a*cos(x)^2 - a + b)*sqrt((a + b)/(a - b))/((a + b)*cos(x)*s
in(x))) + 2*x)/b]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(39) = 78$.
time = 17.70, size = 432, normalized size = 8.31

$$\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0 \wedge b=0 \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases} - \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{2b}-\frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\sin(2x)}{4a} & \text{for } b=0 \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{2bx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*s
qrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) - P
iecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)),
(x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*s
qrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sq
```

```
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))
/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b)))
 + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/
(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b
/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/
(a - b))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(40) = 80.
time = 0.42, size = 141, normalized size = 2.71

$$\frac{\sqrt{a^2-b^2}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a-b|}{2(a^2-2ab+b^2)|b|}-\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)}{2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqr
t(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a^2 - 2*a*b + b^2)
*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16
*(a + b)*(a - b) + 16*a^2))/(a - b))))/abs(b)
```

Mupad [B]
time = 3.43, size = 108, normalized size = 2.08

$$\frac{\mathrm{atan}\left(\frac{2b^3\tan(x)}{2a^2b-2b^3}-\frac{2a^2b\tan(x)}{2a^2b-2b^3}\right)}{2b}+\frac{\mathrm{atanh}\left(\frac{a\tan(x)}{\sqrt{b^2-a^2}}-\frac{b\tan(x)}{\sqrt{b^2-a^2}}\right)\sqrt{b^2-a^2}}{2\left(ab-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a + b*cos(2*x)),x)
```

```
[Out] atan((2*b^3*tan(x))/(2*a^2*b - 2*b^3) - (2*a^2*b*tan(x))/(2*a^2*b - 2*b^3))
/(2*b) + (atanh((a*tan(x))/(b^2 - a^2)^(1/2) - (b*tan(x))/(b^2 - a^2)^(1/2)
)*(b^2 - a^2)^(1/2))/(2*(a*b - b^2))
```

3.849 $\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$

Optimal. Leaf size=52

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

```
[Out] 1/2*x/b-1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {1107, 211}

$$\frac{x}{2b}-\frac{\sqrt{a-b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] x/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b]])/(2*b*Sqrt[
a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = \text{Subst}\left(\int \frac{1}{a+b+2ax^2+(a-b)x^4}\,dx, x, \tan(x)\right)$$
$$= \frac{(a-b)\text{Subst}\left(\int \frac{1}{a-b+(a-b)x^2}\,dx, x, \tan(x)\right)}{2b} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan(x\right.}{2b}$$
$$= \frac{x}{2b} - \frac{\sqrt{a-b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.96

$$\frac{x+\frac{(a-b)\tanh^{-1}\left(\frac{(a-b)\tan(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Cos[2*x]),x]
```

```
[Out] (x + ((a - b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])
/(2*b)
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.98

method	result
default	$\frac{(-a+b)\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}} + \frac{\arctan(\tan(x))}{2b}$
risch	$\frac{x}{2b} + \frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}+\frac{i\sqrt{-(a+b)(a-b)}+a}{b}\right)}{4(a+b)b} - \frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}-\frac{i\sqrt{-(a\right.}{}}{4(a+b)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a+b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(x)/((a+b)*(a-b))^(1/2))+1
/2/b*arctan(tan(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.84, size = 224, normalized size = 4.31

$$\left[\frac{\sqrt{-\frac{a-b}{a+b}}\log\left(\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,(2\,a^2-ab-b^2)\cos(x)^2+4\left(2\,(a^2+ab)\cos(x)^3-(a^2-b^2)\cos(x)\right)\sqrt{-\frac{a-b}{a+b}}\sin(x)+a^2-2\,ab+b^2}{4\,b^2\cos(x)^4+4\,(ab-b^2)\cos(x)^2+a^2-2\,ab+b^2}\right)+4\,x}{8\,b}, -\frac{\sqrt{\frac{a-b}{a+b}}\arctan\left(-\frac{\left(2\,a\cos(x)^2-a+b\right)\sqrt{\frac{a-b}{a+b}}}{2\,(a-b)\cos(x)\sin(x)}\right)-2\,x}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-(a - b)/(a + b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*b
 - b^2)*cos(x)^2 + 4*(2*(a^2 + a*b)*cos(x)^3 - (a^2 - b^2)*cos(x))*sqrt(-(a
 - b)/(a + b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b - b^2)*
cos(x)^2 + a^2 - 2*a*b + b^2)) + 4*x)/b, -1/4*(sqrt((a - b)/(a + b))*arctan
(-1/2*(2*a*cos(x)^2 - a + b)*sqrt((a - b)/(a + b))/((a - b)*cos(x)*sin(x)))
 - 2*x)/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(39) = 78$.
time = 17.69, size = 432, normalized size = 8.31

$$\begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0\wedge b=0 \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases} + \begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x}{2b}-\frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\sin(2x)}{4a} & \text{for } b=0 \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{2bx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*cos(2*x)),x)
```

```
[Out] Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b, 0
)), (1/(4*b*tan(x)), Eq(a, -b)), (tan(x)/(4*b), Eq(a, b)), (log(-sqrt(-a/(a
 - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-
a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*s
qrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)) + P
iecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a, b)),
(x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (2*a*x*s
qrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sq
```

```
rt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x))
/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b)))
 + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(a - b) - b/
(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a/(a - b) - b
/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/
(a - b))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(40) = 80$.
time = 0.41, size = 159, normalized size = 3.06

$$-\frac{\sqrt{a^2-b^2}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a-b|}{2\left((a-b)b^2+(a^2-ab)|b|\right)}-\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)(a-b)}{2\left(b^2-a|b|\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sq
rt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a - b)*b^2 + (a^2
 - a*b)*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - s
qrt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*(a - b)/(b^2 - a*abs(b))
```

Mupad [B]
time = 3.32, size = 684, normalized size = 13.15

$$\frac{\operatorname{atan}\left(\frac{2a^2\tan(x)}{2a^2-4ab+2b^2}+\frac{2b^2\tan(x)}{2a^2-4ab+2b^2}-\frac{4ab\tan(x)}{2a^2-4ab+2b^2}\right)}{2b}+\frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{\tan(x)\left(4a^3-12a^2b+12ab^2-4b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64a^3b^2-128a^2b^3+64ab^4\right)}{16\left(b^2+ab\right)}\right)}{4\left(b^2+ab\right)}\right)\sqrt{b^2-a^2}\,1i}{b^2+ab}+\frac{\left(\frac{\tan(x)\left(4a^3-12a^2b+12ab^2-4b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64a^3b^2-128a^2b^3+64ab^4\right)}{16\left(b^2+ab\right)}\right)}{4\left(b^2+ab\right)}\right)\sqrt{b^2-a^2}\,1i}{b^2+ab}}{\frac{\left(\frac{\tan(x)\left(4a^3-12a^2b+12ab^2-4b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4b^4-8ab^3+4a^2b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64a^3b^2-128a^2b^3+64ab^4\right)}{16\left(b^2+ab\right)}\right)}{4\left(b^2+ab\right)}\right)\sqrt{b^2-a^2}}{b^2+ab}-\frac{\left(\frac{\tan(x)\left(4a^3-12a^2b+12ab^2-4b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(8ab^3-4b^4-4a^2b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64a^3b^2-128a^2b^3+64ab^4\right)}{16\left(b^2+ab\right)}\right)}{4\left(b^2+ab\right)}\right)\sqrt{b^2-a^2}}{b^2+ab}}\right)\sqrt{b^2-a^2}\,1i}{2\left(b^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*cos(2*x)),x)
```

```
[Out] atan((2*a^2*tan(x))/(2*a^2 - 4*a*b + 2*b^2) + (2*b^2*tan(x))/(2*a^2 - 4*a*b
 + 2*b^2) - (4*a*b*tan(x))/(2*a^2 - 4*a*b + 2*b^2))/(2*b) + (atan(((((tan(x
)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 - 8*
a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*
a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(a*b +
b^2) + (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1
/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 12
8*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1
/2)*1i)/(a*b + b^2))/((((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 +
((b^2 - a^2)^(1/2)*(4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)
*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))
*(b^2 - a^2)^(1/2))/(a*b + b^2) - (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 -
```

```
4*b^3))/4 + ((b^2 - a^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2
- a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(
a*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(2*(a*b
 + b^2))
```

3.850 $\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

[Out] arctanh((a*sin(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3284, 65, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]

[Out] ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\sin^2(c+dx)}\right)}{ad}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 1.03

$$\frac{\tanh^{-1}(\sin(c+dx))\sin(c+dx)}{d\sqrt{a\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
[Out] (ArcTanh[Sin[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a*Sin[c + d*x]^2])
```

Maple [A]
time = 0.13, size = 30, normalized size = 1.00

method	result	size
default	$\frac{\sin(dx+c)\operatorname{arctanh}(\sin(dx+c))}{\sqrt{a\left(\sin^2(dx+c)\right)}\,d}$	30
risch	$\frac{2\ln\left(e^{idx}+ie^{-ic}\right)\sin(dx+c)}{d\sqrt{-\left(e^{2i(dx+c)}-1\right)^2 a\,e^{-2i(dx+c)}}} - \frac{2\ln\left(e^{idx}-ie^{-ic}\right)\sin(dx+c)}{d\sqrt{-\left(e^{2i(dx+c)}-1\right)^2 a\,e^{-2i(dx+c)}}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(a*sin(d*x+c)^2)^(1/2)*sin(d*x+c)*arctanh(sin(d*x+c))/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.
time = 0.54, size = 76, normalized size = 2.53

$$\frac{\frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)+1}\right)}{\sqrt{a}}+\frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)-1}\right)}{\sqrt{a}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) + 1))/sqrt(a
) + (-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) - 1))/sqrt(a)
)/d
```

Fricas [A]
time = 2.18, size = 91, normalized size = 3.03

$$\left[\frac{\sqrt{-a\cos\left(dx+c\right)^{2}+a}\,\log\left(-\frac{\sin(dx+c)+1}{\sin(dx+c)-1}\right)}{2\,ad\sin\left(dx+c\right)},-\frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a\cos\left(dx+c\right)^{2}+a}\,\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a*cos(d*x + c)^2 + a)*log(-(sin(d*x + c) + 1)/(sin(d*x + c) - 1)
)/(a*d*sin(d*x + c)), -sqrt(-a)*arctan(sqrt(-a*cos(d*x + c)^2 + a)*sqrt(-a)
/a)/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(c+dx\right)}{\sqrt{a\sin^{2}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)/sqrt(a*sin(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.
time = 0.53, size = 56, normalized size = 1.87

$$\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)-2\right|\right)}{4\,\sqrt{a}\,d\mathrm{sgn}\left(\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*x + c) +
 sin(d*x + c) - 2)))/(sqrt(a)*d*sgn(sin(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(c+d\,x)}{\sqrt{a\sin\left(c+d\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2),x)
```

```
[Out] int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2), x)
```

3.851 $\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -arctanh((a*cos(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3284, 65, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3284

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
+ 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(c+dx)}\right)}{ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

Mathematica [**A**]
time = 0.04, size = 49, normalized size = 1.58

$$\frac{\cos(c+dx)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

```
[Out] (Cos[c + d*x]*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]))/(d*Sqrt[a*C
os[c + d*x]^2])
```

Maple [**A**]
time = 0.13, size = 31, normalized size = 1.00

method	result	size
default	$-\frac{\cos(dx+c)\,\mathrm{arctanh}(\cos(dx+c))}{\sqrt{a\left(\cos^2(dx+c)\right)}\,d}$	31
risch	$\frac{2\ln\left(\mathrm{e}^{idx}-\mathrm{e}^{-ic}\right)\cos(dx+c)}{d\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2 a\,\mathrm{e}^{-2i(dx+c)}}}-\frac{2\ln\left(\mathrm{e}^{idx}+\mathrm{e}^{-ic}\right)\cos(dx+c)}{d\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)^2 a\,\mathrm{e}^{-2i(dx+c)}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*cos(d*x+c)^2)^(1/2)*cos(d*x+c)*arctanh(cos(d*x+c))/d
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

time = 0.33, size = 51, normalized size = 1.65

$$-\frac{\log\left(\frac{2\sqrt{-a\sin\left(dx+c\right)^2+a}\,\sqrt{a}}{|\sin(dx+c)|}+\frac{2\,a}{|\sin(dx+c)|}\right)}{\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(2*sqrt(-a*sin(d*x + c)^2 + a)*sqrt(a)/abs(sin(d*x + c)) + 2*a/abs(sin(
d*x + c)))/(sqrt(a)*d)
```

Fricas [A]
time = 2.16, size = 84, normalized size = 2.71

$$\left[-\frac{\sqrt{a\cos\left(dx+c\right)^2}\,\log\left(-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}\right)}{2\,ad\cos\left(dx+c\right)},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{a\cos\left(dx+c\right)^2}\,\sqrt{-a}}{a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(d*x + c)^2)*log(-(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/(a
*d*cos(d*x + c)), sqrt(-a)*arctan(sqrt(a*cos(d*x + c)^2)*sqrt(-a)/a)/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{\sqrt{a\cos^2\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)/sqrt(a*cos(c + d*x)**2), x)
```

Giac [A]
time = 0.42, size = 27, normalized size = 0.87

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}\,d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] log(abs(tan(1/2*d*x + 1/2*c)))/(sqrt(a)*d*sgn(cos(d*x + c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(c+d\,x)}{\sqrt{a\cos\left(c+d\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2),x)
[Out] int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2), x)
```

3.852 $\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} dx$

Optimal. Leaf size=8

$$\sqrt{\sin(x^2)}$$

[Out] sin(x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3522}

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

[In] Int[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]

[Out] Sqrt[Sin[x^2]]

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} dx = \sqrt{\sin(x^2)}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\sqrt{\sin(x^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]

[Out] Sqrt[Sin[x^2]]

Maple [A]
time = 0.05, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\sqrt{\sin\left(x^2\right)}$	7
default	$\sqrt{\sin\left(x^2\right)}$	7
risch	$\sqrt{\sin\left(x^2\right)}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)/sin(x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x^2)^(1/2)
```

Maxima [A]
time = 0.32, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(sin(x^2))
```

Fricas [A]
time = 2.16, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(sin(x^2))
```

Sympy [A]
time = 0.09, size = 7, normalized size = 0.88

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)/sin(x**2)**(1/2),x)
```

```
[Out] sqrt(sin(x**2))
```

Giac [A]
time = 0.40, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(sin(x^2))
```

Mupad [B]
time = 3.19, size = 6, normalized size = 0.75

$$\sqrt{\sin\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(x^2))/sin(x^2)^(1/2),x)
```

```
[Out] sin(x^2)^(1/2)
```

3.853 $\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{\log(\sin(x))\sin(x)}{\sqrt{2}\sqrt{\sin^2(x)}}$$

```
[Out] 1/2*ln(sin(x))*sin(x)*2^(1/2)/(sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4441, 12, 15, 29}

$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] (Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4441

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{2}\,\sqrt{x^2}}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x^2}}\,dx, x, \sin(x)\right)}{\sqrt{2}}$$
$$= \frac{\sin(x)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{\sqrt{2}\,\sqrt{\sin^2(x)}}$$
$$= \frac{\log(\sin(x))\sin(x)}{\sqrt{2}\,\sqrt{\sin^2(x)}}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.95

$$\frac{\log(\sin(x))\sin(x)}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[1 - Cos[2*x]],x]
[Out] (Log[Sin[x]]*Sin[x])/Sqrt[1 - Cos[2*x]]
```

Maple [A]
time = 0.25, size = 25, normalized size = 1.32

method	result	size
default	$\frac{\sin(x)(\ln(\cos(x)-1)+\ln(1+\cos(x)))\sqrt{2}}{2\sqrt{2-2\cos(2x)}}$	25
risch	$-\frac{i\sqrt{2}\,x\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}+\frac{\sqrt{2}\,\ln\left(e^{2ix}-1\right)\sin(x)}{\sqrt{-\left(e^{2ix}-1\right)^2 e^{-2ix}}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*sin(x)*(ln(cos(x)-1)+ln(1+cos(x)))*2^(1/2)/(sin(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.
time = 0.54, size = 41, normalized size = 2.16

$$\frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A]
time = 2.21, size = 21, normalized size = 1.11

$$\frac{\sqrt{-2\,\cos\left(x\right)^2+2}\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)}{2\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-2*cos(x)^2 + 2)*log(1/2*sin(x))/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(x\right)}{\sqrt{1-\cos\left(2x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))**(1/2),x)
```

```
[Out] Integral(cos(x)/sqrt(1 - cos(2*x)), x)
```

Giac [A]
time = 0.42, size = 14, normalized size = 0.74

$$\frac{\sqrt{2}\,\log\left(\left|\sin\left(x\right)\right|\right)}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(sin(x)))/sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{\cos\left(x\right)}{\sqrt{1-\cos\left(2\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1 - cos(2*x))^(1/2),x)
```

```
[Out] int(cos(x)/(1 - cos(2*x))^(1/2), x)
```

3.854 $\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(x)}{8}+\frac{1}{8}\cos(\log(x))\sin(\log(x))-\frac{1}{4}\cos^3(\log(x))\sin(\log(x))$$

[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2648, 2715, 8}

$$\frac{\log(x)}{8}-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))+\frac{1}{8}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

[In] Int[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]

[Out] Log[x]/8 + (Cos[Log[x]]*Sin[Log[x]])/8 - (Cos[Log[x]]^3*Sin[Log[x]])/4

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \text{Subst}\left(\int \cos^2(x)\sin^2(x)\,dx, x, \log(x)\right)$$
$$= -\frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{4}\text{Subst}\left(\int \cos^2(x)\,dx, x, \log(x)\right)$$
$$= \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x)) + \frac{1}{8}\text{Subst}(\int 1\,dx, x,$$
$$= \frac{\log(x)}{8} + \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x))$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.55

$$\frac{\log(x)}{8} - \frac{1}{32}\sin(4\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
[Out] Log[x]/8 - Sin[4*Log[x]]/32
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\ln(x)}{8} + \frac{\cos(\ln(x))\sin(\ln(x))}{8} - \frac{(\cos^3(\ln(x)))\sin(\ln(x))}{4}$	24
default	$\frac{\ln(x)}{8} + \frac{\cos(\ln(x))\sin(\ln(x))}{8} - \frac{(\cos^3(\ln(x)))\sin(\ln(x))}{4}$	24
risch	$\frac{\ln(x)}{8} + \frac{i x^{4i}}{64} - \frac{i x^{-4i}}{64}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x))^2*sin(ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))
```

Maxima [A]
time = 0.33, size = 12, normalized size = 0.41

$$\frac{1}{8}\log(x) - \frac{1}{32}\sin(4\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="maxima")
```

[Out] 1/8*log(x) - 1/32*sin(4*log(x))

Fricas [A]
time = 2.38, size = 23, normalized size = 0.79

$$-\frac{1}{8} \left(2 \cos \left(\log \left(x\right)\right)^{3} - \cos \left(\log \left(x\right)\right)\right) \sin \left(\log \left(x\right)\right) + \frac{1}{8} \log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="fricas")

[Out] -1/8*(2*cos(log(x))^3 - cos(log(x)))*sin(log(x)) + 1/8*log(x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(29) = 58$.
time = 6.98, size = 476, normalized size = 16.41

$$\frac{\log(x)\tan^{8}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{4\log(x)\tan^{6}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{6\log(x)\tan^{4}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{4\log(x)\tan^{2}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{\log(x)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{2\tan^{7}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} - \frac{14\tan^{5}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} + \frac{14\tan^{3}\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8} - \frac{2\tan\left(\frac{\log(x)}{2}\right)}{8\tan^{8}\left(\frac{\log(x)}{2}\right)+32\tan^{6}\left(\frac{\log(x)}{2}\right)+48\tan^{4}\left(\frac{\log(x)}{2}\right)+32\tan^{2}\left(\frac{\log(x)}{2}\right)+8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(ln(x))**2*sin(ln(x))**2/x,x)

[Out] log(x)*tan(log(x)/2)**8/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/2)**6/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 6*log(x)*tan(log(x)/2)**4/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/2)**2/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + log(x)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 2*tan(log(x)/2)**7/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) - 14*tan(log(x)/2)**5/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 14*tan(log(x)/2)**3/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) - 2*tan(log(x)/2)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8)

Giac [A]
time = 0.41, size = 12, normalized size = 0.41

$$\frac{1}{8} \log \left(x\right) - \frac{1}{32} \sin \left(4 \log \left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="giac")

[Out] 1/8*log(x) - 1/32*sin(4*log(x))

Mupad [B]
time = 3.20, size = 12, normalized size = 0.41

$$\frac{\ln(x)}{8} - \frac{\sin(4\ln(x))}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(log(x))^2*sin(log(x))^2)/x,x)
```

```
[Out] log(x)/8 - sin(4*log(x))/32
```

3.855 $\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{2}-\frac{1}{6}\log(\cos(x)+\sin(x))+\frac{1}{3}\log(2-\sin(2x))$$

```
[Out] 1/2*x-1/6*ln(cos(x)+sin(x))+1/3*ln(2-sin(2*x))
```

Rubi [A]
time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2099, 649, 209, 266, 642}

$$\frac{x}{2}+\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)-\frac{1}{6}\log(\tan(x)+1)+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 + Log[Cos[x]]/2 - Log[1 + Tan[x]]/6 + Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{x^3}{1+x^2+x^3+x^5}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{6(1+x)}+\frac{1-x}{2(1+x^2)}+\frac{-1+2x}{3(1-x+x^2)}\right)dx, x, \tan(x)\right)\\ &= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{1+x^2}\right.\\ &= -\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\right.\\ &= \frac{x}{2}+\frac{1}{2}\log(\cos(x))-\frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 29, normalized size = 1.00

$$\frac{x}{2}-\frac{1}{6}\log(\cos(x)+\sin(x))+\frac{1}{3}\log(2-\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
[Out] x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3
```

Maple [A]
time = 0.16, size = 36, normalized size = 1.24

method	result
default	$-\frac{\ln(\tan^2(x)+1)}{4}+\frac{\arctan(\tan(x))}{2}+\frac{\ln(1-\tan(x)+\tan^2(x))}{3}-\frac{\ln(1+\tan(x))}{6}$
risch	$\frac{x}{2}-\frac{ix}{2}-\frac{\ln(e^{2ix}+i)}{6}+\frac{\ln(e^{4ix}-4ie^{2ix}-1)}{3}$
norman	$\frac{\frac{x}{2}+\frac{3x(\tan^2(\frac{x}{2}))}{2}+\frac{3x(\tan^4(\frac{x}{2}))}{2}+\frac{x(\tan^6(\frac{x}{2}))}{2}}{(1+\tan^2(\frac{x}{2}))^3}-\frac{\ln(1+\tan^2(\frac{x}{2}))}{2}-\frac{\ln(\tan^2(\frac{x}{2})-2\tan(\frac{x}{2})-1)}{6}+\frac{\ln(\tan^4(\frac{x}{2})+2(\tan^3(\frac{x}{2}))+2(\text{ta}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)^3+sin(x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(tan(x)^2+1)+1/2*arctan(tan(x))+1/3*ln(1-tan(x)+tan(x)^2)-1/6*ln(1+t
an(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(23) = 46$.
time = 0.55, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{2\sin(x)^2}{(\cos(x)+1)^2}+\frac{2\sin(x)^3}{(\cos(x)+1)^3}+\frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)-\frac{1}{6}\log\left(-\frac{2\sin(x)}{\cos(x)+1}+\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)-\frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) + 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) -
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) - 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.05, size = 26, normalized size = 0.90

$$\frac{1}{2}x-\frac{1}{12}\log\left(2\cos(x)\sin(x)+1\right)+\frac{1}{3}\log\left(-\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/12*log(2*cos(x)*sin(x) + 1) + 1/3*log(-cos(x)*sin(x) + 1)
```

Sympy [A]
time = 0.12, size = 32, normalized size = 1.10

$$\frac{x}{2}-\frac{\log\left(\sin(x)+\cos(x)\right)}{6}+\frac{\log\left(\sin^2(x)-\sin(x)\cos(x)+\cos^2(x)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/6 + log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

Giac [A]
time = 0.43, size = 34, normalized size = 1.17

$$\frac{1}{2}x+\frac{1}{3}\log\left(\tan(x)^2-\tan(x)+1\right)-\frac{1}{4}\log\left(\tan(x)^2+1\right)-\frac{1}{6}\log\left(|\tan(x)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(ab
s(tan(x) + 1))
```

Mupad [B]
time = 3.31, size = 45, normalized size = 1.55

$$\frac{x}{2} - \frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2} + \frac{\ln\left(\frac{\sin(2x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3} - \frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(cos(x)^3 + sin(x)^3),x)
```

```
[Out] x/2 - log(1/cos(x/2)^2)/2 + log((sin(2*x) - 2)/cos(x/2)^4)/3 - log(sin(x +
pi/4)/cos(x/2)^2)/6
```

3.856 $\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{2}+\frac{1}{6}\log(\cos(x)+\sin(x))-\frac{1}{3}\log(2-\sin(2x))$$

```
[Out] 1/2*x+1/6*ln(cos(x)+sin(x))-1/3*ln(2-sin(2*x))
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2083, 649, 209, 266, 642}

$$\frac{x}{2}-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

```
[Out] x/2 - Log[Cos[x]]/2 + Log[1 + Tan[x]]/6 - Log[1 - Tan[x] + Tan[x]^2]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2+x^3+x^5}\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx,x,\tan(x)\right)\\ &= \frac{1}{6}\log(1+\tan(x))+\frac{1}{3}\text{Subst}\left(\int \frac{1-2x}{1-x+x^2}\,dx,x,\tan(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,d\right.\\ &= \frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(\right.\\ &= \frac{x}{2}-\frac{1}{2}\log(\cos(x))+\frac{1}{6}\log(1+\tan(x))-\frac{1}{3}\log\left(1-\tan(x)+\tan^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 1.00

$$\frac{x}{2}+\frac{1}{6}\log(\cos(x)+\sin(x))-\frac{1}{3}\log(2-\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
[Out] x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3
```

Maple [A]
time = 0.14, size = 36, normalized size = 1.24

method	result
default	$\frac{\ln(\tan^2(x)+1)}{4}+\frac{\arctan(\tan(x))}{2}-\frac{\ln(1-\tan(x)+\tan^2(x))}{3}+\frac{\ln(1+\tan(x))}{6}$
risch	$\frac{x}{2}+\frac{ix}{2}+\frac{\ln(e^{2ix}+i)}{6}-\frac{\ln(e^{4ix}-4ie^{2ix}-1)}{3}$
norman	$\frac{\frac{x}{2}+\frac{3x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{3x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}+\frac{x\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}+\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{6}-\frac{\ln\left(\tan^4\left(\frac{x}{2}\right)+2\left(\tan^3\left(\frac{x}{2}\right)\right)+2(\text{ta}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(cos(x)^3+sin(x)^3),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(tan(x)^2+1)+1/2*arctan(tan(x))-1/3*ln(1-tan(x)+tan(x)^2)+1/6*ln(1+ta
n(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(23) = 46$.
time = 0.57, size = 103, normalized size = 3.55

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) + \frac{1}{6}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/(
cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1) +
1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(sin
(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.30, size = 26, normalized size = 0.90

$$\frac{1}{2}\,x + \frac{1}{12}\,\log\left(2\,\cos(x)\sin(x) + 1\right) - \frac{1}{3}\,\log\left(-\cos(x)\sin(x) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/2*x + 1/12*log(2*cos(x)*sin(x) + 1) - 1/3*log(-cos(x)*sin(x) + 1)
```

Sympy [A]
time = 0.11, size = 32, normalized size = 1.10

$$\frac{x}{2} + \frac{\log(\sin(x) + \cos(x))}{6} - \frac{\log(\sin^2(x) - \sin(x)\cos(x) + \cos^2(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(cos(x)**3+sin(x)**3),x)
```

```
[Out] x/2 + log(sin(x) + cos(x))/6 - log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/3
```

Giac [A]
time = 0.41, size = 34, normalized size = 1.17

$$\frac{1}{2}\,x - \frac{1}{3}\,\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\,\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\,\log\left(|\tan(x) + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(ab
s(tan(x) + 1))
```

Mupad [B]
time = 3.19, size = 45, normalized size = 1.55

$$\frac{x}{2}+\frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2}-\frac{\ln\left(\frac{\sin(2\,x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3}+\frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(cos(x)^3 + sin(x)^3),x)
```

```
[Out] x/2 + log(1/cos(x/2)^2)/2 - log((sin(2*x) - 2)/cos(x/2)^4)/3 + log(sin(x +
pi/4)/cos(x/2)^2)/6
```

3.857 $\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(1+\sin(x))+\frac{1}{3(2-\sin(x))}$$

```
[Out] 1/2*ln(1-sin(x))-4/9*ln(2-sin(x))-1/18*ln(1+sin(x))+1/3/(2-sin(x))
```

Rubi [A]

time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {724, 815}

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

```
[Out] Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 - Sin[x]))
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(2-x)^2\left(-1+x^2\right)}\,dx,x,\sin(x)\right)\\
&= \frac{1}{3(2-\sin(x))}+\frac{1}{3}\text{Subst}\left(\int \frac{2+x}{(2-x)\left(-1+x^2\right)}\,dx,x,\sin(x)\right)\\
&= \frac{1}{3(2-\sin(x))}+\frac{1}{3}\text{Subst}\left(\int \left(-\frac{4}{3(-2+x)}+\frac{3}{2(-1+x)}-\frac{1}{6(1+x)}\right)\,dx,\right.\\
&= \frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(1+\sin(x))+\frac{1}{3(2-\sin(x))}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.86

$$\frac{1}{18}\left(9\log(1-\sin(x))-8\log(2-\sin(x))-\log(1+\sin(x))-\frac{6}{-2+\sin(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
[Out] (9*Log[1 - Sin[x]] - 8*Log[2 - Sin[x]] - Log[1 + Sin[x]] - 6/(-2 + Sin[x]))
/18
```

Maple [A]
time = 0.17, size = 31, normalized size = 0.70

method	result	size
default	$-\frac{1}{3(\sin(x)-2)}-\frac{4\ln(\sin(x)-2)}{9}-\frac{\ln(1+\sin(x))}{18}+\frac{\ln(\sin(x)-1)}{2}$	31
norman	$\frac{\tan(\frac{x}{2})}{6(\tan^2(\frac{x}{2}))-6\tan(\frac{x}{2})+6}-\frac{\ln(\tan(\frac{x}{2})+1)}{9}-\frac{4\ln(\tan^2(\frac{x}{2})-\tan(\frac{x}{2})+1)}{9}+\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$	57
risch	$-\frac{2ie^{ix}}{3(-4ie^{ix}+e^{2ix}-1)}-\frac{\ln(e^{ix}+i)}{9}+\ln\left(e^{ix}-i\right)-\frac{4\ln(-4ie^{ix}+e^{2ix}-1)}{9}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-5+cos(x)^2+4*sin(x)),x,method=_RETURNVERBOSE)
[Out] -1/3/(sin(x)-2)-4/9*ln(sin(x)-2)-1/18*ln(1+sin(x))+1/2*ln(sin(x)-1)
```

Maxima [A]
time = 0.30, size = 30, normalized size = 0.68

$$-\frac{1}{3(\sin(x)-2)}-\frac{1}{18}\log(\sin(x)+1)+\frac{1}{2}\log(\sin(x)-1)-\frac{4}{9}\log(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="maxima")
[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - 4/9*log(si
n(x) - 2)
```

Fricas [A]
time = 2.27, size = 46, normalized size = 1.05

$$-\frac{(\sin(x)-2)\log(\sin(x)+1)+8(\sin(x)-2)\log\left(-\frac{1}{2}\sin(x)+1\right)-9(\sin(x)-2)\log(-\sin(x)+1)+6}{18(\sin(x)-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/18*((sin(x) - 2)*log(sin(x) + 1) + 8*(sin(x) - 2)*log(-1/2*sin(x) + 1) -
 9*(sin(x) - 2)*log(-sin(x) + 1) + 6)/(sin(x) - 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (x)}{4 \sin (x) + \cos^2 (x) - 5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)**2+4*sin(x)),x)
```

```
[Out] Integral(sec(x)/(4*sin(x) + cos(x)**2 - 5), x)
```

Giac [A]
time = 0.41, size = 34, normalized size = 0.77

$$-\frac{1}{3 (\sin (x) - 2)} - \frac{1}{18} \log (\sin (x) + 1) - \frac{4}{9} \log (-\sin (x) + 2) + \frac{1}{2} \log (-\sin (x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="giac")
```

```
[Out] -1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) - 4/9*log(-sin(x) + 2) + 1/2*log(-
sin(x) + 1)
```

Mupad [B]
time = 0.07, size = 32, normalized size = 0.73

$$\frac{\ln (\sin (x) - 1)}{2} - \frac{\ln (\sin (x) + 1)}{18} - \frac{4 \ln (\sin (x) - 2)}{9} - \frac{1}{3 (\sin (x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(4*sin(x) + cos(x)^2 - 5)),x)
```

```
[Out] log(sin(x) - 1)/2 - log(sin(x) + 1)/18 - (4*log(sin(x) - 2))/9 - 1/(3*(sin(
x) - 2))
```

3.858 $\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

```
[Out] 2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(19) = 38.
time = 0.84, antiderivative size = 88, normalized size of antiderivative = 4.63, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1986, 6851, 1077, 8}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

```
[Out] (2*Cos[x/2]^2*(3 + 2*Tan[x/2] - 3*Tan[x/2]^2))/(Sqrt[Cos[x/2]^2*(3 + 2*Tan[
x/2] - 3*Tan[x/2]^2)]*Sqrt[Cos[x/2]^2*(1 - Tan[x/2]^2)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1077

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2
)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(2*a*c
*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*d - a*f)))*x), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*((-c)*e*(2*p + q + 4)))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = 2\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{\frac{3+2x-3x^2}{1+x^2}}\sqrt{\frac{1-x^2}{1+x^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{\left(2\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{3+2x-3x^2}\,(1-x^2)\sqrt{\cdots}}\right.}{\sqrt{\sec^2\left(\frac{x}{2}\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\cdots\right.\right.}}$$

$$= \frac{\left(2\cos^2\left(\frac{x}{2}\right)\sqrt{3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\right)\text{Subst}\left(\cdots\right.}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\cdots}}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\cdots\right.\right.}}$$

$$= \frac{2\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^2\left(\frac{x}{2}\right)\left(3+2\tan\left(\frac{x}{2}\right)-3\tan^2\left(\frac{x}{2}\right)\right)}\sqrt{\cos^2\left(\frac{x}{2}\right)\left(1-\tan^2\left(\frac{x}{2}\cdots\right.\right.}}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 1.00

$$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

```
[Out] (2*Sqrt[3*Cos[x] + Sin[x]])/Sqrt[Cos[x]]
```

Maple [A]
time = 0.48, size = 16, normalized size = 0.84

method	result	size
default	$\frac{2\sqrt{3\cos\left(x\right)+\sin\left(x\right)}}{\sqrt{\cos\left(x\right)}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(15) = 30$.
time = 0.57, size = 145, normalized size = 7.63

$$\frac{2\left(\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{6\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}-\frac{2\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}+\frac{3\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}+3\right)\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)^{2}}{\sqrt{\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{3\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+3}\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)^{\frac{3}{2}}\left(\frac{2\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+\frac{\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(2*sin(x)/(cos(x) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 - 2*sin(x)^3/(cos(x) +
 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 3)*(sin(x)^2/(cos(x) + 1)^2 + 1)^2/(sqr
t(2*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + 3)*(sin(x)/(cos(x) +
1) + 1)^(3/2)*(-sin(x)/(cos(x) + 1) + 1)^(3/2)*(2*sin(x)^2/(cos(x) + 1)^2 +
 sin(x)^4/(cos(x) + 1)^4 + 1))
```

Fricas [A]
time = 1.77, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3\cos\left(x\right)+\sin\left(x\right)}}{\sqrt{\cos\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3*cos(x) + sin(x))/sqrt(cos(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(x) + 3\cos(x)}\,\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(3/2)/(3*cos(x)+sin(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sin(x) + 3*cos(x))*cos(x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(x) + sin(x))*cos(x)^(3/2)), x)
```

Mupad [B]
time = 3.79, size = 15, normalized size = 0.79

$$\frac{2\sqrt{3\cos(x) + \sin(x)}}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^(3/2)*(3*cos(x) + sin(x))^(1/2)),x)
```

```
[Out] (2*(3*cos(x) + sin(x))^(1/2))/cos(x)^(1/2)
```

3.859 $\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$

Optimal. Leaf size=44

$$-\log(\sin(x))+2\log\left(-\sqrt{\cos(x)}+\sqrt{\cos(x)+\sin(x)}\right)+\frac{2\sqrt{\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

```
[Out] -ln(sin(x))+2*ln(-cos(x)^(1/2)+(cos(x)+sin(x))^(1/2))+2*(cos(x)+sin(x))^(1/
2)/cos(x)^(1/2)
```

Rubi [F]
time = 1.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] Defer[Int][(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2), x]
```

Rubi steps

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx=\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$

Mathematica [A]
time = 0.68, size = 68, normalized size = 1.55

$$\frac{2\left(\cos(x)+\sin(x)-\tanh^{-1}\left(\frac{\sqrt{\cos(x)}}{\sqrt{\cos(x)+\sqrt{\sin^2(x)}}}\right)\sqrt{\cos(x)}\sqrt{\cos(x)+\sqrt{\sin^2(x)}}\right)}{\sqrt{\cos(x)}\sqrt{\cos(x)+\sin(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

```
[Out] (2*(Cos[x] + Sin[x] - ArcTanh[Sqrt[Cos[x]]/Sqrt[Cos[x] + Sqrt[Sin[x]^2]]]*S
qrt[Cos[x]]*Sqrt[Cos[x] + Sqrt[Sin[x]^2]]))/(Sqrt[Cos[x]]*Sqrt[Cos[x] + Sin
[x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.74, size = 1911, normalized size = 43.43

method	result	size
default	Expression too large to display	1911

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(cos(x)-1)^2*(1+cos(x))^2*(EllipticPi(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(
1/2))^(1/2)/(2^(1/2)-2),-1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))
*(2+2^(1/2)))^(1/2))*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(
x)*(2+2^(1/2)))^(1/2)*2^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(
1/2))/cos(x)*(2-2^(1/2)))^(1/2)*((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/
2)*sin(x)+EllipticPi(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)
-2),1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*
((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)
*2^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/
2)))^(1/2)*((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*sin(x)-EllipticF((
(1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),I/(2+2^(1/2))*((2-
2^(1/2))*(2+2^(1/2)))^(1/2))*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/
2))/cos(x)*(2+2^(1/2)))^(1/2)*2^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*co
s(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1
/2))^(1/2)*sin(x)-EllipticPi(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/
(2^(1/2)-2),-1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2))
)^(1/2))*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2
)))^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1
/2)))^(1/2)*((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*sin(x)-EllipticPi
(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),1/2*(2^(1/2)-2)*
2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))*((cos(x)*2^(1/2)+sin
(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)*(-(cos(x)*2^(1/2)+s
in(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*((1+sin(x))/cos(x
)*(2-2^(1/2))*2^(1/2))^(1/2)*sin(x)+EllipticF(((1+sin(x))/cos(x)*(2-2^(1/2)
)*2^(1/2))^(1/2)/(2^(1/2)-2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))
*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2
)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1
/2)*((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*sin(x)-((1+sin(x))/cos(x)
*(2-2^(1/2))*2^(1/2))^(1/2)*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2
))/cos(x)*(2+2^(1/2)))^(1/2)*2^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos
(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*EllipticPi(((1+sin(x))/cos(x)*(2-2^(
1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),-1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2
```

```
-2^(1/2))*(2+2^(1/2)))^(1/2))-((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)
*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2
)*2^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1
/2)))^(1/2)*EllipticPi(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/
2)-2),1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2)
)+((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*((cos(x)*2^(1/2)+sin(x)*2^(
1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)*2^(1/2)*(-(cos(x)*2^(1/2)+
sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*EllipticF(((1+si
n(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),I/(2+2^(1/2))*((2-2^(1/
2))*(2+2^(1/2)))^(1/2))+((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*((cos
(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)*(-(c
os(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*El
lipticPi(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),-1/2*(2^
(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))+((1+sin(x))
/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*((cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)
+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)*(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(
x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2)*EllipticPi(((1+sin(x))/cos(x)*(2-2^(1
/2))*2^(1/2))^(1/2)/(2^(1/2)-2),1/2*(2^(1/2)-2)*2^(1/2),I/(2+2^(1/2))*((2-2
^(1/2))*(2+2^(1/2)))^(1/2))-((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)*(
(cos(x)*2^(1/2)+sin(x)*2^(1/2)+2*cos(x)+2^(1/2))/cos(x)*(2+2^(1/2)))^(1/2)*
(-(cos(x)*2^(1/2)+sin(x)*2^(1/2)-2*cos(x)+2^(1/2))/cos(x)*(2-2^(1/2)))^(1/2
)*EllipticF(((1+sin(x))/cos(x)*(2-2^(1/2))*2^(1/2))^(1/2)/(2^(1/2)-2),I/(2+
2^(1/2))*((2-2^(1/2))*(2+2^(1/2)))^(1/2))+2*cos(x)*2^(1/2)+2*sin(x)*2^(1/2)
-4*cos(x)-4*sin(x))/sin(x)^4/(cos(x)+sin(x))^(1/2)/cos(x)^(1/2)/(2+2^(1/2))
/(2^(1/2)-2)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(36) = 72$.
time = 0.74, size = 518, normalized size = 11.77

$$\frac{4\left((2\cos(2x)+\sin(2x))\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^3+(2\cos(2x)+\sin(2x))\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)\sin\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^2-(\cos(2x)-2\sin(2x)+1)\sin\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^3-(\cos(2x)-\sin(2x)-1)\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)-\left((\cos(2x)-2\sin(2x)+1)\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^2+\cos(2x)+\sin(2x)-1\right)\sin\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)\right)}{\left(4(\cos(2x)-\sin(2x))\cos(4x)+2\cos(4x)^2+4\cos(2x)^2+4(\cos(2x)+\sin(2x)+1)\sin(4x)+2\sin(4x)^2+4\sin(2x)^2+4\cos(2x)+4\sin(2x)+2\right)^{\frac{1}{4}}\left(\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^2+\sin\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\sin(4x)+1)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*((2*cos(2*x) + sin(2*x))*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x
) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^3 + (2*cos(2*x) + sin(2*x))*c
os(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x)
 + sin(4*x) + 1))*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 - (cos(2*x) - 2*sin(2*x) + 1)*sin(1/
2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + si
n(4*x) + 1))^3 - (cos(2*x) - sin(2*x) - 1)*cos(1/2*arctan2(-cos(4*x) + sin(
4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1)) - ((cos(2*x)
- 2*sin(2*x) + 1)*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, co
s(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + cos(2*x) + sin(2*x) - 1)*sin(1/2*a
```

```
rctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4
*x) + 1)))/((4*(cos(2*x) - sin(2*x))*cos(4*x) + 2*cos(4*x)^2 + 4*cos(2*x)^2
 + 4*(cos(2*x) + sin(2*x) + 1)*sin(4*x) + 2*sin(4*x)^2 + 4*sin(2*x)^2 + 4*c
os(2*x) + 4*sin(2*x) + 2)^(1/4)*(cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*s
in(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + sin(1/2*arctan2(-co
s(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^
2))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.
time = 2.21, size = 96, normalized size = 2.18

$$-\frac{\cos(x)\log\left((2\cos(x)+\sin(x))\sqrt{\cos(x)+\sin(x)}\sqrt{\cos(x)}+\frac{7}{4}\cos(x)^2+2\cos(x)\sin(x)+\frac{1}{4}\right)-\cos(x)\log\left(-(2\cos(x)+\sin(x))\sqrt{\cos(x)+\sin(x)}\sqrt{\cos(x)}+\frac{7}{4}\cos(x)^2+2\cos(x)\sin(x)+\frac{1}{4}\right)-8\sqrt{\cos(x)+\sin(x)}\sqrt{\cos(x)}}{4\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)*log((2*cos(x) + sin(x))*sqrt(cos(x) + sin(x))*sqrt(cos(x)) + 7
/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - cos(x)*log(-(2*cos(x) + sin(x))*sqrt
(cos(x) + sin(x))*sqrt(cos(x)) + 7/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - 8*
sqrt(cos(x) + sin(x))*sqrt(cos(x)))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin(x)+\cos(x)}\csc(x)}{\cos^{\frac{3}{2}}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))**(1/2)/cos(x)**(3/2),x)
```

```
[Out] Integral(sqrt(sin(x) + cos(x))*csc(x)/cos(x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x) + sin(x))*csc(x)/cos(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(x)+\sin(x)}}{{\cos(x)}^{3/2}\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)),x)
[Out] int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)), x)
```

3.860 $\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$

Optimal. Leaf size=19

$$\frac{x\sqrt{1+\sin(2x)}}{\cos(x)+\sin(x)}$$

```
[Out] x*(1+sin(2*x))^(1/2)/(cos(x)+sin(x))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(19) = 38. time = 1.17, antiderivative size = 72, normalized size of antiderivative = 3.79, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4486, 6851, 1089, 642, 649, 209, 266, 12, 1037}

$$\frac{2\cos^2\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)\text{ArcTan}\left(\tan\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

```
[Out] (2*ArcTan[Tan[x/2]]*Cos[x/2]^2*(1 + 2*Tan[x/2] - Tan[x/2]^2))/Sqrt[Cos[x/2]
^4*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rule 1089

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.
)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}
, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*A*c*f + a^2*C*f + c*((-b)*
C*d + A*b*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 - A*c*d*
f - a*C*d*f + a*A*f^2 - f*((-b)*C*d + A*b*f)*x)/(d + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \int \left(\frac{\cos(x)}{\sqrt{1+\sin(2x)}}+\frac{\sin(x)}{\sqrt{1+\sin(2x)}}\right)dx$$

$$= \int \frac{\cos(x)}{\sqrt{1+\sin(2x)}}\,dx + \int \frac{\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$$

$$= 2\text{Subst}\left(\int \frac{2x}{(1+x^2)^2\sqrt{\frac{(-1-2x+x^2)^2}{(1+x^2)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + 2\text{Subst}\left(\int \frac{}{(1+x^2)}\right.$$

$$= 4\text{Subst}\left(\int \frac{x}{(1+x^2)^2\sqrt{\frac{(-1-2x+x^2)^2}{(1+x^2)^2}}}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{\left(2\cos^2\left(\frac{x}{2}\right)\left(-1-2\right.\right.}{\sqrt{}}$$

$$= \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)\right)\text{Subst}\left(\int \frac{-4+4x}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{4\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-1-2\tan\left(\frac{x}{2}\right)+\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\right.}{}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right)\log\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\left(\cos^2\left(\frac{x}{2}\right)\left(-\right.\right.}{2}$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{2\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}} + \frac{\cos^2\left(\frac{x}{2}\right)\log\left(\cos\left(\frac{x}{2}\right)\right)\left(1+2\tan\left(\right.\right.}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\right.}}$$

$$= \frac{x\cos^2\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(1+2\tan\left(\frac{x}{2}\right)-\tan^2\left(\frac{x}{2}\right)\right)^2}}$$

Mathematica [**A**]
time = 0.01, size = 17, normalized size = 0.89

$$\frac{x(\cos(x)+\sin(x))}{\sqrt{1+\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
[Out] (x*(Cos[x] + Sin[x]))/Sqrt[1 + Sin[2*x]]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.74, size = 12372, normalized size = 651.16

method	result	size
default	Expression too large to display	12372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(17) = 34.
time = 0.56, size = 329, normalized size = 17.32

$$\frac{1}{16}\sqrt{2}\left(2\sqrt{2}\arctan\left(\sin\left(2x\right)+1,\cos\left(2x\right)\right)+\sqrt{2}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\sin\left(2x\right)+1\right)+4\left(\cos\left(4x\right)^2+4\cos\left(2x\right)^2+4\cos\left(2x\right)\sin\left(4x\right)+\sin\left(4x\right)^2-4\cos\left(4x\right)\sin\left(2x\right)+4\sin\left(2x\right)^2\right)^{\frac{1}{4}}\left(\cos\left(\frac{1}{2}\arctan\left(\cos\left(4x\right)-2\sin\left(2x\right),2\cos\left(2x\right)+\sin\left(4x\right)\right)\right)\sin\left(2x\right)+\cos\left(2x\right)\sin\left(\frac{1}{2}\arctan\left(\cos\left(4x\right)-2\sin\left(2x\right),2\cos\left(2x\right)+\sin\left(4x\right)\right)\right)\right)\right)+\frac{1}{16}\sqrt{2}\left(2\sqrt{2}\arctan\left(\sin\left(2x\right)+1,\cos\left(2x\right)\right)-\sqrt{2}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\sin\left(2x\right)+1\right)-4\left(\cos\left(4x\right)^2+4\cos\left(2x\right)^2+4\cos\left(2x\right)\sin\left(4x\right)+\sin\left(4x\right)^2-4\cos\left(4x\right)\sin\left(2x\right)+4\sin\left(2x\right)^2\right)^{\frac{1}{4}}\left(\cos\left(2x\right)\cos\left(\frac{1}{2}\arctan\left(\cos\left(4x\right)-2\sin\left(2x\right),2\cos\left(2x\right)+\sin\left(4x\right)\right)\right)-\sin\left(2x\right)\sin\left(\frac{1}{2}\arctan\left(\cos\left(4x\right)-2\sin\left(2x\right),2\cos\left(2x\right)+\sin\left(4x\right)\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) + sqrt(2)*log(cos(2
*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) + 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos
(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(co
s(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))*sin(2*x) + cos
(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))))) + 1/
16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) - sqrt(2)*log(cos(2*x
)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*cos(2
*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*(cos(
2*x)*cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))) - sin(2
*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))))
```

Fricas [A]
time = 2.65, size = 3, normalized size = 0.16

$$-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(x\right)+\cos\left(x\right)}{\sqrt{\sin\left(2x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))**(1/2),x)
[Out] Integral((sin(x) + cos(x))/sqrt(sin(2*x) + 1), x)
```

Giac [A]
time = 0.43, size = 11, normalized size = 0.58

$$\frac{x}{\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="giac")
[Out] x/sgn(cos(-1/4*pi + x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cos\left(x\right)+\sin\left(x\right)}{\sqrt{\sin\left(2\,x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2),x)
[Out] int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2), x)
```

3.861 $\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=13

$$2\sqrt{\sec(x)(1+\sin(x))}$$

```
[Out] 2*(sec(x)*(1+sin(x)))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4482, 4485, 2784, 2750}

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

```
[Out] 2*Sqrt[Sec[x]*(1 + Sin[x])]
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2784

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]
*(g*Sec[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Cos[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 4485

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rubi steps

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = \int \sec(x)\sqrt{\sec(x)(1+\sin(x))}\,dx$$
$$= \frac{\sqrt{\sec(x)(1+\sin(x))}\int \sec^{\frac{3}{2}}(x)\sqrt{1+\sin(x)}\,dx}{\sqrt{\sec(x)}\sqrt{1+\sin(x)}}$$
$$= \frac{\left(\sqrt{\cos(x)}\sqrt{\sec(x)(1+\sin(x))}\right)\int \frac{\sqrt{1+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{1+\sin(x)}}$$
$$= 2\sqrt{\sec(x)(1+\sin(x))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.
time = 0.03, size = 37, normalized size = 2.85

$$2\sqrt{\frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
[Out] 2*Sqrt[(Cos[x/2] + Sin[x/2])/(Cos[x/2] - Sin[x/2])]
```

Maple [A]
time = 0.18, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$2\sqrt{\sec(x)+\tan(x)}$	10
default	$2\sqrt{\sec(x)+\tan(x)}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(sec(x)+tan(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(sec(x)+tan(x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sec(x) + tan(x))*sec(x), x)
```

Fricas [A]
time = 1.80, size = 21, normalized size = 1.62

$$2\sqrt{\frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt((cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\tan(x)+\sec(x)}\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))**(1/2),x)
[Out] Integral(sqrt(tan(x) + sec(x))*sec(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(11) = 22.
time = 0.53, size = 55, normalized size = 4.23

$$-\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}x\right)^3-\tan\left(\frac{1}{2}x\right)^2-\tan\left(\frac{1}{2}x\right)-1\right)\mathrm{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^2+1}-1}{\tan\left(\frac{1}{2}x\right)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="giac")
[Out] -4*sgn(-tan(1/2*x)^3 - tan(1/2*x)^2 - tan(1/2*x) - 1)*sgn(cos(x))/((sqrt(-t
an(1/2*x)^2 + 1) - 1)/tan(1/2*x) + 1)
```

Mupad [B]
time = 0.29, size = 14, normalized size = 1.08

$$2\sqrt{\frac{1}{\cos(x)}}\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x) + 1/cos(x))^(1/2)/cos(x),x)
[Out] 2*(1/cos(x))^(1/2)*(sin(x) + 1)^(1/2)
```

3.862 $\int \sec(x)\sqrt{4+3\sec(x)}\,\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{2}{9}(4+3\sec(x))^{3/2}$$

[Out] 2/9*(4+3*sec(x))^(3/2)

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4424, 267}

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]

[Out] (2*(4 + 3*Sec[x])^(3/2))/9

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \sec(x)\sqrt{4+3\sec(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{4+\frac{3}{x}}}{x^2}\,dx, x, \cos(x)\right)\\ &= \frac{2}{9}(4+3\sec(x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 14, normalized size = 1.00

$$\frac{2}{9}(4+3\sec(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

```
[Out] (2*(4 + 3*Sec[x])^(3/2))/9
```

Maple [A]
time = 0.05, size = 11, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2(4+3\sec(x))^{\frac{3}{2}}}{9}$	11
default	$\frac{2(4+3\sec(x))^{\frac{3}{2}}}{9}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(4+3*sec(x))^(3/2)
```

Maxima [A]
time = 0.32, size = 10, normalized size = 0.71

$$\frac{2}{9}\left(3\sec\left(x\right)+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*sec(x) + 4)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 1.68, size = 25, normalized size = 1.79

$$\frac{2\sqrt{\frac{4\cos\left(x\right)+3}{\cos\left(x\right)}}\left(4\cos\left(x\right)+3\right)}{9\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt((4*cos(x) + 3)/cos(x))*(4*cos(x) + 3)/cos(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.18, size = 29, normalized size = 2.07

$$\frac{2\sqrt{3\sec{(x)}+4}\sec{(x)}}{3}+\frac{8\sqrt{3\sec{(x)}+4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))**(1/2)*tan(x),x)
```

```
[Out] 2*sqrt(3*sec(x) + 4)*sec(x)/3 + 8*sqrt(3*sec(x) + 4)/9
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(10) = 20$.
time = 0.40, size = 68, normalized size = 4.86

$$\frac{2\left(4\left(\sqrt{4\cos{(x)}^{2}+3\cos{(x)}}-2\cos{(x)}\right)^{2}-6\sqrt{4\cos{(x)}^{2}+3\cos{(x)}}+12\cos{(x)}+3\right)\operatorname{sgn}(\cos{(x)})}{\left(\sqrt{4\cos{(x)}^{2}+3\cos{(x)}}-2\cos{(x)}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] 2*(4*(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^2 - 6*sqrt(4*cos(x)^2 + 3*cos
(x)) + 12*cos(x) + 3)*sgn(cos(x))/(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^
3
```

Mupad [B]
time = 3.23, size = 29, normalized size = 2.07

$$\frac{8\sqrt{\frac{3}{\cos{(x)}}+4}}{9}+\frac{2\sqrt{\frac{3}{\cos{(x)}}+4}}{3\cos{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(3/cos(x) + 4)^(1/2))/cos(x),x)
```

```
[Out] (8*(3/cos(x) + 4)^(1/2))/9 + (2*(3/cos(x) + 4)^(1/2))/(3*cos(x))
```

3.863 $\int \sec(x)\sqrt{1+\sec(x)}\,\tan^3(x)\,dx$

Optimal. Leaf size=25

$$-\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}$$

```
[Out] -4/5*(1+sec(x))^(5/2)+2/7*(1+sec(x))^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4458, 1584, 1483, 641, 45}

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

```
[Out] (-4*(1 + Sec[x])^(5/2))/5 + (2*(1 + Sec[x])^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
```

```
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4458

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rubi steps

$$\begin{aligned}
\int \sec(x)\sqrt{1+\sec(x)}\,\tan^3(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\,(1-x^2)}{x^4}\,dx, x, \cos(x)\right)\\
&= -\text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \cos(x)\right)\\
&= \text{Subst}\left(\int \sqrt{1+x}\,\left(-1+x^2\right)\,dx, x, \sec(x)\right)\\
&= \text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \sec(x)\right)\\
&= \text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \sec(x)\right)\\
&= -\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 30, normalized size = 1.20

$$-\frac{8}{35}\cos^4\left(\frac{x}{2}\right)(-5+9\cos(x))\sec^3(x)\sqrt{1+\sec(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
[Out] (-8*Cos[x/2]^4*(-5 + 9*Cos[x])*Sec[x]^3*Sqrt[1 + Sec[x]])/35
```

Maple [A]
time = 0.12, size = 34, normalized size = 1.36

method	result	size
default	$-\frac{2(9\cos(x)-5)\sqrt{\frac{1+\cos(x)}{\cos(x)}}\,(\sin^4(x))}{35(\cos(x)-1)^2\cos(x)^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(9*cos(x)-5)*((1+cos(x))/cos(x))^(1/2)*sin(x)^4/(cos(x)-1)^2/cos(x)^3
```

Maxima [A]
time = 0.33, size = 21, normalized size = 0.84

$$\frac{2}{7}\left(\frac{1}{\cos(x)}+1\right)^{\frac{7}{2}}-\frac{4}{5}\left(\frac{1}{\cos(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="maxima")
```

```
[Out] 2/7*(1/cos(x) + 1)^(7/2) - 4/5*(1/cos(x) + 1)^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.
time = 3.44, size = 35, normalized size = 1.40

$$-\frac{2\left(9\cos(x)^3+13\cos(x)^2-\cos(x)-5\right)\sqrt{\frac{\cos(x)+1}{\cos(x)}}}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -2/35*(9*cos(x)^3 + 13*cos(x)^2 - cos(x) - 5)*sqrt((cos(x) + 1)/cos(x))/cos
(x)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec(x)+1}\,\tan^3(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))**(1/2)*tan(x)**3,x)
```

```
[Out] Integral(sqrt(sec(x) + 1)*tan(x)**3*sec(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(17) = 34$.
time = 0.45, size = 128, normalized size = 5.12

$$-\frac{2\left(35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^6-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^5-35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^4+105\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^3-91\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^2+35\sqrt{\cos(x)^2+\cos(x)}-35\cos(x)-5\right)\operatorname{sgn}(\cos(x))}{35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^6 - 35*(sqrt(cos(x)^2 + cos(x)
) - cos(x))^5 - 35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^4 + 105*(sqrt(cos(x)^
2 + cos(x)) - cos(x))^3 - 91*(sqrt(cos(x)^2 + cos(x)) - cos(x))^2 + 35*sqrt
(cos(x)^2 + cos(x)) - 35*cos(x) - 5)*sgn(cos(x))/(sqrt(cos(x)^2 + cos(x)) -
 cos(x))^7
```

Mupad [B]
time = 3.33, size = 24, normalized size = 0.96

$$-\frac{2\left(\cos(x)+1\right)^{5/2}\sqrt{\frac{1}{\cos(x)}}\left(9\cos(x)-5\right)}{35\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^3*(1/cos(x) + 1)^(1/2))/cos(x),x)
```

```
[Out] -(2*(cos(x) + 1)^(5/2)*(1/cos(x))^(1/2)*(9*cos(x) - 5))/(35*cos(x)^3)
```

3.864 $\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx$

Optimal. Leaf size=25

$$\frac{4}{5}(1+\csc(x))^{5/2}-\frac{2}{7}(1+\csc(x))^{7/2}$$

```
[Out] 4/5*(1+csc(x))^(5/2)-2/7*(1+csc(x))^(7/2)
```

Rubi [A]
time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4457, 1584, 1483, 641, 45}

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
[Out] (4*(1 + Csc[x])^(5/2))/5 - (2*(1 + Csc[x])^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
```

```
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 4457

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c*d^(n - 1)), Subst[Int[SubstFor[(
1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}\,(1-x^2)}{x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{\left(-1+\frac{1}{x^2}\right)\sqrt{1+\frac{1}{x}}}{x^2}\,dx, x, \sin(x)\right)\\
&= -\text{Subst}\left(\int \sqrt{1+x}\,\left(-1+x^2\right)\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int (-1+x)(1+x)^{3/2}\,dx, x, \csc(x)\right)\\
&= -\text{Subst}\left(\int \left(-2(1+x)^{3/2}+(1+x)^{5/2}\right)\,dx, x, \csc(x)\right)\\
&= \frac{4}{5}(1+\csc(x))^{5/2}-\frac{2}{7}(1+\csc(x))^{7/2}
\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 18, normalized size = 0.72

$$-\frac{2}{35}(1+\csc(x))^{5/2}(-9+5\csc(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

```
[Out] (-2*(1 + Csc[x])^(5/2)*(-9 + 5*Csc[x]))/35
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.16, size = 38, normalized size = 1.52

method	result	size
default	$-\frac{2\left(9\sin(x)\left(\cos^2(x)\right)+13\left(\cos^2(x)\right)-8\sin(x)-8\right)\sqrt{\frac{1+\sin(x)}{\sin(x)}}}{35\sin(x)^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(9*sin(x)*cos(x)^2+13*cos(x)^2-8*sin(x)-8)*((1+sin(x))/sin(x))^(1/2)/
sin(x)^3
```

Maxima [A]
time = 0.30, size = 21, normalized size = 0.84

$$-\frac{2}{7}\left(\frac{1}{\sin(x)}+1\right)^{\frac{7}{2}}+\frac{4}{5}\left(\frac{1}{\sin(x)}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*(1/sin(x) + 1)^(7/2) + 4/5*(1/sin(x) + 1)^(5/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.
time = 3.44, size = 44, normalized size = 1.76

$$\frac{2\left(13\cos(x)^2+\left(9\cos(x)^2-8\right)\sin(x)-8\right)\sqrt{\frac{\sin(x)+1}{\sin(x)}}}{35\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(13*cos(x)^2 + (9*cos(x)^2 - 8)*sin(x) - 8)*sqrt((sin(x) + 1)/sin(x))/
((cos(x)^2 - 1)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\csc(x)+1}\cot^3(x)\csc(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x)*(1+csc(x))**(1/2),x)
```

[Out] Integral(sqrt(csc(x) + 1)*cot(x)**3*csc(x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(17) = 34$.

time = 0.41, size = 128, normalized size = 5.12

$$\frac{2\left(35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^6-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^5-35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^4+105\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^3-91\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^2+35\sqrt{\sin(x)^2+\sin(x)}-35\sin(x)-5\right)\mathrm{sgn}(\sin(x))}{35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="giac")

[Out] 2/35*(35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^6 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^5 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^4 + 105*(sqrt(sin(x)^2 + sin(x)) - sin(x))^3 - 91*(sqrt(sin(x)^2 + sin(x)) - sin(x))^2 + 35*sqrt(sin(x)^2 + sin(x)) - 35*sin(x) - 5)*sgn(sin(x))/(sqrt(sin(x)^2 + sin(x)) - sin(x))^7

Mupad [B]

time = 3.42, size = 24, normalized size = 0.96

$$\frac{2\,{\left(\sin(x)+1\right)}^{5/2}\,\sqrt{\frac{1}{\sin(x)}}\,\left(9\,\sin(x)-5\right)}{35\,{\sin(x)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cot(x)^3*(1/sin(x) + 1)^(1/2))/sin(x),x)

[Out] (2*(sin(x) + 1)^(5/2)*(1/sin(x))^(1/2)*(9*sin(x) - 5))/(35*sin(x)^3)

3.865 $\int \sqrt{\csc(x)}\,(x\cos(x)-4\sec(x)\tan(x))\,dx$

Optimal. Leaf size=20

$$\frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

```
[Out] -4*sec(x)/csc(x)^(3/2)+2*x/csc(x)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {6874, 4298, 3856, 2719, 2706}

$$\frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

```
[Out] (2*x)/Sqrt[Csc[x]] - (4*Sec[x])/Csc[x]^(3/2)
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4298

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p
- 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[
```

```
p, 1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(x)}\,(x\cos(x)-4\sec(x)\tan(x))\,dx &= \int \left(x\cos(x)\sqrt{\csc(x)}-\frac{4\sec^2(x)}{\sqrt{\csc(x)}}\right)dx\\ &= -\left(4\int \frac{\sec^2(x)}{\sqrt{\csc(x)}}\,dx\right)+\int x\cos(x)\sqrt{\csc(x)}\,dx\\ &= \frac{2x}{\sqrt{\csc(x)}}-\frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 17, normalized size = 0.85

$$\frac{2(x\csc(x)-2\sec(x))}{\csc^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
[Out] (2*(x*Csc[x] - 2*Sec[x]))/Csc[x]^(3/2)
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\csc}\,(x)\right)(x\cos(x)-4\sec(x)\tan(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
[Out] int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="maxima")
[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (x\cos(x) - 4\tan(x)\sec(x))\sqrt{\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
[Out] Integral((x*cos(x) - 4*tan(x)*sec(x))*sqrt(csc(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="giac")
[Out] integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

Mupad [B]
time = 3.46, size = 77, normalized size = 3.85

$$\frac{\left(4\cos(x)^3 - 4\,\cos(x) + 2\,x\cos(x)^2\,\sin(x) - \sin(x)\,4\mathrm{i} - x\cos(x)^3\,2\mathrm{i} + \cos(x)^2\,\sin(x)\,4\mathrm{i} + x\,\cos(x)\,2\mathrm{i}\right)\,1\mathrm{i}}{\cos(x)\,\sin(x)\,\sqrt{\frac{1}{\sin(x)}}\,\left(-\sin(x) + \cos(x)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/sin(x))^(1/2)*((4*tan(x))/cos(x) - x*cos(x)),x)
[Out] ((4*cos(x)^3 - sin(x)*4i - x*cos(x)^3*2i - 4*cos(x) + cos(x)^2*sin(x)*4i +
x*cos(x)*2i + 2*x*cos(x)^2*sin(x))*1i)/(cos(x)*sin(x)*(1/sin(x))^(1/2)*(cos
(x)*1i - sin(x)))
```

3.866 $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=76

$$-\frac{35}{16}\sqrt{\cot^2(x)}+\frac{35}{48}\cos^2(x)\sqrt{\cot^2(x)}+\frac{7}{24}\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{6}\cos^6(x)\sqrt{\cot^2(x)}-\frac{35}{16}x\sqrt{\cot^2(x)}\,\tan(x)$$

```
[Out] -35/16*(cot(x)^2)^(1/2)+35/48*cos(x)^2*(cot(x)^2)^(1/2)+7/24*cos(x)^4*(cot(
x)^2)^(1/2)+1/6*cos(x)^6*(cot(x)^2)^(1/2)-35/16*x*(cot(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3254, 4445, 25, 272, 43, 52, 65, 209}

$$\frac{35}{16}\text{ArcTan}\left(\sqrt{\csc^2(x)-1}\right)-\frac{35}{16}\sqrt{\csc^2(x)-1}+\frac{1}{6}\sin^6(x)\left(\csc^2(x)-1\right)^{7/2}+\frac{7}{24}\sin^4(x)\left(\csc^2(x)-1\right)^{5/2}+\frac{35}{48}\sin^2(x)\left(\csc^2(x)-1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (35*ArcTan[Sqrt[-1 + Csc[x]^2]])/16 - (35*Sqrt[-1 + Csc[x]^2])/16 + (35*(-1
 + Csc[x]^2)^(3/2)*Sin[x]^2)/48 + (7*(-1 + Csc[x]^2)^(5/2)*Sin[x]^4)/24 + (
(-1 + Csc[x]^2)^(7/2)*Sin[x]^6)/6
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4445

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \int \cos^6(x)\cot(x)\sqrt{-1+\csc^2(x)}\,dx$$

$$= \text{Subst}\left(\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(1-x^2)^3}{x}\,dx, x, \sin(x)\right)$$

$$= \text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx, x, \sin(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \csc^2(x)\right)\right)$$

$$= \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{7}{12}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \csc^2(x\right.$$

$$= \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\sin^6(x) - \frac{35}{48}\text{Subst}\left(\int \frac{(-\right.$$

$$= \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x) + \frac{1}{6}\cot^2(x)^{7/2}\text{si}$$

$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x$$

$$= -\frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2(x) + \frac{7}{24}\cot^2(x)^{5/2}\sin^4(x$$

$$= \frac{35}{16}\tan^{-1}\left(\sqrt{\cot^2(x)}\right) - \frac{35}{16}\sqrt{\cot^2(x)} + \frac{35}{48}\cot^2(x)^{3/2}\sin^2($$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.53

$$\frac{1}{384}\sqrt{\cot^2(x)}\,\sec(x)(-525\cos(x)+126\cos(3x)+14\cos(5x)+\cos(7x)-840x\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*Sec[x]*(-525*Cos[x] + 126*Cos[3*x] + 14*Cos[5*x] + Cos[7*x]
 - 840*x*Sin[x]))/384
```

Maple [A]
time = 0.28, size = 54, normalized size = 0.71

method	result
default	$-\frac{\left(-8\left(\cos^{7}(x)\right)-14\left(\cos^{5}(x)\right)-35\left(\cos^{3}(x)\right)+105x\sin(x)+105\cos(x)\right)\sqrt{-\frac{\cos^{2}(x)}{\cos^{2}(x)-1}}\,\sqrt{4}}{96\cos(x)}$
risch	$\frac{35i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,\left(e^{2ix}-1\right)x}{16\left(e^{2ix}+1\right)}+\frac{\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{8ix}}{384\,e^{2ix}+384}-\frac{47\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{2ix}}{128\left(e^{2ix}+1\right)}-\frac{47\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,\left(1-e^{-2ix}\right)}{128\left(e^{2ix}+1\right)}-\frac{5\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(-8*cos(x)^7-14*cos(x)^5-35*cos(x)^3+105*x*sin(x)+105*cos(x))*(-cos(x
)^2/(cos(x)^2-1))^(1/2)/cos(x)*4^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.
time = 0.51, size = 136, normalized size = 1.79

$$-\frac{3}{2}\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}\,\sin\left(x\right)^{2}-\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}+\frac{3\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}}{48\left(\left(\frac{1}{\sin(x)^{2}}-1\right)^{3}+3\left(\frac{1}{\sin(x)^{2}}-1\right)^{2}+\frac{3}{\sin(x)^{2}}-2\right)}-\frac{3\left(\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}\right)}{8\left(\left(\frac{1}{\sin(x)^{2}}-1\right)^{2}+\frac{2}{\sin(x)^{2}}-1\right)}+\frac{35}{16}\arctan\left(\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(1/sin(x)^2 - 1)*sin(x)^2 - sqrt(1/sin(x)^2 - 1) + 1/48*(3*(1/sin(
x)^2 - 1)^(5/2) + 8*(1/sin(x)^2 - 1)^(3/2) - 3*sqrt(1/sin(x)^2 - 1))/((1/si
n(x)^2 - 1)^3 + 3*(1/sin(x)^2 - 1)^2 + 3/sin(x)^2 - 2) - 3/8*((1/sin(x)^2 -
 1)^(3/2) - sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^2 + 2/sin(x)^2 - 1) + 3
5/16*arctan(sqrt(1/sin(x)^2 - 1))
```

Fricas [A]
time = 2.41, size = 34, normalized size = 0.45

$$-\frac{8\,\cos\left(x\right)^{7}+14\,\cos\left(x\right)^{5}+35\,\cos\left(x\right)^{3}-105\,x\sin\left(x\right)-105\,\cos\left(x\right)}{48\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^7 + 14*cos(x)^5 + 35*cos(x)^3 - 105*x*sin(x) - 105*cos(x))/
sin(x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [A]
time = 0.44, size = 97, normalized size = 1.28

$$-\frac{1}{48}\left(\left(2\left(4\sin\left(x\right)^{2}-19\right)\sin\left(x\right)^{2}+87\right)\sqrt{-\sin\left(x\right)^{2}+1}\,\sin\left(x\right)-105\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x\right)(-1)^{\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor}+\frac{24\left(\sqrt{-\sin\left(x\right)^{2}+1}-1\right)}{\sin\left(x\right)}-\frac{24\sin\left(x\right)}{\sqrt{-\sin\left(x\right)^{2}+1}-1}\right)\mathrm{sgn}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*((2*(4*sin(x)^2 - 19)*sin(x)^2 + 87)*sqrt(-sin(x)^2 + 1)*sin(x) - 105
*(pi*floor(x/pi + 1/2) - x)*(-1)^floor(x/pi + 1/2) + 24*(sqrt(-sin(x)^2 + 1
) - 1)/sin(x) - 24*sin(x)/(sqrt(-sin(x)^2 + 1) - 1))*sgn(sin(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\cot(x)\,\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}\,\left(\sin\left(x\right)^{2}-1\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
```

```
[Out] int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

3.867 $\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=81

$$\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{7}\cos^6(x)\sqrt{\cot^2(x)}\,\sin(x)-\tanh$$

```
[Out] sin(x)*(cot(x)^2)^(1/2)+1/3*cos(x)^2*sin(x)*(cot(x)^2)^(1/2)+1/5*cos(x)^4*s
in(x)*(cot(x)^2)^(1/2)+1/7*cos(x)^6*sin(x)*(cot(x)^2)^(1/2)-arctanh(cos(x))
*(cot(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3254, 4206, 3739, 2672, 308, 212}

$$\sin(x)\sqrt{\cot^2(x)}+\frac{1}{7}\sin(x)\cos^6(x)\sqrt{\cot^2(x)}+\frac{1}{5}\sin(x)\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{3}\sin(x)\cos^2(x)\sqrt{\cot^2(x)}-\tan(x)\sqrt{\cot^2(x)}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] Sqrt[Cot[x]^2]*Sin[x] + (Cos[x]^2*Sqrt[Cot[x]^2]*Sin[x])/3 + (Cos[x]^4*Sqrt
[Cot[x]^2]*Sin[x])/5 + (Cos[x]^6*Sqrt[Cot[x]^2]*Sin[x])/7 - ArcTanh[Cos[x]]
*Sqrt[Cot[x]^2]*Tan[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 4206

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\sqrt{-1+\csc^2(x)}\,\left(1-\sin^2(x)\right)^3\,dx &= \int \cos^7(x)\sqrt{-1+\csc^2(x)}\,dx\\
&= \int \cos^7(x)\sqrt{\cot^2(x)}\,dx\\
&= \left(\sqrt{\cot^2(x)}\,\tan(x)\right)\int \cos^7(x)\cot(x)\,dx\\
&= -\left(\left(\sqrt{\cot^2(x)}\,\tan(x)\right)\text{Subst}\left(\int \frac{x^8}{1-x^2}\,dx,x,\cos(x)\right)\right)\\
&= -\left(\left(\sqrt{\cot^2(x)}\,\tan(x)\right)\text{Subst}\left(\int \left(-1-x^2-x^4-x^6+\frac{}{1}\right.\right.\right.\\
&= \sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{}\\
&= \sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\,\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.68

$$
\frac{\sqrt{\cot^2(x)}\,\left(9765\cos(x)+1295\cos(3x)+189\cos(5x)+15\cos(7x)-6720\log\left(\cos\left(\frac{x}{2}\right)\right)+6720\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\tan(x)}{6720}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

```
[Out] (Sqrt[Cot[x]^2]*(9765*Cos[x] + 1295*Cos[3*x] + 189*Cos[5*x] + 15*Cos[7*x] -
 6720*Log[Cos[x/2]] + 6720*Log[Sin[x/2]])*Tan[x])/6720
```

Maple [A]
time = 0.24, size = 65, normalized size = 0.80

method	result
default	$\frac{\left(15\left(\cos^{7}(x)\right)+21\left(\cos^{5}(x)\right)+35\left(\cos^{3}(x)\right)+105\cos(x)+105\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)+176\right)\sin(x)\sqrt{-\frac{\cos^{2}(x)}{\cos^{2}(x)-1}}\,\sqrt{4}}{210\cos(x)}$
risch	$-\frac{i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{9ix}}{896\left(e^{2ix}+1\right)}+\frac{121i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{-ix}}{192\left(e^{2ix}+1\right)}-\frac{i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,\ln\left(e^{ix}-1\right)e^{2ix}}{e^{2ix}+1}+\frac{i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,\ln\left(e^{ix}-1\right)}{e^{2ix}+1}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/210*(15*cos(x)^7+21*cos(x)^5+35*cos(x)^3+105*cos(x)+105*ln(-(cos(x)-1)/si
n(x))+176)*sin(x)*(-cos(x)^2/(cos(x)^2-1))^(1/2)/cos(x)*4^(1/2)
```

Maxima [A]
time = 0.30, size = 86, normalized size = 1.06

$$\frac{1}{7}\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{7}{2}}\sin(x)^{7}+\frac{1}{5}\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{5}{2}}\sin(x)^{5}+\frac{1}{3}\left(\frac{1}{\sin(x)^{2}}-1\right)^{\frac{3}{2}}\sin(x)^{3}+\sqrt{\frac{1}{\sin(x)^{2}}-1}\,\sin(x)-\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)^{2}}-1}\,\sin(x)+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)^{2}}-1}\,\sin(x)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(1/sin(x)^2 - 1)^(7/2)*sin(x)^7 + 1/5*(1/sin(x)^2 - 1)^(5/2)*sin(x)^5 +
 1/3*(1/sin(x)^2 - 1)^(3/2)*sin(x)^3 + sqrt(1/sin(x)^2 - 1)*sin(x) - 1/2*lo
g(sqrt(1/sin(x)^2 - 1)*sin(x) + 1) + 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x) -
1)
```

Fricas [A]
time = 2.81, size = 41, normalized size = 0.51

$$-\frac{1}{7}\cos(x)^{7}-\frac{1}{5}\cos(x)^{5}-\frac{1}{3}\cos(x)^{3}-\cos(x)+\frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/7*cos(x)^7 - 1/5*cos(x)^5 - 1/3*cos(x)^3 - cos(x) + 1/2*log(1/2*cos(x) +
 1/2) - 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [**A**]
time = 0.43, size = 44, normalized size = 0.54

$$\frac{1}{210}\left(30\cos\left(x\right)^{7}+42\cos\left(x\right)^{5}+70\cos\left(x\right)^{3}+210\cos\left(x\right)-105\log\left(\cos\left(x\right)+1\right)+105\log\left(-\cos\left(x\right)+1\right)\right)\operatorname{sgn}(\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/210*(30*cos(x)^7 + 42*cos(x)^5 + 70*cos(x)^3 + 210*cos(x) - 105*log(cos(x
) + 1) + 105*log(-cos(x) + 1))*sgn(sin(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int\cos\left(x\right)\,\sqrt{\frac{1}{\sin\left(x\right)^{2}}-1}\,\left(\sin\left(x\right)^{2}-1\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
[Out] -int(cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

3.868 $\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

Optimal. Leaf size=76

$$-\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}+\frac{i\text{PolyLog}\left(2,-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{i\text{PolyLog}\left(2,e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}$$

```
[Out] -2*x*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+I*polylog(2,-exp(I*x))*sec
(x)/(a*sec(x)^2)^(1/2)-I*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6852, 4268, 2317, 2438}

$$\frac{i\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{i\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}-\frac{2x\sec(x)\tanh^{-1}\left(e^{ix}\right)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (I*PolyLog[2, -E^(I*x)]*S
ec[x])/Sqrt[a*Sec[x]^2] - (I*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx &= \frac{\sec(x)\int x\csc(x)\,dx}{\sqrt{a\sec^2(x)}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{\sec(x)\int \log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{\sec(x)\int \log\left(1+e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{(i\sec(x))\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{ix}\right)}{\sqrt{a\sec^2(x)}} - \frac{(i\sec(x))\text{Subst}\left(\int\right.}{\sqrt{a\,\text{s}}}\\ &= -\frac{2x\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{i\text{Li}_2\left(-e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{i\text{Li}_2\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 0.91

$$\frac{\left(x\left(\log\left(1-e^{ix}\right)-\log\left(1+e^{ix}\right)\right)+i\text{PolyLog}\left(2,-e^{ix}\right)-i\text{PolyLog}\left(2,e^{ix}\right)\right)\sec(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + I*PolyLog[2, -E^(I*x)] - I*Poly
Log[2, E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.13, size = 98, normalized size = 1.29

method	result	size
risch	$-\frac{2i\left(-\frac{i e^{ix} x \ln\left(e^{ix}+1\right)}{2}-\frac{e^{ix}\,\text{polylog}\left(2,-e^{ix}\right)}{2}+\frac{i e^{ix} x \ln\left(1-e^{ix}\right)}{2}+\frac{e^{ix}\,\text{polylog}\left(2,e^{ix}\right)}{2}\right)}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(-1/2*I*exp(I*x)*
x*ln(exp(I*x)+1)-1/2*exp(I*x)*polylog(2,-exp(I*x))+1/2*I*exp(I*x)*x*ln(1-ex
p(I*x))+1/2*exp(I*x)*polylog(2,exp(I*x)))
```

Maxima [A]
time = 0.52, size = 79, normalized size = 1.04

$$-\frac{2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\cos\left(x\right)+1\right)-x\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\cos\left(x\right)+1\right)-2i\,\mathrm{Li}_{2}\left(-e^{\left(i\,x\right)}\right)+2i\,\mathrm{Li}_{2}\left(e^{\left(i\,x\right)}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) + 1
) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log(cos(x)^2 + sin(x)^2 -
 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.
time = 2.72, size = 124, normalized size = 1.63

$$-\frac{\left(x\cos\left(x\right)\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+x\cos\left(x\right)\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-x\cos\left(x\right)\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-x\cos\left(x\right)\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+i\,\cos\left(x\right)\mathrm{Li}_{2}(\cos\left(x\right)+i\,\sin\left(x\right))-i\,\cos\left(x\right)\mathrm{Li}_{2}(\cos\left(x\right)-i\,\sin\left(x\right))+i\,\cos\left(x\right)\mathrm{Li}_{2}(-\cos\left(x\right)+i\,\sin\left(x\right))-i\,\cos\left(x\right)\mathrm{Li}_{2}(-\cos\left(x\right)-i\,\sin\left(x\right))\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)))*sqrt(a/cos(x)^2)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\csc\left(x\right)\sec\left(x\right)}{\sqrt{a\sec^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

```
[Out] int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.869 $\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=128

$$-\frac{2x^2 \tanh^{-1}\left(e^{ix}\right) \sec(x)}{\sqrt{a \sec^2(x)}}+\frac{2ix\text{PolyLog}(2,-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}-\frac{2ix\text{PolyLog}(2,e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}-\frac{2\text{PolyLog}(3,-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}$$

```
[Out] -2*x^2*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*I*x*polylog(2,-exp(I*x
))*sec(x)/(a*sec(x)^2)^(1/2)-2*I*x*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2)^
(1/2)-2*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*polylog(3,exp(I*x)
)*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6852, 4268, 2611, 2320, 6724}

$$\frac{2ix\text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}-\frac{2ix\text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}-\frac{2\text{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}+\frac{2\text{Li}_3(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}-\frac{2x^2 \sec(x) \tanh^{-1}\left(e^{ix}\right)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x^2*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((2*I)*x*PolyLog[2, -E^
(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((2*I)*x*PolyLog[2, E^(I*x)]*Sec[x])/Sqrt
[a*Sec[x]^2] - (2*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (2*PolyLo
g[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx &= \frac{\sec(x)\int x^2\csc(x)\,dx}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\int x\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{(2\sec(x))\int x\log\left(1+\right.}{\sqrt{a\sec^2(x)}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2i\sec(x))}{\sqrt{a}}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(2\sec(x))\text{S}}{}\\
&= -\frac{2x^2\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{2ix\text{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2ix\text{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{2\text{Li}_3(-e^{ix})}{\sqrt{a\sec^2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 99, normalized size = 0.77

$$\frac{\left(x^2\log\left(1-e^{ix}\right)-x^2\log\left(1+e^{ix}\right)+2ix\text{PolyLog}(2,-e^{ix})-2ix\text{PolyLog}(2,e^{ix})-2\text{PolyLog}(3,-e^{ix})+2\text{PolyLog}(3,e^{ix})\right)\sec(x)}{\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((x^2*Log[1 - E^(I*x)] - x^2*Log[1 + E^(I*x)] + (2*I)*x*PolyLog[2, -E^(I*x)
] - (2*I)*x*PolyLog[2, E^(I*x)] - 2*PolyLog[3, -E^(I*x)] + 2*PolyLog[3, E^(
I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.12, size = 132, normalized size = 1.03

method	result	si
risch	$-\frac{2\left(\frac{e^{ix} x^2 \ln\left(e^{ix}+1\right)}{2}-i e^{ix} x \,\mathrm{polylog}\left(2,-e^{ix}\right)+e^{ix} \,\mathrm{polylog}\left(3,-e^{ix}\right)-\frac{e^{ix} x^2 \ln\left(1-e^{ix}\right)}{2}+i e^{ix} x \,\mathrm{polylog}\left(2,e^{ix}\right)-e^{ix} \,\mathrm{polylog}\left(3,e^{ix}\right)\right)}{\sqrt{\frac{a\, e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(1/2*exp(I*x)*x^2*l
n(exp(I*x)+1)-I*exp(I*x)*x*polylog(2,-exp(I*x))+exp(I*x)*polylog(3,-exp(I*x
))-1/2*exp(I*x)*x^2*ln(1-exp(I*x))+I*exp(I*x)*x*polylog(2,exp(I*x))-exp(I*x
)*polylog(3,exp(I*x)))
```

Maxima [A]
time = 0.53, size = 107, normalized size = 0.84

$$-\frac{2 i x^2 \arctan\left(\sin(x), \cos(x)+1\right)+2 i x^2 \arctan\left(\sin(x), -\cos(x)+1\right)+x^2 \log\left(\cos(x)^2+\sin(x)^2+2 \cos(x)+1\right)-x^2 \log\left(\cos(x)^2+\sin(x)^2-2 \cos(x)+1\right)-4 i x \mathrm{Li}_2\left(-e^{(ix)}\right)+4 i x \mathrm{Li}_2\left(e^{(ix)}\right)+4 \mathrm{Li}_3\left(-e^{(ix)}\right)-4 \mathrm{Li}_3\left(e^{(ix)}\right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^2*arctan2(sin(x), cos(x) + 1) + 2*I*x^2*arctan2(sin(x), -cos(x)
 + 1) + x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^2*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*I*x*dilog(-e^(I*x)) + 4*I*x*dilog(e^(I*x)) + 4*p
olylog(3, -e^(I*x)) - 4*polylog(3, e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(97) = 194.
time = 2.38, size = 227, normalized size = 1.77

$$\frac{2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \,\mathrm{polylog}(3, \cos(x)+i \sin(x))+2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \,\mathrm{polylog}(3, \cos(x)-i \sin(x))-2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \,\mathrm{polylog}(3,-\cos(x)+i \sin(x))-2 \sqrt{\frac{a}{\cos(x)^2}} \cos(x) \,\mathrm{polylog}(3,-\cos(x)-i \sin(x))-\left(x^2 \cos(x) \log\left(\cos(x)+i \sin(x)+1\right)+x^2 \cos(x) \log\left(\cos(x)-i \sin(x)+1\right)-x^2 \cos(x) \log\left(-\cos(x)+i \sin(x)+1\right)-x^2 \cos(x) \log\left(-\cos(x)-i \sin(x)+1\right)+2 i x \cos(x) \mathrm{Li}_2(\cos(x)+i \sin(x))-2 i x \cos(x) \mathrm{Li}_2(\cos(x)-i \sin(x))+2 i x \cos(x) \mathrm{Li}_2(-\cos(x)+i \sin(x))-2 i x \cos(x) \mathrm{Li}_2(-\cos(x)-i \sin(x))\right) \sqrt{\frac{a}{\cos(x)^2}}}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/cos
(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*pol
ylog(3, -cos(x) + I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x)
```

```
- I*sin(x)) - (x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x
) - I*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log
(-cos(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*co
s(x)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*
I*x*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{{\cos(x)}^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.870 $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=186

$$-\frac{2x^3 \tanh^{-1}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \text{PolyLog}(2, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{PolyLog}(2, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{PolyLog}(3, -e^{ix}) \sec}{\sqrt{a \sec^2(x)}}$$

```
[Out] -2*x^3*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+3*I*x^2*polylog(2,-exp(I
*x))*sec(x)/(a*sec(x)^2)^(1/2)-3*I*x^2*polylog(2,exp(I*x))*sec(x)/(a*sec(x)
^2)^(1/2)-6*x*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+6*x*polylog(3,
exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)-6*I*polylog(4,-exp(I*x))*sec(x)/(a*sec(
x)^2)^(1/2)+6*I*polylog(4,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6852, 4268, 2611, 6744, 2320, 6724}

$$\frac{3ix^2 \text{Li}_2(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{3ix^2 \text{Li}_2(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6x \text{Li}_3(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6x \text{Li}_3(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{6i \text{Li}_4(-e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6i \text{Li}_4(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} - \frac{2x^3 \sec(x) \tanh^{-1}(e^{ix})}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] (-2*x^3*ArcTanh[E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + ((3*I)*x^2*PolyLog[2, -
E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((3*I)*x^2*PolyLog[2, E^(I*x)]*Sec[x])/
Sqrt[a*Sec[x]^2] - (6*x*PolyLog[3, -E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] + (6*
x*PolyLog[3, E^(I*x)]*Sec[x])/Sqrt[a*Sec[x]^2] - ((6*I)*PolyLog[4, -E^(I*x)
]*Sec[x])/Sqrt[a*Sec[x]^2] + ((6*I)*PolyLog[4, E^(I*x)]*Sec[x])/Sqrt[a*Sec[
x]^2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \frac{\sec(x)\int x^3 \csc(x)\,dx}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(3\sec(x))\int x^2\log\left(1-e^{ix}\right)\,dx}{\sqrt{a\sec^2(x)}} + \frac{(3\sec(x))\int x^2\log\left(1+\right.}{\sqrt{a\sec^2(x)}}$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\mathrm{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\mathrm{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{(6i\sec(x))}{\sqrt{}}$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\mathrm{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\mathrm{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\mathrm{Li}_3(-e^{i}}{\sqrt{a\sec}}$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\mathrm{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\mathrm{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\mathrm{Li}_3(-e^{i}}{\sqrt{a\sec}}$$

$$= -\frac{2x^3\tanh^{-1}\left(e^{ix}\right)\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{3ix^2\mathrm{Li}_2(-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{3ix^2\mathrm{Li}_2(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{6x\mathrm{Li}_3(-e^{i}}{\sqrt{a\sec}}$$

Mathematica [A]
time = 0.06, size = 147, normalized size = 0.79

$$-\frac{i\left(\pi^4-2x^4+8ix^3\log\left(1-e^{-ix}\right)-8ix^3\log\left(1+e^{ix}\right)-24x^2\mathrm{PolyLog}(2,e^{-ix})-24x^2\mathrm{PolyLog}(2,-e^{ix})+48ix\mathrm{PolyLog}(3,e^{-ix})-48ix\mathrm{PolyLog}(3,-e^{ix})+48\mathrm{PolyLog}(4,e^{-ix})+48\mathrm{PolyLog}(4,-e^{ix})\right)\sec(x)}{8\sqrt{a\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((-1/8*I)*(Pi^4 - 2*x^4 + (8*I)*x^3*Log[1 - E^((-I)*x)] - (8*I)*x^3*Log[1 +
 E^(I*x)] - 24*x^2*PolyLog[2, E^((-I)*x)] - 24*x^2*PolyLog[2, -E^(I*x)] + (
48*I)*x*PolyLog[3, E^((-I)*x)] - (48*I)*x*PolyLog[3, -E^(I*x)] + 48*PolyLog
[4, E^((-I)*x)] + 48*PolyLog[4, -E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.11, size = 172, normalized size = 0.92

method	result
risch	$\frac{2i\left(\frac{i\,e^{ix}x^3\ln\left(e^{ix}+1\right)}{2}+\frac{3\,e^{ix}x^2\,\mathrm{polylog}\left(2,-e^{ix}\right)}{2}+3i\,e^{ix}x\,\mathrm{polylog}\left(3,-e^{ix}\right)-3\,e^{ix}\,\mathrm{polylog}\left(4,-e^{ix}\right)-\frac{i\,e^{ix}x^3\ln\left(1-e^{ix}\right)}{2}-\frac{3\,e^{ix}x^2\,\mathrm{polylog}\left(2,e^{ix}\right)}{2}\right.}{\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*(1/2*I*exp(I*x)*x^
3*ln(exp(I*x)+1)+3/2*exp(I*x)*x^2*polylog(2,-exp(I*x))+3*I*exp(I*x)*x*polyl
```

```
og(3,-exp(I*x))-3*exp(I*x)*polylog(4,-exp(I*x))-1/2*I*exp(I*x)*x^3*ln(1-exp
(I*x))-3/2*exp(I*x)*x^2*polylog(2,exp(I*x))-3*I*exp(I*x)*x*polylog(3,exp(I*
x))+3*exp(I*x)*polylog(4,exp(I*x)))
```

Maxima [A]
time = 0.56, size = 131, normalized size = 0.70

$$-\frac{2i\,x^{3}\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+2i\,x^{3}\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x^{3}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\cos\left(x\right)+1\right)-x^{3}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\cos\left(x\right)+1\right)-6i\,x^{2}\mathrm{Li}_{2}(-e^{(ix)})+6i\,x^{2}\mathrm{Li}_{2}(e^{(ix)})+12\,x\mathrm{Li}_{3}(-e^{(ix)})-12\,x\mathrm{Li}_{3}(e^{(ix)})+12i\,\mathrm{Li}_{4}(-e^{(ix)})-12i\,\mathrm{Li}_{4}(e^{(ix)})}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*x^3*arctan2(sin(x), cos(x) + 1) + 2*I*x^3*arctan2(sin(x), -cos(x)
 + 1) + x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^3*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 6*I*x^2*dilog(-e^(I*x)) + 6*I*x^2*dilog(e^(I*x)) +
 12*x*polylog(3, -e^(I*x)) - 12*x*polylog(3, e^(I*x)) + 12*I*polylog(4, -e^
(I*x)) - 12*I*polylog(4, e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(141) = 282.
time = 3.14, size = 327, normalized size = 1.76

$$\frac{6\,x\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(3,\cos\left(x\right)+i\,\sin\left(x\right)\right)+6\,x\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(3,\cos\left(x\right)-i\,\sin\left(x\right)\right)-6\,x\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(3,-\cos\left(x\right)+i\,\sin\left(x\right)\right)-6\,x\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(3,-\cos\left(x\right)-i\,\sin\left(x\right)\right)+6i\,\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(4,\cos\left(x\right)+i\,\sin\left(x\right)\right)-6i\,\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(4,\cos\left(x\right)-i\,\sin\left(x\right)\right)+6i\,\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(4,-\cos\left(x\right)+i\,\sin\left(x\right)\right)-6i\,\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\cos\left(x\right){\rm polylog}\left(4,-\cos\left(x\right)-i\,\sin\left(x\right)\right)-\left(x^{3}\cos\left(x\right)\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+x^{3}\cos\left(x\right)\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-x^{3}\cos\left(x\right)\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-x^{3}\cos\left(x\right)\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+3i\,x^{2}\cos\left(x\right){\rm Li}_2\left(\cos\left(x\right)+i\,\sin\left(x\right)\right)-3i\,x^{2}\cos\left(x\right){\rm Li}_2\left(\cos\left(x\right)-i\,\sin\left(x\right)\right)+3i\,x^{2}\cos\left(x\right){\rm Li}_2\left(-\cos\left(x\right)+i\,\sin\left(x\right)\right)-3i\,x^{2}\cos\left(x\right){\rm Li}_2\left(-\cos\left(x\right)-i\,\sin\left(x\right)\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt(a
/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(
x)*polylog(3, -cos(x) + I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3,
-cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin
(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 6*I*sqrt
(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^2)*c
os(x)*polylog(4, -cos(x) - I*sin(x)) - (x^3*cos(x)*log(cos(x) + I*sin(x) +
1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) + I*sin
(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*dilog(co
s(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)
*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt
(a/cos(x)^2))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\csc\left(x\right)\sec\left(x\right)}{\sqrt{a\sec^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{{\cos(x)}^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

```
[Out] int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.871 $\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

Optimal. Leaf size=81

$$-\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}-\frac{i\text{PolyLog}\left(2,e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}$$

```
[Out] -1/2*I*x^2*sec(x)^2/(a*sec(x)^4)^(1/2)+x*ln(1-exp(2*I*x))*sec(x)^2/(a*sec(x
)^4)^(1/2)-1/2*I*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6852, 3798, 2221, 2317, 2438}

$$-\frac{i\text{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}-\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/2*I)*x^2*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x*Log[1 - E^((2*I)*x)]*Sec[x]^2
)/Sqrt[a*Sec[x]^4] - ((I/2)*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]
^4]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx &= \frac{\sec^2(x)\int x\cot(x)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} - \frac{(2i\sec^2(x))\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{\sec^2(x)\int \log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{(i\sec^2(x))\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)}{2\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^2\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{x\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{i\mathrm{Li}_2\left(e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.62

$$
-\frac{i\left(x\left(x+2i\log\left(1-e^{2ix}\right)\right)+\mathrm{PolyLog}\left(2,e^{2ix}\right)\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/2*I)*(x*(x + (2*I)*Log[1 - E^((2*I)*x)]) + PolyLog[2, E^((2*I)*x)])*Se
c[x]^2)/Sqrt[a*Sec[x]^4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(65) = 130.
time = 0.13, size = 147, normalized size = 1.81

method	result	size
risch	$\frac{i\,{\mathrm e}^{2 i x} x^{2}}{2 \sqrt{\frac{a\,{\mathrm e}^{4 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{4}}}\, \left({\mathrm e}^{2 i x}+1\right)^{2}}-\frac{2 i \left(\frac{{\mathrm e}^{2 i x} x^{2}}{2}+\frac{i\,{\mathrm e}^{2 i x} x \ln \left({\mathrm e}^{i x}+1\right)}{2}+\frac{{\mathrm e}^{2 i x} \operatorname{polylog}\left(2,-{\mathrm e}^{i x}\right)}{2}+\frac{i\,{\mathrm e}^{2 i x} x \ln \left(1-{\mathrm e}^{i x}\right)}{2}+\frac{{\mathrm e}^{2 i x} \operatorname{polylog}\left(2,{\mathrm e}^{i x}\right)}{2}\right)}{\sqrt{\frac{a\,{\mathrm e}^{4 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{4}}}\, \left({\mathrm e}^{2 i x}+1\right)^{2}}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^2
-2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/2*exp(2*I*x)
*x^2+1/2*I*exp(2*I*x)*x*ln(exp(I*x)+1)+1/2*exp(2*I*x)*polylog(2,-exp(I*x))+
1/2*I*exp(2*I*x)*x*ln(1-exp(I*x))+1/2*exp(2*I*x)*polylog(2,exp(I*x)))
```

Maxima [A]
time = 0.53, size = 83, normalized size = 1.02

$$\frac{-i x^{2} + 2 i x \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) - 2 i x \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} + 2 \cos\left(x\right) + 1\right) + x \log\left(\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2 \cos\left(x\right) + 1\right) - 2 i \operatorname{Li}_2\left(-e^{\left(i x\right)}\right) - 2 i \operatorname{Li}_2\left(e^{\left(i x\right)}\right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.
time = 2.18, size = 138, normalized size = 1.70

$$\frac{{\left(x \cos\left(x\right)^{2} \log\left(\cos\left(x\right) + i \sin\left(x\right) + 1\right) + x \cos\left(x\right)^{2} \log\left(\cos\left(x\right) - i \sin\left(x\right) + 1\right) + x \cos\left(x\right)^{2} \log\left(-\cos\left(x\right) + i \sin\left(x\right) + 1\right) + x \cos\left(x\right)^{2} \log\left(-\cos\left(x\right) - i \sin\left(x\right) + 1\right) - i \cos\left(x\right)^{2} {\rm Li}_2\left(\cos\left(x\right) + i \sin\left(x\right)\right) + i \cos\left(x\right)^{2} {\rm Li}_2\left(\cos\left(x\right) - i \sin\left(x\right)\right) + i \cos\left(x\right)^{2} {\rm Li}_2\left(-\cos\left(x\right) + i \sin\left(x\right)\right) - i \cos\left(x\right)^{2} {\rm Li}_2\left(-\cos\left(x\right) - i \sin\left(x\right)\right)\right)} \sqrt{\frac{a}{\cos\left(x\right)^{4}}}}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x) -
 I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(cos
(x) - I*sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dilog(-
cos(x) - I*sin(x)))*sqrt(a/cos(x)^4)/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \csc\left(x\right) \sec\left(x\right)}{\sqrt{a \sec^{4}\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{{\cos(x)}^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.872 $\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} dx$

Optimal. Leaf size=109

$$-\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}}+\frac{x^2 \log\left(1-e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}-\frac{ix\text{PolyLog}(2,e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}+\frac{\text{PolyLog}(3,e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}$$

```
[Out] -1/3*I*x^3*sec(x)^2/(a*sec(x)^4)^(1/2)+x^2*ln(1-exp(2*I*x))*sec(x)^2/(a*sec
(x)^4)^(1/2)-I*x*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+1/2*poly
log(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6852, 3798, 2221, 2611, 2320, 6724}

$$-\frac{ix\text{Li}_2(e^{2ix}) \sec^2(x)}{\sqrt{a \sec^4(x)}}+\frac{\text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}-\frac{ix^3 \sec^2(x)}{3\sqrt{a \sec^4(x)}}+\frac{x^2 \log\left(1-e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/3*I)*x^3*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^2*Log[1 - E^((2*I)*x)]*Sec[x]
^2)/Sqrt[a*Sec[x]^4] - (I*x*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]
^4] + (PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx &= \frac{\sec^2(x)\int x^2\cot(x)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} - \frac{(2i\sec^2(x))\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{(2\sec^2(x))\int x\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}}\\
&= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2(e^{2ix})\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{(i\sec^2(x))\int \text{Li}_2(\cdots}{\sqrt{a\sec^4(x\cdots}}\\
&= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2(e^{2ix})\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\sec^2(x)\text{Subst}\left(\int \cdots\right.}{2\sqrt{a\,\text{se}\cdots}}\\
&= -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}} + \frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\text{Li}_2(e^{2ix})\sec^2(x)}{\sqrt{a\sec^4(x)}} + \frac{\text{Li}_3(e^{2ix})\sec^2(x)}{2\sqrt{a\sec^4(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.69

$$\frac{\left(-i\pi^3+8ix^3+24x^2\log\left(1-e^{-2ix}\right)+24ix\text{PolyLog}\left(2,e^{-2ix}\right)+12\text{PolyLog}\left(3,e^{-2ix}\right)\right)\sec^2(x)}{24\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (((-I)*Pi^3 + (8*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] + (24*I)*x*PolyLog[2
, E^((-2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)])*Sec[x]^2)/(24*Sqrt[a*Sec[x]^
4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.
time = 0.12, size = 183, normalized size = 1.68

method	result
risch	$\frac{i e^{2ix} x^3}{3\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(e^{2ix}+1)^2} - \frac{2\left(\frac{i e^{2ix} x^3}{3}-\frac{e^{2ix} x^2 \ln\left(e^{ix}+1\right)}{2}+i e^{2ix} x\,\text{polylog}\left(2,-e^{ix}\right)-e^{2ix}\,\text{polylog}\left(3,-e^{ix}\right)-\frac{e^{2ix} x^2 \ln\left(1-e^{ix}\right)}{2}+i e^{2}\right.}{\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(e^{2ix}+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^3
-2/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(1/3*I*exp(2*I*x)
*x^3-1/2*exp(2*I*x)*x^2*ln(exp(I*x)+1)+I*exp(2*I*x)*x*polylog(2,-exp(I*x))-
exp(2*I*x)*polylog(3,-exp(I*x))-1/2*exp(2*I*x)*x^2*ln(1-exp(I*x))+I*exp(2*I
*x)*x*polylog(2,exp(I*x))-exp(2*I*x)*polylog(3,exp(I*x)))
```

Maxima [A]
time = 0.52, size = 113, normalized size = 1.04

$$\frac{-2ix^3+6ix^2\arctan\left(\sin(x),\cos(x)+1\right)-6ix^2\arctan\left(\sin(x),-\cos(x)+1\right)+3x^2\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+3x^2\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)-12ix\text{Li}_2\left(-e^{(ix)}\right)-12ix\text{Li}_2\left(e^{(ix)}\right)+12\text{Li}_3\left(-e^{(ix)}\right)+12\text{Li}_3\left(e^{(ix)}\right)}{6\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-2*I*x^3 + 6*I*x^2*arctan2(sin(x), cos(x) + 1) - 6*I*x^2*arctan2(sin(x
), -cos(x) + 1) + 3*x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3*x^2*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x*dilog(-e^(I*x)) - 12*I*x*dilo
g(e^(I*x)) + 12*polylog(3, -e^(I*x)) + 12*polylog(3, e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(83) = 166$.
time = 2.71, size = 248, normalized size = 2.28

$$\frac{2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\operatorname{polylog}(3,\cos(x)+i\sin(x))+2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\operatorname{polylog}(3,\cos(x)-i\sin(x))+2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\operatorname{polylog}(3,-\cos(x)+i\sin(x))+2\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\operatorname{polylog}(3,-\cos(x)-i\sin(x))+\left(x^2\cos(x)^2\log(\cos(x)+i\sin(x)+1)+x^2\cos(x)^2\log(\cos(x)-i\sin(x)+1)+x^2\cos(x)^2\log(-\cos(x)+i\sin(x)+1)+x^2\cos(x)^2\log(-\cos(x)-i\sin(x)+1)-2i\,x\cos(x)^2\operatorname{Li}_2(\cos(x)+i\sin(x))+2i\,x\cos(x)^2\operatorname{Li}_2(\cos(x)-i\sin(x))+2i\,x\cos(x)^2\operatorname{Li}_2(-\cos(x)+i\sin(x))-2i\,x\cos(x)^2\operatorname{Li}_2(-\cos(x)-i\sin(x))\right)\sqrt{\frac{a}{\cos(x)^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/c
os(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)
^2*polylog(3, -cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3,
-cos(x) - I*sin(x)) + (x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)
^2*log(cos(x) - I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) + 1) +
x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x) + I*
sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)^2*dilog(-c
os(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)
^4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.873 $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$

Optimal. Leaf size=143

$$-\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}}+\frac{x^3 \log \left(1-e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}-\frac{3ix^2 \text{PolyLog}(2,e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}+\frac{3x \text{PolyLog}(3,e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}+\frac{3i\text{Pol}}{}$$

```
[Out] -1/4*I*x^4*sec(x)^2/(a*sec(x)^4)^(1/2)+x^3*ln(1-exp(2*I*x))*sec(x)^2/(a*sec
(x)^4)^(1/2)-3/2*I*x^2*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+3/
2*x*polylog(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+3/4*I*polylog(4,exp(2
*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6852, 3798, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3ix^2 \text{Li}_2(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}+\frac{3x \text{Li}_3(e^{2ix}) \sec^2(x)}{2\sqrt{a \sec^4(x)}}+\frac{3i \text{Li}_4(e^{2ix}) \sec^2(x)}{4\sqrt{a \sec^4(x)}}-\frac{ix^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}}+\frac{x^3 \log \left(1-e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/4*I)*x^4*Sec[x]^2)/Sqrt[a*Sec[x]^4] + (x^3*Log[1 - E^((2*I)*x)]*Sec[x]
^2)/Sqrt[a*Sec[x]^4] - (((3*I)/2)*x^2*PolyLog[2, E^((2*I)*x)]*Sec[x]^2)/Sqr
t[a*Sec[x]^4] + (3*x*PolyLog[3, E^((2*I)*x)]*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4])
 + (((3*I)/4)*PolyLog[4, E^((2*I)*x)]*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx &= \frac{\sec^2(x)\int x^3\cot(x)\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} - \frac{(2i\sec^2(x))\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{(3\sec^2(x))\int x^2\log\left(1-e^{2ix}\right)\,dx}{\sqrt{a\sec^4(x)}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2(e^{2ix})\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{(3i\sec^2(x))\int}{\sqrt{a\,\text{se}}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2(e^{2ix})\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3(e^{2ix})\,\text{se}}{2\sqrt{a\sec^4(}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2(e^{2ix})\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3(e^{2ix})\,\text{se}}{2\sqrt{a\sec^4(}} \\
&= -\frac{ix^4\sec^2(x)}{4\sqrt{a\sec^4(x)}} + \frac{x^3\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{3ix^2\text{Li}_2(e^{2ix})\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{3x\text{Li}_3(e^{2ix})\,\text{se}}{2\sqrt{a\sec^4(}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 0.61

$$
-\frac{i(\pi^4-16x^4+64ix^3\log\left(1-e^{-2ix}\right)-96x^2\text{PolyLog}(2,e^{-2ix})+96ix\text{PolyLog}(3,e^{-2ix})+48\text{PolyLog}(4,e^{-2ix}))\sec^2(x)}{64\sqrt{a\sec^4(x)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((-1/64*I)*(Pi^4 - 16*x^4 + (64*I)*x^3*Log[1 - E^((-2*I)*x)] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

Maple [A]
time = 0.12, size = 221, normalized size = 1.55

method	result
risch	$\frac{i\,\mathrm{e}^{2ix}x^4}{4\sqrt{\frac{a\,\mathrm{e}^{4ix}}{(\mathrm{e}^{2ix}+1)^4}}\,(\mathrm{e}^{2ix}+1)^2}+\frac{2i\left(-\frac{\mathrm{e}^{2ix}x^4}{4}-\frac{i\mathrm{e}^{2ix}x^3\ln\left(\mathrm{e}^{ix}+1\right)}{2}-\frac{3\,\mathrm{e}^{2ix}x^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{ix}\right)}{2}-3i\mathrm{e}^{2ix}x\,\mathrm{polylog}(3,-\mathrm{e}^{ix})+3\,\mathrm{e}^{2ix}\,\mathrm{polylog}(\right.}{\sqrt{\frac{a\,\mathrm{e}^{4ix}}{(\mathrm{e}^{2ix}+1)^{}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^4
+2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*(-1/4*exp(2*I*x
)*x^4-1/2*I*exp(2*I*x)*x^3*ln(exp(I*x)+1)-3/2*exp(2*I*x)*x^2*polylog(2,-exp
(I*x))-3*I*exp(2*I*x)*x*polylog(3,-exp(I*x))+3*exp(2*I*x)*polylog(4,-exp(I*
x))-1/2*I*exp(2*I*x)*x^3*ln(1-exp(I*x))-3/2*exp(2*I*x)*x^2*polylog(2,exp(I*
x))-3*I*exp(2*I*x)*x*polylog(3,exp(I*x))+3*exp(2*I*x)*polylog(4,exp(I*x)))
```

Maxima [A]
time = 0.52, size = 137, normalized size = 0.96

$$\frac{-i\,x^4 + 4i\,x^3 \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) - 4i\,x^3 \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + 2\,x^3 \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right) + 2\,x^3 \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 - 2\,\cos\left(x\right) + 1\right) - 12i\,x^2 \mathrm{Li}_2(-e^{(i\,x)}) - 12i\,x^2 \mathrm{Li}_2(e^{(i\,x)}) + 24\,x \mathrm{Li}_3(-e^{(i\,x)}) + 24\,x \mathrm{Li}_3(e^{(i\,x)}) + 24i\,\mathrm{Li}_4(-e^{(i\,x)}) + 24i\,\mathrm{Li}_4(e^{(i\,x)})}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-I*x^4 + 4*I*x^3*arctan2(sin(x), cos(x) + 1) - 4*I*x^3*arctan2(sin(x),
 -cos(x) + 1) + 2*x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*x^3*log(c
os(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x^2*dilog(-e^(I*x)) - 12*I*x^2*di
log(e^(I*x)) + 24*x*polylog(3, -e^(I*x)) + 24*x*polylog(3, e^(I*x)) + 24*I*
polylog(4, -e^(I*x)) + 24*I*polylog(4, e^(I*x)))/sqrt(a)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(106) = 212.
time = 2.87, size = 356, normalized size = 2.49

$$\frac{6\,x\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(3, \cos\left(x\right) + i\,\sin\left(x\right)) + 6\,x\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(3, \cos\left(x\right) - i\,\sin\left(x\right)) + 6\,x\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(3, -\cos\left(x\right) + i\,\sin\left(x\right)) + 6\,x\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(3, -\cos\left(x\right) - i\,\sin\left(x\right)) + 6i\,\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(4, \cos\left(x\right) + i\,\sin\left(x\right)) - 6i\,\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(4, \cos\left(x\right) - i\,\sin\left(x\right)) - 6i\,\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(4, -\cos\left(x\right) + i\,\sin\left(x\right)) + 6i\,\sqrt{\frac{a}{\cos\left(x\right)^4}}\cos\left(x\right)^2 \mathrm{polylog}(4, -\cos\left(x\right) - i\,\sin\left(x\right)) + \left(x^3\cos\left(x\right)^2 \log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) + x^3\cos\left(x\right)^2 \log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) + x^3\cos\left(x\right)^2 \log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) + x^3\cos\left(x\right)^2 \log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) - 3i\,x^2\cos\left(x\right)^2 \mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) + 3i\,x^2\cos\left(x\right)^2 \mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right)) + 3i\,x^2\cos\left(x\right)^2 \mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right)) - 3i\,x^2\cos\left(x\right)^2 \mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right))\right)\sqrt{\frac{a}{\cos\left(x\right)^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*
cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*cos(x)^2*pol
ylog(3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(
x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) - I*sin(x)
) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) + I*sin(x)) + 6*I*sqrt
(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) - I*sin(x)) + (x^3*cos(x)^2*log(co
s(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)
^2*log(-cos(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) -
 3*I*x^2*cos(x)^2*dilog(cos(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x)
- I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2
*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \csc\left(x\right) \sec\left(x\right)}{\sqrt{a \sec^4\left(x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\cos\left(x\right)\,\sin\left(x\right)\,\sqrt{\frac{a}{{\cos\left(x\right)}^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

```
[Out] int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.874 $\int x \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=105

$$x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} + i\cos(x)\text{PolyLog}(2, -$$

```
[Out] x*(a*sec(x)^2)^(1/2)-2*x*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-arctan
h(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+I*cos(x)*polylog(2,-exp(I*x))*(a*sec(x)
^2)^(1/2)-I*cos(x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6852, 2702, 327, 213, 4505, 6406, 4268, 2317, 2438, 3855}

$$i\text{Li}_2\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - i\text{Li}_2\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + x\sqrt{a\sec^2(x)} - 2x\cos(x)\tanh^{-1}\left(e^{ix}\right)\sqrt{a\sec^2(x)} - \cos(x)\sqrt{a\sec^2(x)}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x*Sqrt[a*Sec[x]^2] - 2*x*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - ArcTanh
[Sin[x]]*Cos[x]*Sqrt[a*Sec[x]^2] + I*Cos[x]*PolyLog[2, -E^(I*x)]*Sqrt[a*Sec
[x]^2] - I*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6406

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[x*(D[u, x]/(1 - u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x\csc(x)\sec^2(x)\,dx$$
$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \left(\cos(x)\sqrt{a\sec^2(x\right.$$
$$= x\sqrt{a\sec^2(x)} - x\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x\right.$$
$$= x\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos(x)\sqrt{a\sec^2(x)} + \left(\cos(x)\sqrt{a\sec^2(x)}\right.$$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos($$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos($$
$$= x\sqrt{a\sec^2(x)} - 2x\tanh^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - \tanh^{-1}(\sin(x))\cos($$

Mathematica [A]
time = 0.05, size = 108, normalized size = 1.03

$$\left(x + x\cos(x)\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right) + \cos(x)\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \cos(x)\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) + i\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right) - \text{PolyLog}\left(2,e^{ix}\right)\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x + x*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + Cos[x]*Log[Cos[x/2] -
 Sin[x/2]] - Cos[x]*Log[Cos[x/2] + Sin[x/2]] + I*Cos[x]*(PolyLog[2, -E^(I*x
)] - PolyLog[2, E^(I*x)]))*Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.16, size = 86, normalized size = 0.82

method	result	size
risch	$2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,x + 4\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(i\arctan\left(e^{ix}\right) + \frac{i\,\text{dilog}\left(e^{ix}\right)}{2} + \frac{i\,\text{dilog}\left(e^{ix}+1\right)}{2} - \frac{x\ln\left(e^{ix}+1\right)}{2}\right)\cos\left(x\right)$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)
^(1/2)*(I*arctan(exp(I*x))+1/2*I*dilog(exp(I*x))+1/2*I*dilog(exp(I*x)+1)-1/
2*x*ln(exp(I*x)+1))*cos(x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(80) = 160$.
time = 0.51, size = 294, normalized size = 2.80

$$\frac{\left(2\left(\cos\left(2x\right)+i\sin\left(2x\right)+1\right)\arctan\left(\cos\left(x\right),\sin\left(x\right)+1\right)+2\left(\cos\left(2x\right)+i\sin\left(2x\right)+1\right)\arctan\left(\cos\left(x\right),-\sin\left(x\right)+1\right)-2\left(x\cos\left(2x\right)+ix\sin\left(2x\right)+x\right)\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2\left(x\cos\left(2x\right)+ix\sin\left(2x\right)+x\right)\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)-4ix\cos\left(x\right)+2\left(\cos\left(2x\right)+i\sin\left(2x\right)+1\right)\mathrm{Li}_2\left(-e^{\left(ix\right)}\right)-2\left(\cos\left(2x\right)+i\sin\left(2x\right)+1\right)\mathrm{Li}_2\left(e^{\left(ix\right)}\right)-\left(-ix\cos\left(2x\right)+x\sin\left(2x\right)-ix\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)-\left(ix\cos\left(2x\right)-x\sin\left(2x\right)+ix\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)-\left(-i\cos\left(2x\right)+\sin\left(2x\right)-i\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\sin\left(x\right)+1\right)-\left(i\cos\left(2x\right)-\sin\left(2x\right)+i\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\sin\left(x\right)+1\right)+4x\sin\left(x\right)\right)\sqrt{a}}{-2i\cos\left(2x\right)+2\sin\left(2x\right)-2i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (2*(cos(2*x) + I*sin(2*x) + 1)*arctan2(cos(x), sin(x) + 1) + 2*(cos(2*x) +
I*sin(2*x) + 1)*arctan2(cos(x), -sin(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x)
 + x)*arctan2(sin(x), cos(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x) + x)*arcta
n2(sin(x), -cos(x) + 1) - 4*I*x*cos(x) + 2*(cos(2*x) + I*sin(2*x) + 1)*dilo
g(-e^(I*x)) - 2*(cos(2*x) + I*sin(2*x) + 1)*dilog(e^(I*x)) - (-I*x*cos(2*x)
 + x*sin(2*x) - I*x)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (I*x*cos(2*x
) - x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (-I*cos(2*x
) + sin(2*x) - I)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (I*cos(2*x) - s
in(2*x) + I)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*x*sin(x))*sqrt(a)/
(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

Fricas [A]
time = 2.25, size = 140, normalized size = 1.33

$$-\frac{1}{2}\left(x\cos\left(x\right)\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)+x\cos\left(x\right)\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)-x\cos\left(x\right)\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)-x\cos\left(x\right)\log\left(-\cos\left(x\right)-i\sin\left(x\right)+1\right)+i\cos\left(x\right){\rm Li}_2\left(\cos\left(x\right)+i\sin\left(x\right)\right)-i\cos\left(x\right){\rm Li}_2\left(\cos\left(x\right)-i\sin\left(x\right)\right)+i\cos\left(x\right){\rm Li}_2\left(-\cos\left(x\right)+i\sin\left(x\right)\right)-i\cos\left(x\right){\rm Li}_2\left(-\cos\left(x\right)-i\sin\left(x\right)\right)+\cos\left(x\right)\log\left(-\frac{\sin\left(x\right)+1}{\sin\left(x\right)-1}\right)-2x\right)\sqrt{\frac{a}{\cos\left(x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
+ 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*sin(
x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*sin
(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*sin(
x)) + cos(x)*log(-(sin(x) + 1)/(sin(x) - 1)) - 2*x)*sqrt(a/cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\sec^2\left(x\right)}\,\csc\left(x\right)\sec\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(x)^2)*x*csc(x)*sec(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cos(x)^2}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.875 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=225

$$x^2 \sqrt{a \sec^2(x)} + 4ix\text{ArcTan}\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} - 2x^2 \tanh^{-1}\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} + 2ix \cos(x)\text{PolyLo}$$

```
[Out] x^2*(a*sec(x)^2)^(1/2)+4*I*x*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2*x
^2*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+2*I*x*cos(x)*polylog(2,-exp(
I*x))*(a*sec(x)^2)^(1/2)-2*I*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^2)^(1/
2)+2*I*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-2*I*x*cos(x)*polylog
(2,exp(I*x))*(a*sec(x)^2)^(1/2)-2*cos(x)*polylog(3,-exp(I*x))*(a*sec(x)^2)^
(1/2)+2*cos(x)*polylog(3,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6852, 2702, 327, 213, 4505, 14, 6408, 4268, 2611, 2320, 6724, 4266, 2317, 2438}

$$4ix\text{ArcTan}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + 2ix\text{Li}_2\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2ix\text{Li}_2\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2i\text{Li}_2\left(-ie^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + 2i\text{Li}_2\left(ie^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2\text{Li}_3\left(-e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + 2\text{Li}_3\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + x^2\sqrt{a\sec^2(x)} - 2x^2\cos(x)\tanh^{-1}\left(e^{ix}\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^2*Sqrt[a*Sec[x]^2] + (4*I)*x*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] - 2*
x^2*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (2*I)*x*Cos[x]*PolyLog[2, -E
^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt[a*Sec
[x]^2] + (2*I)*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (2*I)*x*Cos[
x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 2*Cos[x]*PolyLog[3, -E^(I*x)]*Sqr
t[a*Sec[x]^2] + 2*Cos[x]*PolyLog[3, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^2\csc(x)\sec^2(x)\,dx$$

$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \Big(2\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \Big(2\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^2\sqrt{a\sec^2(x)} - x^2\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \Big(2\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + \Big(\cos(x)\sqrt{a\sec^2(x}$$

$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\mathrm{co}$$

$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\mathrm{co}$$

$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\mathrm{co}$$

$$= x^2\sqrt{a\sec^2(x)} + 4ix\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^2\tanh^{-1}\left(e^{ix}\right)\mathrm{co}$$

Mathematica [A]
time = 0.09, size = 174, normalized size = 0.77

$$\left(x^2 + x^2\cos(x)\left(\log\left(1-e^{ix}\right) - \log\left(1+e^{ix}\right)\right) - 2\cos(x)\left(x\left(\log\left(1-ie^{ix}\right) - \log\left(1+ie^{ix}\right)\right) + i\left(\text{PolyLog}\left(2,-ie^{ix}\right) - \text{PolyLog}\left(2,ie^{ix}\right)\right)\right) + 2ix\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right) - \text{PolyLog}\left(2,e^{ix}\right)\right) + 2\cos(x)\left(-\text{PolyLog}\left(3,-e^{ix}\right) + \text{PolyLog}\left(3,e^{ix}\right)\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] (x^2 + x^2*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) - 2*Cos[x]*(x*(Log[
1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) + I*(PolyLog[2, (-I)*E^(I*x)] - PolyLo
g[2, I*E^(I*x)])) + (2*I)*x*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^(I*
x)]) + 2*Cos[x]*(-PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)]))*Sqrt[a*Sec[x
]^2]
```

Maple [A]
time = 0.19, size = 200, normalized size = 0.89

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,x^2 - 4i\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\left(2i\left(\frac{x\ln(1+i\mathrm{e}^{ix})}{2} - \frac{x\ln(1-i\mathrm{e}^{ix})}{2} - \frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{ix})}{2} + \frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{ix})}{2}\right) - \frac{i\left(-\frac{ix}{3}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^2-4*I*(a*exp(2*I*x)/(exp(2*I*x)+1
)^2)^(1/2)*(2*I*(1/2*x*ln(1+I*exp(I*x))-1/2*x*ln(1-I*exp(I*x))-1/2*I*dilog(
1+I*exp(I*x))+1/2*I*dilog(1-I*exp(I*x)))-1/2*I*(-1/3*I*x^3+x^2*ln(exp(I*x)+
1)-2*I*x*polylog(2,-exp(I*x))+2*polylog(3,-exp(I*x)))-1/2*I*(1/3*I*x^3-x^2*
ln(1-exp(I*x))+2*I*x*polylog(2,exp(I*x))-2*polylog(3,exp(I*x))))*cos(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*I*x^2*cos(x) - 4*x^2*sin(x) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*a
rctan2(sin(x), cos(x) + 1) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan
2(sin(x), -cos(x) + 1) - 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(-e^(I*x))
+ 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(e^(I*x)) - 8*(I*cos(2*x) - sin(2*
x) + I)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x) + x*cos(x))/(cos(2
*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - 8*(cos(2*x) + I*sin(2*x) + 1)*in
tegrate((x*cos(x)*sin(2*x) - x*cos(2*x)*sin(x) - x*sin(x))/(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1), x) + (-I*x^2*cos(2*x) + x^2*sin(2*x) - I*x^2)*l
og(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(2*x) - x^2*sin(2*x) + I
*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 4*(I*cos(2*x) - sin(2*x) +
I)*polylog(3, -e^(I*x)) - 4*(-I*cos(2*x) + sin(2*x) - I)*polylog(3, e^(I*x)
))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(165) = 330.
time = 2.83, size = 337, normalized size = 1.50

$$\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3, \cos(x) + i\sin(x)) + \sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3, \cos(x) - i\sin(x)) - \sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3, -\cos(x) + i\sin(x)) - \sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3, -\cos(x) - i\sin(x)) - \frac{1}{2}\left(x^2\cos(x)\log(\cos(x) + i\sin(x) + 1) + x^2\cos(x)\log(\cos(x) - i\sin(x) + 1) - x^2\cos(x)\log(-\cos(x) + i\sin(x) + 1) - x^2\cos(x)\log(-\cos(x) - i\sin(x) + 1) + 2ix\cos(x)\mathrm{Li}_2(\cos(x) + i\sin(x)) - 2ix\cos(x)\mathrm{Li}_2(\cos(x) - i\sin(x)) + 2ix\cos(x)\mathrm{Li}_2(-\cos(x) + i\sin(x)) - 2ix\cos(x)\mathrm{Li}_2(-\cos(x) - i\sin(x)) + 2x\cos(x)\log(i\cos(x) + \sin(x) + 1) - 2x\cos(x)\log(i\cos(x) - \sin(x) + 1) + 2x\cos(x)\log(-i\cos(x) + \sin(x) + 1) - 2x\cos(x)\log(-i\cos(x) - \sin(x) + 1) - 2x^2 - 2i\cos(x)\mathrm{Li}_2(i\cos(x) + \sin(x)) - 2i\cos(x)\mathrm{Li}_2(i\cos(x) - \sin(x)) + 2i\cos(x)\mathrm{Li}_2(-i\cos(x) + \sin(x)) + 2i\cos(x)\mathrm{Li}_2(-i\cos(x) - \sin(x))\right)\sqrt{\frac{a}{\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^2)*co
s(x)*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -co
s(x) + I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)) -
 1/2*(x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x) - I*sin
(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log(-cos(x)
- I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*cos(x)*dilo
g(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x
)*dilog(-cos(x) - I*sin(x)) + 2*x*cos(x)*log(I*cos(x) + sin(x) + 1) - 2*x*c
```

```
os(x)*log(I*cos(x) - sin(x) + 1) + 2*x*cos(x)*log(-I*cos(x) + sin(x) + 1) -
 2*x*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^2 - 2*I*cos(x)*dilog(I*cos(x)
 + sin(x)) - 2*I*cos(x)*dilog(I*cos(x) - sin(x)) + 2*I*cos(x)*dilog(-I*cos(
x) + sin(x)) + 2*I*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a\sec^2{(x)}}\,\csc{(x)}\sec{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^2*csc(x)*sec(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\sqrt{\frac{a}{{\cos{(x)}}^2}}}{\cos{(x)}\,\sin{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.876 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)} \, dx$

Optimal. Leaf size=341

$x^3 \sqrt{a \sec^2(x)} + 6ix^2 \text{ArcTan}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 2x^3 \tanh^{-1}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 3ix^2 \cos(x) \text{PolyI}$

```
[Out] x^3*(a*sec(x)^2)^(1/2)+6*I*x^2*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2
*x^3*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+3*I*x^2*cos(x)*polylog(2,-
exp(I*x))*(a*sec(x)^2)^(1/2)-6*I*x*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^
2)^(1/2)+6*I*x*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-3*I*x^2*cos(
x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)-6*x*cos(x)*polylog(3,-exp(I*x))*(
a*sec(x)^2)^(1/2)+6*cos(x)*polylog(3,-I*exp(I*x))*(a*sec(x)^2)^(1/2)-6*cos(
x)*polylog(3,I*exp(I*x))*(a*sec(x)^2)^(1/2)+6*x*cos(x)*polylog(3,exp(I*x))*
(a*sec(x)^2)^(1/2)-6*I*cos(x)*polylog(4,-exp(I*x))*(a*sec(x)^2)^(1/2)+6*I*c
os(x)*polylog(4,exp(I*x))*(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$,
Rules used = {6852, 2702, 327, 213, 4505, 14, 6408, 4268, 2611, 6744, 2320, 6724, 4266}

$$6ix^2 \text{ArcTan}(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 3ix^2 \text{Li}_2(-e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 3ix^2 \text{Li}_2(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 6ix \text{Li}_2(-ie^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 6ix \text{Li}_2(ie^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 6x \text{Li}_3(-e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 6x \text{Li}_3(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 6\text{Li}_3(-ie^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 6\text{Li}_3(ie^{ix}) \cos(x) \sqrt{a \sec^2(x)} - 6i \text{Li}_4(-e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + 6i \text{Li}_4(e^{ix}) \cos(x) \sqrt{a \sec^2(x)} + x^3 \sqrt{a \sec^2(x)} - 2x^3 \cos(x) \tanh^{-1}(e^{ix}) \sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] x^3*Sqrt[a*Sec[x]^2] + (6*I)*x^2*ArcTan[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] -
2*x^3*ArcTanh[E^(I*x)]*Cos[x]*Sqrt[a*Sec[x]^2] + (3*I)*x^2*Cos[x]*PolyLog[2
, -E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*x*Cos[x]*PolyLog[2, (-I)*E^(I*x)]*Sqrt
[a*Sec[x]^2] + (6*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)]*Sqrt[a*Sec[x]^2] - (3*I
)*x^2*Cos[x]*PolyLog[2, E^(I*x)]*Sqrt[a*Sec[x]^2] - 6*x*Cos[x]*PolyLog[3, -
E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*Cos[x]*PolyLog[3, (-I)*E^(I*x)]*Sqrt[a*Sec[x]
^2] - 6*Cos[x]*PolyLog[3, I*E^(I*x)]*Sqrt[a*Sec[x]^2] + 6*x*Cos[x]*PolyLog[
3, E^(I*x)]*Sqrt[a*Sec[x]^2] - (6*I)*Cos[x]*PolyLog[4, -E^(I*x)]*Sqrt[a*Sec
[x]^2] + (6*I)*Cos[x]*PolyLog[4, E^(I*x)]*Sqrt[a*Sec[x]^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6408

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*((a + b*ArcTanh[u])/(d*(m + 1))), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 - u^2)), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(\cos(x)\sqrt{a\sec^2(x)}\right)\int x^3\csc(x)\sec^2(x)\,dx$$

$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \Big(3\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} - \Big(3\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^3\sqrt{a\sec^2(x)} - x^3\tanh^{-1}(\cos(x))\cos(x)\sqrt{a\sec^2(x)} + \Big(3\cos(x)\sqrt{a\,\mathrm{se}}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} + \Big(\cos(x)\sqrt{a\sec^2(}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\mathrm{c}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\mathrm{c}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\mathrm{c}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\mathrm{c}$$

$$= x^3\sqrt{a\sec^2(x)} + 6ix^2\tan^{-1}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)} - 2x^3\tanh^{-1}\left(e^{ix}\right)\mathrm{c}$$

Mathematica [A]
time = 0.28, size = 290, normalized size = 0.85

$$\frac{1}{8}\left(8x^3 - i\pi^4\cos(x) + 2ix^4\cos(x) + 8x^3\cos(x)\log\left(1-e^{-ix}\right) - 24x^2\cos(x)\log\left(1-ie^{ix}\right) + 24x^2\cos(x)\log\left(1+ie^{ix}\right) - 8x^3\cos(x)\log\left(1+e^{ix}\right) + 24ix^2\cos(x)\mathrm{PolyLog}\left(2,e^{-ix}\right) + 24ix^2\cos(x)\mathrm{PolyLog}\left(2,-e^{ix}\right) - 48ix\cos(x)\mathrm{PolyLog}\left(2,-ie^{ix}\right) + 48ix\cos(x)\mathrm{PolyLog}\left(2,ie^{ix}\right) + 48x\cos(x)\mathrm{PolyLog}\left(3,e^{-ix}\right) - 48x\cos(x)\mathrm{PolyLog}\left(3,-e^{ix}\right) + 48\cos(x)\mathrm{PolyLog}\left(3,-ie^{ix}\right) - 48\cos(x)\mathrm{PolyLog}\left(3,ie^{ix}\right) - 48i\cos(x)\mathrm{PolyLog}\left(4,e^{-ix}\right) - 48i\cos(x)\mathrm{PolyLog}\left(4,-e^{ix}\right)\right)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

```
[Out] ((8*x^3 - I*Pi^4*Cos[x] + (2*I)*x^4*Cos[x] + 8*x^3*Cos[x]*Log[1 - E^((-I)*x
)] - 24*x^2*Cos[x]*Log[1 - I*E^(I*x)] + 24*x^2*Cos[x]*Log[1 + I*E^(I*x)] -
8*x^3*Cos[x]*Log[1 + E^(I*x)] + (24*I)*x^2*Cos[x]*PolyLog[2, E^((-I)*x)] +
(24*I)*x^2*Cos[x]*PolyLog[2, -E^(I*x)] - (48*I)*x*Cos[x]*PolyLog[2, (-I)*E^
(I*x)] + (48*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)] + 48*x*Cos[x]*PolyLog[3, E^(
(-I)*x)] - 48*x*Cos[x]*PolyLog[3, -E^(I*x)] + 48*Cos[x]*PolyLog[3, (-I)*E^(
I*x)] - 48*Cos[x]*PolyLog[3, I*E^(I*x)] - (48*I)*Cos[x]*PolyLog[4, E^((-I)*
x)] - (48*I)*Cos[x]*PolyLog[4, -E^(I*x)])*Sqrt[a*Sec[x]^2])/8
```

Maple [A]
time = 0.28, size = 250, normalized size = 0.73

method	result
risch	$2\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,x^3+4\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}\,\left(-\frac{3x^2\ln(1-ie^{ix})}{2}+3ix\,\mathrm{polylog}\left(2,ie^{ix}\right)-3\,\mathrm{polylog}\left(3,ie^{ix}\right)+\frac{3x^2\ln(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^3+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^
2)^(1/2)*(-3/2*x^2*ln(1-I*exp(I*x))+3*I*x*polylog(2,I*exp(I*x))-3*polylog(3
,I*exp(I*x))+3/2*x^2*ln(1+I*exp(I*x))-3*I*x*polylog(2,-I*exp(I*x))+3*polylo
g(3,-I*exp(I*x))+1/2*I*(1/4*x^4+I*x^3*ln(exp(I*x)+1)+3*x^2*polylog(2,-exp(I
*x))+6*I*x*polylog(3,-exp(I*x))-6*polylog(4,-exp(I*x)))+1/2*I*(-1/4*x^4-I*x
^3*ln(1-exp(I*x))-3*x^2*polylog(2,exp(I*x))-6*I*x*polylog(3,exp(I*x))+6*pol
ylog(4,exp(I*x))))*cos(x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(253) = 506$.
time = 0.57, size = 567, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(4*I*x^3*cos(x) - 4*x^3*sin(x) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*a
rctan2(cos(x), sin(x) + 1) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arctan
2(cos(x), -sin(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2(si
n(x), cos(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2(sin(x),
 -cos(x) + 1) - 12*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(I*e^(I*x)) + 12*(x
*cos(2*x) + I*x*sin(2*x) + x)*dilog(-I*e^(I*x)) - 6*(x^2*cos(2*x) + I*x^2*s
in(2*x) + x^2)*dilog(-e^(I*x)) + 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*di
log(e^(I*x)) + (-I*x^3*cos(2*x) + x^3*sin(2*x) - I*x^3)*log(cos(x)^2 + sin(
x)^2 + 2*cos(x) + 1) + (I*x^3*cos(2*x) - x^3*sin(2*x) + I*x^3)*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) - 3*(I*x^2*cos(2*x) - x^2*sin(2*x) + I*x^2)*log
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 3*(-I*x^2*cos(2*x) + x^2*sin(2*x) -
I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 12*(cos(2*x) + I*sin(2*x)
+ 1)*polylog(4, -e^(I*x)) - 12*(cos(2*x) + I*sin(2*x) + 1)*polylog(4, e^(I*
x)) - 12*(I*cos(2*x) - sin(2*x) + I)*polylog(3, I*e^(I*x)) - 12*(-I*cos(2*x
) + sin(2*x) - I)*polylog(3, -I*e^(I*x)) - 12*(I*x*cos(2*x) - x*sin(2*x) +
I*x)*polylog(3, -e^(I*x)) - 12*(-I*x*cos(2*x) + x*sin(2*x) - I*x)*polylog(3
, e^(I*x)))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(253) = 506$.

time = 3.01, size = 539, normalized size = 1.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos(
x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*po
lylog(3, -cos(x) + I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(
x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(x))
- 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/co
s(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)
*polylog(4, -cos(x) - I*sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*co
s(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) - sin(x)) +
3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^
2)*cos(x)*polylog(3, -I*cos(x) - sin(x)) - 1/2*(x^3*cos(x)*log(cos(x) + I*s
in(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x)
 + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)*
dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*x^
2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*sin(x
)) + 3*x^2*cos(x)*log(I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(I*cos(x) -
sin(x) + 1) + 3*x^2*cos(x)*log(-I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*log(-
I*cos(x) - sin(x) + 1) - 2*x^3 - 6*I*x*cos(x)*dilog(I*cos(x) + sin(x)) - 6*
I*x*cos(x)*dilog(I*cos(x) - sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) + sin(x)
) + 6*I*x*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{a\sec^{2}\left(x\right)}\csc\left(x\right)\sec\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^2)*x^3*csc(x)*sec(x), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{\frac{a}{{\cos\left(x\right)}^2}}}{\cos\left(x\right)\,\sin\left(x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.877 $\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

Optimal. Leaf size=142

$$\frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}i\cos^2(x)\text{PolyLog}\left(2,-e^{2ix}\right)\sqrt{a\sec^4(x)} - \frac{1}{2}i\,c$$

```
[Out] 1/2*x*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x
)^4)^(1/2)+1/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*I*c
os(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*cos(x)*sin(x)*(a*sec(x
)^4)^(1/2)+1/2*x*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6852, 2700, 14, 4505, 2628, 4504, 4268, 2317, 2438, 3554, 8}

$$\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x\sin^2(x)\sqrt{a\sec^4(x)} - 2x\cos^2(x)\tanh^{-1}\left(e^{2ix}\right)\sqrt{a\sec^4(x)} - \frac{1}{2}\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*
Sec[x]^4] + (I/2)*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (I/2
)*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - (Cos[x]*Sqrt[a*Sec[x]
^4]*Sin[x])/2 + (x*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x\csc(x)\sec^3(x)\,dx$$
$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) - \Big(\cos^2(x)\sqrt{}$$
$$= x\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) - \frac{1}{2}\Big(\cos^2(x)$$
$$= -\frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\sin^2(x) + \frac{1}{2}\Big(\cos^2(x)\sqrt{a\,s}$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)\sqrt{a\sec^4(x)}\sin(x) + \frac{1}{2}x\sqrt{a\sec^4(x)}\,\mathrm{si}$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\cos(x)$$
$$= \frac{1}{2}x\cos^2(x)\sqrt{a\sec^4(x)} - 2x\tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}i\cos^2($$

Mathematica [A]
time = 0.16, size = 85, normalized size = 0.60

$$\frac{1}{2}\cos^2(x)\sqrt{a\sec^4(x)}\left(2x\log\left(1-e^{2ix}\right)-2x\log\left(1+e^{2ix}\right)+i\text{PolyLog}\left(2,-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+x\sec^2(x)-\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2*x*Log[1 - E^((2*I)*x)] - 2*x*Log[1 + E^((2*I)
*x)] + I*PolyLog[2, -E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + x*Sec[x]^2
- Tan[x]))/2
```

Maple [A]
time = 0.12, size = 165, normalized size = 1.16

method	result
risch	$\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\left(-i+2x-i e^{-2ix}\right)-4i\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\left(e^{2ix}+1\right)^2\left(-\frac{i e^{-2ix}x\ln(e^{2ix}+1)}{4}-\frac{e^{-2ix}\operatorname{polylog}(2,-e^{2ix})}{8}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(-I+2*x-I*exp(-2*I*x))-4*I*(a*exp(4*I
*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-1/4*I*exp(-2*I*x)*x*ln(exp(2
```

```
*I*x)+1)-1/8*exp(-2*I*x)*polylog(2,-exp(2*I*x))+1/4*I*exp(-2*I*x)*x*ln(exp(
I*x)+1)+1/4*exp(-2*I*x)*polylog(2,-exp(I*x))+1/4*I*exp(-2*I*x)*x*ln(1-exp(I
*x))+1/4*exp(-2*I*x)*polylog(2,exp(I*x)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(105) = 210$.
time = 0.58, size = 423, normalized size = 2.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2
(sin(2*x), cos(2*x) + 1) - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) + 2*
I*x*sin(2*x) + x)*arctan2(sin(x), cos(x) + 1) + 2*(x*cos(4*x) + 2*x*cos(2*x
) + I*x*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2(sin(x), -cos(x) + 1) - 2*(-2
*I*x - 1)*cos(2*x) - (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1
)*dilog(-e^(2*I*x)) + 2*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
+ 1)*dilog(-e^(I*x)) + 2*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
 + 1)*dilog(e^(I*x)) + (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*x*s
in(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (I*x*cos(4*x
) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(
2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*(2*x - I)*sin(2*x)
+ 2)*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x) - 2*I)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(105) = 210$.
time = 3.00, size = 270, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin(
x) + 1) - x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(-I*c
os(x) - sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*l
og(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)
^2*dilog(cos(x) - I*sin(x)) - I*cos(x)^2*dilog(I*cos(x) + sin(x)) + I*cos(x
)^2*dilog(I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-I*cos(x) + sin(x)) - I*cos
(x)^2*dilog(-I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*
cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)*sin(x) + x)*sqrt(a/cos(x)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\sec^4(x)}\,\csc(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
[Out] Integral(x*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(x)^4)*x*csc(x)*sec(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
[Out] int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.878 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

Optimal. Leaf size=220

$$\frac{1}{2}x^2 \cos^2(x)\sqrt{a\sec^4(x)} - 2x^2 \tanh^{-1}\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \cos^2(x)\log(\cos(x))\sqrt{a\sec^4(x)} + ix\cos^2(x)$$

```
[Out] 1/2*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x^2*arctanh(exp(2*I*x))*cos(x)^2*(a*s
ec(x)^4)^(1/2)-cos(x)^2*ln(cos(x))*(a*sec(x)^4)^(1/2)+I*x*cos(x)^2*polylog(
2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-I*x*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec
(x)^4)^(1/2)-1/2*cos(x)^2*polylog(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+1/2*cos
(x)^2*polylog(3,exp(2*I*x))*(a*sec(x)^4)^(1/2)-x*cos(x)*sin(x)*(a*sec(x)^4)
^(1/2)+1/2*x^2*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6852, 2700, 14, 4505, 2631, 4504, 4268, 2611, 2320, 6724, 3801, 3556, 30}

$$ix\text{Li}_2\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - ix\text{Li}_2\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} - \frac{1}{2}\text{Li}_3\left(-e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}\text{Li}_3\left(e^{2ix}\right)\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^2\sin^2(x)\sqrt{a\sec^4(x)} - 2x^2\cos^2(x)\tanh^{-1}\left(e^{2ix}\right)\sqrt{a\sec^4(x)} - \cos^2(x)\sqrt{a\sec^4(x)}\log(\cos(x)) - x\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x^2*Cos[x]^2*Sqrt[a*Sec[x]^4])/2 - 2*x^2*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqr
t[a*Sec[x]^4] - Cos[x]^2*Log[Cos[x]]*Sqrt[a*Sec[x]^4] + I*x*Cos[x]^2*PolyLo
g[2, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] - I*x*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*
Sqrt[a*Sec[x]^4] - (Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 +
 (Cos[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - x*Cos[x]*Sqrt[a*Se
c[x]^4]*Sin[x] + (x^2*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \left(\cos^2(x) \sqrt{a \sec^4(x)}\right) \int x^2 \csc(x) \sec^3(x)\, dx$$
$$= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \Big(2 \cos^2(x$$
$$= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \Big(2 \cos^2(x$$
$$= x^2 \cos^2(x) \log(\tan(x)) \sqrt{a \sec^4(x)} + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) - \Big(\cos^2(x)$$
$$= -x \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x) + \Big(\cos^2(x) \sqrt{a \sec$$
$$= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} - x \cos(x) \sqrt{a$$
$$= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x$$
$$= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x$$
$$= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x$$
$$= \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} - 2x^2 \tanh^{-1}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \cos^2(x$$

Mathematica [A]
time = 0.43, size = 138, normalized size = 0.63

$$\frac{1}{24} \cos^2(x) \sqrt{a \sec^4(x)} \left(-i\pi^3 + 16ix^3 + 24x^2 \log\left(1 - e^{-2ix}\right) - 24x^2 \log\left(1 + e^{2ix}\right) - 24 \log(\cos(x)) + 24ix\text{PolyLog}(2, e^{-2ix}) + 24ix\text{PolyLog}(2, -e^{2ix}) + 12\text{PolyLog}(3, e^{-2ix}) - 12\text{PolyLog}(3, -e^{2ix}) + 12x^2 \sec^2(x) - 24x \tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^3 + (16*I)*x^3 + 24*x^2*Log[1 - E^((-2*
I)*x)] - 24*x^2*Log[1 + E^((2*I)*x)] - 24*Log[Cos[x]] + (24*I)*x*PolyLog[2,
 E^((-2*I)*x)] + (24*I)*x*PolyLog[2, -E^((2*I)*x)] + 12*PolyLog[3, E^((-2*I
)*x)] - 12*PolyLog[3, -E^((2*I)*x)] + 12*x^2*Sec[x]^2 - 24*x*Tan[x]))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 254, normalized size = 1.15

method	result
risch	$2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\; x\left(x - i - i e^{-2ix}\right) + 2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\; \left(e^{2ix}+1\right)^2 \Big(-\frac{e^{-2ix} \ln\left(e^{2ix}+1\right)}{2} - e^{-2ix} \Im(x) + e^{-2ix} \ln\big(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x*(x-I-I*exp(-2*I*x))+2*(a*exp(4*I*
x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-1/2*exp(-2*I*x)*ln(exp(2*I*x)
+1)-exp(-2*I*x)*Im(x)+exp(-2*I*x)*ln(exp(I*Re(x)))-1/2*exp(-2*I*x)*x^2*ln(e
xp(2*I*x)+1)+1/2*I*exp(-2*I*x)*x*polylog(2,-exp(2*I*x))-1/4*exp(-2*I*x)*pol
ylog(3,-exp(2*I*x))+1/2*exp(-2*I*x)*x^2*ln(exp(I*x)+1)-I*exp(-2*I*x)*x*poly
log(2,-exp(I*x))+exp(-2*I*x)*polylog(3,-exp(I*x))+1/2*exp(-2*I*x)*x^2*ln(1-
exp(I*x))-I*exp(-2*I*x)*x*polylog(2,exp(I*x))+exp(-2*I*x)*polylog(3,exp(I*x
)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 639 vs. 2(173) = 346.
time = 0.58, size = 639, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*(x^2 + (x^2 + 1)*cos(4*x) + 2*(x^2 + 1)*cos(2*x) - (-I*x^2 - I)*sin(4*x
) - 2*(-I*x^2 - I)*sin(2*x) + 1)*arctan2(sin(2*x), cos(2*x) + 1) - 2*(x^2*c
os(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^2*sin(2*x) + x^2)*arctan2
(sin(x), cos(x) + 1) + 2*(x^2*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) +
2*I*x^2*sin(2*x) + x^2)*arctan2(sin(x), -cos(x) + 1) - 4*x*cos(4*x) - 4*(-I
*x^2 + x)*cos(2*x) - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) + 2*I*x*si
n(2*x) + x)*dilog(-e^(2*I*x)) + 4*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x)
 + 2*I*x*sin(2*x) + x)*dilog(-e^(I*x)) + 4*(x*cos(4*x) + 2*x*cos(2*x) + I*x
*sin(4*x) + 2*I*x*sin(2*x) + x)*dilog(e^(I*x)) + (-I*x^2 + (-I*x^2 - I)*cos
(4*x) - 2*(I*x^2 + I)*cos(2*x) + (x^2 + 1)*sin(4*x) + 2*(x^2 + 1)*sin(2*x)
- I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (I*x^2*cos(4*x) + 2*I*
x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*
x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + (-I*cos(4*x
) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*polylog(3, -e^(2*I*x)) - 4*(-
I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*polylog(3, -e^(I*x))
 - 4*(-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*polylog(3, e^
(I*x)) - 4*I*x*sin(4*x) - 4*(x^2 + I*x)*sin(2*x))*sqrt(a)/(-2*I*cos(4*x) -
4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x) - 2*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(173) = 346.

time = 3.61, size = 550, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")

[Out] sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) - sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 1/2*(x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)^2*log(cos(x) - I*sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) - sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) - 2*I*x*cos(x)^2*dilog(I*cos(x) + sin(x)) + 2*I*x*cos(x)^2*dilog(I*cos(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-I*cos(x) + sin(x)) - 2*I*x*cos(x)^2*dilog(-I*cos(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)^2*log(cos(x) + I*sin(x) + I) - cos(x)^2*log(cos(x) - I*sin(x) + I) - cos(x)^2*log(-cos(x) + I*sin(x) + I) - cos(x)^2*log(-cos(x) - I*sin(x) + I) - 2*x*cos(x)*sin(x) + x^2)*sqrt(a/cos(x)^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a \sec^4(x)} \csc(x) \sec(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)

[Out] Integral(x**2*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(x)^4)*x^2*csc(x)*sec(x), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.879 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx$

Optimal. Leaf size=356

$$\frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \tanh^{-1}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - 3 x \cos^2(x) \log\left(1 +\right.$$

```
[Out] 3/2*I*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)+1/2*x^3*cos(x)^2*(a*sec(x)^4)^(1/2)-2
*x^3*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x)^4)^(1/2)-3*x*cos(x)^2*ln(1+exp(
2*I*x))*(a*sec(x)^4)^(1/2)+3/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^
4)^(1/2)+3/2*I*x^2*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*I
*x^2*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x*cos(x)^2*polyl
og(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+3/2*x*cos(x)^2*polylog(3,exp(2*I*x))*(
a*sec(x)^4)^(1/2)-3/4*I*cos(x)^2*polylog(4,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+
3/4*I*cos(x)^2*polylog(4,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x^2*cos(x)*sin(
x)*(a*sec(x)^4)^(1/2)+1/2*x^3*sin(x)^2*(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 17, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {6852, 2700, 14, 4505, 2631, 4504, 4268, 2611, 6744, 2320, 6724, 3801, 3800, 2221, 2317, 2438, 30}

$$\frac{3}{2} i x^2 \text{Li}_2\left(-e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{3}{2} i x^2 \text{Li}_2\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{3}{2} x \text{Li}_3\left(-e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{3}{2} x \text{Li}_3\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{3}{2} i \text{Li}_2\left(-e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} - \frac{3}{4} i \text{Li}_4\left(-e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{3}{4} i \text{Li}_4\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \sin^2(x) \sqrt{a \sec^4(x)} - 2 x^3 \cos^2(x) \tanh^{-1}\left(e^{2ix}\right) \sqrt{a \sec^4(x)} + \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} - \frac{3}{2} x^2 \sin(x) \cos(x) \sqrt{a \sec^4(x)} - 3 x \log\left(1 + e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] ((3*I)/2)*x^2*Cos[x]^2*Sqrt[a*Sec[x]^4] + (x^3*Cos[x]^2*Sqrt[a*Sec[x]^4])/2
 - 2*x^3*ArcTanh[E^((2*I)*x)]*Cos[x]^2*Sqrt[a*Sec[x]^4] - 3*x*Cos[x]^2*Log[
1 + E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*Cos[x]^2*PolyLog[2, -E^((2*I)
*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, -E^((2*I)*x)]*Sqr
t[a*Sec[x]^4] - ((3*I)/2)*x^2*Cos[x]^2*PolyLog[2, E^((2*I)*x)]*Sqrt[a*Sec[x
]^4] - (3*x*Cos[x]^2*PolyLog[3, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 + (3*x*Co
s[x]^2*PolyLog[3, E^((2*I)*x)]*Sqrt[a*Sec[x]^4])/2 - ((3*I)/4)*Cos[x]^2*Pol
yLog[4, -E^((2*I)*x)]*Sqrt[a*Sec[x]^4] + ((3*I)/4)*Cos[x]^2*PolyLog[4, E^((
2*I)*x)]*Sqrt[a*Sec[x]^4] - (3*x^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x])/2 + (x^3
*Sqrt[a*Sec[x]^4]*Sin[x]^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2631

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)
*(Log[u]/(b*(m + 1))), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a +
b*x)^(m + 1)*(D[u, x]/u), x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3801

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Di
st[b*d*(m/(f*(n - 1))), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4505

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int x^3\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int x^3\csc(x)\sec^3(x)\,dx$$

$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\,\sin^2(x) - \Big(3\cos^2$$

$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\,\sin^2(x) - \Big(3\cos^2$$

$$= x^3\cos^2(x)\log(\tan(x))\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\,\sin^2(x) - \frac{1}{2}\Big(3\cos$$

$$= -\frac{3}{2}x^2\cos(x)\sqrt{a\sec^4(x)}\,\sin(x) + \frac{1}{2}x^3\sqrt{a\sec^4(x)}\,\sin^2(x) + \Big(\cos^2(x)\sqrt{}$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - \frac{3}{2}x^2\cos(x)\sqrt{a\,\text{se}}$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right.$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right.$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right.$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right.$$

$$= \frac{3}{2}ix^2\cos^2(x)\sqrt{a\sec^4(x)} + \frac{1}{2}x^3\cos^2(x)\sqrt{a\sec^4(x)} - 2x^3\tanh^{-1}\left(e^{2ix}\right.$$

Mathematica [A]

time = 0.70, size = 191, normalized size = 0.54

$$\frac{1}{64}\cos^2(x)\sqrt{a\sec^4(x)}\left(-i\pi^4+96ix^2+32ix^4+64x^3\log\left(1-e^{-2ix}\right)-192x\log\left(1+e^{2ix}\right)-64x^3\log\left(1+e^{2ix}\right)+96ix^2\text{PolyLog}\left(2,e^{-2ix}\right)+96i\left(1+x^2\right)\text{PolyLog}\left(2,-e^{2ix}\right)+96x\text{PolyLog}\left(3,e^{-2ix}\right)-96x\text{PolyLog}\left(3,-e^{2ix}\right)-48i\text{PolyLog}\left(4,e^{-2ix}\right)-48i\text{PolyLog}\left(4,-e^{2ix}\right)+32x^3\sec^2(x)-96x^2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^4 + (96*I)*x^2 + (32*I)*x^4 + 64*x^3*Lo
g[1 - E^((-2*I)*x)] - 192*x*Log[1 + E^((2*I)*x)] - 64*x^3*Log[1 + E^((2*I)*
x)] + (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*(1 + x^2)*PolyLog[2, -E^
((2*I)*x)] + 96*x*PolyLog[3, E^((-2*I)*x)] - 96*x*PolyLog[3, -E^((2*I)*x)]
- (48*I)*PolyLog[4, E^((-2*I)*x)] - (48*I)*PolyLog[4, -E^((2*I)*x)] + 32*x^
3*Sec[x]^2 - 96*x^2*Tan[x]))/64
```

Maple [**A**]
time = 0.13, size = 324, normalized size = 0.91

method	result
risch	$\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\;x^2\left(2x-3i-3i\mathrm{e}^{-2ix}\right)-2i\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^4}}\;\left(\mathrm{e}^{2ix}+1\right)^2\left(-\frac{3\,\mathrm{e}^{-2ix}x^2}{2}-\frac{3i\mathrm{e}^{-2ix}x\ln\left(\mathrm{e}^{2ix}+1\right)}{2}-\frac{3\,\mathrm{e}^{-2i}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x^2*(2*x-3*I-3*I*exp(-2*I*x))-2*I*(a*
exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(exp(2*I*x)+1)^2*(-3/2*exp(-2*I*x)*x^2-3
/2*I*exp(-2*I*x)*x*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*polylog(2,-exp(2*I*x))-
1/2*I*exp(-2*I*x)*x^3*ln(exp(2*I*x)+1)-3/4*exp(-2*I*x)*x^2*polylog(2,-exp(2
*I*x))-3/4*I*exp(-2*I*x)*x*polylog(3,-exp(2*I*x))+3/8*exp(-2*I*x)*polylog(4
,-exp(2*I*x))+1/2*I*exp(-2*I*x)*x^3*ln(exp(I*x)+1)+3/2*exp(-2*I*x)*x^2*poly
log(2,-exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,-exp(I*x))-3*exp(-2*I*x)*polyl
og(4,-exp(I*x))+1/2*I*exp(-2*I*x)*x^3*ln(1-exp(I*x))+3/2*exp(-2*I*x)*x^2*po
lylog(2,exp(I*x))+3*I*exp(-2*I*x)*x*polylog(3,exp(I*x))-3*exp(-2*I*x)*polyl
og(4,exp(I*x)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(266) = 532.
time = 0.62, size = 852, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] (18*x^2*cos(4*x) + 18*I*x^2*sin(4*x) - 2*(4*x^3 + (4*x^3 + 9*x)*cos(4*x) +
2*(4*x^3 + 9*x)*cos(2*x) - (-4*I*x^3 - 9*I*x)*sin(4*x) - 2*(-4*I*x^3 - 9*I*
```

```
x)*sin(2*x) + 9*x)*arctan2(sin(2*x), cos(2*x) + 1) + 6*(x^3*cos(4*x) + 2*x^
3*cos(2*x) + I*x^3*sin(4*x) + 2*I*x^3*sin(2*x) + x^3)*arctan2(sin(x), cos(x
) + 1) - 6*(x^3*cos(4*x) + 2*x^3*cos(2*x) + I*x^3*sin(4*x) + 2*I*x^3*sin(2*
x) + x^3)*arctan2(sin(x), -cos(x) + 1) + 6*(-2*I*x^3 + 3*x^2)*cos(2*x) + 3*
(4*x^2 + (4*x^2 + 3)*cos(4*x) + 2*(4*x^2 + 3)*cos(2*x) + (4*I*x^2 + 3*I)*si
n(4*x) + 2*(4*I*x^2 + 3*I)*sin(2*x) + 3)*dilog(-e^(2*I*x)) - 18*(x^2*cos(4*
x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^2*sin(2*x) + x^2)*dilog(-e^(I*
x)) - 18*(x^2*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^2*sin(2*x)
 + x^2)*dilog(e^(I*x)) - (-4*I*x^3 + (-4*I*x^3 - 9*I*x)*cos(4*x) - 2*(4*I*x
^3 + 9*I*x)*cos(2*x) + (4*x^3 + 9*x)*sin(4*x) + 2*(4*x^3 + 9*x)*sin(2*x) -
9*I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 3*(-I*x^3*cos(4*x) -
 2*I*x^3*cos(2*x) + x^3*sin(4*x) + 2*x^3*sin(2*x) - I*x^3)*log(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1) + 3*(-I*x^3*cos(4*x) - 2*I*x^3*cos(2*x) + x^3*sin(4
*x) + 2*x^3*sin(2*x) - I*x^3)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 6*(
cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1)*polylog(4, -e^(2*I*x
)) + 36*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1)*polylog(4,
-e^(I*x)) + 36*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1)*poly
log(4, e^(I*x)) + 12*(I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(
2*x) + I*x)*polylog(3, -e^(2*I*x)) + 36*(-I*x*cos(4*x) - 2*I*x*cos(2*x) + x
*sin(4*x) + 2*x*sin(2*x) - I*x)*polylog(3, -e^(I*x)) + 36*(-I*x*cos(4*x) -
2*I*x*cos(2*x) + x*sin(4*x) + 2*x*sin(2*x) - I*x)*polylog(3, e^(I*x)) + 6*(
2*x^3 + 3*I*x^2)*sin(2*x))*sqrt(a)/(-6*I*cos(4*x) - 12*I*cos(2*x) + 6*sin(4
*x) + 12*sin(2*x) - 6*I)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(266) = 532$.
time = 2.39, size = 740, normalized size = 2.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x
)^2*polylog(3, I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3
, I*cos(x) - sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) +
 sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + 3
*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 3*x*sqrt(a/co
s(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(
x)^2*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) +
 sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - sin(x)) - 3*
I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) + sin(x)) + 3*I*sqrt(a/cos
(x)^4)*cos(x)^2*polylog(4, -I*cos(x) - sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x
```

```
)^2*polylog(4, -cos(x) + I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
4, -cos(x) - I*sin(x)) + 1/2*(x^3*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^3
*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) + I*sin(x)
+ 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 3*I*x^2*cos(x)^2*dilog(co
s(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x) - I*sin(x)) + 3*I*x^2*cos(
x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2*dilog(-cos(x) - I*sin(x))
 - 3*(I*x^2 + I)*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*(-I*x^2 - I)*cos(x)^
2*dilog(I*cos(x) - sin(x)) - 3*(-I*x^2 - I)*cos(x)^2*dilog(-I*cos(x) + sin(
x)) - 3*(I*x^2 + I)*cos(x)^2*dilog(-I*cos(x) - sin(x)) - (x^3 + 3*x)*cos(x)
^2*log(I*cos(x) + sin(x) + 1) - (x^3 + 3*x)*cos(x)^2*log(I*cos(x) - sin(x)
+ 1) - (x^3 + 3*x)*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - (x^3 + 3*x)*cos(x
)^2*log(-I*cos(x) - sin(x) + 1) - 3*x^2*cos(x)*sin(x) + x^3)*sqrt(a/cos(x)^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a \sec^4(x)} \csc(x) \sec(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(x)^4)*x^3*csc(x)*sec(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

```
[Out] int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.880 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2718}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 2.03, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(19) = 38$.
time = 1.07, size = 116, normalized size = 4.64

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{5\sin(x)\sin(3x)\cos(2x)}{24}-\frac{\sin(2x)\sin(3x)\cos(x)}{3}-\frac{3\cos(x)\cos(2x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(3*x)*cos(2*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/3 - 3*cos(x)*cos(2*x)*cos(3*x)/8
```

Giac [A]
time = 0.42, size = 17, normalized size = 0.68

$$\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Mupad [B]
time = 2.94, size = 14, normalized size = 0.56

$$-\frac{\sin(x)^4\left(8\sin(x)^2-9\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*sin(x),x)
```

```
[Out] -(sin(x)^4*(8*sin(x)^2 - 9))/6
```

3.881 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4440, 2717}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A]
time = 0.30, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x+\frac{1}{24}\,\sin\left(6\,x\right)+\frac{1}{16}\,\sin\left(4\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A]
time = 2.55, size = 25, normalized size = 0.83

$$\frac{1}{12}\,\left(16\,\cos\left(x\right)^{5}-10\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.
time = 1.08, size = 114, normalized size = 3.80

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}+\frac{\sin(x)\sin(2x)\sin(3x)}{6}+\frac{\sin(x)\cos(2x)\cos(3x)}{8}+\frac{5\sin(3x)\cos(x)\cos(2x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
6 + sin(x)*cos(2*x)*cos(3*x)/8 + 5*sin(3*x)*cos(x)*cos(2*x)/24
```

Giac [A]
time = 0.40, size = 22, normalized size = 0.73

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B]
time = 3.04, size = 22, normalized size = 0.73

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

3.882 $\int \cos(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

```
[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {4440, 2717}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(2x)\sin(3x)\,dx&=\int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)-\frac{1}{4}\cos(4x)-\frac{1}{4}\cos(6x)\right)\,dx\\&=\frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx-\frac{1}{4}\int \cos(4x)\,dx-\frac{1}{4}\int \cos(6x)\,dx\\&=\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[2*x]*Sin[3*x],x]
[Out] x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

Maple [A]
time = 0.07, size = 23, normalized size = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
[Out] 1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x-\frac{1}{24}\,\sin\left(6\,x\right)-\frac{1}{16}\,\sin\left(4\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A]
time = 2.20, size = 25, normalized size = 0.83

$$-\frac{1}{12}\,\left(16\,\cos\left(x\right)^{5}-10\,\cos\left(x\right)^{3}-3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
[Out] -1/12*(16*cos(x)^5 - 10*cos(x)^3 - 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.
time = 1.06, size = 116, normalized size = 3.87

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}+\frac{\sin(x)\sin(2x)\sin(3x)}{3}+\frac{3\sin(x)\cos(2x)\cos(3x)}{8}-\frac{5\sin(3x)\cos(x)\cos(2x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
3 + 3*sin(x)*cos(2*x)*cos(3*x)/8 - 5*sin(3*x)*cos(x)*cos(2*x)/24
```

Giac [A]
time = 0.42, size = 22, normalized size = 0.73

$$\frac{1}{4}\,x-\frac{1}{24}\,\sin\left(6\,x\right)-\frac{1}{16}\,\sin\left(4\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

Mupad [B]
time = 3.01, size = 22, normalized size = 0.73

$$\frac{x}{4}+\frac{\sin\left(2\,x\right)}{8}-\frac{\sin\left(4\,x\right)}{16}-\frac{\sin\left(6\,x\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(3*x)*cos(x),x)
```

```
[Out] x/4 + sin(2*x)/8 - sin(4*x)/16 - sin(6*x)/24
```

3.883 $\int \cos(2x)\cos(3x)\sin(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

```
[Out] -1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {4440, 2718}

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

```
[Out] -1/8*Cos[2*x] + Cos[4*x]/16 - Cos[6*x]/24
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4440

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(2x)\cos(3x)\sin(x)\,dx &= \int \left(\frac{1}{4}\sin(2x)-\frac{1}{4}\sin(4x)+\frac{1}{4}\sin(6x)\right)\,dx\\
&= \frac{1}{4}\int \sin(2x)\,dx-\frac{1}{4}\int \sin(4x)\,dx+\frac{1}{4}\int \sin(6x)\,dx\\
&= -\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Cos[3*x]*Sin[x],x]
[Out] -1/8*Cos[2*x] + Cos[4*x]/16 - Cos[6*x]/24
```

Maple [A]
time = 0.06, size = 20, normalized size = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{16}-\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{16}-\frac{\cos(6x)}{24}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*sin(x),x,method=_RETURNVERBOSE)
[Out] -1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$-\frac{1}{24}\cos(6x)+\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="maxima")
[Out] -1/24*cos(6*x) + 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A]
time = 2.37, size = 19, normalized size = 0.76

$$-\frac{4}{3}\cos(x)^6+\frac{5}{2}\cos(x)^4-\frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="fricas")
[Out] -4/3*cos(x)^6 + 5/2*cos(x)^4 - 3/2*cos(x)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(19) = 38$.
time = 1.04, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}+\frac{5\sin(x)\sin(3x)\cos(2x)}{24}-\frac{\sin(2x)\sin(3x)\cos(x)}{6}-\frac{\cos(x)\cos(2x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 + 5*sin(x)*sin(3*x)*cos(2*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/6 - cos(x)*cos(2*x)*cos(3*x)/8
```

Giac [A]
time = 0.39, size = 19, normalized size = 0.76

$$-\frac{4}{3}\cos(x)^6+\frac{5}{2}\cos(x)^4-\frac{3}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="giac")
```

```
[Out] -4/3*cos(x)^6 + 5/2*cos(x)^4 - 3/2*cos(x)^2
```

Mupad [B]
time = 3.18, size = 19, normalized size = 0.76

$$\frac{4\sin(x)^6}{3}-\frac{3\sin(x)^4}{2}+\frac{\sin(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*cos(3*x)*sin(x),x)
```

```
[Out] sin(x)^2/2 - (3*sin(x)^4)/2 + (4*sin(x)^6)/3
```

3.884 $\int x \sin(x^2)\, dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\cos\left(x^2\right)$$

```
[Out] -1/2*cos(x^2)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3460, 2718}

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2],x]
```

```
[Out] -1/2*Cos[x^2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3460

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\sin\left(x^2\right)\, dx &= \frac{1}{2}\text{Subst}\left(\int \sin(x)\, dx, x, x^2\right)\\ &= -\frac{1}{2}\cos\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$-\frac{1}{2}\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2],x]
```

```
[Out] -1/2*Cos[x^2]
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\cos\left(x^2\right)}{2}$	7
default	$-\frac{\cos\left(x^2\right)}{2}$	7
risch	$-\frac{\cos\left(x^2\right)}{2}$	7
norman	$-\frac{1}{1+\tan^2\left(\frac{x^2}{2}\right)}$	15
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^2\right)}{\sqrt{\pi}}\right)}{2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(x^2)
```

Maxima [A]
time = 0.30, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x^2)
```

Fricas [A]
time = 2.16, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x^2)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.88

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2),x)
```

```
[Out] -cos(x**2)/2
```

Giac [A]
time = 0.39, size = 6, normalized size = 0.75

$$-\frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*cos(x^2)
```

Mupad [B]
time = 0.05, size = 6, normalized size = 0.75

$$-\frac{\cos\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2),x)
```

```
[Out] -cos(x^2)/2
```

3.885 $\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx$

Optimal. Leaf size=11

$$-\frac{1}{6}(\cos(x)+\sin(x))^6$$

```
[Out] -1/6*(cos(x)+sin(x))^6
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3224}

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] -1/6*(Cos[x] + Sin[x])^6
```

Rule 3224

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[(c*B - b*C)*((b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(b
^2 + c^2))), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = -\frac{1}{6}(\cos(x)+\sin(x))^6$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.05, size = 25, normalized size = 2.27

$$\frac{1}{4}\cos(4x)-\frac{5}{8}\sin(2x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

```
[Out] Cos[4*x]/4 - (5*Sin[2*x])/8 + Sin[6*x]/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(9) = 18$.
time = 0.11, size = 97, normalized size = 8.82

method	result
risch	$\frac{\sin(6x)}{24} + \frac{\cos(4x)}{4} - \frac{5\sin(2x)}{8}$
norman	$\frac{-8\left(\tan^2\left(\frac{x}{2}\right)\right) - \frac{50\left(\tan^3\left(\frac{x}{2}\right)\right)}{3} + 28\left(\tan^5\left(\frac{x}{2}\right)\right) + 16\left(\tan^6\left(\frac{x}{2}\right)\right) - 28\left(\tan^7\left(\frac{x}{2}\right)\right) + \frac{50\left(\tan^9\left(\frac{x}{2}\right)\right)}{3} - 8\left(\tan^{10}\left(\frac{x}{2}\right)\right) + 2\left(\tan^{11}\left(\frac{x}{2}\right)\right) - 2\tan\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^6}$
default	$-\frac{\left(\sin^5(x) + \frac{5\left(\sin^3(x)\right)}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6} + \frac{2\left(\sin^6(x)\right)}{3} - \frac{5\left(\cos^3(x)\right)\left(\sin^3(x)\right)}{6} - \frac{5\left(\cos^3(x)\right)\sin(x)}{8} + \frac{5\cos(x)\sin(x)}{16} + \frac{5(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+2/3*sin(x)^6-5/6*cos(x)^3*s
in(x)^3-5/8*cos(x)^3*sin(x)+5/16*cos(x)*sin(x)+5/6*cos(x)^5*sin(x)-5/24*(co
s(x)^3+3/2*cos(x))*sin(x)+2/3*cos(x)^6-1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(
x))*sin(x)
```

Maxima [A]
time = 0.29, size = 9, normalized size = 0.82

$$-\frac{1}{6}\left(\cos\left(x\right)+\sin\left(x\right)\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(cos(x) + sin(x))^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(9) = 18$.
time = 2.45, size = 34, normalized size = 3.09

$$2\cos\left(x\right)^4 - 2\cos\left(x\right)^2 + \frac{1}{3}\left(4\cos\left(x\right)^5 - 4\cos\left(x\right)^3 - 3\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="fricas")
```

```
[Out] 2*cos(x)^4 - 2*cos(x)^2 + 1/3*(4*cos(x)^5 - 4*cos(x)^3 - 3*cos(x))*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(10) = 20$.
time = 0.30, size = 46, normalized size = 4.18

$$\frac{2\sin^6\left(x\right)}{3} - \sin^5\left(x\right)\cos\left(x\right) - \frac{10\sin^3\left(x\right)\cos^3\left(x\right)}{3} - \sin\left(x\right)\cos^5\left(x\right) + \frac{2\cos^6\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))**5,x)
```

```
[Out] 2*sin(x)**6/3 - sin(x)**5*cos(x) - 10*sin(x)**3*cos(x)**3/3 - sin(x)*cos(x)
**5 + 2*cos(x)**6/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.42, size = 19, normalized size = 1.73

$$\frac{1}{4}\cos(4x) + \frac{1}{24}\sin(6x) - \frac{5}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="giac")
```

```
[Out] 1/4*cos(4*x) + 1/24*sin(6*x) - 5/8*sin(2*x)
```

Mupad [B]
time = 3.19, size = 20, normalized size = 1.82

$$-\frac{\sin(2x)\left(\sin(2x)^2 + 3\sin(2x) + 3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x) + sin(x))^5*(cos(x) - sin(x)),x)
```

```
[Out] -(sin(2*x)*(3*sin(2*x) + sin(2*x)^2 + 3))/6
```

3.886 $\int 2x \sec^2(x) \tan(x) \, dx$

Optimal. Leaf size=11

$$x \sec^2(x) - \tan(x)$$

```
[Out] x*sec(x)^2-tan(x)
```

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {12, 3842, 3852, 8}

$$x \sec^2(x) - \tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[2*x*Sec[x]^2*Tan[x],x]
```

```
[Out] x*Sec[x]^2 - Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int 2x\sec^2(x)\tan(x)\,dx &= 2\int x\sec^2(x)\tan(x)\,dx\\ &= x\sec^2(x) - \int \sec^2(x)\,dx\\ &= x\sec^2(x) + \text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= x\sec^2(x) - \tan(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.64

$$2\left(\frac{1}{2}x\sec^2(x) - \frac{\tan(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x*Sec[x]^2*Tan[x],x]
```

```
[Out] 2*((x*Sec[x]^2)/2 - Tan[x]/2)
```

Maple [A]
time = 0.05, size = 12, normalized size = 1.09

method	result	size
default	$\frac{x}{\cos(x)^2} - \tan(x)$	12
risch	$\frac{-2i e^{2ix} + 4x\, e^{2ix} - 2i}{\left(e^{2ix}+1\right)^2}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x*sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

```
[Out] x/cos(x)^2-tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(11) = 22$.
time = 0.29, size = 133, normalized size = 12.09

$$\frac{2\left(4\,x\cos\left(2\,x\right)^2 + 4\,x\sin\left(2\,x\right)^2 + \left(2\,x\cos\left(2\,x\right) + \sin\left(2\,x\right)\right)\cos\left(4\,x\right) + 2\,x\cos\left(2\,x\right) + \left(2\,x\sin\left(2\,x\right) - \cos\left(2\,x\right) - 1\right)\sin\left(4\,x\right) - \sin\left(2\,x\right)\right)}{2\left(2\,\cos\left(2\,x\right) + 1\right)\cos\left(4\,x\right) + \cos\left(4\,x\right)^2 + 4\,\cos\left(2\,x\right)^2 + \sin\left(4\,x\right)^2 + 4\,\sin\left(4\,x\right)\sin\left(2\,x\right) + 4\,\sin\left(2\,x\right)^2 + 4\,\cos\left(2\,x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 2*(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2
*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(
2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*si
n(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [A]
time = 1.59, size = 15, normalized size = 1.36

$$-\frac{\cos\left(x\right)\sin\left(x\right)-x}{\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="fricas")
[Out] -(cos(x)*sin(x) - x)/cos(x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\int x\tan\left(x\right)\sec^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)**2*tan(x),x)
[Out] 2*Integral(x*tan(x)*sec(x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(11) = 22.
time = 0.43, size = 52, normalized size = 4.73

$$\frac{x\tan\left(\frac{1}{2}x\right)^{4}+2x\tan\left(\frac{1}{2}x\right)^{2}+2\tan\left(\frac{1}{2}x\right)^{3}+x-2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{4}-2\tan\left(\frac{1}{2}x\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x*sec(x)^2*tan(x),x, algorithm="giac")
[Out] (x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(ta
n(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 3.09, size = 16, normalized size = 1.45

$$\frac{2x-\sin\left(2x\right)}{2\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x*tan(x))/cos(x)^2,x)
[Out] (2*x - sin(2*x))/(2*cos(x)^2)
```

3.887 $\int \frac{1+\cos^2(x)}{1+\cos(2x)} \, dx$

Optimal. Leaf size=12

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

```
[Out] 1/2*x+1/2*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {396, 209}

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\cos^2(x)}{1+\cos(2x)} \, dx &= \text{Subst}\left(\int \frac{2+x^2}{2+2x^2} \, dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{2}+\text{Subst}\left(\int \frac{1}{2+2x^2} \, dx, x, \tan(x)\right)\\
&= \frac{x}{2}+\frac{\tan(x)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$\frac{x}{2}+\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

```
[Out] x/2 + Tan[x]/2
```

Maple [A]
time = 0.10, size = 9, normalized size = 0.75

method	result	size
default	$\frac{x}{2}+\frac{\tan(x)}{2}$	9
risch	$\frac{x}{2}+\frac{i}{e^{2ix}+1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(1+cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x+1/2*tan(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.
time = 0.30, size = 18, normalized size = 1.50

$$\frac{1}{2}x+\frac{\sin(2x)}{2(\cos(2x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*sin(2*x)/(cos(2*x) + 1)
```

Fricas [A]
time = 1.84, size = 13, normalized size = 1.08

$$\frac{x\cos(x)+\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x) + sin(x))/cos(x)
```

Sympy [A]
time = 0.79, size = 7, normalized size = 0.58

$$\frac{x}{2} + \frac{\tan{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(1+cos(2*x)),x)
```

```
[Out] x/2 + tan(x)/2
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.67

$$\frac{1}{2}\,x + \frac{1}{2}\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)
```

Mupad [B]
time = 2.93, size = 8, normalized size = 0.67

$$\frac{x}{2} + \frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2 + 1)/(cos(2*x) + 1),x)
```

```
[Out] x/2 + tan(x)/2
```

3.888 $\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx$

Optimal. Leaf size=12

$$\log(\tan(x))+\frac{\tan^2(x)}{2}$$

[Out] ln(tan(x))+1/2*tan(x)^2

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4420, 272, 46}

$$\frac{\sec^2(x)}{2}+\log(\sin(x))-\log(\cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]

[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4420

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x^3(1-x^2)}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx, x, \cos^2(x)\right)\right)$$
$$= -\log(\cos(x))+\log(\sin(x))+\frac{\sec^2(x)}{2}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.42

$$-\log(\cos(x))+\log(\sin(x))+\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
[Out] -Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.07, size = 27, normalized size = 2.25

method	result	size
derivativedivides	$\frac{\ln(1+\cos(x))}{2}+\frac{\ln(\cos(x)-1)}{2}+\frac{1}{2\cos(x)^2}-\ln(\cos(x))$	27
default	$\frac{\ln(1+\cos(x))}{2}+\frac{\ln(\cos(x)-1)}{2}+\frac{1}{2\cos(x)^2}-\ln(\cos(x))$	27
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^2}-\ln(e^{2ix}+1)+\ln(e^{2ix}-1)$	36
norman	$\frac{2(\tan^3(\frac{x}{2}))+2(\tan^5(\frac{x}{2}))}{(1+\tan^2(\frac{x}{2}))(\tan^2(\frac{x}{2})-1)^2\tan(\frac{x}{2})}-\ln(\tan(\frac{x}{2})-1)-\ln(\tan(\frac{x}{2})+1)+\ln(\tan(\frac{x}{2}))$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3-cos(x)^5),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(1+cos(x))+1/2*ln(cos(x)-1)+1/2/cos(x)^2-ln(cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.28, size = 26, normalized size = 2.17

$$\frac{1}{2\cos(x)^2}+\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(\cos(x)-1)-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="maxima")
```

```
[Out] 1/2/cos(x)^2 + 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1) - log(cos(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(10) = 20$.
time = 1.74, size = 33, normalized size = 2.75

$$-\frac{\cos(x)^2\log\left(\cos(x)^2\right)-\cos(x)^2\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)-1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)^2*log(cos(x)^2) - cos(x)^2*log(-1/4*cos(x)^2 + 1/4) - 1)/cos(x
)^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.
time = 0.35, size = 29, normalized size = 2.42

$$\frac{\log(\cos(x)-1)}{2}+\frac{\log(\cos(x)+1)}{2}-\log(\cos(x))+\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3-cos(x)**5),x)
```

```
[Out] log(cos(x) - 1)/2 + log(cos(x) + 1)/2 - log(cos(x)) + 1/(2*cos(x)**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.
time = 0.40, size = 24, normalized size = 2.00

$$\frac{1}{2\cos(x)^2}+\frac{1}{2}\log\left(-\cos(x)^2+1\right)-\log\left(|\cos(x)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2 + 1/2*log(-cos(x)^2 + 1) - log(abs(cos(x)))
```

Mupad [B]
time = 0.09, size = 19, normalized size = 1.58

$$\frac{\ln\left(\sin(x)^2\right)}{2}-\ln(\cos(x))+\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3 - cos(x)^5),x)
```

```
[Out] log(sin(x)^2)/2 - log(cos(x)) + 1/(2*cos(x)^2)
```

3.889 $\int \sec(x) (5-11\sec^5(x))^2 \tan(x)\,dx$

Optimal. Leaf size=19

$$25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

```
[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

Rubi [A]
time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4424, 276}

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
```

```
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}
\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\left(11-5x^5\right)^2}{x^{12}}\,dx,x,\cos(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{121}{x^{12}}-\frac{110}{x^7}+\frac{25}{x^2}\right)\,dx,x,\cos(x)\right)\\
&= 25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
[Out] 25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

Maple [A]
time = 0.09, size = 18, normalized size = 0.95

method	result
derivativedivides	$25\sec(x)-\frac{55(\sec^6(x))}{3}+11(\sec^{11}(x))$
default	$25\sec(x)-\frac{55(\sec^6(x))}{3}+11(\sec^{11}(x))$
risch	$\frac{50\,e^{21ix}+500\,e^{19ix}+2250\,e^{17ix}-\frac{3520\,e^{16ix}}{3}+6000\,e^{15ix}-\frac{17600\,e^{14ix}}{3}+10500\,e^{13ix}-\frac{35200\,e^{12ix}}{3}+35128\,e^{11ix}-\frac{35200\,e^{10ix}}{3}}{(e^{2ix}+1)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*(5-11*sec(x)^5)^2*tan(x),x,method=_RETURNVERBOSE)
[Out] 25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.05

$$\frac{75\cos(x)^{10}-55\cos(x)^5+33}{3\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="maxima")
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

Fricas [A]
time = 1.45, size = 20, normalized size = 1.05

$$\frac{75\cos(x)^{10}-55\cos(x)^5+33}{3\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

Sympy [A]
time = 2.60, size = 19, normalized size = 1.00

$$11\sec^{11}(x) - \frac{55\sec^6(x)}{3} + 25\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)**5)**2*tan(x),x)
[Out] 11*sec(x)**11 - 55*sec(x)**6/3 + 25*sec(x)
```

Giac [A]
time = 0.45, size = 20, normalized size = 1.05

$$\frac{75\,\cos(x)^{10} - 55\,\cos(x)^5 + 33}{3\,\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="giac")
[Out] 1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

Mupad [B]
time = 3.67, size = 19, normalized size = 1.00

$$\frac{25\cos(x)^{10} - \frac{55\cos(x)^5}{3} + 11}{\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*(11/cos(x)^5 - 5)^2)/cos(x),x)
[Out] (25*cos(x)^10 - (55*cos(x)^5)/3 + 11)/cos(x)^11
```

3.890 $\int \sin^3(5x) \tan^3(5x) \, dx$

Optimal. Leaf size=44

$$-\frac{1}{2} \tanh^{-1}(\sin(5x)) + \frac{1}{2} \sin(5x) + \frac{1}{6} \sin^3(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x)$$

```
[Out] -1/2*arctanh(sin(5*x))+1/2*sin(5*x)+1/6*sin(5*x)^3+1/10*sin(5*x)^3*tan(5*x)
^2
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2672, 294, 308, 212}

$$\frac{1}{6} \sin^3(5x) + \frac{1}{2} \sin(5x) + \frac{1}{10} \sin^3(5x) \tan^2(5x) - \frac{1}{2} \tanh^{-1}(\sin(5x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^3,x]
```

```
[Out] -1/2*ArcTanh[Sin[5*x]] + Sin[5*x]/2 + Sin[5*x]^3/6 + (Sin[5*x]^3*Tan[5*x]^2
)/10
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(5x)\tan^3(5x)\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \sin(5x)\right)\\
&= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(5x)\right)\\
&= \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \sin(5x)\right)\\
&= \frac{1}{2}\sin(5x) + \frac{1}{6}\sin^3(5x) + \frac{1}{10}\sin^3(5x)\tan^2(5x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(5x)\right)\\
&= -\frac{1}{2}\tanh^{-1}(\sin(5x)) + \frac{1}{2}\sin(5x) + \frac{1}{6}\sin^3(5x) + \frac{1}{10}\sin^3(5x)\tan^2(5x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 1.18

$$-\frac{1}{2}\tanh^{-1}(\sin(5x)) + \frac{1}{2}\sec(5x)\tan(5x) - \frac{1}{3}\sin(5x)\tan^2(5x) - \frac{1}{15}\sin^3(5x)\tan^2(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^3,x]
[Out] -1/2*ArcTanh[Sin[5*x]] + (Sec[5*x]*Tan[5*x])/2 - (Sin[5*x]*Tan[5*x]^2)/3 -
(Sin[5*x]^3*Tan[5*x]^2)/15
```

Maple [A]
time = 0.10, size = 50, normalized size = 1.14

method	result	size
derivativedivides	$\frac{\sin^7(5x)}{10\cos(5x)^2}+\frac{(\sin^5(5x))}{10}+\frac{(\sin^3(5x))}{6}+\frac{\sin(5x)}{2}-\frac{\ln(\sec(5x)+\tan(5x))}{2}$	50
default	$\frac{\sin^7(5x)}{10\cos(5x)^2}+\frac{(\sin^5(5x))}{10}+\frac{(\sin^3(5x))}{6}+\frac{\sin(5x)}{2}-\frac{\ln(\sec(5x)+\tan(5x))}{2}$	50
risch	$\frac{i e^{15ix}}{120}-\frac{9i e^{5ix}}{40}+\frac{9i e^{-5ix}}{40}-\frac{i e^{-15ix}}{120}-\frac{i(e^{15ix}-e^{5ix})}{5(e^{10ix}+1)^2}-\frac{\ln(e^{5ix}+i)}{2}+\frac{\ln(e^{5ix}-i)}{2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*sin(5*x)^7/cos(5*x)^2+1/10*sin(5*x)^5+1/6*sin(5*x)^3+1/2*sin(5*x)-1/2*
ln(sec(5*x)+tan(5*x))
```

Maxima [A]
time = 0.31, size = 49, normalized size = 1.11

$$\frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x) + 1) + \frac{1}{4}\log(\sin(5x) - 1) + \frac{2}{5}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="maxima")
```

```
[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
1/4*log(sin(5*x) - 1) + 2/5*sin(5*x)
```

Fricas [A]
time = 2.07, size = 65, normalized size = 1.48

$$-\frac{15\cos(5x)^2\log(\sin(5x) + 1) - 15\cos(5x)^2\log(-\sin(5x) + 1) + 2\left(2\cos(5x)^4 - 14\cos(5x)^2 - 3\right)\sin(5x)}{60\cos(5x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(5*x)^2*log(sin(5*x) + 1) - 15*cos(5*x)^2*log(-sin(5*x) + 1) +
 2*(2*cos(5*x)^4 - 14*cos(5*x)^2 - 3)*sin(5*x))/cos(5*x)^2
```

Sympy [A]
time = 0.04, size = 51, normalized size = 1.16

$$\frac{\log(\sin(5x) - 1)}{4} - \frac{\log(\sin(5x) + 1)}{4} + \frac{\sin^3(5x)}{15} + \frac{2\sin(5x)}{5} - \frac{\sin(5x)}{5\cdot(2\sin^2(5x) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**3,x)
```

```
[Out] log(sin(5*x) - 1)/4 - log(sin(5*x) + 1)/4 + sin(5*x)**3/15 + 2*sin(5*x)/5 -
 sin(5*x)/(5*(2*sin(5*x)**2 - 2))
```

Giac [A]
time = 0.45, size = 51, normalized size = 1.16

$$\frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x) + 1) + \frac{1}{4}\log(-\sin(5x) + 1) + \frac{2}{5}\sin(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="giac")
```

```
[Out] 1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
1/4*log(-sin(5*x) + 1) + 2/5*sin(5*x)
```

Mupad [B]
time = 3.11, size = 69, normalized size = 1.57

$$\frac{5\tan\left(\frac{5x}{2}\right)^9+\frac{20\tan\left(\frac{5x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{5x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{5x}{2}\right)^3}{3}+5\tan\left(\frac{5x}{2}\right)}{5\left(\tan\left(\frac{5x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{5x}{2}\right)^2+1\right)^3}-\operatorname{atanh}\left(\tan\left(\frac{5x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^3,x)
```

```
[Out] (5*tan((5*x)/2) + (20*tan((5*x)/2)^3)/3 - (22*tan((5*x)/2)^5)/3 + (20*tan((
5*x)/2)^7)/3 + 5*tan((5*x)/2)^9)/(5*(tan((5*x)/2)^2 - 1)^2*(tan((5*x)/2)^2
+ 1)^3) - atanh(tan((5*x)/2))
```

3.891 $\int \sin^3(5x)\tan^4(5x)\,dx$

Optimal. Leaf size=37

$$-\frac{3}{5}\cos(5x)+\frac{1}{15}\cos^3(5x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)$$

```
[Out] -3/5*cos(5*x)+1/15*cos(5*x)^3-3/5*sec(5*x)+1/15*sec(5*x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2670, 276}

$$\frac{1}{15}\cos^3(5x)-\frac{3}{5}\cos(5x)+\frac{1}{15}\sec^3(5x)-\frac{3}{5}\sec(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-3*Cos[5*x])/5 + Cos[5*x]^3/15 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(5x)\tan^4(5x)\,dx &= -\left(\frac{1}{5}\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx,x,\cos(5x)\right)\right)\\
&= -\left(\frac{1}{5}\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)\,dx,x,\cos(5x)\right)\right)\\
&= -\frac{3}{5}\cos(5x)+\frac{1}{15}\cos^3(5x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.95

$$-\frac{11}{20}\cos(5x)+\frac{1}{60}\cos(15x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]^3*Tan[5*x]^4,x]
```

```
[Out] (-11*Cos[5*x])/20 + Cos[15*x]/60 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.
time = 0.09, size = 60, normalized size = 1.62

method	result	size
derivativedivides	$\frac{\sin^8(5x)}{15\cos(5x)^3}-\frac{\sin^8(5x)}{3\cos(5x)}-\frac{\left(\frac{16}{5}+\sin^6(5x)+\frac{6\left(\sin^4(5x)\right)}{5}+\frac{8\left(\sin^2(5x)\right)}{5}\right)\cos(5x)}{3}$	60
default	$\frac{\sin^8(5x)}{15\cos(5x)^3}-\frac{\sin^8(5x)}{3\cos(5x)}-\frac{\left(\frac{16}{5}+\sin^6(5x)+\frac{6\left(\sin^4(5x)\right)}{5}+\frac{8\left(\sin^2(5x)\right)}{5}\right)\cos(5x)}{3}$	60
risch	$\frac{e^{15ix}}{120}-\frac{11\,e^{5ix}}{40}-\frac{11\,e^{-5ix}}{40}+\frac{e^{-15ix}}{120}-\frac{2(9\,e^{25ix}+14\,e^{15ix}+9\,e^{5ix})}{15(e^{10ix}+1)^3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*sin(5*x)^8/cos(5*x)^3-1/3*sin(5*x)^8/cos(5*x)-1/3*(16/5+sin(5*x)^6+6/5
*sin(5*x)^4+8/5*sin(5*x)^2)*cos(5*x)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.89

$$\frac{1}{15}\cos(5x)^3-\frac{9\cos(5x)^2-1}{15\cos(5x)^3}-\frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="maxima")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

Fricas [A]
time = 1.88, size = 32, normalized size = 0.86

$$\frac{\cos(5x)^6-9\cos(5x)^4-9\cos(5x)^2+1}{15\cos(5x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(cos(5*x)^6 - 9*cos(5*x)^4 - 9*cos(5*x)^2 + 1)/cos(5*x)^3
```

Sympy [A]
time = 0.03, size = 34, normalized size = 0.92

$$\frac{1-9\cos^{2}(5x)}{15\cos^{3}(5x)}+\frac{\cos^{3}(5x)}{15}-\frac{3\cos(5x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)**3*tan(5*x)**4,x)
```

```
[Out] (1 - 9*cos(5*x)**2)/(15*cos(5*x)**3) + cos(5*x)**3/15 - 3*cos(5*x)/5
```

Giac [A]
time = 0.66, size = 33, normalized size = 0.89

$$\frac{1}{15}\cos(5x)^{3}-\frac{9\cos(5x)^{2}-1}{15\cos(5x)^{3}}-\frac{3}{5}\cos(5x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="giac")
```

```
[Out] 1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

Mupad [B]
time = 3.10, size = 30, normalized size = 0.81

$$\frac{(\cos(5x)+1)^{4}\left(\cos(5x)^{2}-4\cos(5x)+1\right)}{15\cos(5x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)^3*tan(5*x)^4,x)
```

```
[Out] ((cos(5*x) + 1)^4*(cos(5*x)^2 - 4*cos(5*x) + 1))/(15*cos(5*x)^3)
```

3.892 $\int \sin^5(6x)\tan^3(6x)\,dx$

Optimal. Leaf size=54

$$-\frac{7}{12}\tanh^{-1}(\sin(6x))+\frac{7}{12}\sin(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{60}\sin^5(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)$$

```
[Out] -7/12*arctanh(sin(6*x))+7/12*sin(6*x)+7/36*sin(6*x)^3+7/60*sin(6*x)^5+1/12*
sin(6*x)^5*tan(6*x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2672, 294, 308, 212}

$$\frac{7}{60}\sin^5(6x)+\frac{7}{36}\sin^3(6x)+\frac{7}{12}\sin(6x)+\frac{1}{12}\sin^5(6x)\tan^2(6x)-\frac{7}{12}\tanh^{-1}(\sin(6x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[6*x]^5*Tan[6*x]^3,x]
```

```
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sin[6*x])/12 + (7*Sin[6*x]^3)/36 + (7*Sin[6*
x]^5)/60 + (Sin[6*x]^5*Tan[6*x]^2)/12
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^5(6x)\tan^3(6x)\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{x^8}{(1-x^2)^2}\,dx, x, \sin(6x)\right) \\
&= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(6x)\right) \\
&= \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx, x, \sin(6x)\right) \\
&= \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x) - \frac{7}{12}\text{Subst}\left(\int \frac{}{1}\right. \\
&= -\frac{7}{12}\tanh^{-1}(\sin(6x)) + \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 1.26

$$-\frac{7}{12}\tanh^{-1}(\sin(6x)) + \frac{7}{12}\sec(6x)\tan(6x) - \frac{7}{18}\sin(6x)\tan^2(6x) - \frac{7}{90}\sin^3(6x)\tan^2(6x) - \frac{1}{30}\sin^5(6x)\tan^2(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[6*x]^5*Tan[6*x]^3,x]
[Out] (-7*ArcTanh[Sin[6*x]])/12 + (7*Sec[6*x]*Tan[6*x])/12 - (7*Sin[6*x]*Tan[6*x]
^2)/18 - (7*Sin[6*x]^3*Tan[6*x]^2)/90 - (Sin[6*x]^5*Tan[6*x]^2)/30
```

Maple [A]
time = 0.10, size = 58, normalized size = 1.07

method	result	s
derivativedivides	$\frac{\sin^9(6x)}{12\cos(6x)^2} + \frac{(\sin^7(6x))}{12} + \frac{7(\sin^5(6x))}{60} + \frac{7(\sin^3(6x))}{36} + \frac{7\sin(6x)}{12} - \frac{7\ln(\sec(6x)+\tan(6x))}{12}$	5
default	$\frac{\sin^9(6x)}{12\cos(6x)^2} + \frac{(\sin^7(6x))}{12} + \frac{7(\sin^5(6x))}{60} + \frac{7(\sin^3(6x))}{36} + \frac{7\sin(6x)}{12} - \frac{7\ln(\sec(6x)+\tan(6x))}{12}$	5
risch	$\frac{11ie^{18ix}}{576} - \frac{29ie^{6ix}}{96} + \frac{29ie^{-6ix}}{96} - \frac{11ie^{-18ix}}{576} - \frac{i(e^{18ix}-e^{6ix})}{6(e^{12ix}+1)^2} + \frac{7\ln(e^{6ix}-i)}{12} - \frac{7\ln(e^{6ix}+i)}{12} + \frac{\sin(30x)}{480}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^5*tan(6*x)^3,x,method=_RETURNVERBOSE)
```

[Out] 1/12*sin(6*x)^9/cos(6*x)^2+1/12*sin(6*x)^7+7/60*sin(6*x)^5+7/36*sin(6*x)^3+7/12*sin(6*x)-7/12*ln(sec(6*x)+tan(6*x))

Maxima [A]
time = 0.29, size = 57, normalized size = 1.06

$$\frac{1}{30}\sin\left(6\,x\right)^{5}+\frac{1}{9}\sin\left(6\,x\right)^{3}-\frac{\sin\left(6\,x\right)}{12\left(\sin\left(6\,x\right)^{2}-1\right)}-\frac{7}{24}\log\left(\sin\left(6\,x\right)+1\right)+\frac{7}{24}\log\left(\sin\left(6\,x\right)-1\right)+\frac{1}{2}\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="maxima")

[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*log(sin(6*x) + 1) + 7/24*log(sin(6*x) - 1) + 1/2*sin(6*x)

Fricas [A]
time = 2.77, size = 73, normalized size = 1.35

$$-\frac{105\cos\left(6\,x\right)^{2}\log\left(\sin\left(6\,x\right)+1\right)-105\cos\left(6\,x\right)^{2}\log\left(-\sin\left(6\,x\right)+1\right)-2\left(6\cos\left(6\,x\right)^{6}-32\cos\left(6\,x\right)^{4}+116\cos\left(6\,x\right)^{2}+15\right)\sin\left(6\,x\right)}{360\cos\left(6\,x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="fricas")

[Out] -1/360*(105*cos(6*x)^2*log(sin(6*x) + 1) - 105*cos(6*x)^2*log(-sin(6*x) + 1) - 2*(6*cos(6*x)^6 - 32*cos(6*x)^4 + 116*cos(6*x)^2 + 15)*sin(6*x))/cos(6*x)^2

Sympy [A]
time = 0.04, size = 61, normalized size = 1.13

$$\frac{7\log\left(\sin\left(6x\right)-1\right)}{24}-\frac{7\log\left(\sin\left(6x\right)+1\right)}{24}+\frac{\sin^{5}\left(6x\right)}{30}+\frac{\sin^{3}\left(6x\right)}{9}+\frac{\sin\left(6x\right)}{2}-\frac{\sin\left(6x\right)}{6\cdot\left(2\sin^{2}\left(6x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)**5*tan(6*x)**3,x)

[Out] 7*log(sin(6*x) - 1)/24 - 7*log(sin(6*x) + 1)/24 + sin(6*x)**5/30 + sin(6*x)**3/9 + sin(6*x)/2 - sin(6*x)/(6*(2*sin(6*x)**2 - 2))

Giac [A]
time = 0.54, size = 59, normalized size = 1.09

$$\frac{1}{30}\sin\left(6\,x\right)^{5}+\frac{1}{9}\sin\left(6\,x\right)^{3}-\frac{\sin\left(6\,x\right)}{12\left(\sin\left(6\,x\right)^{2}-1\right)}-\frac{7}{24}\log\left(\sin\left(6\,x\right)+1\right)+\frac{7}{24}\log\left(-\sin\left(6\,x\right)+1\right)+\frac{1}{2}\sin\left(6\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="giac")

```
[Out] 1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*lo
g(sin(6*x) + 1) + 7/24*log(-sin(6*x) + 1) + 1/2*sin(6*x)
```

Mupad [B]
time = 7.21, size = 85, normalized size = 1.57

$$\frac{7\tan(3x)^{13}+\frac{70\tan(3x)^{11}}{3}+\frac{77\tan(3x)^9}{5}-\frac{412\tan(3x)^7}{15}+\frac{77\tan(3x)^5}{5}+\frac{70\tan(3x)^3}{3}+7\tan(3x)}{6\left(\tan(3x)^2-1\right)^2\left(\tan(3x)^2+1\right)^5}-\frac{7\,\mathrm{atanh}(\tan(3x))}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(6*x)^5*tan(6*x)^3,x)
```

```
[Out] (7*tan(3*x) + (70*tan(3*x)^3)/3 + (77*tan(3*x)^5)/5 - (412*tan(3*x)^7)/15 +
 (77*tan(3*x)^9)/5 + (70*tan(3*x)^11)/3 + 7*tan(3*x)^13)/(6*(tan(3*x)^2 - 1
)^2*(tan(3*x)^2 + 1)^5) - (7*atanh(tan(3*x)))/6
```

3.893 $\int (-1+\sec^2(2x))^3 \sin(2x)\,dx$

Optimal. Leaf size=37

$$\frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

```
[Out] 1/2*cos(2*x)+3/2*sec(2*x)-1/2*sec(2*x)^3+1/10*sec(2*x)^5
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4205, 2670, 276}

$$\frac{1}{2}\cos(2x)+\frac{1}{10}\sec^5(2x)-\frac{1}{2}\sec^3(2x)+\frac{3}{2}\sec(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

```
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \int \sin(2x)\tan^6(2x)\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(2x)\right)\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\,dx, x, \cos(2x)\right)\right)$$
$$= \frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.00

$$\frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
[Out] Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\cos(2x)}{2}+\frac{3}{2\cos(2x)}-\frac{1}{2\cos(2x)^3}+\frac{1}{10\cos(2x)^5}$	32
default	$\frac{\cos(2x)}{2}+\frac{3}{2\cos(2x)}-\frac{1}{2\cos(2x)^3}+\frac{1}{10\cos(2x)^5}$	32
norman	$\frac{-16\left(\tan^4(x)\right)+\frac{64\left(\tan^2(x)\right)}{5}-\frac{16}{5}}{\left(\tan^2(x)-1\right)^5\left(\tan^2(x)+1\right)}$	32
risch	$\frac{e^{2ix}}{4}+\frac{e^{-2ix}}{4}+\frac{15\,e^{18ix}+40\,e^{14ix}+66\,e^{10ix}+40\,e^{6ix}+15\,e^{2ix}}{5\left(e^{4ix}+1\right)^5}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(2*x)^2)^3*sin(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*cos(2*x)+3/2/cos(2*x)-1/2/cos(2*x)^3+1/10/cos(2*x)^5
```

Maxima [A]
time = 0.30, size = 31, normalized size = 0.84

$$\frac{3}{2\,\cos\left(2\,x\right)}-\frac{1}{2\,\cos\left(2\,x\right)^3}+\frac{1}{10\,\cos\left(2\,x\right)^5}+\frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="maxima")
[Out] 3/2/cos(2*x) - 1/2/cos(2*x)^3 + 1/10/cos(2*x)^5 + 1/2*cos(2*x)
```

Fricas [A]
time = 2.89, size = 34, normalized size = 0.92

$$\frac{5\cos(2x)^6+15\cos(2x)^4-5\cos(2x)^2+1}{10\cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="fricas")
[Out] 1/10*(5*cos(2*x)^6 + 15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5
```

Sympy [A]
time = 108.92, size = 42, normalized size = 1.14

$$\frac{\cos(2x)}{2}-\frac{1}{2\left(2\cos^2(x)-1\right)^3}+\frac{1}{10\left(2\cos^2(x)-1\right)^5}+\frac{3}{4\left(\cos^2(x)-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)**2)**3*sin(2*x),x)
[Out] cos(2*x)/2 - 1/(2*(2*cos(x)**2 - 1)**3) + 1/(10*(2*cos(x)**2 - 1)**5) + 3/(
4*(cos(x)**2 - 1/2))
```

Giac [A]
time = 0.43, size = 33, normalized size = 0.89

$$\frac{15\cos(2x)^4-5\cos(2x)^2+1}{10\cos(2x)^5}+\frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="giac")
[Out] 1/10*(15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5 + 1/2*cos(2*x)
```

Mupad [B]
time = 2.94, size = 33, normalized size = 0.89

$$\frac{\cos(2x)}{2}+\frac{3\cos(2x)^4-\cos(2x)^2+\frac{1}{5}}{2\cos(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(1/cos(2*x)^2 - 1)^3,x)
[Out] cos(2*x)/2 + (3*cos(2*x)^4 - cos(2*x)^2 + 1/5)/(2*cos(2*x)^5)
```

3.894 $\int \sin(x)\tan^5(x)\,dx$

Optimal. Leaf size=34

$$\frac{15}{8}\tanh^{-1}(\sin(x))-\frac{15\sin(x)}{8}-\frac{5}{8}\sin(x)\tan^2(x)+\frac{1}{4}\sin(x)\tan^4(x)$$

```
[Out] 15/8*arctanh(sin(x))-15/8*sin(x)-5/8*sin(x)*tan(x)^2+1/4*sin(x)*tan(x)^4
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2672, 294, 327, 212}

$$-\frac{15\sin(x)}{8}+\frac{1}{4}\sin(x)\tan^4(x)-\frac{5}{8}\sin(x)\tan^2(x)+\frac{15}{8}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Tan[x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[x]])/8 - (15*Sin[x])/8 - (5*Sin[x]*Tan[x]^2)/8 + (Sin[x]*Ta
n[x]^4)/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\tan^5(x)\,dx &= \text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \sin(x)\right)\\ &= \frac{1}{4}\sin(x)\tan^4(x) - \frac{5}{4}\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(x)\right)\\ &= -\frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x) + \frac{15}{8}\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{15\sin(x)}{8} - \frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x) + \frac{15}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \frac{15}{8}\tanh^{-1}(\sin(x)) - \frac{15\sin(x)}{8} - \frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.24

$$\frac{15}{8}\tanh^{-1}(\sin(x)) + \frac{15}{8}\sec(x)\tan(x) - \frac{15}{4}\sec^3(x)\tan(x) + 5\sec(x)\tan^3(x) - \sin(x)\tan^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Tan[x]^5,x]
[Out] (15*ArcTanh[Sin[x]])/8 + (15*Sec[x]*Tan[x])/8 - (15*Sec[x]^3*Tan[x])/4 + 5*
Sec[x]*Tan[x]^3 - Sin[x]*Tan[x]^4
```

Maple [A]
time = 0.06, size = 46, normalized size = 1.35

method	result	size
default	$\frac{\sin^7(x)}{4\cos(x)^4} - \frac{3(\sin^7(x))}{8\cos(x)^2} - \frac{3(\sin^5(x))}{8} - \frac{5(\sin^3(x))}{8} - \frac{15\sin(x)}{8} + \frac{15\ln(\sec(x)+\tan(x))}{8}$	46
risch	$\frac{ie^{ix}}{2} - \frac{ie^{-ix}}{2} + \frac{i(9\,e^{7ix}+e^{5ix}-e^{3ix}-9\,e^{ix})}{4(e^{2ix}+1)^4} - \frac{15\ln(e^{ix}-i)}{8} + \frac{15\ln(e^{ix}+i)}{8}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(x)^7/cos(x)^4-3/8*sin(x)^7/cos(x)^2-3/8*sin(x)^5-5/8*sin(x)^3-15/8*
sin(x)+15/8*ln(sec(x)+tan(x))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 1.35

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8 \left(\sin(x)^4 - 2 \sin(x)^2 + 1\right)} + \frac{15}{16} \log(\sin(x) + 1) - \frac{15}{16} \log(\sin(x) - 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 15/16*log(sin(x)
+ 1) - 15/16*log(sin(x) - 1) - sin(x)
```

Fricas [A]
time = 3.04, size = 49, normalized size = 1.44

$$\frac{15 \cos(x)^4 \log(\sin(x) + 1) - 15 \cos(x)^4 \log(-\sin(x) + 1) - 2 \left(8 \cos(x)^4 + 9 \cos(x)^2 - 2\right) \sin(x)}{16 \cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(x)^4*log(sin(x) + 1) - 15*cos(x)^4*log(-sin(x) + 1) - 2*(8*cos
(x)^4 + 9*cos(x)^2 - 2)*sin(x))/cos(x)^4
```

Sympy [A]
time = 0.05, size = 49, normalized size = 1.44

$$-\frac{-9 \sin^3(x) + 7 \sin(x)}{8 \sin^4(x) - 16 \sin^2(x) + 8} - \frac{15 \log(\sin(x) - 1)}{16} + \frac{15 \log(\sin(x) + 1)}{16} - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)**5,x)
```

```
[Out] -(-9*sin(x)**3 + 7*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*log(sin(x)
 - 1)/16 + 15*log(sin(x) + 1)/16 - sin(x)
```

Giac [A]
time = 0.41, size = 42, normalized size = 1.24

$$\frac{9 \sin(x)^3 - 7 \sin(x)}{8 \left(\sin(x)^2 - 1\right)^2} + \frac{15}{16} \log(\sin(x) + 1) - \frac{15}{16} \log(-\sin(x) + 1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^2 - 1)^2 + 15/16*log(sin(x) + 1) - 15/1
6*log(-sin(x) + 1) - sin(x)
```

Mupad [B]
time = 3.04, size = 69, normalized size = 2.03

$$\frac{15\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{4}-\frac{\frac{15\tan\left(\frac{x}{2}\right)^9}{4}-10\tan\left(\frac{x}{2}\right)^7+\frac{9\tan\left(\frac{x}{2}\right)^5}{2}-10\tan\left(\frac{x}{2}\right)^3+\frac{15\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^4\left(\tan\left(\frac{x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*tan(x)^5,x)
```

```
[Out] (15*atanh(tan(x/2)))/4 - ((15*tan(x/2))/4 - 10*tan(x/2)^3 + (9*tan(x/2)^5)/
2 - 10*tan(x/2)^7 + (15*tan(x/2)^9)/4)/((tan(x/2)^2 - 1)^4*(tan(x/2)^2 + 1)
)
```

3.895 $\int \cos^5(2x) \cot^4(2x) \, dx$

Optimal. Leaf size=43

$$2 \csc(2x) - \frac{1}{6} \csc^3(2x) + 3 \sin(2x) - \frac{2}{3} \sin^3(2x) + \frac{1}{10} \sin^5(2x)$$

```
[Out] 2*csc(2*x)-1/6*csc(2*x)^3+3*sin(2*x)-2/3*sin(2*x)^3+1/10*sin(2*x)^5
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2670, 276}

$$\frac{1}{10} \sin^5(2x) - \frac{2}{3} \sin^3(2x) + 3 \sin(2x) - \frac{1}{6} \csc^3(2x) + 2 \csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^5*Cot[2*x]^4,x]
```

```
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(2x) \cot^4(2x) \, dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{(1-x^2)^4}{x^4} \, dx, x, -\sin(2x)\right)\right) \\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(6 + \frac{1}{x^4} - \frac{4}{x^2} - 4x^2 + x^4\right) \, dx, x, -\sin(2x)\right)\right) \\
&= 2 \csc(2x) - \frac{1}{6} \csc^3(2x) + 3 \sin(2x) - \frac{2}{3} \sin^3(2x) + \frac{1}{10} \sin^5(2x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$2\csc(2x)-\frac{1}{6}\csc^3(2x)+3\sin(2x)-\frac{2}{3}\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^5*Cot[2*x]^4,x]
[Out] 2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

Maple [A]
time = 0.09, size = 68, normalized size = 1.58

method	result	size
derivativedivides	$-\frac{\cos^{10}(2x)}{6\sin(2x)^3}+\frac{7(\cos^{10}(2x))}{6\sin(2x)}+\frac{7\left(\frac{128}{35}+\cos^8(2x)+\frac{8(\cos^6(2x))}{7}+\frac{48(\cos^4(2x))}{35}+\frac{64(\cos^2(2x))}{35}\right)\sin(2x)}{6}$	68
default	$-\frac{\cos^{10}(2x)}{6\sin(2x)^3}+\frac{7(\cos^{10}(2x))}{6\sin(2x)}+\frac{7\left(\frac{128}{35}+\cos^8(2x)+\frac{8(\cos^6(2x))}{7}+\frac{48(\cos^4(2x))}{35}+\frac{64(\cos^2(2x))}{35}\right)\sin(2x)}{6}$	68
risch	$-\frac{i e^{10ix}}{320}-\frac{13i e^{6ix}}{192}-\frac{41i e^{2ix}}{32}+\frac{41i e^{-2ix}}{32}+\frac{13i e^{-6ix}}{192}+\frac{i e^{-10ix}}{320}+\frac{4i(3\,e^{10ix}-5\,e^{6ix}+3\,e^{2ix})}{3(e^{4ix}-1)^3}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^5*cot(2*x)^4,x,method=_RETURNVERBOSE)
[Out] -1/6/sin(2*x)^3*cos(2*x)^10+7/6/sin(2*x)*cos(2*x)^10+7/6*(128/35+cos(2*x)^8
+8/7*cos(2*x)^6+48/35*cos(2*x)^4+64/35*cos(2*x)^2)*sin(2*x)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.95

$$\frac{1}{10}\sin(2x)^5-\frac{2}{3}\sin(2x)^3+\frac{12\sin(2x)^2-1}{6\sin(2x)^3}+3\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="maxima")
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

Fricas [A]
time = 3.12, size = 52, normalized size = 1.21

$$-\frac{3\cos(2x)^8+8\cos(2x)^6+48\cos(2x)^4-192\cos(2x)^2+128}{30\left(\cos(2x)^2-1\right)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="fricas")
```

```
[Out] -1/30*(3*cos(2*x)^8 + 8*cos(2*x)^6 + 48*cos(2*x)^4 - 192*cos(2*x)^2 + 128)/
((cos(2*x)^2 - 1)*sin(2*x))
```

Sympy [A]
time = 0.03, size = 42, normalized size = 0.98

$$\frac{12\sin^2{(2x)}-1}{6\sin^3{(2x)}}+\frac{\sin^5{(2x)}}{10}-\frac{2\sin^3{(2x)}}{3}+3\sin{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**5*cot(2*x)**4,x)
```

```
[Out] (12*sin(2*x)**2 - 1)/(6*sin(2*x)**3) + sin(2*x)**5/10 - 2*sin(2*x)**3/3 + 3
*sin(2*x)
```

Giac [A]
time = 0.41, size = 41, normalized size = 0.95

$$\frac{1}{10}\,\sin\left(2\,x\right)^{5}-\frac{2}{3}\,\sin\left(2\,x\right)^{3}+\frac{12\,\sin\left(2\,x\right)^{2}-1}{6\,\sin\left(2\,x\right)^{3}}+3\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="giac")
```

```
[Out] 1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*s
in(2*x)
```

Mupad [B]
time = 3.06, size = 42, normalized size = 0.98

$$\frac{3\sin\left(2\,x\right)^8-20\sin\left(2\,x\right)^6+90\sin\left(2\,x\right)^4+60\sin\left(2\,x\right)^2-5}{30\sin\left(2\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^5*cot(2*x)^4,x)
```

```
[Out] (60*sin(2*x)^2 + 90*sin(2*x)^4 - 20*sin(2*x)^6 + 3*sin(2*x)^8 - 5)/(30*sin(
2*x)^3)
```

3.896 $\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$

Optimal. Leaf size=87

$$-\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{15}\sin^5(3x)+\frac{4}{3}\sin^7(3x)-\frac{8}{27}\sin^9(3x)+\frac{1}{33}\mathrm{si}$$

```
[Out] -28/3*csc(3*x)+8/9*csc(3*x)^3-1/15*csc(3*x)^5-56/3*sin(3*x)+70/9*sin(3*x)^3
-56/15*sin(3*x)^5+4/3*sin(3*x)^7-8/27*sin(3*x)^9+1/33*sin(3*x)^11
```

Rubi [A]
time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3254, 4205, 2670, 276}

$$\frac{1}{33}\sin^{11}(3x)-\frac{8}{27}\sin^9(3x)+\frac{4}{3}\sin^7(3x)-\frac{56}{15}\sin^5(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{3}\sin(3x)-\frac{1}{15}\csc^5(3x)+\frac{8}{9}\csc^3(3x)-\frac{28}{3}\csc(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
```

```
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx=\int \cos^{11}(3x)\left(-1+\csc^2(3x)\right)^3\,dx$$
$$=\int \cos^{11}(3x)\cot^6(3x)\,dx$$
$$=-\left(\frac{1}{3}\text{Subst}\left(\int \frac{(1-x^2)^8}{x^6}\,dx,x,-\sin(3x)\right)\right)$$
$$=-\left(\frac{1}{3}\text{Subst}\left(\int \left(-56+\frac{1}{x^6}-\frac{8}{x^4}+\frac{28}{x^2}+70x^2-56x^4+\right.\right.\right.$$
$$=-\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 1.00

$$-\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{15}\sin^5(3x)+\frac{4}{3}\sin^7(3x)-\frac{8}{27}\sin^9(3x)+\frac{1}{33}\sin^{11}(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

```
[Out] (-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (70
*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/27
+ Sin[3*x]^11/33
```

Maple [A]
time = 0.14, size = 72, normalized size = 0.83

method	result
derivativedivides	$\frac{(\sin^{11}(3x))}{33}-\frac{8(\sin^9(3x))}{27}+\frac{4(\sin^7(3x))}{3}-\frac{56(\sin^5(3x))}{15}+\frac{70(\sin^3(3x))}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9\sin(3}$
default	$\frac{(\sin^{11}(3x))}{33}-\frac{8(\sin^9(3x))}{27}+\frac{4(\sin^7(3x))}{3}-\frac{56(\sin^5(3x))}{15}+\frac{70(\sin^3(3x))}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9\sin(3}$
risch	$\frac{23ie^{27ix}}{55296}+\frac{37ie^{21ix}}{6144}+\frac{1909ie^{15ix}}{30720}+\frac{5197ie^{9ix}}{9216}+\frac{22379ie^{3ix}}{3072}-\frac{22379ie^{-3ix}}{3072}-\frac{5197ie^{-9ix}}{9216}-\frac{1909ie^{-15ix}}{30720}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/33*sin(3*x)^11-8/27*sin(3*x)^9+4/3*sin(3*x)^7-56/15*sin(3*x)^5+70/9*sin(3
*x)^3-56/3*sin(3*x)-28/3/sin(3*x)+8/9/sin(3*x)^3-1/15/sin(3*x)^5
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.84

$$\frac{1}{33}\sin(3x)^{11}-\frac{8}{27}\sin(3x)^{9}+\frac{4}{3}\sin(3x)^{7}-\frac{56}{15}\sin(3x)^{5}+\frac{70}{9}\sin(3x)^{3}-\frac{420\sin(3x)^{4}-40\sin(3x)^{2}+3}{45\sin(3x)^{5}}-\frac{56}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="maxima"
)
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

Fricas [A]
time = 2.47, size = 92, normalized size = 1.06

$$\frac{45\cos(3x)^{16}+80\cos(3x)^{14}+160\cos(3x)^{12}+384\cos(3x)^{10}+1280\cos(3x)^{8}+10240\cos(3x)^{6}-61440\cos(3x)^{4}+81920\cos(3x)^{2}-32768}{1485\left(\cos(3x)^{4}-2\cos(3x)^{2}+1\right)\sin(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="fricas"
)
```

```
[Out] 1/1485*(45*cos(3*x)^16 + 80*cos(3*x)^14 + 160*cos(3*x)^12 + 384*cos(3*x)^10
 + 1280*cos(3*x)^8 + 10240*cos(3*x)^6 - 61440*cos(3*x)^4 + 81920*cos(3*x)^2
 - 32768)/((cos(3*x)^4 - 2*cos(3*x)^2 + 1)*sin(3*x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 73, normalized size = 0.84

$$\frac{1}{33}\sin(3x)^{11}-\frac{8}{27}\sin(3x)^{9}+\frac{4}{3}\sin(3x)^{7}-\frac{56}{15}\sin(3x)^{5}+\frac{70}{9}\sin(3x)^{3}-\frac{420\sin(3x)^{4}-40\sin(3x)^{2}+3}{45\sin(3x)^{5}}-\frac{56}{3}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 70
/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56/3
*sin(3*x)
```

Mupad [B]
time = 2.97, size = 74, normalized size = 0.85

$$-\frac{-45\sin\left(3\,x\right)^{16}+440\sin\left(3\,x\right)^{14}-1980\sin\left(3\,x\right)^{12}+5544\sin\left(3\,x\right)^{10}-11550\sin\left(3\,x\right)^{8}+27720\sin\left(3\,x\right)^{6}+13860\sin\left(3\,x\right)^{4}-1320\sin\left(3\,x\right)^{2}+99}{1485\sin\left(3\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^5,x)
```

```
[Out] -(13860*sin(3*x)^4 - 1320*sin(3*x)^2 + 27720*sin(3*x)^6 - 11550*sin(3*x)^8
+ 5544*sin(3*x)^10 - 1980*sin(3*x)^12 + 440*sin(3*x)^14 - 45*sin(3*x)^16 +
99)/(1485*sin(3*x)^5)
```

3.897 $\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx$

Optimal. Leaf size=42

$$\csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

```
[Out] csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

Rubi [A]
time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {3254, 4445, 272, 45}

$$\frac{1}{8}\sin^4(2x)-\sin^2(2x)-\frac{1}{8}\csc^4(2x)+\csc^2(2x)+3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4445

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx=\int \cos^4(2x)\cot(2x)\left(-1+\csc^2(2x)\right)^2\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int \frac{(1-x^2)^4}{x^5}\,dx,x,\sin(2x)\right)$$
$$=\frac{1}{4}\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx,x,\sin^2(2x)\right)$$
$$=\frac{1}{4}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx,x,\sin^2(2x)\right)$$
$$=\csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\text{sin}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Maple [A]
time = 0.12, size = 37, normalized size = 0.88

method	result	size
derivativedivides	$\frac{(\sin^4(2x))}{8}+\cos^2(2x)+3\ln(\sin(2x))+\frac{1}{\sin(2x)^2}-\frac{1}{8\sin(2x)^4}$	37
default	$\frac{(\sin^4(2x))}{8}+\cos^2(2x)+3\ln(\sin(2x))+\frac{1}{\sin(2x)^2}-\frac{1}{8\sin(2x)^4}$	37
risch	$-6ix+\frac{e^{8ix}}{128}+\frac{7e^{4ix}}{32}+\frac{7e^{-4ix}}{32}+\frac{e^{-8ix}}{128}-\frac{2(2e^{12ix}-3e^{8ix}+2e^{4ix})}{(e^{4ix}-1)^4}+3\ln(e^{4ix}-1)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/8*sin(2*x)^4+cos(2*x)^2+3*ln(sin(2*x))+1/sin(2*x)^2-1/8/sin(2*x)^4
```

Maxima [A]
time = 0.29, size = 44, normalized size = 1.05

$$\frac{1}{8}\sin(2x)^4-\sin(2x)^2+\frac{8\sin(2x)^2-1}{8\sin(2x)^4}+\frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.
time = 2.59, size = 79, normalized size = 1.88

$$\frac{8 \cos(2x)^8 + 32 \cos(2x)^6 - 115 \cos(2x)^4 + 38 \cos(2x)^2 + 192 \left(\cos(2x)^4 - 2 \cos(2x)^2 + 1\right) \log\left(\frac{1}{2} \sin(2x)\right) + 29}{64 \left(\cos(2x)^4 - 2 \cos(2x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)**2)**2*(1-sin(2*x)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.42, size = 52, normalized size = 1.24

$$\frac{1}{8} \cos(2x)^4 + \frac{3}{4} \cos(2x)^2 - \frac{8 \cos(2x)^2 - 7}{8 \left(\cos(2x)^2 - 1\right)^2} + \frac{3}{2} \log\left(-\cos(2x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2
 + 3/2*log(-cos(2*x)^2 + 1)
```

Mupad [B]
time = 3.16, size = 71, normalized size = 1.69

$$3 \ln(\tan(2x)) - \frac{3 \ln\left(\tan(2x)^2 + 1\right)}{2} + \frac{3 \tan(2x)^6 + \frac{9 \tan(2x)^4}{2} + \tan(2x)^2 - \frac{1}{4}}{2 \left(\tan(2x)^8 + 2 \tan(2x)^6 + \tan(2x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)*(1/sin(2*x)^2 - 1)^2*(sin(2*x)^2 - 1)^2,x)
```

```
[Out] 3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)/
2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

3.898 $\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx$

Optimal. Leaf size=63

$$10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x)-\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

```
[Out] 10*csc(2*x)-5/2*csc(2*x)^3+3/5*csc(2*x)^5-1/14*csc(2*x)^7+15/2*sin(2*x)-sin
(2*x)^3+1/10*sin(2*x)^5
```

Rubi [A]
time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3254, 4205, 2670, 276}

$$\frac{1}{10}\sin^5(2x)-\sin^3(2x)+\frac{15}{2}\sin(2x)-\frac{1}{14}\csc^7(2x)+\frac{3}{5}\csc^5(2x)-\frac{5}{2}\csc^3(2x)+10\csc(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4205

```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
b^p, Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx &= \int \cos^5(2x)\left(-1+\csc^2(2x)\right)^4\,dx\\ &= \int \cos^5(2x)\cot^8(2x)\,dx\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x^2)^6}{x^8}\,dx,x,-\sin(2x)\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \left(15+\frac{1}{x^8}-\frac{6}{x^6}+\frac{15}{x^4}-\frac{20}{x^2}-6x^2+x^4\right)\right.\right.\\ &= 10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.00

$$10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x)-\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

```
[Out] 10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Sin
[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

Maple [A]
time = 0.12, size = 56, normalized size = 0.89

method	result
derivativedivides	$\frac{(\sin^5(2x))}{10}-(\sin^3(2x))+\frac{15\sin(2x)}{2}+\frac{10}{\sin(2x)}-\frac{5}{2\sin(2x)^3}+\frac{3}{5\sin(2x)^5}-\frac{1}{14\sin(2x)^7}$
default	$\frac{(\sin^5(2x))}{10}-(\sin^3(2x))+\frac{15\sin(2x)}{2}+\frac{10}{\sin(2x)}-\frac{5}{2\sin(2x)^3}+\frac{3}{5\sin(2x)^5}-\frac{1}{14\sin(2x)^7}$
risch	$-\frac{ie^{10ix}}{320}-\frac{7ie^{6ix}}{64}-\frac{109ie^{2ix}}{32}+\frac{109ie^{-2ix}}{32}+\frac{7ie^{-6ix}}{64}+\frac{ie^{-10ix}}{320}+\frac{4i(175\,e^{26ix}-875\,e^{22ix}+2093\,e^{18ix}-2706\,e}{35(e^{4ix}-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*sin(2*x)^5-sin(2*x)^3+15/2*sin(2*x)+10/sin(2*x)-5/2/sin(2*x)^3+3/5/sin
(2*x)^5-1/14/sin(2*x)^7
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.90

$$\frac{1}{10}\sin\left(2x\right)^{5}-\sin\left(2x\right)^{3}+\frac{700\sin\left(2x\right)^{6}-175\sin\left(2x\right)^{4}+42\sin\left(2x\right)^{2}-5}{70\sin\left(2x\right)^{7}}+\frac{15}{2}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

Fricas [A]
time = 2.21, size = 84, normalized size = 1.33

$$-\frac{7\cos\left(2x\right)^{12}+28\cos\left(2x\right)^{10}+280\cos\left(2x\right)^{8}-2240\cos\left(2x\right)^{6}+4480\cos\left(2x\right)^{4}-3584\cos\left(2x\right)^{2}+1024}{70\left(\cos\left(2x\right)^{6}-3\cos\left(2x\right)^{4}+3\cos\left(2x\right)^{2}-1\right)\sin\left(2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/70*(7*cos(2*x)^12 + 28*cos(2*x)^10 + 280*cos(2*x)^8 - 2240*cos(2*x)^6 +
4480*cos(2*x)^4 - 3584*cos(2*x)^2 + 1024)/((cos(2*x)^6 - 3*cos(2*x)^4 + 3*c
os(2*x)^2 - 1)*sin(2*x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)**2)**4*(1-sin(2*x)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 57, normalized size = 0.90

$$\frac{1}{10}\sin\left(2x\right)^{5}-\sin\left(2x\right)^{3}+\frac{700\sin\left(2x\right)^{6}-175\sin\left(2x\right)^{4}+42\sin\left(2x\right)^{2}-5}{70\sin\left(2x\right)^{7}}+\frac{15}{2}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*s
in(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

Mupad [B]
time = 2.97, size = 57, normalized size = 0.90

$$\frac{\frac{\sin(2x)^{12}}{10}-\sin\left(2x\right)^{10}+\frac{15\sin(2x)^8}{2}+10\sin\left(2x\right)^6-\frac{5\sin(2x)^4}{2}+\frac{3\sin(2x)^2}{5}-\frac{1}{14}}{\sin\left(2x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(1/sin(2*x)^2 - 1)^4*(sin(2*x)^2 - 1)^2,x)
```

```
[Out] ((3*sin(2*x)^2)/5 - (5*sin(2*x)^4)/2 + 10*sin(2*x)^6 + (15*sin(2*x)^8)/2 -
sin(2*x)^10 + sin(2*x)^12/10 - 1/14)/sin(2*x)^7
```

3.899 $\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx$

Optimal. Leaf size=60

$$-\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3x))+\frac{5}{6}\sin^2(3x)-\frac{1}{12}\sin^4(3x)$$

```
[Out] -5/3*csc(3*x)^2+5/12*csc(3*x)^4-1/18*csc(3*x)^6-10/3*ln(sin(3*x))+5/6*sin(3
*x)^2-1/12*sin(3*x)^4
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3254, 4445, 272, 45}

$$-\frac{1}{12}\sin^4(3x)+\frac{5}{6}\sin^2(3x)-\frac{1}{18}\csc^6(3x)+\frac{5}{12}\csc^4(3x)-\frac{5}{3}\csc^2(3x)-\frac{10}{3}\log(\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

```
[Out] (-5*Csc[3*x]^2)/3 + (5*Csc[3*x]^4)/12 - Csc[3*x]^6/18 - (10*Log[Sin[3*x]])/
3 + (5*Sin[3*x]^2)/6 - Sin[3*x]^4/12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 4445

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
```

```
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx &= \int \cos^4(3x)\cot(3x)\left(-1+\csc^2(3x)\right)^3\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{x^7}\,dx, x, \sin(3x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{(1-x)^5}{x^4}\,dx, x, \sin^2(3x)\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \left(5+\frac{1}{x^4}-\frac{5}{x^3}+\frac{10}{x^2}-\frac{10}{x}-x\right)dx, x, \sin^2(3\right.\\
&= -\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.87

$$\frac{1}{36}\left(-60\csc^2(3x)+15\csc^4(3x)-2\csc^6(3x)-120\log(\sin(3x))+30\sin^2(3x)-3\sin^4(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
[Out] (-60*Csc[3*x]^2 + 15*Csc[3*x]^4 - 2*Csc[3*x]^6 - 120*Log[Sin[3*x]] + 30*Sin
[3*x]^2 - 3*Sin[3*x]^4)/36
```

Maple [A]
time = 0.13, size = 49, normalized size = 0.82

method	result
derivativedivides	$-\frac{(\sin^4(3x))}{12}-\frac{5(\cos^2(3x))}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3\sin(3x)^2}+\frac{5}{12\sin(3x)^4}-\frac{1}{18\sin(3x)^6}$
default	$-\frac{(\sin^4(3x))}{12}-\frac{5(\cos^2(3x))}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3\sin(3x)^2}+\frac{5}{12\sin(3x)^4}-\frac{1}{18\sin(3x)^6}$
risch	$10ix-\frac{e^{12ix}}{192}-\frac{3e^{6ix}}{16}-\frac{3e^{-6ix}}{16}-\frac{e^{-12ix}}{192}+\frac{\frac{20e^{30ix}}{3}-20e^{24ix}+\frac{272e^{18ix}}{9}-20e^{12ix}+\frac{20e^{6ix}}{3}}{(e^{6ix}-1)^6}-\frac{10\ln(e^{6ix}-1}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*sin(3*x)^4-5/6*cos(3*x)^2-10/3*ln(sin(3*x))-5/3/sin(3*x)^2+5/12/sin(3
*x)^4-1/18/sin(3*x)^6
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.87

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2-\frac{60\sin(3x)^4-15\sin(3x)^2+2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="maxima"
)
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 - 1/36*(60*sin(3*x)^4 - 15*sin(3*x)^2 + 2
)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(48) = 96$.
time = 3.17, size = 103, normalized size = 1.72

$$-\frac{24\cos(3x)^{10}+120\cos(3x)^8-609\cos(3x)^6+387\cos(3x)^4+333\cos(3x)^2+960\left(\cos(3x)^6-3\cos(3x)^4+3\cos(3x)^2-1\right)\log\left(\frac{1}{2}\sin(3x)\right)-271}{288\left(\cos(3x)^6-3\cos(3x)^4+3\cos(3x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="fricas"
)
```

```
[Out] -1/288*(24*cos(3*x)^10 + 120*cos(3*x)^8 - 609*cos(3*x)^6 + 387*cos(3*x)^4 +
 333*cos(3*x)^2 + 960*(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)*log(1/
2*sin(3*x)) - 271)/(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.46, size = 60, normalized size = 1.00

$$-\frac{1}{12}\sin(3x)^4+\frac{5}{6}\sin(3x)^2+\frac{110\sin(3x)^6-60\sin(3x)^4+15\sin(3x)^2-2}{36\sin(3x)^6}-\frac{5}{3}\log\left(\sin(3x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 + 1/36*(110*sin(3*x)^6 - 60*sin(3*x)^4 +
15*sin(3*x)^2 - 2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

Mupad [B]
time = 4.67, size = 84, normalized size = 1.40

$$\frac{\ln\left(\left(\tan(3\,x)^2+1\right)^5\right)}{3}-\frac{10\,\ln\left(\tan(3\,x)\right)}{3}-\frac{5\tan(3\,x)^8+\frac{15\tan(3\,x)^6}{2}+\frac{5\tan(3\,x)^4}{3}-\frac{5\tan(3\,x)^2}{12}+\frac{1}{6}}{3\,\left(\tan\left(3\,x\right)^{10}+2\tan\left(3\,x\right)^8+\tan\left(3\,x\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^2,x)
```

```
[Out] log((tan(3*x)^2 + 1)^5)/3 - (10*log(tan(3*x)))/3 - ((5*tan(3*x)^4)/3 - (5*t
an(3*x)^2)/12 + (15*tan(3*x)^6)/2 + 5*tan(3*x)^8 + 1/6)/(3*(tan(3*x)^6 + 2*
tan(3*x)^8 + tan(3*x)^10))
```

3.900 $\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx$

Optimal. Leaf size=47

$$-\frac{4}{9}\cot(9x)-\frac{1}{27}\cot^3(9x)+\frac{2}{3}\tan(9x)+\frac{4}{27}\tan^3(9x)+\frac{1}{45}\tan^5(9x)$$

```
[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3738, 2700, 276}

$$\frac{1}{45}\tan^5(9x)+\frac{4}{27}\tan^3(9x)+\frac{2}{3}\tan(9x)-\frac{1}{27}\cot^3(9x)-\frac{4}{9}\cot(9x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-4*Cot[9*x])/9 - Cot[9*x]^3/27 + (2*Tan[9*x])/3 + (4*Tan[9*x]^3)/27 + Tan[
9*x]^5/45
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3738

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rubi steps

$$\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3\, dx = \int \left(1+\cot^2(9x)\right)^2 \sec^6(9x)\, dx$$
$$= \int \csc^4(9x)\sec^6(9x)\, dx$$
$$= \frac{1}{9}\text{Subst}\left(\int \frac{(1+x^2)^4}{x^4}\, dx, x, \tan(9x)\right)$$
$$= \frac{1}{9}\text{Subst}\left(\int \left(6+\frac{1}{x^4}+\frac{4}{x^2}+4x^2+x^4\right)\, dx, x, \tan(9x)\right)$$
$$= -\frac{4}{9}\cot(9x)-\frac{1}{27}\cot^3(9x)+\frac{2}{3}\tan(9x)+\frac{4}{27}\tan^3(9x)+\frac{1}{45}\tan^5(9$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 1.26

$$-\frac{11}{27}\cot(9x)-\frac{1}{27}\cot(9x)\csc^2(9x)+\frac{73}{135}\tan(9x)+\frac{14}{135}\sec^2(9x)\tan(9x)+\frac{1}{45}\sec^4(9x)\tan(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

```
[Out] (-11*Cot[9*x])/27 - (Cot[9*x]*Csc[9*x]^2)/27 + (73*Tan[9*x])/135 + (14*Sec[
9*x]^2*Tan[9*x])/135 + (Sec[9*x]^4*Tan[9*x])/45
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{4\cot(9x)}{9}-\frac{(\cot^3(9x))}{27}+\frac{2\tan(9x)}{3}+\frac{4(\tan^3(9x))}{27}+\frac{(\tan^5(9x))}{45}$	38
default	$-\frac{4\cot(9x)}{9}-\frac{(\cot^3(9x))}{27}+\frac{2\tan(9x)}{3}+\frac{4(\tan^3(9x))}{27}+\frac{(\tan^5(9x))}{45}$	38
norman	$\frac{-\frac{1}{27}-\frac{4\left(\tan^2(9x)\right)}{9}+\frac{2\left(\tan^4(9x)\right)}{3}+\frac{4\left(\tan^6(9x)\right)}{27}+\frac{\left(\tan^8(9x)\right)}{45}}{\tan(9x)^3}$	42
risch	$\frac{256i(6\,e^{54ix}+2\,e^{36ix}-2\,e^{18ix}-1)}{135(e^{18ix}+1)^5(e^{18ix}-1)^3}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

Maxima [A]
time = 0.30, size = 41, normalized size = 0.87

$$\frac{1}{45}\,\tan\left(9\,x\right)^{5}+\frac{4}{27}\,\tan\left(9\,x\right)^{3}-\frac{12\,\tan\left(9\,x\right)^{2}+1}{27\,\tan\left(9\,x\right)^{3}}+\frac{2}{3}\,\tan\left(9\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 + 2
/3*tan(9*x)
```

Fricas [A]
time = 2.10, size = 42, normalized size = 0.89

$$\frac{3\,\tan\left(9\,x\right)^{8}+20\,\tan\left(9\,x\right)^{6}+90\,\tan\left(9\,x\right)^{4}-60\,\tan\left(9\,x\right)^{2}-5}{135\,\tan\left(9\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/135*(3*tan(9*x)^8 + 20*tan(9*x)^6 + 90*tan(9*x)^4 - 60*tan(9*x)^2 - 5)/ta
n(9*x)^3
```

Sympy [A]
time = 3.32, size = 44, normalized size = 0.94

$$\frac{\tan^{5}\left(9x\right)}{45}+\frac{4\tan^{3}\left(9x\right)}{27}+\frac{2\tan\left(9x\right)}{3}-\frac{4}{9\tan\left(9x\right)}-\frac{1}{27\tan^{3}\left(9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)**2)**2*(1+tan(9*x)**2)**3,x)
```

```
[Out] tan(9*x)**5/45 + 4*tan(9*x)**3/27 + 2*tan(9*x)/3 - 4/(9*tan(9*x)) - 1/(27*t
an(9*x)**3)
```

Giac [A]
time = 60.80, size = 41, normalized size = 0.87

$$\frac{1}{45}\,\tan\left(9\,x\right)^{5}+\frac{4}{27}\,\tan\left(9\,x\right)^{3}-\frac{12\,\tan\left(9\,x\right)^{2}+1}{27\,\tan\left(9\,x\right)^{3}}+\frac{2}{3}\,\tan\left(9\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 + 2
/3*tan(9*x)
```

Mupad [B]
time = 5.38, size = 42, normalized size = 0.89

$$\frac{3\tan(9\,x)^8+20\tan(9\,x)^6+90\tan(9\,x)^4-60\tan(9\,x)^2-5}{135\tan\left(9\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(9*x)^2 + 1)^3*(cot(9*x)^2 + 1)^2,x)
```

```
[Out] (90*tan(9*x)^4 - 60*tan(9*x)^2 + 20*tan(9*x)^6 + 3*tan(9*x)^8 - 5)/(135*tan
(9*x)^3)
```

3.901 $\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx$

Optimal. Leaf size=43

$$-2\log(1-\sin(x))+128\log(1+\sin(x))-49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}$$

```
[Out] -2*ln(1-sin(x))+128*ln(1+sin(x))-49*sin(x)+63*sin(x)^2-49/3*sin(x)^3-49/5*s
in(x)^5
```

Rubi [A]
time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3254, 3302, 1824, 647, 31}

$$-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)-2\log(1-\sin(x))+128\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] -2*Log[1 - Sin[x]] + 128*Log[1 + Sin[x]] - 49*Sin[x] + 63*Sin[x]^2 - (49*Si
n[x]^3)/3 - (49*Sin[x]^5)/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3254

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3302

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx &= \int \sec(x)\left(9-7\sin^3(x)\right)^2\,dx\\ &= \text{Subst}\left(\int \frac{(9-7x^3)^2}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(-49+126x-49x^2-49x^4+\frac{2(65-63x)}{1-x^2}\right)dx, x, \sin(x)\right)\\ &= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\text{Subst}\left(\int \frac{65-63x}{1-x^2}\,d\right.\\ &= -49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}+2\text{Subst}\left(\int \frac{1}{1-x}\,dx, x\right.\\ &= -2\log(1-\sin(x))+128\log(1+\sin(x))-49\sin(x)+63\sin^2(x)-\frac{49\sin^3(}{3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.65

$$49\tanh^{-1}(\sin(x))-63\cos^2(x)+126\log(\cos(x))-81\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+81\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-49\sin(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

```
[Out] 49*ArcTanh[Sin[x]] - 63*Cos[x]^2 + 126*Log[Cos[x]] - 81*Log[Cos[x/2] - Sin[
x/2]] + 81*Log[Cos[x/2] + Sin[x/2]] - 49*Sin[x] - (49*Sin[x]^3)/3 - (49*Sin
[x]^5)/5
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.88

method	result
derivativedivides	$-\frac{49(\sin^5(x))}{5}-\frac{49(\sin^3(x))}{3}+63(\sin^2(x))-49\sin(x)-2\ln(\sin(x)-1)+128\ln(1+\sin$

default	$-\frac{49(\sin^5(x))}{5}-\frac{49(\sin^3(x))}{3}+63(\sin^2(x))-49\sin(x)-2\ln(\sin(x)-1)+128\ln(1+\sin(x$
risch	$-126ix+\frac{539i\mathrm{e}^{ix}}{16}-\frac{539i\mathrm{e}^{-ix}}{16}-4\ln(\mathrm{e}^{ix}-i)+256\ln(\mathrm{e}^{ix}+i)-\frac{49\sin(5x)}{80}+\frac{343\sin(3x)}{48}-\frac{63\,\mathrm{co}}{2}$
norman	$\frac{-288(\tan^{14}(\frac{x}{2}))-252(\tan^{12}(\frac{x}{2}))-90(\tan^{16}(\frac{x}{2}))+252(\tan^4(\frac{x}{2}))+288(\tan^2(\frac{x}{2}))+\frac{1862(\tan^3(\frac{x}{2}))}{3}+\frac{7938(\tan^5(\frac{x}{2}))}{5}+\frac{159}{}}{(1+\tan^2(\frac{x}{2}))^7(\tan^2(\frac{x}{2})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -49/5*sin(x)^5-49/3*sin(x)^3+63*sin(x)^2-49*sin(x)-2*ln(sin(x)-1)+128*ln(1+
sin(x))
```

Maxima [A]
time = 0.30, size = 37, normalized size = 0.86

$$-\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(\sin(x)-1)-49\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="maxima")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
sin(x) - 1) - 49*sin(x)
```

Fricas [A]
time = 2.66, size = 41, normalized size = 0.95

$$-63\cos(x)^2-\frac{49}{15}\left(3\cos(x)^4-11\cos(x)^2+23\right)\sin(x)+128\log(\sin(x)+1)-2\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="fricas")
```

```
[Out] -63*cos(x)^2 - 49/15*(3*cos(x)^4 - 11*cos(x)^2 + 23)*sin(x) + 128*log(sin(x
) + 1) - 2*log(-sin(x) + 1)
```

Sympy [A]
time = 0.51, size = 44, normalized size = 1.02

$$-2\log(\sin(x)-1)+128\log(\sin(x)+1)-\frac{49\sin^5(x)}{5}-\frac{49\sin^3(x)}{3}-49\sin(x)-63\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)**3)**2/(1-sin(x)**2),x)
```

```
[Out] -2*log(sin(x) - 1) + 128*log(sin(x) + 1) - 49*sin(x)**5/5 - 49*sin(x)**3/3
- 49*sin(x) - 63*cos(x)**2
```

Giac [A]
time = 0.39, size = 39, normalized size = 0.91

$$-\frac{49}{5}\sin\left(x\right)^{5}-\frac{49}{3}\sin\left(x\right)^{3}+63\sin\left(x\right)^{2}+128\log\left(\sin\left(x\right)+1\right)-2\log\left(-\sin\left(x\right)+1\right)-49\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="giac")
```

```
[Out] -49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log(
-sin(x) + 1) - 49*sin(x)
```

Mupad [B]
time = 0.08, size = 37, normalized size = 0.86

$$128\ln\left(\sin\left(x\right)+1\right)-2\ln\left(\sin\left(x\right)-1\right)-49\sin\left(x\right)+63\sin\left(x\right)^{2}-\frac{49\sin\left(x\right)^{3}}{3}-\frac{49\sin\left(x\right)^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(cos(x)*(7*sin(x)^3 - 9)^2)/(sin(x)^2 - 1),x)
```

```
[Out] 128*log(sin(x) + 1) - 2*log(sin(x) - 1) - 49*sin(x) + 63*sin(x)^2 - (49*sin
(x)^3)/3 - (49*sin(x)^5)/5
```

3.902 $\int \cos^4(2x)\cot^5(2x)\,dx$

Optimal. Leaf size=42

$$\csc^2(2x) - \frac{1}{8}\csc^4(2x) + 3\log(\sin(2x)) - \sin^2(2x) + \frac{1}{8}\sin^4(2x)$$

```
[Out] csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2670, 272, 45}

$$\frac{1}{8}\sin^4(2x) - \sin^2(2x) - \frac{1}{8}\csc^4(2x) + \csc^2(2x) + 3\log(\sin(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^4*Cot[2*x]^5,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(1-x^2)^4}{x^5}\,dx, x, -\sin(2x)\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(2x)\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, \sin^2(2x)\right)$$
$$= \csc^2(2x) - \frac{1}{8}\csc^4(2x) + 3\log(\sin(2x)) - \sin^2(2x) + \frac{1}{8}\sin^4(2x)$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$\csc^2(2x) - \frac{1}{8}\csc^4(2x) + 3\log(\sin(2x)) - \sin^2(2x) + \frac{1}{8}\sin^4(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^4*Cot[2*x]^5,x]
```

```
[Out] Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

Maple [A]
time = 0.09, size = 69, normalized size = 1.64

method	result
derivativedivides	$-\frac{\cos^{10}(2x)}{8\sin(2x)^4}+\frac{3(\cos^{10}(2x))}{8\sin(2x)^2}+\frac{3(\cos^{8}(2x))}{8}+\frac{(\cos^{6}(2x))}{2}+\frac{3(\cos^{4}(2x))}{4}+\frac{3(\cos^{2}(2x))}{2}+3\ln(\sin(2x))$
default	$-\frac{\cos^{10}(2x)}{8\sin(2x)^4}+\frac{3(\cos^{10}(2x))}{8\sin(2x)^2}+\frac{3(\cos^{8}(2x))}{8}+\frac{(\cos^{6}(2x))}{2}+\frac{3(\cos^{4}(2x))}{4}+\frac{3(\cos^{2}(2x))}{2}+3\ln(\sin(2x))$
risch	$-6ix+\frac{e^{8ix}}{128}+\frac{7\,e^{4ix}}{32}+\frac{7\,e^{-4ix}}{32}+\frac{e^{-8ix}}{128}-\frac{2(2\,e^{12ix}-3\,e^{8ix}+2\,e^{4ix})}{(e^{4ix}-1)^4}+3\ln(e^{4ix}-1)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*cot(2*x)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/sin(2*x)^4*cos(2*x)^10+3/8/sin(2*x)^2*cos(2*x)^10+3/8*cos(2*x)^8+1/2*c
os(2*x)^6+3/4*cos(2*x)^4+3/2*cos(2*x)^2+3*ln(sin(2*x))
```

Maxima [A]
time = 0.28, size = 44, normalized size = 1.05

$$\frac{1}{8}\sin(2x)^4 - \sin(2x)^2 + \frac{8\sin(2x)^2 - 1}{8\sin(2x)^4} + \frac{3}{2}\log\left(\sin(2x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(s
in(2*x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.

time = 2.84, size = 79, normalized size = 1.88

$$\frac{8\cos(2x)^8+32\cos(2x)^6-115\cos(2x)^4+38\cos(2x)^2+192\left(\cos(2x)^4-2\cos(2x)^2+1\right)\log\left(\frac{1}{2}\sin(2x)\right)+29}{64\left(\cos(2x)^4-2\cos(2x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*(
cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*cos(
2*x)^2 + 1)
```

Sympy [A]

time = 0.04, size = 41, normalized size = 0.98

$$\frac{8\sin^2(2x)-1}{8\sin^4(2x)}+3\log(\sin(2x))+\frac{\sin^4(2x)}{8}-\sin^2(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**4*cot(2*x)**5,x)
```

```
[Out] (8*sin(2*x)**2 - 1)/(8*sin(2*x)**4) + 3*log(sin(2*x)) + sin(2*x)**4/8 - sin
(2*x)**2
```

Giac [A]

time = 0.41, size = 52, normalized size = 1.24

$$\frac{1}{8}\cos(2x)^4+\frac{3}{4}\cos(2x)^2-\frac{8\cos(2x)^2-7}{8\left(\cos(2x)^2-1\right)^2}+\frac{3}{2}\log\left(-\cos(2x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="giac")
```

```
[Out] 1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^2
 + 3/2*log(-cos(2*x)^2 + 1)
```

Mupad [B]

time = 3.05, size = 71, normalized size = 1.69

$$3\ln(\tan(2x))-\frac{3\ln\left(\tan(2x)^2+1\right)}{2}+\frac{3\tan(2x)^6+\frac{9\tan(2x)^4}{2}+\tan(2x)^2-\frac{1}{4}}{2\left(\tan(2x)^8+2\tan(2x)^6+\tan(2x)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*cot(2*x)^5,x)
```

```
[Out] 3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)/
2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

3.903 $\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$

Optimal. Leaf size=74

$$-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{\tan(x)}{3}$$

```
[Out] -4/9*arctanh(sin(x))-1/9*ln(-sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/9*ln(
sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/3*tan(x)
```

Rubi [A]
time = 0.17, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4482, 2802, 3135, 3080, 3855, 2738, 212}

$$\frac{\tan(x)}{3}-\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{7}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

```
[Out] (-4*ArcTanh[Sin[x]])/9 - (Sqrt[7]*Log[Sqrt[7]*Cos[x/2] - Sin[x/2]])/9 + (Sq
rt[7]*Log[Sqrt[7]*Cos[x/2] + Sin[x/2]])/9 + Tan[x]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
```

```
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx &= \int \frac{\tan^2(x)}{3+4\cos(x)}\,dx\\
&= \int \frac{(1-\cos^2(x))\sec^2(x)}{3+4\cos(x)}\,dx\\
&= \frac{\tan(x)}{3}+\frac{1}{3}\int \frac{(-4-3\cos(x))\sec(x)}{3+4\cos(x)}\,dx\\
&= \frac{\tan(x)}{3}-\frac{4}{9}\int \sec(x)\,dx+\frac{7}{9}\int \frac{1}{3+4\cos(x)}\,dx\\
&= -\frac{4}{9}\tanh^{-1}(\sin(x))+\frac{\tan(x)}{3}+\frac{14}{9}\text{Subst}\left(\int \frac{1}{7-x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\
&= -\frac{4}{9}\tanh^{-1}(\sin(x))-\frac{1}{9}\sqrt{7}\,\log\left(\sqrt{7}\,\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{9}\sqrt{7}\,\log\left(\sqrt{7}\,\cos\left(\frac{x}{2}\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.85

$$\frac{1}{9}\left(2\sqrt{7}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)+4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-4\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+3\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
[Out] (2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]] + 4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[C
os[x/2] + Sin[x/2]] + 3*Tan[x])/9
```

Maple [A]
time = 0.09, size = 55, normalized size = 0.74

method	result	size
default	$-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{4\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{9}+\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{7}}{7}\right)}{9}-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{9}$	55
risch	$\frac{2i}{3\left(e^{2ix}+1\right)}+\frac{4\ln\left(e^{ix}-i\right)}{9}-\frac{\sqrt{7}\,\ln\left(e^{ix}-\frac{i\sqrt{7}}{4}+\frac{3}{4}\right)}{9}+\frac{\sqrt{7}\,\ln\left(e^{ix}+\frac{i\sqrt{7}}{4}+\frac{3}{4}\right)}{9}-\frac{4\ln\left(e^{ix}+i\right)}{9}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2/(4+3*sec(x)),x,method=_RETURNVERBOSE)
[Out] -1/3/(tan(1/2*x)+1)-4/9*ln(tan(1/2*x)+1)+2/9*7^(1/2)*arctanh(1/7*tan(1/2*x)
*7^(1/2))-1/3/(tan(1/2*x)-1)+4/9*ln(tan(1/2*x)-1)
```

Maxima [A]
time = 0.50, size = 91, normalized size = 1.23

$$-\frac{1}{9}\sqrt{7}\log\left(-\frac{\sqrt{7}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{7}+\frac{\sin(x)}{\cos(x)+1}}\right)-\frac{2\sin(x)}{3\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)}-\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="maxima")
[Out] -1/9*sqrt(7)*log(-(sqrt(7) - sin(x)/(cos(x) + 1))/(sqrt(7) + sin(x)/(cos(x)
 + 1))) - 2/3*sin(x)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 4/9*log
(sin(x)/(cos(x) + 1) + 1) + 4/9*log(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A]
time = 2.77, size = 82, normalized size = 1.11

$$\frac{\sqrt{7}\cos(x)\log\left(\frac{2\cos(x)^2+2\left(3\sqrt{7}\cos(x)+4\sqrt{7}\right)\sin(x)+24\cos(x)+23}{16\cos(x)^2+24\cos(x)+9}\right)-4\cos(x)\log(\sin(x)+1)+4\cos(x)\log(-\sin(x)+1)+6\sin(x)}{18\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(7)*cos(x)*log((2*cos(x)^2 + 2*(3*sqrt(7)*cos(x) + 4*sqrt(7))*sin
(x) + 24*cos(x) + 23)/(16*cos(x)^2 + 24*cos(x) + 9)) - 4*cos(x)*log(sin(x)
+ 1) + 4*cos(x)*log(-sin(x) + 1) + 6*sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2/(4+3*sec(x)),x)
```

```
[Out] Integral(tan(x)**2*sec(x)/(3*sec(x) + 4), x)
```

Giac [A]
time = 0.45, size = 72, normalized size = 0.97

$$-\frac{1}{9}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}\right)-\frac{2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}-\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)+\frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(7)*log(abs(-2*sqrt(7) + 2*tan(1/2*x))/abs(2*sqrt(7) + 2*tan(1/2*x
))) - 2/3*tan(1/2*x)/(tan(1/2*x)^2 - 1) - 4/9*log(abs(tan(1/2*x) + 1)) + 4/
9*log(abs(tan(1/2*x) - 1))
```

Mupad [B]
time = 3.14, size = 41, normalized size = 0.55

$$\frac{2\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,\tan\left(\frac{x}{2}\right)}{7}\right)}{9}-\frac{2\tan\left(\frac{x}{2}\right)}{3\left(\tan\left(\frac{x}{2}\right)^2-1\right)}-\frac{8\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(cos(x)*(3/cos(x) + 4)),x)
```

```
[Out] (2*7^(1/2)*atanh((7^(1/2)*tan(x/2))/7))/9 - (2*tan(x/2))/(3*(tan(x/2)^2 - 1
)) - (8*atanh(tan(x/2)))/9
```

3.904 $\int x \sec(1+x) \tan(1+x) \, dx$

Optimal. Leaf size=14

$$-\tanh^{-1}(\sin(1+x))+x\sec(1+x)$$

```
[Out] -arctanh(sin(1+x))+x*sec(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3842, 3855}

$$x\sec(x+1)-\tanh^{-1}(\sin(x+1))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[1 + x]*Tan[1 + x],x]
```

```
[Out] -ArcTanh[Sin[1 + x]] + x*Sec[1 + x]
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sec(1+x)\tan(1+x)\,dx &= x\sec(1+x)-\int \sec(1+x)\,dx\\ &= -\tanh^{-1}(\sin(1+x))+x\sec(1+x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.
time = 0.03, size = 47, normalized size = 3.36

$$\log\left(\cos\left(\frac{1+x}{2}\right)-\sin\left(\frac{1+x}{2}\right)\right)-\log\left(\cos\left(\frac{1+x}{2}\right)+\sin\left(\frac{1+x}{2}\right)\right)+x\sec(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[1 + x]*Tan[1 + x],x]
```

```
[Out] Log[Cos[(1 + x)/2] - Sin[(1 + x)/2]] - Log[Cos[(1 + x)/2] + Sin[(1 + x)/2]]
 + x*Sec[1 + x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.05, size = 32, normalized size = 2.29

method	result	size
derivativedivides	$\frac{1+x}{\cos(1+x)} - \ln\left(\sec\left(1+x\right) + \tan\left(1+x\right)\right) - \frac{1}{\cos(1+x)}$	32
default	$\frac{1+x}{\cos(1+x)} - \ln\left(\sec\left(1+x\right) + \tan\left(1+x\right)\right) - \frac{1}{\cos(1+x)}$	32
risch	$\frac{2x\,\mathrm{e}^{i(1+x)}}{\mathrm{e}^{2i(1+x)}+1} - \ln\left(\mathrm{e}^{i(1+x)} + i\right) + \ln\left(\mathrm{e}^{i(1+x)} - i\right)$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(1+x)*tan(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)/cos(1+x)-ln(sec(1+x)+tan(1+x))-1/cos(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(14) = 28$.
time = 0.52, size = 176, normalized size = 12.57

$$\frac{4\,(x+1)\cos\left(2\,x+2\right)\cos\left(x+1\right) + 4\,(x+1)\sin\left(2\,x+2\right)\sin\left(x+1\right) + 4\,(x+1)\cos\left(x+1\right) - \left(\cos\left(2\,x+2\right)^2 + \sin\left(2\,x+2\right)^2 + 2\,\cos\left(2\,x+2\right) + 1\right)\log\left(\cos\left(x+1\right)^2 + \sin\left(x+1\right)^2 + 2\,\sin\left(x+1\right) + 1\right) + \left(\cos\left(2\,x+2\right)^2 + \sin\left(2\,x+2\right)^2 + 2\,\cos\left(2\,x+2\right) + 1\right)\log\left(\cos\left(x+1\right)^2 + \sin\left(x+1\right)^2 - 2\,\sin\left(x+1\right) + 1\right)}{2\left(\cos\left(2\,x+2\right)^2 + \sin\left(2\,x+2\right)^2 + 2\,\cos\left(2\,x+2\right) + 1\right)} - \frac{1}{\cos\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(x + 1)*cos(2*x + 2)*cos(x + 1) + 4*(x + 1)*sin(2*x + 2)*sin(x + 1)
+ 4*(x + 1)*cos(x + 1) - (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2)
+ 1)*log(cos(x + 1)^2 + sin(x + 1)^2 + 2*sin(x + 1) + 1) + (cos(2*x + 2)^2
+ sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 - 2*
sin(x + 1) + 1))/(cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1) - 1
/cos(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.
time = 2.42, size = 39, normalized size = 2.79

$$-\frac{\cos\left(x+1\right)\log\left(\sin\left(x+1\right)+1\right) - \cos\left(x+1\right)\log\left(-\sin\left(x+1\right)+1\right) - 2\,x}{2\,\cos\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x + 1)*log(sin(x + 1) + 1) - cos(x + 1)*log(-sin(x + 1) + 1) - 2*
x)/cos(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\tan(x+1)\sec(x+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x)
```

```
[Out] Integral(x*tan(x + 1)*sec(x + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. 2(14) = 28.
time = 0.59, size = 1179, normalized size = 84.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(1+x)*tan(1+x),x, algorithm="giac")
```

```
[Out] 1/2*(2*x*tan(1/2)^2*tan(1/2*x)^2 + log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1
/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 +
tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*ta
n(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2*ta
n(1/2*x)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2
*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1
/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2
) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2*tan(1/2*x)^2 + 2*x*tan(1
/2)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(
1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2
*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2
*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)^2 + log(2*(tan(1/2)^2*tan(1/2*x
)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*ta
n(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(
1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*
tan(1/2)^2 - 4*log(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 +
 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan
(1/2)^2*tan(1/2*x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1
/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2)*tan(1/2*x) + 4*log(2*(ta
n(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4
 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*t
```

```
an(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/
(tan(1/2)^2 + 1))*tan(1/2)*tan(1/2*x) + 2*x*tan(1/2*x)^2 - log(2*(tan(1/2)^
2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(1/2*x)^4 + 2*ta
n(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2*
x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/
2)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*tan
(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x
)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^
2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1))*tan(1/2*x)^2 + 2*x + l
og(2*(tan(1/2)^2*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^3 + 2*tan(1/2)*tan(
1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*
x) - 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*
x) + 1)/(tan(1/2)^2 + 1)) - log(2*(tan(1/2)^2*tan(1/2*x)^4 - 2*tan(1/2)^2*t
an(1/2*x)^3 - 2*tan(1/2)*tan(1/2*x)^4 + 2*tan(1/2)^2*tan(1/2*x)^2 + tan(1/2
*x)^4 - 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2*x)^3 + tan(1/2)^2 + 2*tan(1/2*x
)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2 + 1)))/(tan(1/2)^2*tan(1/2
*x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(1/2*x) - tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 3.17, size = 34, normalized size = 2.43

$$\frac{2\,x\,\cos\left(x+1\right)}{\cos\left(2\,x+2\right)+1}+\mathrm{atan}(\cos\left(x+1\right)+\sin\left(x+1\right)\,1\mathrm{i})\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x + 1))/cos(x + 1),x)
[Out] atan(cos(x + 1) + sin(x + 1)*1i)*2i + (2*x*cos(x + 1))/(cos(2*x + 2) + 1)
```

3.905 $\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx$

Optimal. Leaf size=14

$$-2\sqrt{9-\sin^2(x)}$$

```
[Out] -2*(9-sin(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {12, 267}

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)\\ &= 2\text{Subst}\left(\int \frac{x}{\sqrt{9-x^2}}\,dx, x, \sin(x)\right)\\ &= -2\sqrt{9-\sin^2(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-2\sqrt{9-\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

```
[Out] -2*Sqrt[9 - Sin[x]^2]
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$-2\sqrt{9-(\sin^2(x))}$	13
default	$-2\sqrt{9-(\sin^2(x))}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(9-sin(x)^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-sin(x)^2 + 9)
```

Fricas [A]
time = 2.52, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(cos(x)^2 + 8)
```

Sympy [A]
time = 0.74, size = 12, normalized size = 0.86

$$-2\sqrt{9-\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)**2)**(1/2),x)
```

```
[Out] -2*sqrt(9 - sin(x)**2)
```

Giac [A]
time = 0.43, size = 12, normalized size = 0.86

$$-2\sqrt{-\sin(x)^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-sin(x)^2 + 9)
```

Mupad [B]
time = 0.17, size = 10, normalized size = 0.71

$$-2\sqrt{\cos(x)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - sin(x)^2)^(1/2),x)
```

```
[Out] -2*(cos(x)^2 + 8)^(1/2)
```

3.906 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

Optimal. Leaf size=11

$$-\text{ArcSin}\left(\frac{\cos^2(x)}{3}\right)$$

[Out] -arcsin(1/3*cos(x)^2)

Rubi [A]

time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {12, 1121, 633, 222}

$$-\text{ArcSin}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]

[Out] -ArcSin[Cos[x]^2/3]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)$$
$$= 2\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right)$$
$$= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right)$$
$$= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\text{ArcSin}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
[Out] -ArcSin[Cos[x]^2/3]
```

Maple [A]
time = 0.12, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$-\arcsin\left(\frac{(\cos^2(x))}{3}\right)$	10
default	$-\arcsin\left(\frac{(\cos^2(x))}{3}\right)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arcsin(1/3*cos(x)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.
time = 3.02, size = 24, normalized size = 2.18

$$\arctan\left(\frac{\sqrt{-\cos(x)^4+9}\,\cos(x)^2}{\cos(x)^4-9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 0.40, size = 9, normalized size = 0.82

$$-\arcsin\left(\frac{1}{3}\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(1/3*cos(x)^2)
```

Mupad [B]
time = 3.14, size = 18, normalized size = 1.64

$$-\mathrm{atan}\left(\frac{\cos(x)^2}{\sqrt{9-\cos(x)^4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
```

```
[Out] -atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

3.907 $\int \frac{\cos\left(\frac{1}{x}\right)}{x^5} dx$

Optimal. Leaf size=34

$$6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}$$

```
[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3461, 3377, 2718}

$$-\frac{\sin\left(\frac{1}{x}\right)}{x^3} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} + \frac{6\sin\left(\frac{1}{x}\right)}{x} + 6\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x^(-1)]/x^5,x]
```

```
[Out] 6*Cos[x^(-1)] - (3*Cos[x^(-1)])/x^2 - Sin[x^(-1)]/x^3 + (6*Sin[x^(-1)])/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx &= -\text{Subst}\left(\int x^3\cos(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sin\left(\frac{1}{x}\right)}{x^3} + 3\text{Subst}\left(\int x^2\sin(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6\text{Subst}\left(\int x\cos(x)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x} - 6\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right)\\
&= 6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.94

$$\frac{3(-1+2x^2)\cos\left(\frac{1}{x}\right)}{x^2} + \frac{(-1+6x^2)\sin\left(\frac{1}{x}\right)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x^(-1)]/x^5,x]
[Out] (3*(-1 + 2*x^2)*Cos[x^(-1)])/x^2 + ((-1 + 6*x^2)*Sin[x^(-1)])/x^3
```

Maple [A]
time = 0.07, size = 35, normalized size = 1.03

method	result	size
risch	$\frac{3(2x^2-1)\cos\left(\frac{1}{x}\right)}{x^2} + \frac{(6x^2-1)\sin\left(\frac{1}{x}\right)}{x^3}$	33
derivativedivides	$6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}$	35
default	$6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}$	35
meijerg	$-8\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}} - \frac{\left(-\frac{3}{2x^2}+3\right)\cos\left(\frac{1}{x}\right)}{4\sqrt{\pi}} - \frac{\left(-\frac{1}{2x^2}+3\right)\sin\left(\frac{1}{x}\right)}{4\sqrt{\pi}\,x}\right)$	47
norman	$\frac{12x^4-3x^2-2x\tan\left(\frac{1}{2x}\right)+3x^2\left(\tan^2\left(\frac{1}{2x}\right)\right)+12x^3\tan\left(\frac{1}{2x}\right)}{\left(1+\tan^2\left(\frac{1}{2x}\right)\right)x^4}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)/x^5,x,method=_RETURNVERBOSE)
[Out] 6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

Maxima [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 19, normalized size = 0.56

$$\frac{1}{2}\,\Gamma\left(4,\frac{i}{x}\right)+\frac{1}{2}\,\Gamma\left(4,-\frac{i}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="maxima")
[Out] 1/2*gamma(4, I/x) + 1/2*gamma(4, -I/x)
```

Fricas [A]
time = 2.24, size = 32, normalized size = 0.94

$$\frac{3\,(2\,x^3-x)\cos\left(\frac{1}{x}\right)+(6\,x^2-1)\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="fricas")
[Out] (3*(2*x^3 - x)*cos(1/x) + (6*x^2 - 1)*sin(1/x))/x^3
```

Sympy [A]
time = 0.68, size = 32, normalized size = 0.94

$$6\cos\left(\frac{1}{x}\right)+\frac{6\sin\left(\frac{1}{x}\right)}{x}-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x**5,x)
[Out] 6*cos(1/x) + 6*sin(1/x)/x - 3*cos(1/x)/x**2 - sin(1/x)/x**3
```

Giac [A]
time = 0.41, size = 34, normalized size = 1.00

$$\frac{6\,\sin\left(\frac{1}{x}\right)}{x}-\frac{3\,\cos\left(\frac{1}{x}\right)}{x^2}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}+6\,\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/x)/x^5,x, algorithm="giac")
[Out] 6*sin(1/x)/x - 3*cos(1/x)/x^2 - sin(1/x)/x^3 + 6*cos(1/x)
```

Mupad [B]
time = 3.00, size = 33, normalized size = 0.97

$$6\,\cos\left(\frac{1}{x}\right)-\frac{\sin\left(\frac{1}{x}\right)+3\,x\,\cos\left(\frac{1}{x}\right)-6\,x^2\,\sin\left(\frac{1}{x}\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/x)/x^5,x)
[Out] 6*cos(1/x) - (sin(1/x) + 3*x*cos(1/x) - 6*x^2*sin(1/x))/x^3
```

3.908 $\int \cos^3(1+x)\sin^3(1+x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\sin^4(1+x)-\frac{1}{6}\sin^6(1+x)$$

```
[Out] 1/4*sin(1+x)^4-1/6*sin(1+x)^6
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2644, 14}

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

```
[Out] Sin[1 + x]^4/4 - Sin[1 + x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(1+x)\sin^3(1+x)\,dx &= \text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(1+x)\right)\\ &= \text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(1+x)\right)\\ &= \frac{1}{4}\sin^4(1+x)-\frac{1}{6}\sin^6(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.19

$$\frac{1}{8}\left(-\frac{3}{8}\cos(2(1+x))+\frac{1}{24}\cos(6(1+x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[1 + x]^3*Sin[1 + x]^3,x]
[Out] ((-3*Cos[2*(1 + x)])/8 + Cos[6*(1 + x)]/24)/8
```

Maple [A]
time = 0.11, size = 24, normalized size = 1.14

method	result	size
risch	$\frac{\cos(6+6x)}{192}-\frac{3\cos(2+2x)}{64}$	18
derivativedivides	$-\frac{(\cos^4(1+x))(\sin^2(1+x))}{6}-\frac{(\cos^4(1+x))}{12}$	24
default	$-\frac{(\cos^4(1+x))(\sin^2(1+x))}{6}-\frac{(\cos^4(1+x))}{12}$	24
norman	$\frac{6\left(\tan^4\left(\frac{1}{2}+\frac{x}{2}\right)\right)+6\left(\tan^8\left(\frac{1}{2}+\frac{x}{2}\right)\right)+\frac{2\left(\tan^{12}\left(\frac{1}{2}+\frac{x}{2}\right)\right)}{15}+\frac{4\left(\tan^2\left(\frac{1}{2}+\frac{x}{2}\right)\right)}{5}+\frac{4\left(\tan^{10}\left(\frac{1}{2}+\frac{x}{2}\right)\right)}{5}+\frac{2}{15}}{\left(1+\tan^2\left(\frac{1}{2}+\frac{x}{2}\right)\right)^6}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1+x)^3*sin(1+x)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*cos(1+x)^4*sin(1+x)^2-1/12*cos(1+x)^4
```

Maxima [A]
time = 0.31, size = 17, normalized size = 0.81

$$-\frac{1}{6}\sin(x+1)^6+\frac{1}{4}\sin(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="maxima")
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

Fricas [A]
time = 2.60, size = 17, normalized size = 0.81

$$\frac{1}{6}\cos(x+1)^6-\frac{1}{4}\cos(x+1)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x + 1)^6 - 1/4*cos(x + 1)^4
```

Sympy [A]
time = 0.31, size = 22, normalized size = 1.05

$$\frac{\sin^{6}\left(x+1\right)}{12}+\frac{\sin^{4}\left(x+1\right)\cos^{2}\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)**3*sin(1+x)**3,x)
```

```
[Out] sin(x + 1)**6/12 + sin(x + 1)**4*cos(x + 1)**2/4
```

Giac [A]
time = 0.41, size = 17, normalized size = 0.81

$$-\frac{1}{6}\,\sin\left(x+1\right)^{6}+\frac{1}{4}\,\sin\left(x+1\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.86

$$-\frac{\sin\left(x+1\right)^{4}\,\left(2\,\sin\left(x+1\right)^{2}-3\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x + 1)^3*sin(x + 1)^3,x)
```

```
[Out] -(sin(x + 1)^4*(2*sin(x + 1)^2 - 3))/12
```

3.909 $\int (1+2x)^3 \sin^2(1+2x)\,dx$

Optimal. Leaf size=99

$$-\frac{3x}{4}-\frac{3x^2}{4}+\frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x)-\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x)-\frac{3}{16}\sin^2(1+2x)+$$

```
[Out] -3/4*x-3/4*x^2+1/16*(1+2*x)^4+3/8*(1+2*x)*cos(1+2*x)*sin(1+2*x)-1/4*(1+2*x)
^3*cos(1+2*x)*sin(1+2*x)-3/16*sin(1+2*x)^2+3/8*(1+2*x)^2*sin(1+2*x)^2
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$-\frac{3x^2}{4}+\frac{1}{16}(2x+1)^4-\frac{3x}{4}+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{16}\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)+\frac{3}{8}(2x+1)\sin(2x+1)\cos(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*x)/4 - (3*x^2)/4 + (1 + 2*x)^4/16 + (3*(1 + 2*x)*Cos[1 + 2*x]*Sin[1 + 2
*x])/8 - ((1 + 2*x)^3*Cos[1 + 2*x]*Sin[1 + 2*x])/4 - (3*Sin[1 + 2*x]^2)/16
+ (3*(1 + 2*x)^2*Sin[1 + 2*x]^2)/8
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (1+2x)^3\sin^2(1+2x)\,dx = -\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x)+\frac{3}{8}(1+2x)^2\sin^2(1+2x)+\frac{1}{2}\int (1+2x$$
$$= \frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x)-\frac{1}{4}(1+2x)^3\cos(1+2x)\sin$$
$$= -\frac{3x}{4}-\frac{3x^2}{4}+\frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x)-\frac{1}{4}(1+2x)^3$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.56

$$\frac{1}{32}\left(-3\left(1+8x+8x^2\right)\cos(2+4x)+2(1+2x)\left((1+2x)^3+\left(1-8x-8x^2\right)\sin(2+4x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

```
[Out] (-3*(1 + 8*x + 8*x^2)*Cos[2 + 4*x] + 2*(1 + 2*x)*((1 + 2*x)^3 + (1 - 8*x - 
8*x^2)*Sin[2 + 4*x]))/32
```

Maple [A]
time = 0.12, size = 97, normalized size = 0.98

method	result
risch	$x^4+2x^3+\frac{3x^2}{2}+\frac{x}{2}+\frac{1}{16}-\frac{3(8x^2+8x+1)\cos(2+4x)}{32}-\frac{(16x^3+24x^2+6x-1)\sin(2+4x)}{16}$
derivativedivides	$\frac{(1+2x)^3\left(-\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{2}-\frac{3(1+2x)^2(\cos^2(1+2x))}{8}+\frac{3(1+2x)\left(\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{4}-\frac{3(1+2x}{16}$
default	$\frac{(1+2x)^3\left(-\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{2}-\frac{3(1+2x)^2(\cos^2(1+2x))}{8}+\frac{3(1+2x)\left(\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{4}-\frac{3(1+2x}{16}$
norman	$x^4+x^4(\tan^4(\frac{1}{2}+x))-\frac{3(\tan^4(\frac{1}{2}+x))}{8}-\frac{x}{4}+\frac{3x^2}{4}+2x^3-\frac{(\tan^3(\frac{1}{2}+x))}{4}-\frac{3x\tan(\frac{1}{2}+x)}{2}+\frac{11x(\tan^2(\frac{1}{2}+x))}{2}+\frac{3x(\tan^3(\frac{1}{2}+x))}{2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*sin(1+2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+2*x)^3*(-1/2*cos(1+2*x)*sin(1+2*x)+1/2+x)-3/8*(1+2*x)^2*cos(1+2*x)^2
+3/4*(1+2*x)*(1/2*cos(1+2*x)*sin(1+2*x)+1/2+x)-3/16*(1+2*x)^2-3/16*sin(1+2*
x)^2-3/16*(1+2*x)^4
```

Maxima [A]
time = 0.31, size = 51, normalized size = 0.52

$$\frac{1}{16}\,(2\,x+1)^4-\frac{3}{32}\,\left(2\,(2\,x+1)^2-1\right)\cos\left(4\,x+2\right)-\frac{1}{16}\,\left(2\,(2\,x+1)^3-6\,x-3\right)\sin\left(4\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*x + 1)^4 - 3/32*(2*(2*x + 1)^2 - 1)*cos(4*x + 2) - 1/16*(2*(2*x + 1
)^3 - 6*x - 3)*sin(4*x + 2)
```

Fricas [A]
time = 2.22, size = 66, normalized size = 0.67

$$x^4+2x^3-\frac{3}{16}\left(8x^2+8x+1\right)\cos\left(2x+1\right)^2-\frac{1}{8}\left(16x^3+24x^2+6x-1\right)\cos\left(2x+1\right)\sin\left(2x+1\right)+\frac{9}{4}x^2+\frac{5}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] x^4 + 2*x^3 - 3/16*(8*x^2 + 8*x + 1)*cos(2*x + 1)^2 - 1/8*(16*x^3 + 24*x^2
+ 6*x - 1)*cos(2*x + 1)*sin(2*x + 1) + 9/4*x^2 + 5/4*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(94) = 188.
time = 0.23, size = 189, normalized size = 1.91

$$x^4\sin^2\left(2x+1\right)+x^4\cos^2\left(2x+1\right)+2x^3\sin^2\left(2x+1\right)-2x^3\sin\left(2x+1\right)\cos\left(2x+1\right)+2x^3\cos^2\left(2x+1\right)+\frac{9x^2\sin^2\left(2x+1\right)}{4}-3x^2\sin\left(2x+1\right)\cos\left(2x+1\right)+\frac{3x^2\cos^2\left(2x+1\right)}{4}+\frac{5x\sin^2\left(2x+1\right)}{4}-\frac{3x\sin\left(2x+1\right)\cos\left(2x+1\right)}{4}-\frac{x\cos^2\left(2x+1\right)}{4}+\frac{\sin\left(2x+1\right)\cos\left(2x+1\right)}{8}-\frac{3\cos^2\left(2x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*sin(1+2*x)**2,x)
```

```
[Out] x**4*sin(2*x + 1)**2 + x**4*cos(2*x + 1)**2 + 2*x**3*sin(2*x + 1)**2 - 2*x*
*3*sin(2*x + 1)*cos(2*x + 1) + 2*x**3*cos(2*x + 1)**2 + 9*x**2*sin(2*x + 1)
**2/4 - 3*x**2*sin(2*x + 1)*cos(2*x + 1) + 3*x**2*cos(2*x + 1)**2/4 + 5*x*s
in(2*x + 1)**2/4 - 3*x*sin(2*x + 1)*cos(2*x + 1)/4 - x*cos(2*x + 1)**2/4 +
sin(2*x + 1)*cos(2*x + 1)/8 - 3*cos(2*x + 1)**2/16
```

Giac [A]
time = 0.41, size = 58, normalized size = 0.59

$$x^4+2x^3+\frac{3}{2}x^2-\frac{3}{32}\left(8x^2+8x+1\right)\cos\left(4x+2\right)-\frac{1}{16}\left(16x^3+24x^2+6x-1\right)\sin\left(4x+2\right)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="giac")
```

```
[Out] x^4 + 2*x^3 + 3/2*x^2 - 3/32*(8*x^2 + 8*x + 1)*cos(4*x + 2) - 1/16*(16*x^3
+ 24*x^2 + 6*x - 1)*sin(4*x + 2) + 1/2*x
```

Mupad [B]
time = 3.06, size = 69, normalized size = 0.70

$$\frac{3\sin\left(4x+2\right)\,\left(2x+1\right)}{16}-\frac{3\sin\left(2x+1\right)^2}{16}+\frac{{\left(2x+1\right)}^4}{16}-\frac{\sin\left(4x+2\right)\,{\left(2x+1\right)}^3}{8}+\frac{3\,{\left(2x+1\right)}^2\,\left(2\sin\left(2x+1\right)^2-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x + 1)^2*(2*x + 1)^3,x)
```

```
[Out] (3*sin(4*x + 2)*(2*x + 1))/16 - (3*sin(2*x + 1)^2)/16 + (2*x + 1)^4/16 - (s
in(4*x + 2)*(2*x + 1)^3)/8 + (3*(2*x + 1)^2*(2*sin(2*x + 1)^2 - 1))/16
```

3.910 $\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx$

Optimal. Leaf size=37

$$-\frac{x}{2}+\frac{\tanh^{-1}\left(\frac{\cos(x)(1+\tan(x))}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x)-\sin(x))$$

```
[Out] -1/2*x+1/2*ln(cos(x)-sin(x))+1/2*arctanh(1/2*cos(x)*(1+tan(x))*2^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {4486, 3565, 3611, 3590, 212}

$$-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{\tanh^{-1}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sec[x])/(1 - Tan[x]),x]
```

```
[Out] -1/2*x + ArcTanh[(Cos[x]*(1 + Tan[x]))/Sqrt[2]]/Sqrt[2] + Log[Cos[x] - Sin[
x]]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx &= \int \left(\frac{1}{-1+\tan(x)}-\frac{\sec(x)}{-1+\tan(x)}\right)dx\\ &= \int \frac{1}{-1+\tan(x)}\,dx-\int \frac{\sec(x)}{-1+\tan(x)}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\int \frac{1+\tan(x)}{-1+\tan(x)}\,dx+\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\cos(x)(1+\tan(x))\right)\\ &= -\frac{x}{2}+\frac{\tanh^{-1}\left(\frac{\cos(x)(1+\tan(x))}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x)-\sin(x))\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.04, size = 40, normalized size = 1.08

$$\frac{1}{2}\left(-x+(2-2i)\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sec[x])/(1 - Tan[x]),x]
[Out] (-x + (2 - 2*I)*(-1)^(1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + Log[Cos[x] - S
in[x]])/2
```

Maple [A]
time = 0.11, size = 55, normalized size = 1.49

method	result
default	$\frac{\ln\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tan\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)-\frac{\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{2}-\arctan\left(\tan\left(\frac{x}{2}\right)\right)$

risch	$-\frac{x}{2}-\frac{ix}{2}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{i\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)}{2}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{i\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\ln\left(\mathrm{e}^{ix}-\frac{i\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{\ln\left(\mathrm{e}^{ix}-\frac{i\sqrt{2}}{2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sec(x))/(1-tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(tan(1/2*x)^2+2*tan(1/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tan(1/2*x)+2)*2^
(1/2))-1/2*ln(1+tan(1/2*x)^2)-arctan(tan(1/2*x))
```

Maxima [A]
time = 0.52, size = 59, normalized size = 1.59

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}-1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}+1}\right)-\frac{1}{2}x-\frac{1}{4}\log\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\log\left(\tan\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) - 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) + 1)) - 1/2*x - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) - 1)
```

Fricas [A]
time = 2.40, size = 51, normalized size = 1.38

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}+\cos\left(x\right)\right)\sin\left(x\right)+2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)-1}\right)-\frac{1}{2}x+\frac{1}{4}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) + 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) - 1)) - 1/2*x + 1/4*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sec\left(x\right)}{\tan\left(x\right)-1}\,dx-\int\left(-\frac{1}{\tan\left(x\right)-1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x)
```

```
[Out] -Integral(sec(x)/(tan(x) - 1), x) - Integral(-1/(tan(x) - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(31) = 62$.
time = 0.44, size = 70, normalized size = 1.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}\right)-\frac{1}{2}x-\frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sec(x))/(1-tan(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) + 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) + 2)) - 1/2*x - 1/2*log(tan(1/2*x)^2 + 1) + 1/2*log(abs(tan(1/2*x)^2
+ 2*tan(1/2*x) - 1))
```

Mupad [B]
time = 3.13, size = 64, normalized size = 1.73

$$\ln\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)-\ln\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)+\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/cos(x) - 1)/(tan(x) - 1),x)
```

```
[Out] log(tan(x/2) + 2^(1/2) + 1)*(2^(1/2)/2 + 1/2) - log(tan(x/2) + 1i)*(1/2 + 1
i/2) - log(tan(x/2) - 2^(1/2) + 1)*(2^(1/2)/2 - 1/2) - log(tan(x/2) - 1i)*(
1/2 - 1i/2)
```

3.911 $\int x^2 \cos(3x) \cos(5x) \, dx$

Optimal. Leaf size=57

$$\frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)-\frac{1}{8}\sin(2x)+\frac{1}{4}x^2\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{16}x^2\sin(8x)$$

```
[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)
+1/16*x^2*sin(8*x)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4514, 3377, 2717}

$$\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x]*Cos[5*x],x]
```

```
[Out] (x*Cos[2*x])/4 + (x*Cos[8*x])/64 - Sin[2*x]/8 + (x^2*Sin[2*x])/4 - Sin[8*x]
/512 + (x^2*Sin[8*x])/16
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4514

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f
_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cos[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int x^2\cos(3x)\cos(5x)\,dx = \int \left(\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x^2\cos(8x)\right)dx$$
$$= \frac{1}{2}\int x^2\cos(2x)\,dx+\frac{1}{2}\int x^2\cos(8x)\,dx$$
$$= \frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\int x\sin(8x)\,dx-\frac{1}{2}\int x\sin(2x)\,dx$$
$$= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)+\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{64}\int\cos(8x)\,dx-\frac{1}{4}\int$$
$$= \frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)-\frac{1}{8}\sin(2x)+\frac{1}{4}x^2\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{16}x^2\sin(8x$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.86

$$\frac{1}{512}\left(128x\cos(2x)+8x\cos(8x)-64\sin(2x)+128x^2\sin(2x)-\sin(8x)+32x^2\sin(8x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x]*Cos[5*x],x]
[Out] (128*x*Cos[2*x] + 8*x*Cos[8*x] - 64*Sin[2*x] + 128*x^2*Sin[2*x] - Sin[8*x]
+ 32*x^2*Sin[8*x])/512
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.81

method	result
risch	$\frac{x\cos(8x)}{64}+\frac{(32x^2-1)\sin(8x)}{512}+\frac{x\cos(2x)}{4}+\frac{(2x^2-1)\sin(2x)}{8}$
default	$\frac{x\cos(2x)}{4}+\frac{x\cos(8x)}{64}-\frac{\sin(2x)}{8}+\frac{x^2\sin(2x)}{4}-\frac{\sin(8x)}{512}+\frac{x^2\sin(8x)}{16}$
norman	$\frac{\frac{17x}{64}-\frac{17x\left(\tan^2\left(\frac{3x}{2}\right)\right)}{64}-\frac{17x\left(\tan^2\left(\frac{5x}{2}\right)\right)}{64}-\frac{3x^2\tan\left(\frac{3x}{2}\right)}{8}+\frac{5x^2\tan\left(\frac{5x}{2}\right)}{8}-\frac{63\tan\left(\frac{3x}{2}\right)\left(\tan^2\left(\frac{5x}{2}\right)\right)}{256}+\frac{65\left(\tan^2\left(\frac{3x}{2}\right)\right)\tan\left(\frac{5x}{2}\right)}{256}+\frac{15x\tan\left(\frac{3x}{2}\right)\tan(}{16}}{\left(1+\tan^2\left(\frac{3x}{2}\right)\right)\left(1+\tan^2\left(\frac{5}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
[Out] 1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x)
+1/16*x^2*sin(8*x)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.72

$$\frac{1}{64}\,x\cos\left(8\,x\right)+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{512}\,\left(32\,x^2-1\right)\sin\left(8\,x\right)+\frac{1}{8}\,\left(2\,x^2-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="maxima")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

Fricas [A]
time = 2.44, size = 73, normalized size = 1.28

$$2\,x\cos\left(x\right)^{8}-4\,x\cos\left(x\right)^{6}+\frac{5}{2}\,x\cos\left(x\right)^{4}+\frac{1}{64}\,\left(16\,\left(32\,x^{2}-1\right)\cos\left(x\right)^{7}-24\,\left(32\,x^{2}-1\right)\cos\left(x\right)^{5}+10\,\left(32\,x^{2}-1\right)\cos\left(x\right)^{3}-15\,\cos\left(x\right)\right)\sin\left(x\right)-\frac{15}{64}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x)^8 - 4*x*cos(x)^6 + 5/2*x*cos(x)^4 + 1/64*(16*(32*x^2 - 1)*cos(x)
^7 - 24*(32*x^2 - 1)*cos(x)^5 + 10*(32*x^2 - 1)*cos(x)^3 - 15*cos(x))*sin(x
) - 15/64*x
```

Sympy [A]
time = 0.84, size = 90, normalized size = 1.58

$$-\frac{3x^{2}\sin\left(3x\right)\cos\left(5x\right)}{16}+\frac{5x^{2}\sin\left(5x\right)\cos\left(3x\right)}{16}+\frac{15x\sin\left(3x\right)\sin\left(5x\right)}{64}+\frac{17x\cos\left(3x\right)\cos\left(5x\right)}{64}+\frac{63\sin\left(3x\right)\cos\left(5x\right)}{512}-\frac{65\sin\left(5x\right)\cos\left(3x\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x)*cos(5*x),x)
```

```
[Out] -3*x**2*sin(3*x)*cos(5*x)/16 + 5*x**2*sin(5*x)*cos(3*x)/16 + 15*x*sin(3*x)*
sin(5*x)/64 + 17*x*cos(3*x)*cos(5*x)/64 + 63*sin(3*x)*cos(5*x)/512 - 65*sin
(5*x)*cos(3*x)/512
```

Giac [A]
time = 0.43, size = 41, normalized size = 0.72

$$\frac{1}{64}\,x\cos\left(8\,x\right)+\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{512}\,\left(32\,x^{2}-1\right)\sin\left(8\,x\right)+\frac{1}{8}\,\left(2\,x^{2}-1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^2
 - 1)*sin(2*x)
```

Mupad [B]
time = 3.05, size = 45, normalized size = 0.79

$$\frac{x\,\cos\left(2\,x\right)}{4}-\frac{\sin\left(8\,x\right)}{512}-\frac{\sin\left(2\,x\right)}{8}+\frac{x\,\cos\left(8\,x\right)}{64}+\frac{x^2\,\sin\left(2\,x\right)}{4}+\frac{x^2\,\sin\left(8\,x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x)*cos(5*x),x)
```

```
[Out] (x*cos(2*x))/4 - sin(8*x)/512 - sin(2*x)/8 + (x*cos(8*x))/64 + (x^2*sin(2*x
))/4 + (x^2*sin(8*x))/16
```

3.912 $\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=57

$$-\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)+\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

```
[Out] -arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+arctan(1+2^(1/2)*sin(x
)^(1/2)/cos(x)^(1/2))*2^(1/2)
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(57) = 114$.
time = 0.14, antiderivative size = 243, normalized size of antiderivative = 4.26, number of steps used = 22, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3186, 2655, 303, 1176, 631, 210, 1179, 642, 2654}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(x)-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{2\sqrt{2}}+\frac{\log\left(\cot(x)+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 + (Sqrt[
2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]]]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqr
t[2] - Log[1 + Cot[x] - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) +
Log[1 + Cot[x] + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Log[1 -
 (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx = \int \left(\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)dx$$

$$= \int \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\,dx + \int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

$$= -\left(2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right) + 2\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x}}{\sqrt{\cos(x}}\right.$$

$$= \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right) - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) - \text{S}$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+\right.$$

$$= -\frac{\log\left(1+\cot(x)-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}} + \frac{\log\left(1+\cot(x)+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{2\sqrt{2}} + \cdots$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt{}}\right.}{\sqrt{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 68, normalized size = 1.19

$$\frac{2\sqrt[4]{\cos^2(x)}\sqrt{\sin(x)}\left(3\cos(x)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(x)\right)+\sqrt{\cos^2(x)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(x)\right)\sin(x)\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

```
[Out] (2*(Cos[x]^2)^(1/4)*Sqrt[Sin[x]]*(3*Cos[x]*Hypergeometric2F1[1/4, 1/4, 5/4,
 Sin[x]^2] + Sqrt[Cos[x]^2]*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[
x]))/(3*Cos[x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.23, size = 137, normalized size = 2.40

method	result
default	$-\frac{\sqrt{\frac{\sin(x)-\cos(x)+1}{\sin(x)}}\,\sqrt{2}\,\sqrt{\frac{\cos(x)-1+\sin(x)}{\sin(x)}}\,\sqrt{\frac{\cos(x)-1}{\sin(x)}}\,\left(\sin^{\frac{3}{2}}(x)\right)\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{\sin(x)-\cos(x)+1}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\right.}{\sqrt{\cos(x)}\,(\cos(x)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((sin(x)-cos(x)+1)/sin(x))^(1/2)*2^(1/2)*((cos(x)-1+sin(x))/sin(x))^(1/2)*
((cos(x)-1)/sin(x))^(1/2)*sin(x)^(3/2)*(I*EllipticPi(((sin(x)-cos(x)+1)/sin
(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((sin(x)-cos(x)+1)/sin(x))^(
1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticF(((sin(x)-cos(x)+1)/sin(x))^(1/2),1/2*
2^(1/2)))/cos(x)^(1/2)/(cos(x)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.
time = 3.11, size = 85, normalized size = 1.49

$$-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(32\sqrt{2}\cos\left(x\right)^{4}-32\sqrt{2}\cos\left(x\right)^{2}+16\sqrt{2}\cos\left(x\right)\sin\left(x\right)-\sqrt{2}\right)\sqrt{\cos\left(x\right)}\sqrt{\sin\left(x\right)}}{8\left(4\cos\left(x\right)^{5}-3\cos\left(x\right)^{3}-\left(4\cos\left(x\right)^{4}-5\cos\left(x\right)^{2}\right)\sin\left(x\right)-\cos\left(x\right)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(-1/8*(32*sqrt(2)*cos(x)^4 - 32*sqrt(2)*cos(x)^2 + 16*sq
rt(2)*cos(x)*sin(x) - sqrt(2))*sqrt(cos(x))*sqrt(sin(x))/(4*cos(x)^5 - 3*co
s(x)^3 - (4*cos(x)^4 - 5*cos(x)^2)*sin(x) - cos(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(x\right)+\cos\left(x\right)}{\sqrt{\sin\left(x\right)}\sqrt{\cos\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)**(1/2)/sin(x)**(1/2),x)
```

```
[Out] Integral((sin(x) + cos(x))/(sqrt(sin(x))*sqrt(cos(x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

Mupad [B]
time = 4.62, size = 51, normalized size = 0.89

$$-\frac{2\sqrt{\cos(x)}\,\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{3/4}}-\frac{2\cos(x)^{3/2}\,\sqrt{\sin(x)}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\ \frac{7}{4};\ \cos(x)^2\right)}{3\left(\sin(x)^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(cos(x)^(1/2)*sin(x)^(1/2)),x)
```

```
[Out] - (2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x
)^2)^(3/4) - (2*cos(x)^(3/2)*sin(x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(x)
^2))/(3*(sin(x)^2)^(1/4))
```

3.913 $\int \sec^2(x)(1+\sin(x))\,dx$

Optimal. Leaf size=5

$$\sec(x)+\tan(x)$$

```
[Out] sec(x)+tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2748, 3852, 8}

$$\tan(x)+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(x)(1+\sin(x))\,dx &= \sec(x)+\int \sec^2(x)\,dx\\
&= \sec(x)-\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\
&= \sec(x)+\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\sec(x) + \tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*(1 + Sin[x]),x]
```

```
[Out] Sec[x] + Tan[x]
```

Maple [A]
time = 0.05, size = 8, normalized size = 1.60

method	result	size
default	$\tan(x) + \frac{1}{\cos(x)}$	8
risch	$\frac{2}{e^{ix} - i}$	13
norman	$\frac{-2\left(\tan^2\left(\frac{x}{2}\right)\right) - 2\left(\tan^3\left(\frac{x}{2}\right)\right) - 2\tan\left(\frac{x}{2}\right) - 2}{\left(\tan^2\left(\frac{x}{2}\right) - 1\right)\left(1 + \tan^2\left(\frac{x}{2}\right)\right)}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*(1+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(x)+1/cos(x)
```

Maxima [A]
time = 0.30, size = 7, normalized size = 1.40

$$\frac{1}{\cos(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="maxima")
```

```
[Out] 1/cos(x) + tan(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.
time = 2.83, size = 17, normalized size = 3.40

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A]
time = 1.24, size = 7, normalized size = 1.40

$$\tan(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*(1+sin(x)),x)
```

```
[Out] tan(x) + 1/cos(x)
```

Giac [A]
time = 0.42, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{1}{2}x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

Mupad [B]
time = 2.97, size = 10, normalized size = 2.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x) + 1)/cos(x)^2,x)
```

```
[Out] -2/(tan(x/2) - 1)
```

3.914 $\int (10x^9 \cos(x^5 \log(x)) - x^{10}(x^4 + 5x^4 \log(x)) \sin(x^5$

Optimal. Leaf size=11

$$x^{10} \cos(x^5 \log(x))$$

```
[Out] x^10*cos(x^5*ln(x))
```

Rubi [F]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (10x^9 \cos(x^5 \log(x)) - x^{10}(x^4 + 5x^4 \log(x)) \sin(x^5 \log(x)))\, dx$$

Verification is not applicable to the result.

```
[In] Int[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]],x]
```

```
[Out] 10*Defer[Int][x^9*Cos[x^5*Log[x]], x] - Defer[Int][x^14*Sin[x^5*Log[x]], x]
 - 5*Defer[Int][x^14*Log[x]*Sin[x^5*Log[x]], x]
```

Rubi steps

$$\begin{aligned}\int (10x^9 \cos(x^5 \log(x)) - x^{10}(x^4 + 5x^4 \log(x)) \sin(x^5 \log(x)))\, dx &= 10 \int x^9 \cos(x^5 \log(x))\, dx - \int x^{10}(x \\ &= 10 \int x^9 \cos(x^5 \log(x))\, dx - \int x^{14}(1 \\ &= 10 \int x^9 \cos(x^5 \log(x))\, dx - \int (x^{14} s \\ &= -\left(5 \int x^{14} \log(x) \sin(x^5 \log(x))\, dx\right)\end{aligned}$$

Mathematica [A]
time = 0.20, size = 11, normalized size = 1.00

$$x^{10} \cos(x^5 \log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]
],x]
```

```
[Out] x^10*Cos[x^5*Log[x]]
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 30, normalized size = 2.73

method	result	size
risch	$\frac{x^{10}x^{ix^5}}{2}+\frac{x^{10}x^{-ix^5}}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10*x^9*cos(x^5*ln(x))-x^10*(x^4+5*x^4*ln(x))*sin(x^5*ln(x)),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/2*x^10*x^(I*x^5)+1/2*x^10/(x^(I*x^5))
```

Maxima [A]
time = 0.39, size = 11, normalized size = 1.00

$$x^{10}\cos\left(x^5\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="maxima")
```

```
[Out] x^10*cos(x^5*log(x))
```

Fricas [A]
time = 2.33, size = 11, normalized size = 1.00

$$x^{10}\cos\left(x^5\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="fricas")
```

```
[Out] x^10*cos(x^5*log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-10x^9\cos\left(x^5\log\left(x\right)\right)\right)\,dx-\int x^{14}\sin\left(x^5\log\left(x\right)\right)\,dx-\int 5x^{14}\log\left(x\right)\sin\left(x^5\log\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x**9*cos(x**5*ln(x))-x**10*(x**4+5*x**4*ln(x))*sin(x**5*ln(x))
,x)
```

```
[Out] -Integral(-10*x**9*cos(x**5*log(x)), x) - Integral(x**14*sin(x**5*log(x)),
x) - Integral(5*x**14*log(x)*sin(x**5*log(x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.16, size = 11, normalized size = 1.00

$$x^{10}\cos\left(x^5\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10*x^9*cos(x^5*log(x)) - x^10*sin(x^5*log(x))*(5*x^4*log(x) + x^4),x)
```

```
[Out] x^10*cos(x^5*log(x))
```

3.915 $\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=27

$$\frac{x}{2}-\frac{\cos(x)}{2}-\log\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

```
[Out] 1/2*x-1/2*cos(x)-ln(cos(1/4*Pi+1/2*x))
```

Rubi [F]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
[Out] Defer[Int][Cos[x/2]^2*Tan[Pi/4 + x/2], x]
```

Rubi steps

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$

Mathematica [A]
time = 0.10, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(x+2\tanh^{-1}\left(\cot\left(\frac{x}{2}\right)\right)-\cos(x)-\log(\cos(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
[Out] (x + 2*ArcTanh[Cot[x/2]] - Cos[x] - Log[Cos[x]])/2
```

Maple [A]
time = 0.26, size = 22, normalized size = 0.81

method	result	size
default	$\frac{\ln(\sec(x)+\tan(x))}{2}-\frac{\ln(\cos(x))}{2}+\frac{x}{2}-\frac{\cos(x)}{2}$	22
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{e^{ix}}{4}-\frac{e^{-ix}}{4}-\ln\left(e^{ix}-i\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/2*x)^2*tan(1/4*Pi+1/2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(sec(x)+tan(x))-1/2*ln(cos(x))+1/2*x-1/2*cos(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(19) = 38$.
time = 0.52, size = 74, normalized size = 2.74

$$\frac{2\,x\cos\left(x\right)^{2}+2\,x\sin\left(x\right)^{2}-\cos\left(2\,x\right)\cos\left(x\right)-2\,\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\sin\left(x\right)+1\right)-\sin\left(2\,x\right)\sin\left(x\right)-\cos\left(x\right)}{4\,\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x*cos(x)^2 + 2*x*sin(x)^2 - cos(2*x)*cos(x) - 2*(cos(x)^2 + sin(x)^2
)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(2*x)*sin(x) - cos(x))/(cos(
x)^2 + sin(x)^2)
```

Fricas [A]
time = 1.36, size = 27, normalized size = 1.00

$$-\cos\left(\frac{1}{2}\,x\right)^{2}+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(-2\,\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="fricas")
```

```
[Out] -cos(1/2*x)^2 + 1/2*x - 1/2*log(-2*cos(1/2*x)*sin(1/2*x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^{2}\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)**2*tan(1/4*pi+1/2*x),x)
```

```
[Out] Integral(cos(x/2)**2*tan(x/2 + pi/4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(19) = 38$.
time = 0.42, size = 93, normalized size = 3.44

$$\frac{x\tan\left(\frac{1}{2}\,x\right)^{2}-\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{2}+\tan\left(\frac{1}{2}\,x\right)^{2}+x-\log\left(\frac{2\,\left(\tan\left(\frac{1}{2}\,x\right)^{2}-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)-1}{2\,\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2
 + 1))*tan(1/2*x)^2 + tan(1/2*x)^2 + x - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x)
 + 1)/(tan(1/2*x)^2 + 1)) - 1)/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 0.48, size = 38, normalized size = 1.41

$$-2\,\ln\left(\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{2}}\,\mathrm{e}^{x\,1\mathrm{i}}+1\right)\,\sin\left(\frac{\Pi}{4}\right)^2+x\,\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{4}}\,\sin\left(\frac{\Pi}{4}\right)-\frac{\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x/2)^2*tan(Pi/4 + x/2),x)
```

```
[Out] x*sin(Pi/4)*exp((Pi*1i)/4) - 2*sin(Pi/4)^2*log(exp((Pi*1i)/2)*exp(x*1i) + 1
) - cos(x)/2
```

3.916 $\int (2+3x)^2 \sin^3(x)\,dx$

Optimal. Leaf size=65

$$14\cos(x)-\frac{2}{3}(2+3x)^2\cos(x)-\frac{2\cos^3(x)}{3}+4(2+3x)\sin(x)-\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x)+\frac{2}{3}(2+3x)\sin^3(x)$$

```
[Out] 14*cos(x)-2/3*(2+3*x)^2*cos(x)-2/3*cos(x)^3+4*(2+3*x)*sin(x)-1/3*(2+3*x)^2*
cos(x)*sin(x)^2+2/3*(2+3*x)*sin(x)^3
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 3377, 2718, 2713}

$$\frac{2}{3}(3x+2)\sin^3(x)+4(3x+2)\sin(x)-\frac{2}{3}\cos^3(x)-\frac{2}{3}(3x+2)^2\cos(x)+14\cos(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*Sin[x]^3,x]
```

```
[Out] 14*Cos[x] - (2*(2 + 3*x)^2*Cos[x])/3 - (2*Cos[x]^3)/3 + 4*(2 + 3*x)*Sin[x]
- ((2 + 3*x)^2*Cos[x]*Sin[x]^2)/3 + (2*(2 + 3*x)*Sin[x]^3)/3
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (2+3x)^2 \sin^3(x)\,dx = -\frac{1}{3}(2+3x)^2\cos(x)\sin^2(x) + \frac{2}{3}(2+3x)\sin^3(x) + \frac{2}{3}\int (2+3x)^2\sin(x)\,dx - 2\int \sin$$
$$= -\frac{2}{3}(2+3x)^2\cos(x) - \frac{1}{3}(2+3x)^2\cos(x)\sin^2(x) + \frac{2}{3}(2+3x)\sin^3(x) + 2\text{Subst}\left(\int (\right.$$
$$= 2\cos(x) - \frac{2}{3}(2+3x)^2\cos(x) - \frac{2\cos^3(x)}{3} + 4(2+3x)\sin(x) - \frac{1}{3}(2+3x)^2\cos(x)\sin$$
$$= 14\cos(x) - \frac{2}{3}(2+3x)^2\cos(x) - \frac{2\cos^3(x)}{3} + 4(2+3x)\sin(x) - \frac{1}{3}(2+3x)^2\cos(x)\text{s}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.77

$$\frac{1}{12}\left(-9\left(-14+12x+9x^2\right)\cos(x) + \left(2+12x+9x^2\right)\cos(3x) - 2(2+3x)(-27\sin(x)+\sin(3x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*Sin[x]^3,x]
[Out] (-9*(-14 + 12*x + 9*x^2)*Cos[x] + (2 + 12*x + 9*x^2)*Cos[3*x] - 2*(2 + 3*x)
*(-27*Sin[x] + Sin[3*x]))/12
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.95

method	result
risch	$\left(-\frac{27}{4}x^2-9x+\frac{21}{2}\right)\cos(x)+\frac{9(2+3x)\sin(x)}{2}+\left(\frac{3}{4}x^2+x+\frac{1}{6}\right)\cos(3x)-\frac{(2+3x)\sin(3x)}{6}$
default	$-3x^2(2+\sin^2(x))\cos(x)+12\cos(x)+12x\sin(x)+2x(\sin^3(x))-\frac{2(2+\sin^2(x))\cos(x)}{3}-4x(2+\sin$
norman	$\frac{-16(\tan^4(\frac{x}{2}))-\frac{40(\tan^6(\frac{x}{2}))}{3}-8x-6x^2+\frac{128(\tan^3(\frac{x}{2}))}{3}+16(\tan^5(\frac{x}{2}))+24x\tan(\frac{x}{2})-24x(\tan^2(\frac{x}{2}))+64x(\tan^3(\frac{x}{2}))+24x(\tan^4(\frac{x}{2}))+}{(1+\tan^2(\frac{x}{2}))^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*sin(x)^3,x,method=_RETURNVERBOSE)
[Out] -3*x^2*(2+sin(x)^2)*cos(x)+12*cos(x)+12*x*sin(x)+2*x*sin(x)^3-2/3*(2+sin(x)
^2)*cos(x)-4*x*(2+sin(x)^2)*cos(x)+4/3*sin(x)^3+8*sin(x)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 1.02

$$\frac{4}{3}\cos(x)^3+\frac{1}{12}\left(9x^2-2\right)\cos(3x)+x\cos(3x)-\frac{27}{4}\left(x^2-2\right)\cos(x)-9x\cos(x)-\frac{1}{2}x\sin(3x)+\frac{27}{2}x\sin(x)-4\cos(x)-\frac{1}{3}\sin(3x)+9\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 4/3*cos(x)^3 + 1/12*(9*x^2 - 2)*cos(3*x) + x*cos(3*x) - 27/4*(x^2 - 2)*cos(
x) - 9*x*cos(x) - 1/2*x*sin(3*x) + 27/2*x*sin(x) - 4*cos(x) - 1/3*sin(3*x)
+ 9*sin(x)
```

Fricas [A]
time = 3.43, size = 50, normalized size = 0.77

$$\frac{1}{3}\left(9x^2+12x+2\right)\cos(x)^3-\left(9x^2+12x-10\right)\cos(x)-\frac{2}{3}\left((3x+2)\cos(x)^2-21x-14\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 12*x + 2)*cos(x)^3 - (9*x^2 + 12*x - 10)*cos(x) - 2/3*((3*x +
2)*cos(x)^2 - 21*x - 14)*sin(x)
```

Sympy [A]
time = 0.21, size = 100, normalized size = 1.54

$$-9x^2\sin^2(x)\cos(x)-6x^2\cos^3(x)+14x\sin^3(x)-12x\sin^2(x)\cos(x)+12x\sin(x)\cos^2(x)-8x\cos^3(x)+\frac{28\sin^3(x)}{3}+10\sin^2(x)\cos(x)+8\sin(x)\cos^2(x)+\frac{32\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*sin(x)**3,x)
```

```
[Out] -9*x**2*sin(x)**2*cos(x) - 6*x**2*cos(x)**3 + 14*x*sin(x)**3 - 12*x*sin(x)*
*2*cos(x) + 12*x*sin(x)*cos(x)**2 - 8*x*cos(x)**3 + 28*sin(x)**3/3 + 10*sin
(x)**2*cos(x) + 8*sin(x)*cos(x)**2 + 32*cos(x)**3/3
```

Giac [A]
time = 0.38, size = 51, normalized size = 0.78

$$\frac{1}{12}\left(9x^2+12x+2\right)\cos(3x)-\frac{3}{4}\left(9x^2+12x-14\right)\cos(x)-\frac{1}{6}(3x+2)\sin(3x)+\frac{9}{2}(3x+2)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*(9*x^2 + 12*x + 2)*cos(3*x) - 3/4*(9*x^2 + 12*x - 14)*cos(x) - 1/6*(3*
x + 2)*sin(3*x) + 9/2*(3*x + 2)*sin(x)
```

Mupad [B]
time = 3.03, size = 65, normalized size = 1.00

$$10\cos(x)+\frac{28\sin(x)}{3}-9x^2\cos(x)+4x\cos(x)^3+\frac{2\cos(x)^3}{3}+3x^2\cos(x)^3-\frac{4\cos(x)^2\sin(x)}{3}-12x\cos(x)+14x\sin(x)-2x\cos(x)^2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(3*x + 2)^2,x)
```

```
[Out] 10*cos(x) + (28*sin(x))/3 - 9*x^2*cos(x) + 4*x*cos(x)^3 + (2*cos(x)^3)/3 +
3*x^2*cos(x)^3 - (4*cos(x)^2*sin(x))/3 - 12*x*cos(x) + 14*x*sin(x) - 2*x*co
s(x)^2*sin(x)
```

3.917 $\int \sec^{1+m}(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^m(x)}{m}$$

[Out] sec(x)^m/m

Rubi [A]
time = 0.02, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2702, 30}

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^(1 + m)*Sin[x],x]

[Out] Sec[x]^m/m

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \sec^{1+m}(x)\sin(x)\,dx &= \text{Subst}\left(\int x^{-1+m}\,dx, x, \sec(x)\right)\\ &= \frac{\sec^m(x)}{m}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{\sec^m(x)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^(1 + m)*Sin[x],x]
```

```
[Out] Sec[x]^m/m
```

Maple [A]
time = 0.74, size = 11, normalized size = 1.38

method	result
default	$\frac{\left(\frac{1}{\cos(x)}\right)^{m}}{m}$
norman	$\frac{\frac{e^{(1+m)\ln\left(\frac{1+\tan^2\left(\frac{x}{2}\right)}{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)}}{m}-\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)e^{(1+m)\ln\left(\frac{1+\tan^2\left(\frac{x}{2}\right)}{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}\right)}}{m}}{1+\tan^2\left(\frac{x}{2}\right)}$
risch	$2^{m}\left(e^{2ix}+1\right)^{-m}\left(e^{i\Re(x)}\right)^{m}(-1)^{-\frac{\operatorname{csgn}\left(\frac{ie^{ix}}{e^{2ix}+1}\right)\operatorname{csgn}\left(ie^{ix}\right)\operatorname{csgn}\left(\frac{i}{e^{2ix}+1}\right)}{2}}e^{-\Im(x)-m\Im(x)}e^{-ix}e^{-\frac{i\pi\operatorname{csgn}\left(\frac{ie^{ix}}{e^{2ix}+1}\right)^{3}}{2}}e^{\frac{i\pi\operatorname{csgn}\left(\frac{ie^{ix}}{e^{2ix}+1}\right)^{2}\operatorname{csgn}(}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^(1+m)*sin(x),x,method=_RETURNVERBOSE)
```

```
[Out] (1/cos(x))^m/m
```

Maxima [A]
time = 0.30, size = 10, normalized size = 1.25

$$\frac{\cos(x)^{-m}}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="maxima")
```

```
[Out] cos(x)^(-m)/m
```

Fricas [A]
time = 1.90, size = 14, normalized size = 1.75

$$\frac{\frac{1}{\cos(x)}^{m+1}\cos(x)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="fricas")
```

```
[Out] (1/cos(x))^(m + 1)*cos(x)/m
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(x)\sec^{m+1}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**(1+m)*sin(x),x)
[Out] Integral(sin(x)*sec(x)**(m + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(1+m)*sin(x),x, algorithm="giac")
[Out] integrate(sec(x)^(m + 1)*sin(x), x)
```

Mupad [B]
time = 0.14, size = 10, normalized size = 1.25

$$\frac{\left(\frac{1}{\cos(x)}\right)^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1/cos(x))^(m + 1),x)
[Out] (1/cos(x))^m/m
```

3.918 $\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx$

Optimal. Leaf size=32

$$-\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

```
[Out] -cos(b*x+a)^(1+n)*sin(b*x+a)^(-1-n)/b/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2643\}$

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.00

$$-\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
[Out] -((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\cos^{n}\left(b x +a \right)\right) \left(\sin^{-2-n}\left(b x +a \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
[Out] int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(32) = 64$.
time = 0.52, size = 125, normalized size = 3.91

$$\frac{2\left(\frac{\sin\left(bx+a\right)^{2}}{\left(\cos\left(bx+a\right)+1\right)^{2}}-1\right)\left(\cos\left(bx+a\right)+1\right)e^{\left(n\log\left(\frac{\sin\left(bx+a\right)}{\cos\left(bx+a\right)+1}+1\right)-n\log\left(\frac{\sin\left(bx+a\right)}{\cos\left(bx+a\right)+1}\right)+n\log\left(-\frac{\sin\left(bx+a\right)}{\cos\left(bx+a\right)+1}+1\right)\right)}}{\left(2^{n+2}n+2^{n+2}\right)b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="maxima")
[Out] 2*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)*e^(n*log(sin
(b*x + a)/(cos(b*x + a) + 1) + 1) - n*log(sin(b*x + a)/(cos(b*x + a) + 1))
+ n*log(-sin(b*x + a)/(cos(b*x + a) + 1) + 1))/((2^(n + 2)*n + 2^(n + 2))*b
*sin(b*x + a))
```

Fricas [A]
time = 2.29, size = 41, normalized size = 1.28

$$-\frac{\cos\left(bx+a\right)^{n}\sin\left(bx+a\right)^{-n-2}\cos\left(bx+a\right)\sin\left(bx+a\right)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="fricas")
[Out] -cos(b*x + a)^n*sin(b*x + a)^(-n - 2)*cos(b*x + a)*sin(b*x + a)/(b*n + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{-n-2}\left(a+bx\right)\cos^{n}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**n*sin(b*x+a)**(-2-n),x)
```

```
[Out] Integral(sin(a + b*x)**(-n - 2)*cos(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^n*sin(b*x + a)^(-n - 2), x)
```

Mupad [B]
time = 3.43, size = 45, normalized size = 1.41

$$-\frac{\cos\left(a+b\,x\right)^n\,\sin\left(2\,a+2\,b\,x\right)}{2\,b\,\sin\left(a+b\,x\right)^n\,\sin\left(a+b\,x\right)^2\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^n/sin(a + b*x)^(n + 2),x)
```

```
[Out] -(cos(a + b*x)^n*sin(2*a + 2*b*x))/(2*b*sin(a + b*x)^n*sin(a + b*x)^2*(n +
1))
```

3.919 $\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx$

Optimal. Leaf size=3

$$\text{ArcTan}(\sin(x))$$

```
[Out] arctan(sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4482, 3269, 209}

$$\text{ArcTan}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3269

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin^2(x)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(x)\right)\\
&= \tan^{-1}(\sin(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\text{ArcTan}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A]
time = 0.13, size = 4, normalized size = 1.33

method	result	size
default	$\arctan(\sin(x))$	4
risch	$\frac{i\ln(e^{2ix}-2e^{ix}-1)}{2}-\frac{i\ln(e^{2ix}+2e^{ix}-1)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+sin(x)*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(3) = 6$.
time = 0.29, size = 45, normalized size = 15.00

$$\frac{1}{2}\arctan(\sin(2x)+2\sin(x),\cos(2x)+2\cos(x)-1)-\frac{1}{2}\arctan(\sin(2x)-2\sin(x),\cos(2x)-2\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan2(sin(2*x) + 2*sin(x), cos(2*x) + 2*cos(x) - 1) - 1/2*arctan2(sin
(2*x) - 2*sin(x), cos(2*x) - 2*cos(x) - 1)
```

Fricas [A]
time = 2.42, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x)\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x)
```

```
[Out] Integral(1/(sin(x)*tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 3, normalized size = 1.00

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

Mupad [B]
time = 3.19, size = 26, normalized size = 8.67

$$\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2} + \frac{5\tan\left(\frac{x}{2}\right)}{2}\right) - \operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*tan(x) + 1/cos(x)),x)
```

```
[Out] atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

3.920 $\int (a+bx+cx^2)\sin(x)\,dx$

Optimal. Leaf size=35

$$-a\cos(x)+2c\cos(x)-bx\cos(x)-cx^2\cos(x)+b\sin(x)+2cx\sin(x)$$

```
[Out] -a*cos(x)+2*c*cos(x)-b*x*cos(x)-c*x^2*cos(x)+b*sin(x)+2*c*x*sin(x)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6874, 2718, 3377, 2717}

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*Sin[x],x]
```

```
[Out] -(a*Cos[x]) + 2*c*Cos[x] - b*x*Cos[x] - c*x^2*Cos[x] + b*Sin[x] + 2*c*x*Sin
[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx+cx^2\right)\sin(x)\,dx &= \int \left(a\sin(x)+bx\sin(x)+cx^2\sin(x)\right)\,dx \\
&= a\int \sin(x)\,dx + b\int x\sin(x)\,dx + c\int x^2\sin(x)\,dx \\
&= -a\cos(x) - bx\cos(x) - cx^2\cos(x) + b\int \cos(x)\,dx + (2c)\int x\cos(x)\,dx \\
&= -a\cos(x) - bx\cos(x) - cx^2\cos(x) + b\sin(x) + 2cx\sin(x) - (2c)\int \sin(x)\,dx \\
&= -a\cos(x) + 2c\cos(x) - bx\cos(x) - cx^2\cos(x) + b\sin(x) + 2cx\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.91

$$-a\cos(x) - bx\cos(x) - c\left(-2+x^2\right)\cos(x) + b\sin(x) + 2cx\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*Sin[x],x]
[Out] -(a*Cos[x]) - b*x*Cos[x] - c*(-2 + x^2)*Cos[x] + b*Sin[x] + 2*c*x*Sin[x]
```

Maple [A]
time = 0.04, size = 36, normalized size = 1.03

method	result
risch	$\left(-c\,x^2 - bx - a + 2c\right)\cos\left(x\right) + \left(2cx + b\right)\sin\left(x\right)$
default	$c\left(-x^2\cos\left(x\right) + 2\cos\left(x\right) + 2x\sin\left(x\right)\right) + b\left(\sin\left(x\right) - x\cos\left(x\right)\right) - a\cos\left(x\right)$
norman	$\frac{bx\left(\tan^2\left(\frac{x}{2}\right)\right) + c\,x^2\left(\tan^2\left(\frac{x}{2}\right)\right) - bx + 2b\tan\left(\frac{x}{2}\right) - c\,x^2 + 4cx\tan\left(\frac{x}{2}\right) - 2a + 4c}{1+\tan^2\left(\frac{x}{2}\right)}$
meijerg	$4c\sqrt{\pi}\,\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(-\frac{x^2}{2}+1\right)\cos(x)}{2\sqrt{\pi}} + \frac{x\sin(x)}{2\sqrt{\pi}}\right) + 2b\sqrt{\pi}\,\left(-\frac{x\cos(x)}{2\sqrt{\pi}} + \frac{\sin(x)}{2\sqrt{\pi}}\right) + a\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}} - \frac{\cos(x}{\sqrt{\pi}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*sin(x),x,method=_RETURNVERBOSE)
[Out] c*(-x^2*cos(x)+2*cos(x)+2*x*sin(x))+b*(sin(x)-x*cos(x))-a*cos(x)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.00

$$-(x\cos\left(x\right) - \sin\left(x\right))b - \left(\left(x^2-2\right)\cos\left(x\right) - 2\,x\sin\left(x\right)\right)c - a\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="maxima")
[Out] -(x*cos(x) - sin(x))*b - ((x^2 - 2)*cos(x) - 2*x*sin(x))*c - a*cos(x)
```

Fricas [A]
time = 3.83, size = 27, normalized size = 0.77

$$-\left(cx^2+bx+a-2\,c\right)\cos\left(x\right)+\left(2\,cx+b\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="fricas")
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

Sympy [A]
time = 0.09, size = 39, normalized size = 1.11

$$-a\cos\left(x\right)-bx\cos\left(x\right)+b\sin\left(x\right)-cx^2\cos\left(x\right)+2cx\sin\left(x\right)+2c\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*sin(x),x)
[Out] -a*cos(x) - b*x*cos(x) + b*sin(x) - c*x**2*cos(x) + 2*c*x*sin(x) + 2*c*cos(
x)
```

Giac [A]
time = 0.41, size = 27, normalized size = 0.77

$$-\left(cx^2+bx+a-2\,c\right)\cos\left(x\right)+\left(2\,cx+b\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*sin(x),x, algorithm="giac")
[Out] -(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

Mupad [B]
time = 0.06, size = 34, normalized size = 0.97

$$b\,\sin\left(x\right)-\cos\left(x\right)\,\left(a-2\,c\right)-b\,x\,\cos\left(x\right)+2\,c\,x\,\sin\left(x\right)-c\,x^2\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a + b*x + c*x^2),x)
[Out] b*sin(x) - cos(x)*(a - 2*c) - b*x*cos(x) + 2*c*x*sin(x) - c*x^2*cos(x)
```

3.921 $\int \frac{\sin(x^5)}{x} dx$

Optimal. Leaf size=8

$$\frac{\text{Si}(x^5)}{5}$$

[Out] 1/5*Si(x^5)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3456}

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

[In] Int[Sin[x^5]/x,x]

[Out] SinIntegral[x^5]/5

Rule 3456

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rubi steps

$$\int \frac{\sin(x^5)}{x} dx = \frac{\text{Si}(x^5)}{5}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{\text{Si}(x^5)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x^5]/x,x]

[Out] SinIntegral[x^5]/5

Maple [A]
time = 0.05, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\mathrm{sinIntegral}(x^5)}{5}$	7
default	$\frac{\mathrm{sinIntegral}(x^5)}{5}$	7
meijerg	$\frac{\mathrm{sinIntegral}(x^5)}{5}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*Si(x^5)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 17, normalized size = 2.12

$$-\frac{1}{10} i \operatorname{Ei}(i x^5) + \frac{1}{10} i \operatorname{Ei}(-i x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/10*I*Ei(I*x^5) + 1/10*I*Ei(-I*x^5)
```

Fricas [A]
time = 2.48, size = 6, normalized size = 0.75

$$\frac{1}{5} \operatorname{Si}(x^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="fricas")
```

```
[Out] 1/5*sin_integral(x^5)
```

Sympy [A]
time = 0.31, size = 5, normalized size = 0.62

$$\frac{\operatorname{Si}(x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**5)/x,x)
```

```
[Out] Si(x**5)/5
```

Giac [A]
time = 0.40, size = 6, normalized size = 0.75

$$\frac{1}{5}\,\mathrm{Si}\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^5)/x,x, algorithm="giac")
```

```
[Out] 1/5*sin_integral(x^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.12

$$\frac{\mathrm{sinint}(x^5)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^5)/x,x)
```

```
[Out] sinint(x^5)/5
```

3.922 $\int \frac{\sin(2^x)}{1+2^x} \, dx$

Optimal. Leaf size=37

$$\frac{\text{CosIntegral}(1+2^x)\sin(1)}{\log(2)}+\frac{\text{Si}(2^x)}{\log(2)}-\frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

```
[Out] Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)
```

Rubi [A]
time = 0.13, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2320, 6874, 3380, 3384, 3383}

$$\frac{\sin(1)\text{CosIntegral}(2^x+1)}{\log(2)}+\frac{\text{Si}(2^x)}{\log(2)}-\frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2^x]/(1 + 2^x),x]
```

```
[Out] (CosIntegral[1 + 2^x]*Sin[1])/Log[2] + SinIntegral[2^x]/Log[2] - (Cos[1]*Si
nIntegral[1 + 2^x])/Log[2]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(2^x)}{1+2^x}\,dx &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x(1+x)}\,dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{x} - \frac{\sin(x)}{1+x}\right)\,dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, 2^x\right)}{\log(2)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, 2^x\right)}{\log(2)} + \frac{\sin(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, 2^x\right)}{\log(2)} \\
&= \frac{\text{Ci}(1+2^x)\sin(1)}{\log(2)} + \frac{\text{Si}(2^x)}{\log(2)} - \frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.78

$$\frac{\text{CosIntegral}(1+2^x)\sin(1) + \text{Si}(2^x) - \cos(1)\text{Si}(1+2^x)}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2^x]/(1 + 2^x),x]
[Out] (CosIntegral[1 + 2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1 + 2^
x])/Log[2]
```

Maple [A]
time = 0.07, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\text{sinIntegral}(1+2^x)\cos(1)+\text{cosineIntegral}(1+2^x)\sin(1)+\text{sinIntegral}(2^x)}{\ln(2)}$	30
default	$\frac{-\text{sinIntegral}(1+2^x)\cos(1)+\text{cosineIntegral}(1+2^x)\sin(1)+\text{sinIntegral}(2^x)}{\ln(2)}$	30

risch	$-\frac{i\,\mathrm{expIntegral}(1,-i2^x-i)\,\mathrm{e}^{-i}}{2\ln(2)}+\frac{i\,\mathrm{expIntegral}(1,-i2^x)}{2\ln(2)}+\frac{i\,\mathrm{expIntegral}(1,i2^x+i)\,\mathrm{e}^{i}}{2\ln(2)}-\frac{i\,\mathrm{expIntegral}(1,i2^x)}{2\ln(2)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2^x)/(1+2^x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/ln(2)*(-Si(1+2^x)*cos(1)+Ci(1+2^x)*sin(1)+Si(2^x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="maxima")
```

```
[Out] integrate(sin(2^x)/(2^x + 1), x)
```

Fricas [A]
time = 3.51, size = 43, normalized size = 1.16

$$\frac{\mathrm{Ci}\left(2^{x}+1\right)\sin\left(1\right)+\mathrm{Ci}\left(-2^{x}-1\right)\sin\left(1\right)-2\cos\left(1\right)\mathrm{Si}\left(2^{x}+1\right)+2\ \mathrm{Si}\left(2^{x}\right)}{2\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="fricas")
```

```
[Out] 1/2*(cos_integral(2^x + 1)*sin(1) + cos_integral(-2^x - 1)*sin(1) - 2*cos(1
)*sin_integral(2^x + 1) + 2*sin_integral(2^x))/log(2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(2^{x}\right)}{2^{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2**x)/(1+2**x),x)
```

```
[Out] Integral(sin(2**x)/(2**x + 1), x)
```

Giac [A]
time = 0.41, size = 29, normalized size = 0.78

$$\frac{\mathrm{Ci}\left(2^{x}+1\right)\sin\left(1\right)-\cos\left(1\right)\mathrm{Si}\left(2^{x}+1\right)+\mathrm{Si}\left(2^{x}\right)}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2^x)/(1+2^x),x, algorithm="giac")
```

```
[Out] (cos_integral(2^x + 1)*sin(1) - cos(1)*sin_integral(2^x + 1) + sin_integral
(2^x))/log(2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sin(2^x)}{2^x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2^x)/(2^x + 1),x)
```

```
[Out] int(sin(2^x)/(2^x + 1), x)
```

3.923 $\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2)\, dx$

Optimal. Leaf size=14

$$\frac{1}{7}\sin^{\frac{7}{4}}(2x^2)$$

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3522}

$$\frac{1}{7}\sin^{\frac{7}{4}}(2x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

```
[Out] Sin[2*x^2]^(7/4)/7
```

Rule 3522

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(2x^2)\sin^{\frac{3}{4}}(2x^2)\, dx = \frac{1}{7}\sin^{\frac{7}{4}}(2x^2)$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{7}\sin^{\frac{7}{4}}(2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

```
[Out] Sin[2*x^2]^(7/4)/7
```

Maple [A]
time = 0.03, size = 11, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\left(\sin^{\frac{7}{4}}\left(2x^{2}\right)\right)}{7}$	11
default	$\frac{\left(\sin^{\frac{7}{4}}\left(2x^{2}\right)\right)}{7}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.71

$$\frac{1}{7}\sin\left(2\,x^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="maxima")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Fricas [A]
time = 3.26, size = 10, normalized size = 0.71

$$\frac{1}{7}\sin\left(2\,x^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Sympy [A]
time = 6.32, size = 10, normalized size = 0.71

$$\frac{\sin^{\frac{7}{4}}\left(2x^{2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x**2)*sin(2*x**2)**(3/4),x)
```

```
[Out] sin(2*x**2)**(7/4)/7
```

Giac [A]
time = 0.43, size = 10, normalized size = 0.71

$$\frac{1}{7}\sin\left(2\,x^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="giac")
```

```
[Out] 1/7*sin(2*x^2)^(7/4)
```

Mupad [B]
time = 3.14, size = 41, normalized size = 2.93

$$-\frac{\cos\left(2\,x^2\right)^2\,\sin\left(2\,x^2\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{8},1;\ 2;\ \cos\left(2\,x^2\right)^2\right)}{8\,{\left({\sin\left(2\,x^2\right)}^2\right)}^{7/8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)
```

```
[Out] -(cos(2*x^2)^2*sin(2*x^2)^(7/4)*hypergeom([1/8, 1], 2, cos(2*x^2)^2))/(8*(s
in(2*x^2)^2)^(7/8))
```

3.924 $\int x \sec^2(x^2) \tan^2(x^2)\, dx$

Optimal. Leaf size=10

$$\frac{1}{6}\tan^3\left(x^2\right)$$

```
[Out] 1/6*tan(x^2)^3
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {6818}

$$\frac{1}{6}\tan^3\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x^2]^2*Tan[x^2]^2,x]
[Out] Tan[x^2]^3/6
```

Rule 6818

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \sec^2\left(x^2\right)\tan^2\left(x^2\right)\, dx = \frac{1}{6}\tan^3\left(x^2\right)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6}\tan^3\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x^2]^2*Tan[x^2]^2,x]
[Out] Tan[x^2]^3/6
```

Maple [A]
time = 0.08, size = 15, normalized size = 1.50

method	result	size
derivativedivides	$\frac{\sin^3(x^2)}{6\cos(x^2)^3}$	15
default	$\frac{\sin^3(x^2)}{6\cos(x^2)^3}$	15
risch	$-\frac{i\left(3\,e^{4ix^2}+1\right)}{3\left(e^{2ix^2}+1\right)^3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x^2)^2*tan(x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/6*sin(x^2)^3/cos(x^2)^3
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="maxima")
[Out] 1/6*tan(x^2)^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.
time = 2.67, size = 20, normalized size = 2.00

$$-\frac{\left(\cos\left(x^2\right)^2-1\right)\sin\left(x^2\right)}{6\cos\left(x^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="fricas")
[Out] -1/6*(cos(x^2)^2 - 1)*sin(x^2)/cos(x^2)^3
```

Sympy [A]
time = 0.58, size = 7, normalized size = 0.70

$$\frac{\tan^3\left(x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x**2)**2*tan(x**2)**2,x)
[Out] tan(x**2)**3/6
```

Giac [A]
time = 0.39, size = 8, normalized size = 0.80

$$\frac{1}{6}\tan\left(x^2\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*tan(x^2)^3
```

Mupad [B]
time = 3.09, size = 19, normalized size = 1.90

$$\frac{\tan(x^2)}{6\cos\left(x^2\right)^2} - \frac{\tan(x^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*tan(x^2)^2)/cos(x^2)^2,x)
```

```
[Out] tan(x^2)/(6*cos(x^2)^2) - tan(x^2)/6
```

3.925 $\int x^2 \cos^7(a+bx^3) \sin(a+bx^3) \, dx$

Optimal. Leaf size=17

$$-\frac{\cos^8(a+bx^3)}{24b}$$

```
[Out] -1/24*cos(b*x^3+a)^8/b
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3523}

$$-\frac{\cos^8(a+bx^3)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] -1/24*Cos[a + b*x^3]^8/b
```

Rule 3523

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[-Cos[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ
[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x^2 \cos^7(a+bx^3) \sin(a+bx^3) \, dx = -\frac{\cos^8(a+bx^3)}{24b}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$-\frac{\cos^8(a+bx^3)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] -1/24*Cos[a + b*x^3]^8/b
```

Maple [A]
time = 0.13, size = 16, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{\cos^{8}\left(b\,x^{3}+a\right)}{24b}$	16
default	$-\frac{\cos^{8}\left(b\,x^{3}+a\right)}{24b}$	16
risch	$-\frac{\cos\left(8b\,x^{3}+8a\right)}{3072b}-\frac{\cos\left(6b\,x^{3}+6a\right)}{384b}-\frac{7\cos\left(4b\,x^{3}+4a\right)}{768b}-\frac{7\cos\left(2b\,x^{3}+2a\right)}{384b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*cos(b*x^3+a)^8/b
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^{3}+a\right)^{8}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

Fricas [A]
time = 2.66, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^{3}+a\right)^{8}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

Sympy [A]
time = 1.78, size = 27, normalized size = 1.59

$$\begin{cases} -\frac{\cos^{8}\left(a+bx^{3}\right)}{24b} & \text{for}\; b\neq 0\\ \frac{x^{3}\sin\left(a\right)\cos^{7}\left(a\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((-cos(a + b*x**3)**8/(24*b), Ne(b, 0)), (x**3*sin(a)*cos(a)**7/3,
 True))
```

Giac [A]
time = 0.40, size = 15, normalized size = 0.88

$$-\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/24*cos(b*x^3 + a)^8/b
```

Mupad [B]
time = 3.26, size = 56, normalized size = 3.29

$$-\frac{56\,\cos\left(2\,b\,x^3+2\,a\right)+28\,\cos\left(4\,b\,x^3+4\,a\right)+8\,\cos\left(6\,b\,x^3+6\,a\right)+\cos\left(8\,b\,x^3+8\,a\right)}{3072\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

```
[Out] -(56*cos(2*a + 2*b*x^3) + 28*cos(4*a + 4*b*x^3) + 8*cos(6*a + 6*b*x^3) + co
s(8*a + 8*b*x^3))/(3072*b)
```

3.926 $\int x^5 \cos^7(a+bx^3) \sin(a+bx^3)\, dx$

Optimal. Leaf size=129

$$\frac{35x^3}{3072b}-\frac{x^3\cos^8(a+bx^3)}{24b}+\frac{35\cos(a+bx^3)\sin(a+bx^3)}{3072b^2}+\frac{35\cos^3(a+bx^3)\sin(a+bx^3)}{4608b^2}+\frac{7\cos^5(a+bx^3)\sin}{1152b^2}$$

```
[Out] 35/3072*x^3/b-1/24*x^3*cos(b*x^3+a)^8/b+35/3072*cos(b*x^3+a)*sin(b*x^3+a)/b
^2+35/4608*cos(b*x^3+a)^3*sin(b*x^3+a)/b^2+7/1152*cos(b*x^3+a)^5*sin(b*x^3+
a)/b^2+1/192*cos(b*x^3+a)^7*sin(b*x^3+a)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3525, 3461, 2715, 8}

$$\frac{\sin(a+bx^3)\cos^7(a+bx^3)}{192b^2}+\frac{7\sin(a+bx^3)\cos^5(a+bx^3)}{1152b^2}+\frac{35\sin(a+bx^3)\cos^3(a+bx^3)}{4608b^2}+\frac{35\sin(a+bx^3)\cos(a+bx^3)}{3072b^2}-\frac{x^3\cos^8(a+bx^3)}{24b}+\frac{35x^3}{3072b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (35*x^3)/(3072*b) - (x^3*Cos[a + b*x^3]^8)/(24*b) + (35*Cos[a + b*x^3]*Sin[
a + b*x^3])/(3072*b^2) + (35*Cos[a + b*x^3]^3*Sin[a + b*x^3])/(4608*b^2) +
(7*Cos[a + b*x^3]^5*Sin[a + b*x^3])/(1152*b^2) + (Cos[a + b*x^3]^7*Sin[a +
b*x^3])/(192*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3525

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p
 + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 \cos^7\left(a+bx^3\right) \sin\left(a+bx^3\right)\,dx &= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\int x^2\cos^8\left(a+bx^3\right)\,dx}{8b}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\text{Subst}\left(\int\cos^8(a+bx)\,dx,x,x^3\right)}{24b}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)}{192b^2}+\frac{7\text{Subst}\left(\int\cos^6(a\right.}{192}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{7\cos^5\left(a+bx^3\right)\sin\left(a+bx^3\right)}{1152b^2}+\frac{\cos^7\left(a+bx^3\right)\text{si}}{192b^2}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos^3\left(a+bx^3\right)\sin\left(a+bx^3\right)}{4608b^2}+\frac{7\cos^5\left(a+bx^3\right.}{115}\\
&= -\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(a+bx^3\right.}{4608}\\
&= \frac{35x^3}{3072b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}+\frac{35\cos\left(a+bx^3\right)\sin\left(a+bx^3\right)}{3072b^2}+\frac{35\cos^3\left(\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 120, normalized size = 0.93

$$\frac{-1344bx^3\cos\left(2\left(a+bx^3\right)\right)-672bx^3\cos\left(4\left(a+bx^3\right)\right)-192bx^3\cos\left(6\left(a+bx^3\right)\right)-24bx^3\cos\left(8\left(a+bx^3\right)\right)+672\sin\left(2\left(a+bx^3\right)\right)+168\sin\left(4\left(a+bx^3\right)\right)+32\sin\left(6\left(a+bx^3\right)\right)+3\sin\left(8\left(a+bx^3\right)\right)}{73728b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

```
[Out] (-1344*b*x^3*Cos[2*(a + b*x^3)] - 672*b*x^3*Cos[4*(a + b*x^3)] - 192*b*x^3*
Cos[6*(a + b*x^3)] - 24*b*x^3*Cos[8*(a + b*x^3)] + 672*Sin[2*(a + b*x^3)] +
 168*Sin[4*(a + b*x^3)] + 32*Sin[6*(a + b*x^3)] + 3*Sin[8*(a + b*x^3)])/(73
728*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(117) = 234$.
time = 0.68, size = 403, normalized size = 3.12

method	result

risch	$-\frac{x^3\cos(8b\,x^3+8a)}{3072b}+\frac{\sin(8b\,x^3+8a)}{24576b^2}-\frac{x^3\cos(6b\,x^3+6a)}{384b}+\frac{\sin(6b\,x^3+6a)}{2304b^2}-\frac{7x^3\cos(4b\,x^3+4a)}{768b}+\frac{7\sin(4b\,x^3+4a)}{3072b^2}-7x$
default	$\frac{-\frac{4x^3}{3b}+\frac{4\tan(b\,x^3+a)}{3b^2}+\frac{4x^3(\tan^2(b\,x^3+a))}{3b}}{128+128(\tan^2(b\,x^3+a))}+\frac{\frac{\tan(b\,x^3+a)}{b^2}-\frac{x^3}{b}-\frac{\tan^3(b\,x^3+a)}{b^2}+\frac{6x^3(\tan^2(b\,x^3+a))}{b}-\frac{x^3(\tan^4(b\,x^3+a))}{b}}{128(1+\tan^2(b\,x^3+a))^2}+\frac{-18(\cos$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(-4/3*x^3/b+4/3/b^2*tan(b*x^3+a)+4/3*x^3/b*tan(b*x^3+a)^2)/(1+tan(b*x
^3+a)^2)+1/128*(1/b^2*tan(b*x^3+a)-x^3/b-1/b^2*tan(b*x^3+a)^3+6*x^3/b*tan(b
*x^3+a)^2-x^3/b*tan(b*x^3+a)^4)/(1+tan(b*x^3+a)^2)^2+1/1152*(-18*cos(b*x^3+
a)^2*b*x^3-6*x^3*b*cos(3*b*x^3+3*a)^2+12*b*x^3+sin(3*b*x^3+3*a)*cos(3*b*x^3
+3*a)+9*cos(b*x^3+a)*sin(b*x^3+a))/b^2+1/128*(-1/6*x^3/b+1/12/b^2*tan(2*b*x
^3+2*a)+1/6*x^3/b*tan(2*b*x^3+2*a)^2)/(1+tan(2*b*x^3+2*a)^2)+1/128*(-1/24*x
^3/b+1/48/b^2*tan(2*b*x^3+2*a)-1/48/b^2*tan(2*b*x^3+2*a)^3+1/4*x^3/b*tan(2*
b*x^3+2*a)^2-1/24*x^3/b*tan(2*b*x^3+2*a)^4)/(1+tan(2*b*x^3+2*a)^2)^2
```

Maxima [A]
time = 0.31, size = 126, normalized size = 0.98

$$-\frac{24\,bx^3\cos(8\,bx^3+8\,a)+192\,bx^3\cos(6\,bx^3+6\,a)+672\,bx^3\cos(4\,bx^3+4\,a)+1344\,bx^3\cos(2\,bx^3+2\,a)-3\,\sin(8\,bx^3+8\,a)-32\,\sin(6\,bx^3+6\,a)-168\,\sin(4\,bx^3+4\,a)-672\,\sin(2\,bx^3+2\,a)}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672*
b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 +
8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2
*a))/b^2
```

Fricas [A]
time = 2.46, size = 85, normalized size = 0.66

$$-\frac{384\,bx^3\cos(bx^3+a)^8-105\,bx^3-\left(48\,\cos(bx^3+a)^7+56\,\cos(bx^3+a)^5+70\,\cos(bx^3+a)^3+105\,\cos(bx^3+a)\right)\sin(bx^3+a)}{9216\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/9216*(384*b*x^3*cos(b*x^3 + a)^8 - 105*b*x^3 - (48*cos(b*x^3 + a)^7 + 56
*cos(b*x^3 + a)^5 + 70*cos(b*x^3 + a)^3 + 105*cos(b*x^3 + a))*sin(b*x^3 + a
))/b^2
```

Sympy [A]
time = 4.78, size = 241, normalized size = 1.87

$$\begin{cases}\frac{35x^3\sin^8(a+bx^3)}{3072b}+\frac{35x^3\sin^6(a+bx^3)\cos^2(a+bx^3)}{768b}+\frac{35x^3\sin^4(a+bx^3)\cos^4(a+bx^3)}{512b}+\frac{35x^3\sin^2(a+bx^3)\cos^6(a+bx^3)}{768b}-\frac{31x^3\cos^8(a+bx^3)}{1024b}+\frac{35\sin^7(a+bx^3)\cos(a+bx^3)}{3072b^2}+\frac{385\sin^5(a+bx^3)\cos^3(a+bx^3)}{9216b^2}+\frac{511\sin^3(a+bx^3)\cos^5(a+bx^3)}{9216b^2}+\frac{31\sin(a+bx^3)\cos^7(a+bx^3)}{1024b^2} & \text{for}\ b\neq 0\\ \frac{x^6\sin(a)\cos^7(a)}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

```
[Out] Piecewise((35*x**3*sin(a + b*x**3)**8/(3072*b) + 35*x**3*sin(a + b*x**3)**6
*cos(a + b*x**3)**2/(768*b) + 35*x**3*sin(a + b*x**3)**4*cos(a + b*x**3)**4
/(512*b) + 35*x**3*sin(a + b*x**3)**2*cos(a + b*x**3)**6/(768*b) - 31*x**3*
cos(a + b*x**3)**8/(1024*b) + 35*sin(a + b*x**3)**7*cos(a + b*x**3)/(3072*b
**2) + 385*sin(a + b*x**3)**5*cos(a + b*x**3)**3/(9216*b**2) + 511*sin(a +
b*x**3)**3*cos(a + b*x**3)**5/(9216*b**2) + 31*sin(a + b*x**3)*cos(a + b*x*
*3)**7/(1024*b**2), Ne(b, 0)), (x**6*sin(a)*cos(a)**7/6, True))
```

Giac [A]
time = 0.42, size = 155, normalized size = 1.20

$$\frac{a\cos\left(bx^3+a\right)^8}{24\,b^2}-\frac{24\left(bx^3+a\right)\cos\left(8\,bx^3+8\,a\right)+192\left(bx^3+a\right)\cos\left(6\,bx^3+6\,a\right)+672\left(bx^3+a\right)\cos\left(4\,bx^3+4\,a\right)+1344\left(bx^3+a\right)\cos\left(2\,bx^3+2\,a\right)-3\,\sin\left(8\,bx^3+8\,a\right)-32\,\sin\left(6\,bx^3+6\,a\right)-168\,\sin\left(4\,bx^3+4\,a\right)-672\,\sin\left(2\,bx^3+2\,a\right)}{73728\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/24*a*cos(b*x^3 + a)^8/b^2 - 1/73728*(24*(b*x^3 + a)*cos(8*b*x^3 + 8*a) +
192*(b*x^3 + a)*cos(6*b*x^3 + 6*a) + 672*(b*x^3 + a)*cos(4*b*x^3 + 4*a) + 1
344*(b*x^3 + a)*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 + 8*a) - 32*sin(6*b*x^3
+ 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2*a))/b^2
```

Mupad [B]
time = 3.45, size = 147, normalized size = 1.14

$$\frac{168\,\sin\left(2\,b\,x^3+2\,a\right)+42\,\sin\left(4\,b\,x^3+4\,a\right)+8\,\sin\left(6\,b\,x^3+6\,a\right)+\frac{3\,\sin\left(8\,b\,x^3+8\,a\right)}{4}+336\,b\,x^3\,\left(2\sin\left(b\,x^3+a\right)^2-1\right)+168\,b\,x^3\,\left(2\sin\left(2\,b\,x^3+2\,a\right)^2-1\right)+48\,b\,x^3\,\left(2\sin\left(3\,b\,x^3+3\,a\right)^2-1\right)+6\,b\,x^3\,\left(2\sin\left(4\,b\,x^3+4\,a\right)^2-1\right)}{18432\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

```
[Out] (168*sin(2*a + 2*b*x^3) + 42*sin(4*a + 4*b*x^3) + 8*sin(6*a + 6*b*x^3) + (3
*sin(8*a + 8*b*x^3))/4 + 336*b*x^3*(2*sin(a + b*x^3)^2 - 1) + 168*b*x^3*(2*
sin(2*a + 2*b*x^3)^2 - 1) + 48*b*x^3*(2*sin(3*a + 3*b*x^3)^2 - 1) + 6*b*x^3
*(2*sin(4*a + 4*b*x^3)^2 - 1))/(18432*b^2)
```

3.927 $\int x^5 \sec^7(a+bx^3)\tan(a+bx^3)\,dx$

Optimal. Leaf size=110

$$-\frac{5\tanh^{-1}(\sin(a+bx^3))}{336b^2}+\frac{x^3\sec^7(a+bx^3)}{21b}-\frac{5\sec(a+bx^3)\tan(a+bx^3)}{336b^2}-\frac{5\sec^3(a+bx^3)\tan(a+bx^3)}{504b^2}-$$

```
[Out] -5/336*arctanh(sin(b*x^3+a))/b^2+1/21*x^3*sec(b*x^3+a)^7/b-5/336*sec(b*x^3+
a)*tan(b*x^3+a)/b^2-5/504*sec(b*x^3+a)^3*tan(b*x^3+a)/b^2-1/126*sec(b*x^3+a
)^5*tan(b*x^3+a)/b^2
```

Rubi [A]
time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3842, 4289, 3853, 3855}

$$-\frac{5\tanh^{-1}(\sin(a+bx^3))}{336b^2}-\frac{\tan(a+bx^3)\sec^5(a+bx^3)}{126b^2}-\frac{5\tan(a+bx^3)\sec^3(a+bx^3)}{504b^2}-\frac{5\tan(a+bx^3)\sec(a+bx^3)}{336b^2}+\frac{x^3\sec^7(a+bx^3)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x^3]])/(336*b^2) + (x^3*Sec[a + b*x^3]^7)/(21*b) - (5
*Sec[a + b*x^3]*Tan[a + b*x^3])/(336*b^2) - (5*Sec[a + b*x^3]^3*Tan[a + b*x
^3])/(504*b^2) - (Sec[a + b*x^3]^5*Tan[a + b*x^3])/(126*b^2)
```

Rule 3842

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{\int x^2\sec^7\left(a+bx^3\right)\,dx}{7b}\\ &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{\text{Subst}\left(\int \sec^7(a+bx)\,dx,x,x^3\right)}{21b}\\ &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{\sec^5\left(a+bx^3\right)\tan\left(a+bx^3\right)}{126b^2}-\frac{5\text{Subst}\left(\int \sec^5(a+\right.}{126b}\\ &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{5\sec^3\left(a+bx^3\right)\tan\left(a+bx^3\right)}{504b^2}-\frac{\sec^5\left(a+bx^3\right)\tan}{126b^2}\\ &= \frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{5\sec\left(a+bx^3\right)\tan\left(a+bx^3\right)}{336b^2}-\frac{5\sec^3\left(a+bx^3\right)\tan}{504b^2}\\ &= -\frac{5\tanh^{-1}\left(\sin\left(a+bx^3\right)\right)}{336b^2}+\frac{x^3\sec^7\left(a+bx^3\right)}{21b}-\frac{5\sec\left(a+bx^3\right)\tan\left(a\right.}{336b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 352 vs. 2(110) = 220.
time = 0.59, size = 352, normalized size = 3.20

$$\frac{\sec^7\left(a+bx^3\right)\left(3072bx^3+105\cos\left(5(a+bx^3)\right)\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)-\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)+15\cos\left(7(a+bx^3)\right)\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)-\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)+525\cos\left(a+bx^3\right)\left(\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)-\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)+\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)\right)+315\cos\left(3(a+bx^3)\right)\left(\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)-\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)+\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)\right)-105\cos\left(5(a+bx^3)\right)\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)+\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)-15\cos\left(7(a+bx^3)\right)\log\left(\cos\left(\tfrac{1}{2}(a+bx^3)\right)+\sin\left(\tfrac{1}{2}(a+bx^3)\right)\right)-566\sin\left(2(a+bx^3)\right)-200\sin\left(4(a+bx^3)\right)-30\sin\left(6(a+bx^3)\right)\right)}{64512b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

```
[Out] (Sec[a + b*x^3]^7*(3072*b*x^3 + 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/
2] - Sin[(a + b*x^3)/2]] + 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] - S
in[(a + b*x^3)/2]] + 525*Cos[a + b*x^3]*(Log[Cos[(a + b*x^3)/2] - Sin[(a +
b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) + 315*Cos[3*(a +
 b*x^3)]*(Log[Cos[(a + b*x^3)/2] - Sin[(a + b*x^3)/2]] - Log[Cos[(a + b*x^3
)/2] + Sin[(a + b*x^3)/2]]) - 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2]
 + Sin[(a + b*x^3)/2]] - 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] + Sin
[(a + b*x^3)/2]] - 566*Sin[2*(a + b*x^3)] - 200*Sin[4*(a + b*x^3)] - 30*Sin
[6*(a + b*x^3)]))/(64512*b^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 160, normalized size = 1.45

method	result

risch	$\frac{i\left(15\,\mathrm{e}^{13i\left(b\,x^3+a\right)}-3072ix^3b\,\mathrm{e}^{7i\left(b\,x^3+a\right)}+100\,\mathrm{e}^{11i\left(b\,x^3+a\right)}+283\,\mathrm{e}^{9i\left(b\,x^3+a\right)}-283\,\mathrm{e}^{5i\left(b\,x^3+a\right)}-100\,\mathrm{e}^{3i\left(b\,x^3+a\right)}-15\,\mathrm{e}^{i\left(b\,x^3+a\right)}\right)}{504b^2\left(\mathrm{e}^{2i\left(b\,x^3+a\right)}+1\right)^7}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/504*I/b^2/(exp(2*I*(b*x^3+a))+1)^7*(15*exp(13*I*(b*x^3+a))-3072*I*x^3*b*e
xp(7*I*(b*x^3+a))+100*exp(11*I*(b*x^3+a))+283*exp(9*I*(b*x^3+a))-283*exp(5*
I*(b*x^3+a))-100*exp(3*I*(b*x^3+a))-15*exp(I*(b*x^3+a)))+5/336/b^2*ln(exp(I
*(b*x^3+a))-I)-5/336/b^2*ln(exp(I*(b*x^3+a))+I)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. $2(100) = 200$.
time = 0.70, size = 3830, normalized size = 34.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2016*(4*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 15*sin(13*b*x^3 + 13*a) - 100*si
n(11*b*x^3 + 11*a) - 283*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*
sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(14*b*x^3 + 14*a) + 420*(sin(12*
b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x
^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(13*b*x^3 + 13*a)
 + 28*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 100*sin(11*b*x^3 + 11*a) - 283*sin(9
*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*
x^3 + a))*cos(12*b*x^3 + 12*a) + 2800*(3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x
^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a
))*cos(11*b*x^3 + 11*a) + 84*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 283*sin(9*b*x
^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3
+ a))*cos(10*b*x^3 + 10*a) + 7924*(5*sin(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6
*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(9*b*x^3 + 9*a) + 140*(
3072*b*x^3*cos(7*b*x^3 + 7*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 +
3*a) + 15*sin(b*x^3 + a))*cos(8*b*x^3 + 8*a) + 12288*(35*b*x^3*cos(6*b*x^3
+ 6*a) + 21*b*x^3*cos(4*b*x^3 + 4*a) + 7*b*x^3*cos(2*b*x^3 + 2*a) + b*x^3)*
cos(7*b*x^3 + 7*a) + 140*(283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) +
 15*sin(b*x^3 + a))*cos(6*b*x^3 + 6*a) - 7924*(3*sin(4*b*x^3 + 4*a) + sin(2
*b*x^3 + 2*a))*cos(5*b*x^3 + 5*a) + 420*(20*sin(3*b*x^3 + 3*a) + 3*sin(b*x^
3 + a))*cos(4*b*x^3 + 4*a) + 15*(2*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^
3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3
+ 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(14*b*x^3 + 14*a) + cos(14*b*x^3 + 14
*a)^2 + 14*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^
```

```
3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(12*b*x^3 +
 12*a) + 49*cos(12*b*x^3 + 12*a)^2 + 42*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b
*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(10*b*x^
3 + 10*a) + 441*cos(10*b*x^3 + 10*a)^2 + 70*(35*cos(6*b*x^3 + 6*a) + 21*cos
(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos(8*b*x^3 + 8*a) + 1225*cos(8
*b*x^3 + 8*a)^2 + 70*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*cos
(6*b*x^3 + 6*a) + 1225*cos(6*b*x^3 + 6*a)^2 + 42*(7*cos(2*b*x^3 + 2*a) + 1)
*cos(4*b*x^3 + 4*a) + 441*cos(4*b*x^3 + 4*a)^2 + 49*cos(2*b*x^3 + 2*a)^2 +
14*(sin(12*b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) +
5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(14*b*
x^3 + 14*a) + sin(14*b*x^3 + 14*a)^2 + 98*(3*sin(10*b*x^3 + 10*a) + 5*sin(8
*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 +
 2*a))*sin(12*b*x^3 + 12*a) + 49*sin(12*b*x^3 + 12*a)^2 + 294*(5*sin(8*b*x^
3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a)
)*sin(10*b*x^3 + 10*a) + 441*sin(10*b*x^3 + 10*a)^2 + 490*(5*sin(6*b*x^3 +
6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin(8*b*x^3 + 8*a) + 1225
*sin(8*b*x^3 + 8*a)^2 + 490*(3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*sin
(6*b*x^3 + 6*a) + 1225*sin(6*b*x^3 + 6*a)^2 + 441*sin(4*b*x^3 + 4*a)^2 + 29
4*sin(4*b*x^3 + 4*a)*sin(2*b*x^3 + 2*a) + 49*sin(2*b*x^3 + 2*a)^2 + 14*cos(
2*b*x^3 + 2*a) + 1)*log((cos(b*x^3 + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x^3
 + 2*a) + sin(b*x^3 + 2*a)^2 + 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)/(cos(b
*x^3 + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x^3 + 2*a) + sin(b*x^3 + 2*a)^2 -
 2*cos(b*x^3 + 2*a)*sin(a) + sin(a)^2)) + 4*(3072*b*x^3*sin(7*b*x^3 + 7*a)
+ 15*cos(13*b*x^3 + 13*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*
a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*s
in(14*b*x^3 + 14*a) - 60*(7*cos(12*b*x^3 + 12*a) + 21*cos(10*b*x^3 + 10*a)
+ 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^3 + 4*a) + 7
*cos(2*b*x^3 + 2*a) + 1)*sin(13*b*x^3 + 13*a) + 28*(3072*b*x^3*sin(7*b*x^3
+ 7*a) + 100*cos(11*b*x^3 + 11*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5*b*x^
3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(12*b*x^3 + 12*a)
 - 400*(21*cos(10*b*x^3 + 10*a) + 35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 +
6*a) + 21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(11*b*x^3 + 11*
a) + 84*(3072*b*x^3*sin(7*b*x^3 + 7*a) + 283*cos(9*b*x^3 + 9*a) - 283*cos(5
*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*cos(b*x^3 + a))*sin(10*b*x^3 +
10*a) - 1132*(35*cos(8*b*x^3 + 8*a) + 35*cos(6*b*x^3 + 6*a) + 21*cos(4*b*x^
3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*sin(9*b*x^3 + 9*a) + 140*(3072*b*x^3*s
in(7*b*x^3 + 7*a) - 283*cos(5*b*x^3 + 5*a) - 100*cos(3*b*x^3 + 3*a) - 15*co
s(b*x^3 + a))*sin(8*b*x^3 + 8*a) + 86016*(5*b*x^3*sin(6*b*x^3 + 6*a) + 3*b*
x^3*sin(4*b*x^3 + 4*a) + b*x^3*sin(2*b*x^3 + 2*a))*sin(7*b*x^3 + 7*a) - 140
*(283*cos(5*b*x^3 + 5*a) + 100*cos(3*b*x^3 + 3*a) + 15*cos(b*x^3 + a))*sin(
6*b*x^3 + 6*a) + 1132*(21*cos(4*b*x^3 + 4*a) + 7*cos(2*b*x^3 + 2*a) + 1)*si
n(5*b*x^3 + 5*a) - 420*(20*cos(3*b*x^3 + 3*a) + 3*cos(b*x^3 + a))*sin(4*b*x
^3 + 4*a) + 400*(7*cos(2*b*x^3 + 2*a) + 1)*sin(3*b*x^3 + 3*a) - 2800*cos(3*
b*x^3 + 3*a)*sin(2*b*x^3 + 2*a) - 420*cos(b*x^3...
```

Fricas [A]
time = 2.66, size = 115, normalized size = 1.05

$$-\frac{15 \cos \left(bx^3+a\right)^7 \log \left(\sin \left(bx^3+a\right)+1\right)-15 \cos \left(bx^3+a\right)^7 \log \left(-\sin \left(bx^3+a\right)+1\right)-96 bx^3+2 \left(15 \cos \left(bx^3+a\right)^5+10 \cos \left(bx^3+a\right)^3+8 \cos \left(bx^3+a\right)\right) \sin \left(bx^3+a\right)}{2016 b^2 \cos \left(bx^3+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/2016*(15*cos(b*x^3 + a)^7*log(sin(b*x^3 + a) + 1) - 15*cos(b*x^3 + a)^7*
log(-sin(b*x^3 + a) + 1) - 96*b*x^3 + 2*(15*cos(b*x^3 + a)^5 + 10*cos(b*x^3
 + a)^3 + 8*cos(b*x^3 + a))*sin(b*x^3 + a))/(b^2*cos(b*x^3 + a)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \tan \left(a+bx^3\right) \sec^7 \left(a+bx^3\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*sec(b*x**3+a)**7*tan(b*x**3+a),x)
```

```
[Out] Integral(x**5*tan(a + b*x**3)*sec(a + b*x**3)**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. 2(100) = 200.
time = 0.89, size = 1363, normalized size = 12.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/2016*(96*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^14 + 15*log(2*(tan(1/2*b*x^3
 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))
*tan(1/2*b*x^3 + 1/2*a)^14 - 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2
*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)
^14 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^12 - 105*log(2*(tan(1/2*b*x^3
+ 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*
tan(1/2*b*x^3 + 1/2*a)^12 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2
*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)
^12 + 132*tan(1/2*b*x^3 + 1/2*a)^13 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*
a)^10 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)
/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 315*log(2*(tan
(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*
a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 112*tan(1/2*b*x^3 + 1/2*a)^11 + 3360
```

```
*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^8 - 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2
 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*
x^3 + 1/2*a)^8 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/
2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^8 + 340*ta
n(1/2*b*x^3 + 1/2*a)^9 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^6 + 525*lo
g(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^
3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 - 525*log(2*(tan(1/2*b*x^3 + 1/
2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(
1/2*b*x^3 + 1/2*a)^6 + 2016*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^4 - 315*log(
2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3
+ 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^4 + 315*log(2*(tan(1/2*b*x^3 + 1/2*
a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/
2*b*x^3 + 1/2*a)^4 - 340*tan(1/2*b*x^3 + 1/2*a)^5 + 96*b*x^3 + 672*(b*x^3 +
 a)*tan(1/2*b*x^3 + 1/2*a)^2 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(
1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2
*a)^2 - 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)
/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^2 + 112*tan(1/2*b*x
^3 + 1/2*a)^3 + 96*a - 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3
 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1)) + 15*log(2*(tan(1/2*b*x^3 +
1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1)) -
132*tan(1/2*b*x^3 + 1/2*a))/((tan(1/2*b*x^3 + 1/2*a)^14 - 7*tan(1/2*b*x^3 +
 1/2*a)^12 + 21*tan(1/2*b*x^3 + 1/2*a)^10 - 35*tan(1/2*b*x^3 + 1/2*a)^8 + 3
5*tan(1/2*b*x^3 + 1/2*a)^6 - 21*tan(1/2*b*x^3 + 1/2*a)^4 + 7*tan(1/2*b*x^3
+ 1/2*a)^2 - 1)*b^2) - 1/21*a/(b^2*cos(b*x^3 + a)^7)
```

Mupad [B]
time = 13.42, size = 730, normalized size = 6.64

$$-\frac{\frac{8\,e^{1i\,b\,x^3+a\,1i}\,(15\,b\,x^3-8i)}{315\,b^2}-\frac{8\,e^{3i\,b\,x^3+a\,3i}\,(35\,b\,x^3-12i)}{315\,b^2}}{5\,e^{2i\,b\,x^3+a\,2i}+10\,e^{4i\,b\,x^3+a\,4i}+10\,e^{6i\,b\,x^3+a\,6i}+5\,e^{8i\,b\,x^3+a\,8i}+e^{10i\,b\,x^3+a\,10i}+1}+\frac{5\,\ln\left(x^2\left(e^{1i\,b\,x^3+a\,1i}-i\right)\right)}{336\,b^2}-\frac{5\,\ln\left(x^2\left(e^{1i\,b\,x^3+a\,1i}+1i\right)\right)}{336\,b^2}-\frac{\frac{16\,e^{3i\,b\,x^3+a\,3i}\,(5\,b\,x^3-i)}{63\,b^2}-\frac{16\,e^{5i\,b\,x^3+a\,5i}\,(7\,b\,x^3-i)}{63\,b^2}}{6\,e^{2i\,b\,x^3+a\,2i}+15\,e^{4i\,b\,x^3+a\,4i}+20\,e^{6i\,b\,x^3+a\,6i}+15\,e^{8i\,b\,x^3+a\,8i}+6\,e^{10i\,b\,x^3+a\,10i}+e^{12i\,b\,x^3+a\,12i}+1}-\frac{\frac{64\,x^3\,e^{5i\,b\,x^3+a\,5i}}{21\,b}-\frac{64\,x^3\,e^{7i\,b\,x^3+a\,7i}}{21\,b}}{7\,e^{2i\,b\,x^3+a\,2i}+21\,e^{4i\,b\,x^3+a\,4i}+35\,e^{6i\,b\,x^3+a\,6i}+35\,e^{8i\,b\,x^3+a\,8i}+21\,e^{10i\,b\,x^3+a\,10i}+7\,e^{12i\,b\,x^3+a\,12i}+e^{14i\,b\,x^3+a\,14i}+1}+\frac{e^{1i\,b\,x^3+a\,1i}\,1i}{63\,b^2\,(3\,e^{2i\,b\,x^3+a\,2i}+3\,e^{4i\,b\,x^3+a\,4i}+e^{6i\,b\,x^3+a\,6i}+1)}+\frac{e^{1i\,b\,x^3+a\,1i}\,5i}{168\,b^2\,(e^{2i\,b\,x^3+a\,2i}+1)}+\frac{e^{1i\,b\,x^3+a\,1i}\,5i}{252\,b^2\,(2\,e^{2i\,b\,x^3+a\,2i}+e^{4i\,b\,x^3+a\,4i}+1)}+\frac{2\,e^{1i\,b\,x^3+a\,1i}\,(60\,b\,x^3-47i)}{315\,b^2\,(4\,e^{2i\,b\,x^3+a\,2i}+6\,e^{4i\,b\,x^3+a\,4i}+4\,e^{6i\,b\,x^3+a\,6i}+e^{8i\,b\,x^3+a\,8i}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*tan(a + b*x^3))/cos(a + b*x^3)^7,x)
```

```
[Out] (5*log(x^2*(exp(a*1i + b*x^3*1i) - 1i)))/(336*b^2) - ((8*exp(a*1i + b*x^3*1
i)*(15*b*x^3 - 8i))/(315*b^2) - (8*exp(a*3i + b*x^3*3i)*(35*b*x^3 - 12i))/(
315*b^2))/(5*exp(a*2i + b*x^3*2i) + 10*exp(a*4i + b*x^3*4i) + 10*exp(a*6i +
 b*x^3*6i) + 5*exp(a*8i + b*x^3*8i) + exp(a*10i + b*x^3*10i) + 1) - (5*log(
x^2*(exp(a*1i + b*x^3*1i) + 1i)))/(336*b^2) - ((16*exp(a*3i + b*x^3*3i)*(5*
b*x^3 - 1i))/(63*b^2) - (16*exp(a*5i + b*x^3*5i)*(7*b*x^3 - 1i))/(63*b^2))/
(6*exp(a*2i + b*x^3*2i) + 15*exp(a*4i + b*x^3*4i) + 20*exp(a*6i + b*x^3*6i)
 + 15*exp(a*8i + b*x^3*8i) + 6*exp(a*10i + b*x^3*10i) + exp(a*12i + b*x^3*1
2i) + 1) - ((64*x^3*exp(a*5i + b*x^3*5i))/(21*b) - (64*x^3*exp(a*7i + b*x^3
*7i))/(21*b))/(7*exp(a*2i + b*x^3*2i) + 21*exp(a*4i + b*x^3*4i) + 35*exp(a*
6i + b*x^3*6i) + 35*exp(a*8i + b*x^3*8i) + 21*exp(a*10i + b*x^3*10i) + 7*ex
p(a*12i + b*x^3*12i) + exp(a*14i + b*x^3*14i) + 1) + (exp(a*1i + b*x^3*1i)*
```

```
1i)/(63*b^2*(3*exp(a*2i + b*x^3*2i) + 3*exp(a*4i + b*x^3*4i) + exp(a*6i + b
*x^3*6i) + 1)) + (exp(a*1i + b*x^3*1i)*5i)/(168*b^2*(exp(a*2i + b*x^3*2i) +
 1)) + (exp(a*1i + b*x^3*1i)*5i)/(252*b^2*(2*exp(a*2i + b*x^3*2i) + exp(a*4
i + b*x^3*4i) + 1)) + (2*exp(a*1i + b*x^3*1i)*(60*b*x^3 - 47i))/(315*b^2*(4
*exp(a*2i + b*x^3*2i) + 6*exp(a*4i + b*x^3*4i) + 4*exp(a*6i + b*x^3*6i) + e
xp(a*8i + b*x^3*8i) + 1))
```

3.928 $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=6

$$-\tan\left(\frac{1}{x}\right)$$

```
[Out] -tan(1/x)
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4289, 3852, 8}

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x^(-1)]^2/x^2,x]
```

```
[Out] -Tan[x^(-1)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4289

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} dx = -\text{Subst}\left(\int \sec^2(x) dx, x, \frac{1}{x}\right)$$
$$= \text{Subst}\left(\int 1 dx, x, -\tan\left(\frac{1}{x}\right)\right)$$
$$= -\tan\left(\frac{1}{x}\right)$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$-\tan\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x^(-1)]^2/x^2,x]
```

```
[Out] -Tan[x^(-1)]
```

Maple [A]
time = 0.04, size = 7, normalized size = 1.17

method	result	size
derivativedivides	$-\tan\left(\frac{1}{x}\right)$	7
default	$-\tan\left(\frac{1}{x}\right)$	7
risch	$-\frac{2i}{e^{\frac{2i}{x}}+1}$	15
norman	$\frac{2\tan\left(\frac{1}{2x}\right)}{\tan^2\left(\frac{1}{2x}\right)-1}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(1/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -tan(1/x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(6) = 12$.
time = 0.30, size = 36, normalized size = 6.00

$$-\frac{2\sin\left(\frac{2}{x}\right)}{\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\cos\left(\frac{2}{x}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="maxima")
[Out] -2*sin(2/x)/(cos(2/x)^2 + sin(2/x)^2 + 2*cos(2/x) + 1)
```

Fricas [A]
time = 2.73, size = 12, normalized size = 2.00

$$-\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="fricas")
[Out] -sin(1/x)/cos(1/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)**2/x**2,x)
[Out] Integral(sec(1/x)**2/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(6) = 12.
time = 0.41, size = 20, normalized size = 3.33

$$\frac{2\tan\left(\frac{1}{2x}\right)}{\tan\left(\frac{1}{2x}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/x)^2/x^2,x, algorithm="giac")
[Out] 2*tan(1/2/x)/(tan(1/2/x)^2 - 1)
```

Mupad [B]
time = 3.03, size = 14, normalized size = 2.33

$$-\frac{2\mathrm{i}}{\mathrm{e}^{\frac{2\mathrm{i}}{x}}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(1/x)^2),x)
[Out] -2i/(exp(2i/x) + 1)
```

3.929 $\int 3x^2 \cos(x^3) \, dx$

Optimal. Leaf size=4

$$\sin(x^3)$$

[Out] sin(x^3)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 3461, 2717}

$$\sin(x^3)$$

Antiderivative was successfully verified.

[In] Int[3*x^2*Cos[x^3],x]

[Out] Sin[x^3]

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3461

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int 3x^2 \cos(x^3) \, dx &= 3 \int x^2 \cos(x^3) \, dx \\ &= \text{Subst}\left(\int \cos(x) \, dx, x, x^3\right) \\ &= \sin(x^3) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Antiderivative was successfully verified.

```
[In] Integrate[3*x^2*Cos[x^3],x]
```

```
[Out] Sin[x^3]
```

Maple [A]
time = 0.03, size = 5, normalized size = 1.25

method	result	size
derivativedivides	$\sin(x^3)$	5
default	$\sin(x^3)$	5
meijerg	$\sin(x^3)$	5
risch	$\sin(x^3)$	5
norman	$\frac{2\tan\left(\frac{x^3}{2}\right)}{1+\tan^2\left(\frac{x^3}{2}\right)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x^3)
```

Maxima [A]
time = 0.28, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="maxima")
```

```
[Out] sin(x^3)
```

Fricas [A]
time = 2.61, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="fricas")
```

```
[Out] sin(x^3)
```

Sympy [A]
time = 0.08, size = 3, normalized size = 0.75

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x**2*cos(x**3),x)
```

```
[Out] sin(x**3)
```

Giac [A]
time = 0.43, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*x^2*cos(x^3),x, algorithm="giac")
```

```
[Out] sin(x^3)
```

Mupad [B]
time = 2.95, size = 4, normalized size = 1.00

$$\sin(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*x^2*cos(x^3),x)
```

```
[Out] sin(x^3)
```

3.930 $\int (1+2x)\sec^2(1+2x)\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}(1+2x)\tan(1+2x)$$

```
[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4269, 3556}

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\sec^2(1+2x)\,dx &= \frac{1}{2}(1+2x)\tan(1+2x)-\int \tan(1+2x)\,dx\\ &= \frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}(1+2x)\tan(1+2x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.11

$$\frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}\tan(1+2x)+x\tan(1+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

```
[Out] Log[Cos[1 + 2*x]]/2 + Tan[1 + 2*x]/2 + x*Tan[1 + 2*x]
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$	24
default	$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$	24
risch	$-2ix-i+\frac{i(1+2x)}{e^{2i(1+2x)}+1}+\frac{\ln\left(e^{2i(1+2x)}+1\right)}{2}$	43
norman	$\frac{-2x\tan\left(\frac{1}{2}+x\right)-\tan\left(\frac{1}{2}+x\right)}{\tan^2\left(\frac{1}{2}+x\right)-1}+\frac{\ln\left(\tan\left(\frac{1}{2}+x\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{1}{2}+x\right)+1\right)}{2}-\frac{\ln\left(1+\tan^2\left(\frac{1}{2}+x\right)\right)}{2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*sec(1+2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(23) = 46$.
time = 0.50, size = 98, normalized size = 3.63

$$\frac{\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)\log\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)+4(2x+1)\sin(4x+2)}{4\left(\cos(4x+2)^2+\sin(4x+2)^2+2\cos(4x+2)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((cos(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)*log(cos(4*x + 2
)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1) + 4*(2*x + 1)*sin(4*x + 2))/(cos
(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)
```

Fricas [A]
time = 2.22, size = 39, normalized size = 1.44

$$\frac{\cos(2x+1)\log(-\cos(2x+1))+(2x+1)\sin(2x+1)}{2\cos(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(2*x + 1)*log(-cos(2*x + 1)) + (2*x + 1)*sin(2*x + 1))/cos(2*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\sec^2{(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)**2,x)
```

```
[Out] Integral((2*x + 1)*sec(2*x + 1)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 943 vs. 2(23) = 46.
time = 0.52, size = 943, normalized size = 34.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(
x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)
^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2
*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan
(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(
1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1))*tan(1/2)^2*tan(x)^2 - 8*x
*tan(1/2)^2*tan(x) - 8*x*tan(1/2)*tan(x)^2 - log(4*(tan(1/2)^4*tan(x)^8 - 8
*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*ta
n(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8
 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + t
an(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x
)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*ta
n(1/2)^2 + 1))*tan(1/2)^2 - 4*log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan
(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)
^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3
*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*t
an(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^
4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1))*
tan(1/2)*tan(x) - 4*tan(1/2)^2*tan(x) - log(4*(tan(1/2)^4*tan(x)^8 - 8*tan(
1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*tan(1/2)^4*tan(x)^4 - 8*tan(1/2
)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*tan(1/2)*tan(x)^7 + tan(x)^8 + 8*
tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4 + 8*tan(1/2)*tan(x)^5 + tan(1/
2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*tan(x)^2 - 8*tan(1/2)*tan(x)^3 -
 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) + 1)/(tan(1/2)^4 + 2*tan(1/2
)^2 + 1))*tan(x)^2 - 4*tan(1/2)*tan(x)^2 + 8*x*tan(1/2) + 8*x*tan(x) + log(
```

```
4*(tan(1/2)^4*tan(x)^8 - 8*tan(1/2)^3*tan(x)^7 - 2*tan(1/2)^2*tan(x)^8 - 2*
tan(1/2)^4*tan(x)^4 - 8*tan(1/2)^3*tan(x)^5 + 16*tan(1/2)^2*tan(x)^6 + 8*ta
n(1/2)*tan(x)^7 + tan(x)^8 + 8*tan(1/2)^3*tan(x)^3 + 36*tan(1/2)^2*tan(x)^4
 + 8*tan(1/2)*tan(x)^5 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 16*tan(1/2)^2*t
an(x)^2 - 8*tan(1/2)*tan(x)^3 - 2*tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(
x) + 1)/(tan(1/2)^4 + 2*tan(1/2)^2 + 1)) + 4*tan(1/2) + 4*tan(x))/(tan(1/2)
^2*tan(x)^2 - tan(1/2)^2 - 4*tan(1/2)*tan(x) - tan(x)^2 + 1)
```

Mupad [B]
time = 0.10, size = 23, normalized size = 0.85

$$\frac{\ln\left(\cos\left(2\,x+1\right)\right)}{2}+\frac{\tan(2\,x+1)\,\left(2\,x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/cos(2*x + 1)^2,x)
[Out] log(cos(2*x + 1))/2 + (tan(2*x + 1)*(2*x + 1))/2
```

3.931 $\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\right.$

Optimal. Leaf size=26

$$\frac{2x^2\sqrt{x^3+3\sin(a+bx)}}{3b}$$

[Out] 2/3*x^2*(x^3+3*sin(b*x+a))^(1/2)/b

Rubi [F]
time = 0.60, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$

Verification is not applicable to the result.

[In] Int[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Si n[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]

[Out] Defer[Int][x^4/Sqrt[x^3 + 3*Sin[a + b*x]], x]/b + Defer[Int][(x^2*Cos[a + b *x])/Sqrt[x^3 + 3*Sin[a + b*x]], x] + (4*Defer[Int][x*Sqrt[x^3 + 3*Sin[a + b*x]], x])/(3*b)

Rubi steps

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx = \frac{\int \frac{x^4}{\sqrt{x^3+3\sin(a+b}}}{b}$$

Mathematica [A]
time = 0.22, size = 26, normalized size = 1.00

$$\frac{2x^2\sqrt{x^3+3\sin(a+bx)}}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]

[Out] (2*x^2*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b)

Maple [A]
time = 0.64, size = 28, normalized size = 1.08

method	result	size
risch	$\frac{\sqrt{2x^3+6\sin(bx+a)}\,\sqrt{2}\,x^2}{3b}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)+
4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*x^3+6*sin(b*x+a))^(1/2)/b*2^(1/2)*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{7x^4}{\sqrt{x^3+3\sin(a+bx)}}\,dx+\int \frac{12x\sin(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx+\int \frac{3bx^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/b/(x**3+3*sin(b*x+a))**(1/2)+x**2*cos(b*x+a)/(x**3+3*sin(b*x
+a))**(1/2)+4/3*x*(x**3+3*sin(b*x+a))**(1/2)/b,x)
```

```
[Out] (Integral(7*x**4/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(12*x*sin(a + b*
x)/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(3*b*x**2*cos(a + b*x)/sqrt(x*
*3 + 3*sin(a + b*x)), x))/(3*b)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^
(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

Mupad [**B**]
time = 3.47, size = 22, normalized size = 0.85

$$\frac{2\,x^2\,\sqrt{3\,\sin\left(a+b\,x\right)+x^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*(3*sin(a + b*x) + x^3)^(1/2)) + (x^2*cos(a + b*x))/(3*sin(a + b*
x) + x^3)^(1/2) + (4*x*(3*sin(a + b*x) + x^3)^(1/2))/(3*b),x)
```

```
[Out] (2*x^2*(3*sin(a + b*x) + x^3)^(1/2))/(3*b)
```

3.932 $\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$

Optimal. Leaf size=29

$$\mathrm{Int}\left(\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}},x\right)$$

```
[Out] CannotIntegrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
[Out] Defer[Int][(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Rubi steps

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx = \int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Mathematica [A]
time = 5.03, size = 0, normalized size = 0.00

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
[Out] Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\cos\left(bx+a\right)}{\sqrt{x^3+3\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

```
[Out] int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cos{(a + bx)}}{\sqrt{x^3 + 3 \sin{(a + bx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2 \cos (a+b x)}{\sqrt{3 \sin (a+b x)+x^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2),x)
[Out] int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2), x)
```

3.933 $\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx$

Optimal. Leaf size=9

$$\log\left(1+e^x\sin(x)\right)$$

[Out] ln(1+exp(x)*sin(x))

Rubi [F]
time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx$$

Verification is not applicable to the result.

[In] Int[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]

[Out] x + Log[Sin[x]] - Defer[Int][(1 + E^x*Sin[x])^(-1), x] - Defer[Int][Cot[x]/(1 + E^x*Sin[x]), x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx &= \int \left(1+\cot(x)-\frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)}\right) dx \\
&= x+\int \cot(x) \, dx-\int \frac{(1+\cot(x))\csc(x)}{e^x+\csc(x)} \, dx \\
&= x+\log(\sin(x))-\int \left(\frac{1}{1+e^x\sin(x)}+\frac{\cot(x)}{1+e^x\sin(x)}\right) dx \\
&= x+\log(\sin(x))-\int \frac{1}{1+e^x\sin(x)} \, dx-\int \frac{\cot(x)}{1+e^x\sin(x)} \, dx
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 9, normalized size = 1.00

$$\log\left(1+e^x\sin(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]

[Out] Log[1 + E^x*Sin[x]]

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 24, normalized size = 2.67

method	result	size
risch	$-ix+x+\ln\left(\mathrm{e}^{2ix}+2i\mathrm{e}^{(-1+i)x}-1\right)$	24
norman	$\frac{x+x\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}-\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)+\ln\left(\mathrm{e}^{-x}\left(\tan^2\left(\frac{x}{2}\right)\right)+\mathrm{e}^{-x}+2\tan\left(\frac{x}{2}\right)\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(exp(-x)+sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x+x+ln(exp(2*I*x)+2*I*exp((-1+I)*x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(8) = 16$.
time = 0.43, size = 82, normalized size = 9.11

$$x+\frac{1}{2}\log\left(\left(\cos\left(2x\right)^2e^{(2x)}+4\cos\left(x\right)e^x\sin\left(2x\right)+e^{(2x)}\sin\left(2x\right)^2-2\left(2e^x\sin\left(x\right)+e^{(2x)}\right)\cos\left(2x\right)+4\cos\left(x\right)^2+4e^x\sin\left(x\right)+4\sin\left(x\right)^2+e^{(2x)}\right)e^{(-2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="maxima")
```

```
[Out] x + 1/2*log((cos(2*x)^2*e^(2*x) + 4*cos(x)*e^x*sin(2*x) + e^(2*x)*sin(2*x)^
2 - 2*(2*e^x*sin(x) + e^(2*x))*cos(2*x) + 4*cos(x)^2 + 4*e^x*sin(x) + 4*sin
(x)^2 + e^(2*x))*e^(-2*x))
```

Fricas [A]
time = 3.25, size = 10, normalized size = 1.11

$$x+\log\left(e^{(-x)}+\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="fricas")
```

```
[Out] x + log(e^(-x) + sin(x))
```

Sympy [A]
time = 0.14, size = 10, normalized size = 1.11

$$x+\log\left(\sin\left(x\right)+e^{-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

```
[Out] x + log(sin(x) + exp(-x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.
time = 0.40, size = 83, normalized size = 9.22

$$x + \frac{1}{2}\log\left(\frac{4\left(e^{(-2x)}\tan\left(\frac{1}{2}x\right)^4 + 4e^{(-x)}\tan\left(\frac{1}{2}x\right)^3 + 2e^{(-2x)}\tan\left(\frac{1}{2}x\right)^2 + 4e^{(-x)}\tan\left(\frac{1}{2}x\right) + 4\tan\left(\frac{1}{2}x\right)^2 + e^{(-2x)}\right)}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="giac")
```

```
[Out] x + 1/2*log(4*(e^(-2*x)*tan(1/2*x)^4 + 4*e^(-x)*tan(1/2*x)^3 + 2*e^(-2*x)*t
an(1/2*x)^2 + 4*e^(-x)*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(-2*x))/(tan(1/2*x)^
4 + 2*tan(1/2*x)^2 + 1))
```

Mupad [B]
time = 2.95, size = 10, normalized size = 1.11

$$x + \ln\left(\mathrm{e}^{-x} + \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x) + sin(x))/(exp(-x) + sin(x)),x)
```

```
[Out] x + log(exp(-x) + sin(x))
```

3.934 $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx$

Optimal. Leaf size=77

$$\frac{3bx}{8}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 3/8*b*x-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3/8*b*cos(d*x+c)*sin(d*x+c)/d-1
/4*b*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.179$, Rules used = {4478, 2827, 2713, 2715, 8}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 - (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*b*Cos[c + d*
x]*Sin[c + d*x])/(8*d) - (b*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 4478

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx &= \int \sin^3(c+dx)(a+b\sin(c+dx))\,dx\\ &= a\int \sin^3(c+dx)\,dx+b\int \sin^4(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(3b)\int \sin^2(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)\sin(c}{8d}\\ &= \frac{3bx}{8}-\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3b\cos(c+dx)}{8d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 76, normalized size = 0.99

$$\frac{3b(c+dx)}{8d}-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\sin(2(c+dx))}{4d}+\frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12
*d) - (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.10, size = 60, normalized size = 0.78

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$

risch	$\frac{3bx}{8} - \frac{3a\cos(dx+c)}{4d} + \frac{b\sin(4dx+4c)}{32d} + \frac{a\cos(3dx+3c)}{12d} - \frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3bx}{8} - \frac{4a}{3d} - \frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d} - \frac{11b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} + \frac{11b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} + \frac{3b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} + \frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} + \frac{9bx\left(\tan^4\left(\frac{dx}{2}\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.74

$$\frac{32\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a + 3\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) - 8\,\sin(2\,dx+2\,c)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(d*x + c)^3 - 3*cos(d*x + c))*a + 3*(12*d*x + 12*c + sin(4*d*x
 + 4*c) - 8*sin(2*d*x + 2*c))*b)/d
```

Fricas [A]
time = 2.78, size = 60, normalized size = 0.78

$$\frac{8\,a\cos(dx+c)^3 + 9\,bdx - 24\,a\cos(dx+c) + 3\left(2\,b\cos(dx+c)^3 - 5\,b\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x - 24*a*cos(d*x + c) + 3*(2*b*cos(d*x + c
)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(70) = 140$.
time = 0.21, size = 150, normalized size = 1.95

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a\cos^3(c+dx)}{3d} + \frac{3bx\sin^4(c+dx)}{8} + \frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3bx\cos^4(c+dx)}{8} - \frac{5b\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3b\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x\left(a\sin^2(c) + b\sin^3(c)\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) +
3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*c
```

```
os(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b*sin(c + d*x
)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)*sin(c),
True))
```

Giac [A]
time = 0.42, size = 62, normalized size = 0.81

$$\frac{3}{8}\,bx+\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\,a\cos\left(dx+c\right)}{4\,d}+\frac{b\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{b\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/12*a*cos(3*d*x + 3*c)/d - 3/4*a*cos(d*x + c)/d + 1/32*b*sin(4*d
*x + 4*c)/d - 1/4*b*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 6.56, size = 111, normalized size = 1.44

$$\frac{3\,b\,x}{8}-\frac{-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{4\,a}{3}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3),x)
```

```
[Out] (3*b*x)/8 - ((4*a)/3 + (3*b*tan(c/2 + (d*x)/2))/4 + (16*a*tan(c/2 + (d*x)/2
)^2)/3 + 4*a*tan(c/2 + (d*x)/2)^4 + (11*b*tan(c/2 + (d*x)/2)^3)/4 - (11*b*t
an(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^4)
```

3.935 $\int \sin(c+dx) \left(a \sin^2(c+dx)+b \sin^3(c+dx)\right)^2 \, dx$

Optimal. Leaf size=161

$$\frac{5abx}{8}-\frac{(a^2+b^2)\cos(c+dx)}{d}+\frac{(2a^2+3b^2)\cos^3(c+dx)}{3d}-\frac{(a^2+3b^2)\cos^5(c+dx)}{5d}+\frac{b^2\cos^7(c+dx)}{7d}-\frac{5ab\cos(c}{}$$

```
[Out] 5/8*a*b*x-(a^2+b^2)*cos(d*x+c)/d+1/3*(2*a^2+3*b^2)*cos(d*x+c)^3/d-1/5*(a^2+
3*b^2)*cos(d*x+c)^5/d+1/7*b^2*cos(d*x+c)^7/d-5/8*a*b*cos(d*x+c)*sin(d*x+c)/
d-5/12*a*b*cos(d*x+c)*sin(d*x+c)^3/d-1/3*a*b*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.20, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4478, 2868, 2715, 8, 3092, 380}

$$-\frac{(a^2+3b^2)\cos^5(c+dx)}{5d}+\frac{(2a^2+3b^2)\cos^3(c+dx)}{3d}-\frac{(a^2+b^2)\cos(c+dx)}{d}-\frac{ab\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5ab\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{5ab\sin(c+dx)\cos(c+dx)}{8d}+\frac{5abx}{8}+\frac{b^2\cos^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (5*a*b*x)/8 - ((a^2 + b^2)*Cos[c + d*x])/d + ((2*a^2 + 3*b^2)*Cos[c + d*x]^
3)/(3*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^5)/(5*d) + (b^2*Cos[c + d*x]^7)/(7*d
) - (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (5*a*b*Cos[c + d*x]*Sin[c + d
*x]^3)/(12*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
```

```
Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4478

```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx = \int \sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$= (2ab)\int \sin^6(c+dx)\,dx + \int \sin^5(c+dx)\left(a^2+b^2\sin^2 \right.$$

$$= -\frac{ab\cos(c+dx)\sin^5(c+dx)}{3d} + \frac{1}{3}(5ab)\int \sin^4(c+dx)$$

$$= -\frac{5ab\cos(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos(c+dx)\sin^5(c}{3d}$$

$$= -\frac{(a^2+b^2)\cos(c+dx)}{d} + \frac{(2a^2+3b^2)\cos^3(c+dx)}{3d} - ($$

$$= \frac{5abx}{8} - \frac{(a^2+b^2)\cos(c+dx)}{d} + \frac{(2a^2+3b^2)\cos^3(c+d}{3d}$$

Mathematica [A]
time = 0.15, size = 134, normalized size = 0.83

$$\frac{4200abc+4200abdx-525(8a^2+7b^2)\cos(c+dx)+35(20a^2+21b^2)\cos(3(c+dx))-84a^2\cos(5(c+dx))-147b^2\cos(5(c+dx))+15b^2\cos(7(c+dx))-3150ab\sin(2(c+dx))+630ab\sin(4(c+dx))-70ab\sin(6(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x - 525*(8*a^2 + 7*b^2)*Cos[c + d*x] + 35*(20*a^2
+ 21*b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] - 147*b^2*Cos[5*(c + d
```

```
*x)] + 15*b^2*Cos[7*(c + d*x)] - 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*
(c + d*x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A]

time = 0.16, size = 125, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}+2ab\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(}{6}\right.}{d}$
default	$\frac{-\frac{b^2\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)}{7}+2ab\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(}{6}\right.}{d}$
risch	$\frac{5abx}{8}-\frac{5a^2\cos(dx+c)}{8d}-\frac{35\cos(dx+c)b^2}{64d}+\frac{b^2\cos(7dx+7c)}{448d}-\frac{ab\sin(6dx+6c)}{96d}-\frac{\cos(5dx+5c)a^2}{80d}-\frac{7\cos(5dx+}{320d}$
norman	$-\frac{112a^2+96b^2}{105d}+\frac{5abx}{8}-\frac{32a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(80a^2+96b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(112a^2+96b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{\left(112a^2+96b^2\right)(}{5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*b^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x
+c)+2*a*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+
5/16*d*x+5/16*c)-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]

time = 0.27, size = 131, normalized size = 0.81

$$-\frac{224\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)a^2-35\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)ab-96\left(5\cos(dx+c)^7-21\cos(dx+c)^5+35\cos(dx+c)^3-35\cos(dx+c)\right)b^2}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a^2 -
 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*a*b - 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)
^3 - 35*cos(d*x + c))*b^2)/d
```

Fricas [A]

time = 2.48, size = 123, normalized size = 0.76

$$\frac{120b^2\cos(dx+c)^7-168\left(a^2+3b^2\right)\cos(dx+c)^5+525abdx+280\left(2a^2+3b^2\right)\cos(dx+c)^3-840\left(a^2+b^2\right)\cos(dx+c)-35\left(8ab\cos(dx+c)^5-26ab\cos(dx+c)^3+33ab\cos(dx+c)\right)\sin(dx+c)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 3*b^2)*cos(d*x + c)^5 + 525*a*b*
d*x + 280*(2*a^2 + 3*b^2)*cos(d*x + c)^3 - 840*(a^2 + b^2)*cos(d*x + c) - 3
5*(8*a*b*cos(d*x + c)^5 - 26*a*b*cos(d*x + c)^3 + 33*a*b*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(148) = 296$.
time = 0.72, size = 326, normalized size = 2.02

$$\begin{cases} -\frac{a^2\sin^4(c+dx)\cos(c+dx)}{d} - \frac{4a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8a^2\cos^5(c+dx)}{15d} + \frac{5abx\sin^6(c+dx)}{8} + \frac{15abx\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{15abx\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{5abx\cos^6(c+dx)}{8} - \frac{11ab\sin^5(c+dx)\cos(c+dx)}{8d} - \frac{5ab\sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{5ab\sin(c+dx)\cos^5(c+dx)}{8d} - \frac{b^2\sin^6(c+dx)\cos(c+dx)}{d} - \frac{2b^2\sin^4(c+dx)\cos^3(c+dx)}{d} - \frac{8b^2\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{16b^2\cos^7(c+dx)}{35d} & \text{for } d \neq 0 \\ x(a\sin^2(c) + b\sin^3(c))^2\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*a**2*sin(c + d*x)**2*co
s(c + d*x)**3/(3*d) - 8*a**2*cos(c + d*x)**5/(15*d) + 5*a*b*x*sin(c + d*x)*
*6/8 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a*b*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 5*a*b*x*cos(c + d*x)**6/8 - 11*a*b*sin(c + d*x)**5*co
s(c + d*x)/(8*d) - 5*a*b*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*a*b*sin(
c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*b*
*2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**2*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 16*b**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*s
in(c)**3)**2*sin(c), True))
```

Giac [A]
time = 0.43, size = 143, normalized size = 0.89

$$\frac{5}{8}abx + \frac{b^2\cos(7dx+7c)}{448d} - \frac{ab\sin(6dx+6c)}{96d} + \frac{3ab\sin(4dx+4c)}{32d} - \frac{15ab\sin(2dx+2c)}{32d} - \frac{(4a^2+7b^2)\cos(5dx+5c)}{320d} + \frac{(20a^2+21b^2)\cos(3dx+3c)}{192d} - \frac{5(8a^2+7b^2)\cos(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 5/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d + 3/
32*a*b*sin(4*d*x + 4*c)/d - 15/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 + 7
*b^2)*cos(5*d*x + 5*c)/d + 1/192*(20*a^2 + 21*b^2)*cos(3*d*x + 3*c)/d - 5/6
4*(8*a^2 + 7*b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 6.74, size = 210, normalized size = 1.30

$$\frac{5\,a\,b\,x}{8} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{80\,a^2}{3}+32\,b^2\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{112\,a^2}{15}+\frac{32\,b^2}{5}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{112\,a^2}{5}+\frac{96\,b^2}{5}\right) + \frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3} + \frac{16\,a^2}{15} + \frac{32\,b^2}{35} + \frac{25\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} + \frac{283\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} - \frac{283\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{12} - \frac{25\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3} - \frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4} + \frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3)^2,x)
```

```
[Out] (5*a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((80*a^2)/3 + 32*b^2) + tan(c/2 + (d*x)
/2)^2*((112*a^2)/15 + (32*b^2)/5) + tan(c/2 + (d*x)/2)^4*((112*a^2)/5 + (96
*b^2)/5) + (32*a^2*tan(c/2 + (d*x)/2)^8)/3 + (16*a^2)/15 + (32*b^2)/35 + (2
5*a*b*tan(c/2 + (d*x)/2)^3)/3 + (283*a*b*tan(c/2 + (d*x)/2)^5)/12 - (283*a*
b*tan(c/2 + (d*x)/2)^9)/12 - (25*a*b*tan(c/2 + (d*x)/2)^11)/3 - (5*a*b*tan(
c/2 + (d*x)/2)^13)/4 + (5*a*b*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^7)
```

3.936 $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c\right.$

Optimal. Leaf size=89

$$\frac{1}{8}(4a+3c)x-\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}-\frac{(4a+3c)\cos(c+dx)\sin(c+dx)}{8d}-\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*c)*x-b*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d-1/8*(4*a+3*c)*cos(d*x+c
)*sin(d*x+c)/d-1/4*c*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4322, 3102, 2827, 2715, 8, 2713}

$$-\frac{(4a+3c)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3c)+\frac{b\cos^3(c+dx)}{3d}-\frac{b\cos(c+dx)}{d}-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3),x]
```

```
[Out] ((4*a + 3*c)*x)/8 - (b*Cos[c + d*x])/d + (b*Cos[c + d*x]^3)/(3*d) - ((4*a +
 3*c)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (c*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 4322

```
Int[(u_)*((A_.)*sin[(a_.) + (b_.)*(x_)]^(n_.) + (B_.)*sin[(a_.) + (b_.)*(x_
)]^(n1_) + (C_.)*sin[(a_.) + (b_.)*(x_)]^(n2_)), x_Symbol] :> Int[ActivateT
rig[u]*Sin[a + b*x]^n*(A + B*Sin[a + b*x] + C*Sin[a + b*x]^2), x] /; FreeQ[
{a, b, A, B, C, n}, x] && EqQ[n1, n + 1] && EqQ[n2, n + 2]
```

Rubi steps

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx=\int \sin^2(c+dx)\left(a+b\sin(c+dx)+c\,\mathrm{s}\right.$$

$$=-\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\int \mathrm{sin}$$

$$=-\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}+b\int \sin^3$$

$$=-\frac{(4a+3c)\cos(c+dx)\sin(c+dx)}{8d}-$$

$$=\frac{1}{8}(4a+3c)x-\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(}{3}$$

Mathematica [A]
time = 0.11, size = 105, normalized size = 1.18

$$\frac{a(c+dx)}{2d}+\frac{3c(c+dx)}{8d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}-\frac{a\sin(2(c+dx))}{4d}-\frac{c\sin(2(c+dx))}{4d}+\frac{c\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^
3),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (3*c*(c + d*x))/(8*d) - (3*b*Cos[c + d*x])/(4*d) + (b
*Cos[3*(c + d*x)])/(12*d) - (a*Sin[2*(c + d*x)])/(4*d) - (c*Sin[2*(c + d*x)
])/(4*d) + (c*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.13, size = 84, normalized size = 0.94

method	result
risch	$\frac{ax}{2}+\frac{3cx}{8}-\frac{3b\cos(dx+c)}{4d}+\frac{c\sin(4dx+4c)}{32d}+\frac{b\cos(3dx+3c)}{12d}-\frac{\sin(2dx+2c)a}{4d}-\frac{\sin(2dx+2c)c}{4d}$
derivativedivides	$\frac{c\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{c\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(\frac{a}{2}+\frac{3c}{8}\right)x+\left(2a+\frac{3c}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a+\frac{3c}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a+\frac{9c}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a}{2}+\frac{3c}{8}\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a*sin(d*x+c)+sin(d*x+c)^2*b+c*sin(d*x+c)^3),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/d*(c*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*b*
(2+sin(d*x+c)^2)*cos(d*x+c)+a*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 79, normalized size = 0.89

$$\frac{24\,(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a+32\,\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)b+3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right))c}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c - sin(2*d*x + 2*c))*a + 32*(cos(d*x + c)^3 - 3*cos(d*
x + c))*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*c)/d
```

Fricas [A]
time = 2.24, size = 72, normalized size = 0.81

$$\frac{8\,b\cos\left(dx+c\right)^3+3\,(4\,a+3\,c)dx-24\,b\cos\left(dx+c\right)+3\,\left(2\,c\cos\left(dx+c\right)^3-(4\,a+5\,c)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="fricas")
```

```
[Out] 1/24*(8*b*cos(d*x + c)^3 + 3*(4*a + 3*c)*d*x - 24*b*cos(d*x + c) + 3*(2*c*c
os(d*x + c)^3 - (4*a + 5*c)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(76) = 152$.
time = 0.21, size = 201, normalized size = 2.26

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} - \frac{a\sin(c+dx)\cos(c+dx)}{2d} - \frac{b\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b\cos^3(c+dx)}{3d} + \frac{3cx\sin^4(c+dx)}{8} + \frac{3cx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3cx\cos^4(c+dx)}{8} - \frac{5c\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3c\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x(a\sin(c) + b\sin^2(c) + c\sin^3(c))\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c + d*x)**3/
(3*d) + 3*c*x*sin(c + d*x)**4/8 + 3*c*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*c*x*cos(c + d*x)**4/8 - 5*c*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*c*sin
(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2 + c*
sin(c)**3)*sin(c), True))
```

Giac [A]
time = 0.43, size = 70, normalized size = 0.79

$$\frac{1}{8}(4a+3c)x + \frac{b\cos(3dx+3c)}{12d} - \frac{3b\cos(dx+c)}{4d} + \frac{c\sin(4dx+4c)}{32d} - \frac{(a+c)\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algori
thm="giac")
```

```
[Out] 1/8*(4*a + 3*c)*x + 1/12*b*cos(3*d*x + 3*c)/d - 3/4*b*cos(d*x + c)/d + 1/32
*c*sin(4*d*x + 4*c)/d - 1/4*(a + c)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 3.18, size = 73, normalized size = 0.82

$$\frac{2b\cos(3c+3dx) - 18b\cos(c+dx) - 6a\sin(2c+2dx) - 6c\sin(2c+2dx) + \frac{3c\sin(4c+4dx)}{4} + 12adx + 9cdx}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3),x)
```

```
[Out] (2*b*cos(3*c + 3*d*x) - 18*b*cos(c + d*x) - 6*a*sin(2*c + 2*d*x) - 6*c*sin(
2*c + 2*d*x) + (3*c*sin(4*c + 4*d*x))/4 + 12*a*d*x + 9*c*d*x)/(24*d)
```

3.937 $\int \sin(c+dx) \left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c$

Optimal. Leaf size=288

$$\frac{3abx}{4}+\frac{5bcx}{8}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-\frac{(b^2+2ac)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{c^2\cos^3(c+dx)}{d}+\frac{2(b$$

```
[Out] 3/4*a*b*x+5/8*b*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d-(2*a*c+b^2)*cos(d*x+c
)/d+1/3*a^2*cos(d*x+c)^3/d+c^2*cos(d*x+c)^3/d+2/3*(2*a*c+b^2)*cos(d*x+c)^3/
d-3/5*c^2*cos(d*x+c)^5/d-1/5*(2*a*c+b^2)*cos(d*x+c)^5/d+1/7*c^2*cos(d*x+c)^
7/d-3/4*a*b*cos(d*x+c)*sin(d*x+c)/d-5/8*b*c*cos(d*x+c)*sin(d*x+c)/d-1/2*a*b
*cos(d*x+c)*sin(d*x+c)^3/d-5/12*b*c*cos(d*x+c)*sin(d*x+c)^3/d-1/3*b*c*cos(d
*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.31, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {4479, 3337, 2713, 2715, 8}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{(2ac+b^2)\cos^5(c+dx)}{5d}+\frac{2(2ac+b^2)\cos^3(c+dx)}{3d}-\frac{(2ac+b^2)\cos(c+dx)}{d}-\frac{ab\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}-\frac{bc\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5bc\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{5bc\sin(c+dx)\cos(c+dx)}{8d}+\frac{5bcx}{8}+\frac{c^2\cos^7(c+dx)}{7d}-\frac{3c^2\cos^5(c+dx)}{5d}+\frac{c^2\cos^3(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,x
]
```

```
[Out] (3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d - (
(b^2 + 2*a*c)*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) + (c^2*Cos[c + d
*x]^3)/d + (2*(b^2 + 2*a*c)*Cos[c + d*x]^3)/(3*d) - (3*c^2*Cos[c + d*x]^5)/
(5*d) - ((b^2 + 2*a*c)*Cos[c + d*x]^5)/(5*d) + (c^2*Cos[c + d*x]^7)/(7*d) -
 (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (5*b*c*Cos[c + d*x]*Sin[c + d*x]
)/(8*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^3)/(2*d) - (5*b*c*Cos[c + d*x]*Sin
[c + d*x]^3)/(12*d) - (b*c*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3337

```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

Rule 4479

```
Int[(u_)*((a_)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (b_.)*(F_)[(d_.) + (e_.)*(x
_)]^(q_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)]^(r_.))^(n_.), x_Symbol] :> Int[A
ctivateTrig[u*F[d + e*x]^(n*p)*(a + b*F[d + e*x]^(q - p) + c*F[d + e*x]^(r
- p))^n], x] /; FreeQ[{a, b, c, d, e, p, q, r}, x] && InertTrigQ[F] && Inte
gerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx &= \int \sin^3(c+dx)\left(a+b\sin(c+dx)+c\right.\\
&= \int \left(a^2\sin^3(c+dx)+2ab\sin^4(c+dx)\right.\\
&= a^2\int \sin^3(c+dx)\,dx+(2ab)\int \sin^4(c\\
&= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{2d}-\frac{bc\cos}{}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}-\\
&= \frac{3abx}{4}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+d}{d}\\
&= \frac{3abx}{4}+\frac{5bcx}{8}-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos}{}
\end{aligned}
$$

Mathematica [A]
time = 0.31, size = 167, normalized size = 0.58

$$\frac{840b(6a+5c)(c+dx)-105(48a^2+40b^2+80ac+35c^2)\cos(c+dx)+35(16a^2+20b^2+40ac+21c^2)\cos(3(c+dx))-21(4b^2+c(8a+7c))\cos(5(c+dx))+15c^2\cos(7(c+dx))-210b(16a+15c)\sin(2(c+dx))+210b(2a+3c)\sin(4(c+dx))-70bc\sin(6(c+dx))}{6720d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,x]

[Out] (840*b*(6*a + 5*c)*(c + d*x) - 105*(48*a^2 + 40*b^2 + 80*a*c + 35*c^2)*Cos[c + d*x] + 35*(16*a^2 + 20*b^2 + 40*a*c + 21*c^2)*Cos[3*(c + d*x)] - 21*(4*b^2 + c*(8*a + 7*c))*Cos[5*(c + d*x)] + 15*c^2*Cos[7*(c + d*x)] - 210*b*(16*a + 15*c)*Sin[2*(c + d*x)] + 210*b*(2*a + 3*c)*Sin[4*(c + d*x)] - 70*b*c*Sin[6*(c + d*x)])/(6720*d)

Maple [A]
time = 0.20, size = 213, normalized size = 0.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a*sin(d*x+c)+sin(d*x+c)^2*b+c*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/7*c^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)+2*b*c*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c)-2/5*a*c*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/5*b^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+2*a*b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c))

Maxima [A]
time = 0.30, size = 218, normalized size = 0.76

$$\frac{1120 \left(\cos\left(dx+c\right)^{3} - 3\,\cos\left(dx+c\right)\right)a^{2} + 210\left(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) - 8\,\sin\left(2\,dx+2\,c\right)\right)ab - 224\left(3\,\cos\left(dx+c\right)^{5} - 10\,\cos\left(dx+c\right)^{3} + 15\,\cos\left(dx+c\right)\right)b^{2} - 448\left(3\,\cos\left(dx+c\right)^{5} - 10\,\cos\left(dx+c\right)^{3} + 15\,\cos\left(dx+c\right)\right)ac + 35\left(4\,\sin\left(2\,dx+2\,c\right)^{3} + 60\,dx + 60\,c + 9\,\sin\left(4\,dx+4\,c\right) - 48\,\sin\left(2\,dx+2\,c\right)\right)bc + 96\left(5\,\cos\left(dx+c\right)^{7} - 21\,\cos\left(dx+c\right)^{5} + 35\,\cos\left(dx+c\right)^{3} - 35\,\cos\left(dx+c\right)\right)c^{2}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algorithm="maxima")

[Out] 1/3360*(1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 + 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a*b - 224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^2 - 448*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a*c + 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b*c + 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*c^2)/d

Fricas [A]
time = 1.87, size = 162, normalized size = 0.56

$$\frac{120\,c^{2}\cos\left(dx+c\right)^{7} - 168\left(b^{2} + 2\,ac + 3\,c^{2}\right)\cos\left(dx+c\right)^{5} + 280\left(a^{2} + 2\,b^{2} + 4\,ac + 3\,c^{2}\right)\cos\left(dx+c\right)^{3} + 105\left(6\,ab + 5\,bc\right)dx - 840\left(a^{2} + b^{2} + 2\,ac + c^{2}\right)\cos\left(dx+c\right) - 35\left(8\,bc\cos\left(dx+c\right)^{5} - 2\left(6\,ab + 13\,bc\right)\cos\left(dx+c\right)^{3} + 3\left(10\,ab + 11\,bc\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algorithm="fricas")

```
[Out] 1/840*(120*c^2*cos(d*x + c)^7 - 168*(b^2 + 2*a*c + 3*c^2)*cos(d*x + c)^5 +
280*(a^2 + 2*b^2 + 4*a*c + 3*c^2)*cos(d*x + c)^3 + 105*(6*a*b + 5*b*c)*d*x
- 840*(a^2 + b^2 + 2*a*c + c^2)*cos(d*x + c) - 35*(8*b*c*cos(d*x + c)^5 - 2
*(6*a*b + 13*b*c)*cos(d*x + c)^3 + 3*(10*a*b + 11*b*c)*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [A]
time = 0.77, size = 541, normalized size = 1.88

$$\begin{cases} -\frac{a^2 \sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a^2\cos^3(c+dx)}{3d} + \frac{3abx\sin^4(c+dx)}{4} + \frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3abx\cos^4(c+dx)}{4} - \frac{5ab\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{3ab\sin(c+dx)\cos^3(c+dx)}{4d} - \frac{2ac\sin^4(c+dx)\cos(c+dx)}{d} - \frac{8ac\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{16ac\cos^5(c+dx)}{15d} - \frac{b^2\sin^4(c+dx)\cos(c+dx)}{d} - \frac{4b^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8b^2\cos^5(c+dx)}{15d} + \frac{5bcx\sin^6(c+dx)}{8} + \frac{15bcx\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{15bcx\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{5bcx\cos^6(c+dx)}{8} - \frac{11bc\sin^5(c+dx)\cos(c+dx)}{8d} - \frac{5bc\sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{5bc\sin(c+dx)\cos^5(c+dx)}{8d} - \frac{c^2\sin^6(c+dx)\cos(c+dx)}{d} - \frac{2c^2\sin^4(c+dx)\cos^3(c+dx)}{d} - \frac{8c^2\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{16c^2\cos^7(c+dx)}{35d} & \text{for}\ d \neq 0 \\ x(a\sin(c) + b\sin^2(c) + c\sin^3(c))^2\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)/d - 2*a**2*cos(c + d*x)**3/(3
*d) + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2
 + 3*a*b*x*cos(c + d*x)**4/4 - 5*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 3
*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*c*sin(c + d*x)**4*cos(c + d*x
)/d - 8*a*c*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 16*a*c*cos(c + d*x)**5/
(15*d) - b**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**2*sin(c + d*x)**2*cos(c
 + d*x)**3/(3*d) - 8*b**2*cos(c + d*x)**5/(15*d) + 5*b*c*x*sin(c + d*x)**6/
8 + 15*b*c*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*b*c*x*sin(c + d*x)**2*c
os(c + d*x)**4/8 + 5*b*c*x*cos(c + d*x)**6/8 - 11*b*c*sin(c + d*x)**5*cos(c
 + d*x)/(8*d) - 5*b*c*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*b*c*sin(c +
 d*x)*cos(c + d*x)**5/(8*d) - c**2*sin(c + d*x)**6*cos(c + d*x)/d - 2*c**2*
sin(c + d*x)**4*cos(c + d*x)**3/d - 8*c**2*sin(c + d*x)**2*cos(c + d*x)**5/
(5*d) - 16*c**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)*
*2 + c*sin(c)**3)**2*sin(c), True))
```

Giac [A]
time = 0.42, size = 186, normalized size = 0.65

$$\frac{1}{8}(6ab + 5bc)x + \frac{c^2\cos(7dx + 7c)}{448d} - \frac{bc\sin(6dx + 6c)}{96d} - \frac{(4b^2 + 8ac + 7c^2)\cos(5dx + 5c)}{320d} + \frac{(16a^2 + 20b^2 + 40ac + 21c^2)\cos(3dx + 3c)}{192d} - \frac{(48a^2 + 40b^2 + 80ac + 35c^2)\cos(dx + c)}{64d} + \frac{(2ab + 3bc)\sin(4dx + 4c)}{32d} - \frac{(16ab + 15bc)\sin(2dx + 2c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, algo
rithm="giac")
```

```
[Out] 1/8*(6*a*b + 5*b*c)*x + 1/448*c^2*cos(7*d*x + 7*c)/d - 1/96*b*c*sin(6*d*x +
 6*c)/d - 1/320*(4*b^2 + 8*a*c + 7*c^2)*cos(5*d*x + 5*c)/d + 1/192*(16*a^2
+ 20*b^2 + 40*a*c + 21*c^2)*cos(3*d*x + 3*c)/d - 1/64*(48*a^2 + 40*b^2 + 80
*a*c + 35*c^2)*cos(d*x + c)/d + 1/32*(2*a*b + 3*b*c)*sin(4*d*x + 4*c)/d - 1
/32*(16*a*b + 15*b*c)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 4.52, size = 456, normalized size = 1.58

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a+5c)}{4\left(\frac{3ab}{2}+\frac{5bc}{4}\right)}\right)(6a+5c)}{4d} - \frac{b\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)(6a+5c)}{4d} - \frac{\frac{32ac}{15} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{3ab}{2}+\frac{5bc}{4}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(10ab+\frac{25bc}{3}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(10ab+\frac{25bc}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{31ab}{2}+\frac{283bc}{12}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{31ab}{2}+\frac{283bc}{12}\right) + 4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(\frac{52a^2}{3}+\frac{64ca}{3}+\frac{32b^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{88a^2}{3}+\frac{160ac}{3}+\frac{80b^2}{3}+32c^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{28a^2}{3}+\frac{224ac}{15}+\frac{112b^2}{15}+\frac{32c^2}{5}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(24a^2+\frac{224ac}{5}+\frac{112b^2}{5}+\frac{96c^2}{5}\right) + \frac{4a^2}{3} + \frac{16b^2}{15} + \frac{32c^2}{35} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3ab}{2}+\frac{5bc}{4}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} + 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3)^2,x
)
```

```
[Out] (b*atan((b*tan(c/2 + (d*x)/2)*(6*a + 5*c))/(4*((3*a*b)/2 + (5*b*c)/4)))*(6*
a + 5*c))/(4*d) - (b*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(6*a + 5*c))/(4*d
) - ((32*a*c)/15 - tan(c/2 + (d*x)/2)^13*((3*a*b)/2 + (5*b*c)/4) + tan(c/2
+ (d*x)/2)^3*(10*a*b + (25*b*c)/3) - tan(c/2 + (d*x)/2)^11*(10*a*b + (25*b*
c)/3) + tan(c/2 + (d*x)/2)^5*((31*a*b)/2 + (283*b*c)/12) - tan(c/2 + (d*x)/
2)^9*((31*a*b)/2 + (283*b*c)/12) + 4*a^2*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
(d*x)/2)^8*((64*a*c)/3 + (52*a^2)/3 + (32*b^2)/3) + tan(c/2 + (d*x)/2)^6*((
160*a*c)/3 + (88*a^2)/3 + (80*b^2)/3 + 32*c^2) + tan(c/2 + (d*x)/2)^2*((224
*a*c)/15 + (28*a^2)/3 + (112*b^2)/15 + (32*c^2)/5) + tan(c/2 + (d*x)/2)^4*(
(224*a*c)/5 + 24*a^2 + (112*b^2)/5 + (96*c^2)/5) + (4*a^2)/3 + (16*b^2)/15
+ (32*c^2)/35 + tan(c/2 + (d*x)/2)*((3*a*b)/2 + (5*b*c)/4))/(d*(7*tan(c/2 +
 (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/
2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c
/2 + (d*x)/2)^14 + 1))
```

3.938 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right) d$

Optimal. Leaf size=61

$$\frac{cx}{2}-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*c*x-a*cos(d*x+c)/d-2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-1/2*c*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4480, 4486, 2719, 2718, 2715, 8}

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d - (c*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4480

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx = \int \sqrt{\sin(c+dx)}\left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c\right.$$

$$= \int \left(b\sqrt{\sin(c+dx)}+a\sin(c+dx)+c\sin^2(c+\right.$$

$$= a\int \sin(c+dx)\,dx + b\int \sqrt{\sin(c+dx)}\,dx + c\int$$

$$= -\frac{a\cos(c+dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d} - \frac{c\cos}{}$$

$$= \frac{cx}{2} - \frac{a\cos(c+dx)}{d} + \frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d} -$$

Mathematica [A]
time = 0.13, size = 55, normalized size = 0.90

$$\frac{-4a\cos(c+dx)-8bE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+c(2c+2dx-\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

```
[Out] (-4*a*Cos[c + d*x] - 8*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + c*(2*c + 2*d
*x - Sin[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.29, size = 136, normalized size = 2.23

method	result

default	$cx - \frac{a\cos(dx+c)}{d} - \frac{c\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{b\sqrt{\sin(dx+c)+1}\,\sqrt{-2\sin(dx+c)+2}\,\sqrt{-\sin(c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] c*x-a*cos(d*x+c)/d-c/d*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-b*(sin(d*x
+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*(2*EllipticE((sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/c
os(d*x+c)/sin(d*x+c)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(2*c*d*x - 4*a*cos(d*x + c) + 2*d*integrate(-(((b*cos(3/2*d*x + 3/2*c)
- b*cos(1/2*d*x + 1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))
*cos(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (b*cos(3/2*d*x + 3/2*c
) - b*cos(1/2*d*x + 1/2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c
))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(sin(d
*x + c), cos(d*x + c) + 1)) + ((b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x + 1/
2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin
(d*x + c), -cos(d*x + c) + 1)) + (b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*x +
1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2(s
in(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^(1/4)*(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) - c*sin(2*d*x
 + 2*c))/d
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 108, normalized size = 1.77

$$-\frac{c\cos(dx+c)\sin(dx+c) - 2i\sqrt{2}\sqrt{-i}\,b\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))) + 2i\sqrt{2}\sqrt{i}\,b\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))) - c\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right) + 2a\cos(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fric
as")
```

```
[Out] -1/2*(c*cos(d*x + c)*sin(d*x + c) - 2*I*sqrt(2)*sqrt(-I)*b*weierstrassZeta(
4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2*I*sqrt(
2)*sqrt(I)*b*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - c*arctan(sin(d*x + c)/cos(d*x + c)) + 2*a*cos(d*x + c))
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sqrt{\sin\left(c+dx\right)}+b+c\sin^{\frac{3}{2}}\left(c+dx\right)\right)\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2)),x)
```

```
[Out] Integral((a*sqrt(sin(c + d*x)) + b + c*sin(c + d*x)**(3/2))*sqrt(sin(c + d*
x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="giac
")
```

```
[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))*sin(d*x + c), x)
```

Mupad [B]
time = 3.25, size = 51, normalized size = 0.84

$$\frac{c\,x}{2}-\frac{c\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2)),x)
```

```
[Out] (c*x)/2 - (c*sin(2*c + 2*d*x))/(4*d) - (a*cos(c + d*x))/d + (2*b*ellipticE(
c/2 - pi/4 + (d*x)/2, 2))/d
```

3.939 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2$

Optimal. Leaf size=148

$$b^2x+acx-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}+\frac{c^2\cos^3(c+dx)}{3d}+\frac{4abE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}+\frac{4bcF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{3d}$$

```
[Out] b^2*x+a*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d+1/3*c^2*cos(d*x+c)^3/d-4*a*b*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-4/3*b*c*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d
-a*c*cos(d*x+c)*sin(d*x+c)/d-4/3*b*c*cos(d*x+c)*sin(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4480, 4486, 2719, 2718, 2715, 2720, 8, 2713}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x+\frac{4bcF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d}-\frac{4bc\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}+\frac{c^2\cos^3(c+dx)}{3d}-\frac{c^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

```
[Out] b^2*x + a*c*x - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d + (c^2*Cos[c +
d*x]^3)/(3*d) + (4*a*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (4*b*c*Ellipti
cF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]])/
(3*d) - (a*c*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4480

```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx = \int \left(b+a\sqrt{\sin(c+dx)}+c\sin^{\frac{3}{2}}(c+dx)\right)^2 dx$$

$$= \int \left(b^2+2ab\sqrt{\sin(c+dx)}+a^2\sin(c+dx)+2\right.$$

$$= b^2x+a^2\int \sin(c+dx)\,dx+(2ab)\int \sqrt{\sin(c+}$$

$$= b^2x-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}$$

$$= b^2x+acx-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}+$$

Mathematica [A]
time = 0.19, size = 137, normalized size = 0.93

$$\frac{12b^2c+12ac^2+12b^2dx+12acdx-12a^2\cos(c+dx)-9c^2\cos(c+dx)+c^2\cos(3(c+dx))-48abE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)-16bcF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)-16bc\cos(c+dx)\sqrt{\sin(c+dx)}-6ac\sin(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]

[Out] (12*b^2*c + 12*a*c^2 + 12*b^2*d*x + 12*a*c*d*x - 12*a^2*Cos[c + d*x] - 9*c^2*Cos[c + d*x] + c^2*Cos[3*(c + d*x)] - 48*a*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]] - 6*a*c*Sin[2*(c + d*x)])/(12*d)

Maple [**A**]
time = 0.38, size = 266, normalized size = 1.80

method	result
default	$b^2x - \frac{a^2\cos(dx+c)}{d} - \frac{c^2(2+\sin^2(dx+c))\cos(dx+c)}{3d} + \frac{2ac\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{2b\left(3a\sqrt{\sin(dx+c)+1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] b^2*x-a^2*cos(d*x+c)/d-1/3*c^2/d*(2+sin(d*x+c)^2)*cos(d*x+c)+2*a*c/d*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2/3*b*(3*a*(sin(d*x+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+(sin(d*x+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*c-6*a*(sin(d*x+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*EllipticE((sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*c*sin(d*x+c))/cos(d*x+c)/sin(d*x+c)^(1/2)/d

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/12*(12*b^2*c + 12*(b^2 + a*c)*d*x + c^2*cos(3*d*x + 3*c) - 6*a*c*sin(2*d*x + 2*c) - 3*sqrt(2)*d*integrate((((sqrt(2)*b*c*cos(5/2*d*x + 5/2*c) + sqrt(2)*b*c*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*a*b*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*cos(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) - sqrt(2)*b*c*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(sin(d*x +

```
c), cos(d*x + c) + 1)) + ((2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*
a*b*cos(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) - sqrt(2)*b*c*s
in(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(s
in(d*x + c), -cos(d*x + c) + 1)) + (sqrt(2)*b*c*cos(5/2*d*x + 5/2*c) + sqrt
(2)*b*c*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*a*b*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2*c))*sin(1/2*a
rctan2(sin(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^(1
/4)*(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) - 3*s
qrt(2)*d*integrate((((2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*co
s(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) - sqrt(2)*b*c*sin(3/2
*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin(d*x
 + c), -cos(d*x + c) + 1)) + (sqrt(2)*b*c*cos(5/2*d*x + 5/2*c) + sqrt(2)*b*
c*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*a*b
*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2
(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
 c) + 1)) - ((sqrt(2)*b*c*cos(5/2*d*x + 5/2*c) + sqrt(2)*b*c*cos(3/2*d*x +
3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*a*b*sin(3/2*d*x + 3
/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2*c))*cos(1/2*arctan2(sin(d*x + c), -
cos(d*x + c) + 1)) - (2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*co
s(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) - sqrt(2)*b*c*sin(3/2
*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*sin(1/2*arctan2(sin(d*x
 + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c) + 1)
))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^(1/4)*(cos(d*x +
 c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) - 3*(4*a^2 + 3*c^2)
*cos(d*x + c))/d
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 210, normalized size = 1.42

$$\frac{c^2 \cos(dx+c)^3 - 3\,ac\cos(dx+c)\sin(dx+c) + 2\sqrt{2}\,\sqrt{-i}\,bc\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)) + 2\sqrt{2}\,\sqrt{i}\,bc\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)) + 6i\sqrt{2}\,\sqrt{-i}\,ab\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))) - 6i\sqrt{2}\,\sqrt{i}\,ab\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))) - 4\,bc\cos(dx+c)\sqrt{\sin(dx+c)} + 3\,(b^2+ac)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right) - 3\,(a^2+c^2)\cos(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(c^2*cos(d*x + c)^3 - 3*a*c*cos(d*x + c)*sin(d*x + c) + 2*sqrt(2)*sqrt(
-I)*b*c*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*sqrt(2
)*sqrt(I)*b*c*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 6*
I*sqrt(2)*sqrt(-I)*a*b*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
d*x + c) + I*sin(d*x + c))) - 6*I*sqrt(2)*sqrt(I)*a*b*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 4*b*c*cos(d*x
+ c)*sqrt(sin(d*x + c)) + 3*(b^2 + a*c)*arctan(sin(d*x + c)/cos(d*x + c)) -
 3*(a^2 + c^2)*cos(d*x + c))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral((a + b/sqrt(sin(c + d*x)) + c*sin(c + d*x))**2*sin(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))^2*sin(d*x + c), x)
```

Mupad [B]
time = 6.68, size = 129, normalized size = 0.87

$$b^2\,x-\frac{a^2\,\cos\left(c+d\,x\right)}{d}-\frac{a\,c\,\left(\sin\left(2\,c+2\,d\,x\right)-2\,d\,x\right)}{2\,d}+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{c^2\,\cos\left(c+d\,x\right)\,\left({\cos\left(c+d\,x\right)}^2-3\right)}{3\,d}-\frac{2\,b\,c\,\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\left({\sin\left(c+d\,x\right)}^2\right)}^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2))^2,x)
```

```
[Out] b^2*x - (a^2*cos(c + d*x))/d - (a*c*(sin(2*c + 2*d*x) - 2*d*x))/(2*d) + (4*
a*b*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/d + (c^2*cos(c + d*x)*(cos(c + d*x)
^2 - 3))/(3*d) - (2*b*c*cos(c + d*x)*sin(c + d*x)^(5/2)*hypergeom([-1/4, 1/
2], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(5/4))
```

3.940 $\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx$

Optimal. Leaf size=34

$$\frac{\left(e^{i(c+dx)}\right)^n f^{a+bx}}{idn+b\log(f)}$$

```
[Out] exp(I*(d*x+c))^n*f^(b*x+a)/(I*d*n+b*ln(f))
```

Rubi [A]
time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {4710, 2319, 2325, 2225}

$$\frac{f^{a+bx}\left(e^{i(c+dx)}\right)^n}{b\log(f)+idn}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

```
[Out] ((E^(I*(c + d*x)))^n*f^(a + b*x))/(I*d*n + b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4710

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx = \int \left(e^{i(c+dx)}\right)^n f^{a+bx}\,dx$$
$$= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{in(c+dx)}f^{a+bx}\,dx$$
$$= \left(e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\right)\int e^{icn+a\log(f)+x(idn+b\log(f))}\,dx$$
$$= \frac{\left(e^{i(c+dx)}\right)^n f^{a+bx}}{idn+b\log(f)}$$

Mathematica [A]
time = 0.07, size = 43, normalized size = 1.26

$$-\frac{if^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n}{dn-ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
[Out] ((-I)*f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n)/(d*n - I*b*Log[f])
```

Maple [A]
time = 0.31, size = 32, normalized size = 0.94

method	result	size
risch	$\frac{\left(e^{i(dx+c)}\right)^n f^{bx+a}}{idn+b\ln(f)}$	32
norman	$\frac{e^{(bx+a)\ln(f)}e^{n\ln\left(\frac{1-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{idn+b\ln(f)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] exp(I*(d*x+c))^n*f^(b*x+a)/(I*d*n+b*ln(f))
```

Maxima [A]
time = 0.30, size = 50, normalized size = 1.47

$$\frac{-i\,f^{bx}f^{a}\cos\left(dnx+cn\right)+f^{bx}f^{a}\sin\left(dnx+cn\right)}{dn-i\,b\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (-I*f^(b*x)*f^a*cos(d*n*x + c*n) + f^(b*x)*f^a*sin(d*n*x + c*n))/(d*n - I*b
*log(f))
```

Fricas [A]
time = 2.27, size = 30, normalized size = 0.88

$$\frac{f^{bx+a}e^{(i\,dnx+i\,cn)}}{i\,dn+b\log{(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] f^(b*x + a)*e^(I*d*n*x + I*c*n)/(I*d*n + b*log(f))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.
time = 1.42, size = 107, normalized size = 3.15

$$\begin{cases} \frac{f^a f^{bx}(i\sin{(c+dx)}+\cos{(c+dx)})^n}{b\log{(f)}+idn} & \text{for } b \neq -\frac{idn}{\log{(f)}} \\ f^a x(i\sin{(c+dx)}+\cos{(c+dx)})^n e^{-idnx} - \frac{if^a(i\sin{(c+dx)}+\cos{(c+dx)})^n e^{-idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((f**a*f**(b*x)*(I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) + I*d
*n), Ne(b, -I*d*n/log(f))), (f**a*x*(I*sin(c + d*x) + cos(c + d*x))**n*exp(
-I*d*n*x) - I*f**a*(I*sin(c + d*x) + cos(c + d*x))**n*exp(-I*d*n*x)/(d*n),
True))
```

Giac [A]
time = 0.57, size = 31, normalized size = 0.91

$$\frac{f^a e^{(i\,dnx+bx\log{(f)}+i\,cn)}}{i\,dn+b\log{(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] f^a*e^(I*d*n*x + b*x*log(f) + I*c*n)/(I*d*n + b*log(f))
```

Mupad [B]
time = 3.46, size = 35, normalized size = 1.03

$$-\frac{f^{a+b\,x}\,{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\right)}^n\,1\mathrm{i}}{d\,n-b\,\ln\left(f\right)\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*(cos(c + d*x) + sin(c + d*x)*1i)^n,x)
[Out] -(f^(a + b*x)*exp(c*1i + d*x*1i)^n*1i)/(d*n - b*log(f)*1i)
```

3.941 $\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$

Optimal. Leaf size=36

$$-\frac{\left(e^{-i(c+dx)}\right)^n f^{a+bx}}{idn-b\log(f)}$$

```
[Out] -exp(-I*(d*x+c))^n*f^(b*x+a)/(I*d*n-b*ln(f))
```

Rubi [A]
time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {4710, 2319, 2325, 2225}

$$-\frac{f^{a+bx}\left(e^{-i(c+dx)}\right)^n}{-b\log(f)+idn}$$

Antiderivative was successfully verified.

```
[In] Int[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

```
[Out] -(((E^((-I)*(c + d*x)))^n*f^(a + b*x))/(I*d*n - b*Log[f]))
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2319

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 4710

```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx &= \int \left(e^{-i(c+dx)}\right)^n f^{a+bx}\,dx \\
&= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int e^{-in(c+dx)}f^{a+bx}\,dx \\
&= \left(e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\right)\int \exp(-icn+a\log(f)-x(idn-b\log(\\
&= -\frac{\left(e^{-i(c+dx)}\right)^n f^{a+bx}}{idn-b\log(f)}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.19

$$\frac{if^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n}{dn+ib\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

```
[Out] (I*f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n)/(d*n + I*b*Log[f])
```

Maple [A]
time = 0.32, size = 34, normalized size = 0.94

method	result	size
risch	$\frac{f^{bx+a}\left(\mathrm{e}^{i(dx+c)}\right)^{-n}}{-idn+b\ln(f)}$	34
norman	$\frac{\mathrm{e}^{(bx+a)\ln(f)}\mathrm{e}^{n\ln\left(\frac{1-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}}{-idn+b\ln(f)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-I*d*n+b*ln(f))*f^(b*x+a)*exp(I*(d*x+c))^(-n)
```

Maxima [A]
time = 0.30, size = 62, normalized size = 1.72

$$\frac{f^{bx}f^{a}\cos(dnx)-i\,f^{bx}f^{a}\sin(dnx)}{(-i\,dn+b\log(f))\cos(cn)+(dn+i\,b\log(f))\sin(cn)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] (f^(b*x)*f^a*cos(d*n*x) - I*f^(b*x)*f^a*sin(d*n*x))/((-I*d*n + b*log(f))*co
s(c*n) + (d*n + I*b*log(f))*sin(c*n))
```

Fricas [A]
time = 2.44, size = 30, normalized size = 0.83

$$\frac{f^{bx+a}e^{(-i\,dnx-i\,cn)}}{-i\,dn+b\log{(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] f^(b*x + a)*e^(-I*d*n*x - I*c*n)/(-I*d*n + b*log(f))
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(29) = 58.
time = 1.41, size = 105, normalized size = 2.92

$$\begin{cases} \frac{f^a f^{bx}(-i\sin{(c+dx)}+\cos{(c+dx)})^n}{b\log{(f)}-idn} & \text{for}\ b\neq\frac{idn}{\log{(f)}} \\ f^a x(-i\sin{(c+dx)}+\cos{(c+dx)})^n e^{idnx}+\frac{if^a(-i\sin{(c+dx)}+\cos{(c+dx)})^n e^{idnx}}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f**(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))**n,x)
```

```
[Out] Piecewise((f**a*f**(b*x)*(-I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) - I*
d*n), Ne(b, I*d*n/log(f))), (f**a*x*(-I*sin(c + d*x) + cos(c + d*x))**n*exp
(I*d*n*x) + I*f**a*(-I*sin(c + d*x) + cos(c + d*x))**n*exp(I*d*n*x)/(d*n),
True))
```

Giac [A]
time = 0.61, size = 31, normalized size = 0.86

$$\frac{f^a e^{(-i\,dnx+bx\log(f)-i\,cn)}}{-i\,dn+b\log{(f)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="giac")
```

```
[Out] f^a*e^(-I*d*n*x + b*x*log(f) - I*c*n)/(-I*d*n + b*log(f))
```

Mupad [B]
time = 3.35, size = 35, normalized size = 0.97

$$-\frac{f^{a+b\,x}\,{\left(\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\right)}^n}{-b\,\ln\left(f\right)+d\,n\,1\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(f^(a + b*x)*(cos(c + d*x) - sin(c + d*x)*1i)^n,x)
[Out] -(f^(a + b*x)*exp(- c*1i - d*x*1i)^n)/(d*n*1i - b*log(f))
```

3.942 $\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$

Optimal. Leaf size=120

$$\frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{4\log\left(2-\left(1-\sqrt{5}\right)\tan(a+bx)+2\tan^2(a+bx)\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2-\left(\right.\right.}{}$$

```
[Out] ln(cos(b*x+a))/b+1/5*ln(1+tan(b*x+a))/b-4/5*ln(2-(-5^(1/2)+1)*tan(b*x+a)+2*
tan(b*x+a)^2)/b/(-5^(1/2)+1)-4/5*ln(2-(5^(1/2)+1)*tan(b*x+a)+2*tan(b*x+a)^2
)/b/(5^(1/2)+1)
```

Rubi [A]

time = 0.50, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2099, 266, 2111, 642}

$$-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)b}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)b}+\frac{\log(\tan(a+bx)+1)}{5b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^5),x]
```

```
[Out] Log[Cos[a + b*x]]/b + Log[1 + Tan[a + b*x]]/(5*b) - (4*Log[2 - (1 - Sqrt[5]
)*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 - Sqrt[5])*b) - (4*Log[2 - (1 + S
qrt[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 + Sqrt[5])*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2111

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^5}{1+x^2+x^5+x^7}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{5(1+x)}-\frac{x}{1+x^2}+\frac{2(2+x-4x^2+2x^3)}{5(1-x+x^2-x^3+x^4)}\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\log(1+\tan(a+bx))}{5b}+\frac{2\text{Subst}\left(\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx, x, \tan(a+bx)\right)}{5b}-$$
$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{2\text{Subst}\left(\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right.}{2+\left(-1-\sqrt{5}\right)x+}\right.}{5\sqrt{5}}$$
$$= \frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{5b}-\frac{4\log\left(2-\left(1-\sqrt{5}\right)\tan(\right.}{5\left(1-\right.}$$

Mathematica [A]
time = 0.39, size = 73, normalized size = 0.61

$$\frac{\log(\cos(a+bx)+\sin(a+bx))-\left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}+\sin(2(a+bx))\right)+\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+\sin(2(a+bx))\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^
5),x]
```

```
[Out] (Log[Cos[a + b*x] + Sin[a + b*x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + Sin[2*
(a + b*x)]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + Sin[2*(a + b*x)]])/(5*b)
```

Maple [A]
time = 0.71, size = 111, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\frac{4\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-\sqrt{5}\tan(bx+a)+2\left(\tan^2(bx+a)\right)-\tan(bx+a)+2\right)}{5}+\frac{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(\sqrt{5}\tan(bx+a\right.}{5}}{b}$
default	$\frac{-\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\frac{4\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-\sqrt{5}\tan(bx+a)+2\left(\tan^2(bx+a)\right)-\tan(bx+a)+2\right)}{5}+\frac{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(\sqrt{5}\tan(bx+a\right.}{5}}{b}$
risch	$-ix-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}+i\right)}{5b}+\frac{\ln\left(e^{4i(bx+a)}+2i\left(\sqrt{5}+1\right)e^{2i(bx+a)}-1\right)}{5b}+\frac{\ln\left(e^{4i(bx+a)}+2i\left(\sqrt{5}+1\right)e^{2i(bx-\right.}{5b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/b*(-1/2*ln(tan(b*x+a)^2+1)-4/5*(1/4*5^(1/2)-1/4)*ln(-5^(1/2)*tan(b*x+a)+2
*tan(b*x+a)^2-tan(b*x+a)+2)+4/5*(1/4*5^(1/2)+1/4)*ln(5^(1/2)*tan(b*x+a)+2*t
an(b*x+a)^2-tan(b*x+a)+2)+1/5*ln(1+tan(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="maxima")
```

```
[Out] integrate((cos(b*x + a)^5 - sin(b*x + a)^5)/(cos(b*x + a)^5 + sin(b*x + a)^
5), x)
```

Fricas [A]
time = 2.70, size = 150, normalized size = 1.25

$$\frac{2\sqrt{5}\log\left(-\frac{2\cos(bx+a)^4-2\left(\sqrt{5}+1\right)\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-\sqrt{5}-3}{\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1}\right)+2\log\left(\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1\right)+\log\left(2\cos(bx+a)\sin(bx+a)+1\right)}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="fricas")
```

```
[Out] 1/10*(2*sqrt(5)*log(-(2*cos(b*x + a)^4 - 2*(sqrt(5) + 1)*cos(b*x + a)*sin(b
*x + a) - 2*cos(b*x + a)^2 - sqrt(5) - 3)/(cos(b*x + a)^4 - cos(b*x + a)^2
- cos(b*x + a)*sin(b*x + a) + 1)) + 2*log(cos(b*x + a)^4 - cos(b*x + a)^2 -
 cos(b*x + a)*sin(b*x + a) + 1) + log(2*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(100) = 200$.
time = 94.35, size = 379, normalized size = 3.16

$$\begin{cases} -\frac{47\log(\sin(a+bx)+\cos(a+bx))}{-235b+105\sqrt{5}\,b} + \frac{21\sqrt{5}\log(\sin(a+bx)+\cos(a+bx))}{-235b+105\sqrt{5}\,b} - \frac{26\sqrt{5}\log\left(16\sin^2(a+bx)-8\sin(a+bx)\cos(a+bx)+8\sqrt{5}\sin(a+bx)\cos(a+bx)+16\cos^2(a+bx)\right)}{-235b+105\sqrt{5}\,b} + \frac{58\log\left(16\sin^2(a+bx)-8\sin(a+bx)\cos(a+bx)+8\sqrt{5}\sin(a+bx)\cos(a+bx)+16\cos^2(a+bx)\right)}{-235b+105\sqrt{5}\,b} - \frac{152\log\left(16\sin^2(a+bx)-8\sqrt{5}\sin(a+bx)\cos(a+bx)-8\sin(a+bx)\cos(a+bx)+16\cos^2(a+bx)\right)}{-235b+105\sqrt{5}\,b} + \frac{68\sqrt{5}\log\left(16\sin^2(a+bx)-8\sqrt{5}\sin(a+bx)\cos(a+bx)-8\sin(a+bx)\cos(a+bx)+16\cos^2(a+bx)\right)}{-235b+105\sqrt{5}\,b} & \text{for } b \neq 0 \\ \frac{x(-\sin^5(a)+\cos^5(a))}{\sin^5(a)+\cos^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**5-sin(b*x+a)**5)/(cos(b*x+a)**5+sin(b*x+a)**5),x)
```

```
[Out] Piecewise((-47*log(sin(a + b*x) + cos(a + b*x))/(-235*b + 105*sqrt(5)*b) +
21*sqrt(5)*log(sin(a + b*x) + cos(a + b*x))/(-235*b + 105*sqrt(5)*b) - 26*s
qrt(5)*log(16*sin(a + b*x)**2 - 8*sin(a + b*x)*cos(a + b*x) + 8*sqrt(5)*sin
(a + b*x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) + 58*
log(16*sin(a + b*x)**2 - 8*sin(a + b*x)*cos(a + b*x) + 8*sqrt(5)*sin(a + b*
x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) - 152*log(16
*sin(a + b*x)**2 - 8*sqrt(5)*sin(a + b*x)*cos(a + b*x) - 8*sin(a + b*x)*cos
(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) + 68*sqrt(5)*log(1
6*sin(a + b*x)**2 - 8*sqrt(5)*sin(a + b*x)*cos(a + b*x) - 8*sin(a + b*x)*co
s(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b), Ne(b, 0)), (x*(-
sin(a)**5 + cos(a)**5)/(sin(a)**5 + cos(a)**5), True))
```

Giac [A]
time = 0.60, size = 128, normalized size = 1.07

$$-\frac{2\sqrt{5}\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)\tan(bx+a)+\tan(bx+a)^2+1\right) - 2\sqrt{5}\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\tan(bx+a)+\tan(bx+a)^2+1\right) - 2\log\left(\tan(bx+a)^4-\tan(bx+a)^3+\tan(bx+a)^2-\tan(bx+a)+1\right) + 5\log\left(\tan(bx+a)^2+1\right) - 2\log\left(|\tan(bx+a)+1|\right)}{10\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algori
thm="giac")
```

```
[Out] -1/10*(2*sqrt(5)*log(-1/2*(sqrt(5) + 1)*tan(b*x + a) + tan(b*x + a)^2 + 1)
- 2*sqrt(5)*log(1/2*(sqrt(5) - 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*lo
g(tan(b*x + a)^4 - tan(b*x + a)^3 + tan(b*x + a)^2 - tan(b*x + a) + 1) + 5*
log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

Mupad [B]
time = 4.22, size = 226, normalized size = 1.88

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{5b} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)}{b} + \frac{\ln\left(2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+1\right)\left(\sqrt{5}+1\right)}{5b} - \frac{\ln\left(2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4-\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+1\right)\left(\sqrt{5}-1\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^5 - sin(a + b*x)^5)/(cos(a + b*x)^5 + sin(a + b*x)^5),x)
```

```
[Out] log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(5*b) - log(tan(a/2 +
(b*x)/2)^2 + 1)/b + (log(2*tan(a/2 + (b*x)/2)^2 - tan(a/2 + (b*x)/2) + tan(
```

```
a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 5^(1/2)*tan(a/2 + (b*x)/2) - 5^(1
/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) + 1))/(5*b) - (log(2*tan(a/2 + (b*x)
/2)^2 - tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 -
5^(1/2)*tan(a/2 + (b*x)/2) + 5^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) - 1
))/(5*b)
```

3.943 $\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$-\frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}\,b}+\frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}\,b}$$

```
[Out] -1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)+1/4*ln(1+2^(1/2)*tan(b
*x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1179, 642}

$$\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(Sqrt[2]*b) + Log[1 + S
qrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}\,b} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\,x-x^2}\,dx\right.}{2\sqrt{2}\,b}$$
$$= -\frac{\log\left(1-\sqrt{2}\,\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}\,b} + \frac{\log\left(1+\sqrt{2}\,\tan(a+bx\right.}{2\sqrt{2}\,b}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.35

$$\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^
4),x]
```

```
[Out] ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.
time = 0.36, size = 168, normalized size = 2.33

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e^{4i(bx+a)}+2i\sqrt{2}\,e^{2i(bx+a)}-1\right)}{4b} - \frac{\sqrt{2}\,\ln\left(e^{4i(bx+a)}-2i\sqrt{2}\,e^{2i(bx+a)}-1\right)}{4b}$
derivativedivides	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)+2\arctan\left(\sqrt{2}\,\tan(bx+a)+1\right)+2\arctan\left(\sqrt{2}\,\tan(bx+a)-1\right)\right)}{8} - \frac{\sqrt{2}\left(\ln\left(\frac{1-}{1+}\right.\right.}{}}{b}$
default	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\,\tan(bx+a)+\tan^2(bx+a)}\right)+2\arctan\left(\sqrt{2}\,\tan(bx+a)+1\right)+2\arctan\left(\sqrt{2}\,\tan(bx+a)-1\right)\right)}{8} - \frac{\sqrt{2}\left(\ln\left(\frac{1-}{1+}\right.\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/b*(1/8*2^(1/2)*(ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*x
+a)+tan(b*x+a)^2))+2*arctan(2^(1/2)*tan(b*x+a)+1)+2*arctan(2^(1/2)*tan(b*x+
a)-1))-1/8*2^(1/2)*(ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1/2)*tan(b
*x+a)+tan(b*x+a)^2))+2*arctan(2^(1/2)*tan(b*x+a)+1)+2*arctan(2^(1/2)*tan(b*
x+a)-1)))
```

Maxima [A]
time = 0.52, size = 58, normalized size = 0.81

$$\frac{\sqrt{2}\,\log\left(\tan\left(bx+a\right)^{2}+\sqrt{2}\,\tan\left(bx+a\right)+1\right)-\sqrt{2}\,\log\left(\tan\left(bx+a\right)^{2}-\sqrt{2}\,\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="maxima")
[Out] 1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(t
an(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

Fricas [A]
time = 3.39, size = 74, normalized size = 1.03

$$\frac{\sqrt{2}\,\log\left(-\frac{2\,\cos\left(bx+a\right)^{4}-2\,\sqrt{2}\,\cos\left(bx+a\right)\sin\left(bx+a\right)-2\,\cos\left(bx+a\right)^{2}-1}{2\,\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="fricas")
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 - 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

Sympy [A]
time = 1.87, size = 122, normalized size = 1.69

$$\begin{cases} -\frac{\sqrt{2}\log\left(4\sin^{2}\left(a+bx\right)-4\sqrt{2}\sin\left(a+bx\right)\cos\left(a+bx\right)+4\cos^{2}\left(a+bx\right)\right)}{4b}+\frac{\sqrt{2}\log\left(4\sin^{2}\left(a+bx\right)+4\sqrt{2}\sin\left(a+bx\right)\cos\left(a+bx\right)+4\cos^{2}\left(a+bx\right)\right)}{4b} & \text{for}\ b\neq 0\\ \frac{x\left(-\sin^{4}\left(a\right)+\cos^{4}\left(a\right)\right)}{\sin^{4}\left(a\right)+\cos^{4}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**4-sin(b*x+a)**4)/(cos(b*x+a)**4+sin(b*x+a)**4),x)
[Out] Piecewise((-sqrt(2)*log(4*sin(a + b*x)**2 - 4*sqrt(2)*sin(a + b*x)*cos(a +
b*x) + 4*cos(a + b*x)**2)/(4*b) + sqrt(2)*log(4*sin(a + b*x)**2 + 4*sqrt(2)
*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b), Ne(b, 0)), (x*(-sin(
a)**4 + cos(a)**4)/(sin(a)**4 + cos(a)**4), True))
```

Giac [A]
time = 0.47, size = 48, normalized size = 0.67

$$-\frac{\sqrt{2}\,\log\left(\frac{\left|-2\sqrt{2}+2\,\sin\left(2\,bx+2\,a\right)\right|}{\left|2\,\sqrt{2}+2\,\sin\left(2\,bx+2\,a\right)\right|}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algori
thm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin
(2*b*x + 2*a)))/b
```

Mupad [B]
time = 3.17, size = 23, normalized size = 0.32

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sin\left(2\,a+2\,b\,x\right)}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^4 - sin(a + b*x)^4)/(cos(a + b*x)^4 + sin(a + b*x)^4),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

3.944 $\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{3b}-\frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx)\right)}{3b}$$

```
[Out] -ln(cos(b*x+a))/b+1/3*ln(1+tan(b*x+a))/b-2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/
b
```

Rubi [A]
time = 0.29, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2099, 266, 642}

$$-\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}+\frac{\log(\tan(a+bx)+1)}{3b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3),x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) + Log[1 + Tan[a + b*x]]/(3*b) - (2*Log[1 - Tan[a + b
*x] + Tan[a + b*x]^2])/(3*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^3}{1+x^2+x^3+x^5}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{3(1+x)}+\frac{x}{1+x^2}-\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\log(1+\tan(a+bx))}{3b} - \frac{2\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan(a+bx)\right)}{3b} + \frac{\text{Subst}(\int$$
$$= -\frac{\log(\cos(a+bx))}{b} + \frac{\log(1+\tan(a+bx))}{3b} - \frac{2\log\left(1-\tan(a+bx)+\text{t}\right.}{3b}$$

Mathematica [A]
time = 0.15, size = 42, normalized size = 0.76

$$\frac{\log(\cos(a+bx)+\sin(a+bx))}{3b} - \frac{2\log(2-\sin(2(a+bx)))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^
3),x]
```

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) - (2*Log[2 - Sin[2*(a + b*x)]])/(3*b
)
```

Maple [A]
time = 0.60, size = 51, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\frac{2\ln\left(1-\tan(bx+a)+\tan^2(bx+a)\right)}{3}+\frac{\ln(1+\tan(bx+a))}{3}}{b}$
default	$\frac{\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\frac{2\ln\left(1-\tan(bx+a)+\tan^2(bx+a)\right)}{3}+\frac{\ln(1+\tan(bx+a))}{3}}{b}$
risch	$ix+\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}+i\right)}{3b}-\frac{2\ln\left(e^{4i(bx+a)}-4ie^{2i(bx+a)}-1\right)}{3b}$
norman	$\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{\ln\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{3b}-\frac{2\ln\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)+2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\left(\tan^2\left(\frac{bx}{2}+\right.\right.\right.}{3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/b*(1/2*ln(tan(b*x+a)^2+1)-2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)+1/3*ln(1+tan(
b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(51) = 102$.

time = 0.51, size = 154, normalized size = 2.80

$$-\frac{2 \log \left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{2 \sin(bx+a)^2}{(\cos(bx+a)+1)^2} + \frac{2 \sin(bx+a)^3}{(\cos(bx+a)+1)^3} + \frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4} + 1\right) - \log \left(-\frac{2 \sin(bx+a)}{\cos(bx+a)+1} + \frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} - 1\right) - 3 \log \left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2} + 1\right)}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="maxima")
```

```
[Out] -1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x
+ a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b
*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)
^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 +
1))/b
```

Fricas [A]

time = 2.92, size = 42, normalized size = 0.76

$$\frac{\log \left(2 \cos \left(bx+a\right) \sin \left(bx+a\right)+1\right)-4 \log \left(-\cos \left(bx+a\right) \sin \left(bx+a\right)+1\right)}{6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="fricas")
```

```
[Out] 1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x + a
) + 1))/b
```

Sympy [A]

time = 0.42, size = 76, normalized size = 1.38

$$\begin{cases} \frac{\log \left(\sin \left(a+bx\right)+\cos \left(a+bx\right)\right)}{3b} - \frac{2 \log \left(\sin^2 \left(a+bx\right)-\sin \left(a+bx\right) \cos \left(a+bx\right)+\cos^2 \left(a+bx\right)\right)}{3b} & \text{for}\ b \neq 0 \\ \frac{x\left(-\sin^3 \left(a\right)+\cos^3 \left(a\right)\right)}{\sin^3 \left(a\right)+\cos^3 \left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**3-sin(b*x+a)**3)/(cos(b*x+a)**3+sin(b*x+a)**3),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/(3*b) - 2*log(sin(a + b*x)**2 -
 sin(a + b*x)*cos(a + b*x) + cos(a + b*x)**2)/(3*b), Ne(b, 0)), (x*(-sin(a)
**3 + cos(a)**3)/(sin(a)**3 + cos(a)**3), True))
```

Giac [A]

time = 0.50, size = 52, normalized size = 0.95

$$-\frac{4 \log \left(\tan \left(bx+a\right)^2-\tan \left(bx+a\right)+1\right)-3 \log \left(\tan \left(bx+a\right)^2+1\right)-2 \log \left(\left|\tan \left(bx+a\right)+1\right|\right)}{6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algori
thm="giac")
```

```
[Out] -1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1)
- 2*log(abs(tan(b*x + a) + 1)))/b
```

Mupad [B]
time = 3.30, size = 105, normalized size = 1.91

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}{b}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{3\,b}-\frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+1\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3 - sin(a + b*x)^3)/(cos(a + b*x)^3 + sin(a + b*x)^3),x)
```

```
[Out] log(tan(a/2 + (b*x)/2)^2 + 1)/b + log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b
*x)/2) - 1)/(3*b) - (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) +
2*tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b)
```

3.945 $\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} \, dx$

Optimal. Leaf size=16

$$\frac{\cos(a+bx)\sin(a+bx)}{b}$$

```
[Out] cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]

time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4465, 2715, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2),x]
[Out] (Cos[a + b*x]*Sin[a + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4465

```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Dist[(a + c)^p, Int[ActivateTrig[u], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} \, dx &= \int \left(\cos^2(a+bx)-\sin^2(a+bx)\right) \, dx \\
&= \int \cos^2(a+bx) \, dx - \int \sin^2(a+bx) \, dx \\
&= \frac{\cos(a+bx)\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.
time = 0.01, size = 33, normalized size = 2.06

$$\frac{\cos(2bx)\sin(2a)}{2b}+\frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^
2),x]
```

```
[Out] (Cos[2*b*x]*Sin[2*a])/(2*b) + (Cos[2*a]*Sin[2*b*x])/(2*b)
```

Maple [A]
time = 0.20, size = 17, normalized size = 1.06

method	result	size
risch	$\frac{\sin(2bx+2a)}{2b}$	15
derivativedivides	$\frac{\cos(bx+a)\sin(bx+a)}{b}$	17
default	$\frac{\cos(bx+a)\sin(bx+a)}{b}$	17
norman	$\frac{\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x,method=_RETUR
NVERBOSE)
```

```
[Out] cos(b*x+a)*sin(b*x+a)/b
```

Maxima [A]
time = 0.29, size = 22, normalized size = 1.38

$$\frac{\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="maxima")
```

```
[Out] tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

Fricas [A]
time = 2.97, size = 16, normalized size = 1.00

$$\frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="fricas")
```

```
[Out] cos(b*x + a)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.09, size = 32, normalized size = 2.00

$$\frac{\sin(a + bx)\cos(a + bx)}{b\sin^2(a + bx) + b\cos^2(a + bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)**2-sin(b*x+a)**2)/(cos(b*x+a)**2+sin(b*x+a)**2),x)
```

```
[Out] sin(a + b*x)*cos(a + b*x)/(b*sin(a + b*x)**2 + b*cos(a + b*x)**2)
```

Giac [A]
time = 0.42, size = 14, normalized size = 0.88

$$\frac{\sin(2\,bx + 2\,a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algori
thm="giac")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 3.03, size = 14, normalized size = 0.88

$$\frac{\sin(2\,a + 2\,b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2 - sin(a + b*x)^2)/(cos(a + b*x)^2 + sin(a + b*x)^2),x)
```

```
[Out] sin(2*a + 2*b*x)/(2*b)
```

3.946 $\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

[Out] ln(cos(b*x+a)+sin(b*x+a))/b

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {3212}

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx=\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]

```
[Out] Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

Maple [A]
time = 0.27, size = 19, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\ln(\cos(bx+a)+\sin(bx+a))}{b}$	19
default	$\frac{\ln(\cos(bx+a)+\sin(bx+a))}{b}$	19
risch	$-ix-\frac{2ia}{b}+\frac{\ln(\mathrm{e}^{2i(bx+a)}+i)}{b}$	30
norman	$\frac{\ln\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x,method=_RETURNVERBOSE
)
```

```
[Out] ln(cos(b*x+a)+sin(b*x+a))/b
```

Maxima [A]
time = 0.30, size = 18, normalized size = 1.00

$$\frac{\log\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="max
ima")
```

```
[Out] log(cos(b*x + a) + sin(b*x + a))/b
```

Fricas [A]
time = 2.52, size = 22, normalized size = 1.22

$$\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="fri
cas")
```

```
[Out] 1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

time = 0.19, size = 31, normalized size = 1.72

$$\begin{cases} \frac{\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(-\sin\left(a\right)+\cos\left(a\right)\right)}{\sin\left(a\right)+\cos\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

```
[Out] Piecewise((log(sin(a + b*x) + cos(a + b*x))/b, Ne(b, 0)), (x*(-sin(a) + cos
(a))/(sin(a) + cos(a)), True))
```

Giac [A]
time = 0.41, size = 29, normalized size = 1.61

$$-\frac{\log\left(\tan\left(bx+a\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="gia
c")
```

```
[Out] -1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

Mupad [B]
time = 3.14, size = 50, normalized size = 2.78

$$\frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x) - sin(a + b*x))/(cos(a + b*x) + sin(a + b*x)),x)
```

```
[Out] (2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/2
 + (b*x)/2)^2 + 48) - 3))/b
```

3.947 $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

[Out] -ln(cos(b*x+a)+sin(b*x+a))/b

Rubi [A]
time = 0.24, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {815, 266}

$$-\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]

[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x}{(1+x)(1+x^2)}\,dx,x,\tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{x}{1+x^2}\right)\,dx,x,\tan(a+bx)\right)}{b}\\
&= -\frac{\log(1+\tan(a+bx))}{b}+\frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,\tan(a+bx)\right)}{b}\\
&= -\frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{b}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 1.00

$$-\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
[Out] -(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

Maple [A]
time = 0.51, size = 30, normalized size = 1.58

method	result	size
derivativedivides	$\frac{\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\ln(1+\tan(bx+a))}{b}$	30
default	$\frac{\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}-\ln(1+\tan(bx+a))}{b}$	30
risch	$ix+\frac{2ia}{b}-\frac{\ln\left(e^{2i(bx+a)}+i\right)}{b}$	31
norman	$\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x,method=_RETURNVERBOS
E)
[Out] 1/b*(1/2*ln(tan(b*x+a)^2+1)-ln(1+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(19) = 38.
time = 0.51, size = 70, normalized size = 3.68

$$-\frac{\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-\log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="ma
xima")
[Out] -(log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) + 1
)^2 - 1) - log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b
```

Fricas [A]
time = 3.15, size = 22, normalized size = 1.16

$$-\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="fr
icas")
```

```
[Out] -1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\csc(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx - \int \left(-\frac{\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

```
[Out] -Integral(csc(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x) - Integral(-sec(a
+ b*x)/(csc(a + b*x) + sec(a + b*x)), x)
```

Giac [A]
time = 0.46, size = 29, normalized size = 1.53

$$\frac{\log\left(\tan(bx+a)^2+1\right)-2\,\log\left(|\tan(bx+a)+1|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

Mupad [B]
time = 3.37, size = 50, normalized size = 2.63

$$-\frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x) - 1/sin(a + b*x))/(1/cos(a + b*x) + 1/sin(a + b*x)),x)
```

```
[Out] -(2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/
2 + (b*x)/2)^2 + 48) - 3))/b
```

3.948 $\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx$

Optimal. Leaf size=17

$$-\frac{\cos(a+bx)\sin(a+bx)}{b}$$

```
[Out] -cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.12, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {391}

$$-\frac{\sin(a+bx)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2),x]
```

```
[Out] -((Cos[a + b*x]*Sin[a + b*x])/b)
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.94

$$-\frac{\cos(2bx)\sin(2a)}{2b}-\frac{\cos(2a)\sin(2bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]
^2),x]
```

[Out] -1/2*(Cos[2*b*x]*Sin[2*a])/b - (Cos[2*a]*Sin[2*b*x])/(2*b)

Maple [A]
time = 0.29, size = 18, normalized size = 1.06

method	result	size
risch	$-\frac{\sin(2bx+2a)}{2b}$	15
derivativedivides	$-\frac{\cos(bx+a)\sin(bx+a)}{b}$	18
default	$-\frac{\cos(bx+a)\sin(bx+a)}{b}$	18
norman	$\frac{\frac{2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{4\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{2\left(\tan^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$	92

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] -cos(b*x+a)*sin(b*x+a)/b

Maxima [A]
time = 0.30, size = 23, normalized size = 1.35

$$-\frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algorithm="maxima")

[Out] -tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)

Fricas [A]
time = 3.20, size = 17, normalized size = 1.00

$$-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algorithm="fricas")

[Out] -cos(b*x + a)*sin(b*x + a)/b

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx-\int\left(-\frac{\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**2+sec(b*x+a)**2)/(csc(b*x+a)**2+sec(b*x+a)**2),x)
```

```
[Out] -Integral(csc(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x) - Integra
l(-sec(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x)
```

Giac [A]
time = 0.47, size = 14, normalized size = 0.82

$$-\frac{\sin\left(2\,bx+2\,a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algor
ithm="giac")
```

```
[Out] -1/2*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 3.05, size = 14, normalized size = 0.82

$$-\frac{\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^2 - 1/sin(a + b*x)^2)/(1/cos(a + b*x)^2 + 1/sin(a + b*x
)^2),x)
```

```
[Out] -sin(2*a + 2*b*x)/(2*b)
```

3.949 $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx$

Optimal. Leaf size=54

$$\frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx)\right)}{3b}$$

```
[Out] ln(cos(b*x+a))/b-1/3*ln(1+tan(b*x+a))/b+2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/b
```

Rubi [A]
time = 0.39, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {6857, 266, 642}

$$\frac{2\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)}{3b}-\frac{\log(\tan(a+bx)+1)}{3b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]^3),x]
```

```
[Out] Log[Cos[a + b*x]]/b - Log[1 + Tan[a + b*x]]/(3*b) + (2*Log[1 - Tan[a + b*x]
 + Tan[a + b*x]^2])/(3*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^3}{(1+x^2)(1+x^3)}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{3(1+x)}-\frac{x}{1+x^2}+\frac{2(-1+2x)}{3(1-x+x^2)}\right)dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan(a+bx)\right)}{3b} - \frac{\text{Sub}}{}$$
$$= \frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\log\left(1-\tan(a+bx)+\text{t}\right.}{3b}$$

Mathematica [A]
time = 0.17, size = 42, normalized size = 0.78

$$-\frac{\log(\cos(a+bx)+\sin(a+bx))}{3b}+\frac{2\log(2-\sin(2(a+bx)))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]
^3),x]
```

```
[Out] -1/3*Log[Cos[a + b*x] + Sin[a + b*x]]/b + (2*Log[2 - Sin[2*(a + b*x)]])/(3*
b)
```

Maple [A]
time = 0.79, size = 51, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}+\frac{2\ln\left(1-\tan(bx+a)+\tan^2(bx+a)\right)}{3}-\frac{\ln(1+\tan(bx+a))}{3}}{b}$
default	$\frac{-\frac{\ln\left(\tan^2(bx+a)+1\right)}{2}+\frac{2\ln\left(1-\tan(bx+a)+\tan^2(bx+a)\right)}{3}-\frac{\ln(1+\tan(bx+a))}{3}}{b}$
risch	$-ix-\frac{2ia}{b}-\frac{\ln\left(e^{2i(bx+a)}+i\right)}{3b}+\frac{2\ln\left(e^{4i(bx+a)}-4ie^{2i(bx+a)}-1\right)}{3b}$
norman	$-\frac{\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{3b}+\frac{2\ln\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)+2\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\left(\tan^2\left(\right.\right.\right.}{3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/b*(-1/2*ln(tan(b*x+a)^2+1)+2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)-1/3*ln(1+tan
(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(50) = 100$.
time = 0.50, size = 154, normalized size = 2.85

$$\frac{2\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{2\sin(bx+a)^3}{(\cos(bx+a)+1)^3}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1\right)-\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-3\log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="maxima")
```

```
[Out] 1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x +
 a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(b*
x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^
2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1
))/b
```

Fricas [A]
time = 2.30, size = 42, normalized size = 0.78

$$-\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="fricas")
```

```
[Out] -1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x +
a) + 1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^3\left(a+bx\right)}{\csc^3\left(a+bx\right)+\sec^3\left(a+bx\right)}\,dx-\int\left(-\frac{\sec^3\left(a+bx\right)}{\csc^3\left(a+bx\right)+\sec^3\left(a+bx\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**3+sec(b*x+a)**3)/(csc(b*x+a)**3+sec(b*x+a)**3),x)
```

```
[Out] -Integral(csc(a + b*x)**3/(csc(a + b*x)**3 + sec(a + b*x)**3), x) - Integra
l(-sec(a + b*x)**3/(csc(a + b*x)**3 + sec(a + b*x)**3), x)
```

Giac [A]
time = 0.53, size = 52, normalized size = 0.96

$$\frac{4\log\left(\tan\left(bx+a\right)^2-\tan\left(bx+a\right)+1\right)-3\log\left(\tan\left(bx+a\right)^2+1\right)-2\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algor
ithm="giac")
```

```
[Out] 1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1) -
 2*log(abs(tan(b*x + a) + 1)))/b
```

Mupad [B]
time = 3.23, size = 106, normalized size = 1.96

$$\frac{2\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+1\right)}{3\,b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{3\,b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^3 - 1/sin(a + b*x)^3)/(1/cos(a + b*x)^3 + 1/sin(a + b*x
)^3),x)
```

```
[Out] (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) + 2*tan(a/2 + (b*x)/2)
^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b) - log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/
2 + (b*x)/2) - 1)/(3*b) - log(tan(a/2 + (b*x)/2)^2 + 1)/b
```

3.950 $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx$

Optimal. Leaf size=72

$$\frac{\log\left(1-\sqrt{2}\,\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}\,b}-\frac{\log\left(1+\sqrt{2}\,\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}\,b}$$

```
[Out] 1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)-1/4*ln(1+2^(1/2)*tan(b*
x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1179, 642}

$$\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\,\tan(a+bx)+1\right)}{2\sqrt{2}\,b}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\,\tan(a+bx)+1\right)}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]^4),x]
```

```
[Out] Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b) - Log[1 + Sqrt
[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]*b)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^4}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \tan(a+bx)\right)}{2\sqrt{2}b} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx\right.}{2\sqrt{2}b}$$
$$= \frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\sqrt{2}\tan(a+bx\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.36

$$-\frac{\tanh^{-1}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]
^4),x]
```

```
[Out] -(ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(60) = 120$.
time = 0.40, size = 168, normalized size = 2.33

method	result
risch	$\frac{\sqrt{2}\,\ln\left(e^{4i(bx+a)}-2i\sqrt{2}\,e^{2i(bx+a)}-1\right)}{4b}-\frac{\sqrt{2}\,\ln\left(e^{4i(bx+a)}+2i\sqrt{2}\,e^{2i(bx+a)}-1\right)}{4b}$
derivativedivides	$\frac{-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\tan(bx+a)+\tan^2(bx+a)}\right)+2\arctan\left(\sqrt{2}\tan(bx+a)+1\right)+2\arctan\left(\sqrt{2}\tan(bx+a)-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\right.\right.}{}}{b}$
default	$\frac{-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\tan(bx+a)+\tan^2(bx+a)}{1-\sqrt{2}\tan(bx+a)+\tan^2(bx+a)}\right)+2\arctan\left(\sqrt{2}\tan(bx+a)+1\right)+2\arctan\left(\sqrt{2}\tan(bx+a)-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\right.\right.}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/b*(-1/8*2^(1/2)*(ln((1+2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1-2^(1/2)*tan(b*
x+a)+tan(b*x+a)^2))+2*arctan(2^(1/2)*tan(b*x+a)+1)+2*arctan(2^(1/2)*tan(b*x
+a)-1))+1/8*2^(1/2)*(ln((1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/(1+2^(1/2)*tan(
b*x+a)+tan(b*x+a)^2))+2*arctan(2^(1/2)*tan(b*x+a)+1)+2*arctan(2^(1/2)*tan(b
*x+a)-1)))
```

Maxima [**A**]
time = 0.53, size = 58, normalized size = 0.81

$$-\frac{\sqrt{2}\log\left(\tan\left(bx+a\right)^2+\sqrt{2}\tan\left(bx+a\right)+1\right)-\sqrt{2}\log\left(\tan\left(bx+a\right)^2-\sqrt{2}\tan\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(
tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

Fricas [**A**]
time = 3.34, size = 74, normalized size = 1.03

$$\frac{\sqrt{2}\log\left(-\frac{2\cos\left(bx+a\right)^4+2\sqrt{2}\cos\left(bx+a\right)\sin\left(bx+a\right)-2\cos\left(bx+a\right)^2-1}{2\cos\left(bx+a\right)^4-2\cos\left(bx+a\right)^2+1}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 + 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc^4\left(a+bx\right)}{\csc^4\left(a+bx\right)+\sec^4\left(a+bx\right)}\,dx-\int\left(-\frac{\sec^4\left(a+bx\right)}{\csc^4\left(a+bx\right)+\sec^4\left(a+bx\right)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)**4+sec(b*x+a)**4)/(csc(b*x+a)**4+sec(b*x+a)**4),x)
```

```
[Out] -Integral(csc(a + b*x)**4/(csc(a + b*x)**4 + sec(a + b*x)**4), x) - Integra
l(-sec(a + b*x)**4/(csc(a + b*x)**4 + sec(a + b*x)**4), x)
```

Giac [A]
time = 0.53, size = 48, normalized size = 0.67

$$\frac{\sqrt{2}\,\log\left(\frac{\left|-2\sqrt{2}+2\,\sin\left(2\,bx+2\,a\right)\right|}{\left|2\sqrt{2}+2\,\sin\left(2\,bx+2\,a\right)\right|}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algor
ithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin(
2*b*x + 2*a)))/b
```

Mupad [B]
time = 3.09, size = 23, normalized size = 0.32

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sin\left(2\,a+2\,b\,x\right)}{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x)^4 - 1/sin(a + b*x)^4)/(1/cos(a + b*x)^4 + 1/sin(a + b*x
)^4),x)
```

```
[Out] -(2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```