Computer algebra independent integration tests

Summer 2022 edition

3-Logarithms/64-3.5-Logarithm-functions

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [314]. This is test number [64].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (314)	0.00 (0)
Mathematica	100.00 (314)	0.00 (0)
Fricas	88.85 (279)	11.15 (35)
Maple	76.75 (241)	23.25 (73)
Maxima	70.06 (220)	29.94 (94)
Giac	60.51 (190)	39.49 (124)
Mupad	58.28 (183)	41.72 (131)
Sympy	40.76 (128)	59.24 (186)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

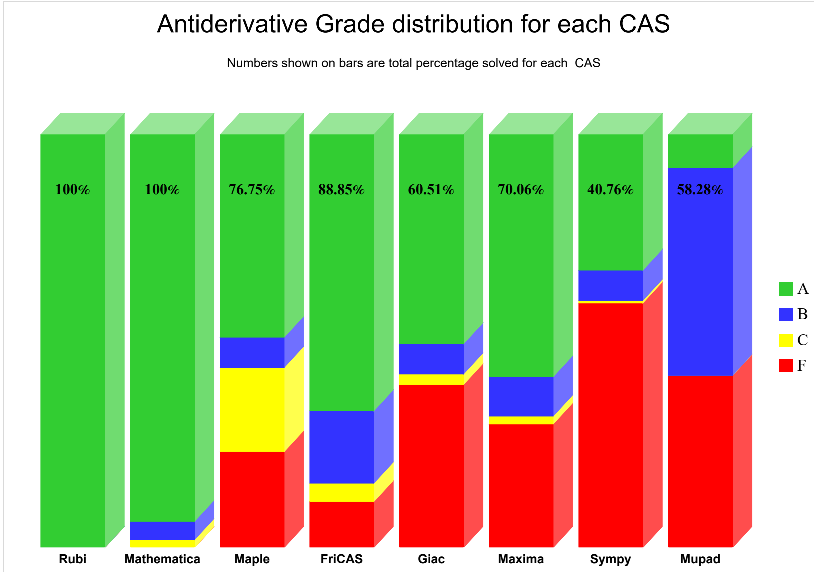
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

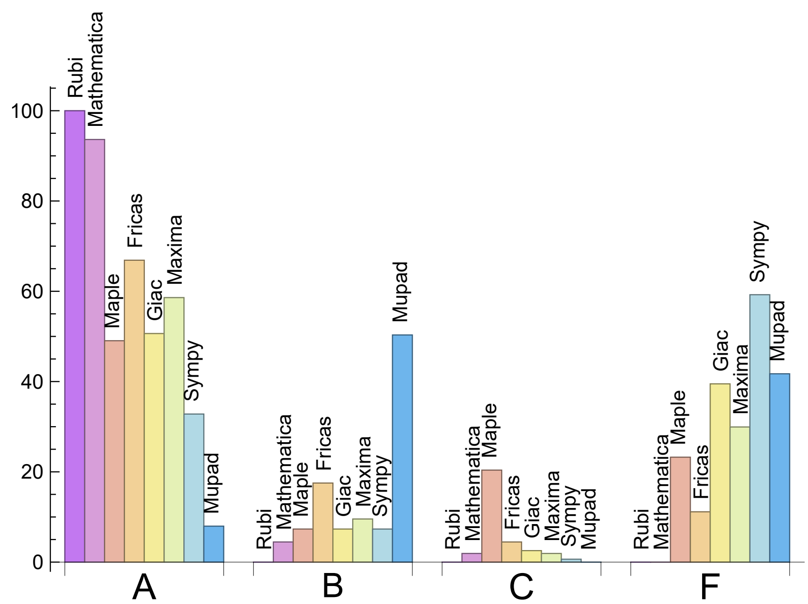
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.63	4.46	1.91	0.00
Fricas	66.88	17.52	4.46	11.15
Maxima	58.60	9.55	1.91	29.94
Giac	50.64	7.32	2.55	39.49
Maple	49.04	7.32	20.38	23.25
Sympy	32.80	7.32	0.64	59.24
Mupad	N/A	50.32	0.00	41.72

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	73	100.00 %	0.00 %	0.00 %
Fricas	35	80.00 %	0.00 %	20.00 %
Giac	124	92.74 %	1.61 %	5.65 %
Maxima	94	55.32 %	0.00 %	44.68 %
Sympy	186	69.35 %	20.43 %	10.22 %
Mupad	131	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

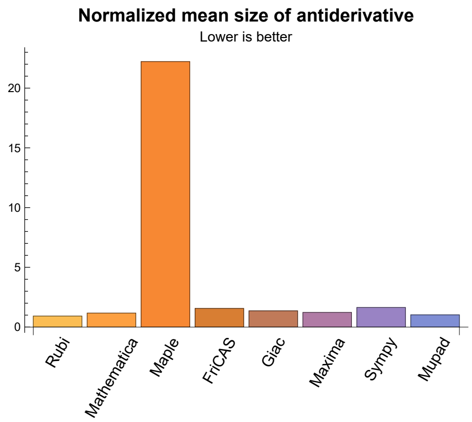
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

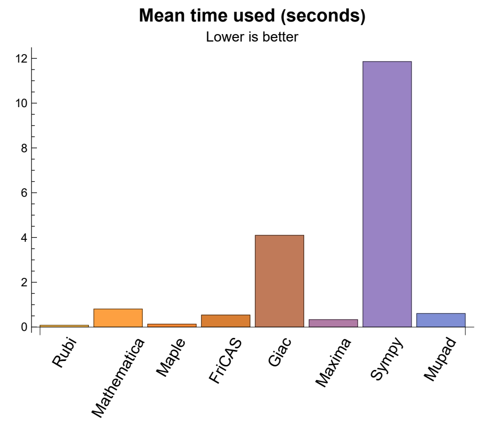
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.08	69.56	0.92	41.00	1.00
Mathematica	0.81	81.48	1.17	42.00	1.00
Maple	0.13	6850.78	22.22	43.00	1.14
Maxima	0.33	53.65	1.22	36.00	1.00
Fricas	0.54	123.41	1.56	46.00	1.06
Sympy	11.86	61.70	1.64	26.00	1.00
Giac	4.10	105.15	1.36	32.50	1.00
Mupad	0.60	97.09	1.02	22.00	0.87

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{40, 100\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

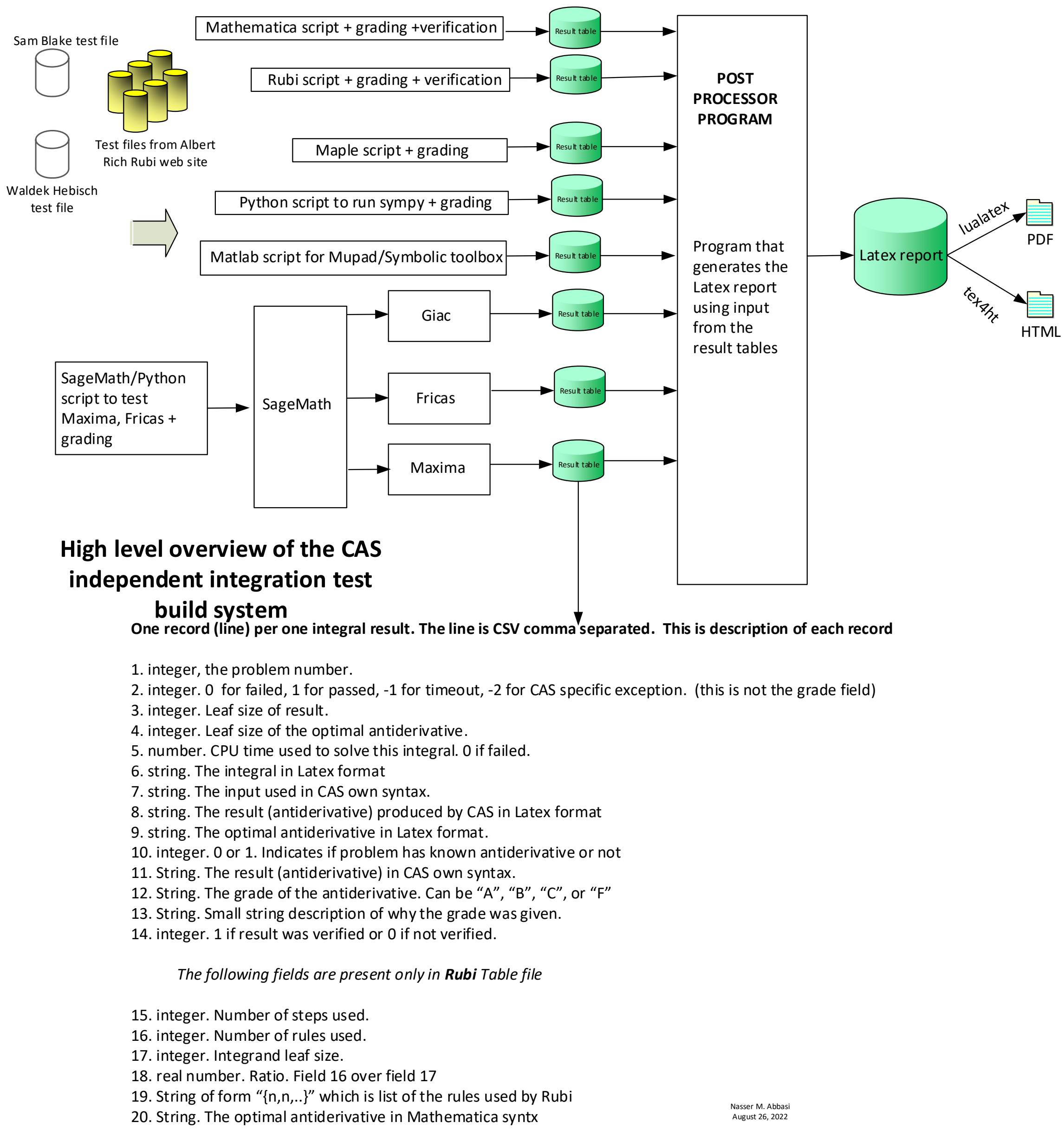
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { 40, 41, 42, 43, 44, 45, 93, 134, 189, 225, 278, 279, 280, 281 }

C grade: { 108, 109, 110, 111, 112, 135 }

F grade: { }

2.1.3 Maple

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 19, 25, 30, 34, 35, 36, 37, 39, 51, 55, 56, 58, 64, 75, 81, 86, 98, 104, 105, 113, 114, 115, 116, 117, 121, 122, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 153, 160, 168, 171, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 207, 209, 210, 211, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 308, 309, 313, 314 }

B grade: { 24, 91, 118, 119, 120, 123, 124, 125, 128, 161, 162, 164, 165, 167, 170, 173, 174, 176, 177, 191, 208, 212, 278 }

C grade: { 9, 10, 11, 17, 18, 20, 21, 22, 23, 26, 27, 28, 29, 38, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 87, 88, 89, 90, 94, 95, 150, 151, 154, 155, 156, 157, 158, 159, 169, 172, 179, 182, 189, 192, 194, 201, 202, 204, 205, 215, 216, 218, 219, 276, 277, 305, 306, 307, 310, 311, 312 }

F grade: { 2, 3, 4, 16, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 92, 93, 96, 97, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 149, 152, 163, 166, 175, 178, 193, 203, 206, 217, 220, 263, 264, 265, 279, 280, 281, 295, 297, 300, 301, 302, 303, 304 }

2.1.4 Maxima

A grade: { 5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 25, 27, 29, 34, 35, 36, 37, 38, 39, 51, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 96, 105, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 254, 259, 262, 267, 268, 270, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 299, 302, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

B grade: { 9, 10, 11, 22, 23, 24, 26, 28, 58, 121, 128, 161, 162, 163, 176, 177, 178, 182, 189, 191, 192, 193, 194, 221, 237, 246, 269, 301, 303, 304 }

C grade: { 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 16, 17, 18, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 149, 150, 152, 231, 232, 233, 234, 251, 252, 253, 255, 256, 257, 258, 260, 261, 263, 264, 265, 266, 277, 278, 279, 280, 281, 293, 300 }

2.1.5 FriCAS

A grade: { 1, 5, 6, 7, 8, 12, 13, 14, 15, 16, 19, 20, 21, 25, 26, 27, 29, 30, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 201, 203, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 271, 272, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 308, 309, 313, 314 }

B grade: { 9, 10, 11, 17, 18, 22, 23, 24, 28, 89, 90, 91, 128, 131, 135, 136, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 191, 195, 196, 197, 198, 199, 200, 202, 218, 219, 220, 221, 222, 225, 246, 305, 306, 307, 310, 311, 312 }

C grade: { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217 }

F grade: { 2, 3, 4, 31, 32, 33, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 193, 263, 264, 265, 266, 267, 269, 270, 277, 278, 279, 301, 302, 303, 304 }

2.1.6 Sympy

A grade: { 6, 9, 10, 11, 13, 14, 15, 19, 20, 23, 24, 25, 34, 35, 38, 39, 55, 60, 61, 62, 63, 64, 66, 67, 68, 69, 81, 105, 129, 130, 132, 137, 138, 139, 141, 142, 144, 145, 146, 147, 148, 153, 160, 182, 184, 187, 188, 190, 194, 222, 224, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 239, 241, 242, 243, 244, 245, 247, 248, 252, 253, 256, 257, 258, 259, 260, 261, 262, 272, 273, 274, 275, 276, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 308, 309, 313, 314 }

B grade: { 12, 18, 74, 75, 77, 85, 86, 131, 133, 140, 189, 192, 223, 235, 238, 240, 246, 249, 250, 251, 254, 255, 283 }

C grade: { 268, 270 }

F grade: { 1, 2, 3, 4, 5, 7, 8, 16, 17, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 65, 70, 71, 72, 73, 76, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135, 136, 143, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 185, 186, 191, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 225, 234, 263, 264, 265, 266, 267, 269, 271, 277, 278, 279, 280, 281, 284, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

2.1.7 Giac

A grade: { 5, 6, 7, 8, 11, 12, 13, 14, 15, 19, 25, 26, 27, 34, 35, 36, 37, 39, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 101, 102, 103, 104, 105, 106, 107, 112, 117, 122, 127, 130, 132, 133, 135, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 160, 179, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 194, 195, 196, 197, 198, 199, 200, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 251, 253, 254, 255, 257, 262, 265, 271, 272, 273, 276, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 297, 298, 301, 302, 304, 308, 309, 313, 314 }

B grade: { 9, 10, 23, 24, 28, 89, 90, 91, 129, 131, 136, 141, 180, 221, 222, 241, 250, 252, 256, 274, 275, 296, 303 }

C grade: { 108, 109, 154, 155, 156, 157, 158, 159 }

F grade: { 1, 2, 3, 4, 16, 17, 18, 20, 21, 22, 29, 30, 31, 32, 33, 38, 40, 41, 42, 43, 44, 45, 52, 53, 54, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 123, 124, 125, 126, 128, 134, 137, 147, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 185, 191, 193, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 249, 258, 259, 260, 261, 263, 264, 266, 267, 268, 269, 270, 277, 278, 279, 280, 281, 293, 299, 300, 305, 306, 307, 310, 311, 312 }

2.1.8 Mupad

A grade: { 1, 6, 7, 8, 13, 14, 15, 30, 35, 36, 37, 39, 105, 117, 122, 127, 286, 287, 288, 289, 290, 308, 309, 313, 314 }

B grade: { 5, 12, 17, 18, 19, 23, 24, 25, 26, 27, 28, 34, 38, 51, 55, 56, 58, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 116, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 194, 207, 211, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

C grade: { }

F grade: { 2, 3, 4, 9, 10, 11, 16, 20, 21, 22, 29, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 57, 59, 65, 70, 76, 87, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 123, 124, 125, 126, 128, 154, 155, 156, 157, 158, 159, 161, 162, 163, 170, 171, 176, 177, 178, 179, 189, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 263, 264, 265, 266, 267, 270, 277, 278, 279, 280, 281, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.976	0.196	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	223	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.480	0.020	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	149	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.288	0.020	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.109	0.021	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	25	0	16	15
N.S.	1	1.00	1.00	1.07	1.00	1.67	0.00	1.07	1.00
time (sec)	N/A	0.014	0.003	0.204	0.299	0.378	0.000	7.349	0.284

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.092	0.025	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.469	0.022	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	1.028	0.029	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	336	61910	1115	655	411	766	-1
N.S.	1	1.00	1.24	227.61	4.10	2.41	1.51	2.82	-0.00
time (sec)	N/A	0.201	0.205	3.892	0.319	0.381	30.898	5.475	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	193	14983	530	267	216	286	-1
N.S.	1	1.00	1.54	119.86	4.24	2.14	1.73	2.29	-0.01
time (sec)	N/A	0.112	0.111	1.050	0.319	0.343	15.721	7.173	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	78	2146	186	81	65	73	-1
N.S.	1	1.00	1.90	52.34	4.54	1.98	1.59	1.78	-0.02
time (sec)	N/A	0.050	0.048	0.209	0.303	0.390	7.015	8.838	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	65	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	4.33	0.87	0.87
time (sec)	N/A	0.004	0.002	0.039	0.288	0.356	1.126	6.678	0.254

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	1.109	0.006	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.225	0.007	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	2.317	0.010	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	42	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	-0.04
time (sec)	N/A	0.115	0.070	0.096	0.000	0.385	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	204	0	65	0	0	20
N.S.	1	1.00	1.00	9.27	0.00	2.95	0.00	0.00	0.91
time (sec)	N/A	0.115	0.031	0.168	0.000	0.347	0.000	0.000	0.705

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	135	0	42	39	0	20
N.S.	1	1.00	1.00	6.14	0.00	1.91	1.77	0.00	0.91
time (sec)	N/A	0.074	0.026	0.154	0.000	0.358	41.298	0.000	0.680

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	28	54	17	16
N.S.	1	1.00	1.00	1.06	1.00	1.75	3.38	1.06	1.00
time (sec)	N/A	0.022	0.014	0.123	0.275	0.359	14.784	6.865	0.304

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	212	22	18	20	0	-1
N.S.	1	1.00	1.00	12.47	1.29	1.06	1.18	0.00	-0.06
time (sec)	N/A	0.124	0.114	0.110	0.418	0.363	120.881	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	68	21	21	0	0	-1
N.S.	1	1.00	1.00	3.40	1.05	1.05	0.00	0.00	-0.05
time (sec)	N/A	0.124	0.033	0.088	0.465	0.360	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	68	49	45	0	0	-1
N.S.	1	1.00	1.00	3.09	2.23	2.05	0.00	0.00	-0.05
time (sec)	N/A	0.126	0.032	2.871	0.574	0.365	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20850	211	195	70	198	52
N.S.	1	1.00	1.00	1042.50	10.55	9.75	3.50	9.90	2.60
time (sec)	N/A	0.098	0.014	1.016	0.308	0.352	8.301	6.447	0.330

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	63	74	89	51	90	18
N.S.	1	1.00	1.90	3.15	3.70	4.45	2.55	4.50	0.90
time (sec)	N/A	0.060	0.009	0.236	0.277	0.371	5.775	2.752	0.293

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	73	20	14
N.S.	1	1.00	1.00	1.07	1.00	1.50	5.21	1.43	1.00
time (sec)	N/A	0.009	0.004	0.041	0.316	0.338	2.085	6.552	0.257

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	428	32	28	0	28	16
N.S.	1	1.00	1.00	28.53	2.13	1.87	0.00	1.87	1.07
time (sec)	N/A	0.053	0.030	0.047	0.317	0.382	0.000	3.530	0.348

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	451	31	31	0	31	18
N.S.	1	1.00	1.00	25.06	1.72	1.72	0.00	1.72	1.00
time (sec)	N/A	0.087	0.012	0.051	0.323	0.348	0.000	5.787	0.255

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	451	95	101	0	306	39
N.S.	1	1.00	1.00	22.55	4.75	5.05	0.00	15.30	1.95
time (sec)	N/A	0.105	0.012	0.066	0.379	0.362	0.000	3.507	0.275

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	23	215	26	23	0	0	-1
N.S.	1	1.00	1.21	11.32	1.37	1.21	0.00	0.00	-0.05
time (sec)	N/A	0.226	0.206	0.128	0.411	0.378	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	1.414	0.094	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	445	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.337	1.022	0.041	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	298	0	0	0	0	0	-1
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	0.598	0.039	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	157	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.259	0.036	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	26	44	49	27	25
N.S.	1	1.00	1.00	1.04	1.04	1.76	1.96	1.08	1.00
time (sec)	N/A	0.024	0.015	0.129	0.300	0.401	21.607	5.082	0.316

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	3.951	0.033	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	22.321	0.037	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	63.421	0.065	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	158	31	30	22	0	26
N.S.	1	1.00	1.00	6.08	1.19	1.15	0.85	0.00	1.00
time (sec)	N/A	0.163	0.166	0.087	0.453	0.341	44.657	0.000	0.325

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	85.792	0.007	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	625	0	0	47	0	0	-1
N.S.	1	1.00	12.76	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.311	0.189	0.000	0.359	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	642	0	0	48	0	0	-1
N.S.	1	1.00	12.84	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.269	0.208	0.000	0.368	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	320	0	0	44	0	0	-1
N.S.	1	1.00	6.04	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.151	0.243	0.000	0.344	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	316	0	0	43	0	0	-1
N.S.	1	1.00	6.08	0.00	0.00	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.135	0.241	0.000	0.349	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	641	0	0	46	0	0	-1
N.S.	1	1.00	13.08	0.00	0.00	0.94	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.223	0.188	0.000	0.364	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	645	0	0	47	0	0	-1
N.S.	1	1.00	12.90	0.00	0.00	0.94	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.183	0.210	0.000	0.355	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	83	0	83	-1
N.S.	1	1.00	0.84	0.00	0.00	1.24	0.00	1.24	-0.01
time (sec)	N/A	0.038	0.140	0.013	0.000	0.372	0.000	5.373	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	0	0	82	0	111	-1
N.S.	1	1.00	0.75	0.00	0.00	1.04	0.00	1.41	-0.01
time (sec)	N/A	0.039	0.108	0.026	0.000	0.408	0.000	4.431	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	-1
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	-0.02
time (sec)	N/A	0.032	0.047	0.009	0.000	0.457	0.000	3.908	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	70	0	56	-1
N.S.	1	1.00	0.89	0.00	0.00	1.27	0.00	1.02	-0.02
time (sec)	N/A	0.022	0.046	0.007	0.000	0.364	0.000	4.185	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	53	0	42	-1
N.S.	1	1.00	0.96	0.00	0.00	1.18	0.00	0.93	-0.02
time (sec)	N/A	0.020	0.038	0.006	0.000	0.357	0.000	5.627	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	43	64	45	0	54	32
N.S.	1	1.00	1.25	1.34	2.00	1.41	0.00	1.69	1.00
time (sec)	N/A	0.011	0.012	0.117	0.280	0.358	0.000	3.396	0.323

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	0	0	46	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.034	0.008	0.000	0.353	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.036	0.009	0.000	0.364	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	0	0	53	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.035	0.007	0.000	0.355	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	23	26	19	26	22
N.S.	1	1.00	1.00	1.18	1.05	1.18	0.86	1.18	1.00
time (sec)	N/A	0.005	0.014	0.027	0.340	0.344	0.571	4.948	0.310

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	23	22	24	0	32	20
N.S.	1	1.00	1.10	1.15	1.10	1.20	0.00	1.60	1.00
time (sec)	N/A	0.013	0.008	0.051	0.283	0.371	0.000	4.183	0.284

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	0	0	45	0	36	-1
N.S.	1	1.00	0.98	0.00	0.00	1.12	0.00	0.90	-0.02
time (sec)	N/A	0.014	0.016	0.007	0.000	0.354	0.000	4.455	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	33	55	37	0	43	27
N.S.	1	1.00	1.26	1.22	2.04	1.37	0.00	1.59	1.00
time (sec)	N/A	0.014	0.010	0.030	0.288	0.345	0.000	4.651	0.295

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.017	0.010	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	87	98	112	89	85
N.S.	1	1.00	0.86	0.00	0.88	0.99	1.13	0.90	0.86
time (sec)	N/A	0.052	0.036	0.007	0.279	0.349	6.228	3.895	0.324

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	74	0	75	86	97	75	73
N.S.	1	1.00	0.87	0.00	0.88	1.01	1.14	0.88	0.86
time (sec)	N/A	0.041	0.028	0.004	0.280	0.345	2.768	5.152	0.338

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	65	74	85	65	61
N.S.	1	1.00	0.89	0.00	0.92	1.04	1.20	0.92	0.86
time (sec)	N/A	0.037	0.023	0.005	0.311	0.359	2.403	5.316	0.316

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	0	51	59	70	51	49
N.S.	1	1.00	0.86	0.00	0.89	1.04	1.23	0.89	0.86
time (sec)	N/A	0.028	0.017	0.019	0.284	0.355	0.733	4.440	0.368

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	37	36	38	44	37	33
N.S.	1	1.00	0.94	1.12	1.09	1.15	1.33	1.12	1.00
time (sec)	N/A	0.012	0.007	0.017	0.269	0.355	0.390	5.511	0.357

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	0	80	0	0	0	-1
N.S.	1	1.00	0.94	0.00	1.51	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.016	0.016	0.301	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	46	46	66	47	43
N.S.	1	1.00	0.96	0.00	0.98	0.98	1.40	1.00	0.91
time (sec)	N/A	0.030	0.010	0.004	0.280	0.369	1.097	3.822	0.796

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	62	70	94	65	54
N.S.	1	1.00	0.90	0.00	0.86	0.97	1.31	0.90	0.75
time (sec)	N/A	0.034	0.024	0.006	0.267	0.360	2.157	5.281	0.476

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	0	75	82	112	80	68
N.S.	1	1.00	0.90	0.00	0.87	0.95	1.30	0.93	0.79
time (sec)	N/A	0.038	0.026	0.003	0.277	0.358	5.495	3.494	0.456

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	86	94	122	92	79
N.S.	1	1.00	0.87	0.00	0.86	0.94	1.22	0.92	0.79
time (sec)	N/A	0.042	0.035	0.006	0.289	0.377	10.185	4.738	0.487

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	137	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.134	0.008	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	190	1621	0	444	0	221	395
N.S.	1	1.00	0.92	7.83	0.00	2.14	0.00	1.07	1.91
time (sec)	N/A	0.143	0.151	0.098	0.000	0.373	0.000	3.749	0.592

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	151	1146	0	364	0	176	288
N.S.	1	1.00	0.90	6.86	0.00	2.18	0.00	1.05	1.72
time (sec)	N/A	0.125	0.107	0.072	0.000	0.374	0.000	3.875	0.544

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	870	0	299	0	146	229
N.S.	1	1.00	0.90	6.40	0.00	2.20	0.00	1.07	1.68
time (sec)	N/A	0.102	0.079	0.063	0.000	0.357	0.000	4.661	0.504

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	510	0	245	359	113	166
N.S.	1	1.00	0.86	4.68	0.00	2.25	3.29	1.04	1.52
time (sec)	N/A	0.076	0.063	0.069	0.000	0.369	114.346	3.321	0.554

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	89	0	190	274	92	120
N.S.	1	1.00	0.99	1.13	0.00	2.41	3.47	1.16	1.52
time (sec)	N/A	0.041	0.043	0.055	0.000	0.376	45.180	4.334	0.342

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	156	315	0	0	0	0	-1
N.S.	1	1.00	1.21	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.119	0.036	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	87	261	0	199	211	99	262
N.S.	1	1.00	1.01	3.03	0.00	2.31	2.45	1.15	3.05
time (sec)	N/A	0.077	0.078	0.052	0.000	0.380	198.612	3.136	0.932

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	105	1178	0	261	0	129	474
N.S.	1	1.00	0.87	9.74	0.00	2.16	0.00	1.07	3.92
time (sec)	N/A	0.102	0.171	0.061	0.000	0.368	0.000	3.151	0.760

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	423	0	318	0	164	505
N.S.	1	1.00	0.89	2.84	0.00	2.13	0.00	1.10	3.39
time (sec)	N/A	0.135	0.261	0.055	0.000	0.402	0.000	4.818	0.933

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	172	505	0	404	0	210	627
N.S.	1	1.00	0.91	2.66	0.00	2.13	0.00	1.11	3.30
time (sec)	N/A	0.155	0.325	0.064	0.000	0.411	0.000	3.160	1.065

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	38	37	33	46	37	39
N.S.	1	1.00	0.83	0.90	0.88	0.79	1.10	0.88	0.93
time (sec)	N/A	0.017	0.011	0.026	0.489	0.356	0.062	4.637	0.064

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	468	31895	0	1274	0	817	1240
N.S.	1	1.00	0.96	65.76	0.00	2.63	0.00	1.68	2.56
time (sec)	N/A	1.359	1.302	0.234	0.000	0.453	0.000	4.796	1.030

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	324	16059	0	882	0	553	775
N.S.	1	1.00	0.96	47.51	0.00	2.61	0.00	1.64	2.29
time (sec)	N/A	0.342	0.357	0.198	0.000	0.388	0.000	3.850	0.837

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	204	7155	0	573	0	349	457
N.S.	1	1.00	0.90	31.66	0.00	2.54	0.00	1.54	2.02
time (sec)	N/A	0.211	0.266	0.151	0.000	0.395	0.000	3.109	0.672

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	123	1706	0	346	379	188	242
N.S.	1	1.00	0.80	11.08	0.00	2.25	2.46	1.22	1.57
time (sec)	N/A	0.124	0.137	0.109	0.000	0.374	116.207	4.152	0.590

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	89	0	190	274	92	120
N.S.	1	1.00	0.99	1.13	0.00	2.41	3.47	1.16	1.52
time (sec)	N/A	0.042	0.043	0.000	0.000	0.394	43.207	3.658	0.002

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	226	493	0	0	0	0	-1
N.S.	1	1.00	0.99	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.217	0.079	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	166	1785	0	425	0	284	590
N.S.	1	1.00	1.01	10.82	0.00	2.58	0.00	1.72	3.58
time (sec)	N/A	0.158	0.224	0.123	0.000	0.422	0.000	6.367	3.336

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	215	14679	0	1337	0	887	1715
N.S.	1	1.00	0.83	56.68	0.00	5.16	0.00	3.42	6.62
time (sec)	N/A	0.248	0.392	0.185	0.000	1.342	0.000	3.347	4.741

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	310	306209	0	3011	0	1963	2707
N.S.	1	1.00	0.87	860.14	0.00	8.46	0.00	5.51	7.60
time (sec)	N/A	0.394	0.767	0.284	0.000	8.926	0.000	6.065	11.302

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	469	1137077	0	5892	0	3759	2500
N.S.	1	1.00	0.90	2190.90	0.00	11.35	0.00	7.24	4.82
time (sec)	N/A	0.628	1.321	0.389	0.000	43.245	0.000	5.149	18.948

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.032	0.033	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	555	0	0	0	0	0	-1
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.243	0.069	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	626	610	0	0	0	0	-1
N.S.	1	1.00	0.82	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.995	0.705	0.129	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	782	908	764	0	0	0	0	-1
N.S.	1	1.00	1.16	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.037	0.405	0.217	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	111	0	123	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.85	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.049	0.014	0.291	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	478	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	0.606	0.015	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	290	279	0	0	0	0	-1
N.S.	1	1.00	0.93	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.136	0.086	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	323	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.355	0.107	0.004	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	826	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.559	0.004	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	0	0	134	0	134	-1
N.S.	1	1.00	0.68	0.00	0.00	0.78	0.00	0.78	-0.01
time (sec)	N/A	0.249	0.109	0.004	0.000	0.364	0.000	3.262	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	107	0	0	124	0	124	-1
N.S.	1	1.00	0.72	0.00	0.00	0.83	0.00	0.83	-0.01
time (sec)	N/A	0.185	0.061	0.003	0.000	0.435	0.000	3.570	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	0	0	114	0	114	-1
N.S.	1	1.00	0.80	0.00	0.00	0.90	0.00	0.90	-0.01
time (sec)	N/A	0.157	0.052	0.002	0.000	0.397	0.000	4.260	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	80	0	101	0	101	-1
N.S.	1	1.00	0.89	0.84	0.00	1.06	0.00	1.06	-0.01
time (sec)	N/A	0.103	0.022	0.026	0.000	0.396	0.000	3.890	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	0.332	0.001	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	115	0	92	-1
N.S.	1	1.00	0.89	0.00	0.00	1.51	0.00	1.21	-0.01
time (sec)	N/A	0.170	0.055	0.002	0.000	0.375	0.000	4.683	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	0	0	138	0	130	-1
N.S.	1	1.00	0.81	0.00	0.00	1.37	0.00	1.29	-0.01
time (sec)	N/A	0.189	0.069	0.001	0.000	0.389	0.000	5.808	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	232	0	0	110	0	128	-1
N.S.	1	1.00	1.24	0.00	0.00	0.59	0.00	0.68	-0.01
time (sec)	N/A	0.349	0.205	0.001	0.000	0.404	0.000	4.006	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	209	0	0	100	0	118	-1
N.S.	1	1.00	1.32	0.00	0.00	0.63	0.00	0.75	-0.01
time (sec)	N/A	0.278	0.160	0.003	0.000	0.386	0.000	3.558	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	186	0	0	84	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.250	0.120	0.003	0.000	0.402	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	84	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.105	0.003	0.000	0.383	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	204	0	0	108	0	181	-1
N.S.	1	1.00	1.35	0.00	0.00	0.72	0.00	1.20	-0.01
time (sec)	N/A	0.312	0.179	0.001	0.000	0.377	0.000	4.905	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	84	82	88	0	0	-1
N.S.	1	1.00	1.00	0.90	0.88	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.007	0.026	0.281	0.345	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	69	67	73	0	0	-1
N.S.	1	1.00	1.00	0.90	0.87	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.005	0.013	0.282	0.373	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	52	50	56	0	0	-1
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.005	0.013	0.289	0.366	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	28	34	40	0	0	35
N.S.	1	1.00	1.00	0.74	0.89	1.05	0.00	0.00	0.92
time (sec)	N/A	0.032	0.003	0.047	0.305	0.359	0.000	0.000	0.381

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.022	0.039	0.009	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	601	200	132	0	0	-1
N.S.	1	1.00	1.00	4.55	1.52	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.011	0.039	0.315	0.354	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	430	162	96	0	0	-1
N.S.	1	1.00	1.00	4.39	1.65	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.007	0.023	0.303	0.375	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	262	124	60	0	0	-1
N.S.	1	1.00	1.00	4.16	1.97	0.95	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.005	0.021	0.304	0.351	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	81	32	0	0	-1
N.S.	1	1.00	1.00	1.03	2.61	1.03	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.002	0.066	0.300	0.357	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	0.209	0.010	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	193	1276	215	251	0	0	-1
N.S.	1	1.00	1.00	6.61	1.11	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.008	0.038	0.302	0.379	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	916	174	210	0	0	-1
N.S.	1	1.00	1.00	5.87	1.12	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.006	0.026	0.316	0.366	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	558	133	173	0	0	-1
N.S.	1	1.00	1.00	4.73	1.13	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.006	0.021	0.304	0.366	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	69	87	109	0	0	-1
N.S.	1	1.00	1.00	0.92	1.16	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.004	0.062	0.294	0.374	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	0.211	0.006	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	96	87	108	0	0	-1
N.S.	1	1.00	1.00	2.46	2.23	2.77	0.00	0.00	-0.03
time (sec)	N/A	0.018	0.007	0.082	0.292	0.370	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	22	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	4.40	1.00
time (sec)	N/A	0.012	0.009	0.006	0.267	0.353	0.029	3.732	0.420

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	8	13
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	0.67	1.08
time (sec)	N/A	0.016	0.004	0.055	0.283	0.339	0.381	4.448	0.386

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	31	39	31	26
N.S.	1	1.00	1.00	0.90	0.80	3.10	3.90	3.10	2.60
time (sec)	N/A	0.011	0.004	0.017	0.291	0.361	0.143	3.489	0.390

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.012	0.003	0.008	0.272	0.329	0.270	5.529	0.071

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	15	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.013	0.012	0.013	0.474	0.358	0.118	4.178	0.525

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	16	16	0	0	12
N.S.	1	1.00	3.00	0.93	1.14	1.14	0.00	0.00	0.86
time (sec)	N/A	0.020	0.015	0.012	0.280	0.363	0.000	0.000	0.467

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	26	8	7	21	0	7	7
N.S.	1	1.00	2.36	0.73	0.64	1.91	0.00	0.64	0.64
time (sec)	N/A	0.022	0.014	0.009	0.492	0.385	0.000	4.549	0.399

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	14	6	5	16	0	16	5
N.S.	1	1.00	2.00	0.86	0.71	2.29	0.00	2.29	0.71
time (sec)	N/A	0.019	0.024	0.007	0.488	0.352	0.000	4.720	0.393

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	87	97	71	17	0	120
N.S.	1	1.00	0.95	0.78	0.87	0.64	0.15	0.00	1.08
time (sec)	N/A	0.070	0.059	0.016	0.488	0.363	0.175	0.000	4.694

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.019	0.004	0.010	0.265	0.350	0.116	3.475	0.521

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.00	1.00
time (sec)	N/A	0.010	0.008	0.039	0.296	0.351	0.228	5.007	0.403

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	16	14	12	13	156	12	12
N.S.	1	1.00	0.94	0.82	0.71	0.76	9.18	0.71	0.71
time (sec)	N/A	0.013	0.013	0.060	0.275	0.357	1.698	7.748	0.376

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	10	27	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.83	2.25	1.00
time (sec)	N/A	0.023	0.014	0.008	0.263	0.341	0.090	6.273	0.375

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	29	24	23	23	27	23	22
N.S.	1	1.00	1.38	1.14	1.10	1.10	1.29	1.10	1.05
time (sec)	N/A	0.024	0.012	0.011	0.276	0.370	0.119	6.116	0.370

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	31	30	36	0	37	26
N.S.	1	1.00	0.74	0.74	0.71	0.86	0.00	0.88	0.62
time (sec)	N/A	0.045	0.018	0.010	0.499	0.350	0.000	4.915	0.395

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	21	20	26	17	34	18
N.S.	1	1.00	0.83	0.88	0.83	1.08	0.71	1.42	0.75
time (sec)	N/A	0.026	0.018	0.013	0.292	0.350	0.044	4.612	0.412

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	17	12	20	17	13
N.S.	1	1.00	0.70	0.78	0.74	0.52	0.87	0.74	0.57
time (sec)	N/A	0.028	0.021	0.019	0.305	0.369	4.273	3.055	0.428

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	20	22	21	16	22	21	15
N.S.	1	1.00	0.69	0.76	0.72	0.55	0.76	0.72	0.52
time (sec)	N/A	0.030	0.021	0.017	0.269	0.338	3.968	5.002	0.439

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	29	29	32	0	18
N.S.	1	1.00	1.00	1.36	1.32	1.32	1.45	0.00	0.82
time (sec)	N/A	0.037	0.028	0.012	0.278	0.367	2.069	0.000	0.407

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	32	32	32	20	18
N.S.	1	1.00	0.81	0.89	1.19	1.19	1.19	0.74	0.67
time (sec)	N/A	0.033	0.021	0.011	0.279	0.345	0.040	13.993	0.424

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	38	0	48	27
N.S.	1	1.00	1.00	0.00	0.00	1.41	0.00	1.78	1.00
time (sec)	N/A	0.017	0.009	0.028	0.000	0.384	0.000	13.246	0.401

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	71	0	27	0	24	27
N.S.	1	1.00	1.00	2.63	0.00	1.00	0.00	0.89	1.00
time (sec)	N/A	0.023	0.006	0.194	0.000	0.395	0.000	4.251	0.384

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	13	0	27	21
N.S.	1	1.00	1.00	0.84	0.80	0.52	0.00	1.08	0.84
time (sec)	N/A	0.013	0.007	0.187	0.285	0.372	0.000	6.777	0.404

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	33	0	35	29
N.S.	1	1.00	1.00	0.00	0.00	1.14	0.00	1.21	1.00
time (sec)	N/A	0.025	0.006	0.030	0.000	0.370	0.000	4.721	0.357

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	29	5	5	3	5	5
N.S.	1	1.00	0.68	0.94	0.16	0.16	0.10	0.16	0.16
time (sec)	N/A	0.011	0.008	0.009	0.280	0.346	0.017	4.710	0.363

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	80	57	37	0	102	-1
N.S.	1	1.00	0.86	2.29	1.63	1.06	0.00	2.91	-0.03
time (sec)	N/A	0.052	0.037	0.141	0.309	0.392	0.000	5.302	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	79	59	0	123	-1
N.S.	1	1.00	0.76	2.00	1.20	0.89	0.00	1.86	-0.02
time (sec)	N/A	0.080	0.061	0.128	0.359	0.433	0.000	4.579	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	162	110	76	0	454	-1
N.S.	1	1.00	0.74	1.82	1.24	0.85	0.00	5.10	-0.01
time (sec)	N/A	0.359	0.082	0.092	0.336	0.398	0.000	3.841	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	79	55	38	0	108	-1
N.S.	1	1.00	0.86	2.26	1.57	1.09	0.00	3.09	-0.03
time (sec)	N/A	0.042	0.036	0.037	0.311	0.367	0.000	3.247	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	132	76	60	0	122	-1
N.S.	1	1.00	0.76	2.00	1.15	0.91	0.00	1.85	-0.02
time (sec)	N/A	0.090	0.071	0.072	0.350	0.384	0.000	5.192	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	162	109	76	0	495	-1
N.S.	1	1.00	0.75	1.84	1.24	0.86	0.00	5.62	-0.01
time (sec)	N/A	0.323	0.115	0.072	0.346	0.405	0.000	7.269	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.021	0.020	0.113	0.321	0.368	12.045	6.440	0.496

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	87	87	104	0	0	-1
N.S.	1	1.00	0.89	1.85	1.85	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.013	0.168	0.613	0.416	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	86	89	109	0	0	-1
N.S.	1	1.00	0.96	1.91	1.98	2.42	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.016	0.132	0.578	0.424	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	91	115	0	0	-1
N.S.	1	1.00	1.00	0.00	1.75	2.21	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.020	0.027	0.650	0.390	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	115	60	104	0	0	37
N.S.	1	1.00	1.00	2.45	1.28	2.21	0.00	0.00	0.79
time (sec)	N/A	0.035	0.007	0.115	0.602	0.381	0.000	0.000	0.390

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	115	62	106	0	0	39
N.S.	1	1.00	0.96	2.56	1.38	2.36	0.00	0.00	0.87
time (sec)	N/A	0.037	0.019	0.122	0.660	0.430	0.000	0.000	0.081

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	65	115	0	0	41
N.S.	1	1.00	1.00	0.00	1.25	2.21	0.00	0.00	0.79
time (sec)	N/A	0.037	0.020	0.027	0.691	0.408	0.000	0.000	0.369

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	85	42	184	0	0	39
N.S.	1	1.00	1.47	1.67	0.82	3.61	0.00	0.00	0.76
time (sec)	N/A	0.030	0.011	0.151	0.524	0.408	0.000	0.000	0.094

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	184	0	0	41
N.S.	1	1.00	1.53	1.67	0.90	3.76	0.00	0.00	0.84
time (sec)	N/A	0.031	0.010	0.158	0.504	0.383	0.000	0.000	0.057

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	81	2356	48	195	0	0	44
N.S.	1	1.00	1.45	42.07	0.86	3.48	0.00	0.00	0.79
time (sec)	N/A	0.032	0.010	1.842	0.501	0.408	0.000	0.000	0.067

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	86	43	147	0	0	-1
N.S.	1	1.00	1.47	1.69	0.84	2.88	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.010	0.155	0.504	0.453	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	82	44	148	0	0	-1
N.S.	1	1.00	1.53	1.67	0.90	3.02	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.010	0.148	0.497	0.398	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	81	2357	49	158	0	0	44
N.S.	1	1.00	1.45	42.09	0.88	2.82	0.00	0.00	0.79
time (sec)	N/A	0.032	0.012	0.615	0.516	0.425	0.000	0.000	0.067

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	108	60	106	0	0	39
N.S.	1	1.00	1.00	2.35	1.30	2.30	0.00	0.00	0.85
time (sec)	N/A	0.033	0.008	0.116	0.632	0.404	0.000	0.000	0.089

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	117	61	102	0	0	39
N.S.	1	1.00	0.96	2.60	1.36	2.27	0.00	0.00	0.87
time (sec)	N/A	0.035	0.017	0.131	0.583	0.461	0.000	0.000	0.381

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	65	117	0	0	43
N.S.	1	1.00	1.00	0.00	1.27	2.29	0.00	0.00	0.84
time (sec)	N/A	0.037	0.021	0.049	0.693	0.382	0.000	0.000	0.383

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	83	87	106	0	0	-1
N.S.	1	1.00	0.89	1.80	1.89	2.30	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.011	0.124	0.637	0.496	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	88	87	107	0	0	-1
N.S.	1	1.00	0.93	1.96	1.93	2.38	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.015	0.118	0.578	0.444	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	91	117	0	0	-1
N.S.	1	1.00	1.00	0.00	1.78	2.29	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.020	0.039	0.638	0.448	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	13	112	17	17	0	13	-1
N.S.	1	1.00	0.62	5.33	0.81	0.81	0.00	0.62	-0.05
time (sec)	N/A	0.013	0.005	0.206	0.279	0.387	0.000	2.632	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	8	7	7	0	24	6
N.S.	1	1.00	1.00	1.33	1.17	1.17	0.00	4.00	1.00
time (sec)	N/A	0.011	0.013	0.088	0.277	0.417	0.000	5.467	0.501

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	35	6	6	0	6	6
N.S.	1	1.00	0.68	0.95	0.16	0.16	0.00	0.16	0.16
time (sec)	N/A	0.018	0.029	0.103	0.272	0.359	0.000	5.209	0.367

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	67	94	22	15	12	35
N.S.	1	1.00	1.00	5.58	7.83	1.83	1.25	1.00	2.92
time (sec)	N/A	0.015	0.014	0.114	0.506	0.390	32.751	5.066	0.579

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	0	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.00	0.78	0.78
time (sec)	N/A	0.009	0.005	0.116	0.285	0.362	0.000	3.470	0.381

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	16	24	17	16	11
N.S.	1	1.00	0.75	0.85	0.80	1.20	0.85	0.80	0.55
time (sec)	N/A	0.025	0.015	0.122	0.274	0.428	1.654	2.656	0.430

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	30	42	65	0	0	29
N.S.	1	1.00	0.66	0.60	0.84	1.30	0.00	0.00	0.58
time (sec)	N/A	0.018	0.010	0.522	0.305	0.392	0.000	0.000	0.521

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	0	7	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.00	1.17	1.00
time (sec)	N/A	0.014	0.008	0.051	0.292	0.386	0.000	6.157	0.399

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.89	0.78	0.78
time (sec)	N/A	0.011	0.004	0.092	0.272	0.382	3.407	5.717	0.433

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	10	10	9
N.S.	1	1.00	1.00	1.10	1.00	1.00	1.00	1.00	0.90
time (sec)	N/A	0.006	0.008	0.070	0.271	0.378	0.478	5.184	0.379

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	43	93	108	27	223	27	-1
N.S.	1	1.00	3.07	6.64	7.71	1.93	15.93	1.93	-0.07
time (sec)	N/A	0.012	0.011	0.094	0.282	0.374	1.617	4.949	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	10	11	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.91	1.00	0.73
time (sec)	N/A	0.006	0.004	0.084	0.264	0.377	0.195	3.561	0.415

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	59	195	104	120	0	0	-1
N.S.	1	1.00	0.80	2.64	1.41	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.045	0.121	0.669	0.412	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	47	214	179	43	456	41	-1
N.S.	1	1.00	1.18	5.35	4.48	1.08	11.40	1.02	-0.02
time (sec)	N/A	0.045	0.031	0.171	0.294	0.396	7.583	3.970	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	88	0	139	0	0	0	-1
N.S.	1	1.00	1.11	0.00	1.76	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.026	0.016	0.288	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	81	81	19	17	19	57
N.S.	1	1.00	1.00	5.40	5.40	1.27	1.13	1.27	3.80
time (sec)	N/A	0.014	0.013	0.144	0.506	0.415	25.445	4.353	0.576

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	36	134	0	52	-1
N.S.	1	1.00	0.86	1.66	1.03	3.83	0.00	1.49	-0.03
time (sec)	N/A	0.053	0.036	0.072	0.345	0.368	0.000	3.660	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	67	313	0	67	-1
N.S.	1	1.00	0.76	1.47	1.02	4.74	0.00	1.02	-0.02
time (sec)	N/A	0.100	0.079	0.068	0.333	0.375	0.000	3.522	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	67	116	110	587	0	102	-1
N.S.	1	1.00	0.75	1.30	1.24	6.60	0.00	1.15	-0.01
time (sec)	N/A	0.348	0.078	0.068	0.354	0.375	0.000	3.466	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	58	37	134	0	54	-1
N.S.	1	1.00	0.86	1.66	1.06	3.83	0.00	1.54	-0.03
time (sec)	N/A	0.046	0.031	0.033	0.331	0.385	0.000	2.463	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	97	67	305	0	67	-1
N.S.	1	1.00	0.76	1.47	1.02	4.62	0.00	1.02	-0.02
time (sec)	N/A	0.091	0.051	0.056	0.328	0.369	0.000	6.458	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	116	111	587	0	104	-1
N.S.	1	1.00	0.75	1.32	1.26	6.67	0.00	1.18	-0.01
time (sec)	N/A	0.324	0.112	0.081	0.367	0.380	0.000	6.430	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	295	43	57	0	0	-1
N.S.	1	1.00	0.92	7.56	1.10	1.46	0.00	0.00	-0.03
time (sec)	N/A	0.038	0.018	0.102	0.316	0.410	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	454	43	69	0	0	-1
N.S.	1	1.00	0.94	12.97	1.23	1.97	0.00	0.00	-0.03
time (sec)	N/A	0.038	0.017	0.223	0.323	0.419	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	47	65	0	0	-1
N.S.	1	1.00	0.98	0.00	1.07	1.48	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.023	0.020	0.335	0.381	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	321	32	65	0	0	-1
N.S.	1	1.00	0.92	8.23	0.82	1.67	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.016	0.098	0.537	0.391	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	478	32	77	0	0	-1
N.S.	1	1.00	0.94	13.66	0.91	2.20	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.015	0.217	0.528	0.382	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	36	73	0	0	-1
N.S.	1	1.00	0.98	0.00	0.82	1.66	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.024	0.023	0.525	0.401	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	54	101	0	0	20
N.S.	1	1.00	0.90	0.62	1.38	2.59	0.00	0.00	0.51
time (sec)	N/A	0.030	0.008	0.068	0.506	0.384	0.000	0.000	0.470

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	76	56	102	0	0	-1
N.S.	1	1.00	1.20	1.85	1.37	2.49	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.009	0.115	0.509	0.406	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	57	129	0	0	-1
N.S.	1	1.00	1.27	1.27	1.54	3.49	0.00	0.00	-0.03
time (sec)	N/A	0.031	0.009	0.143	0.491	0.407	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	-1
N.S.	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.010	0.678	0.508	0.411	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	24	49	101	0	0	22
N.S.	1	1.00	0.90	0.62	1.26	2.59	0.00	0.00	0.56
time (sec)	N/A	0.031	0.008	0.072	0.506	0.384	0.000	0.000	0.414

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	76	51	102	0	0	-1
N.S.	1	1.00	1.20	1.85	1.24	2.49	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.009	0.109	0.509	0.406	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	47	47	59	127	0	0	-1
N.S.	1	1.00	1.27	1.27	1.59	3.43	0.00	0.00	-0.03
time (sec)	N/A	0.031	0.009	0.134	0.515	0.430	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	43	61	116	0	0	-1
N.S.	1	1.00	1.20	0.93	1.33	2.52	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.011	1.267	0.511	0.416	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	314	31	84	0	0	-1
N.S.	1	1.00	0.97	8.26	0.82	2.21	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.013	0.100	0.538	0.400	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	480	32	106	0	0	-1
N.S.	1	1.00	0.94	13.71	0.91	3.03	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.016	0.141	0.535	0.433	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	36	92	0	0	-1
N.S.	1	1.00	1.00	0.00	0.84	2.14	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.019	0.034	0.531	0.410	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	293	37	76	0	0	-1
N.S.	1	1.00	0.97	7.71	0.97	2.00	0.00	0.00	-0.03
time (sec)	N/A	0.036	0.013	0.089	0.338	0.422	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	456	45	97	0	0	-1
N.S.	1	1.00	0.94	13.03	1.29	2.77	0.00	0.00	-0.03
time (sec)	N/A	0.037	0.015	0.134	0.336	0.412	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	47	84	0	0	-1
N.S.	1	1.00	1.00	0.00	1.09	1.95	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.018	0.033	0.339	0.412	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	33	30	112	258	0	94	31
N.S.	1	1.00	0.66	0.60	2.24	5.16	0.00	1.88	0.62
time (sec)	N/A	0.018	0.009	0.221	0.294	0.392	0.000	5.129	0.509

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	12	62	14	37	9
N.S.	1	1.00	1.00	1.08	0.92	4.77	1.08	2.85	0.69
time (sec)	N/A	0.012	0.009	0.122	0.274	0.375	0.484	4.673	0.379

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	11	14	13	9	94	13	9
N.S.	1	1.00	0.65	0.82	0.76	0.53	5.53	0.76	0.53
time (sec)	N/A	0.004	0.003	0.053	0.275	0.360	1.332	2.796	0.034

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	25	24	23	27	24	25
N.S.	1	1.00	0.96	0.93	0.89	0.85	1.00	0.89	0.93
time (sec)	N/A	0.014	0.005	0.011	0.283	0.445	0.043	3.808	0.121

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	16	4	3	20	0	3	3
N.S.	1	1.00	5.33	1.33	1.00	6.67	0.00	1.00	1.00
time (sec)	N/A	0.021	0.014	0.010	0.508	0.380	0.000	4.606	0.388

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	23	17	22	22	22	17
N.S.	1	1.00	1.25	0.96	0.71	0.92	0.92	0.92	0.71
time (sec)	N/A	0.014	0.003	0.023	0.277	0.410	0.035	4.323	0.039

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	26	23	22	29	23	25
N.S.	1	1.00	0.92	1.04	0.92	0.88	1.16	0.92	1.00
time (sec)	N/A	0.006	0.004	0.009	0.279	0.397	0.169	3.443	0.064

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	24	20	22	30	20
N.S.	1	1.00	0.88	0.91	0.71	0.59	0.65	0.88	0.59
time (sec)	N/A	0.009	0.007	0.015	0.267	0.367	0.086	5.524	0.051

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	43	28	24	27	42	22
N.S.	1	1.00	1.00	1.08	0.70	0.60	0.68	1.05	0.55
time (sec)	N/A	0.011	0.008	0.033	0.269	0.392	0.097	7.324	0.377

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	26	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.84	0.81	0.81
time (sec)	N/A	0.020	0.008	0.021	0.482	0.397	0.114	6.005	0.405

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.003	0.013	0.005	0.000	0.371	7.856	3.887	0.079

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.003	0.014	0.006	0.000	0.383	7.131	4.638	0.555

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	22	20	22	22
N.S.	1	1.00	1.00	0.88	0.00	0.85	0.77	0.85	0.85
time (sec)	N/A	0.004	0.015	0.004	0.000	0.382	8.519	5.666	0.486

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	0	28	0	40	37
N.S.	1	1.00	1.00	1.21	0.00	0.65	0.00	0.93	0.86
time (sec)	N/A	0.009	0.018	0.007	0.000	0.377	0.000	5.456	1.079

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	13	14	105	13	9
N.S.	1	1.00	0.71	0.67	0.62	0.67	5.00	0.62	0.43
time (sec)	N/A	0.004	0.003	0.050	0.274	0.383	1.959	3.639	0.340

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	13	24	14	10	14	13
N.S.	1	1.00	0.92	1.00	1.85	1.08	0.77	1.08	1.00
time (sec)	N/A	0.004	0.003	0.016	0.269	0.396	0.083	3.832	0.362

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	14	6	3	6	6
N.S.	1	1.00	1.00	1.17	2.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.007	0.003	0.006	0.272	0.360	0.029	3.152	0.355

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	38	34	33	38	184	33	26
N.S.	1	1.00	1.19	1.06	1.03	1.19	5.75	1.03	0.81
time (sec)	N/A	0.141	0.013	0.012	0.269	0.377	1.090	4.723	0.112

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	15	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.94	1.00	1.00
time (sec)	N/A	0.005	0.003	0.013	0.497	0.425	0.115	4.515	0.050

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	29	23	34	23	19
N.S.	1	1.00	1.00	1.10	1.45	1.15	1.70	1.15	0.95
time (sec)	N/A	0.003	0.008	0.024	0.274	0.388	0.118	3.123	0.387

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	44	33	33	32	139	29
N.S.	1	1.00	0.89	1.26	0.94	0.94	0.91	3.97	0.83
time (sec)	N/A	0.004	0.005	0.047	0.282	0.359	0.064	3.486	0.095

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	12	12	15	12	12
N.S.	1	1.00	1.00	1.17	1.00	1.00	1.25	1.00	1.00
time (sec)	N/A	0.002	0.003	0.014	0.269	0.495	0.035	3.082	0.051

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	39	28	28	27	29	40
N.S.	1	1.00	0.75	1.08	0.78	0.78	0.75	0.81	1.11
time (sec)	N/A	0.010	0.009	0.034	0.297	0.389	0.045	3.413	0.486

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	53	42	42	48	43	56
N.S.	1	1.00	1.00	0.98	0.78	0.78	0.89	0.80	1.04
time (sec)	N/A	0.027	0.011	0.022	0.278	0.429	0.051	3.670	0.585

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	33	30	52	42	41	31	46
N.S.	1	1.00	0.94	0.86	1.49	1.20	1.17	0.89	1.31
time (sec)	N/A	0.010	0.021	0.018	0.284	0.371	0.082	3.531	0.473

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	30	74	64	63	31	73
N.S.	1	1.00	0.69	0.86	2.11	1.83	1.80	0.89	2.09
time (sec)	N/A	0.016	0.010	0.062	0.272	0.359	0.122	3.763	0.461

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87
time (sec)	N/A	0.013	0.003	0.059	0.267	0.407	0.085	2.493	0.546

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	21	30	31	21	26	32	25
N.S.	1	1.00	0.68	0.97	1.00	0.68	0.84	1.03	0.81
time (sec)	N/A	0.014	0.009	0.049	0.288	0.365	0.197	3.453	0.492

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	61	44	47	185	0	52
N.S.	1	1.00	0.68	1.39	1.00	1.07	4.20	0.00	1.18
time (sec)	N/A	0.019	0.014	0.066	0.279	0.393	0.772	0.000	0.460

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	19	31	45	18
N.S.	1	1.00	1.00	1.06	1.33	1.06	1.72	2.50	1.00
time (sec)	N/A	0.007	0.016	0.053	0.288	0.395	0.395	3.761	0.379

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	121	99	26	71
N.S.	1	1.00	1.00	0.75	0.00	3.78	3.09	0.81	2.22
time (sec)	N/A	0.012	0.021	0.144	0.000	0.403	3.634	3.720	0.469

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	115	0	480	175	239	153
N.S.	1	1.00	0.78	0.80	0.00	3.33	1.22	1.66	1.06
time (sec)	N/A	0.058	0.041	0.183	0.000	0.417	40.691	4.088	2.283

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	167	136	0	195	133	170	95
N.S.	1	1.00	0.74	0.60	0.00	0.86	0.59	0.75	0.42
time (sec)	N/A	0.104	0.053	0.424	0.000	0.445	23.381	5.164	2.239

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	33	33	28	116	53	27
N.S.	1	1.00	1.26	1.22	1.22	1.04	4.30	1.96	1.00
time (sec)	N/A	0.016	0.014	0.046	0.275	0.402	2.055	4.032	0.365

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	47	41	0	143	204	38	45
N.S.	1	1.00	1.18	1.02	0.00	3.58	5.10	0.95	1.12
time (sec)	N/A	0.018	0.022	0.083	0.000	0.420	6.142	3.805	0.375

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	132	0	149	245	257	174
N.S.	1	1.00	0.89	0.89	0.00	1.00	1.64	1.72	1.17
time (sec)	N/A	0.070	0.042	0.176	0.000	0.389	44.493	4.888	2.401

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	211	148	0	192	228	178	176
N.S.	1	1.00	0.91	0.64	0.00	0.82	0.98	0.76	0.76
time (sec)	N/A	0.125	0.052	0.420	0.000	0.433	25.520	3.984	2.212

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	20	0	21	22	0	19
N.S.	1	1.00	1.00	0.91	0.00	0.95	1.00	0.00	0.86
time (sec)	N/A	0.013	0.026	0.032	0.000	0.428	0.067	0.000	0.431

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	37	39	22	0	37
N.S.	1	1.00	1.02	0.93	0.90	0.95	0.54	0.00	0.90
time (sec)	N/A	0.037	0.044	0.016	0.561	0.364	0.183	0.000	0.574

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	38	0	39	22	0	37
N.S.	1	1.00	1.02	0.93	0.00	0.95	0.54	0.00	0.90
time (sec)	N/A	0.023	0.043	0.029	0.000	0.373	0.196	0.000	0.354

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	40	0	41	19	0	37
N.S.	1	1.00	1.05	0.95	0.00	0.98	0.45	0.00	0.88
time (sec)	N/A	0.030	0.043	0.022	0.000	0.394	0.171	0.000	0.598

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	17	21	27	22	21
N.S.	1	1.00	1.00	0.84	0.53	0.66	0.84	0.69	0.66
time (sec)	N/A	0.014	0.003	0.011	0.279	0.420	0.121	3.484	0.410

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	0	0	0	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.009	0.006	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	60	0	0	0	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.010	0.004	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	0	0	0	0	32	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.74	-0.02
time (sec)	N/A	0.021	0.009	0.003	0.000	0.000	0.000	3.150	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	102	0	0	0	0	-1
N.S.	1	1.00	0.99	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.060	0.009	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	109	111	0	0	0	-1
N.S.	1	1.00	0.90	1.35	1.37	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.025	0.119	0.288	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	5	12	8	10	0	4
N.S.	1	1.00	1.00	0.56	1.33	0.89	1.11	0.00	0.44
time (sec)	N/A	0.006	0.003	0.068	0.298	0.403	0.918	0.000	0.018

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	47	82	0	0	0	59
N.S.	1	1.00	0.81	1.09	1.91	0.00	0.00	0.00	1.37
time (sec)	N/A	0.047	0.015	0.059	0.312	0.000	0.000	0.000	0.315

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	41	49	0	228	0	-1
N.S.	1	1.00	1.03	1.37	1.63	0.00	7.60	0.00	-0.03
time (sec)	N/A	0.066	0.010	0.071	0.295	0.000	90.980	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	0	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.020	0.088	0.073	0.373	0.411	0.000	5.441	0.417

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	57	57	58	46	59	59
N.S.	1	1.00	0.93	0.84	0.84	0.85	0.68	0.87	0.87
time (sec)	N/A	0.168	0.026	0.043	0.926	0.395	0.092	3.482	0.456

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	23	31	24	23	23
N.S.	1	1.00	1.00	0.83	0.79	1.07	0.83	0.79	0.79
time (sec)	N/A	0.012	0.011	0.007	0.296	0.408	6.355	3.723	0.076

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	28	18	18	14	80	18
N.S.	1	1.00	1.00	1.56	1.00	1.00	0.78	4.44	1.00
time (sec)	N/A	0.002	0.003	0.038	0.308	0.366	0.062	3.726	0.362

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	21	35	25	21	15	103	21
N.S.	1	1.00	0.78	1.30	0.93	0.78	0.56	3.81	0.78
time (sec)	N/A	0.003	0.004	0.026	0.303	0.382	0.102	5.914	0.083

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	112	54	54	41	56	55
N.S.	1	1.00	1.00	1.96	0.95	0.95	0.72	0.98	0.96
time (sec)	N/A	0.039	0.036	0.084	0.501	0.370	0.206	6.302	0.417

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	249	0	0	0	0	-1
N.S.	1	1.00	1.03	4.15	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.006	0.138	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	239	146	0	0	0	0	-1
N.S.	1	1.00	3.92	2.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.037	0.059	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	373	0	0	0	0	0	-1
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.142	180.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	37	0	0	-1
N.S.	1	1.00	4.62	0.00	0.00	1.28	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.483	0.027	0.000	0.376	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	134	0	0	36	0	0	-1
N.S.	1	1.00	4.62	0.00	0.00	1.24	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.386	0.025	0.000	0.377	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	10	10	8	10	17
N.S.	1	1.00	1.00	0.88	0.59	0.59	0.47	0.59	1.00
time (sec)	N/A	0.002	0.005	0.026	0.309	0.414	0.019	3.147	0.065

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	27	16	20	65	16	49
N.S.	1	1.00	0.93	1.00	0.59	0.74	2.41	0.59	1.81
time (sec)	N/A	0.005	0.019	0.016	0.294	0.407	0.475	2.961	0.598

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	31	25	28	26	25	0	26	27
N.S.	1	1.24	1.00	1.12	1.04	1.00	0.00	1.04	1.08
time (sec)	N/A	0.034	0.014	0.025	0.304	0.371	0.000	5.376	0.661

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	26	24	23	26	24	20
N.S.	1	1.00	0.85	1.00	0.92	0.88	1.00	0.92	0.77
time (sec)	N/A	0.024	0.013	0.024	0.335	0.379	3.463	4.308	0.366

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.024	14.938	0.041	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.015	9.027	0.040	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.005	0.006	0.039	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.021	0.033	0.036	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.023	17.064	0.042	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	9	11	10	11	11
N.S.	1	1.00	1.00	0.92	0.69	0.85	0.77	0.85	0.85
time (sec)	N/A	0.016	0.009	0.023	0.320	0.362	0.088	3.652	0.464

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	10	11	10	10	8	14	10
N.S.	1	1.00	1.11	1.22	1.11	1.11	0.89	1.56	1.11
time (sec)	N/A	0.049	0.017	0.079	0.296	0.373	0.097	4.590	0.374

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	20	0	13	15	0	13
N.S.	1	1.00	1.00	1.54	0.00	1.00	1.15	0.00	1.00
time (sec)	N/A	0.094	0.020	0.089	0.000	0.396	2.487	0.000	0.395

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	107	67	48	53	88	63
N.S.	1	1.00	0.79	1.60	1.00	0.72	0.79	1.31	0.94
time (sec)	N/A	0.049	0.026	0.032	0.312	0.390	68.528	5.149	0.644

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	68	49	53	53	38
N.S.	1	1.00	1.06	0.00	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.038	0.030	180.000	0.328	0.429	66.921	5.103	0.448

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	22	18	18	17	47	18
N.S.	1	1.00	0.90	1.05	0.86	0.86	0.81	2.24	0.86
time (sec)	N/A	0.005	0.004	0.036	0.297	0.413	0.030	4.808	0.057

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	68	49	53	53	38
N.S.	1	1.00	1.06	0.00	1.36	0.98	1.06	1.06	0.76
time (sec)	N/A	0.032	0.029	180.000	0.303	0.380	68.145	3.959	0.431

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	107	67	48	53	88	63
N.S.	1	1.00	0.93	1.55	0.97	0.70	0.77	1.28	0.91
time (sec)	N/A	0.035	0.022	0.023	0.275	0.424	68.065	2.739	0.324

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	10	0	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	1.11	0.00	1.00
time (sec)	N/A	0.015	0.008	0.015	0.326	0.400	0.053	0.000	0.366

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	16	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	-0.04
time (sec)	N/A	0.018	0.021	0.016	0.000	0.101	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	0	108	0	0	89	-1
N.S.	1	1.00	1.20	0.00	1.80	0.00	0.00	1.48	-0.02
time (sec)	N/A	0.050	0.184	0.003	0.284	0.000	0.000	8.098	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	94	0	0	74	-1
N.S.	1	1.00	1.11	0.00	1.47	0.00	0.00	1.16	-0.02
time (sec)	N/A	0.048	0.169	0.003	0.297	0.000	0.000	4.795	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	156	0	0	129	-1
N.S.	1	1.00	1.16	0.00	2.26	0.00	0.00	1.87	-0.01
time (sec)	N/A	0.048	0.193	0.007	0.311	0.000	0.000	4.495	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	79	0	130	0	0	106	-1
N.S.	1	1.00	1.11	0.00	1.83	0.00	0.00	1.49	-0.01
time (sec)	N/A	0.051	0.186	0.003	0.353	0.000	0.000	4.569	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	462	94	234	0	0	-1
N.S.	1	1.00	0.97	4.71	0.96	2.39	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.054	0.198	0.578	0.456	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	434	70	174	0	0	-1
N.S.	1	1.00	0.99	5.42	0.88	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.038	0.144	0.590	0.435	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	47	368	43	109	0	0	-1
N.S.	1	1.00	0.90	7.08	0.83	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.025	0.096	0.614	0.437	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.013	1.628	0.038	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.245	1.399	0.032	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	691	94	241	0	0	-1
N.S.	1	1.00	0.97	6.71	0.91	2.34	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.048	0.249	0.593	0.443	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	663	70	181	0	0	-1
N.S.	1	1.00	0.96	7.80	0.82	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.058	0.137	0.602	0.415	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	583	43	116	0	0	-1
N.S.	1	1.00	0.98	10.23	0.75	2.04	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.024	0.116	0.592	0.427	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.015	0.589	0.040	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.044	1.613	0.049	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [38] had the largest ratio of [60]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	0	0	0.00	0	0.000
2	A	10	5	1.00	32	0.156
3	A	8	5	1.00	32	0.156
4	A	6	5	1.00	30	0.167
5	A	2	2	1.00	14	0.143
6	A	0	0	0.00	0	0.000
7	A	0	0	0.00	0	0.000
8	A	0	0	0.00	0	0.000
9	A	13	5	1.00	28	0.179
10	A	8	5	1.00	28	0.179
11	A	5	4	1.00	26	0.154
12	A	1	1	1.00	10	0.100
13	A	0	0	0.00	0	0.000
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	A	1	1	1.00	43	0.023
17	A	1	1	1.00	43	0.023
18	A	1	1	1.00	41	0.024
19	A	4	3	1.00	25	0.120
20	A	1	1	1.00	43	0.023
21	A	1	1	1.00	43	0.023
22	A	1	1	1.00	43	0.023
23	A	3	2	1.00	39	0.051
24	A	3	2	1.00	37	0.054
25	A	2	1	1.00	15	0.067

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	34	0.059
27	A	3	2	1.00	37	0.054
28	A	3	2	1.00	39	0.051
29	A	2	2	1.00	45	0.044
30	A	0	0	0.00	0	0.000
31	A	9	4	1.00	40	0.100
32	A	7	4	1.00	40	0.100
33	A	5	4	1.00	38	0.105
34	A	4	3	1.00	22	0.136
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	1	1	1.00	60	0.017
39	A	0	0	0.00	0	0.000
40	A	1	1	1.00	39	0.026
41	A	1	1	1.00	40	0.025
42	A	1	1	1.00	41	0.024
43	A	1	1	1.00	42	0.024
44	A	1	1	1.00	40	0.025
45	A	1	1	1.00	41	0.024
46	A	3	3	1.00	19	0.158
47	A	3	3	1.00	21	0.143
48	A	3	3	1.00	19	0.158
49	A	3	3	1.00	17	0.176
50	A	4	3	1.00	15	0.200
51	A	1	1	1.00	19	0.053
52	A	3	3	1.00	19	0.158
53	A	3	3	1.00	19	0.158
54	A	3	3	1.00	19	0.158
55	A	2	2	1.00	9	0.222
56	A	1	1	1.00	13	0.077
57	A	3	3	1.00	11	0.273
58	A	1	1	1.00	15	0.067
59	A	3	3	1.00	18	0.167
60	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	18	0.111
62	A	3	2	1.00	18	0.111
63	A	3	2	1.00	16	0.125
64	A	3	2	1.00	14	0.143
65	A	7	6	1.00	18	0.333
66	A	3	2	1.00	18	0.111
67	A	3	2	1.00	18	0.111
68	A	3	2	1.00	18	0.111
69	A	3	2	1.00	18	0.111
70	A	5	3	1.00	19	0.158
71	A	7	6	1.00	19	0.316
72	A	7	6	1.00	19	0.316
73	A	7	6	1.00	19	0.316
74	A	7	6	1.00	17	0.353
75	A	6	6	1.00	15	0.400
76	A	7	4	1.00	19	0.210
77	A	7	6	1.00	19	0.316
78	A	7	6	1.00	19	0.316
79	A	7	6	1.00	19	0.316
80	A	7	6	1.00	19	0.316
81	A	6	6	1.00	7	0.857
82	A	7	6	1.00	23	0.261
83	A	7	6	1.00	23	0.261
84	A	7	6	1.00	23	0.261
85	A	7	6	1.00	21	0.286
86	A	6	6	1.00	15	0.400
87	A	9	5	1.00	23	0.217
88	A	7	6	1.00	23	0.261
89	A	7	6	1.00	23	0.261
90	A	7	6	1.00	23	0.261
91	A	7	6	1.00	23	0.261
92	A	6	7	1.00	25	0.280
93	A	8	9	1.00	32	0.281
94	A	20	6	1.00	25	0.240
95	A	20	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	14	10	1.00	16	0.625
97	A	27	14	1.00	17	0.824
98	A	28	12	1.00	21	0.571
99	A	27	14	1.00	9	1.556
100	A	34	16	1.00	13	1.231
101	A	25	11	1.00	21	0.524
102	A	20	10	1.00	21	0.476
103	A	16	10	1.00	19	0.526
104	A	13	9	1.00	17	0.529
105	A	0	0	0.00	0	0.000
106	A	19	11	1.00	21	0.524
107	A	20	12	1.00	21	0.571
108	A	15	12	1.00	23	0.522
109	A	13	12	1.00	23	0.522
110	A	12	10	1.00	23	0.435
111	A	15	13	1.00	23	0.565
112	A	18	13	1.00	23	0.565
113	A	6	5	1.00	12	0.417
114	A	5	5	1.00	12	0.417
115	A	4	4	1.00	10	0.400
116	A	4	4	1.00	8	0.500
117	A	0	0	0.00	0	0.000
118	A	5	4	1.00	20	0.200
119	A	4	4	1.00	20	0.200
120	A	3	3	1.00	18	0.167
121	A	2	2	1.00	16	0.125
122	A	0	0	0.00	0	0.000
123	A	6	5	1.00	20	0.250
124	A	5	5	1.00	20	0.250
125	A	4	4	1.00	18	0.222
126	A	4	4	1.00	16	0.250
127	A	0	0	0.00	0	0.000
128	A	3	3	1.00	16	0.188
129	A	2	2	1.00	10	0.200
130	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	10	0.200
132	A	2	2	1.00	10	0.200
133	A	2	1	1.00	12	0.083
134	A	3	2	1.00	14	0.143
135	A	2	1	1.00	16	0.062
136	A	2	1	1.00	14	0.071
137	A	7	6	1.00	16	0.375
138	A	2	1	1.00	15	0.067
139	A	2	2	1.00	8	0.250
140	A	3	2	1.00	9	0.222
141	A	3	2	1.00	16	0.125
142	A	3	2	1.00	16	0.125
143	A	4	3	1.00	18	0.167
144	A	3	2	1.00	16	0.125
145	A	3	2	1.00	14	0.143
146	A	3	2	1.00	16	0.125
147	A	4	4	1.00	16	0.250
148	A	3	1	1.00	20	0.050
149	A	3	2	1.00	14	0.143
150	A	3	2	1.00	14	0.143
151	A	3	2	1.00	16	0.125
152	A	3	2	1.00	16	0.125
153	A	4	3	1.00	10	0.300
154	A	5	6	1.00	9	0.667
155	A	5	6	1.00	11	0.546
156	A	15	8	1.00	11	0.727
157	A	5	6	1.00	9	0.667
158	A	7	8	1.00	11	0.727
159	A	15	8	1.00	11	0.727
160	A	4	3	1.00	12	0.250
161	A	5	5	1.00	5	1.000
162	A	6	6	1.00	7	0.857
163	A	6	6	1.00	7	0.857
164	A	5	5	1.00	5	1.000
165	A	6	6	1.00	7	0.857

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	6	1.00	7	0.857
167	A	7	5	1.00	5	1.000
168	A	8	6	1.00	7	0.857
169	A	8	6	1.00	7	0.857
170	A	7	5	1.00	5	1.000
171	A	8	6	1.00	7	0.857
172	A	8	6	1.00	7	0.857
173	A	5	5	1.00	5	1.000
174	A	6	6	1.00	7	0.857
175	A	6	6	1.00	7	0.857
176	A	5	5	1.00	5	1.000
177	A	6	6	1.00	7	0.857
178	A	6	6	1.00	7	0.857
179	A	3	3	1.00	16	0.188
180	A	3	2	1.00	10	0.200
181	A	5	4	1.00	10	0.400
182	A	3	4	1.00	8	0.500
183	A	2	3	1.00	6	0.500
184	A	4	4	1.00	10	0.400
185	A	2	2	1.00	35	0.057
186	A	3	3	1.00	8	0.375
187	A	2	3	1.00	6	0.500
188	A	2	2	1.00	6	0.333
189	A	4	5	1.00	6	0.833
190	A	2	2	1.00	6	0.333
191	A	10	9	1.00	8	1.125
192	A	7	7	1.00	8	0.875
193	A	8	8	1.00	7	1.143
194	A	3	4	1.00	8	0.500
195	A	5	6	1.00	9	0.667
196	A	7	8	1.00	11	0.727
197	A	15	8	1.00	11	0.727
198	A	5	6	1.00	9	0.667
199	A	7	8	1.00	11	0.727
200	A	15	8	1.00	11	0.727

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	5	1.000
202	A	6	6	1.00	7	0.857
203	A	6	6	1.00	7	0.857
204	A	5	5	1.00	5	1.000
205	A	6	6	1.00	7	0.857
206	A	6	6	1.00	7	0.857
207	A	7	5	1.00	3	1.667
208	A	7	5	1.00	5	1.000
209	A	8	6	1.00	7	0.857
210	A	8	6	1.00	7	0.857
211	A	7	5	1.00	3	1.667
212	A	7	5	1.00	5	1.000
213	A	8	6	1.00	7	0.857
214	A	8	6	1.00	7	0.857
215	A	5	5	1.00	5	1.000
216	A	6	6	1.00	7	0.857
217	A	6	6	1.00	7	0.857
218	A	5	5	1.00	5	1.000
219	A	6	6	1.00	7	0.857
220	A	6	6	1.00	7	0.857
221	A	2	2	1.00	35	0.057
222	A	3	3	1.00	8	0.375
223	A	1	1	1.00	8	0.125
224	A	3	3	1.00	10	0.300
225	A	2	1	1.00	16	0.062
226	A	3	3	1.00	9	0.333
227	A	2	2	1.00	10	0.200
228	A	3	2	1.00	10	0.200
229	A	3	2	1.00	12	0.167
230	A	4	3	1.00	8	0.375
231	A	2	2	1.00	12	0.167
232	A	2	2	1.00	12	0.167
233	A	2	2	1.00	14	0.143
234	A	6	6	1.00	14	0.429
235	A	1	1	1.00	8	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	4	0.500
237	A	1	1	1.00	10	0.100
238	A	3	1	1.00	11	0.091
239	A	3	3	1.00	6	0.500
240	A	2	2	1.00	8	0.250
241	A	2	2	1.00	14	0.143
242	A	2	2	1.00	6	0.333
243	A	4	3	1.00	10	0.300
244	A	4	3	1.00	14	0.214
245	A	2	2	1.00	12	0.167
246	A	2	2	1.00	14	0.143
247	A	2	2	1.00	14	0.143
248	A	2	2	1.00	14	0.143
249	A	2	2	1.00	14	0.143
250	A	2	1	1.00	15	0.067
251	A	2	1	1.00	17	0.059
252	A	7	6	1.00	17	0.353
253	A	10	6	1.00	17	0.353
254	A	3	1	1.00	17	0.059
255	A	3	2	1.00	17	0.118
256	A	8	7	1.00	17	0.412
257	A	11	7	1.00	17	0.412
258	A	3	2	1.00	18	0.111
259	A	5	4	1.00	26	0.154
260	A	5	4	1.00	21	0.190
261	A	7	5	1.00	27	0.185
262	A	2	2	1.00	10	0.200
263	A	3	3	1.00	12	0.250
264	A	3	3	1.00	12	0.250
265	A	3	3	1.00	12	0.250
266	A	8	7	1.00	15	0.467
267	A	7	7	1.00	15	0.467
268	A	1	1	1.00	8	0.125
269	A	6	6	1.00	19	0.316
270	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	1	1.00	14	0.071
272	A	8	7	1.00	24	0.292
273	A	4	3	1.00	8	0.375
274	A	2	2	1.00	9	0.222
275	A	2	2	1.00	10	0.200
276	A	8	6	1.00	22	0.273
277	A	5	6	1.00	18	0.333
278	A	5	6	1.00	20	0.300
279	A	5	6	1.00	25	0.240
280	A	1	1	1.00	39	0.026
281	A	1	1	1.00	39	0.026
282	A	2	2	1.00	8	0.250
283	A	3	3	1.00	10	0.300
284	A	5	5	1.24	12	0.417
285	A	3	4	1.00	10	0.400
286	A	0	0	0.00	0	0.000
287	A	0	0	0.00	0	0.000
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	2	2	1.00	14	0.143
292	A	2	2	1.00	16	0.125
293	A	8	6	1.00	14	0.429
294	A	5	3	1.00	14	0.214
295	A	6	4	1.00	14	0.286
296	A	4	4	1.00	12	0.333
297	A	5	3	1.00	14	0.214
298	A	7	4	1.00	14	0.286
299	A	2	1	1.00	14	0.071
300	A	3	3	1.00	6	0.500
301	A	4	4	1.00	13	0.308
302	A	4	4	1.00	14	0.286
303	A	4	4	1.00	17	0.235
304	A	4	4	1.00	18	0.222
305	A	13	13	1.00	10	1.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	12	12	1.00	8	1.500
307	A	7	6	1.00	6	1.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	14	12	1.00	13	0.923
311	A	13	11	1.00	11	1.000
312	A	7	6	1.00	9	0.667
313	A	0	0	0.00	0	0.000
314	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx$

Optimal. Leaf size=76

$$\frac{(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}-\frac{am\text{Int}(x^{-1+m}(ax^m+b\log^q(cx^n))^p,x)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)/b/n/q+(a*x^m+b*ln(
c*x^n)^q)^(1+p)/b/n/(1+p)/q
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(b*n*(1 + p)*q) - (a*m*Defer[Int][x^(-1 +
m)*(a*x^m + b*Log[c*x^n]^q)^p, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx=\frac{(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}-\frac{(am)\int x^{-1+m}(ax^m+b\log^q(cx^n))^p\,d}{bnq}$$

Mathematica [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^p)/x, x]
```

Maple [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{-1+q}(a\,x^m+b\ln(c\,x^n)^q)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="fricas
")
```

```
[Out] integral((a*x^m + b*log(c*x^n)^q)^p*log(c*x^n)^(q - 1)/x, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.46Unable to divide, perhaps due to rou
nding error%%%{1,[0,0,2,5,2,0,5,0,2,1,2,2]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,
2,2]%%%}+
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

3.2 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^3}{x}\,dx$

Optimal. Leaf size=231

$$\frac{b^3\log^{4q}(cx^n)}{4nq}-\frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{n}-\frac{3\,4^{-q}a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(2q,\right.}{}$$

```
[Out] 1/4*b^3*ln(c*x^n)^(4*q)/n/q-3*a*b^2*x^m*GAMMA(3*q,-m*ln(c*x^n)/n)*ln(c*x^n)
^(3*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(3*q))-3*a^2*b*x^(2*m)*GAMMA(2*q
,-2*m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/(4^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)
/n)^(2*q))-a^3*x^(3*m)*GAMMA(q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/n/((c*x^
n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A]

time = 0.23, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2619, 2347, 2212, 2339, 30}

$$-\frac{a^3 3^{-q}x^{3m}(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\text{Gamma}\left(q,-\frac{3m\log(cx^n)}{n}\right)}{n}-\frac{3a^2b4^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\text{Gamma}\left(2q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\text{Gamma}\left(3q,-\frac{m\log(cx^n)}{n}\right)}{n}+\frac{b^3\log^{4q}(cx^n)}{4nq}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (b^3*Log[c*x^n]^(4*q))/(4*n*q) - (3*a*b^2*x^m*Gamma[3*q, -((m*Log[c*x^n])/n
)]*Log[c*x^n]^(3*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q)) - (3*a^2
*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(4^q*n*(c*x^n)
^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^3*x^(3*m)*Gamma[q, (-3*m*Log[c
*x^n])/n]*Log[c*x^n]^q)/(3^q*n*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
```

```
x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx &= \int \left(a^3x^{-1+3m}\log^{-1+q}(cx^n)+3a^2bx^{-1+2m}\log^{-1+2q}(cx^n)+3ab^2\right.\\ &= a^3\int x^{-1+3m}\log^{-1+q}(cx^n)\,dx+\left(3a^2b\right)\int x^{-1+2m}\log^{-1+2q}(cx^n\\ &= \frac{b^3\mathrm{Subst}\left(\int x^{-1+4q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\right)\mathrm{Subst}\left(\right.}{}\\ &= \frac{b^3\log^{4q}(cx^n)}{4nq}-\frac{3ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(\right.}{n}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 223, normalized size = 0.97

$$\frac{\log^q(cx^n)\left(\frac{b^3\log^{3q}(cx^n)}{q}-12ab^2x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}-3\;4^{1-q}a^2bx^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}-4\;3^{-q}a^3x^{3m}(cx^n)^{-\frac{3m}{n}}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^3*Log[c*x^n]^(3*q))/q - (12*a*b^2*x^m*Gamma[3*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q))/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(3*q))
- (3*4^(1 - q)*a^2*b*x^(2*m)*Gamma[2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/
((c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (4*a^3*x^(3*m)*Gamma[q, (
-3*m*Log[c*x^n])/n])/(3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q)))/(4*n
)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln (c\,x^{n})^{-1+q} \left(a\,x^{m}+b\ln (c\,x^{n})^{q}\right)^{3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="fricas
")
```

```
[Out] integral((3*a*b^2*x^m*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1) + 3*a^2*b*x^(2*m)
*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^3*x^(3*m)*log(c*x^n)^(q - 1) + b^3*log
(c*x^n)^(3*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorithm="giac")
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}\,{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^3)/x, x)
```

3.3 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal. Leaf size=156

$$\frac{b^2\log^{3q}(cx^n)}{3nq}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{n}-\frac{2^{-q}a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(q,-\frac{2m\log}{}\right.}{}$$

```
[Out] 1/3*b^2*ln(c*x^n)^(3*q)/n/q-2*a*b*x^m*GAMMA(2*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(
2*q)/n/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*x^(2*m)*GAMMA(q,-2*m*ln
(c*x^n)/n)*ln(c*x^n)^q/(2^q)/n/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A]
time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2619, 2347, 2212, 2339, 30}

$$-\frac{a^2 2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\text{Gamma}\left(q,-\frac{2m\log(cx^n)}{n}\right)}{n}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\text{Gamma}\left(2q,-\frac{m\log(cx^n)}{n}\right)}{n}+\frac{b^2\log^{3q}(cx^n)}{3nq}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (b^2*Log[c*x^n]^(3*q))/(3*n*q) - (2*a*b*x^m*Gamma[2*q, -((m*Log[c*x^n])/n)]
*Log[c*x^n]^(2*q))/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*x^(
2*m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*n*(c*x^n)^((2*m)/n)*(
-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx = \int \left(a^2x^{-1+2m}\log^{-1+q}(cx^n)+2abx^{-1+m}\log^{-1+2q}(cx^n)+\frac{b^2\log\right.$$

$$= a^2\int x^{-1+2m}\log^{-1+q}(cx^n)\,dx+(2ab)\int x^{-1+m}\log^{-1+2q}(cx^n)\,d$$

$$= \frac{b^2\mathrm{Subst}\left(\int x^{-1+3q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\right)\mathrm{Subst}\left(\right.}{}$$

$$= \frac{b^2\log^{3q}(cx^n)}{3nq}-\frac{2abx^m(cx^n)^{-\frac{m}{n}}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\right.}{n}$$

Mathematica [A]
time = 0.29, size = 149, normalized size = 0.96

$$\frac{\log^q(cx^n)\left(\frac{b^2\log^{2q}(cx^n)}{q}-6abx^m(cx^n)^{-\frac{m}{n}}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}-3\,2^{-q}a^2x^{2m}(cx^n)^{-\frac{2m}{n}}\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (Log[c*x^n]^q*((b^2*Log[c*x^n]^(2*q))/q - (6*a*b*x^m*Gamma[2*q, -((m*Log[c*
x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (3*a^
2*x^(2*m)*Gamma[q, (-2*m*Log[c*x^n])/n])/(2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
*x^n])/n))^q)))/(3*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{-1+q}\left(a\,x^m+b\ln(c\,x^n)^q\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="fricas
")
```

```
[Out] integral((2*a*b*x^m*log(c*x^n)^(q - 1)*log(c*x^n)^q + a^2*x^(2*m)*log(c*x^n
)^(q - 1) + b^2*log(c*x^n)^(2*q)*log(c*x^n)^(q - 1))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^m + b\log{(cx^n)}^q)^2 \log{(cx^n)}^{q-1}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)**2*log(c*x**n)**(q - 1)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorithm="giac")
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(c\,x^n)^{q-1}\,\left(a\,x^m+b\,\ln(c\,x^n)^q\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q)^2)/x, x)
```

3.4 $\int \frac{\log^{-1+q}(cx^n)(ax^m+b\log^q(cx^n))}{x} \, dx$

Optimal. Leaf size=81

$$\frac{b\log^{2q}(cx^n)}{2nq} - \frac{ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}}{n}$$

```
[Out] 1/2*b*ln(c*x^n)^(2*q)/n/q-a*x^m*GAMMA(q,-m*ln(c*x^n)/n)*ln(c*x^n)^q/n/((c*x
^n)^(m/n))/((-m*ln(c*x^n)/n)^q)
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2619, 2347, 2212, 2339, 30}

$$\frac{b\log^{2q}(cx^n)}{2nq} - \frac{ax^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\text{Gamma}\left(q,-\frac{m\log(cx^n)}{n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (b*Log[c*x^n]^(2*q))/(2*n*q) - (a*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x
^n]^q)/(n*(c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^{-1+q}(cx^n)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx &= \int \left(ax^{-1+m}\log^{-1+q}(cx^n)+\frac{b\log^{-1+2q}(cx^n)}{x}\right)dx\\ &= a\int x^{-1+m}\log^{-1+q}(cx^n)\,dx+b\int \frac{\log^{-1+2q}(cx^n)}{x}\,dx\\ &= \frac{b\operatorname{Subst}\left(\int x^{-1+2q}\,dx,x,\log(cx^n)\right)}{n}+\frac{\left(ax^m(cx^n)^{-\frac{m}{n}}\right)\operatorname{Subst}\left(\int e^{\frac{m}{n}}\right)}{n}\\ &= \frac{b\log^{2q}(cx^n)}{2nq}-\frac{ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(}{n}\right)}{n}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 0.95

$$\frac{\log^q(cx^n)\left(\frac{b\log^q(cx^n)}{q}-2ax^m(cx^n)^{-\frac{m}{n}}\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]^(-1 + q)*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (Log[c*x^n]^q*((b*Log[c*x^n]^q)/q - (2*a*x^m*Gamma[q, -((m*Log[c*x^n])/n)])
/((c*x^n)^(m/n)*(-((m*Log[c*x^n])/n))^q)))/(2*n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{-1+q}\left(a\,x^m+b\ln(c\,x^n)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^m*log(c*x^n)^(q - 1) + b*log(c*x^n)^(q - 1)*log(c*x^n)^q)/x,
x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax^{m} + b\log\left(cx^{n}\right)^{q}\right)\log\left(cx^{n}\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)*log(c*x**n)**(q - 1)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*log(c*x^n)^(q - 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (c\,x^n)^{q-1}\,(a\,x^m+b\ln (c\,x^n)^q)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x,x)
[Out] int((log(c*x^n)^(q - 1)*(a*x^m + b*log(c*x^n)^q))/x, x)
```

3.5 $\int \frac{\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=15

$$\frac{\log^q(cx^n)}{nq}$$

```
[Out] ln(c*x^n)^q/n/q
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2339, 30}

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^{-1+q}(cx^n)}{x}\,dx &= \frac{\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\log^q(cx^n)}{nq}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/x,x]
```

```
[Out] Log[c*x^n]^q/(n*q)
```

Maple [A]
time = 0.20, size = 16, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\ln(c\,x^{n})^{q}}{nq}$	16
default	$\frac{\ln(c\,x^{n})^{q}}{nq}$	16
risch	$\frac{\left(\ln(c)+\ln(x^{n})-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\right)^{q}}{nq}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*x^n)^q/n/q
```

Maxima [A]
time = 0.30, size = 15, normalized size = 1.00

$$\frac{\log\left(cx^{n}\right)^{q}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="maxima")
```

```
[Out] log(c*x^n)^q/(n*q)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 1.67

$$\frac{(n\log\left(x\right)+\log\left(c\right))(n\log\left(x\right)+\log\left(c\right))^{q-1}}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c))*(n*log(x) + log(c))^(q - 1)/(n*q)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(cx^{n}\right)^{q-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x,x)
```

```
[Out] Integral(log(c*x**n)**(q - 1)/x, x)
```

Giac [A]
time = 7.35, size = 16, normalized size = 1.07

$$\frac{(n\log{(x)}+\log{(c)})^q}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(c))^q/(n*q)
```

Mupad [B]
time = 0.28, size = 15, normalized size = 1.00

$$\frac{\ln{(c\,x^n)}^q}{n\,q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/x,x)
```

```
[Out] log(c*x^n)^q/(n*q)
```

3.6 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal. Leaf size=68

$$\frac{\log\left(ax^m+b\log^q\left(cx^n\right)\right)}{bnq}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)/b/n/q+ln(a*x^m+b*ln(
c*x^n)^q)/b/n/q
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^q), x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\frac{\log\left(ax^m+b\log^q\left(cx^n\right)\right)}{bnq}-\frac{(am)\int\frac{x^{-1+m}}{ax^m+b\log^q(cx^n)}\,dx}{bnq}$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,x^n\right)^{-1+q}}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="maxima")
```

```
[Out] -a*integrate(x^m/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b
^2*x*log(x^n))*(log(c) + log(x^n))^q), x) + log(log(c) + log(x^n))/(b*n)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(cx^{n} \right)}^{q-1}}{x\left(ax^{m}+b\log{\left(cx^{n} \right)}^{q}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Integral(log(c*x**n)**(q - 1)/(x*(a*x**m + b*log(c*x**n)**q)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)),x)
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.7 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)/b/n/q-1/b/n/q/(a*x
^m+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(1/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^q)^2, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{1}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}-\frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^2}\,dx}{bnq}$$

Mathematica [A]
time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^2), x]
```

Maple [**A**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\,x^{n}\right)^{-1+q}}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="maxima
")
```

```
[Out] 1/(a*b*m*x^m*log(x^n) - (n*q - m*log(c))*a*b*x^m + (b^2*m*log(x^n) - (n*q -
 m*log(c))*b^2)*(log(c) + log(x^n))^q) + integrate(-(m*n*(q - 1) - m^2*log(
c) - m^2*log(x^n))/(a*b*m^2*x*x^m*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*a*b*x
*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b*x*x^m + (b^2*
m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(c))*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n
*q*log(c) + m^2*log(c)^2)*b^2*x)*(log(c) + log(x^n))^q), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x^(2*m) + b^2
*x*log(c*x^n)^(2*q)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^q\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.8 $\int \frac{\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal. Leaf size=72

$$-\frac{1}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3},x\right)}{bnq}$$

```
[Out] -a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)/b/n/q-1/2/b/n/q/(a
*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A]
time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/2*1/(b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x
^m + b*Log[c*x^n]^q)^3, x])/(b*n*q)
```

Rubi steps

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{(ax^m+b\log^q(cx^n))^3}\,dx}{bnq}$$

Mathematica [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[Log[c*x^n]^(-1 + q)/(x*(a*x^m + b*Log[c*x^n]^q)^3), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln(c\,x^n)^{-1+q}}{x\,(a\,x^m+b\ln(c\,x^n)^q)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int(ln(c*x^n)^(-1+q)/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a*m^2*x^m*log(x^n)^2 + (2*m^2*log(c) + m*n)*a*x^m*log(x^n) - (n^2*q^2
 - m^2*log(c)^2 - m*n*log(c))*a*x^m + (2*b*m^2*log(x^n)^2 - (m*n*(2*q - 1)
- 4*m^2*log(c))*b*log(x^n) - (m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*b)*(lo
g(c) + log(x^n))^q)/(a^3*b*m^3*x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c)
)*a^3*b*x^(3*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2
)*a^3*b*x^(3*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)
^2 - m^3*log(c)^3)*a^3*b*x^(3*m) + (a*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q -
 m^3*log(c))*a*b^3*x^m*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*l
og(c)^2)*a*b^3*x^m*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log
(c)^2 - m^3*log(c)^3)*a*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^2*b^2*m^3
*x^(2*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^2*x^(2*m)*log(x^n)^2 +
 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^2*x^(2*m)*log(x^n) -
 (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^2*b^2
*x^(2*m))*(log(c) + log(x^n))^q) - integrate(-1/2*(m^3*n*(2*q - 3)*log(c)^2
 - 2*m^4*log(c)^3 - 2*m^4*log(x^n)^3 + 2*(q^2 - 1)*m^2*n^2*log(c) - (2*q^3
- 3*q^2 + q)*m*n^3 + (m^3*n*(2*q - 3) - 6*m^4*log(c))*log(x^n)^2 + 2*(m^3*n
*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 - 1)*m^2*n^2)*log(x^n))/(a^2*b*m^
4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a^2*b*x*x^(2*m)*log(x^n)^
3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a^2*b*x*x^(2*m)*log(x
^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(
c)^3)*a^2*b*x*x^(2*m)*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*
q^2*log(c)^2 - 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a^2*b*x*x^(2*m) + (a*b^2*
m^4*x*x^m*log(x^n)^4 - 4*(m^3*n*q - m^4*log(c))*a*b^2*x*x^m*log(x^n)^3 + 6*
(m^2*n^2*q^2 - 2*m^3*n*q*log(c) + m^4*log(c)^2)*a*b^2*x*x^m*log(x^n)^2 - 4*
(m*n^3*q^3 - 3*m^2*n^2*q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a*b^
```

```
2*x*x^m*log(x^n) + (n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 -
 4*m^3*n*q*log(c)^3 + m^4*log(c)^4)*a*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="fricas
")
```

```
[Out] integral(log(c*x^n)^(q - 1)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3*a^2*b*x*x^(
2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)**(-1+q)/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)^(-1+q)/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)^(q - 1)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

```
[Out] int(log(c*x^n)^(q - 1)/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.9 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=272

$$-\frac{360ab^2n^5x^m}{m^6}-\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2}+\frac{360ab^2n^4x^m\log\left(cx^n\right)}{m^5}+\frac{9a^2bn^2x^{2m}\log\left(cx^n\right)}{4m^3}+\frac{a^3x^{3m}\log\left(cx^n\right)}{3m}-\frac{180ab^2}{}$$

```
[Out] -360*a*b^2*n^5*x^m/m^6-9/8*a^2*b*n^3*x^(2*m)/m^4-1/9*a^3*n*x^(3*m)/m^2+360*
a*b^2*n^4*x^m*ln(c*x^n)/m^5+9/4*a^2*b*n^2*x^(2*m)*ln(c*x^n)/m^3+1/3*a^3*x^(
3*m)*ln(c*x^n)/m-180*a*b^2*n^3*x^m*ln(c*x^n)^2/m^4-9/4*a^2*b*n*x^(2*m)*ln(c
*x^n)^2/m^2+60*a*b^2*n^2*x^m*ln(c*x^n)^3/m^3+3/2*a^2*b*x^(2*m)*ln(c*x^n)^3/
m-15*a*b^2*n*x^m*ln(c*x^n)^4/m^2+3*a*b^2*x^m*ln(c*x^n)^5/m+1/8*b^3*ln(c*x^n
)^8/n
```

Rubi [A]
time = 0.20, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2619, 2341, 2342, 2339, 30}

$$\frac{a^3x^{3m}\log\left(cx^n\right)}{3m}-\frac{a^3nx^{3m}}{9m^2}+\frac{9a^2bn^2x^{2m}\log\left(cx^n\right)}{4m^3}-\frac{9a^2bnx^{2m}\log^2\left(cx^n\right)}{4m^2}+\frac{3a^2bx^{2m}\log^3\left(cx^n\right)}{2m}-\frac{9a^2bn^3x^{2m}}{8m^4}+\frac{360ab^2n^4x^m\log\left(cx^n\right)}{m^5}-\frac{180ab^2n^3x^m\log^2\left(cx^n\right)}{m^4}+\frac{60ab^2n^2x^m\log^3\left(cx^n\right)}{m^3}-\frac{15ab^2nx^m\log^4\left(cx^n\right)}{m^2}+\frac{3ab^2x^m\log^5\left(cx^n\right)}{m}-\frac{360ab^2n^5x^m}{m^6}+\frac{b^3\log^8\left(cx^n\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] (-360*a*b^2*n^5*x^m)/m^6 - (9*a^2*b*n^3*x^(2*m))/(8*m^4) - (a^3*n*x^(3*m))/
(9*m^2) + (360*a*b^2*n^4*x^m*Log[c*x^n])/m^5 + (9*a^2*b*n^2*x^(2*m)*Log[c*x
^n])/(4*m^3) + (a^3*x^(3*m)*Log[c*x^n])/(3*m) - (180*a*b^2*n^3*x^m*Log[c*x^
n]^2)/m^4 - (9*a^2*b*n*x^(2*m)*Log[c*x^n]^2)/(4*m^2) + (60*a*b^2*n^2*x^m*Lo
g[c*x^n]^3)/m^3 + (3*a^2*b*x^(2*m)*Log[c*x^n]^3)/(2*m) - (15*a*b^2*n*x^m*Lo
g[c*x^n]^4)/m^2 + (3*a*b^2*x^m*Log[c*x^n]^5)/m + (b^3*Log[c*x^n]^8)/(8*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^3}{x}\,dx &= \int \Big(a^3x^{-1+3m}\log(cx^n)+3a^2bx^{-1+2m}\log^3(cx^n)+3ab^2x^{-1+m}\log^5(\\
&= a^3\int x^{-1+3m}\log(cx^n)\,dx+\left(3a^2b\right)\int x^{-1+2m}\log^3(cx^n)\,dx+\left(3ab^2\right)\\
&= -\frac{a^3nx^{3m}}{9m^2}+\frac{a^3x^{3m}\log(cx^n)}{3m}+\frac{3a^2bx^{2m}\log^3(cx^n)}{2m}+\frac{3ab^2x^m\log^5(c}{m}\\
&= -\frac{a^3nx^{3m}}{9m^2}+\frac{a^3x^{3m}\log(cx^n)}{3m}-\frac{9a^2bnx^{2m}\log^2(cx^n)}{4m^2}+\frac{3a^2bx^{2m}\log^3}{2m}\\
&= -\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2}+\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3}+\frac{a^3x^{3m}\log(cx^n)}{3m}-\\
&= -\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2}+\frac{9a^2bn^2x^{2m}\log(cx^n)}{4m^3}+\frac{a^3x^{3m}\log(cx^n)}{3m}-\\
&= -\frac{360ab^2n^5x^m}{m^6}-\frac{9a^2bn^3x^{2m}}{8m^4}-\frac{a^3nx^{3m}}{9m^2}+\frac{360ab^2n^4x^m\log(cx^n)}{m^5}+
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 336, normalized size = 1.24

$$-\frac{1}{8}b^3n^7\log^8(x)+b^3n^6\log^7(x)\log(cx^n)-\frac{7}{2}b^3n^5\log^6(x)\log^2(cx^n)+7b^3n^4\log^5(x)\log^3(cx^n)-\frac{35}{4}b^3n^3\log^4(x)\log^4(cx^n)+7b^3n^2\log^3(x)\log^5(cx^n)-\frac{7}{2}b^3n\log^2(x)\log^6(cx^n)+b^3\log(x)\log^7(cx^n)+\frac{ax^m\left(-n\left(25920b^2n^4+81abm^2n^2x^m+8a^2m^4x^{2m}\right)+6m\left(4320b^2n^4+27abm^2n^2x^m+4a^2m^4x^{2m}\right)\log(cx^n)-162bm^2n\left(80bn^2+am^2x^m\right)\log^2(cx^n)+108bm^3\left(40bn^2+am^2x^m\right)\log^3(cx^n)-1080b^2m^4n\log^4(cx^n)+216b^2m^5\log^5(cx^n)\right)}{72m^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^3)/x,x]
```

```
[Out] -1/8*(b^3*n^7*Log[x]^8) + b^3*n^6*Log[x]^7*Log[c*x^n] - (7*b^3*n^5*Log[x]^6
*Log[c*x^n]^2)/2 + 7*b^3*n^4*Log[x]^5*Log[c*x^n]^3 - (35*b^3*n^3*Log[x]^4*L
og[c*x^n]^4)/4 + 7*b^3*n^2*Log[x]^3*Log[c*x^n]^5 - (7*b^3*n*Log[x]^2*Log[c*
x^n]^6)/2 + b^3*Log[x]*Log[c*x^n]^7 + (a*x^m*(-(n*(25920*b^2*n^4 + 81*a*b*m
^2*n^2*x^m + 8*a^2*m^4*x^(2*m))) + 6*m*(4320*b^2*n^4 + 27*a*b*m^2*n^2*x^m +
 4*a^2*m^4*x^(2*m))*Log[c*x^n] - 162*b*m^2*n*(80*b*n^2 + a*m^2*x^m)*Log[c*x
^n]^2 + 108*b*m^3*(40*b*n^2 + a*m^2*x^m)*Log[c*x^n]^3 - 1080*b^2*m^4*n*Log[
c*x^n]^4 + 216*b^2*m^5*Log[c*x^n]^5))/(72*m^6)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 3.89, size = 61910, normalized size = 227.61

method	result	size
risch	Expression too large to display	61910

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. 2(258) = 516.
time = 0.32, size = 1115, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/84*(12*b^3*log(c*x^n)^7/n + 252*a*b^2*x^m*log(c*x^n)^4/m + 126*a^2*b*x^(2
*m)*log(c*x^n)^2/m - 1008*(n*x^m*log(c*x^n)^3/m^2 - 3*(n*x^m*log(c*x^n)^2/m
^2 - 2*n*(n*x^m*log(c*x^n)/m^2 - n^2*x^m/m^3)/m)*n/m)*a*b^2 - 63*a^2*b*(2*n
*x^(2*m)*log(c*x^n)/m^2 - n^2*x^(2*m)/m^3) + 28*a^3*x^(3*m)/m)*log(c*x^n) +
 1/504*(9*b^3*m^6*n^7*log(x)^8 - 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m
^6*n^5*log(c)^2*log(x)^6 - 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*
n^3*log(c)^4*log(x)^4 - 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*l
og(c)^6*log(x)^2 - 72*b^3*m^6*log(c)^7*log(x) - 72*b^3*m^6*log(x)*log(x^n)^
7 - 56*a^3*m^4*n*x^(3*m) + 252*(b^3*m^6*n*log(x)^2 - 2*b^3*m^6*log(c)*log(x
))*log(x^n)^6 - 504*(b^3*m^6*n^2*log(x)^3 - 3*b^3*m^6*n*log(c)*log(x)^2 + 3
*b^3*m^6*log(c)^2*log(x))*log(x^n)^5 - 189*(2*m^4*n*log(c)^2 - 4*m^3*n^2*lo
g(c) + 3*m^2*n^3)*a^2*b*x^(2*m) - 1512*(m^4*n*log(c)^4 - 8*m^3*n^2*log(c)^3
 + 36*m^2*n^3*log(c)^2 - 96*m*n^4*log(c) + 120*n^5)*a*b^2*x^m + 126*(5*b^3*
m^6*n^3*log(x)^4 - 20*b^3*m^6*n^2*log(c)*log(x)^3 + 30*b^3*m^6*n*log(c)^2*l
og(x)^2 - 20*b^3*m^6*log(c)^3*log(x) - 12*a*b^2*m^4*n*x^m)*log(x^n)^4 - 504
*(b^3*m^6*n^4*log(x)^5 - 5*b^3*m^6*n^3*log(c)*log(x)^4 + 10*b^3*m^6*n^2*log
```

```
(c)^2*log(x)^3 - 10*b^3*m^6*n*log(c)^3*log(x)^2 + 5*b^3*m^6*log(c)^4*log(x)
 + 12*(m^4*n*log(c) - 2*m^3*n^2)*a*b^2*x^m)*log(x^n)^3 + 126*(2*b^3*m^6*n^5
*log(x)^6 - 12*b^3*m^6*n^4*log(c)*log(x)^5 + 30*b^3*m^6*n^3*log(c)^2*log(x)
^4 - 40*b^3*m^6*n^2*log(c)^3*log(x)^3 + 30*b^3*m^6*n*log(c)^4*log(x)^2 - 12
*b^3*m^6*log(c)^5*log(x) - 3*a^2*b*m^4*n*x^(2*m) - 72*(m^4*n*log(c)^2 - 4*m
^3*n^2*log(c) + 6*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 - 36*(2*b^3*m^6*n^6*log(x)
^7 - 14*b^3*m^6*n^5*log(c)*log(x)^6 + 42*b^3*m^6*n^4*log(c)^2*log(x)^5 - 70
*b^3*m^6*n^3*log(c)^3*log(x)^4 + 70*b^3*m^6*n^2*log(c)^4*log(x)^3 - 42*b^3*
m^6*n*log(c)^5*log(x)^2 + 14*b^3*m^6*log(c)^6*log(x) + 21*(m^4*n*log(c) - m
^3*n^2)*a^2*b*x^(2*m) + 168*(m^4*n*log(c)^3 - 6*m^3*n^2*log(c)^2 + 18*m^2*n
^3*log(c) - 24*m*n^4)*a*b^2*x^m)*log(x^n))/m^6
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(258) = 516$.
time = 0.38, size = 655, normalized size = 2.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/72*(9*b^3*m^6*n^7*log(x)^8 + 72*b^3*m^6*n^6*log(c)*log(x)^7 + 252*b^3*m^6
*n^5*log(c)^2*log(x)^6 + 504*b^3*m^6*n^4*log(c)^3*log(x)^5 + 630*b^3*m^6*n^
3*log(c)^4*log(x)^4 + 504*b^3*m^6*n^2*log(c)^5*log(x)^3 + 252*b^3*m^6*n*log
(c)^6*log(x)^2 + 72*b^3*m^6*log(c)^7*log(x) + 8*(3*a^3*m^5*n*log(x) + 3*a^3
*m^5*log(c) - a^3*m^4*n)*x^(3*m) + 27*(4*a^2*b*m^5*n^3*log(x)^3 + 4*a^2*b*m
^5*log(c)^3 - 6*a^2*b*m^4*n*log(c)^2 + 6*a^2*b*m^3*n^2*log(c) - 3*a^2*b*m^2
*n^3 + 6*(2*a^2*b*m^5*n^2*log(c) - a^2*b*m^4*n^3)*log(x)^2 + 6*(2*a^2*b*m^5
*n*log(c)^2 - 2*a^2*b*m^4*n^2*log(c) + a^2*b*m^3*n^3)*log(x))*x^(2*m) + 216
*(a*b^2*m^5*n^5*log(x)^5 + a*b^2*m^5*log(c)^5 - 5*a*b^2*m^4*n*log(c)^4 + 20
*a*b^2*m^3*n^2*log(c)^3 - 60*a*b^2*m^2*n^3*log(c)^2 + 120*a*b^2*m*n^4*log(c
) - 120*a*b^2*n^5 + 5*(a*b^2*m^5*n^4*log(c) - a*b^2*m^4*n^5)*log(x)^4 + 10*
(a*b^2*m^5*n^3*log(c)^2 - 2*a*b^2*m^4*n^4*log(c) + 2*a*b^2*m^3*n^5)*log(x)^
3 + 10*(a*b^2*m^5*n^2*log(c)^3 - 3*a*b^2*m^4*n^3*log(c)^2 + 6*a*b^2*m^3*n^4
*log(c) - 6*a*b^2*m^2*n^5)*log(x)^2 + 5*(a*b^2*m^5*n*log(c)^4 - 4*a*b^2*m^4
*n^2*log(c)^3 + 12*a*b^2*m^3*n^3*log(c)^2 - 24*a*b^2*m^2*n^4*log(c) + 24*a*
b^2*m*n^5)*log(x))*x^m)/m^6
```

Sympy [**A**]
time = 30.90, size = 411, normalized size = 1.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**3/x,x)
```

```
[Out] -a**3*n*Piecewise((Piecewise((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))/(3
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**3*Piecewis
e((x**(3*m)/(3*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 3*a**2*b*Piecew
ise((x**(2*m)*log(c*x**n)**3/(2*m) - 3*n*x**(2*m)*log(c*x**n)**2/(4*m**2) +
 3*n**2*x**(2*m)*log(c*x**n)/(4*m**3) - 3*n**3*x**(2*m)/(8*m**4), Ne(m, 0))
, (Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/
(4*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*
meijerg(((), (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg
(((1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True)), True)) +
3*a*b**2*Piecewise((x**m*log(c*x**n)**5/m - 5*n*x**m*log(c*x**n)**4/m**2 +
20*n**2*x**m*log(c*x**n)**3/m**3 - 60*n**3*x**m*log(c*x**n)**2/m**4 + 120*n
**4*x**m*log(c*x**n)/m**5 - 120*n**5*x**m/m**6, Ne(m, 0)), (Piecewise((0, (
Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**6/(6*n), Abs(c*x**n)
 < 1), (log(1/(c*x**n))**6/(6*n), 1/Abs(c*x**n) < 1), (120*meijerg(((), (1,
 1, 1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 120*meijerg((
(1, 1, 1, 1, 1, 1, 1), ()), ((), (0, 0, 0, 0, 0, 0, 0)), c*x**n)/n, True)),
 True)) - b**3*Piecewise((-log(c)**7*log(x), Eq(n, 0)), (-log(c*x**n)**8/(8
*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(258) = 516.
time = 5.48, size = 766, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/8*b^3*n^7*log(x)^8 + b^3*n^6*log(c)*log(x)^7 + 7/2*b^3*n^5*log(c)^2*log(x
)^6 + 7*b^3*n^4*log(c)^3*log(x)^5 + 35/4*b^3*n^3*log(c)^4*log(x)^4 + 7*b^3*
n^2*log(c)^5*log(x)^3 + 3*a*b^2*n^5*x^m*log(x)^5/m + 7/2*b^3*n*log(c)^6*log
(x)^2 + 15*a*b^2*n^4*x^m*log(c)*log(x)^4/m + b^3*log(c)^7*log(x) + 30*a*b^2
*n^3*x^m*log(c)^2*log(x)^3/m - 15*a*b^2*n^5*x^m*log(x)^4/m^2 + 30*a*b^2*n^2
*x^m*log(c)^3*log(x)^2/m - 60*a*b^2*n^4*x^m*log(c)*log(x)^3/m^2 + 15*a*b^2*
n*x^m*log(c)^4*log(x)/m - 90*a*b^2*n^3*x^m*log(c)^2*log(x)^2/m^2 + 3/2*a^2*
b*n^3*x^(2*m)*log(x)^3/m + 60*a*b^2*n^5*x^m*log(x)^3/m^3 + 3*a*b^2*x^m*log(
c)^5/m - 60*a*b^2*n^2*x^m*log(c)^3*log(x)/m^2 + 9/2*a^2*b*n^2*x^(2*m)*log(c
)*log(x)^2/m + 180*a*b^2*n^4*x^m*log(c)*log(x)^2/m^3 - 15*a*b^2*n*x^m*log(c
)^4/m^2 + 9/2*a^2*b*n*x^(2*m)*log(c)^2*log(x)/m + 180*a*b^2*n^3*x^m*log(c)^
2*log(x)/m^3 - 9/4*a^2*b*n^3*x^(2*m)*log(x)^2/m^2 - 180*a*b^2*n^5*x^m*log(x
)^2/m^4 + 3/2*a^2*b*x^(2*m)*log(c)^3/m + 60*a*b^2*n^2*x^m*log(c)^3/m^3 - 9/
2*a^2*b*n^2*x^(2*m)*log(c)*log(x)/m^2 - 360*a*b^2*n^4*x^m*log(c)*log(x)/m^4
 - 9/4*a^2*b*n*x^(2*m)*log(c)^2/m^2 - 180*a*b^2*n^3*x^m*log(c)^2/m^4 + 1/3*
a^3*n*x^(3*m)*log(x)/m + 9/4*a^2*b*n^3*x^(2*m)*log(x)/m^3 + 360*a*b^2*n^5*x
```

```
^m*log(x)/m^5 + 1/3*a^3*x^(3*m)*log(c)/m + 9/4*a^2*b*n^2*x^(2*m)*log(c)/m^3
 + 360*a*b^2*n^4*x^m*log(c)/m^5 - 1/9*a^3*n*x^(3*m)/m^2 - 9/8*a^2*b*n^3*x^(
2*m)/m^4 - 360*a*b^2*n^5*x^m/m^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x,x)
```

```
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^3)/x, x)
```

3.10 $\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx$

Optimal. Leaf size=125

$$-\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log\left(cx^n\right)}{m^3}+\frac{a^2x^{2m}\log\left(cx^n\right)}{2m}-\frac{6abnx^m\log^2\left(cx^n\right)}{m^2}+\frac{2abx^m\log^3\left(cx^n\right)}{m}+\frac{b^2\log^6}{6n}$$

```
[Out] -12*a*b*n^3*x^m/m^4-1/4*a^2*n*x^(2*m)/m^2+12*a*b*n^2*x^m*ln(c*x^n)/m^3+1/2*
a^2*x^(2*m)*ln(c*x^n)/m-6*a*b*n*x^m*ln(c*x^n)^2/m^2+2*a*b*x^m*ln(c*x^n)^3/m
+1/6*b^2*ln(c*x^n)^6/n
```

Rubi [A]
time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2619, 2341, 2342, 2339, 30}

$$\frac{a^2x^{2m}\log\left(cx^n\right)}{2m}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log\left(cx^n\right)}{m^3}-\frac{6abnx^m\log^2\left(cx^n\right)}{m^2}+\frac{2abx^m\log^3\left(cx^n\right)}{m}-\frac{12abn^3x^m}{m^4}+\frac{b^2\log^6\left(cx^n\right)}{6n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] (-12*a*b*n^3*x^m)/m^4 - (a^2*n*x^(2*m))/(4*m^2) + (12*a*b*n^2*x^m*Log[c*x^n
])/m^3 + (a^2*x^(2*m)*Log[c*x^n])/(2*m) - (6*a*b*n*x^m*Log[c*x^n]^2)/m^2 +
(2*a*b*x^m*Log[c*x^n]^3)/m + (b^2*Log[c*x^n]^6)/(6*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)^2}{x}\,dx &= \int \left(a^2x^{-1+2m}\log(cx^n)+2abx^{-1+m}\log^3(cx^n)+\frac{b^2\log^5(cx^n)}{x}\right)dx \\
&= a^2\int x^{-1+2m}\log(cx^n)\,dx+(2ab)\int x^{-1+m}\log^3(cx^n)\,dx+b^2\int \frac{\log \\
&= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}+\frac{2abx^m\log^3(cx^n)}{m}+\frac{b^2\text{Subst}\left(\int x^5\,dx\right.}{n} \\
&= -\frac{a^2nx^{2m}}{4m^2}+\frac{a^2x^{2m}\log(cx^n)}{2m}-\frac{6abnx^m\log^2(cx^n)}{m^2}+\frac{2abx^m\log^3(cx}{m} \\
&= -\frac{12abn^3x^m}{m^4}-\frac{a^2nx^{2m}}{4m^2}+\frac{12abn^2x^m\log(cx^n)}{m^3}+\frac{a^2x^{2m}\log(cx^n)}{2m}-
\end{aligned}
$$

Mathematica [**A**]
time = 0.11, size = 193, normalized size = 1.54

$$
-\frac{1}{6}b^2n^5\log^6(x)+b^2n^4\log^5(x)\log(cx^n)-\frac{5}{2}b^2n^3\log^4(x)\log^2(cx^n)+\frac{10}{3}b^2n^2\log^3(x)\log^3(cx^n)-\frac{5}{2}b^2n\log^2(x)\log^4(cx^n)+b^2\log(x)\log^5(cx^n)+\frac{ax^m\left(-n(48bn^2+am^2x^m)+2m(24bn^2+am^2x^m)\log(cx^n)-24bm^2n\log^2(cx^n)+8bm^3\log^3(cx^n)\right)}{4m^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2)^2)/x,x]
```

```
[Out] -1/6*(b^2*n^5*Log[x]^6) + b^2*n^4*Log[x]^5*Log[c*x^n] - (5*b^2*n^3*Log[x]^4
*Log[c*x^n]^2)/2 + (10*b^2*n^2*Log[x]^3*Log[c*x^n]^3)/3 - (5*b^2*n*Log[x]^2
*Log[c*x^n]^4)/2 + b^2*Log[x]*Log[c*x^n]^5 + (a*x^m*(-(n*(48*b*n^2 + a*m^2*
x^m)) + 2*m*(24*b*n^2 + a*m^2*x^m)*Log[c*x^n] - 24*b*m^2*n*Log[c*x^n]^2 + 8
*b*m^3*Log[c*x^n]^3))/(4*m^4)
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.05, size = 14983, normalized size = 119.86

method	result	size

risch	Expression too large to display	14983

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(119) = 238$.
time = 0.32, size = 530, normalized size = 4.24

$$\frac{1}{10}\left(\frac{2b^2\log(cx^n)^5}{n} + \frac{20abx^m\log(cx^n)^2}{m} - 40ab\left(\frac{nx^m\log(cx^n)}{m^2} - \frac{n^2x^m}{m^3}\right) + \frac{5a^2x^{2m}}{m}\right)\log(cx^n) + \frac{2b^2m^4n^5\log(x)^6 - 12b^2m^4n^4\log(c)\log(x)^5 + 30b^2m^4n^3\log(c)^2\log(x)^4 - 40b^2m^4n^2\log(c)^3\log(x)^3 + 30b^2m^4n\log(c)^4\log(x)^2 - 12b^2m^4\log(c)^5\log(x) - 12b^2m^4\log(x)\log(x^n)^5 - 15a^2m^2nx^{2m} + 30\left(b^2m^4n\log(x)^2 - 2b^2m^4\log(c)\log(x)\right)\log(x^n)^4 - 120\left(m^2n\log(c)^2 - 4mn^2\log(c) + 6n^3\right)abx^m - 40\left(b^2m^4n^2\log(x)^3 - 3b^2m^4n\log(c)\log(x)^2 + 3b^2m^4\log(c)^2\log(x)\right)\log(x^n)^3 + 30\left(b^2m^4n^3\log(x)^4 - 4b^2m^4n^2\log(c)\log(x)^3 + 6b^2m^4n\log(c)^2\log(x)^2 - 4b^2m^4\log(c)^3\log(x) - 4abm^2nx^m\right)\log(x^n)^2 - 12\left(b^2m^4n^4\log(x)^5 - 5b^2m^4n^3\log(c)\log(x)^4 + 10b^2m^4n^2\log(c)^2\log(x)^3 - 10b^2m^4n\log(c)^3\log(x)^2 + 5b^2m^4\log(c)^4\log(x) + 20\left(m^2n\log(c) - 2mn^2\right)abx^m\right)\log(x^n)}{60m^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*(2*b^2*log(c*x^n)^5/n + 20*a*b*x^m*log(c*x^n)^2/m - 40*a*b*(n*x^m*log(
c*x^n)/m^2 - n^2*x^m/m^3) + 5*a^2*x^(2*m)/m)*log(c*x^n) + 1/60*(2*b^2*m^4*n
^5*log(x)^6 - 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*n^3*log(c)^2*log(
x)^4 - 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log(c)^4*log(x)^2 -
12*b^2*m^4*log(c)^5*log(x) - 12*b^2*m^4*log(x)*log(x^n)^5 - 15*a^2*m^2*n*x^
(2*m) + 30*(b^2*m^4*n*log(x)^2 - 2*b^2*m^4*log(c)*log(x))*log(x^n)^4 - 120*
(m^2*n*log(c)^2 - 4*m*n^2*log(c) + 6*n^3)*a*b*x^m - 40*(b^2*m^4*n^2*log(x)^
3 - 3*b^2*m^4*n*log(c)*log(x)^2 + 3*b^2*m^4*log(c)^2*log(x))*log(x^n)^3 + 3
0*(b^2*m^4*n^3*log(x)^4 - 4*b^2*m^4*n^2*log(c)*log(x)^3 + 6*b^2*m^4*n*log(c
)^2*log(x)^2 - 4*b^2*m^4*log(c)^3*log(x) - 4*a*b*m^2*n*x^m)*log(x^n)^2 - 12
*(b^2*m^4*n^4*log(x)^5 - 5*b^2*m^4*n^3*log(c)*log(x)^4 + 10*b^2*m^4*n^2*log
(c)^2*log(x)^3 - 10*b^2*m^4*n*log(c)^3*log(x)^2 + 5*b^2*m^4*log(c)^4*log(x)
 + 20*(m^2*n*log(c) - 2*m*n^2)*a*b*x^m)*log(x^n))/m^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(119) = 238$.
time = 0.34, size = 267, normalized size = 2.14

$$\frac{2b^2m^4n^5\log(x)^6 + 12b^2m^4n^4\log(c)\log(x)^5 + 30b^2m^4n^3\log(c)^2\log(x)^4 + 40b^2m^4n^2\log(c)^3\log(x)^3 + 30b^2m^4n\log(c)^4\log(x)^2 + 12b^2m^4\log(c)^5\log(x) + 3\left(2a^2m^3n\log(x) + 2a^2m^3\log(c) - a^2m^2n\right)x^{2m} + 24\left(abm^3n^3\log(x)^3 + abm^3\log(c)^3 - 3abm^2n\log(c)^2 + 6abmn^2\log(c) - 6abn^3 + 3\left(abm^3n^2\log(c) - abm^2n^3\right)\log(x)^2 + 3\left(abm^3n\log(c)^2 - 2abm^2n^2\log(c) + 2abmn^3\right)\log(x)\right)x^m}{12m^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^2*m^4*n^5*log(x)^6 + 12*b^2*m^4*n^4*log(c)*log(x)^5 + 30*b^2*m^4*
n^3*log(c)^2*log(x)^4 + 40*b^2*m^4*n^2*log(c)^3*log(x)^3 + 30*b^2*m^4*n*log
(c)^4*log(x)^2 + 12*b^2*m^4*log(c)^5*log(x) + 3*(2*a^2*m^3*n*log(x) + 2*a^2
*m^3*log(c) - a^2*m^2*n)*x^(2*m) + 24*(a*b*m^3*n^3*log(x)^3 + a*b*m^3*log(c
)^3 - 3*a*b*m^2*n*log(c)^2 + 6*a*b*m*n^2*log(c) - 6*a*b*n^3 + 3*(a*b*m^3*n^
```

```
2*log(c) - a*b*m^2*n^3)*log(x)^2 + 3*(a*b*m^3*n*log(c)^2 - 2*a*b*m^2*n^2*lo
g(c) + 2*a*b*m*n^3)*log(x))*x^m)/m^4
```

Sympy [A]
time = 15.72, size = 216, normalized size = 1.73

$$-a^{2}n\left(\begin{cases}\frac{\begin{cases}\frac{x^{2m}}{2m} & \text{for}\ m\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{2m} & \text{for}\ m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log(x)^{2}}{2} & \text{otherwise}\end{cases}\right)+a^{2}\left(\begin{cases}\frac{x^{2m}}{2m} & \text{for}\ m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^{n})+2ab\left(\begin{cases}\frac{x^{m}\log(cx^{n})^{3}}{m}-\frac{3nx^{m}\log(cx^{n})^{2}}{m^{2}}+\frac{6n^{2}x^{m}\log(cx^{n})}{m^{3}}-\frac{6n^{3}x^{m}}{m^{4}} & \text{for}\ m\neq 0\\ \begin{cases}0 & \text{for}\ \frac{1}{|cx^{n}|}<1\wedge|cx^{n}|<1\\ \frac{\log(cx^{n})^{4}}{4n} & \text{for}\ |cx^{n}|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^{4}}{4n} & \text{for}\ \frac{1}{|cx^{n}|}<1\\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1\\ 0,0,0,0,0 & \end{matrix}\middle|cx^{n}\right)}{n}+\frac{6G_{5,5}^{0,5}\left(\begin{matrix}1,1,1,1,1 & \\ & 0,0,0,0,0\end{matrix}\middle|cx^{n}\right)}{n} & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}\right)-b^{2}\left(\begin{cases}-\log(c)^{5}\log(x) & \text{for}\ n=0\\ -\frac{\log(cx^{n})^{6}}{6n} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)**2/x,x)
```

```
[Out] -a**2*n*Piecewise((Piecewise((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))/(2
*m), (m > -oo) & (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a**2*Piecewis
e((x**(2*m)/(2*m), Ne(m, 0)), (log(x), True))*log(c*x**n) + 2*a*b*Piecewise
((x**m*log(c*x**n)**3/m - 3*n*x**m*log(c*x**n)**2/m**2 + 6*n**2*x**m*log(c*
x**n)/m**3 - 6*n**3*x**m/m**4, Ne(m, 0)), (Piecewise((0, (Abs(c*x**n) < 1)
& (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Abs(c*x**n) < 1), (log(1/(c*
x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(((), (1, 1, 1, 1, 1)), ((0,
 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1, 1, 1, 1), ()), ((), (0, 0,
 0, 0, 0)), c*x**n)/n, True)), True)) - b**2*Piecewise((-log(c)**5*log(x),
Eq(n, 0)), (-log(c*x**n)**6/(6*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(119) = 238.
time = 7.17, size = 286, normalized size = 2.29

$$\frac{1}{6}b^{2}n^{5}\log(x)^{6}+b^{2}n^{4}\log(c)\log(x)^{5}+\frac{5}{2}b^{2}n^{3}\log(c)^{2}\log(x)^{4}+\frac{10}{3}b^{2}n^{2}\log(c)^{3}\log(x)^{3}+\frac{5}{2}b^{2}n\log(c)^{4}\log(x)^{2}+b^{2}\log(c)^{5}\log(x)+\frac{2abn^{3}x^{m}\log(x)^{3}}{m}+\frac{6abn^{2}x^{m}\log(c)\log(x)^{2}}{m}+\frac{6abnx^{m}\log(c)^{2}\log(x)}{m}-\frac{6abn^{3}x^{m}\log(x)^{2}}{m^{2}}+\frac{2abx^{m}\log(c)^{3}}{m}-\frac{12abn^{2}x^{m}\log(c)\log(x)}{m^{2}}-\frac{6abnx^{m}\log(c)^{2}}{m^{2}}+\frac{a^{2}nx^{2m}\log(x)}{2m}+\frac{12abn^{3}x^{m}\log(x)}{m^{3}}+\frac{a^{2}x^{2m}\log(c)}{2m}+\frac{12abn^{2}x^{m}\log(c)}{m^{3}}-\frac{a^{2}nx^{2m}}{4m^{2}}-\frac{12abn^{3}x^{m}}{m^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*n^5*log(x)^6 + b^2*n^4*log(c)*log(x)^5 + 5/2*b^2*n^3*log(c)^2*log(x
)^4 + 10/3*b^2*n^2*log(c)^3*log(x)^3 + 5/2*b^2*n*log(c)^4*log(x)^2 + b^2*lo
g(c)^5*log(x) + 2*a*b*n^3*x^m*log(x)^3/m + 6*a*b*n^2*x^m*log(c)*log(x)^2/m
+ 6*a*b*n*x^m*log(c)^2*log(x)/m - 6*a*b*n^3*x^m*log(x)^2/m^2 + 2*a*b*x^m*lo
g(c)^3/m - 12*a*b*n^2*x^m*log(c)*log(x)/m^2 - 6*a*b*n*x^m*log(c)^2/m^2 + 1/
2*a^2*n*x^(2*m)*log(x)/m + 12*a*b*n^3*x^m*log(x)/m^3 + 1/2*a^2*x^(2*m)*log(
c)/m + 12*a*b*n^2*x^m*log(c)/m^3 - 1/4*a^2*n*x^(2*m)/m^2 - 12*a*b*n^3*x^m/m
^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln(c\,x^n)\,\left(a\,x^m+b\,\ln(c\,x^n)^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x,x)
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2)^2)/x, x)
```

3.11 $\int \frac{\log(cx^n)(ax^m+b\log^2(cx^n))}{x}\,dx$

Optimal. Leaf size=41

$$-\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\log^4(cx^n)}{4n}$$

```
[Out] -a*n*x^m/m^2+a*x^m*ln(c*x^n)/m+1/4*b*ln(c*x^n)^4/n
```

Rubi [A]
time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2619, 2341, 2339, 30}

$$\frac{ax^m\log(cx^n)}{m}-\frac{anx^m}{m^2}+\frac{b\log^4(cx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -((a*n*x^m)/m^2) + (a*x^m*Log[c*x^n])/m + (b*Log[c*x^n]^4)/(4*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2619

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*
(x_)^(m_.))^(p_.))/(x_), x_Symbol] :> Int[ExpandIntegrand[Log[c*x^n]^r/x, (
a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b, c, m, n, p, q, r}, x] &&
EqQ[r, q - 1] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log(cx^n)\left(ax^m+b\log^2(cx^n)\right)}{x}\,dx = \int \left(ax^{-1+m}\log(cx^n)+\frac{b\log^3(cx^n)}{x}\right)dx$$
$$= a\int x^{-1+m}\log(cx^n)\,dx + b\int \frac{\log^3(cx^n)}{x}\,dx$$
$$= -\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\mathrm{Subst}\left(\int x^3\,dx, x, \log(cx^n)\right)}{n}$$
$$= -\frac{anx^m}{m^2}+\frac{ax^m\log(cx^n)}{m}+\frac{b\log^4(cx^n)}{4n}$$

Mathematica [A]
time = 0.05, size = 78, normalized size = 1.90

$$-\frac{1}{4}bn^3\log^4(x)+bn^2\log^3(x)\log(cx^n)-\frac{3}{2}bn\log^2(x)\log^2(cx^n)+b\log(x)\log^3(cx^n)+\frac{ax^m(-n+m\log(cx^n))}{m^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[c*x^n]*(a*x^m + b*Log[c*x^n]^2))/x,x]
```

```
[Out] -1/4*(b*n^3*Log[x]^4) + b*n^2*Log[x]^3*Log[c*x^n] - (3*b*n*Log[x]^2*Log[c*x
^n]^2)/2 + b*Log[x]*Log[c*x^n]^3 + (a*x^m*(-n + m*Log[c*x^n]))/m^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 2146, normalized size = 52.34

method	result	size
risch	Expression too large to display	2146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)*(a*x^m+b*ln(c*x^n)^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*ln(x)*csgn(I*c*x^n)^3*csgn(I*x^n)*csgn(I*c)^2*b*ln(c)*Pi^2+3/2*ln(x)*cs
gn(I*c*x^n)^3*csgn(I*x^n)^2*csgn(I*c)*b*ln(c)*Pi^2+3/2*csgn(I*c*x^n)^4*csgn
(I*x^n)*csgn(I*c)*n*b*Pi^2*ln(x)^2-3/4*csgn(I*c*x^n)^3*csgn(I*x^n)^2*csgn(I
*c)*n*b*Pi^2*ln(x)^2-1/2*I*Pi*b*n^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(
x)^3+(-3/2*b*n*ln(x)^2-3/2*I*Pi*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(x)
+3/2*I*Pi*b*csgn(I*c)*csgn(I*c*x^n)^2*ln(x)+3/2*I*Pi*b*csgn(I*x^n)*csgn(I*c
*x^n)^2*ln(x)-3/2*I*Pi*b*csgn(I*c*x^n)^3*ln(x)+3*ln(c)*b*ln(x))*ln(x^n)^2+1
/m*ln(c)*a*x^m+ln(c)*b*n^2*ln(x)^3-3/2*ln(x)^2*ln(c)^2*b*n-a*n*x^m/m^2+3/2*
I*ln(x)^2*Pi*ln(c)*b*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/4*(-3*Pi^2*b*c
sgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*ln(x)*m+6*Pi^2*b*csgn(I*c)^2*csgn(
I*x^n)*csgn(I*c*x^n)^3*ln(x)*m-3*Pi^2*b*csgn(I*c)^2*csgn(I*c*x^n)^4*ln(x)*m
+6*Pi^2*b*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3*ln(x)*m-12*Pi^2*b*csgn(I*
```

```
c)*csgn(I*x^n)*csgn(I*c*x^n)^4*ln(x)*m+6*Pi^2*b*csgn(I*c)*csgn(I*c*x^n)^5*l
n(x)*m-3*Pi^2*b*csgn(I*x^n)^2*csgn(I*c*x^n)^4*ln(x)*m+6*Pi^2*b*csgn(I*x^n)*
csgn(I*c*x^n)^5*ln(x)*m-3*Pi^2*b*csgn(I*c*x^n)^6*ln(x)*m+12*I*Pi*ln(c)*b*cs
gn(I*x^n)*csgn(I*c*x^n)^2*ln(x)*m-12*I*Pi*ln(c)*b*csgn(I*c*x^n)^3*ln(x)*m+6
*I*Pi*b*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*ln(x)^2*m+12*I*Pi*ln(c)*b*csg
n(I*c)*csgn(I*c*x^n)^2*ln(x)*m+6*I*Pi*b*n*csgn(I*c*x^n)^3*ln(x)^2*m-6*I*Pi*
b*n*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)^2*m-12*I*Pi*ln(c)*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)*ln(x)*m-6*I*Pi*b*n*csgn(I*c)*csgn(I*c*x^n)^2*ln(x)^2*m+4
*b*n^2*ln(x)^3*m-12*ln(c)*b*n*ln(x)^2*m+12*ln(c)^2*b*ln(x)*m+4*a*x^m)/m*ln(
x^n)+3/8*csgn(I*c*x^n)^6*n*b*Pi^2*ln(x)^2-3/4*ln(x)*csgn(I*c*x^n)^6*b*ln(c)
*Pi^2-3/2*I*ln(x)^2*Pi*ln(c)*b*n*csgn(I*c)*csgn(I*c*x^n)^2-3/2*I*ln(x)^2*Pi
*ln(c)*b*n*csgn(I*x^n)*csgn(I*c*x^n)^2-3/2*I*Pi*ln(c)^2*b*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)*ln(x)-1/2*I/m*Pi*a*csgn(I*c*x^n)^3*x^m-1/2*I*Pi*b*n^2*cs
gn(I*c*x^n)^3*ln(x)^3-3/2*I*Pi*ln(c)^2*b*csgn(I*c*x^n)^3*ln(x)-1/8*I*Pi^3*b
*csgn(I*c)^3*csgn(I*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*c)^2*csgn(I*c*x^n)^7
*ln(x)-3/8*I*Pi^3*b*csgn(I*c)*csgn(I*c*x^n)^8*ln(x)-1/8*I*Pi^3*b*csgn(I*x^n
)^3*csgn(I*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*x^n)^2*csgn(I*c*x^n)^7*ln(x)-
3/8*I*Pi^3*b*csgn(I*x^n)*csgn(I*c*x^n)^8*ln(x)+1/8*I*ln(x)*csgn(I*c*x^n)^3*
csgn(I*x^n)^3*csgn(I*c)^3*b*Pi^3-3/8*I*Pi^3*b*csgn(I*c)^3*csgn(I*x^n)^2*csg
n(I*c*x^n)^4*ln(x)+3/8*I*Pi^3*b*csgn(I*c)^3*csgn(I*x^n)*csgn(I*c*x^n)^5*ln(
x)-3/8*I*Pi^3*b*csgn(I*c)^2*csgn(I*x^n)^3*csgn(I*c*x^n)^4*ln(x)+9/8*I*Pi^3*
b*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^5*ln(x)+1/2*I*Pi*b*n^2*csgn(I*c)*
csgn(I*c*x^n)^2*ln(x)^3+1/2*I*Pi*b*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2*ln(x)^3+
3/2*I*ln(x)^2*Pi*ln(c)*b*n*csgn(I*c*x^n)^3+3/2*I*Pi*ln(c)^2*b*csgn(I*c)*csg
n(I*c*x^n)^2*ln(x)+1/2*I/m*Pi*a*csgn(I*c)*csgn(I*c*x^n)^2*x^m+1/2*I/m*Pi*a*
csgn(I*x^n)*csgn(I*c*x^n)^2*x^m-9/8*I*Pi^3*b*csgn(I*c)^2*csgn(I*x^n)*csgn(I
*c*x^n)^6*ln(x)+3/8*I*Pi^3*b*csgn(I*c)*csgn(I*x^n)^3*csgn(I*c*x^n)^5*ln(x)-
9/8*I*Pi^3*b*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^6*ln(x)+9/8*I*Pi^3*b*csg
n(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^7*ln(x)+3/2*I*Pi*ln(c)^2*b*csgn(I*x^n)*csg
n(I*c*x^n)^2*ln(x)+b*ln(x)*ln(x^n)^3+1/8*I*ln(x)*csgn(I*c*x^n)^9*b*Pi^3-1/2
*I/m*Pi*a*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*x^m+ln(c)^3*ln(x)*b-1/4*b*n^3
*ln(x)^4+3/8*csgn(I*c*x^n)^2*csgn(I*x^n)^2*csgn(I*c)^2*n*b*Pi^2*ln(x)^2-3/4
*csgn(I*c*x^n)^3*csgn(I*x^n)*csgn(I*c)^2*n*b*Pi^2*ln(x)^2-3*ln(x)*csgn(I*c*
x^n)^4*csgn(I*x^n)*csgn(I*c)*b*ln(c)*Pi^2-3/4*ln(x)*csgn(I*c*x^n)^2*csgn(I*
x^n)^2*csgn(I*c)^2*b*ln(c)*Pi^2+3/2*ln(x)*csgn(I*c*x^n)^5*csgn(I*x^n)*b*ln(
c)*Pi^2+3/8*csgn(I*c*x^n)^4*csgn(I*c)^2*n*b*Pi^2*ln(x)^2-3/4*ln(x)*csgn(I*c
*x^n)^4*csgn(I*c)^2*b*ln(c)*Pi^2+3/2*ln(x)*csgn(I*c*x^n)^5*csgn(I*c)*b*ln(c
)*Pi^2-3/4*ln(x)*csgn(I*c*x^n)^4*csgn(I*x^n)^2*b*ln(c)*Pi^2-3/4*csgn(I*c*x^
n)^5*csgn(I*c)*n*b*Pi^2*ln(x)^2+3/8*csgn(I*c*x^n)^4*csgn(I*x^n)^2*n*b*Pi^2*
ln(x)^2-3/4*csgn(I*c*x^n)^5*csgn(I*x^n)*n*b*Pi^2*ln(x)^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(39) = 78$.

time = 0.30, size = 186, normalized size = 4.54

$$\frac{1}{3}\left(\frac{b\log\left(cx^{n}\right)^{3}}{n}+\frac{3\,ax^{m}}{m}\right)\log\left(cx^{n}\right)+\frac{bm^{2}n^{3}\log\left(x\right)^{4}-4\,bm^{2}n^{2}\log\left(c\right)\log\left(x\right)^{3}+6\,bm^{2}n\log\left(c\right)^{2}\log\left(x\right)^{2}-4\,bm^{2}\log\left(c\right)^{3}\log\left(x\right)-4\,bm^{2}\log\left(x\right)\log\left(x^{n}\right)^{3}-12\,anx^{m}+6\left(bm^{2}n\log\left(x\right)^{2}-2\,bm^{2}\log\left(c\right)\log\left(x\right)\right)\log\left(x^{n}\right)^{2}-4\left(bm^{2}n^{2}\log\left(x\right)^{3}-3\,bm^{2}n\log\left(c\right)\log\left(x\right)^{2}+3\,bm^{2}\log\left(c\right)^{2}\log\left(x\right)\right)\log\left(x^{n}\right)}{12\,m^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*log(c*x^n)^3/n + 3*a*x^m/m)*log(c*x^n) + 1/12*(b*m^2*n^3*log(x)^4 -
4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*log(x)^2 - 4*b*m^2*log(c)^
3*log(x) - 4*b*m^2*log(x)*log(x^n)^3 - 12*a*n*x^m + 6*(b*m^2*n*log(x)^2 - 2
*b*m^2*log(c)*log(x))*log(x^n)^2 - 4*(b*m^2*n^2*log(x)^3 - 3*b*m^2*n*log(c)
*log(x)^2 + 3*b*m^2*log(c)^2*log(x))*log(x^n))/m^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(39) = 78.
time = 0.39, size = 81, normalized size = 1.98

$$\frac{bm^2n^3\log(x)^4+4\,bm^2n^2\log(c)\log(x)^3+6\,bm^2n\log(c)^2\log(x)^2+4\,bm^2\log(c)^3\log(x)+4\,(amn\log(x)+am\log(c)-an)x^m}{4\,m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(b*m^2*n^3*log(x)^4 + 4*b*m^2*n^2*log(c)*log(x)^3 + 6*b*m^2*n*log(c)^2*
log(x)^2 + 4*b*m^2*log(c)^3*log(x) + 4*(a*m*n*log(x) + a*m*log(c) - a*n)*x^
m)/m^2
```

Sympy [A]
time = 7.02, size = 65, normalized size = 1.59

$$-an\left(\begin{cases}\frac{\begin{cases}\frac{x^m}{m} & \text{for}\ m\neq 0\\ \log(x) & \text{otherwise}\end{cases}}{m} & \text{for}\ m>-\infty\wedge m<\infty\wedge m\neq 0\\ \frac{\log(x)^2}{2} & \text{otherwise}\end{cases}\right)+a\left(\begin{cases}\frac{x^m}{m} & \text{for}\ m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)\log(cx^n)-b\left(\begin{cases}-\log(c)^3\log(x) & \text{for}\ n=0\\ -\frac{\log(cx^n)^4}{4n} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)*(a*x**m+b*ln(c*x**n)**2)/x,x)
```

```
[Out] -a*n*Piecewise((Piecewise((x**m/m, Ne(m, 0)), (log(x), True))/m, (m > -oo)
& (m < oo) & Ne(m, 0)), (log(x)**2/2, True)) + a*Piecewise((x**m/m, Ne(m, 0
)), (log(x), True))*log(c*x**n) - b*Piecewise((-log(c)**3*log(x), Eq(n, 0))
, (-log(c*x**n)**4/(4*n), True))
```

Giac [A]
time = 8.84, size = 73, normalized size = 1.78

$$\frac{1}{4}\,bn^3\log(x)^4+bn^2\log(c)\log(x)^3+\frac{3}{2}\,bn\log(c)^2\log(x)^2+b\log(c)^3\log(x)+\frac{anx^m\log(x)}{m}+\frac{ax^m\log(c)}{m}-\frac{anx^m}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)*(a*x^m+b*log(c*x^n)^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b*n^3*log(x)^4 + b*n^2*log(c)*log(x)^3 + 3/2*b*n*log(c)^2*log(x)^2 + b*
log(c)^3*log(x) + a*n*x^m*log(x)/m + a*x^m*log(c)/m - a*n*x^m/m^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\,x^n\right)\,\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x,x)
[Out] int((log(c*x^n)*(a*x^m + b*log(c*x^n)^2))/x, x)
```

3.12 $\int \frac{\log(cx^n)}{x} \, dx$

Optimal. Leaf size=15

$$\frac{\log^2(cx^n)}{2n}$$

[Out] 1/2*ln(c*x^n)^2/n

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2338}

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

[In] Int[Log[c*x^n]/x,x]

[Out] Log[c*x^n]^2/(2*n)

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int \frac{\log(cx^n)}{x} \, dx = \frac{\log^2(cx^n)}{2n}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c*x^n]/x,x]

[Out] Log[c*x^n]^2/(2*n)

Maple [A]
time = 0.04, size = 14, normalized size = 0.93

method	result
derivativedivides	$\frac{\ln(c\,x^n)^2}{2n}$
default	$\frac{\ln(c\,x^n)^2}{2n}$
norman	$\frac{\ln\left(c\,e^{n\ln(x)}\right)^2}{2n}$
risch	$\ln(x)\ln(x^n) - \frac{n\ln(x)^2}{2} - \frac{i\pi\ln(x)\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)}{2} + \frac{i\pi\ln(x)\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2}{2} + \frac{i\pi\ln(x)\mathrm{csg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(c*x^n)^2/n
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.87

$$\frac{\log\left(cx^n\right)^2}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="maxima")
[Out] 1/2*log(c*x^n)^2/n
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.87

$$\frac{1}{2}\,n\log\left(x\right)^2 + \log\left(c\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="fricas")
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(10) = 20.
time = 1.13, size = 65, normalized size = 4.33

$$\begin{cases} 0 & \text{for}\ \frac{1}{|cx^n|} < 1 \wedge |cx^n| < 1 \\ \frac{\log\left(cx^n\right)^2}{2n} & \text{for}\ |cx^n| < 1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for}\ \frac{1}{|cx^n|} < 1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x,x)
```

```
[Out] Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2*
n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (meije
rg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), (
(), (0, 0, 0)), c*x**n)/n, True))
```

Giac [A]
time = 6.68, size = 13, normalized size = 0.87

$$\frac{1}{2}\,n\log\left(x\right)^{2}+\log\left(c\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*log(x)^2 + log(c)*log(x)
```

Mupad [B]
time = 0.25, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^n\right)^2}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/x,x)
```

```
[Out] log(c*x^n)^2/(2*n)
```

3.13 $\int \frac{\log(cx^n)}{x(ax^m+b\log^2(cx^n))} dx$

Optimal. Leaf size=67

$$\frac{\log\left(ax^m+b\log^2\left(cx^n\right)\right)}{2bn}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^2(cx^n)},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2),x)/b/n+1/2*ln(a*x^m
+b*ln(c*x^n)^2)/b/n
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^2]/(2*b*n) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m + b
*Log[c*x^n]^2), x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx=\frac{\log\left(ax^m+b\log^2\left(cx^n\right)\right)}{2bn}-\frac{(am)\int \frac{x^{-1+m}}{ax^m+b\log^2(cx^n)}\,dx}{2bn}$$

Mathematica [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,x^n\right)}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b*x*log(c*x^n)^2 + a*x*x^m), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m + b\log\left(cx^n\right)^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2),x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln (c\,x^n)}{x\,\left(a\,x^m+b\ln (c\,x^n)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)),x)
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)), x)
```

3.14 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{2bn\left(ax^m+b\log^2\left(cx^n\right)\right)}-\frac{am\text{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^2,x)/b/n-1/2/b/n/(a
*x^m+b*ln(c*x^n)^2)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] -1/2*1/(b*n*(a*x^m + b*Log[c*x^n]^2)) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m +
 b*Log[c*x^n]^2)^2, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx=-\frac{1}{2bn\left(ax^m+b\log^2\left(cx^n\right)\right)}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^2}\,dx}{2bn}$$

Mathematica [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^2), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(c\,x^{n}\right)}{x \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="maxima")
```

```
[Out] -(m*log(c) + m*log(x^n) + 2*n)/(4*b^2*n^2*log(c)^2 + a^2*m^2*x^(2*m) + (m^2
*log(c)^2 + 4*n^2)*a*b*x^m + (a*b*m^2*x^m + 4*b^2*n^2)*log(x^n)^2 + 2*(a*b*
m^2*x^m*log(c) + 4*b^2*n^2*log(c))*log(x^n)) - integrate((a*m^4*x^m*log(x^n
) + 4*b*m*n^3 + (m^4*log(c) + 3*m^3*n)*a*x^m)/(16*b^3*n^4*x*log(c)^2 + a^3*
m^4*x*x^(3*m) + (m^4*log(c)^2 + 8*m^2*n^2)*a^2*b*x*x^(2*m) + 8*(m^2*n^2*log
(c)^2 + 2*n^4)*a*b^2*x*x^m + (a^2*b*m^4*x*x^(2*m) + 8*a*b^2*m^2*n^2*x*x^m +
 16*b^3*n^4*x)*log(x^n)^2 + 2*(a^2*b*m^4*x*x^(2*m)*log(c) + 8*a*b^2*m^2*n^2
*x*x^m*log(c) + 16*b^3*n^4*x*log(c))*log(x^n)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^2*x*log(c*x^n)^4 + 2*a*b*x*x^m*log(c*x^n)^2 + a^2*x*
x^(2*m)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(cx^{n}\right)}{x \left(ax^{m}+b \log \left(cx^{n}\right)^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**2,x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)}{x\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2),x)
```

```
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^2), x)
```

3.15 $\int \frac{\log(cx^n)}{x\left(ax^m+b\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}-\frac{am\mathrm{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^3},x\right)}{2bn}$$

```
[Out] -1/2*a*m*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^2)^3,x)/b/n-1/4/b/n/(a
*x^m+b*ln(c*x^n)^2)^2
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] -1/4*1/(b*n*(a*x^m + b*Log[c*x^n]^2)^2) - (a*m*Defer[Int][x^(-1 + m)/(a*x^m
 + b*Log[c*x^n]^2)^3, x])/(2*b*n)
```

Rubi steps

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx=-\frac{1}{4bn\left(ax^m+b\log^2\left(cx^n\right)\right)^2}-\frac{(am)\int \frac{x^{-1+m}}{\left(ax^m+b\log^2(cx^n)\right)^3}\,dx}{2bn}$$

Mathematica [A]
time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{\log\left(cx^n\right)}{x\left(ax^m+b\log^2\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3),x]
```

```
[Out] Integrate[Log[c*x^n]/(x*(a*x^m + b*Log[c*x^n]^2)^3), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(c\,x^{n}\right)}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

```
[Out] int(ln(c*x^n)/x/(a*x^m+b*ln(c*x^n)^2)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(24*b^3*m*n^4*log(c)^3 - 5*a^3*m^4*n*x^(3*m) - (m^5*log(c)^3 + 7*m^4*n
*log(c)^2 - 18*m^3*n^2*log(c) - 4*m^2*n^3)*a^2*b*x^(2*m) + 2*(5*m^3*n^2*log
(c)^3 - 6*m^2*n^3*log(c)^2 + 20*m*n^4*log(c) + 16*n^5)*a*b^2*x^m - (a^2*b*m
^5*x^(2*m) - 10*a*b^2*m^3*n^2*x^m - 24*b^3*m*n^4)*log(x^n)^3 + (72*b^3*m*n^
4*log(c) - (3*m^5*log(c) + 7*m^4*n)*a^2*b*x^(2*m) + 6*(5*m^3*n^2*log(c) - 2
*m^2*n^3)*a*b^2*x^m)*log(x^n)^2 + (72*b^3*m*n^4*log(c)^2 - (3*m^5*log(c)^2
+ 14*m^4*n*log(c) - 18*m^3*n^2)*a^2*b*x^(2*m) + 2*(15*m^3*n^2*log(c)^2 - 12
*m^2*n^3*log(c) + 20*m*n^4)*a*b^2*x^m)*log(x^n))/(64*a*b^5*n^6*x^m*log(c)^4
 + a^6*m^6*x^(6*m) + 2*(m^6*log(c)^2 + 6*m^4*n^2)*a^5*b*x^(5*m) + (m^6*log(
c)^4 + 24*m^4*n^2*log(c)^2 + 48*m^2*n^4)*a^4*b^2*x^(4*m) + 4*(3*m^4*n^2*log
(c)^4 + 24*m^2*n^4*log(c)^2 + 16*n^6)*a^3*b^3*x^(3*m) + 16*(3*m^2*n^4*log(c
)^4 + 8*n^6*log(c)^2)*a^2*b^4*x^(2*m) + (a^4*b^2*m^6*x^(4*m) + 12*a^3*b^3*m
^4*n^2*x^(3*m) + 48*a^2*b^4*m^2*n^4*x^(2*m) + 64*a*b^5*n^6*x^m)*log(x^n)^4
+ 4*(a^4*b^2*m^6*x^(4*m)*log(c) + 12*a^3*b^3*m^4*n^2*x^(3*m)*log(c) + 48*a^
2*b^4*m^2*n^4*x^(2*m)*log(c) + 64*a*b^5*n^6*x^m*log(c))*log(x^n)^3 + 2*(192
*a*b^5*n^6*x^m*log(c)^2 + a^5*b*m^6*x^(5*m) + 3*(m^6*log(c)^2 + 4*m^4*n^2)*
a^4*b^2*x^(4*m) + 12*(3*m^4*n^2*log(c)^2 + 4*m^2*n^4)*a^3*b^3*x^(3*m) + 16*
(9*m^2*n^4*log(c)^2 + 4*n^6)*a^2*b^4*x^(2*m))*log(x^n)^2 + 4*(64*a*b^5*n^6*
x^m*log(c)^3 + a^5*b*m^6*x^(5*m)*log(c) + (m^6*log(c)^3 + 12*m^4*n^2*log(c)
)*a^4*b^2*x^(4*m) + 12*(m^4*n^2*log(c)^3 + 4*m^2*n^4*log(c))*a^3*b^3*x^(3*m
) + 16*(3*m^2*n^4*log(c)^3 + 4*n^6*log(c))*a^2*b^4*x^(2*m))*log(x^n)) + int
egrate(1/2*((2*m^8*log(c) + 15*m^7*n)*a^3*x^(3*m) - 2*(17*m^6*n^2*log(c) -
m^5*n^3)*a^2*b*x^(2*m) - 32*(3*m^4*n^4*log(c) + 2*m^3*n^5)*a*b^2*x^m - 96*(
m^2*n^6*log(c) + m*n^7)*b^3 + 2*(a^3*m^8*x^(3*m) - 17*a^2*b*m^6*n^2*x^(2*m)
 - 48*a*b^2*m^4*n^4*x^m - 48*b^3*m^2*n^6)*log(x^n))/(256*a*b^5*n^8*x*x^m*lo
g(c)^2 + a^6*m^8*x*x^(6*m) + (m^8*log(c)^2 + 16*m^6*n^2)*a^5*b*x*x^(5*m) +
```

```
16*(m^6*n^2*log(c)^2 + 6*m^4*n^4)*a^4*b^2*x*x^(4*m) + 32*(3*m^4*n^4*log(c)^
2 + 8*m^2*n^6)*a^3*b^3*x*x^(3*m) + 256*(m^2*n^6*log(c)^2 + n^8)*a^2*b^4*x*x
^(2*m) + (a^5*b*m^8*x*x^(5*m) + 16*a^4*b^2*m^6*n^2*x*x^(4*m) + 96*a^3*b^3*m
^4*n^4*x*x^(3*m) + 256*a^2*b^4*m^2*n^6*x*x^(2*m) + 256*a*b^5*n^8*x*x^m)*log
(x^n)^2 + 2*(a^5*b*m^8*x*x^(5*m)*log(c) + 16*a^4*b^2*m^6*n^2*x*x^(4*m)*log(
c) + 96*a^3*b^3*m^4*n^4*x*x^(3*m)*log(c) + 256*a^2*b^4*m^2*n^6*x*x^(2*m)*lo
g(c) + 256*a*b^5*n^8*x*x^m*log(c))*log(x^n)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="fricas")
```

```
[Out] integral(log(c*x^n)/(b^3*x*log(c*x^n)^6 + 3*a*b^2*x*x^m*log(c*x^n)^4 + 3*a^
2*b*x*x^(2*m)*log(c*x^n)^2 + a^3*x*x^(3*m)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(cx^{n} \right)}}{x \left(ax^{m} + b \log{\left(cx^{n} \right)}^{2}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**n)/x/(a*x**m+b*ln(c*x**n)**2)**3,x)
```

```
[Out] Integral(log(c*x**n)/(x*(a*x**m + b*log(c*x**n)**2)**3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^n)/x/(a*x^m+b*log(c*x^n)^2)^3,x, algorithm="giac")
```

```
[Out] integrate(log(c*x^n)/((b*log(c*x^n)^2 + a*x^m)^3*x), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(c\,x^n\right)}{x\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3),x)
```

```
[Out] int(log(c*x^n)/(x*(a*x^m + b*log(c*x^n)^2)^3), x)
```

3.16 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx$

Optimal. Leaf size=26

$$\frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

```
[Out] (a*x^m+b*ln(c*x^n)^q)^(1+p)/(1+p)
```

Rubi [A]
time = 0.12, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2624}

$$\frac{(ax^m+b\log^q(cx^n))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^p}{x}\,dx = \frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

Mathematica [A]
time = 0.07, size = 26, normalized size = 1.00

$$\frac{(ax^m+b\log^q(cx^n))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p
)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^(1 + p)/(1 + p)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(am\,x^{m}+bnq\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.38, size = 42, normalized size = 1.62

$$\frac{((n\log(x)+\log(c))^{q}b+ax^{m})((n\log(x)+\log(c))^{q}b+ax^{m})^{p}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="fricas")
```

```
[Out] ((n*log(x) + log(c))^q*b + a*x^m)*((n*log(x) + log(c))^q*b + a*x^m)^p/(p +
1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,3,1,2,3]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,2,1,2,3]%%%}+%%%{5,[0,0,2
,4,2,0,4,
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a\,m\,x^m+b\,n\,q\,\ln\left(c\,x^n\right)^{q-1}\right)\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x,x)
```

```
[Out] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^p)/x, x)
```

3.17 $\int \frac{\left(amx^m+bnq\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x} dx$

Optimal. Leaf size=22

$$\frac{1}{3}(ax^m + b\log^q(cx^n))^3$$

```
[Out] 1/3*(a*x^m+b*ln(c*x^n)^q)^3
```

Rubi [A]
time = 0.11, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2624}

$$\frac{1}{3}(ax^m + b\log^q(cx^n))^3$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^3/3
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{\left(amx^m + bnq\log^{-1+q}(cx^n)\right)\left(ax^m + b\log^q(cx^n)\right)^2}{x} dx = \frac{1}{3}(ax^m + b\log^q(cx^n))^3$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$\frac{1}{3}(ax^m + b\log^q(cx^n))^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2
)/x,x]
```

`[Out] (a*x^m + b*Log[c*x^n]^q)^3/3`

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 204, normalized size = 9.27

method	result
risch	$\frac{a^3 x^{3m}}{3}+\frac{b^3\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^{3q}}{3}+a\,b^2x^m\Big(\ln(c)+\ln(x^n)-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3*a^3*(x^m)^3+1/3*b^3*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^3+a*b^2*x^m*((ln(c)+ln(x^n)
-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x
^n)))^q)^2+a^2*b*(x^m)^2*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.
time = 0.35, size = 65, normalized size = 2.95

$$(n\log(x)+\log(c))^q a^2bx^{2m}+(n\log(x)+\log(c))^{2q}ab^2x^m+\frac{1}{3}(n\log(x)+\log(c))^{3q}b^3+\frac{1}{3}a^3x^{3m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a^2*b*x^(2*m) + (n*log(x) + log(c))^(2*q)*a*b^2*x^m +
 1/3*(n*log(x) + log(c))^(3*q)*b^3 + 1/3*a^3*x^(3*m)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)^2/x
, x)
```

Mupad [**B**]
time = 0.70, size = 20, normalized size = 0.91

$$\frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q)^2)/x,x)
```

```
[Out] (a*x^m + b*log(c*x^n)^q)^3/3
```

3.18 $\int \frac{(amx^m+bnq\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}(ax^m + b\log^q(cx^n))^2$$

```
[Out] 1/2*(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A]
time = 0.07, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2624}

$$\frac{1}{2}(ax^m + b\log^q(cx^n))^2$$

Antiderivative was successfully verified.

```
[In] Int[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{(amx^m + bnq\log^{-1+q}(cx^n))(ax^m + b\log^q(cx^n))}{x}\,dx = \frac{1}{2}(ax^m + b\log^q(cx^n))^2$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$\frac{1}{2}(ax^m + b\log^q(cx^n))^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/
x,x]
```

```
[Out] (a*x^m + b*Log[c*x^n]^q)^2/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 135, normalized size = 6.14

method	result
risch	$\frac{a^2x^{2m}}{2} + \frac{b^2\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^{2q}}{2} + ab\,x^m\left(\ln(c)+\ln(x^n)-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2*a^2*(x^m)^2+1/2*b^2*((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x
^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2+a*b*x^m*(ln(c)+ln(x^n)-1/
2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)
))^q
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.
time = 0.36, size = 42, normalized size = 1.91

$$(n\log(x) + \log(c))^q abx^m + \frac{1}{2}(n\log(x) + \log(c))^{2q}b^2 + \frac{1}{2}a^2x^{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="fricas")
```

```
[Out] (n*log(x) + log(c))^q*a*b*x^m + 1/2*(n*log(x) + log(c))^(2*q)*b^2 + 1/2*a^2
*x^(2*m)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.
time = 41.30, size = 39, normalized size = 1.77

$$\frac{a^2x^{2m}}{2} + abx^m\log(cx^n)^q + \frac{b^2\log(cx^n)^{2q}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] a**2*x**(2*m)/2 + a*b*x**m*log(c*x**n)**q + b**2*log(c*x**n)**(2*q)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)*(a*x^m + b*log(c*x^n)^q)/x,
x)
```

Mupad [B]
time = 0.68, size = 20, normalized size = 0.91

$$\frac{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))*(a*x^m + b*log(c*x^n)^q))/x,x)
```

```
[Out] (a*x^m + b*log(c*x^n)^q)^2/2
```

3.19 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=16

$$ax^m + b\log^q(cx^n)$$

```
[Out] a*x^m+b*ln(c*x^n)^q
```

Rubi [A]

time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {14, 2339, 30}

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{amx^m + bnq\log^{-1+q}(cx^n)}{x}\,dx &= \int \left(amx^{-1+m} + \frac{bnq\log^{-1+q}(cx^n)}{x}\right)dx \\
&= ax^m + (bnq)\int \frac{\log^{-1+q}(cx^n)}{x}\,dx \\
&= ax^m + (bq)\text{Subst}\left(\int x^{-1+q}\,dx, x, \log(cx^n)\right) \\
&= ax^m + b\log^q(cx^n)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$ax^m + b\log^q(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] a*x^m + b*Log[c*x^n]^q
```

Maple [A]
time = 0.12, size = 17, normalized size = 1.06

method	result	size
default	$a\,x^m + b\ln(c\,x^n)^q$	17
risch	$a\,x^m + b\left(\ln(c) + \ln(x^n) - \frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^q$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*x^m+b*ln(c*x^n)^q
```

Maxima [A]
time = 0.27, size = 16, normalized size = 1.00

$$ax^m + b\log(cx^n)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

```
[Out] a*x^m + b*log(c*x^n)^q
```

Fricas [A]
time = 0.36, size = 28, normalized size = 1.75

$$(bn\log(x) + b\log(c))(n\log(x) + \log(c))^{q-1} + ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
```

```
[Out] (b*n*log(x) + b*log(c))*(n*log(x) + log(c))^(q - 1) + a*x^m
```

Sympy [A]
time = 14.78, size = 54, normalized size = 3.38

$$am\left(\begin{cases}\frac{x^m}{m} & \text{for } m\neq 0\\ \log(x) & \text{otherwise}\end{cases}\right)+bnq\left(\begin{cases}\frac{\log(x)}{\log(c)} & \text{for } n=0\wedge q=0\\ \frac{\log(\log(cx^n))}{n} & \text{for } q=0\\ \frac{\log(c)^q\log(x)}{\log(c)} & \text{for } n=0\\ \frac{\log(cx^n)^q}{nq} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x,x)
```

```
[Out] a*m*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + b*n*q*Piecewise((log(x)
/log(c), Eq(n, 0) & Eq(q, 0)), (log(log(c*x**n))/n, Eq(q, 0)), (log(c)**q*l
og(x)/log(c), Eq(n, 0)), (log(c*x**n)**q/(n*q), True))
```

Giac [A]
time = 6.86, size = 17, normalized size = 1.06

$$(n\log(x)+\log(c))^q b+ax^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(c))^q*b + a*x^m
```

Mupad [B]
time = 0.30, size = 16, normalized size = 1.00

$$a\,x^m+b\ln(c\,x^n)^q$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/x,x)
```

```
[Out] a*x^m + b*log(c*x^n)^q
```

3.20 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal. Leaf size=17

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

```
[Out] ln(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.12, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2621}

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Mathematica [A]
time = 0.11, size = 17, normalized size = 1.00

$$\log\left(ax^m+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
),x]
```

```
[Out] Log[a*x^m + b*Log[c*x^n]^q]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 212, normalized size = 12.47

method	result
risch	$q \ln\left(-\frac{i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)}{2}+\frac{i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2}{2}+\frac{i\pi\,\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2}{2}-\frac{i\pi\mathrm{csgn}(ic\,x^n)^3}{2}+\ln(c)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x,method=_RETU
RNVERBOSE)
```

```
[Out] q*ln(-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*csgn(I*c)*csgn(
I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*csgn(I*c*x^n)^3+ln
(c)+ln(x^n))-q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn
(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^
n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+1/b*a*x^m)
```

Maxima [A]
time = 0.42, size = 22, normalized size = 1.29

$$\log\left(\frac{ax^m+b(\log(c)+\log(x^n))^q}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="maxima")
```

```
[Out] log((a*x^m + b*(log(c) + log(x^n))^q)/b)
```

Fricas [A]
time = 0.36, size = 18, normalized size = 1.06

$$\log\left((n\log(x)+\log(c))^q b+ax^m\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="fricas")
```

```
[Out] log((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [A]
time = 120.88, size = 20, normalized size = 1.18

$$\begin{cases}\log\left(\frac{ax^m}{b}+\log(cx^n)^q\right) & \text{for}\ b\neq 0\\ m\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Piecewise((log(a*x**m/b + log(c*x**n)**q), Ne(b, 0)), (m*log(x), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, alg
orithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)*x)
, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{a\,m\,x^m+b\,n\,q\,\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.21 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

```
[Out] -1/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.12, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2624}

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx=-\frac{1}{ax^m+b\log^q(cx^n)}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.00

$$-\frac{1}{ax^m+b\log^q(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^2),x]
```

```
[Out] -(a*x^m + b*Log[c*x^n]^q)^(-1)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 68, normalized size = 3.40

method	result	size
risch	$-\frac{1}{a\,x^m+b\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^q}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c)
)*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Maxima [A]
time = 0.46, size = 21, normalized size = 1.05

$$-\frac{1}{ax^m + b(\log(c) + \log(x^n))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="maxima")
```

```
[Out] -1/(a*x^m + b*(log(c) + log(x^n))^q)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 1.05

$$-\frac{1}{(n\log(x) + \log(c))^q b + ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="fricas")
```

```
[Out] -1/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^2*
x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,m\,x^m+b\,n\,q\,\ln\left(c\,x^n\right)^{q-1}}{x\,{\left(a\,x^m+b\,{\ln\left(c\,x^n\right)}^q\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.22 $\int \frac{amx^m+bnq\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

```
[Out] -1/2/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A]
time = 0.13, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{2624\}$

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rubi steps

$$\int \frac{amx^m+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$-\frac{1}{2\left(ax^m+b\log^q\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*m*x^m + b*n*q*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)
^3),x]
```

```
[Out] -1/2*1/(a*x^m + b*Log[c*x^n]^q)^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 2.87, size = 68, normalized size = 3.09

method	result	size
risch	$-\frac{1}{2\left(a\,x^{m}+b\left(\ln(c)+\ln(x^{n})-\frac{i\pi\,\mathrm{csgn}(ic\,x^{n})(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^{n})+\mathrm{csgn}(ix^{n}))}{2}\right)^{q}\right)^{2}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m+b*n*q*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*
c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(20) = 40.
time = 0.57, size = 49, normalized size = 2.23

$$-\frac{1}{2\,\left(a^{2}x^{2\,m}+b^{2}(\log\left(c\right)+\log\left(x^{n}\right))^{2\,q}+2\,abe^{(m\log(x)+q\log(\log(c)+\log(x^{n})))}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="maxima")
```

```
[Out] -1/2/(a^2*x^(2*m) + b^2*(log(c) + log(x^n))^(2*q) + 2*a*b*e^(m*log(x) + q*l
og(log(c) + log(x^n))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(20) = 40.
time = 0.36, size = 45, normalized size = 2.05

$$-\frac{1}{2\,\left(2\,(n\log\left(x\right)+\log\left(c\right))^{q}abx^{m}+(n\log\left(x\right)+\log\left(c\right))^{2\,q}b^{2}+a^{2}x^{2\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="fricas")
```

```
[Out] -1/2/(2*(n*log(x) + log(c))^q*a*b*x^m + (n*log(x) + log(c))^(2*q)*b^2 + a^2
*x^(2*m))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x**m+b*n*q*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*m*x^m+b*n*q*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, a
lgorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*m*x^m)/((a*x^m + b*log(c*x^n)^q)^3*
x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,m\,x^m+b\,n\,q\,\ln\left(c\,x^n\right)^{q-1}}{x\,{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
```

```
[Out] int((a*m*x^m + b*n*q*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.23 $\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2 dx$

Optimal. Leaf size=20

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

```
[Out] 1/3*(a*x+b*ln(c*x^n)^2)^3
```

Rubi [A]
time = 0.10, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2641, 2624}

$$\frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3$$

Antiderivative was successfully verified.

```
[In] Int[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x^2}+\frac{2bn\log(cx^n)}{x^3}\right)\left(ax^2+bx\log^2(cx^n)\right)^2 dx &= \int \frac{(ax+2bn\log(cx^n))\left(ax^2+bx\log^2(cx^n)\right)^2}{x^3}\,dx\\ &= \int \frac{(ax+2bn\log(cx^n))\left(ax+b\log^2(cx^n)\right)^2}{x}\,dx\\ &= \frac{1}{3}\left(ax+b\log^2(cx^n)\right)^3\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{1}{3}\left(ax + b\log^2\left(cx^n\right)\right)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x^2 + (2*b*n*Log[c*x^n])/x^3)*(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^3/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.02, size = 20850, normalized size = 1042.50

method	result	size
risch	Expression too large to display	20850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+2*b*n*ln(c*x^n)/x^3)*(a*x^2+b*x*ln(c*x^n)^2)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(18) = 36.
time = 0.31, size = 211, normalized size = 10.55

$$\frac{1}{3}\,b^3\log\left(cx^n\right)^6 + 4\,ab^2nx\log\left(cx^n\right)^3 + ab^2x\log\left(cx^n\right)^4 - \frac{1}{2}\,a^2bn^2x^2 + a^2bnx^2\log\left(cx^n\right) + a^2bx^2\log\left(cx^n\right)^2 + \frac{1}{3}\,a^3x^3 - 12\left(nx\log\left(cx^n\right)^2 + 2\left(n^2x - nx\log\left(cx^n\right)\right)n\right)ab^2n + \frac{1}{2}\left(n^2x^2 - 2\,nx^2\log\left(cx^n\right)\right)a^2b - 4\left(nx\log\left(cx^n\right)^3 - 3\left(nx\log\left(cx^n\right)^2 + 2\left(n^2x - nx\log\left(cx^n\right)\right)n\right)n\right)ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="maxima")
```

```
[Out] 1/3*b^3*log(c*x^n)^6 + 4*a*b^2*n*x*log(c*x^n)^3 + a*b^2*x*log(c*x^n)^4 - 1/
2*a^2*b*n^2*x^2 + a^2*b*n*x^2*log(c*x^n) + a^2*b*x^2*log(c*x^n)^2 + 1/3*a^3
*x^3 - 12*(n*x*log(c*x^n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*a*b^2*n + 1/2*(
n^2*x^2 - 2*n*x^2*log(c*x^n))*a^2*b - 4*(n*x*log(c*x^n)^3 - 3*(n*x*log(c*x^
n)^2 + 2*(n^2*x - n*x*log(c*x^n))*n)*n)*a*b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(18) = 36.
time = 0.35, size = 195, normalized size = 9.75

$$\frac{1}{3}\,b^3n^6\log\left(x\right)^6 + 2\,b^3n^5\log\left(c\right)\log\left(x\right)^5 + ab^2x\log\left(c\right)^4 + a^2bx^2\log\left(c\right)^2 + \frac{1}{3}\,a^3x^3 + \left(5\,b^3n^4\log\left(c\right)^2 + ab^2n^4x\right)\log\left(x\right)^4 + \frac{4}{3}\left(5\,b^3n^3\log\left(c\right)^3 + 3\,ab^2n^3x\log\left(c\right)\right)\log\left(x\right)^3 + \left(5\,b^3n^2\log\left(c\right)^4 + 6\,ab^2n^2x\log\left(c\right)^2 + a^2bn^2x^2\right)\log\left(x\right)^2 + 2\left(b^3n\log\left(c\right)^5 + 2\,ab^2nx\log\left(c\right)^3 + a^2bnx^2\log\left(c\right)\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="fricas")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + a*b^2*x*log(c)^4 + a^2*b
*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2 + a*b^2*n^4*x)*log(x)^4 +
 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*log(x)^3 + (5*b^3*n^2*log(
c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*log(x)^2 + 2*(b^3*n*log(c)^5
 + 2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Sympy [A]
time = 8.30, size = 70, normalized size = 3.50

$$\frac{a^3x^3}{3} + a^2bx^2\log{(cx^n)}^2 + ab^2x\log{(cx^n)}^4 - 2b^3n\left(\begin{cases} -\log{(c)}^5\log{(x)} & \text{for } n = 0 \\ -\frac{\log{(cx^n)}^6}{6n} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+2*b*n*ln(c*x**n)/x**3)*(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] a**3*x**3/3 + a**2*b*x**2*log(c*x**n)**2 + a*b**2*x*log(c*x**n)**4 - 2*b**3
*n*Piecewise((-log(c)**5*log(x), Eq(n, 0)), (-log(c*x**n)**6/(6*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(18) = 36.
time = 6.45, size = 198, normalized size = 9.90

$$\frac{1}{3}b^3n^6\log{(x)}^6 + 2b^3n^5\log{(c)}\log{(x)}^5 + 2b^3n\log{(c)}^5\log{(x)} + ab^2x\log{(c)}^4 + a^2bx^2\log{(c)}^2 + \frac{1}{3}a^3x^3 + (5b^3n^4\log{(c)}^2 + ab^2n^4x)\log{(x)}^4 + \frac{4}{3}(5b^3n^3\log{(c)}^3 + 3ab^2n^3x\log{(c)})\log{(x)}^3 + (5b^3n^2\log{(c)}^4 + 6ab^2n^2x\log{(c)}^2 + a^2bn^2x^2)\log{(x)}^2 + 2(2ab^2nx\log{(c)}^3 + a^2bnx^2\log{(c)})\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+2*b*n*log(c*x^n)/x^3)*(a*x^2+b*x*log(c*x^n)^2)^2,x, algori
thm="giac")
```

```
[Out] 1/3*b^3*n^6*log(x)^6 + 2*b^3*n^5*log(c)*log(x)^5 + 2*b^3*n*log(c)^5*log(x)
+ a*b^2*x*log(c)^4 + a^2*b*x^2*log(c)^2 + 1/3*a^3*x^3 + (5*b^3*n^4*log(c)^2
 + a*b^2*n^4*x)*log(x)^4 + 4/3*(5*b^3*n^3*log(c)^3 + 3*a*b^2*n^3*x*log(c))*
log(x)^3 + (5*b^3*n^2*log(c)^4 + 6*a*b^2*n^2*x*log(c)^2 + a^2*b*n^2*x^2)*lo
g(x)^2 + 2*(2*a*b^2*n*x*log(c)^3 + a^2*b*n*x^2*log(c))*log(x)
```

Mupad [B]
time = 0.33, size = 52, normalized size = 2.60

$$\frac{a^3\,x^3}{3} + a^2\,b\,x^2\ln{(c\,x^n)}^2 + a\,b^2\,x\ln{(c\,x^n)}^4 + \frac{b^3\ln{(c\,x^n)}^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)^2*(a/x^2 + (2*b*n*log(c*x^n))/x^3),x)
```

```
[Out] (b^3*log(c*x^n)^6)/3 + (a^3*x^3)/3 + a^2*b*x^2*log(c*x^n)^2 + a*b^2*x*log(c
*x^n)^4
```

3.24 $\int \left(\frac{a}{x}+\frac{2bn\log(cx^n)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx$

Optimal. Leaf size=20

$$\frac{1}{2}\left(ax+b\log^2\left(cx^n\right)\right)^2$$

```
[Out] 1/2*(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A]
time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2641, 2624}

$$\frac{1}{2}\left(ax+b\log^2\left(cx^n\right)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a*x + b*Log[c*x^n]^2)^2/2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+\frac{2bn\log\left(cx^n\right)}{x^2}\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)\,dx &= \int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax^2+bx\log^2\left(cx^n\right)\right)}{x^2}\,dx\\ &= \int \frac{\left(ax+2bn\log\left(cx^n\right)\right)\left(ax+b\log^2\left(cx^n\right)\right)}{x}\,dx\\ &= \frac{1}{2}\left(ax+b\log^2\left(cx^n\right)\right)^2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.90

$$\frac{a^2x^2}{2} + abx\log^2(cx^n) + \frac{1}{2}b^2\log^4(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + (2*b*n*Log[c*x^n])/x^2)*(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] (a^2*x^2)/2 + a*b*x*Log[c*x^n]^2 + (b^2*Log[c*x^n]^4)/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.
time = 0.24, size = 63, normalized size = 3.15

method	result	size
default	$\frac{a^2x^2}{2} + abx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2 - 2abnx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right) + \frac{b^2\ln(c\,x^n)^4}{2} + 2\ln\left(c\,x^n\right)abnx$	63
risch	Expression too large to display	2698

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+2*b*n*ln(c*x^n)/x^2)*(a*x^2+b*x*ln(c*x^n)^2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*a^2*x^2+a*b*x*ln(c*exp(n*ln(x)))^2-2*a*b*n*x*ln(c*exp(n*ln(x)))+1/2*b^2
*ln(c*x^n)^4+2*ln(c*x^n)*a*b*n*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(18) = 36$.
time = 0.28, size = 74, normalized size = 3.70

$$\frac{1}{2}\,b^2\log(cx^n)^4 - 2\,abn^2x + 2\,abnx\log(cx^n) + abx\log(cx^n)^2 + \frac{1}{2}\,a^2x^2 + 2\,\left(n^2x - nx\log(cx^n)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"maxima")
```

```
[Out] 1/2*b^2*log(c*x^n)^4 - 2*a*b*n^2*x + 2*a*b*n*x*log(c*x^n) + a*b*x*log(c*x^n
)^2 + 1/2*a^2*x^2 + 2*(n^2*x - n*x*log(c*x^n))*a*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(18) = 36$.
time = 0.37, size = 89, normalized size = 4.45

$$\frac{1}{2}\,b^2n^4\log(x)^4 + 2\,b^2n^3\log(c)\log(x)^3 + abx\log(c)^2 + \frac{1}{2}\,a^2x^2 + \left(3\,b^2n^2\log(c)^2 + abn^2x\right)\log(x)^2 + 2\,\left(b^2n\log(c)^3 + abnx\log(c)\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"fricas")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + a*b*x*log(c)^2 + 1/2*a^2
*x^2 + (3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 2*(b^2*n*log(c)^3 + a*b*
n*x*log(c))*log(x)
```

Sympy [A]
time = 5.77, size = 51, normalized size = 2.55

$$\frac{a^2x^2}{2} + abx\log\left(cx^n\right)^2 - 2b^2n\left(\begin{cases} -\log\left(c\right)^3\log\left(x\right) & \text{for}\ n = 0 \\ -\frac{\log\left(cx^n\right)^4}{4n} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*ln(c*x**n)/x**2)*(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] a**2*x**2/2 + a*b*x*log(c*x**n)**2 - 2*b**2*n*Piecewise((-log(c)**3*log(x),
 Eq(n, 0)), (-log(c*x**n)**4/(4*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(18) = 36.
time = 2.75, size = 90, normalized size = 4.50

$$\frac{1}{2}\,b^2n^4\log\left(x\right)^4 + 2\,b^2n^3\log\left(c\right)\log\left(x\right)^3 + 2\,b^2n\log\left(c\right)^3\log\left(x\right) + 2\,abnx\log\left(c\right)\log\left(x\right) + abx\log\left(c\right)^2 + \frac{1}{2}\,a^2x^2 + \left(3\,b^2n^2\log\left(c\right)^2 + abn^2x\right)\log\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+2*b*n*log(c*x^n)/x^2)*(a*x^2+b*x*log(c*x^n)^2),x, algorithm=
"giac")
```

```
[Out] 1/2*b^2*n^4*log(x)^4 + 2*b^2*n^3*log(c)*log(x)^3 + 2*b^2*n*log(c)^3*log(x)
+ 2*a*b*n*x*log(c)*log(x) + a*b*x*log(c)^2 + 1/2*a^2*x^2 + (3*b^2*n^2*log(c
)^2 + a*b*n^2*x)*log(x)^2
```

Mupad [B]
time = 0.29, size = 18, normalized size = 0.90

$$\frac{{\left(b\ln\left(c\,x^n\right)^2 + a\,x\right)}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*log(c*x^n)^2)*(a/x + (2*b*n*log(c*x^n))/x^2),x)
```

```
[Out] (a*x + b*log(c*x^n)^2)^2/2
```

3.25 $\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx$

Optimal. Leaf size=14

$$ax + b\log^2(cx^n)$$

```
[Out] a*x+b*ln(c*x^n)^2
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2338}

$$ax + b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Int[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{2bn\log(cx^n)}{x}\right) dx &= ax + (2bn)\int \frac{\log(cx^n)}{x}\, dx\\ &= ax + b\log^2(cx^n)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$ax + b\log^2(cx^n)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + (2*b*n*Log[c*x^n])/x,x]
```

```
[Out] a*x + b*Log[c*x^n]^2
```

Maple [A]
time = 0.04, size = 15, normalized size = 1.07

method	result
default	$ax+b\ln\left(c\,x^{n}\right)^{2}$
norman	$ax+b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^{2}$
risch	$ax+2bn\ln\left(x\right)\ln\left(x^{n}\right)-b\,n^{2}\ln\left(x\right)^{2}-ibn\pi\ln\left(x\right)\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})+ibn\pi\ln\left(x\right)\mathrm{csgn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+2*b*n*ln(c*x^n)/x,x,method=_RETURNVERBOSE)
[Out] a*x+b*ln(c*x^n)^2
```

Maxima [A]
time = 0.32, size = 14, normalized size = 1.00

$$b\log\left(cx^{n}\right)^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="maxima")
[Out] b*log(c*x^n)^2 + a*x
```

Fricas [A]
time = 0.34, size = 21, normalized size = 1.50

$$bn^{2}\log\left(x\right)^{2}+2\,bn\log\left(c\right)\log\left(x\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="fricas")
[Out] b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + a*x
```

Sympy [A]
time = 2.08, size = 73, normalized size = 5.21

$$ax+2bn\left(\begin{cases}0 & \text{for}\ \frac{1}{|cx^{n}|}<1\wedge|cx^{n}|<1\\ \frac{\log\left(cx^{n}\right)^{2}}{2n} & \text{for}\ |cx^{n}|<1\\ \frac{\log\left(\frac{x^{-n}}{c}\right)^{2}}{2n} & \text{for}\ \frac{1}{|cx^{n}|}<1\\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1\\ 0,0,0 & \end{matrix}\middle|cx^{n}\right)}{n}+\frac{G_{3,3}^{0,3}\left(\begin{matrix}1,1,1 & \\ & 0,0,0\end{matrix}\middle|cx^{n}\right)}{n} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*ln(c*x**n)/x,x)
```

```
[Out] a*x + 2*b*n*Piecewise((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*
x**n)**2/(2*n), Abs(c*x**n) < 1), (log(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n)
< 1), (meijerg(((), (1, 1, 1)), ((0, 0, 0), ()), c*x**n)/n + meijerg(((1, 1
, 1), ()), ((), (0, 0, 0)), c*x**n)/n, True))
```

Giac [A]
time = 6.55, size = 20, normalized size = 1.43

$$\left(n\log\left(x\right)^{2}+2\,\log\left(c\right)\log\left(x\right)\right)bn+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+2*b*n*log(c*x^n)/x,x, algorithm="giac")
```

```
[Out] (n*log(x)^2 + 2*log(c)*log(x))*b*n + a*x
```

Mupad [B]
time = 0.26, size = 14, normalized size = 1.00

$$b\ln\left(c\,x^{n}\right)^{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + (2*b*n*log(c*x^n))/x,x)
```

```
[Out] a*x + b*log(c*x^n)^2
```

3.26 $\int \frac{ax+2bn\log(cx^n)}{ax^2+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=15

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

```
[Out] ln(a*x+b*ln(c*x^n)^2)
```

Rubi [A]
time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2641, 2621}

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+2bn\log\left(cx^n\right)}{ax^2+bx\log^2\left(cx^n\right)}\,dx &= \int \frac{ax+2bn\log\left(cx^n\right)}{x\left(ax+b\log^2\left(cx^n\right)\right)}\,dx\\ &= \log\left(ax+b\log^2\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 15, normalized size = 1.00

$$\log\left(ax+b\log^2\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + 2*b*n*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2),x]
```

```
[Out] Log[a*x + b*Log[c*x^n]^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 428, normalized size = 28.53

method	result
risch	$\ln\left(\ln\left(x^n\right)^2+\left(-i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+2*b*n*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(ln(x^n)^2+(-I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*csgn(I*c)*csgn
(I*c*x^n)^2+I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*c*x^n)^3+2*ln(c))*
ln(x^n)-1/4*(b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2-2*b*Pi^2*csgn
(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3+b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4-2*b*
Pi^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3+4*b*Pi^2*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)^4-2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5+b*Pi^2*csgn(I*x^n)^2*cs
gn(I*c*x^n)^4-2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5+b*Pi^2*csgn(I*c*x^n)^6+4
*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*b*ln(c)*Pi*csgn(I*c)*
csgn(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(c)*Pi*c
sgn(I*c*x^n)^3-4*b*ln(c)^2-4*a*x)/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.
time = 0.32, size = 32, normalized size = 2.13

$$\log\left(\frac{b\log\left(c\right)^2+2\,b\log\left(c\right)\log\left(x^n\right)+b\log\left(x^n\right)^2+ax}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="max
ima")
```

```
[Out] log((b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)/b)
```

Fricas [A]
time = 0.38, size = 28, normalized size = 1.87

$$\log\left(bn^2\log\left(x\right)^2+2\,bn\log\left(c\right)\log\left(x\right)+b\log\left(c\right)^2+ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="fri
cas")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax + 2bn \log(cx^n)}{x\left(ax + b\log(cx^n)^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2),x)
```

```
[Out] Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)), x)
```

Giac [A]
time = 3.53, size = 28, normalized size = 1.87

$$\log\left(bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+2*b*n*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2),x, algorithm="gia
c")
```

```
[Out] log(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B]
time = 0.35, size = 16, normalized size = 1.07

$$\ln\left(\ln\left(c\,x^n\right)^2 + \frac{a\,x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 2*b*n*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2),x)
```

```
[Out] log(log(c*x^n)^2 + (a*x)/b)
```

3.27 $\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{ax+b\log^2(cx^n)}$$

```
[Out] -1/(a*x+b*ln(c*x^n)^2)
```

Rubi [A]
time = 0.09, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2641, 2624}

$$-\frac{1}{ax+b\log^2(cx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+2bnx\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx &= \int \frac{x(ax+2bn\log(cx^n))}{\left(ax^2+bx\log^2(cx^n)\right)^2}\,dx\\ &= \int \frac{ax+2bn\log(cx^n)}{x\left(ax+b\log^2(cx^n)\right)^2}\,dx\\ &= -\frac{1}{ax+b\log^2(cx^n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{ax+b\log^2\left(cx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + 2*b*n*x*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^2,x]
```

```
[Out] -(a*x + b*Log[c*x^n]^2)^(-1)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 451, normalized size = 25.06

method	result
risch	$-\frac{}{-b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ix^n)^2\mathrm{csgn}(ic\,x^n)^2+2b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^3-b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ic\,x^n)^4+2b\,\pi^2\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)^2\mathrm{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+2*b*n*x*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^2,x,method=_RETURNVER
BOSE)
```

```
[Out] -4/(-b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+2*b*Pi^2*csgn(I*c)^2*
csgn(I*x^n)*csgn(I*c*x^n)^3-b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4+2*b*Pi^2*csg
n(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*b*Pi^2*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)^4+2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x
^n)^4+2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*c*x^n)^6-4*I*b*ln(
x^n)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^3-4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3
-4*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(I*c
)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(x^n
)*Pi*csgn(I*c)*csgn(I*c*x^n)^2+4*b*ln(c)^2+8*b*ln(c)*ln(x^n)+4*b*ln(x^n)^2+
4*a*x)
```

Maxima [A]
time = 0.32, size = 31, normalized size = 1.72

$$-\frac{1}{b\log\left(c\right)^2+2\,b\log\left(c\right)\log\left(x^n\right)+b\log\left(x^n\right)^2+ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="maxima")
```

```
[Out] -1/(b*log(c)^2 + 2*b*log(c)*log(x^n) + b*log(x^n)^2 + a*x)
```

Fricas [A]
time = 0.35, size = 31, normalized size = 1.72

$$-\frac{1}{bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="fricas")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{ax + 2bn\log(cx^n)}{x\left(ax + b\log(cx^n)^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+2*b*n*x*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**2,x)
```

```
[Out] Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)**2), x)
```

Giac [A]
time = 5.79, size = 31, normalized size = 1.72

$$-\frac{1}{bn^2\log(x)^2 + 2\,bn\log(c)\log(x) + b\log(c)^2 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+2*b*n*x*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^2,x, algorith
m="giac")
```

```
[Out] -1/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 + a*x)
```

Mupad [B]
time = 0.26, size = 18, normalized size = 1.00

$$-\frac{1}{b\ln(c\,x^n)^2 + a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + 2*b*n*x*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^2,x)
```

```
[Out] -1/(a*x + b*log(c*x^n)^2)
```

3.28 $\int \frac{ax^3+2bnx^2\log(cx^n)}{\left(ax^2+bx\log^2(cx^n)\right)^3}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

```
[Out] -1/2/(a*x+b*ln(c*x^n)^2)^2
```

Rubi [A]
time = 0.11, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2641, 2624}

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Rule 2624

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] /; FreeQ[{a, b, c, d, e
, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && EqQ[a*e*m - b*d*n*q,
 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^3+2bnx^2\log\left(cx^n\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx &= \int \frac{x^2\left(ax+2bn\log\left(cx^n\right)\right)}{\left(ax^2+bx\log^2\left(cx^n\right)\right)^3}\,dx\\ &= \int \frac{ax+2bn\log\left(cx^n\right)}{x\left(ax+b\log^2\left(cx^n\right)\right)^3}\,dx\\ &= -\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{2\left(ax+b\log^2\left(cx^n\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + 2*b*n*x^2*Log[c*x^n])/(a*x^2 + b*x*Log[c*x^n]^2)^3,x]
```

```
[Out] -1/2*1/(a*x + b*Log[c*x^n]^2)^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 451, normalized size = 22.55

method	result
risch	$-\frac{}{\left(-b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ix^n)^2\mathrm{csgn}(ic\,x^n)^2+2b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^3-b\,\pi^2\mathrm{csgn}(ic)^2\mathrm{csgn}(ic\,x^n)^4+2b\,\pi^2\mathrm{csgn}(ic)\mathrm{csgn}(ix^n\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+2*b*n*x^2*ln(c*x^n))/(a*x^2+b*x*ln(c*x^n)^2)^3,x,method=_RETURNV
ERBOSE)
```

```
[Out] -8/(-b*Pi^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+2*b*Pi^2*csgn(I*c)^2*
csgn(I*x^n)*csgn(I*c*x^n)^3-b*Pi^2*csgn(I*c)^2*csgn(I*c*x^n)^4+2*b*Pi^2*csg
n(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-4*b*Pi^2*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)^4+2*b*Pi^2*csgn(I*c)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*x^n)^2*csgn(I*c*x
^n)^4+2*b*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5-b*Pi^2*csgn(I*c*x^n)^6-4*I*b*ln(
x^n)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-4*I*b*ln(c)*Pi*csgn(I*c*x^n)^3-4*I*b*ln(x^n)*Pi*csgn(I*c*x^n)^3
-4*I*b*ln(c)*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+4*I*b*ln(c)*Pi*csgn(I*c
)*csgn(I*c*x^n)^2+4*I*b*ln(x^n)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*b*ln(x^n
)*Pi*csgn(I*c)*csgn(I*c*x^n)^2+4*b*ln(c)^2+8*b*ln(c)*ln(x^n)+4*b*ln(x^n)^2+
4*a*x)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(18) = 36.
time = 0.38, size = 95, normalized size = 4.75

$$-\frac{1}{2\left(b^2\log\left(c\right)^4+4\,b^2\log\left(c\right)\log\left(x^n\right)^3+b^2\log\left(x^n\right)^4+2\,abx\log\left(c\right)^2+a^2x^2+2\left(3\,b^2\log\left(c\right)^2+abx\right)\log\left(x^n\right)^2+4\left(b^2\log\left(c\right)^3+abx\log\left(c\right)\right)\log\left(x^n\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="maxima")
```

```
[Out] -1/2/(b^2*log(c)^4 + 4*b^2*log(c)*log(x^n)^3 + b^2*log(x^n)^4 + 2*a*b*x*log
(c)^2 + a^2*x^2 + 2*(3*b^2*log(c)^2 + a*b*x)*log(x^n)^2 + 4*(b^2*log(c)^3 +
 a*b*x*log(c))*log(x^n))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(18) = 36$.
time = 0.36, size = 101, normalized size = 5.05

$$-\frac{1}{2\left(b^{2}n^{4}\log\left(x\right)^{4}+4\,b^{2}n^{3}\log\left(c\right)\log\left(x\right)^{3}+b^{2}\log\left(c\right)^{4}+2\,abx\log\left(c\right)^{2}+a^{2}x^{2}+2\left(3\,b^{2}n^{2}\log\left(c\right)^{2}+abn^{2}x\right)\log\left(x\right)^{2}+4\left(b^{2}n\log\left(c\right)^{3}+abnx\log\left(c\right)\right)\log\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="fricas")
```

```
[Out] -1/2/(b^2*n^4*log(x)^4 + 4*b^2*n^3*log(c)*log(x)^3 + b^2*log(c)^4 + 2*a*b*x
*log(c)^2 + a^2*x^2 + 2*(3*b^2*n^2*log(c)^2 + a*b*n^2*x)*log(x)^2 + 4*(b^2*
n*log(c)^3 + a*b*n*x*log(c))*log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax+2bn\log\left(cx^{n}\right)}{x\left(ax+b\log\left(cx^{n}\right)^{2}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+2*b*n*x**2*ln(c*x**n))/(a*x**2+b*x*ln(c*x**n)**2)**3,x)
```

```
[Out] Integral((a*x + 2*b*n*log(c*x**n))/(x*(a*x + b*log(c*x**n)**2)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(18) = 36$.
time = 3.51, size = 306, normalized size = 15.30

$$-\frac{4\,abn^{2}x+a^{2}x^{2}}{2\left(4\,ab^{3}n^{6}x\log\left(x\right)^{4}+16\,ab^{3}n^{5}x\log\left(c\right)\log\left(x\right)^{3}+a^{2}b^{2}n^{4}x^{2}\log\left(x\right)^{4}+24\,ab^{3}n^{4}x\log\left(c\right)^{2}\log\left(x\right)^{2}+4\,a^{2}b^{2}n^{3}x^{2}\log\left(c\right)\log\left(x\right)^{3}+16\,ab^{3}n^{3}x\log\left(c\right)^{3}\log\left(x\right)+8\,a^{2}b^{2}n^{4}x^{2}\log\left(x\right)^{2}+6\,a^{2}b^{2}n^{2}x^{2}\log\left(c\right)^{2}\log\left(x\right)^{2}+4\,ab^{3}n^{2}x\log\left(c\right)^{4}+16\,a^{2}b^{2}n^{3}x^{2}\log\left(c\right)\log\left(x\right)+4\,a^{2}b^{2}nx^{2}\log\left(c\right)^{3}\log\left(x\right)+2\,a^{3}bn^{2}x^{3}\log\left(x\right)^{2}+8\,a^{2}b^{2}n^{2}x^{2}\log\left(c\right)^{2}+a^{2}b^{2}x^{2}\log\left(c\right)^{4}+4\,a^{3}bnx^{3}\log\left(c\right)\log\left(x\right)+4\,a^{3}bn^{2}x^{3}+2\,a^{3}bx^{3}\log\left(c\right)^{2}+a^{4}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+2*b*n*x^2*log(c*x^n))/(a*x^2+b*x*log(c*x^n)^2)^3,x, algori
thm="giac")
```

```
[Out] -1/2*(4*a*b*n^2*x + a^2*x^2)/(4*a*b^3*n^6*x*log(x)^4 + 16*a*b^3*n^5*x*log(c
)*log(x)^3 + a^2*b^2*n^4*x^2*log(x)^4 + 24*a*b^3*n^4*x*log(c)^2*log(x)^2 +
4*a^2*b^2*n^3*x^2*log(c)*log(x)^3 + 16*a*b^3*n^3*x*log(c)^3*log(x) + 8*a^2*
b^2*n^4*x^2*log(x)^2 + 6*a^2*b^2*n^2*x^2*log(c)^2*log(x)^2 + 4*a*b^3*n^2*x*
log(c)^4 + 16*a^2*b^2*n^3*x^2*log(c)*log(x) + 4*a^2*b^2*n*x^2*log(c)^3*log(
x) + 2*a^3*b*n^2*x^3*log(x)^2 + 8*a^2*b^2*n^2*x^2*log(c)^2 + a^2*b^2*x^2*lo
```

```
g(c)^4 + 4*a^3*b*n*x^3*log(c)*log(x) + 4*a^3*b*n^2*x^3 + 2*a^3*b*x^3*log(c)
^2 + a^4*x^4)
```

Mupad [B]
time = 0.27, size = 39, normalized size = 1.95

$$-\frac{1}{2\,a^2\,x^2+4\,a\,b\,x\,\ln\left(c\,x^n\right)^2+2\,b^2\,\ln\left(c\,x^n\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + 2*b*n*x^2*log(c*x^n))/(a*x^2 + b*x*log(c*x^n)^2)^3,x)
[Out] -1/(2*b^2*log(c*x^n)^4 + 2*a^2*x^2 + 4*a*b*x*log(c*x^n)^2)
```

3.29 $\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}(cx^n)}{ax^m+bx\log^q(cx^n)}\,dx$

Optimal. Leaf size=19

$$\log\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)$$

```
[Out] ln(a*x^(-1+m)+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.23, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2641, 2621}

$$\log\left(ax^{m-1}+b\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*Log[c*
x^n]^q),x]
```

```
[Out] Log[a*x^(-1 + m) + b*Log[c*x^n]^q]
```

Rule 2621

```
Int[(Log[(c_.)*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.))/((x_)*(Log[(c_.)
*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))), x_Symbol] :> Simp[e*(Log[a*x^
m + b*Log[c*x^n]^q]/(b*n*q)), x] /; FreeQ[{a, b, c, d, e, m, n, q, r}, x] &
& EqQ[r, q - 1] && EqQ[a*e*m - b*d*n*q, 0]
```

Rule 2641

```
Int[(u_.)*((a_.)*(x_)^(m_.) + Log[(c_.)*(x_)^(n_.)]^(q_.)*(b_.)*(x_)^(r_.))
^(p_.), x_Symbol] :> Int[u*x^(p*r)*(a*x^(m - r) + b*Log[c*x^n]^q)^p, x] /;
FreeQ[{a, b, c, m, n, p, q, r}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{ax^m+bx\log^q\left(cx^n\right)}\,dx &= \int \frac{a(-1+m)x^{-1+m}+bnq\log^{-1+q}\left(cx^n\right)}{x\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)}\,dx\\ &= \log\left(ax^{-1+m}+b\log^q\left(cx^n\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.21, size = 23, normalized size = 1.21

$$-\log(x)+\log\left(ax^m+bx\log^q\left(cx^n\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(-1 + m)*x^(-1 + m) + b*n*q*Log[c*x^n]^(-1 + q))/(a*x^m + b*x*
Log[c*x^n]^q),x]
```

```
[Out] -Log[x] + Log[a*x^m + b*x*Log[c*x^n]^q]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 215, normalized size = 11.32

method	result
risch	$q \ln \left(-\frac{i\pi \,\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})}{2}+\frac{i\pi \,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^{n})^{2}}{2}+\frac{i\pi \,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})^{2}}{2}-\frac{i\pi \mathrm{csgn}(ic\,x^{n})^{3}}{2}+\ln (c) -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(-1+m)*x^(-1+m)+b*n*q*ln(c*x^n)^(-1+q))/(a*x^m+b*x*ln(c*x^n)^q),x,me
thod=_RETURNVERBOSE)
```

```
[Out] q*ln(-1/2*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+1/2*I*Pi*csgn(I*c)*csgn(
I*c*x^n)^2+1/2*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-1/2*I*Pi*csgn(I*c*x^n)^3+ln
(c)+ln(x^n))-q*ln(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn
(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))+ln((ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^
n)*(-csgn(I*c*x^n)+csgn(I*c))*(-csgn(I*c*x^n)+csgn(I*x^n)))^q+1/x*a*x^m/b)
```

Maxima [A]
time = 0.41, size = 26, normalized size = 1.37

$$\log \left(\frac{bx(\log (c)+\log (x^{n}))^{q}+ax^{m}}{bx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="maxima")
```

```
[Out] log((b*x*(log(c) + log(x^n))^q + a*x^m)/(b*x))
```

Fricas [A]
time = 0.38, size = 23, normalized size = 1.21

$$\log \left(\frac{(n\log (x)+\log (c))^{q}bx+ax^{m}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="fricas")
```

```
[Out] log(((n*log(x) + log(c))^q*b*x + a*x^m)/x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x**(-1+m)+b*n*q*ln(c*x**n)**(-1+q))/(a*x**m+b*x*ln(c*x*
*n)**q),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(-1+m)*x^(-1+m)+b*n*q*log(c*x^n)^(-1+q))/(a*x^m+b*x*log(c*x^n)
^q),x, algorithm="giac")
```

```
[Out] integrate((b*n*q*log(c*x^n)^(q - 1) + a*(m - 1)*x^(m - 1))/(b*x*log(c*x^n)^
q + a*x^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a\,x^{m-1}\,(m-1)+b\,n\,q\ln\left(c\,x^n\right)^{q-1}}{a\,x^m+b\,x\ln\left(c\,x^n\right)^q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q),x)
```

```
[Out] int((a*x^(m - 1)*(m - 1) + b*n*q*log(c*x^n)^(q - 1))/(a*x^m + b*x*log(c*x^n
)^q), x)
```

3.30 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$

Optimal. Leaf size=81

$$\frac{e(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}+\left(d-\frac{aem}{bnq}\right)\text{Int}\left(x^{-1+m}(ax^m+b\log^q(cx^n))^p,x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)*(a*x^m+b*ln(c*x^n)^q)^p,x)+e*(a*x^
m+b*ln(c*x^n)^q)^(1+p)/b/n/(1+p)/q
```

Rubi [A]
time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] (e*(a*x^m + b*Log[c*x^n]^q)^(1 + p))/(b*n*(1 + p)*q) + (d - (a*e*m)/(b*n*q)
)*Defer[Int][x^(-1 + m)*(a*x^m + b*Log[c*x^n]^q)^p, x]
```

Rubi steps

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx=\frac{e(ax^m+b\log^q(cx^n))^{1+p}}{bn(1+p)q}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}($$

Mathematica [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^p}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x,x]
```

```
[Out] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^p)/x, x
]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^m+e\ln(c\,x^n)^{-1+q}\right)\left(a\,x^m+b\ln(c\,x^n)^q\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^p/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + log(c*x^n)^(q - 1)*e)*(a*x^m + b*log(c*x^n)^q)^p/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**p/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^p/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0,0
,2,5,2,0,5,0,2,1,2,2,1]%%%}+%%%{-2,[0,0,2,4,2,1,5,0,1,1,2,2,1]%%%}+%%%{5,[0
,0,2,4,2,
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,x^m+b\ln\left(c\,x^n\right)^q)^p\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
[Out] int(((a*x^m + b*log(c*x^n)^q)^p*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.31 $\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^3}{x}\,dx$

Optimal. Leaf size=331

$$-\frac{a^3(aem-bdnq)x^{4m}}{4bmnq}-\frac{b^2(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}}{mnq}-\frac{3\,2^{-}\cdots}{\cdots}$$

```
[Out] -1/4*a^3*(-b*d*n*q+a*e*m)*x^(4*m)/b/m/n/q-b^2*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+
3*q,-m*ln(c*x^n)/n)*ln(c*x^n)^(3*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)
^(3*q))-3*2^(-1-2*q)*a*b*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+2*q,-2*m*ln(c*x^n
)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^(2*q))-a^2*(
-b*d*n*q+a*e*m)*x^(3*m)*GAMMA(1+q,-3*m*ln(c*x^n)/n)*ln(c*x^n)^q/(3^q)/m/n/q
/((c*x^n)^(3*m/n))/((-m*ln(c*x^n)/n)^q)+1/4*e*(a*x^m+b*ln(c*x^n)^q)^4/b/n/q
```

Rubi [A]
time = 0.34, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$,
Rules used = {2625, 6874, 2347, 2212}

$$-\frac{a^2 3^{-q} x^{3m}(cx^n)^{-\frac{3m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\text{Gamma}\left(q+1,-\frac{3m\log(cx^n)}{n}\right)}{mnq}-\frac{b^2x^m(cx^n)^{-\frac{m}{n}}\log^{3q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}(aem-bdnq)\text{Gamma}\left(3q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}-\frac{3ab2^{-2q-1}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\text{Gamma}\left(2q+1,-\frac{2m\log(cx^n)}{n}\right)}{mnq}-\frac{a^3x^{4m}(aem-bdnq)}{4bmnq}+\frac{e\left(ax^m+b\log^q(cx^n)\right)^4}{4bnq}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] -1/4*(a^3*(a*e*m - b*d*n*q)*x^(4*m))/(b*m*n*q) - (b^2*(a*e*m - b*d*n*q)*x^m
*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q))/(m*n*q*(c*x^n)^(m/n)
*(-((m*Log[c*x^n])/n))^(3*q)) - (3*2^(-1 - 2*q)*a*b*(a*e*m - b*d*n*q)*x^(2*
m)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^((2
*m)/n)*(-((m*Log[c*x^n])/n))^(2*q)) - (a^2*(a*e*m - b*d*n*q)*x^(3*m)*Gamma[
1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((
m*Log[c*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^4)/(4*b*n*q)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(dx^m + e\log^{-1+q}(cx^n)\right)(ax^m + b\log^q(cx^n))^3}{x}\,dx &= \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int x^{-1+m}(ax \\
&= \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \left(-d + \frac{aem}{bnq}\right)\int (a^3x^{-1+4r} \\
&= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \Big(3a^2b \\
&= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} + \frac{e(ax^m + b\log^q(cx^n))^4}{4bnq} - \frac{\Big(3a^2b(}{} \\
&= \frac{a^3\left(d - \frac{aem}{bnq}\right)x^{4m}}{4m} - \frac{b^2(aem - bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\Big(1}{}
\end{aligned}$$

Mathematica [A]
time = 1.02, size = 445, normalized size = 1.34

$$\frac{3^{-q}4^{-1-q}(cx^n)^{-\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-3q}\left(-12^{1+q}ab^2emqx^m(cx^n)^{\frac{2m}{n}}\Gamma\left(3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)+3^q4^{1+q}b^3dnqx^m(cx^n)^{\frac{2m}{n}}\Gamma\left(1+3q,-\frac{m\log(cx^n)}{n}\right)\log^{3q}(cx^n)+\left(-\frac{m\log(cx^n)}{n}\right)^q\left(-4\,3^{1+q}a^2bemqx^{2m}(cx^n)^{m/n}\Gamma\left(2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)+2\,3^{1+q}ab^2dnqx^{2m}(cx^n)^{m/n}\Gamma\left(1+2q,-\frac{2m\log(cx^n)}{n}\right)\log^{2q}(cx^n)+4^q\left(-\frac{m\log(cx^n)}{n}\right)^q\left(-4a^3emqx^{3m}\Gamma\left(q,-\frac{3m\log(cx^n)}{n}\right)\log^q(cx^n)+4a^2bdnqx^{3m}\Gamma\left(1+q,-\frac{3m\log(cx^n)}{n}\right)\log^q(cx^n)+3^q(cx^n)^{\frac{3m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^q\left(a^3dnqx^{4m}+b^3em\log^{4q}(cx^n)\right)\right)\right)\right)}{mnq}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^3)/x,x]
```

```
[Out] (4^(-1 - q)*(-(12^(1 + q)*a*b^2*e*m*q*x^m*(c*x^n)^((2*m)/n)*Gamma[3*q, -((m
*Log[c*x^n])/n)]*Log[c*x^n]^(3*q)) + 3^q*4^(1 + q)*b^3*d*n*q*x^m*(c*x^n)^((
2*m)/n)*Gamma[1 + 3*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(3*q) + (-((m*Log[c*
x^n])/n))^q*(-4*3^(1 + q)*a^2*b*e*m*q*x^(2*m)*(c*x^n)^(m/n)*Gamma[2*q, (-2*
```

```
m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 2*3^(1 + q)*a*b^2*d*n*q*x^(2*m)*(c*x^n)
^(m/n)*Gamma[1 + 2*q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^(2*q) + 4^q*(-((m*Log
[c*x^n])/n))^q*(-4*a^3*e*m*q*x^(3*m)*Gamma[q, (-3*m*Log[c*x^n])/n]*Log[c*x^
n]^q + 4*a^2*b*d*n*q*x^(3*m)*Gamma[1 + q, (-3*m*Log[c*x^n])/n]*Log[c*x^n]^q
 + 3^q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))^q*(a^3*d*n*q*x^(4*m) + b^3*e
*m*Log[c*x^n]^(4*q))))))/(3^q*m*n*q*(c*x^n)^((3*m)/n)*(-((m*Log[c*x^n])/n))
^(3*q))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{m}+e \ln \left(c\,x^{n}\right)^{-1+q}\right) \left(a\,x^{m}+b \ln \left(c\,x^{n}\right)^{q}\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^3/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^3*x^(3*m)*log(c*x^n)^(q - 1)*e + a^3*d*x^(4*m) + (b^3*d*x^m + b
^3*log(c*x^n)^(q - 1)*e)*log(c*x^n)^(3*q) + 3*(a*b^2*x^m*log(c*x^n)^(q - 1)
*e + a*b^2*d*x^(2*m))*log(c*x^n)^(2*q) + 3*(a^2*b*x^(2*m)*log(c*x^n)^(q - 1
)*e + a^2*b*d*x^(3*m))*log(c*x^n)^q)/x, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**3/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^3/x,x, algorit
hm="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + log(c*x^n)^(q - 1)*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3\,\left(d\,x^m+e\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)^3*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.32 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))^2}{x}\,dx$

Optimal. Leaf size=235

$$-\frac{a^2(aem-bdnq)x^{3m}}{3bmnq}-\frac{b(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}}{mnq}-\frac{2^{-q}a(\ldots}{\ldots}$$

```
[Out] -1/3*a^2*(-b*d*n*q+a*e*m)*x^(3*m)/b/m/n/q-b*(-b*d*n*q+a*e*m)*x^m*GAMMA(1+2*
q,-m*ln(c*x^n)/n)*ln(c*x^n)^(2*q)/m/n/q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^(
2*q))-a*(-b*d*n*q+a*e*m)*x^(2*m)*GAMMA(1+q,-2*m*ln(c*x^n)/n)*ln(c*x^n)^q/(2
^q)/m/n/q/((c*x^n)^(2*m/n))/((-m*ln(c*x^n)/n)^q)+1/3*e*(a*x^m+b*ln(c*x^n)^q
)^3/b/n/q
```

Rubi [A]

time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2625, 6874, 2347, 2212}

$$-\frac{a2^{-q}x^{2m}(cx^n)^{-\frac{2m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}(aem-bdnq)\text{Gamma}\left(q+1,-\frac{2m\log(cx^n)}{n}\right)}{mnq}-\frac{bx^m(cx^n)^{-\frac{m}{n}}\log^{2q}(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}(aem-bdnq)\text{Gamma}\left(2q+1,-\frac{m\log(cx^n)}{n}\right)}{mnq}-\frac{a^2x^{3m}(aem-bdnq)}{3bmnq}+\frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] -1/3*(a^2*(a*e*m - b*d*n*q)*x^(3*m))/(b*m*n*q) - (b*(a*e*m - b*d*n*q)*x^m*G
amma[1 + 2*q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^(2*q))/(m*n*q*(c*x^n)^(m/n)*(
-((m*Log[c*x^n])/n))^(2*q)) - (a*(a*e*m - b*d*n*q)*x^(2*m)*Gamma[1 + q, (-2
*m*Log[c*x^n])/n]*Log[c*x^n]^q)/(2^q*m*n*q*(c*x^n)^((2*m)/n)*(-((m*Log[c*x^
n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^3)/(3*b*n*q)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)^2}{x}\,dx &= \frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}(ax\\ &= \frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}-\left(-d+\frac{aem}{bnq}\right)\int \left(a^2x^{-1+3r}\right.\\ &= \frac{a^2\left(d-\frac{aem}{bnq}\right)x^{3m}}{3m}+\frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}-\left(2ab\left(\right.\right.\\ &= \frac{a^2\left(d-\frac{aem}{bnq}\right)x^{3m}}{3m}+\frac{e(ax^m+b\log^q(cx^n))^3}{3bnq}-\frac{\left(2ab\left(\right.\right.}{}\\ &= \frac{a^2\left(d-\frac{aem}{bnq}\right)x^{3m}}{3m}-\frac{b(aem-bdnq)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 298, normalized size = 1.27

$$\frac{2^{-q}(cx^n)^{-\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-2q}\left(-32^{1+q}abemqx^m(cx^n)^{m/n}\Gamma\left(2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)+3\,2^qb^2dnqx^m(cx^n)^{m/n}\Gamma\left(1+2q,-\frac{m\log(cx^n)}{n}\right)\log^{2q}(cx^n)+\left(-\frac{m\log(cx^n)}{n}\right)^q\left(-3a^2emqx^{2m}\Gamma\left(q,-\frac{2m\log(cx^n)}{n}\right)\log^q(cx^n)+3abdnqx^{2m}\Gamma\left(1+q,-\frac{2m\log(cx^n)}{n}\right)\log^q(cx^n)+2^q(cx^n)^{\frac{2m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^q\left(a^2dnqx^{3m}+b^2em\log^{3q}(cx^n)\right)\right)\right)}{3mnq}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q)^2)/x,x]
```

```
[Out] (-3*2^(1 + q)*a*b*e*m*q*x^m*(c*x^n)^(m/n)*Gamma[2*q, -((m*Log[c*x^n])/n)]*L
og[c*x^n]^(2*q) + 3*2^q*b^2*d*n*q*x^m*(c*x^n)^(m/n)*Gamma[1 + 2*q, -((m*Log
[c*x^n])/n)]*Log[c*x^n]^(2*q) + (-((m*Log[c*x^n])/n))^q*(-3*a^2*e*m*q*x^(2*
m)*Gamma[q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 3*a*b*d*n*q*x^(2*m)*Gamma[1
 + q, (-2*m*Log[c*x^n])/n]*Log[c*x^n]^q + 2^q*(c*x^n)^((2*m)/n)*(-((m*Log[c
```

```
*x^n])/n))^q*(a^2*d*n*q*x^(3*m) + b^2*e*m*Log[c*x^n]^(3*q))))/(3*2^q*m*n*q*
(c*x^n)^((2*m)/n)*(-((m*Log[c*x^n])/n))^(2*q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}\right)\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)^2/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="fricas")
```

```
[Out] integral((a^2*x^(2*m)*log(c*x^n)^(q - 1)*e + a^2*d*x^(3*m) + (b^2*d*x^m + b
^2*log(c*x^n)^(q - 1)*e)*log(c*x^n)^(2*q) + 2*(a*b*x^m*log(c*x^n)^(q - 1)*e
 + a*b*d*x^(2*m))*log(c*x^n)^q)/x, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)**2/x,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)^2/x,x, algorit
hm="giac")
[Out] integrate((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + log(c*x^n)^(q - 1)*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)^2\,\left(d\,x^m+e\,\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
[Out] int(((a*x^m + b*log(c*x^n)^q)^2*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.33 $\int \frac{(dx^m+e\log^{-1+q}(cx^n))(ax^m+b\log^q(cx^n))}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{a(aem-bdnq)x^{2m}}{2bmnq}+\left(\frac{bd}{m}-\frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\,\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}+\frac{e(ax$$

```
[Out] -1/2*a*(-b*d*n*q+a*e*m)*x^(2*m)/b/m/n/q+(b*d/m-a*e/n/q)*x^m*GAMMA(1+q,-m*ln
(c*x^n)/n)*ln(c*x^n)^q/((c*x^n)^(m/n))/((-m*ln(c*x^n)/n)^q)+1/2*e*(a*x^m+b*
ln(c*x^n)^q)^2/b/n/q
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2625, 14, 2347, 2212}

$$x^m(cx^n)^{-\frac{m}{n}}\log^q(cx^n)\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(\frac{bd}{m}-\frac{ae}{nq}\right)\text{Gamma}\left(q+1,-\frac{m\log(cx^n)}{n}\right)+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\frac{ax^{2m}(aem-bdnq)}{2bmnq}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] -1/2*(a*(a*e*m - b*d*n*q)*x^(2*m))/(b*m*n*q) + (((b*d)/m - (a*e)/(n*q))*x^m
*Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q)/((c*x^n)^(m/n)*(-((m*Log[c
*x^n])/n))^q) + (e*(a*x^m + b*Log[c*x^n]^q)^2)/(2*b*n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2625

```
Int[((Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x_)^(m_.))^(p_.)*(Log[(c_.)
*(x_)^(n_.)]^(r_.)*(e_.) + (d_.)*(x_)^(m_.)))/(x_), x_Symbol] :> Simp[e*((a
*x^m + b*Log[c*x^n]^q)^(p + 1)/(b*n*q*(p + 1))), x] - Dist[(a*e*m - b*d*n*q
)/(b*n*q), Int[x^(m - 1)*(a*x^m + b*Log[c*x^n]^q)^p, x], x] /; FreeQ[{a, b,
 c, d, e, m, n, p, q, r}, x] && EqQ[r, q - 1] && NeQ[p, -1] && NeQ[a*e*m -
b*d*n*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(dx^m+e\log^{-1+q}(cx^n)\right)\left(ax^m+b\log^q(cx^n)\right)}{x}\,dx &= \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(-d+\frac{aem}{bnq}\right)\int x^{-1+m}(ax^{m}\\
&= \frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(-d+\frac{aem}{bnq}\right)\int\left(ax^{-1+2m}\right.\\
&= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m}+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\left(b\left(-d\right.\right.\\
&= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m}+\frac{e(ax^m+b\log^q(cx^n))^2}{2bnq}-\frac{\left(b\left(-d\right.\right.}{}\\
&= \frac{a\left(d-\frac{aem}{bnq}\right)x^{2m}}{2m}+\left(\frac{bd}{m}-\frac{ae}{nq}\right)x^m(cx^n)^{-\frac{m}{n}}\Gamma\left(1+q,\right.
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 157, normalized size = 1.13

$$\frac{(cx^n)^{-\frac{m}{n}}\left(-\frac{m\log(cx^n)}{n}\right)^{-q}\left(-2aemqx^m\Gamma\left(q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)+2bdnqx^m\Gamma\left(1+q,-\frac{m\log(cx^n)}{n}\right)\log^q(cx^n)+(cx^n)^{m/n}\left(-\frac{m\log(cx^n)}{n}\right)^q\left(adnqx^{2m}+bem\log^{2q}(cx^n)\right)\right)}{2mnq}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x^m + e*Log[c*x^n]^(-1 + q))*(a*x^m + b*Log[c*x^n]^q))/x,x]
```

```
[Out] (-2*a*e*m*q*x^m*Gamma[q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + 2*b*d*n*q*x^m*
Gamma[1 + q, -((m*Log[c*x^n])/n)]*Log[c*x^n]^q + (c*x^n)^(m/n)*(-((m*Log[c*
x^n])/n))^q*(a*d*n*q*x^(2*m) + b*e*m*Log[c*x^n]^(2*q)))/(2*m*n*q*(c*x^n)^(m
/n)*(-((m*Log[c*x^n])/n))^q)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^m+e\ln\left(c\,x^n\right)^{-1+q}\right)\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))*(a*x^m+b*ln(c*x^n)^q)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="fricas")
```

```
[Out] integral((a*x^m*log(c*x^n)^(q - 1)*e + a*d*x^(2*m) + (b*d*x^m + b*log(c*x^n
)^(q - 1)*e)*log(c*x^n)^q)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^m + b\log{(cx^n)}^q)\left(dx^m + \frac{e\log{(cx^n)}^q}{\log{(cx^n)}}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))*(a*x**m+b*ln(c*x**n)**q)/x,x)
```

```
[Out] Integral((a*x**m + b*log(c*x**n)**q)*(d*x**m + e*log(c*x**n)**q/log(c*x**n)
)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))*(a*x^m+b*log(c*x^n)^q)/x,x, algorithm
="giac")
```

```
[Out] integrate((a*x^m + b*log(c*x^n)^q)*(d*x^m + log(c*x^n)^(q - 1)*e)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^m+b\,\ln\left(c\,x^n\right)^q\right)\,\left(d\,x^m+e\,\ln\left(c\,x^n\right)^{q-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x,x)
```

```
[Out] int(((a*x^m + b*log(c*x^n)^q)*(d*x^m + e*log(c*x^n)^(q - 1)))/x, x)
```

3.34 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

```
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {14, 2339, 30}

$$\frac{e\log^q(cx^n)}{nq}+\frac{dx^m}{m}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
```

```
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x}\,dx=\int\left(dx^{-1+m}+\frac{e\log^{-1+q}(cx^n)}{x}\right)dx$$
$$=\frac{dx^m}{m}+e\int\frac{\log^{-1+q}(cx^n)}{x}\,dx$$
$$=\frac{dx^m}{m}+\frac{e\text{Subst}\left(\int x^{-1+q}\,dx,x,\log(cx^n)\right)}{n}$$
$$=\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{dx^m}{m}+\frac{e\log^q(cx^n)}{nq}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/x,x]
[Out] (d*x^m)/m + (e*Log[c*x^n]^q)/(n*q)
```

Maple [A]
time = 0.13, size = 26, normalized size = 1.04

method	result	size
default	$\frac{d\,x^m}{m}+\frac{e\ln(c\,x^n)^q}{nq}$	26
risch	$\frac{d\,x^m}{m}+\frac{e\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\text{csgn}(ic\,x^n)(-\text{csgn}(ic\,x^n)+\text{csgn}(ic))(-\text{csgn}(ic\,x^n)+\text{csgn}(ix^n))}{2}\right)^q}{nq}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x,x,method=_RETURNVERBOSE)
[Out] d*x^m/m+e*ln(c*x^n)^q/n/q
```

Maxima [A]
time = 0.30, size = 26, normalized size = 1.04

$$\frac{dx^m}{m}+\frac{\log(cx^n)^q e}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="maxima")
```

```
[Out] d*x^m/m + log(c*x^n)^q*e/(n*q)
```

Fricas [A]
time = 0.40, size = 44, normalized size = 1.76

$$\frac{dnqx^m + (mne\log(x) + me\log(c))(n\log(x) + \log(c))^{q-1}}{mnq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="fricas")
[Out] (d*n*q*x^m + (m*n*e*log(x) + m*e*log(c))*(n*log(x) + log(c))^(q - 1))/(m*n*
q)
```

Sympy [A]
time = 21.61, size = 49, normalized size = 1.96

$$d\left(\begin{cases}\frac{x^m}{m} & \text{for}\ m \neq 0\\ \log(x) & \text{otherwise}\end{cases}\right) + e\left(\begin{cases}\frac{\log(x)}{\log(c)} & \text{for}\ n = 0 \wedge q = 0\\ \frac{\log(\log(cx^n))}{n} & \text{for}\ q = 0\\ \frac{\log(c)^q\log(x)}{\log(c)} & \text{for}\ n = 0\\ \frac{\log(cx^n)^q}{nq} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x,x)
[Out] d*Piecewise((x**m/m, Ne(m, 0)), (log(x), True)) + e*Piecewise((log(x)/log(c
), Eq(n, 0) & Eq(q, 0)), (log(log(c*x**n))/n, Eq(q, 0)), (log(c)**q*log(x)/
log(c), Eq(n, 0)), (log(c*x**n)**q/(n*q), True))
```

Giac [A]
time = 5.08, size = 27, normalized size = 1.08

$$\frac{dx^m}{m} + \frac{(n\log(x) + \log(c))^q e}{nq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x,x, algorithm="giac")
[Out] d*x^m/m + (n*log(x) + log(c))^q*e/(n*q)
```

Mupad [B]
time = 0.32, size = 25, normalized size = 1.00

$$\frac{d\,x^m}{m} + \frac{e\ln(c\,x^n)^q}{n\,q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/x,x)
[Out] (d*x^m)/m + (e*log(c*x^n)^q)/(n*q)
```

3.35 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))}\,dx$

Optimal. Leaf size=73

$$\frac{e\log\left(ax^m+b\log^q\left(cx^n\right)\right)}{bnq}+\left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{-1+m}}{ax^m+b\log^q\left(cx^n\right)},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q),x)+e*ln(a*x^
m+b*ln(c*x^n)^q)/b/n/q
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] (e*Log[a*x^m + b*Log[c*x^n]^q])/(b*n*q) + (d - (a*e*m)/(b*n*q))*Defer[Int][
x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q), x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx=\frac{e\log\left(ax^m+b\log^q\left(cx^n\right)\right)}{bnq}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{ax^m+b\log^q\left(cx^n\right)}\,dx$$

Mathematica [A]
time = 3.95, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{d\,x^m+e\ln\left(c\,x^n\right)^{-1+q}}{x\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="maxima")
```

```
[Out] e*log(log(c) + log(x^n))/(b*n) + integrate((b*d*x^m*log(x^n) + (b*d*log(c)
- a*e)*x^m)/(a*b*x*x^m*log(c) + a*b*x*x^m*log(x^n) + (b^2*x*log(c) + b^2*x*
log(x^n))*(log(c) + log(x^n))^q), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="fricas")
```

```
[Out] integral((d*x^m + log(c*x^n)^(q - 1)*e)/(a*x*x^m + b*x*log(c*x^n)^q), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^m + \frac{e \log{(cx^n)}^q}{\log{(cx^n)}}}{x\left(ax^m + b\log{(cx^n)}^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q),x)
```

```
[Out] Integral((d*x**m + e*log(c*x**n)**q/log(c*x**n))/(x*(a*x**m + b*log(c*x**n)
**q)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q),x, algorithm
="giac")
```

```
[Out] integrate((d*x^m + log(c*x^n)^(q - 1)*e)/((a*x^m + b*log(c*x^n)^q)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^m+e\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)),x)
```

```
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)), x)
```

3.36 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=75

$$-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}+\left(d-\frac{aem}{bnq}\right)\operatorname{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^2,x)-e/b/n/q
/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] -(e/(b*n*q*(a*x^m + b*Log[c*x^n]^q))) + (d - (a*e*m)/(b*n*q))*Defer[Int][x^
(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^2, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=-\frac{e}{bnq\left(ax^m+b\log^q\left(cx^n\right)\right)}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Mathematica [A]
time = 22.32, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^2), x
]
```

Maple [**A**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="maxima")
```

```
[Out] -(b*d*log(c) + b*d*log(x^n) - a*e)/(a^2*b*m*x^m*log(x^n) - (n*q - m*log(c))
*a^2*b*x^m + (a*b^2*m*log(x^n) - (n*q - m*log(c))*a*b^2)*(log(c) + log(x^n)
)^q) + integrate(-((m*n*(q - 1) - m^2*log(c))*a*e + (d*m*n*q*log(c) - (q^2
- q)*d*n^2)*b + (b*d*m*n*q - a*m^2*e)*log(x^n))/(a^2*b*m^2*x*x^m*log(x^n)^2
 - 2*(m*n*q - m^2*log(c))*a^2*b*x*x^m*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c)
+ m^2*log(c)^2)*a^2*b*x*x^m + (a*b^2*m^2*x*log(x^n)^2 - 2*(m*n*q - m^2*log(
c))*a*b^2*x*log(x^n) + (n^2*q^2 - 2*m*n*q*log(c) + m^2*log(c)^2)*a*b^2*x)*(
log(c) + log(x^n))^q), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + log(c*x^n)^(q - 1)*e)/(2*a*b*x*x^m*log(c*x^n)^q + a^2*x*x
^(2*m) + b^2*x*log(c*x^n)^(2*q)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + log(c*x^n)^(q - 1)*e)/((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^m+e\ln\left(c\,x^n\right)^{q-1}}{x\,{\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^2), x)
```

3.37 $\int \frac{dx^m+e\log^{-1+q}(cx^n)}{x(ax^m+b\log^q(cx^n))^3}\,dx$

Optimal. Leaf size=77

$$-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}+\left(d-\frac{aem}{bnq}\right)\text{Int}\left(\frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3},x\right)$$

```
[Out] (d-a*e*m/b/n/q)*CannotIntegrate(x^(-1+m)/(a*x^m+b*ln(c*x^n)^q)^3,x)-1/2*e/b
/n/q/(a*x^m+b*ln(c*x^n)^q)^2
```

Rubi [A]
time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] -1/2*e/(b*n*q*(a*x^m + b*Log[c*x^n]^q)^2) + (d - (a*e*m)/(b*n*q))*Defer[Int
][x^(-1 + m)/(a*x^m + b*Log[c*x^n]^q)^3, x]
```

Rubi steps

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx=-\frac{e}{2bnq\left(ax^m+b\log^q\left(cx^n\right)\right)^2}-\left(-d+\frac{aem}{bnq}\right)\int \frac{x^{-1+m}}{\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Mathematica [A]
time = 63.42, size = 0, normalized size = 0.00

$$\int \frac{dx^m+e\log^{-1+q}\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3),x]
```

```
[Out] Integrate[(d*x^m + e*Log[c*x^n]^(-1 + q))/(x*(a*x^m + b*Log[c*x^n]^q)^3), x
]
```

Maple [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{d\,x^{m}+e\ln\left(c\,x^{n}\right)^{-1+q}}{x\left(a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

```
[Out] int((d*x^m+e*ln(c*x^n)^(-1+q))/x/(a*x^m+b*ln(c*x^n)^q)^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="maxima")
```

```
[Out] -1/2*(a*b*d*m^2*x^m*log(x^n)^3 + (a^2*m^2*e - (4*d*m*n*q - 3*d*m^2*log(c))*
a*b)*x^m*log(x^n)^2 + ((2*m^2*log(c) + m*n)*a^2*e - (8*d*m*n*q*log(c) - 3*d
*m^2*log(c)^2 - (3*q^2 - q)*d*n^2)*a*b)*x^m*log(x^n) - ((n^2*q^2 - m^2*log(
c)^2 - m*n*log(c))*a^2*e + (4*d*m*n*q*log(c)^2 - d*m^2*log(c)^3 - (3*q^2 -
q)*d*n^2*log(c))*a*b)*x^m - ((m*n*(2*q - 1)*log(c) - 2*m^2*log(c)^2)*a*b*e
+ (2*d*m*n*q*log(c)^2 - (2*q^2 - q)*d*n^2*log(c))*b^2 + 2*(b^2*d*m*n*q - a*
b*m^2*e)*log(x^n)^2 + ((m*n*(2*q - 1) - 4*m^2*log(c))*a*b*e + (4*d*m*n*q*lo
g(c) - (2*q^2 - q)*d*n^2)*b^2)*log(x^n))*(log(c) + log(x^n))^q)/(a^4*b*m^3*
x^(3*m)*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^4*b*x^(3*m)*log(x^n)^2 + 3*
(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^4*b*x^(3*m)*log(x^n) - (n^3
*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^4*b*x^(3*m
) + (a^2*b^3*m^3*x^m*log(x^n)^3 - 3*(m^2*n*q - m^3*log(c))*a^2*b^3*x^m*log(
x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n*q*log(c) + m^3*log(c)^2)*a^2*b^3*x^m*log(x^
n) - (n^3*q^3 - 3*m*n^2*q^2*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^2
*b^3*x^m)*(log(c) + log(x^n))^(2*q) + 2*(a^3*b^2*m^3*x^(2*m)*log(x^n)^3 - 3
*(m^2*n*q - m^3*log(c))*a^3*b^2*x^(2*m)*log(x^n)^2 + 3*(m*n^2*q^2 - 2*m^2*n
*q*log(c) + m^3*log(c)^2)*a^3*b^2*x^(2*m)*log(x^n) - (n^3*q^3 - 3*m*n^2*q^2
*log(c) + 3*m^2*n*q*log(c)^2 - m^3*log(c)^3)*a^3*b^2*x^(2*m))*(log(c) + log
(x^n))^q) - integrate(-1/2*(2*(b*d*m^3*n*q - a*m^4*e)*log(x^n)^3 + (m^3*n*(
2*q - 3)*log(c)^2 - 2*m^4*log(c)^3 + 2*(q^2 - 1)*m^2*n^2*log(c) - (2*q^3 -
3*q^2 + q)*m*n^3)*a*e + ((m^3*n*(2*q - 3) - 6*m^4*log(c))*a*e + (6*d*m^3*n*
q*log(c) - (2*q^2 - 3*q)*d*m^2*n^2)*b)*log(x^n)^2 + (2*d*m^3*n*q*log(c)^3 -
 (2*q^2 - 3*q)*d*m^2*n^2*log(c)^2 - 2*(q^3 - q)*d*m*n^3*log(c) + (2*q^4 - 3
*q^3 + q^2)*d*n^4)*b + 2*((m^3*n*(2*q - 3)*log(c) - 3*m^4*log(c)^2 + (q^2 -
 1)*m^2*n^2)*a*e + (3*d*m^3*n*q*log(c)^2 - (2*q^2 - 3*q)*d*m^2*n^2*log(c) -
```

```
 (q^3 - q)*d*m*n^3)*b)*log(x^n))/(a^3*b*m^4*x*x^(2*m)*log(x^n)^4 - 4*(m^3*n
*q - m^4*log(c))*a^3*b*x*x^(2*m)*log(x^n)^3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*lo
g(c) + m^4*log(c)^2)*a^3*b*x*x^(2*m)*log(x^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*
q^2*log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^3*b*x*x^(2*m)*log(x^n) +
(n^4*q^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 - 4*m^3*n*q*log(c)^3
 + m^4*log(c)^4)*a^3*b*x*x^(2*m) + (a^2*b^2*m^4*x*x^m*log(x^n)^4 - 4*(m^3*n
*q - m^4*log(c))*a^2*b^2*x*x^m*log(x^n)^3 + 6*(m^2*n^2*q^2 - 2*m^3*n*q*log(
c) + m^4*log(c)^2)*a^2*b^2*x*x^m*log(x^n)^2 - 4*(m*n^3*q^3 - 3*m^2*n^2*q^2*
log(c) + 3*m^3*n*q*log(c)^2 - m^4*log(c)^3)*a^2*b^2*x*x^m*log(x^n) + (n^4*q
^4 - 4*m*n^3*q^3*log(c) + 6*m^2*n^2*q^2*log(c)^2 - 4*m^3*n*q*log(c)^3 + m^4
*log(c)^4)*a^2*b^2*x*x^m)*(log(c) + log(x^n))^q), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="fricas")
```

```
[Out] integral((d*x^m + log(c*x^n)^(q - 1)*e)/(3*a*b^2*x*x^m*log(c*x^n)^(2*q) + 3
*a^2*b*x*x^(2*m)*log(c*x^n)^q + a^3*x*x^(3*m) + b^3*x*log(c*x^n)^(3*q)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**m+e*ln(c*x**n)**(-1+q))/x/(a*x**m+b*ln(c*x**n)**q)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^m+e*log(c*x^n)^(-1+q))/x/(a*x^m+b*log(c*x^n)^q)^3,x, algorit
hm="giac")
```

```
[Out] integrate((d*x^m + log(c*x^n)^(q - 1)*e)/((a*x^m + b*log(c*x^n)^q)^3*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^m+e\ln\left(c\,x^n\right)^{q-1}}{x\,\left(a\,x^m+b\ln\left(c\,x^n\right)^q\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3),x)
[Out] int((d*x^m + e*log(c*x^n)^(q - 1))/(x*(a*x^m + b*log(c*x^n)^q)^3), x)
```

3.38 $\int \frac{adnx^m-admx^m\log(cx^n)-bdn(-1+q)\log^q(cx^n)}{x(ax^m+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=26

$$\frac{d\log\left(cx^n\right)}{ax^m+b\log^q\left(cx^n\right)}$$

```
[Out] d*ln(c*x^n)/(a*x^m+b*ln(c*x^n)^q)
```

Rubi [A]
time = 0.16, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$, Rules used = {2626}

$$\frac{d\log\left(cx^n\right)}{ax^m+b\log^q\left(cx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/(x*(a*
x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Rule 2626

```
Int[(Log[(c_.)*(x_)^(n_.)]^(q_.)*(f_.) + (d_.)*(x_)^(m_.) + Log[(c_.)*(x_)^
(n_.)]*(e_.)*(x_)^(m_.))/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (a_.)*(x
_)^(m_.))^2), x_Symbol] :> Simp[d*(Log[c*x^n]/(a*n*(a*x^m + b*Log[c*x^n]^q)
)), x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[e*n + d*m, 0] && EqQ
[a*f + b*d*(q - 1), 0]
```

Rubi steps

$$\int \frac{adnx^m-admx^m\log\left(cx^n\right)-bdn(-1+q)\log^q\left(cx^n\right)}{x\left(ax^m+b\log^q\left(cx^n\right)\right)^2}\,dx=\frac{d\log\left(cx^n\right)}{ax^m+b\log^q\left(cx^n\right)}$$

Mathematica [A]
time = 0.17, size = 26, normalized size = 1.00

$$\frac{d\log\left(cx^n\right)}{ax^m+b\log^q\left(cx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*n*x^m - a*d*m*x^m*Log[c*x^n] - b*d*n*(-1 + q)*Log[c*x^n]^q)/
(x*(a*x^m + b*Log[c*x^n]^q)^2),x]
```

```
[Out] (d*Log[c*x^n])/(a*x^m + b*Log[c*x^n]^q)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.09, size = 158, normalized size = 6.08

method	result	size
risch	$\frac{\left(2\ln(c)+2\ln(x^n)-i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)+i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2+i\pi\,\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2-i\pi\mathrm{csgn}(ic\,x^n)^3\right)d}{2a\,x^m+2b\left(\ln(c)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic\,x^n)(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^q}$	158

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*n*x^m-a*d*m*x^m*ln(c*x^n)-b*d*n*(-1+q)*ln(c*x^n)^q)/x/(a*x^m+b*ln(
c*x^n)^q)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*ln(c)+2*ln(x^n)-I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*csgn(I
*c)*csgn(I*c*x^n)^2+I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*c*x^n)^3)*
d/(a*x^m+b*(ln(c)+ln(x^n)-1/2*I*Pi*csgn(I*c*x^n)*(-csgn(I*c*x^n)+csgn(I*c))
*(-csgn(I*c*x^n)+csgn(I*x^n)))^q)
```

Maxima [A]
time = 0.45, size = 31, normalized size = 1.19

$$\frac{d\log\left(c\right)+d\log\left(x^n\right)}{ax^m+b(\log\left(c\right)+\log\left(x^n\right))^q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] (d*log(c) + d*log(x^n))/(a*x^m + b*(log(c) + log(x^n))^q)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 1.15

$$\frac{dn\log\left(x\right)+d\log\left(c\right)}{(n\log\left(x\right)+\log\left(c\right))^qb+ax^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] (d*n*log(x) + d*log(c))/((n*log(x) + log(c))^q*b + a*x^m)
```

Sympy [A]
time = 44.66, size = 22, normalized size = 0.85

$$\frac{d\log\left(cx^{n}\right)}{ax^{m}+b\log\left(cx^{n}\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x**m-a*d*m*x**m*ln(c*x**n)-b*d*n*(-1+q)*ln(c*x**n)**q)/x/(
a*x**m+b*ln(c*x**n)**q)**2,x)
```

```
[Out] d*log(c*x**n)/(a*x**m + b*log(c*x**n)**q)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*n*x^m-a*d*m*x^m*log(c*x^n)-b*d*n*(-1+q)*log(c*x^n)^q)/x/(a*x
^m+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate(-(b*d*n*(q - 1)*log(c*x^n)^q + a*d*m*x^m*log(c*x^n) - a*d*n*x^m)/
((a*x^m + b*log(c*x^n)^q)^2*x), x)
```

Mupad [B]
time = 0.33, size = 26, normalized size = 1.00

$$\frac{d\,\ln\left(c\,x^{n}\right)}{a\,x^{m}+b\ln\left(c\,x^{n}\right)^{q}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d*m*x^m*log(c*x^n) - a*d*n*x^m + b*d*n*log(c*x^n)^q*(q - 1))/(x*(a*
x^m + b*log(c*x^n)^q)^2),x)
```

```
[Out] (d*log(c*x^n))/(a*x^m + b*log(c*x^n)^q)
```

3.39 $\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$

Optimal. Leaf size=61

$$\frac{\log(cx^n)}{a(ax+b\log^q(cx^n))}-\frac{n(1-q)\operatorname{Int}\left(\frac{1}{x(ax+b\log^q(cx^n))},x\right)}{a}$$

```
[Out] -n*(1-q)*CannotIntegrate(1/x/(a*x+b*ln(c*x^n)^q),x)/a+ln(c*x^n)/a/(a*x+b*ln
(c*x^n)^q)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Log[c*x^n]/(a*(a*x + b*Log[c*x^n]^q)) - (n*(1 - q)*Defer[Int][1/(x*(a*x + b
*Log[c*x^n]^q)), x])/a
```

Rubi steps

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx=\frac{\log(cx^n)}{a(ax+b\log^q(cx^n))}-\frac{(n(1-q))\int \frac{1}{x(ax+b\log^q(cx^n))}\,dx}{a}$$

Mathematica [A]
time = 85.79, size = 0, normalized size = 0.00

$$\int \frac{nq-\log(cx^n)}{(ax+b\log^q(cx^n))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2,x]
```

```
[Out] Integrate[(n*q - Log[c*x^n])/(a*x + b*Log[c*x^n]^q)^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{nq-\ln(c\,x^n)}{(ax+b\ln(c\,x^n)^q)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

```
[Out] int((n*q-ln(c*x^n))/(a*x+b*ln(c*x^n)^q)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="maxima")
```

```
[Out] n*(q - 1)*integrate(1/(a^2*x^2 + a*b*x*(log(c) + log(x^n))^q), x) + (log(c)
 + log(x^n))/(a^2*x + a*b*(log(c) + log(x^n))^q)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="fricas")
```

```
[Out] integral((n*q - log(c*x^n))/(a^2*x^2 + 2*a*b*x*log(c*x^n)^q + b^2*log(c*x^n
)^(2*q)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{nq - \log (cx^n)}{(ax + b\log (cx^n)^q)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-ln(c*x**n))/(a*x+b*ln(c*x**n)**q)**2,x)
```

```
[Out] Integral((n*q - log(c*x**n))/(a*x + b*log(c*x**n)**q)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((n*q-log(c*x^n))/(a*x+b*log(c*x^n)^q)^2,x, algorithm="giac")
```

```
[Out] integrate((n*q - log(c*x^n))/(a*x + b*log(c*x^n)^q)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\ln\left(c\,x^n\right)-n\,q}{{\left(b\,{\ln\left(c\,x^n\right)}^q+a\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2,x)
[Out] int(-(log(c*x^n) - n*q)/(b*log(c*x^n)^q + a*x)^2, x)
```

3.40 $\int \frac{\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

```
[Out] -1/2*polylog(2,1-2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2497}

$$-\frac{\sqrt{-\frac{e}{d}}\,\text{PolyLog}\left(2,1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*(Sqrt[-(e/d)]*PolyLog[2, 1 - (2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]
)/e
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log \left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\sqrt{-\frac{e}{d}}\,\text{Li}_2\left(1-\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 625 vs. 2(49) = 98.
time = 0.31, size = 625, normalized size = 12.76

$$\frac{-2\log\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}\,x\right)+2\log\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}\,x\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{d\sqrt{-\frac{e}{d}}+ex}{\sqrt{-d}\,\sqrt{e}+d\sqrt{-\frac{e}{d}}}\right)+2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{e+d\left(-\frac{e}{d}\right)^{3/2}x}{e+\sqrt{-d}\,\sqrt{e}\,\sqrt{-\frac{e}{d}}}\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{2x\left(d\sqrt{-\frac{e}{d}}+ex\right)}{d+ex^2}\right)+2\text{Li}_2\left(\frac{\sqrt{-d}+\sqrt{e}\,x}{\sqrt{-d}+\frac{\sqrt{e}}{\sqrt{-\frac{e}{d}}}}\right)+2\text{Li}_2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)-2\text{Li}_2\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)+2\text{Li}_2\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\text{Li}_2\left(1+\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)-2\text{Li}_2\left(\frac{\sqrt{-d}\,\sqrt{e}-ex}{\sqrt{-d}\,\sqrt{e}+d\sqrt{-\frac{e}{d}}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d*Sqrt[-(e/d)] + e*x)/(Sqrt[-d]*
Sqrt[e] + d*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(e + d*(-(e/d)
)^(3/2)*x)/(e + Sqrt[-d]*Sqrt[e]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d] - Sqrt[e]*
x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] + Sqrt[e]
*x]*Log[(2*x*(d*Sqrt[-(e/d)] + e*x))/(d + e*x^2)] + 2*PolyLog[2, (Sqrt[-d]
+ Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e]/Sqrt[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]
*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2,
 (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2
)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[-d]*Sqrt[e] + d*Sqrt[-(e/d
)])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(\frac{2x\left(ex+d\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

[Out] int(ln(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="m
axima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.36, size = 47, normalized size = 0.96

$$-\frac{1}{2}\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e+dx\sqrt{-\frac{e}{d}}\right)}{x^2e+d}+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="f
ricas")
```

```
[Out] -1/2*sqrt(-e/d)*dilog(-2*(x^2*e + d*x*sqrt(-e/d))/(x^2*e + d) + 1)*e^(-1)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm="g
iac")
```

```
[Out] integrate(log(2*(x*e + d*sqrt(-e/d))*x/(x^2*e + d))/(x^2*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.41 $\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

```
[Out] 1/2*polylog(2,1+2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))*(-e/d)^(1/2)/e
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2497}

$$\frac{\sqrt{-\frac{e}{d}}\,\mathrm{PolyLog}\left(2,\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (Sqrt[-(e/d)]*PolyLog[2, 1 + (2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)])/(2*
e)
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(1+\frac{2x\left(d\sqrt{-\frac{e}{d}}-ex\right)}{d+ex^2}\right)}{2e}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 642 vs. $2(50) = 100$.
time = 0.27, size = 642, normalized size = 12.84

$$\frac{-2\log\left(\frac{\sqrt{e}x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}x\right)+2\log\left(\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}x\right)+2\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)-2\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)+2\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(1+\sqrt{-\frac{e}{d}}x\right)}{\sqrt{e}-\sqrt{-d}\sqrt{-\frac{e}{d}}}\right)-2\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{\sqrt{e}\left(1+\sqrt{-\frac{e}{d}}x\right)}{\sqrt{e}+\sqrt{-d}\sqrt{-\frac{e}{d}}}\right)+2\log\left(\sqrt{-d}-\sqrt{e}x\right)\log\left(\frac{2ex\left(\frac{1}{\sqrt{-\frac{e}{d}}}+x\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}x\right)\log\left(\frac{2ex\left(\frac{1}{\sqrt{-\frac{e}{d}}}+x\right)}{d+ex^2}\right)-2\mathrm{Li}_2\left(\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{e}+\sqrt{-d}\sqrt{-\frac{e}{d}}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt{-\frac{e}{d}}\left(\sqrt{-d}+\sqrt{e}x\right)}{-\sqrt{e}+\sqrt{-d}\sqrt{-\frac{e}{d}}}\right)+2\mathrm{Li}_2\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\sqrt{e}x}{2d}\right)+2\mathrm{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}x}{2d}\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{e}x}{(-d)^{3/2}}\right)}{4\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(d*Sqrt[-(e/d)] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[e]*(1 + Sqrt[-(e/d)]*x))/(S
qrt[e] - Sqrt[-d]*Sqrt[-(e/d)])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[e]
*(1 + Sqrt[-(e/d)]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] + 2*Log[Sqrt[-d]
- Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*Log[Sqrt[-d]
 + Sqrt[e]*x]*Log[(2*e*x*(1/Sqrt[-(e/d)] + x))/(d + e*x^2)] - 2*PolyLog[2,
(Sqrt[-(e/d)]*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e] + Sqrt[-d]*Sqrt[-(e/d)])] +
2*PolyLog[2, (Sqrt[-(e/d)]*(Sqrt[-d] + Sqrt[e]*x))/(-Sqrt[e] + Sqrt[-d]*Sqr
t[-(e/d)])] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sq
rt[-d]*Sqrt[e]*x)/(2*d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2
*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2x\left(-ex+d\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.37, size = 48, normalized size = 0.96

$$\frac{1}{2}\sqrt{-\frac{e}{d}}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e-dx\sqrt{-\frac{e}{d}}\right)}{x^2e+d}+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"fricas")
```

```
[Out] 1/2*sqrt(-e/d)*dilog(-2*(x^2*e - d*x*sqrt(-e/d))/(x^2*e + d) + 1)*e^(-1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*(-e/d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*(-e/d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorithm=
"giac")
```

```
[Out] integrate(log(2*(x*e - d*sqrt(-e/d))*x/(x^2*e + d))/(x^2*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-d\,\sqrt{-\frac{e}{d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
```

```
[Out] int(log((2*x*(e*x - d*(-e/d)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.42 $\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\mathrm{Li}_2\left(1+\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

```
[Out] -1/2*polylog(2,1+2*x*e^(1/2)*((-d)^(1/2)-x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2497}

$$-\frac{\text{PolyLog}\left(2,\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{d+ex^2}+1\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*PolyLog[2, 1 + (2*Sqrt[e]*x*(Sqrt[-d] - Sqrt[e]*x))/(d + e*x^2)]/(Sqrt
[-d]*Sqrt[e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = -\frac{\mathrm{Li}_2\left(1+\frac{2\sqrt{e}\,x\left(\sqrt{-d}-\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(53) = 106.
time = 0.15, size = 320, normalized size = 6.04

$$\frac{-2\log\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+2\log\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}\,x\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{d-\sqrt{-d}\sqrt{e}\,x}{2d}\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{2\left(-\sqrt{-d}\sqrt{e}\,x+ex^2\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{2\left(-\sqrt{-d}\sqrt{e}\,x+ex^2\right)}{d+ex^2}\right)+2\mathrm{Li}_2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)+2\mathrm{Li}_2\left(\frac{d+\sqrt{-d}\sqrt{e}\,x}{2d}\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*((d*Sqrt[e])/Sqrt[-d] + e*x))/(d + e*x^2)]/(d + e*x^2),x
]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + 2*Log[(d*Sqrt[e]*
x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Log[Sqrt[-d] + Sqrt[e]*x]^2 + 2*
Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2)] - 2*L
og[Sqrt[-d] + Sqrt[e]*x]*Log[(2*(-(Sqrt[-d]*Sqrt[e]*x) + e*x^2))/(d + e*x^2
)] + 2*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] + 2*PolyLog[2, (d + Sqrt[-d]*Sq
rt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*S
qrt[e])
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{2x\left(ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.83

$$\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e-\sqrt{-d}\,xe^{\frac{1}{2}}\right)}{x^2e+d}+1\right)e^{\left(-\frac{1}{2}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-d)*dilog(-2*(x^2*e - sqrt(-d)*x*e^(1/2))/(x^2*e + d) + 1)*e^(-1/2
)/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] integrate(log(2*(x*e + d*e^(1/2)/sqrt(-d))*x/(x^2*e + d))/(x^2*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.43 $\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=52

$$\frac{\text{Li}_2\left(1-\frac{2\sqrt{e}\,x\left(\sqrt{-d}+\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

```
[Out] 1/2*polylog(2,1-2*x*e^(1/2)*((-d)^(1/2)+x*e^(1/2))/(e*x^2+d))/(-d)^(1/2)/e^
(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2497}

$$\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{e}\,x\left(\sqrt{-d}+\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*Sqrt[e]*x*(Sqrt[-d] + Sqrt[e]*x))/(d + e*x^2)]/(2*Sqrt[-d
]*Sqrt[e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\frac{d\sqrt{e}}{\sqrt{-d}}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx = \frac{\mathrm{Li}_2\left(1-\frac{2\sqrt{e}\,x\left(\sqrt{-d}+\sqrt{e}\,x\right)}{d+ex^2}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(52) = 104.
time = 0.13, size = 316, normalized size = 6.08

$$\frac{-2\log\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}\,x\right)+2\log\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{2\left(\sqrt{-d}\,\sqrt{e}\,x+ex^2\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{2\left(\sqrt{-d}\,\sqrt{e}\,x+ex^2\right)}{d+ex^2}\right)+2\mathrm{Li}_2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*((d*Sqrt[e])/Sqrt[-d] - e*x))/(d + e*x^2)]/(d + e*x^2),
x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - 2*
Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] + 2*Log[Sqrt[
-d] - Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] - 2*Log[
Sqrt[-d] + Sqrt[e]*x]*Log[(2*(Sqrt[-d]*Sqrt[e]*x + e*x^2))/(d + e*x^2)] + 2
*PolyLog[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*
x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*x)/(-d)^(3/2)])/(4*Sqrt[-d]*Sqrt[e]
)
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2x\left(-ex+\frac{d\sqrt{e}}{\sqrt{-d}}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 0.35, size = 43, normalized size = 0.83

$$-\frac{\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e+\sqrt{-d}\,xe^{\frac{1}{2}}\right)}{x^2e+d}+1\right)e^{\left(-\frac{1}{2}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="fricas")
```

```
[Out] -1/2*sqrt(-d)*dilog(-2*(x^2*e + sqrt(-d)*x*e^(1/2))/(x^2*e + d) + 1)*e^(-1/
2)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d*e**(1/2)/(-d)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d*e^(1/2)/(-d)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algo
rithm="giac")
```

```
[Out] integrate(log(2*(x*e - d*e^(1/2)/sqrt(-d))*x/(x^2*e + d))/(x^2*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{-d}\,\sqrt{e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + (-d)^(1/2)*e^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.44 $\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=49

$$\frac{\text{Li}_2\left(1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] 1/2*polylog(2,1-2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2497}

$$\frac{\text{PolyLog}\left(2,1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] PolyLog[2, 1 - (2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(2*Sqrt[d]*Sqrt[
-e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=\frac{\text{Li}_2\left(1-\frac{2x\left(\sqrt{d}\sqrt{-e}+ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 641 vs. 2(49) = 98.
time = 0.22, size = 641, normalized size = 13.08

$$\frac{-2\log\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}\,x\right)+2\log\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}\,x\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)+2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{\sqrt{d}\,\sqrt{-e}+ex}{\sqrt{d}\,\sqrt{-e}-\sqrt{-d}\,\sqrt{e}}\right)-2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{\sqrt{d}\,\sqrt{-e}+ex}{\sqrt{d}\,\sqrt{-e}+\sqrt{-d}\,\sqrt{e}}\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{2x\left(\sqrt{d}\,\sqrt{-e}+ex\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{2x\left(\sqrt{d}\,\sqrt{-e}+ex\right)}{d+ex^2}\right)+2\mathrm{Li}_2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)+2\mathrm{Li}_2\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)-2\mathrm{Li}_2\left(\frac{\sqrt{-d}\,\sqrt{e}-ex}{\sqrt{d}\,\sqrt{-e}+\sqrt{-d}\,\sqrt{e}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt{-d}\,\sqrt{e}+ex}{-\sqrt{d}\,\sqrt{-e}+\sqrt{-d}\,\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] + e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] + e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d] +
 Sqrt[e]*x]*Log[(2*x*(Sqrt[d]*Sqrt[-e] + e*x))/(d + e*x^2)] + 2*PolyLog[2,
1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*d)] +
2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sqrt[e]*
x)/(-d)^(3/2)] - 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] - e*x)/(Sqrt[d]*Sqrt[-e] +
Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(-(Sqrt[d]*Sqrt[
-e]) + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{2x\left(ex+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 46, normalized size = 0.94

$$-\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e+\sqrt{d}\,x\sqrt{-e}\right)}{x^2e+d}+1\right)e^{(-1)}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="fricas")
```

```
[Out] -1/2*sqrt(-e)*dilog(-2*(x^2*e + sqrt(d)*x*sqrt(-e))/(x^2*e + d) + 1)*e^(-1)
/sqrt(d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2*x*(e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2*x*(e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x + d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.45 $\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx$

Optimal. Leaf size=50

$$-\frac{\mathrm{Li}_2\left(1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

```
[Out] -1/2*polylog(2,1+2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/d^(1/2)/(-e)^(1/2
)
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2497}

$$-\frac{\text{PolyLog}\left(2,\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}+1\right)}{2\sqrt{d}\sqrt{-e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] -1/2*PolyLog[2, 1 + (2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(Sqrt[d]*Sq
rt[-e])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\log\left(-\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{d+ex^2}\,dx=-\frac{\mathrm{Li}_2\left(1+\frac{2x\left(\sqrt{d}\sqrt{-e}-ex\right)}{d+ex^2}\right)}{2\sqrt{d}\sqrt{-e}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 645 vs. 2(50) = 100.
time = 0.18, size = 645, normalized size = 12.90

$$\frac{-2\log\left(\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+\log^2\left(\sqrt{-d}-\sqrt{e}\,x\right)+2\log\left(\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)-\log^2\left(\sqrt{-d}+\sqrt{e}\,x\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{\sqrt{d}\,\sqrt{-e}-ex}{\sqrt{d}\,\sqrt{-e}-\sqrt{-d}\,\sqrt{e}}\right)+2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{\sqrt{d}\,\sqrt{-e}-ex}{\sqrt{d}\,\sqrt{-e}+\sqrt{-d}\,\sqrt{e}}\right)+2\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{2x\left(-\sqrt{d}\,\sqrt{-e}+ex\right)}{d+ex^2}\right)-2\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(\frac{2x\left(-\sqrt{d}\,\sqrt{-e}+ex\right)}{d+ex^2}\right)+2\mathrm{Li}_2\left(1+\frac{\sqrt{e}\,x}{\sqrt{-d}}\right)-2\mathrm{Li}_2\left(\frac{d-\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)+2\mathrm{Li}_2\left(\frac{d+\sqrt{-d}\,\sqrt{e}\,x}{2d}\right)-2\mathrm{Li}_2\left(1+\frac{d\sqrt{e}\,x}{(-d)^{3/2}}\right)-2\mathrm{Li}_2\left(\frac{-\sqrt{-d}\,\sqrt{e}+ex}{\sqrt{d}\,\sqrt{-e}-\sqrt{-d}\,\sqrt{e}}\right)+2\mathrm{Li}_2\left(\frac{\sqrt{-d}\,\sqrt{e}+ex}{\sqrt{d}\,\sqrt{-e}+\sqrt{-d}\,\sqrt{e}}\right)}{4\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-2*x*(Sqrt[d]*Sqrt[-e] - e*x))/(d + e*x^2)]/(d + e*x^2),x]
```

```
[Out] (-2*Log[(Sqrt[e]*x)/Sqrt[-d]]*Log[Sqrt[-d] - Sqrt[e]*x] + Log[Sqrt[-d] - Sq
rt[e]*x]^2 + 2*Log[(d*Sqrt[e]*x)/(-d)^(3/2)]*Log[Sqrt[-d] + Sqrt[e]*x] - Lo
g[Sqrt[-d] + Sqrt[e]*x]^2 + 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(d - Sqrt[-d]*S
qrt[e]*x)/(2*d)] - 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(d + Sqrt[-d]*Sqrt[e]*x)
/(2*d)] - 2*Log[Sqrt[-d] - Sqrt[e]*x]*Log[(Sqrt[d]*Sqrt[-e] - e*x)/(Sqrt[d]
*Sqrt[-e] - Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] + Sqrt[e]*x]*Log[(Sqrt[d]*S
qrt[-e] - e*x)/(Sqrt[d]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])] + 2*Log[Sqrt[-d] - Sq
rt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] - 2*Log[Sqrt[-d
] + Sqrt[e]*x]*Log[(2*x*(-(Sqrt[d]*Sqrt[-e]) + e*x))/(d + e*x^2)] + 2*PolyL
og[2, 1 + (Sqrt[e]*x)/Sqrt[-d]] - 2*PolyLog[2, (d - Sqrt[-d]*Sqrt[e]*x)/(2*
d)] + 2*PolyLog[2, (d + Sqrt[-d]*Sqrt[e]*x)/(2*d)] - 2*PolyLog[2, 1 + (d*Sq
rt[e]*x)/(-d)^(3/2)] - 2*PolyLog[2, (-(Sqrt[-d]*Sqrt[e]) + e*x)/(Sqrt[d]*Sq
rt[-e] - Sqrt[-d]*Sqrt[e])] + 2*PolyLog[2, (Sqrt[-d]*Sqrt[e] + e*x)/(Sqrt[d
]*Sqrt[-e] + Sqrt[-d]*Sqrt[e])])/(4*Sqrt[-d]*Sqrt[e])
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-\frac{2x\left(-ex+\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

```
[Out] int(ln(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.36, size = 47, normalized size = 0.94

$$\frac{\sqrt{-e}\,\mathrm{Li}_2\left(-\frac{2\left(x^2e-\sqrt{d}\,x\sqrt{-e}\right)}{x^2e+d}+1\right)e^{(-1)}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="fricas")
```

```
[Out] 1/2*sqrt(-e)*dilog(-2*(x^2*e - sqrt(d)*x*sqrt(-e))/(x^2*e + d) + 1)*e^(-1)/
sqrt(d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2*x*(-e*x+d**(1/2)*(-e)**(1/2))/(e*x**2+d))/(e*x**2+d),x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2*x*(-e*x+d^(1/2)*(-e)^(1/2))/(e*x^2+d))/(e*x^2+d),x, algori
thm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{2\,x\,\left(e\,x-\sqrt{d}\,\sqrt{-e}\right)}{e\,x^2+d}\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2),x)
[Out] int(log((2*x*(e*x - d^(1/2)*(-e)^(1/2)))/(d + e*x^2))/(d + e*x^2), x)
```

3.46 $\int (ex)^m (a+b\log (c\log^p(dx)))\, dx$

Optimal. Leaf size=67

$$-\frac{bp(dx)^{-1-m}(ex)^{1+m}\text{Ei}((1+m)\log(dx))}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(1+m)}$$

```
[Out] -b*p*(d*x)^(-1-m)*(e*x)^(1+m)*Ei((1+m)*ln(d*x))/e/(1+m)+(e*x)^(1+m)*(a+b*ln
(c*ln(d*x)^p))/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2602, 2347, 2209}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p(dx)\right)\right)}{e(m+1)}-\frac{bp(dx)^{-m-1}(ex)^{m+1}\text{Ei}((m+1)\log(dx))}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] -((b*p*(d*x)^(-1 - m)*(e*x)^(1 + m)*ExpIntegralEi[(1 + m)*Log[d*x]])/(e*(1
+ m))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x]^p]))/(e*(1 + m))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a + b\log(c\log^p(dx)))\,dx = \frac{(ex)^{1+m}(a + b\log(c\log^p(dx)))}{e(1+m)} - \frac{(bp)\int \frac{(ex)^m}{\log(dx)}\,dx}{1+m}$$
$$= \frac{(ex)^{1+m}(a + b\log(c\log^p(dx)))}{e(1+m)} - \frac{(bp(dx)^{-1-m}(ex)^{1+m})\,\text{Subst}\left(\int \frac{e^{(1+m)}}{x}\right.}{e(1+m)}$$
$$= -\frac{bp(dx)^{-1-m}(ex)^{1+m}\text{Ei}((1+m)\log(dx))}{e(1+m)} + \frac{(ex)^{1+m}(a + b\log(c\log^p(\right.}{e(1+m)}$$

Mathematica [A]
time = 0.14, size = 56, normalized size = 0.84

$$\frac{(dx)^{-m}(ex)^m(-bp\text{Ei}((1+m)\log(dx)) + dx(dx)^m(a + b\log(c\log^p(dx))))}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x]^p]),x]
```

```
[Out] ((e*x)^m*(-(b*p*ExpIntegralEi[(1 + m)*Log[d*x]]) + d*x*(d*x)^m*(a + b*Log[c
*Log[d*x]^p])))/(d*(1 + m)*(d*x)^m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\ln(c\ln(dx)^p))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x)^p)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="maxima")
```

```
[Out] -(p*integrate(e^(m*log(x) + m)/((m^2 + 2*m + 1)*log(d)^2 + 2*(m^2 + 2*m + 1
)*log(d)*log(x) + (m^2 + 2*m + 1)*log(x)^2), x) - (((m*e^m + e^m)*x*log(d)
+ (m*e^m + e^m)*x*log(x))*x^m*log((log(d) + log(x))^p) + ((m*e^m + e^m)*x*l
```

```
og(c)*log(x) + ((m*e^m + e^m)*log(c)*log(d) - p*e^m)*x)*x^m)/((m^2 + 2*m +
1)*log(d) + (m^2 + 2*m + 1)*log(x)))*b + (x*e)^(m + 1)*a*e^(-1)/(m + 1)
```

Fricas [A]
time = 0.37, size = 83, normalized size = 1.24

$$\frac{bdpxe^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}\log\left(\log\left(dx\right)\right)-bp\left(\frac{e}{d}\right)^{m}\operatorname{Ei}\left((m+1)\log\left(dx\right)\right)+\left(bdx\log\left(c\right)+adx\right)e^{\left(m\log(dx)+m\log\left(\frac{e}{d}\right)\right)}}{dm+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="fricas")
[Out] (b*d*p*x*e^(m*log(d*x) + m*log(e/d))*log(log(d*x)) - b*p*(e/d)^m*Ei((m + 1)
*log(d*x)) + (b*d*x*log(c) + a*d*x)*e^(m*log(d*x) + m*log(e/d)))/(d*m + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(ex)^{m}\left(a+b\log\left(c\log\left(dx\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x)**p)),x)
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x)**p)), x)
```

Giac [A]
time = 5.37, size = 83, normalized size = 1.24

$$\frac{bpxx^{m}e^{m}\log\left(\log\left(d\right)+\log\left(x\right)\right)}{m+1}+\frac{bxx^{m}e^{m}\log\left(c\right)}{m+1}+\frac{axx^{m}e^{m}}{m+1}-\frac{bp\operatorname{Ei}\left(m\log\left(d\right)+m\log\left(x\right)+\log\left(d\right)+\log\left(x\right)\right)e^{m}}{dd^{m}m+dd^{m}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x)^p)),x, algorithm="giac")
[Out] b*p*x*x^m*e^m*log(log(d) + log(x))/(m + 1) + b*x*x^m*e^m*log(c)/(m + 1) + a
*x*x^m*e^m/(m + 1) - b*p*Ei(m*log(d) + m*log(x) + log(d) + log(x))*e^m/(d*d
^m*m + d*d^m)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\,\ln\left(c\ln\left(d\,x\right)^{p}\right)\right)\,{\left(e\,x\right)}^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x)^p))*(e*x)^m,x)
[Out] int((a + b*log(c*log(d*x)^p))*(e*x)^m, x)
```

3.47 $\int (ex)^m \left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)\,dx$

Optimal. Leaf size=79

$$-\frac{bp(ex)^{1+m}\left(dx^n\right)^{-\frac{1+m}{n}}\operatorname{Ei}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)}+\frac{(ex)^{1+m}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(1+m)}$$

```
[Out] -b*p*(e*x)^(1+m)*Ei((1+m)*ln(d*x^n)/n)/e/(1+m)/((d*x^n)^((1+m)/n))+(e*x)^(1
+m)*(a+b*ln(c*ln(d*x^n)^p))/e/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2602, 2347, 2209}

$$\frac{(ex)^{m+1}\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{e(m+1)}-\frac{bp(ex)^{m+1}\left(dx^n\right)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)\log(dx^n)}{n}\right)}{e(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -((b*p*(e*x)^(1 + m)*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(e*(1 + m)*(d*x
^n)^((1 + m)/n))) + ((e*x)^(1 + m)*(a + b*Log[c*Log[d*x^n]^p]))/(e*(1 + m))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a + b\log(c\log^p(dx^n)))\, dx = \frac{(ex)^{1+m}(a + b\log(c\log^p(dx^n)))}{e(1+m)} - \frac{(bnp)\int \frac{(ex)^m}{\log(dx^n)}\,dx}{1+m}$$

$$= \frac{(ex)^{1+m}(a + b\log(c\log^p(dx^n)))}{e(1+m)} - \frac{\left(bp(ex)^{1+m}(dx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\Big(}{e(1+m}$$

$$= -\frac{bp(ex)^{1+m}(dx^n)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log(dx^n)}{n}\right)}{e(1+m)} + \frac{(ex)^{1+m}(a + b\log(c\log}{e(1+m)}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.75

$$\frac{x(ex)^m\left(a - bp(dx^n)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\log(dx^n)}{n}\right) + b\log(c\log^p(dx^n))\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x*(e*x)^m*(a - (b*p*ExpIntegralEi[((1 + m)*Log[d*x^n])/n])/(d*x^n)^((1 + m
)/n) + b*Log[c*Log[d*x^n]^p]))/(1 + m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\ln(c\ln(d\,x^n)^p))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int((e*x)^m*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] -(n*p*integrate(e^(m*log(x) + m)/((m + 1)*log(d) + (m + 1)*log(x^n)), x) -
(x*e^(m*log(x) + m)*log(c) + x*e^(m*log(x) + m)*log((log(d) + log(x^n))^p))
/(m + 1))*b + (x*e)^(m + 1)*a*e^(-1)/(m + 1)
```

Fricas [A]
time = 0.41, size = 82, normalized size = 1.04

$$\frac{bpxe^{(m\log(x)+m)}\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\mathrm{Ei}\left(\frac{(m+1)n\log(x)+(m+1)\log(d)}{n}\right)e^{\left(\frac{mn-(m+1)\log(d)}{n}\right)}+\left(bx\log\left(c\right)+ax\right)e^{(m\log(x)+m)}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] (b*p*x*e^(m*log(x) + m)*log(n*log(x) + log(d)) - b*p*Ei(((m + 1)*n*log(x) +
 (m + 1)*log(d))/n)*e^((m*n - (m + 1)*log(d))/n) + (b*x*log(c) + a*x)*e^(m*
log(x) + m))/(m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ex\right)^{m}\left(a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral((e*x)**m*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A]
time = 4.43, size = 111, normalized size = 1.41

$$\frac{bpxx^{m}e^{m}\log\left(n\log\left(x\right)+\log\left(d\right)\right)}{m+1}-\frac{bnp\mathrm{Ei}\left(m\log\left(x\right)+\frac{m\log(d)}{n}+\frac{\log(d)}{n}+\log\left(x\right)\right)e^{m}}{d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}mn+d^{\frac{m}{n}}d^{\left(\frac{1}{n}\right)}n}+\frac{bxx^{m}e^{m}\log\left(c\right)}{m+1}+\frac{axx^{m}e^{m}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] b*p*x*x^m*e^m*log(n*log(x) + log(d))/(m + 1) - b*n*p*Ei(m*log(x) + m*log(d)
/n + log(d)/n + log(x))*e^m/(d^(m/n)*d^(1/n)*m*n + d^(m/n)*d^(1/n)*n) + b*x
*x^m*e^m*log(c)/(m + 1) + a*x*x^m*e^m/(m + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(e\,x\right)}^m\,\left(a+b\,\ln\left(c\,{\ln\left(d\,x^n\right)}^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int((e*x)^m*(a + b*log(c*log(d*x^n)^p)), x)
```

3.48 $\int x^2(a+b\log(c\log^p(dx^n)))\,dx$

Optimal. Leaf size=55

$$-\frac{1}{3}bpx^3(dx^n)^{-3/n}\operatorname{Ei}\left(\frac{3\log(dx^n)}{n}\right)+\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))$$

```
[Out] -1/3*b*p*x^3*Ei(3*ln(d*x^n)/n)/((d*x^n)^(3/n))+1/3*x^3*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.158$, Rules used = {2602, 2347, 2209}

$$\frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))-\frac{1}{3}bpx^3(dx^n)^{-3/n}\operatorname{Ei}\left(\frac{3\log(dx^n)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -1/3*(b*p*x^3*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + (x^3*(a + b*
Log[c*Log[d*x^n]^p]))/3
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{3}x^3(a+b\log(c\log^p(dx^n))) - \frac{1}{3}(bnp)\int \frac{x^2}{\log(dx^n)}\,dx$$
$$= \frac{1}{3}x^3(a+b\log(c\log^p(dx^n))) - \frac{1}{3}\left(bpx^3(dx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{x}\,dx, x, \log$$
$$= -\frac{1}{3}bpx^3(dx^n)^{-3/n}\,\text{Ei}\left(\frac{3\log(dx^n)}{n}\right) + \frac{1}{3}x^3(a+b\log(c\log^p(dx^n)))$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.89

$$\frac{1}{3}x^3\left(a - bp(dx^n)^{-3/n}\,\text{Ei}\left(\frac{3\log(dx^n)}{n}\right) + b\log(c\log^p(dx^n))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*Log[d*x^n]^p]),x]
[Out] (x^3*(a - (b*p*ExpIntegralEi[(3*Log[d*x^n])/n])/(d*x^n)^(3/n) + b*Log[c*Log
[d*x^n]^p]))/3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2(a+b\ln(c\ln(d\,x^n)^p))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
[Out] int(x^2*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/3*(x^3*log(c) + x^3*log((log(d) + log(x^n))^p) - 3*n*p*integr
ate(1/3*x^2/(log(d) + log(x^n)), x))*b
```

Fricas [A]
time = 0.46, size = 70, normalized size = 1.27

$$\frac{bd^{\frac{3}{n}}px^3\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\mathrm{log\_integral}\left(d^{\frac{3}{n}}x^3\right)+\left(bx^3\log\left(c\right)+ax^3\right)d^{\frac{3}{n}}}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log(n*log(x) + log(d)) - b*p*log_integral(d^(3/n)*x^3)
 + (b*x^3*log(c) + a*x^3)*d^(3/n))/d^(3/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\log\left(c\log\left(dx^n\right)^p\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x**2*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A]
time = 3.91, size = 56, normalized size = 1.02

$$\frac{1}{3}\,bpx^3\log\left(n\log\left(x\right)+\log\left(d\right)\right)+\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3-\frac{bp\mathrm{Ei}\left(\frac{3\,\log\left(d\right)}{n}+3\,\log\left(x\right)\right)}{3\,d^{\frac{3}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/3*b*p*x^3*log(n*log(x) + log(d)) + 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/3*b*p
*Ei(3*log(d)/n + 3*log(x))/d^(3/n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int(x^2*(a + b*log(c*log(d*x^n)^p)), x)
```

3.49 $\int x(a+b\log(c\log^p(dx^n)))\,dx$

Optimal. Leaf size=55

$$-\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\mathrm{Ei}\left(\frac{2\log(dx^n)}{n}\right)+\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))$$

```
[Out] -1/2*b*p*x^2*Ei(2*ln(d*x^n)/n)/((d*x^n)^(2/n))+1/2*x^2*(a+b*ln(c*ln(d*x^n)^
p))
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2602, 2347, 2209}

$$\frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))-\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\mathrm{Ei}\left(\frac{2\log(dx^n)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] -1/2*(b*p*x^2*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + (x^2*(a + b*
Log[c*Log[d*x^n]^p]))/2
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(a+b\log(c\log^p(dx^n)))\,dx = \frac{1}{2}x^2(a+b\log(c\log^p(dx^n))) - \frac{1}{2}(bnp)\int \frac{x}{\log(dx^n)}\,dx$$
$$= \frac{1}{2}x^2(a+b\log(c\log^p(dx^n))) - \frac{1}{2}\left(bpx^2(dx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{x}\,dx, x, \text{lo}\right.$$
$$= -\frac{1}{2}bpx^2(dx^n)^{-2/n}\,\text{Ei}\left(\frac{2\log(dx^n)}{n}\right) + \frac{1}{2}x^2(a+b\log(c\log^p(dx^n)))$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.89

$$\frac{1}{2}x^2\left(a - bp(dx^n)^{-2/n}\,\text{Ei}\left(\frac{2\log(dx^n)}{n}\right) + b\log(c\log^p(dx^n))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Log[c*Log[d*x^n]^p]),x]
```

```
[Out] (x^2*(a - (b*p*ExpIntegralEi[(2*Log[d*x^n])/n])/(d*x^n)^(2/n) + b*Log[c*Log
[d*x^n]^p]))/2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x(a+b\ln(c\ln(d\,x^n)^p))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

```
[Out] int(x*(a+b*ln(c*ln(d*x^n)^p)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 - 1/2*(2*n*p*integrate(1/2*x/(log(d) + log(x^n)), x) - x^2*log(c)
 - x^2*log((log(d) + log(x^n))^p))*b
```

Fricas [A]
time = 0.36, size = 70, normalized size = 1.27

$$\frac{bd^{\frac{2}{n}}px^2\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\mathrm{log\_integral}\left(d^{\frac{2}{n}}x^2\right)+\left(bx^2\log\left(c\right)+ax^2\right)d^{\frac{2}{n}}}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log(n*log(x) + log(d)) - b*p*log_integral(d^(2/n)*x^2)
 + (b*x^2*log(c) + a*x^2)*d^(2/n))/d^(2/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\log\left(c\log\left(dx^n\right)^p\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*ln(c*ln(d*x**n)**p)),x)
```

```
[Out] Integral(x*(a + b*log(c*log(d*x**n)**p)), x)
```

Giac [A]
time = 4.18, size = 56, normalized size = 1.02

$$\frac{1}{2}\,bpx^2\log\left(n\log\left(x\right)+\log\left(d\right)\right)+\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{2}\,ax^2-\frac{bp\mathrm{Ei}\left(\frac{2\,\log(d)}{n}+2\,\log\left(x\right)\right)}{2\,d^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*log(d*x^n)^p)),x, algorithm="giac")
```

```
[Out] 1/2*b*p*x^2*log(n*log(x) + log(d)) + 1/2*b*x^2*log(c) + 1/2*a*x^2 - 1/2*b*p
*Ei(2*log(d)/n + 2*log(x))/d^(2/n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\left(a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*log(c*log(d*x^n)^p)),x)
```

```
[Out] int(x*(a + b*log(c*log(d*x^n)^p)), x)
```

3.50 $\int (a+b\log(c\log^p(dx^n)))\,dx$

Optimal. Leaf size=45

$$ax-bpx(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right)+bx\log(c\log^p(dx^n))$$

```
[Out] a*x-b*p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+b*x*ln(c*ln(d*x^n)^p)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2600, 2337, 2209}

$$ax+bx\log(c\log^p(dx^n))-bpx(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] a*x - (b*p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*x*Log[c*Log[d*
x^n]^p]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\log(c\log^p(dx^n)))\,dx &= ax+b\int \log(c\log^p(dx^n))\,dx\\
&= ax+bx\log(c\log^p(dx^n))-(bnp)\int \frac{1}{\log(dx^n)}\,dx\\
&= ax+bx\log(c\log^p(dx^n))-\left(bpx(dx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}\,dx,x,\log(dx^n)\right)\\
&= ax-bpx(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right)+bx\log(c\log^p(dx^n))
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 43, normalized size = 0.96

$$
x\left(a-bp(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log(dx^n)}{n}\right)+b\log(c\log^p(dx^n))\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Log[c*Log[d*x^n]^p],x]
```

```
[Out] x*(a - (b*p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1) + b*Log[c*Log[d*x^n
]^p])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$
\int a+b\ln\left(c\ln(d\,x^n)^p\right)dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*ln(c*ln(d*x^n)^p),x)
```

```
[Out] int(a+b*ln(c*ln(d*x^n)^p),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="maxima")
```

```
[Out] -(n*p*integrate(1/(log(d) + log(x^n)), x) - x*log(c) - x*log((log(d) + log(
x^n))^p))*b + a*x
```

Fricas [A]
time = 0.36, size = 53, normalized size = 1.18

$$\frac{bd^{\left(\frac{1}{n}\right)}px\log\left(n\log\left(x\right)+\log\left(d\right)\right)-bp\,\text{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)+\left(bx\log\left(c\right)+ax\right)d^{\left(\frac{1}{n}\right)}}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log(n*log(x) + log(d)) - b*p*log_integral(d^(1/n)*x) + (b*x*
log(c) + a*x)*d^(1/n))/d^(1/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*ln(c*ln(d*x**n)**p),x)
```

```
[Out] Integral(a + b*log(c*log(d*x**n)**p), x)
```

Giac [A]
time = 5.63, size = 42, normalized size = 0.93

$$\left(px\log\left(n\log\left(x\right)+\log\left(d\right)\right)+x\log\left(c\right)-\frac{p\text{Ei}\left(\frac{\log\left(d\right)}{n}+\log\left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*log(c*log(d*x^n)^p),x, algorithm="giac")
```

```
[Out] (p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n))*b
 + a*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int a+b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*log(c*log(d*x^n)^p),x)
```

```
[Out] int(a + b*log(c*log(d*x^n)^p), x)
```

3.51 $\int \frac{a+b\log(c\log^p(dx^n))}{x}\,dx$

Optimal. Leaf size=32

$$-bp\log(x)+\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}$$

[Out] -b*p*ln(x)+ln(d*x^n)*(a+b*ln(c*ln(d*x^n)^p))/n

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = {2601}

$$\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}-bp\log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x,x]

[Out] -(b*p*Log[x]) + (Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=-bp\log(x)+\frac{\log\left(dx^n\right)\left(a+b\log\left(c\log^p\left(dx^n\right)\right)\right)}{n}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.25

$$a\log(x)-\frac{bp\log\left(dx^n\right)}{n}+\frac{b\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x,x]

[Out] a*Log[x] - (b*p*Log[d*x^n])/n + (b*Log[d*x^n]*Log[c*Log[d*x^n]^p])/n

Maple [A]
time = 0.12, size = 43, normalized size = 1.34

method	result	size
derivativedivides	$\frac{\ln(d\,x^n)a+\ln(d\,x^n)\ln(c\ln(d\,x^n)^p)b-bp\ln(d\,x^n)}{n}$	43
default	$\frac{\ln(d\,x^n)a+\ln(d\,x^n)\ln(c\ln(d\,x^n)^p)b-bp\ln(d\,x^n)}{n}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(d*x^n)*a+ln(d*x^n)*ln(c*ln(d*x^n)^p)*b-b*p*ln(d*x^n))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 2.00

$$b\log\left(c\log\left(dx^n\right)^p\right)\log\left(x\right)-\left(p\log\left(x\right)\log\left(\log\left(dx^n\right)\right)-\frac{\left(\log\left(dx^n\right)\log\left(\log\left(dx^n\right)\right)-\log\left(dx^n\right)\right)p}{n}\right)b+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="maxima")
```

```
[Out] b*log(c*log(d*x^n)^p)*log(x) - (p*log(x)*log(log(d*x^n)) - (log(d*x^n)*log(
log(d*x^n)) - log(d*x^n))*p/n)*b + a*log(x)
```

Fricas [A]
time = 0.36, size = 45, normalized size = 1.41

$$\frac{\left(bnp\log\left(x\right)+bp\log\left(d\right)\right)\log\left(n\log\left(x\right)+\log\left(d\right)\right)-\left(bnp-bn\log\left(c\right)-an\right)\log\left(x\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="fricas")
```

```
[Out] ((b*n*p*log(x) + b*p*log(d))*log(n*log(x) + log(d)) - (b*n*p - b*n*log(c) -
 a*n)*log(x))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\log\left(c\log\left(dx^n\right)^p\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x, x)
```

Giac [A]
time = 3.40, size = 54, normalized size = 1.69

$$\frac{((n\log(x)+\log(d))\log(n\log(x)+\log(d))-n\log(x)-\log(d))bp+(n\log(x)+\log(d))b\log(c)+(n\log(x)+\log(d))a}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x,x, algorithm="giac")
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*b*p + (n*
log(x) + log(d))*b*log(c) + (n*log(x) + log(d))*a)/n
```

Mupad [B]
time = 0.32, size = 32, normalized size = 1.00

$$\ln(x)\ (a-b\,p)+\frac{b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)\,\ln\left(d\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x,x)
[Out] log(x)*(a - b*p) + (b*log(c*log(d*x^n)^p)*log(d*x^n))/n
```

3.52 $\int \frac{a+b\log(c\log^p(dx^n))}{x^2}\,dx$

Optimal. Leaf size=48

$$\frac{bp(dx^n)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

[Out] b*p*(d*x^n)^(1/n)*Ei(-ln(d*x^n)/n)/x+(-a-b*ln(c*ln(d*x^n)^p))/x

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\frac{bp(dx^n)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]

[Out] (b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)])/x - (a + b*Log[c*Log[d*x^n]^p])/x

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^2}\,dx = -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x} + (bnp)\int \frac{1}{x^2\log\left(dx^n\right)}\,dx$$
$$= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x} + \frac{\left(bp(dx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{x}\,dx, x, \log\left(dx^n\right)\right)}{x}$$
$$= \frac{bp(dx^n)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{\log(dx^n)}{n}\right)}{x} - \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

Mathematica [**A**]
time = 0.03, size = 45, normalized size = 0.94

$$-\frac{a-bp(dx^n)^{\frac{1}{n}}\,\text{Ei}\left(-\frac{\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^2,x]
```

```
[Out] -((a - b*p*(d*x^n)^n^(-1)*ExpIntegralEi[-(Log[d*x^n]/n)] + b*Log[c*Log[d*x^
n]^p])/x)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="maxima")
```

```
[Out] (n*p*integrate(1/(x^2*log(d) + x^2*log(x^n)), x) - (log(c) + log((log(d) +
log(x^n))^p))/x)*b - a/x
```

Fricas [A]
time = 0.35, size = 46, normalized size = 0.96

$$\frac{bd^{\left(\frac{1}{n}\right)}px\,\mathrm{log\_integral}\left(\frac{1}{d^{\left(\frac{1}{n}\right)}x}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="fricas")
```

```
[Out] (b*d^(1/n)*p*x*log_integral(1/(d^(1/n)*x)) - b*p*log(n*log(x) + log(d)) - b
*log(c) - a)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\log\left(c\log\left(dx^{n}\right)^{p}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**2,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^2,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^2, x)
```

3.53 $\int \frac{a+b\log(c\log^p(dx^n))}{x^3}\,dx$

Optimal. Leaf size=55

$$\frac{bp(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

```
[Out] 1/2*b*p*(d*x^n)^(2/n)*Ei(-2*ln(d*x^n)/n)/x^2+1/2*(-a-b*ln(c*ln(d*x^n)^p))/x
^2
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\frac{bp(dx^n)^{2/n}\,\text{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] (b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n])/(2*x^2) - (a + b*Log[c
*Log[d*x^n]^p])/(2*x^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{x^3}\,dx = -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{1}{2}(bnp)\int \frac{1}{x^3\log\left(dx^n\right)}\,dx$$
$$= -\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}+\frac{\left(bp(dx^n)^{2/n}\right)\mathrm{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{x}\,dx,x,\log\left(dx^n\right)\right)}{2x^2}$$
$$= \frac{bp(dx^n)^{2/n}\,\mathrm{Ei}\left(-\frac{2\log(dx^n)}{n}\right)}{2x^2}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.89

$$-\frac{a-bp(dx^n)^{2/n}\,\mathrm{Ei}\left(-\frac{2\log(dx^n)}{n}\right)+b\log\left(c\log^p\left(dx^n\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^3,x]
```

```
[Out] -1/2*(a - b*p*(d*x^n)^(2/n)*ExpIntegralEi[(-2*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*n*p*integrate(1/2/(x^3*log(d) + x^3*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.96

$$\frac{bd^{\frac{2}{n}}px^2 \operatorname{log\_integral}\left(\frac{1}{d^{\frac{2}{n}}x^2}\right) - bp\log\left(n\log\left(x\right) + \log\left(d\right)\right) - b\log\left(c\right) - a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^(2/n)*p*x^2*log_integral(1/(d^(2/n)*x^2)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\log\left(c\log\left(dx^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**3,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^3,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^3, x)
```

3.54 $\int \frac{a+b\log(c\log^p(dx^n))}{x^4}\,dx$

Optimal. Leaf size=55

$$\frac{bp(dx^n)^{3/n}\operatorname{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}$$

```
[Out] 1/3*b*p*(d*x^n)^(3/n)*Ei(-3*ln(d*x^n)/n)/x^3+1/3*(-a-b*ln(c*ln(d*x^n)^p))/x
^3
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2602, 2347, 2209}

$$\frac{bp(dx^n)^{3/n}\operatorname{Ei}\left(-\frac{3\log(dx^n)}{n}\right)}{3x^3}-\frac{a+b\log\left(c\log^p\left(dx^n\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] (b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n])/(3*x^3) - (a + b*Log[c
*Log[d*x^n]^p])/(3*x^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2602

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)*((a + b*Log[c*Log[d*x^n]^p])/(e*(m + 1)
)), x] - Dist[b*n*(p/(m + 1)), Int[(e*x)^m/Log[d*x^n], x], x] /; FreeQ[{a,
b, c, d, e, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{x^4}\,dx=-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}+\frac{1}{3}(bnp)\int \frac{1}{x^4\log \left(dx^n\right)}\,dx$$
$$=-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}+\frac{\left(bp\left(dx^n\right)^{3/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{x}\,dx,x,\log \left(dx^n\right)\right)}{3x^3}$$
$$=\frac{bp\left(dx^n\right)^{3/n}\text{Ei}\left(-\frac{3\log \left(dx^n\right)}{n}\right)}{3x^3}-\frac{a+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.89

$$-\frac{a-bp\left(dx^n\right)^{3/n}\text{Ei}\left(-\frac{3\log \left(dx^n\right)}{n}\right)+b\log \left(c\log ^p\left(dx^n\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*Log[d*x^n]^p])/x^4,x]
```

```
[Out] -1/3*(a - b*p*(d*x^n)^(3/n)*ExpIntegralEi[(-3*Log[d*x^n])/n] + b*Log[c*Log[
d*x^n]^p])/x^3
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\ln \left(c\ln \left(d\,x^n\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

```
[Out] int((a+b*ln(c*ln(d*x^n)^p))/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*n*p*integrate(1/3/(x^4*log(d) + x^4*log(x^n)), x) - (log(c) + log((l
og(d) + log(x^n))^p))/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.35, size = 53, normalized size = 0.96

$$\frac{bd^{\frac{3}{n}}px^3\,\mathrm{log\_integral}\left(\frac{1}{d^{\frac{3}{n}}x^3}\right)-bp\log\left(n\log\left(x\right)+\log\left(d\right)\right)-b\log\left(c\right)-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^(3/n)*p*x^3*log_integral(1/(d^(3/n)*x^3)) - b*p*log(n*log(x) + log
(d)) - b*log(c) - a)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\log\left(c\log\left(dx^n\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*ln(d*x**n)**p))/x**4,x)
```

```
[Out] Integral((a + b*log(c*log(d*x**n)**p))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*log(d*x^n)^p))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*log(d*x^n)^p) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\ln\left(c\ln\left(d\,x^n\right)^p\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*log(c*log(d*x^n)^p))/x^4,x)
```

```
[Out] int((a + b*log(c*log(d*x^n)^p))/x^4, x)
```

3.55 $\int \log(c\log^p(dx))\,dx$

Optimal. Leaf size=22

$$x\log(c\log^p(dx)) - \frac{p\mathrm{li}(dx)}{d}$$

```
[Out] -p*Li(d*x)/d+x*ln(c*ln(d*x)^p)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2600, 2335}

$$x\log(c\log^p(dx)) - \frac{p\mathrm{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p],x]
```

```
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log(c\log^p(dx))\,dx &= x\log(c\log^p(dx)) - p\int \frac{1}{\log(dx)}\,dx\\ &= x\log(c\log^p(dx)) - \frac{p\mathrm{li}(dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$x\log(c\log^p(dx)) - \frac{p\mathrm{li}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p],x]
[Out] x*Log[c*Log[d*x]^p] - (p*LogIntegral[d*x])/d
```

Maple [A]
time = 0.03, size = 26, normalized size = 1.18

method	result	size
default	$x \ln \left(c \ln \left(d x\right)^{p}\right)+\frac{p \,\mathrm{expIntegral}\left(1,-\ln \left(d x\right)\right)}{d}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*ln(d*x)^p)+p/d*Ei(1,-ln(d*x))
```

Maxima [A]
time = 0.34, size = 23, normalized size = 1.05

$$x \log \left(c \log \left(dx\right)^{p}\right) - \frac{p\mathrm{Ei}(\log \left(dx\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="maxima")
[Out] x*log(c*log(d*x)^p) - p*Ei(log(d*x))/d
```

Fricas [A]
time = 0.34, size = 26, normalized size = 1.18

$$\frac{dpx \log \left(\log \left(dx\right)\right) + dx \log \left(c\right) - p \operatorname{log\_integral}\left(dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="fricas")
[Out] (d*p*x*log(log(d*x)) + d*x*log(c) - p*log_integral(d*x))/d
```

Sympy [A]
time = 0.57, size = 19, normalized size = 0.86

$$x \log \left(c \log \left(dx\right)^{p}\right) - \frac{p \operatorname{li}\left(dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p),x)
[Out] x*log(c*log(d*x)**p) - p*li(d*x)/d
```

Giac [**A**]
time = 4.95, size = 26, normalized size = 1.18

$$px\log\left(\log\left(d\right)+\log\left(x\right)\right)+x\log\left(c\right)-\frac{p\mathrm{Ei}(\log\left(d\right)+\log\left(x\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p),x, algorithm="giac")
```

```
[Out] p*x*log(log(d) + log(x)) + x*log(c) - p*Ei(log(d) + log(x))/d
```

Mupad [**B**]
time = 0.31, size = 22, normalized size = 1.00

$$x\,\ln\left(c\ln\left(d\,x\right)^p\right)-\frac{p\,\mathrm{logint}(d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x)^p),x)
```

```
[Out] x*log(c*log(d*x)^p) - (p*logint(d*x))/d
```

3.56 $\int \frac{\log(c\log^p(dx))}{x}\,dx$

Optimal. Leaf size=20

$$-p\log(x)+\log(dx)\log\left(c\log^p(dx)\right)$$

```
[Out] -p*ln(x)+ln(d*x)*ln(c*ln(d*x)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2601}

$$\log(dx)\log\left(c\log^p(dx)\right)-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x]^p]/x,x]
```

```
[Out] -(p*Log[x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log\left(c\log^p(dx)\right)}{x}\,dx=-p\log(x)+\log(dx)\log\left(c\log^p(dx)\right)$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.10

$$-p\log(dx)+\log(dx)\log\left(c\log^p(dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x]^p]/x,x]
```

```
[Out] -(p*Log[d*x]) + Log[d*x]*Log[c*Log[d*x]^p]
```

Maple [A]
time = 0.05, size = 23, normalized size = 1.15

method	result	size
derivativedivides	$\ln(dx)\ln(c\ln(dx)^p) - \ln(dx)\,p$	23
default	$\ln(dx)\ln(c\ln(dx)^p) - \ln(dx)\,p$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(d*x)*ln(c*ln(d*x)^p)-ln(d*x)*p
```

Maxima [A]
time = 0.28, size = 22, normalized size = 1.10

$$-p\log(dx) + \log(dx)\log(c\log(dx)^p)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="maxima")
```

```
[Out] -p*log(d*x) + log(d*x)*log(c*log(d*x)^p)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 1.20

$$p\log(dx)\log(\log(dx)) - (p - \log(c))\log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="fricas")
```

```
[Out] p*log(d*x)*log(log(d*x)) - (p - log(c))*log(d*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log(c\log(dx)^p)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x)**p)/x,x)
```

```
[Out] Integral(log(c*log(d*x)**p)/x, x)
```

Giac [A]
time = 4.18, size = 32, normalized size = 1.60

$$((\log(d) + \log(x))\log(\log(d) + \log(x)) - \log(d) - \log(x))p + (\log(d) + \log(x))\log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x)^p)/x,x, algorithm="giac")
```

```
[Out] ((log(d) + log(x))*log(log(d) + log(x)) - log(d) - log(x))*p + (log(d) + lo
g(x))*log(c)
```

Mupad [B]
time = 0.28, size = 20, normalized size = 1.00

$$\ln\left(c\ln\left(d\,x\right)^p\right)\,\ln\left(d\,x\right)-p\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x)^p)/x,x)
```

```
[Out] log(c*log(d*x)^p)*log(d*x) - p*log(x)
```

3.57 $\int \log (c\log^p (dx^n))\, dx$

Optimal. Leaf size=40

$$-px(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log (dx^n)}{n}\right)+x\log (c\log^p (dx^n))$$

```
[Out] -p*x*Ei(ln(d*x^n)/n)/((d*x^n)^(1/n))+x*ln(c*ln(d*x^n)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2600, 2337, 2209}

$$x\log (c\log^p (dx^n))-px(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log (dx^n)}{n}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p],x]
[Out] -((p*x*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + x*Log[c*Log[d*x^n]^p]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2600

```
Int[Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)], x_Symbol] :> Simp[x*Log[c*Log[d
*x^n]^p], x] - Dist[n*p, Int[1/Log[d*x^n], x], x] /; FreeQ[{c, d, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \log (c\log^p (dx^n))\, dx &= x\log (c\log^p (dx^n))-(np)\int \frac{1}{\log (dx^n)}\, dx\\
&= x\log (c\log^p (dx^n))-\left(px(dx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{x}\, dx,x,\log (dx^n)\right)\\
&= -px(dx^n)^{-1/n}\,\text{Ei}\left(\frac{\log (dx^n)}{n}\right)+x\log (c\log^p (dx^n))
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.98

$$x\left(-p(dx^n)^{-1/n}\,\mathrm{Ei}\left(\frac{\log(dx^n)}{n}\right)+\log\left(c\log^p(dx^n)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p],x]
[Out] x*(-((p*ExpIntegralEi[Log[d*x^n]/n])/(d*x^n)^n^(-1)) + Log[c*Log[d*x^n]^p])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \ln\left(c\ln\left(d\,x^{n}\right)^{p}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p),x)
[Out] int(ln(c*ln(d*x^n)^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="maxima")
[Out] -n*p*integrate(1/(log(d) + log(x^n)), x) + x*log(c) + x*log((log(d) + log(x
^n))^p)
```

Fricas [A]
time = 0.35, size = 45, normalized size = 1.12

$$\frac{d^{\left(\frac{1}{n}\right)}px\log\left(n\log(x)+\log(d)\right)+d^{\left(\frac{1}{n}\right)}x\log(c)-p\,\mathrm{log\_integral}\left(d^{\left(\frac{1}{n}\right)}x\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="fricas")
[Out] (d^(1/n)*p*x*log(n*log(x) + log(d)) + d^(1/n)*x*log(c) - p*log_integral(d^(
1/n)*x))/d^(1/n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(c \log \left(dx^{n}\right)^{p}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p),x)
[Out] Integral(log(c*log(d*x**n)**p), x)
```

Giac [A]
time = 4.46, size = 36, normalized size = 0.90

$$px \log \left(n \log \left(x\right) + \log \left(d\right)\right) + x \log \left(c\right) - \frac{p \mathrm{Ei}\left(\frac{\log\left(d\right)}{n} + \log \left(x\right)\right)}{d^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p),x, algorithm="giac")
[Out] p*x*log(n*log(x) + log(d)) + x*log(c) - p*Ei(log(d)/n + log(x))/d^(1/n)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(c \ln \left(d\, x^{n}\right)^{p}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x^n)^p),x)
[Out] int(log(c*log(d*x^n)^p), x)
```

3.58 $\int \frac{\log(c\log^p(dx^n))}{x}\,dx$

Optimal. Leaf size=27

$$-p\log(x)+\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

```
[Out] -p*ln(x)+ln(d*x^n)*ln(c*ln(d*x^n)^p)/n
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{2601\}$

$$\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}-p\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*Log[d*x^n]^p]/x,x]
```

```
[Out] -(p*Log[x]) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Rule 2601

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[Log[d*x^n]*((a + b*Log[c*Log[d*x^n]^p])/n), x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int\frac{\log\left(c\log^p\left(dx^n\right)\right)}{x}\,dx=-p\log(x)+\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.26

$$-\frac{p\log\left(dx^n\right)}{n}+\frac{\log\left(dx^n\right)\log\left(c\log^p\left(dx^n\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*Log[d*x^n]^p]/x,x]
```

```
[Out] -((p*Log[d*x^n])/n) + (Log[d*x^n]*Log[c*Log[d*x^n]^p])/n
```

Maple [A]
time = 0.03, size = 33, normalized size = 1.22

method	result	size
derivativedivides	$\frac{\ln(c\ln(d\,x^n)^p)\ln(d\,x^n)-\ln(d\,x^n)p}{n}$	33
default	$\frac{\ln(c\ln(d\,x^n)^p)\ln(d\,x^n)-\ln(d\,x^n)p}{n}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*ln(d*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(c*ln(d*x^n)^p)*ln(d*x^n)-ln(d*x^n)*p)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(27) = 54.
time = 0.29, size = 55, normalized size = 2.04

$$-p\log(x)\log(\log(dx^n))+\log(c\log(dx^n)^p)\log(x)+\frac{(\log(dx^n)\log(\log(dx^n))-\log(dx^n))p}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -p*log(x)*log(log(d*x^n)) + log(c*log(d*x^n)^p)*log(x) + (log(d*x^n)*log(lo
g(d*x^n)) - log(d*x^n))*p/n
```

Fricas [A]
time = 0.34, size = 37, normalized size = 1.37

$$\frac{(np\log(x)+p\log(d))\log(n\log(x)+\log(d))-(np-n\log(c))\log(x)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] ((n*p*log(x) + p*log(d))*log(n*log(x) + log(d)) - (n*p - n*log(c))*log(x))/
n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log(c\log(dx^n)^p)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*ln(d*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*log(d*x**n)**p)/x, x)
```

Giac [A]
time = 4.65, size = 43, normalized size = 1.59

$$\frac{((n\log{(x)}+\log{(d)})\log{(n\log{(x)}+\log{(d)})}-n\log{(x)}-\log{(d)})p+(n\log{(x)}+\log{(d)})\log{(c)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*log(d*x^n)^p)/x,x, algorithm="giac")
[Out] (((n*log(x) + log(d))*log(n*log(x) + log(d)) - n*log(x) - log(d))*p + (n*lo
g(x) + log(d))*log(c))/n
```

Mupad [B]
time = 0.30, size = 27, normalized size = 1.00

$$\frac{\ln{\left(c\ln{\left(d\,x^n\right)}^p\right)}\,\ln{\left(d\,x^n\right)}}{n}-p\,\ln{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*log(d*x^n)^p)/x,x)
[Out] (log(c*log(d*x^n)^p)*log(d*x^n))/n - p*log(x)
```

3.59 $\int x^m \log \left(d(bx + cx^2)^n\right) \, dx$

Optimal. Leaf size=66

$$-\frac{2nx^{1+m}}{(1+m)^2}+\frac{nx^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)}{(1+m)^2}+\frac{x^{1+m}\log\left(d(bx+cx^2)^n\right)}{1+m}$$

```
[Out] -2*n*x^(1+m)/(1+m)^2+n*x^(1+m)*hypergeom([1, 1+m],[2+m],-c*x/b)/(1+m)^2+x^(
1+m)*ln(d*(c*x^2+b*x)^n)/(1+m)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2605, 81, 66}

$$\frac{x^{m+1}\log\left(d(bx+cx^2)^n\right)}{m+1}+\frac{nx^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{cx}{b}\right)}{(m+1)^2}-\frac{2nx^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (-2*n*x^(1 + m))/(1 + m)^2 + (n*x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m
, -((c*x)/b)])/(1 + m)^2 + (x^(1 + m)*Log[d*(b*x + c*x^2)^n])/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
```

```
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} - \frac{n\int\frac{x^m(b+2cx)}{b+cx}\,dx}{1+m}$$
$$= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m} + \frac{(bn)\int\frac{x^m}{b+cx}\,dx}{1+m}$$
$$= -\frac{2nx^{1+m}}{(1+m)^2} + \frac{nx^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)}{(1+m)^2} + \frac{x^{1+m}\log\left(d\left(bx+cx^2\right)^n\right)}{1+m}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.73

$$\frac{x^{1+m}\left(-2n+n\,{}_2F_1\left(1,1+m;2+m;-\frac{cx}{b}\right)+(1+m)\log\left(d(x(b+cx))^n\right)\right)}{(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(b*x + c*x^2)^n],x]
[Out] (x^(1 + m)*(-2*n + n*Hypergeometric2F1[1, 1 + m, 2 + m, -((c*x)/b)] + (1 +
m)*Log[d*(x*(b + c*x))^n]))/(1 + m)^2
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m \ln\left(d\left(c\,x^2+bx\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x)^n),x)
[Out] int(x^m*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] (x*x^m*log((c*x + b)^n) + x*x^m*log(x^n))/(m + 1) + integrate((((m + 1)*log
(d) - 2*n)*c*x + ((m + 1)*log(d) - n)*b)*x^m/(c*(m + 1)*x + b*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
[Out] integral(x^m*log((c*x^2 + b*x)^n*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \log \left(d\left(bx+cx^2\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x)**n),x)
[Out] Integral(x**m*log(d*(b*x + c*x**2)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
[Out] integrate(x^m*log((c*x^2 + b*x)^n*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m \ln \left(d\left(c\,x^2+b\,x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(d*(b*x + c*x^2)^n),x)
[Out] int(x^m*log(d*(b*x + c*x^2)^n), x)
```

3.60 $\int x^4 \log\left(d(bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=99

$$-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{b^5n\log(b+cx)}{5c^5}+\frac{1}{5}x^5\log\left(d(bx+cx^2)^n\right)$$

```
[Out] -1/5*b^4*n*x/c^4+1/10*b^3*n*x^2/c^3-1/15*b^2*n*x^3/c^2+1/20*b*n*x^4/c-2/25*
n*x^5+1/5*b^5*n*ln(c*x+b)/c^5+1/5*x^5*ln(d*(c*x^2+b*x)^n)
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2605, 78}

$$\frac{b^5n\log(b+cx)}{5c^5}-\frac{b^4nx}{5c^4}+\frac{b^3nx^2}{10c^3}-\frac{b^2nx^3}{15c^2}+\frac{1}{5}x^5\log\left(d(bx+cx^2)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/5*(b^4*n*x)/c^4 + (b^3*n*x^2)/(10*c^3) - (b^2*n*x^3)/(15*c^2) + (b*n*x^4
)/(20*c) - (2*n*x^5)/25 + (b^5*n*Log[b + c*x])/(5*c^5) + (x^5*Log[d*(b*x +
c*x^2)^n])/5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^4 \log\left(d\left(bx+cx^2\right)^n\right)\,dx = \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{5}n\int \frac{x^4(b+2cx)}{b+cx}\,dx$$
$$= \frac{1}{5}x^5\log\left(d\left(bx+cx^2\right)^n\right) - \frac{1}{5}n\int\left(\frac{b^4}{c^4} - \frac{b^3x}{c^3} + \frac{b^2x^2}{c^2} - \frac{bx^3}{c} + 2x^4 - \frac{b^5}{c^4(b+cx)}\right.$$
$$= -\frac{b^4nx}{5c^4} + \frac{b^3nx^2}{10c^3} - \frac{b^2nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{b^5n\log(b+cx)}{5c^5} + \frac{1}{5}x^5\log\left(d\left(bx+\right.\right.$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 0.86

$$\frac{cnx(-60b^4+30b^3cx-20b^2c^2x^2+15bc^3x^3-24c^4x^4)+60b^5n\log(b+cx)+60c^5x^5\log\left(d(x(b+cx))^n\right)}{300c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c^2*x^2 + 15*b*c^3*x^3 - 24*c^4*x^4)
+ 60*b^5*n*Log[b + c*x] + 60*c^5*x^5*Log[d*(x*(b + c*x))^n])/(300*c^5)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4 \ln\left(d\left(c\,x^2+bx\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^4*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.88

$$\frac{1}{5}\,x^5\log\left(\left(cx^2+bx\right)^n d\right) + \frac{1}{300}\,n\left(\frac{60\,b^5\log\left(cx+b\right)}{c^5} - \frac{24\,c^4x^5-15\,bc^3x^4+20\,b^2c^2x^3-30\,b^3cx^2+60\,b^4x}{c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((c*x^2 + b*x)^n*d) + 1/300*n*(60*b^5*log(c*x + b)/c^5 - (24*c^4
*x^5 - 15*b*c^3*x^4 + 20*b^2*c^2*x^3 - 30*b^3*c*x^2 + 60*b^4*x)/c^4)
```

Fricas [A]
time = 0.35, size = 98, normalized size = 0.99

$$\frac{60\,c^5nx^5\log\left(cx^2+bx\right)-24\,c^5nx^5+60\,c^5x^5\log\left(d\right)+15\,bc^4nx^4-20\,b^2c^3nx^3+30\,b^3c^2nx^2-60\,b^4cnx+60\,b^5n\log\left(cx+b\right)}{300\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/300*(60*c^5*n*x^5*log(c*x^2 + b*x) - 24*c^5*n*x^5 + 60*c^5*x^5*log(d) + 1
5*b*c^4*n*x^4 - 20*b^2*c^3*n*x^3 + 30*b^3*c^2*n*x^2 - 60*b^4*c*n*x + 60*b^5
*n*log(c*x + b))/c^5
```

Sympy [A]
time = 6.23, size = 112, normalized size = 1.13

$$\begin{cases} \frac{b^5 n \log{(b+cx)}}{5c^5} - \frac{b^4 nx}{5c^4} + \frac{b^3 nx^2}{10c^3} - \frac{b^2 nx^3}{15c^2} + \frac{bnx^4}{20c} - \frac{2nx^5}{25} + \frac{x^5 \log{(d(bx+cx^2)^n)}}{5} & \text{for } c \neq 0 \\ -\frac{nx^5}{25} + \frac{x^5 \log{(d(bx)^n)}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**5*n*log(b + c*x)/(5*c**5) - b**4*n*x/(5*c**4) + b**3*n*x**2/(
10*c**3) - b**2*n*x**3/(15*c**2) + b*n*x**4/(20*c) - 2*n*x**5/25 + x**5*log
(d*(b*x + c*x**2)**n)/5, Ne(c, 0)), (-n*x**5/25 + x**5*log(d*(b*x)**n)/5, T
rue))
```

Giac [A]
time = 3.89, size = 89, normalized size = 0.90

$$\frac{1}{5}nx^5\log\left(cx^2+bx\right) - \frac{1}{25}\left(2n - 5\log\left(d\right)\right)x^5 + \frac{bnx^4}{20c} - \frac{b^2nx^3}{15c^2} + \frac{b^3nx^2}{10c^3} - \frac{b^4nx}{5c^4} + \frac{b^5n\log\left(cx+b\right)}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c - 1
/15*b^2*n*x^3/c^2 + 1/10*b^3*n*x^2/c^3 - 1/5*b^4*n*x/c^4 + 1/5*b^5*n*log(c*
x + b)/c^5
```

Mupad [B]
time = 0.32, size = 85, normalized size = 0.86

$$\frac{x^5\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{5} - \frac{2\,n\,x^5}{25} - \frac{b^2\,n\,x^3}{15\,c^2} + \frac{b^3\,n\,x^2}{10\,c^3} + \frac{b^5\,n\,\ln\left(b+c\,x\right)}{5\,c^5} + \frac{b\,n\,x^4}{20\,c} - \frac{b^4\,n\,x}{5\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^5*log(d*(b*x + c*x^2)^n))/5 - (2*n*x^5)/25 - (b^2*n*x^3)/(15*c^2) + (b^3
*n*x^2)/(10*c^3) + (b^5*n*log(b + c*x))/(5*c^5) + (b*n*x^4)/(20*c) - (b^4*n
*x)/(5*c^4)
```

3.61 $\int x^3 \log\left(d(bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=85

$$\frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b^4n\log(b+cx)}{4c^4}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] 1/4*b^3*n*x/c^3-1/8*b^2*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*x^4-1/4*b^4*n*ln(c*x
+b)/c^4+1/4*x^4*ln(d*(c*x^2+b*x)^n)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2605, 78}

$$-\frac{b^4n\log(b+cx)}{4c^4}+\frac{b^3nx}{4c^3}-\frac{b^2nx^2}{8c^2}+\frac{1}{4}x^4\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (b^3*n*x)/(4*c^3) - (b^2*n*x^2)/(8*c^2) + (b*n*x^3)/(12*c) - (n*x^4)/8 - (b
^4*n*Log[b + c*x])/(4*c^4) + (x^4*Log[d*(b*x + c*x^2)^n])/4
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \log \left(d\left(bx+cx^2\right)^n\right) \, dx = \frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right) - \frac{1}{4}n \int \frac{x^3(b+2cx)}{b+cx} \, dx$$
$$= \frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right) - \frac{1}{4}n \int \left(-\frac{b^3}{c^3} + \frac{b^2x}{c^2} - \frac{bx^2}{c} + 2x^3 + \frac{b^4}{c^3(b+cx)}\right) \, dx$$
$$= \frac{b^3nx}{4c^3} - \frac{b^2nx^2}{8c^2} + \frac{bnx^3}{12c} - \frac{nx^4}{8} - \frac{b^4n\log(b+cx)}{4c^4} + \frac{1}{4}x^4 \log \left(d\left(bx+cx^2\right)^n\right)$$

Mathematica [A]
time = 0.03, size = 74, normalized size = 0.87

$$\frac{cnx(6b^3 - 3b^2cx + 2bc^2x^2 - 3c^3x^3) - 6b^4n\log(b+cx) + 6c^4x^4\log\left(d(x(b+cx))^n\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(b*x + c*x^2)^n],x]
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*x^3) - 6*b^4*n*Log[b + c*x]
 + 6*c^4*x^4*Log[d*(x*(b + c*x))^n])/(24*c^4)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \ln \left(d\left(c\,x^2+bx\right)^n\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x)^n),x)
[Out] int(x^3*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.88

$$\frac{1}{4}\,x^4\log\left(\left(cx^2+bx\right)^n d\right) - \frac{1}{24}\,n\left(\frac{6\,b^4\log\left(cx+b\right)}{c^4} + \frac{3\,c^3x^4 - 2\,bc^2x^3 + 3\,b^2cx^2 - 6\,b^3x}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
[Out] 1/4*x^4*log((c*x^2 + b*x)^n*d) - 1/24*n*(6*b^4*log(c*x + b)/c^4 + (3*c^3*x^
4 - 2*b*c^2*x^3 + 3*b^2*c*x^2 - 6*b^3*x)/c^3)
```

Fricas [A]
time = 0.35, size = 86, normalized size = 1.01

$$\frac{6\,c^{4}nx^{4}\log\left(cx^{2}+bx\right)-3\,c^{4}nx^{4}+6\,c^{4}x^{4}\log\left(d\right)+2\,bc^{3}nx^{3}-3\,b^{2}c^{2}nx^{2}+6\,b^{3}cnx-6\,b^{4}n\log\left(cx+b\right)}{24\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/24*(6*c^4*n*x^4*log(c*x^2 + b*x) - 3*c^4*n*x^4 + 6*c^4*x^4*log(d) + 2*b*c
^3*n*x^3 - 3*b^2*c^2*n*x^2 + 6*b^3*c*n*x - 6*b^4*n*log(c*x + b))/c^4
```

Sympy [A]
time = 2.77, size = 97, normalized size = 1.14

$$\begin{cases} -\frac{b^{4}n\log\left(b+cx\right)}{4c^{4}}+\frac{b^{3}nx}{4c^{3}}-\frac{b^{2}nx^{2}}{8c^{2}}+\frac{bnx^{3}}{12c}-\frac{nx^{4}}{8}+\frac{x^{4}\log\left(d\left(bx+cx^{2}\right)^{n}\right)}{4} & \text{for}\: c\neq 0 \\ -\frac{nx^{4}}{16}+\frac{x^{4}\log\left(d\left(bx\right)^{n}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**4*n*log(b + c*x)/(4*c**4) + b**3*n*x/(4*c**3) - b**2*n*x**2/
(8*c**2) + b*n*x**3/(12*c) - n*x**4/8 + x**4*log(d*(b*x + c*x**2)**n)/4, Ne
(c, 0)), (-n*x**4/16 + x**4*log(d*(b*x)**n)/4, True))
```

Giac [A]
time = 5.15, size = 75, normalized size = 0.88

$$\frac{1}{4}\,nx^{4}\log\left(cx^{2}+bx\right)-\frac{1}{8}\,(n-2\,\log\left(d\right))x^{4}+\frac{bnx^{3}}{12\,c}-\frac{b^{2}nx^{2}}{8\,c^{2}}+\frac{b^{3}nx}{4\,c^{3}}-\frac{b^{4}n\log\left(cx+b\right)}{4\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c - 1/8*
b^2*n*x^2/c^2 + 1/4*b^3*n*x/c^3 - 1/4*b^4*n*log(c*x + b)/c^4
```

Mupad [B]
time = 0.34, size = 73, normalized size = 0.86

$$\frac{x^4\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{4}-\frac{n\,x^4}{8}-\frac{b^2\,n\,x^2}{8\,c^2}-\frac{b^4\,n\,\ln\left(b+c\,x\right)}{4\,c^4}+\frac{b\,n\,x^3}{12\,c}+\frac{b^3\,n\,x}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^4*log(d*(b*x + c*x^2)^n))/4 - (n*x^4)/8 - (b^2*n*x^2)/(8*c^2) - (b^4*n*l
og(b + c*x))/(4*c^4) + (b*n*x^3)/(12*c) + (b^3*n*x)/(4*c^3)
```

3.62 $\int x^2 \log \left(d(bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=71

$$-\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{b^3n\log(b+cx)}{3c^3}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] -1/3*b^2*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/3*b^3*n*ln(c*x+b)/c^3+1/3*x^3*ln
(d*(c*x^2+b*x)^n)
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2605, 78}

$$\frac{b^3n\log(b+cx)}{3c^3}-\frac{b^2nx}{3c^2}+\frac{1}{3}x^3\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/3*(b^2*n*x)/c^2 + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (b^3*n*Log[b + c*x])/(
3*c^3) + (x^3*Log[d*(b*x + c*x^2)^n])/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log \left(d(bx+cx^2)^n\right)\,dx = \frac{1}{3}x^3 \log \left(d(bx+cx^2)^n\right) - \frac{1}{3}n \int \frac{x^2(b+2cx)}{b+cx}\,dx$$
$$= \frac{1}{3}x^3 \log \left(d(bx+cx^2)^n\right) - \frac{1}{3}n \int \left(\frac{b^2}{c^2} - \frac{bx}{c} + 2x^2 - \frac{b^3}{c^2(b+cx)}\right)dx$$
$$= -\frac{b^2nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{b^3n\log(b+cx)}{3c^3} + \frac{1}{3}x^3 \log \left(d(bx+cx^2)^n\right)$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 0.89

$$\frac{cnx(-6b^2+3bcx-4c^2x^2)+6b^3n\log(b+cx)+6c^3x^3\log\left(d(x(b+cx))^n\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c^2*x^2) + 6*b^3*n*Log[b + c*x] + 6*c^3*x^3*Lo
g[d*(x*(b + c*x))^n])/(18*c^3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \ln \left(d(c\,x^2+bx)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x^2*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A]
time = 0.31, size = 65, normalized size = 0.92

$$\frac{1}{3}\,x^3 \log \left((cx^2+bx)^n d\right) + \frac{1}{18}\,n\left(\frac{6\,b^3\log(cx+b)}{c^3} - \frac{4\,c^2x^3 - 3\,bcx^2 + 6\,b^2x}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((c*x^2 + b*x)^n*d) + 1/18*n*(6*b^3*log(c*x + b)/c^3 - (4*c^2*x^
3 - 3*b*c*x^2 + 6*b^2*x)/c^2)
```

Fricas [A]
time = 0.36, size = 74, normalized size = 1.04

$$\frac{6\,c^3nx^3\log\left(cx^2+bx\right)-4\,c^3nx^3+6\,c^3x^3\log\left(d\right)+3\,bc^2nx^2-6\,b^2cnx+6\,b^3n\log\left(cx+b\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/18*(6*c^3*n*x^3*log(c*x^2 + b*x) - 4*c^3*n*x^3 + 6*c^3*x^3*log(d) + 3*b*c
^2*n*x^2 - 6*b^2*c*n*x + 6*b^3*n*log(c*x + b))/c^3
```

Sympy [A]
time = 2.40, size = 85, normalized size = 1.20

$$\begin{cases} \frac{b^3n\log\left(b+cx\right)}{3c^3}-\frac{b^2nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{x^3\log\left(d(bx+cx^2)^n\right)}{3} & \text{for } c\neq 0 \\ -\frac{nx^3}{9}+\frac{x^3\log\left(d(bx)^n\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b**3*n*log(b + c*x)/(3*c**3) - b**2*n*x/(3*c**2) + b*n*x**2/(6*c
) - 2*n*x**3/9 + x**3*log(d*(b*x + c*x**2)**n)/3, Ne(c, 0)), (-n*x**3/9 + x
**3*log(d*(b*x)**n)/3, True))
```

Giac [A]
time = 5.32, size = 65, normalized size = 0.92

$$\frac{1}{3}\,nx^3\log\left(cx^2+bx\right)-\frac{1}{9}\,(2\,n-3\,\log\left(d\right))x^3+\frac{bnx^2}{6\,c}-\frac{b^2nx}{3\,c^2}+\frac{b^3n\log\left(cx+b\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c - 1/3
*b^2*n*x/c^2 + 1/3*b^3*n*log(c*x + b)/c^3
```

Mupad [B]
time = 0.32, size = 61, normalized size = 0.86

$$\frac{x^3\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{3}-\frac{2\,n\,x^3}{9}+\frac{b^3\,n\,\ln\left(b+c\,x\right)}{3\,c^3}+\frac{b\,n\,x^2}{6\,c}-\frac{b^2\,n\,x}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^3*log(d*(b*x + c*x^2)^n))/3 - (2*n*x^3)/9 + (b^3*n*log(b + c*x))/(3*c^3)
 + (b*n*x^2)/(6*c) - (b^2*n*x)/(3*c^2)
```

3.63 $\int x \log \left(d(bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=57

$$\frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b^2n\log(b+cx)}{2c^2}+\frac{1}{2}x^2\log\left(d(bx+cx^2)^n\right)$$

[Out] 1/2*b*n*x/c-1/2*n*x^2-1/2*b^2*n*ln(c*x+b)/c^2+1/2*x^2*ln(d*(c*x^2+b*x)^n)

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2605, 78}

$$-\frac{b^2n\log(b+cx)}{2c^2}+\frac{1}{2}x^2\log\left(d(bx+cx^2)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Log[d*(b*x + c*x^2)^n],x]

[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b^2*n*Log[b + c*x])/(2*c^2) + (x^2*Log[d*(b*x + c*x^2)^n])/2

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x \log \left(d\left(bx+cx^2\right)^n\right) dx = \frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right) - \frac{1}{2}n \int \frac{x(b+2cx)}{b+cx} dx$$
$$= \frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right) - \frac{1}{2}n \int \left(-\frac{b}{c}+2x+\frac{b^2}{c(b+cx)}\right) dx$$
$$= \frac{bnx}{2c} - \frac{nx^2}{2} - \frac{b^2 n \log(b+cx)}{2c^2} + \frac{1}{2}x^2 \log \left(d\left(bx+cx^2\right)^n\right)$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.86

$$-\frac{1}{2}n\left(-\frac{bx}{c}+x^2+\frac{b^2\log(b+cx)}{c^2}\right)+\frac{1}{2}x^2\log\left(d(x(b+cx))^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -1/2*(n*(-((b*x)/c) + x^2 + (b^2*Log[b + c*x])/c^2)) + (x^2*Log[d*(x*(b + c
*x))^n])/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x \ln\left(d\left(c\,x^2+bx\right)^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x)^n),x)
```

```
[Out] int(x*ln(d*(c*x^2+b*x)^n),x)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.89

$$\frac{1}{2}\,x^2\log\left(\left(cx^2+bx\right)^n d\right)-\frac{1}{2}\,n\left(\frac{b^2\log\left(cx+b\right)}{c^2}+\frac{cx^2-bx}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((c*x^2 + b*x)^n*d) - 1/2*n*(b^2*log(c*x + b)/c^2 + (c*x^2 - b*x
)/c)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 1.04

$$\frac{c^2nx^2\log\left(cx^2+bx\right)-c^2nx^2+c^2x^2\log\left(d\right)+bcnx-b^2n\log\left(cx+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*n*x^2*log(c*x^2 + b*x) - c^2*n*x^2 + c^2*x^2*log(d) + b*c*n*x - b^
2*n*log(c*x + b))/c^2
```

Sympy [A]
time = 0.73, size = 70, normalized size = 1.23

$$\begin{cases} -\frac{b^2n\log\left(b+cx\right)}{2c^2}+\frac{bnx}{2c}-\frac{nx^2}{2}+\frac{x^2\log\left(d\left(bx+cx^2\right)^n\right)}{2} & \text{for } c\neq 0\\ -\frac{nx^2}{4}+\frac{x^2\log\left(d(bx)^n\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((-b**2*n*log(b + c*x)/(2*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*lo
g(d*(b*x + c*x**2)**n)/2, Ne(c, 0)), (-n*x**2/4 + x**2*log(d*(b*x)**n)/2, T
rue))
```

Giac [A]
time = 4.44, size = 51, normalized size = 0.89

$$\frac{1}{2}\,nx^2\log\left(cx^2+bx\right)-\frac{1}{2}\,(n-\log\left(d\right))x^2+\frac{bnx}{2\,c}-\frac{b^2n\log\left(cx+b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/2*b^2*n
*log(c*x + b)/c^2
```

Mupad [B]
time = 0.37, size = 49, normalized size = 0.86

$$\frac{x^2\,\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{2}-\frac{n\,x^2}{2}+\frac{b\,n\,x}{2\,c}-\frac{b^2\,n\,\ln\left(b+c\,x\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(b*x + c*x^2)^n))/2 - (n*x^2)/2 + (b*n*x)/(2*c) - (b^2*n*log(b +
 c*x))/(2*c^2)
```

3.64 $\int \log \left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=33

$$-2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d\left(bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+b*n*ln(c*x+b)/c+x*ln(d*(c*x^2+b*x)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2603, 45}

$$x\log\left(d\left(bx+cx^2\right)^n\right)+\frac{bn\log(b+cx)}{c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(b*x + c*x^2)^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(d\left(bx+cx^2\right)^n\right)\,dx&=x\log\left(d\left(bx+cx^2\right)^n\right)-n\int\frac{b+2cx}{b+cx}\,dx\\&=x\log\left(d\left(bx+cx^2\right)^n\right)-n\int\left(2-\frac{b}{b+cx}\right)\,dx\\&=-2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d\left(bx+cx^2\right)^n\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.94

$$-2nx+\frac{bn\log(b+cx)}{c}+x\log\left(d(x(b+cx))^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (b*n*Log[b + c*x])/c + x*Log[d*(x*(b + c*x))^n]
```

Maple [A]
time = 0.02, size = 37, normalized size = 1.12

method	result	size
default	$x\ln\left(d(c\,x^2+bx)^n\right)-n\left(2x-\frac{b\ln(cx+b)}{c}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*(c*x^2+b*x)^n)-n*(2*x-b/c*ln(c*x+b))
```

Maxima [A]
time = 0.27, size = 36, normalized size = 1.09

$$-n\left(2\,x-\frac{b\log\left(cx+b\right)}{c}\right)+x\log\left(\left(cx^2+bx\right)^nd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="maxima")
```

```
[Out] -n*(2*x - b*log(c*x + b)/c) + x*log((c*x^2 + b*x)^n*d)
```

Fricas [A]
time = 0.36, size = 38, normalized size = 1.15

$$\frac{cnx\log\left(cx^2+bx\right)-2\,cnx+bn\log\left(cx+b\right)+cx\log\left(d\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="fricas")
```

```
[Out] (c*n*x*log(c*x^2 + b*x) - 2*c*n*x + b*n*log(c*x + b) + c*x*log(d))/c
```

Sympy [A]
time = 0.39, size = 44, normalized size = 1.33

$$\begin{cases} \frac{bn\log{(b+cx)}}{c} - 2nx + x\log{\left(d(bx+cx^2)^n\right)} & \text{for } c \neq 0 \\ -nx + x\log{(d(bx)^n)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n),x)
```

```
[Out] Piecewise((b*n*log(b + c*x)/c - 2*n*x + x*log(d*(b*x + c*x**2)**n), Ne(c, 0
)), (-n*x + x*log(d*(b*x)**n), True))
```

Giac [A]
time = 5.51, size = 37, normalized size = 1.12

$$nx\log\left(cx^2+bx\right) - (2\,n - \log\left(d\right))x + \frac{bn\log\left(cx+b\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x) - (2*n - log(d))*x + b*n*log(c*x + b)/c
```

Mupad [B]
time = 0.36, size = 33, normalized size = 1.00

$$x\,\ln\left(d\left(c\,x^2+b\,x\right)^n\right) - 2\,n\,x + \frac{b\,n\,\ln\left(b+c\,x\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(b*x + c*x^2)^n) - 2*n*x + (b*n*log(b + c*x))/c
```

3.65 $\int \frac{\log \left(d\left(bx+cx^2\right)^n\right)}{x} \, dx$

Optimal. Leaf size=53

$$-\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\text{Li}_2\left(-\frac{cx}{b}\right)$$

```
[Out] -1/2*n*ln(x)^2-n*ln(x)*ln(1+c*x/b)+ln(x)*ln(d*(c*x^2+b*x)^n)-n*polylog(2,-c
*x/b)
```

Rubi [A]
time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2604, 1607, 2404, 2338, 2354, 2438}

$$-n\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\log(x)\log\left(\frac{cx}{b}+1\right)-\frac{1}{2}n\log^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x,x]
```

```
[Out] -1/2*(n*Log[x]^2) - n*Log[x]*Log[1 + (c*x)/b] + Log[x]*Log[d*(b*x + c*x^2)^
n] - n*PolyLog[2, -((c*x)/b)]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
```

```
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}\,dx &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{bx+cx^2}\,dx\\ &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx\\ &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx\\ &= \log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\int \frac{\log(x)}{x}\,dx-(cn)\int \frac{\log(x)}{b+cx}\,dx\\ &= -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)+n\int \frac{\log\left(1\right.}{x}\\ &= -\frac{1}{2}n\log^2(x)-n\log(x)\log\left(1+\frac{cx}{b}\right)+\log(x)\log\left(d\left(bx+cx^2\right)^n\right)-n\mathrm{Li}_2\left(-\frac{cx}{b}\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.94

$$\log(x)\log\left(d(x(b+cx))^n\right)-n\left(\frac{\log^2(x)}{2}+\log(x)\log\left(\frac{b+cx}{b}\right)+\mathrm{Li}_2\left(-\frac{cx}{b}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x,x]
[Out] Log[x]*Log[d*(x*(b + c*x))^n] - n*(Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + P
olyLog[2, -((c*x)/b)])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(d(c\,x^{2}+bx)^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x,x)
[Out] int(ln(d*(c*x^2+b*x)^n)/x,x)
```

Maxima [A]
time = 0.30, size = 80, normalized size = 1.51

$$-n\log\left(cx^{2}+bx\right)\log\left(x\right)+\frac{1}{2}\left(2\,\log\left(cx^{2}+bx\right)\log\left(x\right)-2\,\log\left(\frac{cx}{b}+1\right)\log\left(x\right)-\log\left(x\right)^{2}-2\,\mathrm{Li}_{2}\left(-\frac{cx}{b}\right)\right)n+\log\left(\left(cx^{2}+bx\right)^{n}d\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="maxima")
[Out] -n*log(c*x^2 + b*x)*log(x) + 1/2*(2*log(c*x^2 + b*x)*log(x) - 2*log(c*x/b +
 1)*log(x) - log(x)^2 - 2*dilog(-c*x/b))*n + log((c*x^2 + b*x)^n*d)*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="fricas")
[Out] integral(log((c*x^2 + b*x)^n*d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(d(bx+cx^{2})^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x,x)
[Out] Integral(log(d*(b*x + c*x**2)**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x)^n*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x,x)
```

```
[Out] int(log(d*(b*x + c*x^2)^n)/x, x)
```

3.66 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{n}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}$$

[Out] -n/x+c*n*ln(x)/b-c*n*ln(c*x+b)/b-ln(d*(c*x^2+b*x)^n)/x

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2605, 78}

$$-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x}+\frac{cn\log(x)}{b}-\frac{cn\log(b+cx)}{b}-\frac{n}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[d*(b*x + c*x^2)^n]/x^2,x]

[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(b*x + c*x^2)^n]/x

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^2}\,dx = -\frac{\log\left(d(bx+cx^2)^n\right)}{x} + n\int \frac{b+2cx}{x^2(b+cx)}\,dx$$
$$= -\frac{\log\left(d(bx+cx^2)^n\right)}{x} + n\int \left(\frac{1}{x^2}+\frac{c}{bx}-\frac{c^2}{b(b+cx)}\right)dx$$
$$= -\frac{n}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{\log\left(d(bx+cx^2)^n\right)}{x}$$

Mathematica [**A**]
time = 0.01, size = 45, normalized size = 0.96

$$-\frac{n}{x} + \frac{cn\log(x)}{b} - \frac{cn\log(b+cx)}{b} - \frac{\log\left(d(x(b+cx))^n\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^2,x]
[Out] -(n/x) + (c*n*Log[x])/b - (c*n*Log[b + c*x])/b - Log[d*(x*(b + c*x))^n]/x
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+bx)^n\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
[Out] int(ln(d*(c*x^2+b*x)^n)/x^2,x)
```

Maxima [**A**]
time = 0.28, size = 46, normalized size = 0.98

$$-n\left(\frac{c\log(cx+b)}{b} - \frac{c\log(x)}{b} + \frac{1}{x}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="maxima")
[Out] -n*(c*log(c*x + b)/b - c*log(x)/b + 1/x) - log((c*x^2 + b*x)^n*d)/x
```

Fricas [**A**]
time = 0.37, size = 46, normalized size = 0.98

$$-\frac{cnx\log(cx+b) - cnx\log(x) + bn\log(cx^2+bx) + bn + b\log(d)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="fricas")
```

```
[Out] -(c*n*x*log(c*x + b) - c*n*x*log(x) + b*n*log(c*x^2 + b*x) + b*n + b*log(d)
)/(b*x)
```

Sympy [A]
time = 1.10, size = 66, normalized size = 1.40

$$\begin{cases} -\frac{n}{x} - \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x} - \frac{2cn\log\left(b+cx\right)}{b} + \frac{c\log\left(d\left(bx+cx^2\right)^n\right)}{b} & \text{for}\ b \neq 0 \\ -\frac{2n}{x} - \frac{\log\left(d\left(cx^2\right)^n\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**2,x)
```

```
[Out] Piecewise((-n/x - log(d*(b*x + c*x**2)**n)/x - 2*c*n*log(b + c*x)/b + c*log
(d*(b*x + c*x**2)**n)/b, Ne(b, 0)), (-2*n/x - log(d*(c*x**2)**n)/x, True))
```

Giac [A]
time = 3.82, size = 47, normalized size = 1.00

$$-\frac{cn\log\left(cx+b\right)}{b} + \frac{cn\log\left(x\right)}{b} - \frac{n\log\left(cx^2+bx\right)}{x} - \frac{n+\log\left(d\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^2,x, algorithm="giac")
```

```
[Out] -c*n*log(c*x + b)/b + c*n*log(x)/b - n*log(c*x^2 + b*x)/x - (n + log(d))/x
```

Mupad [B]
time = 0.80, size = 43, normalized size = 0.91

$$-\frac{\ln\left(d\left(c\,x^2+b\,x\right)^n\right)}{x} - \frac{n}{x} - \frac{2\,c\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^2,x)
```

```
[Out] - log(d*(b*x + c*x^2)^n)/x - n/x - (2*c*n*atanh((2*c*x)/b + 1))/b
```

3.67 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=72

$$-\frac{n}{4x^2}-\frac{cn}{2bx}-\frac{c^2n\log(x)}{2b^2}+\frac{c^2n\log(b+cx)}{2b^2}-\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}$$

```
[Out] -1/4*n/x^2-1/2*c*n/b/x-1/2*c^2*n*ln(x)/b^2+1/2*c^2*n*ln(c*x+b)/b^2-1/2*ln(d
*(c*x^2+b*x)^n)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2605, 78}

$$-\frac{c^2n\log(x)}{2b^2}+\frac{c^2n\log(b+cx)}{2b^2}-\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}-\frac{cn}{2bx}-\frac{n}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/4*n/x^2 - (c*n)/(2*b*x) - (c^2*n*Log[x])/(2*b^2) + (c^2*n*Log[b + c*x])/
(2*b^2) - Log[d*(b*x + c*x^2)^n]/(2*x^2)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^3}\,dx = -\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2} + \frac{1}{2}n\int \frac{b+2cx}{x^3(b+cx)}\,dx$$
$$= -\frac{\log\left(d(bx+cx^2)^n\right)}{2x^2} + \frac{1}{2}n\int \left(\frac{1}{x^3}+\frac{c}{bx^2}-\frac{c^2}{b^2x}+\frac{c^3}{b^2(b+cx)}\right)dx$$
$$= -\frac{n}{4x^2} - \frac{cn}{2bx} - \frac{c^2n\log(x)}{2b^2} + \frac{c^2n\log(b+cx)}{2b^2} - \frac{\log\left(d(bx+cx^2)^n\right)}{2x^2}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.90

$$\frac{1}{2}n\left(-\frac{1}{2x^2}-\frac{c}{bx}-\frac{c^2\log(x)}{b^2}+\frac{c^2\log(b+cx)}{b^2}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^3,x]
```

```
[Out] (n*(-1/2*1/x^2 - c/(b*x) - (c^2*Log[x])/b^2 + (c^2*Log[b + c*x])/b^2))/2 -
Log[d*(x*(b + c*x))^n]/(2*x^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+bx)^n\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^3,x)
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.86

$$\frac{1}{4}n\left(\frac{2\,c^2\log(cx+b)}{b^2}-\frac{2\,c^2\log(x)}{b^2}-\frac{2\,cx+b}{bx^2}\right)-\frac{\log\left((cx^2+bx)^n d\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*n*(2*c^2*log(c*x + b)/b^2 - 2*c^2*log(x)/b^2 - (2*c*x + b)/(b*x^2)) - 1
/2*log((c*x^2 + b*x)^n*d)/x^2
```

Fricas [A]
time = 0.36, size = 70, normalized size = 0.97

$$\frac{2\,c^2nx^2\log\left(cx+b\right)-2\,c^2nx^2\log\left(x\right)-2\,bcnx-2\,b^2n\log\left(cx^2+bx\right)-b^2n-2\,b^2\log\left(d\right)}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^2*n*x^2*log(c*x + b) - 2*c^2*n*x^2*log(x) - 2*b*c*n*x - 2*b^2*n*lo
g(c*x^2 + b*x) - b^2*n - 2*b^2*log(d))/(b^2*x^2)
```

Sympy [A]
time = 2.16, size = 94, normalized size = 1.31

$$\begin{cases} -\frac{n}{4x^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{2x^2}-\frac{cn}{2bx}+\frac{c^2n\log\left(b+cx\right)}{b^2}-\frac{c^2\log\left(d\left(bx+cx^2\right)^n\right)}{2b^2} & \text{for}\ b\neq 0\\ -\frac{n}{2x^2}-\frac{\log\left(d\left(cx^2\right)^n\right)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**3,x)
```

```
[Out] Piecewise((-n/(4*x**2) - log(d*(b*x + c*x**2)**n)/(2*x**2) - c*n/(2*b*x) +
c**2*n*log(b + c*x)/b**2 - c**2*log(d*(b*x + c*x**2)**n)/(2*b**2), Ne(b, 0)
), (-n/(2*x**2) - log(d*(c*x**2)**n)/(2*x**2), True))
```

Giac [A]
time = 5.28, size = 65, normalized size = 0.90

$$\frac{c^2n\log\left(cx+b\right)}{2\,b^2}-\frac{c^2n\log\left(x\right)}{2\,b^2}-\frac{n\log\left(cx^2+bx\right)}{2\,x^2}-\frac{2\,cnx+bn+2\,b\log\left(d\right)}{4\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c^2*n*log(c*x + b)/b^2 - 1/2*c^2*n*log(x)/b^2 - 1/2*n*log(c*x^2 + b*x)/
x^2 - 1/4*(2*c*n*x + b*n + 2*b*log(d))/(b*x^2)
```

Mupad [B]
time = 0.48, size = 54, normalized size = 0.75

$$\frac{c^2\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2}-\frac{\frac{n}{2}+\frac{c\,n\,x}{b}}{2\,x^2}-\frac{\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (c^2*n*atanh((2*c*x)/b + 1))/b^2 - (n/2 + (c*n*x)/b)/(2*x^2) - log(d*(b*x +
 c*x^2)^n)/(2*x^2)
```

3.68 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{n}{9x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}+\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}$$

```
[Out] -1/9*n/x^3-1/6*c*n/b/x^2+1/3*c^2*n/b^2/x+1/3*c^3*n*ln(x)/b^3-1/3*c^3*n*ln(c
*x+b)/b^3-1/3*ln(d*(c*x^2+b*x)^n)/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2605, 78}

$$\frac{c^3n\log(x)}{3b^3}-\frac{c^3n\log(b+cx)}{3b^3}+\frac{c^2n}{3b^2x}-\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}-\frac{cn}{6bx^2}-\frac{n}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/9*n/x^3 - (c*n)/(6*b*x^2) + (c^2*n)/(3*b^2*x) + (c^3*n*Log[x])/(3*b^3) -
 (c^3*n*Log[b + c*x])/(3*b^3) - Log[d*(b*x + c*x^2)^n]/(3*x^3)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^4}\,dx = -\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3} + \frac{1}{3}n\int \frac{b+2cx}{x^4(b+cx)}\,dx$$
$$= -\frac{\log\left(d(bx+cx^2)^n\right)}{3x^3} + \frac{1}{3}n\int \left(\frac{1}{x^4}+\frac{c}{bx^3}-\frac{c^2}{b^2x^2}+\frac{c^3}{b^3x}-\frac{c^4}{b^3(b+cx)}\right)dx$$
$$= -\frac{n}{9x^3} - \frac{cn}{6bx^2} + \frac{c^2n}{3b^2x} + \frac{c^3n\log(x)}{3b^3} - \frac{c^3n\log(b+cx)}{3b^3} - \frac{\log\left(d(bx+cx^2)^n\right)}{3x^3}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.90

$$\frac{1}{3}n\left(-\frac{1}{3x^3}-\frac{c}{2bx^2}+\frac{c^2}{b^2x}+\frac{c^3\log(x)}{b^3}-\frac{c^3\log(b+cx)}{b^3}\right)-\frac{\log\left(d(x(b+cx))^n\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^4,x]
```

```
[Out] (n*(-1/3*1/x^3 - c/(2*b*x^2) + c^2/(b^2*x) + (c^3*Log[x])/b^3 - (c^3*Log[b
+ c*x]])/b^3))/3 - Log[d*(x*(b + c*x))^n]/(3*x^3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+bx)^n\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^4,x)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.87

$$-\frac{1}{18}n\left(\frac{6\,c^3\log(cx+b)}{b^3}-\frac{6\,c^3\log(x)}{b^3}-\frac{6\,c^2x^2-3\,bcx-2\,b^2}{b^2x^3}\right)-\frac{\log\left((cx^2+bx)^n d\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*n*(6*c^3*log(c*x + b)/b^3 - 6*c^3*log(x)/b^3 - (6*c^2*x^2 - 3*b*c*x -
 2*b^2)/(b^2*x^3)) - 1/3*log((c*x^2 + b*x)^n*d)/x^3
```

Fricas [A]
time = 0.36, size = 82, normalized size = 0.95

$$-\frac{6\,c^3nx^3\log\left(cx+b\right)-6\,c^3nx^3\log\left(x\right)-6\,bc^2nx^2+3\,b^2cnx+6\,b^3n\log\left(cx^2+bx\right)+2\,b^3n+6\,b^3\log\left(d\right)}{18\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*c^3*n*x^3*log(c*x + b) - 6*c^3*n*x^3*log(x) - 6*b*c^2*n*x^2 + 3*b^
2*c*n*x + 6*b^3*n*log(c*x^2 + b*x) + 2*b^3*n + 6*b^3*log(d))/(b^3*x^3)
```

Sympy [A]
time = 5.50, size = 112, normalized size = 1.30

$$\begin{cases} -\frac{n}{9x^3}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{3x^3}-\frac{cn}{6bx^2}+\frac{c^2n}{3b^2x}-\frac{2c^3n\log\left(b+cx\right)}{3b^3}+\frac{c^3\log\left(d\left(bx+cx^2\right)^n\right)}{3b^3} & \text{for}\: b\neq 0 \\ -\frac{2n}{9x^3}-\frac{\log\left(d\left(cx^2\right)^n\right)}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**4,x)
```

```
[Out] Piecewise((-n/(9*x**3) - log(d*(b*x + c*x**2)**n)/(3*x**3) - c*n/(6*b*x**2)
 + c**2*n/(3*b**2*x) - 2*c**3*n*log(b + c*x)/(3*b**3) + c**3*log(d*(b*x + c
*x**2)**n)/(3*b**3), Ne(b, 0)), (-2*n/(9*x**3) - log(d*(c*x**2)**n)/(3*x**3
), True))
```

Giac [A]
time = 3.49, size = 80, normalized size = 0.93

$$-\frac{c^3n\log\left(cx+b\right)}{3\,b^3}+\frac{c^3n\log\left(x\right)}{3\,b^3}-\frac{n\log\left(cx^2+bx\right)}{3\,x^3}+\frac{6\,c^2nx^2-3\,bcnx-2\,b^2n-6\,b^2\log\left(d\right)}{18\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*c^3*n*log(c*x + b)/b^3 + 1/3*c^3*n*log(x)/b^3 - 1/3*n*log(c*x^2 + b*x)
/x^3 + 1/18*(6*c^2*n*x^2 - 3*b*c*n*x - 2*b^2*n - 6*b^2*log(d))/(b^2*x^3)
```

Mupad [B]
time = 0.46, size = 68, normalized size = 0.79

$$-\frac{\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{3\,x^3}-\frac{\frac{n}{3}-\frac{c^2\,n\,x^2}{b^2}+\frac{c\,n\,x}{2\,b}}{3\,x^3}-\frac{2\,c^3\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^4,x)
```

```
[Out] - log(d*(b*x + c*x^2)^n)/(3*x^3) - (n/3 - (c^2*n*x^2)/b^2 + (c*n*x)/(2*b))/
(3*x^3) - (2*c^3*n*atanh((2*c*x)/b + 1))/(3*b^3)
```

3.69 $\int \frac{\log\left(d\left(bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal. Leaf size=100

$$-\frac{n}{16x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}-\frac{c^4n\log(x)}{4b^4}+\frac{c^4n\log(b+cx)}{4b^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}$$

```
[Out] -1/16*n/x^4-1/12*c*n/b/x^3+1/8*c^2*n/b^2/x^2-1/4*c^3*n/b^3/x-1/4*c^4*n*ln(x
)/b^4+1/4*c^4*n*ln(c*x+b)/b^4-1/4*ln(d*(c*x^2+b*x)^n)/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2605, 78}

$$-\frac{c^4n\log(x)}{4b^4}+\frac{c^4n\log(b+cx)}{4b^4}-\frac{c^3n}{4b^3x}+\frac{c^2n}{8b^2x^2}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}-\frac{cn}{12bx^3}-\frac{n}{16x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/16*n/x^4 - (c*n)/(12*b*x^3) + (c^2*n)/(8*b^2*x^2) - (c^3*n)/(4*b^3*x) -
(c^4*n*Log[x])/(4*b^4) + (c^4*n*Log[b + c*x])/(4*b^4) - Log[d*(b*x + c*x^2)
^n]/(4*x^4)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(bx+cx^2)^n\right)}{x^5}\,dx = -\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4} + \frac{1}{4}n\int \frac{b+2cx}{x^5(b+cx)}\,dx$$
$$= -\frac{\log\left(d(bx+cx^2)^n\right)}{4x^4} + \frac{1}{4}n\int \left(\frac{1}{x^5} + \frac{c}{bx^4} - \frac{c^2}{b^2x^3} + \frac{c^3}{b^3x^2} - \frac{c^4}{b^4x} + \frac{c^5}{b^4(b+cx)}\right) dx$$
$$= -\frac{n}{16x^4} - \frac{cn}{12bx^3} + \frac{c^2n}{8b^2x^2} - \frac{c^3n}{4b^3x} - \frac{c^4n\log(x)}{4b^4} + \frac{c^4n\log(b+cx)}{4b^4} - \frac{\log\left(d(bx+c\right.}{4x^4}$$

Mathematica [A]
time = 0.04, size = 87, normalized size = 0.87

$$-\frac{bn(3b^3+4b^2cx-6bc^2x^2+12c^3x^3)+12c^4nx^4\log(x)-12c^4nx^4\log(b+cx)+12b^4\log\left(d(x(b+cx))^n\right)}{48b^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/48*(b*n*(3*b^3 + 4*b^2*c*x - 6*b*c^2*x^2 + 12*c^3*x^3) + 12*c^4*n*x^4*Lo
g[x] - 12*c^4*n*x^4*Log[b + c*x] + 12*b^4*Log[d*(x*(b + c*x))^n])/(b^4*x^4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+bx)^n\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)/x^5,x)
```

Maxima [A]
time = 0.29, size = 86, normalized size = 0.86

$$\frac{1}{48}n\left(\frac{12\,c^4\log\left(cx+b\right)}{b^4} - \frac{12\,c^4\log\left(x\right)}{b^4} - \frac{12\,c^3x^3-6\,bc^2x^2+4\,b^2cx+3\,b^3}{b^3x^4}\right) - \frac{\log\left((cx^2+bx)^n d\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*n*(12*c^4*log(c*x + b)/b^4 - 12*c^4*log(x)/b^4 - (12*c^3*x^3 - 6*b*c^2
*x^2 + 4*b^2*c*x + 3*b^3)/(b^3*x^4)) - 1/4*log((c*x^2 + b*x)^n*d)/x^4
```

Fricas [A]
time = 0.38, size = 94, normalized size = 0.94

$$\frac{12\,c^4nx^4\log\left(cx+b\right)-12\,c^4nx^4\log\left(x\right)-12\,bc^3nx^3+6\,b^2c^2nx^2-4\,b^3cnx-12\,b^4n\log\left(cx^2+bx\right)-3\,b^4n-12\,b^4\log\left(d\right)}{48\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(12*c^4*n*x^4*log(c*x + b) - 12*c^4*n*x^4*log(x) - 12*b*c^3*n*x^3 + 6*
b^2*c^2*n*x^2 - 4*b^3*c*n*x - 12*b^4*n*log(c*x^2 + b*x) - 3*b^4*n - 12*b^4*
log(d))/(b^4*x^4)
```

Sympy [A]
time = 10.18, size = 122, normalized size = 1.22

$$\begin{cases} -\frac{n}{16x^4}-\frac{\log\left(d\left(bx+cx^2\right)^n\right)}{4x^4}-\frac{cn}{12bx^3}+\frac{c^2n}{8b^2x^2}-\frac{c^3n}{4b^3x}+\frac{c^4n\log\left(b+cx\right)}{2b^4}-\frac{c^4\log\left(d\left(bx+cx^2\right)^n\right)}{4b^4} & \text{for}\ b\neq 0 \\ -\frac{n}{8x^4}-\frac{\log\left(d\left(cx^2\right)^n\right)}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)/x**5,x)
```

```
[Out] Piecewise((-n/(16*x**4) - log(d*(b*x + c*x**2)**n)/(4*x**4) - c*n/(12*b*x**
3) + c**2*n/(8*b**2*x**2) - c**3*n/(4*b**3*x) + c**4*n*log(b + c*x)/(2*b**4
) - c**4*log(d*(b*x + c*x**2)**n)/(4*b**4), Ne(b, 0)), (-n/(8*x**4) - log(d
*(c*x**2)**n)/(4*x**4), True))
```

Giac [A]
time = 4.74, size = 92, normalized size = 0.92

$$\frac{c^4n\log\left(cx+b\right)}{4\,b^4}-\frac{c^4n\log\left(x\right)}{4\,b^4}-\frac{n\log\left(cx^2+bx\right)}{4\,x^4}-\frac{12\,c^3nx^3-6\,bc^2nx^2+4\,b^2cnx+3\,b^3n+12\,b^3\log\left(d\right)}{48\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^4*n*log(c*x + b)/b^4 - 1/4*c^4*n*log(x)/b^4 - 1/4*n*log(c*x^2 + b*x)/
x^4 - 1/48*(12*c^3*n*x^3 - 6*b*c^2*n*x^2 + 4*b^2*c*n*x + 3*b^3*n + 12*b^3*l
og(d))/(b^3*x^4)
```

Mupad [B]
time = 0.49, size = 79, normalized size = 0.79

$$\frac{c^4\,n\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{2\,b^4}-\frac{\ln\left(d\,{\left(c\,x^2+b\,x\right)}^n\right)}{4\,x^4}-\frac{\frac{n}{4}-\frac{c^2\,n\,x^2}{2\,b^2}+\frac{c^3\,n\,x^3}{b^3}+\frac{c\,n\,x}{3\,b}}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (c^4*n*atanh((2*c*x)/b + 1))/(2*b^4) - log(d*(b*x + c*x^2)^n)/(4*x^4) - (n/
4 - (c^2*n*x^2)/(2*b^2) + (c^3*n*x^3)/b^3 + (c*n*x)/(3*b))/(4*x^4)
```

3.70 $\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=157

$$-\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)}-\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)(1+m)(2+m)}+\frac{x^{1+m}}{}$$

```
[Out] x^(1+m)*ln(d*(c*x^2+b*x+a)^n)/(1+m)-2*c*n*x^(2+m)*hypergeom([1, 2+m],[3+m],
-2*c*x/(b-(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b-(-4*a*c+b^2)^(1/2))-2*c*n*x^(
2+m)*hypergeom([1, 2+m],[3+m],-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(2+m)/(b
+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2605, 844, 66}

$$-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cnx^{m+2}\,{}_2F_1\left(1,m+2;m+3;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)(m+2)\left(\sqrt{b^2-4ac}+b\right)}+\frac{x^{m+1}\log\left(d\left(a+bx+cx^2\right)^n\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*c*n*x^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b - Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) - (2*c*n*x^(2 + m)*H
ypergeometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/((b +
Sqrt[b^2 - 4*a*c])*(1 + m)*(2 + m)) + (x^(1 + m)*Log[d*(a + b*x + c*x^2)^n]
)/(1 + m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \frac{x^{1+m}(b+2cx)}{a+bx+cx^2}\,dx}{1+m}$$

$$= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{n\int \left(\frac{2cx^{1+m}}{b-\sqrt{b^2-4ac}+2cx} + \frac{2cx^{1+m}}{b+\sqrt{b^2-4ac}}\right.}{1+m}$$

$$= \frac{x^{1+m}\log\left(d\left(a+bx+cx^2\right)^n\right)}{1+m} - \frac{(2cn)\int \frac{x^{1+m}}{b-\sqrt{b^2-4ac}+2cx}\,dx}{1+m} - \frac{(2cn)\int}{}$$

$$= -\frac{2cnx^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m)(2+m)} - \frac{2cnx^{2+m}\,{}_2F_1\left(1,2\right.}{\left(b+\sqrt{b^2-\right.}}$$

Mathematica [A]
time = 0.13, size = 137, normalized size = 0.87

$$-\frac{x^{1+m}\left(\left(b+\sqrt{b^2-4ac}\right)nx\,{}_2F_1\left(1,2+m;3+m;\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)+\left(b-\sqrt{b^2-4ac}\right)nx\,{}_2F_1\left(1,2+m;3+m;-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)-2a(2+m)\log\left(d(a+x(b+cx))^n\right)\right)}{2a\left(2+3m+m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/2*(x^(1 + m)*((b + Sqrt[b^2 - 4*a*c])*n*x*Hypergeometric2F1[1, 2 + m, 3
+ m, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*n*x*Hyperg
eometric2F1[1, 2 + m, 3 + m, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])] - 2*a*(2 + m
)*Log[d*(a + x*(b + c*x))^n]))/(a*(2 + 3*m + m^2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^m \ln\left(d\left(c\,x^2+bx+a\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

```
[Out] int(x^m*ln(d*(c*x^2+b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] x*x^m*log((c*x^2 + b*x + a)^n)/(m + 1) + integrate((((m + 1)*log(d) - 2*n)*
c*x^2 + ((m + 1)*log(d) - n)*b*x + a*(m + 1)*log(d))*x^m/(c*(m + 1)*x^2 + b
*(m + 1)*x + a*(m + 1)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*log((c*x^2 + b*x + a)^n*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \ln \left(d\left(c\,x^2+b\,x+a\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*log(d*(a + b*x + c*x^2)^n),x)
[Out] int(x^m*log(d*(a + b*x + c*x^2)^n), x)
```

3.71 $\int x^4 \log\left(d(a+bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=207

$$-\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-\frac{2nx^5}{25}+\frac{\sqrt{b^2-4ac}\,(b^4-3ab^2c+a^2c^2)\,n}{5c^5}$$

```
[Out] -1/5*(2*a^2*c^2-4*a*b^2*c+b^4)*n*x/c^4+1/10*b*(-3*a*c+b^2)*n*x^2/c^3-1/15*(
-2*a*c+b^2)*n*x^3/c^2+1/20*b*n*x^4/c-2/25*n*x^5+1/10*b*(5*a^2*c^2-5*a*b^2*c
+b^4)*n*ln(c*x^2+b*x+a)/c^5+1/5*x^5*ln(d*(c*x^2+b*x+a)^n)+1/5*(a^2*c^2-3*a*
b^2*c+b^4)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A]
time = 0.14, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{bn(5a^2c^2-5ab^2c+b^4)\log(a+bx+cx^2)}{10c^5}+\frac{n\sqrt{b^2-4ac}\,(a^2c^2-3ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5}-\frac{nx(2a^2c^2-4ab^2c+b^4)}{5c^4}+\frac{bnx^2(b^2-3ac)}{10c^3}-\frac{nx^3(b^2-2ac)}{15c^2}+\frac{1}{5}x^5\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bnx^4}{20c}-\frac{2nx^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/5*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x)/c^4 + (b*(b^2 - 3*a*c)*n*x^2)/(10*
c^3) - ((b^2 - 2*a*c)*n*x^3)/(15*c^2) + (b*n*x^4)/(20*c) - (2*n*x^5)/25 + (
Sqrt[b^2 - 4*a*c]*(b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(5*c^5) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + b*x + c*x
^2])/(10*c^5) + (x^5*Log[d*(a + b*x + c*x^2)^n])/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4 \log \left(d\left(a+bx+cx^2\right)^n\right)\, dx &= \frac{1}{5}x^5 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{5}n\int \frac{x^5(b+2cx)}{a+bx+cx^2}\, dx \\
&= \frac{1}{5}x^5 \log \left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{5}n\int \left(\frac{b^4-4ab^2c+2a^2c^2}{c^4}-\frac{b(b^2-3ac)\,
\end{aligned}
$$

$$= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-$$

$$= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-$$

$$= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-$$

$$= -\frac{(b^4-4ab^2c+2a^2c^2)\,nx}{5c^4}+\frac{b(b^2-3ac)\,nx^2}{10c^3}-\frac{(b^2-2ac)\,nx^3}{15c^2}+\frac{bnx^4}{20c}-$$

Mathematica [A]
time = 0.15, size = 190, normalized size = 0.92

$$\frac{cnx(-60b^4+30b^3cx-20b^2c(-12a+cx^2)+15bc^2x(-6a+cx^2)-8c^2(15a^2-5acx^2+3c^2x^4))+60\sqrt{b^2-4ac}\,(b^4-3ab^2c+a^2c^2)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+30b(b^4-5ab^2c+5a^2c^2)\,n\log(a+x(b+cx))+60c^5x^5\log\left(d(a+x(b+cx))^n\right)}{300c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(-60*b^4 + 30*b^3*c*x - 20*b^2*c*(-12*a + c*x^2) + 15*b*c^2*x*(-6*a
+ c*x^2) - 8*c^2*(15*a^2 - 5*a*c*x^2 + 3*c^2*x^4)) + 60*Sqrt[b^2 - 4*a*c]*(
b^4 - 3*a*b^2*c + a^2*c^2)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 30*b*
(b^4 - 5*a*b^2*c + 5*a^2*c^2)*n*Log[a + x*(b + c*x)] + 60*c^5*x^5*Log[d*(a
+ x*(b + c*x))^n])/(300*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 1621, normalized size = 7.83

method	result	size
risch	Expression too large to display	1621

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln((c*x^2+b*x+a)^n)+1/10*I*Pi*x^5*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^
n)^2-1/10*I*Pi*x^5*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/10*I*Pi*x^5*csgn(I*d)*csgn
(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/10*I*Pi*x^5*csgn(I*(c*x^2+b
*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/5*ln(d)*x^5-2/25*n*x^5+1/20*b*n*x^4/
c+2/15/c*a*n*x^3-1/15*b^2*n*x^3/c^2-3/10/c^2*a*b*n*x^2+1/10*b^3*n*x^2/c^3+1
/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b^2
*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5+2
5*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a^2*b
-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b
^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5
+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*a*b
^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
b^5+1/2/c^3*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*
c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*
a^2*b-1/2/c^4*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*
a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^
5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b
)*a*b^3+1/10/c^5*n*ln(4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+
25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4
*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2
)*b)*b^5-2/5/c^2*a^2*n*x+4/5/c^3*a*b^2*n*x-1/5*b^4*n*x/c^4-1/10/c^5*n*ln(4*
a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6+2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*b^
4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x+(-4*a^5*c^5+25*a^4*b^2*c^4-
50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^4*
```

```
b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)+1/10/c^5*n*ln(
4*a^3*c^3-13*a^2*b^2*c^2+7*a*b^4*c-b^6-2*(-4*a^5*c^5+25*a^4*b^2*c^4-50*a^3*
b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*c*x-(-4*a^5*c^5+25*a^4*b^2*c^
4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)*b)*(-4*a^5*c^5+25*a^
4*b^2*c^4-50*a^3*b^4*c^3+35*a^2*b^6*c^2-10*a*b^8*c+b^10)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.37, size = 444, normalized size = 2.14

$$\left[-\frac{24\,c^{5}nx^{5}-60\,c^{5}x^{5}\log\left(d\right)-15\,bc^{4}nx^{4}+20\,(b^{2}c^{3}-2\,ac^{4})nx^{3}-30\,(b^{3}c^{2}-3\,abc^{3})nx^{2}-30\,(b^{4}-3\,ab^{2}c+a^{2}c^{2})\sqrt{b^{2}-4\,ac}\,n\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+60\,(b^{4}c-4\,ab^{2}c^{2}+2\,a^{2}c^{3})nx-30\,(2\,c^{5}nx^{5}+(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2})n)\log\left(cx^{2}+bx+a\right)}{300\,c^{5}},\ -\frac{24\,c^{5}nx^{5}-60\,c^{5}x^{5}\log\left(d\right)-15\,bc^{4}nx^{4}+20\,(b^{2}c^{3}-2\,ac^{4})nx^{3}-30\,(b^{3}c^{2}-3\,abc^{3})nx^{2}-60\,(b^{4}-3\,ab^{2}c+a^{2}c^{2})\sqrt{-b^{2}+4\,ac}\,n\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+60\,(b^{4}c-4\,ab^{2}c^{2}+2\,a^{2}c^{3})nx-30\,(2\,c^{5}nx^{5}+(b^{5}-5\,ab^{3}c+5\,a^{2}bc^{2})n)\log\left(cx^{2}+bx+a\right)}{300\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4 + 20*(b^2*c^3 -
2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 30*(b^4 - 3*a*b^2*c + a^2
*c^2)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2
*c^3)*n*x - 30*(2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2
+ b*x + a))/c^5, -1/300*(24*c^5*n*x^5 - 60*c^5*x^5*log(d) - 15*b*c^4*n*x^4
+ 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 - 60*(b^4 -
 3*a*b^2*c + a^2*c^2)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x - 30*(2*c^
5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*log(c*x^2 + b*x + a))/c^5]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(d*(c*x**2+b*x+a)**n),x)
```

[Out] Timed out

Giac [A]
time = 3.75, size = 221, normalized size = 1.07

$$\frac{1}{5}nx^5\log\left(cx^2+bx+a\right)-\frac{1}{25}\left(2n-5\log\left(d\right)\right)x^5+\frac{bnx^4}{20c}-\frac{(b^2n-2acn)x^3}{15c^2}+\frac{(b^3n-3abcn)x^2}{10c^3}-\frac{(b^4n-4ab^2cn+2a^2c^2n)x}{5c^4}+\frac{(b^5n-5ab^3cn+5a^2bc^2n)\log\left(cx^2+bx+a\right)}{10c^5}-\frac{(b^6n-7ab^4cn+13a^2b^2c^2n-4a^3c^3n)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{5\sqrt{-b^2+4ac}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/5*n*x^5*log(c*x^2 + b*x + a) - 1/25*(2*n - 5*log(d))*x^5 + 1/20*b*n*x^4/c
 - 1/15*(b^2*n - 2*a*c*n)*x^3/c^2 + 1/10*(b^3*n - 3*a*b*c*n)*x^2/c^3 - 1/5*
(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*x/c^4 + 1/10*(b^5*n - 5*a*b^3*c*n + 5*a
^2*b*c^2*n)*log(c*x^2 + b*x + a)/c^5 - 1/5*(b^6*n - 7*a*b^4*c*n + 13*a^2*b^
2*c^2*n - 4*a^3*c^3*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^5)
```

Mupad [B]
time = 0.59, size = 395, normalized size = 1.91

$$x^2\left(\frac{b\left(\frac{b^2n}{5c^2}-\frac{2an}{5c}\right)}{2c}-\frac{abn}{10c^2}\right)-\frac{2nx^5}{25}+x\left(\frac{a\left(\frac{b^2n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{b\left(\frac{b\left(\frac{b^2n}{5c^2}-\frac{2an}{5c}\right)}{c}-\frac{abn}{5c^2}\right)}{c}\right)+\frac{x^5\ln\left(d\left(cx^2+bx+a\right)^n\right)}{5}-x^3\left(\frac{b^2n}{15c^2}-\frac{2an}{15c}\right)+\frac{\ln\left(b\sqrt{b^2-4ac}-4ac+b^2+2cx\sqrt{b^2-4ac}\right)\left(\frac{b^5n}{10}+c^2\left(\frac{a^2n\sqrt{b^2-4ac}}{10}+\frac{a^2bn}{2}\right)-c\left(\frac{ab^3n}{2}+\frac{3ab^2n\sqrt{b^2-4ac}}{10}\right)+\frac{b^4n\sqrt{b^2-4ac}}{10}\right)}{c^5}-\frac{\ln\left(4ac+b\sqrt{b^2-4ac}-b^2+2cx\sqrt{b^2-4ac}\right)\left(c^2\left(\frac{a^2n\sqrt{b^2-4ac}}{10}-\frac{a^2bn}{2}\right)-\frac{b^5n}{10}+c\left(\frac{ab^3n}{2}-\frac{3ab^2n\sqrt{b^2-4ac}}{10}\right)+\frac{b^4n\sqrt{b^2-4ac}}{10}\right)}{c^5}+\frac{bnx^4}{20c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x^2*((b*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/(2*c) - (a*b*n)/(10*c^2)) - (2*n
*x^5)/25 + x*((a*((b^2*n)/(5*c^2) - (2*a*n)/(5*c)))/c - (b*((b*((b^2*n)/(5*
c^2) - (2*a*n)/(5*c)))/c - (a*b*n)/(5*c^2)))/c) + (x^5*log(d*(a + b*x + c*x
^2)^n))/5 - x^3*((b^2*n)/(15*c^2) - (2*a*n)/(15*c)) + (log(b*(b^2 - 4*a*c)^
(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^5*n)/10 + c^2*((a^2*n*
(b^2 - 4*a*c)^(1/2))/10 + (a^2*b*n)/2) - c*((a*b^3*n)/2 + (3*a*b^2*n*(b^2 -
 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 - (log(4*a*c + b*
(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c^2*((a^2*n*(b^2 -
4*a*c)^(1/2))/10 - (a^2*b*n)/2) - (b^5*n)/10 + c*((a*b^3*n)/2 - (3*a*b^2*n*
(b^2 - 4*a*c)^(1/2))/10) + (b^4*n*(b^2 - 4*a*c)^(1/2))/10))/c^5 + (b*n*x^4)
/(20*c)
```

3.72 $\int x^3 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=167

$$\frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b\sqrt{b^2-4ac}\,(b^2-2ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}-\frac{(b^4-4ab^2c$$

```
[Out] 1/4*b*(-3*a*c+b^2)*n*x/c^3-1/8*(-2*a*c+b^2)*n*x^2/c^2+1/12*b*n*x^3/c-1/8*n*
x^4-1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x^2+b*x+a)/c^4+1/4*x^4*ln(d*(c*x^2
+b*x+a)^n)-1/4*b*(-2*a*c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a
*c+b^2)^(1/2)/c^4
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{n(2a^2c^2-4ab^2c+b^4)\log\left(a+bx+cx^2\right)}{8c^4}-\frac{bn\sqrt{b^2-4ac}\,(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}+\frac{bnx(b^2-3ac)}{4c^3}-\frac{nx^2(b^2-2ac)}{8c^2}+\frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bnx^3}{12c}-\frac{nx^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (b*(b^2 - 3*a*c)*n*x)/(4*c^3) - ((b^2 - 2*a*c)*n*x^2)/(8*c^2) + (b*n*x^3)/(
12*c) - (n*x^4)/8 - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*
x + c*x^2])/(8*c^4) + (x^4*Log[d*(a + b*x + c*x^2)^n])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int \frac{x^4(b+2cx)}{a+bx+cx^2}\,dx$$

$$= \frac{1}{4}x^4\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{4}n\int\left(-\frac{b(b^2-3ac)}{c^3}+\frac{(b^2-2ac)\,x}{c^2}-\frac{bx^2}{c}\right.$$

$$= \frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}+\frac{1}{4}x^4\log\left(d\left(a+bx+cx\right.\right.$$

$$= \frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}+\frac{1}{4}x^4\log\left(d\left(a+bx+cx\right.\right.$$

$$= \frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{(b^4-4ab^2c+2a^2c^2)\,n}{8c^4}$$

$$= \frac{b(b^2-3ac)\,nx}{4c^3}-\frac{(b^2-2ac)\,nx^2}{8c^2}+\frac{bnx^3}{12c}-\frac{nx^4}{8}-\frac{b\sqrt{b^2-4ac}\,(b^2-2ac)}{4}$$

Mathematica [A]

time = 0.11, size = 151, normalized size = 0.90

$$\frac{cnx(6b^3-3b^2cx+2bc(-9a+cx^2)-3c^2x(-2a+cx^2))-6b\sqrt{b^2-4ac}\,(b^2-2ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-3(b^4-4ab^2c+2a^2c^2)\,n\log(a+x(b+cx))+6c^4x^4\log\left(d(a+x(b+cx))^n\right)}{24c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (c*n*x*(6*b^3 - 3*b^2*c*x + 2*b*c*(-9*a + c*x^2) - 3*c^2*x*(-2*a + c*x^2))
- 6*b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]] - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + x*(b + c*x)] + 6*c^4*x^4*Lo
g[d*(a + x*(b + c*x))^n])/(24*c^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 1146, normalized size = 6.86

method	result
risch	$\frac{x^4 \ln\left(\left(c\,x^2+bx+a\right)^n\right)}{4}+\frac{i\pi\,x^4 \mathrm{csgn}(id)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2}{8}+\frac{i\pi\,x^4 \mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2}{8}-\frac{i\pi\,x^4 \mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln((c*x^2+b*x+a)^n)+1/8*I*Pi*x^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n
)^2+1/8*I*Pi*x^4*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/8*I*
Pi*x^4*csgn(I*d*(c*x^2+b*x+a)^n)^3-1/8*I*Pi*x^4*csgn(I*d)*csgn(I*(c*x^2+b*x
+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/4*ln(d)*x^4-1/8*n*x^4+1/12*b*n*x^3/c+1/4
/c*a*n*x^2-1/8*b^2*n*x^2/c^2-1/4/c^2*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5+2*(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^3+20*a^2
*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5
+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c^
3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/8/c^4*n*ln(8*a^2*b*c^2-6*a
*b^3*c+b^5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*
a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*b^4-1/4/c^2*n*ln(8*a^2*b
*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c
*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a^2+1/2/c^3*n*ln
(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)
^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*a*b^2-1/
8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*
b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b
)*b^4-3/4/c^2*a*b*n*x+1/4*b^3*n*x/c^3-1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^
5+2*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x+(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8
*a*b^6*c+b^8)^(1/2)+1/8/c^4*n*ln(8*a^2*b*c^2-6*a*b^3*c+b^5-2*(-16*a^3*b^2*c
^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2)*c*x-(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-
8*a*b^6*c+b^8)^(1/2)*b)*(-16*a^3*b^2*c^3+20*a^2*b^4*c^2-8*a*b^6*c+b^8)^(1/2
)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.37, size = 364, normalized size = 2.18

$$\left[-\frac{3\,c^4nx^4-6\,c^4x^4\log\left(d\right)-2\,bc^3nx^3+3\,(b^2c^2-2\,ac^3)nx^2+3\,(b^3-2\,abc)\sqrt{b^2-4\,ac}\,n\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-6\,(b^3c-3\,abc^2)nx-3\,(2\,c^4nx^4-(b^4-4\,ab^2c+2\,a^2c^2)n)\log\left(cx^2+bx+a\right)}{24\,c^4},-\frac{3\,c^4nx^4-6\,c^4x^4\log\left(d\right)-2\,bc^3nx^3+3\,(b^2c^2-2\,ac^3)nx^2+6\,(b^3-2\,abc)\sqrt{-b^2+4\,ac}\,n\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-6\,(b^3c-3\,abc^2)nx-3\,(2\,c^4nx^4-(b^4-4\,ab^2c+2\,a^2c^2)n)\log\left(cx^2+bx+a\right)}{24\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c
^3)*n*x^2 + 3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 6*(b^3*
c - 3*a*b*c^2)*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(
c*x^2 + b*x + a))/c^4, -1/24*(3*c^4*n*x^4 - 6*c^4*x^4*log(d) - 2*b*c^3*n*x^
3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 + 6*(b^3 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*ar
ctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 6*(b^3*c - 3*a*b*c^2)
*n*x - 3*(2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*log(c*x^2 + b*x +
a))/c^4]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.87, size = 176, normalized size = 1.05

$$\frac{1}{4}\,nx^4\log\left(cx^2+bx+a\right)-\frac{1}{8}\,(n-2\,\log\left(d\right))x^4+\frac{bnx^3}{12\,c}-\frac{(b^2n-2\,acn)x^2}{8\,c^2}+\frac{(b^3n-3\,abcn)x}{4\,c^3}-\frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log\left(cx^2+bx+a\right)}{8\,c^4}+\frac{(b^5n-6\,ab^3cn+8\,a^2bc^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/4*n*x^4*log(c*x^2 + b*x + a) - 1/8*(n - 2*log(d))*x^4 + 1/12*b*n*x^3/c -
1/8*(b^2*n - 2*a*c*n)*x^2/c^2 + 1/4*(b^3*n - 3*a*b*c*n)*x/c^3 - 1/8*(b^4*n
- 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/c^4 + 1/4*(b^5*n - 6*a*b^
3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^4)
```

Mupad [B]

time = 0.54, size = 288, normalized size = 1.72

$$x\left(\frac{b\left(\frac{b^2\,n}{4\,c^2}-\frac{a\,n}{2\,c}\right)}{c}-\frac{a\,b\,n}{4\,c^2}\right)-\frac{n\,x^4}{8}+\frac{x^4\,\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{4}-x^2\,\left(\frac{b^2\,n}{8\,c^2}-\frac{a\,n}{4\,c}\right)+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,b^2\,n}{2}-\frac{a\,b\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)-\frac{b^4\,n}{8}+\frac{b^3\,n\,\sqrt{b^2-4\,a\,c}}{8}-\frac{a^2\,c^2\,n}{4}\right)}{c^4}-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^4\,n}{8}-c\,\left(\frac{a\,b^2\,n}{2}+\frac{a\,b\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)+\frac{b^3\,n\,\sqrt{b^2-4\,a\,c}}{8}+\frac{a^2\,c^2\,n}{4}\right)}{c^4}+\frac{b\,n\,x^3}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*((b*((b^2*n)/(4*c^2) - (a*n)/(2*c)))/c - (a*b*n)/(4*c^2)) - (n*x^4)/8 + (
x^4*log(d*(a + b*x + c*x^2)^n))/4 - x^2*((b^2*n)/(8*c^2) - (a*n)/(4*c)) + (
log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c*((a
*b^2*n)/2 - (a*b*n*(b^2 - 4*a*c)^(1/2))/4) - (b^4*n)/8 + (b^3*n*(b^2 - 4*a*
c)^(1/2))/8 - (a^2*c^2*n)/4))/c^4 - (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^
2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - c*((a*b^2*n)/2 + (a*b*n*(b^2 -
4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4))/c^4 + (b
*n*x^3)/(12*c)
```

3.73 $\int x^2 \log\left(d(a+bx+cx^2)^n\right)\, dx$

Optimal. Leaf size=136

$$-\frac{(b^2-2ac)\,nx}{3c^2}+\frac{bnx^2}{6c}-\frac{2nx^3}{9}+\frac{\sqrt{b^2-4ac}\,(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}+\frac{b(b^2-3ac)\,n\log\left(a+bx+cx\right.}{6c^3}$$

```
[Out] -1/3*(-2*a*c+b^2)*n*x/c^2+1/6*b*n*x^2/c-2/9*n*x^3+1/6*b*(-3*a*c+b^2)*n*ln(c
*x^2+b*x+a)/c^3+1/3*x^3*ln(d*(c*x^2+b*x+a)^n)+1/3*(-a*c+b^2)*n*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^3
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{bn(b^2-3ac)\log\left(a+bx+cx^2\right)}{6c^3}+\frac{n\sqrt{b^2-4ac}\,(b^2-ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}-\frac{nx(b^2-2ac)}{3c^2}+\frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bnx^2}{6c}-\frac{2nx^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/3*((b^2 - 2*a*c)*n*x)/c^2 + (b*n*x^2)/(6*c) - (2*n*x^3)/9 + (Sqrt[b^2 -
4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) + (b*(
b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*c^3) + (x^3*Log[d*(a + b*x + c*x^2)
^n])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx = \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int\frac{x^3(b+2cx)}{a+bx+cx^2}\,dx$$

$$= \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) - \frac{1}{3}n\int\left(\frac{b^2-2ac}{c^2} - \frac{bx}{c} + 2x^2 - \frac{a(b^2-2ac}{c^2\,(a}\right.$$

$$= -\frac{(b^2-2ac)\,nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{n\int\frac{a(b^2}{}}{}$$

$$= -\frac{(b^2-2ac)\,nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{1}{3}x^3\log\left(d\left(a+bx+cx^2\right)^n\right) + \frac{(b(b^2-}{}$$

$$= -\frac{(b^2-2ac)\,nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{b(b^2-3ac)\,n\log\left(a+bx+cx^2\right)}{6c^3} + \frac{1}{3}x$$

$$= -\frac{(b^2-2ac)\,nx}{3c^2} + \frac{bnx^2}{6c} - \frac{2nx^3}{9} + \frac{\sqrt{b^2-4ac}\,(b^2-ac)\,n\tanh^{-1}\left(\frac{b+}{\sqrt{b^2-}}\right.}{3c^3}$$

Mathematica [A]
time = 0.08, size = 122, normalized size = 0.90

$$\frac{cnx(-6b^2+3bcx-4c(-3a+cx^2)) + 6\sqrt{b^2-4ac}\,(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) + 3b(b^2-3ac)\,n\log(a+x(b+cx)) + 6c^3x^3\log\left(d(a+x(b+cx))^n\right)}{18c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Log[d*(a + b*x + c*x^2)^n],x]

[Out] (c*n*x*(-6*b^2 + 3*b*c*x - 4*c*(-3*a + c*x^2)) + 6*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*b*(b^2 - 3*a*c)*n*Log[a + x*(b + c*x)] + 6*c^3*x^3*Log[d*(a + x*(b + c*x))^n])/(18*c^3)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 870, normalized size = 6.40

method	result
risch	$\frac{x^3 \ln((c x^2+bx+a)^n)}{3} - \frac{i\pi x^3 \operatorname{csgn}(id(c x^2+bx+a)^n)^3}{6} + \frac{i\pi x^3 \operatorname{csgn}(i(c x^2+bx+a)^n)\operatorname{csgn}(id(c x^2+bx+a)^n)^2}{6} + \frac{i\pi x^3 \operatorname{csgn}(id)\operatorname{csg}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*ln((c*x^2+b*x+a)^n)-1/6*I*Pi*x^3*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/6*I*Pi*x^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/6*I*Pi*x^3*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/6*I*Pi*x^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/3*ln(d)*x^3-2/9*n*x^3+1/6*b*n*x^2/c-1/2/c^2*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3-1/2/c^2*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*a*b+1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*b^3+2/3/c*a*n*x-1/3*b^2*n*x/c^2-1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4+2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x+(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)+1/6/c^3*n*ln(-4*a^2*c^2+5*a*b^2*c-b^4-2*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*c*x-(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)*b)*(-4*a^3*c^3+9*a^2*b^2*c^2-6*a*b^4*c+b^6)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 299, normalized size = 2.20

$$\left[-\frac{4\,c^{3}nx^{3}-6\,c^{3}x^{3}\log\left(d\right)-3\,bc^{2}nx^{2}+3\,(b^{2}-ac)\sqrt{b^{2}-4\,ac}\,n\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+6\,(b^{2}c-2\,ac^{2})nx-3\,(2\,c^{3}nx^{3}+(b^{3}-3\,abc)n)\log\left(cx^{2}+bx+a\right)}{18\,c^{3}},-\frac{4\,c^{3}nx^{3}-6\,c^{3}x^{3}\log\left(d\right)-3\,bc^{2}nx^{2}-6\,(b^{2}-ac)\sqrt{-b^{2}+4\,ac}\,n\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+6\,(b^{2}c-2\,ac^{2})nx-3\,(2\,c^{3}nx^{3}+(b^{3}-3\,abc)n)\log\left(cx^{2}+bx+a\right)}{18\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(4*c^3*n*x^3 - 6*c^3*x^3*log(d) - 3*b*c^2*n*x^2 + 3*(b^2 - a*c)*sqrt
(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2*c^3*n*x^3
+ (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3, -1/18*(4*c^3*n*x^3 - 6*c^3*
x^3*log(d) - 3*b*c^2*n*x^2 - 6*(b^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*arctan(-sqr
t(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^2*c - 2*a*c^2)*n*x - 3*(2
*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*log(c*x^2 + b*x + a))/c^3]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.66, size = 146, normalized size = 1.07

$$\frac{1}{3}\,nx^{3}\log\left(cx^{2}+bx+a\right)-\frac{1}{9}\,(2\,n-3\,\log\left(d\right))x^{3}+\frac{bnx^{2}}{6\,c}-\frac{(b^{2}n-2\,acn)x}{3\,c^{2}}+\frac{(b^{3}n-3\,abcn)\log\left(cx^{2}+bx+a\right)}{6\,c^{3}}-\frac{(b^{4}n-5\,ab^{2}cn+4\,a^{2}c^{2}n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{3\,\sqrt{-b^{2}+4\,ac}\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/3*n*x^3*log(c*x^2 + b*x + a) - 1/9*(2*n - 3*log(d))*x^3 + 1/6*b*n*x^2/c -
 1/3*(b^2*n - 2*a*c*n)*x/c^2 + 1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)
/c^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B]
time = 0.50, size = 229, normalized size = 1.68

$$\frac{x^{3}\,\ln\left(d\,{\left(c\,x^{2}+b\,x+a\right)}^{n}\right)}{3}-\frac{2\,n\,x^{3}}{9}-x\,\left(\frac{b^{2}\,n}{3\,c^{2}}-\frac{2\,a\,n}{3\,c}\right)-\frac{\ln\left(4\,a\,c+b\,\sqrt{b^{2}-4\,a\,c}-b^{2}+2\,c\,x\,\sqrt{b^{2}-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,b\,n}{2}-\frac{a\,n\,\sqrt{b^{2}-4\,a\,c}}{6}\right)-\frac{b^{3}\,n}{6}+\frac{b^{2}\,n\,\sqrt{b^{2}-4\,a\,c}}{6}\right)}{c^{3}}+\frac{\ln\left(b\,\sqrt{b^{2}-4\,a\,c}-4\,a\,c+b^{2}+2\,c\,x\,\sqrt{b^{2}-4\,a\,c}\right)\,\left(\frac{b^{3}\,n}{6}-c\,\left(\frac{a\,b\,n}{2}+\frac{a\,n\,\sqrt{b^{2}-4\,a\,c}}{6}\right)+\frac{b^{2}\,n\,\sqrt{b^{2}-4\,a\,c}}{6}\right)}{c^{3}}+\frac{b\,n\,x^{2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*log(d*(a + b*x + c*x^2)^n),x)

[Out] (x^3*log(d*(a + b*x + c*x^2)^n))/3 - (2*n*x^3)/9 - x*((b^2*n)/(3*c^2) - (2*a*n)/(3*c)) - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(c*((a*b*n)/2 - (a*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - c*((a*b*n)/2 + (a*n*(b^2 - 4*a*c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/c^3 + (b*n*x^2)/(6*c)

3.74 $\int x \log \left(d(a+bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=109

$$\frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right.\right.$$

[Out] 1/2*b*n*x/c-1/2*n*x^2-1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a)/c^2+1/2*x^2*ln(d*(c*x^2+b*x+a)^n)-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^2

Rubi [A]

time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{n(b^2-2ac)\log\left(a+bx+cx^2\right)}{4c^2}-\frac{bn\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bnx}{2c}-\frac{nx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*Log[d*(a + b*x + c*x^2)^n],x]

[Out] (b*n*x)/(2*c) - (n*x^2)/2 - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((b^2 - 2*a*c)*n*Log[a + b*x + c*x^2])/(4*c^2) + (x^2*Log[d*(a + b*x + c*x^2)^n])/2

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\frac{x^2(b+2cx)}{a+bx+cx^2}\,dx\\
&= \frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{1}{2}n\int\left(-\frac{b}{c}+2x+\frac{ab+\left(b^2-2ac\right)x}{c\left(a+bx+cx^2\right)}\right)dx\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{ab+\left(b^2-2ac\right)x}{a+bx+cx^2}\,dx}{2c}\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(b\left(b^2-4ac\right)n\right)\int\frac{1}{a+bx+cx^2}\,dx}{4c^2}-\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4c^2}+\frac{1}{2}x^2\log\left(d\left(a+bx+cx^2\right)^n\right)\\
&= \frac{bnx}{2c}-\frac{nx^2}{2}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{\left(b^2-2ac\right)n\log\left(a+\right.}{4c^2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 94, normalized size = 0.86

$$-\frac{2b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+\left(b^2-2ac\right)n\log(a+x(b+cx))-2cx\left(n(b-cx)+cx\log\left(d(a+x(b+cx))^n\right)\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/4*(2*b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + (b^2
 - 2*a*c)*n*Log[a + x*(b + c*x)] - 2*c*x*(n*(b - c*x) + c*x*Log[d*(a + x*(b
 + c*x))^n]))/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 510, normalized size = 4.68

method	result
risch	$\frac{x^2 \ln\left(\left(c\,x^2+bx+a\right)^n\right)}{2}-\frac{i\pi\,x^2 \mathrm{csgn}(id)\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)}{4}+\frac{i\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2\mathrm{csgn}(id)x^2\pi}{4}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln((c*x^2+b*x+a)^n)-1/4*I*Pi*x^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*
csgn(I*d*(c*x^2+b*x+a)^n)+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*d)*x^2*P
i+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*x^2+b*x+a)^n)*x^2*Pi-1/4*I*Pi
*x^2*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/2*ln(d)*x^2-1/2*n*x^2+1/2/c*n*ln(-2*(-4*
a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*a-1/4/c^2*n*ln
(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*b^2+1/
2/c*n*ln(2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)
*a-1/4/c^2*n*ln(2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(
1/2)*b)*b^2+1/2*b*n*x/c+1/4/c^2*n*ln(-2*(-4*a*b^2*c+b^4)^(1/2)*c*x-4*a*b*c+
b^3-(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^(1/2)-1/4/c^2*n*ln(2*(-4*a*b
^2*c+b^4)^(1/2)*c*x-4*a*b*c+b^3+(-4*a*b^2*c+b^4)^(1/2)*b)*(-4*a*b^2*c+b^4)^
(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 245, normalized size = 2.25

$$\left[-\frac{2\,c^2nx^2-2\,c^2x^2\log\left(d\right)-2\,bcnx-\sqrt{b^2-4\,ac}\,bn\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-\left(2\,c^2nx^2-\left(b^2-2\,ac\right)n\right)\log\left(cx^2+bx+a\right)}{4\,c^2},\ -\frac{2\,c^2nx^2-2\,c^2x^2\log\left(d\right)-2\,bcnx+2\,\sqrt{-b^2+4\,ac}\,bn\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-\left(2\,c^2nx^2-\left(b^2-2\,ac\right)n\right)\log\left(cx^2+bx+a\right)}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x - sqrt(b^2 - 4*a*c)*b*n*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x
^2 + b*x + a)) - (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2,
 -1/4*(2*c^2*n*x^2 - 2*c^2*x^2*log(d) - 2*b*c*n*x + 2*sqrt(-b^2 + 4*a*c)*b*
n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c^2*n*x^2 - (b
^2 - 2*a*c)*n)*log(c*x^2 + b*x + a))/c^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(102) = 204$.
time = 114.35, size = 359, normalized size = 3.29

$$\begin{cases} -\frac{a^2\log\left(d(a+bx)^n\right)}{2b^2} + \frac{anx}{2b} - \frac{nx^2}{4} + \frac{x^2\log\left(d(a+bx)^n\right)}{2} & \text{for } c = 0 \\ -\frac{b^2\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{8c^2} + \frac{bnx}{2c} - \frac{nx^2}{2} + \frac{x^2\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2} & \text{for } a = \frac{b^2}{4c} \\ \frac{2abn\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{ab\log\left(d(a+bx+cx^2)^n\right)}{c\sqrt{-4ac+b^2}} + \frac{a\log\left(d(a+bx+cx^2)^n\right)}{2c} - \frac{b^3n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2\sqrt{-4ac+b^2}} + \frac{b^3\log\left(d(a+bx+cx^2)^n\right)}{4c^2\sqrt{-4ac+b^2}} - \frac{b^2\log\left(d(a+bx+cx^2)^n\right)}{4c^2} + \frac{bnx}{2c} - \frac{nx^2}{2} + \frac{x^2\log\left(d(a+bx+cx^2)^n\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((-a**2*log(d*(a + b*x)**n)/(2*b**2) + a*n*x/(2*b) - n*x**2/4 + x*
*2*log(d*(a + b*x)**n)/2, Eq(c, 0)), (-b**2*log(d*(b**2/(4*c) + b*x + c*x**
2)**n)/(8*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*(b**2/(4*c) + b*x + c
*x**2)**n)/2, Eq(a, b**2/(4*c))), (2*a*b*n*log(b/(2*c) + x + sqrt(-4*a*c +
b**2)/(2*c))/(c*sqrt(-4*a*c + b**2)) - a*b*log(d*(a + b*x + c*x**2)**n)/(c*
sqrt(-4*a*c + b**2)) + a*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**3*n*log(b/
(2*c) + x + sqrt(-4*a*c + b**2)/(2*c))/(2*c**2*sqrt(-4*a*c + b**2)) + b**3*
log(d*(a + b*x + c*x**2)**n)/(4*c**2*sqrt(-4*a*c + b**2)) - b**2*log(d*(a +
 b*x + c*x**2)**n)/(4*c**2) + b*n*x/(2*c) - n*x**2/2 + x**2*log(d*(a + b*x
+ c*x**2)**n)/2, True))
```

Giac [A]
time = 3.32, size = 113, normalized size = 1.04

$$\frac{1}{2}\,nx^2\log\left(cx^2+bx+a\right) - \frac{1}{2}\,(n-\log\left(d\right))x^2 + \frac{bnx}{2\,c} - \frac{(b^2n-2\,acn)\log\left(cx^2+bx+a\right)}{4\,c^2} + \frac{(b^3n-4\,abcn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*n*x^2*log(c*x^2 + b*x + a) - 1/2*(n - log(d))*x^2 + 1/2*b*n*x/c - 1/4*(
b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/c^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 0.55, size = 166, normalized size = 1.52

$$\frac{x^2\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{2}-\frac{n\,x^2}{2}-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,n-2\,a\,c\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(2\,a\,c\,n-b^2\,n+b\,n\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2}+\frac{b\,n\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] (x^2*log(d*(a + b*x + c*x^2)^n))/2 - (n*x^2)/2 - (log(b*(b^2 - 4*a*c)^(1/2)
 - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4
*a*c)^(1/2)))/(4*c^2) + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b
^2 - 4*a*c)^(1/2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*c^2) + (
b*n*x)/(2*c)
```

3.75 $\int \log\left(d(a+bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=79

$$-2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2603, 787, 648, 632, 212, 642}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int \frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int \frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int \frac{}{}}{2c}\\
&= -2nx+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(\left(b^2-4ac\right)n\right)}{}\\
&= -2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.99

$$\frac{2\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+bn\log(a+x(b+cx))+2cx(-2n+\log\left(d(a+x(b+cx))^n\right))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A]
time = 0.06, size = 89, normalized size = 1.13

method	result
default	$x \ln \left(d(c\,x^{2}+bx+a)^{n}\right)-n \left(2x-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}\right)$
risch	$x\ln\left((c\,x^{2}+bx+a)^{n}\right)-\frac{i\pi x\,\mathrm{csgn}(id)\mathrm{csgn}\left(i(c\,x^{2}+bx+a)^{n}\right)\mathrm{csgn}\left(id(c\,x^{2}+bx+a)^{n}\right)}{2}+\frac{i\mathrm{csgn}\left(id(c\,x^{2}+bx+a)^{n}\right)^{2}\mathrm{csgn}(id)x\pi}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 190, normalized size = 2.41

$$\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^{2}-4\,ac}\,n\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-(2\,cnx+bn)\log\left(cx^{2}+bx+a\right)}{2\,c},-\frac{4\,cnx-2\,cx\log\left(d\right)-2\,\sqrt{-b^{2}+4\,ac}\,n\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-(2\,cnx+bn)\log\left(cx^{2}+bx+a\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.
time = 45.18, size = 274, normalized size = 3.47

$$\begin{cases} \frac{a\log\left(d(a+bx)^n\right)}{b} - nx + x\log\left(d(a+bx)^n\right) & \text{for } c = 0 \\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c} - 2nx + x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right) & \text{for } a = \frac{b^2}{4c} \\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}} + \frac{2a\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{-4ac+b^2}} + \frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{b^2\log\left(d(a+bx+cx^2)^n\right)}{2c\sqrt{-4ac+b^2}} + \frac{b\log\left(d(a+bx+cx^2)^n\right)}{2c} - 2nx + x\log\left(d(a+bx+cx^2)^n\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0))
, (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/(4
*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + sqrt
(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**2)**
n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sqrt(-4*
a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(a +
b*x + c*x**2)**n), True))
```

Giac [A]
time = 4.33, size = 92, normalized size = 1.16

$$nx\log\left(cx^2+bx+a\right) - (2\,n - \log(d))x + \frac{bn\log\left(cx^2+bx+a\right)}{2\,c} - \frac{\left(b^2n - 4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

Mupad [B]
time = 0.34, size = 120, normalized size = 1.52

$$x\,\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right) - 2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)} - \frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

3.76 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}\,dx$

Optimal. Leaf size=129

$$-n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)+\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)-n$$

```
[Out] ln(x)*ln(d*(c*x^2+b*x+a)^n)-n*ln(x)*ln(1+2*c*x/(b-(-4*a*c+b^2)^(1/2)))-n*ln
(x)*ln(1+2*c*x/(b+(-4*a*c+b^2)^(1/2)))-n*polylog(2,-2*c*x/(b-(-4*a*c+b^2)^(
1/2)))-n*polylog(2,-2*c*x/(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2604, 2404, 2354, 2438}

$$-n\text{PolyLog}\left(2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)-n\text{PolyLog}\left(2,-\frac{2cx}{\sqrt{b^2-4ac}+b}\right)-n\log(x)\log\left(\frac{2cx}{b-\sqrt{b^2-4ac}}+1\right)-n\log(x)\log\left(\frac{2cx}{\sqrt{b^2-4ac}+b}+1\right)+\log(x)\log\left(d\left(a+bx+cx^2\right)^n\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[1 + (2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) - n*Log[x]*Log[1 + (2*
c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log[x]*Log[d*(a + b*x + c*x^2)^n] - n*PolyL
og[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b
^2 - 4*a*c])]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x}\,dx = \log(x)\log\left(d(a+bx+cx^2)^n\right) - n\int \frac{(b+2cx)\log(x)}{a+bx+cx^2}\,dx$$
$$= \log(x)\log\left(d(a+bx+cx^2)^n\right) - n\int \left(\frac{2c\log(x)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log}{b+\sqrt{b^2-}}\right.$$
$$= \log(x)\log\left(d(a+bx+cx^2)^n\right) - (2cn)\int \frac{\log(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx - (2cn)$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)$$
$$= -n\log(x)\log\left(1+\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(1+\frac{2cx}{b+\sqrt{b^2-4ac}}\right)$$

Mathematica [A]
time = 0.12, size = 156, normalized size = 1.21

$$-n\log(x)\log\left(\frac{b-\sqrt{b^2-4ac}+2cx}{b-\sqrt{b^2-4ac}}\right) - n\log(x)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{b+\sqrt{b^2-4ac}}\right) + \log(x)\log\left(d(a+x(b+cx))^n\right) - n\mathrm{Li}_2\left(-\frac{2cx}{b-\sqrt{b^2-4ac}}\right) - n\mathrm{Li}_2\left(-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x,x]
```

```
[Out] -(n*Log[x]*Log[(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/(b - Sqrt[b^2 - 4*a*c])]) -
n*Log[x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(b + Sqrt[b^2 - 4*a*c])] + Log
[x]*Log[d*(a + x*(b + c*x))^n] - n*PolyLog[2, (-2*c*x)/(b - Sqrt[b^2 - 4*a*
c])] - n*PolyLog[2, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 315, normalized size = 2.44

method	result
risch	$\ln(x)\ln\left((cx^2+bx+a)^n\right) - \ln\left(\frac{-b-2cx+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}\right)\ln(x)n - \ln\left(\frac{b+2cx+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}\right)\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((c*x^2+b*x+a)^n)-ln((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)
^(1/2)))*ln(x)*n-ln((b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))*ln
(x)*n-dilog((-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2)))*n-dilog(
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))*n-1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/2*I*ln(x)*Pi*csgn(
I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/2*I*ln(x)*Pi*csgn(I*(c*x^2+b*x+a)^n)*csg
n(I*d*(c*x^2+b*x+a)^n)^2-1/2*I*ln(x)*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(x)*l
n(d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x,x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x,x)
[Out] int(log(d*(a + b*x + c*x^2)^n)/x, x)
```

3.77 $\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx$

Optimal. Leaf size=86

$$\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}+\frac{bn\log(x)}{a}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}$$

```
[Out] b*n*ln(x)/a-1/2*b*n*ln(c*x^2+b*x+a)/a-ln(d*(c*x^2+b*x+a)^n)/x+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a
```

Rubi [A]

time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x}-\frac{bn\log\left(a+bx+cx^2\right)}{2a}+\frac{bn\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/a + (b*n*Log[x
])/a - (b*n*Log[a + b*x + c*x^2])/(2*a) - Log[d*(a + b*x + c*x^2)^n]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^2}\,dx &= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + n\int \frac{b+2cx}{x\,(a+bx+cx^2)}\,dx \\
&= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + n\int \left(\frac{b}{ax} + \frac{-b^2+2ac-bcx}{a\,(a+bx+cx^2)}\right)dx \\
&= \frac{bn\log(x)}{a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + \frac{n\int \frac{-b^2+2ac-bcx}{a+bx+cx^2}\,dx}{a} \\
&= \frac{bn\log(x)}{a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} - \frac{(bn)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a} - \frac{((b^2-4ac)\,n)\int}{2a} \\
&= \frac{bn\log(x)}{a} - \frac{bn\log\left(a+bx+cx^2\right)}{2a} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + \frac{((b^2-4ac)\,n}{} \\
&= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a} + \frac{bn\log(x)}{a} - \frac{bn\log\left(a+bx+cx^2\right)}{2a} -
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 87, normalized size = 1.01

$$\frac{2\sqrt{-b^2+4ac}\,n\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right) + 2bn\log(x) - bn\log(a+x(b+cx)) - \frac{2a\log(d(a+x(b+cx))^n)}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^2,x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*b*n*Log[
x] - b*n*Log[a + x*(b + c*x)] - (2*a*Log[d*(a + x*(b + c*x))^n])/x)/(2*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 261, normalized size = 3.03

method	result
risch	$-\frac{\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{x}+\frac{i\pi a\,\mathrm{csgn}(id)\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)-i\pi a\,\mathrm{csgn}(id)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}-i\pi a\,\mathrm{csgn}(\ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln((c*x^2+b*x+a)^n)+1/2*(I*Pi*a*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn
(I*d*(c*x^2+b*x+a)^n)-I*Pi*a*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a*c
sgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+I*Pi*a*csgn(I*d*(c*x^2+b
*x+a)^n)^3+2*b*n*ln(x)*x+2*sum(_R*ln(((6*a*c-2*b^2)*_R^2+_R*b*c*n+4*c^2*n^2
)*x-a*b*_R^2+(-2*a*c*n+b^2*n)*_R+2*b*c*n^2),_R=RootOf(_Z^2*a+_Z*b*n+c*n^2))
*a*x-2*ln(d)*a)/a/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 199, normalized size = 2.31

$$\left[\frac{2\,bnx\log\left(x\right)+\sqrt{b^{2}-4\,ac}\,nx\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-(bnx+2\,an)\log\left(cx^{2}+bx+a\right)-2\,a\log\left(d\right)}{2\,ax},\ \frac{2\,bnx\log\left(x\right)+2\,\sqrt{-b^{2}+4\,ac}\,nx\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-(bnx+2\,an)\log\left(cx^{2}+bx+a\right)-2\,a\log\left(d\right)}{2\,ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*n*x*log(x) + sqrt(b^2 - 4*a*c)*n*x*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b*n*x + 2*a
*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x), 1/2*(2*b*n*x*log(x) + 2*sqrt(
-b^2 + 4*a*c)*n*x*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
b*n*x + 2*a*n)*log(c*x^2 + b*x + a) - 2*a*log(d))/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(78) = 156$.
time = 198.61, size = 211, normalized size = 2.45

$$\begin{cases} -\frac{n}{x} - \frac{\log\left(d(bx)^n\right)}{x} & \text{for}\ a = 0 \wedge c = 0 \\ -\frac{n}{x} - \frac{\log\left(d(bx+cx^2)^n\right)}{x} - \frac{2cn\log(b+cx)}{b} + \frac{c\log\left(d(bx+cx^2)^n\right)}{b} & \text{for}\ a = 0 \\ -\frac{\log\left(d(a+bx)^n\right)}{x} + \frac{bn\log(x)}{a} - \frac{b\log\left(d(a+bx)^n\right)}{a} & \text{for}\ c = 0 \\ -\frac{\log\left(d(a+bx+cx^2)^n\right)}{x} + \frac{bn\log(x)}{a} - \frac{b\log\left(d(a+bx+cx^2)^n\right)}{2a} + \frac{n\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{a} - \frac{\sqrt{-4ac+b^2}\log\left(d(a+bx+cx^2)^n\right)}{2a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**2,x)
```

```
[Out] Piecewise((-n/x - log(d*(b*x)**n)/x, Eq(a, 0) & Eq(c, 0)), (-n/x - log(d*(b
*x + c*x**2)**n)/x - 2*c*n*log(b + c*x)/b + c*log(d*(b*x + c*x**2)**n)/b, E
q(a, 0)), (-log(d*(a + b*x)**n)/x + b*n*log(x)/a - b*log(d*(a + b*x)**n)/a,
 Eq(c, 0)), (-log(d*(a + b*x + c*x**2)**n)/x + b*n*log(x)/a - b*log(d*(a +
b*x + c*x**2)**n)/(2*a) + n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a
*c + b**2)/(2*c))/a - sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(2*a
), True))
```

Giac [A]
time = 3.14, size = 99, normalized size = 1.15

$$-\frac{bn\log\left(cx^2+bx+a\right)}{2\,a} + \frac{bn\log(x)}{a} - \frac{n\log\left(cx^2+bx+a\right)}{x} - \frac{\left(b^2n-4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a} - \frac{\log(d)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*n*log(c*x^2 + b*x + a)/a + b*n*log(x)/a - n*log(c*x^2 + b*x + a)/x -
 (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - log(d)/x
```

Mupad [B]
time = 0.93, size = 262, normalized size = 3.05

$$\frac{b\,n\,\ln(x)}{a} - \frac{\ln\left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\left(b-\sqrt{b^2-4\,a\,c}\right)\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\left(b\,n-n\,\sqrt{b^2-4\,a\,c}\right)}{2\,a} - \frac{\ln\left(2\,b\,c^2\,n^2+4\,c^3\,n^2\,x-\frac{n\left(b+\sqrt{b^2-4\,a\,c}\right)\left(b^2\,c\,n-2\,a\,c^2\,n+b\,c^2\,n\,x+\frac{c\,n\left(b+\sqrt{b^2-4\,a\,c}\right)\left(2\,x\,b^2+a\,b-6\,a\,c\,x\right)}{2\,a}\right)}{2\,a}\right)\left(b\,n+n\,\sqrt{b^2-4\,a\,c}\right)}{2\,a} - \frac{\ln\left(d\,\left(c\,x^2+b\,x+a\right)^n\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^2,x)
```

```
[Out] (b*n*log(x))/a - (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b - (b^2 - 4*a*c)^(1/
2))*(b^2*c*n - 2*a*c^2*n + b*c^2*n*x + (c*n*(b - (b^2 - 4*a*c)^(1/2))*(a*b
+ 2*b^2*x - 6*a*c*x))/(2*a)))/(2*a))*(b*n - n*(b^2 - 4*a*c)^(1/2)))/(2*a) -
 (log(2*b*c^2*n^2 + 4*c^3*n^2*x - (n*(b + (b^2 - 4*a*c)^(1/2))*(b^2*c*n - 2
*a*c^2*n + b*c^2*n*x + (c*n*(b + (b^2 - 4*a*c)^(1/2))*(a*b + 2*b^2*x - 6*a*
c*x))/(2*a)))/(2*a))*(b*n + n*(b^2 - 4*a*c)^(1/2)))/(2*a) - log(d*(a + b*x
+ c*x^2)^n)/x
```

3.78 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^3}\,dx$

Optimal. Leaf size=121

$$-\frac{bn}{2ax}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{(b^2-2ac)\,n\log(x)}{2a^2}+\frac{(b^2-2ac)\,n\log\left(a+bx+cx^2\right)}{4a^2}-\frac{\log\left(d\right.}{}$$

```
[Out] -1/2*b*n/a/x-1/2*(-2*a*c+b^2)*n*ln(x)/a^2+1/4*(-2*a*c+b^2)*n*ln(c*x^2+b*x+a
)/a^2-1/2*ln(d*(c*x^2+b*x+a)^n)/x^2-1/2*b*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))*(-4*a*c+b^2)^(1/2)/a^2
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n(b^2-2ac)\log\left(a+bx+cx^2\right)}{4a^2}-\frac{n\log(x)\left(b^2-2ac\right)}{2a^2}-\frac{bn\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2x^2}-\frac{bn}{2ax}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/2*(b*n)/(a*x) - (b*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(2*a^2) - ((b^2 - 2*a*c)*n*Log[x])/(2*a^2) + ((b^2 - 2*a*c)*n*Log[a
+ b*x + c*x^2])/(4*a^2) - Log[d*(a + b*x + c*x^2)^n]/(2*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^3}\,dx &= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}+\frac{1}{2}n\int \frac{b+2cx}{x^2\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}+\frac{1}{2}n\int \left(\frac{b}{ax^2}+\frac{-b^2+2ac}{a^2x}+\frac{b(b^2-3ac)+c(b^2-\cdots}{a^2\left(a+bx+cx\cdots\right.}\right.\\
&= -\frac{bn}{2ax}-\frac{\left(b^2-2ac\right)n\log(x)}{2a^2}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}+\frac{n\int \frac{b(b^2-3ac)+c(b^2-2a\cdots}{a+bx+cx^2}}{2a^2}\cdots\\
&= -\frac{bn}{2ax}-\frac{\left(b^2-2ac\right)n\log(x)}{2a^2}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{2x^2}+\frac{\left(b(b^2-4ac)n\right)\int \frac{\cdots}{a+\cdots}}{4a^2}\cdots\\
&= -\frac{bn}{2ax}-\frac{\left(b^2-2ac\right)n\log(x)}{2a^2}+\frac{\left(b^2-2ac\right)n\log\left(a+bx+cx^2\right)}{4a^2}-\frac{\log\left(d(a+b\cdots\right.}{2x\cdots}\\
&= -\frac{bn}{2ax}-\frac{b\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2a^2}-\frac{\left(b^2-2ac\right)n\log(x)}{2a^2}+\frac{\left(b^2-\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 105, normalized size = 0.87

$$-\frac{\frac{nx\left(2ab+2b\sqrt{b^2-4ac}\,x\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+2(b^2-2ac)x\log(x)-(b^2-2ac)x\log(a+x(b+cx))\right)}{a^2}+2\log\left(d(a+x(b+cx))^n\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^3,x]
```

```
[Out] -1/4*((n*x*(2*a*b + 2*b*Sqrt[b^2 - 4*a*c]*x*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]] + 2*(b^2 - 2*a*c)*x*Log[x] - (b^2 - 2*a*c)*x*Log[a + x*(b + c*x)]))
/a^2 + 2*Log[d*(a + x*(b + c*x))^n])/x^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 1178, normalized size = 9.74

method	result
risch	$-\frac{\ln\left((c x^2+bx+a)^n\right)}{2x^2}-\frac{-i\pi a^2\mathrm{csgn}(id)\mathrm{csgn}\left(i(c x^2+bx+a)^n\right)\mathrm{csgn}\left(id(c x^2+bx+a)^n\right)+i\pi a^2\mathrm{csgn}(id)\mathrm{csgn}\left(id(c x^2+bx+a)^n\right)^2+i\pi a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln((c*x^2+b*x+a)^n)-1/4*(-I*Pi*a^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^2*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^2*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^2*csgn
(I*d*(c*x^2+b*x+a)^n)^3+2*n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^
2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^
(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2
*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4
*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c-
2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5+5*(-4*a*b
^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*b^2*x^2+2*n*ln((-16
*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*
c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*
a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3
)*c*a*x^2-n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*b^2*c+b^4)^(1/2)*a
^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*
a^3*b*c^2+11*b^3*c*a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c+2*(-4*a*b^2
*c+b^4)^(1/2)*a*b^3)*b^2*x^2-4*n*ln(x)*c*a*x^2+2*n*ln(x)*b^2*x^2-n*ln((-16*
a^2*b^2*c^2+12*a*b^4*c-2*b^6-6*(-4*a*b^2*c+b^4)^(1/2)*a^2*c^2+8*(-4*a*b^2*c
+b^4)^(1/2)*a*b^2*c-2*(-4*a*b^2*c+b^4)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a
^2-2*a*b^5+5*(-4*a*b^2*c+b^4)^(1/2)*a^2*b*c-2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)
*(-4*a*b^2*c+b^4)^(1/2)*x^2+n*ln((-16*a^2*b^2*c^2+12*a*b^4*c-2*b^6+6*(-4*a*
b^2*c+b^4)^(1/2)*a^2*c^2-8*(-4*a*b^2*c+b^4)^(1/2)*a*b^2*c+2*(-4*a*b^2*c+b^4
```

```
)^(1/2)*b^4)*x-12*a^3*b*c^2+11*b^3*c*a^2-2*a*b^5-5*(-4*a*b^2*c+b^4)^(1/2)*a
^2*b*c+2*(-4*a*b^2*c+b^4)^(1/2)*a*b^3)*(-4*a*b^2*c+b^4)^(1/2)*x^2+2*a*b*n*x
+2*ln(d)*a^2)/a^2/x^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.37, size = 261, normalized size = 2.16

$$\left[\frac{\sqrt{b^2-4\,ac}\,bnx^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2-2\,ac)nx^2\log\left(x\right)-2\,abnx-2\,a^2\log\left(d\right)+((b^2-2\,ac)nx^2-2\,a^2n)\log\left(cx^2+bx+a\right)}{4\,a^2x^2},-\frac{2\,\sqrt{-b^2+4\,ac}\,bnx^2\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2-2\,ac)nx^2\log\left(x\right)+2\,abnx+2\,a^2\log\left(d\right)-((b^2-2\,ac)nx^2-2\,a^2n)\log\left(cx^2+bx+a\right)}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*n*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2 - 2*a*c)*n*x^2*log
(x) - 2*a*b*n*x - 2*a^2*log(d) + ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2
+ b*x + a))/(a^2*x^2), -1/4*(2*sqrt(-b^2 + 4*a*c)*b*n*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2 - 2*a*c)*n*x^2*log(x) + 2*a*b
*n*x + 2*a^2*log(d) - ((b^2 - 2*a*c)*n*x^2 - 2*a^2*n)*log(c*x^2 + b*x + a))
/(a^2*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.15, size = 129, normalized size = 1.07

$$\frac{(b^2n-2\,acn)\log\left(cx^2+bx+a\right)}{4\,a^2}-\frac{n\log\left(cx^2+bx+a\right)}{2\,x^2}-\frac{(b^2n-2\,acn)\log\left(x\right)}{2\,a^2}+\frac{(b^3n-4\,abcn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{bnx+a\log\left(d\right)}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*n - 2*a*c*n)*log(c*x^2 + b*x + a)/a^2 - 1/2*n*log(c*x^2 + b*x + a)
/x^2 - 1/2*(b^2*n - 2*a*c*n)*log(x)/a^2 + 1/2*(b^3*n - 4*a*b*c*n)*arctan((2
*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/2*(b*n*x + a*log
(d))/(a*x^2)
```

Mupad [B]
time = 0.76, size = 474, normalized size = 3.92

$$\frac{\ln\left(\frac{b^3c^2n^2-2abc^3n^2}{4a^2}+\frac{\left(b^2n-2acn+bn\sqrt{b^2-4ac}\right)\left(\frac{\left(\frac{x\left(24a^3c^2-8a^2b^2c\right)}{4a^2}-abc\right)\left(b^2n-2acn+bn\sqrt{b^2-4ac}\right)}{4a^2}-\frac{2ab^3cn-6a^2bc^2n}{4a^2}+\frac{x\left(12a^2c^3n-4ab^2c^2n\right)}{4a^2}\right)}{4a^2}+\frac{b^2c^3n^2x}{4a^2}\right)\left(b^2n-2acn+bn\sqrt{b^2-4ac}\right)}{4a^2}-\frac{\ln(x)\,\left(b^2n-2acn\right)}{2a^2}-\frac{\ln\left(d\left(cx^2+bx+a\right)^n\right)}{2x^2}-\frac{\ln\left(\frac{b^3c^2n^2-2abc^3n^2}{4a^2}+\frac{\left(2acn-b^2n+bn\sqrt{b^2-4ac}\right)\left(\frac{2ab^3cn-6a^2bc^2n}{4a^2}+\frac{\left(\frac{x\left(24a^3c^2-8a^2b^2c\right)}{4a^2}-abc\right)\left(2acn-b^2n+bn\sqrt{b^2-4ac}\right)}{4a^2}-\frac{x\left(12a^2c^3n-4ab^2c^2n\right)}{4a^2}\right)}{4a^2}+\frac{b^2c^3n^2x}{4a^2}\right)\left(2acn-b^2n+bn\sqrt{b^2-4ac}\right)}{4a^2}-\frac{bn}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^3,x)
```

```
[Out] (log((b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((b^2*n - 2*a*c*n + b*n*(b^2 -
 4*a*c)^(1/2))*((((x*(24*a^3*c^2 - 8*a^2*b^2*c))/(4*a^2) - a*b*c)*(b^2*n -
2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (2*a*b^3*c*n - 6*a^2*b*c^2*n)
/(4*a^2) + (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*
n^2*x)/(4*a^2))*(b^2*n - 2*a*c*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (log
(x)*(b^2*n - 2*a*c*n))/(2*a^2) - log(d*(a + b*x + c*x^2)^n)/(2*x^2) - (log(
(b^3*c^2*n^2 - 2*a*b*c^3*n^2)/(4*a^2) + ((2*a*c*n - b^2*n + b*n*(b^2 - 4*a*
c)^(1/2))*((2*a*b^3*c*n - 6*a^2*b*c^2*n)/(4*a^2) + (((x*(24*a^3*c^2 - 8*a^2
*b^2*c))/(4*a^2) - a*b*c)*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a
^2) - (x*(12*a^2*c^3*n - 4*a*b^2*c^2*n))/(4*a^2)))/(4*a^2) + (b^2*c^3*n^2*x
)/(4*a^2))*(2*a*c*n - b^2*n + b*n*(b^2 - 4*a*c)^(1/2)))/(4*a^2) - (b*n)/(2*
a*x)
```

3.79 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{\sqrt{b^2-4ac}\,(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{b(b^2-3ac)\,n\log}{6a}$$

```
[Out] -1/6*b*n/a/x^2+1/3*(-2*a*c+b^2)*n/a^2/x+1/3*b*(-3*a*c+b^2)*n*ln(x)/a^3-1/6*
b*(-3*a*c+b^2)*n*ln(c*x^2+b*x+a)/a^3-1/3*ln(d*(c*x^2+b*x+a)^n)/x^3+1/3*(-a*
c+b^2)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^3
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{bn(b^2-3ac)\log\left(a+bx+cx^2\right)}{6a^3}+\frac{bn\log(x)\left(b^2-3ac\right)}{3a^3}+\frac{n\sqrt{b^2-4ac}\,(b^2-ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{n(b^2-2ac)}{3a^2x}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3x^3}-\frac{bn}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/6*(b*n)/(a*x^2) + ((b^2 - 2*a*c)*n)/(3*a^2*x) + (Sqrt[b^2 - 4*a*c]*(b^2
 - a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*a^3) + (b*(b^2 - 3*a*c)
*n*Log[x])/(3*a^3) - (b*(b^2 - 3*a*c)*n*Log[a + b*x + c*x^2])/(6*a^3) - Log
[d*(a + b*x + c*x^2)^n]/(3*x^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^4}\,dx &= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\frac{b+2cx}{x^3\left(a+bx+cx^2\right)}\,dx\\
&= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{1}{3}n\int\left(\frac{b}{ax^3}+\frac{-b^2+2ac}{a^2x^2}+\frac{b^3-3abc}{a^3x}+\frac{-b^4+
\end{aligned}$$

$$= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}+\frac{n}{}$$

$$= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{\log\left(d(a+bx+cx^2)^n\right)}{3x^3}-\frac{(}{}$$

$$= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{b(b^2-3ac)\,n\log(x)}{3a^3}-\frac{b(b^2-3ac)\,n\log\left(a+bx+\right.}{6a^3}$$

$$= -\frac{bn}{6ax^2}+\frac{(b^2-2ac)\,n}{3a^2x}+\frac{\sqrt{b^2-4ac}\,(b^2-ac)\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3a^3}+\frac{b}{}$$

Mathematica [A]

time = 0.26, size = 132, normalized size = 0.89

$$-\frac{\frac{nx\left(a^2b-2a(b^2-2ac)x-2\sqrt{b^2-4ac}\,(b^2-ac)x^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2b(b^2-3ac)x^2\log(x)+b(b^2-3ac)x^2\log(a+x(b+cx))\right)}{a^3}+2\log\left(d(a+x(b+cx))^n\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^4,x]
```

```
[Out] -1/6*((n*x*(a^2*b - 2*a*(b^2 - 2*a*c)*x - 2*Sqrt[b^2 - 4*a*c]*(b^2 - a*c)*x
^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*b*(b^2 - 3*a*c)*x^2*Log[x] +
b*(b^2 - 3*a*c)*x^2*Log[a + x*(b + c*x)]))/a^3 + 2*Log[d*(a + x*(b + c*x))^
n])/x^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 423, normalized size = 2.84

method	result
risch	$-\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{3x^3}-\frac{-i\pi\,a^3\mathrm{csgn}(id)\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)+i\pi\,a^3\mathrm{csgn}(id)\mathrm{csgn}\left(id\left(c\,x^2+bx+a\right)^n\right)^2+i\pi\,a^3\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3*ln((c*x^2+b*x+a)^n)-1/6*(-I*Pi*a^3*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a^3*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2
+I*Pi*a^3*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a^3*csgn
(I*d*(c*x^2+b*x+a)^n)^3+6*ln(x)*a*b*c*n*x^3-2*ln(x)*b^3*n*x^3-2*sum(_R*ln((
(6*a^5*c-2*a^4*b^2)*_R^2+(-7*a^3*b*c^2*n+2*a^2*b^3*c*n)*_R+4*a^2*c^4*n^2-4*
a*b^2*c^3*n^2+b^4*c^2*n^2)*x-a^5*b*_R^2+(2*a^4*c^2*n-4*a^3*b^2*c*n+a^2*b^4*
n)*_R+6*a^2*b*c^3*n^2-5*a*b^3*c^2*n^2+b^5*c*n^2),_R=RootOf(a^3*_Z^2+(-3*a*b
*c*n+b^3*n)*_Z+c^3*n^2))*a^3*x^3+4*a^2*c*n*x^2-2*a*b^2*n*x^2+a^2*b*n*x+2*ln
(d)*a^3)/a^3/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 318, normalized size = 2.13

$$\left[-\frac{(b^2-ac)\sqrt{b^2-4ac}\,nx^3\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}\,(2cx+b)}{cx^2+bx+a}\right)-2\,(b^3-3abc)nx^3\log(x)+a^2bnx-2\,(ab^2-2a^2c)nx^2+2a^3\log(d)+((b^3-3abc)nx^3+2a^3n)\log(cx^2+bx+a)}{6a^3x^3},\ \frac{2\,(b^2-ac)\sqrt{-b^2+4ac}\,nx^3\arctan\left(-\frac{\sqrt{-b^2+4ac}\,(2cx+b)}{b^2-4ac}\right)+2\,(b^3-3abc)nx^3\log(x)-a^2bnx+2\,(ab^2-2a^2c)nx^2-2a^3\log(d)-((b^3-3abc)nx^3+2a^3n)\log(cx^2+bx+a)}{6a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - a*c)*sqrt(b^2 - 4*a*c)*n*x^3*log((2*c^2*x^2 + 2*b*c*x + b^2 -
 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3 - 3*a*b
*c)*n*x^3*log(x) + a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2 + 2*a^3*log(d) + (
(b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 + b*x + a))/(a^3*x^3), 1/6*(2*(b
^2 - a*c)*sqrt(-b^2 + 4*a*c)*n*x^3*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) + 2*(b^3 - 3*a*b*c)*n*x^3*log(x) - a^2*b*n*x + 2*(a*b^2 - 2*a
^2*c)*n*x^2 - 2*a^3*log(d) - ((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*log(c*x^2 +
b*x + a))/(a^3*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.82, size = 164, normalized size = 1.10

$$-\frac{(b^3n-3\,abcn)\log(cx^2+bx+a)}{6\,a^3}-\frac{n\log(cx^2+bx+a)}{3\,x^3}+\frac{(b^3n-3\,abcn)\log(x)}{3\,a^3}-\frac{(b^4n-5\,ab^2cn+4\,a^2c^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^3}+\frac{2\,b^2nx^2-4\,acnx^2-abnx-2\,a^2\log(d)}{6\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*n - 3*a*b*c*n)*log(c*x^2 + b*x + a)/a^3 - 1/3*n*log(c*x^2 + b*x +
 a)/x^3 + 1/3*(b^3*n - 3*a*b*c*n)*log(x)/a^3 - 1/3*(b^4*n - 5*a*b^2*c*n + 4
*a^2*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3)
 + 1/6*(2*b^2*n*x^2 - 4*a*c*n*x^2 - a*b*n*x - 2*a^2*log(d))/(a^2*x^3)
```

Mupad [B]
time = 0.93, size = 505, normalized size = 3.39

$$\frac{\ln\left(2ab^4\sqrt{b^2-4ac}-2b^6x-2ab^5+2b^5x\sqrt{b^2-4ac}+13a^2b^3c-20a^3bc^2+4a^3c^3x+2a^3c^2\sqrt{b^2-4ac}-25a^2b^2c^2x+14ab^4cx-7a^2b^2c\sqrt{b^2-4ac}-10ab^3cx\sqrt{b^2-4ac}+11a^2bc^2x\sqrt{b^2-4ac}\right)\left(a\left(\frac{bcn}{2}-\frac{cn\sqrt{b^2-4ac}}{6}\right)-\frac{b^3n}{6}+\frac{b^2n\sqrt{b^2-4ac}}{6}\right)}{a^3}-\frac{\ln\left(d\,(cx^2+bx+a)^n\right)}{3x^3}-\frac{\frac{bn}{2a}+\frac{nx\,(2ac-b^2)}{a^2}}{3x^2}-\frac{\ln\left(2ab^5+2b^6x+2ab^4\sqrt{b^2-4ac}+2b^5x\sqrt{b^2-4ac}-13a^2b^3c+20a^3bc^2-4a^3c^3x+2a^3c^2\sqrt{b^2-4ac}+25a^2b^2c^2x-14ab^4cx-7a^2b^2c\sqrt{b^2-4ac}-10ab^3cx\sqrt{b^2-4ac}+11a^2bc^2x\sqrt{b^2-4ac}\right)\left(\frac{b^3n}{6}-a\left(\frac{bcn}{2}+\frac{cn\sqrt{b^2-4ac}}{6}\right)+\frac{b^2n\sqrt{b^2-4ac}}{6}\right)}{a^3}+\frac{\ln(x)\,(b^3n-3abcn)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^4,x)
```

```
[Out] (log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 13*a^2*b^3*c - 20*a^3*b*c^2 + 4*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*
c)^(1/2) - 25*a^2*b^2*c^2*x + 14*a*b^4*c*x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*
(a*((b*c*n)/2 - (c*n*(b^2 - 4*a*c)^(1/2))/6) - (b^3*n)/6 + (b^2*n*(b^2 - 4*
a*c)^(1/2))/6))/a^3 - log(d*(a + b*x + c*x^2)^n)/(3*x^3) - ((b*n)/(2*a) + (
n*x*(2*a*c - b^2))/a^2)/(3*x^2) - (log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 - 4
*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b^3*c + 20*a^3*b*c^2 - 4
*a^3*c^3*x + 2*a^3*c^2*(b^2 - 4*a*c)^(1/2) + 25*a^2*b^2*c^2*x - 14*a*b^4*c*
x - 7*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 10*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 11
*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*n)/6 - a*((b*c*n)/2 + (c*n*(b^2 - 4
*a*c)^(1/2))/6) + (b^2*n*(b^2 - 4*a*c)^(1/2))/6))/a^3 + (log(x)*(b^3*n - 3*
a*b*c*n))/(3*a^3)
```

3.80 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{x^5}\,dx$

Optimal. Leaf size=190

$$-\frac{bn}{12ax^3}+\frac{\left(b^2-2ac\right)n}{8a^2x^2}-\frac{b\left(b^2-3ac\right)n}{4a^3x}-\frac{b\sqrt{b^2-4ac}\,\left(b^2-2ac\right)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4}-\frac{\left(b^4-4ab^2c+2a^2\right.}{4a^4}$$

```
[Out] -1/12*b*n/a/x^3+1/8*(-2*a*c+b^2)*n/a^2/x^2-1/4*b*(-3*a*c+b^2)*n/a^3/x-1/4*(
2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(x)/a^4+1/8*(2*a^2*c^2-4*a*b^2*c+b^4)*n*ln(c*x
^2+b*x+a)/a^4-1/4*ln(d*(c*x^2+b*x+a)^n)/x^4-1/4*b*(-2*a*c+b^2)*n*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a^4
```

Rubi [A]
time = 0.15, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{bn\sqrt{b^2-4ac}\,\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4a^4}-\frac{bn\left(b^2-3ac\right)}{4a^3x}+\frac{n\left(b^2-2ac\right)}{8a^2x^2}+\frac{n\left(2a^2c^2-4ab^2c+b^4\right)\log\left(a+bx+cx^2\right)}{8a^4}-\frac{n\log(x)\left(2a^2c^2-4ab^2c+b^4\right)}{4a^4}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4x^4}-\frac{bn}{12ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/12*(b*n)/(a*x^3) + ((b^2 - 2*a*c)*n)/(8*a^2*x^2) - (b*(b^2 - 3*a*c)*n)/(
4*a^3*x) - (b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(4*a^4) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[x])/(4*a^4) + (
(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*Log[a + b*x + c*x^2])/(8*a^4) - Log[d*(a +
b*x + c*x^2)^n]/(4*x^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{x^5}\,dx = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\frac{b+2cx}{x^4\left(a+bx+cx^2\right)}\,dx$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4x^4}+\frac{1}{4}n\int\left(\frac{b}{ax^4}+\frac{-b^2+2ac}{a^2x^3}+\frac{b^3-3abc}{a^3x^2}+\frac{-b^4+4\right.$$

$$= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}-\frac{\text{lo}}{}$$

$$= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}-\frac{\text{lo}}{}$$

$$= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\,n\log(x)}{4a^4}+\frac{(b}{}$$

$$= -\frac{bn}{12ax^3}+\frac{(b^2-2ac)\,n}{8a^2x^2}-\frac{b(b^2-3ac)\,n}{4a^3x}-\frac{b\sqrt{b^2-4ac}\,(b^2-2ac)\,n\tanh^{-1}\left(\right.}{4a^4}$$

Mathematica [A]
time = 0.33, size = 172, normalized size = 0.91

$$-\frac{\frac{nx\left(2a^3b-3a^2(b^2-2ac)x+6ab(b^2-3ac)x^2+6b\sqrt{b^2-4ac}\,(b^2-2ac)x^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+6(b^4-4ab^2c+2a^2c^2)x^3\log(x)-3(b^4-4ab^2c+2a^2c^2)x^3\log(a+x(b+cx))\right)}{a^4}+6\log\left(d(a+x(b+cx))^n\right)}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/x^5,x]
```

```
[Out] -1/24*((n*x*(2*a^3*b - 3*a^2*(b^2 - 2*a*c)*x + 6*a*b*(b^2 - 3*a*c)*x^2 + 6*
b*Sqrt[b^2 - 4*a*c]*(b^2 - 2*a*c)*x^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
] + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*Log[x] - 3*(b^4 - 4*a*b^2*c + 2*a^2
*c^2)*x^3*Log[a + x*(b + c*x)]))/a^4 + 6*Log[d*(a + x*(b + c*x))^n])/x^4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 505, normalized size = 2.66

method	result
risch	$-\frac{\ln\left(\left(c\,x^2+bx+a\right)^n\right)}{4x^4}-\frac{12\ln(x)a^2c^2n\,x^4-24\ln(x)a\,b^2cn\,x^4+6\ln(x)b^4n\,x^4-3i\pi\,a^4\mathrm{csgn}(id)\mathrm{csgn}\left(i\left(c\,x^2+bx+a\right)^n\right)\mathrm{csgn}\left(id\left(c\,x^2+bx\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*ln((c*x^2+b*x+a)^n)-1/24*(12*ln(x)*a^2*c^2*n*x^4-24*ln(x)*a*b^2*c*
n*x^4+6*ln(x)*b^4*n*x^4-3*I*Pi*a^4*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*d*(c*x^2+b*x+a)^n)+3*I*Pi*a^4*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+3*I*Pi
*a^4*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-3*I*Pi*a^4*csgn(I*
d*(c*x^2+b*x+a)^n)^3-6*sum(_R*ln(((6*a^7*c-2*a^6*b^2)*_R^2+(-6*a^5*c^3*n+9*
a^4*b^2*c^2*n-2*a^3*b^4*c*n)*_R+9*a^2*b^2*c^4*n^2-6*a*b^4*c^3*n^2+b^6*c^2*n
^2)*x-a^7*b*_R^2+(-5*a^5*b*c^2*n+5*a^4*b^3*c*n-a^3*b^5*n)*_R-6*a^3*b*c^4*n^
2+14*a^2*b^3*c^3*n^2-7*a*b^5*c^2*n^2+b^7*c*n^2),_R=RootOf(a^4*_Z^2+(-2*a^2*
c^2*n+4*a*b^2*c*n-b^4*n)*_Z+c^4*n^2))*a^4*x^4-18*a^2*b*c*n*x^3+6*a*b^3*n*x^
3+6*a^3*c*n*x^2-3*a^2*b^2*n*x^2+2*a^3*b*n*x+6*ln(d)*a^4)/a^4/x^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 404, normalized size = 2.13

$$\left[-\frac{3\,(b^3-2\,abc)\sqrt{b^2-4\,ac}\,nx^4\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(x)+2\,a^3bnx+6\,(ab^3-3\,a^2bc)nx^3+6\,a^4\log(d)-3\,(a^2b^2-2\,a^3c)nx^2-3\,((b^4-4\,ab^2c+2\,a^2c^2)nx^4-2\,a^4n)\log(cx^2+bx+a)}{24\,a^4x^4},\ -\frac{6\,(b^3-2\,abc)\sqrt{-b^2+4\,ac}\,nx^4\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+6\,(b^4-4\,ab^2c+2\,a^2c^2)nx^4\log(x)+2\,a^3bnx+6\,(ab^3-3\,a^2bc)nx^3+6\,a^4\log(d)-3\,(a^2b^2-2\,a^3c)nx^2-3\,((b^4-4\,ab^2c+2\,a^2c^2)nx^4-2\,a^4n)\log(cx^2+bx+a)}{24\,a^4x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*(b^3 - 2*a*b*c)*sqrt(b^2 - 4*a*c)*n*x^4*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4
- 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)
*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^2 - 3*((b^4 - 4*a*b^2*c +
 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))/(a^4*x^4), -1/24*(6*(b^3
 - 2*a*b*c)*sqrt(-b^2 + 4*a*c)*n*x^4*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4*log(x) + 2*a^3*b*n*
x + 6*(a*b^3 - 3*a^2*b*c)*n*x^3 + 6*a^4*log(d) - 3*(a^2*b^2 - 2*a^3*c)*n*x^
2 - 3*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*log(c*x^2 + b*x + a))
/(a^4*x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.16, size = 210, normalized size = 1.11

$$\frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log(cx^2+bx+a)}{8\,a^4}-\frac{n\log(cx^2+bx+a)}{4\,x^4}-\frac{(b^4n-4\,ab^2cn+2\,a^2c^2n)\log(x)}{4\,a^4}+\frac{(b^5n-6\,ab^3cn+8\,a^2bc^2n)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a^4}-\frac{6\,b^3nx^3-18\,abcnx^3-3\,ab^2nx^2+6\,a^2cnx^2+2\,a^2bnx+6\,a^3\log(d)}{24\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(c*x^2 + b*x + a)/a^4 - 1/4*n*lo
g(c*x^2 + b*x + a)/x^4 - 1/4*(b^4*n - 4*a*b^2*c*n + 2*a^2*c^2*n)*log(x)/a^4
 + 1/4*(b^5*n - 6*a*b^3*c*n + 8*a^2*b*c^2*n)*arctan((2*c*x + b)/sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) - 1/24*(6*b^3*n*x^3 - 18*a*b*c*n*x^3 - 3*
a*b^2*n*x^2 + 6*a^2*c*n*x^2 + 2*a^2*b*n*x + 6*a^3*log(d))/(a^3*x^4)
```

Mupad [B]
time = 1.07, size = 627, normalized size = 3.30

$$\frac{\ln\left(2ab^6+2b^7x-12a^4c^3+2ab^5\sqrt{b^2-4ac}+2b^6x\sqrt{b^2-4ac}-15a^2b^4c+31a^3b^2c^2+37a^2b^3c^2x-16ab^5cx-20a^3bc^3x-9a^2b^3c\sqrt{b^2-4ac}+7a^3bc^2\sqrt{b^2-4ac}-6a^3c^3x\sqrt{b^2-4ac}-12ab^4cx\sqrt{b^2-4ac}+19a^2b^2c^2x\sqrt{b^2-4ac}\right)\left(\frac{b^4n}{8}-a\left(\frac{b^2cn}{2}+\frac{bcn\sqrt{b^2-4ac}}{4}\right)+\frac{b^3n\sqrt{b^2-4ac}}{8}+\frac{a^2c^2n}{4}\right)}{a^4}-\frac{\ln\left(d\left(cx^2+bx+a\right)^n\right)}{4x^4}-\frac{\ln(x)\left(2na^2c^2-4nab^2c+nb^4\right)}{4a^4}-\frac{\ln\left(12a^4c^3-2b^7x-2ab^6+2ab^5\sqrt{b^2-4ac}+2b^6x\sqrt{b^2-4ac}+15a^2b^4c-31a^3b^2c^2-37a^2b^3c^2x+16ab^5cx+20a^3bc^3x-9a^2b^3c\sqrt{b^2-4ac}+7a^3bc^2\sqrt{b^2-4ac}-6a^3c^3x\sqrt{b^2-4ac}-12ab^4cx\sqrt{b^2-4ac}+19a^2b^2c^2x\sqrt{b^2-4ac}\right)\left(a\left(\frac{b^2cn}{2}-\frac{bcn\sqrt{b^2-4ac}}{4}\right)-\frac{b^4n}{8}+\frac{b^3n\sqrt{b^2-4ac}}{8}-\frac{a^2c^2n}{4}\right)}{a^4}-\frac{\frac{bn}{3a}+\frac{nx\left(2ac-b^2\right)}{2a^2}-\frac{bnx^2\left(3ac-b^2\right)}{a^3}}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/x^5,x)
```

```
[Out] (log(2*a*b^6 + 2*b^7*x - 12*a^4*c^3 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x
*(b^2 - 4*a*c)^(1/2) - 15*a^2*b^4*c + 31*a^3*b^2*c^2 + 37*a^2*b^3*c^2*x - 1
6*a*b^5*c*x - 20*a^3*b*c^3*x - 9*a^2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^
2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2
 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*(b^2 - 4*a*c)^(1/2))*((b^4*n)/8 - a*((b^
2*c*n)/2 + (b*c*n*(b^2 - 4*a*c)^(1/2))/4) + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 +
 (a^2*c^2*n)/4))/a^4 - log(d*(a + b*x + c*x^2)^n)/(4*x^4) - (log(x)*(b^4*n
+ 2*a^2*c^2*n - 4*a*b^2*c*n))/(4*a^4) - (log(12*a^4*c^3 - 2*b^7*x - 2*a*b^6
 + 2*a*b^5*(b^2 - 4*a*c)^(1/2) + 2*b^6*x*(b^2 - 4*a*c)^(1/2) + 15*a^2*b^4*c
 - 31*a^3*b^2*c^2 - 37*a^2*b^3*c^2*x + 16*a*b^5*c*x + 20*a^3*b*c^3*x - 9*a^
2*b^3*c*(b^2 - 4*a*c)^(1/2) + 7*a^3*b*c^2*(b^2 - 4*a*c)^(1/2) - 6*a^3*c^3*x
*(b^2 - 4*a*c)^(1/2) - 12*a*b^4*c*x*(b^2 - 4*a*c)^(1/2) + 19*a^2*b^2*c^2*x*
(b^2 - 4*a*c)^(1/2))*(a*((b^2*c*n)/2 - (b*c*n*(b^2 - 4*a*c)^(1/2))/4) - (b^
4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 - (a^2*c^2*n)/4))/a^4 - ((b*n)/(3*a)
 + (n*x*(2*a*c - b^2))/(2*a^2) - (b*n*x^2*(3*a*c - b^2))/a^3)/(4*x^3)
```

3.81 $\int \log(1+x+x^2)\, dx$

Optimal. Leaf size=42

$$-2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2603, 787, 648, 632, 210, 642}

$$\sqrt{3}\,\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)+x\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2],x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2 + x*Log[1 + x
 + x^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(1+x+x^2\right)\,dx &= x\log\left(1+x+x^2\right)-\int \frac{x(1+2x)}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)-\int \frac{-2-x}{1+x+x^2}\,dx\\
&= -2x+x\log\left(1+x+x^2\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{3}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= -2x+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)-3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2
\\
&= -2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.83

$$-2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\left(\frac{1}{2}+x\right)\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + x^2],x]
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (1/2 + x)*Log[1 + x + x^2]
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.90

method	result	size
default	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln\left(x^2+x+1\right)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	38
risch	$x\ln\left(x^2+x+1\right)-2x+\frac{\ln(4x^2+4x+4)}{2}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.88

$$x\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.79

$$\frac{1}{2}\left(2x+1\right)\log\left(x^2+x+1\right)+\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 1)*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*
x
```

Sympy [A]
time = 0.06, size = 46, normalized size = 1.10

$$x\log\left(x^2+x+1\right)-2x+\frac{\log\left(x^2+x+1\right)}{2}+\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1),x)
```

[Out] x*log(x**2 + x + 1) - 2*x + log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/
3 + sqrt(3)/3)

Giac [A]
time = 4.64, size = 37, normalized size = 0.88

$$x\log\left(x^2+x+1\right)+\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-2x+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1),x, algorithm="giac")
```

```
[Out] x*log(x^2 + x + 1) + sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 1/2*log(
x^2 + x + 1)
```

Mupad [B]
time = 0.06, size = 39, normalized size = 0.93

$$\frac{\ln\left(x^2+x+1\right)}{2}-2x+\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)+x\,\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 + 1),x)
```

```
[Out] log(x + x^2 + 1)/2 - 2*x + 3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3) + x*lo
g(x + x^2 + 1)
```

3.82 $\int (d+ex)^4 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=485

$$-\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10b^2d^2+15abde+2a^2e^2))\,nx}{5c^4}-\frac{e(20c^3d^3-b$$

```
[Out] -1/5*(10*c^4*d^4+b^4*e^4-10*c^3*d^2*e*(2*a*e+b*d)-b^2*c*e^3*(4*a*e+5*b*d)+c
^2*e^2*(2*a^2*e^2+15*a*b*d*e+10*b^2*d^2))*n*x/c^4-1/10*e*(20*c^3*d^3-b^3*e^
3-10*c^2*d*e*(a*e+b*d)+b*c*e^2*(3*a*e+5*b*d))*n*x^2/c^3-1/15*e^2*(20*c^2*d^
2+b^2*e^2-c*e*(2*a*e+5*b*d))*n*x^3/c^2-1/20*e^3*(-b*e+10*c*d)*n*x^4/c-2/25*
e^4*n*x^5-1/10*(-b*e+2*c*d)*(c^4*d^4+b^4*e^4-2*c^3*d^2*e*(5*a*e+b*d)-b^2*c*
e^3*(5*a*e+3*b*d)+c^2*e^2*(5*a^2*e^2+10*a*b*d*e+4*b^2*d^2))*n*ln(c*x^2+b*x+
a)/c^5/e+1/5*(e*x+d)^5*ln(d*(c*x^2+b*x+a)^n)/e+1/5*(5*c^4*d^4+b^4*e^4-10*c^
3*d^2*e*(a*e+b*d)-b^2*c*e^3*(3*a*e+5*b*d)+c^2*e^2*(a^2*e^2+10*a*b*d*e+10*b^
2*d^2))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^5
```

Rubi [A]
time = 1.36, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{nx(c^2e^2(2a^2e^2+15abde+10b^2d^2)-b^2ce^3(4ae+5bd)-10c^3d^2e(2ae+bd)+b^4e^4+10c^4d^4)}{5c^4}-\frac{n(2cd-be)\left(c^2e^2(5a^2e^2+10abde+4b^2d^2)-b^2ce^3(5ae+3bd)-2c^3d^2e(5ae+bd)+b^4e^4+c^4d^4\right)\log\left(a+bx+cx^2\right)}{10c^5e}+\frac{n\sqrt{b^2-4ac}\left(c^2e^2(a^2e^2+10abde+10b^2d^2)-b^2ce^3(3ae+5bd)-10c^3d^2e(ae+bd)+b^4e^4+5c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{5c^5}-\frac{enx^2(-10c^2de(ae+bd)+bce^2(3ae+5bd)-b^3e^3+20c^3d^3)}{10c^3}-\frac{e^2nx^3(-ce(2ae+5bd)+b^2e^2+20c^2d^2)}{15c^2}+\frac{(d+ex)^5\log\left(d(a+bx+cx^2)^n\right)}{5e}-\frac{e^3nx^4(10cd-be)}{20c}-\frac{2}{25}e^4nx^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/5*((10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*c*e^3*(5*b*d
 + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*n*x)/c^4 - (e*(2
0*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d + 3*a*e))*n*x
^2)/(10*c^3) - (e^2*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*n*x^3)/(15
*c^2) - (e^3*(10*c*d - b*e)*n*x^4)/(20*c) - (2*e^4*n*x^5)/25 + (Sqrt[b^2 -
4*a*c]*(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d +
 3*a*e) + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*n*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(5*c^5) - ((2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d
^2*e*(b*d + 5*a*e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*
b*d*e + 5*a^2*e^2))*n*Log[a + b*x + c*x^2])/(10*c^5*e) + ((d + e*x)^5*Log[d
*(a + b*x + c*x^2)^n])/(5*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^4 \log\left(d\left(a+bx+cx^2\right)^n\right)\, dx = \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \frac{(b+2cx)(d+ex)^5}{a+bx+cx^2}\,dx}{5e}$$

$$= \frac{(d+ex)^5 \log\left(d\left(a+bx+cx^2\right)^n\right)}{5e} - \frac{n\int \left(\frac{e(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-}{\cdots}\right.}{\cdots}$$

$$= -\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10}{5c^4}$$

$$= -\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10}{5c^4}$$

$$= -\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10}{5c^4}$$

$$= -\frac{(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2(10}{5c^4}$$

Mathematica [A]
time = 1.30, size = 468, normalized size = 0.96

$$-\frac{\frac{n\left(60ce\left(10c^4d^4+b^4e^4-10c^3d^2e(bd+2ae)-b^2ce^3(5bd+4ae)+c^2e^2\left(10b^2d^2+15abde+2a^2e^2\right)\right)x+30c^2e^2\left(20c^3d^3-b^3e^3-10c^2de(bd+ae)+bce^2(5bd+3ae)\right)x^2+20c^3e^3\left(20c^2d^2+b^2e^2-ce(5bd+2ae)\right)x^3+15c^4e^4(10cd-be)x^4+24c^5e^5x^5-60\sqrt{b^2-4ac}\,e\left(5c^4d^4+b^4e^4-10c^3d^2e(bd+ae)-b^2ce^3(5bd+3ae)+c^2e^2\left(10b^2d^2+10abde+a^2e^2\right)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+30(2cd-be)\left(c^4d^4+b^4e^4-2c^3d^2e(bd+5ae)-b^2ce^3(3bd+5ae)+c^2e^2\left(4b^2d^2+10abde+5a^2e^2\right)\right)\log(a+x(b+cx))\right)}{60c^5}+(d+ex)^5\log\left(d(a+x(b+cx))^n\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/60*(n*(60*c*e*(10*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + 2*a*e) - b^2*
c*e^3*(5*b*d + 4*a*e) + c^2*e^2*(10*b^2*d^2 + 15*a*b*d*e + 2*a^2*e^2))*x +
30*c^2*e^2*(20*c^3*d^3 - b^3*e^3 - 10*c^2*d*e*(b*d + a*e) + b*c*e^2*(5*b*d
+ 3*a*e))*x^2 + 20*c^3*e^3*(20*c^2*d^2 + b^2*e^2 - c*e*(5*b*d + 2*a*e))*x^3
 + 15*c^4*e^4*(10*c*d - b*e)*x^4 + 24*c^5*e^5*x^5 - 60*Sqrt[b^2 - 4*a*c]*e*
(5*c^4*d^4 + b^4*e^4 - 10*c^3*d^2*e*(b*d + a*e) - b^2*c*e^3*(5*b*d + 3*a*e)
 + c^2*e^2*(10*b^2*d^2 + 10*a*b*d*e + a^2*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]] + 30*(2*c*d - b*e)*(c^4*d^4 + b^4*e^4 - 2*c^3*d^2*e*(b*d + 5*a*
e) - b^2*c*e^3*(3*b*d + 5*a*e) + c^2*e^2*(4*b^2*d^2 + 10*a*b*d*e + 5*a^2*e^
2))*Log[a + x*(b + c*x)]))/c^5 + (d + e*x)^5*Log[d*(a + x*(b + c*x))^n])/(5
*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 31895, normalized size = 65.76

method	result	size
risch	Expression too large to display	31895

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.45, size = 1274, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/300*(600*c^5*d^4*n*x - 30*(5*c^4*d^4*n - 10*b*c^3*d^3*n*e + 10*(b^2*c^2
 - a*c^3)*d^2*n*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*n*e^3 + (b^4 - 3*a*b^2*c + a^
2*c^2)*n*e^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sq
rt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (24*c^5*n*x^5 - 15*b*c^4*
n*x^4 + 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^2 - 3*a*b*c^3)*n*x^2 + 60*
(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x)*e^4 + 50*(3*c^5*d*n*x^4 - 2*b*c^4*d*
n*x^3 + 3*(b^2*c^3 - 2*a*c^4)*d*n*x^2 - 6*(b^3*c^2 - 3*a*b*c^3)*d*n*x)*e^3
+ 100*(4*c^5*d^2*n*x^3 - 3*b*c^4*d^2*n*x^2 + 6*(b^2*c^3 - 2*a*c^4)*d^2*n*x)
*e^2 + 600*(c^5*d^3*n*x^2 - b*c^4*d^3*n*x)*e - 30*(10*c^5*d^4*n*x + 5*b*c^4
*d^4*n + (2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*n)*e^4 + 5*(2*c^5*d
*n*x^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*n)*e^3 + 10*(2*c^5*d^2*n*x^3 +
 (b^3*c^2 - 3*a*b*c^3)*d^2*n)*e^2 + 10*(2*c^5*d^3*n*x^2 - (b^2*c^3 - 2*a*c^
4)*d^3*n)*e)*log(c*x^2 + b*x + a) - 60*(c^5*x^5*e^4 + 5*c^5*d*x^4*e^3 + 10*
c^5*d^2*x^3*e^2 + 10*c^5*d^3*x^2*e + 5*c^5*d^4*x)*log(d))/c^5, -1/300*(600*
c^5*d^4*n*x - 60*(5*c^4*d^4*n - 10*b*c^3*d^3*n*e + 10*(b^2*c^2 - a*c^3)*d^2
*n*e^2 - 5*(b^3*c - 2*a*b*c^2)*d*n*e^3 + (b^4 - 3*a*b^2*c + a^2*c^2)*n*e^4)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
```

```
(24*c^5*n*x^5 - 15*b*c^4*n*x^4 + 20*(b^2*c^3 - 2*a*c^4)*n*x^3 - 30*(b^3*c^
2 - 3*a*b*c^3)*n*x^2 + 60*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*n*x)*e^4 + 50*(
3*c^5*d*n*x^4 - 2*b*c^4*d*n*x^3 + 3*(b^2*c^3 - 2*a*c^4)*d*n*x^2 - 6*(b^3*c^
2 - 3*a*b*c^3)*d*n*x)*e^3 + 100*(4*c^5*d^2*n*x^3 - 3*b*c^4*d^2*n*x^2 + 6*(b
^2*c^3 - 2*a*c^4)*d^2*n*x)*e^2 + 600*(c^5*d^3*n*x^2 - b*c^4*d^3*n*x)*e - 30
*(10*c^5*d^4*n*x + 5*b*c^4*d^4*n + (2*c^5*n*x^5 + (b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*n)*e^4 + 5*(2*c^5*d*n*x^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*n)*e
^3 + 10*(2*c^5*d^2*n*x^3 + (b^3*c^2 - 3*a*b*c^3)*d^2*n)*e^2 + 10*(2*c^5*d^3
*n*x^2 - (b^2*c^3 - 2*a*c^4)*d^3*n)*e)*log(c*x^2 + b*x + a) - 60*(c^5*x^5*e
^4 + 5*c^5*d*x^4*e^3 + 10*c^5*d^2*x^3*e^2 + 10*c^5*d^3*x^2*e + 5*c^5*d^4*x)
*log(d))/c^5]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.80, size = 817, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/300*(60*c^4*n*x^5*e^4*log(c*x^2 + b*x + a) + 300*c^4*d*n*x^4*e^3*log(c*x^
2 + b*x + a) + 600*c^4*d^2*n*x^3*e^2*log(c*x^2 + b*x + a) + 600*c^4*d^3*n*x
^2*e*log(c*x^2 + b*x + a) - 24*c^4*n*x^5*e^4 - 150*c^4*d*n*x^4*e^3 - 400*c^
4*d^2*n*x^3*e^2 - 600*c^4*d^3*n*x^2*e + 300*c^4*d^4*n*x*log(c*x^2 + b*x + a
) + 60*c^4*x^5*e^4*log(d) + 300*c^4*d*x^4*e^3*log(d) + 600*c^4*d^2*x^3*e^2*
log(d) + 600*c^4*d^3*x^2*e*log(d) - 600*c^4*d^4*n*x + 15*b*c^3*n*x^4*e^4 +
100*b*c^3*d*n*x^3*e^3 + 300*b*c^3*d^2*n*x^2*e^2 + 600*b*c^3*d^3*n*x*e + 300
*c^4*d^4*x*log(d) - 20*b^2*c^2*n*x^3*e^4 + 40*a*c^3*n*x^3*e^4 - 150*b^2*c^2
*d*n*x^2*e^3 + 300*a*c^3*d*n*x^2*e^3 - 600*b^2*c^2*d^2*n*x*e^2 + 1200*a*c^3
*d^2*n*x*e^2 + 30*b^3*c*n*x^2*e^4 - 90*a*b*c^2*n*x^2*e^4 + 300*b^3*c*d*n*x*
e^3 - 900*a*b*c^2*d*n*x*e^3 - 60*b^4*n*x*e^4 + 240*a*b^2*c*n*x*e^4 - 120*a^
2*c^2*n*x*e^4)/c^4 + 1/10*(5*b*c^4*d^4*n - 10*b^2*c^3*d^3*n*e + 20*a*c^4*d^
3*n*e + 10*b^3*c^2*d^2*n*e^2 - 30*a*b*c^3*d^2*n*e^2 - 5*b^4*c*d*n*e^3 + 20*
a*b^2*c^2*d*n*e^3 - 10*a^2*c^3*d*n*e^3 + b^5*n*e^4 - 5*a*b^3*c*n*e^4 + 5*a^
2*b*c^2*n*e^4)*log(c*x^2 + b*x + a)/c^5 - 1/5*(5*b^2*c^4*d^4*n - 20*a*c^5*d
^4*n - 10*b^3*c^3*d^3*n*e + 40*a*b*c^4*d^3*n*e + 10*b^4*c^2*d^2*n*e^2 - 50*
```

```
a*b^2*c^3*d^2*n*e^2 + 40*a^2*c^4*d^2*n*e^2 - 5*b^5*c*d*n*e^3 + 30*a*b^3*c^2
*d*n*e^3 - 40*a^2*b*c^3*d*n*e^3 + b^6*n*e^4 - 7*a*b^4*c*n*e^4 + 13*a^2*b^2*
c^2*n*e^4 - 4*a^3*c^3*n*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-
b^2 + 4*a*c)*c^5)
```

Mupad [B]
time = 1.03, size = 1240, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^4,x)
```

```
[Out] x^3*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(3*c) + (2*a*e^
4*n)/(15*c) - (d*e^2*n*(b*e + 4*c*d))/(3*c)) - x*((a*((b*((e^3*n*(b*e + 10*
c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4
*c*d))/c))/c - (b*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)
))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/c - (a*((e^3*n*(b*e
+ 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/c + (2*d^2*e*n*(b*e + 2*c*d))/c))/c
+ (2*d^3*n*(b*e + c*d))/c) - x^2*((b*((b*((e^3*n*(b*e + 10*c*d))/(5*c) - (2
*b*e^4*n)/(5*c)))/c + (2*a*e^4*n)/(5*c) - (d*e^2*n*(b*e + 4*c*d))/c))/(2*c)
 - (a*((e^3*n*(b*e + 10*c*d))/(5*c) - (2*b*e^4*n)/(5*c)))/(2*c) + (d^2*e*n*
(b*e + 2*c*d))/c) - x^4*((e^3*n*(b*e + 10*c*d))/(20*c) - (b*e^4*n)/(10*c))
+ log(d*(a + b*x + c*x^2)^n)*(d^4*x + (e^4*x^5)/5 + 2*d^3*e*x^2 + d*e^3*x^4
 + 2*d^2*e^2*x^3) + (log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 -
 4*a*c)^(1/2))*(b^5*e^4*n + 5*b*c^4*d^4*n + b^4*e^4*n*(b^2 - 4*a*c)^(1/2) +
 5*c^4*d^4*n*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b
^4*c*d*e^3*n + 5*a^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n
+ a^2*c^2*e^4*n*(b^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n - 10*a*c^3*d^2*e
^2*n*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) - 3*a*b
^2*c*e^4*n*(b^2 - 4*a*c)^(1/2) - 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) - 5*b
^3*c*d*e^3*n*(b^2 - 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^
3*n + 10*a*b*c^2*d*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5) - (2*e^4*n*x^5)/25
+ (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^
5*e^4*n + 5*b*c^4*d^4*n - b^4*e^4*n*(b^2 - 4*a*c)^(1/2) - 5*c^4*d^4*n*(b^2
- 4*a*c)^(1/2) - 5*a*b^3*c*e^4*n + 20*a*c^4*d^3*e*n - 5*b^4*c*d*e^3*n + 5*a
^2*b*c^2*e^4*n - 10*a^2*c^3*d*e^3*n - 10*b^2*c^3*d^3*e*n - a^2*c^2*e^4*n*(b
^2 - 4*a*c)^(1/2) + 10*b^3*c^2*d^2*e^2*n + 10*a*c^3*d^2*e^2*n*(b^2 - 4*a*c)
^(1/2) - 10*b^2*c^2*d^2*e^2*n*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c*e^4*n*(b^2 -
4*a*c)^(1/2) + 10*b*c^3*d^3*e*n*(b^2 - 4*a*c)^(1/2) + 5*b^3*c*d*e^3*n*(b^2
- 4*a*c)^(1/2) - 30*a*b*c^3*d^2*e^2*n + 20*a*b^2*c^2*d*e^3*n - 10*a*b*c^2*d
*e^3*n*(b^2 - 4*a*c)^(1/2)))/(10*c^5)
```

3.83 $\int (d+ex)^3 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=338

$$-\frac{(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae))\,nx}{4c^3}-\frac{e(12c^2d^2+b^2e^2-2ce(2bd+ae))\,nx^2}{8c^2}-\frac{e^2(8cd-$$

```
[Out] -1/4*(8*c^3*d^3-b^3*e^3+b*c*e^2*(3*a*e+4*b*d)-2*c^2*d*e*(4*a*e+3*b*d))*n*x/
c^3-1/8*e*(12*c^2*d^2+b^2*e^2-2*c*e*(a*e+2*b*d))*n*x^2/c^2-1/12*e^2*(-b*e+8
*c*d)*n*x^3/c-1/8*e^3*n*x^4-1/8*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*
c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b
*x+a)/c^4/e+1/4*(e*x+d)^4*ln(d*(c*x^2+b*x+a)^n)/e+1/4*(-b*e+2*c*d)*(2*c^2*d
^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c
+b^2)^(1/2)/c^4
```

Rubi [A]

time = 0.34, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{n(2c^2e^2(a^2e^2+6abde+3b^2d^2)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4)\log(a+bx+cx^2)}{8c^4e}-\frac{nx(-2c^2de(4ae+3bd)+bce^2(3ae+4bd)-b^3e^3+8c^3d^3)}{4c^3}-\frac{enx^2(-2ce(ae+2bd)+b^2e^2+12c^2d^2)}{8c^2}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\,(-2ce(ae+bd)+b^2e^2+2c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4c^4}+\frac{(d+ex)^4\log\left(d(a+bx+cx^2)^n\right)}{4e}-\frac{e^2nx^3(8cd-be)}{12c}-\frac{1}{8}e^3nx^4$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/4*((8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*(3*b*d + 4
*a*e))*n*x)/c^3 - (e*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))*n*x^2)/(8
*c^2) - (e^2*(8*c*d - b*e)*n*x^3)/(12*c) - (e^3*n*x^4)/8 + (Sqrt[b^2 - 4*a*
c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(4*c^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*
d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a
^2*e^2))*n*Log[a + b*x + c*x^2])/(8*c^4*e) + ((d + e*x)^4*Log[d*(a + b*x +
c*x^2)^n])/(4*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{(d+ex)^4 \log \left(d(a+bx+cx^2)^n\right)}{4e} - \frac{n \int \frac{(b+2cx)(d+ex)^4}{a+bx+cx^2} \, dx}{4e}$$

$$= \frac{(d+ex)^4 \log \left(d(a+bx+cx^2)^n\right)}{4e} - \frac{n \int \left(\frac{e(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2}{c^3}\right.}{}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd+3ae) - 2c^2de(3bd+4ae))\,nx}{4c^3} - \frac{e(12c^2}{}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd+3ae) - 2c^2de(3bd+4ae))\,nx}{4c^3} - \frac{e(12c^2}{}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd+3ae) - 2c^2de(3bd+4ae))\,nx}{4c^3} - \frac{e(12c^2}{}$$

$$= -\frac{(8c^3d^3 - b^3e^3 + bce^2(4bd+3ae) - 2c^2de(3bd+4ae))\,nx}{4c^3} - \frac{e(12c^2}{}$$

Mathematica [A]
time = 0.36, size = 324, normalized size = 0.96

$$\frac{-\frac{n\left(6ce\left(8c^3d^3-b^3e^3+bce^2(4bd+3ae)-2c^2de(3bd+4ae)\right)x+3c^2e^2\left(12c^2d^2+b^2e^2-2ce(2bd+ae)\right)x^2+2c^3e^3(8cd-be)x^3+3c^4e^4x^4-6\sqrt{b^2-4ac}\,e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3\left(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2\left(3b^2d^2+6abde+a^2e^2\right)\right)\log(a+x(b+cx))\right)}{6c^4}+(d+ex)^4\log\left(d(a+x(b+cx))^n\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/6*(n*(6*c*e*(8*c^3*d^3 - b^3*e^3 + b*c*e^2*(4*b*d + 3*a*e) - 2*c^2*d*e*
(3*b*d + 4*a*e))*x + 3*c^2*e^2*(12*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d + a*e))
*x^2 + 2*c^3*e^3*(8*c*d - b*e)*x^3 + 3*c^4*e^4*x^4 - 6*Sqrt[b^2 - 4*a*c]*e*
(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]] + 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*
c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*Log[
a + x*(b + c*x)]))/c^4 + (d + e*x)^4*Log[d*(a + x*(b + c*x))^n])/(4*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 16059, normalized size = 47.51

method	result	size
risch	Expression too large to display	16059

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

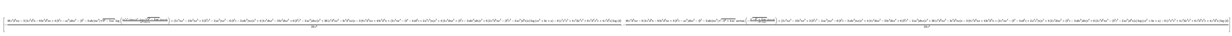
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 882, normalized size = 2.61



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/24*(48*c^4*d^3*n*x - 3*(4*c^3*d^3*n - 6*b*c^2*d^2*n*e + 4*(b^2*c - a*c^
2)*d*n*e^2 - (b^3 - 2*a*b*c)*n*e^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*
c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (3*
c^4*n*x^4 - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 - 6*(b^3*c - 3*a*b*
c^2)*n*x)*e^3 + 4*(4*c^4*d*n*x^3 - 3*b*c^3*d*n*x^2 + 6*(b^2*c^2 - 2*a*c^3)*
d*n*x)*e^2 + 36*(c^4*d^2*n*x^2 - b*c^3*d^2*n*x)*e - 3*(8*c^4*d^3*n*x + 4*b*
c^3*d^3*n + (2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*e^3 + 4*(2*c^4*
d*n*x^3 + (b^3*c - 3*a*b*c^2)*d*n)*e^2 + 6*(2*c^4*d^2*n*x^2 - (b^2*c^2 - 2*
a*c^3)*d^2*n)*e)*log(c*x^2 + b*x + a) - 6*(c^4*x^4*e^3 + 4*c^4*d*x^3*e^2 +
6*c^4*d^2*x^2*e + 4*c^4*d^3*x)*log(d))/c^4, -1/24*(48*c^4*d^3*n*x - 6*(4*c^
3*d^3*n - 6*b*c^2*d^2*n*e + 4*(b^2*c - a*c^2)*d*n*e^2 - (b^3 - 2*a*b*c)*n*e
^3)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (3*c^4*n*x^4 - 2*b*c^3*n*x^3 + 3*(b^2*c^2 - 2*a*c^3)*n*x^2 - 6*(b^3*c -
 3*a*b*c^2)*n*x)*e^3 + 4*(4*c^4*d*n*x^3 - 3*b*c^3*d*n*x^2 + 6*(b^2*c^2 - 2*
a*c^3)*d*n*x)*e^2 + 36*(c^4*d^2*n*x^2 - b*c^3*d^2*n*x)*e - 3*(8*c^4*d^3*n*x
 + 4*b*c^3*d^3*n + (2*c^4*n*x^4 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*n)*e^3 + 4*
(2*c^4*d*n*x^3 + (b^3*c - 3*a*b*c^2)*d*n)*e^2 + 6*(2*c^4*d^2*n*x^2 - (b^2*c
^2 - 2*a*c^3)*d^2*n)*e)*log(c*x^2 + b*x + a) - 6*(c^4*x^4*e^3 + 4*c^4*d*x^3
*e^2 + 6*c^4*d^2*x^2*e + 4*c^4*d^3*x)*log(d))/c^4]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.85, size = 553, normalized size = 1.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/24*(6*c^3*n*x^4*e^3*log(c*x^2 + b*x + a) + 24*c^3*d*n*x^3*e^2*log(c*x^2 +
 b*x + a) + 36*c^3*d^2*n*x^2*e*log(c*x^2 + b*x + a) - 3*c^3*n*x^4*e^3 - 16*
c^3*d*n*x^3*e^2 - 36*c^3*d^2*n*x^2*e + 24*c^3*d^3*n*x*log(c*x^2 + b*x + a)
+ 6*c^3*x^4*e^3*log(d) + 24*c^3*d*x^3*e^2*log(d) + 36*c^3*d^2*x^2*e*log(d)
- 48*c^3*d^3*n*x + 2*b*c^2*n*x^3*e^3 + 12*b*c^2*d*n*x^2*e^2 + 36*b*c^2*d^2*
n*x*e + 24*c^3*d^3*x*log(d) - 3*b^2*c*n*x^2*e^3 + 6*a*c^2*n*x^2*e^3 - 24*b^
2*c*d*n*x*e^2 + 48*a*c^2*d*n*x*e^2 + 6*b^3*n*x*e^3 - 18*a*b*c*n*x*e^3)/c^3
+ 1/8*(4*b*c^3*d^3*n - 6*b^2*c^2*d^2*n*e + 12*a*c^3*d^2*n*e + 4*b^3*c*d*n*e
^2 - 12*a*b*c^2*d*n*e^2 - b^4*n*e^3 + 4*a*b^2*c*n*e^3 - 2*a^2*c^2*n*e^3)*lo
g(c*x^2 + b*x + a)/c^4 - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*
d^2*n*e + 24*a*b*c^3*d^2*n*e + 4*b^4*c*d*n*e^2 - 20*a*b^2*c^2*d*n*e^2 + 16*
a^2*c^3*d*n*e^2 - b^5*n*e^3 + 6*a*b^3*c*n*e^3 - 8*a^2*b*c^2*n*e^3)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B]
time = 0.84, size = 775, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^3,x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x
^3) - x^3*((e^2*n*(b*e + 8*c*d))/(12*c) - (b*e^3*n)/(6*c)) - x*((b*((b*((e^
2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c + (a*e^3*n)/(2*c) - (d*e*n*(
b*e + 3*c*d))/c))/c - (a*((e^2*n*(b*e + 8*c*d))/(4*c) - (b*e^3*n)/(2*c)))/c
 + (d^2*n*(3*b*e + 4*c*d))/(2*c)) + x^2*((b*((e^2*n*(b*e + 8*c*d))/(4*c) -
(b*e^3*n)/(2*c)))/(2*c) + (a*e^3*n)/(4*c) - (d*e*n*(b*e + 3*c*d))/(2*c)) -
(log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(b^4*
e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n + b^3*e^3*n*(b^2 - 4*a*c)^(1/2) - 4
*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n - 4*b^3
*c*d*e^2*n + 6*b^2*c^2*d^2*e*n - 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 12*a*b
```

```
*c^2*d*e^2*n + 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) + 6*b*c^2*d^2*e*n*(b^2 -
 4*a*c)^(1/2) - 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4) - (e^3*n*x^4)
/8 - (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*
(b^4*e^3*n + 2*a^2*c^2*e^3*n - 4*b*c^3*d^3*n - b^3*e^3*n*(b^2 - 4*a*c)^(1/2
) + 4*c^3*d^3*n*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e^3*n - 12*a*c^3*d^2*e*n -
4*b^3*c*d*e^2*n + 6*b^2*c^2*d^2*e*n + 2*a*b*c*e^3*n*(b^2 - 4*a*c)^(1/2) + 1
2*a*b*c^2*d*e^2*n - 4*a*c^2*d*e^2*n*(b^2 - 4*a*c)^(1/2) - 6*b*c^2*d^2*e*n*(
b^2 - 4*a*c)^(1/2) + 4*b^2*c*d*e^2*n*(b^2 - 4*a*c)^(1/2)))/(8*c^4)
```

3.84 $\int (d+ex)^2 \log \left(d(a+bx+cx^2)^n\right) \, dx$

Optimal. Leaf size=226

$$-\frac{(6c^2d^2+b^2e^2-ce(3bd+2ae))\,nx}{3c^2}-\frac{e(6cd-be)nx^2}{6c}-\frac{2}{9}e^2nx^3+\frac{\sqrt{b^2-4ac}\,(3c^2d^2+b^2e^2-ce(3bd+ae))\,n\,t}{3c^3}$$

```
[Out] -1/3*(6*c^2*d^2+b^2*e^2-c*e*(2*a*e+3*b*d))*n*x/c^2-1/6*e*(-b*e+6*c*d)*n*x^2
/c-2/9*e^2*n*x^3-1/6*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*
x^2+b*x+a)/c^3/e+1/3*(e*x+d)^3*ln(d*(c*x^2+b*x+a)^n)/e+1/3*(3*c^2*d^2+b^2*e
^2-c*e*(a*e+3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1
/2)/c^3
```

Rubi [A]
time = 0.21, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{nx(-ce(2ae+3bd)+b^2e^2+6c^2d^2)}{3c^2}-\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6c^3e}+\frac{n\sqrt{b^2-4ac}\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3c^3}+\frac{(d+ex)^3\log\left(d(a+bx+cx^2)^n\right)}{3e}-\frac{enx^2(6cd-be)}{6c}-\frac{2}{9}e^2nx^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/3*((6*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + 2*a*e))*n*x)/c^2 - (e*(6*c*d - b*
e)*n*x^2)/(6*c) - (2*e^2*n*x^3)/9 + (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2
 - c*e*(3*b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*c^3) - (
(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[a + b*x + c*x^2
])/(6*c^3*e) + ((d + e*x)^3*Log[d*(a + b*x + c*x^2)^n])/(3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \log \left(d\left(a+bx+cx^2\right)^n\right) \, dx = \frac{(d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n \int \frac{(b+2cx)(d+ex)^3}{a+bx+cx^2} \, dx}{3e}$$

$$= \frac{(d+ex)^3 \log \left(d\left(a+bx+cx^2\right)^n\right)}{3e} - \frac{n \int \left(\frac{e\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)}{c^2} + \right.}{}$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3$$

$$= -\frac{\left(6c^2d^2+b^2e^2-ce(3bd+2ae)\right)nx}{3c^2} - \frac{e(6cd-be)nx^2}{6c} - \frac{2}{9}e^2nx^3$$

Mathematica [A]
time = 0.27, size = 204, normalized size = 0.90

$$\frac{-\frac{n\left(cex\left(6b^2e^2-3ce(6bd+4ae+bex)+2c^2\left(18d^2+9dex+2e^2x^2\right)\right)-6\sqrt{b^2-4ac}\,e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+3(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\log(a+x(b+cx))\right)}{6c^3}+(d+ex)^3\log\left(d(a+x(b+cx))^n\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-1/6*(n*(c*e*x*(6*b^2*e^2 - 3*c*e*(6*b*d + 4*a*e + b*e*x) + 2*c^2*(18*d^2
+ 9*d*e*x + 2*e^2*x^2)) - 6*Sqrt[b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*
(3*b*d + a*e))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + 3*(2*c*d - b*e)*(c^
2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*Log[a + x*(b + c*x)]))/c^3 + (d + e*x)
^3*Log[d*(a + x*(b + c*x))^n])/(3*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 7155, normalized size = 31.66

method	result	size
risch	Expression too large to display	7155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 573, normalized size = 2.54

$$\left[-\frac{36c^3d^2nx+3\left(3c^2d^2n-3bcdne+(b^2-ac)ne^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)+\left(4c^3nx^3-3bc^2nx^2+6(b^2c-2ac^2)nx\right)e^2+18\left(c^3dnx^2-bc^2dnx\right)e-3\left(6c^3d^2nx+3bc^2d^2n+\left(2c^3nx^3+(b^3-3abc)n\right)e^2+3\left(2c^3dnx^2-(b^2c-2ac^2)dn\right)e\right)\log\left(cx^2+bx+a\right)-6\left(c^3x^3e^2+3c^3dx^2e+3c^3d^2x\right)\log(d)}{18c^3},\ -\frac{36c^3d^2nx-6\left(3c^2d^2n-3bcdne+(b^2-ac)ne^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)+\left(4c^3nx^3-3bc^2nx^2+6(b^2c-2ac^2)nx\right)e^2+18\left(c^3dnx^2-bc^2dnx\right)e-3\left(6c^3d^2nx+3bc^2d^2n+\left(2c^3nx^3+(b^3-3abc)n\right)e^2+3\left(2c^3dnx^2-(b^2c-2ac^2)dn\right)e\right)\log\left(cx^2+bx+a\right)-6\left(c^3x^3e^2+3c^3dx^2e+3c^3d^2x\right)\log(d)}{18c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/18*(36*c^3*d^2*n*x + 3*(3*c^2*d^2*n - 3*b*c*d*n*e + (b^2 - a*c)*n*e^2)*
sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c
)*(2*c*x + b))/(c*x^2 + b*x + a)) + (4*c^3*n*x^3 - 3*b*c^2*n*x^2 + 6*(b^2*c
 - 2*a*c^2)*n*x)*e^2 + 18*(c^3*d*n*x^2 - b*c^2*d*n*x)*e - 3*(6*c^3*d^2*n*x
+ 3*b*c^2*d^2*n + (2*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*e^2 + 3*(2*c^3*d*n*x^2
- (b^2*c - 2*a*c^2)*d*n)*e)*log(c*x^2 + b*x + a) - 6*(c^3*x^3*e^2 + 3*c^3*d
*x^2*e + 3*c^3*d^2*x)*log(d))/c^3, -1/18*(36*c^3*d^2*n*x - 6*(3*c^2*d^2*n -
 3*b*c*d*n*e + (b^2 - a*c)*n*e^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*
a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (4*c^3*n*x^3 - 3*b*c^2*n*x^2 + 6*(b^2*c -
 2*a*c^2)*n*x)*e^2 + 18*(c^3*d*n*x^2 - b*c^2*d*n*x)*e - 3*(6*c^3*d^2*n*x +
3*b*c^2*d^2*n + (2*c^3*n*x^3 + (b^3 - 3*a*b*c)*n)*e^2 + 3*(2*c^3*d*n*x^2 -
(b^2*c - 2*a*c^2)*d*n)*e)*log(c*x^2 + b*x + a) - 6*(c^3*x^3*e^2 + 3*c^3*d*x
^2*e + 3*c^3*d^2*x)*log(d))/c^3]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.11, size = 349, normalized size = 1.54

$$\frac{6\,c^2nx^3e^2\log\left(cx^2+bx+a\right)+18\,c^2dnx^2e\log\left(cx^2+bx+a\right)-4\,c^2nx^3e^2-18\,c^2dnx^2e+18\,c^2d^2nx\log\left(cx^2+bx+a\right)+6\,c^2x^3e^2\log\left(d\right)+18\,c^2dx^2e\log\left(d\right)-36\,c^2d^2nx+3\,bcnx^2e^2+18\,bcdnxe+18\,c^2d^2x\log\left(d\right)-6\,b^2nxe^2+12\,acnxe^2}{18\,c^2}+\frac{\left(3\,bc^2d^2n-3\,b^2cdne+6\,ac^2dne+b^3ne^2-3\,abcne^2\right)\log\left(cx^2+bx+a\right)}{6\,c^3}-\frac{\left(3\,b^2c^2d^2n-12\,ac^3d^2n-3\,b^3cdne+12\,abc^2dne+b^4ne^2-5\,ab^2cne^2+4\,a^2c^2ne^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/18*(6*c^2*n*x^3*e^2*log(c*x^2 + b*x + a) + 18*c^2*d*n*x^2*e*log(c*x^2 + b
*x + a) - 4*c^2*n*x^3*e^2 - 18*c^2*d*n*x^2*e + 18*c^2*d^2*n*x*log(c*x^2 + b
*x + a) + 6*c^2*x^3*e^2*log(d) + 18*c^2*d*x^2*e*log(d) - 36*c^2*d^2*n*x + 3
*b*c*n*x^2*e^2 + 18*b*c*d*n*x*e + 18*c^2*d^2*x*log(d) - 6*b^2*n*x*e^2 + 12*
a*c*n*x*e^2)/c^2 + 1/6*(3*b*c^2*d^2*n - 3*b^2*c*d*n*e + 6*a*c^2*d*n*e + b^3
*n*e^2 - 3*a*b*c*n*e^2)*log(c*x^2 + b*x + a)/c^3 - 1/3*(3*b^2*c^2*d^2*n - 1
2*a*c^3*d^2*n - 3*b^3*c*d*n*e + 12*a*b*c^2*d*n*e + b^4*n*e^2 - 5*a*b^2*c*n*
e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*c^3)
```

Mupad [**B**]
time = 0.67, size = 457, normalized size = 2.02

$$\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{\frac{d^2\,n\,\sqrt{b^2-4\,a\,c}}{2}+\frac{b\,d^2\,n}{2}+a\,d\,e\,n}{c}-\frac{\frac{a\,b\,e^2\,n}{2}+\frac{b^2\,d\,e\,n}{2}+\frac{a\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6}+\frac{b\,d\,e\,n\,\sqrt{b^2-4\,a\,c}}{2}}{c^2}+\frac{b^3\,e^2\,n}{6\,c^3}+\frac{b^2\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6\,c^3}\right)+x\,\left(\frac{b\,\left(\frac{e\,n\,\left(b\,e+6\,c\,d\right)}{3\,c}-\frac{2\,b\,e^2\,n}{3\,c}\right)}{c}-\frac{d\,n\,\left(b\,e+2\,c\,d\right)}{c}+\frac{2\,a\,e^2\,n}{3\,c}\right)-\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{\frac{a\,b\,e^2\,n}{2}+\frac{b^2\,d\,e\,n}{2}-\frac{a\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6}-\frac{b\,d\,e\,n\,\sqrt{b^2-4\,a\,c}}{2}}{c^2}-\frac{\frac{b\,d^2\,n}{2}-\frac{d^2\,n\,\sqrt{b^2-4\,a\,c}}{2}+a\,d\,e\,n}{c}-\frac{b^3\,e^2\,n}{6\,c^3}+\frac{b^2\,e^2\,n\,\sqrt{b^2-4\,a\,c}}{6\,c^3}\right)+\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-x^2\,\left(\frac{e\,n\,\left(b\,e+6\,c\,d\right)}{6\,c}-\frac{b\,e^2\,n}{3\,c}\right)-\frac{2\,e^2\,n\,x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x)^2,x)
```

```
[Out] log(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*(((d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + (b*d^2*n)/2 + a*d*e*n)/c - ((a*b*e^2*n)/2 + (b^
2*d*e*n)/2 + (a*e^2*n*(b^2 - 4*a*c)^(1/2))/6 + (b*d*e*n*(b^2 - 4*a*c)^(1/2)
)/2)/c^2 + (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(b^2 - 4*a*c)^(1/2))/(6*c^3)) +
 x*((b*((e*n*(b*e + 6*c*d))/(3*c) - (2*b*e^2*n)/(3*c)))/c - (d*n*(b*e + 2*c
*d))/c + (2*a*e^2*n)/(3*c)) - log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c
*x*(b^2 - 4*a*c)^(1/2))*(((a*b*e^2*n)/2 + (b^2*d*e*n)/2 - (a*e^2*n*(b^2 - 4
*a*c)^(1/2))/6 - (b*d*e*n*(b^2 - 4*a*c)^(1/2))/2)/c^2 - ((b*d^2*n)/2 - (d^2
*n*(b^2 - 4*a*c)^(1/2))/2 + a*d*e*n)/c - (b^3*e^2*n)/(6*c^3) + (b^2*e^2*n*(
b^2 - 4*a*c)^(1/2))/(6*c^3)) + log(d*(a + b*x + c*x^2)^n)*(d^2*x + (e^2*x^3
)/3 + d*e*x^2) - x^2*((e*n*(b*e + 6*c*d))/(6*c) - (b*e^2*n)/(3*c)) - (2*e^2
*n*x^3)/9
```

3.85 $\int (d+ex)\log\left(d(a+bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=154

$$-\frac{1}{2}\left(4d-\frac{be}{c}\right)nx-\frac{1}{2}enx^2+\frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}-\frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n}{4c^2e}$$

```
[Out] -1/2*(4*d-b*e/c)*n*x-1/2*e*n*x^2-1/4*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*
ln(c*x^2+b*x+a)/c^2/e+1/2*(e*x+d)^2*ln(d*(c*x^2+b*x+a)^n)/e+1/2*(-b*e+2*c*d
)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2605, 814, 648, 632, 212, 642}

$$-\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4c^2e}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2c^2}+\frac{(d+ex)^2\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e}-\frac{1}{2}nx\left(4d-\frac{be}{c}\right)-\frac{1}{2}enx^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -1/2*((4*d - (b*e)/c)*n*x) - (e*n*x^2)/2 + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)
*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*c^2) - ((2*c^2*d^2 + b^2*e^2
- 2*c*e*(b*d + a*e))*n*Log[a + b*x + c*x^2])/(4*c^2*e) + ((d + e*x)^2*Log[d
*(a + b*x + c*x^2)^n])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e} - \frac{n\int \frac{(b+2cx)(d+ex)^2}{a+bx+cx^2}\,dx}{2e} \\
&= \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e} - \frac{n\int \left(e\left(4d-\frac{be}{c}\right)+2e^2x+\frac{bcd^2-4\cdots}{\cdots}\right.}{\cdots} \\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e} - \frac{n\cdots}{\cdots} \\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{(d+ex)^2\log\left(d(a+bx+cx^2)^n\right)}{2e} - \frac{((\cdots}{\cdots} \\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 - \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n\log(a\cdots}{4c^2e} \\
&= -\frac{1}{2}\left(4d-\frac{be}{c}\right)nx - \frac{1}{2}enx^2 + \frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b\cdots}{\sqrt{b^2\cdots}}\right.}{2c^2}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 123, normalized size = 0.80

$$\frac{-2\sqrt{b^2-4ac}\,(-2cd+be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+(2bcd-b^2e+2ace)\,n\log(a+x(b+cx))+2cx(ben-cn(4d+ex)+c(2d+ex)\log\left(d(a+x(b+cx))^n\right))}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (-2*Sqrt[b^2 - 4*a*c]*(-2*c*d + b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]] + (2*b*c*d - b^2*e + 2*a*c*e)*n*Log[a + x*(b + c*x)] + 2*c*x*(b*e*n - c*
n*(4*d + e*x) + c*(2*d + e*x)*Log[d*(a + x*(b + c*x))^n]))/(4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.11, size = 1706, normalized size = 11.08

method	result	size
risch	Expression too large to display	1706

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*e*x^2+d*x)*ln((c*x^2+b*x+a)^n)-1/4*I*Pi*e*x^2*csgn(I*d*(c*x^2+b*x+a)^n
)^3+1/2*I*Pi*d*x*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-1/2*I*
Pi*d*x*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+1/2*I*Pi
*d*x*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^
2*csgn(I*d)*x^2*e*Pi+1/4*I*csgn(I*d*(c*x^2+b*x+a)^n)^2*csgn(I*(c*x^2+b*x+a)
^n)*x^2*e*Pi-1/4*I*Pi*e*x^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x
^2+b*x+a)^n)-1/2*I*Pi*d*x*csgn(I*d*(c*x^2+b*x+a)^n)^3+1/2*ln(d)*e*x^2-1/2*e
*n*x^2+ln(d)*d*x+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^
2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2
)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2
*c^2*d^2)^(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*
(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d
^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d
*e+4*b^2*c^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^
2*c*d+2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b
^2*c^2*d^2)^(1/2)*c*x+(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4
*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b*d+1/2/c*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*
e-2*b^2*c*d-2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d
*e+4*b^2*c^2*d^2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4
*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*a*e-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^
2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4
*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3
*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b^2*e+1/2/c*n*ln(-4*a*b*c*
e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b
^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-
16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*b)*b*d+1/2/c*b*e*n*x-
2*d*n*x-1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d+2*(-4*a*b^2*c*e^2
+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)*c*x+(
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
```

```
2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e
+4*b^2*c^2*d^2)^(1/2)+1/4/c^2*n*ln(-4*a*b*c*e+8*a*c^2*d+b^3*e-2*b^2*c*d-2*(
-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*e+4*b^2*c^2*d^
2)^(1/2)*c*x-(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e^2-4*b^3*c*d*
e+4*b^2*c^2*d^2)^(1/2)*b)*(-4*a*b^2*c*e^2+16*a*b*c^2*d*e-16*a*c^3*d^2+b^4*e
^2-4*b^3*c*d*e+4*b^2*c^2*d^2)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.37, size = 346, normalized size = 2.25

$$\left[-\frac{8\,c^2dnx + (2\,cdn - bne)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right) + 2\,(c^2nx^2-bcnx)e - (4\,c^2dnx+2\,bcdn+(2\,c^2nx^2-(b^2-2\,ac)n)e)\log\left(cx^2+bx+a\right) - 2\,(c^2x^2e+2\,c^2dx)\log\left(d\right)}{4\,c^2}, -\frac{8\,c^2dnx - 2\,(2\,cdn - bne)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right) + 2\,(c^2nx^2-bcnx)e - (4\,c^2dnx+2\,bcdn+(2\,c^2nx^2-(b^2-2\,ac)n)e)\log\left(cx^2+bx+a\right) - 2\,(c^2x^2e+2\,c^2dx)\log\left(d\right)}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/4*(8*c^2*d*n*x + (2*c*d*n - b*n*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
2*(c^2*n*x^2 - b*c*n*x)*e - (4*c^2*d*n*x + 2*b*c*d*n + (2*c^2*n*x^2 - (b^2
- 2*a*c)*n)*e)*log(c*x^2 + b*x + a) - 2*(c^2*x^2*e + 2*c^2*d*x)*log(d))/c^2
, -1/4*(8*c^2*d*n*x - 2*(2*c*d*n - b*n*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(c^2*n*x^2 - b*c*n*x)*e - (4*c^
2*d*n*x + 2*b*c*d*n + (2*c^2*n*x^2 - (b^2 - 2*a*c)*n)*e)*log(c*x^2 + b*x +
a) - 2*(c^2*x^2*e + 2*c^2*d*x)*log(d))/c^2]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(139) = 278$.
time = 116.21, size = 379, normalized size = 2.46

$$\begin{cases} \frac{ae\log\left(d(a+bx+cx^2)^n\right)}{2c} - \frac{b^2e\log\left(d(a+bx+cx^2)^n\right)}{4c^2} + \frac{bd\log\left(d(a+bx+cx^2)^n\right)}{2c} + \frac{benx}{2c} - \frac{ben\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2c^2} + \frac{be\sqrt{-4ac+b^2}\log\left(d(a+bx+cx^2)^n\right)}{4c^2} - 2dnx + dx\log\left(d(a+bx+cx^2)^n\right) - \frac{enx^2}{2} + \frac{ex^2\log\left(d(a+bx+cx^2)^n\right)}{2} + \frac{dn\sqrt{-4ac+b^2}\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c} - \frac{d\sqrt{-4ac+b^2}\log\left(d(a+bx+cx^2)^n\right)}{2c} & \text{for}\: c \neq 0 \\ -\frac{a^2e\log\left(d(a+bx)^n\right)}{2b^2} + \frac{ad\log\left(d(a+bx)^n\right)}{b} + \frac{aenx}{2b} - dnx + dx\log\left(d(a+bx)^n\right) - \frac{enx^2}{4} + \frac{ex^2\log\left(d(a+bx)^n\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*e*log(d*(a + b*x + c*x**2)**n)/(2*c) - b**2*e*log(d*(a + b*x +
 c*x**2)**n)/(4*c**2) + b*d*log(d*(a + b*x + c*x**2)**n)/(2*c) + b*e*n*x/(2
*c) - b*e*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(2*c**2) + b*e*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(4*c**2)
- 2*d*n*x + d*x*log(d*(a + b*x + c*x**2)**n) - e*n*x**2/2 + e*x**2*log(d*(a
 + b*x + c*x**2)**n)/2 + d*n*sqrt(-4*a*c + b**2)*log(b/(2*c) + x + sqrt(-4*
a*c + b**2)/(2*c))/c - d*sqrt(-4*a*c + b**2)*log(d*(a + b*x + c*x**2)**n)/(
2*c), Ne(c, 0)), (-a**2*e*log(d*(a + b*x)**n)/(2*b**2) + a*d*log(d*(a + b*x
)**n)/b + a*e*n*x/(2*b) - d*n*x + d*x*log(d*(a + b*x)**n) - e*n*x**2/4 + e*
x**2*log(d*(a + b*x)**n)/2, True))
```

Giac [A]
time = 4.15, size = 188, normalized size = 1.22

$$\frac{cnx^2e\log\left(cx^2+bx+a\right)-cnx^2e+2\,cdnx\log\left(cx^2+bx+a\right)+cx^2e\log\left(d\right)-4\,cdnx+bnxe+2\,cdx\log\left(d\right)}{2\,c}+\frac{\left(2\,bcdn-b^2ne+2\,acne\right)\log\left(cx^2+bx+a\right)}{4\,c^2}-\frac{\left(2\,b^2cdn-8\,ac^2dn-b^3ne+4\,abcne\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] 1/2*(c*n*x^2*e*log(c*x^2 + b*x + a) - c*n*x^2*e + 2*c*d*n*x*log(c*x^2 + b*x
 + a) + c*x^2*e*log(d) - 4*c*d*n*x + b*n*x*e + 2*c*d*x*log(d))/c + 1/4*(2*b
*c*d*n - b^2*n*e + 2*a*c*n*e)*log(c*x^2 + b*x + a)/c^2 - 1/2*(2*b^2*c*d*n -
 8*a*c^2*d*n - b^3*n*e + 4*a*b*c*n*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 0.59, size = 242, normalized size = 1.57

$$\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(\frac{n\,\left(b\,e+4\,c\,d\right)}{2\,c}-\frac{b\,e\,n}{c}\right)-\frac{e\,n\,x^2}{2}+\frac{\ln\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}-\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)-\frac{b^2\,e\,n}{4}+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}-\frac{\ln\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2+2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^2\,e\,n}{4}-c\,\left(\frac{a\,e\,n}{2}+\frac{b\,d\,n}{2}+\frac{d\,n\,\sqrt{b^2-4\,a\,c}}{2}\right)+\frac{b\,e\,n\,\sqrt{b^2-4\,a\,c}}{4}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)*(d + e*x),x)
```

```
[Out] log(d*(a + b*x + c*x^2)^n)*(d*x + (e*x^2)/2) - x*((n*(b*e + 4*c*d))/(2*c) -
 (b*e*n)/c) - (e*n*x^2)/2 + (log(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2 + 2*c*
x*(b^2 - 4*a*c)^(1/2))*(c*((a*e*n)/2 + (b*d*n)/2 - (d*n*(b^2 - 4*a*c)^(1/2)
)/2) - (b^2*e*n)/4 + (b*e*n*(b^2 - 4*a*c)^(1/2))/4))/c^2 - (log(b*(b^2 - 4*
a*c)^(1/2) - 4*a*c + b^2 + 2*c*x*(b^2 - 4*a*c)^(1/2))*((b^2*e*n)/4 - c*((a*
e*n)/2 + (b*d*n)/2 + (d*n*(b^2 - 4*a*c)^(1/2))/2) + (b*e*n*(b^2 - 4*a*c)^(1
/2))/4))/c^2
```

3.86 $\int \log \left(d(a+bx+cx^2)^n\right)\,dx$

Optimal. Leaf size=79

$$-2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)$$

```
[Out] -2*n*x+1/2*b*n*ln(c*x^2+b*x+a)/c+x*ln(d*(c*x^2+b*x+a)^n)+n*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2603, 787, 648, 632, 212, 642}

$$\frac{n\sqrt{b^2-4ac}\,\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{bn\log\left(a+bx+cx^2\right)}{2c}-2nx$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] -2*n*x + (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c + (
b*n*Log[a + b*x + c*x^2])/(2*c) + x*Log[d*(a + b*x + c*x^2)^n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(d\left(a+bx+cx^2\right)^n\right)\,dx &= x\log\left(d\left(a+bx+cx^2\right)^n\right)-n\int\frac{x(b+2cx)}{a+bx+cx^2}\,dx\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{n\int\frac{-2ac-bcx}{a+bx+cx^2}\,dx}{c}\\
&= -2nx+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{(bn)\int\frac{b+2cx}{a+bx+cx^2}\,dx}{2c}-\frac{\left(\left(b^2-4ac\right)n\right)\int \frac{}{a}}{2c}\\
&= -2nx+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(\left(b^2-4ac\right)n\right)}{}\\
&= -2nx+\frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+\frac{bn\log\left(a+bx+cx^2\right)}{2c}+x\log
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.99

$$\frac{2\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)+bn\log(a+x(b+cx))+2cx(-2n+\log\left(d(a+x(b+cx))^n\right))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n],x]
```

```
[Out] (2*Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] + b*n*Log[a +
 x*(b + c*x)] + 2*c*x*(-2*n + Log[d*(a + x*(b + c*x))^n]))/(2*c)
```

Maple [A]
time = 0.00, size = 89, normalized size = 1.13

method	result
default	$x\ln\left(d\left(c\,x^{2}+bx+a\right)^{n}\right)-n\left(2x-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-2a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}\right)$
risch	$x\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)-\frac{i\pi x\,\mathrm{csgn}\left(id\right)\mathrm{csgn}\left(i\left(c\,x^{2}+bx+a\right)^{n}\right)\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)}{2}+\frac{i\mathrm{csgn}\left(id\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}\mathrm{csgn}\left(id\right)x\pi}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(d*(c*x^2+b*x+a)^n)-n*(2*x-1/2*b/c*ln(c*x^2+b*x+a)+2*(-2*a+1/2*b^2/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 190, normalized size = 2.41

$$\left[-\frac{4\,cnx-2\,cx\log\left(d\right)-\sqrt{b^{2}-4\,ac}\,n\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-(2\,cnx+bn)\log\left(cx^{2}+bx+a\right)}{2\,c},-\frac{4\,cnx-2\,cx\log\left(d\right)-2\,\sqrt{-b^{2}+4\,ac}\,n\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-(2\,cnx+bn)\log\left(cx^{2}+bx+a\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*c*n*x - 2*c*x*log(d) - sqrt(b^2 - 4*a*c)*n*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (2*c*
n*x + b*n)*log(c*x^2 + b*x + a))/c, -1/2*(4*c*n*x - 2*c*x*log(d) - 2*sqrt(-
b^2 + 4*a*c)*n*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*c
*n*x + b*n)*log(c*x^2 + b*x + a))/c]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(75) = 150$.
time = 43.21, size = 274, normalized size = 3.47

$$\begin{cases} \frac{a\log\left(d(a+bx)^n\right)}{b} - nx + x\log\left(d(a+bx)^n\right) & \text{for } c = 0 \\ \frac{b\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right)}{2c} - 2nx + x\log\left(d\left(\frac{b^2}{4c}+bx+cx^2\right)^n\right) & \text{for } a = \frac{b^2}{4c} \\ -\frac{4an\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{\sqrt{-4ac+b^2}} + \frac{2a\log\left(d(a+bx+cx^2)^n\right)}{\sqrt{-4ac+b^2}} + \frac{b^2n\log\left(\frac{b}{2c}+x+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{c\sqrt{-4ac+b^2}} - \frac{b^2\log\left(d(a+bx+cx^2)^n\right)}{2c\sqrt{-4ac+b^2}} + \frac{b\log\left(d(a+bx+cx^2)^n\right)}{2c} - 2nx + x\log\left(d(a+bx+cx^2)^n\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n),x)
```

```
[Out] Piecewise((a*log(d*(a + b*x)**n)/b - n*x + x*log(d*(a + b*x)**n), Eq(c, 0))
, (b*log(d*(b**2/(4*c) + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(b**2/(4
*c) + b*x + c*x**2)**n), Eq(a, b**2/(4*c))), (-4*a*n*log(b/(2*c) + x + sqrt
(-4*a*c + b**2)/(2*c))/sqrt(-4*a*c + b**2) + 2*a*log(d*(a + b*x + c*x**2)**
n)/sqrt(-4*a*c + b**2) + b**2*n*log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)
)/(c*sqrt(-4*a*c + b**2)) - b**2*log(d*(a + b*x + c*x**2)**n)/(2*c*sqrt(-4*
a*c + b**2)) + b*log(d*(a + b*x + c*x**2)**n)/(2*c) - 2*n*x + x*log(d*(a +
b*x + c*x**2)**n), True))
```

Giac [A]
time = 3.66, size = 92, normalized size = 1.16

$$nx\log\left(cx^2+bx+a\right) - (2\,n - \log(d))x + \frac{bn\log\left(cx^2+bx+a\right)}{2\,c} - \frac{(b^2n - 4\,acn)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n),x, algorithm="giac")
```

```
[Out] n*x*log(c*x^2 + b*x + a) - (2*n - log(d))*x + 1/2*b*n*log(c*x^2 + b*x + a)/
c - (b^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4
*a*c)*c)
```

Mupad [B]
time = 0.00, size = 120, normalized size = 1.52

$$x\,\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right) - 2\,n\,x - \frac{n\,\mathrm{atan}\left(\frac{b\,n\,\sqrt{4\,a\,c-b^2}}{2\,\left(\frac{b^2\,n}{2}-2\,a\,c\,n\right)} - \frac{n\,x\,\sqrt{4\,a\,c-b^2}}{2\,a\,n-\frac{b^2\,n}{2\,c}}\right)\,\sqrt{4\,a\,c-b^2}}{c} + \frac{b\,n\,\ln\left(c\,x^2+b\,x+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n),x)
```

```
[Out] x*log(d*(a + b*x + c*x^2)^n) - 2*n*x - (n*atan((b*n*(4*a*c - b^2)^(1/2))/(2
*((b^2*n)/2 - 2*a*c*n)) - (n*x*(4*a*c - b^2)^(1/2))/(2*a*n - (b^2*n)/(2*c))
)*(4*a*c - b^2)^(1/2))/c + (b*n*log(a + b*x + c*x^2))/(2*c)
```

3.87 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{d+ex}\,dx$

Optimal. Leaf size=228

$$-\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e}-\frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e}+\frac{\log(d+ex)\log}{}$$

```
[Out] ln(e*x+d)*ln(d*(c*x^2+b*x+a)^n)/e-n*ln(e*x+d)*ln(-e*(b+2*c*x-(-4*a*c+b^2)^(
1/2))/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*ln(e*x+d)*ln(-e*(b+2*c*x+(-4*a*
c+b^2)^(1/2))/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))/e-n*polylog(2,2*c*(e*x+d)/(2*c*d-e*(b+(-4*
a*c+b^2)^(1/2))))/e
```

Rubi [A]
time = 0.27, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2604, 2465, 2441, 2440, 2438}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\text{PolyLog}\left(2,\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\right)}{e}-\frac{n\log(d+ex)\log\left(-\frac{e\left(\sqrt{b^2-4ac}+b+2cx\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\right)}{e}+\frac{\log(d+ex)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + b*x + c*x^2)^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\int \frac{(b+2cx)\log(d+ex)}{a+bx+cx^2}\,dx}{e}$$

$$= \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{n\int \left(\frac{2c\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx} + \frac{2c\log(d+ex}{b+\sqrt{b^2-4ac}\right.}{e}$$

$$= \frac{\log(d+ex)\log\left(d(a+bx+cx^2)^n\right)}{e} - \frac{(2cn)\int \frac{\log(d+ex)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{e} - \frac{(2cn)\int}{}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2c\right.}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{e}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2c\right.}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{e}$$

$$= -\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2c\right.}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{e}$$

Mathematica [A]
time = 0.22, size = 226, normalized size = 0.99

$$-\frac{n\log\left(-\frac{e\left(b-\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} - \frac{n\log\left(-\frac{e\left(b+\sqrt{b^2-4ac}+2cx\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(d(a+x(b+cx))^n\right)}{e} - \frac{n\mathrm{Li}_2\left(\frac{2c(d+ex)}{2cd-be+\sqrt{b^2-4ac}\,e}\right)}{e} - \frac{n\mathrm{Li}_2\left(\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x),x]
```

```
[Out] -((n*Log[-((e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e))]*Log[d + e*x])/e) - (n*Log[-((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e))]*Log[d + e*x])/e + (Log[d + e*x]*Log[d*
(a + x*(b + c*x))^n])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)])/e - (n*PolyLog[2, (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 493, normalized size = 2.16

method	result
risch	$\frac{\ln(ex+d)\ln\left(\left(c\,x^{2}+bx+a\right)^{n}\right)}{e}-\frac{n\ln(ex+d)\ln\left(\frac{-2c(ex+d)-be+2cd+\sqrt{-4ac\,e^{2}+b^{2}e^{2}}}{-be+2cd+\sqrt{-4ac\,e^{2}+b^{2}e^{2}}}\right)}{e}-\frac{n\ln(ex+d)\ln\left(\frac{2c(ex+d)+be-2cd\cdots}{be-2cd+\sqrt{\cdots}}\right)}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e*ln((c*x^2+b*x+a)^n)-1/e*n*ln(e*x+d)*ln((-2*c*(e*x+d)-b*e+2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*
ln(e*x+d)*ln((2*c*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+
(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((-2*c*(e*x+d)-b*e+2*c*d+(-4*a*c*e^
2+b^2*e^2)^(1/2))/(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))-1/e*n*dilog((2*c
*(e*x+d)+b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/(b*e-2*c*d+(-4*a*c*e^2+b^2*e
^2)^(1/2)))-1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d
*(c*x^2+b*x+a)^n)+1/2*I*ln(e*x+d)/e*Pi*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^
2+1/2*I*ln(e*x+d)/e*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-
1/2*I*ln(e*x+d)/e*Pi*csgn(I*d*(c*x^2+b*x+a)^n)^3+ln(e*x+d)/e*ln(d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(x*e + d), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)/(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x),x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x), x)
```

3.88 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^2} dx$

Optimal. Leaf size=165

$$\frac{\sqrt{b^2-4ac}\, n \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2}-\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}+\frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)}-\frac{\log\left(d(a+\right.}{e(d}$$

```
[Out] -(-b*e+2*c*d)*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/2*(-b*e+2*c*d)*n*ln(c*x^2
+b*x+a)/e/(a*e^2-b*d*e+c*d^2)-ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)+n*arctanh((2*
c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)
```

Rubi [A]
time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{ae^2-bde+cd^2}+\frac{n(2cd-be)\log\left(a+bx+cx^2\right)}{2e\left(ae^2-bde+cd^2\right)}-\frac{n(2cd-be)\log(d+ex)}{e\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] (Sqrt[b^2 - 4*a*c]*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*d^2 - b*d*e
 + a*e^2) - ((2*c*d - b*e)*n*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^2)) + ((
2*c*d - b*e)*n*Log[a + b*x + c*x^2])/(2*e*(c*d^2 - b*d*e + a*e^2)) - Log[d*
(a + b*x + c*x^2)^n]/(e*(d + e*x))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^2}\,dx = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int \frac{b+2cx}{(d+ex)(a+bx+cx^2)}\,dx}{e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int \left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)} + \frac{bcd-b^2e+2ace+c(2cd-be)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right) \cdots}{e}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} + \frac{n\int \frac{bcd-b^2e+2ace+c(2cd-b\cdots}{a+bx+cx^2}}{e\left(cd^2-bde+ae^{\cdots}\right.}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx+cx^2)^n\right)}{e(d+ex)} - \frac{\left(\left(b^2-4ac\right)n\right)\int \frac{1}{a+bx+\cdots}}{2\left(cd^2-bde+ae^{\cdots}\right.}$$

$$= -\frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)n\log\left(a+bx+cx^2\right)}{2e\left(cd^2-bde+ae^2\right)} - \frac{\log\left(d(a+bx\cdots\right.}{e(d+e\cdots}$$

$$= \frac{\sqrt{b^2-4ac}\,n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd^2-bde+ae^2} - \frac{(2cd-be)n\log(d+ex)}{e\left(cd^2-bde+ae^2\right)} + \frac{(2cd-be)\cdots}{2e\left(c\cdots\right.}$$

Mathematica [A]
time = 0.22, size = 166, normalized size = 1.01

$$-\frac{\sqrt{-b^2+4ac}\, n \tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{-cd^2+e(bd-ae)}+\frac{(-2cd+be)n\log(d+ex)}{e\left(cd^2+e(-bd+ae)\right)}-\frac{(-2cd+be)n\log(a+x(b+cx))}{2e\left(cd^2+e(-bd+ae)\right)}-\frac{\log\left(d(a+x(b+cx))^n\right)}{e(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[-b^2 + 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-(c*d^2) +
 e*(b*d - a*e))) + ((-2*c*d + b*e)*n*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d) +
a*e))) - ((-2*c*d + b*e)*n*Log[a + x*(b + c*x)])/(2*e*(c*d^2 + e*(-(b*d) +
a*e))) - Log[d*(a + x*(b + c*x))^n]/(e*(d + e*x))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 1785, normalized size = 10.82

method	result	size
risch	Expression too large to display	1785

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/(e*x+d)*ln((c*x^2+b*x+a)^n)+1/2*(-2*sum(_R*ln(((6*a*c*e^4-2*b^2*e^4+2*
b*c*d*e^3-2*c^2*d^2*e^2)*_R^2+(b*c*e^2*n-2*c^2*d*e*n)*_R+4*c^2*n^2)*x+(-a*b
*e^4+8*a*c*d*e^3-b^2*d*e^3-b*c*d^2*e^2)*_R^2+(-2*a*c*e^2*n+b^2*e^2*n-b*c*d*
e*n)*_R+2*b*c*n^2),_R=RootOf((a*e^4-b*d*e^3+c*d^2*e^2)*_Z^2+(b*e^2*n-2*c*d*
e*n)*_Z+c*n^2))*b*d*e^3*x-4*ln(e*x+d)*c*d*e*n*x-I*Pi*b*e*csgn(I*d*(c*x^2+b*
x+a)^n)^3*d+2*sum(_R*ln(((6*a*c*e^4-2*b^2*e^4+2*b*c*d*e^3-2*c^2*d^2*e^2)*_R
^2+(b*c*e^2*n-2*c^2*d*e*n)*_R+4*c^2*n^2)*x+(-a*b*e^4+8*a*c*d*e^3-b^2*d*e^3-
b*c*d^2*e^2)*_R^2+(-2*a*c*e^2*n+b^2*e^2*n-b*c*d*e*n)*_R+2*b*c*n^2),_R=RootO
f((a*e^4-b*d*e^3+c*d^2*e^2)*_Z^2+(b*e^2*n-2*c*d*e*n)*_Z+c*n^2))*c*d^3*e-4*l
n(e*x+d)*c*d^2*n+2*ln(d)*b*e*d-I*Pi*b*e*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*c
sgn(I*d*(c*x^2+b*x+a)^n)*d+2*sum(_R*ln(((6*a*c*e^4-2*b^2*e^4+2*b*c*d*e^3-2*
c^2*d^2*e^2)*_R^2+(b*c*e^2*n-2*c^2*d*e*n)*_R+4*c^2*n^2)*x+(-a*b*e^4+8*a*c*d
*e^3-b^2*d*e^3-b*c*d^2*e^2)*_R^2+(-2*a*c*e^2*n+b^2*e^2*n-b*c*d*e*n)*_R+2*b*
c*n^2),_R=RootOf((a*e^4-b*d*e^3+c*d^2*e^2)*_Z^2+(b*e^2*n-2*c*d*e*n)*_Z+c*n^
2))*a*e^4*x+2*sum(_R*ln(((6*a*c*e^4-2*b^2*e^4+2*b*c*d*e^3-2*c^2*d^2*e^2)*_R
^2+(b*c*e^2*n-2*c^2*d*e*n)*_R+4*c^2*n^2)*x+(-a*b*e^4+8*a*c*d*e^3-b^2*d*e^3-
b*c*d^2*e^2)*_R^2+(-2*a*c*e^2*n+b^2*e^2*n-b*c*d*e*n)*_R+2*b*c*n^2),_R=RootO
f((a*e^4-b*d*e^3+c*d^2*e^2)*_Z^2+(b*e^2*n-2*c*d*e*n)*_Z+c*n^2))*a*d*e^3-2*s
um(_R*ln(((6*a*c*e^4-2*b^2*e^4+2*b*c*d*e^3-2*c^2*d^2*e^2)*_R^2+(b*c*e^2*n-2
*c^2*d*e*n)*_R+4*c^2*n^2)*x+(-a*b*e^4+8*a*c*d*e^3-b^2*d*e^3-b*c*d^2*e^2)*_R
^2+(-2*a*c*e^2*n+b^2*e^2*n-b*c*d*e*n)*_R+2*b*c*n^2),_R=RootOf((a*e^4-b*d*e^
3+c*d^2*e^2)*_Z^2+(b*e^2*n-2*c*d*e*n)*_Z+c*n^2))*b*d^2*e^2-2*ln(d)*a*e^2-2*
```

```
ln(d)*c*d^2+2*ln(e*x+d)*b*d*e*n-I*Pi*c*d^2*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)
^n)^2-I*Pi*c*d^2*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a
*e^2*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2-I*Pi*a*e^2*csgn(I*(c*x^2+b*x+a)^
n)*csgn(I*d*(c*x^2+b*x+a)^n)^2+I*Pi*a*e^2*csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)
*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*b*e*csgn(I*d)*csgn(I*d*(c*x^2+b*x+a)^n)^2*d
+I*Pi*b*e*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)^2*d+I*Pi*c*d^2*
csgn(I*d)*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*d*(c*x^2+b*x+a)^n)+I*Pi*a*e^2*csgn
(I*d*(c*x^2+b*x+a)^n)^3+I*Pi*c*d^2*csgn(I*d*(c*x^2+b*x+a)^n)^3+2*sum(_R*ln(
((6*a*c*e^4-2*b^2*e^4+2*b*c*d*e^3-2*c^2*d^2*e^2)*_R^2+(b*c*e^2*n-2*c^2*d*e*
n)*_R+4*c^2*n^2)*x+(-a*b*e^4+8*a*c*d*e^3-b^2*d*e^3-b*c*d^2*e^2)*_R^2+(-2*a*
c*e^2*n+b^2*e^2*n-b*c*d*e*n)*_R+2*b*c*n^2),_R=RootOf((a*e^4-b*d*e^3+c*d^2*e
^2)*_Z^2+(b*e^2*n-2*c*d*e*n)*_Z+c*n^2))*c*d^2*e^2*x+2*ln(e*x+d)*b*e^2*n*x)/
e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.42, size = 425, normalized size = 2.58

$$\left[\frac{(nxe^2+dne)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-((bnx+2\,an)e^2-(2\,cdnx+bdn)e)\log\left(cx^2+bx+a\right)-2\,(2\,cd^2n-bnxe^2+(2\,cdnx-bdn)e)\log\left(xe+d\right)-2\,(cd^2-bde+ae^2)\log\left(d\right)}{2\,(cd^3e+axe^4-(bdx-ad)e^3+(cd^2x-bd^2)e^2)}, \frac{2\,(nxe^2+dne)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-((bnx+2\,an)e^2-(2\,cdnx+bdn)e)\log\left(cx^2+bx+a\right)-2\,(2\,cd^2n-bnxe^2+(2\,cdnx-bdn)e)\log\left(xe+d\right)-2\,(cd^2-bde+ae^2)\log\left(d\right)}{2\,(cd^3e+axe^4-(bdx-ad)e^3+(cd^2x-bd^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((n*x*e^2 + d*n*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - ((b*n*x + 2*a*n
)*e^2 - (2*c*d*n*x + b*d*n)*e)*log(c*x^2 + b*x + a) - 2*(2*c*d^2*n - b*n*x*
e^2 + (2*c*d*n*x - b*d*n)*e)*log(x*e + d) - 2*(c*d^2 - b*d*e + a*e^2)*log(d
))/(c*d^3*e + a*x*e^4 - (b*d*x - a*d)*e^3 + (c*d^2*x - b*d^2)*e^2), 1/2*(2*
(n*x*e^2 + d*n*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) - ((b*n*x + 2*a*n)*e^2 - (2*c*d*n*x + b*d*n)*e)*log(c*x^2 +
 b*x + a) - 2*(2*c*d^2*n - b*n*x*e^2 + (2*c*d*n*x - b*d*n)*e)*log(x*e + d)
- 2*(c*d^2 - b*d*e + a*e^2)*log(d))/(c*d^3*e + a*x*e^4 - (b*d*x - a*d)*e^3
+ (c*d^2*x - b*d^2)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.37, size = 284, normalized size = 1.72

$$\frac{(2\,cdn - bne)\log\left(cx^2 + bx + a\right)}{2\left(cd^2e - bde^2 + ae^3\right)} - \frac{\left(b^2n - 4\,acn\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(cd^2 - bde + ae^2\right)\sqrt{-b^2+4\,ac}} - \frac{2\,cdnxe\log\left(xe+d\right) + cd^2n\log\left(cx^2+bx+a\right) - bdne\log\left(cx^2+bx+a\right) + 2\,cd^2n\log\left(xe+d\right) - bnxe^2\log\left(xe+d\right) - bdne\log\left(xe+d\right) + ane^2\log\left(cx^2+bx+a\right) + cd^2\log\left(d\right) - bde\log\left(d\right) + ae^2\log\left(d\right)}{cd^2xe^2 + cd^3e - bdxe^3 - bd^2e^2 + axe^4 + ade^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*c*d*n - b*n*e)*log(c*x^2 + b*x + a)/(c*d^2*e - b*d*e^2 + a*e^3) - (b
^2*n - 4*a*c*n)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*
e^2)*sqrt(-b^2 + 4*a*c)) - (2*c*d*n*x*e*log(x*e + d) + c*d^2*n*log(c*x^2 +
b*x + a) - b*d*n*e*log(c*x^2 + b*x + a) + 2*c*d^2*n*log(x*e + d) - b*n*x*e^
2*log(x*e + d) - b*d*n*e*log(x*e + d) + a*n*e^2*log(c*x^2 + b*x + a) + c*d^
2*log(d) - b*d*e*log(d) + a*e^2*log(d))/(c*d^2*x*e^2 + c*d^3*e - b*d*x*e^3
- b*d^2*e^2 + a*x*e^4 + a*d*e^3)
```

Mupad [B]
time = 3.34, size = 590, normalized size = 3.58

$$\frac{\ln\left(d+e\,x\right)\,\left(b\,e\,n-2\,c\,d\,n\right)}{c\,d^2\,e-b\,d\,e^2+a\,e^3}-\frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{e\,\left(d+e\,x\right)}-\frac{\ln\left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c^2\,n\,x\,\left(b\,e-2\,c\,d\right)-c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)+\frac{c\,e\,n\,\left(b\,e-2\,c\,d+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+2\,x\,b^2\,e^2+b\,c\,d^2-2\,x\,b\,c\,d\,e+a\,b\,e^2+2\,x\,c^2\,d^2-8\,a\,c\,d\,e-6\,a\,x\,c\,e^2\right)}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right)}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right)\,\left(e\,\left(\frac{b\,n}{2}+\frac{n\,\sqrt{b^2-4\,a\,c}}{2}\right)-c\,d\,n\right)}{c\,d^2\,e-b\,d\,e^2+a\,e^3}-\frac{\ln\left(\frac{2\,b\,c^2\,n^2}{e}+\frac{4\,c^3\,n^2\,x}{e}-\frac{n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(c\,n\,\left(-e\,b^2+c\,d\,b+2\,a\,c\,e\right)-c^2\,n\,x\,\left(b\,e-2\,c\,d\right)+\frac{c\,e\,n\,\left(2\,c\,d-b\,e+e\,\sqrt{b^2-4\,a\,c}\right)\,\left(b^2\,d\,e+2\,x\,b^2\,e^2+b\,c\,d^2-2\,x\,b\,c\,d\,e+a\,b\,e^2+2\,x\,c^2\,d^2-8\,a\,c\,d\,e-6\,a\,x\,c\,e^2\right)}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right)}{2\,\left(c\,d^2\,e-b\,d\,e^2+a\,e^3\right)}\right)\,\left(e\,\left(\frac{b\,n}{2}-\frac{n\,\sqrt{b^2-4\,a\,c}}{2}\right)-c\,d\,n\right)}{c\,d^2\,e-b\,d\,e^2+a\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^2,x)
```

```
[Out] (log(d + e*x)*(b*e*n - 2*c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e) - log(d*(a + b
*x + c*x^2)^n)/(e*(d + e*x)) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*
(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(c^2*n*x*(b*e - 2*c*d) - c*n*(2*a*c*e
 - b^2*e + b*c*d) + (c*e*n*(b*e - 2*c*d + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2
*x + 2*c^2*d^2*x + a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e -
2*b*c*d*e*x))/(2*(a*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2
*e)))*(e*((b*n)/2 + (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 +
 c*d^2*e) - (log((2*b*c^2*n^2)/e + (4*c^3*n^2*x)/e - (n*(2*c*d - b*e + e*(b
^2 - 4*a*c)^(1/2))*(c*n*(2*a*c*e - b^2*e + b*c*d) - c^2*n*x*(b*e - 2*c*d) +
 (c*e*n*(2*c*d - b*e + e*(b^2 - 4*a*c)^(1/2))*(2*b^2*e^2*x + 2*c^2*d^2*x +
a*b*e^2 + b*c*d^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*b*c*d*e*x))/(2*(a
*e^3 - b*d*e^2 + c*d^2*e))))/(2*(a*e^3 - b*d*e^2 + c*d^2*e)))*(e*((b*n)/2 -
 (n*(b^2 - 4*a*c)^(1/2))/2) - c*d*n))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

3.89 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^3}\,dx$

Optimal. Leaf size=259

$$\frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(cd^2-bde+ae^2\right)^2}-\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)}{2e\left(cd^2-bde+ae\right.}$$

```
[Out] 1/2*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)-1/2*(2*c^2*d^2+b^2*e^2-2*c
*e*(a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^2+1/4*(2*c^2*d^2+b^2*e^2-2*
c*e*(a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^2-1/2*ln(d*(c*x^2+b*
x+a)^n)/e/(e*x+d)^2+1/2*(-b*e+2*c*d)*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^2
```

Rubi [A]
time = 0.25, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\log\left(a+bx+cx^2\right)}{4e\left(ae^2-bde+cd^2\right)^2}-\frac{n\log(d+ex)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{2e\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{2\left(ae^2-bde+cd^2\right)^2}+\frac{n(2cd-be)}{2e(d+ex)\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{2e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((2*c*d - b*e)*n)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)) + (Sqrt[b^2 - 4*a
*c]*(2*c*d - b*e)*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(2*(c*d^2 - b*d
*e + a*e^2)^2) - ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*Log[d + e*x])
/(2*e*(c*d^2 - b*d*e + a*e^2)^2) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e
))*n*Log[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)^2) - Log[d*(a + b*x
 + c*x^2)^n]/(2*e*(d + e*x)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{(d+ex)^3}\,dx &= -\frac{\log \left(d(a+bx+cx^2)^n\right)}{2e(d+ex)^2} + \frac{n\int \frac{b+2cx}{(d+ex)^2(a+bx+cx^2)}\,dx}{2e} \\ &= -\frac{\log \left(d(a+bx+cx^2)^n\right)}{2e(d+ex)^2} + \frac{n\int \left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^2} + \frac{e(-2c^2d^2-b^2e^2+2ce(bd+ae))}{(cd^2-bde+ae^2)^2(d+ex)}\right.}{} \\ &= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} \\ &= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} \\ &= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} - \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n\log(d+ex)}{2e\left(cd^2-bde+ae^2\right)^2} \\ &= \frac{(2cd-be)n}{2e\left(cd^2-bde+ae^2\right)(d+ex)} + \frac{\sqrt{b^2-4ac}\,(2cd-be)n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{2\left(cd^2-bde+ae^2\right)^2}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 215, normalized size = 0.83

$$\frac{\frac{n(d+ex)\left(2(2cd-be)(cd^2+e(-bd+ae))-2\sqrt{b^2-4ac}\,e(-2cd+be)(d+ex)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2(2c^2d^2+b^2e^2-2ce(bd+ae))(d+ex)\log(d+ex)+(2c^2d^2+b^2e^2-2ce(bd+ae))(d+ex)\log(a+x(b+cx))\right)}{(cd^2+e(-bd+ae))^2}-2\log\left(d(a+x(b+cx))^n\right)}{4e(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^3,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e)) - 2*Sqrt[b^2 - 4*
a*c]*e*(-2*c*d + b*e)*(d + e*x)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[d + e*x] + (2*c^2*d
^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x)*Log[a + x*(b + c*x)]))/(c*d^2 +
 e*(-(b*d) + a*e))^2 - 2*Log[d*(a + x*(b + c*x))^n])/(4*e*(d + e*x)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 14679, normalized size = 56.68

method	result	size
risch	Expression too large to display	14679

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(248) = 496$.
time = 1.34, size = 1337, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c^2*d^4*n - 2*a*b*n*x*e^4 - (2*c*d^3*n*e - b*n*x^2*e^4 + 2*(c*d*n*x
^2 - b*d*n*x)*e^3 + (4*c*d^2*n*x - b*d^2*n)*e^2)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*
x + a)) - 2*(a*b*d*n - (b^2 + 2*a*c)*d*n*x)*e^3 - 2*(3*b*c*d^2*n*x - (b^2 +
 2*a*c)*d^2*n)*e^2 + 2*(2*c^2*d^3*n*x - 3*b*c*d^3*n)*e + (((b^2 - 2*a*c)*n*
x^2 - 2*a^2*n)*e^4 - 2*(b*c*d*n*x^2 - 2*a*b*d*n - (b^2 - 2*a*c)*d*n*x)*e^3
+ (2*c^2*d^2*n*x^2 - 4*b*c*d^2*n*x - (b^2 + 6*a*c)*d^2*n)*e^2 + 2*(2*c^2*d^
3*n*x + b*c*d^3*n)*e)*log(c*x^2 + b*x + a) - 2*(2*c^2*d^4*n + (b^2 - 2*a*c)
*n*x^2*e^4 - 2*(b*c*d*n*x^2 - (b^2 - 2*a*c)*d*n*x)*e^3 + (2*c^2*d^2*n*x^2 -
 4*b*c*d^2*n*x + (b^2 - 2*a*c)*d^2*n)*e^2 + 2*(2*c^2*d^3*n*x - b*c*d^3*n)*e
)*log(x*e + d) - 2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2 + 2*a*c)*d^2
*e^2 + a^2*e^4)*log(d))/(c^2*d^6*e + a^2*x^2*e^7 - 2*(a*b*d*x^2 - a^2*d*x)*
e^6 - (4*a*b*d^2*x - (b^2 + 2*a*c)*d^2*x^2 - a^2*d^2)*e^5 - 2*(b*c*d^3*x^2
+ a*b*d^3 - (b^2 + 2*a*c)*d^3*x)*e^4 + (c^2*d^4*x^2 - 4*b*c*d^4*x + (b^2 +
2*a*c)*d^4)*e^3 + 2*(c^2*d^5*x - b*c*d^5)*e^2), 1/4*(4*c^2*d^4*n - 2*a*b*n*
x*e^4 + 2*(2*c*d^3*n*e - b*n*x^2*e^4 + 2*(c*d*n*x^2 - b*d*n*x)*e^3 + (4*c*d
^2*n*x - b*d^2*n)*e^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) - 2*(a*b*d*n - (b^2 + 2*a*c)*d*n*x)*e^3 - 2*(3*b*c*d^2
*n*x - (b^2 + 2*a*c)*d^2*n)*e^2 + 2*(2*c^2*d^3*n*x - 3*b*c*d^3*n)*e + (((b^
2 - 2*a*c)*n*x^2 - 2*a^2*n)*e^4 - 2*(b*c*d*n*x^2 - 2*a*b*d*n - (b^2 - 2*a*c
)*d*n*x)*e^3 + (2*c^2*d^2*n*x^2 - 4*b*c*d^2*n*x - (b^2 + 6*a*c)*d^2*n)*e^2
+ 2*(2*c^2*d^3*n*x + b*c*d^3*n)*e)*log(c*x^2 + b*x + a) - 2*(2*c^2*d^4*n +
(b^2 - 2*a*c)*n*x^2*e^4 - 2*(b*c*d*n*x^2 - (b^2 - 2*a*c)*d*n*x)*e^3 + (2*c^
```

```
2*d^2*n*x^2 - 4*b*c*d^2*n*x + (b^2 - 2*a*c)*d^2*n)*e^2 + 2*(2*c^2*d^3*n*x -
 b*c*d^3*n)*e)*log(x*e + d) - 2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2
 + 2*a*c)*d^2*e^2 + a^2*e^4)*log(d))/(c^2*d^6*e + a^2*x^2*e^7 - 2*(a*b*d*x^
2 - a^2*d*x)*e^6 - (4*a*b*d^2*x - (b^2 + 2*a*c)*d^2*x^2 - a^2*d^2)*e^5 - 2*
(b*c*d^3*x^2 + a*b*d^3 - (b^2 + 2*a*c)*d^3*x)*e^4 + (c^2*d^4*x^2 - 4*b*c*d^
4*x + (b^2 + 2*a*c)*d^4)*e^3 + 2*(c^2*d^5*x - b*c*d^5)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(248) = 496.
time = 3.35, size = 887, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^2*d^2*n - 2*b*c*d*n*e + b^2*n*e^2 - 2*a*c*n*e^2)*log(c*x^2 + b*x +
 a)/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4
+ a^2*e^5) - 1/2*(2*b^2*c*d*n - 8*a*c^2*d*n - b^3*n*e + 4*a*b*c*n*e)*arctan
((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*
a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d^2*n
*x^2*e^2*log(x*e + d) + 4*c^2*d^3*n*x*e*log(x*e + d) - 2*c^2*d^3*n*x*e + c^
2*d^4*n*log(c*x^2 + b*x + a) - 2*b*c*d^3*n*e*log(c*x^2 + b*x + a) + 2*c^2*d
^4*n*log(x*e + d) - 2*b*c*d*n*x^2*e^3*log(x*e + d) - 4*b*c*d^2*n*x*e^2*log(
x*e + d) - 2*b*c*d^3*n*e*log(x*e + d) - 2*c^2*d^4*n + 3*b*c*d^2*n*x*e^2 + 3
*b*c*d^3*n*e + b^2*d^2*n*e^2*log(c*x^2 + b*x + a) + 2*a*c*d^2*n*e^2*log(c*x
^2 + b*x + a) + b^2*n*x^2*e^4*log(x*e + d) - 2*a*c*n*x^2*e^4*log(x*e + d) +
 2*b^2*d*n*x*e^3*log(x*e + d) - 4*a*c*d*n*x*e^3*log(x*e + d) + b^2*d^2*n*e^
2*log(x*e + d) - 2*a*c*d^2*n*e^2*log(x*e + d) + c^2*d^4*log(d) - 2*b*c*d^3*
e*log(d) - b^2*d*n*x*e^3 - 2*a*c*d*n*x*e^3 - b^2*d^2*n*e^2 - 2*a*c*d^2*n*e^
2 - 2*a*b*d*n*e^3*log(c*x^2 + b*x + a) + b^2*d^2*e^2*log(d) + 2*a*c*d^2*e^2
*log(d) + a*b*n*x*e^4 + a*b*d*n*e^3 + a^2*n*e^4*log(c*x^2 + b*x + a) - 2*a*
b*d*e^3*log(d) + a^2*e^4*log(d))/(c^2*d^4*x^2*e^3 + 2*c^2*d^5*x*e^2 + c^2*d
^6*e - 2*b*c*d^3*x^2*e^4 - 4*b*c*d^4*x*e^3 - 2*b*c*d^5*e^2 + b^2*d^2*x^2*e^
5 + 2*a*c*d^2*x^2*e^5 + 2*b^2*d^3*x*e^4 + 4*a*c*d^3*x*e^4 + b^2*d^4*e^3 + 2
```

```
*a*c*d^4*e^3 - 2*a*b*d*x^2*e^6 - 4*a*b*d^2*x*e^5 - 2*a*b*d^3*e^4 + a^2*x^2*
e^7 + 2*a^2*d*x*e^6 + a^2*d^2*e^5)
```

Mupad [B]
time = 4.74, size = 1715, normalized size = 6.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^3,x)
```

```
[Out] (log(3*b^2*c^3*d^4 - 12*a*c^4*d^4 - 2*b^5*e^4*x - 12*a^3*c^2*e^4 - 2*a*b^4*
e^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) + 1
1*a^2*b^2*c*e^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2 + 40*a^2*c^3*d^2*e^2 + 2*
a*b^3*e^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^3
*d^3*e + 6*a*b^3*c*d*e^3 + 12*a*b^3*c*e^4*x - 32*a*c^4*d^3*e*x + 8*b^4*c*d*
e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1
/2) - 24*a^2*b*c^2*d*e^3 - 16*a^2*b*c^2*e^4*x + 32*a^2*c^3*d*e^3*x + 8*b^2*
c^3*d^3*e*x + 16*a^2*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 -
 4*a*c)^(1/2) + b^3*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 -
4*a*c)^(1/2) - 14*a*b^2*c^2*d^2*e^2 - 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2
*e^2*(b^2 - 4*a*c)^(1/2) - 20*a*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*
c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) -
8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2)
- 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 48*a*b*c^3*d^2*e^2*x - 40*a*b^2*c^2
*d*e^3*x + 20*a*b*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2))*(e*((c*d*n*(b^2 - 4*a*c)
^(1/2))/2 - (b*c*d*n)/2) - e^2*((a*c*n)/2 - (b^2*n)/4 + (b*n*(b^2 - 4*a*c)^
(1/2))/4) + (c^2*d^2*n)/2))/(a^2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^
4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e^2) - (log(d + e*x)*(e^2*(b^2*n - 2*a*c*n) +
 2*c^2*d^2*n - 2*b*c*d*e*n))/(2*a^2*e^5 + 2*c^2*d^4*e + 2*b^2*d^2*e^3 - 4*a
*b*d*e^4 + 4*a*c*d^2*e^3 - 4*b*c*d^3*e^2) + (log(2*a*b^4*e^4 + 12*a*c^4*d^4
 + 2*b^5*e^4*x + 12*a^3*c^2*e^4 - 3*b^2*c^3*d^4 + 2*b^4*e^4*x*(b^2 - 4*a*c)
^(1/2) + 6*c^4*d^4*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^2*c*e^4 + 2*b^3*c^2*d^3
*e - b^4*c*d^2*e^2 - 40*a^2*c^3*d^2*e^2 + 2*a*b^3*e^4*(b^2 - 4*a*c)^(1/2) +
 3*b*c^3*d^4*(b^2 - 4*a*c)^(1/2) - 8*a*b*c^3*d^3*e - 6*a*b^3*c*d*e^3 - 12*a
*b^3*c*e^4*x + 32*a*c^4*d^3*e*x - 8*b^4*c*d*e^3*x - 5*a^2*b*c*e^4*(b^2 - 4*
a*c)^(1/2) - 16*a*c^3*d^3*e*(b^2 - 4*a*c)^(1/2) + 24*a^2*b*c^2*d*e^3 + 16*a
^2*b*c^2*e^4*x - 32*a^2*c^3*d*e^3*x - 8*b^2*c^3*d^3*e*x + 16*a^2*c^2*d*e^3*
(b^2 - 4*a*c)^(1/2) - 2*b^2*c^2*d^3*e*(b^2 - 4*a*c)^(1/2) + b^3*c*d^2*e^2*(
b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 14*a*b^2*c^2*d^2
*e^2 + 12*b^3*c^2*d^2*e^2*x + 14*a*b*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 20*a
*c^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 14*b^2*c^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/
2) - 10*a*b^2*c*d*e^3*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*e^4*x*(b^2 - 4*a*c)^(
1/2) - 12*b*c^3*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 8*b^3*c*d*e^3*x*(b^2 - 4*a*c)
^(1/2) - 48*a*b*c^3*d^2*e^2*x + 40*a*b^2*c^2*d*e^3*x + 20*a*b*c^2*d*e^3*x*(
```

```
b^2 - 4*a*c)^(1/2))*(e^2*((b^2*n)/4 - (a*c*n)/2 + (b*n*(b^2 - 4*a*c)^(1/2))
/4) - e*((c*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b*c*d*n)/2) + (c^2*d^2*n)/2))/(a^
2*e^5 + c^2*d^4*e + b^2*d^2*e^3 - 2*a*b*d*e^4 + 2*a*c*d^2*e^3 - 2*b*c*d^3*e
^2) - log(d*(a + b*x + c*x^2)^n)/(2*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (n*(b*e
- 2*c*d))/((2*d*e + 2*e^2*x)*(a*e^2 + c*d^2 - b*d*e))
```

3.90 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^4}\,dx$

Optimal. Leaf size=356

$$\frac{(2cd-be)n}{6e\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{3e\left(cd^2-bde+ae^2\right)^2(d+ex)}+\frac{\sqrt{b^2-4ac}\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)}{3\left(cd^2-bde+ae^2\right.}$$

```
[Out] 1/6*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+1/3*(2*c^2*d^2+b^2*e^2-2
*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-1/3*(-b*e+2*c*d)*(c^2*d^2
+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^3+1/6*(-b*e+2*c
*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^
2)^3-1/3*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^3+1/3*(3*c^2*d^2+b^2*e^2-c*e*(a*e+
3*b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b
*d*e+c*d^2)^3
```

Rubi [A]
time = 0.39, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)\log\left(a+bx+cx^2\right)}{6e\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{3e(d+ex)\left(ae^2-bde+cd^2\right)^2}-\frac{n(2cd-be)\log(d+ex)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{3e\left(ae^2-bde+cd^2\right)^3}+\frac{n\sqrt{b^2-4ac}\left(-ce(ae+3bd)+b^2e^2+3c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{3\left(ae^2-bde+cd^2\right)^3}+\frac{n(2cd-be)}{6e(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{3e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((2*c*d - b*e)*n)/(6*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) + ((2*c^2*d^2 +
 b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(3*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x))
+ (Sqrt[b^2 - 4*a*c]*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*n*ArcTanh[(b
 + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(3*(c*d^2 - b*d*e + a*e^2)^3) - ((2*c*d - b*e
)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*Log[d + e*x])/(3*e*(c*d^2 - b*d
*e + a*e^2)^3) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n*L
og[a + b*x + c*x^2])/(6*e*(c*d^2 - b*d*e + a*e^2)^3) - Log[d*(a + b*x + c*x
^2)^n]/(3*e*(d + e*x)^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^4}\,dx = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3} + \frac{n\int \frac{b+2cx}{(d+ex)^3(a+bx+cx^2)}\,dx}{3e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{3e(d+ex)^3} + \frac{n\int\Big(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^3} + \frac{e(-2c^2d^2-b^2e^2+2ce(bd+ae)}{(cd^2-bde+ae^2)^2(d+ex)^2}}{}$$

$$= \frac{(2cd-be)n}{6e\,(cd^2-bde+ae^2)\,(d+ex)^2} + \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n}{3e\,(cd^2-bde+ae^2)^2\,(d+ex)} - \frac{(2cd-}{}$$

$$= \frac{(2cd-be)n}{6e\,(cd^2-bde+ae^2)\,(d+ex)^2} + \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n}{3e\,(cd^2-bde+ae^2)^2\,(d+ex)} - \frac{(2cd-}{}$$

$$= \frac{(2cd-be)n}{6e\,(cd^2-bde+ae^2)\,(d+ex)^2} + \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n}{3e\,(cd^2-bde+ae^2)^2\,(d+ex)} - \frac{(2cd-}{}$$

$$= \frac{(2cd-be)n}{6e\,(cd^2-bde+ae^2)\,(d+ex)^2} + \frac{(2c^2d^2+b^2e^2-2ce(bd+ae))\,n}{3e\,(cd^2-bde+ae^2)^2\,(d+ex)} + \frac{\sqrt{b^2-}}{}$$

Mathematica [A]
time = 0.77, size = 310, normalized size = 0.87

$$\frac{\frac{n(d+ex)\left((2cd-be)\left(cd^2+e(-bd+ae)\right)^2+2\left(cd^2+e(-bd+ae)\right)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)(d+ex)+2\sqrt{b^2-4ac}\,e\left(3c^2d^2+b^2e^2-ce(3bd+ae)\right)(d+ex)^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-2(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)(d+ex)^2\log(d+ex)+(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)(d+ex)^2\log(a+x(b+cx))\right)}{\left(cd^2+e(-bd+ae)\right)^3}-2\log\left(d(a+x(b+cx))^n\right)}{6e(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^4,x]
```

```
[Out] ((n*(d + e*x)*((2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^2 + 2*(c*d^2 + e*(-
(b*d) + a*e))*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 2*Sqrt[
b^2 - 4*a*c]*e*(3*c^2*d^2 + b^2*e^2 - c*e*(3*b*d + a*e))*(d + e*x)^2*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - 2*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e
*(b*d + 3*a*e))*(d + e*x)^2*Log[d + e*x] + (2*c*d - b*e)*(c^2*d^2 + b^2*e^2
 - c*e*(b*d + 3*a*e))*(d + e*x)^2*Log[a + x*(b + c*x)]))/(c*d^2 + e*(-(b*d)
 + a*e))^3 - 2*Log[d*(a + x*(b + c*x))^n])/(6*e*(d + e*x)^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.28, size = 306209, normalized size = 860.14

method	result	size
risch	Expression too large to display	306209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(346) = 692$.
time = 8.93, size = 3011, normalized size = 8.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] [1/6*(6*c^3*d^6*n - (3*c^2*d^5*n*e + (b^2 - a*c)*n*x^3*e^6 - 3*(b*c*d*n*x^3
 - (b^2 - a*c)*d*n*x^2)*e^5 + 3*(c^2*d^2*n*x^3 - 3*b*c*d^2*n*x^2 + (b^2 - a
*c)*d^2*n*x)*e^4 + (9*c^2*d^3*n*x^2 - 9*b*c*d^3*n*x + (b^2 - a*c)*d^3*n)*e^
3 + 3*(3*c^2*d^4*n*x - b*c*d^4*n)*e^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) -
(a^2*b*n*x - 2*(a*b^2 - 2*a^2*c)*n*x^2)*e^6 - (2*b^3*d*n*x^2 + a^2*b*d*n -
6*(a*b^2 - a^2*c)*d*n*x)*e^5 + (6*b^2*c*d^2*n*x^2 - (5*b^3 + 6*a*b*c)*d^2*n
*x + 2*(2*a*b^2 - a^2*c)*d^2*n)*e^4 - (8*b*c^2*d^3*n*x^2 - 4*(4*b^2*c + a*c
^2)*d^3*n*x + 3*(b^3 + 2*a*b*c)*d^3*n)*e^3 + (4*c^3*d^4*n*x^2 - 21*b*c^2*d^
4*n*x + 2*(5*b^2*c + 2*a*c^2)*d^4*n)*e^2 + (10*c^3*d^5*n*x - 13*b*c^2*d^5*n
)*e - (((b^3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*e^6 - 3*((b^2*c - 2*a*c^2)*d*n*x^3
 + 2*a^2*b*d*n - (b^3 - 3*a*b*c)*d*n*x^2)*e^5 + 3*(b*c^2*d^2*n*x^3 - 3*(b^2
*c - 2*a*c^2)*d^2*n*x^2 + (b^3 - 3*a*b*c)*d^2*n*x + 2*(a*b^2 + a^2*c)*d^2*n
)*e^4 - (2*c^3*d^3*n*x^3 - 9*b*c^2*d^3*n*x^2 + 9*(b^2*c - 2*a*c^2)*d^3*n*x
+ (b^3 + 15*a*b*c)*d^3*n)*e^3 - 3*(2*c^3*d^4*n*x^2 - 3*b*c^2*d^4*n*x - (b^2
*c + 4*a*c^2)*d^4*n)*e^2 - 3*(2*c^3*d^5*n*x + b*c^2*d^5*n)*e)*log(c*x^2 + b
*x + a) - 2*(2*c^3*d^6*n - (b^3 - 3*a*b*c)*n*x^3*e^6 + 3*((b^2*c - 2*a*c^2)
*d*n*x^3 - (b^3 - 3*a*b*c)*d*n*x^2)*e^5 - 3*(b*c^2*d^2*n*x^3 - 3*(b^2*c - 2
*a*c^2)*d^2*n*x^2 + (b^3 - 3*a*b*c)*d^2*n*x)*e^4 + (2*c^3*d^3*n*x^3 - 9*b*c
^2*d^3*n*x^2 + 9*(b^2*c - 2*a*c^2)*d^3*n*x - (b^3 - 3*a*b*c)*d^3*n)*e^3 + 3
*(2*c^3*d^4*n*x^2 - 3*b*c^2*d^4*n*x + (b^2*c - 2*a*c^2)*d^4*n)*e^2 + 3*(2*c
^3*d^5*n*x - b*c^2*d^5*n)*e)*log(x*e + d) - 2*(c^3*d^6 - 3*b*c^2*d^5*e + 3*
```

```
(b^2*c + a*c^2)*d^4*e^2 - 3*a^2*b*d*e^5 - (b^3 + 6*a*b*c)*d^3*e^3 + a^3*e^6
 + 3*(a*b^2 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e + a^3*x^3*e^10 - 3*(a^2*b*
d*x^3 - a^3*d*x^2)*e^9 - 3*(3*a^2*b*d^2*x^2 - a^3*d^2*x - (a*b^2 + a^2*c)*d
^2*x^3)*e^8 - (9*a^2*b*d^3*x + (b^3 + 6*a*b*c)*d^3*x^3 - a^3*d^3 - 9*(a*b^2
 + a^2*c)*d^3*x^2)*e^7 + 3*((b^2*c + a*c^2)*d^4*x^3 - a^2*b*d^4 - (b^3 + 6*
a*b*c)*d^4*x^2 + 3*(a*b^2 + a^2*c)*d^4*x)*e^6 - 3*(b*c^2*d^5*x^3 - 3*(b^2*c
 + a*c^2)*d^5*x^2 + (b^3 + 6*a*b*c)*d^5*x - (a*b^2 + a^2*c)*d^5)*e^5 + (c^3
*d^6*x^3 - 9*b*c^2*d^6*x^2 + 9*(b^2*c + a*c^2)*d^6*x - (b^3 + 6*a*b*c)*d^6)
*e^4 + 3*(c^3*d^7*x^2 - 3*b*c^2*d^7*x + (b^2*c + a*c^2)*d^7)*e^3 + 3*(c^3*d
^8*x - b*c^2*d^8)*e^2), 1/6*(6*c^3*d^6*n + 2*(3*c^2*d^5*n*e + (b^2 - a*c)*n
*x^3*e^6 - 3*(b*c*d*n*x^3 - (b^2 - a*c)*d*n*x^2)*e^5 + 3*(c^2*d^2*n*x^3 - 3
*b*c*d^2*n*x^2 + (b^2 - a*c)*d^2*n*x)*e^4 + (9*c^2*d^3*n*x^2 - 9*b*c*d^3*n*
x + (b^2 - a*c)*d^3*n)*e^3 + 3*(3*c^2*d^4*n*x - b*c*d^4*n)*e^2)*sqrt(-b^2 +
 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (a^2*b*n*x
- 2*(a*b^2 - 2*a^2*c)*n*x^2)*e^6 - (2*b^3*d*n*x^2 + a^2*b*d*n - 6*(a*b^2 -
a^2*c)*d*n*x)*e^5 + (6*b^2*c*d^2*n*x^2 - (5*b^3 + 6*a*b*c)*d^2*n*x + 2*(2*a
*b^2 - a^2*c)*d^2*n)*e^4 - (8*b*c^2*d^3*n*x^2 - 4*(4*b^2*c + a*c^2)*d^3*n*x
 + 3*(b^3 + 2*a*b*c)*d^3*n)*e^3 + (4*c^3*d^4*n*x^2 - 21*b*c^2*d^4*n*x + 2*(
5*b^2*c + 2*a*c^2)*d^4*n)*e^2 + (10*c^3*d^5*n*x - 13*b*c^2*d^5*n)*e - (((b^
3 - 3*a*b*c)*n*x^3 + 2*a^3*n)*e^6 - 3*((b^2*c - 2*a*c^2)*d*n*x^3 + 2*a^2*b*
d*n - (b^3 - 3*a*b*c)*d*n*x^2)*e^5 + 3*(b*c^2*d^2*n*x^3 - 3*(b^2*c - 2*a*c^
2)*d^2*n*x^2 + (b^3 - 3*a*b*c)*d^2*n*x + 2*(a*b^2 + a^2*c)*d^2*n)*e^4 - (2*
c^3*d^3*n*x^3 - 9*b*c^2*d^3*n*x^2 + 9*(b^2*c - 2*a*c^2)*d^3*n*x + (b^3 + 15
*a*b*c)*d^3*n)*e^3 - 3*(2*c^3*d^4*n*x^2 - 3*b*c^2*d^4*n*x - (b^2*c + 4*a*c^
2)*d^4*n)*e^2 - 3*(2*c^3*d^5*n*x + b*c^2*d^5*n)*e)*log(c*x^2 + b*x + a) - 2
*(2*c^3*d^6*n - (b^3 - 3*a*b*c)*n*x^3*e^6 + 3*((b^2*c - 2*a*c^2)*d*n*x^3 -
(b^3 - 3*a*b*c)*d*n*x^2)*e^5 - 3*(b*c^2*d^2*n*x^3 - 3*(b^2*c - 2*a*c^2)*d^2
*n*x^2 + (b^3 - 3*a*b*c)*d^2*n*x)*e^4 + (2*c^3*d^3*n*x^3 - 9*b*c^2*d^3*n*x^
2 + 9*(b^2*c - 2*a*c^2)*d^3*n*x - (b^3 - 3*a*b*c)*d^3*n)*e^3 + 3*(2*c^3*d^4
*n*x^2 - 3*b*c^2*d^4*n*x + (b^2*c - 2*a*c^2)*d^4*n)*e^2 + 3*(2*c^3*d^5*n*x
- b*c^2*d^5*n)*e)*log(x*e + d) - 2*(c^3*d^6 - 3*b*c^2*d^5*e + 3*(b^2*c + a*
c^2)*d^4*e^2 - 3*a^2*b*d*e^5 - (b^3 + 6*a*b*c)*d^3*e^3 + a^3*e^6 + 3*(a*b^2
 + a^2*c)*d^2*e^4)*log(d))/(c^3*d^9*e + a^3*x^3*e^10 - 3*(a^2*b*d*x^3 - a^3
*d*x^2)*e^9 - 3*(3*a^2*b*d^2*x^2 - a^3*d^2*x - (a*b^2 + a^2*c)*d^2*x^3)*e^8
 - (9*a^2*b*d^3*x + (b^3 + 6*a*b*c)*d^3*x^3 - a^3*d^3 - 9*(a*b^2 + a^2*c)*d
^3*x^2)*e^7 + 3*((b^2*c + a*c^2)*d^4*x^3 - a^2*b*d^4 - (b^3 + 6*a*b*c)*d^4*
x^2 + 3*(a*b^2 + a^2*c)*d^4*x)*e^6 - 3*(b*c^2*d^5*x^3 - 3*(b^2*c + a*c^2)*d
^5*x^2 + (b^3 + 6*a*b*c)*d^5*x - (a*b^2 + a^2*c)*d^5)*e^5 + (c^3*d^6*x^3 -
9*b*c^2*d^6*x^2 + 9*(b^2*c + a*c^2)*d^6*x - (b^3 + 6*a*b*c)*d^6)*e^4 + 3*(c
^3*d^7*x^2 - 3*b*c^2*d^7*x + (b^2*c + a*c^2)*d^7)*e^3 + 3*(c^3*d^8*x - b*c^
2*d^8)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(346) = 692$.
time = 6.07, size = 1963, normalized size = 5.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*c^3*d^3*n - 3*b*c^2*d^2*n*e + 3*b^2*c*d*n*e^2 - 6*a*c^2*d*n*e^2 - b^
3*n*e^3 + 3*a*b*c*n*e^3)*log(c*x^2 + b*x + a)/(c^3*d^6*e - 3*b*c^2*d^5*e^2
+ 3*b^2*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b
^2*d^2*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) - 1/3*(3*b^2*c^2*d^
2*n - 12*a*c^3*d^2*n - 3*b^3*c*d*n*e + 12*a*b*c^2*d*n*e + b^4*n*e^2 - 5*a*b
^2*c*n*e^2 + 4*a^2*c^2*n*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*
d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a
*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)
*sqrt(-b^2 + 4*a*c)) - 1/6*(4*c^3*d^3*n*x^3*e^3*log(x*e + d) + 12*c^3*d^4*n
*x^2*e^2*log(x*e + d) + 12*c^3*d^5*n*x*e*log(x*e + d) - 4*c^3*d^4*n*x^2*e^2
 - 10*c^3*d^5*n*x*e + 2*c^3*d^6*n*log(c*x^2 + b*x + a) - 6*b*c^2*d^5*n*e*lo
g(c*x^2 + b*x + a) + 4*c^3*d^6*n*log(x*e + d) - 6*b*c^2*d^2*n*x^3*e^4*log(x
*e + d) - 18*b*c^2*d^3*n*x^2*e^3*log(x*e + d) - 18*b*c^2*d^4*n*x*e^2*log(x*
e + d) - 6*b*c^2*d^5*n*e*log(x*e + d) - 6*c^3*d^6*n + 8*b*c^2*d^3*n*x^2*e^3
 + 21*b*c^2*d^4*n*x*e^2 + 13*b*c^2*d^5*n*e + 6*b^2*c*d^4*n*e^2*log(c*x^2 +
b*x + a) + 6*a*c^2*d^4*n*e^2*log(c*x^2 + b*x + a) + 6*b^2*c*d*n*x^3*e^5*log
(x*e + d) - 12*a*c^2*d*n*x^3*e^5*log(x*e + d) + 18*b^2*c*d^2*n*x^2*e^4*log(
x*e + d) - 36*a*c^2*d^2*n*x^2*e^4*log(x*e + d) + 18*b^2*c*d^3*n*x*e^3*log(x
*e + d) - 36*a*c^2*d^3*n*x*e^3*log(x*e + d) + 6*b^2*c*d^4*n*e^2*log(x*e + d
) - 12*a*c^2*d^4*n*e^2*log(x*e + d) + 2*c^3*d^6*log(d) - 6*b*c^2*d^5*e*log(
d) - 6*b^2*c*d^2*n*x^2*e^4 - 16*b^2*c*d^3*n*x*e^3 - 4*a*c^2*d^3*n*x*e^3 - 1
0*b^2*c*d^4*n*e^2 - 4*a*c^2*d^4*n*e^2 - 2*b^3*d^3*n*e^3*log(c*x^2 + b*x + a
) - 12*a*b*c*d^3*n*e^3*log(c*x^2 + b*x + a) - 2*b^3*n*x^3*e^6*log(x*e + d)
+ 6*a*b*c*n*x^3*e^6*log(x*e + d) - 6*b^3*d*n*x^2*e^5*log(x*e + d) + 18*a*b*
c*d*n*x^2*e^5*log(x*e + d) - 6*b^3*d^2*n*x*e^4*log(x*e + d) + 18*a*b*c*d^2*
n*x*e^4*log(x*e + d) - 2*b^3*d^3*n*e^3*log(x*e + d) + 6*a*b*c*d^3*n*e^3*log
(x*e + d) + 6*b^2*c*d^4*e^2*log(d) + 6*a*c^2*d^4*e^2*log(d) + 2*b^3*d*n*x^2
*e^5 + 5*b^3*d^2*n*x*e^4 + 6*a*b*c*d^2*n*x*e^4 + 3*b^3*d^3*n*e^3 + 6*a*b*c*
d^3*n*e^3 + 6*a*b^2*d^2*n*e^4*log(c*x^2 + b*x + a) + 6*a^2*c*d^2*n*e^4*log(
c*x^2 + b*x + a) - 2*b^3*d^3*e^3*log(d) - 12*a*b*c*d^3*e^3*log(d) - 2*a*b^2
*n*x^2*e^6 + 4*a^2*c*n*x^2*e^6 - 6*a*b^2*d*n*x*e^5 + 6*a^2*c*d*n*x*e^5 - 4*
```

```
a*b^2*d^2*n*e^4 + 2*a^2*c*d^2*n*e^4 - 6*a^2*b*d*n*e^5*log(c*x^2 + b*x + a)
+ 6*a*b^2*d^2*e^4*log(d) + 6*a^2*c*d^2*e^4*log(d) + a^2*b*n*x*e^6 + a^2*b*d
*n*e^5 + 2*a^3*n*e^6*log(c*x^2 + b*x + a) - 6*a^2*b*d*e^5*log(d) + 2*a^3*e^
6*log(d))/(c^3*d^6*x^3*e^4 + 3*c^3*d^7*x^2*e^3 + 3*c^3*d^8*x*e^2 + c^3*d^9*
e - 3*b*c^2*d^5*x^3*e^5 - 9*b*c^2*d^6*x^2*e^4 - 9*b*c^2*d^7*x*e^3 - 3*b*c^2
*d^8*e^2 + 3*b^2*c*d^4*x^3*e^6 + 3*a*c^2*d^4*x^3*e^6 + 9*b^2*c*d^5*x^2*e^5
+ 9*a*c^2*d^5*x^2*e^5 + 9*b^2*c*d^6*x*e^4 + 9*a*c^2*d^6*x*e^4 + 3*b^2*c*d^7
*e^3 + 3*a*c^2*d^7*e^3 - b^3*d^3*x^3*e^7 - 6*a*b*c*d^3*x^3*e^7 - 3*b^3*d^4*
x^2*e^6 - 18*a*b*c*d^4*x^2*e^6 - 3*b^3*d^5*x*e^5 - 18*a*b*c*d^5*x*e^5 - b^3
*d^6*e^4 - 6*a*b*c*d^6*e^4 + 3*a*b^2*d^2*x^3*e^8 + 3*a^2*c*d^2*x^3*e^8 + 9*
a*b^2*d^3*x^2*e^7 + 9*a^2*c*d^3*x^2*e^7 + 9*a*b^2*d^4*x*e^6 + 9*a^2*c*d^4*x
*e^6 + 3*a*b^2*d^5*e^5 + 3*a^2*c*d^5*e^5 - 3*a^2*b*d*x^3*e^9 - 9*a^2*b*d^2*
x^2*e^8 - 9*a^2*b*d^3*x*e^7 - 3*a^2*b*d^4*e^6 + a^3*x^3*e^10 + 3*a^3*d*x^2*
e^9 + 3*a^3*d^2*x*e^8 + a^3*d^3*e^7)
```

Mupad [B]
time = 11.30, size = 2707, normalized size = 7.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^4,x)
```

```
[Out] (log(d + e*x)*(e^3*(b^3*n - 3*a*b*c*n) + e^2*(6*a*c^2*d*n - 3*b^2*c*d*n) -
2*c^3*d^3*n + 3*b*c^2*d^2*e*n))/(3*a^3*e^7 + 3*c^3*d^6*e - 3*b^3*d^3*e^4 +
9*a*b^2*d^2*e^5 + 9*a*c^2*d^4*e^3 + 9*a^2*c*d^2*e^5 - 9*b*c^2*d^5*e^2 + 9*b
^2*c*d^4*e^3 - 9*a^2*b*d*e^6 - 18*a*b*c*d^3*e^4) - (log(32*a*b^5*e^5 - 2*a*
e^5*(b^2 - 4*a*c)^(5/2) - 192*a*c^5*d^5 + 32*b^6*e^5*x + 48*b^2*c^4*d^5 - 1
8*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) - 3*b^5*e^5*x*(b^2 - 4*a*c)^(1/2) + 96*c^5*
d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320*a^3*b*c^2*e^5 - 704*a^3
*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 - 64*a^3*c^3*e^5*x + 1152*
a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 - 33*b*d*e^4*(b^2 - 4*a*c)^(5/2) - 11*
b*e^5*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^2*e^5*(b^2 - 4*a*c)^(3/2) - 6*a*b^4*e^
5*(b^2 - 4*a*c)^(1/2) + 48*b*c^4*d^5*(b^2 - 4*a*c)^(1/2) + 18*b^3*d*e^4*(b^
2 - 4*a*c)^(3/2) + 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2) + 44*c*d^2*e^3*(b^2 - 4
*a*c)^(5/2) + 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) + 22*c*d*e^4*x*(b^2 - 4*a*c)
^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 + 120*b^3*c^2*d^3*e^2*(b^2 -
 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e*x - 160*b^5*c*d*e^4*x +
 144*b^2*c^4*d^4*e*x - 72*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 120*b^2*c^3*d
^4*e*(b^2 - 4*a*c)^(1/2) - 60*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 144*c^3*d
^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^2 + 320*a*b^3*c^2*d^2*e^
3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4 + 400*a^2*b^2*c^2*e^5*x
+ 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x + 304*b^4*c^2*d^2*e^3*x -
216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^2*c^3*d^2*e^3*x + 240*b^
2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 120*b^3*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(
```

```
1/2) - 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 108*b^2*c*d*e^4*x*(b^2 - 4*a
*c)^(3/2) + 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 1152*a*b*c^4*d^3*e^2*x +
 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e^3*((b^3*n)/6 - (b^2*n*(
b^2 - 4*a*c)^(1/2))/6 + (a*c*n*(b^2 - 4*a*c)^(1/2))/6 - (a*b*c*n)/2) + e^2*
(a*c^2*d*n - (b^2*c*d*n)/2 + (b*c*d*n*(b^2 - 4*a*c)^(1/2))/2) + e*((b*c^2*d
^2*n)/2 - (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3*d^3*n)/3))/(a^3*e^7 + c
^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^
5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e^6 - 6*a*b*c*d^3*e^4) -
(log(2*a*e^5*(b^2 - 4*a*c)^(5/2) + 32*a*b^5*e^5 - 192*a*c^5*d^5 + 32*b^6*e^
5*x + 48*b^2*c^4*d^5 + 18*b^3*e^5*x*(b^2 - 4*a*c)^(3/2) + 3*b^5*e^5*x*(b^2
- 4*a*c)^(1/2) - 96*c^5*d^5*x*(b^2 - 4*a*c)^(1/2) - 208*a^2*b^3*c*e^5 + 320
*a^3*b*c^2*e^5 - 704*a^3*c^3*d*e^4 - 48*b^3*c^3*d^4*e - 16*b^5*c*d^2*e^3 -
64*a^3*c^3*e^5*x + 1152*a^2*c^4*d^3*e^2 + 48*b^4*c^2*d^3*e^2 + 33*b*d*e^4*(
b^2 - 4*a*c)^(5/2) + 11*b*e^5*x*(b^2 - 4*a*c)^(5/2) + 24*a*b^2*e^5*(b^2 - 4
*a*c)^(3/2) + 6*a*b^4*e^5*(b^2 - 4*a*c)^(1/2) - 48*b*c^4*d^5*(b^2 - 4*a*c)^
(1/2) - 18*b^3*d*e^4*(b^2 - 4*a*c)^(3/2) - 15*b^5*d*e^4*(b^2 - 4*a*c)^(1/2)
 - 44*c*d^2*e^3*(b^2 - 4*a*c)^(5/2) - 72*c^3*d^4*e*(b^2 - 4*a*c)^(3/2) - 22
*c*d*e^4*x*(b^2 - 4*a*c)^(5/2) + 192*a*b*c^4*d^4*e - 128*a*b^4*c*d*e^4 - 12
0*b^3*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 224*a*b^4*c*e^5*x - 576*a*c^5*d^4*e
*x - 160*b^5*c*d*e^4*x + 144*b^2*c^4*d^4*e*x + 72*b*c^2*d^3*e^2*(b^2 - 4*a*
c)^(3/2) + 120*b^2*c^3*d^4*e*(b^2 - 4*a*c)^(1/2) + 60*b^4*c*d^2*e^3*(b^2 -
4*a*c)^(1/2) - 144*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(3/2) - 480*a*b^2*c^3*d^3*e^
2 + 320*a*b^3*c^2*d^2*e^3 - 1024*a^2*b*c^3*d^2*e^3 + 688*a^2*b^2*c^2*d*e^4
+ 400*a^2*b^2*c^2*e^5*x + 1408*a^2*c^4*d^2*e^3*x - 288*b^3*c^3*d^3*e^2*x +
304*b^4*c^2*d^2*e^3*x + 216*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(3/2) - 1568*a*b^
2*c^3*d^2*e^3*x - 240*b^2*c^3*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 120*b^3*c^2*d
^2*e^3*x*(b^2 - 4*a*c)^(1/2) + 240*b*c^4*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 108*
b^2*c*d*e^4*x*(b^2 - 4*a*c)^(3/2) - 30*b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) +
1152*a*b*c^4*d^3*e^2*x + 992*a*b^3*c^2*d*e^4*x - 1408*a^2*b*c^3*d*e^4*x)*(e
^3*((b^3*n)/6 + (b^2*n*(b^2 - 4*a*c)^(1/2))/6 - (a*c*n*(b^2 - 4*a*c)^(1/2))
/6 - (a*b*c*n)/2) - e^2*((b^2*c*d*n)/2 - a*c^2*d*n + (b*c*d*n*(b^2 - 4*a*c)
^(1/2))/2) + e*((b*c^2*d^2*n)/2 + (c^2*d^2*n*(b^2 - 4*a*c)^(1/2))/2) - (c^3
*d^3*n)/3))/(a^3*e^7 + c^3*d^6*e - b^3*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a*c^2*
d^4*e^3 + 3*a^2*c*d^2*e^5 - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - 3*a^2*b*d*e
^6 - 6*a*b*c*d^3*e^4) - ((a*b*e^3*n - 6*c^2*d^3*n - 3*b^2*d*e^2*n + 2*a*c*d
*e^2*n + 7*b*c*d^2*e*n)/(2*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 -
 2*b*c*d^3*e + 2*a*c*d^2*e^2)) - (n*x*(b^2*e^3 + 2*c^2*d^2*e - 2*a*c*e^3 -
2*b*c*d*e^2))/(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e
+ 2*a*c*d^2*e^2))/(3*d^2*e + 3*e^3*x^2 + 6*d*e^2*x) - log(d*(a + b*x + c*x^
2)^n)/(3*e*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x))
```

3.91 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{(d+ex)^5}\,dx$

Optimal. Leaf size=519

$$\frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3}+\frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2}+\frac{(2cd-be)\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)n}{4e\left(cd^2-bde+ae^2\right)^3(d+ex)}$$

[Out] 1/12*(-b*e+2*c*d)*n/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^3+1/8*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^2+1/4*(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))*n/e/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)-1/4*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)^4+1/8*(2*c^4*d^4+b^4*e^4-4*b^2*c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))*n*ln(c*x^2+b*x+a)/e/(a*e^2-b*d*e+c*d^2)^4-1/4*ln(d*(c*x^2+b*x+a)^n)/e/(e*x+d)^4+1/4*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^4

Rubi [A]
time = 0.63, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2605, 814, 648, 632, 212, 642}

$$\frac{n\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)\log\left(a+bx+cx^2\right)}{8e\left(ae^2-bde+cd^2\right)^4}-\frac{n\log(d+ex)\left(2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4\right)}{4e\left(ae^2-bde+cd^2\right)^4}+\frac{n(2cd-be)\left(-ce(3ae+bd)+b^2e^2+c^2d^2\right)}{4e(d+ex)\left(ae^2-bde+cd^2\right)^3}+\frac{n\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)}{8e(d+ex)^2\left(ae^2-bde+cd^2\right)^2}+\frac{n\sqrt{b^2-4ac}\,(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{4\left(ae^2-bde+cd^2\right)^4}+\frac{n(2cd-be)}{12e(d+ex)^3\left(ae^2-bde+cd^2\right)}-\frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{4e(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Int[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]

[Out] ((2*c*d - b*e)*n)/(12*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^3) + ((2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n)/(8*e*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^2) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e))*n)/(4*e*(c*d^2 - b*d*e + a*e^2)^3*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(4*(c*d^2 - b*d*e + a*e^2)^4) - ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*n*Log[d + e*x])/(4*e*(c*d^2 - b*d*e + a*e^2)^4) + ((2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*n*Log[a + b*x + c*x^2])/(8*e*(c*d^2 - b*d*e + a*e^2)^4) - Log[d*(a + b*x + c*x^2)^n]/(4*e*(d + e*x)^4)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(d(a+bx+cx^2)^n\right)}{(d+ex)^5}\,dx = -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4} + \frac{n\int \frac{b+2cx}{(d+ex)^4(a+bx+cx^2)}\,dx}{4e}$$

$$= -\frac{\log\left(d(a+bx+cx^2)^n\right)}{4e(d+ex)^4} + \frac{n\int \left(\frac{e(-2cd+be)}{(cd^2-bde+ae^2)(d+ex)^4} + \frac{e(-2c^2d^2-b^2e^2+2ce(bd+ae)}{(cd^2-bde+ae^2)^2(d+ex)^3}\right.}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd -}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd -}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd -}{}$$

$$= \frac{(2cd-be)n}{12e\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)n}{8e\left(cd^2-bde+ae^2\right)^2(d+ex)^2} + \frac{(2cd -}{}$$

Mathematica [**A**]
time = 1.32, size = 469, normalized size = 0.90

$$\frac{\frac{n(d+ex)\left(2(2cd-be)(cd^2+e(-bd+ae))^3+3(cd^2+e(-bd+ae))^2(2c^2d^2+b^2e^2-2ce(bd+ae))(d+ex)+6(2cd-be)(cd^2+e(-bd+ae))(c^2d^2+b^2e^2-ce(bd+3ae))(d+ex)^2+6\sqrt{b^2-4ac}\,e(2cd-be)(2c^2d^2+b^2e^2-2ce(bd+ae))(d+ex)^3\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-6(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2))(d+ex)^3\log(d+ex)+3(2c^4d^4+b^4e^4-4b^2ce^3(bd+ae)-4c^3d^2e(bd+3ae)+2c^2e^2(3b^2d^2+6abde+a^2e^2))(d+ex)^3\log(a+x(b+cx))\right)}{(cd^2+e(-bd+ae))^4}-6\log\left(d(a+x(b+cx))^n\right)}{24e(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(d + e*x)^5,x]
```

```
[Out] ((n*(d + e*x)*(2*(2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))^3 + 3*(c*d^2 + e*
(-(b*d) + a*e))^2*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))*(d + e*x) + 6*(
2*c*d - b*e)*(c*d^2 + e*(-(b*d) + a*e))*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a
*e))*(d + e*x)^2 + 6*Sqrt[b^2 - 4*a*c]*e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2
 - 2*c*e*(b*d + a*e))*(d + e*x)^3*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] -
6*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*e
) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[d + e*x] +
 3*(2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*d^2*e*(b*d + 3*a*
e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))*(d + e*x)^3*Log[a + x*(b
+ c*x)]))/(c*d^2 + e*(-(b*d) + a*e))^4 - 6*Log[d*(a + x*(b + c*x))^n])/(24*
e*(d + e*x)^4)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 0.39, size = 1137077, normalized size = 2190.90

method	result	size

risch	Expression too large to display	1137077

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2936 vs. $2(506) = 1012$.
time = 43.25, size = 5892, normalized size = 11.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="fricas")
```

```
[Out] [1/24*(22*c^4*d^8*n + 3*(4*c^3*d^7*n*e - (b^3 - 2*a*b*c)*n*x^4*e^8 + 4*((b^
2*c - a*c^2)*d*n*x^4 - (b^3 - 2*a*b*c)*d*n*x^3)*e^7 - 2*(3*b*c^2*d^2*n*x^4
- 8*(b^2*c - a*c^2)*d^2*n*x^3 + 3*(b^3 - 2*a*b*c)*d^2*n*x^2)*e^6 + 4*(c^3*d
^3*n*x^4 - 6*b*c^2*d^3*n*x^3 + 6*(b^2*c - a*c^2)*d^3*n*x^2 - (b^3 - 2*a*b*c
)*d^3*n*x)*e^5 + (16*c^3*d^4*n*x^3 - 36*b*c^2*d^4*n*x^2 + 16*(b^2*c - a*c^2
)*d^4*n*x - (b^3 - 2*a*b*c)*d^4*n)*e^4 + 4*(6*c^3*d^5*n*x^2 - 6*b*c^2*d^5*n
*x + (b^2*c - a*c^2)*d^5*n)*e^3 + 2*(8*c^3*d^6*n*x - 3*b*c^2*d^6*n)*e^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*
(2*c*x + b))/(c*x^2 + b*x + a)) - (2*a^3*b*n*x + 6*(a*b^3 - 3*a^2*b*c)*n*x^
3 - 3*(a^2*b^2 - 2*a^3*c)*n*x^2)*e^8 - 2*(a^3*b*d*n - 3*(b^4 - 6*a^2*c^2)*d
*n*x^3 + 6*(2*a*b^3 - 5*a^2*b*c)*d*n*x^2 - 2*(3*a^2*b^2 - 2*a^3*c)*d*n*x)*e
^7 - (12*(2*b^3*c - 3*a*b*c^2)*d^2*n*x^3 - 3*(7*b^4 + 4*a*b^2*c - 38*a^2*c^
2)*d^2*n*x^2 + 12*(3*a*b^3 - 4*a^2*b*c)*d^2*n*x - (9*a^2*b^2 - 2*a^3*c)*d^2
*n)*e^6 + 2*(6*(3*b^2*c^2 - 2*a*c^3)*d^3*n*x^3 - 6*(7*b^3*c - 8*a*b*c^2)*d^
3*n*x^2 + (13*b^4 + 24*a*b^2*c - 54*a^2*c^2)*d^3*n*x - 3*(3*a*b^3 - a^2*b*c
```

```
)*d^3*n)*e^5 - (30*b*c^3*d^4*n*x^3 - 3*(43*b^2*c^2 - 22*a*c^3)*d^4*n*x^2 +
2*(53*b^3*c - 27*a*b*c^2)*d^4*n*x - (11*b^4 + 36*a*b^2*c - 30*a^2*c^2)*d^4*
n)*e^4 + 2*(6*c^4*d^5*n*x^3 - 54*b*c^3*d^5*n*x^2 + 12*(7*b^2*c^2 - 2*a*c^3)
*d^5*n*x - (23*b^3*c + 3*a*b*c^2)*d^5*n)*e^3 + (42*c^4*d^6*n*x^2 - 140*b*c^
3*d^6*n*x + 3*(25*b^2*c^2 - 2*a*c^3)*d^6*n)*e^2 + 2*(26*c^4*d^7*n*x - 31*b*
c^3*d^7*n)*e + 3*(((b^4 - 4*a*b^2*c + 2*a^2*c^2)*n*x^4 - 2*a^4*n)*e^8 - 4*(
(b^3*c - 3*a*b*c^2)*d*n*x^4 - 2*a^3*b*d*n - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d
*n*x^3)*e^7 + 2*(3*(b^2*c^2 - 2*a*c^3)*d^2*n*x^4 - 8*(b^3*c - 3*a*b*c^2)*d^
2*n*x^3 + 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*n*x^2 - 2*(3*a^2*b^2 + 2*a^3*
c)*d^2*n)*e^6 - 4*(b*c^3*d^3*n*x^4 - 6*(b^2*c^2 - 2*a*c^3)*d^3*n*x^3 + 6*(b
^3*c - 3*a*b*c^2)*d^3*n*x^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*n*x - 2*(a*
b^3 + 3*a^2*b*c)*d^3*n)*e^5 + (2*c^4*d^4*n*x^4 - 16*b*c^3*d^4*n*x^3 + 36*(b
^2*c^2 - 2*a*c^3)*d^4*n*x^2 - 16*(b^3*c - 3*a*b*c^2)*d^4*n*x - (b^4 + 28*a*
b^2*c + 10*a^2*c^2)*d^4*n)*e^4 + 4*(2*c^4*d^5*n*x^3 - 6*b*c^3*d^5*n*x^2 + 6
*(b^2*c^2 - 2*a*c^3)*d^5*n*x + (b^3*c + 9*a*b*c^2)*d^5*n)*e^3 + 2*(6*c^4*d^
6*n*x^2 - 8*b*c^3*d^6*n*x - (3*b^2*c^2 + 10*a*c^3)*d^6*n)*e^2 + 4*(2*c^4*d^
7*n*x + b*c^3*d^7*n)*e)*log(c*x^2 + b*x + a) - 6*(2*c^4*d^8*n + (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*n*x^4*e^8 - 4*((b^3*c - 3*a*b*c^2)*d*n*x^4 - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*d*n*x^3)*e^7 + 2*(3*(b^2*c^2 - 2*a*c^3)*d^2*n*x^4 - 8*(b
^3*c - 3*a*b*c^2)*d^2*n*x^3 + 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*n*x^2)*e^
6 - 4*(b*c^3*d^3*n*x^4 - 6*(b^2*c^2 - 2*a*c^3)*d^3*n*x^3 + 6*(b^3*c - 3*a*b
*c^2)*d^3*n*x^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*n*x)*e^5 + (2*c^4*d^4*n
*x^4 - 16*b*c^3*d^4*n*x^3 + 36*(b^2*c^2 - 2*a*c^3)*d^4*n*x^2 - 16*(b^3*c -
3*a*b*c^2)*d^4*n*x + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^4*n)*e^4 + 4*(2*c^4*d^
5*n*x^3 - 6*b*c^3*d^5*n*x^2 + 6*(b^2*c^2 - 2*a*c^3)*d^5*n*x - (b^3*c - 3*a*
b*c^2)*d^5*n)*e^3 + 2*(6*c^4*d^6*n*x^2 - 8*b*c^3*d^6*n*x + 3*(b^2*c^2 - 2*a
*c^3)*d^6*n)*e^2 + 4*(2*c^4*d^7*n*x - b*c^3*d^7*n)*e)*log(x*e + d) - 6*(c^4
*d^8 - 4*b*c^3*d^7*e + 2*(3*b^2*c^2 + 2*a*c^3)*d^6*e^2 - 4*(b^3*c + 3*a*b*c
^2)*d^5*e^3 - 4*a^3*b*d*e^7 + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*e^4 + a^4*
e^8 - 4*(a*b^3 + 3*a^2*b*c)*d^3*e^5 + 2*(3*a^2*b^2 + 2*a^3*c)*d^2*e^6)*log(
d))/(c^4*d^12*e + a^4*x^4*e^13 - 4*(a^3*b*d*x^4 - a^4*d*x^3)*e^12 - 2*(8*a^
3*b*d^2*x^3 - 3*a^4*d^2*x^2 - (3*a^2*b^2 + 2*a^3*c)*d^2*x^4)*e^11 - 4*(6*a^
3*b*d^3*x^2 - a^4*d^3*x + (a*b^3 + 3*a^2*b*c)*d^3*x^4 - 2*(3*a^2*b^2 + 2*a^
3*c)*d^3*x^3)*e^10 - (16*a^3*b*d^4*x - (b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^4*x
^4 - a^4*d^4 + 16*(a*b^3 + 3*a^2*b*c)*d^4*x^3 - 12*(3*a^2*b^2 + 2*a^3*c)*d^
4*x^2)*e^9 - 4*((b^3*c + 3*a*b*c^2)*d^5*x^4 + a^3*b*d^5 - (b^4 + 12*a*b^2*c
 + 6*a^2*c^2)*d^5*x^3 + 6*(a*b^3 + 3*a^2*b*c)*d^5*x^2 - 2*(3*a^2*b^2 + 2*a^
3*c)*d^5*x)*e^8 + 2*((3*b^2*c^2 + 2*a*c^3)*d^6*x^4 - 8*(b^3*c + 3*a*b*c^2)*
d^6*x^3 + 3*(b^4 + 12*a*b^2*c + 6*a^2*c^2)*d^6*x^2 - 8*(a*b^3 + 3*a^2*b*c)*
d^6*x + (3*a^2*b^2 + 2*a^3*c)*d^6)*e^7 - 4*(b*c^3*d^7*x^4 - 2*(3*b^2*c^2 +
2*a*c^3)*d^7*x^3 + 6*(b^3*c + 3*a*b*c^2)*d^7*x^2 - (b^4 + 12*a*b^2*c + 6*a^
2*c^2)*d^7*x + (a*b^3 + 3*a^2*b*c)*d^7)*e^6 + (c^4*d^8*x^4 - 16*b*c^3*d^8*x
^3 + 12*(3*b^2*c^2 + 2*a*c^3)*d^8*x^2 - 16*(b^3*c + 3*a*b*c^2)*d^8*x + (b^4
 + 12*a*b^2*c + 6*a^2*c^2)*d^8)*e^5 + 4*(c^4*d^9*x^3 - 6*b*c^3*d^9*x^2 + 2*
(3*b^2*c^2 + 2*a*c^3)*d^9*x - (b^3*c + 3*a*b*c^2)*d^9)*e^4 + 2*(3*c^4*d^10*
```

```
x^2 - 8*b*c^3*d^10*x + (3*b^2*c^2 + 2*a*c^3)*d^10)*e^3 + 4*(c^4*d^11*x - b*
c^3*d^11)*e^2), 1/24*(22*c^4*d^8*n + 6*(4*c^3*d^7*n*e - (b^3 - 2*a*b*c)*n*x
^4*e^8 + 4*((b^2*c - a*c^2)*d*n*x^4 - (b^3 - 2*a*b*c)*d*n*x^3)*e^7 - 2*(3*b
*c^2*d^2*n*x^4 - 8*(b^2*c - a*c^2)*d^2*n*x^3 + 3*(b^3 - 2*a*b*c)*d^2*n*x^2)
*e^6 + 4*(c^3*d^3*n*x^4 - 6*b*c^2*d^3*n*x^3 + 6*(b^2*c - a*c^2)*d^3*n*x^2 -
 (b^3 - 2*a*b*c)*d^3*n*x)*e^5 + (16*c^3*d^4*n*x^3 - 36*b*c^2*d^4*n*x^2 + 16
*(b^2*c - a*c^2)*d^4*n*x - (b^3 - 2*a*b*c)*d^4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(e*x+d)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3759 vs. 2(506) = 1012.
time = 5.15, size = 3759, normalized size = 7.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] 1/8*(2*c^4*d^4*n - 4*b*c^3*d^3*n*e + 6*b^2*c^2*d^2*n*e^2 - 12*a*c^3*d^2*n*e
^2 - 4*b^3*c*d*n*e^3 + 12*a*b*c^2*d*n*e^3 + b^4*n*e^4 - 4*a*b^2*c*n*e^4 + 2
*a^2*c^2*n*e^4)*log(c*x^2 + b*x + a)/(c^4*d^8*e - 4*b*c^3*d^7*e^2 + 6*b^2*c
^2*d^6*e^3 + 4*a*c^3*d^6*e^3 - 4*b^3*c*d^5*e^4 - 12*a*b*c^2*d^5*e^4 + b^4*d
^4*e^5 + 12*a*b^2*c*d^4*e^5 + 6*a^2*c^2*d^4*e^5 - 4*a*b^3*d^3*e^6 - 12*a^2*
b*c*d^3*e^6 + 6*a^2*b^2*d^2*e^7 + 4*a^3*c*d^2*e^7 - 4*a^3*b*d*e^8 + a^4*e^9
) - 1/4*(4*b^2*c^3*d^3*n - 16*a*c^4*d^3*n - 6*b^3*c^2*d^2*n*e + 24*a*b*c^3*
d^2*n*e + 4*b^4*c*d*n*e^2 - 20*a*b^2*c^2*d*n*e^2 + 16*a^2*c^3*d*n*e^2 - b^5
*n*e^3 + 6*a*b^3*c*n*e^3 - 8*a^2*b*c^2*n*e^3)*arctan((2*c*x + b)/sqrt(-b^2
+ 4*a*c))/((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 + 4*a*c^3*d^6*e^2 -
 4*b^3*c*d^5*e^3 - 12*a*b*c^2*d^5*e^3 + b^4*d^4*e^4 + 12*a*b^2*c*d^4*e^4 +
6*a^2*c^2*d^4*e^4 - 4*a*b^3*d^3*e^5 - 12*a^2*b*c*d^3*e^5 + 6*a^2*b^2*d^2*e^
6 + 4*a^3*c*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*sqrt(-b^2 + 4*a*c)) - 1/24*(
12*c^4*d^4*n*x^4*e^4*log(x*e + d) + 48*c^4*d^5*n*x^3*e^3*log(x*e + d) + 72*
c^4*d^6*n*x^2*e^2*log(x*e + d) + 48*c^4*d^7*n*x*e*log(x*e + d) - 12*c^4*d^5
*n*x^3*e^3 - 42*c^4*d^6*n*x^2*e^2 - 52*c^4*d^7*n*x*e + 6*c^4*d^8*n*log(c*x^
2 + b*x + a) - 24*b*c^3*d^7*n*e*log(c*x^2 + b*x + a) + 12*c^4*d^8*n*log(x*e
 + d) - 24*b*c^3*d^3*n*x^4*e^5*log(x*e + d) - 96*b*c^3*d^4*n*x^3*e^4*log(x*
e + d) - 144*b*c^3*d^5*n*x^2*e^3*log(x*e + d) - 96*b*c^3*d^6*n*x*e^2*log(x*
```

e + d) - 24*b*c^3*d^7*n*e*log(x*e + d) - 22*c^4*d^8*n + 30*b*c^3*d^4*n*x^3*
e^4 + 108*b*c^3*d^5*n*x^2*e^3 + 140*b*c^3*d^6*n*x*e^2 + 62*b*c^3*d^7*n*e +
36*b^2*c^2*d^6*n*e^2*log(c*x^2 + b*x + a) + 24*a*c^3*d^6*n*e^2*log(c*x^2 +
b*x + a) + 36*b^2*c^2*d^2*n*x^4*e^6*log(x*e + d) - 72*a*c^3*d^2*n*x^4*e^6*l
og(x*e + d) + 144*b^2*c^2*d^3*n*x^3*e^5*log(x*e + d) - 288*a*c^3*d^3*n*x^3*
e^5*log(x*e + d) + 216*b^2*c^2*d^4*n*x^2*e^4*log(x*e + d) - 432*a*c^3*d^4*n
*x^2*e^4*log(x*e + d) + 144*b^2*c^2*d^5*n*x*e^3*log(x*e + d) - 288*a*c^3*d^
5*n*x*e^3*log(x*e + d) + 36*b^2*c^2*d^6*n*e^2*log(x*e + d) - 72*a*c^3*d^6*n
*e^2*log(x*e + d) + 6*c^4*d^8*log(d) - 24*b*c^3*d^7*e*log(d) - 36*b^2*c^2*d
^3*n*x^3*e^5 + 24*a*c^3*d^3*n*x^3*e^5 - 129*b^2*c^2*d^4*n*x^2*e^4 + 66*a*c^
3*d^4*n*x^2*e^4 - 168*b^2*c^2*d^5*n*x*e^3 + 48*a*c^3*d^5*n*x*e^3 - 75*b^2*c
^2*d^6*n*e^2 + 6*a*c^3*d^6*n*e^2 - 24*b^3*c*d^5*n*e^3*log(c*x^2 + b*x + a)
- 72*a*b*c^2*d^5*n*e^3*log(c*x^2 + b*x + a) - 24*b^3*c*d*n*x^4*e^7*log(x*e
+ d) + 72*a*b*c^2*d*n*x^4*e^7*log(x*e + d) - 96*b^3*c*d^2*n*x^3*e^6*log(x*e
 + d) + 288*a*b*c^2*d^2*n*x^3*e^6*log(x*e + d) - 144*b^3*c*d^3*n*x^2*e^5*lo
g(x*e + d) + 432*a*b*c^2*d^3*n*x^2*e^5*log(x*e + d) - 96*b^3*c*d^4*n*x*e^4*
log(x*e + d) + 288*a*b*c^2*d^4*n*x*e^4*log(x*e + d) - 24*b^3*c*d^5*n*e^3*lo
g(x*e + d) + 72*a*b*c^2*d^5*n*e^3*log(x*e + d) + 36*b^2*c^2*d^6*e^2*log(d)
+ 24*a*c^3*d^6*e^2*log(d) + 24*b^3*c*d^2*n*x^3*e^6 - 36*a*b*c^2*d^2*n*x^3*e
^6 + 84*b^3*c*d^3*n*x^2*e^5 - 96*a*b*c^2*d^3*n*x^2*e^5 + 106*b^3*c*d^4*n*x*
e^4 - 54*a*b*c^2*d^4*n*x*e^4 + 46*b^3*c*d^5*n*e^3 + 6*a*b*c^2*d^5*n*e^3 + 6
*b^4*d^4*n*e^4*log(c*x^2 + b*x + a) + 72*a*b^2*c*d^4*n*e^4*log(c*x^2 + b*x
+ a) + 36*a^2*c^2*d^4*n*e^4*log(c*x^2 + b*x + a) + 6*b^4*n*x^4*e^8*log(x*e
+ d) - 24*a*b^2*c*n*x^4*e^8*log(x*e + d) + 12*a^2*c^2*n*x^4*e^8*log(x*e + d
) + 24*b^4*d*n*x^3*e^7*log(x*e + d) - 96*a*b^2*c*d*n*x^3*e^7*log(x*e + d) +
 48*a^2*c^2*d*n*x^3*e^7*log(x*e + d) + 36*b^4*d^2*n*x^2*e^6*log(x*e + d) -
144*a*b^2*c*d^2*n*x^2*e^6*log(x*e + d) + 72*a^2*c^2*d^2*n*x^2*e^6*log(x*e +
 d) + 24*b^4*d^3*n*x*e^5*log(x*e + d) - 96*a*b^2*c*d^3*n*x*e^5*log(x*e + d)
 + 48*a^2*c^2*d^3*n*x*e^5*log(x*e + d) + 6*b^4*d^4*n*e^4*log(x*e + d) - 24*
a*b^2*c*d^4*n*e^4*log(x*e + d) + 12*a^2*c^2*d^4*n*e^4*log(x*e + d) - 24*b^3
*c*d^5*e^3*log(d) - 72*a*b*c^2*d^5*e^3*log(d) - 6*b^4*d*n*x^3*e^7 + 36*a^2*
c^2*d*n*x^3*e^7 - 21*b^4*d^2*n*x^2*e^6 - 12*a*b^2*c*d^2*n*x^2*e^6 + 114*a^2
*c^2*d^2*n*x^2*e^6 - 26*b^4*d^3*n*x*e^5 - 48*a*b^2*c*d^3*n*x*e^5 + 108*a^2*
c^2*d^3*n*x*e^5 - 11*b^4*d^4*n*e^4 - 36*a*b^2*c*d^4*n*e^4 + 30*a^2*c^2*d^4*
n*e^4 - 24*a*b^3*d^3*n*e^5*log(c*x^2 + b*x + a) - 72*a^2*b*c*d^3*n*e^5*log(
c*x^2 + b*x + a) + 6*b^4*d^4*e^4*log(d) + 72*a*b^2*c*d^4*e^4*log(d) + 36*a^
2*c^2*d^4*e^4*log(d) + 6*a*b^3*n*x^3*e^8 - 18*a^2*b*c*n*x^3*e^8 + 24*a*b^3*
d*n*x^2*e^7 - 60*a^2*b*c*d*n*x^2*e^7 + 36*a*b^3*d^2*n*x*e^6 - 48*a^2*b*c*d^
2*n*x*e^6 + 18*a*b^3*d^3*n*e^5 - 6*a^2*b*c*d^3*n*e^5 + 36*a^2*b^2*d^2*n*e^6
*log(c*x^2 + b*x + a) + 24*a^3*c*d^2*n*e^6*log(c*x^2 + b*x + a) - 24*a*b^3*
d^3*e^5*log(d) - 72*a^2*b*c*d^3*e^5*log(d) - 3*a^2*b^2*n*x^2*e^8 + 6*a^3*c*
n*x^2*e^8 - 12*a^2*b^2*d*n*x*e^7 + 8*a^3*c*d*n*x*e^7 - 9*a^2*b^2*d^2*n*e^6
+ 2*a^3*c*d^2*n*e^6 - 24*a^3*b*d*n*e^7*log(c*x^2 + b*x + a) + 36*a^2*b^2*d^
2*e^6*log(d) + 24*a^3*c*d^2*e^6*log(d) + 2*a^3*b*n*x*e^8 + 2*a^3*b*d*n*e^7
+ 6*a^4*n*e^8*log(c*x^2 + b*x + a) - 24*a^3*b*d*e^7*log(d) + 6*a^4*e^8*log(

```
d))/(c^4*d^8*x^4*e^5 + 4*c^4*d^9*x^3*e^4 + 6*c^4*d^10*x^2*e^3 + 4*c^4*d^11*
x*e^2 + c^4*d^12*e - 4*b*c^3*d^7*x^4*e^6 - 16*b*c^3*d^8*x^3*e^5 - 24*b*c^3*
d^9*x^2*e^4 - 16*b*c^3*d^10*x*e^3 - 4*b*c^3*d^1...
```

Mupad [B]
time = 18.95, size = 2500, normalized size = 4.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(d + e*x)^5,x)
```

```
[Out] (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) - 6*a*e^6*(
4*a*c - b^2)^3 + 96*c^5*d^6*(4*a*c - b^2) - 10*b*e^6*x*(4*a*c - b^2)^3 - 10
*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^4*e^6*x*
(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6*x*(b^2
- 4*a*c)^(1/2) + 44*a*b^2*e^6*(4*a*c - b^2)^2 - 16*b^3*d*e^5*(4*a*c - b^2)^
2 + 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5/2) + 44
*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) - 232*c^3*d^4*e
^2*(4*a*c - b^2)^2 - 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2 - 4*a*c
)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a*c)^(1/2
) - 48*b*d*e^5*(4*a*c - b^2)^3 + 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^2*d*e^5*
(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e^5*(b^2
- 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) + 288*b*c^2*d^3*e^3*(4*a
*c - b^2)^2 - 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 - 40*b^2*c^3*d^4*e^2*(4*a*c
- b^2) + 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^2 - 4*a*
c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 - 128*b*c^4*d^5*e*(4*
a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*c*d*e^5*x*(4*a*c - b^
2)^3 + 320*c^5*d^5*e*x*(4*a*c - b^2) - 110*b^4*c*d^2*e^4*(4*a*c - b^2) - 16
8*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*c)^(1/2)
 + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 - 4*a*c)^(
1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b^2 - 4*a
*c)^(3/2) - 264*b^2*c*d*e^5*x*(4*a*c - b^2)^2 - 800*b*c^4*d^4*e^2*x*(4*a*c
- b^2) - 928*b*c^3*d^3*e^3*x*(b^2 - 4*a*c)^(3/2) - 116*b*c*d*e^5*x*(b^2 - 4
*a*c)^(5/2) + 528*b*c^2*d^2*e^4*x*(4*a*c - b^2)^2 + 800*b^2*c^3*d^3*e^3*x*(
4*a*c - b^2) - 400*b^3*c^2*d^2*e^4*x*(4*a*c - b^2) + 696*b^2*c^2*d^2*e^4*x*
(b^2 - 4*a*c)^(3/2) + 720*b^2*c^4*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 480*b^3*c
^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 180*b^4*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2
) + 100*b^4*c*d*e^5*x*(4*a*c - b^2) - 576*b*c^5*d^5*e*x*(b^2 - 4*a*c)^(1/2)
 - 232*b^3*c*d*e^5*x*(b^2 - 4*a*c)^(3/2) - 36*b^5*c*d*e^5*x*(b^2 - 4*a*c)^(
1/2))*(e^4*((b^4*n)/8 + (b^3*n*(b^2 - 4*a*c)^(1/2))/8 + (a^2*c^2*n)/4 - (a*
b^2*c*n)/2 - (a*b*c*n*(b^2 - 4*a*c)^(1/2))/4) - e^3*((b^3*c*d*n)/2 - (3*a*b
*c^2*d*n)/2 - (a*c^2*d*n*(b^2 - 4*a*c)^(1/2))/2 + (b^2*c*d*n*(b^2 - 4*a*c)^
(1/2))/2) + e^2*((3*b^2*c^2*d^2*n)/4 - (3*a*c^3*d^2*n)/2 + (3*b*c^2*d^2*n*(
```

```
b^2 - 4*a*c)^(1/2))/4) - e*((b*c^3*d^3*n)/2 + (c^3*d^3*n*(b^2 - 4*a*c)^(1/2
))/2) + (c^4*d^4*n)/4))/(a^4*e^9 + c^4*d^8*e + b^4*d^4*e^5 - 4*a*b^3*d^3*e^
6 + 4*a*c^3*d^6*e^3 + 4*a^3*c*d^2*e^7 - 4*b*c^3*d^7*e^2 - 4*b^3*c*d^5*e^4 +
 6*a^2*b^2*d^2*e^7 + 6*a^2*c^2*d^4*e^5 + 6*b^2*c^2*d^6*e^3 - 4*a^3*b*d*e^8
- 12*a*b*c^2*d^5*e^4 + 12*a*b^2*c*d^4*e^5 - 12*a^2*b*c*d^3*e^6) - (log(d +
e*x)*(e^2*(6*b^2*c^2*d^2*n - 12*a*c^3*d^2*n) - e^3*(4*b^3*c*d*n - 12*a*b*c^
2*d*n) + e^4*(b^4*n + 2*a^2*c^2*n - 4*a*b^2*c*n) + 2*c^4*d^4*n - 4*b*c^3*d^
3*e*n))/(4*a^4*e^9 + 4*c^4*d^8*e + 4*b^4*d^4*e^5 - 16*a*b^3*d^3*e^6 + 16*a*
c^3*d^6*e^3 + 16*a^3*c*d^2*e^7 - 16*b*c^3*d^7*e^2 - 16*b^3*c*d^5*e^4 + 24*a
^2*b^2*d^2*e^7 + 24*a^2*c^2*d^4*e^5 + 24*b^2*c^2*d^6*e^3 - 16*a^3*b*d*e^8 -
 48*a*b*c^2*d^5*e^4 + 48*a*b^2*c*d^4*e^5 - 48*a^2*b*c*d^3*e^6) - log(d*(a +
 b*x + c*x^2)^n)/(4*e*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*
e*x)) - (log(10*d*e^5*(b^2 - 4*a*c)^(7/2) + 3*e^6*x*(b^2 - 4*a*c)^(7/2) + 6
*a*e^6*(4*a*c - b^2)^3 - 96*c^5*d^6*(4*a*c - b^2) + 10*b*e^6*x*(4*a*c - b^2
)^3 + 10*b^5*e^6*x*(4*a*c - b^2) + 29*b^2*e^6*x*(b^2 - 4*a*c)^(5/2) + 29*b^
4*e^6*x*(b^2 - 4*a*c)^(3/2) + 3*b^6*e^6*x*(b^2 - 4*a*c)^(1/2) + 192*c^6*d^6
*x*(b^2 - 4*a*c)^(1/2) - 44*a*b^2*e^6*(4*a*c - b^2)^2 + 16*b^3*d*e^5*(4*a*c
 - b^2)^2 - 58*c*d^2*e^4*(4*a*c - b^2)^3 + 176*c^2*d^3*e^3*(b^2 - 4*a*c)^(5
/2) - 44*b^3*e^6*x*(4*a*c - b^2)^2 + 14*a*b*e^6*(b^2 - 4*a*c)^(5/2) + 232*c
^3*d^4*e^2*(4*a*c - b^2)^2 + 14*a*b^4*e^6*(4*a*c - b^2) + 44*a*b^3*e^6*(b^2
 - 4*a*c)^(3/2) + 6*a*b^5*e^6*(b^2 - 4*a*c)^(1/2) + 96*b*c^5*d^6*(b^2 - 4*a
*c)^(1/2) + 48*b*d*e^5*(4*a*c - b^2)^3 - 32*b^5*d*e^5*(4*a*c - b^2) + 74*b^
2*d*e^5*(b^2 - 4*a*c)^(5/2) - 66*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 18*b^6*d*e
^5*(b^2 - 4*a*c)^(1/2) + 160*c^4*d^5*e*(b^2 - 4*a*c)^(3/2) - 288*b*c^2*d^3*
e^3*(4*a*c - b^2)^2 + 84*b^2*c*d^2*e^4*(4*a*c - b^2)^2 + 40*b^2*c^3*d^4*e^2
*(4*a*c - b^2) - 160*b^3*c^2*d^3*e^3*(4*a*c - b^2) - 64*b^2*c^2*d^3*e^3*(b^
2 - 4*a*c)^(3/2) + 360*b^3*c^3*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 240*b^4*c^2*d^
3*e^3*(b^2 - 4*a*c)^(1/2) + 352*c^3*d^3*e^3*x*(4*a*c - b^2)^2 + 128*b*c^4*d
^5*e*(4*a*c - b^2) - 206*b*c*d^2*e^4*(b^2 - 4*a*c)^(5/2) - 20*c*d*e^5*x*(4*
a*c - b^2)^3 - 320*c^5*d^5*e*x*(4*a*c - b^2) + 110*b^4*c*d^2*e^4*(4*a*c - b
^2) - 168*b*c^3*d^4*e^2*(b^2 - 4*a*c)^(3/2) - 288*b^2*c^4*d^5*e*(b^2 - 4*a*
c)^(1/2) + 148*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 90*b^5*c*d^2*e^4*(b^2 -
4*a*c)^(1/2) + 116*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(5/2) + 464*c^4*d^4*e^2*x*(b
^2 - 4*a*c)^(3/2) + 264*b^2*c*d*e^5*x*(4*a*c - ...
```

3.92 $\int \frac{\log\left(d\left(a+cx^2\right)^n\right)}{ae+cex^2}\,dx$

Optimal. Leaf size=175

$$\frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e}+\frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{in\text{Li}_2\left(1-\frac{}{\sqrt{}}\right.}{\sqrt{a}\,\sqrt{}}$$

```
[Out] I*n*arctan(x*c^(1/2)/a^(1/2))^2/e/a^(1/2)/c^(1/2)+arctan(x*c^(1/2)/a^(1/2))
*ln(d*(c*x^2+a)^n)/e/a^(1/2)/c^(1/2)+2*n*arctan(x*c^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^
(1/2)+I*x*c^(1/2)))/e/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {211, 2520, 12, 5040, 4964, 2449, 2352}

$$\frac{in\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d\left(a+cx^2\right)^n\right)}{\sqrt{a}\,\sqrt{c}\,e}+\frac{in\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e}+\frac{2n\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[c]*e) + (2*n*ArcTan[(Sqrt
[c]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*
e) + (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*Log[d*(a + c*x^2)^n])/(Sqrt[a]*Sqrt[c]*e)
 + (I*n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[c]*x)])/(Sqrt[a]*Sqrt[
c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(d(a+cx^2)^n\right)}{ae+cex^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} - (2cn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}\,e\,(a+cx^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} - \frac{\left(2\sqrt{c}\,n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{a+cx^2}\,dx}{\sqrt{a}\,e}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(d(a+cx^2)^n\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{i-\frac{\sqrt{c}\,x}{\sqrt{a}}}\,dx}{ae}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{a}\,\sqrt{c}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{a}\,\sqrt{c}}$$

$$= \frac{in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{c}\,e} + \frac{2n\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e} + \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\log\left(\right.}{\sqrt{a}\,\sqrt{c}}$$

Mathematica [A]
time = 0.03, size = 131, normalized size = 0.75

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(in\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+2n\log\left(\frac{2i}{i-\frac{\sqrt{c}\,x}{\sqrt{a}}}\right)+\log\left(d(a+cx^2)^n\right)\right)+in\mathrm{Li}_2\left(\frac{i\sqrt{a}+\sqrt{c}\,x}{-i\sqrt{a}+\sqrt{c}\,x}\right)}{\sqrt{a}\,\sqrt{c}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + c*x^2)^n]/(a*e + c*e*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[c]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[c]*x)/Sqrt[a])] + Log[d*(a + c*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[c]*x)/((-I)*Sqrt[a] + Sqrt[c]*x)])/(Sqrt[a]*Sqrt[c]*e)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+a)^n\right)}{ce\,x^2+ea}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

```
[Out] int(ln(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="maxima")
```

```
[Out] integrate(log((c*x^2 + a)^n*d)/(c*x^2*e + a*e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="fricas")
```

```
[Out] integral(e^(-1)*log((c*x^2 + a)^n*d)/(c*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\log\left(d(a+cx^2)^n\right)}{a+cx^2}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+a)**n)/(c*e*x**2+a*e),x)
```

```
[Out] Integral(log(d*(a + c*x**2)**n)/(a + c*x**2), x)/e
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+a)^n)/(c*e*x^2+a*e),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + a)^n*d)/(c*x^2*e + a*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(d\left(c\,x^2+a\right)^n\right)}{c\,e\,x^2+a\,e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2),x)
```

```
[Out] int(log(d*(a + c*x^2)^n)/(a*e + c*e*x^2), x)
```

3.93 $\int \frac{\log\left(d\left(a+bx+cx^2\right)^n\right)}{ae+bex+cex^2}\,dx$

Optimal. Leaf size=258

$$\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e}-\frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}-\frac{2cx}{\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,e}-\frac{2\tanh^{-1}\left(\frac{b}{\sqrt{b^2}}\right.}{}$$

```
[Out] 2*n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))^2/e/(-4*a*c+b^2)^(1/2)-2*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(d*(c*x^2+b*x+a)^n)/e/(-4*a*c+b^2)^(1/2)-4*
n*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*ln(2/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/
(-4*a*c+b^2)^(1/2)))/e/(-4*a*c+b^2)^(1/2)-2*n*polylog(2,(-1-b/(-4*a*c+b^2)^
(1/2)-2*c*x/(-4*a*c+b^2)^(1/2))/(1-b/(-4*a*c+b^2)^(1/2)-2*c*x/(-4*a*c+b^2)^
(1/2)))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {632, 212, 2607, 12, 6256, 6131, 6055, 2449, 2352}

$$-\frac{2n\text{PolyLog}\left(2,-\frac{\frac{2cx}{\sqrt{b^2-4ac}}+\frac{b}{\sqrt{b^2-4ac}}+1}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)}{e\sqrt{b^2-4ac}}-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{e\sqrt{b^2-4ac}}+\frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{e\sqrt{b^2-4ac}}-\frac{4n\log\left(\frac{2}{-\frac{2cx}{\sqrt{b^2-4ac}}-\frac{b}{\sqrt{b^2-4ac}}+1}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] (2*n*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]^2)/(Sqrt[b^2 - 4*a*c]*e) - (4*n
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]*Log[2/(1 - b/Sqrt[b^2 - 4*a*c] - (2
*c*x)/Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*e) - (2*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[d*(a + b*x + c*x^2)^n])/(Sqrt[b^2 - 4*a*c]*e) - (2*n*
PolyLog[2, -((1 + b/Sqrt[b^2 - 4*a*c] + (2*c*x)/Sqrt[b^2 - 4*a*c])/(1 - b/S
qrt[b^2 - 4*a*c] - (2*c*x)/Sqrt[b^2 - 4*a*c]))])/(Sqrt[b^2 - 4*a*c]*e)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2607

```
Int[Log[(c_.)*(Px_)^(n_.)]/(Qx_), x_Symbol] :> With[{u = IntHide[1/Qx, x]},
 Simp[u*Log[c*Px^n], x] - Dist[n, Int[SimplifyIntegrand[u*(D[Px, x]/Px), x]
, x], x]] /; FreeQ[{c, n}, x] && QuadraticQ[{Qx, Px}, x] && EqQ[D[Px/Qx, x]
, 0]
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6256

```
Int[((a_.) + ArcTanh[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(
m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)^(q_.), x_Symbol] :> Dist[1/d, Subs
t[Int[((d*e - c*f)/d + f*(x/d))^m*(-C/d^2 + (C/d^2)*x^2)^q*(a + b*ArcTanh[x
])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, p, q}, x]
 && EqQ[B*(1 - c^2) + 2*A*c*d, 0] && EqQ[2*c*C - B*d, 0]
```

Rubi steps

$$\int \frac{\log \left(d(a+bx+cx^2)^n\right)}{ae+bex+cex^2}\,dx = -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}\,e} - n\int \frac{2(-b-2cx)\tanh^{}}{\sqrt{b^2-}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}\,e} - \frac{(2n)\int \frac{(-b-2cx)\tanh^{-1}\left(\right.}{}}{\sqrt{b^2}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}\,e} + \frac{n\text{Subst}\left(\int \frac{\sqrt{b^2-4ac}}{-\frac{b^2-4ac}{4c}+\frac{(b}{}}\right.}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)^n\right)}{\sqrt{b^2-4ac}\,e} + \frac{\left(\sqrt{b^2-4ac}\,n\right)\text{Subst}\left(\right.}{}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log \left(d(a+bx+cx^2)\right.}{\sqrt{b^2-4ac}\,e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,e}$$

$$= \frac{2n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)^2}{\sqrt{b^2-4ac}\,e} - \frac{4n\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\log\left(\frac{2}{1-\frac{b}{\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\,e}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 555 vs. 2(258) = 516.
time = 0.24, size = 555, normalized size = 2.15

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]/(a*e + b*e*x + c*e*x^2),x]
```

```
[Out] -1/2*(4*Sqrt[b^2 - 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]*Log[(b -
 Sqrt[b^2 - 4*a*c])/(2*c) + x] - Sqrt[-b^2 + 4*a*c]*n*Log[(b - Sqrt[b^2 - 4
```

```
*a*c])/(2*c) + x]^2 + 4*Sqrt[b^2 - 4*a*c]*n*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*c)] - 2*Sqrt[-b^2 + 4*a*c]*n
*Log[(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]*Log[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x)/(2*c)] + Sqrt[-b^2 + 4*a*c]*n*Log[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x)/(2*c)]^2 + 2*Sqrt[-b^2 + 4*a*c]*n*Log[(b - Sqrt[b^2 - 4*a*c])/
(2*c) + x]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] - 4*S
qrt[b^2 - 4*a*c]*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]*Log[d*(a + x*(b + c
*x))^n] + 2*Sqrt[-b^2 + 4*a*c]*n*PolyLog[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x
)/(2*Sqrt[b^2 - 4*a*c])] - 2*Sqrt[-b^2 + 4*a*c]*n*PolyLog[2, (b + Sqrt[b^2
- 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[-(b^2 - 4*a*c)^2]*e)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d(c\,x^2+bx+a)^n\right)}{ce\,x^2+bex+ea}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
[Out] int(ln(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="fricas")
[Out] integral(e^(-1)*log((c*x^2 + b*x + a)^n*d)/(c*x^2 + b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)/(c*e*x**2+b*e*x+a*e),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)/(c*e*x^2+b*e*x+a*e),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)/(c*x^2*e + b*x*e + a*e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(d\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{c\,e\,x^2+b\,e\,x+a\,e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2),x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)/(a*e + b*e*x + c*e*x^2), x)
```

3.94 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex^2}\,dx$

Optimal. Leaf size=762

$$-\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{\right.}}{2\sqrt{-d}\,\sqrt{e}}$$

```
[Out] 1/2*ln(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)-x*e^(1/2))/(-d)^(1/2)/e^(1/2)-1/2*l
n(g*(c*x^2+b*x+a)^n)*ln((-d)^(1/2)+x*e^(1/2))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((
-d)^(1/2)+x*e^(1/2))*ln(-(b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/
2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-
x*e^(1/2))*ln((b+2*c*x-(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b-(-4*a
*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*n*ln((-d)^(1/2)+x*e^(1/2))*
ln(-(b+2*c*x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)-(b+(-4*a*c+b^2)^(1
/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*ln((-d)^(1/2)-x*e^(1/2))*ln((b+2*c*
x+(-4*a*c+b^2)^(1/2))*e^(1/2)/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2
)))/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)
^(1/2)-(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,
2*c*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b-(-4*a*c+b^2)^(1/2))*e^(1/2)))
/(-d)^(1/2)/e^(1/2)+1/2*n*polylog(2,2*c*((-d)^(1/2)+x*e^(1/2))/(2*c*(-d)^(1
/2)-(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*n*polylog(2,2*c
*((-d)^(1/2)-x*e^(1/2))/(2*c*(-d)^(1/2)+(b+(-4*a*c+b^2)^(1/2))*e^(1/2)))/(-
d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2608, 2604, 2465, 2441, 2440, 2438}

$$-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{n\text{PolyLog}\left(2,\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{2c\sqrt{-d}-\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{n\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{n\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)+2c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{n\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(-\frac{\sqrt{e}\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(b-\sqrt{b^2-4ac}\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{n\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(-\frac{\sqrt{e}\left(\sqrt{b^2-4ac}+b+2cx\right)}{2c\sqrt{-d}-\sqrt{e}\left(\sqrt{b^2-4ac}+b\right)}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] -1/2*(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b -
Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(Sqrt[-d]*Sqrt[e])
- (n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e]) +
 (n*Log[-((Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b - Sq
rt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e])
 + (n*Log[-((Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] - (b +
Sqrt[b^2 - 4*a*c])*Sqrt[e]))]*Log[Sqrt[-d] + Sqrt[e]*x])/(2*Sqrt[-d]*Sqrt[e
```

```
]) + (Log[Sqrt[-d] - Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*Sqr
t[e]) - (Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a + b*x + c*x^2)^n])/(2*Sqrt[-d]*
Sqrt[e]) - (n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b -
Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (n*PolyLog[2, (2*c*(Sq
rt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*
Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d
] - (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) + (n*PolyLog[2,
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2 - 4*a*c])*Sqrt[
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
```

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\log\left(g(a+bx+cx^2)^n\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\log\left(g(a+bx+cx^2)^n\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g(a\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g(a\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g(a+bx+cx^2)^n\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g(a\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{}}{2c\sqrt{-}}\right.}{}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{}}{2c\sqrt{-}}\right.}{}$$

$$= -\frac{n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{n\log\left(\frac{\sqrt{}}{2c\sqrt{-}}\right.}{}$$

Mathematica [A]

time = 0.70, size = 626, normalized size = 0.82

$$\frac{-n\log\left(\frac{\sqrt{e}\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)-n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}-\sqrt{e}\,x\right)+n\log\left(\frac{\sqrt{e}\left(-b+\sqrt{b^2-4ac}-2cx\right)}{2c\sqrt{-d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)+n\log\left(\frac{\sqrt{e}\left(b+\sqrt{b^2-4ac}+2cx\right)}{-2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)\log\left(\sqrt{-d}+\sqrt{e}\,x\right)+\log\left(\sqrt{-d}-\sqrt{e}\,x\right)\log\left(g(a+x(b+cx))^n\right)-\log\left(\sqrt{-d}+\sqrt{e}\,x\right)\log\left(g(a+x(b+cx))^n\right)-n\mathrm{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2c\sqrt{-d}+\left(b-\sqrt{b^2-4ac}\right)\sqrt{e}}\right)-n\mathrm{Li}_2\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}\,x\right)}{2c\sqrt{-d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)+n\mathrm{Li}_2\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{2c\sqrt{-d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)+n\mathrm{Li}_2\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}\,x\right)}{2c\sqrt{-d}-\left(b+\sqrt{b^2-4ac}\right)\sqrt{e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x^2),x]
```

```
[Out] (-(n*Log[(Sqrt[e]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b - Sqr
t[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] - Sqrt[e]*x]) - n*Log[(Sqrt[e]*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e]
)]*Log[Sqrt[-d] - Sqrt[e]*x] + n*Log[(Sqrt[e]*(-b + Sqrt[b^2 - 4*a*c] - 2*c
*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt
[e]*x] + n*Log[(Sqrt[e]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*Sqrt[-d] + (
b + Sqrt[b^2 - 4*a*c])*Sqrt[e])]*Log[Sqrt[-d] + Sqrt[e]*x] + Log[Sqrt[-d] -
 Sqrt[e]*x]*Log[g*(a + x*(b + c*x))^n] - Log[Sqrt[-d] + Sqrt[e]*x]*Log[g*(a
 + x*(b + c*x))^n] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[e]*x))/(2*c*Sqrt[-d
] + (b - Sqrt[b^2 - 4*a*c])*Sqrt[e])] - n*PolyLog[2, (2*c*(Sqrt[-d] - Sqrt[
e]*x))/(2*c*Sqrt[-d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[e])] + n*PolyLog[2, (2*
c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[e])
] + n*PolyLog[2, (2*c*(Sqrt[-d] + Sqrt[e]*x))/(2*c*Sqrt[-d] - (b + Sqrt[b^2
 - 4*a*c])*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 610, normalized size = 0.80

method	result
risch	$\frac{\left(\ln\left(\left(c\,x^2+bx+a\right)^n\right)-n\ln\left(c\,x^2+bx+a\right)\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{n\left(\sum_{\_\_\alpha=\mathrm{RootOf}\left(e\_\_Z^2+d\right)}\frac{\ln(x-\_\_\alpha)\ln\left(c\,x^2+bx+a\right)-\ln(x-\_\_\alpha)\ln\left(\frac{\mathrm{Root}\ldots}{\ldots}\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
+1/2*n/e*sum(1/_alpha*(ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((RootOf
(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1)-x+_alpha)/RootO
f(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1))-ln(x-_alpha)*
ln((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2))-dil
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=1))-dil
```

```
og((RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2)-x+_al
pha)/RootOf(_Z^2*c*e+(2*_alpha*c*e+b*e)*_Z+b*_alpha*e+e*a-c*d,index=2))),_a
lpha=RootOf(_Z^2*e+d))-1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*
(c*x^2+b*x+a)^n)*csgn(I*g)*csgn(I*g*(c*x^2+b*x+a)^n)+1/2*I/(d*e)^(1/2)*arct
an(e*x/(d*e)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2+
1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*g)*csgn(I*g*(c*x^2+b*x+
a)^n)^2-1/2*I/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*Pi*csgn(I*g*(c*x^2+b*x+a)
^n)^3+1/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*ln(g)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(x^2*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(x^2*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(g\,{\left(c\,x^2+b\,x+a\right)}^n\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2),x)
```

```
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x^2), x)
```

3.95 $\int \frac{\log\left(g\left(a+bx+cx^2\right)^n\right)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=782

$$-\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}}-\frac{n\log\left(\frac{f\left(b+\sqrt{b^2-4ac}+\right.}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e-\sqrt{\right.}}\right.}{}$$

[Out] ln(g*(c*x^2+b*x+a)^n)*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*l
n(f*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2
)^(1/2))))*ln(e+2*f*x-(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-ln(g*(c*x^2+b*
x+a)^n)*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)-n*ln(e+2*f*x-(-4*
d*f+e^2)^(1/2))*ln(-f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(c*e-b*f+f*(-4*a*c+b^2)^
(1/2)-c*(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(
1/2))*ln(f*(b+2*c*x-(-4*a*c+b^2)^(1/2))/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*
d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*ln(e+2*f*x+(-4*d*f+e^2)^(1/2))*ln(f*
(b+2*c*x+(-4*a*c+b^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1
/2))))/(-4*d*f+e^2)^(1/2)-n*polylog(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b
-(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)-n*polylo
g(2,-c*(e+2*f*x-(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e-(-4*d*f+
e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n*polylog(2,-c*(e+2*f*x+(-4*d*f+e^2)^(1/2)
)/(f*(b-(-4*a*c+b^2)^(1/2))-c*(e+(-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)+n
polylog(2,-c(e+2*f*x+(-4*d*f+e^2)^(1/2))/(f*(b+(-4*a*c+b^2)^(1/2))-c*(e+(
-4*d*f+e^2)^(1/2))))/(-4*d*f+e^2)^(1/2)

Rubi [A]
time = 1.04, antiderivative size = 782, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2608, 2604, 2465, 2441, 2440, 2438}

$$-\frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(-\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(\sqrt{e^2-4df}+e+2fx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\,\mathrm{PolyLog}\left(2,-\frac{c\left(\sqrt{e^2-4df}+e+2fx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}-\frac{n\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(-\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\sqrt{b^2-4ac}-bf-c\sqrt{e^2-4df}+ce}\right)}{\sqrt{e^2-4df}}-\frac{n\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(e-\sqrt{e^2-4df}\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(-\sqrt{b^2-4ac}+b+2cx\right)}{f\left(b-\sqrt{b^2-4ac}\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{n\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(\frac{f\left(\sqrt{b^2-4ac}+b+2cx\right)}{f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)}\right)}{\sqrt{e^2-4df}}+\frac{\log\left(-\sqrt{e^2-4df}+e+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}}-\frac{\log\left(\sqrt{e^2-4df}+e+2fx\right)\log\left(g\left(a+bx+cx^2\right)^n\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Int[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]

[Out] -((n*Log[-((f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(c*e - b*f + Sqrt[b^2 - 4*a*
c]*f - c*Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sqrt[e^2
 - 4*d*f]) - (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*a
*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))]*Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x])/Sq
rt[e^2 - 4*d*f] + (n*Log[(f*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b - Sqrt[b^2
 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*
x])/Sqrt[e^2 - 4*d*f] + (n*Log[(f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sq
rt[b^2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))]*Log[e + Sqrt[e^2 - 4*d*f]

```
+ 2*f*x])/Sqrt[e^2 - 4*d*f] + (Log[e - Sqrt[e^2 - 4*d*f] + 2*f*x]*Log[g*(a
+ b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x]*
Log[g*(a + b*x + c*x^2)^n])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d
*f]))))])/Sqrt[e^2 - 4*d*f] - (n*PolyLog[2, -((c*(e - Sqrt[e^2 - 4*d*f] + 2*
f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(e - Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 -
 4*d*f] + (n*PolyLog[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b - Sqrt[b^
2 - 4*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*d*f] + (n*PolyLo
g[2, -((c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f - c*(
e + Sqrt[e^2 - 4*d*f]))))])/Sqrt[e^2 - 4*d*f]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
```

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{d+ex+fx^2}\,dx = \int \left(\frac{2f\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}\left(e-\sqrt{e^2-4df}+2fx\right)} - \frac{2f\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}\left(e+\sqrt{e^2-4df}\right.}\right.$$

$$= \frac{(2f)\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{e-\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{\log\left(g(a+bx+cx^2)^n\right)}{e+\sqrt{e^2-4df}+2fx}\,dx}{\sqrt{e^2-4df}}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\right.}{}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\right.}{}$$

$$= \frac{\log\left(e-\sqrt{e^2-4df}+2fx\right)\log\left(g(a+bx+cx^2)^n\right)}{\sqrt{e^2-4df}} - \frac{\log\left(e+\sqrt{e^2-4df}\right.}{}$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} -$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} -$$

$$= -\frac{n\log\left(-\frac{f\left(b-\sqrt{b^2-4ac}+2cx\right)}{ce-bf+\sqrt{b^2-4ac}\,f-c\sqrt{e^2-4df}}\right)\log\left(e-\sqrt{e^2-4df}+2fx\right)}{\sqrt{e^2-4df}} -$$

Mathematica [A]

time = 0.41, size = 908, normalized size = 1.16

$$\frac{-2\sqrt{e^2-4df}\,n\tan^{-1}\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)-2\sqrt{e^2-4df}\,n\tan^{-1}\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2c}\right)+\sqrt{-e^2+4df}\,n\log\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)\log\left(\frac{c\left(-e+\sqrt{e^2-4df}-2fx\right)}{-ce+bf-\sqrt{b^2-4ac}\,f+c\sqrt{e^2-4df}}\right)+\sqrt{-e^2+4df}\,n\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2c}\right)\log\left(\frac{c\left(-e+\sqrt{e^2-4df}-2fx\right)}{\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)}\right)-\sqrt{-e^2+4df}\,n\log\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x\right)\log\left(\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(-b+\sqrt{b^2-4ac}\right)f+c\left(e+\sqrt{e^2-4df}\right)}\right)-\sqrt{-e^2+4df}\,n\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2c}\right)\log\left(\frac{c\left(e+\sqrt{e^2-4df}+2fx\right)}{-\left(\left(b+\sqrt{b^2-4ac}\right)f\right)+c\left(e+\sqrt{e^2-4df}\right)}\right)+2\sqrt{e^2-4df}\tan^{-1}\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)\log\left(g(a+x(b+cx))^n\right)+\sqrt{-e^2+4df}\,n\,\mathrm{Li}_2\left(\frac{f\left(-b+\sqrt{b^2-4ac}-2cx\right)}{\left(-b+\sqrt{b^2-4ac}\right)f+c\left(e-\sqrt{e^2-4df}\right)}\right)-\sqrt{-e^2+4df}\,n\,\mathrm{Li}_2\left(\frac{f\left(-b+\sqrt{b^2-4ac}-2cx\right)}{\left(-b+\sqrt{b^2-4ac}\right)f+c\left(e+\sqrt{e^2-4df}\right)}\right)+\sqrt{-e^2+4df}\,n\,\mathrm{Li}_2\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f+c\left(-e+\sqrt{e^2-4df}\right)}\right)-\sqrt{-e^2+4df}\,n\,\mathrm{Li}_2\left(\frac{f\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)}\right)}{\sqrt{-\left(e^2-4df\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[g*(a + b*x + c*x^2)^n]/(d + e*x + f*x^2),x]
```

```
[Out] (-2*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]]*Log[(b - Sqr
t[b^2 - 4*a*c])/(2*c) + x] - 2*Sqrt[e^2 - 4*d*f]*n*ArcTan[(e + 2*f*x)/Sqrt[
-e^2 + 4*d*f]]*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*c)] + Sqrt[-e^2 + 4*d
*f]*n*Log[(b - Sqrt[b^2 - 4*a*c])/(2*c) + x]*Log[(c*(-e + Sqrt[e^2 - 4*d*f]
 - 2*f*x))/(-(c*e) + b*f - Sqrt[b^2 - 4*a*c]*f + c*Sqrt[e^2 - 4*d*f])] + Sq
rt[-e^2 + 4*d*f]*n*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*c)]*Log[(c*(-e +
Sqrt[e^2 - 4*d*f] - 2*f*x))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 -
 4*d*f]))] - Sqrt[-e^2 + 4*d*f]*n*Log[(b - Sqrt[b^2 - 4*a*c])/(2*c) + x]*Lo
g[(c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e +
Sqrt[e^2 - 4*d*f]))] - Sqrt[-e^2 + 4*d*f]*n*Log[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/(2*c)]*Log[(c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(-((b + Sqrt[b^2 - 4*a*
c])*f) + c*(e + Sqrt[e^2 - 4*d*f]))] + 2*Sqrt[e^2 - 4*d*f]*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]]*Log[g*(a + x*(b + c*x))^n] + Sqrt[-e^2 + 4*d*f]*n*Po
lyLog[2, (f*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f +
 c*(e - Sqrt[e^2 - 4*d*f]))] - Sqrt[-e^2 + 4*d*f]*n*PolyLog[2, (f*(-b + Sqr
t[b^2 - 4*a*c] - 2*c*x))/((-b + Sqrt[b^2 - 4*a*c])*f + c*(e + Sqrt[e^2 - 4*
d*f]))] + Sqrt[-e^2 + 4*d*f]*n*PolyLog[2, (f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
))/((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))] - Sqrt[-e^2 +
4*d*f]*n*PolyLog[2, (f*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + Sqrt[b^2 - 4*
a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))])/Sqrt[-(e^2 - 4*d*f)^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 764, normalized size = 0.98

method	result	size
risch	Expression too large to display	764

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(ln((c*x^2+b*x+a)^n)-n*ln(c*x^2+b*x+a))/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e
)/(4*d*f-e^2)^(1/2))+n*sum((ln(x-_alpha)*ln(c*x^2+b*x+a)-ln(x-_alpha)*ln((R
ootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)
-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-
c*d,index=1))-ln(x-_alpha)*ln((RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alp
ha*f-_alpha*c*e+a*f-c*d,index=2)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*
f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2))-dilog((RootOf(_Z^2*c*f+(2*_al
pha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1)-x+_alpha)/RootOf(_Z^
```

```
2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=1))-dilog((
RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f-c*d,index=2
)-x+_alpha)/RootOf(_Z^2*c*f+(2*_alpha*c*f+b*f)*_Z+b*_alpha*f-_alpha*c*e+a*f
-c*d,index=2)))/(2*_alpha*f+e),_alpha=RootOf(_Z^2*f+_Z*e+d))-I/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I
*g)*csgn(I*g*(c*x^2+b*x+a)^n)+I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e
^2)^(1/2))*Pi*csgn(I*(c*x^2+b*x+a)^n)*csgn(I*g*(c*x^2+b*x+a)^n)^2+I/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csgn(I*g)*csgn(I*g*(c*x^2
+b*x+a)^n)^2-I/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*Pi*csg
n(I*g*(c*x^2+b*x+a)^n)^3+2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*ln(g)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-%e^2>0)', see `assume?` for m
ore det
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + x*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(g*(c*x**2+b*x+a)**n)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(g*(c*x^2+b*x+a)^n)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate(log((c*x^2 + b*x + a)^n*g)/(f*x^2 + x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(g\left(c\,x^2+b\,x+a\right)^n\right)}{f\,x^2+e\,x+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2),x)
[Out] int(log(g*(a + b*x + c*x^2)^n)/(d + e*x + f*x^2), x)
```

3.96 $\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=144

$$8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{bn^2\log^2(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+$$

```
[Out] 8*n^2*x-4*b*n^2*ln(c*x+b)/c-2*b*n^2*ln(-c*x/b)*ln(c*x+b)/c-b*n^2*ln(c*x+b)^
2/c-4*n*x*ln(d*(c*x^2+b*x)^n)+2*b*n*ln(c*x+b)*ln(d*(c*x^2+b*x)^n)/c+x*ln(d*
(c*x^2+b*x)^n)^2-2*b*n^2*polylog(2,1+c*x/b)/c
```

Rubi [A]
time = 0.19, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2603, 2608, 45, 2604, 1607, 2465, 2441, 2352, 2437, 2338}

$$-\frac{2bn^2\text{PolyLog}\left(2,\frac{cx}{b}+1\right)}{c}+x\log^2\left(d\left(bx+cx^2\right)^n\right)-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}-\frac{bn^2\log^2(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{4bn^2\log(b+cx)}{c}+8n^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*b*n^2*Log[b + c*x])/c - (2*b*n^2*Log[-((c*x)/b)]*Log[b + c*x])
/c - (b*n^2*Log[b + c*x]^2)/c - 4*n*x*Log[d*(b*x + c*x^2)^n] + (2*b*n*Log[b
 + c*x]*Log[d*(b*x + c*x^2)^n])/c + x*Log[d*(b*x + c*x^2)^n]^2 - (2*b*n^2*P
olyLog[2, 1 + (c*x)/b])/c
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(d\left(bx+cx^2\right)^n\right)\,dx = x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int \frac{(b+2cx)\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}\,dx$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(2n)\int\left(2\log\left(d\left(bx+cx^2\right)^n\right)-\frac{b\log\left(d\left(bx+cx^2\right)^n\right)}{b+cx}\right)$$

$$= x\log^2\left(d\left(bx+cx^2\right)^n\right)-(4n)\int \log\left(d\left(bx+cx^2\right)^n\right)\,dx+(2bn)\int \frac{\log\left(d\left(bx+cx\right.\right.}{b+cx}$$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx+\right.\right.$$

$$= -4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+cx^2\right)^n\right)}{c}+x\log^2\left(d\left(bx+\right.\right.$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+\right.\right.}{c}$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)+\frac{2bn\log(b+cx)\log\left(d\left(bx+\right.\right.}{c}$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-4nx\log\left(d\left(bx+cx^2\right)^n\right)$$

$$= 8n^2x-\frac{4bn^2\log(b+cx)}{c}-\frac{2bn^2\log\left(-\frac{cx}{b}\right)\log(b+cx)}{c}-\frac{bn^2\log^2(b+cx)}{c}-4nx$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 0.77

$$\frac{-bn^2\log^2(b+cx)-2bn\log(b+cx)\left(2n+n\log\left(-\frac{cx}{b}\right)-\log\left(d(x(b+cx))^n\right)\right)+cx\left(8n^2-4n\log\left(d(x(b+cx))^n\right)+\log^2\left(d(x(b+cx))^n\right)\right)-2bn^2\text{Li}_2\left(1+\frac{cx}{b}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(b*x + c*x^2)^n]^2,x]
```

```
[Out] (-(b*n^2*Log[b + c*x]^2) - 2*b*n*Log[b + c*x]*(2*n + n*Log[-((c*x)/b)] - Lo
g[d*(x*(b + c*x))^n]) + c*x*(8*n^2 - 4*n*Log[d*(x*(b + c*x))^n] + Log[d*(x*
(b + c*x))^n]^2) - 2*b*n^2*PolyLog[2, 1 + (c*x)/b])/c
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \ln\left(d\left(c\,x^2+bx\right)^n\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x)^n)^2,x)
```

Maxima [A]
time = 0.29, size = 123, normalized size = 0.85

$$-\left(\frac{2\left(\log\left(cx+b\right)\log\left(-\frac{cx+b}{b}+1\right)+\mathrm{Li}_2\left(\frac{cx+b}{b}\right)\right)b}{c}+\frac{b\log\left(cx+b\right)^2-8\,cx+4\,b\log\left(cx+b\right)}{c}\right)n^2-2\,n\left(2\,x-\frac{b\log\left(cx+b\right)}{c}\right)\log\left(\left(cx^2+bx\right)^n d\right)+x\log\left(\left(cx^2+bx\right)^n d\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="maxima")
```

```
[Out] -(2*(log(c*x + b)*log(-(c*x + b)/b + 1) + dilog((c*x + b)/b))*b/c + (b*log(
c*x + b)^2 - 8*c*x + 4*b*log(c*x + b))/c)*n^2 - 2*n*(2*x - b*log(c*x + b)/c
)*log((c*x^2 + b*x)^n*d) + x*log((c*x^2 + b*x)^n*d)^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x)^n*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(d\left(bx+cx^2\right)^n\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x)**n)**2,x)
```

```
[Out] Integral(log(d*(b*x + c*x**2)**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x)^n*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(d \left(c x^2 + b x\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(b*x + c*x^2)^n)^2,x)
[Out] int(log(d*(b*x + c*x^2)^n)^2, x)
```

3.97 $\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx$

Optimal. Leaf size=587

$$8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2\left(b-\sqrt{b^2-4ac}+2cx\right)}{2c}-\frac{\left(b+\sqrt{\right.}$$

```
[Out] 8*n^2*x-2*b*n^2*ln(c*x^2+b*x+a)/c-4*n*x*ln(d*(c*x^2+b*x+a)^n)+x*ln(d*(c*x^2
+b*x+a)^n)^2+n*ln(d*(c*x^2+b*x+a)^n)*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*(b-(-4*
a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))^2*(b-(-4*a*c+b^2)^
(1/2))/c-n^2*ln(b+2*c*x-(-4*a*c+b^2)^(1/2))*ln(1/2*(b+2*c*x+(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))/c-4*n^2*a
rctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/c+n*ln(d*(c*x^2+b*x
+a)^n)*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c-n^2*ln(1/2*(
-b-2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*ln(b+2*c*x+(-4*a*c+b^2)^(1
/2))*(b+(-4*a*c+b^2)^(1/2))/c-1/2*n^2*ln(b+2*c*x+(-4*a*c+b^2)^(1/2))^2*(b+(
-4*a*c+b^2)^(1/2))/c-n^2*polylog(2,1/2*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c
+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2))/c
```

Rubi [A]

time = 0.61, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {2603, 2608, 787, 648, 632, 212, 642, 2604, 2465, 2437, 2338, 2441, 2440, 2438}

$$-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\text{PolyLog}\left(2,-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\text{PolyLog}\left(2,\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}+\frac{n\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c}+\frac{n\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right)\log\left(d\left(a+bx+cx^2\right)^n\right)}{c}-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\log^2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{2c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\log^2\left(\sqrt{b^2-4ac}+b+2cx\right)}{2c}-\frac{n^2\left(\sqrt{b^2-4ac}+b\right)\log\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right)}{c}-\frac{n^2\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}+b+2cx\right)\log\left(\frac{\sqrt{b^2-4ac}+b+2cx}{2\sqrt{b^2-4ac}}\right)}{c}-\frac{4n^2\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-4nx\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}+8n^2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] 8*n^2*x - (4*Sqrt[b^2 - 4*a*c]*n^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
c - ((b - Sqrt[b^2 - 4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c
) - ((b + Sqrt[b^2 - 4*a*c])*n^2*Log[-1/2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/c - ((b + Sqrt[b^2 -
4*a*c])*n^2*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]^2)/(2*c) - ((b - Sqrt[b^2 -
4*a*c])*n^2*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c - (2*b*n^2*Log[a + b*x + c*x^2])/c - 4*n*
x*Log[d*(a + b*x + c*x^2)^n] + ((b - Sqrt[b^2 - 4*a*c])*n*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + ((b + Sqrt[b^2 - 4*a*c])*
n*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + b*x + c*x^2)^n])/c + x*Log[
d*(a + b*x + c*x^2)^n]^2 - ((b - Sqrt[b^2 - 4*a*c])*n^2*PolyLog[2, -1/2*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]])/c - ((b + Sqrt[b^2 - 4*a*c
])*n^2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/c
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(d\left(a+bx+cx^2\right)^n\right)\,dx = x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-(2n)\int \frac{x(b+2cx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right)-(2n)\int \left(2\log\left(d\left(a+bx+cx^2\right)^n\right)-\frac{(2a+bx)}{}\right.$$

$$= x\log^2\left(d\left(a+bx+cx^2\right)^n\right)+(2n)\int \frac{(2a+bx)\log\left(d\left(a+bx+cx^2\right)^n\right)}{a+bx+cx^2}\,dx-($$

$$= -4nx\log\left(d\left(a+bx+cx^2\right)^n\right)+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)+(2n)\int\left(\frac{\left(b-\right.}{}\right.$$

$$= 8n^2x-4nx\log\left(d\left(a+bx+cx^2\right)^n\right)+x\log^2\left(d\left(a+bx+cx^2\right)^n\right)+\left(2\left(b-\right.\right.$$

$$= 8n^2x-4nx\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(b-\sqrt{b^2-4ac}\right)n\log\left(b-\sqrt{b^2-4\right.}{c}$$

$$= 8n^2x-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}-4nx\log\left(d\left(a+bx+cx^2\right)^n\right)+\frac{\left(b-\sqrt{b^2-\right.}{}$$

$$= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{2bn^2\log\left(a+bx+cx^2\right)}{c}-$$

$$= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{\left(b+\sqrt{b^2-4ac}\right)n^2\log\left(\right.}{}$$

$$= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2}{2c}$$

$$= 8n^2x-\frac{4\sqrt{b^2-4ac}\,n^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c}-\frac{\left(b-\sqrt{b^2-4ac}\right)n^2\log^2}{2c}$$

Mathematica [A]
time = 0.61, size = 478, normalized size = 0.81

$$x\log^2\left(d(a+x(b+cx))^n\right)+\frac{n\left(4n\left(4cx-2\sqrt{b^2-4ac}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)-b\log(a+x(b+cx))\right)-8cx\log\left(d(a+x(b+cx))^n\right)+2\left(b-\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+x(b+cx))^n\right)+2\left(b+\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx\right)\log\left(d(a+x(b+cx))^n\right)+\left(-b+\sqrt{b^2-4ac}\right)n\left(\log\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\log\left(b-\sqrt{b^2-4ac}+2cx\right)+2\log\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\right)+2\text{Li}_2\left(\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)\right)-\left(b+\sqrt{b^2-4ac}\right)n\left(\log\left(b+\sqrt{b^2-4ac}+2cx\right)\left(2\log\left(\frac{-b+\sqrt{b^2-4ac}-2cx}{2\sqrt{b^2-4ac}}\right)+\log\left(b+\sqrt{b^2-4ac}+2cx\right)\right)+2\text{Li}_2\left(\frac{b+\sqrt{b^2-4ac}+2cx}{2\sqrt{b^2-4ac}}\right)\right)\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d*(a + b*x + c*x^2)^n]^2,x]
```

```
[Out] x*Log[d*(a + x*(b + c*x))^n]^2 + (n*(4*n*(4*c*x - 2*Sqrt[b^2 - 4*a*c]*ArcTa
nh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]] - b*Log[a + x*(b + c*x)]) - 8*c*x*Log[d*(
a + x*(b + c*x))^n] + 2*(b - Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] +
 2*c*x]*Log[d*(a + x*(b + c*x))^n] + 2*(b + Sqrt[b^2 - 4*a*c])*Log[b + Sqrt
[b^2 - 4*a*c] + 2*c*x]*Log[d*(a + x*(b + c*x))^n] + (-b + Sqrt[b^2 - 4*a*c]
)*n*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*(Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x]
 + 2*Log[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) + 2*PolyLo
g[2, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])]) - (b + Sqrt[b
^2 - 4*a*c])*n*(Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*(2*Log[(-b + Sqrt[b^2 -
4*a*c] - 2*c*x)/(2*Sqrt[b^2 - 4*a*c])] + Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x]
) + 2*PolyLog[2, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])))/
(2*c)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \ln \left(d\left(c\,x^{2}+bx+a\right)^{n}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

```
[Out] int(ln(d*(c*x^2+b*x+a)^n)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(d\left(a+bx+cx^2\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d*(c*x**2+b*x+a)**n)**2,x)
```

```
[Out] Integral(log(d*(a + b*x + c*x**2)**n)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(c*x^2+b*x+a)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((c*x^2 + b*x + a)^n*d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln \left(d\left(c\,x^2+b\,x+a\right)^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d*(a + b*x + c*x^2)^n)^2,x)
```

```
[Out] int(log(d*(a + b*x + c*x^2)^n)^2, x)
```

3.98 $\int \frac{x^2 \log(1+x+x^2)}{2+3x+x^2} \, dx$

Optimal. Leaf size=311

$$-2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right)+4\log(4+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{3+i\sqrt{3}}\right)-\log$$

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+ln(2+2*x)*ln(x^2+x+1)-4*ln(4+2*x)*ln(x^2
+x+1)-ln(2+2*x)*ln((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))+4*ln(4+2*x)*ln((-1-2*x
+I*3^(1/2))/(3+I*3^(1/2)))-ln(2+2*x)*ln((-1-2*x-I*3^(1/2))/(1-I*3^(1/2)))+4
*ln(4+2*x)*ln((-1-2*x-I*3^(1/2))/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1-I*3^(1
/2)))+4*polylog(2,2*(2+x)/(3-I*3^(1/2)))-polylog(2,2*(1+x)/(1+I*3^(1/2)))+4
*polylog(2,2*(2+x)/(3+I*3^(1/2)))+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2608, 2603, 787, 648, 632, 210, 642, 2604, 2465, 2441, 2440, 2438}

$$-\text{PolyLog}\left(2,\frac{2(x+1)}{1-i\sqrt{3}}\right)-\text{PolyLog}\left(2,\frac{2(x+1)}{1+i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3-i\sqrt{3}}\right)+4\text{PolyLog}\left(2,\frac{2(x+2)}{3+i\sqrt{3}}\right)+\sqrt{3}\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)+x\log\left(x^2+x+1\right)+\log(2x+2)\log\left(x^2+x+1\right)-4\log(2x+4)\log\left(x^2+x+1\right)+\frac{1}{2}\log\left(x^2+x+1\right)-2x-\log(2x+2)\log\left(-\frac{2x-i\sqrt{3}+1}{1+i\sqrt{3}}\right)+4\log(2x+4)\log\left(-\frac{2x-i\sqrt{3}+1}{3+i\sqrt{3}}\right)-\log(2x+2)\log\left(-\frac{2x+i\sqrt{3}+1}{1-i\sqrt{3}}\right)+4\log(2x+4)\log\left(-\frac{2x+i\sqrt{3}+1}{3-i\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[2 + 2*x]*Log[-((1 - I*Sqrt[3
] + 2*x)/(1 + I*Sqrt[3]))] + 4*Log[4 + 2*x]*Log[-((1 - I*Sqrt[3] + 2*x)/(3
+ I*Sqrt[3]))] - Log[2 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(1 - I*Sqrt[3]))]
 + 4*Log[4 + 2*x]*Log[-((1 + I*Sqrt[3] + 2*x)/(3 - I*Sqrt[3]))] + Log[1 + x
 + x^2]/2 + x*Log[1 + x + x^2] + Log[2 + 2*x]*Log[1 + x + x^2] - 4*Log[4 +
2*x]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/(1 - I*Sqrt[3])] - PolyLog[2
, (2*(1 + x))/(1 + I*Sqrt[3])] + 4*PolyLog[2, (2*(2 + x))/(3 - I*Sqrt[3])]
+ 4*PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \log\left(1+x+x^2\right)}{2+3x+x^2}\,dx &= \int \left(\log\left(1+x+x^2\right) - \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\right)dx \\ &= \int \log\left(1+x+x^2\right)dx - \int \frac{(2+3x)\log\left(1+x+x^2\right)}{2+3x+x^2}\,dx \\ &= x\log\left(1+x+x^2\right) - \int \frac{x(1+2x)}{1+x+x^2}\,dx - \int\left(-\frac{2\log\left(1+x+x^2\right)}{2+2x} + \frac{8\log\left(1+x\right.}{4+2x}\right. \\ &= -2x + x\log\left(1+x+x^2\right) + 2\int \frac{\log\left(1+x+x^2\right)}{2+2x}\,dx - 8\int \frac{\log\left(1+x+x^2\right)}{4+2x}\,dx - \\ &= -2x + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - 4\log(4+2x)\log\left(1+\right. \\ &= -2x + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+2x)\log\left(1+x+x^2\right) - \\ &= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) + \frac{1}{2}\log\left(1+x+x^2\right) + x\log\left(1+x+x^2\right) + \log(2+ \\ &= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2 \\ &= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2 \\ &= -2x + \sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \log(2+2x)\log\left(-\frac{1-i\sqrt{3}+2x}{1+i\sqrt{3}}\right) + 4\log(4+2\end{aligned}$$

Mathematica [A]
time = 0.14, size = 290, normalized size = 0.93

$$-2x+\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log\left(\frac{-i+\sqrt{3}-2ix}{i+\sqrt{3}}\right)\log(2(1+x))-\log\left(\frac{i+\sqrt{3}+2ix}{-i+\sqrt{3}}\right)\log(2(1+x))+\frac{1}{2}\log\left(1+x+x^2\right)+x\log\left(1+x+x^2\right)+\log(2(1+x))\log\left(1+x+x^2\right)-4\log(2(2+x))\log\left(1+x+x^2\right)-\text{Li}_2\left(\frac{2(1+x)}{1+i\sqrt{3}}\right)-\text{Li}_2\left(\frac{2i(1+x)}{i+\sqrt{3}}\right)+4\left(\left(\log\left(\frac{-i+\sqrt{3}-2ix}{3i+\sqrt{3}}\right)+\log\left(\frac{i+\sqrt{3}+2ix}{-3i+\sqrt{3}}\right)\right)\log(2(2+x))+\text{Li}_2\left(\frac{2(2+x)}{3+i\sqrt{3}}\right)+\text{Li}_2\left(\frac{2i(2+x)}{3i+\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Log[1 + x + x^2])/(2 + 3*x + x^2),x]
```

```
[Out] -2*x + Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[(-I + Sqrt[3] - (2*I)*x)/(I
+ Sqrt[3])]*Log[2*(1 + x)] - Log[(I + Sqrt[3] + (2*I)*x)/(-I + Sqrt[3])]*Lo
g[2*(1 + x)] + Log[1 + x + x^2]/2 + x*Log[1 + x + x^2] + Log[2*(1 + x)]*Log
[1 + x + x^2] - 4*Log[2*(2 + x)]*Log[1 + x + x^2] - PolyLog[2, (2*(1 + x))/
```

```
(1 + I*Sqrt[3])] - PolyLog[2, ((2*I)*(1 + x))/(I + Sqrt[3])] + 4*((Log[(-I
+ Sqrt[3] - (2*I)*x)/(3*I + Sqrt[3])] + Log[(I + Sqrt[3] + (2*I)*x)/(-3*I +
 Sqrt[3])])*Log[2*(2 + x)] + PolyLog[2, (2*(2 + x))/(3 + I*Sqrt[3])] + Poly
Log[2, ((2*I)*(2 + x))/(3*I + Sqrt[3])])
```

Maple [A]
time = 0.09, size = 279, normalized size = 0.90

method	result
default	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln(x^2+x+1)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}+\ln(1+x)\ln(x^2+x+1)-\mathrm{l}$
risch	$-2x+\frac{\ln(x^2+x+1)}{2}+x\ln(x^2+x+1)+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}+\ln(1+x)\ln(x^2+x+1)-\mathrm{l}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x^2+x+1)/(x^2+3*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x+1/2*ln(x^2+x+1)+x*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+ln(1
+x)*ln(x^2+x+1)-ln(1+x)*ln((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-ln(1+x)*ln((1+
2*x+I*3^(1/2))/(-1+I*3^(1/2)))-dilog((-1-2*x+I*3^(1/2))/(1+I*3^(1/2)))-dilo
g((1+2*x+I*3^(1/2))/(-1+I*3^(1/2)))-4*ln(2+x)*ln(x^2+x+1)+4*ln(2+x)*ln((-1-
2*x+I*3^(1/2))/(3+I*3^(1/2)))+4*ln(2+x)*ln((1+2*x+I*3^(1/2))/(I*3^(1/2)-3))
+4*dilog((-1-2*x+I*3^(1/2))/(3+I*3^(1/2)))+4*dilog((1+2*x+I*3^(1/2))/(I*3^(
1/2)-3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \log \left(x^2+x+1\right)}{(x+1)(x+2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x**2+x+1)/(x**2+3*x+2),x)
```

```
[Out] Integral(x**2*log(x**2 + x + 1)/((x + 1)*(x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x^2+x+1)/(x^2+3*x+2),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(x^2 + x + 1)/(x^2 + 3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\, \ln \left(x^2+x+1\right)}{x^2+3\,x+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2),x)
```

```
[Out] int((x^2*log(x + x^2 + 1))/(3*x + x^2 + 2), x)
```

3.99 $\int \log^2\left(1+x+x^2\right)\,dx$

Optimal. Leaf size=371

$$8x-4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right)-\left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\text{lo}$$

```
[Out] 8*x-2*ln(x^2+x+1)-4*x*ln(x^2+x+1)+x*ln(x^2+x+1)^2+ln(x^2+x+1)*ln(1+2*x-I*3^
(1/2))*(1-I*3^(1/2))-1/2*ln(1+2*x-I*3^(1/2))^2*(1-I*3^(1/2))-ln(1+2*x-I*3^(
1/2))*ln(-1/6*I*(1+2*x+I*3^(1/2))*3^(1/2))*(1-I*3^(1/2))-polylog(2,1/6*(I+2
*I*x+3^(1/2))*3^(1/2))*(1-I*3^(1/2))+ln(x^2+x+1)*ln(1+2*x+I*3^(1/2))*(1+I*3
^(1/2))-1/2*ln(1+2*x+I*3^(1/2))^2*(1+I*3^(1/2))-ln(1+2*x+I*3^(1/2))*ln(1/6*
I*(1+2*x-I*3^(1/2))*3^(1/2))*(1+I*3^(1/2))-polylog(2,1/6*(-I-2*I*x+3^(1/2))
*3^(1/2))*(1+I*3^(1/2))-4*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.556$, Rules used = {2603, 2608, 787, 648, 632, 210, 642, 2604, 2465, 2437, 2338, 2441, 2440, 2438}

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2ix-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2ix+\sqrt{3}+i}{2\sqrt{3}}\right)-4\sqrt{3}\,\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)+x\log^2\left(x^2+x+1\right)+\left(1-i\sqrt{3}\right)\log\left(x^2+x+1\right)\log\left(2x-i\sqrt{3}+1\right)-4x\log\left(x^2+x+1\right)+\left(1+i\sqrt{3}\right)\log\left(2x+i\sqrt{3}+1\right)\log\left(x^2+x+1\right)-2\log\left(x^2+x+1\right)+8x-\frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(2x-i\sqrt{3}+1\right)-\frac{1}{2}\left(1+i\sqrt{3}\right)\log^2\left(2x+i\sqrt{3}+1\right)-\left(1-i\sqrt{3}\right)\log\left(-\frac{i\left(2x+i\sqrt{3}+1\right)}{2\sqrt{3}}\right)\log\left(2x-i\sqrt{3}+1\right)-\left(1+i\sqrt{3}\right)\log\left(\frac{i\left(2x-i\sqrt{3}+1\right)}{2\sqrt{3}}\right)\log\left(2x+i\sqrt{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqrt
[3] + 2*x]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*x))/Sqrt[3]
]*Log[1 + I*Sqrt[3] + 2*x] - ((1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]^2)/2
 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[((-1/2*I)*(1 + I*Sqrt[3] +
2*x))/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1 + x + x^2] + (1 - I*Sqrt[3]
)*Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqr
t[3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2]^2 - (1 + I*Sqrt[3])*PolyL
og[2, -1/2*(I - Sqrt[3] + (2*I)*x)/Sqrt[3]] - (1 - I*Sqrt[3])*PolyLog[2, (I
 + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \log^2\left(1+x+x^2\right)\,dx = x\log^2\left(1+x+x^2\right) - 2\int \frac{x(1+2x)\log\left(1+x+x^2\right)}{1+x+x^2}\,dx$$

$$= x\log^2\left(1+x+x^2\right) - 2\int \left(2\log\left(1+x+x^2\right) - \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\right)\,dx$$

$$= x\log^2\left(1+x+x^2\right) + 2\int \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\,dx - 4\int \log\left(1+x+x^2\right)\,dx$$

$$= -4x\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) + 2\int \left(\frac{\left(1-i\sqrt{3}\right)\log\left(1+x+x^2\right)}{1-i\sqrt{3}+2x}\right.$$

$$= 8x - 4x\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) + 4\int \frac{-2-x}{1+x+x^2}\,dx + \left(2\left(1-i\sqrt{}\right.\right.$$

$$= 8x - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + \left(\right.$$

$$= 8x - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{}\right.$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2x\right)}{2\sqrt{3}}\right)\log\left(1+i\sqrt{}\right.$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) - \left(1+i\sqrt{3}\right)$$

$$= 8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2x\right) - \left(1+i\sqrt{3}\right)$$

Mathematica [A]
time = 0.11, size = 323, normalized size = 0.87

$$8x - 4\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log\left(1+x+x^2\right) - 4x\log\left(1+x+x^2\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + \left(1+i\sqrt{3}\right)\log\left(1+i\sqrt{3}+2x\right)\log\left(1+x+x^2\right) + x\log^2\left(1+x+x^2\right) - \frac{1}{2}i\left(-i+\sqrt{3}\right)\left(\log\left(1+i\sqrt{3}+2x\right)\left(2\log\left(\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right) + \log\left(1+i\sqrt{3}+2x\right)\right) + 2\operatorname{Li}_2\left(\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right)\right) + \frac{1}{2}i\left(i+\sqrt{3}\right)\left(\log\left(1-i\sqrt{3}+2x\right)\left(2\log\left(\frac{-i+\sqrt{3}-2ix}{2\sqrt{3}}\right) + \log\left(1-i\sqrt{3}+2x\right)\right) + 2\operatorname{Li}_2\left(\frac{i+\sqrt{3}+2ix}{2\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + x^2]^2,x]
```

```
[Out] 8*x - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 + x + x^2] - 4*x*Log[1
+ x + x^2] + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + (1
 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*x]*Log[1 + x + x^2] + x*Log[1 + x + x^2
]^2 - (I/2)*(-I + Sqrt[3])*(Log[1 + I*Sqrt[3] + 2*x]*(2*Log[(I + Sqrt[3] +
(2*I)*x)/(2*Sqrt[3])] + Log[1 + I*Sqrt[3] + 2*x]) + 2*PolyLog[2, (-I + Sqrt
[3] - (2*I)*x)/(2*Sqrt[3])]) + (I/2)*(I + Sqrt[3])*(Log[1 - I*Sqrt[3] + 2*x
]*(2*Log[(-I + Sqrt[3] - (2*I)*x)/(2*Sqrt[3])] + Log[1 - I*Sqrt[3] + 2*x])
+ 2*PolyLog[2, (I + Sqrt[3] + (2*I)*x)/(2*Sqrt[3])])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \ln \left(x^2+x+1\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x+1)^2,x)
```

```
[Out] int(ln(x^2+x+1)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] x*log(x^2 + x + 1)^2 - integrate(2*(2*x^2 + x)*log(x^2 + x + 1)/(x^2 + x +
1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x + 1)^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x+1)**2,x)
```

```
[Out] Exception raised: RecursionError >> maximum recursion depth exceeded
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \ln\left(x^2+x+1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 + 1)^2,x)
```

```
[Out] int(log(x + x^2 + 1)^2, x)
```

3.100 $\int \frac{\log^2(-1+x+x^2)}{x^3} \, dx$

Optimal. Leaf size=443

$$\log(x) - \frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + 3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right) - \frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(\right.$$

[Out] ln(x)+ln(x^2+x-1)/x-3*ln(x)*ln(x^2+x-1)-1/2*ln(x^2+x-1)^2/x^2+3*ln(1+2*x-5^(1/2))*ln(1/2*5^(1/2)-1/2)+3*ln(x)*ln(1+2*x/(5^(1/2)+1))-3*polylog(2,1+2*x/(-5^(1/2)+1))+3*polylog(2,-2*x/(5^(1/2)+1))-1/2*ln(1+2*x+5^(1/2))*(-5^(1/2)+1)+1/2*ln(x^2+x-1)*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/2*ln(1/10*(-1-2*x+5^(1/2))*5^(1/2))*ln(1+2*x+5^(1/2))*(3-5^(1/2))-1/4*ln(1+2*x+5^(1/2))^2*(3-5^(1/2))-1/2*polylog(2,1/10*(1+2*x+5^(1/2))*5^(1/2))*(3-5^(1/2))-1/2*ln(1+2*x-5^(1/2))*(5^(1/2)+1)+1/2*ln(x^2+x-1)*ln(1+2*x-5^(1/2))*(3+5^(1/2))-1/4*ln(1+2*x-5^(1/2))^2*(3+5^(1/2))-1/2*ln(1+2*x-5^(1/2))*ln(1/10*(1+2*x+5^(1/2))*5^(1/2))*(3+5^(1/2))-1/2*polylog(2,1/10*(-1-2*x+5^(1/2))*5^(1/2))*(3+5^(1/2))

Rubi [A]
time = 0.44, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 16, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.231$, Rules used = {2605, 2608, 814, 646, 31, 2604, 2404, 2353, 2352, 2354, 2438, 2465, 2437, 2338, 2441, 2440}

$$3\,\text{PolyLog}\left(2,-\frac{2x}{1+\sqrt{5}}\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)-\frac{1}{2}\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)-3\,\text{PolyLog}\left(2,\frac{2x}{1-\sqrt{5}}+1\right)-\frac{\log^2\left(x^2+x-1\right)}{2x^2}+\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(x^2+x-1\right)\log\left(2x-\sqrt{5}+1\right)-3\log(x)\log\left(x^2+x-1\right)+\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\log\left(x^2+x-1\right)+\frac{\log\left(x^2+x-1\right)}{x}-\frac{1}{4}\left(3+\sqrt{5}\right)\log^2\left(2x-\sqrt{5}+1\right)-\frac{1}{4}\left(3-\sqrt{5}\right)\log^2\left(2x+\sqrt{5}+1\right)-\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(\frac{2x+\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x-\sqrt{5}+1\right)+3\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\log(x)-\frac{1}{2}\left(3-\sqrt{5}\right)\log\left(-\frac{2x-\sqrt{5}+1}{2\sqrt{5}}\right)\log\left(2x+\sqrt{5}+1\right)-\frac{1}{2}\left(1-\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)+3\log(x)\log\left(\frac{2x}{1+\sqrt{5}}+1\right)$$

Antiderivative was successfully verified.

[In] Int[Log[-1 + x + x^2]^2/x^3,x]

[Out] Log[x] - ((1 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/2 + 3*Log[(-1 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*x] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]^2)/4 - ((1 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[-1/2*(1 - Sqrt[5] + 2*x)/Sqrt[5]]*Log[1 + Sqrt[5] + 2*x])/2 - ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]^2)/4 - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 + 3*Log[x]*Log[1 + (2*x)/(1 + Sqrt[5])] + Log[-1 + x + x^2]/x - 3*Log[x]*Log[-1 + x + x^2] + ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*x]*Log[-1 + x + x^2])/2 + ((3 - Sqrt[5])*Log[1 + Sqrt[5] + 2*x]*Log[-1 + x + x^2])/2 - Log[-1 + x + x^2]^2/(2*x^2) + 3*PolyLog[2, (-2*x)/(1 + Sqrt[5])] - ((3 + Sqrt[5])*PolyLog[2, -1/2*(1 - Sqrt[5] + 2*x)/Sqrt[5]])/2 - ((3 - Sqrt[5])*PolyLog[2, (1 + Sqrt[5] + 2*x)/(2*Sqrt[5])])/2 - 3*PolyLog[2, 1 + (2*x)/(1 - Sqrt[5])]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2465

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2608

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx = -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+\int \frac{(1+2x)\log\left(-1+x+x^2\right)}{x^2\left(-1+x+x^2\right)}\,dx$$

$$= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+\int \left(-\frac{\log\left(-1+x+x^2\right)}{x^2}-\frac{3\log\left(-1+x+x^2\right)}{x}+\frac{(4+3$$

$$= -\frac{\log^2\left(-1+x+x^2\right)}{2x^2}-3\int \frac{\log\left(-1+x+x^2\right)}{x}\,dx-\int \frac{\log\left(-1+x+x^2\right)}{x^2}\,dx+\int$$

$$= \frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right)-\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\int \frac{(1-}{-}$$

$$= \frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right)-\frac{\log^2\left(-1+x+x^2\right)}{2x^2}+3\int \left(\frac{}{1}\right.$$

$$= \log(x)+\frac{\log\left(-1+x+x^2\right)}{x}-3\log(x)\log\left(-1+x+x^2\right)+\frac{1}{2}\left(3+\sqrt{5}\right)\log\left(1-\right.$$

$$= \log(x)+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}+2x\right)+3\log(x)\log\left(1+\frac{2x}{1+\sqrt{5}}\right.$$

$$= \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}\right.$$

$$= \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}\right.$$

$$= \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}\right.$$

$$= \log(x)-\frac{1}{2}\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)+3\log\left(\frac{1}{2}\left(-1+\sqrt{5}\right)\right)\log\left(1-\sqrt{5}\right.$$

Mathematica [A]
time = 0.56, size = 826, normalized size = 1.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[-1 + x + x^2]^2/x^3,x]
```

```
[Out] (-2*Log[-1 + x + x^2]^2 + x*(4*x*Log[x] - 12*x*Log[(1 + Sqrt[5])/2]*Log[x]
- 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] - 2*Sqrt[5]*x*Log[-1
 + Sqrt[5] - 2*x]*Log[1/2 - Sqrt[5]/2 + x] + 12*x*Log[x]*Log[1/2 - Sqrt[5]/
2 + x] - 12*x*Log[(2*x)/(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + x] + 3*x*Log[
1/2 - Sqrt[5]/2 + x]^2 + Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]^2 - 6*x*Log[-1
+ Sqrt[5] - 2*x]*Log[(1 + Sqrt[5])/2 + x] - 2*Sqrt[5]*x*Log[-1 + Sqrt[5] -
2*x]*Log[(1 + Sqrt[5])/2 + x] + 12*x*Log[x]*Log[(1 + Sqrt[5])/2 + x] + 3*x*
Log[(1 + Sqrt[5])/2 + x]^2 - Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]^2 - 2*x*Log
[1 - Sqrt[5] + 2*x] - 2*Sqrt[5]*x*Log[1 - Sqrt[5] + 2*x] + 3*x*Log[5]*Log[1
 - Sqrt[5] + 2*x] + Sqrt[5]*x*Log[5]*Log[1 - Sqrt[5] + 2*x] - 2*x*Log[1 + S
qrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x] - 6*x*Log[1/2 - Sqrt[5]/
2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*Sqrt[5]*x*Log[1/2 - Sqrt[5]/2 + x]*Log[1
+ Sqrt[5] + 2*x] - 6*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 2*
Sqrt[5]*x*Log[(1 + Sqrt[5])/2 + x]*Log[1 + Sqrt[5] + 2*x] + 6*x*Log[1/2 - S
qrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] - 2*Sqrt[5]*x*Log[1/2 -
Sqrt[5]/2 + x]*Log[(1 + Sqrt[5] + 2*x)/(2*Sqrt[5])] + 4*Log[-1 + x + x^2] +
 6*x*Log[-1 + Sqrt[5] - 2*x]*Log[-1 + x + x^2] + 2*Sqrt[5]*x*Log[-1 + Sqrt[
5] - 2*x]*Log[-1 + x + x^2] - 12*x*Log[x]*Log[-1 + x + x^2] + 6*x*Log[1 + S
qrt[5] + 2*x]*Log[-1 + x + x^2] - 2*Sqrt[5]*x*Log[1 + Sqrt[5] + 2*x]*Log[-1
 + x + x^2] - 4*Sqrt[5]*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(2*Sqrt[5])] - 12
*x*PolyLog[2, (-1 + Sqrt[5] - 2*x)/(-1 + Sqrt[5])] + 12*x*PolyLog[2, (-2*x)
/(1 + Sqrt[5])]))/(4*x^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(x^{2}+x-1\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+x-1)^2/x^3,x)
```

```
[Out] int(ln(x^2+x-1)^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 + x - 1)^2/x^2 + integrate((2*x + 1)*log(x^2 + x - 1)/(x^4 + x
^3 - x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(log(x^2 + x - 1)^2/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+x-1)**2/x**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+x-1)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(log(x^2 + x - 1)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\ln\left(x^2+x-1\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^2 - 1)^2/x^3,x)
```

```
[Out] int(log(x + x^2 - 1)^2/x^3, x)
```

3.101 $\int x^3 \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal. Leaf size=172

$$\frac{x}{4096}-\frac{x^2}{1024}+\frac{x^3}{192}-\frac{x^4}{32}-\frac{683\sqrt{-x+x^2}}{4096}+\frac{149(1-2x)\sqrt{-x+x^2}}{2048}-\frac{1}{12}\left(-x+x^2\right)^{3/2}-\frac{1}{32}x\left(-x+x^2\right)^{3/2}+\frac{\text{ta}}{}$$

```
[Out] 1/4096*x-1/1024*x^2+1/192*x^3-1/32*x^4-1/12*(x^2-x)^(3/2)-1/32*x*(x^2-x)^(3
/2)+1/32768*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-1537/16384*arctanh(x/(x^2-x
)^(1/2))-1/32768*ln(1+8*x)+1/4*x^4*ln(-1+4*x+4*(x^2-x)^(1/2))-683/4096*(x^2
-x)^(1/2)+149/2048*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738, 684}

$$-\frac{x^4}{32}+\frac{x^3}{192}-\frac{x^2}{1024}-\frac{1}{32}\left(x^2-x\right)^{3/2}x-\frac{1}{12}\left(x^2-x\right)^{3/2}+\frac{149(1-2x)\sqrt{x^2-x}}{2048}-\frac{683\sqrt{x^2-x}}{4096}+\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{32768}-\frac{1537\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)}{16384}+\frac{1}{4}x^4\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{x}{4096}-\frac{\log(8x+1)}{32768}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/4096 - x^2/1024 + x^3/192 - x^4/32 - (683*Sqrt[-x + x^2])/4096 + (149*(1
- 2*x)*Sqrt[-x + x^2])/2048 - (-x + x^2)^(3/2)/12 - (x*(-x + x^2)^(3/2))/32
 + ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/32768 - (1537*ArcTanh[x/Sqrt[-x +
 x^2]])/16384 - Log[1 + 8*x]/32768 + (x^4*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])
/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
```

```
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$
$$= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \frac{x^4}{-4(1+2x)\sqrt{-x+x}}$$
$$= \frac{1}{4}x^4 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\int \left(\frac{1}{8192} - \frac{x}{1024} + \frac{x}{12}\right.$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{\log(1+8x)}{32768} + \frac{1}{4}x^4 \log\left(-1+4x\right.$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{85(1-2x)\sqrt{-}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{129(1-2x)\sqrt{-}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-}}{2048}$$
$$= \frac{x}{4096} - \frac{x^2}{1024} + \frac{x^3}{192} - \frac{x^4}{32} - \frac{683\sqrt{-x+x^2}}{4096} + \frac{149(1-2x)\sqrt{-}}{2048}$$

Mathematica [A]
time = 0.11, size = 117, normalized size = 0.68

$$\frac{24x-96x^2+512x^3-3072x^4-8\sqrt{(-1+x)x}\,(1155+764x+640x^2+384x^3)-6\log(1+8x)-4611\log\left(1-2x-2\sqrt{(-1+x)x}\right)+24576x^4\log\left(-1+4x+4\sqrt{(-1+x)x}\right)+3\log\left(1-10x+6\sqrt{(-1+x)x}\right)}{98304}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (24*x - 96*x^2 + 512*x^3 - 3072*x^4 - 8*Sqrt[(-1 + x)*x]*(1155 + 764*x + 64
0*x^2 + 384*x^3) - 6*Log[1 + 8*x] - 4611*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]]
+ 24576*x^4*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + 3*Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/98304
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^3*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A]
time = 0.36, size = 134, normalized size = 0.78

$$-\frac{1}{32}x^4+\frac{1}{192}x^3-\frac{1}{1024}x^2+\frac{1}{4}(x^4-1)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{12288}(384x^3+640x^2+764x+1155)\sqrt{x^2-x}+\frac{1}{4096}x+\frac{4095}{32768}\log(8x+1)-\frac{2559}{32768}\log\left(-2x+2\sqrt{x^2-x}+1\right)+\frac{4095}{32768}\log\left(-2x+2\sqrt{x^2-x}-1\right)-\frac{4095}{32768}\log\left(-4x+4\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/32*x^4 + 1/192*x^3 - 1/1024*x^2 + 1/4*(x^4 - 1)*log(4*x + 4*sqrt(x^2 - x
) - 1) - 1/12288*(384*x^3 + 640*x^2 + 764*x + 1155)*sqrt(x^2 - x) + 1/4096*
x + 4095/32768*log(8*x + 1) - 2559/32768*log(-2*x + 2*sqrt(x^2 - x) + 1) +
```

```
4095/32768*log(-2*x + 2*sqrt(x^2 - x) - 1) - 4095/32768*log(-4*x + 4*sqrt(x
^2 - x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.26, size = 134, normalized size = 0.78

$$\frac{1}{4}\,x^{4}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{32}\,x^{4}+\frac{1}{192}\,x^{3}-\frac{1}{1024}\,x^{2}-\frac{1}{12288}\,(4\,(32\,(3\,x+5)x+191)x+1155)\sqrt{x^{2}-x}+\frac{1}{4096}\,x-\frac{1}{32768}\,\log\left(|8\,x+1|\right)+\frac{1537}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^{2}-x}+1\right|\right)-\frac{1}{32768}\,\log\left(\left|-2\,x+2\,\sqrt{x^{2}-x}-1\right|\right)+\frac{1}{32768}\,\log\left(\left|-4\,x+4\,\sqrt{x^{2}-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/32*x^4 + 1/192*x^3 - 1/1024*x^
2 - 1/12288*(4*(32*(3*x + 5)*x + 191)*x + 1155)*sqrt(x^2 - x) + 1/4096*x -
1/32768*log(abs(8*x + 1)) + 1537/32768*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
 - 1/32768*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/32768*log(abs(-4*x + 4*
sqrt(x^2 - x) + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^3*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.102 $\int x^2 \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=149

$$-\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}-\frac{1}{18}\left(-x+x^2\right)^{3/2}-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)}{3072}-\frac{223\tan}{}$$

```
[Out] -1/384*x+1/96*x^2-1/18*x^3-1/18*(x^2-x)^(3/2)-1/3072*arctanh(1/6*(1-10*x)/(
x^2-x)^(1/2))-223/1536*arctanh(x/(x^2-x)^(1/2))+1/3072*ln(1+8*x)+1/3*x^3*ln
(-1+4*x+4*(x^2-x)^(1/2))-85/384*(x^2-x)^(1/2)+5/64*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738}

$$-\frac{x^3}{18}+\frac{x^2}{96}-\frac{1}{18}\left(x^2-x\right)^{3/2}+\frac{5}{64}(1-2x)\sqrt{x^2-x}-\frac{85\sqrt{x^2-x}}{384}-\frac{\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)}{3072}-\frac{223\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)}{1536}+\frac{1}{3}x^3\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{x}{384}+\frac{\log(8x+1)}{3072}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/384*x + x^2/96 - x^3/18 - (85*Sqrt[-x + x^2])/384 + (5*(1 - 2*x)*Sqrt[-x
 + x^2])/64 - (-x + x^2)^(3/2)/18 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/
3072 - (223*ArcTanh[x/Sqrt[-x + x^2]])/1536 + Log[1 + 8*x]/3072 + (x^3*Log[
-1 + 4*x + 4*Sqrt[-x + x^2]])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
```

```
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$

$$= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \frac{x^3}{-4(1+2x)\sqrt{-x+x^2}$$

$$= \frac{1}{3}x^3 \log\left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{8}{3}\int \left(-\frac{1}{1024}+\frac{x}{128}-\frac{x^2}{16}\right.$$

$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}+\frac{\log(1+8x)}{3072}+\frac{1}{3}x^3\log\left(-1+4x+4\sqrt{-x+\right.$$

$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{11}{192}(1-2x)\sqrt{-x+x^2}\ -$$

$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}\ -$$

$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}\ -$$

$$= -\frac{x}{384}+\frac{x^2}{96}-\frac{x^3}{18}-\frac{85}{384}\sqrt{-x+x^2}+\frac{5}{64}(1-2x)\sqrt{-x+x^2}\ -$$

Mathematica [A]
time = 0.06, size = 107, normalized size = 0.72

$$\frac{-24x+96x^2-512x^3-8\sqrt{(-1+x)x}\,(165+116x+64x^2)+6\log(1+8x)-669\log\left(1-2x-2\sqrt{(-1+x)x}\right)+3072x^3\log\left(-1+4x+4\sqrt{(-1+x)x}\right)-3\log\left(1-10x+6\sqrt{(-1+x)x}\right)}{9216}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-24*x + 96*x^2 - 512*x^3 - 8*Sqrt[(-1 + x)*x]*(165 + 116*x + 64*x^2) + 6*L
og[1 + 8*x] - 669*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 3072*x^3*Log[-1 + 4*x
 + 4*Sqrt[(-1 + x)*x]] - 3*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/9216
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \ln \left(-1+4 x+4 \sqrt{\left(-1+x\right) x}\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^2*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A]
time = 0.44, size = 124, normalized size = 0.83

$$-\frac{1}{18} \, x^{3} + \frac{1}{96} \, x^{2} + \frac{1}{3} \, {\left(x^{3} + 1\right)} \log\left(4 \, x + 4 \, \sqrt{x^{2} - x} - 1\right) - \frac{1}{1152} \, {\left(64 \, x^{2} + 116 \, x + 165\right)} \sqrt{x^{2} - x} - \frac{1}{384} \, x - \frac{511}{3072} \, \log\left(8 \, x + 1\right) + \frac{245}{1024} \, \log\left(-2 \, x + 2 \, \sqrt{x^{2} - x} + 1\right) - \frac{511}{3072} \, \log\left(-2 \, x + 2 \, \sqrt{x^{2} - x} - 1\right) + \frac{511}{3072} \, \log\left(-4 \, x + 4 \, \sqrt{x^{2} - x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/18*x^3 + 1/96*x^2 + 1/3*(x^3 + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/115
2*(64*x^2 + 116*x + 165)*sqrt(x^2 - x) - 1/384*x - 511/3072*log(8*x + 1) +
245/1024*log(-2*x + 2*sqrt(x^2 - x) + 1) - 511/3072*log(-2*x + 2*sqrt(x^2 -
 x) - 1) + 511/3072*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.57, size = 124, normalized size = 0.83

$$\frac{1}{3} \, x^{3} \log\left(4 \, x + 4 \, \sqrt{{\left(x - 1\right)} x} - 1\right) - \frac{1}{18} \, x^{3} + \frac{1}{96} \, x^{2} - \frac{1}{1152} \, {\left(4 \, {\left(16 \, x + 29\right)} x + 165\right)} \sqrt{x^{2} - x} - \frac{1}{384} \, x + \frac{1}{3072} \, \log\left({\left| 8 \, x + 1 \right|}\right) + \frac{223}{3072} \, \log\left({\left| -2 \, x + 2 \, \sqrt{x^{2} - x} + 1 \right|}\right) + \frac{1}{3072} \, \log\left({\left| -2 \, x + 2 \, \sqrt{x^{2} - x} - 1 \right|}\right) - \frac{1}{3072} \, \log\left({\left| -4 \, x + 4 \, \sqrt{x^{2} - x} + 1 \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/18*x^3 + 1/96*x^2 - 1/1152*(4*
(16*x + 29)*x + 165)*sqrt(x^2 - x) - 1/384*x + 1/3072*log(abs(8*x + 1)) + 2
23/3072*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/3072*log(abs(-2*x + 2*sqrt
(x^2 - x) - 1)) - 1/3072*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^2*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.103 $\int x \log \left(-1+4x+4\sqrt{(-1+x)x}\right)\, dx$

Optimal. Leaf size=127

$$\frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{33}{128}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)-$$

```
[Out] 1/32*x-1/8*x^2+1/256*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-33/128*arctanh(x/(
x^2-x)^(1/2))-1/256*ln(1+8*x)+1/2*x^2*ln(-1+4*x+4*(x^2-x)^(1/2))-11/32*(x^2
-x)^(1/2)+1/16*(1-2*x)*(x^2-x)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2617, 2615, 6874, 654, 634, 212, 626, 748, 857, 738}

$$-\frac{x^2}{8}+\frac{1}{16}(1-2x)\sqrt{x^2-x}-\frac{11\sqrt{x^2-x}}{32}+\frac{1}{2}x^2\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{1}{256}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{33}{128}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)+\frac{x}{32}-\frac{1}{256}\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] x/32 - x^2/8 - (11*Sqrt[-x + x^2])/32 + ((1 - 2*x)*Sqrt[-x + x^2])/16 + Arc
Tanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/256 - (33*ArcTanh[x/Sqrt[-x + x^2]])/12
8 - Log[1 + 8*x]/256 + (x^2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int x \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx &= \int x \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx \\
&= \frac{1}{2}x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int \frac{x^2}{-4(1+2x)\sqrt{-x+x^2}} \\
&= \frac{1}{2}x^2 \log\left(-1+4x+4\sqrt{-x+x^2}\right)+4\int \left(\frac{1}{128}-\frac{x}{16}-\frac{1}{128(1} \right. \\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{1}{256}\log(1+8x)+\frac{1}{2}x^2\log\left(-1+4x+4\sqrt{-x+x^2}\right. \\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1 \\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{1}{256}\log(1 \\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}-\frac{13}{48}\tanh^{-} \\
&= \frac{x}{32}-\frac{x^2}{8}-\frac{11}{32}\sqrt{-x+x^2}+\frac{1}{16}(1-2x)\sqrt{-x+x^2}+\frac{1}{256}\tanh
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 102, normalized size = 0.80

$$\frac{1}{256}\left(8x-32x^2-72\sqrt{(-1+x)x}-32x\sqrt{(-1+x)x}-2\log(1+8x)-33\log\left(1-2x-2\sqrt{(-1+x)x}\right)+128x^2\log\left(-1+4x+4\sqrt{(-1+x)x}\right)+\log\left(1-10x+6\sqrt{(-1+x)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
[Out] (8*x - 32*x^2 - 72*Sqrt[(-1 + x)*x] - 32*x*Sqrt[(-1 + x)*x] - 2*Log[1 + 8*x
] - 33*Log[1 - 2*x - 2*Sqrt[(-1 + x)*x]] + 128*x^2*Log[-1 + 4*x + 4*Sqrt[(-
1 + x)*x]] + Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]])/256
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x*log(4*x + 4*sqrt((x - 1)*x) - 1), x)
```

Fricas [A]
time = 0.40, size = 114, normalized size = 0.90

$$-\frac{1}{8}x^2+\frac{1}{2}(x^2-1)\log\left(4x+4\sqrt{x^2-x}-1\right)-\frac{1}{32}\sqrt{x^2-x}(4x+9)+\frac{1}{32}x+\frac{63}{256}\log(8x+1)-\frac{31}{256}\log\left(-2x+2\sqrt{x^2-x}+1\right)+\frac{63}{256}\log\left(-2x+2\sqrt{x^2-x}-1\right)-\frac{63}{256}\log\left(-4x+4\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*x^2 + 1/2*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/32*sqrt(x^2 - x
)*(4*x + 9) + 1/32*x + 63/256*log(8*x + 1) - 31/256*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 63/256*log(-2*x + 2*sqrt(x^2 - x) - 1) - 63/256*log(-4*x + 4*sqr
t(x^2 - x) + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.26, size = 114, normalized size = 0.90

$$\frac{1}{2}x^2\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{8}x^2-\frac{1}{32}\sqrt{x^2-x}(4x+9)+\frac{1}{32}x-\frac{1}{256}\log(|8x+1|)+\frac{33}{256}\log\left(\left|-2x+2\sqrt{x^2-x}+1\right|\right)-\frac{1}{256}\log\left(\left|-2x+2\sqrt{x^2-x}-1\right|\right)+\frac{1}{256}\log\left(\left|-4x+4\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/8*x^2 - 1/32*sqrt(x^2 - x)*(4*
x + 9) + 1/32*x - 1/256*log(abs(8*x + 1)) + 33/256*log(abs(-2*x + 2*sqrt(x^
```

```
2 - x) + 1)) - 1/256*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 1/256*log(abs(-
4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x \ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
[Out] int(x*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.104 $\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx$

Optimal. Leaf size=95

$$-\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+\right.$$

```
[Out] -1/2*x-1/16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-7/8*arctanh(x/(x^2-x)^(1/2)
)+1/16*ln(1+8*x)+x*ln(-1+4*x+4*(x^2-x)^(1/2))-1/2*(x^2-x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2617, 2613, 6874, 654, 634, 212, 748, 857, 738}

$$-\frac{\sqrt{x^2-x}}{2}+x\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{x}{2}+\frac{1}{16}\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/2*x - Sqrt[-x + x^2]/2 - ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])]/16 - (7*
ArcTanh[x/Sqrt[-x + x^2]])/8 + Log[1 + 8*x]/16 + x*Log[-1 + 4*x + 4*Sqrt[-x
 + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2613

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
, x_Symbol] :> Simp[x*Log[d + e*x + f*Sqrt[a + b*x + c*x^2]], x] + Dist[f^2
*((b^2 - 4*a*c)/2), Int[x/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a
*e + (2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log\left(-1+4x+4\sqrt{(-1+x)x}\right)\,dx = \int \log\left(-1+4x+4\sqrt{-x+x^2}\right)\,dx$$
$$= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int \frac{x}{-4(1+2x)\sqrt{-x+x^2}+8 \cdots}$$
$$= x\log\left(-1+4x+4\sqrt{-x+x^2}\right)+8\int\left(-\frac{1}{16}+\frac{1}{16(1+8x)}-\frac{\cdots}{12\sqrt{\cdots}}\right.$$
$$= -\frac{x}{2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+x^2}\right)-\frac{2}{3}\int\frac{\cdots}{\sqrt{\cdots}}$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+\cdots}\right.$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}+\frac{1}{16}\log(1+8x)+x\log\left(-1+4x+4\sqrt{-x+\cdots}\right.$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{2}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+\frac{1}{16}\log(1+8x)+\cdots$$
$$= -\frac{x}{2}-\frac{1}{2}\sqrt{-x+x^2}-\frac{1}{16}\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-\frac{7}{8}\tanh^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.89

$$\frac{1}{16}\left(-8x-8\sqrt{(-1+x)x}+2\log(1+8x)-7\log\left(1-2x-2\sqrt{(-1+x)x}\right)+16x\log\left(-1+4x+4\sqrt{(-1+x)x}\right)-\log\left(1-10x+6\sqrt{(-1+x)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-8*x - 8*Sqrt[(-1 + x)*x] + 2*Log[1 + 8*x] - 7*Log[1 - 2*x - 2*Sqrt[(-1 +
x)*x]] + 16*x*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - Log[1 - 10*x + 6*Sqrt[(-
1 + x)*x]])/16
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.84

method	result
default	$x\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)-\frac{7\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)}{16}-\frac{\operatorname{arctanh}\left(\frac{\frac{4}{3}-\frac{40x}{3}}{\sqrt{64\left(x+\frac{1}{8}\right)^2-80x-1}}\right)}{16}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-1+4*x+4*((-1+x)*x)^(1/2))-7/16*ln(-1/2+x+(x^2-x)^(1/2))-1/16*arctanh(
32/3*(1/8-5/4*x)/(64*(x+1/8)^2-80*x-1)^(1/2))-1/2*(x^2-x)^(1/2)-1/2*x+1/16*
ln(1+8*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 1/2*x + integrate(1/2*(2*x^2 + x)/
(4*x^3 - 5*x^2 + 4*(x^(5/2) - x^(3/2))*sqrt(x - 1) + x), x) - 1/2*log(sqrt(
x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.40, size = 101, normalized size = 1.06

$$(x+1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-x}-\frac{7}{16}\,\log\left(8\,x+1\right)+\frac{15}{16}\,\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)-\frac{7}{16}\,\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)+\frac{7}{16}\,\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) - 7/16*l
og(8*x + 1) + 15/16*log(-2*x + 2*sqrt(x^2 - x) + 1) - 7/16*log(-2*x + 2*sqr
t(x^2 - x) - 1) + 7/16*log(-4*x + 4*sqrt(x^2 - x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(4x+4\sqrt{x\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Integral(log(4*x + 4*sqrt(x*(x - 1)) - 1), x)
```

Giac [A]
time = 3.89, size = 101, normalized size = 1.06

$$x\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-x}+\frac{1}{16}\,\log\left(|8\,x+1|\right)+\frac{7}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)+\frac{1}{16}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)-\frac{1}{16}\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/2*x - 1/2*sqrt(x^2 - x) + 1/16*log(a
bs(8*x + 1)) + 7/16*log(abs(-2*x + 2*sqrt(x^2 - x) + 1)) + 1/16*log(abs(-2*
x + 2*sqrt(x^2 - x) - 1)) - 1/16*log(abs(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.105 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(-1+4*x+4*(x^2-x)^(1/2))/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
[Out] Defer[Int][Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x, x]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx=\int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}\,dx$$

Mathematica [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x,x]
[Out] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x, x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(-1+4x+4\sqrt{\left(-1+x\right)x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="maxima")
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="fricas")
[Out] integral(log(4*x + 4*sqrt(x^2 - x) - 1)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(4x+4\sqrt{x^{2}-x}-1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x,x)
[Out] Integral(log(4*x + 4*sqrt(x**2 - x) - 1)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x, x)
```

3.106 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx$

Optimal. Leaf size=76

$$\frac{4\sqrt{-x+x^2}}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}$$

```
[Out] 4*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))+4*ln(x)-4*ln(1+8*x)-ln(-1+4*x+4*(x^2-
x)^(1/2))/x+4*(x^2-x)^(1/2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2617, 2615, 6874, 654, 634, 212, 676, 678, 748, 857, 738}

$$\frac{4\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)+4\log(x)-4\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[-x + x^2])/x + 4*ArcTanh[(1 - 10*x)/(6*Sqrt[-x + x^2])] + 4*Log[x]
- 4*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[-x + x^2]]/x
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
```

```
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^2}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^2}\,dx$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int \frac{1}{x\left(-4(1+2x)\sqrt{-x+x^2}\right.}$$

$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x} - 8\int\left(-\frac{1}{2x}+\frac{4}{1+8x}-\frac{}{12\sqrt{-}}\right.$$

$$= 4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x}+\frac{2}{3}\int$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x\right.}}{x}$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\log(x)-4\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{-x\right.}}{x}$$

$$= \frac{4\sqrt{-x+x^2}}{x}-\frac{40}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right)+4\log(x)-4\log(1+8$$

$$= \frac{4\sqrt{-x+x^2}}{x}+4\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)+4\log(x)-4\log(1+8x$$

Mathematica [**A**]
time = 0.06, size = 68, normalized size = 0.89

$$\frac{4\sqrt{(-1+x)x}}{x}+4\log(x)-8\log(1+8x)-\frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x}+4\log\left(1-10x+6\sqrt{(-1+x)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^2,x]
```

```
[Out] (4*Sqrt[(-1 + x)*x])/x + 4*Log[x] - 8*Log[1 + 8*x] - Log[-1 + 4*x + 4*Sqrt[
(-1 + x)*x]]/x + 4*Log[1 - 10*x + 6*Sqrt[(-1 + x)*x]]
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2, x)
```

Fricas [**A**]
time = 0.38, size = 115, normalized size = 1.51

$$-\frac{7\,x\log\left(8\,x+1\right)+2\,(x+1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-8\,x\log\left(x\right)+x\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)+7\,x\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)-7\,x\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)-8\,x-8\,\sqrt{x^2-x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(7*x*log(8*x + 1) + 2*(x + 1)*log(4*x + 4*sqrt(x^2 - x) - 1) - 8*x*log
(x) + x*log(-2*x + 2*sqrt(x^2 - x) + 1) + 7*x*log(-2*x + 2*sqrt(x^2 - x) -
1) - 7*x*log(-4*x + 4*sqrt(x^2 - x) + 1) - 8*x - 8*sqrt(x^2 - x))/x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.68, size = 92, normalized size = 1.21

$$-\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{x}+\frac{4}{x-\sqrt{x^2-x}}-4\,\log\left(|8\,x+1|\right)+4\,\log\left(|x|\right)-4\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)+4\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] -log(4*x + 4*sqrt((x - 1)*x) - 1)/x + 4/(x - sqrt(x^2 - x)) - 4*log(abs(8*x
 + 1)) + 4*log(abs(x)) - 4*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) + 4*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^2, x)
```

3.107 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{x}-\frac{10\sqrt{-x+x^2}}{x}-\frac{2(-x+x^2)^{3/2}}{3x^3}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right)-16\log(x)+16\log(1+8x)-\frac{\log\left(-1+4x\ \cdots\right.}{2\cdots}$$

```
[Out] -2/x-2/3*(x^2-x)^(3/2)/x^3-16*arctanh(1/6*(1-10*x)/(x^2-x)^(1/2))-16*ln(x)+
16*ln(1+8*x)-1/2*ln(-1+4*x+4*(x^2-x)^(1/2))/x^2-10*(x^2-x)^(1/2)/x
```

Rubi [A]
time = 0.19, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2617, 2615, 6874, 654, 634, 212, 748, 857, 738, 664, 676, 678}

$$-\frac{10\sqrt{x^2-x}}{x}-\frac{\log\left(4\sqrt{x^2-x}+4x-1\right)}{2x^2}-16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{x^2-x}}\right)-\frac{2(x^2-x)^{3/2}}{3x^3}-\frac{2}{x}-16\log(x)+16\log(8x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (10*Sqrt[-x + x^2])/x - (2*(-x + x^2)^(3/2))/(3*x^3) - 16*ArcTanh[(1
 - 10*x)/(6*Sqrt[-x + x^2])] - 16*Log[x] + 16*Log[1 + 8*x] - Log[-1 + 4*x +
 4*Sqrt[-x + x^2]]/(2*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 676

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Dist[c*(p/(e^2*(m + p + 1))), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
- 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0])
&& NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 678

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^3}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^3}\,dx$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \frac{1}{x^2\left(-4(1+2x)\sqrt{-x+x^2}\right.}$$
$$= -\frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2} - 4\int \left(-\frac{1}{2x^2}+\frac{4}{x}-\frac{32}{1+8x}-\right.$$
$$= -\frac{2}{x} - 16\log(x) + 16\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{2x^2}$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} - 16\log(x) + 16\log(1+8x$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} + \frac{160}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x+x^2}}\right.$$
$$= -\frac{2}{x} - \frac{10\sqrt{-x+x^2}}{x} - \frac{2(-x+x^2)^{3/2}}{3x^3} - 16\tanh^{-1}\left(\frac{1-10x}{6\sqrt{-x+x^2}}\right.$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.81

$$-\frac{2}{x} - \frac{2\sqrt{(-1+x)x}\,(-1+16x)}{3x^2} - 16\log(x) + 32\log(1+8x) - \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{2x^2} - 16\log\left(1-10x+6\sqrt{(-1+x)x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^3,x]
```

```
[Out] -2/x - (2*Sqrt[(-1 + x)*x]*(-1 + 16*x))/(3*x^2) - 16*Log[x] + 32*Log[1 + 8*
x] - Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/(2*x^2) - 16*Log[1 - 10*x + 6*Sqrt[
(-1 + x)*x]]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/x^3, x)
```

Fricas [A]
time = 0.39, size = 138, normalized size = 1.37

$$\frac{189\,x^2\log\left(8\,x+1\right)-192\,x^2\log\left(x\right)+3\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)+189\,x^2\log\left(-2\,x+2\,\sqrt{x^2-x}-1\right)-189\,x^2\log\left(-4\,x+4\,\sqrt{x^2-x}+1\right)-128\,x^2+6\,(x^2-1)\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-8\,\sqrt{x^2-x}\,(16\,x-1)-24\,x}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(189*x^2*log(8*x + 1) - 192*x^2*log(x) + 3*x^2*log(-2*x + 2*sqrt(x^2 -
 x) + 1) + 189*x^2*log(-2*x + 2*sqrt(x^2 - x) - 1) - 189*x^2*log(-4*x + 4*s
qrt(x^2 - x) + 1) - 128*x^2 + 6*(x^2 - 1)*log(4*x + 4*sqrt(x^2 - x) - 1) -
8*sqrt(x^2 - x)*(16*x - 1) - 24*x)/x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.81, size = 130, normalized size = 1.29

$$-\frac{2}{x}-\frac{\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)}{2\,x^2}-\frac{2\left(18\left(x-\sqrt{x^2-x}\right)^2-3\,x+3\,\sqrt{x^2-x}+1\right)}{3\left(x-\sqrt{x^2-x}\right)^3}+16\,\log\left(|8\,x+1|\right)-16\,\log\left(|x|\right)+16\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}-1\right|\right)-16\,\log\left(\left|-4\,x+4\,\sqrt{x^2-x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] -2/x - 1/2*log(4*x + 4*sqrt((x - 1)*x) - 1)/x^2 - 2/3*(18*(x - sqrt(x^2 - x
))^2 - 3*x + 3*sqrt(x^2 - x) + 1)/(x - sqrt(x^2 - x))^3 + 16*log(abs(8*x +
1)) - 16*log(abs(x)) + 16*log(abs(-2*x + 2*sqrt(x^2 - x) - 1)) - 16*log(abs
(-4*x + 4*sqrt(x^2 - x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3,x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^3, x)
```

3.108 $\int x^{3/2} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) \, dx$

Optimal. Leaf size=187

$$-\frac{\sqrt{x}}{160}+\frac{x^{3/2}}{60}-\frac{2x^{5/2}}{25}-\frac{17\sqrt{-x+x^2}}{32\sqrt{x}}-\frac{71(-x+x^2)^{3/2}}{300x^{3/2}}-\frac{2(-x+x^2)^{3/2}}{25\sqrt{x}}-\frac{\sqrt{-x+x^2}\,\tan^{-1}\left(\frac{2}{3}\sqrt{2}\,\sqrt{-1+x}\right.}{320\sqrt{2}\,\sqrt{-1+x}\,\sqrt{x}}$$

```
[Out] 1/60*x^(3/2)-2/25*x^(5/2)-71/300*(x^2-x)^(3/2)/x^(3/2)+2/5*x^(5/2)*ln(-1+4*
x+4*(x^2-x)^(1/2))+1/640*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2/25*(x^2-x)^(3/
2)/x^(1/2)-1/160*x^(1/2)-17/32/x^(1/2)*(x^2-x)^(1/2)-1/640*arctan(2/3*2^(1/
2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2025, 2041, 1160, 455, 52, 65}

$$-\frac{\sqrt{x^2-x}\,\text{ArcTan}\left(\frac{2}{3}\sqrt{2}\,\sqrt{x-1}\right)}{320\sqrt{2}\,\sqrt{x-1}\,\sqrt{x}}+\frac{\text{ArcTan}\left(2\sqrt{2}\,\sqrt{x}\right)}{320\sqrt{2}}-\frac{2x^{5/2}}{25}+\frac{x^{3/2}}{60}-\frac{2(x^2-x)^{3/2}}{25\sqrt{x}}-\frac{17\sqrt{x^2-x}}{32\sqrt{x}}-\frac{71(x^2-x)^{3/2}}{300x^{3/2}}+\frac{2}{5}x^{5/2}\log\left(4\sqrt{x^2-x}+4x-1\right)-\frac{\sqrt{x}}{160}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] -1/160*Sqrt[x] + x^(3/2)/60 - (2*x^(5/2))/25 - (17*Sqrt[-x + x^2])/(32*Sqrt
[x]) - (71*(-x + x^2)^(3/2))/(300*x^(3/2)) - (2*(-x + x^2)^(3/2))/(25*Sqrt[
x]) - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(320*Sqrt[2]*Sqrt
[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x]]/(320*Sqrt[2]) + (2*x^(5/2)*Lo
g[-1 + 4*x + 4*Sqrt[-x + x^2]])/5
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
```

```
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^{3/2} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int x^{3/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) dx$$
$$= \frac{2}{5}x^{5/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{16}{5} \int \frac{}{-4(1+2x)\sqrt{-x}}$$
$$= \frac{2}{5}x^{5/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{5}\text{Subst}\left(\int \frac{}{-4\left(1+2\right.}\right.$$
$$= \frac{2}{5}x^{5/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{5}\text{Subst}\left(\int \left(-\frac{1}{1024}\right.\right.$$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} + \frac{2}{5}x^{5/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right)$$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{8\sqrt{-x+x^2}}{15\sqrt{x}} - \frac{11(-x+x^2)^{3/2}}{60x^{3/2}} - 2($$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{8\sqrt{-x+x^2}}{15\sqrt{x}} - \frac{71(-x+x^2)^{3/2}}{300x^{3/2}} - 2($$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{17\sqrt{-x+x^2}}{32\sqrt{x}} - \frac{71(-x+x^2)^{3/2}}{300x^{3/2}} - 2$$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{17\sqrt{-x+x^2}}{32\sqrt{x}} - \frac{71(-x+x^2)^{3/2}}{300x^{3/2}} - 2$$
$$= -\frac{\sqrt{x}}{160} + \frac{x^{3/2}}{60} - \frac{2x^{5/2}}{25} - \frac{17\sqrt{-x+x^2}}{32\sqrt{x}} - \frac{71(-x+x^2)^{3/2}}{300x^{3/2}} - 2$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 0.21, size = 232, normalized size = 1.24

$$\frac{-240\sqrt{x}+640x^{3/2}-3072x^{5/2}-\frac{11312\sqrt{(-1+x)x}}{\sqrt{x}}-6016\sqrt{x}\sqrt{(-1+x)x}-3072x^{3/2}\sqrt{(-1+x)x}+60\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)-60\sqrt{2}\tan^{-1}\left(\frac{2\sqrt{2}\sqrt{(-1+x)x}}{3\sqrt{x}}\right)-30i\sqrt{2}\log\left(4(1+8x)^2\right)+15i\sqrt{2}\log\left((1+8x)\left(1-10x-6\sqrt{(-1+x)x}\right)\right)+15360x^{5/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)+15i\sqrt{2}\log\left((1+8x)\left(1-10x+6\sqrt{(-1+x)x}\right)\right)}{38400}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (-240*Sqrt[x] + 640*x^(3/2) - 3072*x^(5/2) - (11312*Sqrt[(-1 + x)*x])/Sqrt[
x] - 6016*Sqrt[x]*Sqrt[(-1 + x)*x] - 3072*x^(3/2)*Sqrt[(-1 + x)*x] + 60*Sqr
t[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 60*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)
*x])/(3*Sqrt[x])] - (30*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] + (15*I)*Sqrt[2]*Log[
```

```
(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 15360*x^(5/2)*Log[-1 + 4*x + 4
*Sqrt[(-1 + x)*x]] + (15*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 +
x)*x])])/38400
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \ln \left(-1+4 x+4 \sqrt{(-1+x) x}\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(3/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 2/25*(2*x^2 + 5)*sqrt(x)
 - 2/15*x^(3/2) + integrate(1/5*(2*x^(5/2) + x^(3/2))/(4*x^2 + 4*(x^(3/2) -
 sqrt(x))*sqrt(x - 1) - 5*x + 1), x) + 1/5*log(sqrt(x) + 1) - 1/5*log(sqrt(
x) - 1)
```

Fricas [A]
time = 0.40, size = 110, normalized size = 0.59

$$\frac{3840\,x^{\frac{7}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)+15\,\sqrt{2}\,x\arctan\left(2\,\sqrt{2}\,\sqrt{x}\right)+15\,\sqrt{2}\,x\arctan\left(\frac{3\,\sqrt{2}\,\sqrt{x}}{4\,\sqrt{x^2-x}}\right)-4\,(192\,x^2+376\,x+707)\sqrt{x^2-x}\,\sqrt{x}-4\,(192\,x^3-40\,x^2+15\,x)\sqrt{x}}{9600\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/9600*(3840*x^(7/2)*log(4*x + 4*sqrt(x^2 - x) - 1) + 15*sqrt(2)*x*arctan(2
*sqrt(2)*sqrt(x)) + 15*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x))
- 4*(192*x^2 + 376*x + 707)*sqrt(x^2 - x)*sqrt(x) - 4*(192*x^3 - 40*x^2 + 1
5*x)*sqrt(x))/x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [C] Result contains complex when optimal does not.
time = 4.01, size = 128, normalized size = 0.68

$$\frac{2}{5}\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{(x-1)x}-1\right)-\frac{2}{25}\,x^{\frac{5}{2}}+\frac{1}{1280}i\,\sqrt{2}\,\pi+\frac{1}{1280}\,\sqrt{2}\left(\pi-2\,\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{1}{2400}\,(8\,(24\,x+47)x+707)\sqrt{x-1}+\frac{1}{60}\,x^{\frac{3}{2}}+\frac{1}{640}\,\sqrt{2}\,\arctan\left(2\,\sqrt{2}\,\sqrt{x}\right)+\frac{1}{640}\,\sqrt{2}\,\arctan\left(\frac{2}{3}i\,\sqrt{2}\right)-\frac{1}{160}\,\sqrt{x}+\frac{707}{2400}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 2/25*x^(5/2) + 1/1280*I*sqrt
(2)*pi + 1/1280*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))
^2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 1/2400*(8*(24*x + 47)*x + 707)*sqrt(x -
 1) + 1/60*x^(3/2) + 1/640*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) + 1/640*sqrt(2
)*arctan(2/3*I*sqrt(2)) - 1/160*sqrt(x) + 707/2400*I
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(3/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.109 $\int \sqrt{x} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx$

Optimal. Leaf size=158

$$\frac{\sqrt{x}}{12}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{-x+x^2}}{12\sqrt{x}}-\frac{2(-x+x^2)^{3/2}}{9x^{3/2}}+\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{24\sqrt{2}\sqrt{-1+x}\sqrt{x}}-\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}+$$

[Out] -2/9*x^(3/2)-2/9*(x^2-x)^(3/2)/x^(3/2)+2/3*x^(3/2)*ln(-1+4*x+4*(x^2-x)^(1/2))-1/48*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)+1/12*x^(1/2)-11/12/x^(1/2)*(x^2-x)^(1/2)+1/48*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)

Rubi [A]
time = 0.28, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2025, 1160, 455, 52, 65, 210}

$$\frac{\sqrt{x^2-x}\,\text{ArcTan}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{24\sqrt{2}\sqrt{x-1}\sqrt{x}}-\frac{\text{ArcTan}\left(2\sqrt{2}\sqrt{x}\right)}{24\sqrt{2}}-\frac{2x^{3/2}}{9}-\frac{11\sqrt{x^2-x}}{12\sqrt{x}}-\frac{2(x^2-x)^{3/2}}{9x^{3/2}}+\frac{2}{3}x^{3/2}\log\left(4\sqrt{x^2-x}+4x-1\right)+\frac{\sqrt{x}}{12}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]

[Out] Sqrt[x]/12 - (2*x^(3/2))/9 - (11*Sqrt[-x + x^2])/(12*Sqrt[x]) - (2*(-x + x^2)^(3/2))/(9*x^(3/2)) + (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(24*Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) - ArcTan[2*Sqrt[2]*Sqrt[x]]/(24*Sqrt[2]) + (2*x^(3/2)*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/3

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt{x} \log \left(-1+4x+4\sqrt{(-1+x)x}\right) dx = \int \sqrt{x} \log \left(-1+4x+4\sqrt{-x+x^2}\right) dx$$

$$= \frac{2}{3}x^{3/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{16}{3} \int \frac{}{-4(1+2x)\sqrt{-x}}$$

$$= \frac{2}{3}x^{3/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{3}\text{Subst}\left(\int \frac{}{-4\,(1+2}\right.$$

$$= \frac{2}{3}x^{3/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) + \frac{32}{3}\text{Subst}\left(\int \left(\frac{1}{128} - \right.\right.$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} + \frac{2}{3}x^{3/2} \log \left(-1+4x+4\sqrt{-x+x^2}\right) - \frac{1}{12}\text{Sub}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{8\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{2}\right.}{24\sqrt{2}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{8\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{2}\right.}{24\sqrt{2}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{}\right.}{24\sqrt{}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} - \frac{\tan^{-1}\left(2\sqrt{}\right.}{24\sqrt{}}$$

$$= \frac{\sqrt{x}}{12} - \frac{2x^{3/2}}{9} - \frac{11\sqrt{-x+x^2}}{12\sqrt{x}} - \frac{2(-x+x^2)^{3/2}}{9x^{3/2}} + \frac{\sqrt{-x+x^2}}{24}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 209, normalized size = 1.32

$$\frac{1}{576}\left(48\sqrt{x} - 128x^{3/2} - \frac{400\sqrt{(-1+x)x}}{\sqrt{x}} - 128\sqrt{x}\sqrt{(-1+x)x} - 12\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + 12\sqrt{2}\tan^{-1}\left(\frac{2\sqrt{2}\sqrt{(-1+x)x}}{3\sqrt{x}}\right) + 6i\sqrt{2}\log\left(4(1+8x)^2\right) - 3i\sqrt{2}\log\left((1+8x)\left(1-10x-6\sqrt{(-1+x)x}\right)\right) + 384x^{3/2}\log\left(-1+4x+4\sqrt{(-1+x)x}\right) - 3i\sqrt{2}\log\left((1+8x)\left(1-10x+6\sqrt{(-1+x)x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]],x]
```

```
[Out] (48*Sqrt[x] - 128*x^(3/2) - (400*Sqrt[(-1 + x)*x])/Sqrt[x] - 128*Sqrt[x]*Sq
rt[(-1 + x)*x] - 12*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 12*Sqrt[2]*ArcTan[(
2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (6*I)*Sqrt[2]*Log[4*(1 + 8*x)^2]
 - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] + 384*x^(3/
```

```
2)*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] - (3*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10
*x + 6*Sqrt[(-1 + x)*x])])/576
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\, \ln \left(-1+4x+4\sqrt{\left(-1+x\right)x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

```
[Out] int(x^(1/2)*ln(-1+4*x+4*((-1+x)*x)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4/9*x^(3/2) - 2/3*sqrt(x
) + integrate(1/3*(2*x^2 + x)/(4*x^(5/2) + 4*(x^2 - x)*sqrt(x - 1) - 5*x^(3
/2) + sqrt(x)), x) + 1/3*log(sqrt(x) + 1) - 1/3*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.39, size = 100, normalized size = 0.63

$$\frac{96\,x^{\frac{5}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}-1\right)-3\,\sqrt{2}\,x\arctan\left(2\,\sqrt{2}\,\sqrt{x}\right)-3\,\sqrt{2}\,x\arctan\left(\frac{3\,\sqrt{2}\,\sqrt{x}}{4\,\sqrt{x^2-x}}\right)-4\,\sqrt{x^2-x}\,(8\,x+25)\sqrt{x}-4\,(8\,x^2-3\,x)\sqrt{x}}{144\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/144*(96*x^(5/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 3*sqrt(2)*x*arctan(2*sqr
t(2)*sqrt(x)) - 3*sqrt(2)*x*arctan(3/4*sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 4*s
qrt(x^2 - x)*(8*x + 25)*sqrt(x) - 4*(8*x^2 - 3*x)*sqrt(x))/x
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*ln(-1+4*x+4*((-1+x)*x)**(1/2)),x)
```

[Out] Timed out

Giac [C] Result contains complex when optimal does not.
time = 3.56, size = 118, normalized size = 0.75

$$\frac{2}{3}x^{\frac{3}{2}}\log\left(4x+4\sqrt{(x-1)x}-1\right)-\frac{1}{96}i\sqrt{2}\pi-\frac{1}{96}\sqrt{2}\left(\pi-2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^2-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{1}{36}(8x+25)\sqrt{x-1}-\frac{2}{9}x^{\frac{3}{2}}-\frac{1}{48}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)-\frac{1}{48}\sqrt{2}\arctan\left(\frac{2}{3}i\sqrt{2}\right)+\frac{1}{12}\sqrt{x}+\frac{25}{36}i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*log(-1+4*x+4*((-1+x)*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*log(4*x + 4*sqrt((x - 1)*x) - 1) - 1/96*I*sqrt(2)*pi - 1/96*sqr
t(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^2 - 1)/(sqrt(x - 1
) - sqrt(x)))) - 1/36*(8*x + 25)*sqrt(x - 1) - 2/9*x^(3/2) - 1/48*sqrt(2)*a
rctan(2*sqrt(2)*sqrt(x)) - 1/48*sqrt(2)*arctan(2/3*I*sqrt(2)) + 1/12*sqrt(x
) + 25/36*I
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\ \ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1),x)
```

```
[Out] int(x^(1/2)*log(4*x + 4*(x*(x - 1))^(1/2) - 1), x)
```

3.110 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=118

$$-2\sqrt{x}-\frac{2\sqrt{-x+x^2}}{\sqrt{x}}-\frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}}+\frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+2\sqrt{x}\log\left(-1+4x+4\right.$$

```
[Out] 1/2*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*x^(1/2)+2*ln(-1+4*x+4*(x^2-x)^(1/2)
)*x^(1/2)-2/x^(1/2)*(x^2-x)^(1/2)-1/2*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2
-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 1160, 455, 52, 65}

$$-\frac{\sqrt{x^2-x}\,\text{ArcTan}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{2}\sqrt{x-1}\sqrt{x}}+\frac{\text{ArcTan}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{2\sqrt{x^2-x}}{\sqrt{x}}+2\sqrt{x}\log\left(4\sqrt{x^2-x}+4x-1\right)-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] - (2*Sqrt[-x + x^2])/Sqrt[x] - (Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]
*Sqrt[-1 + x])/3])/(Sqrt[2]*Sqrt[-1 + x]*Sqrt[x]) + ArcTan[2*Sqrt[2]*Sqrt[x
]]/Sqrt[2] + 2*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[-x + x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}}\,dx$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 16\int \frac{\sqrt{x}}{-4(1+2x)\sqrt{-x+x}}$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 32\text{Subst}\left(\int \frac{}{-4\left(1+2x^2\right)}\right.$$

$$= 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 32\text{Subst}\left(\int\left(-\frac{1}{16}+\frac{}{16(}\right.\right.$$

$$= -2\sqrt{x} + 2\sqrt{x}\log\left(-1+4x+4\sqrt{-x+x^2}\right) + 2\text{Subst}\left(\int \frac{1}{1+}\right.$$

$$= -2\sqrt{x} - \frac{8\sqrt{-x+x^2}}{3\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+\right.$$

$$= -2\sqrt{x} - \frac{8\sqrt{-x+x^2}}{3\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+\right.$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+\right.$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} + \frac{\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + 2\sqrt{x}\log\left(-1+\right.$$

$$= -2\sqrt{x} - \frac{2\sqrt{-x+x^2}}{\sqrt{x}} - \frac{\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{2}\sqrt{-1+x}\sqrt{x}} +$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.12, size = 186, normalized size = 1.58

$$\frac{1}{8}\left(-16\sqrt{x}-\frac{16\sqrt{(-1+x)x}}{\sqrt{x}}+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)-4\sqrt{2}\tan^{-1}\left(\frac{2\sqrt{2}\sqrt{(-1+x)x}}{3\sqrt{x}}\right)-2i\sqrt{2}\log\left(4(1+8x)^2\right)+i\sqrt{2}\log\left((1+8x)\left(1-10x-6\sqrt{(-1+x)x}\right)\right)+16\sqrt{x}\log\left(-1+4x+4\sqrt{(-1+x)x}\right)+i\sqrt{2}\log\left((1+8x)\left(1-10x+6\sqrt{(-1+x)x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/Sqrt[x],x]
```

```
[Out] (-16*Sqrt[x] - (16*Sqrt[(-1 + x)*x])/Sqrt[x] + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*S
qrt[x]] - 4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I
)*Sqrt[2]*Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(
-1 + x)*x])] + 16*Sqrt[x]*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]] + I*Sqrt[2]*Lo
g[(1 + 8*x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])])/8
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(-1+4x+4\sqrt{(-1+x)\,x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1) - 4*sqrt(x) + integrate((2*x
^2 + x)/(4*x^(7/2) - 5*x^(5/2) + 4*(x^3 - x^2)*sqrt(x - 1) + x^(3/2)), x) +
 log(sqrt(x) + 1) - log(sqrt(x) - 1)
```

Fricas [A]
time = 0.40, size = 84, normalized size = 0.71

$$\frac{\sqrt{2}\,x\arctan\left(2\,\sqrt{2}\,\sqrt{x}\right)+\sqrt{2}\,x\arctan\left(\frac{3\,\sqrt{2}\,\sqrt{x}}{4\,\sqrt{x^2-x}}\right)+4\,x^{\frac{3}{2}}\log\left(4\,x+4\,\sqrt{x^2-x}\,-1\right)-4\,x^{\frac{3}{2}}-4\,\sqrt{x^2-x}\,\sqrt{x}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + sqrt(2)*x*arctan(3/4*sqrt(2)*sqr
t(x)/sqrt(x^2 - x)) + 4*x^(3/2)*log(4*x + 4*sqrt(x^2 - x) - 1) - 4*x^(3/2)
- 4*sqrt(x^2 - x)*sqrt(x))/x
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log(4*x + 4*sqrt((x - 1)*x) - 1)/sqrt(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(1/2), x)
```

3.111 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{4\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}}+4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)-8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right)-\frac{2\log\left(-1+\right.}{}$$

```
[Out] -8*arctan(x^(1/2)/(x^2-x)^(1/2))+4*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-2*ln(-
1+4*x+4*(x^2-x)^(1/2))/x^(1/2)-4*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(
1/2)*2^(1/2)/(-1+x)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2046, 2033, 1160, 455, 52, 65, 210}

$$-\frac{4\sqrt{2}\sqrt{x^2-x}\,\text{ArcTan}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{\sqrt{x-1}\sqrt{x}}-8\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)+4\sqrt{2}\,\text{ArcTan}\left(2\sqrt{2}\sqrt{x}\right)-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[-x + x^2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(Sqrt[-1 + x
]*Sqrt[x]) + 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] - 8*ArcTan[Sqrt[x]/Sqrt[-x
 + x^2]] - (2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/Sqrt[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{3/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 16\int \frac{}{\sqrt{x}\,\left(-4(1+2x)\sqrt{-x\right.}}$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\mathrm{Subst}\left(\int \frac{}{-4\left(1+2x^2\right)\sqrt{}}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} - 32\mathrm{Subst}\left(\int\left(-\frac{1}{2\left(1+8x^2\right)}\right.\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{\sqrt{x}} + \frac{8}{3}\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{-x^2+x^4}}\,dx\right.$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\right.}{\sqrt{x}}$$

$$= -\frac{16\sqrt{-x+x^2}}{3\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - \frac{2\log\left(-1+4x+4\right.}{\sqrt{x}}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+\right.}{}$$

$$= 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) - 8\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{-x+x^2}}\right) - \frac{2\log\left(-1+\right.}{}$$

$$= -\frac{4\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{\sqrt{-1+x}\sqrt{x}} + 4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 177, normalized size = 1.55

$$4\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + 8\tan^{-1}\left(\frac{\sqrt{(-1+x)x}}{\sqrt{x}}\right) - 4\sqrt{2}\tan^{-1}\left(\frac{2\sqrt{2}\sqrt{(-1+x)x}}{3\sqrt{x}}\right) - 2i\sqrt{2}\log\left(4(1+8x)^2\right) + i\sqrt{2}\log\left((1+8x)\left(1-10x-6\sqrt{(-1+x)x}\right)\right) - \frac{2\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{\sqrt{x}} + i\sqrt{2}\log\left((1+8x)\left(1-10x+6\sqrt{(-1+x)x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(3/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTan[2*Sqrt[2]*Sqrt[x]] + 8*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] -
4*Sqrt[2]*ArcTan[(2*Sqrt[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] - (2*I)*Sqrt[2]*
```

```
Log[4*(1 + 8*x)^2] + I*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x]
)] - (2*Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]])/Sqrt[x] + I*Sqrt[2]*Log[(1 + 8*
x)*(1 - 10*x + 6*Sqrt[(-1 + x)*x])]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(-1+4 x+4 \sqrt{\left(-1+x\right) x}\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/sqrt(x) - 2/sqrt(x) - integrate((2*
x^2 + x)/(4*x^(9/2) - 5*x^(7/2) + x^(5/2) + 4*(x^4 - x^3)*sqrt(x - 1)), x)
- log(sqrt(x) + 1) + log(sqrt(x) - 1)
```

Fricas [A]
time = 0.38, size = 84, normalized size = 0.74

$$\frac{2 \left(2 \sqrt{2}\, x \arctan \left(2 \sqrt{2}\, \sqrt{x}\right) + 2 \sqrt{2}\, x \arctan \left(\frac{3 \sqrt{2}\, \sqrt{x}}{4 \sqrt{x^{2}-x}}\right) - 4 \, x \arctan \left(\frac{\sqrt{x}}{\sqrt{x^{2}-x}}\right) - \sqrt{x} \log \left(4 \, x + 4 \sqrt{x^{2}-x} - 1\right)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(2)*x*arctan(2*sqrt(2)*sqrt(x)) + 2*sqrt(2)*x*arctan(3/4*sqrt(2)*s
qrt(x)/sqrt(x^2 - x)) - 4*x*arctan(sqrt(x)/sqrt(x^2 - x)) - sqrt(x)*log(4*x
 + 4*sqrt(x^2 - x) - 1))/x
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(2*
sqrt(2)*atan(4*sqrt(sageVARx)/sqrt(2))-2*(-2*(1/2*pi*sign(-sqrt(sageVARx)+s
qrt(sageVARx-1))+atan(1/2*((-sqrt(sageVARx)+sqrt(sageVARx-1))^2-1)/(-sqrt(s
ageVARx)+sqrt(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(3/2), x)
```

3.112 $\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{16}{3\sqrt{x}}+\frac{4\sqrt{-x+x^2}}{3x^{3/2}}+\frac{32\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+x}\right)}{3\sqrt{-1+x}\sqrt{x}}-\frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)+\frac{44}{3}\tan^{-1}\left(\right.$$

```
[Out] 44/3*arctan(x^(1/2)/(x^2-x)^(1/2))-2/3*ln(-1+4*x+4*(x^2-x)^(1/2))/x^(3/2)-3
2/3*arctan(2*2^(1/2)*x^(1/2))*2^(1/2)-16/3/x^(1/2)+4/3*(x^2-x)^(1/2)/x^(3/2
)+32/3*arctan(2/3*2^(1/2)*(-1+x)^(1/2))*(x^2-x)^(1/2)*2^(1/2)/(-1+x)^(1/2)/
x^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2617, 2615, 6865, 6874, 209, 1602, 2045, 2033, 2046, 1160, 455, 52, 65}

$$\frac{32\sqrt{2}\sqrt{x^2-x}\,\text{ArcTan}\left(\frac{2}{3}\sqrt{2}\sqrt{x-1}\right)}{3\sqrt{x-1}\sqrt{x}}+\frac{44}{3}\text{ArcTan}\left(\frac{\sqrt{x}}{\sqrt{x^2-x}}\right)-\frac{32}{3}\sqrt{2}\,\text{ArcTan}\left(2\sqrt{2}\sqrt{x}\right)+\frac{4\sqrt{x^2-x}}{3x^{3/2}}-\frac{2\log\left(4\sqrt{x^2-x}+4x-1\right)}{3x^{3/2}}-\frac{16}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] -16/(3*Sqrt[x]) + (4*Sqrt[-x + x^2])/(3*x^(3/2)) + (32*Sqrt[2]*Sqrt[-x + x^
2]*ArcTan[(2*Sqrt[2]*Sqrt[-1 + x])/3])/(3*Sqrt[-1 + x]*Sqrt[x]) - (32*Sqrt[
2]*ArcTan[2*Sqrt[2]*Sqrt[x]])/3 + (44*ArcTan[Sqrt[x]/Sqrt[-x + x^2]])/3 - (
2*Log[-1 + 4*x + 4*Sqrt[-x + x^2]])/(3*x^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
```

```
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2615

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]]
*((g_.)*(x_))^(m_.), x_Symbol] :> Simp[(g*x)^(m + 1)*(Log[d + e*x + f*Sqrt[
a + b*x + c*x^2]]/(g*(m + 1))), x] + Dist[f^2*((b^2 - 4*a*c)/(2*g*(m + 1)))
, Int[(g*x)^(m + 1)/((2*d*e - b*f^2)*(a + b*x + c*x^2) - f*(b*d - 2*a*e + (
2*c*d - b*e)*x)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 2617

```
Int[Log[(d_.) + (f_.)*Sqrt[u_] + (e_.)*(x_)]*(v_.), x_Symbol] :> Int[v*Log[
d + e*x + f*Sqrt[ExpandToSum[u, x]]], x] /; FreeQ[{d, e, f}, x] && Quadrati
cQ[u, x] &&  !QuadraticMatchQ[u, x] && (EqQ[v, 1] || MatchQ[v, ((g_.)*x)^(m
_.) /; FreeQ[{g, m}, x]])
```

Rule 6865

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{5/2}}\,dx = \int \frac{\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{x^{5/2}}\,dx$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{16}{3}\int \frac{}{x^{3/2}\left(-4(1+2x)\sqrt{-\right.}}$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\text{Subst}\left(\int \frac{}{x^2\left(-4\left(1+2\right.\right.}\right.$$

$$= -\frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} - \frac{32}{3}\text{Subst}\left(\int\left(-\frac{1}{2x^2}+\frac{4}{1+}\right.\right.$$

$$= -\frac{16}{3\sqrt{x}} - \frac{2\log\left(-1+4x+4\sqrt{-x+x^2}\right)}{3x^{3/2}} + \frac{8}{9}\text{Subst}\left(\int \frac{x^2}{\sqrt{-x^2}}\right.$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\right.$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{128\sqrt{-x+x^2}}{9\sqrt{x}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\right.$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} - \frac{32}{3}\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right) + \frac{44}{3}\tan^{-1}$$

$$= -\frac{16}{3\sqrt{x}} + \frac{4\sqrt{-x+x^2}}{3x^{3/2}} + \frac{32\sqrt{2}\sqrt{-x+x^2}\tan^{-1}\left(\frac{2}{3}\sqrt{2}\sqrt{-1+\right.}}{3\sqrt{-1+x}\sqrt{x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 204, normalized size = 1.35

$$\frac{2}{3}\left(-\frac{8}{\sqrt{x}}+\frac{2\sqrt{(-1+x)x}}{x^{3/2}}-16\sqrt{2}\tan^{-1}\left(2\sqrt{2}\sqrt{x}\right)-22\tan^{-1}\left(\frac{\sqrt{(-1+x)x}}{\sqrt{x}}\right)+16\sqrt{2}\tan^{-1}\left(\frac{2\sqrt{2}\sqrt{(-1+x)x}}{3\sqrt{x}}\right)+8i\sqrt{2}\log\left(4(1+8x)^2\right)-4i\sqrt{2}\log\left((1+8x)\left(1-10x-6\sqrt{(-1+x)x}\right)\right)-\frac{\log\left(-1+4x+4\sqrt{(-1+x)x}\right)}{x^{3/2}}-4i\sqrt{2}\log\left((1+8x)\left(1-10x+6\sqrt{(-1+x)x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + 4*x + 4*Sqrt[(-1 + x)*x]]/x^(5/2),x]
```

```
[Out] (2*(-8/Sqrt[x] + (2*Sqrt[(-1 + x)*x])/x^(3/2) - 16*Sqrt[2]*ArcTan[2*Sqrt[2]
*Sqrt[x]] - 22*ArcTan[Sqrt[(-1 + x)*x]/Sqrt[x]] + 16*Sqrt[2]*ArcTan[(2*Sqrt
```

```
[2]*Sqrt[(-1 + x)*x])/(3*Sqrt[x])] + (8*I)*Sqrt[2]*Log[4*(1 + 8*x)^2] - (4*
I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x - 6*Sqrt[(-1 + x)*x])] - Log[-1 + 4*x +
4*Sqrt[(-1 + x)*x]]/x^(3/2) - (4*I)*Sqrt[2]*Log[(1 + 8*x)*(1 - 10*x + 6*Sqr
t[(-1 + x)*x])]))/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(-1+4 x +4 \sqrt{\left(-1+x \right) x}\right)}{x^{\frac{5}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
```

```
[Out] int(ln(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/sqrt(x) - 2/3*log(4*sqrt(x - 1)*sqrt(x) + 4*x - 1)/x^(3/2) - 2/9/x^(3/2
) - integrate(1/3*(2*x^2 + x)/(4*x^(11/2) - 5*x^(9/2) + x^(7/2) + 4*(x^5 -
x^4)*sqrt(x - 1)), x) - 1/3*log(sqrt(x) + 1) + 1/3*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.38, size = 108, normalized size = 0.72

$$-\frac{2 \left(16 \sqrt{2}\, x^{2} \arctan\left(2 \sqrt{2} \sqrt{x}\right) + 16 \sqrt{2}\, x^{2} \arctan\left(\frac{3 \sqrt{2} \sqrt{x}}{4 \sqrt{x^{2} - x}}\right) - 22 \, x^{2} \arctan\left(\frac{\sqrt{x}}{\sqrt{x^{2} - x}}\right) + 8 \, x^{\frac{3}{2}} + \sqrt{x} \log\left(4 \, x + 4 \sqrt{x^{2} - x} - 1\right) - 2 \sqrt{x^{2} - x} \sqrt{x}\right)}{3 \, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*sqrt(2)*x^2*arctan(2*sqrt(2)*sqrt(x)) + 16*sqrt(2)*x^2*arctan(3/4*
sqrt(2)*sqrt(x)/sqrt(x^2 - x)) - 22*x^2*arctan(sqrt(x)/sqrt(x^2 - x)) + 8*x
^(3/2) + sqrt(x)*log(4*x + 4*sqrt(x^2 - x) - 1) - 2*sqrt(x^2 - x)*sqrt(x))/
x^2
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+4*x+4*((-1+x)*x)**(1/2))/x**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.90, size = 181, normalized size = 1.20

$$\frac{22}{3}\pi-\frac{16}{3}\sqrt{2}\left(\pi-2\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1\right)}{3\left(\sqrt{x-1}-\sqrt{x}\right)}\right)\right)-\frac{32}{3}\sqrt{2}\arctan\left(2\sqrt{2}\sqrt{x}\right)+\frac{8\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)}{3\left(\left(\sqrt{x-1}-\sqrt{x}-\frac{1}{\sqrt{x-1}-\sqrt{x}}\right)^{2}+4\right)}-\frac{16}{3\sqrt{x}}-\frac{2\log\left(4x+4\sqrt{x^{2}-x}-1\right)}{3x^{\frac{3}{2}}}-\frac{44}{3}\arctan\left(\frac{\left(\sqrt{x-1}-\sqrt{x}\right)^{2}-1}{2\left(\sqrt{x-1}-\sqrt{x}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+4*x+4*((-1+x)*x)^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] 22/3*pi - 16/3*sqrt(2)*(pi - 2*arctan(1/3*sqrt(2)*((sqrt(x - 1) - sqrt(x))^
2 - 1)/(sqrt(x - 1) - sqrt(x)))) - 32/3*sqrt(2)*arctan(2*sqrt(2)*sqrt(x)) +
 8/3*(sqrt(x - 1) - sqrt(x) - 1/(sqrt(x - 1) - sqrt(x)))/((sqrt(x - 1) - sq
rt(x) - 1/(sqrt(x - 1) - sqrt(x)))^2 + 4) - 16/3/sqrt(x) - 2/3*log(4*x + 4*
sqrt(x^2 - x) - 1)/x^(3/2) - 44/3*arctan(1/2*((sqrt(x - 1) - sqrt(x))^2 - 1
)/(sqrt(x - 1) - sqrt(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(4\,x+4\,\sqrt{x\,\left(x-1\right)}-1\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2),x)
```

```
[Out] int(log(4*x + 4*(x*(x - 1))^(1/2) - 1)/x^(5/2), x)
```

3.113 $\int x^3 \log(a+be^x)\,dx$

Optimal. Leaf size=93

$$\frac{1}{4}x^4\log(a+be^x)-\frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right)-x^3\text{Li}_2\left(-\frac{be^x}{a}\right)+3x^2\text{Li}_3\left(-\frac{be^x}{a}\right)-6x\text{Li}_4\left(-\frac{be^x}{a}\right)+6\text{Li}_5\left(-\frac{be^x}{a}\right)$$

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2612, 2611, 6744, 2320, 6724}

$$-x^3\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+3x^2\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-6x\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+6\text{PolyLog}\left(5,-\frac{be^x}{a}\right)+\frac{1}{4}x^4\log(a+be^x)-\frac{1}{4}x^4\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
```

```
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \log(a+be^x)\,dx &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) + \int x^3\log\left(1+\frac{be^x}{a}\right)dx\\ &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3\int x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)dx\\ &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6\int x\\ &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6x\mathrm{Li}_4\\ &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6x\mathrm{Li}_4\\ &= \frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6x\mathrm{Li}_4\end{aligned}$$

Mathematica [A]
time = 0.01, size = 93, normalized size = 1.00

$$\frac{1}{4}x^4\log(a+be^x) - \frac{1}{4}x^4\log\left(1+\frac{be^x}{a}\right) - x^3\mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3x^2\mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6x\mathrm{Li}_4\left(-\frac{be^x}{a}\right) + 6\mathrm{Li}_5\left(-\frac{be^x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[a + b*E^x],x]
```

```
[Out] (x^4*Log[a + b*E^x])/4 - (x^4*Log[1 + (b*E^x)/a])/4 - x^3*PolyLog[2, -((b*E
^x)/a)] + 3*x^2*PolyLog[3, -((b*E^x)/a)] - 6*x*PolyLog[4, -((b*E^x)/a)] + 6
*PolyLog[5, -((b*E^x)/a)]
```

Maple [A]
time = 0.03, size = 84, normalized size = 0.90

method	result
default	$\frac{x^4 \ln(a+b\,\mathrm{e}^x)}{4} - \frac{x^4 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{4} - x^3 \operatorname{polylog}\left(2, -\frac{b\,\mathrm{e}^x}{a}\right) + 3x^2 \operatorname{polylog}\left(3, -\frac{b\,\mathrm{e}^x}{a}\right) - 6x \operatorname{polylog}\left(4, -\frac{b\,\mathrm{e}^x}{a}\right) +$
risch	$\frac{x^4 \ln(a+b\,\mathrm{e}^x)}{4} - \frac{x^4 \ln\left(1+\frac{b\,\mathrm{e}^x}{a}\right)}{4} - x^3 \operatorname{polylog}\left(2, -\frac{b\,\mathrm{e}^x}{a}\right) + 3x^2 \operatorname{polylog}\left(3, -\frac{b\,\mathrm{e}^x}{a}\right) - 6x \operatorname{polylog}\left(4, -\frac{b\,\mathrm{e}^x}{a}\right) +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(a+b*exp(x))-1/4*x^4*ln(1+b*exp(x)/a)-x^3*polylog(2,-b*exp(x)/a)+
3*x^2*polylog(3,-b*exp(x)/a)-6*x*polylog(4,-b*exp(x)/a)+6*polylog(5,-b*exp(
x)/a)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.88

$$\frac{1}{4}\,x^4 \log\left(be^x + a\right) - \frac{1}{4}\,x^4 \log\left(\frac{be^x}{a} + 1\right) - x^3 \mathrm{Li}_2\left(-\frac{be^x}{a}\right) + 3\,x^2 \mathrm{Li}_3\left(-\frac{be^x}{a}\right) - 6\,x \mathrm{Li}_4\left(-\frac{be^x}{a}\right) + 6\,\mathrm{Li}_5\left(-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log(b*e^x/a + 1) - x^3*dilog(-b*e^x/a) + 3
*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylog(5, -b*e^x/
a)
```

Fricas [A]
time = 0.35, size = 88, normalized size = 0.95

$$\frac{1}{4}\,x^4 \log\left(be^x + a\right) - \frac{1}{4}\,x^4 \log\left(\frac{be^x + a}{a}\right) - x^3 \mathrm{Li}_2\left(-\frac{be^x + a}{a} + 1\right) + 3\,x^2 \mathrm{polylog}\left(3, -\frac{be^x}{a}\right) - 6\,x \mathrm{polylog}\left(4, -\frac{be^x}{a}\right) + 6\,\mathrm{polylog}\left(5, -\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(b*e^x + a) - 1/4*x^4*log((b*e^x + a)/a) - x^3*dilog(-(b*e^x + a
)/a + 1) + 3*x^2*polylog(3, -b*e^x/a) - 6*x*polylog(4, -b*e^x/a) + 6*polylo
g(5, -b*e^x/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^4e^x}{a+be^x}\,dx}{4}+\frac{x^4\log\left(a+be^x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(a+b*exp(x)),x)
[Out] -b*Integral(x**4*exp(x)/(a + b*exp(x)), x)/4 + x**4*log(a + b*exp(x))/4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x^3*log(b*e^x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(a + b*exp(x)),x)
[Out] int(x^3*log(a + b*exp(x)), x)
```

3.114 $\int x^2 \log(a+be^x)\, dx$

Optimal. Leaf size=77

$$\frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\text{Li}_2\left(-\frac{be^x}{a}\right)+2x\text{Li}_3\left(-\frac{be^x}{a}\right)-2\text{Li}_4\left(-\frac{be^x}{a}\right)$$

```
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2612, 2611, 6744, 2320, 6724}

$$-x^2\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+2x\text{PolyLog}\left(3,-\frac{be^x}{a}\right)-2\text{PolyLog}\left(4,-\frac{be^x}{a}\right)+\frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
```

```
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\log(a+be^x)\,dx &= \frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)+\int x^2\log\left(1+\frac{be^x}{a}\right)dx\\ &= \frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2\int x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)dx\\ &= \frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right)-2\int \mathrm{L}\\ &= \frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right)-2\mathrm{Subs}\\ &= \frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right)-2\mathrm{Li}_4\Big(\end{aligned}$$

Mathematica [A]
time = 0.01, size = 77, normalized size = 1.00

$$\frac{1}{3}x^3\log(a+be^x)-\frac{1}{3}x^3\log\left(1+\frac{be^x}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2x\mathrm{Li}_3\left(-\frac{be^x}{a}\right)-2\mathrm{Li}_4\left(-\frac{be^x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a + b*E^x],x]
```

```
[Out] (x^3*Log[a + b*E^x])/3 - (x^3*Log[1 + (b*E^x)/a])/3 - x^2*PolyLog[2, -((b*E
^x)/a)] + 2*x*PolyLog[3, -((b*E^x)/a)] - 2*PolyLog[4, -((b*E^x)/a)]
```

Maple [A]
time = 0.01, size = 69, normalized size = 0.90

method	result	siz
default	$\frac{x^3\ln(a+b\,e^x)}{3}-\frac{x^3\ln\left(1+\frac{b\,e^x}{a}\right)}{3}-x^2\,\mathrm{polylog}\left(2,-\frac{b\,e^x}{a}\right)+2x\,\mathrm{polylog}\left(3,-\frac{b\,e^x}{a}\right)-2\,\mathrm{polylog}\left(4,-\frac{b\,e^x}{a}\right)$	69
risch	$\frac{x^3\ln(a+b\,e^x)}{3}-\frac{x^3\ln\left(1+\frac{b\,e^x}{a}\right)}{3}-x^2\,\mathrm{polylog}\left(2,-\frac{b\,e^x}{a}\right)+2x\,\mathrm{polylog}\left(3,-\frac{b\,e^x}{a}\right)-2\,\mathrm{polylog}\left(4,-\frac{b\,e^x}{a}\right)$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(a+b*exp(x))-1/3*x^3*ln(1+b*exp(x)/a)-x^2*polylog(2,-b*exp(x)/a)+
2*x*polylog(3,-b*exp(x)/a)-2*polylog(4,-b*exp(x)/a)
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.87

$$\frac{1}{3}\,x^3\log\left(be^x+a\right)-\frac{1}{3}\,x^3\log\left(\frac{be^x}{a}+1\right)-x^2\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+2\,x\mathrm{Li}_3(-\frac{be^x}{a})-2\,\mathrm{Li}_4(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log(b*e^x/a + 1) - x^2*dilog(-b*e^x/a) + 2
*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Fricas [A]
time = 0.37, size = 73, normalized size = 0.95

$$\frac{1}{3}\,x^3\log\left(be^x+a\right)-\frac{1}{3}\,x^3\log\left(\frac{be^x+a}{a}\right)-x^2\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+2\,x\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)-2\,\mathrm{polylog}\left(4,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(b*e^x + a) - 1/3*x^3*log((b*e^x + a)/a) - x^2*dilog(-(b*e^x + a
)/a + 1) + 2*x*polylog(3, -b*e^x/a) - 2*polylog(4, -b*e^x/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^3e^x}{a+be^x}\,dx}{3}+\frac{x^3\log\left(a+be^x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a+b*exp(x)),x)
[Out] -b*Integral(x**3*exp(x)/(a + b*exp(x)), x)/3 + x**3*log(a + b*exp(x))/3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(x^2*log(b*e^x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(a+b\,\mathrm{e}^x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(a + b*exp(x)),x)
[Out] int(x^2*log(a + b*exp(x)), x)
```

3.115 $\int x \log (a+be^x)\, dx$

Optimal. Leaf size=59

$$\frac{1}{2}x^2\log (a+be^x)-\frac{1}{2}x^2\log \left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)$$

```
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2612, 2611, 2320, 6724}

$$-x\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+\text{PolyLog}\left(3,-\frac{be^x}{a}\right)+\frac{1}{2}x^2\log (a+be^x)-\frac{1}{2}x^2\log \left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
```

```
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\log\left(a+be^x\right)\,dx &= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)+\int x\log\left(1+\frac{be^x}{a}\right)\,dx\\ &= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\int \text{Li}_2\left(-\frac{be^x}{a}\right)\,dx\\ &= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Subst}\left(\int\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{x}\,dx,x\right.\\ &= \frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.00

$$\frac{1}{2}x^2\log\left(a+be^x\right)-\frac{1}{2}x^2\log\left(1+\frac{be^x}{a}\right)-x\text{Li}_2\left(-\frac{be^x}{a}\right)+\text{Li}_3\left(-\frac{be^x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + b*E^x],x]
```

```
[Out] (x^2*Log[a + b*E^x])/2 - (x^2*Log[1 + (b*E^x)/a])/2 - x*PolyLog[2, -((b*E^x
)/a)] + PolyLog[3, -((b*E^x)/a)]
```

Maple [A]
time = 0.01, size = 52, normalized size = 0.88

method	result	size
default	$\frac{x^2\ln(a+b\,e^x)}{2}-\frac{x^2\ln\left(1+\frac{b\,e^x}{a}\right)}{2}-x\,\text{polylog}\left(2,-\frac{b\,e^x}{a}\right)+\text{polylog}\left(3,-\frac{b\,e^x}{a}\right)$	52
risch	$\frac{x^2\ln(a+b\,e^x)}{2}-\frac{x^2\ln\left(1+\frac{b\,e^x}{a}\right)}{2}-x\,\text{polylog}\left(2,-\frac{b\,e^x}{a}\right)+\text{polylog}\left(3,-\frac{b\,e^x}{a}\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(a+b*exp(x))-1/2*x^2*ln(1+b*exp(x)/a)-x*polylog(2,-b*exp(x)/a)+po
lylog(3,-b*exp(x)/a)
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.85

$$\frac{1}{2}x^2\log\left(be^x+a\right)-\frac{1}{2}x^2\log\left(\frac{be^x}{a}+1\right)-x\mathrm{Li}_2\left(-\frac{be^x}{a}\right)+\mathrm{Li}_3(-\frac{be^x}{a})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log(b*e^x/a + 1) - x*dilog(-b*e^x/a) + pol
ylog(3, -b*e^x/a)
```

Fricas [A]
time = 0.37, size = 56, normalized size = 0.95

$$\frac{1}{2}x^2\log\left(be^x+a\right)-\frac{1}{2}x^2\log\left(\frac{be^x+a}{a}\right)-x\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)+\mathrm{polylog}\left(3,-\frac{be^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(b*e^x + a) - 1/2*x^2*log((b*e^x + a)/a) - x*dilog(-(b*e^x + a)/
a + 1) + polylog(3, -b*e^x/a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{b\int\frac{x^2e^x}{a+be^x}\,dx}{2}+\frac{x^2\log\left(a+be^x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a+b*exp(x)),x)
```

```
[Out] -b*Integral(x**2*exp(x)/(a + b*exp(x)), x)/2 + x**2*log(a + b*exp(x))/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(b*e^x + a), x)
```

<u>**Mupad**</u> **[F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int x \ln \left(a + b \, \mathrm{e}^{x}\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(a + b*exp(x)),x)
[Out] int(x*log(a + b*exp(x)), x)
```

3.116 $\int \log(a+be^x)\,dx$

Optimal. Leaf size=38

$$x\log(a+be^x)-x\log\left(1+\frac{be^x}{a}\right)-\text{Li}_2\left(-\frac{be^x}{a}\right)$$

```
[Out] x*ln(a+b*exp(x))-x*ln(1+b*exp(x)/a)-polylog(2,-b*exp(x)/a)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2318, 2221, 2317, 2438}

$$-\text{PolyLog}\left(2,-\frac{be^x}{a}\right)+x\log(a+be^x)-x\log\left(\frac{be^x}{a}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*E^x],x]
```

```
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2318

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[x*(
(F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \log(a+be^x)\,dx &= x\log(a+be^x) - b\int \frac{e^x x}{a+be^x}\,dx \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \int \log\left(1+\frac{be^x}{a}\right)\,dx \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) + \text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx, x, e^x\right) \\
&= x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{Li}_2\left(-\frac{be^x}{a}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.00

$$x\log(a+be^x) - x\log\left(1+\frac{be^x}{a}\right) - \text{Li}_2\left(-\frac{be^x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*E^x],x]
[Out] x*Log[a + b*E^x] - x*Log[1 + (b*E^x)/a] - PolyLog[2, -((b*E^x)/a)]
```

Maple [A]
time = 0.05, size = 28, normalized size = 0.74

method	result	size
derivativedivides	$\text{dilog}\left(-\frac{b\,e^{x}}{a}\right)+\ln\left(a+b\,e^{x}\right)\ln\left(-\frac{b\,e^{x}}{a}\right)$	28
default	$\text{dilog}\left(-\frac{b\,e^{x}}{a}\right)+\ln\left(a+b\,e^{x}\right)\ln\left(-\frac{b\,e^{x}}{a}\right)$	28
risch	$x\ln\left(a+b\,e^{x}\right)-\ln\left(\frac{a+b\,e^{x}}{a}\right)x-\text{dilog}\left(\frac{a+b\,e^{x}}{a}\right)$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
[Out] dilog(-b*exp(x)/a)+ln(a+b*exp(x))*ln(-b*exp(x)/a)
```

Maxima [A]
time = 0.31, size = 34, normalized size = 0.89

$$\log(be^x+a)\log\left(-\frac{be^x+a}{a}+1\right) + \text{Li}_2\left(\frac{be^x+a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="maxima")
[Out] log(b*e^x + a)*log(-(b*e^x + a)/a + 1) + dilog((b*e^x + a)/a)
```

Fricas [A]
time = 0.36, size = 40, normalized size = 1.05

$$x\log\left(be^x+a\right)-x\log\left(\frac{be^x+a}{a}\right)-\mathrm{Li}_2\left(-\frac{be^x+a}{a}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="fricas")
[Out] x*log(b*e^x + a) - x*log((b*e^x + a)/a) - dilog(-(b*e^x + a)/a + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-b\int\frac{xe^x}{a+be^x}\,dx+x\log\left(a+be^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x)),x)
[Out] -b*Integral(x*exp(x)/(a + b*exp(x)), x) + x*log(a + b*exp(x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x)),x, algorithm="giac")
[Out] integrate(log(b*e^x + a), x)
```

Mupad [B]
time = 0.38, size = 35, normalized size = 0.92

$$x\,\ln\left(a+b\,\mathrm{e}^x\right)-x\,\ln\left(\frac{b\,\mathrm{e}^x}{a}+1\right)-\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*exp(x)),x)
[Out] x*log(a + b*exp(x)) - x*log((b*exp(x))/a + 1) - polylog(2, -(b*exp(x))/a)
```

3.117 $\int \frac{\log(a+be^x)}{x}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\log\left(a+be^x\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(a+b*exp(x))/x,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(a+be^x\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[a + b*E^x]/x,x]
```

```
[Out] Defer[Int][Log[a + b*E^x]/x, x]
```

Rubi steps

$$\int \frac{\log\left(a+be^x\right)}{x}\,dx = \int \frac{\log\left(a+be^x\right)}{x}\,dx$$

Mathematica [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\log\left(a+be^x\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[a + b*E^x]/x,x]
```

```
[Out] Integrate[Log[a + b*E^x]/x, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(a+b\,\mathrm{e}^{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a+b*exp(x))/x,x)
[Out] int(ln(a+b*exp(x))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="maxima")
[Out] integrate(log(b*e^x + a)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="fricas")
[Out] integral(log(b*e^x + a)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a+b*exp(x))/x,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a+b*exp(x))/x,x, algorithm="giac")
[Out] integrate(log(b*e^x + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\ln\left(a+b\,\mathrm{e}^x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*exp(x))/x,x)
[Out] int(log(a + b*exp(x))/x, x)
```

3.118 $\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=132

$$-\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Li}_5\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}$$

```
[Out] -x^3*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(
b*x+a)))^n)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3
/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n)/b^4/c^4/n^4/ln(f)^4
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2611, 6744, 2320, 6724}

$$\frac{6\text{PolyLog}\left(5,-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}-\frac{6x\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{3x^2\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x^3\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx = -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3\int x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{bcn\log(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6\int x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right.\right.}{b^2c^2n^2\log^2(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

$$= -\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

Mathematica [**A**]
time = 0.01, size = 132, normalized size = 1.00

$$-\frac{x^3\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Li}_5\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^4c^4n^4\log^4(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^3*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (3*x^2*PolyL
og[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -
(e*(f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -(e*(f^(c*
(a + b*x)))^n)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(132) = 264$.
time = 0.04, size = 601, normalized size = 4.55

method	result
risch	$\frac{x^{4} \ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{4}+\frac{3 \operatorname{polylog}\left(3,-e\, f^{bcnx} f^{-bcnx} \left(f^{c(bx+a)}\right)^{n}\right) x^{2}}{c^{2} b^{2} \ln(f)^{2} n^{2}}-\frac{3 \operatorname{dilog}\left(1+e\, f^{bcnx} f^{-bcnx} \left(f^{c(bx+a)}\right)^{n}\right) \ln\left(f^{c(bx+a)}\right)^{2}}{c^{3} b^{3} \ln(f)^{3} n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n)+3/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x^2-3/c^3/b^3/ln(f)^3/n*dilog(1+e*f^(
b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2*x+3/c^2/b^2/ln
(f)^2/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+
a)))*x^2+6/c^4/b^4/ln(f)^4/n^4*polylog(5,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*
(b*x+a)))^n)-1/4*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x^4-6/c
^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)
*x-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a))
)^n)*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*x)*f^(
-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2*x-1/c/b/ln(f)/n*dilog(1+e*
f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x^3+1/c^4/b^4/ln(f)^4/n*dilog(1
+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^3-1/c^4/b^
4/ln(f)^4/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(
c*(b*x+a)))^3
```

Maxima [A]

time = 0.31, size = 200, normalized size = 1.52

$$\frac{1}{4} x^{4} \log\left(f^{(bx+a)cn} e + 1\right) - \frac{b^{4} c^{4} n^{4} x^{4} \log\left(f^{acn} e^{(bcnx \log(f)+1)} + 1\right) \log(f)^{4} + 4 b^{3} c^{3} n^{3} x^{3} \mathrm{Li}_2(-f^{acn} e^{(bcnx \log(f)+1)}) \log(f)^{3} - 12 b^{2} c^{2} n^{2} x^{2} \log(f)^{2} \mathrm{Li}_3(-f^{acn} e^{(bcnx \log(f)+1)}) + 24 bcnx \log(f) \mathrm{Li}_4(-f^{acn} e^{(bcnx \log(f)+1)}) - 24 \mathrm{Li}_5(-f^{acn} e^{(bcnx \log(f)+1)})}{4 b^{4} c^{4} n^{4} \log(f)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(f^((b*x + a)*c*n)*e + 1) - 1/4*(b^4*c^4*n^4*x^4*log(f^(a*c*n)*e
^(b*c*n*x*log(f) + 1) + 1)*log(f)^4 + 4*b^3*c^3*n^3*x^3*dilog(-f^(a*c*n)*e^
(b*c*n*x*log(f) + 1))*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -f^
(a*c*n)*e^(b*c*n*x*log(f) + 1)) + 24*b*c*n*x*log(f)*polylog(4, -f^(a*c*n)*e
^(b*c*n*x*log(f) + 1)) - 24*polylog(5, -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)))/
(b^4*c^4*n^4*log(f)^4)
```

Fricas [A]

time = 0.35, size = 132, normalized size = 1.00

$$-\frac{b^{3} c^{3} n^{3} x^{3} \mathrm{Li}_2(-f^{bcnx+acn} e) \log(f)^{3} - 3 b^{2} c^{2} n^{2} x^{2} \log(f)^{2} \operatorname{polylog}(3, -f^{bcnx+acn} e) + 6 bcnx \log(f) \operatorname{polylog}(4, -f^{bcnx+acn} e) - 6 \operatorname{polylog}(5, -f^{bcnx+acn} e)}{b^{4} c^{4} n^{4} \log(f)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^3*c^3*n^3*x^3*dilog(-f^(b*c*n*x + a*c*n)*e)*log(f)^3 - 3*b^2*c^2*n^2*x^
2*log(f)^2*polylog(3, -f^(b*c*n*x + a*c*n)*e) + 6*b*c*n*x*log(f)*polylog(4,
 -f^(b*c*n*x + a*c*n)*e) - 6*polylog(5, -f^(b*c*n*x + a*c*n)*e))/(b^4*c^4*n
^4*log(f)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^4e^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx}{4}+\frac{x^4\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/4 + x**4*log(e*(f**(c*(a + b*x))
)**n + 1)/4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((f^((b*x + a)*c))^n*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(x^3*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.119 $\int x^2 \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=98

$$-\frac{x^2 \text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

```
[Out] -x^2*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*
x+a)))^n)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2611, 6744, 2320, 6724}

$$-\frac{2\text{PolyLog}\left(4,-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}+\frac{2x\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x^2\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)dx &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2\int x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)dx}{bcn\log(f)}\\ &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\int\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\\ &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Subst}\left(\int\frac{\text{Li}_3(-ex^n)}{x}dx\right.}{b^3c^3n^2\log^3(}\\ &= -\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 98, normalized size = 1.00

$$-\frac{x^2\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^3c^3n^3\log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x^2*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + (2*x*PolyLog
[3, -(e*(f^(c*(a + b*x)))^n)])/(b^2*c^2*n^2*Log[f]^2) - (2*PolyLog[4, -(e*(
f^(c*(a + b*x)))^n)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(98) = 196$.
time = 0.02, size = 430, normalized size = 4.39

method	result

risch	$\frac{x^3 \ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{3} - \frac{\ln\left(1+e\, f^{bcnx} f^{-bcnx} \left(f^{c(bx+a)}\right)^n\right) x^3}{3} - \frac{2\, \mathrm{polylog}\left(4, -e\, f^{bcnx} f^{-bcnx} \left(f^{c(bx+a)}\right)^n\right)}{c^3 b^3 \ln(f)^3 n^3} - \frac{\mathrm{dilog}\left(1+e\, f^{bc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n)-1/3*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c
*(b*x+a)))^n)*x^3-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x)*f^(-b*c*n*
x)*(f^(c*(b*x+a)))^n)-1/c/b/ln(f)/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(
c*(b*x+a)))^n)*x^2+2/c^2/b^2/ln(f)^2/n*dilog(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(
f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x-1/c^3/b^3/ln(f)^3/n*dilog(1+e*f^(b*c*
n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2-2/c^2/b^2/ln(f)^2/
n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a))
)*x+1/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)
))^n)*ln(f^(c*(b*x+a)))^2+2/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*f^
(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x
```

Maxima [A]
time = 0.30, size = 162, normalized size = 1.65

$$\frac{1}{3}\, x^3 \log\left(f^{(bx+a)cn} e + 1\right) - \frac{b^3 c^3 n^3 x^3 \log\left(f^{acn} e^{(bcnx \log(f)+1)} + 1\right) \log(f)^3 + 3\, b^2 c^2 n^2 x^2 \mathrm{Li}_2\left(-f^{acn} e^{(bcnx \log(f)+1)}\right) \log(f)^2 - 6\, bcnx \log(f)\, \mathrm{Li}_3\left(-f^{acn} e^{(bcnx \log(f)+1)}\right) + 6\, \mathrm{Li}_4\left(-f^{acn} e^{(bcnx \log(f)+1)}\right)}{3\, b^3 c^3 n^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(f^((b*x + a)*c*n)*e + 1) - 1/3*(b^3*c^3*n^3*x^3*log(f^(a*c*n)*e
^(b*c*n*x*log(f) + 1) + 1)*log(f)^3 + 3*b^2*c^2*n^2*x^2*dilog(-f^(a*c*n)*e^
(b*c*n*x*log(f) + 1))*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -f^(a*c*n)*e^(
b*c*n*x*log(f) + 1)) + 6*polylog(4, -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)))/(b^
3*c^3*n^3*log(f)^3)
```

Fricas [A]
time = 0.38, size = 96, normalized size = 0.98

$$-\frac{b^2 c^2 n^2 x^2 \mathrm{Li}_2\left(-f^{bcnx+acn} e\right) \log(f)^2 - 2\, bcnx \log(f)\, \mathrm{polylog}\left(3, -f^{bcnx+acn} e\right) + 2\, \mathrm{polylog}\left(4, -f^{bcnx+acn} e\right)}{b^3 c^3 n^3 \log(f)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b^2*c^2*n^2*x^2*dilog(-f^(b*c*n*x + a*c*n)*e)*log(f)^2 - 2*b*c*n*x*log(f)
*polylog(3, -f^(b*c*n*x + a*c*n)*e) + 2*polylog(4, -f^(b*c*n*x + a*c*n)*e))
/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^3e^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx}{3}+\frac{x^3\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/3 + x**3*log(e*(f**(c*(a + b*x))
)**n + 1)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((f^((b*x + a)*c))^n*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(x^2*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.120 $\int x \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) dx$

Optimal. Leaf size=63

$$-\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}$$

```
[Out] -x*polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))
^n)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2611, 2320, 6724}

$$\frac{\text{PolyLog}\left(3,-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int x\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)\,dx &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\int \text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)\,dx}{bcn\log(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Subst}\left(\int\frac{\text{Li}_2(-ex^n)}{x}\,dx,x,f^{c(a+bx)}\right)}{b^2c^2n\log^2(f)}\\ &= -\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 1.00

$$-\frac{x\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-e\left(f^{c(a+bx)}\right)^n\right)}{b^2c^2n^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -((x*PolyLog[2, -(e*(f^(c*(a + b*x)))^n)])/(b*c*n*Log[f])) + PolyLog[3, -(e
*(f^(c*(a + b*x)))^n)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(63) = 126$.
time = 0.02, size = 262, normalized size = 4.16

method	result
risch	$\frac{x^2\ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{2}-\frac{\ln\left(1+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^n\right)x^2}{2}-\frac{\text{polylog}\left(2,-e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^n\right)\ln\left(f^{c(bx+a)}\right)}{c^2b^2\ln(f)^2n}+\text{poly}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n)-1/2*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c
*(b*x+a)))^n)*x^2-1/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)
*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))+1/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^
(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)-1/c/b/ln(f)/n*dilog(1+e*f^(b*c*n*
x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x+1/c^2/b^2/ln(f)^2/n*dilog(1+e*f^(b*c*n
*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.97

$$\frac{1}{2}x^2\log\left(f^{(bx+a)cn}e+1\right)-\frac{b^2c^2n^2x^2\log\left(f^{acn}e^{(bcnx\log(f)+1)}+1\right)\log(f)^2+2\,bcnx\text{Li}_2\left(-f^{acn}e^{(bcnx\log(f)+1)}\right)\log(f)-2\,\text{Li}_3\left(-f^{acn}e^{(bcnx\log(f)+1)}\right)}{2\,b^2c^2n^2\log(f)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(f^((b*x + a)*c*n)*e + 1) - 1/2*(b^2*c^2*n^2*x^2*log(f^(a*c*n)*e
^(b*c*n*x*log(f) + 1) + 1)*log(f)^2 + 2*b*c*n*x*dilog(-f^(a*c*n)*e^(b*c*n*x
*log(f) + 1))*log(f) - 2*polylog(3, -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)))/(b^
2*c^2*n^2*log(f)^2)
```

Fricas [A]
time = 0.35, size = 60, normalized size = 0.95

$$-\frac{bcnx\mathrm{Li}_2\left(-f^{bcnx+acn}e\right)\log\left(f\right)-\mathrm{polylog}\left(3,-f^{bcnx+acn}e\right)}{b^2c^2n^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -(b*c*n*x*dilog(-f^(b*c*n*x + a*c*n)*e)*log(f) - polylog(3, -f^(b*c*n*x + a
*c*n)*e))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^2e^{bcnx\log\left(f\right)}}{ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}+1}\,dx}{2}+\frac{x^2\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(e*exp(
a*c*n*log(f))*exp(b*c*n*x*log(f)) + 1), x)/2 + x**2*log(e*(f**(c*(a + b*x))
)**n + 1)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log((f^((b*x + a)*c))^n*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\ln\left(e\,{\left(f^{c\,(a+b\,x)}\right)}^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(e*(f^(c*(a + b*x)))^n + 1),x)
[Out] int(x*log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.121 $\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal. Leaf size=31

$$-\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

```
[Out] -polylog(2,-e*(f^(c*(b*x+a)))^n)/b/c/n/ln(f)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2317, 2438}

$$-\frac{\text{PolyLog}\left(2,-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \log \left(1+e\left(f^{c(a+bx)}\right)^n\right) \, dx &= \frac{\text{Subst}\left(\int \frac{\log(1+ex)}{x} \, dx, x, \left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\\ &= -\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\text{Li}_2\left(-e\left(f^{c(a+bx)}\right)^n\right)}{bcn\log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] -(PolyLog[2, -(e*(f^(c*(a + b*x)))^n)]/(b*c*n*Log[f]))
```

Maple [A]
time = 0.07, size = 32, normalized size = 1.03

method	result
derivativedivides	$-\frac{\mathrm{dilog}\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{cb\ln(f)n}$
default	$-\frac{\mathrm{dilog}\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)}{cb\ln(f)n}$
risch	$x\ln\left(1+e\left(f^{c(bx+a)}\right)^{n}\right)-\frac{\mathrm{dilog}\left(1+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^{n}\right)}{cb\ln(f)n}-\ln\left(1+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^{n}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/b/ln(f)/n*dilog(1+e*(f^(c*(b*x+a)))^n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(31) = 62.
time = 0.30, size = 81, normalized size = 2.61

$$x\log\left(f^{(bx+a)cn}e+1\right)-\frac{bcnx\log\left(f^{acn}e^{(bcnx\log(f)+1)}+1\right)\log(f)+\mathrm{Li}_2\left(-f^{acn}e^{(bcnx\log(f)+1)}\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] x*log(f^((b*x + a)*c*n)*e + 1) - (b*c*n*x*log(f^(a*c*n)*e^(b*c*n*x*log(f) +
 1) + 1)*log(f) + dilog(-f^(a*c*n)*e^(b*c*n*x*log(f) + 1)))/(b*c*n*log(f))
```

Fricas [A]
time = 0.36, size = 32, normalized size = 1.03

$$-\frac{\mathrm{Li}_2\left(-f^{bcnx+acn}e\right)}{bcn\log(f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -dilog(-f^(b*c*n*x + a*c*n)*e)/(b*c*n*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-bcene^{acn\log(f)}\log(f)\int\frac{xe^{bcnx\log(f)}}{ee^{acn\log(f)}e^{bcnx\log(f)}+1}\,dx+x\log\left(e\left(f^{c(a+bx)}\right)^n+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(e*exp(a*c
*n*log(f))*exp(b*c*n*x*log(f)) + 1), x) + x*log(e*(f**(c*(a + b*x)))**n + 1
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(log((f^((b*x + a)*c))^n*e + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(e\left(f^{c\,(a+b\,x)}\right)^n+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1),x)
```

```
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1), x)
```

3.122 $\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Defer[Int][Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\log\left(1+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[1 + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+e\left(f^{c(bx+a)}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

```
[Out] int(ln(1+e*(f^(c*(b*x+a)))^n)/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log(f^((b*x + a)*c*n)*e + 1)/x, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((f^(b*c*x + a*c))^n*e + 1)/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+e*(f**(c*(b*x+a)))**n)/x,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((f^((b*x + a)*c))^n*e + 1)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(e\left(f^{c\,(a+b\,x)}\right)^n+1\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(e*(f^(c*(a + b*x)))^n + 1)/x,x)
[Out] int(log(e*(f^(c*(a + b*x)))^n + 1)/x, x)
```

3.123 $\int x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal. Leaf size=193

$$\frac{1}{4}x^4 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4 \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^3 \text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)}+\frac{3x^2 \text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2 \log^2(f)}-\frac{6x}{}$$

[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)-1/4*x^4*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^3*polylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+3*x^2*polylog(3,-e*(f^(c*(b*x+a)))^n/d)/b^2/c^2/n^2/ln(f)^2-6*x*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3/n^3/ln(f)^3+6*polylog(5,-e*(f^(c*(b*x+a)))^n/d)/b^4/c^4/n^4/ln(f)^4

Rubi [A]

time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2612, 2611, 6744, 2320, 6724}

$$\frac{6\text{PolyLog}\left(5,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}-\frac{6x\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{3x^2\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^3\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{4}x^4\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{4}x^4\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

[In] Int[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]

[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + (3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*PolyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int x^3 \log\left(1+\right.$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a}\right.}{}\right.}{bcn\log(}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a}\right.}{}\right.}{bcn\log(}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a}\right.}{}\right.}{bcn\log(}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a}\right.}{}\right.}{bcn\log(}$$

$$= \frac{1}{4}x^4 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{4}x^4 \log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^3 \mathrm{Li}_2\left(-\frac{e\left(f^{c(a}\right.}{}\right.}{bcn\log(}$$

Mathematica [A]
time = 0.01, size = 193, normalized size = 1.00

$$\frac{1}{4}x^4\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{4}x^4\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^3\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{3x^2\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{6x\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{6\text{Li}_5\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4n^4\log^4(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^4*Log[d + e*(f^(c*(a + b*x)))^n])/4 - (x^4*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/4 - (x^3*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(3*x^2*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (
6*x*PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3) + (6*P
olyLog[5, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^4*c^4*n^4*Log[f]^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(189) = 378$.
time = 0.04, size = 1276, normalized size = 6.61

method	result	size
risch	Expression too large to display	1276

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(d+e*(f^(c*(b*x+a)))^n)+6/c^4/b^4/ln(f)^4/n^4*polylog(5,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)+3/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*
x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x^2-3/c^3/b^3/ln(
f)^3*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)
))^3*x-3/c^2/b^2/ln(f)^2/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x
+a)))^n/d)*ln(f^(c*(b*x+a)))*x^2+3/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*
x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^2*x-3/2/c^2/b^2/ln(f
)^2*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2*
x^2+1/c^3/b^3/ln(f)^3*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln
(f^(c*(b*x+a)))^3*x-3/2/c^2/b^2/ln(f)^2*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^
(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^2*x^2+2/c^3/b^3/ln(f)^3*ln(1+e*f^(b*c*n
*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^3*x-1/c/b/ln(f)*ln(
(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))*x^3+3
/c^2/b^2/ln(f)^2/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d
)*ln(f^(c*(b*x+a)))*x^2-3/c^3/b^3/ln(f)^3/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*
n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x+1/c/b/ln(f)*ln(d+e*f^(b*c*
n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x^3-6/c^3/b^3/ln(f)^
3/n^3*polylog(4,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*x+3/c^2/b^
2/ln(f)^2/n^2*polylog(3,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*x^
2-3/4/c^4/b^4/ln(f)^4*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*
```

```
ln(f^(c*(b*x+a)))^4+1/c^4/b^4/ln(f)^4*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(
c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^4-1/c^4/b^4/ln(f)^4/n*polylog(2,-e*f^(b
*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^3-1/c/b/ln(f)/n
*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*x^3+1/c^4/b^4/ln
(f)^4/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(
b*x+a)))^3-1/4*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x^4-1/4/c
^4/b^4/ln(f)^4*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(
b*x+a)))^4
```

Maxima [A]
time = 0.30, size = 215, normalized size = 1.11

$$\frac{1}{4}\,x^4\log\left(f^{(bx+a)cn}e+d\right)-\frac{b^4c^4n^4x^4\log\left(f\right)^4\log\left(\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}+1\right)+4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)\log\left(f\right)^3-12\,b^2c^2n^2x^2\log\left(f\right)^2\mathrm{Li}_3(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d})+24\,bcnx\log\left(f\right)\mathrm{Li}_4(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d})-24\,\mathrm{Li}_5(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d})}{4\,b^4c^4n^4\log\left(f\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(f^((b*x + a)*c*n)*e + d) - 1/4*(b^4*c^4*n^4*x^4*log(f)^4*log(f^
(a*c*n)*e^(b*c*n*x*log(f) + 1)/d + 1) + 4*b^3*c^3*n^3*x^3*dilog(-f^(a*c*n)*
e^(b*c*n*x*log(f) + 1)/d)*log(f)^3 - 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3,
 -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)/d) + 24*b*c*n*x*log(f)*polylog(4, -f^(a*
c*n)*e^(b*c*n*x*log(f) + 1)/d) - 24*polylog(5, -f^(a*c*n)*e^(b*c*n*x*log(f)
 + 1)/d))/(b^4*c^4*n^4*log(f)^4)
```

Fricas [A]
time = 0.38, size = 251, normalized size = 1.30

$$-\frac{4\,b^3c^3n^3x^3\mathrm{Li}_2\left(-\frac{f^{bcnx+acn}e+d}{d}+1\right)\log\left(f\right)^3-12\,b^2c^2n^2x^2\log\left(f\right)^2\mathrm{polylog}\left(3,-\frac{f^{bcnx+acn}e}{d}\right)-(b^4c^4n^4x^4-a^4c^4n^4)\log\left(f^{bcnx+acn}e+d\right)\log\left(f\right)^4+(b^4c^4n^4x^4-a^4c^4n^4)\log\left(f\right)^4\log\left(\frac{f^{bcnx+acn}e+d}{d}\right)+24\,bcnx\log\left(f\right)\mathrm{polylog}\left(4,-\frac{f^{bcnx+acn}e}{d}\right)-24\,\mathrm{polylog}\left(5,-\frac{f^{bcnx+acn}e}{d}\right)}{4\,b^4c^4n^4\log\left(f\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*c^3*n^3*x^3*dilog(-(f^(b*c*n*x + a*c*n)*e + d)/d + 1)*log(f)^3
- 12*b^2*c^2*n^2*x^2*log(f)^2*polylog(3, -f^(b*c*n*x + a*c*n)*e/d) - (b^4*c
^4*n^4*x^4 - a^4*c^4*n^4)*log(f^(b*c*n*x + a*c*n)*e + d)*log(f)^4 + (b^4*c^
4*n^4*x^4 - a^4*c^4*n^4)*log(f)^4*log((f^(b*c*n*x + a*c*n)*e + d)/d) + 24*b
*c*n*x*log(f)*polylog(4, -f^(b*c*n*x + a*c*n)*e/d) - 24*polylog(5, -f^(b*c*
n*x + a*c*n)*e/d))/(b^4*c^4*n^4*log(f)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log{(f)}}\log{(f)}\int\frac{x^4e^{bcnx\log{(f)}}}{d+ee^{acn\log{(f)}}e^{bcnx\log{(f)}}}\,dx}{4}+\frac{x^4\log{\left(d+e\left(f^{c(a+bx)}\right)^n\right)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**4*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/4 + x**4*log(d + e*(f**(c*(a + b
*x)))**n)/4
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^3*log((f^((b*x + a)*c))^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\ln\left(d+e\,{\left(f^{c\,(a+b\,x)}\right)}^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x^3*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.124 $\int x^2 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) \, dx$

Optimal. Leaf size=156

$$\frac{1}{3}x^3 \log \left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{3}x^3 \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{2\text{Li}_4}{b^3}$$

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)-1/3*x^3*ln(1+e*(f^(c*(b*x+a)))^n/d)-x^2*p
olylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+2*x*polylog(3,-e*(f^(c*(b*x+a)
))^n/d)/b^2/c^2/n^2/ln(f)^2-2*polylog(4,-e*(f^(c*(b*x+a)))^n/d)/b^3/c^3/n^3
/ln(f)^3
```

Rubi [A]

time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2612, 2611, 6744, 2320, 6724}

$$-\frac{2\text{PolyLog}\left(4,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}+\frac{2x\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x^2\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{3}x^3\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{3}x^3\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^2 \log\left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int x^2 \log\left(1\right.$$

$$= \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2\text{Li}_2\left(-\frac{e(f^{c}}{}\right.}{bcn\log}$$

$$= \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2\text{Li}_2\left(-\frac{e(f^{c}}{}\right.}{bcn\log}$$

$$= \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2\text{Li}_2\left(-\frac{e(f^{c}}{}\right.}{bcn\log}$$

$$= \frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2\text{Li}_2\left(-\frac{e(f^{c}}{}\right.}{bcn\log}$$

Mathematica [A]
time = 0.01, size = 156, normalized size = 1.00

$$\frac{1}{3}x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right) - \frac{1}{3}x^3\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{x^2\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)} + \frac{2x\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)} - \frac{2\text{Li}_4\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3n^3\log^3(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^3*Log[d + e*(f^(c*(a + b*x)))^n])/3 - (x^3*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/3 - (x^2*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) +
(2*x*PolyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[f]^2) - (2*
PolyLog[4, -((e*(f^(c*(a + b*x)))^n)/d)])/(b^3*c^3*n^3*Log[f]^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(152) = 304$.
time = 0.03, size = 916, normalized size = 5.87

method	result
risch	$\frac{x^3 \ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)}{3}+\frac{2\,\mathrm{polylog}\left(3,-\frac{e\,f^{bcnx} f^{-bcnx}\left(f^{c(bx+a)}\right)^n}{d}\right)x}{c^2 b^2 \ln(f)^2 n^2}-\frac{2\,\mathrm{polylog}\left(2,-\frac{e\,f^{bcnx} f^{-bcnx}\left(f^{c(bx+a)}\right)^n}{d}\right)\ln\left(f^{c(bx+a)}\right.}{c^2 b^2 \ln(f)^2 n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(d+e*(f^(c*(b*x+a)))^n)+2/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*x-2/c^2/b^2/ln(f)^2/n*polylog(2,-e*
f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))*x-1/c^2/b^2
/ln(f)^2*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*x*ln(f^(c*(b*
x+a)))^2-1/c/b/ln(f)*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)
*ln(f^(c*(b*x+a)))*x^2+2/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(
f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2*x+1/c/b/ln(f)*ln(d+e*f^(b*c*n*x)*f
^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x^2-1/c^2/b^2/ln(f)^2*ln(d
+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2*x+2/c^2/
b^2/ln(f)^2/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(
f^(c*(b*x+a)))*x-2/c^3/b^3/ln(f)^3/n^3*polylog(4,-e*f^(b*c*n*x)*f^(-b*c*n*x
)*(f^(c*(b*x+a)))^n/d)-1/c^3/b^3/ln(f)^3*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(
f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^3-1/3*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x
)*(f^(c*(b*x+a)))^n)*x^3+1/3/c^3/b^3/ln(f)^3*ln(d+e*f^(b*c*n*x)*f^(-b*c*n*x
)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^3-1/c/b/ln(f)/n*dilog((d+e*f^(b*c*n*
x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*x^2-1/c^3/b^3/ln(f)^3/n*dilog((d+e*f^
(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+a)))^2+2/3/c^3/b^
3/ln(f)^3*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x
+a)))^3+1/c^3/b^3/ln(f)^3/n*polylog(2,-e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*
x+a)))^n/d)*ln(f^(c*(b*x+a)))^2
```

Maxima [A]
time = 0.32, size = 174, normalized size = 1.12

$$\frac{1}{3}\,x^3\log\left(f^{(bx+a)cn}e+d\right)-\frac{b^3c^3n^3x^3\log\left(f\right)^3\log\left(\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}+1\right)+3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)\log\left(f\right)^2-6\,bcnx\log\left(f\right)\mathrm{Li}_3\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)+6\,\mathrm{Li}_4\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)}{3\,b^3c^3n^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(f^((b*x + a)*c*n)*e + d) - 1/3*(b^3*c^3*n^3*x^3*log(f)^3*log(f^
(a*c*n)*e^(b*c*n*x*log(f) + 1)/d + 1) + 3*b^2*c^2*n^2*x^2*dilog(-f^(a*c*n)*
e^(b*c*n*x*log(f) + 1)/d)*log(f)^2 - 6*b*c*n*x*log(f)*polylog(3, -f^(a*c*n)
*e^(b*c*n*x*log(f) + 1)/d) + 6*polylog(4, -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)
/d))/(b^3*c^3*n^3*log(f)^3)
```

Fricas [A]
time = 0.37, size = 210, normalized size = 1.35

$$-\frac{3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{f^{bcnx+acn}e+d}{d}+1\right)\log\left(f\right)^2-6\,bcnx\log\left(f\right)\mathrm{polylog}\left(3,-\frac{f^{bcnx+acn}e}{d}\right)-\left(b^3c^3n^3x^3+a^3c^3n^3\right)\log\left(f^{bcnx+acn}e+d\right)\log\left(f\right)^3+\left(b^3c^3n^3x^3+a^3c^3n^3\right)\log\left(f\right)^3\log\left(\frac{f^{bcnx+acn}e+d}{d}\right)+6\,\mathrm{polylog}\left(4,-\frac{f^{bcnx+acn}e}{d}\right)}{3\,b^3c^3n^3\log\left(f\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*c^2*n^2*x^2*dilog(-(f^(b*c*n*x + a*c*n)*e + d)/d + 1)*log(f)^2
- 6*b*c*n*x*log(f)*polylog(3, -f^(b*c*n*x + a*c*n)*e/d) - (b^3*c^3*n^3*x^3
+ a^3*c^3*n^3)*log(f^(b*c*n*x + a*c*n)*e + d)*log(f)^3 + (b^3*c^3*n^3*x^3 +
 a^3*c^3*n^3)*log(f)^3*log((f^(b*c*n*x + a*c*n)*e + d)/d) + 6*polylog(4, -f
^(b*c*n*x + a*c*n)*e/d))/(b^3*c^3*n^3*log(f)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{x^3e^{bcnx\log\left(f\right)}}{d+ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}}\,dx}{3}+\frac{x^3\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**3*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/3 + x**3*log(d + e*(f**(c*(a + b
*x)))**n)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((f^((b*x + a)*c))^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \ln\left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(d + e*(f^(c*(a + b*x)))^n),x)
[Out] int(x^2*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.125 $\int x \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=118

$$\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\text{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

```
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)-1/2*x^2*ln(1+e*(f^(c*(b*x+a)))^n/d)-x*pol
ylog(2,-e*(f^(c*(b*x+a)))^n/d)/b/c/n/ln(f)+polylog(3,-e*(f^(c*(b*x+a)))^n/d
)/b^2/c^2/n^2/ln(f)^2
```

Rubi [A]

time = 0.04, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2612, 2611, 2320, 6724}

$$\frac{\text{PolyLog}\left(3,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}-\frac{x\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{1}{2}x^2\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-\frac{1}{2}x^2\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2612

```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
```

```
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)+\int x\log\left(1+\right.$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+b}\right.}{d}\right.}{bcn\log(f)}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+b}\right.}{d}\right.}{bcn\log(f)}$$

$$=\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+b}\right.}{d}\right.}{bcn\log(f)}$$

Mathematica [**A**]
time = 0.01, size = 118, normalized size = 1.00

$$\frac{1}{2}x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-\frac{1}{2}x^2\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{x\mathrm{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+\frac{\mathrm{Li}_3\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] (x^2*Log[d + e*(f^(c*(a + b*x)))^n])/2 - (x^2*Log[1 + (e*(f^(c*(a + b*x)))^
n)/d])/2 - (x*PolyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[f]) + Po
lyLog[3, -((e*(f^(c*(a + b*x)))^n)/d)]/(b^2*c^2*n^2*Log[f]^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(114) = 228.
time = 0.02, size = 558, normalized size = 4.73

method	result
risch	$\frac{x^2\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)}{2}+\frac{\operatorname{polylog}\left(3,-\frac{e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^n}{d}\right)}{c^2b^2\ln(f)^2n^2}+\frac{\ln\left(d+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^n\right)\ln\left(f^{c(bx+a)}\right)x}{cb\ln(f)}-\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(d+e*(f^(c*(b*x+a)))^n)+1/c^2/b^2/ln(f)^2/n^2*polylog(3,-e*f^(b*c
*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)+1/c/b/ln(f)*ln(d+e*f^(b*c*n*x)*f^(-
b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))*x-1/c/b/ln(f)*ln((d+e*f^(b*c*
n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*x*ln(f^(c*(b*x+a)))-1/c/b/ln(f)/n*d
ilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*x+1/c^2/b^2/ln(f)^
2/n*dilog((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/d)*ln(f^(c*(b*x+
a)))+1/c^2/b^2/ln(f)^2*ln((d+e*f^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)/
d)*ln(f^(c*(b*x+a)))^2-1/2/c^2/b^2/ln(f)^2*ln(1+e*f^(b*c*n*x)*f^(-b*c*n*x)*
(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))^2-1/c^2/b^2/ln(f)^2/n*polylog(2,-e*f
^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n/d)*ln(f^(c*(b*x+a)))-1/2*ln(d+e*f
^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*x^2-1/2/c^2/b^2/ln(f)^2*ln(d+e*f
^(b*c*n*x)*f^(-b*c*n*x)*(f^(c*(b*x+a)))^n)*ln(f^(c*(b*x+a)))^2
```

Maxima [A]
time = 0.30, size = 133, normalized size = 1.13

$$\frac{1}{2}\,x^2\log\left(f^{(bx+a)cn}e+d\right)-\frac{b^2c^2n^2x^2\log\left(f\right)^2\log\left(\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}+1\right)+2\,bcnx\mathrm{Li}_2\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)\log\left(f\right)-2\,\mathrm{Li}_3\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)}{2\,b^2c^2n^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(f^((b*x + a)*c*n)*e + d) - 1/2*(b^2*c^2*n^2*x^2*log(f)^2*log(f^
(a*c*n)*e^(b*c*n*x*log(f) + 1)/d + 1) + 2*b*c*n*x*dilog(-f^(a*c*n)*e^(b*c*n
*x*log(f) + 1)/d)*log(f) - 2*polylog(3, -f^(a*c*n)*e^(b*c*n*x*log(f) + 1)/d
))/(b^2*c^2*n^2*log(f)^2)
```

Fricas [A]
time = 0.37, size = 173, normalized size = 1.47

$$-\frac{2\,bcnx\mathrm{Li}_2\left(-\frac{f^{bcnx+acn}e+d}{d}+1\right)\log\left(f\right)-\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(f^{bcnx+acn}e+d\right)\log\left(f\right)^2+\left(b^2c^2n^2x^2-a^2c^2n^2\right)\log\left(f\right)^2\log\left(\frac{f^{bcnx+acn}e+d}{d}\right)-2\,\mathrm{polylog}\left(3,-\frac{f^{bcnx+acn}e}{d}\right)}{2\,b^2c^2n^2\log\left(f\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*n*x*dilog(-(f^(b*c*n*x + a*c*n)*e + d)/d + 1)*log(f) - (b^2*c^2
*n^2*x^2 - a^2*c^2*n^2)*log(f^(b*c*n*x + a*c*n)*e + d)*log(f)^2 + (b^2*c^2*
```

```
n^2*x^2 - a^2*c^2*n^2)*log(f)^2*log((f^(b*c*n*x + a*c*n)*e + d)/d) - 2*poly
log(3, -f^(b*c*n*x + a*c*n)*e/d))/(b^2*c^2*n^2*log(f)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bcene^{acn\log(f)}\log(f)\int\frac{x^2e^{bcnx\log(f)}}{d+ee^{acn\log(f)}e^{bcnx\log(f)}}\,dx}{2}+\frac{x^2\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x**2*exp(b*c*n*x*log(f))/(d + e*
exp(a*c*n*log(f))*exp(b*c*n*x*log(f))), x)/2 + x**2*log(d + e*(f**(c*(a + b
*x)))**n)/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
```

```
[Out] integrate(x*log((f^((b*x + a)*c))^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(d+e\,{\left(f^{c\,(a+b\,x)}\right)}^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(d + e*(f^(c*(a + b*x)))^n),x)
```

```
[Out] int(x*log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.126 $\int \log\left(d+e\left(f^{c(a+bx)}\right)^n\right)\,dx$

Optimal. Leaf size=75

$$x\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)-x\log\left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)-\frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}$$

```
[Out] x*ln(d+e*(f^(c*(b*x+a)))^n)-x*ln(1+e*(f^(c*(b*x+a)))^n/d)-polylog(2,-e*(f^(
c*(b*x+a)))^n/d)/b/c/n/ln(f)
```

Rubi [A]
time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2318, 2221, 2317, 2438}

$$-\frac{\text{PolyLog}\left(2,-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(f)}+x\log\left(e\left(f^{c(a+bx)}\right)^n+d\right)-x\log\left(\frac{e\left(f^{c(a+bx)}\right)^n}{d}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n],x]
```

```
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2318

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[x*Log[a + b*(F^(e*(c + d*x)))^n], x] - Dist[b*d*e*n*Log[F], Int[x*(
(F^(e*(c + d*x)))^n/(a + b*(F^(e*(c + d*x)))^n)), x], x] /; FreeQ[{F, a, b,
 c, d, e, n}, x] &&  !GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) dx = x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - (bcen \log(f)) \int \frac{\left(f^{c(a+bx)}\right)^n x}{d+e\left(f^{c(a+bx)}\right)^n} dx$$

$$= x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - x \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \int \log \left(1+\frac{e\left(f^{c(a+bx}\right.}{d}\right.$$

$$= x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - x \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{d}\right)}{x} dx,\right.}{bcn \log(}$$

$$= x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - x \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)}$$

Mathematica [A]
time = 0.00, size = 75, normalized size = 1.00

$$x \log \left(d+e\left(f^{c(a+bx)}\right)^n\right) - x \log \left(1+\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right) - \frac{\text{Li}_2\left(-\frac{e\left(f^{c(a+bx)}\right)^n}{d}\right)}{bcn \log(f)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n],x]
[Out] x*Log[d + e*(f^(c*(a + b*x)))^n] - x*Log[1 + (e*(f^(c*(a + b*x)))^n)/d] - P
olyLog[2, -((e*(f^(c*(a + b*x)))^n)/d)]/(b*c*n*Log[f])
```

Maple [A]
time = 0.06, size = 69, normalized size = 0.92

method	result
derivativedivides	$\frac{\text{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)}{cb\ln(f)n}$
default	$\frac{\text{dilog}\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)+\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)\ln\left(-\frac{e\left(f^{c(bx+a)}\right)^n}{d}\right)}{cb\ln(f)n}$

risch	$x\ln\left(d+e\left(f^{c(bx+a)}\right)^{n}\right)-\frac{\operatorname{dilog}\left(\frac{d+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^{n}}{d}\right)}{cb\ln(f)n}-\frac{\ln\left(\frac{d+e\,f^{bcnx}f^{-bcnx}\left(f^{c(bx+a)}\right)^{n}}{d}\right)\ln(f^{\cdots}}{cb\ln(f)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/b/ln(f)/n*(dilog(-e*(f^(c*(b*x+a)))^n/d)+ln(d+e*(f^(c*(b*x+a)))^n)*ln(-
e*(f^(c*(b*x+a)))^n/d))
```

Maxima [A]
time = 0.29, size = 87, normalized size = 1.16

$$x\log\left(f^{(bx+a)cn}e+d\right)-\frac{bcnx\log\left(f\right)\log\left(\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}+1\right)+\mathrm{Li}_2\left(-\frac{f^{acn}e^{(bcnx\log(f)+1)}}{d}\right)}{bcn\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="maxima")
```

```
[Out] x*log(f^((b*x + a)*c*n)*e + d) - (b*c*n*x*log(f)*log(f^(a*c*n)*e^(b*c*n*x*l
og(f) + 1)/d + 1) + dilog(-f^(a*c*n)*e^(b*c*n*x*log(f) + 1)/d))/(b*c*n*log(
f))
```

Fricas [A]
time = 0.37, size = 109, normalized size = 1.45

$$\frac{(bcnx+acn)\log\left(f^{bcnx+acn}e+d\right)\log\left(f\right)-(bcnx+acn)\log\left(f\right)\log\left(\frac{f^{bcnx+acn}e+d}{d}\right)-\mathrm{Li}_2\left(-\frac{f^{bcnx+acn}e+d}{d}+1\right)}{bcn\log\left(f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="fricas")
```

```
[Out] ((b*c*n*x + a*c*n)*log(f^(b*c*n*x + a*c*n)*e + d)*log(f) - (b*c*n*x + a*c*n
)*log(f)*log((f^(b*c*n*x + a*c*n)*e + d)/d) - dilog(-(f^(b*c*n*x + a*c*n)*e
 + d)/d + 1))/(b*c*n*log(f))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-bcene^{acn\log\left(f\right)}\log\left(f\right)\int\frac{xe^{bcnx\log\left(f\right)}}{d+ee^{acn\log\left(f\right)}e^{bcnx\log\left(f\right)}}\,dx+x\log\left(d+e\left(f^{c(a+bx)}\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n),x)
```

```
[Out] -b*c*e*n*exp(a*c*n*log(f))*log(f)*Integral(x*exp(b*c*n*x*log(f))/(d + e*exp
(a*c*n*log(f))*exp(b*c*n*x*log(f))), x) + x*log(d + e*(f**(c*(a + b*x)))**n
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n),x, algorithm="giac")
[Out] integrate(log((f^((b*x + a)*c))^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(d+e\,{\left(f^{c\,\left(a+b\,x\right)}\right)}^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n),x)
[Out] int(log(d + e*(f^(c*(a + b*x)))^n), x)
```

3.127 $\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x},x\right)$$

```
[Out] CannotIntegrate(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Defer[Int][Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Rubi steps

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx = \int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\log\left(d+e\left(f^{c(a+bx)}\right)^n\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x,x]
[Out] Integrate[Log[d + e*(f^(c*(a + b*x)))^n]/x, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(d+e\left(f^{c(bx+a)}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

```
[Out] int(ln(d+e*(f^(c*(b*x+a)))^n)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="maxima")
```

```
[Out] integrate(log(f^((b*x + a)*c*n)*e + d)/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="fricas")
```

```
[Out] integral(log((f^(b*c*x + a*c))^n*e + d)/x, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(d+e*(f**(c*(b*x+a)))**n)/x,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d+e*(f^(c*(b*x+a)))^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log((f^((b*x + a)*c))^n*e + d)/x, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln \left(d+e\left(f^{c\,(a+b\,x)}\right)^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(d + e*(f^(c*(a + b*x)))^n)/x,x)
[Out] int(log(d + e*(f^(c*(a + b*x)))^n)/x, x)
```

3.128 $\int \log \left(b\left(F^{e(c+dx)}\right)^n+\pi \right) \, dx$

Optimal. Leaf size=39

$$x \log(\pi) - \frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

[Out] x*ln(Pi)-polylog(2,-b*(F^(e*(d*x+c)))^n/Pi)/d/e/n/ln(F)

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2317, 2439, 2438}

$$x \log(\pi) - \frac{\text{PolyLog}\left(2, -\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den \log(F)}$$

Antiderivative was successfully verified.

[In] Int[Log[b*(F^(e*(c + d*x)))^n + Pi],x]

[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rubi steps

$$\int \log\left(b\left(F^{e(c+dx)}\right)^n+\pi\right)\,dx = \frac{\text{Subst}\left(\int \frac{\log(\pi+bx)}{x}\,dx, x, \left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}$$
$$= x\log(\pi)+\frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{\pi}\right)}{x}\,dx, x, \left(F^{e(c+dx)}\right)^n\right)}{den\log(F)}$$
$$= x\log(\pi)-\frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$x\log(\pi)-\frac{\text{Li}_2\left(-\frac{b\left(F^{e(c+dx)}\right)^n}{\pi}\right)}{den\log(F)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[b*(F^(e*(c + d*x)))^n + Pi],x]
```

```
[Out] x*Log[Pi] - PolyLog[2, -((b*(F^(e*(c + d*x)))^n)/Pi)]/(d*e*n*Log[F])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(39) = 78$.
time = 0.08, size = 96, normalized size = 2.46

method	result
derivativedivides	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)-\text{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{de\ln(F)n}$
default	$\frac{\left(\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\ln\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)\right)\ln\left(-\frac{b\left(F^{e(dx+c)}\right)^n}{\pi}\right)-\text{dilog}\left(\frac{b\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{de\ln(F)n}$
risch	$x\ln\left(b\left(F^{e(dx+c)}\right)^n+\pi\right)-\frac{\text{dilog}\left(\frac{b\,F^{xned}F^{-xned}\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)}{\ln(F)den}-\frac{\ln\left(\frac{b\,F^{xned}F^{-xned}\left(F^{e(dx+c)}\right)^n+\pi}{\pi}\right)\,\mathrm{l}}{\ln(F)de}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*(F^(e*(d*x+c)))^n+Pi),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/e/ln(F)/n*((ln(b*(F^(e*(d*x+c)))^n+Pi)-ln((b*(F^(e*(d*x+c)))^n+Pi)/Pi))
*ln(-b*(F^(e*(d*x+c)))^n/Pi)-dilog((b*(F^(e*(d*x+c)))^n+Pi)/Pi))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(38) = 76$.
time = 0.29, size = 87, normalized size = 2.23

$$x \log\left(\pi + F^{(dx+c)ne}b\right) - \frac{\left(dnxe \log\left(\frac{F^{dnxe}F^{cne}b}{\pi} + 1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dnxe}F^{cne}b}{\pi}\right)\right)e^{(-1)}}{dn\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="maxima")
```

```
[Out] x*log(pi + F^((d*x + c)*n*e)*b) - (d*n*x*e*log(F^(d*n*x*e)*F^(c*n*e)*b/pi +
 1)*log(F) + dilog(-F^(d*n*x*e)*F^(c*n*e)*b/pi))*e^(-1)/(d*n*log(F))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(38) = 76$.
time = 0.37, size = 108, normalized size = 2.77

$$\frac{\left((dnx+cn)e\log\left(\pi + F^{(dnx+cn)e}b\right)\log(F) - (dnx+cn)e\log(F)\log\left(\frac{\pi + F^{(dnx+cn)e}b}{\pi}\right) - \mathrm{Li}_2\left(-\frac{\pi + F^{(dnx+cn)e}b}{\pi} + 1\right)\right)e^{(-1)}}{dn\log(F)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="fricas")
```

```
[Out] ((d*n*x + c*n)*e*log(pi + F^((d*n*x + c*n)*e)*b)*log(F) - (d*n*x + c*n)*e*l
og(F)*log((pi + F^((d*n*x + c*n)*e)*b)/pi) - dilog(-(pi + F^((d*n*x + c*n)*
e)*b)/pi + 1))*e^(-1)/(d*n*log(F))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-bdene^{cen\log(F)}\log(F)\int \frac{xe^{denx\log(F)}}{be^{cen\log(F)}e^{denx\log(F)} + \pi}\,dx + x\log\left(b\left(F^{e(c+dx)}\right)^n + \pi\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*(F**(e*(d*x+c)))**n+pi),x)
```

```
[Out] -b*d*e*n*exp(c*e*n*log(F))*log(F)*Integral(x*exp(d*e*n*x*log(F))/(b*exp(c*e
*n*log(F))*exp(d*e*n*x*log(F)) + pi), x) + x*log(b*(F**(e*(c + d*x)))**n +
pi)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*(F^(e*(d*x+c)))^n+pi),x, algorithm="giac")
```

```
[Out] integrate(log(pi + (F^((d*x + c)*e))^n*b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\Pi + b\left(F^{e\,(c+d\,x)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(Pi + b*(F^(e*(c + d*x)))^n),x)
```

```
[Out] int(log(Pi + b*(F^(e*(c + d*x)))^n), x)
```

3.129 $\int \frac{1}{x(3+\log(x))} dx$

Optimal. Leaf size=5

$$\log(3 + \log(x))$$

```
[Out] ln(3+ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2339, 29}

$$\log(\log(x) + 3)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(3+\log(x))} dx &= \text{Subst}\left(\int \frac{1}{x} dx, x, 3 + \log(x)\right) \\ &= \log(3 + \log(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 5, normalized size = 1.00

$$\log(3 + \log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + Log[x])),x]
```

```
[Out] Log[3 + Log[x]]
```

Maple [A]
time = 0.01, size = 6, normalized size = 1.20

method	result	size
derivativedivides	$\ln(3+\ln(x))$	6
default	$\ln(3+\ln(x))$	6
norman	$\ln(3+\ln(x))$	6
risch	$\ln(3+\ln(x))$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(3+ln(x)),x,method=_RETURNVERBOSE)
[Out] ln(3+ln(x))
```

Maxima [A]
time = 0.27, size = 5, normalized size = 1.00

$$\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="maxima")
[Out] log(log(x) + 3)
```

Fricas [A]
time = 0.35, size = 5, normalized size = 1.00

$$\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="fricas")
[Out] log(log(x) + 3)
```

Sympy [A]
time = 0.03, size = 5, normalized size = 1.00

$$\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+ln(x)),x)
[Out] log(log(x) + 3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5) = 10$.
time = 3.73, size = 22, normalized size = 4.40

$$\frac{1}{2}\,\log\left(\frac{1}{4}\,\pi^2(\mathrm{sgn}(x)-1)^2+(\log\left(|x|\right)+3)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(3+log(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2)
```

Mupad [B]
time = 0.42, size = 5, normalized size = 1.00

$$\ln\left(\ln\left(x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x) + 3)),x)
[Out] log(log(x) + 3)
```

3.130 $\int \frac{\sqrt{1+\log(x)}}{x}\,dx$

Optimal. Leaf size=12

$$\frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2339, 30}

$$\frac{2}{3}(\log(x)+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\log(x)}}{x}\,dx &= \text{Subst}\left(\int \sqrt{x}\,dx, x, 1+\log(x)\right)\\ &= \frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$\frac{2}{3}(1+\log(x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/x,x]
```

```
[Out] (2*(1 + Log[x])^(3/2))/3
```

Maple [A]
time = 0.06, size = 9, normalized size = 0.75

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+ln(x))^(3/2)
```

Maxima [A]
time = 0.28, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

Fricas [A]
time = 0.34, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(log(x) + 1)^(3/2)
```

Sympy [A]
time = 0.38, size = 10, normalized size = 0.83

$$\frac{2(\log\left(x\right)+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x,x)
[Out] 2*(log(x) + 1)**(3/2)/3
```

Giac [A]
time = 4.45, size = 8, normalized size = 0.67

$$\frac{2}{3}\left(\log\left(x\right)+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x,x, algorithm="giac")
[Out] 2/3*(log(x) + 1)^(3/2)
```

Mupad [B]
time = 0.39, size = 13, normalized size = 1.08

$$\sqrt{\ln\left(x\right)+1}\,\left(\frac{2\,\ln\left(x\right)}{3}+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^(1/2)/x,x)
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 + 2/3)
```

3.131 $\int \frac{(1+\log(x))^5}{x}\,dx$

Optimal. Leaf size=10

$$\frac{1}{6}(1+\log(x))^6$$

```
[Out] 1/6*(1+ln(x))^6
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2339, 30}

$$\frac{1}{6}(\log(x)+1)^6$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Log[x])^5/x,x]
```

```
[Out] (1 + Log[x])^6/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+\log(x))^5}{x}\,dx &= \text{Subst}\left(\int x^5\,dx, x, 1+\log(x)\right)\\ &= \frac{1}{6}(1+\log(x))^6\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{6}(1+\log(x))^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])^5/x,x]
[Out] (1 + Log[x])^6/6
```

Maple [A]
time = 0.02, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\frac{(1+\ln(x))^6}{6}$	9
default	$\frac{(1+\ln(x))^6}{6}$	9
norman	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32
risch	$\ln(x)^5+\ln(x)+\frac{5\ln(x)^2}{2}+\frac{10\ln(x)^3}{3}+\frac{5\ln(x)^4}{2}+\frac{\ln(x)^6}{6}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^5/x,x,method=_RETURNVERBOSE)
[Out] 1/6*(1+ln(x))^6
```

Maxima [A]
time = 0.29, size = 8, normalized size = 0.80

$$\frac{1}{6}\left(\log\left(x\right)+1\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="maxima")
[Out] 1/6*(log(x) + 1)^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.
time = 0.36, size = 31, normalized size = 3.10

$$\frac{1}{6}\log\left(x\right)^{6}+\log\left(x\right)^{5}+\frac{5}{2}\log\left(x\right)^{4}+\frac{10}{3}\log\left(x\right)^{3}+\frac{5}{2}\log\left(x\right)^{2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="fricas")
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.
time = 0.14, size = 39, normalized size = 3.90

$$\frac{\log\left(x\right)^{6}}{6}+\log\left(x\right)^{5}+\frac{5\log\left(x\right)^{4}}{2}+\frac{10\log\left(x\right)^{3}}{3}+\frac{5\log\left(x\right)^{2}}{2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**5/x,x)
```

```
[Out] log(x)**6/6 + log(x)**5 + 5*log(x)**4/2 + 10*log(x)**3/3 + 5*log(x)**2/2 +
log(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.
time = 3.49, size = 31, normalized size = 3.10

$$\frac{1}{6}\,\log\left(x\right)^{6}+\log\left(x\right)^{5}+\frac{5}{2}\,\log\left(x\right)^{4}+\frac{10}{3}\,\log\left(x\right)^{3}+\frac{5}{2}\,\log\left(x\right)^{2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^5/x,x, algorithm="giac")
```

```
[Out] 1/6*log(x)^6 + log(x)^5 + 5/2*log(x)^4 + 10/3*log(x)^3 + 5/2*log(x)^2 + log
(x)
```

Mupad [B]
time = 0.39, size = 26, normalized size = 2.60

$$\frac{\ln\left(x\right)\,\left(\ln\left(x\right)+2\right)\,\left(\ln\left(x\right)^{2}+\ln\left(x\right)+1\right)\,\left(\ln\left(x\right)^{2}+3\,\ln\left(x\right)+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^5/x,x)
```

```
[Out] (log(x)*(log(x) + 2)*(log(x) + log(x)^2 + 1)*(3*log(x) + log(x)^2 + 3))/6
```

3.132 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

```
[Out] 2*ln(x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2339, 30}

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\log(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
[Out] 2*Sqrt[Log[x]]
```

Maple [A]
time = 0.01, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2\sqrt{\ln(x)}$	7
default	$2\sqrt{\ln(x)}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(x)^(1/2)
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(log(x))
```

Fricas [A]
time = 0.33, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(log(x))
```

Sympy [A]
time = 0.27, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)**(1/2),x)
[Out] 2*sqrt(log(x))
```

Giac [A]
time = 5.53, size = 6, normalized size = 0.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(log(x))
```

Mupad [B]
time = 0.07, size = 6, normalized size = 0.75

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(x)^(1/2)),x)
```

```
[Out] 2*log(x)^(1/2)
```

3.133 $\int \frac{1}{x(1+\log^2(x))}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] arctan(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {209}

$$\text{ArcTan}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 3, normalized size = 1.00

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arctan(\ln(x))$	4
default	$\arctan(\ln(x))$	4
risch	$\frac{i\ln(\ln(x)+i)}{2}-\frac{i\ln(\ln(x)-i)}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(ln(x))
```

Maxima [A]
time = 0.47, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A]
time = 0.36, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.
time = 0.12, size = 15, normalized size = 5.00

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log(2i+\log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A]
time = 4.18, size = 3, normalized size = 1.00

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

Mupad [B]
time = 0.53, size = 3, normalized size = 1.00

$$\operatorname{atan}(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 1)),x)
```

```
[Out] atan(log(x))
```

3.134 $\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

```
[Out] arctanh(ln(x)/(-3+ln(x)^2)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {223, 212}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-3 + Log[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{-3+\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{-3+x^2}}\,dx, x, \log(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$
$$= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.
time = 0.01, size = 42, normalized size = 3.00

$$-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)+\frac{1}{2}\log\left(1+\frac{\log(x)}{\sqrt{-3+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 + Log[x]^2]),x]
```

```
[Out] -1/2*Log[1 - Log[x]/Sqrt[-3 + Log[x]^2]] + Log[1 + Log[x]/Sqrt[-3 + Log[x]^
2]]/2
```

Maple [**A**]
time = 0.01, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^2}\right)$	13
default	$\ln\left(\ln(x)+\sqrt{-3+\ln(x)^2}\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-3+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(ln(x)+(-3+ln(x)^2)^(1/2))
```

Maxima [**A**]
time = 0.28, size = 16, normalized size = 1.14

$$\log\left(2\sqrt{\log(x)^2-3}+2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(log(x)^2 - 3) + 2*log(x))
```

Fricas [**A**]
time = 0.36, size = 16, normalized size = 1.14

$$-\log\left(\sqrt{\log(x)^2-3}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(log(x)^2 - 3) - log(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{\log(x)^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(log(x)**2 - 3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-3+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 0.47, size = 12, normalized size = 0.86

$$\ln\left(\ln(x)+\sqrt{\ln(x)^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 - 3)^(1/2)),x)
```

```
[Out] log(log(x) + (log(x)^2 - 3)^(1/2))
```

3.135 $\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{3\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
```

```
[Out] ArcSin[(3*Log[x])/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{4-9\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4-9x^2}}\,dx, x, \log(x)\right)\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3\log(x)}{2}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.01, size = 26, normalized size = 2.36

$$\frac{1}{3}i\log\left(-3i\log(x)+\sqrt{4-9\log^2(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 - 9*Log[x]^2]),x]
```

```
[Out] (I/3)*Log[(-3*I)*Log[x] + Sqrt[4 - 9*Log[x]^2]]
```

Maple [A]
time = 0.01, size = 8, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8
default	$\frac{\arcsin\left(\frac{3\ln(x)}{2}\right)}{3}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4-9*ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsin(3/2*ln(x))
```

Maxima [A]
time = 0.49, size = 7, normalized size = 0.64

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*log(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.
time = 0.38, size = 21, normalized size = 1.91

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9\log\left(x\right)^{2}+4}-2}{3\log\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*log(x)^2 + 4) - 2)/log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-\left(3\log\left(x\right)-2\right)\left(3\log\left(x\right)+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(3*log(x) - 2)*(3*log(x) + 2))), x)
```

Giac [A]
time = 4.55, size = 7, normalized size = 0.64

$$\frac{1}{3} \arcsin\left(\frac{3}{2} \log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4-9*log(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*arcsin(3/2*log(x))
```

Mupad [B]
time = 0.40, size = 7, normalized size = 0.64

$$\frac{\operatorname{asin}\left(\frac{3\,\ln(x)}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4 - 9*log(x)^2)^(1/2)),x)
[Out] asin((3*log(x))/2)/3
```

3.136 $\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx$

Optimal. Leaf size=7

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

```
[Out] arcsinh(1/2*ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{221\}$

$$\sinh^{-1}\left(\frac{\log(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[4 + Log[x]^2]),x]
```

```
[Out] ArcSinh[Log[x]/2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{4+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \log(x)\right)\\ &= \sinh^{-1}\left(\frac{\log(x)}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 14, normalized size = 2.00

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{4+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[4 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[4 + Log[x]^2]]
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6
default	$\operatorname{arcsinh}\left(\frac{\ln(x)}{2}\right)$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(1/2*ln(x))
```

Maxima [A]
time = 0.49, size = 5, normalized size = 0.71

$$\operatorname{arsinh}\left(\frac{1}{2}\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*log(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.
time = 0.35, size = 16, normalized size = 2.29

$$-\log\left(\sqrt{\log\left(x\right)^{2}+4}-\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{\log\left(x\right)^{2}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(log(x)**2 + 4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.
time = 4.72, size = 16, normalized size = 2.29

$$-\log\left(\sqrt{\log\left(x\right)^2 + 4} - \log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+log(x)^2)^(1/2),x, algorithm="giac")
[Out] -log(sqrt(log(x)^2 + 4) - log(x))
```

Mupad [B]
time = 0.39, size = 5, normalized size = 0.71

$$\operatorname{asinh}\left(\frac{\ln\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(log(x)^2 + 4)^(1/2)),x)
[Out] asinh(log(x)/2)
```

3.137 $\int \frac{1}{x(2+3\log^3(6x))}\,dx$

Optimal. Leaf size=111

$$-\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}+\frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}}-\frac{\log\left(2^{2/3}-\sqrt[3]{6}\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}}$$

```
[Out] 1/6*arctan(1/3*2^(2/3)*ln(6*x)*3^(5/6)-1/3*3^(1/2))*2^(1/3)*3^(1/6)+1/18*ln
(2^(1/3)+3^(1/3)*ln(6*x))*2^(1/3)*3^(2/3)-1/36*ln(2^(2/3)-6^(1/3)*ln(6*x)+3
^(2/3)*ln(6*x)^2)*2^(1/3)*3^(2/3)
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\log(6x)}{\sqrt[6]{3}}\right)}{2^{2/3}3^{5/6}}-\frac{\log\left(3^{2/3}\log^2(6x)-\sqrt[3]{6}\log(6x)+2^{2/3}\right)}{6\ 2^{2/3}\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}\log(6x)+\sqrt[3]{2}\right)}{3\ 2^{2/3}\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] - (2^(2/3)*Log[6*x])/3^(1/6)]/(2^(2/3)*3^(5/6))) + Log[2
^(1/3) + 3^(1/3)*Log[6*x]]/(3*2^(2/3)*3^(1/3)) - Log[2^(2/3) - 6^(1/3)*Log[
6*x] + 3^(2/3)*Log[6*x]^2]/(6*2^(2/3)*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x\left(2+3\log^3(6x)\right)}\,dx = \text{Subst}\left(\int \frac{1}{2+3x^3}\,dx, x, \log(6x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}+\sqrt[3]{3}\,x}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{2}-\sqrt[3]{3}\,x}{2^{2/3}-\sqrt[3]{6}\,x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{3\ 2^{2/3}}$$
$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\,\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{2^{2/3}-\sqrt[3]{6}\,x+3^{2/3}x^2}\,dx, x, \log(6x)\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}}{}$$
$$= \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\,\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\,\log(6x)+3^{2/3}\log^2(6x)\right)}{6\ 2^{2/3}\sqrt[3]{3}} + \frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{\tan^{-1}\left(\frac{1-2^{2/3}\sqrt[3]{3}\,\log(6x)}{\sqrt{3}}\right)}{2^{2/3}3^{5/6}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[3]{3}\,\log(6x)\right)}{3\ 2^{2/3}\sqrt[3]{3}} - \frac{\log\left(2^{2/3}-\sqrt[3]{6}\,\log(6\right.}{6\ 2^{2/3}}$$

Mathematica [A]
time = 0.06, size = 106, normalized size = 0.95

$$\frac{6\tan^{-1}\left(\frac{-1+2^{2/3}\sqrt[3]{3}\,\log(6x)}{\sqrt{3}}\right)+\sqrt{3}\left(2\log\left(2+2^{2/3}\sqrt[3]{3}\,\log(6x)\right)-\log\left(2-2^{2/3}\sqrt[3]{3}\,\log(6x)+\sqrt[3]{2}\,3^{2/3}\log^2(6x)\right)\right)}{6\ 2^{2/3}3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*Log[6*x]^3)),x]
```

```
[Out] (6*ArcTan[(-1 + 2^(2/3)*3^(1/3)*Log[6*x])/Sqrt[3]] + Sqrt[3]*(2*Log[2 + 2^(
2/3)*3^(1/3)*Log[6*x]] - Log[2 - 2^(2/3)*3^(1/3)*Log[6*x] + 2^(1/3)*3^(2/3)
*Log[6*x]^2]))/(6*2^(2/3)*3^(5/6))
```

Maple [A]
time = 0.02, size = 87, normalized size = 0.78

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(324\_Z^3-1)} \_R\ln\left(\ln(6x)+6\_R\right)$
derivativedivides	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$
default	$\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)+\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}}{3}\right)}{18}-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln\left(\ln(6x)^2-\frac{2^{\frac{1}{3}}3^{\frac{2}{3}}\ln(6x)}{3}+\frac{2^{\frac{2}{3}}3^{\frac{1}{3}}}{3}\right)}{36}+\frac{2^{\frac{1}{3}}3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(2^{\frac{2}{3}}3^{\frac{1}{3}}\ln(6x)-1\right)}{3}\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*ln(6*x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*2^(1/3)*3^(2/3)*ln(ln(6*x)+1/3*2^(1/3)*3^(2/3))-1/36*2^(1/3)*3^(2/3)*l
n(ln(6*x)^2-1/3*2^(1/3)*3^(2/3)*ln(6*x)+1/3*2^(2/3)*3^(1/3))+1/6*2^(1/3)*3^
(1/6)*arctan(1/3*3^(1/2)*(2^(2/3)*3^(1/3)*ln(6*x)-1))
```

Maxima [A]
time = 0.49, size = 97, normalized size = 0.87

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(3^{\frac{2}{3}}\log(6\,x)^2-3^{\frac{1}{3}}2^{\frac{1}{3}}\log(6\,x)+2^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}2^{\frac{1}{3}}\log\left(\frac{1}{3}\cdot 3^{\frac{2}{3}}\left(3^{\frac{1}{3}}\log(6\,x)+2^{\frac{1}{3}}\right)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{1}{6}}2^{\frac{2}{3}}\left(2\cdot 3^{\frac{2}{3}}\log(6\,x)-3^{\frac{1}{3}}2^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*2^(1/3)*log(3^(2/3)*log(6*x)^2 - 3^(1/3)*2^(1/3)*log(6*x) + 2
^(2/3)) + 1/18*3^(2/3)*2^(1/3)*log(1/3*3^(2/3)*(3^(1/3)*log(6*x) + 2^(1/3))
) + 1/6*3^(1/6)*2^(1/3)*arctan(1/6*3^(1/6)*2^(2/3)*(2*3^(2/3)*log(6*x) - 3^
(1/3)*2^(1/3)))
```

Fricas [A]
time = 0.36, size = 71, normalized size = 0.64

$$-\frac{1}{72}\cdot 12^{\frac{2}{3}}\log\left(6\,\log(6\,x)^2-12^{\frac{2}{3}}\log(6\,x)+2\cdot 12^{\frac{1}{3}}\right)+\frac{1}{36}\cdot 12^{\frac{2}{3}}\log\left(12^{\frac{2}{3}}+6\,\log(6\,x)\right)+\frac{1}{6}\cdot 12^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{1}{6}}\left(12^{\frac{2}{3}}\log(6\,x)-12^{\frac{1}{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="fricas")
```

```
[Out] -1/72*12^(2/3)*log(6*log(6*x)^2 - 12^(2/3)*log(6*x) + 2*12^(1/3)) + 1/36*12
^(2/3)*log(12^(2/3) + 6*log(6*x)) + 1/6*12^(1/6)*arctan(1/6*12^(1/6)*(12^(2
/3)*log(6*x) - 12^(1/3)))
```

Sympy [A]
time = 0.17, size = 17, normalized size = 0.15

$$\operatorname{RootSum}\left(324z^3 - 1, \left(i \mapsto i \log\left(6i + \log\left(6x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*ln(6*x)**3),x)
```

```
[Out] RootSum(324*_z**3 - 1, Lambda(_i, _i*log(6*_i + log(6*x))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*log(6*x)^3),x, algorithm="giac")
```

```
[Out] integrate(1/((3*log(6*x)^3 + 2)*x), x)
```

Mupad [B]
time = 4.69, size = 120, normalized size = 1.08

$$\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}}{3\,x^2}\right)}{18}+\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}+\frac{2^{1/3}\,3^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}-\frac{2^{1/3}\,3^{2/3}\,\ln\left(\frac{\ln(6\,x)}{x^2}-\frac{2^{1/3}\,3^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,x^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*log(6*x)^3 + 2)),x)
```

```
[Out] (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3))/(3*x^2)))/18 + (2^(1/
3)*3^(2/3)*log(log(6*x)/x^2 + (2^(1/3)*3^(2/3)*((3^(1/2)*1i)/2 - 1/2))/(3*x
^2))*((3^(1/2)*1i)/2 - 1/2))/18 - (2^(1/3)*3^(2/3)*log(log(6*x)/x^2 - (2^(1
/3)*3^(2/3)*((3^(1/2)*1i)/2 + 1/2))/(3*x^2))*((3^(1/2)*1i)/2 + 1/2))/18
```

3.138 $\int \frac{\log(\log(6x))}{x\log(6x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\log^2(\log(6x))$$

```
[Out] 1/2*ln(ln(6*x))^2
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2338}

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[6*x]]/(x*Log[6*x]),x]
[Out] Log[Log[6*x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(\log(6x))}{x\log(6x)}\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \log(6x)\right)\\ &= \frac{1}{2}\log^2(\log(6x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{2}\log^2(\log(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[6*x]]/(x*Log[6*x]),x]
[Out] Log[Log[6*x]]^2/2
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\ln(\ln(6x))^2}{2}$	10
default	$\frac{\ln(\ln(6x))^2}{2}$	10
norman	$\frac{\ln(\ln(6x))^2}{2}$	10
risch	$\frac{\ln(\ln(6x))^2}{2}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(6*x))/x/ln(6*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(ln(6*x))^2
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\log\left(6\,x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="maxima")
[Out] 1/2*log(log(6*x))^2
```

Fricas [A]
time = 0.35, size = 9, normalized size = 0.82

$$\frac{1}{2}\log\left(\log\left(6\,x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="fricas")
[Out] 1/2*log(log(6*x))^2
```

Sympy [A]
time = 0.12, size = 8, normalized size = 0.73

$$\frac{\log\left(\log\left(6x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(6*x))/x/ln(6*x),x)
```

```
[Out] log(log(6*x))**2/2
```

Giac [A]
time = 3.47, size = 9, normalized size = 0.82

$$\frac{1}{2}\,\log\left(\log\left(6\,x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(6*x))/x/log(6*x),x, algorithm="giac")
```

```
[Out] 1/2*log(log(6*x))^2
```

Mupad [B]
time = 0.52, size = 9, normalized size = 0.82

$$\frac{\ln\left(\ln\left(6\,x\right)\right)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(6*x))/(x*log(6*x)),x)
```

```
[Out] log(log(6*x))^2/2
```

3.139 $\int \frac{2^{\log(x)}}{x} \, dx$

Optimal. Leaf size=9

$$\frac{2^{\log(x)}}{\log(2)}$$

```
[Out] 2^ln(x)/ln(2)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2306, 30}

$$\frac{x^{\log(2)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x]/x,x]
```

```
[Out] x^Log[2]/Log[2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{2^{\log(x)}}{x} \, dx &= \int x^{-1+\log(2)} \, dx \\ &= \frac{x^{\log(2)}}{\log(2)} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x]/x,x]
```

```
[Out] 2^Log[x]/Log[2]
```

Maple [A]
time = 0.04, size = 10, normalized size = 1.11

method	result	size
gosper	$\frac{2^{\ln(x)}}{\ln(2)}$	10
derivativedivides	$\frac{2^{\ln(x)}}{\ln(2)}$	10
default	$\frac{2^{\ln(x)}}{\ln(2)}$	10
risch	$\frac{x^{\ln(2)}}{\ln(2)}$	10
norman	$\frac{e^{\ln(x)\ln(2)}}{\ln(2)}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2^ln(x)/ln(2)
```

Maxima [A]
time = 0.30, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="maxima")
```

```
[Out] 2^log(x)/log(2)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 1.22

$$\frac{e^{(\log(2)\log(x))}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="fricas")
```

```
[Out] e^(log(2)*log(x))/log(2)
```

Sympy [A]
time = 0.23, size = 7, normalized size = 0.78

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x)/x,x)
```

```
[Out] 2**log(x)/log(2)
```

Giac [A]
time = 5.01, size = 9, normalized size = 1.00

$$\frac{2^{\log(x)}}{\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x)/x,x, algorithm="giac")
```

```
[Out] 2^log(x)/log(2)
```

Mupad [B]
time = 0.40, size = 9, normalized size = 1.00

$$\frac{x^{\ln(2)}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(x)/x,x)
```

```
[Out] x^log(2)/log(2)
```

3.140 $\int \frac{\sin^2(\log(x))}{x} \, dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x))$$

```
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2715, 8}

$$\frac{\log(x)}{2} - \frac{1}{2}\sin(\log(x))\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]]^2/x,x]
```

```
[Out] Log[x]/2 - (Cos[Log[x]]*Sin[Log[x]])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(\log(x))}{x} \, dx &= \text{Subst}\left(\int \sin^2(x) \, dx, x, \log(x)\right)\\
&= -\frac{1}{2}\cos(\log(x))\sin(\log(x)) + \frac{1}{2}\text{Subst}\left(\int 1 \, dx, x, \log(x)\right)\\
&= \frac{\log(x)}{2} - \frac{1}{2}\cos(\log(x))\sin(\log(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.94

$$\frac{\log(x)}{2} - \frac{1}{4}\sin(2\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]]^2/x,x]
[Out] Log[x]/2 - Sin[2*Log[x]]/4
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\ln(x)}{2}-\frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
default	$\frac{\ln(x)}{2}-\frac{\cos(\ln(x))\sin(\ln(x))}{2}$	14
risch	$\frac{\ln(x)}{2}+\frac{i x^{2i}}{8}-\frac{i x^{-2i}}{8}$	24
norman	$\frac{\tan^{3}\left(\frac{\ln(x)}{2}\right)+\frac{\ln(x)}{2}+\ln(x)\left(\tan^{2}\left(\frac{\ln(x)}{2}\right)\right)+\frac{\ln(x)\left(\tan^{4}\left(\frac{\ln(x)}{2}\right)\right)}{2}-\tan\left(\frac{\ln(x)}{2}\right)}{\left(1+\tan^{2}\left(\frac{\ln(x)}{2}\right)\right)^{2}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)-1/2*cos(ln(x))*sin(ln(x))
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.71

$$\frac{1}{2}\log\left(x\right)-\frac{1}{4}\sin\left(2\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="maxima")
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.76

$$-\frac{1}{2}\cos\left(\log\left(x\right)\right)\sin\left(\log\left(x\right)\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="fricas")
[Out] -1/2*cos(log(x))*sin(log(x)) + 1/2*log(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(15) = 30$.
time = 1.70, size = 156, normalized size = 9.18

$$\frac{\log\left(x\right)\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)+4\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)+2}+\frac{2\log\left(x\right)\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)+4\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)+2}+\frac{\log\left(x\right)}{2\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)+4\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)+2}+\frac{2\tan^{3}\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)+4\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)+2}-\frac{2\tan\left(\frac{\log\left(x\right)}{2}\right)}{2\tan^{4}\left(\frac{\log\left(x\right)}{2}\right)+4\tan^{2}\left(\frac{\log\left(x\right)}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x))**2/x,x)
```

```
[Out] log(x)*tan(log(x)/2)**4/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*l
og(x)*tan(log(x)/2)**2/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + log(
x)/(2*tan(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) + 2*tan(log(x)/2)**3/(2*ta
n(log(x)/2)**4 + 4*tan(log(x)/2)**2 + 2) - 2*tan(log(x)/2)/(2*tan(log(x)/2)
**4 + 4*tan(log(x)/2)**2 + 2)
```

Giac [A]
time = 7.75, size = 12, normalized size = 0.71

$$\frac{1}{2}\log\left(x\right)-\frac{1}{4}\sin\left(2\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x))^2/x,x, algorithm="giac")
```

```
[Out] 1/2*log(x) - 1/4*sin(2*log(x))
```

Mupad [B]
time = 0.38, size = 12, normalized size = 0.71

$$\frac{\ln\left(x\right)}{2}-\frac{\sin\left(2\ln\left(x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(log(x))^2/x,x)
```

```
[Out] log(x)/2 - sin(2*log(x))/4
```

3.141 $\int \frac{7-\log(x)}{x(3+\log(x))} \, dx$

Optimal. Leaf size=12

$$-\log(x)+10\log(3+\log(x))$$

```
[Out] -ln(x)+10*ln(3+ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2412, 45}

$$10\log(\log(x)+3)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{7-\log(x)}{x(3+\log(x))} \, dx &= \text{Subst}\left(\int \frac{7-x}{3+x} \, dx, x, \log(x)\right) \\
&= \text{Subst}\left(\int \left(-1+\frac{10}{3+x}\right) \, dx, x, \log(x)\right) \\
&= -\log(x)+10\log(3+\log(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\log(x)+10\log(3+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - Log[x])/(x*(3 + Log[x])),x]
```

```
[Out] -Log[x] + 10*Log[3 + Log[x]]
```

Maple [A]
time = 0.01, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\ln(x)+10\ln(3+\ln(x))$	13
default	$-\ln(x)+10\ln(3+\ln(x))$	13
norman	$-\ln(x)+10\ln(3+\ln(x))$	13
risch	$-\ln(x)+10\ln(3+\ln(x))$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7-ln(x))/x/(3+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+10*ln(3+ln(x))
```

Maxima [A]
time = 0.26, size = 12, normalized size = 1.00

$$-\log(x)+10\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="maxima")
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Fricas [A]
time = 0.34, size = 12, normalized size = 1.00

$$-\log(x)+10\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="fricas")
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Sympy [A]
time = 0.09, size = 10, normalized size = 0.83

$$-\log(x)+10\log(\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-ln(x))/x/(3+ln(x)),x)
```

```
[Out] -log(x) + 10*log(log(x) + 3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 6.27, size = 27, normalized size = 2.25

$$5 \log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(x)-1)^2 + (\log(|x|)+3)^2\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7-log(x))/x/(3+log(x)),x, algorithm="giac")
```

```
[Out] 5*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) + 3)^2) - log(x)
```

Mupad [B]
time = 0.37, size = 12, normalized size = 1.00

$$10 \ln(\ln(x)+3) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 7)/(x*(log(x) + 3)),x)
```

```
[Out] 10*log(log(x) + 3) - log(x)
```

3.142 $\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx$

Optimal. Leaf size=21

$$\frac{5}{3}(3+\log(x))^3-\frac{1}{4}(3+\log(x))^4$$

```
[Out] 5/3*(3+ln(x))^3-1/4*(3+ln(x))^4
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2412, 45}

$$\frac{5}{3}(\log(x)+3)^3-\frac{1}{4}(\log(x)+3)^4$$

Antiderivative was successfully verified.

```
[In] Int[((2 - Log[x])*(3 + Log[x])^2)/x,x]
```

```
[Out] (5*(3 + Log[x])^3)/3 - (3 + Log[x])^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-\log(x))(3+\log(x))^2}{x}\,dx &= \text{Subst}\left(\int (2-x)(3+x)^2\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(5(3+x)^2-(3+x)^3\right)\,dx, x, \log(x)\right)\\ &= \frac{5}{3}(3+\log(x))^3-\frac{1}{4}(3+\log(x))^4\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.38

$$18\log(x)+\frac{3\log^2(x)}{2}-\frac{4\log^3(x)}{3}-\frac{\log^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - Log[x])*(3 + Log[x])^2)/x,x]
[Out] 18*Log[x] + (3*Log[x]^2)/2 - (4*Log[x]^3)/3 - Log[x]^4/4
```

Maple [A]
time = 0.01, size = 24, normalized size = 1.14

method	result	size
derivativedivides	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
default	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
norman	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24
risch	$-\frac{\ln(x)^4}{4}-\frac{4\ln(x)^3}{3}+\frac{3\ln(x)^2}{2}+18\ln(x)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-ln(x))*(3+ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] -1/4*ln(x)^4-4/3*ln(x)^3+3/2*ln(x)^2+18*ln(x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4-\frac{4}{3}\log(x)^3+\frac{3}{2}\log(x)^2+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="maxima")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Fricas [A]
time = 0.37, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4-\frac{4}{3}\log(x)^3+\frac{3}{2}\log(x)^2+18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Sympy [A]
time = 0.12, size = 27, normalized size = 1.29

$$-\frac{\log(x)^4}{4} - \frac{4\log(x)^3}{3} + \frac{3\log(x)^2}{2} + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-ln(x))*(3+ln(x))**2/x,x)
[Out] -log(x)**4/4 - 4*log(x)**3/3 + 3*log(x)**2/2 + 18*log(x)
```

Giac [A]
time = 6.12, size = 23, normalized size = 1.10

$$-\frac{1}{4}\log(x)^4 - \frac{4}{3}\log(x)^3 + \frac{3}{2}\log(x)^2 + 18\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-log(x))*(3+log(x))^2/x,x, algorithm="giac")
[Out] -1/4*log(x)^4 - 4/3*log(x)^3 + 3/2*log(x)^2 + 18*log(x)
```

Mupad [B]
time = 0.37, size = 22, normalized size = 1.05

$$\frac{\ln(x)\left(-3\ln(x)^3 - 16\ln(x)^2 + 18\ln(x) + 216\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((log(x) - 2)*(log(x) + 3)^2)/x,x)
[Out] (log(x)*(18*log(x) - 16*log(x)^2 - 3*log(x)^3 + 216))/12
```

3.143 $\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{8}\sinh^{-1}(\log(x))+\frac{1}{8}\log(x)\sqrt{1+\log^2(x)}+\frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}$$

```
[Out] -1/8*arcsinh(ln(x))+1/8*ln(x)*(1+ln(x)^2)^(1/2)+1/4*ln(x)^3*(1+ln(x)^2)^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {285, 327, 221}

$$\frac{1}{8}\sqrt{\log^2(x)+1}\log(x)+\frac{1}{4}\sqrt{\log^2(x)+1}\log^3(x)-\frac{1}{8}\sinh^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
```

```
[Out] -1/8*ArcSinh[Log[x]] + (Log[x]*Sqrt[1 + Log[x]^2])/8 + (Log[x]^3*Sqrt[1 + L
og[x]^2])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\log^2(x)\sqrt{1+\log^2(x)}}{x}\,dx = \text{Subst}\left(\int x^2\sqrt{1+x^2}\,dx, x, \log(x)\right)$$
$$= \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)} + \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^2}}\,dx, x, \log(x)\right)$$
$$= \frac{1}{8}\log(x)\sqrt{1+\log^2(x)} + \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx\right.$$
$$= -\frac{1}{8}\sinh^{-1}(\log(x)) + \frac{1}{8}\log(x)\sqrt{1+\log^2(x)} + \frac{1}{4}\log^3(x)\sqrt{1+\log^2(x)}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.74

$$\frac{1}{8}\left(-\sinh^{-1}(\log(x)) + \log(x)\sqrt{1+\log^2(x)}\,(1+2\log^2(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^2*Sqrt[1 + Log[x]^2])/x,x]
[Out] (-ArcSinh[Log[x]] + Log[x]*Sqrt[1 + Log[x]^2]*(1 + 2*Log[x]^2))/8
```

Maple [A]
time = 0.01, size = 31, normalized size = 0.74

method	result	size
derivativedivides	$\frac{\ln(x)\left(1+\ln(x)^2\right)^{\frac{3}{2}}}{4} - \frac{\ln(x)\sqrt{1+\ln(x)^2}}{8} - \frac{\text{arcsinh}(\ln(x))}{8}$	31
default	$\frac{\ln(x)\left(1+\ln(x)^2\right)^{\frac{3}{2}}}{4} - \frac{\ln(x)\sqrt{1+\ln(x)^2}}{8} - \frac{\text{arcsinh}(\ln(x))}{8}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2*(1+ln(x)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x)*(1+ln(x)^2)^(3/2)-1/8*ln(x)*(1+ln(x)^2)^(1/2)-1/8*arcsinh(ln(x))
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.71

$$\frac{1}{4}\left(\log(x)^2+1\right)^{\frac{3}{2}}\log(x) - \frac{1}{8}\sqrt{\log(x)^2+1}\,\log(x) - \frac{1}{8}\,\text{arsinh}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(log(x)^2 + 1)^(3/2)*log(x) - 1/8*sqrt(log(x)^2 + 1)*log(x) - 1/8*arcsi
nh(log(x))
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.86

$$\frac{1}{8}\left(2\log\left(x\right)^{3}+\log\left(x\right)\right)\sqrt{\log\left(x\right)^{2}+1}+\frac{1}{8}\log\left(\sqrt{\log\left(x\right)^{2}+1}-\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*log(x)^3 + log(x))*sqrt(log(x)^2 + 1) + 1/8*log(sqrt(log(x)^2 + 1) -
 log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\log\left(x\right)^{2}+1}\,\log\left(x\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2*(1+ln(x)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(log(x)**2 + 1)*log(x)**2/x, x)
```

Giac [A]
time = 4.92, size = 37, normalized size = 0.88

$$\frac{1}{8}\left(2\log\left(x\right)^{2}+1\right)\sqrt{\log\left(x\right)^{2}+1}\,\log\left(x\right)+\frac{1}{8}\log\left(\sqrt{\log\left(x\right)^{2}+1}-\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2*(1+log(x)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*log(x)^2 + 1)*sqrt(log(x)^2 + 1)*log(x) + 1/8*log(sqrt(log(x)^2 + 1)
 - log(x))
```

Mupad [B]
time = 0.40, size = 26, normalized size = 0.62

$$\left(\frac{\ln\left(x\right)^{3}}{4}+\frac{\ln\left(x\right)}{8}\right)\sqrt{\ln\left(x\right)^{2}+1}-\frac{\mathrm{asinh}(\ln\left(x\right))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2*(log(x)^2 + 1)^(1/2))/x,x)
```

```
[Out] (log(x)/8 + log(x)^3/4)*(log(x)^2 + 1)^(1/2) - asinh(log(x))/8
```

3.144 $\int \frac{1+\log(x)}{x(3+2\log(x))^2} dx$

Optimal. Leaf size=24

$$\frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))$$

```
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2412, 45}

$$\frac{1}{4}\log(2\log(x)+3)+\frac{1}{4(2\log(x)+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
```

```
[Out] 1/(4*(3 + 2*Log[x])) + Log[3 + 2*Log[x]]/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\log(x)}{x(3+2\log(x))^2}\,dx &= \text{Subst}\left(\int \frac{1+x}{(3+2x)^2}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{2(3+2x)^2}+\frac{1}{2(3+2x)}\right)\,dx,x,\log(x)\right)\\ &= \frac{1}{4(3+2\log(x))}+\frac{1}{4}\log(3+2\log(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.83

$$\frac{1}{4}\left(\frac{1}{3+2\log(x)}+\log(3+2\log(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Log[x])/(x*(3 + 2*Log[x])^2),x]
[Out] ((3 + 2*Log[x])^(-1) + Log[3 + 2*Log[x]])/4
```

Maple [A]
time = 0.01, size = 21, normalized size = 0.88

method	result	size
risch	$\frac{1}{12+8\ln(x)}+\frac{\ln\left(\frac{3}{2}+\ln(x)\right)}{4}$	19
derivativedivides	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
default	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21
norman	$\frac{1}{12+8\ln(x)}+\frac{\ln(3+2\ln(x))}{4}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))/x/(3+2*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4/(3+2*ln(x))+1/4*ln(3+2*ln(x))
```

Maxima [A]
time = 0.29, size = 20, normalized size = 0.83

$$\frac{1}{4\,(2\,\log(x)+3)}+\frac{1}{4}\,\log(2\,\log(x)+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4/(2*log(x) + 3) + 1/4*log(2*log(x) + 3)
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.08

$$\frac{(2\,\log(x)+3)\log(2\,\log(x)+3)+1}{4\,(2\,\log(x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*log(x) + 3)*log(2*log(x) + 3) + 1)/(2*log(x) + 3)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.71

$$\frac{\log\left(\log\left(x\right)+\frac{3}{2}\right)}{4}+\frac{1}{8\log\left(x\right)+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))/x/(3+2*ln(x))**2,x)
```

```
[Out] log(log(x) + 3/2)/4 + 1/(8*log(x) + 12)
```

Giac [A]
time = 4.61, size = 34, normalized size = 1.42

$$\frac{1}{4\left(2\,\log\left(x\right)+3\right)}+\frac{1}{8}\,\log\left(\pi^2(\mathrm{sgn}(x)-1)^2+\left(2\,\log\left(|x|\right)+3\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))/x/(3+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/4/(2*log(x) + 3) + 1/8*log(pi^2*(sgn(x) - 1)^2 + (2*log(abs(x)) + 3)^2)
```

Mupad [B]
time = 0.41, size = 18, normalized size = 0.75

$$\frac{\ln\left(2\,\ln\left(x\right)+3\right)}{4}+\frac{1}{4\,\left(2\,\ln\left(x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)/(x*(2*log(x) + 3)^2),x)
```

```
[Out] log(2*log(x) + 3)/4 + 1/(4*(2*log(x) + 3))
```

3.145 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$-2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}$$

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2412, 45}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx,x,\log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)\,dx,x,\log(x)\right)\\ &= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 0.70

$$\frac{2}{3}(-2+\log(x))\sqrt{1+\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18
default	$\frac{2(1+\ln(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\ln(x)}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Maxima [A]
time = 0.30, size = 17, normalized size = 0.74

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Fricas [A]
time = 0.37, size = 12, normalized size = 0.52

$$\frac{2}{3}\sqrt{\log(x)+1}(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A]
time = 4.27, size = 20, normalized size = 0.87

$$\frac{2(\log(x)+1)^{\frac{3}{2}}}{3}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Giac [A]
time = 3.05, size = 17, normalized size = 0.74

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}}-2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Mupad [B]
time = 0.43, size = 13, normalized size = 0.57

$$\sqrt{\ln(x)+1}\left(\frac{2\ln(x)}{3}-\frac{4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(log(x) + 1)^(1/2)),x)
```

```
[Out] (log(x) + 1)^(1/2)*((2*log(x))/3 - 4/3)
```

3.146 $\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx$

Optimal. Leaf size=29

$$\frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}$$

[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2412, 45}

$$\frac{1}{24}(4\log(x)-1)^{3/2}+\frac{1}{8}\sqrt{4\log(x)-1}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]

[Out] Sqrt[-1 + 4*Log[x]]/8 + (-1 + 4*Log[x])^(3/2)/24

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x\sqrt{-1+4\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{-1+4x}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{4\sqrt{-1+4x}}+\frac{1}{4}\sqrt{-1+4x}\right)\,dx, x, \log(x)\right)\\
&= \frac{1}{8}\sqrt{-1+4\log(x)}+\frac{1}{24}(-1+4\log(x))^{3/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.69

$$\frac{1}{12}(1+2\log(x))\sqrt{-1+4\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[-1 + 4*Log[x]]),x]
[Out] ((1 + 2*Log[x])*Sqrt[-1 + 4*Log[x]])/12
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.76

method	result	size
derivativedivides	$\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}+\frac{\sqrt{-1+4\ln(x)}}{8}$	22
default	$\frac{(-1+4\ln(x))^{\frac{3}{2}}}{24}+\frac{\sqrt{-1+4\ln(x)}}{8}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(-1+4*ln(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/24*(-1+4*ln(x))^(3/2)+1/8*(-1+4*ln(x))^(1/2)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.72

$$\frac{1}{24}\,(4\,\log\left(x\right)-1)^{\frac{3}{2}}+\frac{1}{8}\,\sqrt{4\,\log\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="maxima")
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.55

$$\frac{1}{12}\,\sqrt{4\,\log\left(x\right)-1}\,(2\,\log\left(x\right)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(4*log(x) - 1)*(2*log(x) + 1)
```

Sympy [A]
time = 3.97, size = 22, normalized size = 0.76

$$\frac{(4\log{(x)}-1)^{\frac{3}{2}}}{24}+\frac{\sqrt{4\log{(x)}-1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(-1+4*ln(x))**(1/2),x)
```

```
[Out] (4*log(x) - 1)**(3/2)/24 + sqrt(4*log(x) - 1)/8
```

Giac [A]
time = 5.00, size = 21, normalized size = 0.72

$$\frac{1}{24}\,(4\,\log{(x)}-1)^{\frac{3}{2}}+\frac{1}{8}\,\sqrt{4\,\log{(x)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(-1+4*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(4*log(x) - 1)^(3/2) + 1/8*sqrt(4*log(x) - 1)
```

Mupad [B]
time = 0.44, size = 15, normalized size = 0.52

$$\sqrt{4\,\ln{(x)}-1}\,\left(\frac{\ln{(x)}}{6}+\frac{1}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x*(4*log(x) - 1)^(1/2)),x)
```

```
[Out] (4*log(x) - 1)^(1/2)*(log(x)/6 + 1/12)
```

3.147 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$-2\tanh^{-1}\left(\sqrt{1+\log(x)}\right)+2\sqrt{1+\log(x)}$$

```
[Out] -2*arctanh((1+ln(x))^(1/2))+2*(1+ln(x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2412, 52, 65, 213}

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2412

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right) \\
&= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right) \\
&= 2\sqrt{1+\log(x)} + 2\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right) \\
&= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$-2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.36

method	result	size
derivativedivides	$2\sqrt{1+\ln(x)} + \ln\left(\sqrt{1+\ln(x)} - 1\right) - \ln\left(\sqrt{1+\ln(x)} + 1\right)$	30
default	$2\sqrt{1+\ln(x)} + \ln\left(\sqrt{1+\ln(x)} - 1\right) - \ln\left(\sqrt{1+\ln(x)} + 1\right)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x/ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+ln(x))^(1/2)+ln((1+ln(x))^(1/2)-1)-ln((1+ln(x))^(1/2)+1)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1} + 1\right) + \log\left(\sqrt{\log(x)+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Fricas [**A**]
time = 0.37, size = 29, normalized size = 1.32

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [**A**]
time = 2.07, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1} + \log\left(\sqrt{\log(x)+1}-1\right) - \log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

```
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 0.41, size = 18, normalized size = 0.82

$$2\sqrt{\ln(x)+1} - 2\,\mathrm{atanh}\left(\sqrt{\ln(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x) + 1)^(1/2)/(x*log(x)),x)
```

```
[Out] 2*(log(x) + 1)^(1/2) - 2*atanh((log(x) + 1)^(1/2))
```

3.148 $\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}$$

```
[Out] -2/3/(1-ln(x))^3+1/(1-ln(x))+1/(-1+ln(x))^2
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {712}

$$\frac{1}{(\log(x)-1)^2}+\frac{1}{1-\log(x)}-\frac{2}{3(1-\log(x))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
```

```
[Out] -2/(3*(1 - Log[x])^3) + (1 - Log[x])^(-1) + (-1 + Log[x])^(-2)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-4\log(x)+\log^2(x)}{x(-1+\log(x))^4}\,dx &= \text{Subst}\left(\int \frac{1-4x+x^2}{(-1+x)^4}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{2}{(-1+x)^4}-\frac{2}{(-1+x)^3}+\frac{1}{(-1+x)^2}\right)dx, x, \log(x)\right)\\ &= -\frac{2}{3(1-\log(x))^3}+\frac{1}{1-\log(x)}+\frac{1}{(-1+\log(x))^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.81

$$\frac{-4+9\log(x)-3\log^2(x)}{3(-1+\log(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*Log[x] + Log[x]^2)/(x*(-1 + Log[x])^4),x]
[Out] (-4 + 9*Log[x] - 3*Log[x]^2)/(3*(-1 + Log[x])^3)
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.89

method	result	size
norman	$\frac{-\ln(x)^2+3\ln(x)-\frac{4}{3}}{(-1+\ln(x))^3}$	20
risch	$-\frac{3\ln(x)^2-9\ln(x)+4}{3(-1+\ln(x))^3}$	21
derivativedivides	$\frac{1}{(-1+\ln(x))^2}+\frac{2}{3(-1+\ln(x))^3}-\frac{1}{-1+\ln(x)}$	24
default	$\frac{1}{(-1+\ln(x))^2}+\frac{2}{3(-1+\ln(x))^3}-\frac{1}{-1+\ln(x)}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-4*ln(x)+ln(x)^2)/x/(-1+ln(x))^4,x,method=_RETURNVERBOSE)
[Out] 1/(-1+ln(x))^2+2/3/(-1+ln(x))^3-1/(-1+ln(x))
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.19

$$-\frac{3\log(x)^2-9\log(x)+4}{3\left(\log(x)^3-3\log(x)^2+3\log(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="maxima")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 1.19

$$-\frac{3\log(x)^2-9\log(x)+4}{3\left(\log(x)^3-3\log(x)^2+3\log(x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="fricas")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x)^3 - 3*log(x)^2 + 3*log(x) - 1)
```

Sympy [A]
time = 0.04, size = 32, normalized size = 1.19

$$\frac{-3\log\left(x\right)^{2}+9\log\left(x\right)-4}{3\log\left(x\right)^{3}-9\log\left(x\right)^{2}+9\log\left(x\right)-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*ln(x)+ln(x)**2)/x/(-1+ln(x))**4,x)
[Out] (-3*log(x)**2 + 9*log(x) - 4)/(3*log(x)**3 - 9*log(x)**2 + 9*log(x) - 3)
```

Giac [A]
time = 13.99, size = 20, normalized size = 0.74

$$-\frac{3\,\log\left(x\right)^{2}-9\,\log\left(x\right)+4}{3\,(\log\left(x\right)-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-4*log(x)+log(x)^2)/x/(-1+log(x))^4,x, algorithm="giac")
[Out] -1/3*(3*log(x)^2 - 9*log(x) + 4)/(log(x) - 1)^3
```

Mupad [B]
time = 0.42, size = 18, normalized size = 0.67

$$-\frac{\ln\left(x\right)^{2}-3\,\ln\left(x\right)+\frac{4}{3}}{(\ln\left(x\right)-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^2 - 4*log(x) + 1)/(x*(log(x) - 1)^4),x)
[Out] -(log(x)^2 - 3*log(x) + 4/3)/(log(x) - 1)^3
```

3.149 $\int \frac{\log^2(ax^n)^p}{x} \, dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

[Out] ln(a*x^n)*(ln(a*x^n)^2)^p/n/(1+2*p)

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(Log[a*x^n]^2)^p/x,x]

[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(ax^n)^p}{x} \, dx &= \frac{\text{Subst}\left(\int (x^2)^p \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-2p}(ax^n)\log^2(ax^n)^p\right)\text{Subst}\left(\int x^{2p} \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(ax^n)\log^2(ax^n)^p}{n(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^2)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^2)^p)/(n*(1 + 2*p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(\ln \left(a\,x^{n}\right)^{2}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^p/x,x)
```

```
[Out] int((ln(a*x^n)^2)^p/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.38, size = 38, normalized size = 1.41

$$\frac{\left(n\log\left(x\right)+\log\left(a\right)\right)\left(n^{2}\log\left(x\right)^{2}+2\,n\log\left(a\right)\log\left(x\right)+\log\left(a\right)^{2}\right)^{p}}{2\,np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)^p/(2*n*p
+ n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\log\left(ax^{n}\right)^{2}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**p/x,x)
```

```
[Out] Integral((log(a*x**n)**2)**p/x, x)
```

Giac [A]
time = 13.25, size = 48, normalized size = 1.78

$$\frac{(n\log(x)\operatorname{sgn}(\log(ax^n)) + \log(a)\operatorname{sgn}(\log(ax^n)))^{2p+1}}{n(2p+1)\operatorname{sgn}(\log(ax^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x)*sgn(log(a*x^n)) + log(a)*sgn(log(a*x^n)))^(2*p + 1)/(n*(2*p + 1)*
sgn(log(a*x^n)))
```

Mupad [B]
time = 0.40, size = 27, normalized size = 1.00

$$\frac{\ln(a\,x^n)\,\left(\ln(a\,x^n)^2\right)^p}{n\,(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^2)^p/x,x)
```

```
[Out] (log(a*x^n)*(log(a*x^n)^2)^p)/(n*(2*p + 1))
```

3.150 $\int \frac{\log^m(ax^n)^p}{x} dx$

Optimal. Leaf size=27

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

```
[Out] ln(a*x^n)*(ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {15, 30}

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^m(ax^n)^p}{x} dx &= \frac{\text{Subst}\left(\int (x^m)^p dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(\log^{-mp}(ax^n)\log^m(ax^n)^p\right)\text{Subst}\left(\int x^{mp} dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(ax^n)\log^m(ax^n)^p}{n(1+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[a*x^n]^m)^p/x,x]
[Out] (Log[a*x^n]*(Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 71, normalized size = 2.63

method	result	size
risch	$\frac{\left(\ln(a)+\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ia\,x^n)(-\mathrm{csgn}(ia\,x^n)+\mathrm{csgn}(ia))(-\mathrm{csgn}(ia\,x^n)+\mathrm{csgn}(ix^n))}{2}\right)^{mp+1}}{n(mp+1)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^m)^p/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(ln(a)+ln(x^n)-1/2*I*Pi*csgn(I*a*x^n)*(-csgn(I*a*x^n)+csgn(I*a))*(-csgn
(I*a*x^n)+csgn(I*x^n)))^(m*p+1)/(m*p+1)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.40, size = 27, normalized size = 1.00

$$\frac{(n\log{(x)}+\log{(a)})(n\log{(x)}+\log{(a)})^{mp}}{mnp+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="fricas")
[Out] (n*log(x) + log(a))*(n*log(x) + log(a))^(m*p)/(m*n*p + n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(\log{(ax^n)}^m)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**m)**p/x,x)
[Out] Integral((log(a*x**n)**m)**p/x, x)
```

Giac [A]
time = 4.25, size = 24, normalized size = 0.89

$$\frac{(n\log\left(x\right)+\log\left(a\right))^{mp+1}}{(mp+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^m)^p/x,x, algorithm="giac")
[Out] (n*log(x) + log(a))^(m*p + 1)/((m*p + 1)*n)
```

Mupad [B]
time = 0.38, size = 27, normalized size = 1.00

$$\frac{\ln\left(a\,x^n\right)\,{\left({\ln\left(a\,x^n\right)}^m\right)}^p}{n\,\left(m\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^m)^p/x,x)
[Out] (log(a*x^n)*(log(a*x^n)^m)^p)/(n*(m*p + 1))
```

3.151 $\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx$

Optimal. Leaf size=25

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

[Out] 1/2*ln(a*x^n)*(ln(a*x^n)^2)^(1/2)/n

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 30}

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a*x^n]^2]/x,x]

[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\log^2(ax^n)}}{x} \, dx &= \frac{\text{Subst}\left(\int \sqrt{x^2} \, dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\sqrt{\log^2(ax^n)}\,\text{Subst}\left(\int x \, dx, x, \log(ax^n)\right)}{n \log(ax^n)} \\
&= \frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{\log(ax^n)\sqrt{\log^2(ax^n)}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Log[a*x^n]^2]/x,x]
```

```
[Out] (Log[a*x^n]*Sqrt[Log[a*x^n]^2])/(2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 21, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\mathrm{csgn}(\ln(a\,x^n))\ln(a\,x^n)^2}{2n}$	21
default	$\frac{\mathrm{csgn}(\ln(a\,x^n))\ln(a\,x^n)^2}{2n}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(a*x^n)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*csgn(ln(a*x^n))*ln(a*x^n)^2
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.80

$$-\frac{1}{2}\,n\log(x)^2+\log(a)\log(x)+\log(x)\log(x^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + log(a)*log(x) + log(x)*log(x^n)
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.52

$$\frac{1}{2}\,n\log(x)^2+\log(a)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*n*log(x)^2 + log(a)*log(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\log (ax^n)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(a*x**n)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(log(a*x**n)**2)/x, x)
```

Giac [**A**]
time = 6.78, size = 27, normalized size = 1.08

$$\frac{1}{2}\,n\log (x)^2\,\mathrm{sgn}(\log (ax^n)) + \log (a)\log (x)\,\mathrm{sgn}(\log (ax^n))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(a*x^n)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*n*log(x)^2*sgn(log(a*x^n)) + log(a)*log(x)*sgn(log(a*x^n))
```

Mupad [**B**]
time = 0.40, size = 21, normalized size = 0.84

$$\frac{\ln (a\,x^n)\,\sqrt{\ln (a\,x^n)^2}}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(a*x^n)^2)^(1/2)/x,x)
```

```
[Out] (log(a*x^n)*(log(a*x^n)^2)^(1/2))/(2*n)
```

3.152 $\int \frac{(b\log^m(ax^n))^p}{x}\,dx$

Optimal. Leaf size=29

$$\frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}$$

```
[Out] ln(a*x^n)*(b*ln(a*x^n)^m)^p/n/(m*p+1)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 30}

$$\frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\log^m(ax^n))^p}{x}\,dx &= \frac{\text{Subst}\left(\int (bx^m)^p\,dx, x, \log(ax^n)\right)}{n}\\
&= \frac{\left(\log^{-mp}(ax^n)\,(b\log^m(ax^n))^p\right)\text{Subst}\left(\int x^{mp}\,dx, x, \log(ax^n)\right)}{n}\\
&= \frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\frac{\log(ax^n)\,(b\log^m(ax^n))^p}{n(1+mp)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Log[a*x^n]^m)^p/x,x]
```

```
[Out] (Log[a*x^n]*(b*Log[a*x^n]^m)^p)/(n*(1 + m*p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(b \ln \left(a \,x^{n}\right)^{m}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*ln(a*x^n)^m)^p/x,x)
```

```
[Out] int((b*ln(a*x^n)^m)^p/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Fricas [A]
time = 0.37, size = 33, normalized size = 1.14

$$\frac{(n \log(x) + \log(a))e^{(mp \log(n \log(x) + \log(a)) + p \log(b))}}{mnp + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/(m*n*p + n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \log{\left(a x^{n} \right)}^{m}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*ln(a*x**n)**m)**p/x,x)
```

```
[Out] Integral((b*log(a*x**n)**m)**p/x, x)
```

Giac [A]
time = 4.72, size = 35, normalized size = 1.21

$$\frac{(n\log(x)+\log(a))e^{(mp\log(n\log(x)+\log(a))+p\log(b))}}{(mp+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*log(a*x^n)^m)^p/x,x, algorithm="giac")
```

```
[Out] (n*log(x) + log(a))*e^(m*p*log(n*log(x) + log(a)) + p*log(b))/((m*p + 1)*n)
```

Mupad [B]
time = 0.36, size = 29, normalized size = 1.00

$$\frac{\ln(a\,x^n)\,\left(b\ln(a\,x^n)^m\right)^p}{n\,\left(m\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*log(a*x^n)^m)^p/x,x)
```

```
[Out] (log(a*x^n)*(b*log(a*x^n)^m)^p)/(n*(m*p + 1))
```

3.153 $\int \frac{1}{x\log(e^x)}\,dx$

Optimal. Leaf size=31

$$-\frac{\log(x)}{x-\log\left(e^x\right)}+\frac{\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}$$

```
[Out] -ln(x)/(x-ln(exp(x)))+ln(ln(exp(x)))/(x-ln(exp(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2191, 2188, 29}

$$\frac{\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}-\frac{\log(x)}{x-\log\left(e^x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[E^x]),x]
```

```
[Out] -(Log[x]/(x - Log[E^x])) + Log[Log[E^x]]/(x - Log[E^x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log\left(e^x\right)}\,dx&=-\frac{\int \frac{1}{x}\,dx}{x-\log\left(e^x\right)}+\frac{\int \frac{1}{\log(e^x)}\,dx}{x-\log\left(e^x\right)}\\&=-\frac{\log(x)}{x-\log\left(e^x\right)}+\frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}\\&=-\frac{\log(x)}{x-\log\left(e^x\right)}+\frac{\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.68

$$\frac{-\log(x)+\log\left(\log\left(e^x\right)\right)}{x-\log\left(e^x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[E^x]),x]
[Out] (-Log[x] + Log[Log[E^x]])/(x - Log[E^x])
```

Maple [A]
time = 0.01, size = 29, normalized size = 0.94

method	result	size
default	$-\frac{\ln(\ln(e^x))}{\ln(e^x)-x}+\frac{\ln(x)}{\ln(e^x)-x}$	29
risch	$-\frac{\ln(\ln(e^x))}{\ln(e^x)-x}+\frac{\ln(x)}{\ln(e^x)-x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(exp(x)),x,method=_RETURNVERBOSE)
[Out] -1/(ln(exp(x))-x)*ln(ln(exp(x)))+1/(ln(exp(x))-x)*ln(x)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="maxima")
[Out] -1/x
```

Fricas [A]
time = 0.35, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="fricas")
[Out] -1/x
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.10

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(exp(x)),x)
```

```
[Out] -1/x
```

Giac [A]
time = 4.71, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(exp(x)),x, algorithm="giac")
```

```
[Out] -1/x
```

Mupad [B]
time = 0.36, size = 5, normalized size = 0.16

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*log(exp(x))),x)
```

```
[Out] -1/x
```

3.154 $\int \log(x)\sin(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\cos(a)\text{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}$$

```
[Out] Ci(b*x)*cos(a)/b-cos(b*x+a)*ln(x)/b-Si(b*x)*sin(a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2718, 2634, 12, 3384, 3380, 3383}

$$\frac{\cos(a)\text{CosIntegral}(bx)}{b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x],x]
```

```
[Out] (Cos[a]*CosIntegral[b*x])/b - (Cos[a + b*x]*Log[x])/b - (Sin[a]*SinIntegral
[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(x)\sin(a+bx)\,dx &= -\frac{\cos(a+bx)\log(x)}{b}+\int \frac{\cos(a+bx)}{bx}\,dx\\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\int \frac{\cos(a+bx)}{x}\,dx}{b}\\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos(a)\int \frac{\cos(bx)}{x}\,dx}{b}-\frac{\sin(a)\int \frac{\sin(bx)}{x}\,dx}{b}\\
&= \frac{\cos(a)\mathrm{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}-\frac{\sin(a)\mathrm{Si}(bx)}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 30, normalized size = 0.86

$$\frac{\cos(a)\mathrm{Ci}(bx)-\cos(a+bx)\log(x)-\sin(a)\mathrm{Si}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x],x]
[Out] (Cos[a]*CosIntegral[b*x] - Cos[a + b*x]*Log[x] - Sin[a]*SinIntegral[b*x])/b
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 80, normalized size = 2.29

method	result	size
risch	$-\frac{\cos(bx+a)\ln(x)}{b}+\frac{i\mathrm{e}^{-ia}\pi\,\mathrm{csgn}(bx)}{2b}-\frac{i\mathrm{e}^{-ia}\,\mathrm{sinIntegral}(bx)}{b}-\frac{\mathrm{e}^{-ia}\,\mathrm{expIntegral}(1,-ibx)}{2b}-\frac{\mathrm{e}^{ia}\,\mathrm{expIntegral}(1,-ibx)}{2b}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] -cos(b*x+a)*ln(x)/b+1/2*I/b*exp(-I*a)*Pi*csgn(b*x)-I/b*exp(-I*a)*Si(b*x)-1/
2/b*exp(-I*a)*Ei(1,-I*b*x)-1/2/b*exp(I*a)*Ei(1,-I*b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 57, normalized size = 1.63

$$-\frac{\cos(bx+a)\log(x)}{b}-\frac{(E_1(i\,bx)+E_1(-i\,bx))\cos(a)-(i\,E_1(i\,bx)-i\,E_1(-i\,bx))\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*((exp_integral_e(1, I*b*x) + exp_integral_e(1,
 -I*b*x))*cos(a) - (I*exp_integral_e(1, I*b*x) - I*exp_integral_e(1, -I*b*x
))*sin(a))/b
```

Fricas [A]
time = 0.39, size = 37, normalized size = 1.06

$$\frac{(\mathrm{Ci}(bx)+\mathrm{Ci}(-bx))\cos(a)-2\,\cos(bx+a)\log(x)-2\,\sin(a)\,\mathrm{Si}(bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((cos_integral(b*x) + cos_integral(-b*x))*cos(a) - 2*cos(b*x + a)*log(x
) - 2*sin(a)*sin_integral(b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a),x)
```

```
[Out] Integral(log(x)*sin(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.30, size = 102, normalized size = 2.91

$$-\frac{\cos(bx+a)\log(x)}{b}-\frac{\Re(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}\,a\right)^2+\Re(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}\,a\right)^2+2\,\Im(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}\,a\right)-2\,\Im(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}\,a\right)+4\,\mathrm{Si}(bx)\tan\left(\frac{1}{2}\,a\right)-\Re(\mathrm{Ci}(bx))-\Re(\mathrm{Ci}(-bx))}{2\left(b\tan\left(\frac{1}{2}\,a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)*log(x)/b - 1/2*(real_part(cos_integral(b*x))*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(b*x))*
```

```
tan(1/2*a) - 2*imag_part(cos_integral(-b*x))*tan(1/2*a) + 4*sin_integral(b*
x)*tan(1/2*a) - real_part(cos_integral(b*x)) - real_part(cos_integral(-b*x)
))/(b*tan(1/2*a)^2 + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sin (a+b\,x)\,\ln (x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*log(x),x)
```

```
[Out] int(sin(a + b*x)*log(x), x)
```

3.155 $\int \log(x)\sin^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\mathrm{Ci}(2bx)\sin(2a)}{4b}-\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)+1/4*cos(2*a)*Si(2*b*x)/b+1/4*Ci(2*b*x)*sin(2*a)/b-1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A]
time = 0.08, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2715, 8, 2634, 3384, 3380, 3383}

$$\frac{\sin(2a)\mathrm{CosIntegral}(2bx)}{4b}+\frac{\cos(2a)\mathrm{Si}(2bx)}{4b}-\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*x + (x*Log[x])/2 + (CosIntegral[2*b*x]*Sin[2*a])/(4*b) - (Cos[a + b*x]
*Log[x]*Sin[a + b*x])/(2*b) + (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sin^2(a+bx)\,dx &= \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} - \int \left(\frac{1}{2} - \frac{\sin(2a+2bx)}{4bx}\right)dx\\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\int \frac{\sin(2a+2bx)}{x}\,dx}{4b}\\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\int \frac{\sin(2bx)}{x}\,dx}{4b} + \sin(2\\ &= -\frac{x}{2} + \frac{1}{2}x\log(x) + \frac{\text{Ci}(2bx)\sin(2a)}{4b} - \frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b} + \frac{\cos(2a)\text{S}}{4b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.76

$$\frac{-2bx + 2bx\log(x) + \text{Ci}(2bx)\sin(2a) - \log(x)\sin(2(a+bx)) + \cos(2a)\text{Si}(2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^2,x]
```

```
[Out] (-2*b*x + 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a + b*
x)] + Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 132, normalized size = 2.00

method	result
risch	$\frac{x\ln(x)}{2} - \frac{\ln(x)\sin(2bx+2a)}{4b} - \frac{e^{-2ia}\pi\,\text{csgn}(bx)}{8b} + \frac{e^{-2ia}\,\text{sinIntegral}(2bx)}{4b} - \frac{i e^{-2ia}\,\text{expIntegral}(1,-2ibx)}{8b} + \frac{a\ln(ibx)}{2b} - \frac{x}{2} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*ln(x)-1/4*ln(x)/b*sin(2*b*x+2*a)-1/8/b*exp(-2*I*a)*Pi*csgn(b*x)+1/4/b
*exp(-2*I*a)*Si(2*b*x)-1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2*x-1/2*a/b-1/2/b*a*ln(a+I*(I*b*x+I*a))+1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.36, size = 79, normalized size = 1.20

$$\frac{(2\,bx+2\,a-\sin(2\,bx+2\,a))\log(x)}{4\,b}-\frac{4\,bx+(i\,\mathrm{Ei}(2i\,bx)-i\,\mathrm{Ei}(-2i\,bx))\cos(2\,a)+4\,a\log(x)-(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}(-2i\,bx))\sin(2\,a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (I*Ei(2*I*b*x)
 - I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) - (Ei(2*I*b*x) + Ei(-2*I*b*x))*sin
(2*a))/b
```

Fricas [A]
time = 0.43, size = 59, normalized size = 0.89

$$\frac{4\,bx\log(x)-4\,\cos(bx+a)\log(x)\sin(bx+a)-4\,bx+(\mathrm{Ci}(2\,bx)+\mathrm{Ci}(-2\,bx))\sin(2\,a)+2\,\cos(2\,a)\,\mathrm{Si}(2\,bx)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) - 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x + (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) + 2*cos(2*a)*sin_integral(2*b*x
))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log(x)\sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**2,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.58, size = 123, normalized size = 1.86

$$\frac{1}{4}\left(2\,x-\frac{\sin(2\,bx+2\,a)}{b}\right)\log(x)-\frac{4\,bx\tan(a)^2+\Im(\mathrm{Ci}(2\,bx))\tan(a)^2-\Im(\mathrm{Ci}(-2\,bx))\tan(a)^2+2\,\mathrm{Si}(2\,bx)\tan(a)^2+4\,bx-2\,\Re(\mathrm{Ci}(2\,bx))\tan(a)-2\,\Re(\mathrm{Ci}(-2\,bx))\tan(a)-\Im(\mathrm{Ci}(2\,bx))+\Im(\mathrm{Ci}(-2\,bx))-2\,\mathrm{Si}(2\,bx)}{8\,(b\tan(a)^2+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x - sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 + imag_part(cos
_integral(2*b*x))*tan(a)^2 - imag_part(cos_integral(-2*b*x))*tan(a)^2 + 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x - 2*real_part(cos_integral(2*b*x))*tan(
a) - 2*real_part(cos_integral(-2*b*x))*tan(a) - imag_part(cos_integral(2*b*
x)) + imag_part(cos_integral(-2*b*x)) - 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin{(a+b\,x)}^2\,\ln{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*log(x),x)
```

```
[Out] int(sin(a + b*x)^2*log(x), x)
```

3.156 $\int \log(x)\sin^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cos(a)\text{Ci}(bx)}{4b}-\frac{\cos(3a)\text{Ci}(3bx)}{12b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}$$

```
[Out] 3/4*Ci(b*x)*cos(a)/b-1/12*Ci(3*b*x)*cos(3*a)/b-cos(b*x+a)*ln(x)/b+1/3*cos(b
*x+a)^3*ln(x)/b-3/4*Si(b*x)*sin(a)/b+1/12*Si(3*b*x)*sin(3*a)/b
```

Rubi [A]
time = 0.36, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3380, 3383, 3393}

$$\frac{3\cos(a)\text{CosIntegral}(bx)}{4b}-\frac{\cos(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\sin(a)\text{Si}(bx)}{4b}+\frac{\sin(3a)\text{Si}(3bx)}{12b}+\frac{\log(x)\cos^3(a+bx)}{3b}-\frac{\log(x)\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x])/(4*b) - (Cos[3*a]*CosIntegral[3*b*x])/(12*b) -
(Cos[a + b*x]*Log[x])/b + (Cos[a + b*x]^3*Log[x])/(3*b) - (3*Sin[a]*SinInte
gral[b*x])/(4*b) + (Sin[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \log(x)\sin^3(a+bx)\,dx &= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx\right)}{3bx} \\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cos(a+bx)\left(-3+\cos^2(a+bx)\right)}{x}\,dx}{3b} \\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \left(-\frac{3\cos(a+bx)}{x}+\frac{\cos^3(a+bx)}{x}\right)dx}{3b} \\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cos^3(a+bx)}{x}\,dx}{3b}+\frac{\int \frac{\cos(a+bx)}{x}\,dx}{b} \\
&= -\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\int \left(\frac{3\cos(a+bx)}{4x}+\frac{\cos(3a+3bx)}{4x}\right)dx}{3b}+ \\
&= \frac{\cos(a)\text{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\int \frac{\text{co}}{}}{} \\
&= \frac{\cos(a)\text{Ci}(bx)}{b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-\frac{\sin(a)\text{Si}(bx)}{b}-\frac{\cos(}{} \\
&= \frac{3\cos(a)\text{Ci}(bx)}{4b}-\frac{\cos(3a)\text{Ci}(3bx)}{12b}-\frac{\cos(a+bx)\log(x)}{b}+\frac{\cos^3(a+bx)\log(x)}{3b}-
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.74

$$\frac{9\cos(a)\text{Ci}(bx)-\cos(3a)\text{Ci}(3bx)-9\cos(a+bx)\log(x)+\cos(3(a+bx))\log(x)-9\sin(a)\text{Si}(bx)+\sin(3a)\text{Si}(3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sin[a + b*x]^3,x]
```

```
[Out] (9*Cos[a]*CosIntegral[b*x] - Cos[3*a]*CosIntegral[3*b*x] - 9*Cos[a + b*x]*L
og[x] + Cos[3*(a + b*x)]*Log[x] - 9*Sin[a]*SinIntegral[b*x] + Sin[3*a]*SinI
ntegral[3*b*x])/(12*b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 162, normalized size = 1.82

method	result
risch	$-\frac{3\cos(bx+a)\ln(x)}{4b}+\frac{\ln(x)\cos(3bx+3a)}{12b}-\frac{i\mathrm{e}^{-3ia}\pi\,\text{csgn}(bx)}{24b}+\frac{i\mathrm{e}^{-3ia}\,\text{sinIntegral}(3bx)}{12b}+\frac{\mathrm{e}^{-3ia}\,\text{expIntegral}(1,-3ibx)}{24b}+\frac{3i\mathrm{e}^{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*cos(b*x+a)*ln(x)/b+1/12*ln(x)/b*cos(3*b*x+3*a)-1/24*I/b*exp(-3*I*a)*Pi
*csgn(b*x)+1/12*I/b*exp(-3*I*a)*Si(3*b*x)+1/24/b*exp(-3*I*a)*Ei(1,-3*I*b*x)
+3/8*I/b*exp(-I*a)*Pi*csgn(b*x)-3/4*I/b*exp(-I*a)*Si(b*x)-3/8/b*exp(-I*a)*E
i(1,-I*b*x)-3/8/b*exp(I*a)*Ei(1,-I*b*x)+1/24/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.34, size = 110, normalized size = 1.24

$$\frac{\left(\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\log\left(x\right)}{3\,b}+\frac{\left(E_{1}(3i\,bx)+E_{1}(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(E_{1}(i\,bx)+E_{1}(-i\,bx)\right)\cos\left(a\right)-\left(i\,E_{1}(3i\,bx)-i\,E_{1}(-3i\,bx)\right)\sin\left(3\,a\right)+9\left(i\,E_{1}(i\,bx)-i\,E_{1}(-i\,bx)\right)\sin\left(a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))*log(x)/b + 1/24*((exp_integral_e(1, 3
*I*b*x) + exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(exp_integral_e(1, I*b*
x) + exp_integral_e(1, -I*b*x))*cos(a) - (I*exp_integral_e(1, 3*I*b*x) - I*
exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(I*exp_integral_e(1, I*b*x) - I*e
xp_integral_e(1, -I*b*x))*sin(a))/b
```

Fricas [A]
time = 0.40, size = 76, normalized size = 0.85

$$-\frac{\left(\text{Ci}\left(3\,bx\right)+\text{Ci}\left(-3\,bx\right)\right)\cos\left(3\,a\right)-9\left(\text{Ci}\left(bx\right)+\text{Ci}\left(-bx\right)\right)\cos\left(a\right)-8\left(\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\log\left(x\right)-2\,\sin\left(3\,a\right)\text{Si}\left(3\,bx\right)+18\,\sin\left(a\right)\text{Si}\left(bx\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*((cos_integral(3*b*x) + cos_integral(-3*b*x))*cos(3*a) - 9*(cos_integ
ral(b*x) + cos_integral(-b*x))*cos(a) - 8*(cos(b*x + a)^3 - 3*cos(b*x + a))
*log(x) - 2*sin(3*a)*sin_integral(3*b*x) + 18*sin(a)*sin_integral(b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sin^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sin(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sin(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.84, size = 454, normalized size = 5.10

$$\frac{1}{3}\left(\frac{\cos(bx+a)^3}{b}-\frac{3\cos(bx+a)}{b}\right)\log(x)+\frac{\Re(\mathrm{Ci}(3bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)^2-9\Re(\mathrm{Ci}(bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)^2-9\Re(\mathrm{Ci}(-bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)^2+\Re(\mathrm{Ci}(-3bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)^2-18\Im(\mathrm{Ci}(bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)+18\Im(\mathrm{Ci}(-bx))\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)-36\,\mathrm{Si}(bx)\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)+2\Im(\mathrm{Ci}(3bx))\tan\left(\frac{3}{2}a\right)\tan\left(\frac{1}{2}a\right)^2-2\Im(\mathrm{Ci}(-3bx))\tan\left(\frac{3}{2}a\right)\tan\left(\frac{1}{2}a\right)^2+4\,\mathrm{Si}(3bx)\tan\left(\frac{3}{2}a\right)\tan\left(\frac{1}{2}a\right)^2+\Re(\mathrm{Ci}(3bx))\tan\left(\frac{3}{2}a\right)^2+9\Re(\mathrm{Ci}(bx))\tan\left(\frac{3}{2}a\right)^2+9\Re(\mathrm{Ci}(-bx))\tan\left(\frac{3}{2}a\right)^2+\Re(\mathrm{Ci}(-3bx))\tan\left(\frac{3}{2}a\right)^2-\Re(\mathrm{Ci}(3bx))\tan\left(\frac{1}{2}a\right)^2-9\Re(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right)^2-9\Re(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}a\right)^2-\Re(\mathrm{Ci}(-3bx))\tan\left(\frac{1}{2}a\right)^2+2\Im(\mathrm{Ci}(3bx))\tan\left(\frac{3}{2}a\right)-2\Im(\mathrm{Ci}(-3bx))\tan\left(\frac{3}{2}a\right)+4\,\mathrm{Si}(3bx)\tan\left(\frac{3}{2}a\right)-18\Im(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}a\right)+18\Im(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}a\right)-36\,\mathrm{Si}(bx)\tan\left(\frac{1}{2}a\right)-\Re(\mathrm{Ci}(3bx))+9\Re(\mathrm{Ci}(bx))+9\Re(\mathrm{Ci}(-bx))-\Re(\mathrm{Ci}(-3bx))}{24\left(b\tan\left(\frac{3}{2}a\right)^2\tan\left(\frac{1}{2}a\right)^2+b\tan\left(\frac{3}{2}a\right)^2+b\tan\left(\frac{1}{2}a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(cos(b*x + a)^3/b - 3*cos(b*x + a)/b)*log(x) + 1/24*(real_part(cos_inte
gral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan
(3/2*a)^2*tan(1/2*a)^2 - 9*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1
/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 - 18*im
ag_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) + 18*imag_part(cos_integ
ral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 36*sin_integral(b*x)*tan(3/2*a)^2*tan(
1/2*a) + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*imag_
part(cos_integral(-3*b*x))*tan(3/2*a)*tan(1/2*a)^2 + 4*sin_integral(3*b*x)*
tan(3/2*a)*tan(1/2*a)^2 + real_part(cos_integral(3*b*x))*tan(3/2*a)^2 + 9*r
eal_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*real_part(cos_integral(-b*x))*
tan(3/2*a)^2 + real_part(cos_integral(-3*b*x))*tan(3/2*a)^2 - real_part(cos
_integral(3*b*x))*tan(1/2*a)^2 - 9*real_part(cos_integral(b*x))*tan(1/2*a)^
2 - 9*real_part(cos_integral(-b*x))*tan(1/2*a)^2 - real_part(cos_integral(-
3*b*x))*tan(1/2*a)^2 + 2*imag_part(cos_integral(3*b*x))*tan(3/2*a) - 2*imag
_part(cos_integral(-3*b*x))*tan(3/2*a) + 4*sin_integral(3*b*x)*tan(3/2*a) -
 18*imag_part(cos_integral(b*x))*tan(1/2*a) + 18*imag_part(cos_integral(-b*
x))*tan(1/2*a) - 36*sin_integral(b*x)*tan(1/2*a) - real_part(cos_integral(3
*b*x)) + 9*real_part(cos_integral(b*x)) + 9*real_part(cos_integral(-b*x)) -
 real_part(cos_integral(-3*b*x)))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*
a)^2 + b*tan(1/2*a)^2 + b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (a+b\,x)^3\,\ln (x)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*log(x),x)
```

```
[Out] int(sin(a + b*x)^3*log(x), x)
```

3.157 $\int \cos(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\text{Ci}(bx)\sin(a)}{b}+\frac{\log(x)\sin(a+bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}$$

```
[Out] -cos(a)*Si(b*x)/b-Ci(b*x)*sin(a)/b+ln(x)*sin(b*x+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2717, 2634, 12, 3384, 3380, 3383}

$$-\frac{\sin(a)\text{CosIntegral}(bx)}{b}-\frac{\cos(a)\text{Si}(bx)}{b}+\frac{\log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a])/b) + (Log[x]*Sin[a + b*x])/b - (Cos[a]*SinInteg
ral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\log(x)\,dx &= \frac{\log(x)\sin(a+bx)}{b} - \int \frac{\sin(a+bx)}{bx}\,dx\\ &= \frac{\log(x)\sin(a+bx)}{b} - \frac{\int \frac{\sin(a+bx)}{x}\,dx}{b}\\ &= \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\int \frac{\sin(bx)}{x}\,dx}{b} - \frac{\sin(a)\int \frac{\cos(bx)}{x}\,dx}{b}\\ &= -\frac{\text{Ci}(bx)\sin(a)}{b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\cos(a)\text{Si}(bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.86

$$-\frac{\text{Ci}(bx)\sin(a) - \log(x)\sin(a+bx) + \cos(a)\text{Si}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[x],x]
```

```
[Out] -((CosIntegral[b*x]*Sin[a] - Log[x]*Sin[a + b*x] + Cos[a]*SinIntegral[b*x])
/b)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 79, normalized size = 2.26

method	result	size
risch	$\frac{\ln(x)\sin(bx+a)}{b} + \frac{e^{-ia}\pi\,\text{csgn}(bx)}{2b} - \frac{e^{-ia}\,\text{sinIntegral}(bx)}{b} + \frac{i e^{-ia}\,\text{expIntegral}(1,-ibx)}{2b} - \frac{i e^{ia}\,\text{expIntegral}(1,-ibx)}{2b}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*sin(b*x+a)/b+1/2/b*exp(-I*a)*Pi*csgn(b*x)-1/b*exp(-I*a)*Si(b*x)+1/2*I
/b*exp(-I*a)*Ei(1,-I*b*x)-1/2*I/b*exp(I*a)*Ei(1,-I*b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 55, normalized size = 1.57

$$\frac{\log(x)\sin(bx+a)}{b}+\frac{(i\,E_1(i\,bx)-i\,E_1(-i\,bx))\cos(a)+(E_1(i\,bx)+E_1(-i\,bx))\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*((I*exp_integral_e(1, I*b*x) - I*exp_integral_e
(1, -I*b*x))*cos(a) + (exp_integral_e(1, I*b*x) + exp_integral_e(1, -I*b*x)
)*sin(a))/b
```

Fricas [A]
time = 0.37, size = 38, normalized size = 1.09

$$\frac{2\,\log(x)\sin(bx+a)-(\mathrm{Ci}(bx)+\mathrm{Ci}(-bx))\sin(a)-2\,\cos(a)\,\mathrm{Si}(bx)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*log(x)*sin(b*x + a) - (cos_integral(b*x) + cos_integral(-b*x))*sin(a
) - 2*cos(a)*sin_integral(b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log(x)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x), x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.25, size = 108, normalized size = 3.09

$$\frac{\log(x)\sin(bx+a)}{b}+\frac{\Im(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}\,a\right)^2-\Im(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}\,a\right)^2+2\,\mathrm{Si}(bx)\tan\left(\frac{1}{2}\,a\right)^2-2\,\Re(\mathrm{Ci}(bx))\tan\left(\frac{1}{2}\,a\right)-2\,\Re(\mathrm{Ci}(-bx))\tan\left(\frac{1}{2}\,a\right)-\Im(\mathrm{Ci}(bx))+\Im(\mathrm{Ci}(-bx))-2\,\mathrm{Si}(bx)}{2\left(b\tan\left(\frac{1}{2}\,a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] log(x)*sin(b*x + a)/b + 1/2*(imag_part(cos_integral(b*x))*tan(1/2*a)^2 - im
ag_part(cos_integral(-b*x))*tan(1/2*a)^2 + 2*sin_integral(b*x)*tan(1/2*a)^2
 - 2*real_part(cos_integral(b*x))*tan(1/2*a) - 2*real_part(cos_integral(-b*
x))*tan(1/2*a) - imag_part(cos_integral(b*x)) + imag_part(cos_integral(-b*x
)) - 2*sin_integral(b*x))/(b*tan(1/2*a)^2 + b)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.03

$$\int \cos(a+b\,x)\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*log(x),x)
```

```
[Out] int(cos(a + b*x)*log(x), x)
```

3.158 $\int \cos^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Ci}(2bx)\sin(2a)}{4b}+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cos(2*a)*Si(2*b*x)/b-1/4*Ci(2*b*x)*sin(2*a)/b+1/2*co
s(b*x+a)*ln(x)*sin(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.727$, Rules used = {2715, 8, 2634, 12, 3408, 3384, 3380, 3383}

$$-\frac{\sin(2a)\text{CosIntegral}(2bx)}{4b}-\frac{\cos(2a)\text{Si}(2bx)}{4b}+\frac{\log(x)\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -1/2*x + (x*Log[x])/2 - (CosIntegral[2*b*x]*Sin[2*a])/(4*b) + (Cos[a + b*x]
*Log[x]*Sin[a + b*x])/(2*b) - (Cos[2*a]*SinIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3408

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[ExpandToSum[
u, x]^m*(a + b*Sin[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] && L
inearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\int \frac{1}{4}\left(2+\frac{\sin(2(a+bx))}{bx}\right)dx\\
&= \frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{1}{4}\int\left(2+\frac{\sin(2(a+bx))}{bx}\right)dx\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\int\frac{\sin(2(a+bx))}{x}\,dx}{4b}\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\int\frac{\sin(2a+2bx)}{x}\,dx}{4b}\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\int\frac{\sin(2bx)}{x}\,dx}{4b}-\frac{\sin(2a
\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Ci}(2bx)\sin(2a)}{4b}+\frac{\cos(a+bx)\log(x)\sin(a+bx)}{2b}-\frac{\cos(2a)\text{Si}}{4b}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.76

$$
-\frac{2bx-2bx\log(x)+\text{Ci}(2bx)\sin(2a)-\log(x)\sin(2(a+bx))+\cos(2a)\text{Si}(2bx)}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Log[x],x]
```

```
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CosIntegral[2*b*x]*Sin[2*a] - Log[x]*Sin[2*(a
+ b*x)] + Cos[2*a]*SinIntegral[2*b*x])/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 132, normalized size = 2.00

method	result
risch	$\frac{x\ln(x)}{2}+\frac{\ln(x)\sin(2bx+2a)}{4b}+\frac{\mathrm{e}^{-2ia}\pi\,\mathrm{csgn}(bx)}{8b}-\frac{\mathrm{e}^{-2ia}\,\mathrm{sinIntegral}(2bx)}{4b}+\frac{i\mathrm{e}^{-2ia}\,\mathrm{expIntegral}(1,-2ibx)}{8b}+\frac{a\ln(ibx)}{2b}-\frac{x}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*ln(x)+1/4*ln(x)/b*sin(2*b*x+2*a)+1/8/b*exp(-2*I*a)*Pi*csgn(b*x)-1/4/b
*exp(-2*I*a)*Si(2*b*x)+1/8*I/b*exp(-2*I*a)*Ei(1,-2*I*b*x)+1/2/b*a*ln(I*b*x)
-1/2*x-1/2*a/b-1/2/b*a*ln(a+I*(I*b*x+I*a))-1/8*I/b*exp(2*I*a)*Ei(1,-2*I*b*x
)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 76, normalized size = 1.15

$$\frac{(2\,bx+2\,a+\sin(2\,bx+2\,a))\log(x)}{4\,b}-\frac{4\,bx+(-i\,\mathrm{Ei}(2i\,bx)+i\,\mathrm{Ei}(-2i\,bx))\cos(2\,a)+4\,a\log(x)+(\mathrm{Ei}(2i\,bx)+\mathrm{Ei}(-2i\,bx))\sin(2\,a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))*log(x)/b - 1/8*(4*b*x + (-I*Ei(2*I*b*x
) + I*Ei(-2*I*b*x))*cos(2*a) + 4*a*log(x) + (Ei(2*I*b*x) + Ei(-2*I*b*x))*si
n(2*a))/b
```

Fricas [A]
time = 0.38, size = 60, normalized size = 0.91

$$\frac{4\,bx\log(x)+4\,\cos(bx+a)\log(x)\sin(bx+a)-4\,bx-(\mathrm{Ci}(2\,bx)+\mathrm{Ci}(-2\,bx))\sin(2\,a)-2\,\cos(2\,a)\,\mathrm{Si}(2\,bx)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*log(x) + 4*cos(b*x + a)*log(x)*sin(b*x + a) - 4*b*x - (cos_integ
ral(2*b*x) + cos_integral(-2*b*x))*sin(2*a) - 2*cos(2*a)*sin_integral(2*b*x
))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**2, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.19, size = 122, normalized size = 1.85

$$\frac{1}{4}\left(2\,x+\frac{\sin(2\,bx+2\,a)}{b}\right)\log(x)-\frac{4\,bx\tan(a)^2-\Im(\mathrm{Ci}(2\,bx))\tan(a)^2+\Im(\mathrm{Ci}(-2\,bx))\tan(a)^2-2\,\mathrm{Si}(2\,bx)\tan(a)^2+4\,bx+2\,\Re(\mathrm{Ci}(2\,bx))\tan(a)+2\,\Re(\mathrm{Ci}(-2\,bx))\tan(a)+\Im(\mathrm{Ci}(2\,bx))-\Im(\mathrm{Ci}(-2\,bx))+2\,\mathrm{Si}(2\,bx)}{8\,(b\tan(a)^2+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + sin(2*b*x + 2*a)/b)*log(x) - 1/8*(4*b*x*tan(a)^2 - imag_part(cos
_integral(2*b*x))*tan(a)^2 + imag_part(cos_integral(-2*b*x))*tan(a)^2 - 2*s
in_integral(2*b*x)*tan(a)^2 + 4*b*x + 2*real_part(cos_integral(2*b*x))*tan(
a) + 2*real_part(cos_integral(-2*b*x))*tan(a) + imag_part(cos_integral(2*b*
x)) - imag_part(cos_integral(-2*b*x)) + 2*sin_integral(2*b*x))/(b*tan(a)^2
+ b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(a+b\,x)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*log(x),x)
```

```
[Out] int(cos(a + b*x)^2*log(x), x)
```

3.159 $\int \cos^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\text{Ci}(bx)\sin(a)}{4b}-\frac{\text{Ci}(3bx)\sin(3a)}{12b}+\frac{\log(x)\sin(a+bx)}{b}-\frac{\log(x)\sin^3(a+bx)}{3b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}$$

```
[Out] -3/4*cos(a)*Si(b*x)/b-1/12*cos(3*a)*Si(3*b*x)/b-3/4*Ci(b*x)*sin(a)/b-1/12*C
i(3*b*x)*sin(3*a)/b+ln(x)*sin(b*x+a)/b-1/3*ln(x)*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.32, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3380, 3383, 4515}

$$-\frac{3\sin(a)\text{CosIntegral}(bx)}{4b}-\frac{\sin(3a)\text{CosIntegral}(3bx)}{12b}-\frac{3\cos(a)\text{Si}(bx)}{4b}-\frac{\cos(3a)\text{Si}(3bx)}{12b}-\frac{\log(x)\sin^3(a+bx)}{3b}+\frac{\log(x)\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CosIntegral[b*x]*Sin[a])/(4*b) - (CosIntegral[3*b*x]*Sin[3*a])/(12*b) +
 (Log[x]*Sin[a + b*x])/b - (Log[x]*Sin[a + b*x]^3)/(3*b) - (3*Cos[a]*SinInt
egral[b*x])/(4*b) - (Cos[3*a]*SinIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4515

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sin[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IGtQ[q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \cos^3(a+bx)\log(x)\,dx = \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{6bx}\,dx$$

$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{(5+\cos(2(a+bx)))\sin(a+bx)}{x}\,dx}{6b}$$

$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(\frac{5\sin(a+bx)}{x} + \frac{\cos(2a+2bx)\sin(a+bx)}{x}\right) dx}{6b}$$

$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \frac{\cos(2a+2bx)\sin(a+bx)}{x}\,dx}{6b} - \frac{5\int \frac{\sin(a+bx)}{x}}{6b}$$

$$= \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{\int \left(-\frac{\sin(a+bx)}{2x} + \frac{\sin(3a+3bx)}{2x}\right) dx}{6b} - \frac{(5\,c}{}$$

$$= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} +$$

$$= -\frac{5\mathrm{Ci}(bx)\sin(a)}{6b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} - \frac{5\cos(a)\mathrm{Si}(bx)}{6b} +$$

$$= -\frac{3\mathrm{Ci}(bx)\sin(a)}{4b} - \frac{\mathrm{Ci}(3bx)\sin(3a)}{12b} + \frac{\log(x)\sin(a+bx)}{b} - \frac{\log(x)\sin^3(a+bx)}{3b} -$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 0.75

$$-\frac{9\text{Ci}(bx)\sin(a)+\text{Ci}(3bx)\sin(3a)-9\log(x)\sin(a+bx)-\log(x)\sin(3(a+bx))+9\cos(a)\text{Si}(bx)+\cos(3a)\text{Si}(3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Log[x],x]
```

```
[Out] -1/12*(9*CosIntegral[b*x]*Sin[a] + CosIntegral[3*b*x]*Sin[3*a] - 9*Log[x]*S
in[a + b*x] - Log[x]*Sin[3*(a + b*x)] + 9*Cos[a]*SinIntegral[b*x] + Cos[3*a
]*SinIntegral[3*b*x])/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 162, normalized size = 1.84

method	result
risch	$\frac{3\ln(x)\sin(bx+a)}{4b}+\frac{\ln(x)\sin(3bx+3a)}{12b}+\frac{\mathrm{e}^{-3ia}\pi\,\text{csgn}(bx)}{24b}-\frac{\mathrm{e}^{-3ia}\,\text{sinIntegral}(3bx)}{12b}+\frac{i\mathrm{e}^{-3ia}\,\text{expIntegral}(1,-3ibx)}{24b}+\frac{3\,\mathrm{e}^{-ia}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*ln(x)*sin(b*x+a)/b+1/12*ln(x)/b*sin(3*b*x+3*a)+1/24/b*exp(-3*I*a)*Pi*cs
gn(b*x)-1/12/b*exp(-3*I*a)*Si(3*b*x)+1/24*I/b*exp(-3*I*a)*Ei(1,-3*I*b*x)+3/
8/b*exp(-I*a)*Pi*csgn(b*x)-3/4/b*exp(-I*a)*Si(b*x)+3/8*I/b*exp(-I*a)*Ei(1,-
I*b*x)-3/8*I/b*exp(I*a)*Ei(1,-I*b*x)-1/24*I/b*exp(3*I*a)*Ei(1,-3*I*b*x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.35, size = 109, normalized size = 1.24

$$-\frac{\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)\log\left(x\right)}{3\,b}+\frac{\left(i\,E_{1}(3i\,bx)-i\,E_{1}(-3i\,bx)\right)\cos\left(3\,a\right)-9\left(-i\,E_{1}(i\,bx)+i\,E_{1}(-i\,bx)\right)\cos\left(a\right)+\left(E_{1}(3i\,bx)+E_{1}(-3i\,bx)\right)\sin\left(3\,a\right)+9\left(E_{1}(i\,bx)+E_{1}(-i\,bx)\right)\sin\left(a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*((I*exp_integral_e(1
, 3*I*b*x) - I*exp_integral_e(1, -3*I*b*x))*cos(3*a) - 9*(-I*exp_integral_e
(1, I*b*x) + I*exp_integral_e(1, -I*b*x))*cos(a) + (exp_integral_e(1, 3*I*b
*x) + exp_integral_e(1, -3*I*b*x))*sin(3*a) + 9*(exp_integral_e(1, I*b*x) +
 exp_integral_e(1, -I*b*x))*sin(a))/b
```

Fricas [A]
time = 0.41, size = 76, normalized size = 0.86

$$\frac{8\left(\cos\left(bx+a\right)^{2}+2\right)\log\left(x\right)\sin\left(bx+a\right)-\left(\text{Ci}\left(3\,bx\right)+\text{Ci}\left(-3\,bx\right)\right)\sin\left(3\,a\right)-9\left(\text{Ci}\left(bx\right)+\text{Ci}\left(-bx\right)\right)\sin\left(a\right)-2\,\cos\left(3\,a\right)\text{Si}\left(3\,bx\right)-18\,\cos\left(a\right)\text{Si}\left(bx\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(cos(b*x + a)^2 + 2)*log(x)*sin(b*x + a) - (cos_integral(3*b*x) + c
os_integral(-3*b*x))*sin(3*a) - 9*(cos_integral(b*x) + cos_integral(-b*x))*
sin(a) - 2*cos(3*a)*sin_integral(3*b*x) - 18*cos(a)*sin_integral(b*x))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*ln(x),x)
```

```
[Out] Integral(log(x)*cos(a + b*x)**3, x)
```

Giac [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 7.27, size = 495, normalized size = 5.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))*log(x)/b + 1/24*(imag_part(cos_integ
ral(3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(
3/2*a)^2*tan(1/2*a)^2 - 9*imag_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/
2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2*tan(1/2*a)^2 + 2*sin_
integral(3*b*x)*tan(3/2*a)^2*tan(1/2*a)^2 + 18*sin_integral(b*x)*tan(3/2*a)
^2*tan(1/2*a)^2 - 18*real_part(cos_integral(b*x))*tan(3/2*a)^2*tan(1/2*a) -
 18*real_part(cos_integral(-b*x))*tan(3/2*a)^2*tan(1/2*a) - 2*real_part(cos
_integral(3*b*x))*tan(3/2*a)*tan(1/2*a)^2 - 2*real_part(cos_integral(-3*b*x
))*tan(3/2*a)*tan(1/2*a)^2 + imag_part(cos_integral(3*b*x))*tan(3/2*a)^2 -
9*imag_part(cos_integral(b*x))*tan(3/2*a)^2 + 9*imag_part(cos_integral(-b*x
))*tan(3/2*a)^2 - imag_part(cos_integral(-3*b*x))*tan(3/2*a)^2 + 2*sin_inte
gral(3*b*x)*tan(3/2*a)^2 - 18*sin_integral(b*x)*tan(3/2*a)^2 - imag_part(co
s_integral(3*b*x))*tan(1/2*a)^2 + 9*imag_part(cos_integral(b*x))*tan(1/2*a)
^2 - 9*imag_part(cos_integral(-b*x))*tan(1/2*a)^2 + imag_part(cos_integral(
-3*b*x))*tan(1/2*a)^2 - 2*sin_integral(3*b*x)*tan(1/2*a)^2 + 18*sin_integra
l(b*x)*tan(1/2*a)^2 - 2*real_part(cos_integral(3*b*x))*tan(3/2*a) - 2*real_
part(cos_integral(-3*b*x))*tan(3/2*a) - 18*real_part(cos_integral(b*x))*tan
(1/2*a) - 18*real_part(cos_integral(-b*x))*tan(1/2*a) - imag_part(cos_integ
ral(3*b*x)) - 9*imag_part(cos_integral(b*x)) + 9*imag_part(cos_integral(-b*
x)) + imag_part(cos_integral(-3*b*x)) - 2*sin_integral(3*b*x) - 18*sin_inte
```

```
gral(b*x))/(b*tan(3/2*a)^2*tan(1/2*a)^2 + b*tan(3/2*a)^2 + b*tan(1/2*a)^2 +
 b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*log(x),x)
[Out] int(cos(a + b*x)^3*log(x), x)
```

3.160 $\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx$

Optimal. Leaf size=5

$$\log(x)\sin(x)$$

[Out] ln(x)*sin(x)

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2717, 2634, 3380}

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Log[x] + Sin[x]/x,x]

[Out] Log[x]*Sin[x]

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\cos(x)\log(x)+\frac{\sin(x)}{x}\right)\,dx &= \int \cos(x)\log(x)\,dx+\int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x)+\text{Si}(x)-\int \frac{\sin(x)}{x}\,dx\\ &= \log(x)\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[x] + Sin[x]/x,x]
```

```
[Out] Log[x]*Sin[x]
```

Maple [A]
time = 0.11, size = 6, normalized size = 1.20

method	result	size
risch	$\ln(x)\sin(x)$	6
norman	$\frac{2\ln(x)\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(x)+sin(x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*sin(x)
```

Maxima [A]
time = 0.32, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="maxima")
```

```
[Out] log(x)*sin(x)
```

Fricas [A]
time = 0.37, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="fricas")
```

```
[Out] log(x)*sin(x)
```

Sympy [A]
time = 12.05, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(x)+sin(x)/x,x)
```

```
[Out] log(x)*sin(x)
```

Giac [A]
time = 6.44, size = 5, normalized size = 1.00

$$\log(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(x)+sin(x)/x,x, algorithm="giac")
```

```
[Out] log(x)*sin(x)
```

Mupad [B]
time = 0.50, size = 5, normalized size = 1.00

$$\ln(x)\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*log(x) + sin(x)/x,x)
```

```
[Out] log(x)*sin(x)
```

3.161 $\int \log(a\sin(x))\,dx$

Optimal. Leaf size=47

$$\frac{ix^2}{2} - x\log\left(1 - e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-x*ln(1-exp(2*I*x))+x*ln(a*sin(x))+1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 3798, 2221, 2317, 2438}

$$\frac{1}{2}i\text{PolyLog}\left(2, e^{2ix}\right) + x\log(a\sin(x)) + \frac{ix^2}{2} - x\log\left(1 - e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]] + (I/2)*PolyLog[2, E^(
(2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\sin(x))\,dx &= x\log(a\sin(x)) - \int x\cot(x)\,dx\\ &= \frac{ix^2}{2} + x\log(a\sin(x)) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \int \log\left(1-e^{2ix}\right)\,dx\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)\\ &= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.89

$$-x\log\left(1-e^{2ix}\right) + x\log(a\sin(x)) + \frac{1}{2}i\left(x^2 + \text{Li}_2\left(e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]],x]
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[a*Sin[x]] + (I/2)*(x^2 + PolyLog[2, E^((2
*I)*x)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.
time = 0.17, size = 87, normalized size = 1.85

method	result
default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(ia\left(1-\mathrm{e}^{2ix}\right)\mathrm{e}^{-ix}\right)+\frac{\ln\left(\mathrm{e}^{ix}\right)^2}{2}-\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+\mathrm{dilog}\left(\mathrm{e}^{ix}\right)-\ln\right.$
risch	$-x\ln\left(\mathrm{e}^{ix}\right)+\frac{ix\pi\,\mathrm{csgn}(a\sin(x))\mathrm{csgn}(ia\sin(x))}{2}-\frac{ix\pi}{2}+\frac{ix\pi\mathrm{csgn}(\sin(x))^3}{2}+\frac{ix\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)\mathrm{csgn}(\sin(x))^2}{2}+i\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(I*a*(-exp(I*x)^2+1)/exp(I*x))+1/2*ln(exp(I*x))^2-ln(exp
(I*x))*ln(exp(I*x)+1)-dilog(exp(I*x)+1)+dilog(exp(I*x))-ln(2)*ln(exp(I*x)))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(32) = 64$.
time = 0.61, size = 87, normalized size = 1.85

$$\frac{1}{2}i\,x^2 - i\,x\arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) + i\,x\arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 - 2\,\cos\left(x\right) + 1\right) + x\log\left(a\sin\left(x\right)\right) + i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(x), cos(x) + 1) + I*x*arctan2(sin(x), -cos(x) +
 1) - 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/2*x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) + x*log(a*sin(x)) + I*dilog(-e^(I*x)) + I*dilog(e^
(I*x))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(32) = 64$.
time = 0.42, size = 104, normalized size = 2.21

$$x\log\left(a\sin\left(x\right)\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-\cos\left(x\right) + i\,\sin\left(x\right) + 1\right) - \frac{1}{2}\,x\log\left(-\cos\left(x\right) - i\,\sin\left(x\right) + 1\right) + \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) + i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right) - i\,\sin\left(x\right)) - \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) + i\,\sin\left(x\right)) + \frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right) - i\,\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x) - I*s
in(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I*sin(
x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(x)) -
 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)),x)
```

```
[Out] Integral(log(a*sin(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(a \sin \left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)),x)
```

```
[Out] int(log(a*sin(x)), x)
```

3.162 $\int \log\left(a\sin^2(x)\right)\,dx$

Optimal. Leaf size=45

$$ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^2)+I*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+ix^2-2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^2] + I*PolyLog[2, E^((2*I
)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\sin^2(x)\right)\,dx &= x\log\left(a\sin^2(x)\right)-\int 2x\cot(x)\,dx\\ &= x\log\left(a\sin^2(x)\right)-2\int x\cot(x)\,dx\\ &= ix^2+x\log\left(a\sin^2(x)\right)+4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+2\int \log\left(1-e^{2ix}\right)\,dx\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)-i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)\\ &= ix^2-2x\log\left(1-e^{2ix}\right)+x\log\left(a\sin^2(x)\right)+i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.96

$$x\left(ix-2\log\left(1-e^{2ix}\right)+\log\left(a\sin^2(x)\right)\right)+i\text{Li}_2\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 - E^((2*I)*x)] + Log[a*Sin[x]^2]) + I*PolyLog[2, E^((2*I)*
x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.
time = 0.13, size = 86, normalized size = 1.91

method	result

default	$-i\left(\ln\left(e^{ix}\right)\ln\left(-a\left(e^{2ix}-1\right)^{2}e^{-2ix}\right)+\ln\left(e^{ix}\right)^{2}-2\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-2\,\mathrm{dilog}\left(e^{ix}+1\right)+2\,\mathrm{dilo}$
risch	$-2i\ln\left(e^{ix}\right)\ln\left(e^{2ix}-1\right)+\frac{ix\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(ia\left(e^{2ix}-1\right)^{2}e^{-2ix}\right)^{2}}{2}+\frac{ix\pi\,\mathrm{csgn}\left(ie^{-2ix}\right)\mathrm{csgn}\left(ie^{-2ix}\left(e^{2ix}-1\right)^{2}\right)^{2}}{2}-\frac{ix\pi\,\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(-a*(exp(I*x)^2-1)^2/exp(I*x)^2)+ln(exp(I*x))^2-2*ln(exp
(I*x))*ln(exp(I*x)+1)-2*dilog(exp(I*x)+1)+2*dilog(exp(I*x))-2*ln(2)*ln(exp(
I*x)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.
time = 0.58, size = 89, normalized size = 1.98

$$i\,x^2 - 2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) + 2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right) + x\log\left(a\sin\left(x\right)^2\right) - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right) - x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right) + 2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + 2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) +
 1) + x*log(a*sin(x)^2) - x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I
*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(34) = 68$.
time = 0.42, size = 109, normalized size = 2.42

$$x\log\left(-a\cos\left(x\right)^2+a\right) - x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right) - x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right) - x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right) - x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right) + i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right)) - i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right)) - i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right)) + i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a*cos(x)^2 + a) - x*log(cos(x) + I*sin(x) + 1) - x*log(cos(x) - I*si
n(x) + 1) - x*log(-cos(x) + I*sin(x) + 1) - x*log(-cos(x) - I*sin(x) + 1) +
 I*dilog(cos(x) + I*sin(x)) - I*dilog(cos(x) - I*sin(x)) - I*dilog(-cos(x)
+ I*sin(x)) + I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sin^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**2),x)
```

```
[Out] Integral(log(a*sin(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sin\left(x\right)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)^2),x)
```

```
[Out] int(log(a*sin(x)^2), x)
```

3.163 $\int \log(a\sin^n(x))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}inx^2 - nx\log\left(1 - e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1-exp(2*I*x))+x*ln(a*sin(x)^n)+1/2*I*n*polylog(2,exp(2*I
*x))
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$\frac{1}{2}in\text{PolyLog}\left(2, e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + \frac{1}{2}inx^2 - nx\log\left(1 - e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\sin^n(x)\right)\,dx &= x\log\left(a\sin^n(x)\right) - \int nx\cot(x)\,dx \\
&= x\log\left(a\sin^n(x)\right) - n\int x\cot(x)\,dx \\
&= \frac{1}{2}inx^2 + x\log\left(a\sin^n(x)\right) + (2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + n\int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) - \frac{1}{2}(in)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 52, normalized size = 1.00

$$\frac{1}{2}inx^2 - nx\log\left(1-e^{2ix}\right) + x\log\left(a\sin^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sin[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Sin[x]^n] + (I/2)*n*PolyLo
g[2, E^((2*I)*x)]
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \ln\left(a(\sin^n(x))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sin(x)^n),x)
```

```
[Out] int(ln(a*sin(x)^n),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(37) = 74$.
time = 0.65, size = 91, normalized size = 1.75

$$-\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)-2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)-2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\right)n+x\log\left(a\sin\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*l
og(a*sin(x)^n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(37) = 74$.
time = 0.39, size = 115, normalized size = 2.21

$$-\frac{1}{2}nx\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+nx\log\left(\sin\left(x\right)\right)+\frac{1}{2}i\,n\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,n\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))-\frac{1}{2}i\,n\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{2}i\,n\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))+x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(cos(x) + I*sin(x) + 1) - 1/2*n*x*log(cos(x) - I*sin(x) + 1) -
1/2*n*x*log(-cos(x) + I*sin(x) + 1) - 1/2*n*x*log(-cos(x) - I*sin(x) + 1) +
 n*x*log(sin(x)) + 1/2*I*n*dilog(cos(x) + I*sin(x)) - 1/2*I*n*dilog(cos(x)
- I*sin(x)) - 1/2*I*n*dilog(-cos(x) + I*sin(x)) + 1/2*I*n*dilog(-cos(x) - I
*sin(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sin^n\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sin(x)**n),x)
```

```
[Out] Integral(log(a*sin(x)**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sin(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sin(x)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sin\left(x\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sin(x)^n),x)
```

```
[Out] int(log(a*sin(x)^n), x)
```

3.164 $\int \log(a\cos(x))\,dx$

Optimal. Leaf size=47

$$\frac{ix^2}{2} - x\log\left(1+e^{2ix}\right) + x\log(a\cos(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-x*ln(1+exp(2*I*x))+x*ln(a*cos(x))+1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 3800, 2221, 2317, 2438}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right) + x\log(a\cos(x)) + \frac{ix^2}{2} - x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log(a\cos(x))\,dx &= x\log(a\cos(x))+\int x\tan(x)\,dx\\ &= \frac{ix^2}{2}+x\log(a\cos(x))-2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\ &= \frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\int \log\left(1+e^{2ix}\right)\,dx\\ &= \frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))-\frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\ &= \frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+x\log(a\cos(x))+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]],x]
[Out] (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]] + (I/2)*PolyLog[2, -E^
((2*I)*x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(37) = 74.
time = 0.12, size = 115, normalized size = 2.45

method	result
default	$-i\Big(\ln\left(e^{ix}\right)\ln\left(a(1+e^{2ix})\,e^{-ix}\right)+\frac{\ln(e^{ix})^2}{2}-\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)-\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-\text{dilog}\left(1+\right.$
risch	$-x\ln\left(e^{ix}\right)+\frac{ix\pi\,\text{csgn}(ia)\text{csgn}(ia\cos(x))^2}{2}+i\,\text{dilog}\left(1-ie^{ix}\right)+i\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)+\frac{ix\pi\,\text{csgn}(i\cos(x))\text{csgn}(i}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)/exp(I*x))+1/2*ln(exp(I*x))^2-ln(exp(I*
x))*ln(1+I*exp(I*x))-ln(exp(I*x))*ln(1-I*exp(I*x))-dilog(1+I*exp(I*x))-dilo
g(1-I*exp(I*x))-ln(2)*ln(exp(I*x)))
```

Maxima [A]
time = 0.60, size = 60, normalized size = 1.28

$$\frac{1}{2} i x^2 - i x \arctan\left(\sin\left(2 x\right), \cos\left(2 x\right) + 1\right) - \frac{1}{2} x \log\left(\cos\left(2 x\right)^2 + \sin\left(2 x\right)^2 + 2 \cos\left(2 x\right) + 1\right) + x \log\left(a \cos\left(x\right)\right) + \frac{1}{2} i \operatorname{Li}_2\left(-e^{(2 i x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) - 1/2*x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*cos(x)) + 1/2*I*dilog(-e^(2*I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(32) = 64.
time = 0.38, size = 104, normalized size = 2.21

$$x \log\left(a \cos\left(x\right)\right) - \frac{1}{2} x \log\left(i \cos\left(x\right) + \sin\left(x\right) + 1\right) - \frac{1}{2} x \log\left(i \cos\left(x\right) - \sin\left(x\right) + 1\right) - \frac{1}{2} x \log\left(-i \cos\left(x\right) + \sin\left(x\right) + 1\right) - \frac{1}{2} x \log\left(-i \cos\left(x\right) - \sin\left(x\right) + 1\right) - \frac{1}{2} i \operatorname{Li}_2(i \cos\left(x\right) + \sin\left(x\right)) + \frac{1}{2} i \operatorname{Li}_2(i \cos\left(x\right) - \sin\left(x\right)) + \frac{1}{2} i \operatorname{Li}_2(-i \cos\left(x\right) + \sin\left(x\right)) - \frac{1}{2} i \operatorname{Li}_2(-i \cos\left(x\right) - \sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)) - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - s
in(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(
x) + 1) - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)) +
 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \cos\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)),x)
```

```
[Out] Integral(log(a*cos(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)), x)
```

Mupad [B]
time = 0.39, size = 37, normalized size = 0.79

$$x\,\ln\left(a\,\cos\left(x\right)\right)+\frac{\mathrm{polylog}(2,-{\mathrm{e}}^{x\,2{}\mathrm{i}})\,1{}\mathrm{i}}{2}+\frac{x\,\left(x+\ln\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)\,2{}\mathrm{i}\right)\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)),x)
```

```
[Out] (polylog(2, -exp(x*2i))*1i)/2 + (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 + x*lo
g(a*cos(x))
```

3.165 $\int \log \left(a\cos^2(x)\right)\, dx$

Optimal. Leaf size=45

$$ix^2-2x\log\left(1+e^{2ix}\right)+x\log\left(a\cos^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] I*x^2-2*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^2)+I*polylog(2,-exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\cos^2(x)\right)+ix^2-2x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^2],x]
```

```
[Out] I*x^2 - 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^2] + I*PolyLog[2, -E^((2*
I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cos^2(x)\right)\,dx &= x\log\left(a\cos^2(x)\right) - \int -2x\tan(x)\,dx\\
&= x\log\left(a\cos^2(x)\right) + 2\int x\tan(x)\,dx\\
&= ix^2 + x\log\left(a\cos^2(x)\right) - 4i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + 2\int \log\left(1+e^{2ix}\right)\,dx\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) - i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right)\\
&= ix^2 - 2x\log\left(1+e^{2ix}\right) + x\log\left(a\cos^2(x)\right) + i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.96

$$x\left(ix - 2\log\left(1+e^{2ix}\right) + \log\left(a\cos^2(x)\right)\right) + i\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^2],x]
```

```
[Out] x*(I*x - 2*Log[1 + E^((2*I)*x)] + Log[a*Cos[x]^2]) + I*PolyLog[2, -E^((2*I)
*x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(39) = 78.
time = 0.12, size = 115, normalized size = 2.56

method	result

default	$-i\left(\ln\left(e^{ix}\right)\ln\left(a(1+e^{2ix})^{2}e^{-2ix}\right)+\ln\left(e^{ix}\right)^{2}-2\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)-2\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-2$
risch	$\frac{ix\pi\,\mathrm{csgn}\left(ie^{-2ix}\right)\mathrm{csgn}\left(ie^{-2ix}\left(1+e^{2ix}\right)^{2}\right)^{2}}{2}+\frac{ix\pi\mathrm{csgn}\left(ie^{ix}\right)^{2}\mathrm{csgn}\left(ie^{2ix}\right)}{2}-\frac{ix\pi\mathrm{csgn}\left(ie^{-2ix}\left(1+e^{2ix}\right)^{2}\right)^{3}}{2}-\frac{ix\pi\mathrm{csgn}\left(ia\left(1+e^{2ix}\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(a*(exp(I*x)^2+1)^2/exp(I*x)^2)+ln(exp(I*x))^2-2*ln(exp(
I*x))*ln(1+I*exp(I*x))-2*ln(exp(I*x))*ln(1-I*exp(I*x))-2*dilog(1+I*exp(I*x)
)-2*dilog(1-I*exp(I*x))-2*ln(2)*ln(exp(I*x)))
```

Maxima [A]
time = 0.66, size = 62, normalized size = 1.38

$$i\,x^{2}-2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(a\cos\left(x\right)^{2}\right)-x\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)+i\,\mathrm{Li}_{2}\left(-e^{(2i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="maxima")
```

```
[Out] I*x^2 - 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*cos(x)^2) - x*log(c
os(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + I*dilog(-e^(2*I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(34) = 68.
time = 0.43, size = 106, normalized size = 2.36

$$x\log\left(a\cos\left(x\right)^{2}\right)-x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-i\,\mathrm{Li}_{2}(i\,\cos\left(x\right)+\sin\left(x\right))+i\,\mathrm{Li}_{2}(i\,\cos\left(x\right)-\sin\left(x\right))+i\,\mathrm{Li}_{2}(-i\,\cos\left(x\right)+\sin\left(x\right))-i\,\mathrm{Li}_{2}(-i\,\cos\left(x\right)-\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*cos(x)^2) - x*log(I*cos(x) + sin(x) + 1) - x*log(I*cos(x) - sin(x)
+ 1) - x*log(-I*cos(x) + sin(x) + 1) - x*log(-I*cos(x) - sin(x) + 1) - I*di
log(I*cos(x) + sin(x)) + I*dilog(I*cos(x) - sin(x)) + I*dilog(-I*cos(x) + s
in(x)) - I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cos^{2}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**2),x)
```

```
[Out] Integral(log(a*cos(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^2), x)
```

Mupad [B]
time = 0.08, size = 39, normalized size = 0.87

$$x\,\ln\left(a\cos\left(x\right)^2\right)+\mathrm{polylog}\left(2,-{\mathrm{e}}^{x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}+x\,\left(x+\ln\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)\,2{}\mathrm{i}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)^2),x)
```

```
[Out] polylog(2, -exp(x*2i))*1i + x*(x + log(exp(x*2i) + 1)*2i)*1i + x*log(a*cos(
x)^2)
```

3.166 $\int \log (a \cos^n(x)) \, dx$

Optimal. Leaf size=52

$$\frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right) + x\log\left(a\cos^n(x)\right) + \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] 1/2*I*n*x^2-n*x*ln(1+exp(2*I*x))+x*ln(a*cos(x)^n)+1/2*I*n*polylog(2,-exp(2*
I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right) + x\log\left(a\cos^n(x)\right) + \frac{1}{2}inx^2 - nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\cos^n(x)\right)\,dx &= x\log\left(a\cos^n(x)\right)+\int nx\tan(x)\,dx\\
&= x\log\left(a\cos^n(x)\right)+n\int x\tan(x)\,dx\\
&= \frac{1}{2}inx^2+x\log\left(a\cos^n(x)\right)-(2in)\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+n\int\log\left(1+e^{2ix}\right)\,dx\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)-\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.00

$$\frac{1}{2}inx^2-nx\log\left(1+e^{2ix}\right)+x\log\left(a\cos^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cos[x]^n],x]
```

```
[Out] (I/2)*n*x^2 - n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Cos[x]^n] + (I/2)*n*PolyLo
g[2, -E^((2*I)*x)]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\cos^n(x)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cos(x)^n),x)
```

```
[Out] int(ln(a*cos(x)^n),x)
```

Maxima [A]
time = 0.69, size = 65, normalized size = 1.25

$$-\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)-i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)\right)n+x\log\left(a\cos\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*cos(x)^n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(37) = 74.
time = 0.41, size = 115, normalized size = 2.21

$$-\frac{1}{2}nx\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-\frac{1}{2}nx\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+nx\log\left(\cos\left(x\right)\right)-\frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))+\frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))+x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(I*cos(x) + sin(x) + 1) - 1/2*n*x*log(I*cos(x) - sin(x) + 1) -
1/2*n*x*log(-I*cos(x) + sin(x) + 1) - 1/2*n*x*log(-I*cos(x) - sin(x) + 1) +
 n*x*log(cos(x)) - 1/2*I*n*dilog(I*cos(x) + sin(x)) + 1/2*I*n*dilog(I*cos(x
) - sin(x)) + 1/2*I*n*dilog(-I*cos(x) + sin(x)) - 1/2*I*n*dilog(-I*cos(x) -
 sin(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cos^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cos(x)**n),x)
```

```
[Out] Integral(log(a*cos(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cos(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cos(x)^n), x)
```

Mupad [B]
time = 0.37, size = 41, normalized size = 0.79

$$x\,\ln\left(a\cos\left(x\right)^n\right)+\frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\,\,1\mathrm{i}}{2}+\frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cos(x)^n),x)
```

```
[Out] x*log(a*cos(x)^n) + (n*polylog(2, -exp(x*2i))*1i)/2 + (n*x*(x + log(exp(x*2
i) + 1)*2i)*1i)/2
```

3.167 $\int \log(a\tan(x))\,dx$

Optimal. Leaf size=51

$$2x\tanh^{-1}\left(e^{2ix}\right)+x\log(a\tan(x))-\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)+\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] 2*x*arctanh(exp(2*I*x))+x*ln(a*tan(x))-1/2*I*polylog(2,-exp(2*I*x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 4504, 4268, 2317, 2438}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\tan(x))+2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]],x]
```

```
[Out] 2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]] - (I/2)*PolyLog[2, -E^((2*I)*x)]
 + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \log(a\tan(x))\,dx &= x\log(a\tan(x)) - \int x\csc(x)\sec(x)\,dx\\ &= x\log(a\tan(x)) - 2\int x\csc(2x)\,dx\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) + \int \log\left(1-e^{2ix}\right)\,dx - \int \log\left(1+e^{2ix}\right)\,dx\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) + \frac{1}{2}i\text{Subst}\left(\int\right.\\ &= 2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\tan(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) + \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.47

$$-\frac{1}{2}i\log(-i(i-\tan(x)))\log(a\tan(x)) + \frac{1}{2}i\log(a\tan(x))\log(-i(i+\tan(x))) - \frac{1}{2}i\text{Li}_2(-i\tan(x)) + \frac{1}{2}i\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]] + (I/2)*Log[a*Tan[x]]*Log[(-I
)*(I + Tan[x])] - (I/2)*PolyLog[2, (-I)*Tan[x]] + (I/2)*PolyLog[2, I*Tan[x]
]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(39) = 78.
time = 0.15, size = 85, normalized size = 1.67

method	result
derivativedivides	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{i\tan(x)a+a}{a}\right)-\ln\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\tan(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}\right)$
default	$a\left(-\frac{i\ln(a\tan(x))\left(\ln\left(\frac{i\tan(x)a+a}{a}\right)-\ln\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}-\frac{i\left(\text{dilog}\left(\frac{i\tan(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\tan(x)a-a}{a}\right)\right)}{2a}\right)$

risch	$-x\ln\left(1+\mathrm{e}^{2ix}\right)+\frac{ix\pi\,\mathrm{csgn}\left(\frac{i\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)\mathrm{csgn}\left(\frac{ia\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)^{2}}{2}-\frac{ix\pi\mathrm{csgn}\left(\frac{a\left(\mathrm{e}^{2ix}-1\right)}{1+\mathrm{e}^{2ix}}\right)^{3}}{2}-i\,\mathrm{dilog}\left(1-i\mathrm{e}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a*(-1/2*I*ln(a*tan(x))*(ln((I*tan(x)*a+a)/a)-ln(-(I*tan(x)*a-a)/a))/a-1/2*I
*(dilog((I*tan(x)*a+a)/a)-dilog(-(I*tan(x)*a-a)/a))/a)
```

Maxima [A]
time = 0.52, size = 42, normalized size = 0.82

$$x\log\left(a\tan\left(x\right)\right)+\frac{1}{4}\,\pi\log\left(\tan\left(x\right)^{2}+1\right)-x\log\left(\tan\left(x\right)\right)+\frac{1}{2}i\,\mathrm{Li}_{2}(i\,\tan\left(x\right)+1)-\frac{1}{2}i\,\mathrm{Li}_{2}(-i\,\tan\left(x\right)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)) + 1/4*pi*log(tan(x)^2 + 1) - x*log(tan(x)) + 1/2*I*dilog(I*
tan(x) + 1) - 1/2*I*dilog(-I*tan(x) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(32) = 64.
time = 0.41, size = 184, normalized size = 3.61

$$x\log\left(a\tan\left(x\right)\right)-\frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^{2}+i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}\,x\log\left(\frac{2\,\left(\tan\left(x\right)^{2}-i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}\,x\log\left(-\frac{2\,\left(i\,\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}\,x\log\left(-\frac{2\,\left(-i\,\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{4}i\,\mathrm{Li}_{2}\left(-\frac{2\,\left(\tan\left(x\right)^{2}+i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}+1\right)+\frac{1}{4}i\,\mathrm{Li}_{2}\left(-\frac{2\,\left(\tan\left(x\right)^{2}-i\,\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}+1\right)+\frac{1}{4}i\,\mathrm{Li}_{2}\left(\frac{2\,\left(i\,\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)-\frac{1}{4}i\,\mathrm{Li}_{2}\left(\frac{2\,\left(-i\,\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)) - 1/2*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*x
*log(2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*x*log(-2*(I*tan(x) - 1)/
(tan(x)^2 + 1)) + 1/2*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/4*I*dilo
g(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(-2*(tan(x)^2 -
 I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1
) + 1) - 1/4*I*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tan\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)),x)
```

```
[Out] Integral(log(a*tan(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)),x, algorithm="giac")
[Out] integrate(log(a*tan(x)), x)
```

Mupad [B]
time = 0.09, size = 39, normalized size = 0.76

$$2\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)+x\,\ln\left(a\tan(x)\right)-\frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2}+\frac{\mathrm{polylog}(2,\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)),x)
[Out] 2*x*atanh(exp(x*2i)) - (polylog(2, -exp(x*2i))*1i)/2 + (polylog(2, exp(x*2i
))*1i)/2 + x*log(a*tan(x))
```

3.168 $\int \log \left(a \tan^2(x)\right) dx$

Optimal. Leaf size=49

$$4x \tanh^{-1}\left(e^{2ix}\right) + x \log\left(a \tan^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right) + i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] 4*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^2)-I*polylog(2,-exp(2*I*x))+I*polylog
(2,exp(2*I*x))
```

Rubi [A]

time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$-i\text{PolyLog}\left(2, -e^{2ix}\right) + i\text{PolyLog}\left(2, e^{2ix}\right) + x \log\left(a \tan^2(x)\right) + 4x \tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^2] - I*PolyLog[2, -E^((2*I)*x)] +
 I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tan^2(x)\right)\,dx &= x\log\left(a\tan^2(x)\right)-\int 2x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^2(x)\right)-2\int x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^2(x)\right)-4\int x\csc(2x)\,dx\\ &= 4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^2(x)\right)+2\int\log\left(1-e^{2ix}\right)\,dx-2\int\log\left(1+e^{2ix}\right)\,dx\\ &= 4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^2(x)\right)-i\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)+i\text{Subst}\Bigg(\\ &= 4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^2(x)\right)-i\text{Li}_2\left(-e^{2ix}\right)+i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.53

$$-\frac{1}{2}i\log(-i(i-\tan(x)))\log\left(a\tan^2(x)\right)+\frac{1}{2}i\log\left(a\tan^2(x)\right)\log(-i(i+\tan(x)))-i\text{Li}_2(-i\tan(x))+i\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^2],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^2] + (I/2)*Log[a*Tan[x]^2]*Log
[(-I)*(I + Tan[x])] - I*PolyLog[2, (-I)*Tan[x]] + I*PolyLog[2, I*Tan[x]]
```

Maple [A]
time = 0.16, size = 82, normalized size = 1.67

method	result
derivativedivides	$-\frac{i(\ln(\tan(x)-i)\ln(a(\tan^2(x)))-2\,\text{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x)))}{2}+\frac{i(\ln(\tan(x)+i)\ln(a(\tan^2(x)))-}{}$

default	$-\frac{i(\ln(\tan(x)-i)\ln(a(\tan^2(x)))-2\operatorname{dilog}(-i\tan(x))-2\ln(\tan(x)-i)\ln(-i\tan(x)))}{2}+\frac{i(\ln(\tan(x)+i)\ln(a(\tan^2(x)))$
risch	$-2i\operatorname{dilog}(1-ie^{ix})-2i\operatorname{dilog}(1+ie^{ix})+x\ln(a)-\frac{ix\pi\operatorname{csgn}\left(i(e^{2ix}-1)^2\right)^3}{2}+\frac{ix\pi\operatorname{csgn}\left(i(1+e^{2ix}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tan(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(ln(tan(x)-I)*ln(a*tan(x)^2)-2*dilog(-I*tan(x))-2*ln(tan(x)-I)*ln(-I
*tan(x)))+1/2*I*(ln(tan(x)+I)*ln(a*tan(x)^2)-2*dilog(I*tan(x))-2*ln(tan(x)+
I)*ln(I*tan(x)))
```

Maxima [A]
time = 0.50, size = 44, normalized size = 0.90

$$x\log\left(a\tan(x)^2\right)+\frac{1}{2}\pi\log\left(\tan(x)^2+1\right)-2x\log(\tan(x))+i\operatorname{Li}_2(i\tan(x)+1)-i\operatorname{Li}_2(-i\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tan(x)^2) + 1/2*pi*log(tan(x)^2 + 1) - 2*x*log(tan(x)) + I*dilog(I*
tan(x) + 1) - I*dilog(-I*tan(x) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(34) = 68.
time = 0.38, size = 184, normalized size = 3.76

$$x\log\left(a\tan(x)^2\right)-x\log\left(\frac{2(\tan(x)^2+i\tan(x))}{\tan(x)^2+1}\right)-x\log\left(\frac{2(\tan(x)^2-i\tan(x))}{\tan(x)^2+1}\right)+x\log\left(-\frac{2(i\tan(x)-1)}{\tan(x)^2+1}\right)+x\log\left(-\frac{2(-i\tan(x)-1)}{\tan(x)^2+1}\right)-\frac{1}{2}i\operatorname{Li}_2\left(-\frac{2(\tan(x)^2+i\tan(x))}{\tan(x)^2+1}+1\right)+\frac{1}{2}i\operatorname{Li}_2\left(-\frac{2(\tan(x)^2-i\tan(x))}{\tan(x)^2+1}+1\right)+\frac{1}{2}i\operatorname{Li}_2\left(\frac{2(i\tan(x)-1)}{\tan(x)^2+1}+1\right)-\frac{1}{2}i\operatorname{Li}_2\left(\frac{2(-i\tan(x)-1)}{\tan(x)^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a*tan(x)^2) - x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - x*log(2
*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1)) + x*log(-2*(I*tan(x) - 1)/(tan(x)^2
+ 1)) + x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*I*dilog(-2*(tan(x)^2
 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/2*I*dilog(-2*(tan(x)^2 - I*tan(x))/(ta
n(x)^2 + 1) + 1) + 1/2*I*dilog(2*(I*tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/2*I
*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tan^2(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**2),x)
```

```
[Out] Integral(log(a*tan(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^2), x)
```

Mupad [B]
time = 0.06, size = 41, normalized size = 0.84

$$x\,\ln\left(a\tan(x)^2\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}+4\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)+\mathrm{polylog}\left(2,\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)^2),x)
```

```
[Out] x*log(a*tan(x)^2) - polylog(2, -exp(x*2i))*1i + 4*x*atanh(exp(x*2i)) + poly
log(2, exp(x*2i))*1i
```

3.169 $\int \log(a\tan^n(x))\,dx$

Optimal. Leaf size=56

$$2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)+\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*I*x))+x*ln(a*tan(x)^n)-1/2*I*n*polylog(2,-exp(2*I*x))+1
/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\tan^n(x)\right)+2nx\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tan[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Tan[x]^n] - (I/2)*n*PolyLog[2, -E^((2*
I)*x)] + (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tan^n(x)\right)\,dx &= x\log\left(a\tan^n(x)\right)-\int nx\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^n(x)\right)-n\int x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\tan^n(x)\right)-(2n)\int x\csc(2x)\,dx\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^n(x)\right)+n\int\log\left(1-e^{2ix}\right)\,dx-n\int\log\left(1+e^{2ix}\right)\,dx\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^n(x)\right)-\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)+\frac{1}{2}(i\\ &= 2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\tan^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)+\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.45

$$-\frac{1}{2}i\log(-i(i-\tan(x)))\log\left(a\tan^n(x)\right)+\frac{1}{2}i\log\left(a\tan^n(x)\right)\log(-i(i+\tan(x)))-\frac{1}{2}in\text{Li}_2(-i\tan(x))+\frac{1}{2}in\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tan[x]^n],x]
```

```
[Out] (-1/2*I)*Log[(-I)*(I - Tan[x])]*Log[a*Tan[x]^n] + (I/2)*Log[a*Tan[x]^n]*Log
[(-I)*(I + Tan[x])] - (I/2)*n*PolyLog[2, (-I)*Tan[x]] + (I/2)*n*PolyLog[2,
I*Tan[x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.84, size = 2356, normalized size = 42.07

method	result	size
risch	Expression too large to display	2356

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a*tan(x)^n),x,method=_RETURNVERBOSE)

```
[Out] x*ln(exp(-1/2*n*(I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp
(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)
-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+
exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp
(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn(
(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))
*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)
))^2*Pi+I*Pi-2*ln(exp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))+1/2*I*x*Pi*csgn(I*a)*
csgn(I*a*exp(-1/2*n*(I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*
(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*
I*x)-1)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*
(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*c
sgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*
x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*
I*x)))^2*Pi+I*Pi-2*ln(exp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))^2+1/2*I*x*Pi*csgn
(I*exp(-1/2*n*(I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp(2
*I*x)-1)/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)-1
)/(1+exp(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+ex
p(2*I*x)))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp(2
*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn((e
xp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*c
sgn((exp(2*I*x)-1)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))
^2*Pi+I*Pi-2*ln(exp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))*csgn(I*a*exp(-1/2*n*(I*
csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2
*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))
^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I
*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2
*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn((exp(2*I*x)-1)/(1+e
xp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1
)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*Pi-2*ln(e
xp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))^2+x*ln(a)-I*n*dilog(exp(I*x))+I*n*dilog(
exp(I*x)+1)-n*x*ln(exp(I*x)+1)+n*x*ln(1+I*exp(I*x))+n*x*ln(1-I*exp(I*x))-I*
n*dilog(1+I*exp(I*x))-I*n*dilog(1-I*exp(I*x))-1/2*I*x*Pi*csgn(I*a*exp(-1/2*
n*(I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+
exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I
*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*c
```

```
sgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+
exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn((exp(2*I*x)-1)
/(1+exp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I
*x)-1)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*Pi-2
*ln(exp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))^3-1/2*I*x*Pi*csgn(I*a)*csgn(I*exp(-
1/2*n*(I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))^2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp
(2*I*x)))^2*csgn(I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)
))*csgn(I*(exp(2*I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp(2*I*x)-1)
/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn((exp(2*I*x
)-1)/(1+exp(2*I*x)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp
(2*I*x)-1)/(1+exp(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*
Pi-2*ln(exp(2*I*x)-1)+2*ln(1+exp(2*I*x)))))*csgn(I*a*exp(-1/2*n*(I*csgn(I*(
exp(2*I*x)-1)/(1+exp(2*I*x)))^3*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^
2*csgn(I*(exp(2*I*x)-1))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))^2*csgn(
I/(1+exp(2*I*x)))*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn(I*(exp(2*
I*x)-1))*csgn(I/(1+exp(2*I*x)))*Pi-I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*
csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x
)))^3*Pi+I*csgn(I*(exp(2*I*x)-1)/(1+exp(2*I*x)))*csgn((exp(2*I*x)-1)/(1+exp
(2*I*x)))*Pi-I*csgn((exp(2*I*x)-1)/(1+exp(2*I*x)))^2*Pi+I*Pi-2*ln(exp(2*I*x
)-1)+2*ln(1+exp(2*I*x)))))
```

Maxima [A]
time = 0.50, size = 48, normalized size = 0.86

$$-nx\log\left(\tan\left(x\right)\right)+\frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^{2}+1\right)+2i\operatorname{Li}_{2}(i\tan\left(x\right)+1)-2i\operatorname{Li}_{2}(-i\tan\left(x\right)+1)\right)n+x\log\left(a\tan\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="maxima")
```

```
[Out] -n*x*log(tan(x)) + 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*
I*dilog(-I*tan(x) + 1))*n + x*log(a*tan(x)^n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(37) = 74.
time = 0.41, size = 195, normalized size = 3.48

$$-\frac{1}{2}nx\log\left(\frac{2\left(\tan\left(x\right)^{2}+i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{2}nx\log\left(\frac{2\left(\tan\left(x\right)^{2}-i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}nx\log\left(-\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+\frac{1}{2}nx\log\left(-\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}\right)+nx\log\left(\tan\left(x\right)\right)-\frac{1}{4}in\operatorname{Li}_{2}\left(-\frac{2\left(\tan\left(x\right)^{2}+i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}+1\right)+\frac{1}{4}in\operatorname{Li}_{2}\left(-\frac{2\left(\tan\left(x\right)^{2}-i\tan\left(x\right)\right)}{\tan\left(x\right)^{2}+1}+1\right)+\frac{1}{4}in\operatorname{Li}_{2}\left(\frac{2\left(i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)-\frac{1}{4}in\operatorname{Li}_{2}\left(\frac{2\left(-i\tan\left(x\right)-1\right)}{\tan\left(x\right)^{2}+1}+1\right)+x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x*log(2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1)) - 1/2*n*x*log(2*(tan(x
)^2 - I*tan(x))/(tan(x)^2 + 1)) + 1/2*n*x*log(-2*(I*tan(x) - 1)/(tan(x)^2 +
 1)) + 1/2*n*x*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + n*x*log(tan(x)) - 1
```

```
/4*I*n*dilog(-2*(tan(x)^2 + I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(-
2*(tan(x)^2 - I*tan(x))/(tan(x)^2 + 1) + 1) + 1/4*I*n*dilog(2*(I*tan(x) - 1
)/(tan(x)^2 + 1) + 1) - 1/4*I*n*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1) + 1)
 + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(a\tan^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tan(x)**n),x)
```

```
[Out] Integral(log(a*tan(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tan(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*tan(x)^n), x)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 0.79

$$\frac{n\,\mathrm{polylog}(2,\mathrm{e}^{x\,2\mathrm{i}})\;1\mathrm{i}}{2}+x\,\ln\left(a\tan(x)^n\right)-\frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\;1\mathrm{i}}{2}+2\,n\,x\,\mathrm{atanh}\left(\mathrm{e}^{x\,2\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tan(x)^n),x)
```

```
[Out] (n*polylog(2, exp(x*2i))*1i)/2 + x*log(a*tan(x)^n) - (n*polylog(2, -exp(x*2
i))*1i)/2 + 2*n*x*atanh(exp(x*2i))
```

3.170 $\int \log(a\cot(x))\,dx$

Optimal. Leaf size=51

$$-2x\tanh^{-1}\left(e^{2ix}\right)+x\log(a\cot(x))+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -2*x*arctanh(exp(2*I*x))+x*ln(a*cot(x))+1/2*I*polylog(2,-exp(2*I*x))-1/2*I*
polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 4504, 4268, 2317, 2438}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\cot(x))-2x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]],x]
```

```
[Out] -2*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]] + (I/2)*PolyLog[2, -E^((2*I)*x)
] - (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\int \log(a\cot(x))\,dx &= x\log(a\cot(x)) + \int x\csc(x)\sec(x)\,dx \\
&= x\log(a\cot(x)) + 2\int x\csc(2x)\,dx \\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) - \int \log\left(1-e^{2ix}\right)\,dx + \int \log\left(1+e^{2ix}\right)\,dx \\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right) - \frac{1}{2}i\text{Subst}(\\
&= -2x\tanh^{-1}\left(e^{2ix}\right) + x\log(a\cot(x)) + \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.47

$$\frac{1}{2}i\log(-i(i-\cot(x)))\log(a\cot(x)) - \frac{1}{2}i\log(a\cot(x))\log(-i(i+\cot(x))) + \frac{1}{2}i\text{Li}_2(-i\cot(x)) - \frac{1}{2}i\text{Li}_2(i\cot(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]],x]
[Out] (I/2)*Log[(-I)*(I - Cot[x])]*Log[a*Cot[x]] - (I/2)*Log[a*Cot[x]]*Log[(-I)*(
I + Cot[x])] + (I/2)*PolyLog[2, (-I)*Cot[x]] - (I/2)*PolyLog[2, I*Cot[x]]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.
time = 0.16, size = 86, normalized size = 1.69

method	result
derivativedivides	$-a\left(\frac{i\ln(a\cot(x))\left(-\ln\left(\frac{i\cot(x)a+a}{a}\right)+\ln\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a} - \frac{i\left(\text{dilog}\left(\frac{i\cot(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}\right)$
default	$-a\left(\frac{i\ln(a\cot(x))\left(-\ln\left(\frac{i\cot(x)a+a}{a}\right)+\ln\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a} - \frac{i\left(\text{dilog}\left(\frac{i\cot(x)a+a}{a}\right)-\text{dilog}\left(-\frac{i\cot(x)a-a}{a}\right)\right)}{2a}\right)$

risch	$x\ln\left(1+\mathrm{e}^{2ix}\right)-\frac{ix\pi\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ix}-1}\right)^{3}}{2}-\frac{ix\pi\operatorname{csgn}\left(\frac{a\left(1+\mathrm{e}^{2ix}\right)}{\mathrm{e}^{2ix}-1}\right)^{2}}{2}+i\operatorname{dilog}\left(1-i\mathrm{e}^{ix}\right)-\frac{ix\pi\operatorname{csgn}\left(\frac{ia\left(1+\right.}{\mathrm{e}^{2ix}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a*(1/2*I*ln(a*cot(x))*(-ln((I*cot(x)*a+a)/a)+ln(-(I*cot(x)*a-a)/a))/a-1/2*
I*(dilog((I*cot(x)*a+a)/a)-dilog(-(I*cot(x)*a-a)/a))/a)
```

Maxima [A]
time = 0.50, size = 43, normalized size = 0.84

$$-\frac{1}{4}\,\pi\log\left(\tan\left(x\right)^{2}+1\right)+x\log\left(\frac{a}{\tan\left(x\right)}\right)+x\log\left(\tan\left(x\right)\right)-\frac{1}{2}i\,\mathrm{Li}_{2}(i\,\tan\left(x\right)+1)+\frac{1}{2}i\,\mathrm{Li}_{2}(-i\,\tan\left(x\right)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="maxima")
```

```
[Out] -1/4*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)) + x*log(tan(x)) - 1/2*I*dilog(I
*tan(x) + 1) + 1/2*I*dilog(-I*tan(x) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(32) = 64.
time = 0.45, size = 147, normalized size = 2.88

$$x\log\left(\frac{a\cos\left(2\,x\right)+a}{\sin\left(2\,x\right)}\right)-\frac{1}{2}\,x\log\left(\cos\left(2\,x\right)+i\,\sin\left(2\,x\right)+1\right)-\frac{1}{2}\,x\log\left(\cos\left(2\,x\right)-i\,\sin\left(2\,x\right)+1\right)+\frac{1}{2}\,x\log\left(-\cos\left(2\,x\right)+i\,\sin\left(2\,x\right)+1\right)+\frac{1}{2}\,x\log\left(-\cos\left(2\,x\right)-i\,\sin\left(2\,x\right)+1\right)-\frac{1}{4}i\,\mathrm{Li}_{2}(\cos\left(2\,x\right)+i\,\sin\left(2\,x\right))+\frac{1}{4}i\,\mathrm{Li}_{2}(\cos\left(2\,x\right)-i\,\sin\left(2\,x\right))-\frac{1}{4}i\,\mathrm{Li}_{2}(-\cos\left(2\,x\right)+i\,\sin\left(2\,x\right))+\frac{1}{4}i\,\mathrm{Li}_{2}(-\cos\left(2\,x\right)-i\,\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="fricas")
```

```
[Out] x*log((a*cos(2*x) + a)/sin(2*x)) - 1/2*x*log(cos(2*x) + I*sin(2*x) + 1) - 1
/2*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*x*log(-cos(2*x) + I*sin(2*x) + 1)
 + 1/2*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*dilog(cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*dilog(-cos(2*x) + I*sin(2*x
)) + 1/4*I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cot\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)),x)
```

```
[Out] Integral(log(a*cot(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)),x, algorithm="giac")
[Out] integrate(log(a*cot(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\cot(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)),x)
[Out] int(log(a*cot(x)), x)
```

3.171 $\int \log\left(a\cot^2(x)\right)\,dx$

Optimal. Leaf size=49

$$-4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)-i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -4*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^2)+I*polylog(2,-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$i\text{PolyLog}\left(2,-e^{2ix}\right)-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-4x\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^2] + I*PolyLog[2, -E^((2*I)*x)]
- I*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\cot^2(x)\right)\,dx &= x\log\left(a\cot^2(x)\right)-\int -2x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\cot^2(x)\right)+2\int x\csc(x)\sec(x)\,dx\\ &= x\log\left(a\cot^2(x)\right)+4\int x\csc(2x)\,dx\\ &= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)-2\int \log\left(1-e^{2ix}\right)\,dx+2\int \log\left(1+e^{2ix}\right)\,d\\ &= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)+i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)-i\text{Subst}\\ &= -4x\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^2(x)\right)+i\text{Li}_2\left(-e^{2ix}\right)-i\text{Li}_2\left(e^{2ix}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.53

$$-\frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i-\tan(x)))+\frac{1}{2}i\log\left(a\cot^2(x)\right)\log(-i(i+\tan(x)))+i\text{Li}_2(-i\tan(x))-i\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^2],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]^2]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^2]*Log
[(-I)*(I + Tan[x])] + I*PolyLog[2, (-I)*Tan[x]] - I*PolyLog[2, I*Tan[x]]
```

Maple [A]
time = 0.15, size = 82, normalized size = 1.67

method	result
derivativedivides	$\frac{i(\ln(\cot(x)-i)\ln(a(\cot^2(x)))-2\,\text{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x)))}{2}-\frac{i(\ln(\cot(x)+i)\ln(a(\cot^2(x)))-2\,\text{c}}{}$

default	$\frac{i\left(\ln(\cot(x)-i)\ln\left(a\left(\cot^2(x)\right)\right)-2\,\mathrm{dilog}(-i\cot(x))-2\ln(\cot(x)-i)\ln(-i\cot(x))\right)}{2}-\frac{i\left(\ln(\cot(x)+i)\ln\left(a\left(\cot^2(x)\right)\right)-2\,\mathrm{dil}\right.}{}$
risch	$\frac{ix\pi\,\mathrm{csgn}\left(\frac{i\left(1+e^{2ix}\right)^2}{\left(e^{2ix}-1\right)^2}\right)\mathrm{csgn}\left(\frac{ia\left(1+e^{2ix}\right)^2}{\left(e^{2ix}-1\right)^2}\right)^2}{2}+\frac{ix\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(\frac{ia\left(1+e^{2ix}\right)^2}{\left(e^{2ix}-1\right)^2}\right)^2}{2}+\frac{ix\pi\,\mathrm{csgn}\left(i\left(1+e^{2ix}\right)^2\right)\mathrm{csgn}\left(\frac{i(1}{(e}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(ln(cot(x)-I)*ln(a*cot(x)^2)-2*dilog(-I*cot(x))-2*ln(cot(x)-I)*ln(-I*
cot(x)))-1/2*I*(ln(cot(x)+I)*ln(a*cot(x)^2)-2*dilog(I*cot(x))-2*ln(cot(x)+I
)*ln(I*cot(x)))
```

Maxima [A]
time = 0.50, size = 44, normalized size = 0.90

$$-\frac{1}{2}\pi\log\left(\tan(x)^2+1\right)+x\log\left(\frac{a}{\tan(x)^2}\right)+2\,x\log(\tan(x))-i\,\mathrm{Li}_2(i\,\tan(x)+1)+i\,\mathrm{Li}_2(-i\,\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(tan(x)^2 + 1) + x*log(a/tan(x)^2) + 2*x*log(tan(x)) - I*dilog(I
*tan(x) + 1) + I*dilog(-I*tan(x) + 1)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(34) = 68.
time = 0.40, size = 148, normalized size = 3.02

$$x\log\left(-\frac{a\cos(2\,x)+a}{\cos(2\,x)-1}\right)-x\log(\cos(2\,x)+i\,\sin(2\,x)+1)-x\log(\cos(2\,x)-i\,\sin(2\,x)+1)+x\log(-\cos(2\,x)+i\,\sin(2\,x)+1)+x\log(-\cos(2\,x)-i\,\sin(2\,x)+1)-\frac{1}{2}i\,\mathrm{Li}_2(\cos(2\,x)+i\,\sin(2\,x))+\frac{1}{2}i\,\mathrm{Li}_2(\cos(2\,x)-i\,\sin(2\,x))-\frac{1}{2}i\,\mathrm{Li}_2(-\cos(2\,x)+i\,\sin(2\,x))+\frac{1}{2}i\,\mathrm{Li}_2(-\cos(2\,x)-i\,\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-(a*cos(2*x) + a)/(cos(2*x) - 1)) - x*log(cos(2*x) + I*sin(2*x) + 1)
- x*log(cos(2*x) - I*sin(2*x) + 1) + x*log(-cos(2*x) + I*sin(2*x) + 1) + x*
log(-cos(2*x) - I*sin(2*x) + 1) - 1/2*I*dilog(cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(cos(2*x) - I*sin(2*x)) - 1/2*I*dilog(-cos(2*x) + I*sin(2*x)) + 1/2*
I*dilog(-cos(2*x) - I*sin(2*x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\cot^2(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**2),x)
```

```
[Out] Integral(log(a*cot(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(a \cot(x)^2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)^2),x)
```

```
[Out] int(log(a*cot(x)^2), x)
```

3.172 $\int \log\left(a\cot^n(x)\right)\,dx$

Optimal. Leaf size=56

$$-2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -2*n*x*arctanh(exp(2*I*x))+x*ln(a*cot(x)^n)+1/2*I*n*polylog(2,-exp(2*I*x))-
1/2*I*n*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 4504, 4268, 2317, 2438}

$$\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\cot^n(x)\right)-2nx\tanh^{-1}\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cot[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^((2*I)*x)] + x*Log[a*Cot[x]^n] + (I/2)*n*PolyLog[2, -E^((2
*I)*x)] - (I/2)*n*PolyLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4504

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\cot^n(x)\right)\,dx &= x\log\left(a\cot^n(x)\right)+\int nx\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+n\int x\csc(x)\sec(x)\,dx\\
&= x\log\left(a\cot^n(x)\right)+(2n)\int x\csc(2x)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)-n\int \log\left(1-e^{2ix}\right)\,dx+n\int \log\left(1+e^{2ix}\right)
\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}(in)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2ix}\right)-
\\
&= -2nx\tanh^{-1}\left(e^{2ix}\right)+x\log\left(a\cot^n(x)\right)+\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.45

$$-\frac{1}{2}i\log\left(a\cot^n(x)\right)\log(-i(i-\tan(x)))+\frac{1}{2}i\log\left(a\cot^n(x)\right)\log(-i(i+\tan(x)))+\frac{1}{2}in\text{Li}_2(-i\tan(x))-\frac{1}{2}in\text{Li}_2(i\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cot[x]^n],x]
```

```
[Out] (-1/2*I)*Log[a*Cot[x]^n]*Log[(-I)*(I - Tan[x])] + (I/2)*Log[a*Cot[x]^n]*Log
[(-I)*(I + Tan[x])] + (I/2)*n*PolyLog[2, (-I)*Tan[x]] - (I/2)*n*PolyLog[2,
I*Tan[x]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 2357, normalized size = 42.09

method	result	size
risch	Expression too large to display	2357

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cot(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi-I*csgn(I/(exp
(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi-I*csgn(I/(exp(2*I*x)
-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn(I/(exp(2*I*x)-1)*(1+
exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))*Pi-I*csgn(I/(exp
(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*Pi-I*csg
n(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*
x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1/(exp(2*I*x)-1)*(1+ex
p(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I*x)))))+1/2*I*x*Pi*cs
gn(I*a)*csgn(I*a*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi-I
*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi-I*csgn(I
/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn(I/(exp(2
*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))*Pi-I
*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x))
)^2*Pi-I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(2*I*x)-1)
*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1/(exp(2*I
*x)-1)*(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I*x)))))^2
+1/2*I*x*Pi*csgn(I*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi
-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi-I*csgn
(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn(I/(exp
(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))*Pi
-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x
)))^2*Pi-I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(2*I*x)-
1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1/(exp(2
*I*x)-1)*(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I*x)))))
*csgn(I*a*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi-I*csgn(I
/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi-I*csgn(I/(exp(2
*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn(I/(exp(2*I*x)-1
)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x)))*Pi-I*csgn(I
/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*Pi-
I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(2*I*x)-1)*(1+exp
(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1/(exp(2*I*x)-1)*
(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I*x)))))^2+x*ln(a
)-n*x*ln(1+I*exp(I*x))-n*x*ln(1-I*exp(I*x))+I*n*dilog(1+I*exp(I*x))+I*n*dil
og(1-I*exp(I*x))+I*n*dilog(exp(I*x))-I*n*dilog(exp(I*x)+1)+n*x*ln(exp(I*x)+
1)-1/2*I*x*Pi*csgn(I*a*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^
3*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi-I*
csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn(I/
```

```
(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*x))
)*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2
*I*x)))^2*Pi-I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(2*I
*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1/(e
xp(2*I*x)-1)*(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I*x)
))))^3-1/2*I*x*Pi*csgn(I*a)*csgn(I*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+e
xp(2*I*x)))^3*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*
x)-1))*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*
Pi+I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1
+exp(2*I*x)))*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)
-1)*(1+exp(2*I*x)))^2*Pi-I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csg
n(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi
+I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(
1+exp(2*I*x)))))*csgn(I*a*exp(-1/2*n*(I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)
))^3*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I/(exp(2*I*x)-1))*Pi
-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*csgn(I*(1+exp(2*I*x)))*Pi+I*csgn
(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(I/(exp(2*I*x)-1))*csgn(I*(1+exp(2*I*
x)))*Pi-I*csgn(I/(exp(2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+ex
p(2*I*x)))^2*Pi-I*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))^3*Pi+I*csgn(I/(exp(
2*I*x)-1)*(1+exp(2*I*x)))*csgn(1/(exp(2*I*x)-1)*(1+exp(2*I*x)))*Pi+I*csgn(1
/(exp(2*I*x)-1)*(1+exp(2*I*x)))^2*Pi-I*Pi+2*ln(exp(2*I*x)-1)-2*ln(1+exp(2*I
*x)))))
```

Maxima [**A**]
time = 0.52, size = 49, normalized size = 0.88

$$nx\log\left(\tan\left(x\right)\right) - \frac{1}{4}\left(\pi\log\left(\tan\left(x\right)^{2} + 1\right) + 2i\,\mathrm{Li}_{2}(i\,\tan\left(x\right) + 1) - 2i\,\mathrm{Li}_{2}(-i\,\tan\left(x\right) + 1)\right)n + x\log\left(a\frac{1}{\tan\left(x\right)}^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="maxima")
```

```
[Out] n*x*log(tan(x)) - 1/4*(pi*log(tan(x)^2 + 1) + 2*I*dilog(I*tan(x) + 1) - 2*I
*dilog(-I*tan(x) + 1))*n + x*log(a*(1/tan(x))^n)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(37) = 74.
time = 0.43, size = 158, normalized size = 2.82

$$nx\log\left(\frac{\cos\left(2\,x\right) + 1}{\sin\left(2\,x\right)}\right) - \frac{1}{2}\,nx\log\left(\cos\left(2\,x\right) + i\,\sin\left(2\,x\right) + 1\right) - \frac{1}{2}\,nx\log\left(\cos\left(2\,x\right) - i\,\sin\left(2\,x\right) + 1\right) + \frac{1}{2}\,nx\log\left(-\cos\left(2\,x\right) + i\,\sin\left(2\,x\right) + 1\right) + \frac{1}{2}\,nx\log\left(-\cos\left(2\,x\right) - i\,\sin\left(2\,x\right) + 1\right) - \frac{1}{4}i\,n\mathrm{Li}_{2}(\cos\left(2\,x\right) + i\,\sin\left(2\,x\right)) + \frac{1}{4}i\,n\mathrm{Li}_{2}(\cos\left(2\,x\right) - i\,\sin\left(2\,x\right)) - \frac{1}{4}i\,n\mathrm{Li}_{2}(-\cos\left(2\,x\right) + i\,\sin\left(2\,x\right)) + \frac{1}{4}i\,n\mathrm{Li}_{2}(-\cos\left(2\,x\right) - i\,\sin\left(2\,x\right)) + x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log((cos(2*x) + 1)/sin(2*x)) - 1/2*n*x*log(cos(2*x) + I*sin(2*x) + 1) -
 1/2*n*x*log(cos(2*x) - I*sin(2*x) + 1) + 1/2*n*x*log(-cos(2*x) + I*sin(2*x
```

```
) + 1) + 1/2*n*x*log(-cos(2*x) - I*sin(2*x) + 1) - 1/4*I*n*dilog(cos(2*x) +
 I*sin(2*x)) + 1/4*I*n*dilog(cos(2*x) - I*sin(2*x)) - 1/4*I*n*dilog(-cos(2*
x) + I*sin(2*x)) + 1/4*I*n*dilog(-cos(2*x) - I*sin(2*x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(a \cot^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cot(x)**n),x)
```

```
[Out] Integral(log(a*cot(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cot(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cot(x)^n), x)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 0.79

$$x\,\ln\left(a\,{\mathrm{cot}\left(x\right)}^n\right)-\frac{n\,\mathrm{polylog}\left(2,{\mathrm{e}}^{x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{n\,\mathrm{polylog}\left(2,-{\mathrm{e}}^{x\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}-2\,n\,x\,\mathrm{atanh}\left({\mathrm{e}}^{x\,2{}\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cot(x)^n),x)
```

```
[Out] x*log(a*cot(x)^n) - (n*polylog(2, exp(x*2i))*1i)/2 + (n*polylog(2, -exp(x*2
i))*1i)/2 - 2*n*x*atanh(exp(x*2i))
```

3.173 $\int \log(a\sec(x))\,dx$

Optimal. Leaf size=46

$$-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)+x\log(a\sec(x))-\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1+exp(2*I*x))+x*ln(a*sec(x))-1/2*I*polylog(2,-exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3800, 2221, 2317, 2438}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log(a\sec(x))-\frac{ix^2}{2}+x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
-E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\sec(x))\,dx &= x\log(a\sec(x)) - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} + x\log(a\sec(x)) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \int \log\left(1+e^{2ix}\right)\,dx \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= -\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$
-\frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) + x\log(a\sec(x)) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]] - (I/2)*PolyLog[2,
-E^((2*I)*x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(36) = 72.
time = 0.12, size = 108, normalized size = 2.35

method	result
default	$-i\left(\ln(2)\ln\left(e^{ix}\right) + \ln\left(e^{ix}\right)\ln\left(\frac{a\,e^{ix}}{1+e^{2ix}}\right) - \frac{\ln\left(e^{ix}\right)^2}{2} + \ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right) + \ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right) + \mathrm{dil}\right.$
risch	$x\ln\left(e^{ix}\right) - \frac{ix\pi\,\mathrm{csgn}\left(ie^{ix}\right)\mathrm{csgn}\left(\frac{i}{1+e^{2ix}}\right)\mathrm{csgn}\left(\frac{ie^{ix}}{1+e^{2ix}}\right)}{2} - i\,\mathrm{dilog}\left(1-ie^{ix}\right) + \frac{ix\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(\frac{ia\,e^{ix}}{1+e^{2ix}}\right)^2}{2} - \frac{ix\pi\mathrm{csgn}\left(\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(a*exp(I*x)/(exp(I*x)^2+1))-1/2*ln(ex
p(I*x))^2+ln(exp(I*x))*ln(1+I*exp(I*x))+ln(exp(I*x))*ln(1-I*exp(I*x))+dilog
(1+I*exp(I*x))+dilog(1-I*exp(I*x)))
```

Maxima [A]
time = 0.63, size = 60, normalized size = 1.30

$$-\frac{1}{2}i\,x^2+i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+\frac{1}{2}\,x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)+x\log\left(a\sec\left(x\right)\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/2*x*log(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1) + x*log(a*sec(x)) - 1/2*I*dilog(-e^(2*I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(31) = 62.
time = 0.40, size = 106, normalized size = 2.30

$$x\log\left(\frac{a}{\cos\left(x\right)}\right)+\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)) + 1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - s
in(x) + 1) + 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(
x) + 1) + 1/2*I*dilog(I*cos(x) + sin(x)) - 1/2*I*dilog(I*cos(x) - sin(x)) -
 1/2*I*dilog(-I*cos(x) + sin(x)) + 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\sec\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)),x)
```

```
[Out] Integral(log(a*sec(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)), x)
```

Mupad [**B**]
time = 0.09, size = 39, normalized size = 0.85

$$x\ln\left(\frac{a}{\cos\left(x\right)}\right)-\frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\ 1\mathrm{i}}{2}-\frac{x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\ 2\mathrm{i}\right)\ 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cos(x)),x)
```

```
[Out] x*log(a/cos(x)) - (x*(x + log(exp(x*2i) + 1)*2i)*1i)/2 - (polylog(2, -exp(x
*2i))*1i)/2
```

3.174 $\int \log\left(a\sec^2(x)\right)\,dx$

Optimal. Leaf size=45

$$-ix^2+2x\log\left(1+e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-i\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^2)-I*polylog(2,-exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$-i\text{PolyLog}\left(2,-e^{2ix}\right)+x\log\left(a\sec^2(x)\right)-ix^2+2x\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^2] - I*PolyLog[2, -E^(
(2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log \left(a \sec^2(x)\right)\, dx &= x \log \left(a \sec^2(x)\right) - \int 2x \tan(x)\, dx \\
&= x \log \left(a \sec^2(x)\right) - 2 \int x \tan(x)\, dx \\
&= -ix^2 + x \log \left(a \sec^2(x)\right) + 4i \int \frac{e^{2ix} x}{1+e^{2ix}}\, dx \\
&= -ix^2 + 2x \log \left(1+e^{2ix}\right) + x \log \left(a \sec^2(x)\right) - 2 \int \log \left(1+e^{2ix}\right)\, dx \\
&= -ix^2 + 2x \log \left(1+e^{2ix}\right) + x \log \left(a \sec^2(x)\right) + i\text{Subst}\left(\int \frac{\log(1+x)}{x}\, dx, x, e^{2ix}\right) \\
&= -ix^2 + 2x \log \left(1+e^{2ix}\right) + x \log \left(a \sec^2(x)\right) - i\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.96

$$x\left(-ix + 2\log \left(1+e^{2ix}\right) + \log \left(a \sec^2(x)\right)\right) - i\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^2],x]
```

```
[Out] x*((-I)*x + 2*Log[1 + E^((2*I)*x)] + Log[a*Sec[x]^2]) - I*PolyLog[2, -E^((2
*I)*x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(39) = 78.
time = 0.13, size = 117, normalized size = 2.60

method	result

default	$-i\left(\ln\left(e^{ix}\right)\ln\left(\frac{a\,e^{2ix}}{\left(1+e^{2ix}\right)^{2}}\right)+2\ln\left(e^{ix}\right)\ln\left(1+ie^{ix}\right)+2\ln\left(e^{ix}\right)\ln\left(1-ie^{ix}\right)-\ln\left(e^{ix}\right)^{2}+2\,\mathrm{dilog}\left(1-\right.\right.$
risch	$\frac{ix\pi\,\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)^{2}\mathrm{csgn}\left(i\left(1+e^{2ix}\right)^{2}\right)}{2}-2i\,\mathrm{dilog}\left(1-ie^{ix}\right)-\frac{ix\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(\frac{ie^{2ix}}{\left(1+e^{2ix}\right)^{2}}\right)\mathrm{csgn}\left(\frac{ia\,e^{2ix}}{\left(1+e^{2ix}\right)^{2}}\right)}{2}+ix\pi$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(a*exp(I*x)^2/(exp(I*x)^2+1)^2)+2*ln(exp(I*x))*ln(1+I*ex
p(I*x))+2*ln(exp(I*x))*ln(1-I*exp(I*x))-ln(exp(I*x))^2+2*dilog(1+I*exp(I*x)
)+2*dilog(1-I*exp(I*x))+2*ln(2)*ln(exp(I*x)))
```

Maxima [A]
time = 0.58, size = 61, normalized size = 1.36

$$-i\,x^{2}+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(a\sec\left(x\right)^{2}\right)+x\log\left(\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1\right)-i\,\mathrm{Li}_{2}\left(-e^{(2i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(a*sec(x)^2) + x*log(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(34) = 68.
time = 0.46, size = 102, normalized size = 2.27

$$x\log\left(\frac{a}{\cos\left(x\right)^{2}}\right)+x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+i\,\mathrm{Li}_{2}(i\,\cos\left(x\right)+\sin\left(x\right))-i\,\mathrm{Li}_{2}(i\,\cos\left(x\right)-\sin\left(x\right))-i\,\mathrm{Li}_{2}(-i\,\cos\left(x\right)+\sin\left(x\right))+i\,\mathrm{Li}_{2}(-i\,\cos\left(x\right)-\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(a/cos(x)^2) + x*log(I*cos(x) + sin(x) + 1) + x*log(I*cos(x) - sin(x)
+ 1) + x*log(-I*cos(x) + sin(x) + 1) + x*log(-I*cos(x) - sin(x) + 1) + I*di
log(I*cos(x) + sin(x)) - I*dilog(I*cos(x) - sin(x)) - I*dilog(-I*cos(x) + s
in(x)) + I*dilog(-I*cos(x) - sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sec^{2}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**2),x)
```

```
[Out] Integral(log(a*sec(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^2), x)
```

Mupad [B]
time = 0.38, size = 39, normalized size = 0.87

$$x\ln\left(\frac{a}{\cos\left(x\right)^2}\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x\,2\mathrm{i}}\right)\,1\mathrm{i}-x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\,2\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cos(x)^2),x)
```

```
[Out] x*log(a/cos(x)^2) - x*(x + log(exp(x*2i) + 1)*2i)*1i - polylog(2, -exp(x*2i
))*1i
```

3.175 $\int \log(a\sec^n(x))\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right) + x\log\left(a\sec^n(x)\right) - \frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1+exp(2*I*x))+x*ln(a*sec(x)^n)-1/2*I*n*polylog(2,-exp(2
*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3800, 2221, 2317, 2438}

$$-\frac{1}{2}in\text{PolyLog}\left(2,-e^{2ix}\right) + x\log\left(a\sec^n(x)\right) - \frac{1}{2}inx^2 + nx\log\left(1+e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sec[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Pol
yLog[2, -E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\sec^n(x)\right)\,dx &= x\log\left(a\sec^n(x)\right)-\int nx\tan(x)\,dx\\
&= x\log\left(a\sec^n(x)\right)-n\int x\tan(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\sec^n(x)\right)+(2in)\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-n\int\log\left(1+e^{2ix}\right)\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)+\frac{1}{2}(in)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2i
\\
&= -\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.00

$$-\frac{1}{2}inx^2+nx\log\left(1+e^{2ix}\right)+x\log\left(a\sec^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sec[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 + E^((2*I)*x)] + x*Log[a*Sec[x]^n] - (I/2)*n*Pol
yLog[2, -E^((2*I)*x)]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\sec^n\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sec(x)^n),x)
```

```
[Out] int(ln(a*sec(x)^n),x)
```

Maxima [A]
time = 0.69, size = 65, normalized size = 1.27

$$\frac{1}{2}\left(-i\,x^2+2i\,x\arctan\left(\sin\left(2\,x\right),\cos\left(2\,x\right)+1\right)+x\log\left(\cos\left(2\,x\right)^2+\sin\left(2\,x\right)^2+2\,\cos\left(2\,x\right)+1\right)-i\,\mathrm{Li}_2\left(-e^{(2i\,x)}\right)\right)n+x\log\left(a\sec\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(2*x), cos(2*x) + 1) + x*log(cos(2*x)^2 + si
n(2*x)^2 + 2*cos(2*x) + 1) - I*dilog(-e^(2*I*x)))*n + x*log(a*sec(x)^n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(36) = 72$.
time = 0.38, size = 117, normalized size = 2.29

$$nx\log\left(\frac{1}{\cos\left(x\right)}\right)+\frac{1}{2}\,nx\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+\frac{1}{2}\,nx\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+\frac{1}{2}\,nx\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+\frac{1}{2}\,nx\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+\frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,n\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right))-\frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,n\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))+x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/cos(x)) + 1/2*n*x*log(I*cos(x) + sin(x) + 1) + 1/2*n*x*log(I*cos(
x) - sin(x) + 1) + 1/2*n*x*log(-I*cos(x) + sin(x) + 1) + 1/2*n*x*log(-I*cos
(x) - sin(x) + 1) + 1/2*I*n*dilog(I*cos(x) + sin(x)) - 1/2*I*n*dilog(I*cos(
x) - sin(x)) - 1/2*I*n*dilog(-I*cos(x) + sin(x)) + 1/2*I*n*dilog(-I*cos(x)
- sin(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sec^{n}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sec(x)**n),x)
```

```
[Out] Integral(log(a*sec(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sec(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sec(x)^n), x)
```

Mupad [B]
time = 0.38, size = 43, normalized size = 0.84

$$x\,\ln\left(a\,{\left(\frac{1}{\cos\left(x\right)}\right)}^n\right)-\frac{n\,\mathrm{polylog}(2,-\mathrm{e}^{x\,2\mathrm{i}})\;1\mathrm{i}}{2}-\frac{n\,x\,\left(x+\ln\left(\mathrm{e}^{x\,2\mathrm{i}}+1\right)\;2\mathrm{i}\right)\;1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/cos(x))^n),x)
```

```
[Out] x*log(a*(1/cos(x))^n) - (n*polylog(2, -exp(x*2i))*1i)/2 - (n*x*(x + log(exp
(x*2i) + 1)*2i)*1i)/2
```

3.176 $\int \log(a\csc(x))\,dx$

Optimal. Leaf size=46

$$-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)+x\log(a\csc(x))-\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -1/2*I*x^2+x*ln(1-exp(2*I*x))+x*ln(a*csc(x))-1/2*I*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3798, 2221, 2317, 2438}

$$-\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)+x\log(a\csc(x))-\frac{ix^2}{2}+x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]],x]
```

```
[Out] (-1/2*I)*x^2 + x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*PolyLog[2,
E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(a\csc(x))\,dx &= x\log(a\csc(x)) + \int x\cot(x)\,dx\\
&= -\frac{ix^2}{2} + x\log(a\csc(x)) - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \int \log\left(1-e^{2ix}\right)\,dx\\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)\\
&= -\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.89

$$x\log\left(1-e^{2ix}\right) + x\log(a\csc(x)) - \frac{1}{2}i\left(x^2 + \text{Li}_2\left(e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]],x]
```

```
[Out] x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]] - (I/2)*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.
time = 0.12, size = 83, normalized size = 1.80

method	result
default	$-i\left(\ln(2)\ln\left(e^{ix}\right) + \ln\left(e^{ix}\right)\ln\left(\frac{ia\,e^{ix}}{e^{2ix}-1}\right) - \frac{\ln\left(e^{ix}\right)^2}{2} + \ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right) + \text{dilog}\left(e^{ix}+1\right) - \text{dilog}\left(e^{ix}\right.\right.$
risch	$x\ln\left(e^{ix}\right) + \frac{ix\pi\,\text{csgn}\left(\frac{i}{e^{2ix}-1}\right)\text{csgn}\left(\frac{ie^{ix}}{e^{2ix}-1}\right)^2}{2} - \frac{ix\pi\text{csgn}\left(\frac{ia\,e^{ix}}{e^{2ix}-1}\right)^3}{2} - \frac{ix\pi\text{csgn}\left(\frac{a\,e^{ix}}{e^{2ix}-1}\right)^2}{2} + i\ln\left(e^{ix}\right)\ln\left(e^{2ix}-1\right) +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(2)*ln(exp(I*x))+ln(exp(I*x))*ln(I*a*exp(I*x)/(exp(I*x)^2-1))-1/2*ln(
exp(I*x))^2+ln(exp(I*x))*ln(exp(I*x)+1)+dilog(exp(I*x)+1)-dilog(exp(I*x)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.
time = 0.64, size = 87, normalized size = 1.89

$$-\frac{1}{2} i x^2 + i x \arctan\left(\sin(x), \cos(x) + 1\right) - i x \arctan\left(\sin(x), -\cos(x) + 1\right) + \frac{1}{2} x \log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{2} x \log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right) + x \log\left(a \csc(x)\right) - i \operatorname{Li}_2\left(-e^{(ix)}\right) - i \operatorname{Li}_2\left(e^{(ix)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2 + I*x*arctan2(sin(x), cos(x) + 1) - I*x*arctan2(sin(x), -cos(x)
+ 1) + 1/2*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/2*x*log(cos(x)^2 +
 sin(x)^2 - 2*cos(x) + 1) + x*log(a*csc(x)) - I*dilog(-e^(I*x)) - I*dilog(e
^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(31) = 62$.
time = 0.50, size = 106, normalized size = 2.30

$$x \log\left(\frac{a}{\sin(x)}\right) + \frac{1}{2} x \log\left(\cos(x) + i \sin(x) + 1\right) + \frac{1}{2} x \log\left(\cos(x) - i \sin(x) + 1\right) + \frac{1}{2} x \log\left(-\cos(x) + i \sin(x) + 1\right) + \frac{1}{2} x \log\left(-\cos(x) - i \sin(x) + 1\right) - \frac{1}{2} i \operatorname{Li}_2(\cos(x) + i \sin(x)) + \frac{1}{2} i \operatorname{Li}_2(\cos(x) - i \sin(x)) + \frac{1}{2} i \operatorname{Li}_2(-\cos(x) + i \sin(x)) - \frac{1}{2} i \operatorname{Li}_2(-\cos(x) - i \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="fricas")
```

```
[Out] x*log(a/sin(x)) + 1/2*x*log(cos(x) + I*sin(x) + 1) + 1/2*x*log(cos(x) - I*s
in(x) + 1) + 1/2*x*log(-cos(x) + I*sin(x) + 1) + 1/2*x*log(-cos(x) - I*sin(
x) + 1) - 1/2*I*dilog(cos(x) + I*sin(x)) + 1/2*I*dilog(cos(x) - I*sin(x)) +
 1/2*I*dilog(-cos(x) + I*sin(x)) - 1/2*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a \csc(x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)),x)
```

```
[Out] Integral(log(a*csc(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\frac{a}{\sin\left(x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sin(x)),x)
```

```
[Out] int(log(a/sin(x)), x)
```

3.177 $\int \log \left(a \csc^2(x)\right) \, dx$

Optimal. Leaf size=45

$$-ix^2+2x\log\left(1-e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -I*x^2+2*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^2)-I*polylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^2(x)\right)-ix^2+2x\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^2],x]
```

```
[Out] (-I)*x^2 + 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*PolyLog[2, E^((
2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\csc^2(x)\right)\,dx &= x\log\left(a\csc^2(x)\right) - \int -2x\cot(x)\,dx\\
&= x\log\left(a\csc^2(x)\right) + 2\int x\cot(x)\,dx\\
&= -ix^2 + x\log\left(a\csc^2(x)\right) - 4i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -ix^2 + 2x\log\left(1-e^{2ix}\right) + x\log\left(a\csc^2(x)\right) - 2\int \log\left(1-e^{2ix}\right)\,dx\\
&= -ix^2 + 2x\log\left(1-e^{2ix}\right) + x\log\left(a\csc^2(x)\right) + i\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2ix}\right)\\
&= -ix^2 + 2x\log\left(1-e^{2ix}\right) + x\log\left(a\csc^2(x)\right) - i\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.93

$$2x\log\left(1-e^{2ix}\right) + x\log\left(a\csc^2(x)\right) - i\left(x^2 + \text{Li}_2\left(e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^2],x]
```

```
[Out] 2*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^2] - I*(x^2 + PolyLog[2, E^((2*I)
*x)])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(39) = 78.
time = 0.12, size = 88, normalized size = 1.96

method	result

default	$-i\left(\ln\left(\mathrm{e}^{ix}\right)\ln\left(-\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)+2\ln\left(\mathrm{e}^{ix}\right)\ln\left(\mathrm{e}^{ix}+1\right)-\ln\left(\mathrm{e}^{ix}\right)^{2}-2\,\mathrm{dilog}\left(\mathrm{e}^{ix}\right)+2\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+\right.$
risch	$\frac{ix\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2ix}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)^{2}}{2}-ix\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\right)\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)^{2}\right)^{2}+\frac{ix\pi\mathrm{csgn}\left(\frac{ia\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}-1\right)^{2}}\right)^{3}}{2}+ix\pi$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(ln(exp(I*x))*ln(-a*exp(I*x)^2/(exp(I*x)^2-1)^2)+2*ln(exp(I*x))*ln(exp(I
*x)+1)-ln(exp(I*x))^2-2*dilog(exp(I*x))+2*dilog(exp(I*x)+1)+2*ln(2)*ln(exp(
I*x)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(34) = 68$.
time = 0.58, size = 87, normalized size = 1.93

$$-i\,x^2+2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)-2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(a\csc\left(x\right)^2\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right)-2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)-2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="maxima")
```

```
[Out] -I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -cos(x)
+ 1) + x*log(a*csc(x)^2) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*lo
g(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(
I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(34) = 68$.
time = 0.44, size = 107, normalized size = 2.38

$$x\log\left(-\frac{a}{\cos\left(x\right)^2-1}\right)+x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)+x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))+i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))+i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))-i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="fricas")
```

```
[Out] x*log(-a/(cos(x)^2 - 1)) + x*log(cos(x) + I*sin(x) + 1) + x*log(cos(x) - I*
sin(x) + 1) + x*log(-cos(x) + I*sin(x) + 1) + x*log(-cos(x) - I*sin(x) + 1)
 - I*dilog(cos(x) + I*sin(x)) + I*dilog(cos(x) - I*sin(x)) + I*dilog(-cos(x
) + I*sin(x)) - I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\csc^2\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**2),x)
```

```
[Out] Integral(log(a*csc(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csc(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\frac{a}{\sin\left(x\right)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sin(x)^2),x)
```

```
[Out] int(log(a/sin(x)^2), x)
```

3.178 $\int \log(a\csc^n(x))\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] -1/2*I*n*x^2+n*x*ln(1-exp(2*I*x))+x*ln(a*csc(x)^n)-1/2*I*n*polylog(2,exp(2*
I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3798, 2221, 2317, 2438}

$$-\frac{1}{2}in\text{PolyLog}\left(2,e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csc[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Pol
yLog[2, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\csc^n(x)\right)\,dx &= x\log\left(a\csc^n(x)\right)+\int nx\cot(x)\,dx\\
&= x\log\left(a\csc^n(x)\right)+n\int x\cot(x)\,dx\\
&= -\frac{1}{2}inx^2+x\log\left(a\csc^n(x)\right)-(2in)\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-n\int \log\left(1-e^{2ix}\right)\,dx\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)+\frac{1}{2}(in)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2i
\\
&= -\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.00

$$-\frac{1}{2}inx^2+nx\log\left(1-e^{2ix}\right)+x\log\left(a\csc^n(x)\right)-\frac{1}{2}in\text{Li}_2\left(e^{2ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csc[x]^n],x]
```

```
[Out] (-1/2*I)*n*x^2 + n*x*Log[1 - E^((2*I)*x)] + x*Log[a*Csc[x]^n] - (I/2)*n*Pol
yLog[2, E^((2*I)*x)]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \ln\left(a(\csc^n(x))\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csc(x)^n),x)
```

```
[Out] int(ln(a*csc(x)^n),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(36) = 72$.
time = 0.64, size = 91, normalized size = 1.78

$$\frac{1}{2}\left(-i\,x^2 + 2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) - 2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right) + x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right) + x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right) - 2i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) - 2i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)\right)n + x\log\left(a\csc\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -co
s(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))*n + x*lo
g(a*csc(x)^n)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(36) = 72$.
time = 0.45, size = 117, normalized size = 2.29

$$nx\log\left(\frac{1}{\sin\left(x\right)}\right) + \frac{1}{2}nx\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right) + \frac{1}{2}nx\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right) + \frac{1}{2}nx\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right) + \frac{1}{2}nx\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right) - \frac{1}{2}i\,n\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right)) + \frac{1}{2}i\,n\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right)) + \frac{1}{2}i\,n\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right)) - \frac{1}{2}i\,n\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right)) + x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(1/sin(x)) + 1/2*n*x*log(cos(x) + I*sin(x) + 1) + 1/2*n*x*log(cos(x)
 - I*sin(x) + 1) + 1/2*n*x*log(-cos(x) + I*sin(x) + 1) + 1/2*n*x*log(-cos(x
) - I*sin(x) + 1) - 1/2*I*n*dilog(cos(x) + I*sin(x)) + 1/2*I*n*dilog(cos(x)
 - I*sin(x)) + 1/2*I*n*dilog(-cos(x) + I*sin(x)) - 1/2*I*n*dilog(-cos(x) -
I*sin(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\csc^n\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csc(x)**n),x)
```

```
[Out] Integral(log(a*csc(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csc(x)^n),x, algorithm="giac")
[Out] integrate(log(a*csc(x)^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\left(\frac{1}{\sin\left(x\right)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/sin(x))^n),x)
[Out] int(log(a*(1/sin(x))^n), x)
```

3.179 $\int \cos(x) \log\left(\frac{1}{2}(1-\cos(2x))\right) \, dx$

Optimal. Leaf size=21

$$-2\sin(x)+\log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)$$

```
[Out] -2*sin(x)+ln(1/2-1/2*cos(2*x))*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2717, 2634, 12}

$$\sin(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)-2\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[(1 - Cos[2*x])/2],x]
[Out] -2*Sin[x] + Log[(1 - Cos[2*x])/2]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(x)\log\left(\frac{1}{2}(1-\cos(2x))\right)\,dx &= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)-\int 2\cos(x)\,dx\\
&= \log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)-2\int \cos(x)\,dx\\
&= -2\sin(x)+\log\left(\frac{1}{2}(1-\cos(2x))\right)\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 0.62

$$-2\sin(x) + \log\left(\sin^2(x)\right)\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[(1 - Cos[2*x])/2],x]
```

```
[Out] -2*Sin[x] + Log[Sin[x]^2]*Sin[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 112, normalized size = 5.33

method	result
default	$-\frac{i\left(\mathrm{e}^{ix}\ln\left(\left(-\mathrm{e}^{4ix}+2\,\mathrm{e}^{2ix}-1\right)\mathrm{e}^{-2ix}\right)-2\,\mathrm{e}^{ix}-\mathrm{e}^{-ix}\ln\left(\left(-\mathrm{e}^{4ix}+2\,\mathrm{e}^{2ix}-1\right)\mathrm{e}^{-2ix}\right)+2\,\mathrm{e}^{-ix}-2\ln(2)\mathrm{e}^{ix}+2\,\mathrm{e}^{-ix}\ln(2)\right)}{2}$
risch	$-\frac{\mathrm{e}^{-ix}\pi\,\mathrm{csgn}\left(-i\mathrm{e}^{4ix}+2i\mathrm{e}^{2ix}-i\right)\mathrm{csgn}\left(i\mathrm{e}^{-2ix}\right)\mathrm{csgn}(-2i+2i\cos(2x))}{4}+\frac{\mathrm{e}^{ix}\pi\,\mathrm{csgn}\left(-i\mathrm{e}^{4ix}+2i\mathrm{e}^{2ix}-i\right)\mathrm{csgn}\left(i\mathrm{e}^{-2ix}\right)\mathrm{csgn}(-2i+2i\cos(2x}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(1/2-1/2*cos(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(exp(I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)-2*exp(I*x)-exp
(-I*x)*ln((-exp(I*x)^4+2*exp(I*x)^2-1)/exp(I*x)^2)+2/exp(I*x)-2*ln(2)*exp(I
*x)+2/exp(I*x)*ln(2))
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$\log\left(-\frac{1}{2}\,\cos\left(2\,x\right) + \frac{1}{2}\right)\sin\left(x\right) - 2\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="maxima")
```

```
[Out] log(-1/2*cos(2*x) + 1/2)*sin(x) - 2*sin(x)
```

Fricas [A]
time = 0.39, size = 17, normalized size = 0.81

$$\log\left(-\cos\left(x\right)^2 + 1\right)\sin\left(x\right) - 2\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="fricas")
```

```
[Out] log(-cos(x)^2 + 1)*sin(x) - 2*sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\frac{1}{2} - \frac{\cos(2x)}{2}\right)\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(1/2-1/2*cos(2*x)),x)
[Out] Integral(log(1/2 - cos(2*x)/2)*cos(x), x)
```

Giac [A]
time = 2.63, size = 13, normalized size = 0.62

$$\log\left(\sin(x)^2\right)\sin(x) - 2\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(1/2-1/2*cos(2*x)),x, algorithm="giac")
[Out] log(sin(x)^2)*sin(x) - 2*sin(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \ln\left(\frac{1}{2} - \frac{\cos(2\,x)}{2}\right)\,\cos(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/2 - cos(2*x)/2)*cos(x),x)
[Out] int(log(1/2 - cos(2*x)/2)*cos(x), x)
```

3.180 $\int \frac{\cot(x)}{\log(e\sin(x))}\,dx$

Optimal. Leaf size=6

$$\log(\log(e\sin(x)))$$

```
[Out] ln(ln(exp(1)*sin(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4423, 31}

$$\log(\log(\sin(x))+1)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[E*Sin[x]],x]
```

```
[Out] Log[1 + Log[Sin[x]]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFactors[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\log(e\sin(x))}\,dx &= \text{Subst}\left(\int \frac{1}{x+x\log(x)}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\log(\sin(x))\right)\\ &= \log(1+\log(\sin(x)))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\log(1+\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E*Sin[x]],x]
[Out] Log[1 + Log[Sin[x]]]
```

Maple [A]
time = 0.09, size = 8, normalized size = 1.33

method	result
derivativedivides	$\ln(\ln(\mathrm{e}\sin(x)))$
default	$\ln(\ln(\mathrm{e}\sin(x)))$
risch	$\ln\left(-\frac{i\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\mathrm{csgn}(i\mathrm{e}^{-ix})\mathrm{csgn}(\sin(x))}{2}-\frac{i\pi\,\mathrm{csgn}(i(\mathrm{e}^{2ix}-1))\mathrm{csgn}(\sin(x))^2}{2}-\frac{i\pi\,\mathrm{csgn}(i\mathrm{e}^{-ix})\mathrm{csgn}(\sin(x\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(exp(1)*sin(x)),x,method=_RETURNVERBOSE)
[Out] ln(ln(exp(1)*sin(x)))
```

Maxima [A]
time = 0.28, size = 7, normalized size = 1.17

$$\log(\log(e\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="maxima")
[Out] log(log(e*sin(x)))
```

Fricas [A]
time = 0.42, size = 7, normalized size = 1.17

$$\log(\log(e\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="fricas")
[Out] log(log(e*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log(\sin(x))+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(exp(1)*sin(x)),x)
[Out] Integral(cot(x)/(log(sin(x)) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(7) = 14$.
time = 5.47, size = 24, normalized size = 4.00

$$\frac{1}{2}\log\left(\frac{1}{4}\pi^2(\mathrm{sgn}(\sin(x))-1)^2+(\log(|\sin(x)|)+1)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(1)*sin(x)),x, algorithm="giac")
[Out] 1/2*log(1/4*pi^2*(sgn(sin(x)) - 1)^2 + (log(abs(sin(x))) + 1)^2)
```

Mupad [B]
time = 0.50, size = 6, normalized size = 1.00

$$\ln(\ln(\sin(x))+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/log(exp(1)*sin(x)),x)
[Out] log(log(sin(x)) + 1)
```

3.181 $\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)} dx$

Optimal. Leaf size=37

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)}-\frac{\log(\sin(x))}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$

```
[Out] ln(ln(exp(sin(x))))/(-ln(exp(sin(x)))+sin(x))-ln(sin(x))/(-ln(exp(sin(x)))+
sin(x))
```

Rubi [A]

time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4423, 2191, 2188, 29}

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\sin(x)-\log\left(e^{\sin(x)}\right)}-\frac{\log(\sin(x))}{\sin(x)-\log\left(e^{\sin(x)}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[E^Sin[x]],x]
```

```
[Out] Log[Log[E^Sin[x]]]/(-Log[E^Sin[x]] + Sin[x]) - Log[Sin[x]]/(-Log[E^Sin[x]]
+ Sin[x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rule 2191

```
Int[1/((u_)*(v_)), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D
[v, x]]}, Dist[b/(b*u - a*v), Int[1/v, x], x] - Dist[a/(b*u - a*v), Int[1/u
, x], x] /; NeQ[b*u - a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\log\left(e^{\sin(x)}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{x\log\left(e^{x}\right)}\,dx, x, \sin(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{\log(e^{x})}\,dx, x, \sin(x)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)}\\
&= \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log\left(e^{\sin(x)}\right)\right)}{-\log\left(e^{\sin(x)}\right)+\sin(x)}\\
&= -\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)}{\log\left(e^{\sin(x)}\right)-\sin(x)} + \frac{\log(\sin(x))}{\log\left(e^{\sin(x)}\right)-\sin(x)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.68

$$\frac{\log\left(\log\left(e^{\sin(x)}\right)\right)-\log(\sin(x))}{-\log\left(e^{\sin(x)}\right)+\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[E^Sin[x]],x]
[Out] (Log[Log[E^Sin[x]]] - Log[Sin[x]])/(-Log[E^Sin[x]] + Sin[x])
```

Maple [A]
time = 0.10, size = 35, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{\ln\left(\ln\left(e^{\sin(x)}\right)\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\ln(\sin(x))}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$	35
default	$-\frac{\ln\left(\ln\left(e^{\sin(x)}\right)\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\ln(\sin(x))}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$	35
risch	$\frac{\ln(e^{ix}+1)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}+\frac{\ln(e^{ix}-1)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}-\frac{\ln\left(2ie^{ix}\ln\left(e^{-\frac{ie^{ix}}{2}}e^{\frac{ie^{-ix}}{2}}\right)\right)}{\ln\left(e^{\sin(x)}\right)-\sin(x)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(exp(sin(x))),x,method=_RETURNVERBOSE)
[Out] -1/(ln(exp(sin(x)))-sin(x))*ln(ln(exp(sin(x))))+1/(ln(exp(sin(x)))-sin(x))*
ln(sin(x))
```

Maxima [A]
time = 0.27, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A]
time = 0.36, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\log(e^{\sin(x)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(exp(sin(x))),x)
```

```
[Out] Integral(cot(x)/log(exp(sin(x))), x)
```

Giac [A]
time = 5.21, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(exp(sin(x))),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

Mupad [B]
time = 0.37, size = 6, normalized size = 0.16

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/log(exp(sin(x))),x)
```

```
[Out] -1/sin(x)
```

3.182 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x + \tan(x) + \log(\cos(x)) \tan(x)$$

```
[Out] -x+tan(x)+ln(cos(x))*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8, 2634, 3554}

$$-x + \tan(x) + \tan(x) \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \log(\cos(x))\sec^2(x)\,dx = \log(\cos(x))\tan(x) + \int \tan^2(x)\,dx$$
$$= \tan(x) + \log(\cos(x))\tan(x) - \int 1\,dx$$
$$= -x + \tan(x) + \log(\cos(x))\tan(x)$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-x + \tan(x) + \log(\cos(x))\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 67, normalized size = 5.58

method	result
norman	$\frac{x-x\left(\tan^2\left(\frac{x}{2}\right)\right)-2\tan\left(\frac{x}{2}\right)\ln\left(\frac{1-\left(\tan^2\left(\frac{x}{2}\right)\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$
default	$-4i\left(\frac{\frac{e^{2ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)}{2}-\frac{1}{2}}{1+e^{2ix}}-\frac{\ln\left(1+e^{2ix}\right)}{4}+\frac{\ln(2)}{2+2\,e^{2ix}}\right)$
risch	$-\frac{2i\ln\left(e^{ix}\right)}{1+e^{2ix}}+\frac{\pi\,\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)\mathrm{csgn}\left(i\cos(x)\right)-\pi\,\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}\left(i\cos(x)\right)^2-\pi\,\mathrm{csgn}\left(i\left(1+e^{2ix}\right)\right)\mathrm{csgn}\left(i\cos(x)\right)^2+\pi\mathrm{cs}}{1+e^{2ix}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I*((1/2*exp(2*I*x)*ln((exp(I*x)^2+1)/exp(I*x))-1/2)/(1+exp(2*I*x))-1/4*l
n(1+exp(2*I*x))+1/2*ln(2)/(exp(I*x)^2+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(12) = 24.
time = 0.51, size = 94, normalized size = 7.83

$$-\frac{2\,\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.39, size = 22, normalized size = 1.83

$$-\frac{x\cos(x) - \log(\cos(x))\sin(x) - \sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A]
time = 32.75, size = 15, normalized size = 1.25

$$-x + \log(\cos(x))\tan(x) + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Giac [A]
time = 5.07, size = 12, normalized size = 1.00

$$\log(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

Mupad [B]
time = 0.58, size = 35, normalized size = 2.92

$$\tan(x) - 2\,x + \ln\left(\cos(x)\right)\,\tan(x) + \ln\left(\cos(x)\right)\,1\mathrm{i} - \ln\left(\cos(2\,x) + 1 + \sin(2\,x)\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))/cos(x)^2,x)
```

```
[Out] log(cos(x))*1i - 2*x - log(cos(2*x) + sin(2*x)*1i + 1)*1i + tan(x) + log(co
s(x))*tan(x)
```

3.183 $\int \cot(x)\log(\sin(x))\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\sin(x))$$

```
[Out] 1/2*ln(sin(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4423, 2338}

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Log[Sin[x]],x]
```

```
[Out] Log[Sin[x]]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \cot(x)\log(\sin(x))\,dx &= \text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\log^2(\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{1}{2}\log^2(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Log[Sin[x]],x]
[Out] Log[Sin[x]]^2/2
```

Maple [A]
time = 0.12, size = 8, normalized size = 0.89

method	result
derivativedivides	$\frac{\ln(\sin(x))^2}{2}$
default	$\frac{\ln(\sin(x))^2}{2}$
risch	$-\frac{i\pi\ln(e^{2ix}-1)\,\mathrm{csgn}(\sin(x))\mathrm{csgn}(i\sin(x))^2}{2}+\frac{\pi\,\mathrm{csgn}(i(e^{2ix}-1))\mathrm{csgn}(ie^{-ix})\mathrm{csgn}(\sin(x))x}{2}+\frac{\pi\,\mathrm{csgn}(i(e^{2ix}-1))\mathrm{csgn}(}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sin(x))^2
```

Maxima [A]
time = 0.28, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="maxima")
[Out] 1/2*log(sin(x))^2
```

Fricas [A]
time = 0.36, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="fricas")
[Out] 1/2*log(sin(x))^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*ln(sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.47, size = 7, normalized size = 0.78

$$\frac{1}{2}\log\left(\sin\left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x))^2
```

Mupad [**B**]
time = 0.38, size = 7, normalized size = 0.78

$$\frac{\ln\left(\sin\left(x\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cot(x),x)
```

```
[Out] log(sin(x))^2/2
```

3.184 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{9}\sin^3(x)+\frac{1}{3}\log(\sin(x))\sin^3(x)$$

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2644, 30, 2634, 12}

$$\frac{1}{3}\sin^3(x)\log(\sin(x))-\frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -1/9*Sin[x]^3 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\log(\sin(x))\sin^2(x)\,dx &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx \\
&= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) \\
&= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.75

$$\frac{1}{9}(-1+3\log(\sin(x)))\sin^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A]
time = 0.12, size = 17, normalized size = 0.85

method	result
derivativedivides	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$
default	$-\frac{(\sin^3(x))}{9}+\frac{\ln(\sin(x))(\sin^3(x))}{3}$
risch	$-\frac{e^{-ix}\pi \mathrm{csgn}(\sin(x))^3}{16}+\frac{e^{ix}\pi \mathrm{csgn}(\sin(x))^3}{16}+2i\Big(\frac{\mathrm{csgn}(i\sin(x))^3\pi}{48}-\frac{\pi \mathrm{csgn}(i\sin(x))^2}{48}-\frac{\pi\,\mathrm{csgn}(ie^{2ix}-i)\,\mathrm{csgn}(ie}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.80

$$\frac{1}{3}\log(\sin(x))\sin(x)^3 - \frac{1}{9}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Fricas [A]
time = 0.43, size = 24, normalized size = 1.20

$$-\frac{1}{3}\left(\cos\left(x\right)^{2}-1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+\frac{1}{9}\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A]
time = 1.65, size = 17, normalized size = 0.85

$$\frac{\log\left(\sin\left(x\right)\right)\sin^{3}\left(x\right)}{3}-\frac{\sin^{3}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Giac [A]
time = 2.66, size = 16, normalized size = 0.80

$$\frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^{3}-\frac{1}{9}\sin\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Mupad [B]
time = 0.43, size = 11, normalized size = 0.55

$$\frac{\sin\left(x\right)^{3}\left(\ln\left(\sin\left(x\right)\right)-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x)*sin(x)^2,x)
[Out] (sin(x)^3*(log(sin(x)) - 1/3))/3
```

3.185 $\int \cos(a+bx) \log \left(\cos \left(\frac{a}{2}+\frac{bx}{2}\right) \sin \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \, dx$

Optimal. Leaf size=50

$$-\frac{\sin(a+bx)}{b}+\frac{\log \left(\cos \left(\frac{a}{2}+\frac{bx}{2}\right) \sin \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sin(a+bx)}{b}$$

```
[Out] -sin(b*x+a)/b+ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x))*sin(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2717, 2634}

$$\frac{\sin(a+bx) \log \left(\sin \left(\frac{a}{2}+\frac{bx}{2}\right) \cos \left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]]*Sin[a + b*x
])/b
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx) \log \left(\cos \left(\frac{a}{2}+\frac{bx}{2}\right) \sin \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \, dx &= \frac{\log \left(\cos \left(\frac{a}{2}+\frac{bx}{2}\right) \sin \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sin(a+bx)}{b}-\int \cos\\ &= -\frac{\sin(a+bx)}{b}+\frac{\log \left(\cos \left(\frac{a}{2}+\frac{bx}{2}\right) \sin \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sin}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.66

$$-\frac{\sin(a+bx)}{b}+\frac{\log \left(\frac{1}{2} \sin(a+bx)\right) \sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Log[Cos[a/2 + (b*x)/2]*Sin[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sin[a + b*x]/b) + (Log[Sin[a + b*x]/2]*Sin[a + b*x])/b
```

Maple [A]
time = 0.52, size = 30, normalized size = 0.60

method	result	size
default	$\frac{\ln\left(\frac{\sin(bx+a)}{2}\right)\sin(bx+a)-\sin(bx+a)}{b}$	30
risch	Expression too large to display	1389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/b*(ln(1/2*sin(b*x+a))*sin(b*x+a)-sin(b*x+a))
```

Maxima [A]
time = 0.30, size = 42, normalized size = 0.84

$$\frac{\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(bx+a\right)}{b}-\frac{\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="maxima")
```

```
[Out] log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(b*x + a)/b - sin(b*x + a
)/b
```

Fricas [A]
time = 0.39, size = 65, normalized size = 1.30

$$\frac{2\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\log\left(\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="fricas")
```

```
[Out] 2*(cos(1/2*b*x + 1/2*a)*log(cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))*sin(
1/2*b*x + 1/2*a) - cos(1/2*b*x + 1/2*a)*sin(1/2*b*x + 1/2*a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*ln(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x)
```

```
[Out] Integral(log(sin(a/2 + b*x/2)*cos(a/2 + b*x/2))*cos(a + b*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*log(cos(1/2*a+1/2*b*x)*sin(1/2*a+1/2*b*x)),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 0.52, size = 29, normalized size = 0.58

$$-\frac{\sin\left(a+b\,x\right)-\ln\left(\frac{\sin\left(a+b\,x\right)}{2}\right)\,\sin\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(a/2 + (b*x)/2)*sin(a/2 + (b*x)/2))*cos(a + b*x),x)
```

```
[Out] -(sin(a + b*x) - log(sin(a + b*x)/2)*sin(a + b*x))/b
```

3.186 $\int \frac{\tan(x)}{\log(\cos(x))} \, dx$

Optimal. Leaf size=6

$$-\log(\log(\cos(x)))$$

```
[Out] -ln(ln(cos(x)))
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4424, 2339, 29}

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Log[Cos[x]],x]
```

```
[Out] -Log[Log[Cos[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\log(\cos(x))} \, dx &= -\text{Subst}\left(\int \frac{1}{x \log(x)} \, dx, x, \cos(x)\right) \\
&= -\text{Subst}\left(\int \frac{1}{x} \, dx, x, \log(\cos(x))\right) \\
&= -\log(\log(\cos(x)))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$-\log(\log(\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Log[Cos[x]],x]
[Out] -Log[Log[Cos[x]]]
```

Maple [A]
time = 0.05, size = 7, normalized size = 1.17

method	result
derivativedivides	$-\ln(\ln(\cos(x)))$
default	$-\ln(\ln(\cos(x)))$
risch	$-\ln\left(\frac{i\pi\,\mathrm{csgn}(ie^{-ix})\mathrm{csgn}(i(1+e^{2ix}))\mathrm{csgn}(i\cos(x))}{2}-\frac{i\pi\,\mathrm{csgn}(ie^{-ix})\mathrm{csgn}(i\cos(x))^2}{2}-\frac{i\pi\,\mathrm{csgn}(i(1+e^{2ix}))\mathrm{csgn}(i}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/ln(cos(x)),x,method=_RETURNVERBOSE)
[Out] -ln(ln(cos(x)))
```

Maxima [A]
time = 0.29, size = 6, normalized size = 1.00

$$-\log(\log(\cos(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="maxima")
[Out] -log(log(cos(x)))
```

Fricas [A]
time = 0.39, size = 6, normalized size = 1.00

$$-\log(\log(\cos(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="fricas")
[Out] -log(log(cos(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan (x)}{\log (\cos (x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/ln(cos(x)),x)
[Out] Integral(tan(x)/log(cos(x)), x)
```

Giac [A]
time = 6.16, size = 7, normalized size = 1.17

$$-\log (|\log (\cos (x))|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/log(cos(x)),x, algorithm="giac")
[Out] -log(abs(log(cos(x))))
```

Mupad [B]
time = 0.40, size = 6, normalized size = 1.00

$$-\ln (\ln (\cos (x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/log(cos(x)),x)
[Out] -log(log(cos(x)))
```

3.187 $\int \log(\cos(x))\tan(x)\,dx$

Optimal. Leaf size=9

$$-\frac{1}{2}\log^2(\cos(x))$$

```
[Out] -1/2*ln(cos(x))^2
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3556, 4424, 2338}

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Tan[x],x]
```

```
[Out] -1/2*Log[Cos[x]]^2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4424

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, ta
n])
```

Rubi steps

$$\begin{aligned}\int \log(\cos(x))\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, \cos(x)\right)\\ &= -\frac{1}{2}\log^2(\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\frac{1}{2}\log^2(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Tan[x],x]
[Out] -1/2*Log[Cos[x]]^2
```

Maple [A]
time = 0.09, size = 8, normalized size = 0.89

method	result
derivativedivides	$-\frac{\ln(\cos(x))^2}{2}$
default	$-\frac{\ln(\cos(x))^2}{2}$
risch	$-i(i\ln(1+\mathrm{e}^{2ix})+x)\ln(\mathrm{e}^{ix})+\frac{\pi\,\mathrm{csgn}(i\mathrm{e}^{-ix})\mathrm{csgn}(i(1+\mathrm{e}^{2ix}))\mathrm{csgn}(i\cos(x))x}{2}-\frac{\pi\,\mathrm{csgn}(i\mathrm{e}^{-ix})\mathrm{csgn}(i\cos(x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*tan(x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(cos(x))^2
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.78

$$-\frac{1}{2}\log\left(\cos\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="maxima")
[Out] -1/2*log(cos(x))^2
```

Fricas [A]
time = 0.38, size = 7, normalized size = 0.78

$$-\frac{1}{2}\log\left(\cos\left(x\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="fricas")
[Out] -1/2*log(cos(x))^2
```

Sympy [A]
time = 3.41, size = 8, normalized size = 0.89

$$-\frac{\log{(\cos{(x)})}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*tan(x),x)
```

```
[Out] -log(cos(x))**2/2
```

Giac [A]
time = 5.72, size = 7, normalized size = 0.78

$$-\frac{1}{2}\,\log{(\cos{(x)})}^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x))^2
```

Mupad [B]
time = 0.43, size = 7, normalized size = 0.78

$$-\frac{\ln{(\cos{(x)})}^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*tan(x),x)
```

```
[Out] -log(cos(x))^2/2
```

3.188 $\int \log(\cos(x)) \sin(x) \, dx$

Optimal. Leaf size=10

$$\cos(x) - \cos(x)\log(\cos(x))$$

```
[Out] cos(x)-cos(x)*ln(cos(x))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2718, 2634}

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sin[x],x]
```

```
[Out] Cos[x] - Cos[x]*Log[Cos[x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \log(\cos(x))\sin(x)\,dx &= -\cos(x)\log(\cos(x)) - \int \sin(x)\,dx \\ &= \cos(x) - \cos(x)\log(\cos(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\cos(x) - \cos(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sin[x],x]
```

[Out] Cos[x] - Cos[x]*Log[Cos[x]]

Maple [A]
time = 0.07, size = 11, normalized size = 1.10

method	result
derivativedivides	$\cos \left(x \right)-\cos \left(x \right) \ln \left(\cos \left(x \right)\right)$
default	$\cos \left(x \right)-\cos \left(x \right) \ln \left(\cos \left(x \right)\right)$
norman	$\frac{\left(\tan^{2}\left(\frac{x}{2}\right)\right) \ln \left(\frac{1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)-\ln \left(\frac{1-\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)+2}{1+\tan^{2}\left(\frac{x}{2}\right)}$
risch	$\ln \left({\mathrm e}^{i x}\right) \cos \left(x \right)+\frac{i \pi \,{\mathrm e}^{i x} \mathrm{csgn}\left(i {\mathrm e}^{2 i x}+i \right) \mathrm{csgn}\left(i {\mathrm e}^{-i x}\right) \mathrm{csgn}\left(i \cos \left(x \right)\right)}{4}-\frac{i \pi \,{\mathrm e}^{i x} \mathrm{csgn}\left(i {\mathrm e}^{2 i x}+i \right) \mathrm{csgn}\left(i \cos \left(x \right)\right)^{2}}{4}-\frac{i \pi \,{\mathrm e}^{i x}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(cos(x))*sin(x),x,method=_RETURNVERBOSE)

[Out] cos(x)-cos(x)*ln(cos(x))

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$-\cos\left(x\right)\log\left(\cos\left(x\right)\right)+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(cos(x))*sin(x),x, algorithm="maxima")

[Out] -cos(x)*log(cos(x)) + cos(x)

Fricas [A]
time = 0.38, size = 10, normalized size = 1.00

$$-\cos\left(x\right)\log\left(\cos\left(x\right)\right)+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(cos(x))*sin(x),x, algorithm="fricas")

[Out] -cos(x)*log(cos(x)) + cos(x)

Sympy [A]
time = 0.48, size = 10, normalized size = 1.00

$$-\log\left(\cos\left(x\right)\right)\cos\left(x\right)+\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(cos(x))*sin(x),x)

```
[Out] -log(cos(x))*cos(x) + cos(x)
```

Giac [**A**]
time = 5.18, size = 10, normalized size = 1.00

$$-\cos(x)\log(\cos(x)) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)*log(cos(x)) + cos(x)
```

Mupad [**B**]
time = 0.38, size = 9, normalized size = 0.90

$$-\cos(x)\ (\ln(\cos(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*sin(x),x)
```

```
[Out] -cos(x)*(log(cos(x)) - 1)
```

3.189 $\int \cos(x)\log(\cos(x))\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)$$

```
[Out] arctanh(sin(x))-sin(x)+ln(cos(x))*sin(x)
```

Rubi [A]

time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {2717, 2634, 2672, 327, 212}

$$-\sin(x) + \tanh^{-1}(\sin(x)) + \sin(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Cos[x]],x]
```

```
[Out] ArcTanh[Sin[x]] - Sin[x] + Log[Cos[x]]*Sin[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\cos(x))\,dx &= \log(\cos(x))\sin(x) + \int \sin(x)\tan(x)\,dx\\ &= \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(x)\right)\\ &= -\sin(x) + \log(\cos(x))\sin(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \tanh^{-1}(\sin(x)) - \sin(x) + \log(\cos(x))\sin(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(14) = 28.
time = 0.01, size = 43, normalized size = 3.07

$$-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\sin(x)+\log(\cos(x))\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Cos[x]],x]
```

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] - Sin[x] + Log[Cos[x]]
*Sin[x]
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 93, normalized size = 6.64

method	result
default	$-\frac{i\left(e^{ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)-e^{ix}+4\arctan\left(e^{ix}\right)-e^{-ix}\ln\left(\left(1+e^{2ix}\right)e^{-ix}\right)+e^{-ix}-\ln(2)e^{ix}+e^{-ix}\ln(2)\right)}{2}$
risch	$-\ln\left(e^{ix}\right)\sin\left(x\right)-\frac{e^{ix}\pi\,\text{csgn}\left(ie^{-ix}\right)\text{csgn}\left(ie^{2ix}+i\right)\text{csgn}\left(i\cos(x)\right)}{4}+\frac{e^{ix}\pi\,\text{csgn}\left(ie^{-ix}\right)\text{csgn}\left(i\cos(x)\right)^2}{4}+\frac{e^{ix}\pi\,\text{csgn}\left(ie^{2ix}+i\right)\text{cs}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(cos(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(exp(I*x)*ln((exp(I*x)^2+1)/exp(I*x))-exp(I*x)+4*arctan(exp(I*x))-ex
p(-I*x)*ln((exp(I*x)^2+1)/exp(I*x))+exp(-I*x)-ln(2)*exp(I*x)+1/exp(I*x)*ln(
2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(14) = 28$.
time = 0.28, size = 108, normalized size = 7.71

$$\frac{2 \log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="maxima")
[Out] 2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)/
((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
+ 1)^2 + 1)*(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(
x) + 1) - 1)
```

Fricas [A]
time = 0.37, size = 27, normalized size = 1.93

$$\log(\cos(x))\sin(x) + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="fricas")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(15) = 30$.
time = 1.62, size = 223, normalized size = 15.93

$$-\frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(-\frac{\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}+\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} + \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)+1} - \frac{2\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(cos(x)),x)
[Out] -log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))*tan(x/2)**2/(tan
(x/2)**2 + 1) + 2*log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x/2)**2 + 1))
*tan(x/2)/(tan(x/2)**2 + 1) - log(-tan(x/2)**2/(tan(x/2)**2 + 1) + 1/(tan(x
/2)**2 + 1))/(tan(x/2)**2 + 1) + 2*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)*
*2 + 1) + 2*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)*tan(
x/2)**2/(tan(x/2)**2 + 1) - log(tan(x/2)**2 + 1)/(tan(x/2)**2 + 1) - 2*tan(
x/2)/(tan(x/2)**2 + 1)
```

Giac [A]
time = 4.95, size = 27, normalized size = 1.93

$$\log(\cos(x))\sin(x) + \frac{1}{2}\log(\sin(x)+1) - \frac{1}{2}\log(-\sin(x)+1) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(cos(x)),x, algorithm="giac")
[Out] log(cos(x))*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.07

$$\int \ln(\cos(x)) \cos(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cos(x))*cos(x),x)
[Out] int(log(cos(x))*cos(x), x)
```

3.190 $\int \cos(x)\log(\sin(x))\,dx$

Optimal. Leaf size=11

$$-\sin(x)+\log(\sin(x))\sin(x)$$

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Rubi [A]

time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2717, 2634}

$$\sin(x)\log(\sin(x))-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x)-\int \cos(x)\,dx\\ &= -\sin(x)+\log(\sin(x))\sin(x)\end{aligned}$$

Mathematica [A]

time = 0.00, size = 11, normalized size = 1.00

$$-\sin(x)+\log(\sin(x))\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A]
time = 0.08, size = 12, normalized size = 1.09

method	result
derivativedivides	$-\sin\left(x\right)+\ln\left(\sin\left(x\right)\right)\sin\left(x\right)$
default	$-\sin\left(x\right)+\ln\left(\sin\left(x\right)\right)\sin\left(x\right)$
norman	$\frac{2\tan\left(\frac{x}{2}\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}\right)-2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}$
risch	$-\frac{e^{-ix}\pi\mathrm{csgn}(\sin(x))^{3}}{4}+\frac{e^{ix}\pi\mathrm{csgn}(\sin(x))^{3}}{4}+\frac{e^{-ix}\pi\mathrm{csgn}(i\sin(x))^{3}}{4}+\frac{e^{ix}\pi\mathrm{csgn}(i\sin(x))^{2}}{4}-\frac{e^{-ix}\pi\mathrm{csgn}(i\sin(x))^{2}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x)),x,method=_RETURNVERBOSE)
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 1.00

$$\log\left(\sin\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
[Out] log(sin(x))*sin(x) - sin(x)
```

Fricas [A]
time = 0.38, size = 11, normalized size = 1.00

$$\log\left(\sin\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A]
time = 0.19, size = 10, normalized size = 0.91

$$\log\left(\sin\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Giac [**A**]
time = 3.56, size = 11, normalized size = 1.00

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
[Out] log(sin(x))*sin(x) - sin(x)
```

Mupad [**B**]
time = 0.41, size = 8, normalized size = 0.73

$$\sin(x)\ (\ln(\sin(x)) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*cos(x),x)
[Out] sin(x)*(log(sin(x)) - 1)
```

3.191 $\int \log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=74

$$\frac{x}{4}+\frac{ix^2}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}i\text{Li}_2\left(e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}\cos(x)\log(\sin(x))\sin(x)$$

```
[Out] 1/4*x+1/4*I*x^2-1/2*x*ln(1-exp(2*I*x))+1/2*x*ln(sin(x))+1/4*I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)-1/2*cos(x)*ln(sin(x))*sin(x)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {2715, 8, 2634, 12, 6874, 3798, 2221, 2317, 2438}

$$\frac{1}{4}i\text{PolyLog}\left(2,e^{2ix}\right)+\frac{ix^2}{4}+\frac{x}{4}-\frac{1}{2}x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\log(\sin(x))+\frac{1}{4}\sin(x)\cos(x)-\frac{1}{2}\sin(x)\cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] x/4 + (I/4)*x^2 - (x*Log[1 - E^((2*I)*x)])/2 + (x*Log[Sin[x]])/2 + (I/4)*Po
lyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (Cos[x]*Log[Sin[x]]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log(\sin(x))\sin^2(x)\,dx = \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \int \frac{1}{2}\cot(x)(x-\cos(x)\sin(x))\,dx$$
$$= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \cot(x)(x-\cos(x)\sin(x))\,dx$$
$$= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) - \frac{1}{2}\int \left(-\cos^2(x)+x\cot(x)\right)\,dx$$
$$= \frac{1}{2}x\log(\sin(x)) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + \frac{1}{2}\int \cos^2(x)\,dx - \frac{1}{2}\int x\cot(x)\,dx$$
$$= \frac{ix^2}{4} + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin(x))\sin(x) + i\int \frac{e^{2ix}x}{1-e^{2i}$$
$$= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin$$
$$= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}\cos(x)\log(\sin$$
$$= \frac{x}{4} + \frac{ix^2}{4} - \frac{1}{2}x\log\left(1-e^{2ix}\right) + \frac{1}{2}x\log(\sin(x)) + \frac{1}{4}i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}$$

Mathematica [**A**]
time = 0.04, size = 59, normalized size = 0.80

$$\frac{1}{8}\left(2x\left(1+ix-2\log\left(1-e^{2ix}\right)+2\log(\sin(x))\right)+2i\text{Li}_2\left(e^{2ix}\right)+(1-2\log(\sin(x)))\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] (2*x*(1 + I*x - 2*Log[1 - E^((2*I)*x)] + 2*Log[Sin[x]]) + (2*I)*PolyLog[2,
E^((2*I)*x)] + (1 - 2*Log[Sin[x]])*Sin[2*x])/8
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(54) = 108.
time = 0.12, size = 195, normalized size = 2.64

method	result
default	$\frac{i\left(\frac{\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)e^{2ix}}{2}-\frac{e^{2ix}}{4}-2\ln\left(e^{ix}\right)\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)-\ln\left(e^{ix}\right)^2+2\ln\left(e^{ix}\right)\ln\left(e^{ix}+1\right)-2\,\text{dilog}\left(e^{ix}\right)+2\,\text{dilog}\left(e^{ix}+1\right)-\frac{e^{-2ix}\ln(}{}\right.}{4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I*(1/2*ln(I*(-exp(I*x)^2+1)/exp(I*x))*exp(2*I*x)-1/4*exp(I*x)^2-2*ln(ex
p(I*x))*ln(I*(-exp(I*x)^2+1)/exp(I*x))-ln(exp(I*x))^2+2*ln(exp(I*x))*ln(exp
(I*x)+1)-2*dilog(exp(I*x))+2*dilog(exp(I*x)+1)-1/2*exp(-2*I*x)*ln(I*(-exp(I
*x)^2+1)/exp(I*x))+1/4/exp(I*x)^2-ln(exp(I*x))-1/2*ln(2)*exp(I*x)^2+1/2*ln(
2)/exp(I*x)^2+2*ln(2)*ln(exp(I*x)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.
time = 0.67, size = 104, normalized size = 1.41

$$\frac{1}{4}i\,x^2 - \frac{1}{2}i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right) + \frac{1}{2}i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right) - \frac{1}{4}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\cos\left(x\right)+1\right) - \frac{1}{4}\,x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\cos\left(x\right)+1\right) + \frac{1}{4}\left(2\,x-\sin\left(2\,x\right)\right)\log\left(\sin\left(x\right)\right) + \frac{1}{4}\,x + \frac{1}{2}i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(e^{(i\,x)}\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*I*x^2 - 1/2*I*x*arctan2(sin(x), cos(x) + 1) + 1/2*I*x*arctan2(sin(x), -
cos(x) + 1) - 1/4*x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 1/4*x*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 1/4*(2*x - sin(2*x))*log(sin(x)) + 1/4*x
 + 1/2*I*dilog(-e^(I*x)) + 1/2*I*dilog(e^(I*x)) + 1/8*sin(2*x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(49) = 98$.
time = 0.41, size = 120, normalized size = 1.62

$$-\frac{1}{4}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right) - \frac{1}{4}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right) - \frac{1}{4}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right) - \frac{1}{4}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right) - \frac{1}{2}\left(\cos\left(x\right)\sin\left(x\right)-x\right)\log\left(\sin\left(x\right)\right) + \frac{1}{4}\,\cos\left(x\right)\sin\left(x\right) + \frac{1}{4}\,x + \frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right)) - \frac{1}{4}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right)) - \frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right)) + \frac{1}{4}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*x*log(cos(x) + I*sin(x) + 1) - 1/4*x*log(cos(x) - I*sin(x) + 1) - 1/4*
x*log(-cos(x) + I*sin(x) + 1) - 1/4*x*log(-cos(x) - I*sin(x) + 1) - 1/2*(co
s(x)*sin(x) - x)*log(sin(x)) + 1/4*cos(x)*sin(x) + 1/4*x + 1/4*I*dilog(cos(
x) + I*sin(x)) - 1/4*I*dilog(cos(x) - I*sin(x)) - 1/4*I*dilog(-cos(x) + I*s
in(x)) + 1/4*I*dilog(-cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(x\right)\right)\sin^2\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**2,x)
```

```
[Out] Integral(log(sin(x))*sin(x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(sin(x))*sin(x)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln\left(\sin\left(x\right)\right)\,\sin\left(x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*sin(x)^2,x)
```

```
[Out] int(log(sin(x))*sin(x)^2, x)
```

3.192 $\int \log(\sin(x)) \sin^3(x) \, dx$

Optimal. Leaf size=40

$$-\frac{2}{3} \tanh^{-1}(\cos(x)) + \frac{2\cos(x)}{3} - \frac{\cos^3(x)}{9} - \cos(x)\log(\sin(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x))$$

```
[Out] -2/3*arctanh(cos(x))+2/3*cos(x)-1/9*cos(x)^3-cos(x)*ln(sin(x))+1/3*cos(x)^3
*ln(sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {2713, 2634, 12, 4451, 470, 327, 212}

$$-\frac{\cos^3(x)}{9} + \frac{2\cos(x)}{3} - \frac{2}{3}\tanh^{-1}(\cos(x)) + \frac{1}{3}\cos^3(x)\log(\sin(x)) - \cos(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (-2*ArcTanh[Cos[x]])/3 + (2*Cos[x])/3 - Cos[x]^3/9 - Cos[x]*Log[Sin[x]] + (
Cos[x]^3*Log[Sin[x]])/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \log(\sin(x))\sin^3(x)\,dx &= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\int \frac{1}{6}\cos(x)(-5+\cos(2x))\cot(x)\,d\\ &= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{1}{6}\int \cos(x)(-5+\cos(2x))\cot(x)\,d\\ &= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))+\frac{1}{6}\mathrm{Subst}\left(\int \frac{2x^2(-3+x^2)}{1-x^2}\,dx,x,\cos\right.\\ &= -\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))+\frac{1}{3}\mathrm{Subst}\left(\int \frac{x^2(-3+x^2)}{1-x^2}\,dx,x,\cos(\right.\\ &= -\frac{1}{9}\cos^3(x)-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{2}{3}\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,\right.\\ &= \frac{2\cos(x)}{3}-\frac{\cos^3(x)}{9}-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin(x))-\frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{1-}\right.\\ &= -\frac{2}{3}\tanh^{-1}(\cos(x))+\frac{2\cos(x)}{3}-\frac{\cos^3(x)}{9}-\cos(x)\log(\sin(x))+\frac{1}{3}\cos^3(x)\log(\sin\end{aligned}$$

Mathematica [A]

time = 0.03, size = 47, normalized size = 1.18

$$\frac{1}{36}\left(24\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+\cos(3x)(-1+3\log(\sin(x)))-3\cos(x)(-7+9\log(\sin(x)))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[x]]*Sin[x]^3,x]
```

```
[Out] (24*(-Log[Cos[x/2]] + Log[Sin[x/2]]) + Cos[3*x]*(-1 + 3*Log[Sin[x]]) - 3*Co
s[x]*(-7 + 9*Log[Sin[x]]))/36
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 214, normalized size = 5.35

method	result
default	$\frac{e^{3ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{24}-\frac{e^{3ix}}{72}+\frac{7e^{ix}}{24}+\frac{2\ln\left(e^{ix}-1\right)}{3}-\frac{2\ln\left(e^{ix}+1\right)}{3}-\frac{3e^{ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{8}-\frac{3e^{-ix}\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)}{8}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x))*sin(x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*exp(3*I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))-1/72*exp(I*x)^3+7/24*exp(I*x
)+2/3*ln(exp(I*x)-1)-2/3*ln(exp(I*x)+1)-3/8*exp(I*x)*ln(I*(-exp(I*x)^2+1)/e
xp(I*x))-3/8*exp(-I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))+7/24/exp(I*x)+1/24*ex
p(-3*I*x)*ln(I*(-exp(I*x)^2+1)/exp(I*x))-1/72/exp(I*x)^3-1/24*ln(2)*exp(I*x
)^3+3/8*ln(2)*exp(I*x)-1/24*ln(2)/exp(I*x)^3+3/8/exp(I*x)*ln(2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(32) = 64.
time = 0.29, size = 179, normalized size = 4.48

$$-\frac{4\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+1\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)}{3\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}+\frac{2\left(\frac{12\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+5\right)}{9\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\sin(x)^4}{(\cos(x)+1)^4}+\frac{\sin(x)^6}{(\cos(x)+1)^6}+1\right)}-\frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)+\frac{2}{3}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -4/3*(3*sin(x)^2/(cos(x) + 1)^2 + 1)*log(2*sin(x)/((sin(x)^2/(cos(x) + 1)^2
 + 1)*(cos(x) + 1)))/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1)^4
 + sin(x)^6/(cos(x) + 1)^6 + 1) + 2/9*(12*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x
)^4/(cos(x) + 1)^4 + 5)/(3*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^4/(cos(x) + 1
)^4 + sin(x)^6/(cos(x) + 1)^6 + 1) - 2/3*log(sin(x)^2/(cos(x) + 1)^2 + 1) +
 2/3*log(sin(x)^2/(cos(x) + 1)^2)
```

Fricas [A]
time = 0.40, size = 43, normalized size = 1.08

$$-\frac{1}{9}\cos\left(x\right)^{3}+\frac{1}{3}\left(\cos\left(x\right)^{3}-3\cos\left(x\right)\right)\log\left(\sin\left(x\right)\right)+\frac{2}{3}\cos\left(x\right)-\frac{1}{3}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{3}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(1/2*cos(x) + 1/2) + 1/3*log(-1/2*cos(x) + 1/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(41) = 82.
time = 7.58, size = 456, normalized size = 11.40

$$\frac{12\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}\right)\tan^{6}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{36\log\left(\frac{\tan\left(\frac{x}{2}\right)}{\tan^{2}\left(\frac{x}{2}\right)+1}\right)\tan^{4}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{6\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)\tan^{6}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{18\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)\tan^{4}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{18\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)\tan^{2}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{6\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{12\log\left(2\right)\tan^{6}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{6\tan^{4}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{36\log\left(2\right)\tan^{4}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{24\tan^{2}\left(\frac{x}{2}\right)}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}+\frac{10}{9\tan^{6}\left(\frac{x}{2}\right)+27\tan^{4}\left(\frac{x}{2}\right)+27\tan^{2}\left(\frac{x}{2}\right)+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x))*sin(x)**3,x)
```

```
[Out] 12*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)
**4 + 27*tan(x/2)**2 + 9) + 36*log(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**4/
(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 +
 1)*tan(x/2)**6/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 18*
log(tan(x/2)**2 + 1)*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan(x
/2)**2 + 9) + 18*log(tan(x/2)**2 + 1)*tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(x
/2)**4 + 27*tan(x/2)**2 + 9) + 6*log(tan(x/2)**2 + 1)/(9*tan(x/2)**6 + 27*t
an(x/2)**4 + 27*tan(x/2)**2 + 9) + 12*log(2)*tan(x/2)**6/(9*tan(x/2)**6 + 2
7*tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 6*tan(x/2)**4/(9*tan(x/2)**6 + 27*tan
(x/2)**4 + 27*tan(x/2)**2 + 9) + 36*log(2)*tan(x/2)**4/(9*tan(x/2)**6 + 27*
tan(x/2)**4 + 27*tan(x/2)**2 + 9) + 24*tan(x/2)**2/(9*tan(x/2)**6 + 27*tan(
x/2)**4 + 27*tan(x/2)**2 + 9) + 10/(9*tan(x/2)**6 + 27*tan(x/2)**4 + 27*tan
(x/2)**2 + 9)
```

Giac [A]
time = 3.97, size = 41, normalized size = 1.02

$$-\frac{1}{9}\cos\left(x\right)^{3}+\frac{1}{3}\left(\cos\left(x\right)^{3}-3\cos\left(x\right)\right)\log\left(\sin\left(x\right)\right)+\frac{2}{3}\cos\left(x\right)-\frac{1}{3}\log\left(\cos\left(x\right)+1\right)+\frac{1}{3}\log\left(-\cos\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x))*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/9*cos(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*log(sin(x)) + 2/3*cos(x) - 1/3*lo
g(cos(x) + 1) + 1/3*log(-cos(x) + 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln(\sin(x)) \sin(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))*sin(x)^3,x)
```

```
[Out] int(log(sin(x))*sin(x)^3, x)
```

3.193 $\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$

Optimal. Leaf size=79

$$\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)$$

```
[Out] 1/3*I*x^(3/2)-x*ln(1-exp(2*I*x^(1/2)))+x*ln(sin(x^(1/2)))-1/2*polylog(3,exp
(2*I*x^(1/2)))+I*polylog(2,exp(2*I*x^(1/2)))*x^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 1.143$, Rules used = {2628, 12, 3833, 3798, 2221, 2611, 2320, 6724}

$$i\sqrt{x}\,\text{PolyLog}\left(2,e^{2i\sqrt{x}}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2i\sqrt{x}}\right)+\frac{1}{3}ix^{3/2}-x\log\left(1-e^{2i\sqrt{x}}\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sin[Sqrt[x]]],x]
```

```
[Out] (I/3)*x^(3/2) - x*Log[1 - E^((2*I)*Sqrt[x])] + x*Log[Sin[Sqrt[x]]] + I*Sqrt
[x]*PolyLog[2, E^((2*I)*Sqrt[x])] - PolyLog[3, E^((2*I)*Sqrt[x])]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3833

```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx = x\log\left(\sin\left(\sqrt{x}\right)\right) - \int \frac{1}{2}\sqrt{x}\cot\left(\sqrt{x}\right)\,dx$$
$$= x\log\left(\sin\left(\sqrt{x}\right)\right) - \frac{1}{2}\int \sqrt{x}\cot\left(\sqrt{x}\right)\,dx$$
$$= x\log\left(\sin\left(\sqrt{x}\right)\right) - \text{Subst}\left(\int x^2\cot(x)\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}ix^{3/2} + x\log\left(\sin\left(\sqrt{x}\right)\right) + 2i\text{Subst}\left(\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + 2\text{Subst}\left(\int x\log\left(1-e^{2ix}\right)\,dx, x,\right.$$
$$= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right) - i\text{Subst}\left(\int \text{Li}\right.$$
$$= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{\text{L}}{}\right.$$
$$= \frac{1}{3}ix^{3/2} - x\log\left(1-e^{2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) + i\sqrt{x}\,\text{Li}_2\left(e^{2i\sqrt{x}}\right) - \frac{1}{2}\text{Li}_3\left(e^{2i\sqrt{x}}\right)$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.11

$$\frac{i\pi^3}{24} - \frac{1}{3}ix^{3/2} - x\log\left(1-e^{-2i\sqrt{x}}\right) + x\log\left(\sin\left(\sqrt{x}\right)\right) - i\sqrt{x}\,\text{Li}_2\left(e^{-2i\sqrt{x}}\right) - \frac{1}{2}\text{Li}_3\left(e^{-2i\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sin[Sqrt[x]]],x]
[Out] (I/24)*Pi^3 - (I/3)*x^(3/2) - x*Log[1 - E^((-2*I)*Sqrt[x])] + x*Log[Sin[Sqr
t[x]]] - I*Sqrt[x]*PolyLog[2, E^((-2*I)*Sqrt[x])] - PolyLog[3, E^((-2*I)*Sq
rt[x])]/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \ln\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(sin(x^(1/2))),x)
[Out] int(ln(sin(x^(1/2))),x)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.
time = 0.29, size = 139, normalized size = 1.76

$$-i\,x\arctan\left(\sin\left(\sqrt{x}\right),\cos\left(\sqrt{x}\right)+1\right)+i\,x\arctan\left(\sin\left(\sqrt{x}\right),-\cos\left(\sqrt{x}\right)+1\right)-\frac{1}{2}\,x\log\left(\cos\left(\sqrt{x}\right)^{2}+\sin\left(\sqrt{x}\right)^{2}+2\,\cos\left(\sqrt{x}\right)+1\right)-\frac{1}{2}\,x\log\left(\cos\left(\sqrt{x}\right)^{2}+\sin\left(\sqrt{x}\right)^{2}-2\,\cos\left(\sqrt{x}\right)+1\right)+x\log\left(\sin\left(\sqrt{x}\right)\right)+\frac{1}{3}i\,x^{\frac{3}{2}}+2i\,\sqrt{x}\,\mathrm{Li}_2\left(-e^{\left(i\,\sqrt{x}\right)}\right)+2i\,\sqrt{x}\,\mathrm{Li}_2\left(e^{\left(i\,\sqrt{x}\right)}\right)-2\,\mathrm{Li}_3\left(-e^{\left(i\,\sqrt{x}\right)}\right)-2\,\mathrm{Li}_3\left(e^{\left(i\,\sqrt{x}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="maxima")
```

```
[Out] -I*x*arctan2(sin(sqrt(x)), cos(sqrt(x)) + 1) + I*x*arctan2(sin(sqrt(x)), -c
os(sqrt(x)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 + 2*cos(sqrt(x
)) + 1) - 1/2*x*log(cos(sqrt(x))^2 + sin(sqrt(x))^2 - 2*cos(sqrt(x)) + 1) +
 x*log(sin(sqrt(x))) + 1/3*I*x^(3/2) + 2*I*sqrt(x)*dilog(-e^(I*sqrt(x))) +
2*I*sqrt(x)*dilog(e^(I*sqrt(x))) - 2*polylog(3, -e^(I*sqrt(x))) - 2*polylog
(3, e^(I*sqrt(x)))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(log(sin(sqrt(x))), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sin\left(\sqrt{x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(sin(x**(1/2))),x)
```

```
[Out] Integral(log(sin(sqrt(x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(sin(x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(log(sin(sqrt(x))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \ln \left(\sin \left(\sqrt{x}\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x^(1/2))),x)
```

```
[Out] int(log(sin(x^(1/2))), x)
```

3.194 $\int \csc^2(x) \log(\sin(x)) \, dx$

Optimal. Leaf size=15

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

```
[Out] -x-cot(x)-cot(x)*ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3852, 8, 2634, 3554}

$$-x - \cot(x) - \cot(x) \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^2(x)\log(\sin(x))\,dx = -\cot(x)\log(\sin(x)) + \int \cot^2(x)\,dx$$
$$= -\cot(x) - \cot(x)\log(\sin(x)) - \int 1\,dx$$
$$= -x - \cot(x) - \cot(x)\log(\sin(x))$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-x - \cot(x) - \cot(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Log[Sin[x]],x]
```

```
[Out] -x - Cot[x] - Cot[x]*Log[Sin[x]]
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 81, normalized size = 5.40

method	result
norman	$\frac{-\frac{1}{2}+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}-x\tan\left(\frac{x}{2}\right)+\frac{\left(\tan^2\left(\frac{x}{2}\right)\right)\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}{2}-\frac{\ln\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}\right)}{2}}{\tan\left(\frac{x}{2}\right)}$
default	$4i\left(\frac{-\frac{\ln\left(i\left(1-e^{2ix}\right)e^{-ix}\right)e^{2ix}}{2}-\frac{1}{2}}{e^{2ix}-1}+\frac{\ln(e^{ix}-1)}{4}+\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(2)}{2\,e^{2ix}-2}\right)$
risch	$\frac{2i\ln(e^{ix})}{e^{2ix}-1} - \frac{i\ln(e^{2ix}-1)e^{2ix}-\pi\,\mathrm{csgn}\left(i\left(e^{2ix}-1\right)\right)\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(\sin(x))-\pi\,\mathrm{csgn}\left(i\left(e^{2ix}-1\right)\right)\mathrm{csgn}(\sin(x))^2-\pi\,\mathrm{csgn}\left(ie^{-ix}\right)\mathrm{csgn}(\sin}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*ln(sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*I*((-1/2*ln(I*(-exp(I*x)^2+1)/exp(I*x))*exp(2*I*x)-1/2)/(exp(2*I*x)-1)+1/
4*ln(exp(I*x)-1)+1/4*ln(exp(I*x)+1)+1/2*ln(2)/(exp(I*x)^2-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(15) = 30.
time = 0.51, size = 81, normalized size = 5.40

$$-\frac{1}{2}\left(\frac{\cos(x)+1}{\sin(x)}-\frac{\sin(x)}{\cos(x)+1}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right)-\frac{\cos(x)+1}{2\sin(x)}+\frac{\sin(x)}{2(\cos(x)+1)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(x) + 1)/sin(x) - sin(x)/(cos(x) + 1))*log(2*sin(x)/((sin(x)^2/(c
os(x) + 1)^2 + 1)*(cos(x) + 1))) - 1/2*(cos(x) + 1)/sin(x) + 1/2*sin(x)/(co
s(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 0.41, size = 19, normalized size = 1.27

$$-\frac{\cos(x)\log(\sin(x)) + x\sin(x) + \cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x)*log(sin(x)) + x*sin(x) + cos(x))/sin(x)
```

Sympy [A]
time = 25.44, size = 17, normalized size = 1.13

$$-x - \log(\sin(x))\cot(x) - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*ln(sin(x)),x)
```

```
[Out] -x - log(sin(x))*cot(x) - cos(x)/sin(x)
```

Giac [A]
time = 4.35, size = 19, normalized size = 1.27

$$-x - \frac{\log(\sin(x))}{\tan(x)} - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*log(sin(x)),x, algorithm="giac")
```

```
[Out] -x - log(sin(x))/tan(x) - 1/tan(x)
```

Mupad [B]
time = 0.58, size = 57, normalized size = 3.80

$$-2x - \ln\left(e^{x\,2i} - 1\right)\,1i - \frac{\ln\left(\frac{e^{-x\,1i}\,1i}{2} - \frac{e^{x\,1i}\,1i}{2}\right)\,2i}{e^{x\,2i} - 1} - \frac{2i}{e^{x\,2i} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(sin(x))/sin(x)^2,x)
```

```
[Out] - 2*x - log(exp(x*2i) - 1)*1i - (log((exp(-x*1i)*1i)/2 - (exp(x*1i)*1i)/2)*
2i)/(exp(x*2i) - 1) - 2i/(exp(x*2i) - 1)
```

3.195 $\int \log(x)\sinh(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{\cosh(a)\mathrm{Chi}(bx)}{b}+\frac{\cosh(a+bx)\log(x)}{b}-\frac{\sinh(a)\mathrm{Shi}(bx)}{b}$$

```
[Out] -Chi(b*x)*cosh(a)/b+cosh(b*x+a)*ln(x)/b-Shi(b*x)*sinh(a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2718, 2634, 12, 3384, 3379, 3382}

$$-\frac{\cosh(a)\mathrm{Chi}(bx)}{b}-\frac{\sinh(a)\mathrm{Shi}(bx)}{b}+\frac{\log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x],x]
```

```
[Out] -((Cosh[a]*CoshIntegral[b*x])/b) + (Cosh[a + b*x]*Log[x])/b - (Sinh[a]*Sinh
Integral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sinh(a+bx)\,dx &= \frac{\cosh(a+bx)\log(x)}{b} - \int \frac{\cosh(a+bx)}{bx}\,dx\\ &= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\int \frac{\cosh(a+bx)}{x}\,dx}{b}\\ &= \frac{\cosh(a+bx)\log(x)}{b} - \frac{\cosh(a)\int \frac{\cosh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\sinh(bx)}{x}\,dx}{b}\\ &= -\frac{\cosh(a)\text{Chi}(bx)}{b} + \frac{\cosh(a+bx)\log(x)}{b} - \frac{\sinh(a)\text{Shi}(bx)}{b}\end{aligned}$$

Mathematica [A]

time = 0.04, size = 30, normalized size = 0.86

$$-\frac{\cosh(a)\text{Chi}(bx) - \cosh(a+bx)\log(x) + \sinh(a)\text{Shi}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x],x]
[Out] -((Cosh[a]*CoshIntegral[b*x] - Cosh[a + b*x]*Log[x] + Sinh[a]*SinhIntegral[
b*x])/b)
```

Maple [A]

time = 0.07, size = 58, normalized size = 1.66

method	result
risch	$\left(\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}\right)\ln(x)+\frac{e^{-a}\,\text{expIntegral}(1,bx)}{2b}+\frac{e^{a}\,\text{expIntegral}(1,-bx)}{2b}$
meijerg	$-\frac{\sinh(a)\sinh(bx)}{b}+\frac{\sinh(a)\ln(x)\sinh(bx)}{b}+\frac{\sinh(a)b^2\left(\frac{9\sinh(bx)}{b^3}-\frac{9\,\text{hyperbolicSineIntegral}(bx)}{b^3}\right)}{9}-\frac{\cosh(a)b\left(-\frac{2}{b^2}+\frac{2\cosh(bx)}{b^2}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2/b*exp(b*x+a)+1/2/b*exp(-b*x-a))*ln(x)+1/2/b*exp(-a)*Ei(1,b*x)+1/2/b*ex
p(a)*Ei(1,-b*x)
```

Maxima [A]
time = 0.35, size = 36, normalized size = 1.03

$$\frac{\cosh\left(bx+a\right)\log\left(x\right)}{b}-\frac{\operatorname{Ei}(-bx)\,e^{(-a)}+\operatorname{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] cosh(b*x + a)*log(x)/b - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x)*e^a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(35) = 70.
time = 0.37, size = 134, normalized size = 3.83

$$-\frac{\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\cosh\left(bx+a\right)\cosh\left(a\right)-\log\left(x\right)\sinh\left(bx+a\right)^{2}+\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\cosh\left(bx+a\right)\sinh\left(a\right)-\left(\cosh\left(bx+a\right)^{2}+1\right)\log\left(x\right)+\left(\left(\operatorname{Ei}(bx)+\operatorname{Ei}(-bx)\right)\cosh\left(a\right)-2\,\cosh\left(bx+a\right)\log\left(x\right)+\left(\operatorname{Ei}(bx)-\operatorname{Ei}(-bx)\right)\sinh\left(a\right)\right)\sinh\left(bx+a\right)}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 + 1)*log(x)
+ ((Ei(b*x) + Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) - Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(x\right)\sinh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a),x)
```

```
[Out] Integral(log(x)*sinh(a + b*x), x)
```

Giac [A]
time = 3.66, size = 52, normalized size = 1.49

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)\log\left(x\right)-\frac{\operatorname{Ei}(-bx)\,e^{(-a)}+\operatorname{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b + e^(-b*x - a)/b)*log(x) - 1/2*(Ei(-b*x)*e^(-a) + Ei(b*x
)*e^a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sinh(a+b\,x)\,\ln\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)*log(x),x)
```

```
[Out] int(sinh(a + b*x)*log(x), x)
```

3.196 $\int \log(x)\sinh^2(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{x}{2}-\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}$$

```
[Out] 1/2*x-1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/2
*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2715, 8, 2634, 12, 5382, 3384, 3379, 3382}

$$-\frac{\sinh(2a)\text{Chi}(2bx)}{4b}-\frac{\cosh(2a)\text{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}-\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] x/2 - (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b*x]
*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5382

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}\int \log(x)\sinh^2(a+bx)\,dx &= -\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\int \frac{1}{4}\left(-2+\frac{\sinh(2(a+bx))}{bx}\right.\\ &= -\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{1}{4}\int \left(-2+\frac{\sinh(2(a+bx))}{bx}\right.\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2(a+bx))}{x}\,dx}{4b}\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2a+2bx)}{x}\,dx}{4b}\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx}{4b}-\\ &= \frac{x}{2}-\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\text{cosh}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.76

$$-\frac{-2bx+2bx\log(x)+\text{Chi}(2bx)\sinh(2a)-\log(x)\sinh(2(a+bx))+\cosh(2a)\text{Shi}(2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*b*x + 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2
*(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Maple [A]
time = 0.07, size = 97, normalized size = 1.47

method	result
risch	$\left(-\frac{x}{2}+\frac{\mathrm{e}^{2bx+2a}}{8b}-\frac{\mathrm{e}^{-2bx-2a}}{8b}\right)\ln(x)+\frac{\mathrm{e}^{2a}\,\mathrm{expIntegral}(1,-2bx)}{8b}-\frac{a\ln(bx)}{2b}+\frac{a\ln(-bx)}{2b}+\frac{x}{2}+\frac{a}{2b}-\frac{\mathrm{e}^{-2a}\,\mathrm{expIntegral}(}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei
(1,-2*b*x)-1/2/b*a*ln(b*x)+1/2/b*a*ln(-b*x)+1/2*x+1/2*a/b-1/8/b*exp(-2*a)*E
i(1,2*b*x)
```

Maxima [A]
time = 0.33, size = 67, normalized size = 1.02

$$-\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)+\frac{1}{2}\,x-\frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/2*x - 1/8*Ei
(2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(56) = 112.
time = 0.38, size = 313, normalized size = 4.74

$$\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\sinh(2\,a)+(4\,bx-(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a))\cosh(bx+a)^2+\left(4\,bx-(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a)-2\left(2\,bx-3\cosh(bx+a)^2\right)\log(x)-(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\sinh(2\,a)\right)\sinh(bx+a)^2-\left(4\,bx\cosh(bx+a)^2-\cosh(bx+a)^4+1\right)\log(x)-2\left((\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)\sinh(2\,a)-(4\,bx-(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a))\cosh(bx+a)+2\left(2\,bx\cosh(bx+a)-\cosh(bx+a)^3\right)\log(x)\right)\sinh(bx+a)}{8\left(b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) + (4*b*x - (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 + (4*b*x - (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x - 3*cosh(b*x + a)^2)*log(x) - (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 - (4*b*x*cosh(b*x + a)^2 - cosh(b*x + a)^4 + 1)*log
```

```
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) - (4*b*x - (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) + 2*(2*b*x*cosh(b*x + a) - cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sinh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**2,x)
[Out] Integral(log(x)*sinh(a + b*x)**2, x)
```

Giac [A]
time = 3.52, size = 67, normalized size = 1.02

$$-\frac{1}{8}\left(4\,x-\frac{e^{(2\,bx+2\,a)}}{b}+\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)+\frac{4\,bx-\mathrm{Ei}(2\,bx)\,e^{(2\,a)}+\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/8*(4*x - e^(2*b*x + 2*a)/b + e^(-2*b*x - 2*a)/b)*log(x) + 1/8*(4*b*x - E
i(2*b*x)*e^(2*a) + Ei(-2*b*x)*e^(-2*a))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sinh(a+b\,x)^2\,\ln(x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^2*log(x),x)
[Out] int(sinh(a + b*x)^2*log(x), x)
```

3.197 $\int \log(x)\sinh^3(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{3\cosh(a)\mathrm{Chi}(bx)}{4b}-\frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}+\frac{3\sinh(a)\mathrm{Shi}(bx)}{4b}-\frac{\sinh(3\cdots}{}$$

```
[Out] 3/4*Chi(b*x)*cosh(a)/b-1/12*Chi(3*b*x)*cosh(3*a)/b-cosh(b*x+a)*ln(x)/b+1/3*
cosh(b*x+a)^3*ln(x)/b+3/4*Shi(b*x)*sinh(a)/b-1/12*Shi(3*b*x)*sinh(3*a)/b
```

Rubi [A]
time = 0.35, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3379, 3382, 3393}

$$\frac{3\cosh(a)\mathrm{Chi}(bx)}{4b}-\frac{\cosh(3a)\mathrm{Chi}(3bx)}{12b}+\frac{3\sinh(a)\mathrm{Shi}(bx)}{4b}-\frac{\sinh(3a)\mathrm{Shi}(3bx)}{12b}+\frac{\log(x)\cosh^3(a+bx)}{3b}-\frac{\log(x)\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x])/(4*b) - (Cosh[3*a]*CoshIntegral[3*b*x])/(12*b
) - (Cosh[a + b*x]*Log[x])/b + (Cosh[a + b*x]^3*Log[x])/(3*b) + (3*Sinh[a]*
SinhIntegral[b*x])/(4*b) - (Sinh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \log(x)\sinh^3(a+bx)\,dx = -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\int \frac{\cosh(a+bx)\left(-3+\cosh^2(\right.}{3bx}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cosh(a+bx)\left(-3+\cosh^2(a+bx)\right)}{x}\,dx}{3b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \left(-\frac{3\cosh(a+bx)}{x}+\frac{\cosh^3(a+bx)}{x}\right)}{3b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \frac{\cosh^3(a+bx)}{x}\,dx}{3b}+\frac{\int \frac{\cosh(a+bx)}{x}}{b}$$

$$= -\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}-\frac{\int \left(\frac{3\cosh(a+bx)}{4x}+\frac{\cosh(3a+3bx)}{4x}\right)}{3b}$$

$$= \frac{\cosh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}+\frac{\sinh(a)\text{Shi}(b}{b}$$

$$= \frac{\cosh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)\log(x)}{3b}+\frac{\sinh(a)\text{Shi}(b}{b}$$

$$= \frac{3\cosh(a)\text{Chi}(bx)}{4b}-\frac{\cosh(3a)\text{Chi}(3bx)}{12b}-\frac{\cosh(a+bx)\log(x)}{b}+\frac{\cosh^3(a+bx)}{3b}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.75

$$\frac{9\cosh(a)\mathrm{Chi}(bx)-\cosh(3a)\mathrm{Chi}(3bx)-9\cosh(a+bx)\log(x)+\cosh(3(a+bx))\log(x)+9\sinh(a)\mathrm{Shi}(bx)-\sinh(3a)\mathrm{Shi}(3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]*Sinh[a + b*x]^3,x]
```

```
[Out] (9*Cosh[a]*CoshIntegral[b*x] - Cosh[3*a]*CoshIntegral[3*b*x] - 9*Cosh[a + b
*x]*Log[x] + Cosh[3*(a + b*x)]*Log[x] + 9*Sinh[a]*SinhIntegral[b*x] - Sinh[
3*a]*SinhIntegral[3*b*x])/(12*b)
```

Maple [A]
time = 0.07, size = 116, normalized size = 1.30

method	result
risch	$\left(\frac{e^{3bx+3a}}{24b}-\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}\right)\ln(x)+\frac{e^{-3a}\,\mathrm{expIntegral}(1,3bx)}{24b}+\frac{e^{3a}\,\mathrm{expIntegral}(1,-3bx)}{24b}-\frac{3\,e^{-a}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)-3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)+1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(-3*a)*Ei(1,3*b*x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)-3/8/b*exp(a)*Ei(1,-b*x)
```

Maxima [A]
time = 0.35, size = 110, normalized size = 1.24

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(-bx)\,e^{(-a)}}{8\,b}-\frac{\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b}+\frac{3\,\mathrm{Ei}(bx)\,e^{a}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b - 1/24*Ei
(-3*b*x)*e^(-3*a)/b + 3/8*Ei(b*x)*e^a/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(79) = 158$.
time = 0.37, size = 587, normalized size = 6.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) - Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 - 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) - Ei(-
3*b*x))*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) - 9*(Ei(b*x) - Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 - 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 - 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 + 1)*log(x) - 3*((Ei(3*b*x) - Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) - 9*(Ei(b*x) - Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) + Ei(-3*b
*x))*cosh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 - 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\sinh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*sinh(b*x+a)**3,x)
```

```
[Out] Integral(log(x)*sinh(a + b*x)**3, x)
```

Giac [A]
time = 3.47, size = 102, normalized size = 1.15

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}-\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}+\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(-a)}+\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}-9\,\mathrm{Ei}(bx)\,e^{a}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b - 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b + e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) + Ei(-3*b*x)*e
^(-3*a) - 9*Ei(b*x)*e^a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\sinh(a+b\,x)}^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(a + b*x)^3*log(x),x)
[Out] int(sinh(a + b*x)^3*log(x), x)
```

3.198 $\int \cosh(a+bx)\log(x)\,dx$

Optimal. Leaf size=35

$$-\frac{\text{Chi}(bx)\sinh(a)}{b}+\frac{\log(x)\sinh(a+bx)}{b}-\frac{\cosh(a)\text{Shi}(bx)}{b}$$

```
[Out] -cosh(a)*Shi(b*x)/b-Chi(b*x)*sinh(a)/b+ln(x)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2717, 2634, 12, 3384, 3379, 3382}

$$-\frac{\sinh(a)\text{Chi}(bx)}{b}-\frac{\cosh(a)\text{Shi}(bx)}{b}+\frac{\log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a])/b) + (Log[x]*Sinh[a + b*x])/b - (Cosh[a]*Sinh
Integral[b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh(a+bx)\log(x)\,dx &= \frac{\log(x)\sinh(a+bx)}{b} - \int \frac{\sinh(a+bx)}{bx}\,dx\\ &= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}\\ &= \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\int \frac{\sinh(bx)}{x}\,dx}{b} - \frac{\sinh(a)\int \frac{\cosh(bx)}{x}\,dx}{b}\\ &= -\frac{\mathrm{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} - \frac{\cosh(a)\mathrm{Shi}(bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.86

$$-\frac{\mathrm{Chi}(bx)\sinh(a) - \log(x)\sinh(a+bx) + \cosh(a)\mathrm{Shi}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[x],x]
```

```
[Out] -((CoshIntegral[b*x]*Sinh[a] - Log[x]*Sinh[a + b*x] + Cosh[a]*SinhIntegral[
b*x])/b)
```

Maple [A]
time = 0.03, size = 58, normalized size = 1.66

method	result
risch	$\left(\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b}\right)\ln(x) + \frac{e^{a}\,\mathrm{expIntegral}(1,-bx)}{2b} - \frac{e^{-a}\,\mathrm{expIntegral}(1,bx)}{2b}$
meijerg	$-\frac{\cosh(a)\sinh(bx)}{b} + \frac{\cosh(a)\ln(x)\sinh(bx)}{b} + \frac{\cosh(a)b^{2}\left(\frac{9\sinh(bx)}{b^{3}} - \frac{9\,\mathrm{hyperbolicSineIntegral}(bx)}{b^{3}}\right)}{9} - \frac{\sinh(a)b\left(-\frac{2}{b^{2}} + \frac{2\cosh(bx)}{b^{2}}\right)}{4} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2/b*exp(b*x+a)-1/2/b*exp(-b*x-a))*ln(x)+1/2/b*exp(a)*Ei(1,-b*x)-1/2/b*ex
p(-a)*Ei(1,b*x)
```

Maxima [A]
time = 0.33, size = 37, normalized size = 1.06

$$\frac{\log\left(x\right)\sinh\left(bx+a\right)}{b}+\frac{\mathrm{Ei}(-bx)\,e^{(-a)}-\mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] log(x)*sinh(b*x + a)/b + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x)*e^a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(35) = 70$.
time = 0.39, size = 134, normalized size = 3.83

$$-\frac{(\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\cosh\left(bx+a\right)\cosh\left(a\right)-\log\left(x\right)\sinh\left(bx+a\right)^{2}+(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\cosh\left(bx+a\right)\sinh\left(a\right)-\left(\cosh\left(bx+a\right)^{2}-1\right)\log\left(x\right)+((\mathrm{Ei}(bx)-\mathrm{Ei}(-bx))\cosh\left(a\right)-2\cosh\left(bx+a\right)\log\left(x\right)+(\mathrm{Ei}(bx)+\mathrm{Ei}(-bx))\sinh\left(a\right))\sinh\left(bx+a\right)}{2\left(b\cosh\left(bx+a\right)+b\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/2*((Ei(b*x) - Ei(-b*x))*cosh(b*x + a)*cosh(a) - log(x)*sinh(b*x + a)^2 +
 (Ei(b*x) + Ei(-b*x))*cosh(b*x + a)*sinh(a) - (cosh(b*x + a)^2 - 1)*log(x)
+ ((Ei(b*x) - Ei(-b*x))*cosh(a) - 2*cosh(b*x + a)*log(x) + (Ei(b*x) + Ei(-b
*x))*sinh(a))*sinh(b*x + a))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(x\right)\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x), x)
```

Giac [A]
time = 2.46, size = 54, normalized size = 1.54

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(x\right)+\frac{\mathrm{Ei}(-bx)\,e^{(-a)}-\mathrm{Ei}(bx)\,e^{a}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(x) + 1/2*(Ei(-b*x)*e^(-a) - Ei(b*x
)*e^a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cosh(a+b\,x)\,\ln\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*log(x),x)
```

```
[Out] int(cosh(a + b*x)*log(x), x)
```

3.199 $\int \cosh^2(a+bx)\log(x)\,dx$

Optimal. Leaf size=66

$$-\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\mathrm{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}$$

```
[Out] -1/2*x+1/2*x*ln(x)-1/4*cosh(2*a)*Shi(2*b*x)/b-1/4*Chi(2*b*x)*sinh(2*a)/b+1/
2*cosh(b*x+a)*ln(x)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.727$, Rules used = {2715, 8, 2634, 12, 5382, 3384, 3379, 3382}

$$-\frac{\sinh(2a)\mathrm{Chi}(2bx)}{4b}-\frac{\cosh(2a)\mathrm{Shi}(2bx)}{4b}+\frac{\log(x)\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}+\frac{1}{2}x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -1/2*x + (x*Log[x])/2 - (CoshIntegral[2*b*x]*Sinh[2*a])/(4*b) + (Cosh[a + b
*x]*Log[x]*Sinh[a + b*x])/(2*b) - (Cosh[2*a]*SinhIntegral[2*b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5382

```
Int[(u_)^(m_.)*((a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[ExpandToSum
[u, x]^m*(a + b*Sinh[ExpandToSum[v, x]])^n, x] /; FreeQ[{a, b, m, n}, x] &&
 LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rubi steps

$$\begin{aligned}
\int \cosh^2(a+bx)\log(x)\,dx &= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\int \frac{1}{4}\left(2+\frac{\sinh(2(a+bx))}{bx}\right)\,dx\\
&= \frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{1}{4}\int \left(2+\frac{\sinh(2(a+bx))}{bx}\right)\,dx\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2(a+bx))}{x}\,dx}{4b}\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\int \frac{\sinh(2a+2bx)}{x}\,dx}{4b}\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cosh(2a)\int \frac{\sinh(2bx)}{x}\,dx}{4b}-\\
&= -\frac{x}{2}+\frac{1}{2}x\log(x)-\frac{\text{Chi}(2bx)\sinh(2a)}{4b}+\frac{\cosh(a+bx)\log(x)\sinh(a+bx)}{2b}-\frac{\cos
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.76

$$-\frac{2bx-2bx\log(x)+\text{Chi}(2bx)\sinh(2a)-\log(x)\sinh(2(a+bx))+\cosh(2a)\text{Shi}(2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2*Log[x],x]
```

```
[Out] -1/4*(2*b*x - 2*b*x*Log[x] + CoshIntegral[2*b*x]*Sinh[2*a] - Log[x]*Sinh[2*
(a + b*x)] + Cosh[2*a]*SinhIntegral[2*b*x])/b
```

Maple [A]
time = 0.06, size = 97, normalized size = 1.47

method	result
risch	$\left(\frac{x}{2}+\frac{\mathrm{e}^{2bx+2a}}{8b}-\frac{\mathrm{e}^{-2bx-2a}}{8b}\right)\ln(x)+\frac{\mathrm{e}^{2a}\,\mathrm{expIntegral}(1,-2bx)}{8b}+\frac{a\ln(bx)}{2b}-\frac{a\ln(-bx)}{2b}-\frac{x}{2}-\frac{a}{2b}-\frac{\mathrm{e}^{-2a}\,\mathrm{expIntegral}(}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*x+1/8/b*exp(2*b*x+2*a)-1/8/b*exp(-2*b*x-2*a))*ln(x)+1/8/b*exp(2*a)*Ei(
1,-2*b*x)+1/2/b*a*ln(b*x)-1/2/b*a*ln(-b*x)-1/2*x-1/2*a/b-1/8/b*exp(-2*a)*Ei
(1,2*b*x)
```

Maxima [A]
time = 0.33, size = 67, normalized size = 1.02

$$\frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)-\frac{1}{2}\,x-\frac{\mathrm{Ei}(2\,bx)\,e^{(2\,a)}}{8\,b}+\frac{\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/2*x - 1/8*Ei(
2*b*x)*e^(2*a)/b + 1/8*Ei(-2*b*x)*e^(-2*a)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(56) = 112$.
time = 0.37, size = 305, normalized size = 4.62

$$\frac{4\cosh(bx+a)\log(x)\sinh(bx+a)^3+\log(x)\sinh(bx+a)^4-(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)^2\sinh(2\,a)-(4\,bx+(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a))\cosh(bx+a)^2-\left(4\,bx+(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a)-2\left(2\,bx+3\cosh(bx+a)^2\right)\log(x)+(\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\sinh(2\,a)\right)\sinh(bx+a)^2+\left(4\,bx\cosh(bx+a)^2+\cosh(bx+a)^4-1\right)\log(x)-2\left((\mathrm{Ei}(2\,bx)+\mathrm{Ei}(-2\,bx))\cosh(bx+a)\sinh(2\,a)+(4\,bx+(\mathrm{Ei}(2\,bx)-\mathrm{Ei}(-2\,bx))\cosh(2\,a))\cosh(bx+a)-2\left(2\,bx\cosh(bx+a)+\cosh(bx+a)^3\right)\log(x)\right)\sinh(bx+a)}{8\left(b\cosh(bx+a)^2+2\,b\cosh(bx+a)\sinh(bx+a)+b\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(b*x + a)*log(x)*sinh(b*x + a)^3 + log(x)*sinh(b*x + a)^4 - (Ei(
2*b*x) + Ei(-2*b*x))*cosh(b*x + a)^2*sinh(2*a) - (4*b*x + (Ei(2*b*x) - Ei(-
2*b*x))*cosh(2*a))*cosh(b*x + a)^2 - (4*b*x + (Ei(2*b*x) - Ei(-2*b*x))*cosh
(2*a) - 2*(2*b*x + 3*cosh(b*x + a)^2)*log(x) + (Ei(2*b*x) + Ei(-2*b*x))*sin
h(2*a))*sinh(b*x + a)^2 + (4*b*x*cosh(b*x + a)^2 + cosh(b*x + a)^4 - 1)*log
```

```
(x) - 2*((Ei(2*b*x) + Ei(-2*b*x))*cosh(b*x + a)*sinh(2*a) + (4*b*x + (Ei(2*
b*x) - Ei(-2*b*x))*cosh(2*a))*cosh(b*x + a) - 2*(2*b*x*cosh(b*x + a) + cosh
(b*x + a)^3)*log(x))*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*
sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**2, x)
```

Giac [**A**]
time = 6.46, size = 67, normalized size = 1.02

$$\frac{1}{8}\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)\log(x)-\frac{4\,bx+\mathrm{Ei}(2\,bx)\,e^{(2\,a)}-\mathrm{Ei}(-2\,bx)\,e^{(-2\,a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/8*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)*log(x) - 1/8*(4*b*x + Ei
(2*b*x)*e^(2*a) - Ei(-2*b*x)*e^(-2*a))/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cosh(a+b\,x)^2\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*log(x),x)
```

```
[Out] int(cosh(a + b*x)^2*log(x), x)
```

3.200 $\int \cosh^3(a+bx)\log(x)\,dx$

Optimal. Leaf size=88

$$-\frac{3\text{Chi}(bx)\sinh(a)}{4b}-\frac{\text{Chi}(3bx)\sinh(3a)}{12b}+\frac{\log(x)\sinh(a+bx)}{b}+\frac{\log(x)\sinh^3(a+bx)}{3b}-\frac{3\cosh(a)\text{Shi}(bx)}{4b}-\frac{\cos}{}$$

```
[Out] -3/4*cosh(a)*Shi(b*x)/b-1/12*cosh(3*a)*Shi(3*b*x)/b-3/4*Chi(b*x)*sinh(a)/b-
1/12*Chi(3*b*x)*sinh(3*a)/b+ln(x)*sinh(b*x+a)/b+1/3*ln(x)*sinh(b*x+a)^3/b
```

Rubi [A]
time = 0.32, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2713, 2634, 12, 6874, 3384, 3379, 3382, 3393}

$$-\frac{3\sinh(a)\text{Chi}(bx)}{4b}-\frac{\sinh(3a)\text{Chi}(3bx)}{12b}-\frac{3\cosh(a)\text{Shi}(bx)}{4b}-\frac{\cosh(3a)\text{Shi}(3bx)}{12b}+\frac{\log(x)\sinh^3(a+bx)}{3b}+\frac{\log(x)\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] (-3*CoshIntegral[b*x]*Sinh[a])/(4*b) - (CoshIntegral[3*b*x]*Sinh[3*a])/(12*
b) + (Log[x]*Sinh[a + b*x])/b + (Log[x]*Sinh[a + b*x]^3)/(3*b) - (3*Cosh[a]
*SinhIntegral[b*x])/(4*b) - (Cosh[3*a]*SinhIntegral[3*b*x])/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \cosh^3(a+bx)\log(x)\,dx = \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx\right.}{3bx}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh(a+bx)\left(3+\sinh^2(a+bx)\right)}{x}\,dx}{3b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \left(\frac{3\sinh(a+bx)}{x} + \frac{\sinh^3(a+bx)}{x}\right)dx}{3b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\int \frac{\sinh^3(a+bx)}{x}\,dx}{3b} - \frac{\int \frac{\sinh(a+bx)}{x}\,dx}{b}$$

$$= \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{i\int \left(\frac{3i\sinh(a+bx)}{4x} - \frac{i\sinh(3a+3bx)}{4x}\right) d\text{:}}{3b}$$

$$= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\text{Shi}(b}{b}$$

$$= -\frac{\text{Chi}(bx)\sinh(a)}{b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a+bx)}{3b} - \frac{\cosh(a)\text{Shi}(b}{b}$$

$$= -\frac{3\text{Chi}(bx)\sinh(a)}{4b} - \frac{\text{Chi}(3bx)\sinh(3a)}{12b} + \frac{\log(x)\sinh(a+bx)}{b} + \frac{\log(x)\sinh^3(a}{3b}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 0.75

$$-\frac{9\text{Chi}(bx)\sinh(a)+\text{Chi}(3bx)\sinh(3a)-9\log(x)\sinh(a+bx)-\log(x)\sinh(3(a+bx))+9\cosh(a)\text{Shi}(bx)+\cosh(3a)\text{Shi}(3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3*Log[x],x]
```

```
[Out] -1/12*(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] - 9*Log[
x]*Sinh[a + b*x] - Log[x]*Sinh[3*(a + b*x)] + 9*Cosh[a]*SinhIntegral[b*x] +
 Cosh[3*a]*SinhIntegral[3*b*x])/b
```

Maple [A]
time = 0.08, size = 116, normalized size = 1.32

method	result
risch	$\left(\frac{e^{3bx+3a}}{24b}+\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}\right)\ln(x)+\frac{e^{3a}\,\text{expIntegral}(1,-3bx)}{24b}-\frac{e^{-3a}\,\text{expIntegral}(1,3bx)}{24b}-\frac{3\,e^{-a}\,\text{exp}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] (1/24/b*exp(3*b*x+3*a)+3/8/b*exp(b*x+a)-3/8/b*exp(-b*x-a)-1/24/b*exp(-3*b*x
-3*a))*ln(x)+1/24/b*exp(3*a)*Ei(1,-3*b*x)-1/24/b*exp(-3*a)*Ei(1,3*b*x)-3/8/
b*exp(-a)*Ei(1,b*x)+3/8/b*exp(a)*Ei(1,-b*x)
```

Maxima [A]
time = 0.37, size = 111, normalized size = 1.26

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\text{Ei}(3\,bx)\,e^{(3\,a)}}{24\,b}+\frac{3\,\text{Ei}(-bx)\,e^{(-a)}}{8\,b}+\frac{\text{Ei}(-3\,bx)\,e^{(-3\,a)}}{24\,b}-\frac{3\,\text{Ei}(bx)\,e^{a}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*Ei(3*b*x)*e^(3*a)/b + 3/8*Ei(-b*x)*e^(-a)/b + 1/24*Ei
(-3*b*x)*e^(-3*a)/b - 3/8*Ei(b*x)*e^a/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(78) = 156$.
time = 0.38, size = 587, normalized size = 6.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/24*(6*cosh(b*x + a)*log(x)*sinh(b*x + a)^5 + log(x)*sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 + 3)*log(x)*sinh(b*x + a)^4 - (Ei(3*b*x) + Ei(-3*b*x))*co
sh(b*x + a)^3*sinh(3*a) - 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^3*sinh(a) -
((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(
b*x + a)^3 - ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*c
osh(a) - 4*(5*cosh(b*x + a)^3 + 9*cosh(b*x + a))*log(x) + (Ei(3*b*x) + Ei(-
3*b*x))*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*sinh(a))*sinh(b*x + a)^3 - 3*((E
i(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)*sinh(3*a) + 9*(Ei(b*x) + Ei(-b*x))*cos
h(b*x + a)*sinh(a) + ((Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(
-b*x))*cosh(a))*cosh(b*x + a) - (5*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 - 3
)*log(x))*sinh(b*x + a)^2 + (cosh(b*x + a)^6 + 9*cosh(b*x + a)^4 - 9*cosh(b
*x + a)^2 - 1)*log(x) - 3*((Ei(3*b*x) + Ei(-3*b*x))*cosh(b*x + a)^2*sinh(3*
a) + 9*(Ei(b*x) + Ei(-b*x))*cosh(b*x + a)^2*sinh(a) + ((Ei(3*b*x) - Ei(-3*b
*x))*cosh(3*a) + 9*(Ei(b*x) - Ei(-b*x))*cosh(a))*cosh(b*x + a)^2 - 2*(cosh(
b*x + a)^5 + 6*cosh(b*x + a)^3 - 3*cosh(b*x + a))*log(x))*sinh(b*x + a))/(b
*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*si
nh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log(x)\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*ln(x),x)
```

```
[Out] Integral(log(x)*cosh(a + b*x)**3, x)
```

Giac [A]
time = 6.43, size = 104, normalized size = 1.18

$$\frac{1}{24}\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)\log(x)-\frac{\mathrm{Ei}(3\,bx)\,e^{(3\,a)}-9\,\mathrm{Ei}(-bx)\,e^{(-a)}-\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)}+9\,\mathrm{Ei}(bx)\,e^{a}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/24*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x -
3*a)/b)*log(x) - 1/24*(Ei(3*b*x)*e^(3*a) - 9*Ei(-b*x)*e^(-a) - Ei(-3*b*x)*e
^(-3*a) + 9*Ei(b*x)*e^a)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)^3\,\ln(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*log(x),x)
```

```
[Out] int(cosh(a + b*x)^3*log(x), x)
```

3.201 $\int \log(a\sinh(x))\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{2} - x\log\left(1 - e^{2x}\right) + x\log(a\sinh(x)) - \frac{\text{Li}_2(e^{2x})}{2}$$

```
[Out] 1/2*x^2-x*ln(1-exp(2*x))+x*ln(a*sinh(x))-1/2*polylog(2,exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 3797, 2221, 2317, 2438}

$$-\frac{1}{2}\text{PolyLog}\left(2, e^{2x}\right) + x\log(a\sinh(x)) + \frac{x^2}{2} - x\log\left(1 - e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]] - PolyLog[2, E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(a\sinh(x))\,dx &= x\log(a\sinh(x)) - \int x\coth(x)\,dx \\
&= \frac{x^2}{2} + x\log(a\sinh(x)) + 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{x^2}{2} - x\log\left(1-e^{2x}\right) + x\log(a\sinh(x)) - \frac{\text{Li}_2(e^{2x})}{2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(-x\left(x+2\log\left(1-e^{-2x}\right)-2\log(a\sinh(x))\right)+\text{Li}_2\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]])) + PolyLog[2, E^(-2*x)])/
2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 295, normalized size = 7.56

method	result
risch	$-x\ln\left(\mathrm{e}^{x}\right)-\frac{i x\pi \mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\right)^{3}}{2}-\frac{i x\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}-1\right)\right)\mathrm{csgn}\left(ia\left(\mathrm{e}^{2x}-1\right)\mathrm{e}^{-x}\right)}{2}-\frac{i x\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2x}-1\right)\right)\mathrm{csgn}(i\mathrm{e}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*ln(exp(x))-1/2*I*x*Pi*csgn(I*exp(-x)*(exp(2*x)-1))^3-1/2*I*x*Pi*csgn(I*a
)*csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x))-1/2*I*x*Pi*cs
```

```
gn(I*(exp(2*x)-1))*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)-1))+1/2*I*x*Pi*
csgn(I*exp(-x)*(exp(2*x)-1))*csgn(I*a*(exp(2*x)-1)*exp(-x))^2-1/2*I*x*Pi*cs
gn(I*a*(exp(2*x)-1)*exp(-x))^3+1/2*I*x*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)-1)*e
xp(-x))^2-x*ln(2)+x*ln(a)+1/2*x^2+1/2*I*x*Pi*csgn(I*(exp(2*x)-1))*csgn(I*ex
p(-x)*(exp(2*x)-1))^2+1/2*I*x*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)-1
))^2+ln(exp(x))*ln(exp(2*x)-1)+dilog(exp(x))-dilog(1+exp(x))-ln(exp(x))*ln(
1+exp(x))
```

Maxima [A]
time = 0.32, size = 43, normalized size = 1.10

$$\frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(e^x+1\right) - x\log\left(-e^x+1\right) - \mathrm{Li}_2(-e^x) - \mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="maxima")
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(e^x + 1) - x*log(-e^x + 1) - dilog(-e^x)
 - dilog(e^x)
```

Fricas [A]
time = 0.41, size = 57, normalized size = 1.46

$$\frac{1}{2}x^2 + x\log\left(a\sinh\left(x\right)\right) - x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) - x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right) - \mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right)) - \mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="fricas")
[Out] 1/2*x^2 + x*log(a*sinh(x)) - x*log(cosh(x) + sinh(x) + 1) - x*log(-cosh(x)
- sinh(x) + 1) - dilog(cosh(x) + sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sinh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)),x)
[Out] Integral(log(a*sinh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)),x, algorithm="giac")
[Out] integrate(log(a*sinh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln(a\sinh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)),x)
[Out] int(log(a*sinh(x)), x)
```

3.202 $\int \log\left(a\sinh^2(x)\right)\,dx$

Optimal. Leaf size=35

$$x^2-2x\log\left(1-e^{2x}\right)+x\log\left(a\sinh^2(x)\right)-\text{Li}_2\left(e^{2x}\right)$$

```
[Out] x^2-2*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^2)-polylog(2,exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$-\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^2(x)\right)+x^2-2x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^2] - PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\sinh^2(x)\right)\,dx &= x\log\left(a\sinh^2(x)\right) - \int 2x\coth(x)\,dx \\
&= x\log\left(a\sinh^2(x)\right) - 2\int x\coth(x)\,dx \\
&= x^2 + x\log\left(a\sinh^2(x)\right) + 4\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + 2\int \log\left(1-e^{2x}\right)\,dx \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) + \operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= x^2 - 2x\log\left(1-e^{2x}\right) + x\log\left(a\sinh^2(x)\right) - \operatorname{Li}_2\left(e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.94

$$x\left(-x-2\log\left(1-e^{-2x}\right)+\log\left(a\sinh^2(x)\right)\right)+\operatorname{Li}_2\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 - E^(-2*x)] + Log[a*Sinh[x]^2]) + PolyLog[2, E^(-2*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 454, normalized size = 12.97

method	result
risch	$\frac{ix\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}-1\right)^2\right)\mathrm{csgn}\left(ia\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}\right)^2}{2}+x^2+\frac{ix\pi\,\mathrm{csgn}(ia)\mathrm{csgn}\left(ia\left(\mathrm{e}^{2x}-1\right)^2\mathrm{e}^{-2x}\right)^2}{2}+\frac{ix\pi\mathrm{csgn}\left(i\mathrm{e}^{2x}\right)^3}{2}-\frac{ix\pi\mathrm{cs}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*x*Pi*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*
x))^2+x^2+1/2*I*x*Pi*csgn(I*a)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))^2+1/2*I*x
*Pi*csgn(I*exp(2*x))^3-1/2*I*x*Pi*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))^3-1/2*
I*x*Pi*csgn(I*(exp(2*x)-1)^2)^3+1/2*I*x*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x)
)-1/2*I*x*Pi*csgn(I*(exp(2*x)-1))^2*csgn(I*(exp(2*x)-1)^2)+1/2*I*x*Pi*csgn(
I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^2-2*x*ln(exp(x))+2*ln(exp(x))
*ln(exp(2*x)-1)-2*ln(exp(x))*ln(1+exp(x))+2*dilog(exp(x))-2*dilog(1+exp(x))
-1/2*I*x*Pi*csgn(I*(exp(2*x)-1)^2)*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(exp(
2*x)-1)^2)-I*x*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2-1/2*I*x*Pi*csgn(I*a)*cs
gn(I*exp(-2*x)*(exp(2*x)-1)^2)*csgn(I*a*(exp(2*x)-1)^2*exp(-2*x))-1/2*I*x*P
i*csgn(I*exp(-2*x)*(exp(2*x)-1)^2)^3-2*x*ln(2)+x*ln(a)+I*x*Pi*csgn(I*(exp(2
*x)-1))*csgn(I*(exp(2*x)-1)^2)^2+1/2*I*x*Pi*csgn(I*(exp(2*x)-1)^2)*csgn(I*e
xp(-2*x)*(exp(2*x)-1)^2)^2
```

Maxima [A]
time = 0.32, size = 43, normalized size = 1.23

$$x^2 + x\log\left(a\sinh\left(x\right)^2\right) - 2\,x\log\left(e^x+1\right) - 2\,x\log\left(-e^x+1\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x^2 + x*log(a*sinh(x)^2) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-
e^x) - 2*dilog(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(32) = 64.
time = 0.42, size = 69, normalized size = 1.97

$$x^2 + x\log\left(\frac{1}{2}\,a\cosh\left(x\right)^2 + \frac{1}{2}\,a\sinh\left(x\right)^2 - \frac{1}{2}\,a\right) - 2\,x\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - 2\,x\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) - 2\,\mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - 2\,\mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 - 1/2*a) - 2*x*log(cosh(x) +
sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x) + sinh(x))
 - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sinh^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**2),x)
```

```
[Out] Integral(log(a*sinh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\sinh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)^2),x)
```

```
[Out] int(log(a*sinh(x)^2), x)
```

3.203 $\int \log(a\sinh^n(x))\,dx$

Optimal. Leaf size=44

$$\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)+x\log\left(a\sinh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)$$

```
[Out] 1/2*n*x^2-n*x*ln(1-exp(2*x))+x*ln(a*sinh(x)^n)-1/2*n*polylog(2,exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\sinh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sinh[x]^n],x]
```

```
[Out] (n*x^2)/2 - n*x*Log[1 - E^(2*x)] + x*Log[a*Sinh[x]^n] - (n*PolyLog[2, E^(2*
x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(a\sinh^n(x))\,dx &= x\log(a\sinh^n(x)) - \int nx\coth(x)\,dx \\
&= x\log(a\sinh^n(x)) - n\int x\coth(x)\,dx \\
&= \frac{nx^2}{2} + x\log(a\sinh^n(x)) + (2n)\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log(a\sinh^n(x)) + n\int \log\left(1-e^{2x}\right)\,dx \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log(a\sinh^n(x)) + \frac{1}{2}n\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= \frac{nx^2}{2} - nx\log\left(1-e^{2x}\right) + x\log(a\sinh^n(x)) - \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(-x\left(nx+2n\log\left(1-e^{-2x}\right)-2\log(a\sinh^n(x))\right)+n\text{Li}_2\left(e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sinh[x]^n],x]
```

```
[Out] (-(x*(n*x + 2*n*Log[1 - E^(-2*x)] - 2*Log[a*Sinh[x]^n])) + n*PolyLog[2, E^(
-2*x)])/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\sinh^n(x)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sinh(x)^n),x)
```

```
[Out] int(ln(a*sinh(x)^n),x)
```

Maxima [A]
time = 0.34, size = 47, normalized size = 1.07

$$\frac{1}{2}\left(x^2 - 2\,x\log\left(e^x + 1\right) - 2\,x\log\left(-e^x + 1\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)\right)n + x\log\left(a\sinh\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(e
^x))*n + x*log(a*sinh(x)^n)
```

Fricas [A]
time = 0.38, size = 65, normalized size = 1.48

$$\frac{1}{2}nx^2 - nx\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - nx\log\left(-\cosh\left(x\right) - \sinh\left(x\right) + 1\right) + nx\log\left(\sinh\left(x\right)\right) - n\mathrm{Li}_2(\cosh\left(x\right) + \sinh\left(x\right)) - n\mathrm{Li}_2(-\cosh\left(x\right) - \sinh\left(x\right)) + x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*n*x^2 - n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(-cosh(x) - sinh(x) + 1
) + n*x*log(sinh(x)) - n*dilog(cosh(x) + sinh(x)) - n*dilog(-cosh(x) - sinh
(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\sinh^n\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sinh(x)**n),x)
```

```
[Out] Integral(log(a*sinh(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sinh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sinh(x)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\sinh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*sinh(x)^n),x)
```

```
[Out] int(log(a*sinh(x)^n), x)
```

3.204 $\int \log(a\cosh(x))\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)$$

```
[Out] 1/2*x^2-x*ln(1+exp(2*x))+x*ln(a*cosh(x))-1/2*polylog(2,-exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 3799, 2221, 2317, 2438}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right) + x\log(a\cosh(x)) + \frac{x^2}{2} - x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]],x]
```

```
[Out] x^2/2 - x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]] - PolyLog[2, -E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\cosh(x))\,dx &= x\log(a\cosh(x)) - \int x\tanh(x)\,dx\\
&= \frac{x^2}{2} + x\log(a\cosh(x)) - 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \int \log\left(1+e^{2x}\right)\,dx\\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right)\\
&= \frac{x^2}{2} - x\log\left(1+e^{2x}\right) + x\log(a\cosh(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 0.92

$$\frac{1}{2}\left(-x\left(x+2\log\left(1+e^{-2x}\right)-2\log(a\cosh(x))\right)+\text{Li}_2\left(-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]],x]
```

```
[Out] (-(x*(x + 2*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]])) + PolyLog[2, -E^(-2*x)])
/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 321, normalized size = 8.23

method	result
risch	$-x\ln\left(e^{x}\right)+\frac{ix\pi\,\text{csgn}(ia)\text{csgn}\left(ia\left(1+e^{2x}\right)e^{-x}\right)^{2}}{2}+\frac{ix\pi\,\text{csgn}\left(ie^{-x}\right)\text{csgn}\left(ie^{-x}\left(1+e^{2x}\right)\right)^{2}}{2}-\frac{ix\pi\,\text{csgn}\left(ie^{-x}\right)\text{csgn}\left(i\left(1+e^{2x}\right)\right)\text{csg}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*ln(exp(x))+1/2*I*x*Pi*csgn(I*a)*csgn(I*a*(1+exp(2*x))*exp(-x))^2+1/2*I*x
*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(1+exp(2*x)))^2-1/2*I*x*Pi*csgn(I*exp(-x
```

```
))*csgn(I*(1+exp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)))+1/2*I*x*Pi*csgn(I*(1+e
xp(2*x)))*csgn(I*exp(-x)*(1+exp(2*x)))^2-1/2*I*x*Pi*csgn(I*a*(1+exp(2*x))*e
xp(-x))^3-1/2*I*x*Pi*csgn(I*exp(-x)*(1+exp(2*x)))^3-x*ln(2)+x*ln(a)+1/2*x^2
+1/2*I*x*Pi*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*exp(-x))^2-1
/2*I*x*Pi*csgn(I*a)*csgn(I*exp(-x)*(1+exp(2*x)))*csgn(I*a*(1+exp(2*x))*exp(
-x))+ln(exp(x))*ln(1+exp(2*x))-ln(exp(x))*ln(1+I*exp(x))-ln(exp(x))*ln(1-I*
exp(x))-dilog(1+I*exp(x))-dilog(1-I*exp(x))
```

Maxima [A]
time = 0.54, size = 32, normalized size = 0.82

$$\frac{1}{2}\,x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(e^{\left(2\,x\right)} + 1\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-e^{\left(2\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(e^(2*x) + 1) - 1/2*dilog(-e^(2*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.39, size = 65, normalized size = 1.67

$$\frac{1}{2}\,x^2 + x\log\left(a\cosh\left(x\right)\right) - x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - \mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - \mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x*log(a*cosh(x)) - x*log(I*cosh(x) + I*sinh(x) + 1) - x*log(-I*co
sh(x) - I*sinh(x) + 1) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I*cosh(x) -
I*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)),x)
```

```
[Out] Integral(log(a*cosh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)),x, algorithm="giac")
[Out] integrate(log(a*cosh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\cosh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)),x)
[Out] int(log(a*cosh(x)), x)
```

3.205 $\int \log \left(a \cosh^2(x)\right) dx$

Optimal. Leaf size=35

$$x^2 - 2x \log \left(1 + e^{2x}\right) + x \log \left(a \cosh^2(x)\right) - \text{Li}_2\left(-e^{2x}\right)$$

```
[Out] x^2-2*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^2)-polylog(2,-exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$-\text{PolyLog}\left(2, -e^{2x}\right) + x \log \left(a \cosh^2(x)\right) + x^2 - 2x \log \left(e^{2x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Cosh[x]^2],x]
```

```
[Out] x^2 - 2*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^2] - PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\cosh^2(x)\right)\,dx &= x\log\left(a\cosh^2(x)\right)-\int 2x\tanh(x)\,dx\\ &= x\log\left(a\cosh^2(x)\right)-2\int x\tanh(x)\,dx\\ &= x^2+x\log\left(a\cosh^2(x)\right)-4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+2\int \log\left(1+e^{2x}\right)\,dx\\ &= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)+\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\ &= x^2-2x\log\left(1+e^{2x}\right)+x\log\left(a\cosh^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.94

$$x\left(-x-2\log\left(1+e^{-2x}\right)+\log\left(a\cosh^2(x)\right)\right)+\text{Li}_2\left(-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^2],x]
```

```
[Out] x*(-x - 2*Log[1 + E^(-2*x)] + Log[a*Cosh[x]^2]) + PolyLog[2, -E^(-2*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 478, normalized size = 13.66

method	result
risch	$x^2+\frac{ix\pi\,\text{csgn}\left(i\mathrm{e}^{-2x}\left(1+\mathrm{e}^{2x}\right)^2\right)\text{csgn}\left(ia\left(1+\mathrm{e}^{2x}\right)^2\mathrm{e}^{-2x}\right)^2}{2}+ix\pi\,\text{csgn}(i(1+\mathrm{e}^{2x}))\,\text{csgn}\left(i(1+\mathrm{e}^{2x})^2\right)^2+2\ln\left(\mathrm{e}^{x}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2+1/2*I*x*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*csgn(I*a*(1+exp(2*x))^2*exp
(-2*x))^2+I*x*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1+exp(2*x))^2)^2+2*ln(exp(x))
*ln(1+exp(2*x))-2*ln(exp(x))*ln(1+I*exp(x))-2*ln(exp(x))*ln(1-I*exp(x))-2*d
ilog(1+I*exp(x))-2*dilog(1-I*exp(x))-2*x*ln(exp(x))-1/2*I*x*Pi*csgn(I*exp(-
2*x))*csgn(I*(1+exp(2*x))^2)*csgn(I*exp(-2*x)*(1+exp(2*x))^2)-1/2*I*x*Pi*cs
gn(I*(1+exp(2*x))^2)^3-1/2*I*x*Pi*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))^3+1/2*
I*x*Pi*csgn(I*exp(2*x))^3-2*x*ln(2)+x*ln(a)+1/2*I*x*Pi*csgn(I*(1+exp(2*x))^
2)*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^2-1/2*I*x*Pi*csgn(I*(1+exp(2*x)))^2*csg
n(I*(1+exp(2*x))^2)-I*x*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2-1/2*I*x*Pi*csg
n(I*a)*csgn(I*exp(-2*x)*(1+exp(2*x))^2)*csgn(I*a*(1+exp(2*x))^2*exp(-2*x))+
1/2*I*x*Pi*csgn(I*exp(x))^2*csgn(I*exp(2*x))+1/2*I*x*Pi*csgn(I*a)*csgn(I*a*
(1+exp(2*x))^2*exp(-2*x))^2+1/2*I*x*Pi*csgn(I*exp(-2*x))*csgn(I*exp(-2*x)*(
1+exp(2*x))^2)^2-1/2*I*x*Pi*csgn(I*exp(-2*x)*(1+exp(2*x))^2)^3
```

Maxima [A]
time = 0.53, size = 32, normalized size = 0.91

$$x^2 + x\log\left(a\cosh\left(x\right)^2\right) - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] x^2 + x*log(a*cosh(x)^2) - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 77, normalized size = 2.20

$$x^2 + x\log\left(\frac{1}{2}\,a\cosh\left(x\right)^2 + \frac{1}{2}\,a\sinh\left(x\right)^2 + \frac{1}{2}\,a\right) - 2\,x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - 2\,x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - 2\,\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - 2\,\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] x^2 + x*log(1/2*a*cosh(x)^2 + 1/2*a*sinh(x)^2 + 1/2*a) - 2*x*log(I*cosh(x)
+ I*sinh(x) + 1) - 2*x*log(-I*cosh(x) - I*sinh(x) + 1) - 2*dilog(I*cosh(x)
+ I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**2),x)
```

```
[Out] Integral(log(a*cosh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\cosh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)^2),x)
```

```
[Out] int(log(a*cosh(x)^2), x)
```

3.206 $\int \log(a\cosh^n(x))\,dx$

Optimal. Leaf size=44

$$\frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)$$

[Out] 1/2*n*x^2-n*x*ln(1+exp(2*x))+x*ln(a*cosh(x)^n)-1/2*n*polylog(2,-exp(2*x))

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{nx^2}{2}-nx\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[Log[a*Cosh[x]^n],x]

[Out] (n*x^2)/2 - n*x*Log[1 + E^(2*x)] + x*Log[a*Cosh[x]^n] - (n*PolyLog[2, -E^(2*x)])/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log\left(a\cosh^n(x)\right)\,dx &= x\log\left(a\cosh^n(x)\right)-\int nx\tanh(x)\,dx\\
&= x\log\left(a\cosh^n(x)\right)-n\int x\tanh(x)\,dx\\
&= \frac{nx^2}{2}+x\log\left(a\cosh^n(x)\right)-(2n)\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+n\int \log\left(1+e^{2x}\right)\,dx\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)+\frac{1}{2}n\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= \frac{nx^2}{2}-nx\log\left(1+e^{2x}\right)+x\log\left(a\cosh^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(-x\left(nx+2n\log\left(1+e^{-2x}\right)-2\log\left(a\cosh^n(x)\right)\right)+n\text{Li}_2\left(-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Cosh[x]^n],x]
[Out] (-(x*(n*x + 2*n*Log[1 + E^(-2*x)] - 2*Log[a*Cosh[x]^n])) + n*PolyLog[2, -E^
(-2*x)])/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \ln\left(a\left(\cosh^n\left(x\right)\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*cosh(x)^n),x)
```

```
[Out] int(ln(a*cosh(x)^n),x)
```

Maxima [A]
time = 0.53, size = 36, normalized size = 0.82

$$\frac{1}{2}\left(x^2 - 2\,x\log\left(e^{(2\,x)} + 1\right) - \mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n + x\log\left(a\cosh\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*cosh(x)^n)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 73, normalized size = 1.66

$$\frac{1}{2}\,nx^2 - nx\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - nx\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) + nx\log\left(\cosh\left(x\right)\right) - n\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) - n\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right)) + x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="fricas")
```

```
[Out] 1/2*n*x^2 - n*x*log(I*cosh(x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sin
h(x) + 1) + n*x*log(cosh(x)) - n*dilog(I*cosh(x) + I*sinh(x)) - n*dilog(-I*
cosh(x) - I*sinh(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\cosh^n\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*cosh(x)**n),x)
```

```
[Out] Integral(log(a*cosh(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*cosh(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*cosh(x)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(a \cosh (x)^n\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*cosh(x)^n),x)
[Out] int(log(a*cosh(x)^n), x)
```

3.207 $\int \log(\tanh(x))\,dx$

Optimal. Leaf size=39

$$2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))+\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)-\frac{\text{Li}_2\left(e^{2x}\right)}{2}$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(2,
exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+2x\tanh^{-1}\left(e^{2x}\right)+x\log(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[2,
 E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log(\tanh(x))\,dx &= x\log(\tanh(x)) - \int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx \\ &= x\log(\tanh(x)) - 2\int x\operatorname{csch}(2x)\,dx \\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx \\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}\text{Subst}\left(\int \right. \\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.90

$$\frac{1}{2}\log(\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\text{Li}_2(1-\tanh(x)) + \frac{1}{2}\text{Li}_2(-\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Tanh[x]],x]
[Out] (Log[Tanh[x]]*Log[1 + Tanh[x]])/2 + PolyLog[2, 1 - Tanh[x]]/2 + PolyLog[2,
-Tanh[x]]/2
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.62

method	result
derivativedivides	$\frac{\text{dilog}(\tanh(x))}{2} + \frac{\text{dilog}(\tanh(x)+1)}{2} + \frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
default	$\frac{\text{dilog}(\tanh(x))}{2} + \frac{\text{dilog}(\tanh(x)+1)}{2} + \frac{\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$
risch	$-x\ln\left(1+e^{2x}\right) + \frac{i x\pi\,\text{csgn}\left(i\left(e^{2x}-1\right)\right)\text{csgn}\left(\frac{i\left(e^{2x}-1\right)}{1+e^{2x}}\right)^2}{2} + \frac{i x\pi\,\text{csgn}\left(\frac{i}{1+e^{2x}}\right)\text{csgn}\left(\frac{i\left(e^{2x}-1\right)}{1+e^{2x}}\right)^2}{2} + x\ln(1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*dilog(tanh(x))+1/2*dilog(tanh(x)+1)+1/2*ln(tanh(x))*ln(tanh(x)+1)
```

Maxima [A]
time = 0.51, size = 54, normalized size = 1.38

$$x\log\left(e^{(2x)}+1\right)-x\log\left(e^{x}+1\right)-x\log\left(-e^{x}+1\right)+x\log\left(\tanh\left(x\right)\right)+\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)-\mathrm{Li}_2(-e^{x})-\mathrm{Li}_2(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) + x*log(tanh(x)) + 1/
2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 101, normalized size = 2.59

$$x\log\left(\frac{\sinh\left(x\right)}{\cosh\left(x\right)}\right)-x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)+x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)-x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)-\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))+\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))+\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))-\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) + I
*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh(x
) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I
*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\tanh\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tanh(x)),x)
```

```
[Out] Integral(log(tanh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(tanh(x)), x)
```

Mupad [B]
time = 0.47, size = 20, normalized size = 0.51

$$x \ln(\tanh(x)) - \frac{\operatorname{polylog}(2, \tanh(x))}{2} + \frac{\operatorname{polylog}(2, -\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(tanh(x)),x)
[Out] x*log(tanh(x)) - polylog(2, tanh(x))/2 + polylog(2, -tanh(x))/2
```

3.208 $\int \log(a\tanh(x))\,dx$

Optimal. Leaf size=41

$$2x\tanh^{-1}\left(e^{2x}\right)+x\log(a\tanh(x))+\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)-\frac{\text{Li}_2\left(e^{2x}\right)}{2}$$

```
[Out] 2*x*arctanh(exp(2*x))+x*ln(a*tanh(x))+1/2*polylog(2,-exp(2*x))-1/2*polylog(
2,exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}=1.000$, Rules used = {2628, 5569, 4267, 2317, 2438}

$$\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\tanh(x))+2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]],x]
```

```
[Out] 2*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]] + PolyLog[2, -E^(2*x)]/2 - PolyLog[
2, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log(a\tanh(x))\,dx &= x\log(a\tanh(x)) - \int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log(a\tanh(x)) - 2\int x\text{csch}(2x)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \int \log\left(1-e^{2x}\right)\,dx - \int \log\left(1+e^{2x}\right)\,dx\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}\text{Subst}\Bigg(\\ &= 2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\tanh(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) - \frac{\text{Li}_2(e^{2x})}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.20

$$-\frac{1}{2}\log(1-\tanh(x))\log(a\tanh(x)) + \frac{1}{2}\log(a\tanh(x))\log(1+\tanh(x)) + \frac{1}{2}\text{Li}_2(-\tanh(x)) - \frac{\text{Li}_2(\tanh(x))}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]],x]
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]]) + (Log[a*Tanh[x]]*Log[1 + Tanh[x]])/
2 + PolyLog[2, -Tanh[x]]/2 - PolyLog[2, Tanh[x]]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.
time = 0.12, size = 76, normalized size = 1.85

method	result
derivativedivides	$\frac{-\frac{\left(\text{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)a}{2}+\frac{\left(\text{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)a}{2}}{a}$
default	$\frac{-\frac{\left(\text{dilog}\left(-\frac{a\tanh(x)-a}{a}\right)+\ln(a\tanh(x))\ln\left(-\frac{a\tanh(x)-a}{a}\right)\right)a}{2}+\frac{\left(\text{dilog}\left(\frac{a\tanh(x)+a}{a}\right)+\ln(a\tanh(x))\ln\left(\frac{a\tanh(x)+a}{a}\right)\right)a}{2}}{a}$

risch	$-x\ln\left(1+\mathrm{e}^{2x}\right)+\frac{i x \pi \operatorname{csgn}(i a) \operatorname{csgn}\left(\frac{i a\left(\mathrm{e}^{2x}-1\right)}{1+\mathrm{e}^{2x}}\right)^{2}}{2}+\frac{i x \pi \operatorname{csgn}\left(i\left(\mathrm{e}^{2x}-1\right)\right) \operatorname{csgn}\left(\frac{i\left(\mathrm{e}^{2x}-1\right)}{1+\mathrm{e}^{2x}}\right)^{2}}{2}+\frac{i x \pi \operatorname{csgn}\left(\frac{i\left(\mathrm{e}^{2x}\right.}{1+\mathrm{e}^{\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*(dilog(-(a*tanh(x)-a)/a)+ln(a*tanh(x))*ln(-(a*tanh(x)-a)/a))*a+1/
2*(dilog((a*tanh(x)+a)/a)+ln(a*tanh(x))*ln((a*tanh(x)+a)/a))*a)
```

Maxima [A]
time = 0.51, size = 56, normalized size = 1.37

$$x\log\left(a\tanh\left(x\right)\right)+x\log\left(e^{(2x)}+1\right)-x\log\left(e^{x}+1\right)-x\log\left(-e^{x}+1\right)+\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)-\mathrm{Li}_2(-e^{x})-\mathrm{Li}_2(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)) + x*log(e^(2*x) + 1) - x*log(e^x + 1) - x*log(-e^x + 1) +
1/2*dilog(-e^(2*x)) - dilog(-e^x) - dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 102, normalized size = 2.49

$$x\log\left(\frac{a\sinh\left(x\right)}{\cosh\left(x\right)}\right)-x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)+x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)-x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)-\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))+\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))+\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))-\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*sinh(x)/cosh(x)) - x*log(cosh(x) + sinh(x) + 1) + x*log(I*cosh(x) +
 I*sinh(x) + 1) + x*log(-I*cosh(x) - I*sinh(x) + 1) - x*log(-cosh(x) - sinh
(x) + 1) - dilog(cosh(x) + sinh(x)) + dilog(I*cosh(x) + I*sinh(x)) + dilog(
-I*cosh(x) - I*sinh(x)) - dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tanh\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)),x)
```

```
[Out] Integral(log(a*tanh(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*tanh(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\tanh(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)),x)
```

```
[Out] int(log(a*tanh(x)), x)
```

3.209 $\int \log \left(a \tanh^2(x)\right) \, dx$

Optimal. Leaf size=37

$$4x \tanh^{-1}\left(e^{2x}\right) + x \log \left(a \tanh^2(x)\right) + \mathrm{Li}_2\left(-e^{2x}\right) - \mathrm{Li}_2\left(e^{2x}\right)$$

```
[Out] 4*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^2)+polylog(2,-exp(2*x))-polylog(2,exp(
2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\text{PolyLog}\left(2, -e^{2x}\right) - \text{PolyLog}\left(2, e^{2x}\right) + x \log \left(a \tanh^2(x)\right) + 4x \tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^2],x]
```

```
[Out] 4*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^2] + PolyLog[2, -E^(2*x)] - PolyLog[
2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\tanh^2(x)\right)\,dx &= x\log\left(a\tanh^2(x)\right)-\int 2x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\tanh^2(x)\right)-2\int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\tanh^2(x)\right)-4\int x\text{csch}(2x)\,dx\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+2\int\log\left(1-e^{2x}\right)\,dx-2\int\log\left(1+e^{2x}\right)\,d\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)-\text{Subst}\Big(\\ &= 4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^2(x)\right)+\text{Li}_2\left(-e^{2x}\right)-\text{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.27

$$-\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^2(x)\right)+\frac{1}{2}\log\left(a\tanh^2(x)\right)\log(1+\tanh(x))+\text{Li}_2(-\tanh(x))-\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^2],x]
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^2]) + (Log[a*Tanh[x]^2]*Log[1 + Tanh[x
]])/2 + PolyLog[2, -Tanh[x]] - PolyLog[2, Tanh[x]]
```

Maple [A]
time = 0.14, size = 47, normalized size = 1.27

method	result
derivativedivides	$-\frac{\ln(\tanh(x)-1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2}+\text{dilog}\left(\tanh\left(x\right)\right)+\ln\left(\tanh\left(x\right)-1\right)\ln\left(\tanh\left(x\right)\right)+\frac{\ln(\tanh(x}{}$

default	$-\frac{\ln(\tanh(x)-1)\ln\left(a\left(\tanh^2(x)\right)\right)}{2}+\operatorname{dilog}\left(\tanh\left(x\right)\right)+\ln\left(\tanh\left(x\right)-1\right)\ln\left(\tanh\left(x\right)\right)+\frac{\ln(\tanh(x)+}{}$
risch	$-2x\ln\left(1+\mathrm{e}^{2x}\right)-ix\pi\operatorname{csgn}(i(1+\mathrm{e}^{2x}))\operatorname{csgn}\left(i(1+\mathrm{e}^{2x})^2\right)^2-\frac{ix\pi\operatorname{csgn}\left(\frac{i\left(\mathrm{e}^{2x}-1\right)^2}{\left(1+\mathrm{e}^{2x}\right)^2}\right)^3}{2}-\frac{ix\pi\operatorname{csgn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(tanh(x)-1)*ln(a*tanh(x)^2)+dilog(tanh(x))+ln(tanh(x)-1)*ln(tanh(x))
+1/2*ln(tanh(x)+1)*ln(a*tanh(x)^2)+dilog(tanh(x)+1)
```

Maxima [A]
time = 0.49, size = 57, normalized size = 1.54

$$x\log\left(a\tanh\left(x\right)^2\right)+2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^x+1\right)-2\,x\log\left(-e^x+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2(-e^x)-2\,\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*tanh(x)^2) + 2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x
 + 1) + dilog(-e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 129, normalized size = 3.49

$$x\log\left(\frac{a\cosh\left(x\right)^2+a\sinh\left(x\right)^2-a}{\cosh\left(x\right)^2+\sinh\left(x\right)^2+1}\right)-2\,x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+2\,x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)+2\,x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)-2\,x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)-2\,\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))+2\,\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))+2\,\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))-2\,\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 - a)/(cosh(x)^2 + sinh(x)^2 + 1)) - 2*x*lo
g(cosh(x) + sinh(x) + 1) + 2*x*log(I*cosh(x) + I*sinh(x) + 1) + 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) - 2*x*log(-cosh(x) - sinh(x) + 1) - 2*dilog(cosh(x
) + sinh(x)) + 2*dilog(I*cosh(x) + I*sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh
(x)) - 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tanh^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**2),x)
```

```
[Out] Integral(log(a*tanh(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^2),x, algorithm="giac")
[Out] integrate(log(a*tanh(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\tanh(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)^2),x)
[Out] int(log(a*tanh(x)^2), x)
```

3.210 $\int \log(a\tanh^n(x))\,dx$

Optimal. Leaf size=46

$$2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)-\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)$$

```
[Out] 2*n*x*arctanh(exp(2*x))+x*ln(a*tanh(x)^n)+1/2*n*polylog(2,-exp(2*x))-1/2*n*
polylog(2,exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)-\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\tanh^n(x)\right)+2nx\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Tanh[x]^n],x]
```

```
[Out] 2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Tanh[x]^n] + (n*PolyLog[2, -E^(2*x)])/2 -
(n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\tanh^n(x)\right)\,dx &= x\log\left(a\tanh^n(x)\right) - \int nx\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\tanh^n(x)\right) - n\int x\text{csch}(x)\text{sech}(x)\,dx \\
&= x\log\left(a\tanh^n(x)\right) - (2n)\int x\text{csch}(2x)\,dx \\
&= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + n\int \log\left(1-e^{2x}\right)\,dx - n\int \log\left(1+e^{2x}\right) \\
&= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + \frac{1}{2}n\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) - \frac{1}{2}n \\
&= 2nx\tanh^{-1}\left(e^{2x}\right) + x\log\left(a\tanh^n(x)\right) + \frac{1}{2}n\text{Li}_2\left(-e^{2x}\right) - \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 55, normalized size = 1.20

$$-\frac{1}{2}\log(1-\tanh(x))\log\left(a\tanh^n(x)\right) + \frac{1}{2}\log\left(a\tanh^n(x)\right)\log(1+\tanh(x)) + \frac{1}{2}n\text{Li}_2(-\tanh(x)) - \frac{1}{2}n\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Tanh[x]^n],x]
```

```
[Out] -1/2*(Log[1 - Tanh[x]]*Log[a*Tanh[x]^n]) + (Log[a*Tanh[x]^n]*Log[1 + Tanh[x
]])/2 + (n*PolyLog[2, -Tanh[x]])/2 - (n*PolyLog[2, Tanh[x]])/2
```

Maple [**A**]
time = 0.68, size = 43, normalized size = 0.93

method	result	s
default	$x(\ln(a(\tanh^n(x))) - n\ln(\tanh(x))) + \frac{n\,\mathrm{dilog}(\tanh(x))}{2} + \frac{n\,\mathrm{dilog}(\tanh(x)+1)}{2} + \frac{n\ln(\tanh(x))\ln(\tanh(x)+1)}{2}$	4
risch	Expression too large to display	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*tanh(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(a*tanh(x)^n)-n*ln(tanh(x)))+1/2*n*dilog(tanh(x))+1/2*n*dilog(tanh(x)+
1)+1/2*n*ln(tanh(x))*ln(tanh(x)+1)
```

Maxima [A]
time = 0.51, size = 61, normalized size = 1.33

$$\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right)-2\,x\log\left(e^{x}+1\right)-2\,x\log\left(-e^{x}+1\right)+\mathrm{Li}_2\left(-e^{(2\,x)}\right)-2\,\mathrm{Li}_2(-e^{x})-2\,\mathrm{Li}_2(e^{x})\right)n+x\log\left(a\tanh\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-e
^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*tanh(x)^n)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 116, normalized size = 2.52

$$nx\log\left(\frac{\sinh(x)}{\cosh(x)}\right)-nx\log(\cosh(x)+\sinh(x)+1)+nx\log(i\cosh(x)+i\sinh(x)+1)+nx\log(-i\cosh(x)-i\sinh(x)+1)-nx\log(-\cosh(x)-\sinh(x)+1)-n\mathrm{Li}_2(\cosh(x)+\sinh(x))+n\mathrm{Li}_2(i\cosh(x)+i\sinh(x))+n\mathrm{Li}_2(-i\cosh(x)-i\sinh(x))-n\mathrm{Li}_2(-\cosh(x)-\sinh(x))+x\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(sinh(x)/cosh(x)) - n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(I*cosh(
x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x) - I*sinh(x) + 1) - n*x*log(-cosh(x
) - sinh(x) + 1) - n*dilog(cosh(x) + sinh(x)) + n*dilog(I*cosh(x) + I*sinh(
x)) + n*dilog(-I*cosh(x) - I*sinh(x)) - n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\tanh^{n}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*tanh(x)**n),x)
```

```
[Out] Integral(log(a*tanh(x)**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*tanh(x)^n),x, algorithm="giac")
[Out] integrate(log(a*tanh(x)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\tanh(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*tanh(x)^n),x)
[Out] int(log(a*tanh(x)^n), x)
```

3.211 $\int \log(\coth(x))\,dx$

Optimal. Leaf size=39

$$-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))-\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)+\frac{\text{Li}_2\left(e^{2x}\right)}{2}$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog(2
,exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}=1.667$, Rules used = {2628, 5569, 4267, 2317, 2438}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)-2x\tanh^{-1}\left(e^{2x}\right)+x\log(\coth(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog[2
, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log(\coth(x))\,dx &= x\log(\coth(x)) + \int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log(\coth(x)) + 2\int x\text{csch}(2x)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \int \log\left(1-e^{2x}\right)\,dx + \int \log\left(1+e^{2x}\right)\,dx\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int\right.\\ &= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.90

$$\frac{1}{2}\log(\coth(x))\log(1+\coth(x)) + \frac{1}{2}\text{Li}_2(1-\coth(x)) + \frac{1}{2}\text{Li}_2(-\coth(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Coth[x]],x]
```

```
[Out] (Log[Coth[x]]*Log[1 + Coth[x]])/2 + PolyLog[2, 1 - Coth[x]]/2 + PolyLog[2,
-Coth[x]]/2
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.62

method	result
derivativedivides	$\frac{\text{dilog}(\coth(x)+1)}{2} + \frac{\ln(\coth(x))\ln(\coth(x)+1)}{2} + \frac{\text{dilog}(\coth(x))}{2}$
default	$\frac{\text{dilog}(\coth(x)+1)}{2} + \frac{\ln(\coth(x))\ln(\coth(x)+1)}{2} + \frac{\text{dilog}(\coth(x))}{2}$
risch	$x\ln\left(1+e^{2x}\right) + \frac{i x\pi\,\text{csgn}\left(i\left(1+e^{2x}\right)\right)\text{csgn}\left(\frac{i\left(1+e^{2x}\right)}{e^{2x}-1}\right)^2}{2} + \frac{i x\pi\,\text{csgn}\left(\frac{i}{e^{2x}-1}\right)\text{csgn}\left(\frac{i\left(1+e^{2x}\right)}{e^{2x}-1}\right)^2}{2} - x\ln(1+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*dilog(coth(x)+1)+1/2*ln(coth(x))*ln(coth(x)+1)+1/2*dilog(coth(x))
```

Maxima [A]
time = 0.51, size = 49, normalized size = 1.26

$$-x\log\left(e^{(2x)}+1\right)+x\log\left(e^{x}+1\right)+x\log\left(-e^{x}+1\right)+x\log\left(\coth\left(x\right)\right)-\frac{1}{2}\operatorname{Li}_2\left(-e^{(2x)}\right)+\operatorname{Li}_2(-e^{x})+\operatorname{Li}_2(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="maxima")
```

```
[Out] -x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) + x*log(coth(x)) - 1
/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.38, size = 101, normalized size = 2.59

$$x\log\left(\frac{\cosh\left(x\right)}{\sinh\left(x\right)}\right)+x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-x\log\left(i\cosh\left(x\right)+i\sinh\left(x\right)+1\right)-x\log\left(-i\cosh\left(x\right)-i\sinh\left(x\right)+1\right)+x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+\operatorname{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\operatorname{Li}_2(i\cosh\left(x\right)+i\sinh\left(x\right))-\operatorname{Li}_2(-i\cosh\left(x\right)-i\sinh\left(x\right))+\operatorname{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) + I
*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh(x
) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(-I
*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\coth\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(coth(x)),x)
```

```
[Out] Integral(log(coth(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(coth(x)), x)
```

Mupad [B]
time = 0.41, size = 22, normalized size = 0.56

$$\frac{\text{polylog}(2,-\coth(x))}{2}-\frac{\text{polylog}(2,\coth(x))}{2}+\text{atanh}(\coth(x))\,\ln\left(\coth(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(coth(x)),x)
```

```
[Out] polylog(2, -coth(x))/2 - polylog(2, coth(x))/2 + atanh(coth(x))*log(coth(x)
)
```

3.212 $\int \log(a\coth(x))\,dx$

Optimal. Leaf size=41

$$-2x\tanh^{-1}\left(e^{2x}\right)+x\log(a\coth(x))-\frac{1}{2}\text{Li}_2\left(-e^{2x}\right)+\frac{\text{Li}_2\left(e^{2x}\right)}{2}$$

```
[Out] -2*x*arctanh(exp(2*x))+x*ln(a*coth(x))-1/2*polylog(2,-exp(2*x))+1/2*polylog
(2,exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2628, 5569, 4267, 2317, 2438}

$$-\frac{1}{2}\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\coth(x))-2x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]],x]
```

```
[Out] -2*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]] - PolyLog[2, -E^(2*x)]/2 + PolyLog
[2, E^(2*x)]/2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \log(a\coth(x))\,dx &= x\log(a\coth(x)) + \int x\operatorname{csch}(x)\operatorname{sech}(x)\,dx\\
&= x\log(a\coth(x)) + 2\int x\operatorname{csch}(2x)\,dx\\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \int \log\left(1-e^{2x}\right)\,dx + \int \log\left(1+e^{2x}\right)\,dx\\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\Big(\\
&= -2x\tanh^{-1}\left(e^{2x}\right) + x\log(a\coth(x)) - \frac{1}{2}\text{Li}_2\left(-e^{2x}\right) + \frac{\text{Li}_2\left(e^{2x}\right)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.20

$$-\frac{1}{2}\log(1-\coth(x))\log(a\coth(x)) + \frac{1}{2}\log(a\coth(x))\log(1+\coth(x)) + \frac{1}{2}\text{Li}_2(-\coth(x)) - \frac{\text{Li}_2(\coth(x))}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]],x]
```

```
[Out] -1/2*(Log[1 - Coth[x]]*Log[a*Coth[x]]) + (Log[a*Coth[x]]*Log[1 + Coth[x]])/
2 + PolyLog[2, -Coth[x]]/2 - PolyLog[2, Coth[x]]/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.
time = 0.11, size = 76, normalized size = 1.85

method	result
derivativedivides	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)a}{2}}{a}$
default	$\frac{\frac{\left(\operatorname{dilog}\left(\frac{a\coth(x)+a}{a}\right)+\ln(a\coth(x))\ln\left(\frac{a\coth(x)+a}{a}\right)\right)a}{2}-\frac{\left(\operatorname{dilog}\left(-\frac{a\coth(x)-a}{a}\right)+\ln(a\coth(x))\ln\left(-\frac{a\coth(x)-a}{a}\right)\right)a}{2}}{a}$

risch	$x\ln\left(1+\mathrm{e}^{2x}\right)+\frac{i x\pi\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2x}-1}\right)\mathrm{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}\right)^{2}}{2}+\frac{i x\pi\,\mathrm{csgn}\left(i\left(1+\mathrm{e}^{2x}\right)\right)\mathrm{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}\right)^{2}}{2}-\frac{i x\pi\mathrm{csgn}\left(\frac{i\left(1+\right.}{\mathrm{e}^{2x}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*(dilog((a*coth(x)+a)/a)+ln(a*coth(x))*ln((a*coth(x)+a)/a))*a-1/2*(
dilog(-(a*coth(x)-a)/a)+ln(a*coth(x))*ln(-(a*coth(x)-a)/a))*a)
```

Maxima [A]
time = 0.51, size = 51, normalized size = 1.24

$$x\log\left(a\coth\left(x\right)\right)-x\log\left(e^{(2x)}+1\right)+x\log\left(e^{x}+1\right)+x\log\left(-e^{x}+1\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2x)}\right)+\mathrm{Li}_2(-e^{x})+\mathrm{Li}_2(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)) - x*log(e^(2*x) + 1) + x*log(e^x + 1) + x*log(-e^x + 1) -
1/2*dilog(-e^(2*x)) + dilog(-e^x) + dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 102, normalized size = 2.49

$$x\log\left(\frac{a\cosh\left(x\right)}{\sinh\left(x\right)}\right)+x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-x\log\left(i\cosh\left(x\right)+i\sinh\left(x\right)+1\right)-x\log\left(-i\cosh\left(x\right)-i\sinh\left(x\right)+1\right)+x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-\mathrm{Li}_2(i\cosh\left(x\right)+i\sinh\left(x\right))-\mathrm{Li}_2(-i\cosh\left(x\right)-i\sinh\left(x\right))+\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="fricas")
```

```
[Out] x*log(a*cosh(x)/sinh(x)) + x*log(cosh(x) + sinh(x) + 1) - x*log(I*cosh(x) +
 I*sinh(x) + 1) - x*log(-I*cosh(x) - I*sinh(x) + 1) + x*log(-cosh(x) - sinh
(x) + 1) + dilog(cosh(x) + sinh(x)) - dilog(I*cosh(x) + I*sinh(x)) - dilog(
-I*cosh(x) - I*sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\coth\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)),x)
```

```
[Out] Integral(log(a*coth(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*coth(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\coth(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)),x)
```

```
[Out] int(log(a*coth(x)), x)
```

3.213 $\int \log\left(a\coth^2(x)\right)\,dx$

Optimal. Leaf size=37

$$-4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)+\text{Li}_2\left(e^{2x}\right)$$

```
[Out] -4*x*arctanh(exp(2*x))+x*ln(a*coth(x)^2)-polylog(2,-exp(2*x))+polylog(2,exp
(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$-\text{PolyLog}\left(2,-e^{2x}\right)+\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^2(x)\right)-4x\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^2],x]
```

```
[Out] -4*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^2] - PolyLog[2, -E^(2*x)] + PolyLog
[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
```

```
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\coth^2(x)\right)\,dx &= x\log\left(a\coth^2(x)\right)-\int -2x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\coth^2(x)\right)+2\int x\text{csch}(x)\text{sech}(x)\,dx\\ &= x\log\left(a\coth^2(x)\right)+4\int x\text{csch}(2x)\,dx\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-2\int \log\left(1-e^{2x}\right)\,dx+2\int \log\left(1+e^{2x}\right)\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)+\text{Subst}\\ &= -4x\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^2(x)\right)-\text{Li}_2\left(-e^{2x}\right)+\text{Li}_2\left(e^{2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.27

$$-\frac{1}{2}\log\left(a\coth^2(x)\right)\log(1-\tanh(x))+\frac{1}{2}\log\left(a\coth^2(x)\right)\log(1+\tanh(x))-\text{Li}_2(-\tanh(x))+\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^2],x]
```

```
[Out] -1/2*(Log[a*Coth[x]^2]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^2]*Log[1 + Tanh[x
]])/2 - PolyLog[2, -Tanh[x]] + PolyLog[2, Tanh[x]]
```

Maple [A]
time = 0.13, size = 47, normalized size = 1.27

method	result
derivativedivides	$\frac{\ln(\coth(x)+1)\ln\left(a\left(\coth^2(x)\right)\right)}{2}+\text{dilog}\left(\coth\left(x\right)+1\right)-\frac{\ln(\coth(x)-1)\ln\left(a\left(\coth^2(x)\right)\right)}{2}+\text{dilog}\left(\coth\left(\right.\right.$

default	$\frac{\ln(\coth(x)+1)\ln\left(a\left(\coth^2(x)\right)\right)}{2}+\operatorname{dilog}\left(\coth\left(x\right)+1\right)-\frac{\ln(\coth(x)-1)\ln\left(a\left(\coth^2(x)\right)\right)}{2}+\operatorname{dilog}\left(\coth\left(x\right.\right.$
risch	$2x\ln\left(1+\mathrm{e}^{2x}\right)-\frac{i x\pi\operatorname{csgn}(ia)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)^2}{\left(\mathrm{e}^{2x}-1\right)^2}\right)\operatorname{csgn}\left(\frac{ia\left(1+\mathrm{e}^{2x}\right)^2}{\left(\mathrm{e}^{2x}-1\right)^2}\right)}{2}+\frac{i x\pi\operatorname{csgn}\left(\frac{i}{\left(\mathrm{e}^{2x}-1\right)^2}\right)\operatorname{csgn}\left(\frac{i\left(1+\mathrm{e}^{2x}\right)^2}{\left(\mathrm{e}^{2x}-1\right)^2}\right)^2}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(coth(x)+1)*ln(a*coth(x)^2)+dilog(coth(x)+1)-1/2*ln(coth(x)-1)*ln(a*c
oth(x)^2)+dilog(coth(x))+ln(coth(x)-1)*ln(coth(x))
```

Maxima [A]
time = 0.52, size = 59, normalized size = 1.59

$$x\log\left(a\coth\left(x\right)^2\right)-2\,x\log\left(e^{(2\,x)}+1\right)+2\,x\log\left(e^x+1\right)+2\,x\log\left(-e^x+1\right)-\mathrm{Li}_2\left(-e^{(2\,x)}\right)+2\,\mathrm{Li}_2(-e^x)+2\,\mathrm{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="maxima")
```

```
[Out] x*log(a*coth(x)^2) - 2*x*log(e^(2*x) + 1) + 2*x*log(e^x + 1) + 2*x*log(-e^x
 + 1) - dilog(-e^(2*x)) + 2*dilog(-e^x) + 2*dilog(e^x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 127, normalized size = 3.43

$$x\log\left(\frac{a\cosh\left(x\right)^2+a\sinh\left(x\right)^2+a}{\cosh\left(x\right)^2+\sinh\left(x\right)^2-1}\right)+2\,x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-2\,x\log\left(i\,\cosh\left(x\right)+i\,\sinh\left(x\right)+1\right)-2\,x\log\left(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right)+1\right)+2\,x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right)+2\,\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right))-2\,\mathrm{Li}_2(i\,\cosh\left(x\right)+i\,\sinh\left(x\right))-2\,\mathrm{Li}_2(-i\,\cosh\left(x\right)-i\,\sinh\left(x\right))+2\,\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="fricas")
```

```
[Out] x*log((a*cosh(x)^2 + a*sinh(x)^2 + a)/(cosh(x)^2 + sinh(x)^2 - 1)) + 2*x*lo
g(cosh(x) + sinh(x) + 1) - 2*x*log(I*cosh(x) + I*sinh(x) + 1) - 2*x*log(-I*
cosh(x) - I*sinh(x) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x
) + sinh(x)) - 2*dilog(I*cosh(x) + I*sinh(x)) - 2*dilog(-I*cosh(x) - I*sinh
(x)) + 2*dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\coth^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**2),x)
```

```
[Out] Integral(log(a*coth(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^2),x, algorithm="giac")
[Out] integrate(log(a*coth(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(a\coth(x)^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)^2),x)
[Out] int(log(a*coth(x)^2), x)
```

3.214 $\int \log(a\coth^n(x))\,dx$

Optimal. Leaf size=46

$$-2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)+\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)$$

```
[Out] -2*n*x*arctanh(exp(2*x))+x*ln(a*coth(x)^n)-1/2*n*polylog(2,-exp(2*x))+1/2*n
*polylog(2,exp(2*x))
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 5569, 4267, 2317, 2438}

$$-\frac{1}{2}n\text{PolyLog}\left(2,-e^{2x}\right)+\frac{1}{2}n\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\coth^n(x)\right)-2nx\tanh^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Coth[x]^n],x]
```

```
[Out] -2*n*x*ArcTanh[E^(2*x)] + x*Log[a*Coth[x]^n] - (n*PolyLog[2, -E^(2*x)])/2 +
 (n*PolyLog[2, E^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5569

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\coth^n(x)\right)\,dx &= x\log\left(a\coth^n(x)\right)+\int nx\text{csch}(x)\text{sech}(x)\,dx\\
&= x\log\left(a\coth^n(x)\right)+n\int x\text{csch}(x)\text{sech}(x)\,dx\\
&= x\log\left(a\coth^n(x)\right)+(2n)\int x\text{csch}(2x)\,dx\\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx+n\int\log\left(1+e^{2x}\right.\\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}\\
&= -2nx\tanh^{-1}\left(e^{2x}\right)+x\log\left(a\coth^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)+\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.20

$$-\frac{1}{2}\log\left(a\coth^n(x)\right)\log(1-\tanh(x))+\frac{1}{2}\log\left(a\coth^n(x)\right)\log(1+\tanh(x))-\frac{1}{2}n\text{Li}_2(-\tanh(x))+\frac{1}{2}n\text{Li}_2(\tanh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Coth[x]^n],x]
```

```
[Out] -1/2*(Log[a*Coth[x]^n]*Log[1 - Tanh[x]]) + (Log[a*Coth[x]^n]*Log[1 + Tanh[x
]])/2 - (n*PolyLog[2, -Tanh[x]])/2 + (n*PolyLog[2, Tanh[x]])/2
```

Maple [A]
time = 1.27, size = 43, normalized size = 0.93

method	result	siz
default	$x(\ln(a(\coth^n(x))) - n\ln(\coth(x))) + \frac{n\,\mathrm{dilog}(\coth(x))}{2} + \frac{n\,\mathrm{dilog}(\coth(x)+1)}{2} + \frac{n\ln(\coth(x))\ln(\coth(x)+1)}{2}$	43
risch	Expression too large to display	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*coth(x)^n),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(a*coth(x)^n)-n*ln(coth(x)))+1/2*n*dilog(coth(x))+1/2*n*dilog(coth(x)+
1)+1/2*n*ln(coth(x))*ln(coth(x)+1)
```

Maxima [A]
time = 0.51, size = 61, normalized size = 1.33

$$-\frac{1}{2}\left(2\,x\log\left(e^{(2\,x)}+1\right) - 2\,x\log\left(e^{x}+1\right) - 2\,x\log\left(-e^{x}+1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right) - 2\,\mathrm{Li}_2\left(-e^{x}\right) - 2\,\mathrm{Li}_2\left(e^{x}\right)\right)n + x\log\left(a\coth\left(x\right)^{n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x*log(e^(2*x) + 1) - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) + dilog(-
e^(2*x)) - 2*dilog(-e^x) - 2*dilog(e^x))*n + x*log(a*coth(x)^n)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.42, size = 116, normalized size = 2.52

$$nx\log\left(\frac{\cosh(x)}{\sinh(x)}\right) + nx\log(\cosh(x)+\sinh(x)+1) - nx\log(i\,\cosh(x)+i\,\sinh(x)+1) - nx\log(-i\,\cosh(x)-i\,\sinh(x)+1) + nx\log(-\cosh(x)-\sinh(x)+1) + n\mathrm{Li}_2(\cosh(x)+\sinh(x)) - n\mathrm{Li}_2(i\,\cosh(x)+i\,\sinh(x)) - n\mathrm{Li}_2(-i\,\cosh(x)-i\,\sinh(x)) + n\mathrm{Li}_2(-\cosh(x)-\sinh(x)) + x\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="fricas")
```

```
[Out] n*x*log(cosh(x)/sinh(x)) + n*x*log(cosh(x) + sinh(x) + 1) - n*x*log(I*cosh(
x) + I*sinh(x) + 1) - n*x*log(-I*cosh(x) - I*sinh(x) + 1) + n*x*log(-cosh(x
) - sinh(x) + 1) + n*dilog(cosh(x) + sinh(x)) - n*dilog(I*cosh(x) + I*sinh(
x)) - n*dilog(-I*cosh(x) - I*sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\coth^{n}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*coth(x)**n),x)
```

```
[Out] Integral(log(a*coth(x)**n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*coth(x)^n),x, algorithm="giac")
[Out] integrate(log(a*coth(x)^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\,\mathrm{coth}(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*coth(x)^n),x)
[Out] int(log(a*coth(x)^n), x)
```

3.215 $\int \log(a\,\mathbf{sech}(x))\,dx$

Optimal. Leaf size=38

$$-\frac{x^2}{2}+x\log\left(1+e^{2x}\right)+x\log(a\mathrm{sech}(x))+\frac{1}{2}\mathrm{Li}_2\left(-e^{2x}\right)$$

```
[Out] -1/2*x^2+x*ln(1+exp(2*x))+x*ln(a*sech(x))+1/2*polylog(2,-exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3799, 2221, 2317, 2438}

$$\frac{1}{2}\mathrm{PolyLog}\left(2,-e^{2x}\right)+x\log(a\mathrm{sech}(x))-\frac{x^2}{2}+x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]],x]
```

```
[Out] -1/2*x^2 + x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]] + PolyLog[2, -E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\operatorname{sech}(x))\,dx &= x\log(a\operatorname{sech}(x)) + \int x\tanh(x)\,dx \\
&= -\frac{x^2}{2} + x\log(a\operatorname{sech}(x)) + 2\int \frac{e^{2x}x}{1+e^{2x}}\,dx \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\operatorname{sech}(x)) - \int \log\left(1+e^{2x}\right)\,dx \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\operatorname{sech}(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2x}\right) \\
&= -\frac{x^2}{2} + x\log\left(1+e^{2x}\right) + x\log(a\operatorname{sech}(x)) + \frac{1}{2}\text{Li}_2\left(-e^{2x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$\frac{1}{2}\left(x\left(x+2\log\left(1+e^{-2x}\right)+2\log(a\operatorname{sech}(x))\right)-\text{Li}_2\left(-e^{-2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]],x]
[Out] (x*(x + 2*Log[1 + E^(-2*x)] + 2*Log[a*Sech[x]]) - PolyLog[2, -E^(-2*x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 314, normalized size = 8.26

method	result
risch	$x\ln\left(\mathrm{e}^{x}\right)+\frac{i x\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)\mathrm{csgn}\left(\frac{i a\,\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{2}}{2}-\frac{i x\pi\mathrm{csgn}\left(\frac{i\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{3}}{2}-\frac{i x\pi\mathrm{csgn}\left(\frac{i a\,\mathrm{e}^{x}}{1+\mathrm{e}^{2x}}\right)^{3}}{2}-\frac{i x\pi\,\mathrm{csgn}\left(i\mathrm{e}^{x}\right)\mathrm{csgn}\left(\frac{i}{1+\mathrm{e}^{2x}}\right)\mathrm{csgn}(}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)),x,method=_RETURNVERBOSE)
[Out] x*ln(exp(x))+1/2*I*x*Pi*csgn(I*exp(x)/(1+exp(2*x)))*csgn(I*a/(1+exp(2*x))*e
xp(x))^2-1/2*I*x*Pi*csgn(I*exp(x)/(1+exp(2*x)))^3-1/2*I*x*Pi*csgn(I*a/(1+ex
p(2*x))*exp(x))^3-1/2*I*x*Pi*csgn(I*exp(x))*csgn(I/(1+exp(2*x)))*csgn(I*exp
```

```
(x)/(1+exp(2*x)))+1/2*I*x*Pi*csgn(I*a)*csgn(I*a/(1+exp(2*x))*exp(x))^2-1/2*
I*x*Pi*csgn(I*a)*csgn(I*exp(x)/(1+exp(2*x)))*csgn(I*a/(1+exp(2*x))*exp(x))+
x*ln(2)+x*ln(a)-1/2*x^2+1/2*I*x*Pi*csgn(I*exp(x))*csgn(I*exp(x)/(1+exp(2*x)
))^2+1/2*I*x*Pi*csgn(I/(1+exp(2*x)))*csgn(I*exp(x)/(1+exp(2*x)))^2-ln(exp(x
))*ln(1+exp(2*x))+ln(exp(x))*ln(1+I*exp(x))+ln(exp(x))*ln(1-I*exp(x))+dilog
(1+I*exp(x))+dilog(1-I*exp(x))
```

Maxima [A]
time = 0.54, size = 31, normalized size = 0.82

$$-\frac{1}{2}\,x^2 + x\log\left(a\operatorname{sech}\left(x\right)\right) + x\log\left(e^{(2\,x)} + 1\right) + \frac{1}{2}\,\mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="maxima")
[Out] -1/2*x^2 + x*log(a*sech(x)) + x*log(e^(2*x) + 1) + 1/2*dilog(-e^(2*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.40, size = 84, normalized size = 2.21

$$-\frac{1}{2}\,x^2 + x\log\left(\frac{2\,\left(a\cosh\left(x\right) + a\sinh\left(x\right)\right)}{\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 1}\right) + x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) + x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) + \mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + \mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="fricas")
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + x*log(I*cosh(x) + I*sinh(x) + 1) + x*log(-I*cosh(x) - I*
sinh(x) + 1) + dilog(I*cosh(x) + I*sinh(x)) + dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{sech}\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)),x)
[Out] Integral(log(a*sech(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \ln(\cosh(x)) - \ln(a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cosh(x)),x)
```

```
[Out] -int(log(cosh(x)) - log(a), x)
```

3.216 $\int \log \left(a\mathbf{sech}^2(x)\right) \, dx$

Optimal. Leaf size=35

$$-x^2 + 2x \log \left(1 + e^{2x}\right) + x \log \left(a\text{sech}^2(x)\right) + \text{Li}_2\left(-e^{2x}\right)$$

```
[Out] -x^2+2*x*ln(1+exp(2*x))+x*ln(a*sech(x)^2)+polylog(2,-exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\text{PolyLog}\left(2, -e^{2x}\right) + x \log \left(a\text{sech}^2(x)\right) - x^2 + 2x \log \left(e^{2x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^2] + PolyLog[2, -E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(a\text{sech}^2(x)\right)\,dx &= x\log\left(a\text{sech}^2(x)\right)-\int -2x\tanh(x)\,dx\\ &= x\log\left(a\text{sech}^2(x)\right)+2\int x\tanh(x)\,dx\\ &= -x^2+x\log\left(a\text{sech}^2(x)\right)+4\int \frac{e^{2x}x}{1+e^{2x}}\,dx\\ &= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^2(x)\right)-2\int \log\left(1+e^{2x}\right)\,dx\\ &= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^2(x)\right)-\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\ &= -x^2+2x\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^2(x)\right)+\text{Li}_2\left(-e^{2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.94

$$x\left(x+2\log\left(1+e^{-2x}\right)+\log\left(a\text{sech}^2(x)\right)\right)-\text{Li}_2\left(-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^2],x]
[Out] x*(x + 2*Log[1 + E^(-2*x)] + Log[a*Sech[x]^2]) - PolyLog[2, -E^(-2*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 480, normalized size = 13.71

method	result
risch	$-ix\pi\,\text{csgn}(i(1+e^{2x}))\,\text{csgn}\left(i(1+e^{2x})^2\right)^2+\frac{ix\pi\,\text{csgn}(ie^{2x})\text{csgn}\left(\frac{ie^{2x}}{(1+e^{2x})^2}\right)^2}{2}-x^2-2\ln(e^x)\ln(1+e^{2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*x*Pi*csgn(I*(1+exp(2*x)))*csgn(I*(1+exp(2*x))^2)^2+1/2*I*x*Pi*csgn(I*exp
(2*x))*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2-x^2-2*ln(exp(x))*ln(1+exp(2*x))+2*
ln(exp(x))*ln(1+I*exp(x))+2*ln(exp(x))*ln(1-I*exp(x))+2*dilog(1+I*exp(x))+2
*dilog(1-I*exp(x))+2*x*ln(exp(x))-1/2*I*x*Pi*csgn(I*exp(2*x))^3+1/2*I*x*Pi*
csgn(I*(1+exp(2*x))^2)^3+1/2*I*x*Pi*csgn(I*a)*csgn(I*a/(1+exp(2*x))^2*exp(2
*x))^2+2*x*ln(2)+x*ln(a)-1/2*I*x*Pi*csgn(I*a)*csgn(I*exp(2*x)/(1+exp(2*x))^
2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x))-1/2*I*x*Pi*csgn(I*exp(2*x))*csgn(I/(1+
exp(2*x))^2)*csgn(I*exp(2*x)/(1+exp(2*x))^2)+I*x*Pi*csgn(I*exp(x))*csgn(I*e
xp(2*x))^2-1/2*I*x*Pi*csgn(I*exp(2*x)/(1+exp(2*x))^2)^3+1/2*I*x*Pi*csgn(I*(
1+exp(2*x)))^2*csgn(I*(1+exp(2*x))^2)-1/2*I*x*Pi*csgn(I*exp(x))^2*csgn(I*ex
p(2*x))+1/2*I*x*Pi*csgn(I/(1+exp(2*x))^2)*csgn(I*exp(2*x)/(1+exp(2*x))^2)^2
+1/2*I*x*Pi*csgn(I*exp(2*x)/(1+exp(2*x))^2)*csgn(I*a/(1+exp(2*x))^2*exp(2*x
))^2-1/2*I*x*Pi*csgn(I*a/(1+exp(2*x))^2*exp(2*x))^3
```

Maxima [A]
time = 0.54, size = 32, normalized size = 0.91

$$-x^2 + x\log\left(a\,\mathrm{sech}\left(x\right)^2\right) + 2\,x\log\left(e^{(2\,x)} + 1\right) + \mathrm{Li}_2\left(-e^{(2\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*sech(x)^2) + 2*x*log(e^(2*x) + 1) + dilog(-e^(2*x))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 106, normalized size = 3.03

$$-x^2 + x\log\left(\frac{4\left(a\cosh\left(x\right) + a\sinh\left(x\right)\right)}{\cosh\left(x\right)^3 + 3\,\cosh\left(x\right)\sinh\left(x\right)^2 + \sinh\left(x\right)^3 + \left(3\,\cosh\left(x\right)^2 + 1\right)\sinh\left(x\right) + 3\,\cosh\left(x\right)}\right) + 2\,x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) + 2\,x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) + 2\,\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + 2\,\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + (3*cosh(x)^2 + 1)*sinh(x) + 3*cosh(x))) + 2*x*log(I*cosh(x) + I*
sinh(x) + 1) + 2*x*log(-I*cosh(x) - I*sinh(x) + 1) + 2*dilog(I*cosh(x) + I*
sinh(x)) + 2*dilog(-I*cosh(x) - I*sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\,\mathrm{sech}^2\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**2),x)
```

```
[Out] Integral(log(a*sech(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int 2\ln\left(\cosh(x)\right)-\ln\left(a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/cosh(x)^2),x)
```

```
[Out] -int(2*log(cosh(x)) - log(a), x)
```

3.217 $\int \log(a\mathbf{sech}^n(x))\, dx$

Optimal. Leaf size=43

$$-\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\mathrm{sech}^n(x)\right)+\frac{1}{2}n\mathrm{Li}_2\left(-e^{2x}\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1+exp(2*x))+x*ln(a*sech(x)^n)+1/2*n*polylog(2,-exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2628, 12, 3799, 2221, 2317, 2438}

$$\frac{1}{2}n\mathrm{PolyLog}\left(2,-e^{2x}\right)+x\log\left(a\mathrm{sech}^n(x)\right)-\frac{nx^2}{2}+nx\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Sech[x]^n],x]
```

```
[Out] -1/2*(n*x^2) + n*x*Log[1 + E^(2*x)] + x*Log[a*Sech[x]^n] + (n*PolyLog[2, -E
^(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\text{sech}^n(x)\right)\,dx &= x\log\left(a\text{sech}^n(x)\right)+\int nx\tanh(x)\,dx\\
&= x\log\left(a\text{sech}^n(x)\right)+n\int x\tanh(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\text{sech}^n(x)\right)+(2n)\int\frac{e^{2x}x}{1+e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^n(x)\right)-n\int\log\left(1+e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^n(x)\right)-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1+e^{2x}\right)+x\log\left(a\text{sech}^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(-e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$\frac{nx^2}{2}+nx\log\left(1+e^{-2x}\right)+x\log\left(a\text{sech}^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(-e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Sech[x]^n],x]
[Out] (n*x^2)/2 + n*x*Log[1 + E^(-2*x)] + x*Log[a*Sech[x]^n] - (n*PolyLog[2, -E^(
-2*x)])/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \ln\left(a\,\text{sech}(x)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*sech(x)^n),x)
```

```
[Out] int(ln(a*sech(x)^n),x)
```

Maxima [A]
time = 0.53, size = 36, normalized size = 0.84

$$-\frac{1}{2}\left(x^2-2\,x\log\left(e^{(2\,x)}+1\right)-\mathrm{Li}_2\left(-e^{(2\,x)}\right)\right)n+x\log\left(a\,\mathrm{sech}\,(x)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x*log(e^(2*x) + 1) - dilog(-e^(2*x)))*n + x*log(a*sech(x)^n)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.41, size = 92, normalized size = 2.14

$$-\frac{1}{2}\,nx^2+nx\log\left(\frac{2\,(\cosh\,(x)+\sinh\,(x))}{\cosh\,(x)^2+2\,\cosh\,(x)\sinh\,(x)+\sinh\,(x)^2+1}\right)+nx\log\,(i\,\cosh\,(x)+i\,\sinh\,(x)+1)+nx\log\,(-i\,\cosh\,(x)-i\,\sinh\,(x)+1)+n\mathrm{Li}_2(i\,\cosh\,(x)+i\,\sinh\,(x))+n\mathrm{Li}_2(-i\,\cosh\,(x)-i\,\sinh\,(x))+x\log\,(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 + 1)) + n*x*log(I*cosh(x) + I*sinh(x) + 1) + n*x*log(-I*cosh(x)
- I*sinh(x) + 1) + n*dilog(I*cosh(x) + I*sinh(x)) + n*dilog(-I*cosh(x) - I*
sinh(x)) + x*log(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(a\,\mathrm{sech}^n\,(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*sech(x)**n),x)
```

```
[Out] Integral(log(a*sech(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*sech(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*sech(x)^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln \left(a \left(\frac{1}{\cosh (x)}\right)^n\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/cosh(x))^n),x)
```

```
[Out] int(log(a*(1/cosh(x))^n), x)
```

3.218 $\int \log(a\mathbf{csch}(x))\,dx$

Optimal. Leaf size=38

$$-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)+x\log(a\operatorname{csch}(x))+\frac{\operatorname{Li}_2\left(e^{2x}\right)}{2}$$

```
[Out] -1/2*x^2+x*ln(1-exp(2*x))+x*ln(a*csch(x))+1/2*polylog(2,exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2628, 3797, 2221, 2317, 2438}

$$\frac{1}{2}\text{PolyLog}\left(2,e^{2x}\right)+x\log(a\operatorname{csch}(x))-\frac{x^2}{2}+x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]],x]
```

```
[Out] -1/2*x^2 + x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]] + PolyLog[2, E^(2*x)]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \log(a\operatorname{csch}(x))\,dx &= x\log(a\operatorname{csch}(x)) + \int x\coth(x)\,dx \\
&= -\frac{x^2}{2} + x\log(a\operatorname{csch}(x)) - 2\int \frac{e^{2x}x}{1-e^{2x}}\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \int \log\left(1-e^{2x}\right)\,dx \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2x}\right) \\
&= -\frac{x^2}{2} + x\log\left(1-e^{2x}\right) + x\log(a\operatorname{csch}(x)) + \frac{\text{Li}_2(e^{2x})}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$
\frac{1}{2}\left(x\left(x+2\log\left(1-e^{-2x}\right)+2\log(a\operatorname{csch}(x))\right)-\text{Li}_2\left(e^{-2x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]],x]
[Out] (x*(x + 2*Log[1 - E^(-2*x)] + 2*Log[a*Csch[x]]) - PolyLog[2, E^(-2*x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 293, normalized size = 7.71

method	result
risch	$x\ln\left(e^{x}\right)+\frac{i x\pi\,\text{csgn}\left(\frac{i e^{x}}{e^{2x}-1}\right)\text{csgn}\left(\frac{i a\,e^{x}}{e^{2x}-1}\right)^{2}}{2}-\frac{i x\pi\,\text{csgn}(i a)\text{csgn}\left(\frac{i e^{x}}{e^{2x}-1}\right)\text{csgn}\left(\frac{i a\,e^{x}}{e^{2x}-1}\right)}{2}+\frac{i x\pi\,\text{csgn}(i a)\text{csgn}\left(\frac{i a\,e^{x}}{e^{2x}-1}\right)^{2}}{2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)),x,method=_RETURNVERBOSE)
[Out] x*ln(exp(x))+1/2*I*x*Pi*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x)-1)*e
xp(x))^2-1/2*I*x*Pi*csgn(I*a)*csgn(I*exp(x)/(exp(2*x)-1))*csgn(I*a/(exp(2*x
```

```
)-1)*exp(x))+1/2*I*x*Pi*csgn(I*a)*csgn(I*a/(exp(2*x)-1)*exp(x))^2+1/2*I*x*P
i*csgn(I*exp(x))*csgn(I*exp(x)/(exp(2*x)-1))^2-1/2*I*x*Pi*csgn(I*a/(exp(2*x
)-1)*exp(x))^3+1/2*I*x*Pi*csgn(I/(exp(2*x)-1))*csgn(I*exp(x)/(exp(2*x)-1))^
2+x*ln(2)+x*ln(a)-1/2*x^2-1/2*I*x*Pi*csgn(I*exp(x))*csgn(I/(exp(2*x)-1))*cs
gn(I*exp(x)/(exp(2*x)-1))-1/2*I*x*Pi*csgn(I*exp(x)/(exp(2*x)-1))^3-ln(exp(x
))*ln(exp(2*x)-1)-dilog(exp(x))+dilog(1+exp(x))+ln(exp(x))*ln(1+exp(x))
```

Maxima [A]
time = 0.34, size = 37, normalized size = 0.97

$$-\frac{1}{2}x^2 + x\log\left(a\operatorname{csch}\left(x\right)\right) + x\log\left(e^x+1\right) + x\log\left(-e^x+1\right) + \operatorname{Li}_2(-e^x) + \operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + x*log(a*csch(x)) + x*log(e^x + 1) + x*log(-e^x + 1) + dilog(-e^x
) + dilog(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(31) = 62$.
time = 0.42, size = 76, normalized size = 2.00

$$-\frac{1}{2}x^2 + x\log\left(\frac{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)\right)}{\cosh\left(x\right)^2+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1}\right) + x\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) + x\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right) + \operatorname{Li}_2(\cosh\left(x\right)+\sinh\left(x\right)) + \operatorname{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(2*(a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + x*log(cosh(x) + sinh(x) + 1) + x*log(-cosh(x) - sinh(x)
+ 1) + dilog(cosh(x) + sinh(x)) + dilog(-cosh(x) - sinh(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{csch}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)),x)
```

```
[Out] Integral(log(a*csch(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\frac{a}{\sinh\left(x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sinh(x)),x)
```

```
[Out] int(log(a/sinh(x)), x)
```

3.219 $\int \log\left(a\text{csch}^2(x)\right)\, dx$

Optimal. Leaf size=35

$$-x^2+2x\log\left(1-e^{2x}\right)+x\log\left(a\text{csch}^2(x)\right)+\text{Li}_2\left(e^{2x}\right)$$

```
[Out] -x^2+2*x*ln(1-exp(2*x))+x*ln(a*csch(x)^2)+polylog(2,exp(2*x))
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\text{PolyLog}\left(2,e^{2x}\right)+x\log\left(a\text{csch}^2(x)\right)-x^2+2x\log\left(1-e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^2],x]
```

```
[Out] -x^2 + 2*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^2] + PolyLog[2, E^(2*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log \left(a\operatorname{csch}^2(x)\right)\, dx &= x\log \left(a\operatorname{csch}^2(x)\right)-\int -2x\coth(x)\,dx\\
&= x\log \left(a\operatorname{csch}^2(x)\right)+2\int x\coth(x)\,dx\\
&= -x^2+x\log \left(a\operatorname{csch}^2(x)\right)-4\int \frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -x^2+2x\log \left(1-e^{2x}\right)+x\log \left(a\operatorname{csch}^2(x)\right)-2\int \log \left(1-e^{2x}\right)\, dx\\
&= -x^2+2x\log \left(1-e^{2x}\right)+x\log \left(a\operatorname{csch}^2(x)\right)-\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -x^2+2x\log \left(1-e^{2x}\right)+x\log \left(a\operatorname{csch}^2(x)\right)+\operatorname{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.94

$$x\left(x+2\log \left(1-e^{-2x}\right)+\log \left(a\operatorname{csch}^2(x)\right)\right)-\operatorname{Li}_2\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^2],x]
[Out] x*(x + 2*Log[1 - E^(-2*x)] + Log[a*Csch[x]^2]) - PolyLog[2, E^(-2*x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 456, normalized size = 13.03

method	result

risch	$ix\pi \operatorname{csgn}(ie^{x}) \operatorname{csgn}(ie^{2x})^2 - \frac{ix\pi \operatorname{csgn}(ie^{2x}) \operatorname{csgn}\left(\frac{i}{(e^{2x}-1)^2}\right) \operatorname{csgn}\left(\frac{ie^{2x}}{(e^{2x}-1)^2}\right)}{2} - x^2 + 2x\ln(e^x) - 2\ln(e^x)\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] I*x*Pi*csgn(I*exp(x))*csgn(I*exp(2*x))^2-1/2*I*x*Pi*csgn(I*exp(2*x))*csgn(I
/(exp(2*x)-1)^2)*csgn(I*exp(2*x)/(exp(2*x)-1)^2)-x^2+2*x*ln(exp(x))-2*ln(ex
p(x))*ln(exp(2*x)-1)+2*ln(exp(x))*ln(1+exp(x))-2*dilog(exp(x))+2*dilog(1+ex
p(x))-1/2*I*x*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^3+1/2*I*x*Pi*csgn(I*a)*csg
n(I*a/(exp(2*x)-1)^2*exp(2*x))^2-1/2*I*x*Pi*csgn(I*a)*csgn(I*exp(2*x)/(exp(
2*x)-1)^2)*csgn(I*a/(exp(2*x)-1)^2*exp(2*x))-1/2*I*x*Pi*csgn(I*a/(exp(2*x)-
1)^2*exp(2*x))^3-1/2*I*x*Pi*csgn(I*exp(2*x))^3+2*x*ln(2)+x*ln(a)+1/2*I*x*Pi
*csgn(I*exp(2*x))*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^2-I*x*Pi*csgn(I*(exp(2*x)
-1))*csgn(I*(exp(2*x)-1)^2)^2+1/2*I*x*Pi*csgn(I*(exp(2*x)-1))^2*csgn(I*(exp
(2*x)-1)^2)+1/2*I*x*Pi*csgn(I*exp(2*x)/(exp(2*x)-1)^2)*csgn(I*a/(exp(2*x)-1
)^2*exp(2*x))^2+1/2*I*x*Pi*csgn(I*(exp(2*x)-1)^2)^3+1/2*I*x*Pi*csgn(I/(exp(
2*x)-1)^2)*csgn(I*exp(2*x)/(exp(2*x)-1)^2)^2-1/2*I*x*Pi*csgn(I*exp(x))^2*cs
gn(I*exp(2*x))
```

Maxima [A]
time = 0.34, size = 45, normalized size = 1.29

$$-x^2 + x\log\left(a\operatorname{csch}(x)^2\right) + 2x\log\left(e^x + 1\right) + 2x\log\left(-e^x + 1\right) + 2\operatorname{Li}_2(-e^x) + 2\operatorname{Li}_2(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="maxima")
```

```
[Out] -x^2 + x*log(a*csch(x)^2) + 2*x*log(e^x + 1) + 2*x*log(-e^x + 1) + 2*dilog(
-e^x) + 2*dilog(e^x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(32) = 64.
time = 0.41, size = 97, normalized size = 2.77

$$-x^2 + x\log\left(\frac{4\,(a\cosh(x) + a\sinh(x))}{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + 3\left(\cosh(x)^2 - 1\right)\sinh(x) - \cosh(x)}\right) + 2x\log\left(\cosh(x) + \sinh(x) + 1\right) + 2x\log\left(-\cosh(x) - \sinh(x) + 1\right) + 2\operatorname{Li}_2(\cosh(x) + \sinh(x)) + 2\operatorname{Li}_2(-\cosh(x) - \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="fricas")
```

```
[Out] -x^2 + x*log(4*(a*cosh(x) + a*sinh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + s
inh(x)^3 + 3*(cosh(x)^2 - 1)*sinh(x) - cosh(x))) + 2*x*log(cosh(x) + sinh(x
) + 1) + 2*x*log(-cosh(x) - sinh(x) + 1) + 2*dilog(cosh(x) + sinh(x)) + 2*d
ilog(-cosh(x) - sinh(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(a \operatorname{csch}^2 (x)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**2),x)
```

```
[Out] Integral(log(a*csch(x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^2),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \ln\left(\frac{a}{\sinh (x)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a/sinh(x)^2),x)
```

```
[Out] int(log(a/sinh(x)^2), x)
```

3.220 $\int \log (a\text{csch}^n(x)) \, dx$

Optimal. Leaf size=43

$$-\frac{nx^2}{2} + nx\log \left(1 - e^{2x}\right) + x\log \left(a\text{csch}^n(x)\right) + \frac{1}{2}n\text{Li}_2\left(e^{2x}\right)$$

```
[Out] -1/2*n*x^2+n*x*ln(1-exp(2*x))+x*ln(a*csch(x)^n)+1/2*n*polylog(2,exp(2*x))
```

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {2628, 12, 3797, 2221, 2317, 2438}

$$\frac{1}{2}n\text{PolyLog}\left(2, e^{2x}\right) + x\log \left(a\text{csch}^n(x)\right) - \frac{nx^2}{2} + nx\log \left(1 - e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[a*Csch[x]^n],x]
```

```
[Out] -1/2*(n*x^2) + n*x*Log[1 - E^(2*x)] + x*Log[a*Csch[x]^n] + (n*PolyLog[2, E^
(2*x)])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log\left(a\text{csch}^n(x)\right)\,dx &= x\log\left(a\text{csch}^n(x)\right)+\int nx\coth(x)\,dx\\
&= x\log\left(a\text{csch}^n(x)\right)+n\int x\coth(x)\,dx\\
&= -\frac{nx^2}{2}+x\log\left(a\text{csch}^n(x)\right)-(2n)\int\frac{e^{2x}x}{1-e^{2x}}\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\text{csch}^n(x)\right)-n\int\log\left(1-e^{2x}\right)\,dx\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\text{csch}^n(x)\right)-\frac{1}{2}n\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2x}\right)\\
&= -\frac{nx^2}{2}+nx\log\left(1-e^{2x}\right)+x\log\left(a\text{csch}^n(x)\right)+\frac{1}{2}n\text{Li}_2\left(e^{2x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$\frac{nx^2}{2}+nx\log\left(1-e^{-2x}\right)+x\log\left(a\text{csch}^n(x)\right)-\frac{1}{2}n\text{Li}_2\left(e^{-2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a*Csch[x]^n],x]
```

```
[Out] (n*x^2)/2 + n*x*Log[1 - E^(-2*x)] + x*Log[a*Csch[x]^n] - (n*PolyLog[2, E^(-
2*x)])/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \ln\left(a\,\text{csch}(x)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a*csch(x)^n),x)
```

```
[Out] int(ln(a*csch(x)^n),x)
```

Maxima [A]
time = 0.34, size = 47, normalized size = 1.09

$$-\frac{1}{2}\left(x^2 - 2\,x\log\left(e^x+1\right) - 2\,x\log\left(-e^x+1\right) - 2\,\mathrm{Li}_2(-e^x) - 2\,\mathrm{Li}_2(e^x)\right)n + x\log\left(a\operatorname{csch}\left(x\right)^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 - 2*x*log(e^x + 1) - 2*x*log(-e^x + 1) - 2*dilog(-e^x) - 2*dilog(
e^x))*n + x*log(a*csch(x)^n)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.
time = 0.41, size = 84, normalized size = 1.95

$$-\frac{1}{2}\,nx^2 + nx\log\left(\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 - 1}\right) + nx\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) + nx\log\left(-\cosh\left(x\right)-\sinh\left(x\right)+1\right) + n\mathrm{Li}_2(\cosh\left(x\right)+\sinh\left(x\right)) + n\mathrm{Li}_2(-\cosh\left(x\right)-\sinh\left(x\right)) + x\log\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="fricas")
```

```
[Out] -1/2*n*x^2 + n*x*log(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2 - 1)) + n*x*log(cosh(x) + sinh(x) + 1) + n*x*log(-cosh(x) - sinh
(x) + 1) + n*dilog(cosh(x) + sinh(x)) + n*dilog(-cosh(x) - sinh(x)) + x*log
(a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(a\operatorname{csch}^{n}\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*csch(x)**n),x)
```

```
[Out] Integral(log(a*csch(x)**n), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*csch(x)^n),x, algorithm="giac")
```

```
[Out] integrate(log(a*csch(x)^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(a\left(\frac{1}{\sinh\left(x\right)}\right)^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a*(1/sinh(x))^n),x)
```

```
[Out] int(log(a*(1/sinh(x))^n), x)
```

3.221 $\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) dx$

Optimal. Leaf size=50

$$-\frac{\sinh(a+bx)}{b}+\frac{\log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sinh(a+bx)}{b}$$

```
[Out] -sinh(b*x+a)/b+ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x))*sinh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$,
Rules used = {2717, 2634}

$$\frac{\sinh(a+bx) \log \left(\sinh \left(\frac{a}{2}+\frac{bx}{2}\right) \cosh \left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]]*Sinh[a +
 b*x])/b
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(a+bx) \log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) dx = \frac{\log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\frac{bx}{2}\right)\right) \sinh(a+bx)}{b}$$
$$= -\frac{\sinh(a+bx)}{b}+\frac{\log \left(\cosh \left(\frac{a}{2}+\frac{bx}{2}\right) \sinh \left(\frac{a}{2}+\right.\right.}{b}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.66

$$-\frac{\sinh(a+bx)}{b}+\frac{\log \left(\frac{1}{2} \sinh(a+bx)\right) \sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]*Log[Cosh[a/2 + (b*x)/2]*Sinh[a/2 + (b*x)/2]],x]
```

```
[Out] -(Sinh[a + b*x]/b) + (Log[Sinh[a + b*x]/2]*Sinh[a + b*x])/b
```

Maple [A]
time = 0.22, size = 30, normalized size = 0.60

method	result	size
default	$\frac{\ln\left(\frac{\sinh(bx+a)}{2}\right)\sinh(bx+a)-\sinh(bx+a)}{b}$	30
risch	Expression too large to display	1098

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/b*(ln(1/2*sinh(b*x+a))*sinh(b*x+a)-sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(42) = 84.
time = 0.29, size = 112, normalized size = 2.24

$$\frac{\log\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)\sinh\left(bx+a\right)}{b}-\frac{b\left(\frac{2\,(bx+a)}{b}+\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)-b\left(\frac{2\,(bx+a)}{b}-\frac{e^{(bx+a)}}{b}+\frac{e^{(-bx-a)}}{b}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="maxima")
```

```
[Out] log(cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x + 1/2*a))*sinh(b*x + a)/b - 1/4*(b*(
2*(b*x + a)/b + e^(b*x + a)/b - e^(-b*x - a)/b) - b*(2*(b*x + a)/b - e^(b*x
 + a)/b + e^(-b*x - a)/b))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(42) = 84.
time = 0.39, size = 258, normalized size = 5.16

$$-\frac{\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}+\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+6\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+4\,\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{3}+\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{4}-1\right)\log\left(\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)-1}{2\left(b\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}+2\,b\cosh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)+b\sinh\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="fricas")
```

```
[Out] -1/2*(cosh(1/2*b*x + 1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/
2*a) + 6*cosh(1/2*b*x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x +
```

```
1/2*a)*sinh(1/2*b*x + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - (cosh(1/2*b*x +
1/2*a)^4 + 4*cosh(1/2*b*x + 1/2*a)^3*sinh(1/2*b*x + 1/2*a) + 6*cosh(1/2*b*
x + 1/2*a)^2*sinh(1/2*b*x + 1/2*a)^2 + 4*cosh(1/2*b*x + 1/2*a)*sinh(1/2*b*x
 + 1/2*a)^3 + sinh(1/2*b*x + 1/2*a)^4 - 1)*log(cosh(1/2*b*x + 1/2*a)*sinh(1
/2*b*x + 1/2*a)) - 1)/(b*cosh(1/2*b*x + 1/2*a)^2 + 2*b*cosh(1/2*b*x + 1/2*a
)*sinh(1/2*b*x + 1/2*a) + b*sinh(1/2*b*x + 1/2*a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\sinh\left(\frac{a}{2}+\frac{bx}{2}\right)\cosh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*ln(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x)
```

```
[Out] Integral(log(sinh(a/2 + b*x/2)*cosh(a/2 + b*x/2))*cosh(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 5.13, size = 94, normalized size = 1.88

$$\frac{1}{2}\left(\frac{e^{(bx+a)}}{b}-\frac{e^{(-bx-a)}}{b}\right)\log\left(\frac{1}{4}\left(e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}+e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}\right)\left(e^{\left(\frac{1}{2}bx+\frac{1}{2}a\right)}-e^{\left(-\frac{1}{2}bx-\frac{1}{2}a\right)}\right)\right)-\frac{e^{(bx+a)}-e^{(-bx-a)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*log(cosh(1/2*a+1/2*b*x)*sinh(1/2*a+1/2*b*x)),x, algor
ithm="giac")
```

```
[Out] 1/2*(e^(b*x + a)/b - e^(-b*x - a)/b)*log(1/4*(e^(1/2*b*x + 1/2*a) + e^(-1/2
*b*x - 1/2*a))*(e^(1/2*b*x + 1/2*a) - e^(-1/2*b*x - 1/2*a))) - 1/2*(e^(b*x
+ a) - e^(-b*x - a))/b
```

Mupad [B]
time = 0.51, size = 31, normalized size = 0.62

$$\frac{\ln\left(\frac{\sinh(a+b\,x)}{2}\right)\,\sinh(a+b\,x)}{b}-\frac{\sinh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(a/2 + (b*x)/2)*sinh(a/2 + (b*x)/2))*cosh(a + b*x),x)
```

```
[Out] (log(sinh(a + b*x)/2)*sinh(a + b*x))/b - sinh(a + b*x)/b
```

3.222 $\int \log(\cosh^2(x)) \sinh(x)\,dx$

Optimal. Leaf size=13

$$-2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)$$

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2718, 2634, 12}

$$\cosh(x)\log\left(\cosh^2(x)\right) - 2\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\cosh^2(x)\right)\sinh(x)\,dx &= \cosh(x)\log\left(\cosh^2(x)\right) - \int 2\sinh(x)\,dx\\
&= \cosh(x)\log\left(\cosh^2(x)\right) - 2\int \sinh(x)\,dx\\
&= -2\cosh(x) + \cosh(x)\log\left(\cosh^2(x)\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-2\cosh(x)+\cosh(x)\log\left(\cosh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]^2]*Sinh[x],x]
```

```
[Out] -2*Cosh[x] + Cosh[x]*Log[Cosh[x]^2]
```

Maple [A]
time = 0.12, size = 14, normalized size = 1.08

method	result
derivativedivides	$-2\cosh(x)+\cosh(x)\ln\left(\cosh^2(x)\right)$
default	$-2\cosh(x)+\cosh(x)\ln\left(\cosh^2(x)\right)$
risch	$-(1+e^{2x})e^{-x}\ln(e^x)+\frac{\left(-4-4e^{2x}-i\pi\operatorname{csgn}\left(ie^{-2x}(1+e^{2x})^2\right)^3+i\pi\operatorname{csgn}(ie^{2x})^3-i\pi\operatorname{csgn}\left(i(1+e^{2x})^2\right)^3-4\ln(2)e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x)^2)*sinh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cosh(x)+cosh(x)*ln(cosh(x)^2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.92

$$2\cosh(x)\log(\cosh(x))-2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="maxima")
```

```
[Out] 2*cosh(x)*log(cosh(x)) - 2*cosh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(13) = 26.
time = 0.37, size = 62, normalized size = 4.77

$$-\frac{2\cosh(x)^2-\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{1}{2}\cosh(x)^2+\frac{1}{2}\sinh(x)^2+\frac{1}{2}\right)+4\cosh(x)\sinh(x)+2\sinh(x)^2+2}{2(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(1/2
*cosh(x)^2 + 1/2*sinh(x)^2 + 1/2) + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 + 2)/(c
osh(x) + sinh(x))
```

Sympy [A]
time = 0.48, size = 14, normalized size = 1.08

$$\log\left(\cosh^2(x)\right)\cosh(x) - 2\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x)**2)*sinh(x),x)
[Out] log(cosh(x)**2)*cosh(x) - 2*cosh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.
time = 4.67, size = 37, normalized size = 2.85

$$(e^{(2x)} + 1)e^{(-x)}\log\left(\frac{1}{2}\left(e^{(2x)} + 1\right)e^{(-x)}\right) - \left(e^{(2x)} + 1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x)^2)*sinh(x),x, algorithm="giac")
[Out] (e^(2*x) + 1)*e^(-x)*log(1/2*(e^(2*x) + 1)*e^(-x)) - (e^(2*x) + 1)*e^(-x)
```

Mupad [B]
time = 0.38, size = 9, normalized size = 0.69

$$2\cosh(x)\,\left(\ln\left(\cosh(x)\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(cosh(x)^2)*sinh(x),x)
[Out] 2*cosh(x)*(log(cosh(x)) - 1)
```

3.223 $\int \frac{\log(x)}{\sqrt{x}} dx$

Optimal. Leaf size=17

$$-4\sqrt{x} + 2\sqrt{x}\log(x)$$

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[x],x]
```

```
[Out] -4*Sqrt[x] + 2*Sqrt[x]*Log[x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{\sqrt{x}} dx = -4\sqrt{x} + 2\sqrt{x}\log(x)$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.65

$$2\sqrt{x}(-2 + \log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*(-2 + Log[x])
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$-4\sqrt{x}+2\ln(x)\sqrt{x}$	14
default	$-4\sqrt{x}+2\ln(x)\sqrt{x}$	14
risch	$-4\sqrt{x}+2\ln(x)\sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*x^(1/2)+2*ln(x)*x^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$2\sqrt{x}\log(x)-4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

Fricas [A]
time = 0.36, size = 9, normalized size = 0.53

$$2\sqrt{x}(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*(log(x) - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(15) = 30$.
time = 1.33, size = 94, normalized size = 5.53

$$\begin{cases} -2\sqrt{x}\log\left(\frac{1}{x}\right)+2\sqrt{x}\log(x)-8\sqrt{x} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ 2\sqrt{x}\log(x)-4\sqrt{x} & \text{for } |x|<1 \\ -2\sqrt{x}\log\left(\frac{1}{x}\right)-4\sqrt{x} & \text{for } \frac{1}{|x|}<1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{3}{2},\frac{3}{2} \\ \frac{1}{2},\frac{1}{2} & 0 \end{matrix}\middle| x\right)+G_{3,3}^{0,3}\left(\begin{matrix} \frac{3}{2},\frac{3}{2},1 & \\ & \frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(x)*log(1/x) + 2*sqrt(x)*log(x) - 8*sqrt(x), (Abs(x) < 1)
 & (1/Abs(x) < 1)), (2*sqrt(x)*log(x) - 4*sqrt(x), Abs(x) < 1), (-2*sqrt(x)
*log(1/x) - 4*sqrt(x), 1/Abs(x) < 1), (-meijerg(((1,), (3/2, 3/2)), ((1/2,
1/2), (0,)), x) + meijerg(((3/2, 3/2, 1), ()), ((), (1/2, 1/2, 0)), x), Tru
e))
```

Giac [A]
time = 2.80, size = 13, normalized size = 0.76

$$2\sqrt{x}\log(x) - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*log(x) - 4*sqrt(x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.53

$$2\sqrt{x}\,(\ln(x) - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)*(log(x) - 2)
```

3.224 $\int x \log(2-3x^2)\,dx$

Optimal. Leaf size=27

$$-\frac{x^2}{2}-\frac{1}{6}(2-3x^2)\log(2-3x^2)$$

[Out] -1/2*x^2-1/6*(-3*x^2+2)*ln(-3*x^2+2)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2504, 2436, 2332}

$$-\frac{x^2}{2}-\frac{1}{6}(2-3x^2)\log(2-3x^2)$$

Antiderivative was successfully verified.

[In] Int[x*Log[2 - 3*x^2],x]

[Out] -1/2*x^2 - ((2 - 3*x^2)*Log[2 - 3*x^2])/6

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\int x\log(2-3x^2)\,dx = \frac{1}{2}\text{Subst}\left(\int \log(2-3x)\,dx, x, x^2\right)$$
$$= -\left(\frac{1}{6}\text{Subst}\left(\int \log(x)\,dx, x, 2-3x^2\right)\right)$$
$$= -\frac{x^2}{2} - \frac{1}{6}(2-3x^2)\log(2-3x^2)$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{1}{6}\left(-3x^2+(-2+3x^2)\log(2-3x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[2 - 3*x^2],x]
[Out] (-3*x^2 + (-2 + 3*x^2)*Log[2 - 3*x^2])/6
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.93

method	result	size
derivativedivides	$-\frac{(-3x^2+2)\ln(-3x^2+2)}{6} - \frac{x^2}{2} + \frac{1}{3}$	25
default	$-\frac{(-3x^2+2)\ln(-3x^2+2)}{6} - \frac{x^2}{2} + \frac{1}{3}$	25
norman	$-\frac{x^2}{2} + \frac{\ln(-3x^2+2)x^2}{2} - \frac{\ln(-3x^2+2)}{3}$	30
risch	$\frac{\ln(-3x^2+2)x^2}{2} - \frac{x^2}{2} - \frac{\ln(3x^2-2)}{3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-3*x^2+2),x,method=_RETURNVERBOSE)
[Out] -1/6*(-3*x^2+2)*ln(-3*x^2+2)-1/2*x^2+1/3
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.89

$$-\frac{1}{2}x^2 + \frac{1}{6}(3x^2-2)\log(-3x^2+2) + \frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Fricas [A]
time = 0.44, size = 23, normalized size = 0.85

$$-\frac{1}{2}\,x^2+\frac{1}{6}\,\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="fricas")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2)
```

Sympy [A]
time = 0.04, size = 27, normalized size = 1.00

$$\frac{x^2\log\left(2-3x^2\right)}{2}-\frac{x^2}{2}-\frac{\log\left(3x^2-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-3*x**2+2),x)
[Out] x**2*log(2 - 3*x**2)/2 - x**2/2 - log(3*x**2 - 2)/3
```

Giac [A]
time = 3.81, size = 24, normalized size = 0.89

$$-\frac{1}{2}\,x^2+\frac{1}{6}\,\left(3\,x^2-2\right)\log\left(-3\,x^2+2\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-3*x^2+2),x, algorithm="giac")
[Out] -1/2*x^2 + 1/6*(3*x^2 - 2)*log(-3*x^2 + 2) + 1/3
```

Mupad [B]
time = 0.12, size = 25, normalized size = 0.93

$$x^2\left(\frac{\ln\left(2-3\,x^2\right)}{2}-\frac{1}{2}\right)-\frac{\ln\left(x^2-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(2 - 3*x^2),x)
[Out] x^2*(log(2 - 3*x^2)/2 - 1/2) - log(x^2 - 2/3)/3
```

3.225 $\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx$

Optimal. Leaf size=3

$$\sin^{-1}(\log(x))$$

```
[Out] arcsin(ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {222}

$$\text{ArcSin}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcSin[Log[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \log(x)\right)\\ &= \sin^{-1}(\log(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.
time = 0.01, size = 16, normalized size = 5.33

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{1-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[1 - Log[x]^2]]
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size
derivativedivides	$\arcsin(\ln(x))$	4
default	$\arcsin(\ln(x))$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1-ln(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsin(ln(x))
```

Maxima [A]
time = 0.51, size = 3, normalized size = 1.00

$$\arcsin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="maxima")
[Out] arcsin(log(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(3) = 6.
time = 0.38, size = 20, normalized size = 6.67

$$-2\arctan\left(\frac{\sqrt{-\log(x)^2+1}-1}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(-log(x)^2 + 1) - 1)/log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-(\log(x)-1)(\log(x)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-ln(x)**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(log(x) - 1)*(log(x) + 1))), x)
```

Giac [**A**]
time = 4.61, size = 3, normalized size = 1.00

$$\arcsin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(log(x))
```

Mupad [**B**]
time = 0.39, size = 3, normalized size = 1.00

$$\mathrm{asin}(\ln\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - log(x)^2)^(1/2)),x)
```

```
[Out] asin(log(x))
```

3.226 $\int 16x^3 \log^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x^4}{2} - 2x^4 \log(x) + 4x^4 \log^2(x)$$

[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 2342, 2341}

$$\frac{x^4}{2} + 4x^4 \log^2(x) - 2x^4 \log(x)$$

Antiderivative was successfully verified.

[In] Int[16*x^3*Log[x]^2,x]

[Out] x^4/2 - 2*x^4*Log[x] + 4*x^4*Log[x]^2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int 16x^3 \log^2(x)\,dx &= 16 \int x^3 \log^2(x)\,dx \\
&= 4x^4 \log^2(x) - 8 \int x^3 \log(x)\,dx \\
&= \frac{x^4}{2} - 2x^4 \log(x) + 4x^4 \log^2(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.25

$$16\left(\frac{x^4}{32}-\frac{1}{8}x^4\log(x)+\frac{1}{4}x^4\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[16*x^3*Log[x]^2,x]
[Out] 16*(x^4/32 - (x^4*Log[x])/8 + (x^4*Log[x]^2)/4)
```

Maple [A]
time = 0.02, size = 23, normalized size = 0.96

method	result	size
default	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
norman	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23
risch	$\frac{x^4}{2}-2x^4\ln(x)+4x^4\ln(x)^2$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*x^3*ln(x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^4-2*x^4*ln(x)+4*x^4*ln(x)^2
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.71

$$\frac{1}{2}\left(8\,\log\left(x\right)^{2}-4\,\log\left(x\right)+1\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="maxima")
[Out] 1/2*(8*log(x)^2 - 4*log(x) + 1)*x^4
```

Fricas [A]
time = 0.41, size = 22, normalized size = 0.92

$$4\,x^{4}\log\left(x\right)^{2}-2\,x^{4}\log\left(x\right)+\frac{1}{2}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="fricas")
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.92

$$4x^4\log{(x)}^2 - 2x^4\log{(x)} + \frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x**3*ln(x)**2,x)
```

```
[Out] 4*x**4*log(x)**2 - 2*x**4*log(x) + x**4/2
```

Giac [A]
time = 4.32, size = 22, normalized size = 0.92

$$4\,x^4\log{(x)}^2 - 2\,x^4\log{(x)} + \frac{1}{2}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(16*x^3*log(x)^2,x, algorithm="giac")
```

```
[Out] 4*x^4*log(x)^2 - 2*x^4*log(x) + 1/2*x^4
```

Mupad [B]
time = 0.04, size = 17, normalized size = 0.71

$$\frac{x^4\left(8\ln{(x)}^2 - 4\,\ln{(x)} + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(16*x^3*log(x)^2,x)
```

```
[Out] (x^4*(8*log(x)^2 - 4*log(x) + 1))/2
```

3.227 $\int \log\left(\sqrt{a+bx}\right)\,dx$

Optimal. Leaf size=25

$$-\frac{x}{2}+\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}$$

```
[Out] -1/2*x+1/2*(b*x+a)*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2332}

$$\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[a + b*x]],x]
```

```
[Out] -1/2*x + ((a + b*x)*Log[Sqrt[a + b*x]])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\sqrt{a+bx}\right)\,dx &= \frac{\text{Subst}\left(\int \log\left(\sqrt{x}\right)\,dx,x,a+bx\right)}{b}\\ &= -\frac{x}{2}+\frac{(a+bx)\log\left(\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.92

$$\frac{1}{2}\left(-x+\frac{(a+bx)\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[a + b*x]],x]
```

```
[Out] (-x + ((a + b*x)*Log[a + b*x])/b)/2
```

Maple [A]
time = 0.01, size = 26, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
default	$\frac{(bx+a)\ln(bx+a)-bx-a}{2b}$	26
norman	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26
risch	$-\frac{x}{2}+\frac{x\ln(bx+a)}{2}+\frac{a\ln(bx+a)}{2b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((b*x+a)*ln(b*x+a)-b*x-a)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.92

$$-\frac{bx-(bx+a)\log(bx+a)+a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Fricas [A]
time = 0.40, size = 22, normalized size = 0.88

$$-\frac{bx-(bx+a)\log(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a))/b
```

Sympy [A]
time = 0.17, size = 29, normalized size = 1.16

$$-b\left(-\frac{a\log(a+bx)}{2b^2}+\frac{x}{2b}\right)+\frac{x\log(a+bx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(b*x+a),x)
```

```
[Out] -b*(-a*log(a + b*x)/(2*b**2) + x/(2*b)) + x*log(a + b*x)/2
```

Giac [A]
time = 3.44, size = 23, normalized size = 0.92

$$-\frac{bx - (bx + a)\log(bx + a) + a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*x - (b*x + a)*log(b*x + a) + a)/b
```

Mupad [B]
time = 0.06, size = 25, normalized size = 1.00

$$\frac{x\,\ln(a + b\,x)}{2} - \frac{x}{2} + \frac{a\,\ln(a + b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/2,x)
```

```
[Out] (x*log(a + b*x))/2 - x/2 + (a*log(a + b*x))/(2*b)
```

3.228 $\int x \log\left(\sqrt{2+x}\right)\,dx$

Optimal. Leaf size=34

$$\frac{x}{2}-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\log(2+x)$$

```
[Out] 1/2*x-1/8*x^2-ln(2+x)+1/4*x^2*ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2442, 45}

$$-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{x+2}\right)+\frac{x}{2}-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Sqrt[2 + x]],x]
```

```
[Out] x/2 - x^2/8 + (x^2*Log[Sqrt[2 + x]])/2 - Log[2 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(\sqrt{2+x}\right)\,dx &= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\frac{1}{4}\int\frac{x^2}{2+x}\,dx\\
&= \frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\frac{1}{4}\int\left(-2+x+\frac{4}{2+x}\right)\,dx\\
&= \frac{x}{2}-\frac{x^2}{8}+\frac{1}{2}x^2\log\left(\sqrt{2+x}\right)-\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.88

$$\frac{1}{2}\left(x-\frac{x^2}{4}-2\log(2+x)+\frac{1}{2}x^2\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Sqrt[2 + x]],x]
[Out] (x - x^2/4 - 2*Log[2 + x] + (x^2*Log[2 + x])/2)/2
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.91

method	result	size
norman	$\frac{x}{2}-\frac{x^2}{8}-\ln\left(2+x\right)+\frac{x^2\ln(2+x)}{4}$	25
risch	$\frac{x}{2}-\frac{x^2}{8}-\ln\left(2+x\right)+\frac{x^2\ln(2+x)}{4}$	25
derivativedivides	$\frac{(2+x)^2\ln(2+x)}{4}-\frac{(2+x)^2}{8}-\ln\left(2+x\right)\left(2+x\right)+2+x$	31
default	$\frac{(2+x)^2\ln(2+x)}{4}-\frac{(2+x)^2}{8}-\ln\left(2+x\right)\left(2+x\right)+2+x$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x*ln(2+x),x,method=_RETURNVERBOSE)
[Out] 1/4*(2+x)^2*ln(2+x)-1/8*(2+x)^2-ln(2+x)*(2+x)+2+x
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.71

$$\frac{1}{4}\,x^2\log\left(x+2\right)-\frac{1}{8}\,x^2+\frac{1}{2}\,x-\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="maxima")
[Out] 1/4*x^2*log(x + 2) - 1/8*x^2 + 1/2*x - log(x + 2)
```

Fricas [A]
time = 0.37, size = 20, normalized size = 0.59

$$-\frac{1}{8}\,x^2+\frac{1}{4}\left(x^2-4\right)\log\left(x+2\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="fricas")
```

```
[Out] -1/8*x^2 + 1/4*(x^2 - 4)*log(x + 2) + 1/2*x
```

Sympy [A]
time = 0.09, size = 22, normalized size = 0.65

$$\frac{x^2\log(x+2)}{4} - \frac{x^2}{8} + \frac{x}{2} - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*ln(2+x),x)
[Out] x**2*log(x + 2)/4 - x**2/8 + x/2 - log(x + 2)
```

Giac [A]
time = 5.52, size = 30, normalized size = 0.88

$$\frac{1}{4}(x+2)^2\log(x+2) - \frac{1}{8}(x+2)^2 - (x+2)\log(x+2) + x + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x*log(2+x),x, algorithm="giac")
[Out] 1/4*(x + 2)^2*log(x + 2) - 1/8*(x + 2)^2 - (x + 2)*log(x + 2) + x + 2
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.59

$$\frac{x}{2} - \frac{x^2}{8} + \frac{\ln(x+2)\,(x^2-4)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(x + 2))/2,x)
[Out] x/2 - x^2/8 + (log(x + 2)*(x^2 - 4))/4
```

3.229 $\int x \log\left(\sqrt[3]{1+3x}\right)\,dx$

Optimal. Leaf size=40

$$\frac{x}{18}-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{54}\log(1+3x)$$

```
[Out] 1/18*x-1/12*x^2+1/6*x^2*ln(1+3*x)-1/54*ln(1+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2442, 45}

$$-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{3x+1}\right)+\frac{x}{18}-\frac{1}{54}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[(1 + 3*x)^(1/3)],x]
```

```
[Out] x/18 - x^2/12 + (x^2*Log[(1 + 3*x)^(1/3)])/2 - Log[1 + 3*x]/54
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int x\log\left(\sqrt[3]{1+3x}\right)\,dx &= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\frac{x^2}{1+3x}\,dx\\
&= \frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{2}\int\left(-\frac{1}{9}+\frac{x}{3}+\frac{1}{9(1+3x)}\right)dx\\
&= \frac{x}{18}-\frac{x^2}{12}+\frac{1}{2}x^2\log\left(\sqrt[3]{1+3x}\right)-\frac{1}{54}\log(1+3x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.00

$$\frac{1}{3}\left(\frac{x}{6}-\frac{x^2}{4}-\frac{1}{18}\log(1+3x)+\frac{1}{2}x^2\log(1+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + 3*x)^(1/3)],x]
[Out] (x/6 - x^2/4 - Log[1 + 3*x]/18 + (x^2*Log[1 + 3*x])/2)/3
```

Maple [A]
time = 0.03, size = 43, normalized size = 1.08

method	result	size
meijerg	$\frac{x(-9x+6)}{108}-\frac{(-27x^2+3)\ln(1+3x)}{162}$	25
norman	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
risch	$\frac{x}{18}-\frac{x^2}{12}+\frac{x^2\ln(1+3x)}{6}-\frac{\ln(1+3x)}{54}$	29
derivativedivides	$-\frac{\ln(1+3x)(1+3x)}{27}+\frac{1}{27}+\frac{x}{9}+\frac{(1+3x)^2\ln(1+3x)}{54}-\frac{(1+3x)^2}{108}$	43
default	$-\frac{\ln(1+3x)(1+3x)}{27}+\frac{1}{27}+\frac{x}{9}+\frac{(1+3x)^2\ln(1+3x)}{54}-\frac{(1+3x)^2}{108}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/3*x*ln(1+3*x),x,method=_RETURNVERBOSE)
[Out] -1/27*ln(1+3*x)*(1+3*x)+1/27+1/9*x+1/54*(1+3*x)^2*ln(1+3*x)-1/108*(1+3*x)^2
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.70

$$\frac{1}{6}x^2\log(3x+1)-\frac{1}{12}x^2+\frac{1}{18}x-\frac{1}{54}\log(3x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="maxima")
[Out] 1/6*x^2*log(3*x + 1) - 1/12*x^2 + 1/18*x - 1/54*log(3*x + 1)
```

Fricas [A]
time = 0.39, size = 24, normalized size = 0.60

$$-\frac{1}{12}x^2+\frac{1}{54}(9x^2-1)\log(3x+1)+\frac{1}{18}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="fricas")
```

```
[Out] -1/12*x^2 + 1/54*(9*x^2 - 1)*log(3*x + 1) + 1/18*x
```

Sympy [A]
time = 0.10, size = 27, normalized size = 0.68

$$\frac{x^2 \log(3x+1)}{6} - \frac{x^2}{12} + \frac{x}{18} - \frac{\log(3x+1)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*ln(1+3*x),x)
```

```
[Out] x**2*log(3*x + 1)/6 - x**2/12 + x/18 - log(3*x + 1)/54
```

Giac [A]
time = 7.32, size = 42, normalized size = 1.05

$$\frac{1}{54} (3x+1)^2 \log(3x+1) - \frac{1}{108} (3x+1)^2 - \frac{1}{27} (3x+1) \log(3x+1) + \frac{1}{9} x + \frac{1}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/3*x*log(1+3*x),x, algorithm="giac")
```

```
[Out] 1/54*(3*x + 1)^2*log(3*x + 1) - 1/108*(3*x + 1)^2 - 1/27*(3*x + 1)*log(3*x
+ 1) + 1/9*x + 1/27
```

Mupad [B]
time = 0.38, size = 22, normalized size = 0.55

$$\frac{x}{18} + \frac{\ln(3x+1) \left(x^2 - \frac{1}{9}\right)}{6} - \frac{x^2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(3*x + 1))/3,x)
```

```
[Out] x/18 + (log(3*x + 1)*(x^2 - 1/9))/6 - x^2/12
```

3.230 $\int x \log(x+x^3)\, dx$

Optimal. Leaf size=31

$$-\frac{3x^2}{4}+\frac{1}{2}\log\left(1+x^2\right)+\frac{1}{2}x^2\log\left(x+x^3\right)$$

```
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2605, 455, 45}

$$-\frac{3x^2}{4}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\log\left(x^3+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$\int x \log(x+x^3)\,dx = \frac{1}{2}x^2\log(x+x^3) - \frac{1}{2}\int \frac{x(1+3x^2)}{1+x^2}\,dx$$
$$= \frac{1}{2}x^2\log(x+x^3) - \frac{1}{4}\text{Subst}\left(\int \frac{1+3x}{1+x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}x^2\log(x+x^3) - \frac{1}{4}\text{Subst}\left(\int \left(3-\frac{2}{1+x}\right)\,dx, x, x^2\right)$$
$$= -\frac{3x^2}{4} + \frac{1}{2}\log(1+x^2) + \frac{1}{2}x^2\log(x+x^3)$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{3x^2}{4} + \frac{1}{2}\log(1+x^2) + \frac{1}{2}x^2\log(x+x^3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + x^3],x]
```

```
[Out] (-3*x^2)/4 + Log[1 + x^2]/2 + (x^2*Log[x + x^3])/2
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.84

method	result	size
default	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26
risch	$-\frac{3x^2}{4}+\frac{\ln(x^2+1)}{2}+\frac{x^2\ln(x^3+x)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*x^2+1/2*ln(x^2+1)+1/2*x^2*ln(x^3+x)
```

Maxima [A]
time = 0.48, size = 25, normalized size = 0.81

$$\frac{1}{2}x^2\log(x^3+x) - \frac{3}{4}x^2 + \frac{1}{2}\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Fricas [A]
time = 0.40, size = 25, normalized size = 0.81

$$\frac{1}{2}\,x^2\log\left(x^3+x\right)-\frac{3}{4}\,x^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="fricas")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.11, size = 26, normalized size = 0.84

$$\frac{x^2\log\left(x^3+x\right)}{2}-\frac{3x^2}{4}+\frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**3+x),x)
[Out] x**2*log(x**3 + x)/2 - 3*x**2/4 + log(x**2 + 1)/2
```

Giac [A]
time = 6.01, size = 25, normalized size = 0.81

$$\frac{1}{2}\,x^2\log\left(x^3+x\right)-\frac{3}{4}\,x^2+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^3+x),x, algorithm="giac")
[Out] 1/2*x^2*log(x^3 + x) - 3/4*x^2 + 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.41, size = 25, normalized size = 0.81

$$\frac{\ln\left(x^2+1\right)}{2}+\frac{x^2\,\ln\left(x^3+x\right)}{2}-\frac{3\,x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(x + x^3),x)
[Out] log(x^2 + 1)/2 + (x^2*log(x + x^3))/2 - (3*x^2)/4
```

3.231 $\int \log \left(x+\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=26

$$-\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right)$$

```
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2614, 267}

$$x \log \left(\sqrt{x^2+1}+x\right)-\sqrt{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned} \int \log \left(x+\sqrt{1+x^2}\right) dx &= x \log \left(x+\sqrt{1+x^2}\right)-\int \frac{x}{\sqrt{1+x^2}} dx \\ &= -\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\sqrt{1+x^2}+x \log \left(x+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -Sqrt[1 + x^2] + x*Log[x + Sqrt[1 + x^2]]
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.88

method	result	size
default	$x \ln \left(x+\sqrt{x^{2}+1}\right)-\sqrt{x^{2}+1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x+(x^2+1)^(1/2))-(x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - x + arctan(x) - integrate(x/(x^3 + (x^2 + 1)^(3/
2) + x), x)
```

Fricas [A]
time = 0.37, size = 22, normalized size = 0.85

$$x \log \left(x+\sqrt{x^{2}+1}\right)-\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Sympy [A]
time = 7.86, size = 20, normalized size = 0.77

$$x \log \left(x+\sqrt{x^{2}+1}\right)-\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] x*log(x + sqrt(x**2 + 1)) - sqrt(x**2 + 1)
```

Giac [A]
time = 3.89, size = 22, normalized size = 0.85

$$x \log \left(x + \sqrt{x^2 + 1}\right) - \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 + 1)) - sqrt(x^2 + 1)
```

Mupad [B]
time = 0.08, size = 22, normalized size = 0.85

$$x \ln \left(x + \sqrt{x^2 + 1}\right) - \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 + 1)^(1/2)),x)
[Out] x*log(x + (x^2 + 1)^(1/2)) - (x^2 + 1)^(1/2)
```

3.232 $\int \log \left(x+\sqrt{-1+x^2}\right) \, dx$

Optimal. Leaf size=26

$$-\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)$$

```
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2614, 267}

$$x \log \left(\sqrt{x^2-1}+x\right)-\sqrt{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \log \left(x+\sqrt{-1+x^2}\right) \, dx &= x \log \left(x+\sqrt{-1+x^2}\right)-\int \frac{x}{\sqrt{-1+x^2}} \, dx\\ &= -\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$-\sqrt{-1+x^2}+x \log \left(x+\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[-1 + x^2]],x]
```

```
[Out] -Sqrt[-1 + x^2] + x*Log[x + Sqrt[-1 + x^2]]
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.88

method	result	size
default	$x \ln \left(x+\sqrt{x^{2}-1}\right)-\sqrt{x^{2}-1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x+(x^2-1)^(1/2))-(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(sqrt(x + 1)*sqrt(x - 1) + x) - x + integrate(x/(x^3 + (x^2 - 1)*e^(1/
2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 0.38, size = 22, normalized size = 0.85

$$x \log \left(x+\sqrt{x^{2}-1}\right)-\sqrt{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Sympy [A]
time = 7.13, size = 20, normalized size = 0.77

$$x \log \left(x+\sqrt{x^{2}-1}\right)-\sqrt{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(x**2-1)**(1/2)),x)
```

```
[Out] x*log(x + sqrt(x**2 - 1)) - sqrt(x**2 - 1)
```

Giac [A]
time = 4.64, size = 22, normalized size = 0.85

$$x \log \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x^2 - 1)) - sqrt(x^2 - 1)
```

Mupad [B]
time = 0.55, size = 22, normalized size = 0.85

$$x \ln \left(x + \sqrt{x^2 - 1}\right) - \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x^2 - 1)^(1/2)),x)
[Out] x*log(x + (x^2 - 1)^(1/2)) - (x^2 - 1)^(1/2)
```

3.233 $\int \log \left(x-\sqrt{-1+x^2}\right) \, dx$

Optimal. Leaf size=26

$$\sqrt{-1+x^2}+x \log \left(x-\sqrt{-1+x^2}\right)$$

[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2614, 267}

$$\sqrt{x^2-1}+x \log \left(x-\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

[In] Int[Log[x - Sqrt[-1 + x^2]],x]

[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2614

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned} \int \log \left(x-\sqrt{-1+x^2}\right) \, dx &= x \log \left(x-\sqrt{-1+x^2}\right)+\int \frac{x}{\sqrt{-1+x^2}} \, dx \\ &= \sqrt{-1+x^2}+x \log \left(x-\sqrt{-1+x^2}\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\sqrt{-1+x^2}+x \log \left(x-\sqrt{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x - Sqrt[-1 + x^2]],x]
```

```
[Out] Sqrt[-1 + x^2] + x*Log[x - Sqrt[-1 + x^2]]
```

Maple [A]
time = 0.00, size = 23, normalized size = 0.88

method	result	size
default	$x \ln\left(x-\sqrt{x^{2}-1}\right)+\sqrt{x^{2}-1}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(x-(x^2-1)^(1/2))+(x^2-1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(-sqrt(x + 1)*sqrt(x - 1) + x) - x - integrate(-x/(x^3 - (x^2 - 1)*e^(
1/2*log(x + 1) + 1/2*log(x - 1)) - x), x) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [A]
time = 0.38, size = 22, normalized size = 0.85

$$x \log\left(x-\sqrt{x^{2}-1}\right)+\sqrt{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Sympy [A]
time = 8.52, size = 20, normalized size = 0.77

$$x \log\left(x-\sqrt{x^{2}-1}\right)+\sqrt{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x-(x**2-1)**(1/2)),x)
```

```
[Out] x*log(x - sqrt(x**2 - 1)) + sqrt(x**2 - 1)
```

Giac [A]
time = 5.67, size = 22, normalized size = 0.85

$$x \log \left(x - \sqrt{x^2 - 1}\right) + \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2-1)^(1/2)),x, algorithm="giac")
[Out] x*log(x - sqrt(x^2 - 1)) + sqrt(x^2 - 1)
```

Mupad [B]
time = 0.49, size = 22, normalized size = 0.85

$$x \ln \left(x - \sqrt{x^2 - 1}\right) + \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x - (x^2 - 1)^(1/2)),x)
[Out] x*log(x - (x^2 - 1)^(1/2)) + (x^2 - 1)^(1/2)
```

3.234 $\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\sqrt{x}\,\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)$$

```
[Out] 1/2*arcsinh(x^(1/2))+x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2628, 12, 1978, 52, 56, 221}

$$-\frac{1}{2}\sqrt{x}\,\sqrt{x+1}+x\log\left(\sqrt{x}+\sqrt{x+1}\right)+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 +
x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(\sqrt{x}+\sqrt{1+x}\right)\,dx &= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\int \frac{1}{2}\sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \sqrt{\frac{x}{1+x}}\,dx\\
&= x\log\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{4}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+x\log\left(\sqrt{x}+\sqrt{1+x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.00

$$-\frac{1}{2}\sqrt{x}\sqrt{1+x}+\frac{1}{2}\sinh^{-1}\left(\sqrt{x}\right)+x\log\left(\sqrt{x}+\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + Sqrt[1 + x]],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[1 + x]) + ArcSinh[Sqrt[x]]/2 + x*Log[Sqrt[x] + Sqrt[1 +
x]]
```

Maple [A]
time = 0.01, size = 52, normalized size = 1.21

method	result	size

default	$x\ln\left(\sqrt{x}+\sqrt{1+x}\right)-\frac{\sqrt{x}\,\sqrt{1+x}}{2}+\frac{\sqrt{x\,(1+x)}\,\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{4\sqrt{x}\,\sqrt{1+x}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] x*ln(x^(1/2)+(1+x)^(1/2))-1/2*x^(1/2)*(1+x)^(1/2)+1/4*(x*(1+x))^(1/2)/x^(1/
2)/(1+x)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*x - integrate(1/2*x/(x^2 + (x^(3/2) + sq
rt(x))*sqrt(x + 1) + x), x) + 1/2*log(x + 1)
```

Fricas [A]
time = 0.38, size = 28, normalized size = 0.65

$$\frac{1}{2}\,(2\,x+1)\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x+1}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(2*x + 1)*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x + 1)*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\log\left(\sqrt{x}+\sqrt{x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**(1/2)+(1+x)**(1/2)),x)
[Out] Integral(log(sqrt(x) + sqrt(x + 1)), x)
```

Giac [A]
time = 5.46, size = 40, normalized size = 0.93

$$x\log\left(\sqrt{x+1}+\sqrt{x}\right)-\frac{1}{2}\,\sqrt{x^{2}+x}-\frac{1}{4}\,\log\left(\left|-2\,x+2\,\sqrt{x^{2}+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt(x + 1) + sqrt(x)) - 1/2*sqrt(x^2 + x) - 1/4*log(abs(-2*x + 2*sqr
t(x^2 + x) - 1))
```

Mupad [B]
time = 1.08, size = 37, normalized size = 0.86

$$\operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)-\frac{\sqrt{x}\,\sqrt{x+1}}{2}+x\,\ln\left(\sqrt{x+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] atanh(x^(1/2)/((x + 1)^(1/2) - 1)) - (x^(1/2)*(x + 1)^(1/2))/2 + x*log((x +
 1)^(1/2) + x^(1/2))
```

3.235 $\int \sqrt[3]{x} \log(x) \, dx$

Optimal. Leaf size=21

$$-\frac{9x^{4/3}}{16}+\frac{3}{4}x^{4/3}\log(x)$$

[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\frac{3}{4}x^{4/3}\log(x)-\frac{9x^{4/3}}{16}$$

Antiderivative was successfully verified.

[In] Int[x^(1/3)*Log[x],x]

[Out] (-9*x^(4/3))/16 + (3*x^(4/3)*Log[x])/4

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x} \log(x) \, dx = -\frac{9x^{4/3}}{16}+\frac{3}{4}x^{4/3}\log(x)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.71

$$\frac{3}{16}x^{4/3}(-3+4\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[x^(1/3)*Log[x],x]

[Out] (3*x^(4/3)*(-3 + 4*Log[x]))/16

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
derivativedivides	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14
default	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14
risch	$-\frac{9x^{\frac{4}{3}}}{16}+\frac{3x^{\frac{4}{3}}\ln(x)}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*ln(x),x,method=_RETURNVERBOSE)
[Out] -9/16*x^(4/3)+3/4*x^(4/3)*ln(x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{3}{4}\,x^{\frac{4}{3}}\log\left(x\right)-\frac{9}{16}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="maxima")
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Fricas [A]
time = 0.38, size = 14, normalized size = 0.67

$$\frac{3}{16}\,(4\,x\log\left(x\right)-3\,x)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="fricas")
[Out] 3/16*(4*x*log(x) - 3*x)*x^(1/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(19) = 38.
time = 1.96, size = 105, normalized size = 5.00

$$\begin{cases} -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4}+\frac{3x^{\frac{4}{3}}\log\left(x\right)}{4}-\frac{9x^{\frac{4}{3}}}{8} & \text{for}\ \frac{1}{|x|}<1\wedge|x|<1 \\ \frac{3x^{\frac{4}{3}}\log\left(x\right)}{4}-\frac{9x^{\frac{4}{3}}}{16} & \text{for}\ |x|<1 \\ -\frac{3x^{\frac{4}{3}}\log\left(\frac{1}{x}\right)}{4}-\frac{9x^{\frac{4}{3}}}{16} & \text{for}\ \frac{1}{|x|}<1 \\ -G_{3,3}^{2,1}\left(\begin{matrix}1 & \frac{7}{3},\frac{7}{3} \\ \frac{4}{3},\frac{4}{3} & 0\end{matrix}\middle|x\right)+G_{3,3}^{0,3}\left(\begin{matrix}\frac{7}{3},\frac{7}{3},1 & \\ & \frac{4}{3},\frac{4}{3},0\end{matrix}\middle|x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*ln(x),x)
```

```
[Out] Piecewise((-3*x**(4/3)*log(1/x)/4 + 3*x**(4/3)*log(x)/4 - 9*x**(4/3)/8, (Ab
s(x) < 1) & (1/Abs(x) < 1)), (3*x**(4/3)*log(x)/4 - 9*x**(4/3)/16, Abs(x) <
 1), (-3*x**(4/3)*log(1/x)/4 - 9*x**(4/3)/16, 1/Abs(x) < 1), (-meijerg(((1,
), (7/3, 7/3)), ((4/3, 4/3), (0,)), x) + meijerg(((7/3, 7/3, 1), ()), ((),
(4/3, 4/3, 0)), x), True))
```

Giac [**A**]
time = 3.64, size = 13, normalized size = 0.62

$$\frac{3}{4}\,x^{\frac{4}{3}}\log\left(x\right)-\frac{9}{16}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*log(x),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)*log(x) - 9/16*x^(4/3)
```

Mupad [**B**]
time = 0.34, size = 9, normalized size = 0.43

$$\frac{3\,x^{4/3}\,\left(\ln\left(x\right)-\frac{3}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*log(x),x)
```

```
[Out] (3*x^(4/3)*(log(x) - 3/4))/4
```

3.236 $\int 2^{\log(x)} \, dx$

Optimal. Leaf size=13

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

```
[Out] x^(1+ln(2))/(1+ln(2))
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2306, 30}

$$\frac{x^{1+\log(2)}}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[x],x]
```

```
[Out] x^(1 + Log[2])/(1 + Log[2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(x)} \, dx &= \int x^{\log(2)} \, dx \\ &= \frac{x^{1+\log(2)}}{1+\log(2)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.92

$$\frac{2^{\log(x)} x}{1+\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[x],x]
```

```
[Out] (2^Log[x]*x)/(1 + Log[2])
```

Maple [A]
time = 0.02, size = 13, normalized size = 1.00

method	result	size
gosper	$\frac{x 2^{\ln(x)}}{1+\ln(2)}$	13
risch	$\frac{x\, x^{\ln(2)}}{1+\ln(2)}$	13
norman	$\frac{x\, \mathrm{e}^{\ln(x)\ln(2)}}{1+\ln(2)}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] x/(1+ln(2))*2^ln(x)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 1.85

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(x)}}{\left(\frac{1}{\log(2)}+1\right)\log(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="maxima")
```

```
[Out] 2^((1/log(2) + 1)*log(x))/((1/log(2) + 1)*log(2))
```

Fricas [A]
time = 0.40, size = 14, normalized size = 1.08

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="fricas")
```

```
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Sympy [A]
time = 0.08, size = 10, normalized size = 0.77

$$\frac{2^{\log(x)}x}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(x),x)
```

```
[Out] 2**log(x)*x/(log(2) + 1)
```

Giac [A]
time = 3.83, size = 14, normalized size = 1.08

$$\frac{xe^{(\log(2)\log(x))}}{\log(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(x),x, algorithm="giac")
```

```
[Out] x*e^(log(2)*log(x))/(log(2) + 1)
```

Mupad [B]
time = 0.36, size = 13, normalized size = 1.00

$$\frac{x^{\ln(2)+1}}{\ln(2)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(x),x)
```

```
[Out] x^(log(2) + 1)/(log(2) + 1)
```

3.237 $\int \frac{1-\log(x)}{x^2}\,dx$

Optimal. Leaf size=6

$$\frac{\log(x)}{x}$$

```
[Out] ln(x)/x
```

Rubi [A]
time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2340}

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Log[x])/x^2,x]
```

```
[Out] Log[x]/x
```

Rule 2340

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[b*(d*x)^(m + 1)*(Log[c*x^n]/(d*(m + 1))), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Rubi steps

$$\int \frac{1-\log(x)}{x^2}\,dx = \frac{\log(x)}{x}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[x])/x^2,x]
```

```
[Out] Log[x]/x
```

Maple [A]
time = 0.01, size = 7, normalized size = 1.17

method	result	size
default	$\frac{\ln(x)}{x}$	7
norman	$\frac{\ln(x)}{x}$	7
risch	$\frac{\ln(x)}{x}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.27, size = 14, normalized size = 2.33

$$\frac{\log(x) + 1}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="maxima")
```

```
[Out] (log(x) + 1)/x - 1/x
```

Fricas [A]
time = 0.36, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="fricas")
```

```
[Out] log(x)/x
```

Sympy [A]
time = 0.03, size = 3, normalized size = 0.50

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x**2,x)
```

```
[Out] log(x)/x
```

Giac [A]
time = 3.15, size = 6, normalized size = 1.00

$$\frac{\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x^2,x, algorithm="giac")
```

```
[Out] log(x)/x
```

Mupad [B]
time = 0.35, size = 6, normalized size = 1.00

$$\frac{\ln(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 1)/x^2,x)
```

```
[Out] log(x)/x
```

3.238 $\int \log \left(1+x+\sqrt{1+x}\right) \, dx$

Optimal. Leaf size=32

$$-x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log\left(1+x+\sqrt{1+x}\right)$$

```
[Out] -x+1/2*ln(1+x)+x*ln(1+x+(1+x)^(1/2))+(1+x)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2628}

$$-x+\sqrt{x+1}+x\log\left(x+\sqrt{x+1}+1\right)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] + Log[1 + x]/2 + x*Log[1 + x + Sqrt[1 + x]]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \log\left(1+x+\sqrt{1+x}\right) \, dx &= x\log\left(1+x+\sqrt{1+x}\right)-\int \frac{x\left(1+\frac{1}{2\sqrt{1+x}}\right)}{1+x+\sqrt{1+x}} \, dx \\
&= x\log\left(1+x+\sqrt{1+x}\right)-2\text{Subst}\left(\int \left(-\frac{1}{2}-\frac{1}{2x}+x\right) \, dx, x, \sqrt{1+x}\right) \\
&= -x+\sqrt{1+x}+\frac{1}{2}\log(1+x)+x\log\left(1+x+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.19

$$-x+\sqrt{1+x}-\log\left(1+\sqrt{1+x}\right)+(1+x)\log\left(1+x+\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x + Sqrt[1 + x]],x]
```

```
[Out] -x + Sqrt[1 + x] - Log[1 + Sqrt[1 + x]] + (1 + x)*Log[1 + x + Sqrt[1 + x]]
```

Maple [A]
time = 0.01, size = 34, normalized size = 1.06

method	result	size
derivativedivides	$(1+x)\ln\left(1+x+\sqrt{1+x}\right)-1-x+\sqrt{1+x}-\ln\left(\sqrt{1+x}+1\right)$	34
default	$(1+x)\ln\left(1+x+\sqrt{1+x}\right)-1-x+\sqrt{1+x}-\ln\left(\sqrt{1+x}+1\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)*ln(1+x+(1+x)^(1/2))-1-x+(1+x)^(1/2)-ln((1+x)^(1/2)+1)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 1.03

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

Fricas [A]
time = 0.38, size = 38, normalized size = 1.19

$$(x-1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}+\log\left(\sqrt{x+1}+1\right)+2\log\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x - 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) + log(sqrt(x + 1) + 1) +
 2*log(sqrt(x + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(27) = 54.
time = 1.09, size = 184, normalized size = 5.75

$$\frac{x\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{x\sqrt{x+1}}{\sqrt{x+1}+1}+\frac{x\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{\sqrt{x+1}\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}+\frac{\sqrt{x+1}\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{\sqrt{x+1}}{\sqrt{x+1}+1}-\frac{\log\left(\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}+\frac{\log\left(x+\sqrt{x+1}+1\right)}{\sqrt{x+1}+1}-\frac{1}{\sqrt{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x+(1+x)**(1/2)),x)
```

```
[Out] x*sqrt(x + 1)*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - x*sqrt(x + 1)/(s
qrt(x + 1) + 1) + x*log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - sqrt(x + 1
)*log(sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) + sqrt(x + 1)*log(x + sqrt(x + 1)
+ 1)/(sqrt(x + 1) + 1) - sqrt(x + 1)/(sqrt(x + 1) + 1) - log(sqrt(x + 1) +
1)/(sqrt(x + 1) + 1) + log(x + sqrt(x + 1) + 1)/(sqrt(x + 1) + 1) - 1/(sqrt
(x + 1) + 1)
```

Giac [A]
time = 4.72, size = 33, normalized size = 1.03

$$(x+1)\log\left(x+\sqrt{x+1}+1\right)-x+\sqrt{x+1}-\log\left(\sqrt{x+1}+1\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] (x + 1)*log(x + sqrt(x + 1) + 1) - x + sqrt(x + 1) - log(sqrt(x + 1) + 1) -
 1
```

Mupad [B]
time = 0.11, size = 26, normalized size = 0.81

$$\ln\left(\sqrt{x+1}\right)-x+\sqrt{x+1}+x\,\ln\left(x+\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + (x + 1)^(1/2) + 1),x)
```

```
[Out] log((x + 1)^(1/2)) - x + (x + 1)^(1/2) + x*log(x + (x + 1)^(1/2) + 1)
```

3.239 $\int \log(x+x^3)\,dx$

Optimal. Leaf size=16

$$-3x+2\tan^{-1}(x)+x\log(x+x^3)$$

```
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2603, 396, 209}

$$2\text{ArcTan}(x)+x\log(x^3+x)-3x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + x^3],x]
```

```
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2603

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[x*(a + b*Log[c*R
Fx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \log(x+x^3)\,dx &= x\log(x+x^3)-\int \frac{1+3x^2}{1+x^2}\,dx\\ &= -3x+x\log(x+x^3)+2\int \frac{1}{1+x^2}\,dx\\ &= -3x+2\tan^{-1}(x)+x\log(x+x^3)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$-3x+2\tan^{-1}(x)+x\log\left(x+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + x^3],x]
[Out] -3*x + 2*ArcTan[x] + x*Log[x + x^3]
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.06

method	result	size
default	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17
risch	$-3x+2\arctan\left(x\right)+x\ln\left(x^{3}+x\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^3+x),x,method=_RETURNVERBOSE)
[Out] -3*x+2*arctan(x)+x*ln(x^3+x)
```

Maxima [A]
time = 0.50, size = 16, normalized size = 1.00

$$x\log\left(x^3+x\right)-3\,x+2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="maxima")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Fricas [A]
time = 0.43, size = 16, normalized size = 1.00

$$x\log\left(x^3+x\right)-3\,x+2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="fricas")
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Sympy [A]
time = 0.12, size = 15, normalized size = 0.94

$$x\log\left(x^3+x\right)-3x+2\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**3+x),x)
```

```
[Out] x*log(x**3 + x) - 3*x + 2*atan(x)
```

Giac [A]
time = 4.52, size = 16, normalized size = 1.00

$$x \log \left(x^{3} + x\right) - 3 \, x + 2 \, \arctan \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^3+x),x, algorithm="giac")
```

```
[Out] x*log(x^3 + x) - 3*x + 2*arctan(x)
```

Mupad [B]
time = 0.05, size = 16, normalized size = 1.00

$$2\,\mathrm{atan}(x)-3\,x+x\,\ln\left(x^3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^3),x)
```

```
[Out] 2*atan(x) - 3*x + x*log(x + x^3)
```

3.240 $\int 2^{\log(-8+7x)}\,dx$

Optimal. Leaf size=20

$$\frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}$$

```
[Out] 1/7*(-8+7*x)^(1+ln(2))/(1+ln(2))
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2306, 32}

$$\frac{(7x-8)^{1+\log(2)}}{7(1+\log(2))}$$

Antiderivative was successfully verified.

```
[In] Int[2^Log[-8 + 7*x],x]
```

```
[Out] (-8 + 7*x)^(1 + Log[2])/(7*(1 + Log[2]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2306

```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Rubi steps

$$\begin{aligned}\int 2^{\log(-8+7x)}\,dx &= \int (-8+7x)^{\log(2)}\,dx\\ &= \frac{(-8+7x)^{1+\log(2)}}{7(1+\log(2))}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{2^{\log(-8+7x)}(-8+7x)}{7+\log(128)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Log[-8 + 7*x],x]
[Out] (2^Log[-8 + 7*x]*(-8 + 7*x))/(7 + Log[128])
```

Maple [A]
time = 0.02, size = 22, normalized size = 1.10

method	result	size
gosper	$\frac{2^{\ln(-8+7x)}(-8+7x)}{7\ln(2)+7}$	22
risch	$\frac{(-8+7x)(-8+7x)^{\ln(2)}}{7\ln(2)+7}$	22
norman	$\frac{x\,{\mathrm e}^{\ln(-8+7x)\ln(2)}}{1+\ln(2)}-\frac{8\,{\mathrm e}^{\ln(-8+7x)\ln(2)}}{7(1+\ln(2))}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^ln(-8+7*x),x,method=_RETURNVERBOSE)
[Out] 1/7*(-8+7*x)/(1+ln(2))*2^ln(-8+7*x)
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.45

$$\frac{2^{\left(\frac{1}{\log(2)}+1\right)\log(7\,x-8)}}{7\left(\frac{1}{\log(2)}+1\right)\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="maxima")
[Out] 1/7*2^((1/log(2) + 1)*log(7*x - 8))/((1/log(2) + 1)*log(2))
```

Fricas [A]
time = 0.39, size = 23, normalized size = 1.15

$$\frac{(7\,x-8)e^{(\log(2)\log(7\,x-8))}}{7\left(\log\left(2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="fricas")
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.12, size = 34, normalized size = 1.70

$$\frac{7\cdot 2^{\log\left(7x-8\right)}x}{7\log\left(2\right)+7}-\frac{8\cdot 2^{\log\left(7x-8\right)}}{7\log\left(2\right)+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**ln(-8+7*x),x)
```

```
[Out] 7*2**log(7*x - 8)*x/(7*log(2) + 7) - 8*2**log(7*x - 8)/(7*log(2) + 7)
```

Giac [A]
time = 3.12, size = 23, normalized size = 1.15

$$\frac{(7\,x-8)e^{(\log(2)\log(7\,x-8))}}{7\,(\log(2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^log(-8+7*x),x, algorithm="giac")
```

```
[Out] 1/7*(7*x - 8)*e^(log(2)*log(7*x - 8))/(log(2) + 1)
```

Mupad [B]
time = 0.39, size = 19, normalized size = 0.95

$$\frac{(7\,x-8)^{\ln(2)+1}}{7\,(\ln(2)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^log(7*x - 8),x)
```

```
[Out] (7*x - 8)^(log(2) + 1)/(7*(log(2) + 1))
```

3.241 $\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx$

Optimal. Leaf size=35

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)$$

```
[Out] -1/5*(11-5*x)*ln((-11+5*x)/(5+76*x))-861/380*ln(5+76*x)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2535, 31}

$$-\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{76x+5}\right)-\frac{861}{380}\log(76x+5)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] -1/5*((11 - 5*x)*Log[-((11 - 5*x)/(5 + 76*x))]) - (861*Log[5 + 76*x])/380
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{-11+5x}{5+76x}\right)\,dx &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{5}\int\frac{1}{5+76x}\,dx\\ &= -\frac{1}{5}(11-5x)\log\left(-\frac{11-5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\left(-\frac{11}{5}+x\right)\log\left(\frac{-11+5x}{5+76x}\right)-\frac{861}{380}\log(5+76x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-11 + 5*x)/(5 + 76*x)],x]
```

```
[Out] (-11/5 + x)*Log[(-11 + 5*x)/(5 + 76*x)] - (861*Log[5 + 76*x])/380
```

Maple [A]
time = 0.05, size = 44, normalized size = 1.26

method	result	size
risch	$x\ln\left(\frac{-11+5x}{5+76x}\right)-\frac{11\ln(-11+5x)}{5}-\frac{5\ln(5+76x)}{76}$	34
derivativedivides	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44
default	$\frac{861\ln\left(-\frac{861}{5+76x}\right)}{380}+\frac{\ln\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)\left(\frac{5}{76}-\frac{861}{76(5+76x)}\right)(5+76x)}{5}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-11+5*x)/(5+76*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 861/380*ln(-861/(5+76*x))+1/5*ln(5/76-861/76/(5+76*x))*(5/76-861/76/(5+76*x
))*(5+76*x)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.94

$$x\log\left(\frac{5\,x-11}{76\,x+5}\right)-\frac{5}{76}\,\log\left(76\,x+5\right)-\frac{11}{5}\,\log\left(5\,x-11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="maxima")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.94

$$x\log\left(\frac{5\,x-11}{76\,x+5}\right)-\frac{5}{76}\,\log\left(76\,x+5\right)-\frac{11}{5}\,\log\left(5\,x-11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="fricas")
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 5/76*log(76*x + 5) - 11/5*log(5*x - 11)
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.91

$$x\log\left(\frac{5x-11}{76x+5}\right)-\frac{11\log\left(x-\frac{11}{5}\right)}{5}-\frac{5\log\left(x+\frac{5}{76}\right)}{76}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-11+5*x)/(5+76*x)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - 11*log(x - 11/5)/5 - 5*log(x + 5/76)/76
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(30) = 60.
time = 3.49, size = 139, normalized size = 3.97

$$-\frac{861\log\left(\frac{\frac{5\left(\frac{5(5x-11)}{76x+5}+11\right)}{\frac{76(5x-11)}{76x+5}-5}+11}{\frac{76\left(\frac{5(5x-11)}{76x+5}+11\right)}{\frac{76(5x-11)}{76x+5}-5}-5}\right)}{76\left(\frac{76(5x-11)}{76x+5}-5\right)}-\frac{861}{380}\log\left(\frac{|5x-11|}{|76x+5|}\right)+\frac{861}{380}\log\left(\left|\frac{76(5x-11)}{76x+5}-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-11+5*x)/(5+76*x)),x, algorithm="giac")
```

```
[Out] -861/76*log((5*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5
) + 11)/(76*(5*(5*x - 11)/(76*x + 5) + 11)/(76*(5*x - 11)/(76*x + 5) - 5) -
 5))/(76*(5*x - 11)/(76*x + 5) - 5) - 861/380*log(abs(5*x - 11)/abs(76*x +
5)) + 861/380*log(abs(76*(5*x - 11)/(76*x + 5) - 5))
```

Mupad [B]
time = 0.10, size = 29, normalized size = 0.83

$$x\ln\left(\frac{5x-11}{76x+5}\right)-\frac{5\ln\left(x+\frac{5}{76}\right)}{76}-\frac{11\ln\left(x-\frac{11}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((5*x - 11)/(76*x + 5)),x)
```

```
[Out] x*log((5*x - 11)/(76*x + 5)) - (5*log(x + 5/76))/76 - (11*log(x - 11/5))/5
```

3.242 $\int \log\left(\frac{1}{13+x}\right)\,dx$

Optimal. Leaf size=12

$$x+(13+x)\log\left(\frac{1}{13+x}\right)$$

```
[Out] x+(13+x)*ln(1/(13+x))
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2436, 2332}

$$x+(x+13)\log\left(\frac{1}{x+13}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(13 + x)^(-1)],x]
```

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{1}{13+x}\right)\,dx &= \text{Subst}\left(\int \log\left(\frac{1}{x}\right)\,dx,x,13+x\right)\\ &= x+(13+x)\log\left(\frac{1}{13+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$x+(13+x)\log\left(\frac{1}{13+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(13 + x)^(-1)],x]
```

```
[Out] x + (13 + x)*Log[(13 + x)^(-1)]
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.17

method	result	size
derivativedivides	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
default	$(13+x)\ln\left(\frac{1}{13+x}\right)+13+x$	14
risch	$x\ln\left(\frac{1}{13+x}\right)+x-13\ln\left(13+x\right)$	17
norman	$x+x\ln\left(\frac{1}{13+x}\right)+13\ln\left(\frac{1}{13+x}\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/(13+x)),x,method=_RETURNVERBOSE)
```

```
[Out] (13+x)*ln(1/(13+x))+13+x
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.00

$$-(x+13)\log\left(x+13\right)+x+13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="maxima")
```

```
[Out] -(x + 13)*log(x + 13) + x + 13
```

Fricas [A]
time = 0.50, size = 12, normalized size = 1.00

$$(x+13)\log\left(\frac{1}{x+13}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="fricas")
```

```
[Out] (x + 13)*log(1/(x + 13)) + x
```

Sympy [A]
time = 0.03, size = 15, normalized size = 1.25

$$x\log\left(\frac{1}{x+13}\right)+x-13\log\left(x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/(13+x)),x)
```

```
[Out] x*log(1/(x + 13)) + x - 13*log(x + 13)
```

Giac [A]
time = 3.08, size = 12, normalized size = 1.00

$$-(x + 13)\log(x + 13) + x + 13$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/(13+x)),x, algorithm="giac")
```

```
[Out] -(x + 13)*log(x + 13) + x + 13
```

Mupad [B]
time = 0.05, size = 12, normalized size = 1.00

$$\left(\ln\left(\frac{1}{x+13}\right)+1\right)\,(x+13)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/(x + 13)),x)
```

```
[Out] (log(1/(x + 13)) + 1)*(x + 13)
```

3.243 $\int x \log\left(\frac{1+x}{x^2}\right) dx$

Optimal. Leaf size=36

$$\frac{x}{2}+\frac{x^2}{4}-\frac{1}{2}\log(1+x)+\frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)$$

[Out] 1/2*x+1/4*x^2-1/2*ln(1+x)+1/2*x^2*ln((1+x)/x^2)

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2581, 30, 45}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log\left(\frac{x+1}{x^2}\right)+\frac{x}{2}-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x*Log[(1 + x)/x^2],x]

[Out] x/2 + x^2/4 - Log[1 + x]/2 + (x^2*Log[(1 + x)/x^2])/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x\log\left(\frac{1+x}{x^2}\right)dx = \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right) - \frac{1}{2}\int\frac{x^2}{1+x}\,dx + \int x\,dx$$
$$= \frac{x^2}{2} + \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right) - \frac{1}{2}\int\left(-1+x+\frac{1}{1+x}\right)dx$$
$$= \frac{x}{2} + \frac{x^2}{4} - \frac{1}{2}\log(1+x) + \frac{1}{2}x^2\log\left(\frac{1+x}{x^2}\right)$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.75

$$\frac{1}{4}\left(-2\log(1+x) + x\left(2+x+2x\log\left(\frac{1+x}{x^2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[(1 + x)/x^2],x]
[Out] (-2*Log[1 + x] + x*(2 + x + 2*x*Log[(1 + x)/x^2]))/4
```

Maple [A]
time = 0.03, size = 39, normalized size = 1.08

method	result	size
risch	$\frac{x}{2}+\frac{x^2}{4}-\frac{\ln(1+x)}{2}+\frac{x^2\ln\left(\frac{1+x}{x^2}\right)}{2}$	29
derivativedivides	$\frac{x^2\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}+\frac{x^2}{4}+\frac{x}{2}+\frac{\ln\left(\frac{1}{x}\right)}{2}-\frac{\ln\left(1+\frac{1}{x}\right)}{2}$	39
default	$\frac{x^2\ln\left(\frac{1+\frac{1}{x}}{x}\right)}{2}+\frac{x^2}{4}+\frac{x}{2}+\frac{\ln\left(\frac{1}{x}\right)}{2}-\frac{\ln\left(1+\frac{1}{x}\right)}{2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln((1+x)/x^2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(1/x*(1+1/x))+1/4*x^2+1/2*x+1/2*ln(1/x)-1/2*ln(1+1/x)
```

Maxima [A]
time = 0.30, size = 28, normalized size = 0.78

$$\frac{1}{2}\,x^2\log\left(\frac{x+1}{x^2}\right) + \frac{1}{4}\,x^2 + \frac{1}{2}\,x - \frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.78

$$\frac{1}{2}\,x^{2}\log\left(\frac{x+1}{x^{2}}\right)+\frac{1}{4}\,x^{2}+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(x + 1)
```

Sympy [A]
time = 0.05, size = 27, normalized size = 0.75

$$\frac{x^{2}\log\left(\frac{x+1}{x^{2}}\right)}{2}+\frac{x^{2}}{4}+\frac{x}{2}-\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln((1+x)/x**2),x)
```

```
[Out] x**2*log((x + 1)/x**2)/2 + x**2/4 + x/2 - log(x + 1)/2
```

Giac [A]
time = 3.41, size = 29, normalized size = 0.81

$$\frac{1}{2}\,x^{2}\log\left(\frac{x+1}{x^{2}}\right)+\frac{1}{4}\,x^{2}+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log((1+x)/x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log((x + 1)/x^2) + 1/4*x^2 + 1/2*x - 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.49, size = 40, normalized size = 1.11

$$\frac{x}{2}-\frac{\ln\left(x\,\left(x+1\right)\right)}{3}-\frac{\ln\left(\frac{x+1}{x^{2}}\right)}{6}+\frac{x^{2}\,\ln\left(\frac{x+1}{x^{2}}\right)}{2}+\frac{x^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log((x + 1)/x^2),x)
```

```
[Out] x/2 - log(x*(x + 1))/3 - log((x + 1)/x^2)/6 + (x^2*log((x + 1)/x^2))/2 + x^
2/4
```

3.244 $\int x^3 \log\left(\frac{7+5x}{x^2}\right) dx$

Optimal. Leaf size=54

$$\frac{343x}{500} - \frac{49x^2}{200} + \frac{7x^3}{60} + \frac{x^4}{16} - \frac{2401\log(7+5x)}{2500} + \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)$$

```
[Out] 343/500*x-49/200*x^2+7/60*x^3+1/16*x^4-2401/2500*ln(7+5*x)+1/4*x^4*ln((7+5*
x)/x^2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2581, 30, 45}

$$\frac{x^4}{16} + \frac{7x^3}{60} - \frac{49x^2}{200} + \frac{1}{4}x^4\log\left(\frac{5x+7}{x^2}\right) + \frac{343x}{500} - \frac{2401\log(5x+7)}{2500}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2581

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(Lo
g[e*(f*(a + b*x)^p*(c + d*x)^q)^r]/(h*(m + 1))), x] + (-Dist[b*p*(r/(h*(m +
 1))), Int[(g + h*x)^(m + 1)/(a + b*x), x], x] - Dist[d*q*(r/(h*(m + 1))),
Int[(g + h*x)^(m + 1)/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h,
m, p, q, r}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\log\left(\frac{7+5x}{x^2}\right)dx = \frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)+\frac{\int x^3\,dx}{2}-\frac{5}{4}\int\frac{x^4}{7+5x}\,dx$$
$$=\frac{x^4}{8}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)-\frac{5}{4}\int\left(-\frac{343}{625}+\frac{49x}{125}-\frac{7x^2}{25}+\frac{x^3}{5}+\frac{2401}{625(7+5x)}\right)dx$$
$$=\frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\log(7+5x)}{2500}+\frac{1}{4}x^4\log\left(\frac{7+5x}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[(7 + 5*x)/x^2],x]
```

```
[Out] (343*x)/500 - (49*x^2)/200 + (7*x^3)/60 + x^4/16 - (2401*Log[7 + 5*x])/2500
 + (x^4*Log[(7 + 5*x)/x^2])/4
```

Maple [A]
time = 0.02, size = 53, normalized size = 0.98

method	result	size
risch	$\frac{343x}{500}-\frac{49x^2}{200}+\frac{7x^3}{60}+\frac{x^4}{16}-\frac{2401\ln(7+5x)}{2500}+\frac{x^4\ln\left(\frac{7+5x}{x^2}\right)}{4}$	43
derivativedivides	$\frac{x^4\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}-\frac{2401\ln\left(\frac{7}{x}+5\right)}{2500}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}+\frac{2401\ln\left(\frac{1}{x}\right)}{2500}$	53
default	$\frac{x^4\ln\left(\frac{\frac{7}{x}+5}{x}\right)}{4}-\frac{2401\ln\left(\frac{7}{x}+5\right)}{2500}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}+\frac{2401\ln\left(\frac{1}{x}\right)}{2500}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln((7+5*x)/x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln((7/x+5)/x)-2401/2500*ln(7/x+5)+1/16*x^4+7/60*x^3-49/200*x^2+343/
500*x+2401/2500*ln(1/x)
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.78

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Fricas [A]
time = 0.43, size = 42, normalized size = 0.78

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(5\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(5*x + 7)
```

Sympy [A]
time = 0.05, size = 48, normalized size = 0.89

$$\frac{x^4\log\left(\frac{5x+7}{x^2}\right)}{4}+\frac{x^4}{16}+\frac{7x^3}{60}-\frac{49x^2}{200}+\frac{343x}{500}-\frac{2401\log\left(5x+7\right)}{2500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln((7+5*x)/x**2),x)
```

```
[Out] x**4*log((5*x + 7)/x**2)/4 + x**4/16 + 7*x**3/60 - 49*x**2/200 + 343*x/500
- 2401*log(5*x + 7)/2500
```

Giac [A]
time = 3.67, size = 43, normalized size = 0.80

$$\frac{1}{4}\,x^4\log\left(\frac{5\,x+7}{x^2}\right)+\frac{1}{16}\,x^4+\frac{7}{60}\,x^3-\frac{49}{200}\,x^2+\frac{343}{500}\,x-\frac{2401}{2500}\,\log\left(|5\,x+7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log((7+5*x)/x^2),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log((5*x + 7)/x^2) + 1/16*x^4 + 7/60*x^3 - 49/200*x^2 + 343/500*x -
 2401/2500*log(abs(5*x + 7))
```

Mupad [B]
time = 0.59, size = 56, normalized size = 1.04

$$\frac{343\,x}{500}-\frac{2401\,\ln\left(x\,\left(5\,x+7\right)\right)}{3750}-\frac{2401\,\ln\left(\frac{5\,x+7}{x^2}\right)}{7500}+\frac{x^4\,\ln\left(\frac{5\,x+7}{x^2}\right)}{4}-\frac{49\,x^2}{200}+\frac{7\,x^3}{60}+\frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*log((5*x + 7)/x^2),x)
```

```
[Out] (343*x)/500 - (2401*log(x*(5*x + 7)))/3750 - (2401*log((5*x + 7)/x^2))/7500
 + (x^4*log((5*x + 7)/x^2))/4 - (49*x^2)/200 + (7*x^3)/60 + x^4/16
```

3.245 $\int (a+bx)\log(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}$$

```
[Out] -1/4*(b*x+a)^2/b+1/2*(b*x+a)^2*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2437, 2341}

$$\frac{(a+bx)^2\log(a+bx)}{2b}-\frac{(a+bx)^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[a + b*x],x]
```

```
[Out] -1/4*(a + b*x)^2/b + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\log(x)\,dx, x, a+bx\right)}{b}\\ &= -\frac{(a+bx)^2}{4b}+\frac{(a+bx)^2\log(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.94

$$-\frac{1}{4}x(2a+bx)+\frac{(a+bx)^2\log(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[a + b*x],x]
[Out] -1/4*(x*(2*a + b*x)) + ((a + b*x)^2*Log[a + b*x])/(2*b)
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{(bx+a)^2\ln(bx+a)}{2}-\frac{(bx+a)^2}{4}}{b}$	30
default	$\frac{\frac{(bx+a)^2\ln(bx+a)}{2}-\frac{(bx+a)^2}{4}}{b}$	30
risch	$\left(\frac{1}{2}b\,x^2+ax\right)\ln\left(bx+a\right)-\frac{b\,x^2}{4}-\frac{ax}{2}+\frac{a^2\ln(bx+a)}{2b}$	43
norman	$ax\ln\left(bx+a\right)-\frac{ax}{2}-\frac{b\,x^2}{4}+\frac{a^2\ln(bx+a)}{2b}+\frac{b\,x^2\ln(bx+a)}{2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*(b*x+a)^2*ln(b*x+a)-1/4*(b*x+a)^2)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.49

$$\frac{1}{4}\,b\left(\frac{2\,a^2\log\left(bx+a\right)}{b^2}-\frac{bx^2+2\,ax}{b}\right)+\frac{1}{2}\left(bx^2+2\,ax\right)\log\left(bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="maxima")
[Out] 1/4*b*(2*a^2*log(b*x + a)/b^2 - (b*x^2 + 2*a*x)/b) + 1/2*(b*x^2 + 2*a*x)*lo
g(b*x + a)
```

Fricas [A]
time = 0.37, size = 42, normalized size = 1.20

$$-\frac{b^2x^2+2\,abx-2\left(b^2x^2+2\,abx+a^2\right)\log\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="fricas")
[Out] -1/4*(b^2*x^2 + 2*a*b*x - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/b
```

Sympy [A]
time = 0.08, size = 41, normalized size = 1.17

$$\frac{a^2\log\left(a+bx\right)}{2b}-\frac{ax}{2}-\frac{bx^2}{4}+\left(ax+\frac{bx^2}{2}\right)\log\left(a+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(b*x+a),x)
[Out] a**2*log(a + b*x)/(2*b) - a*x/2 - b*x**2/4 + (a*x + b*x**2/2)*log(a + b*x)
```

Giac [A]
time = 3.53, size = 31, normalized size = 0.89

$$\frac{(bx+a)^2\log\left(bx+a\right)}{2\,b}-\frac{(bx+a)^2}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(b*x+a),x, algorithm="giac")
[Out] 1/2*(b*x + a)^2*log(b*x + a)/b - 1/4*(b*x + a)^2/b
```

Mupad [B]
time = 0.47, size = 46, normalized size = 1.31

$$\frac{a^2\,\ln\left(a+b\,x\right)}{2\,b}-\frac{b\,x^2}{4}-\frac{a\,x}{2}+a\,x\,\ln\left(a+b\,x\right)+\frac{b\,x^2\,\ln\left(a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x),x)
[Out] (a^2*log(a + b*x))/(2*b) - (b*x^2)/4 - (a*x)/2 + a*x*log(a + b*x) + (b*x^2*
log(a + b*x))/2
```

3.246 $\int (a+bx)^2 \log(a+bx)\, dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}$$

```
[Out] -1/9*(b*x+a)^3/b+1/3*(b*x+a)^3*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2341}

$$\frac{(a+bx)^3\log(a+bx)}{3b}-\frac{(a+bx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Log[a + b*x],x]
```

```
[Out] -1/9*(a + b*x)^3/b + ((a + b*x)^3*Log[a + b*x])/(3*b)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\log(x)\,dx,x,a+bx\right)}{b}\\ &= -\frac{(a+bx)^3}{9b}+\frac{(a+bx)^3\log(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.69

$$\frac{(a+bx)^3(-1+3\log(a+bx))}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Log[a + b*x],x]
[Out] ((a + b*x)^3*(-1 + 3*Log[a + b*x]))/(9*b)
```

Maple [A]
time = 0.06, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{(bx+a)^3\ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
default	$\frac{\frac{(bx+a)^3\ln(bx+a)}{3}-\frac{(bx+a)^3}{9}}{b}$	30
risch	$-\frac{b^2x^3}{9}-\frac{ab\,x^2}{3}-\frac{x\,a^2}{3}-\frac{a^3}{9b}+\frac{(bx+a)^3\ln(bx+a)}{3b}$	49
norman	$x\,a^2\ln(bx+a)+ab\,x^2\ln(bx+a)-\frac{b^2x^3}{9}-\frac{x\,a^2}{3}-\frac{ab\,x^2}{3}+\frac{a^3\ln(bx+a)}{3b}+\frac{b^2x^3\ln(bx+a)}{3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/3*(b*x+a)^3*ln(b*x+a)-1/9*(b*x+a)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.
time = 0.27, size = 74, normalized size = 2.11

$$\frac{1}{9}\left(\frac{3\,a^3\log(bx+a)}{b^2}-\frac{b^2x^3+3\,abx^2+3\,a^2x}{b}\right)b+\frac{1}{3}\left(b^2x^3+3\,abx^2+3\,a^2x\right)\log(bx+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="maxima")
[Out] 1/9*(3*a^3*log(b*x + a)/b^2 - (b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/b)*b + 1/3*(b
^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*log(b*x + a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(31) = 62$.
time = 0.36, size = 64, normalized size = 1.83

$$-\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx-3\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)\log(bx+a)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="fricas")
[Out] -1/9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.12, size = 63, normalized size = 1.80

$$\frac{a^3 \log(a+bx)}{3b} - \frac{a^2x}{3} - \frac{abx^2}{3} - \frac{b^2x^3}{9} + \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\log(a+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*ln(b*x+a),x)
```

```
[Out] a**3*log(a + b*x)/(3*b) - a**2*x/3 - a*b*x**2/3 - b**2*x**3/9 + (a**2*x + a
*b*x**2 + b**2*x**3/3)*log(a + b*x)
```

Giac [A]
time = 3.76, size = 31, normalized size = 0.89

$$\frac{(bx+a)^3 \log(bx+a)}{3\,b} - \frac{(bx+a)^3}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*log(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3*log(b*x + a)/b - 1/9*(b*x + a)^3/b
```

Mupad [B]
time = 0.46, size = 73, normalized size = 2.09

$$\frac{a^3\,\ln(a+b\,x)}{3\,b} - \frac{b^2\,x^3}{9} - \frac{a^2\,x}{3} + \frac{b^2\,x^3\,\ln(a+b\,x)}{3} - \frac{a\,b\,x^2}{3} + a^2\,x\,\ln(a+b\,x) + a\,b\,x^2\,\ln(a+b\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x)^2,x)
```

```
[Out] (a^3*log(a + b*x))/(3*b) - (b^2*x^3)/9 - (a^2*x)/3 + (b^2*x^3*log(a + b*x))
/3 - (a*b*x^2)/3 + a^2*x*log(a + b*x) + a*b*x^2*log(a + b*x)
```

3.247 $\int \frac{\log(a+bx)}{a+bx}\,dx$

Optimal. Leaf size=15

$$\frac{\log^2(a+bx)}{2b}$$

```
[Out] 1/2*ln(b*x+a)^2/b
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2437, 2338}

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(a+bx)}{a+bx}\,dx &= \frac{\text{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, a+bx\right)}{b}\\ &= \frac{\log^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x),x]
```

```
[Out] Log[a + b*x]^2/(2*b)
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\ln(bx+a)^2}{2b}$	14
default	$\frac{\ln(bx+a)^2}{2b}$	14
norman	$\frac{\ln(bx+a)^2}{2b}$	14
risch	$\frac{\ln(bx+a)^2}{2b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(b*x+a)^2/b
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$\frac{\log(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.87

$$\frac{\log(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

Sympy [A]
time = 0.08, size = 10, normalized size = 0.67

$$\frac{\log(a+bx)^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a),x)
```

```
[Out] log(a + b*x)**2/(2*b)
```

Giac [A]
time = 2.49, size = 13, normalized size = 0.87

$$\frac{\log\left(bx+a\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(b*x + a)^2/b
```

Mupad [B]
time = 0.55, size = 13, normalized size = 0.87

$$\frac{\ln\left(a+b\,x\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/(a + b*x),x)
```

```
[Out] log(a + b*x)^2/(2*b)
```

3.248 $\int \frac{\log(a+bx)}{(a+bx)^2} dx$

Optimal. Leaf size=31

$$-\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)-ln(b*x+a)/b/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2437, 2341}

$$-\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -(1/(b*(a + b*x))) - Log[a + b*x]/(b*(a + b*x))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(a+bx)}{(a+bx)^2}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\log(x)}{x^2}\,dx, x, a+bx\right)}{b}\\ &= -\frac{1}{b(a+bx)} - \frac{\log(a+bx)}{b(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.68

$$-\frac{1+\log(a+bx)}{ab+b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a + b*x]/(a + b*x)^2,x]
```

```
[Out] -((1 + Log[a + b*x])/(a*b + b^2*x))
```

Maple [A]
time = 0.05, size = 30, normalized size = 0.97

method	result	size
norman	$\frac{\frac{x}{a}-\frac{\ln(bx+a)}{b}}{bx+a}$	26
derivativedivides	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
default	$\frac{-\frac{\ln(bx+a)}{bx+a}-\frac{1}{bx+a}}{b}$	30
risch	$-\frac{1}{b(bx+a)}-\frac{\ln(bx+a)}{b(bx+a)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(b*x+a)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-ln(b*x+a)/(b*x+a)-1/(b*x+a))
```

Maxima [A]
time = 0.29, size = 31, normalized size = 1.00

$$-\frac{\log(bx+a)}{(bx+a)b}-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/((b*x + a)*b) - 1/((b*x + a)*b)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 0.68

$$-\frac{\log(bx+a)+1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) + 1)/(b^2*x + a*b)
```

Sympy [A]
time = 0.20, size = 26, normalized size = 0.84

$$-\frac{\log(a+bx)}{ab+b^2x}-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(b*x+a)/(b*x+a)**2,x)
```

```
[Out] -log(a + b*x)/(a*b + b**2*x) - 1/(a*b + b**2*x)
```

Giac [A]
time = 3.45, size = 32, normalized size = 1.03

$$-b\left(\frac{\log\left(bx+a\right)}{(bx+a)b^2}+\frac{1}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(b*x+a)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -b*(log(b*x + a)/((b*x + a)*b^2) + 1/((b*x + a)*b^2))
```

Mupad [B]
time = 0.49, size = 25, normalized size = 0.81

$$-\frac{a+a\,\ln\left(a+b\,x\right)}{a\,b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)/(a + b*x)^2,x)
```

```
[Out] -(a + a*log(a + b*x))/(a*b*(a + b*x))
```

3.249 $\int (a+bx)^n \log(a+bx)\,dx$

Optimal. Leaf size=44

$$-\frac{(a+bx)^{1+n}}{b(1+n)^2}+\frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}$$

```
[Out] -(b*x+a)^(1+n)/b/(1+n)^2+(b*x+a)^(1+n)*ln(b*x+a)/b/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2437, 2341}

$$\frac{(a+bx)^{n+1}\log(a+bx)}{b(n+1)}-\frac{(a+bx)^{n+1}}{b(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*Log[a + b*x],x]
```

```
[Out] -((a + b*x)^(1 + n)/(b*(1 + n)^2)) + ((a + b*x)^(1 + n)*Log[a + b*x])/(b*(1
 + n))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n\log(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^n\log(x)\,dx,x,a+bx\right)}{b}\\ &= -\frac{(a+bx)^{1+n}}{b(1+n)^2}+\frac{(a+bx)^{1+n}\log(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.68

$$\frac{(a+bx)^{1+n}(-1+(1+n)\log(a+bx))}{b(1+n)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*Log[a + b*x],x]
[Out] ((a + b*x)^(1 + n)*(-1 + (1 + n)*Log[a + b*x]))/(b*(1 + n)^2)
```

Maple [A]
time = 0.07, size = 61, normalized size = 1.39

method	result	size
risch	$\frac{(bnx\ln(bx+a)+an\ln(bx+a)+bx\ln(bx+a)+a\ln(bx+a)-bx-a)(bx+a)^n}{(1+n)^2 b}$	61
norman	$\frac{x\ln(bx+a)\mathrm{e}^{\ln(bx+a)n}}{1+n}+\frac{a\ln(bx+a)\mathrm{e}^{\ln(bx+a)n}}{b(1+n)}-\frac{x\,\mathrm{e}^{\ln(bx+a)n}}{n^2+2n+1}-\frac{a\,\mathrm{e}^{\ln(bx+a)n}}{b(n^2+2n+1)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*ln(b*x+a),x,method=_RETURNVERBOSE)
[Out] (b*n*x*ln(b*x+a)+a*n*ln(b*x+a)+b*x*ln(b*x+a)+a*ln(b*x+a)-b*x-a)/(1+n)^2/b*(
b*x+a)^n
```

Maxima [A]
time = 0.28, size = 44, normalized size = 1.00

$$\frac{(bx+a)^{n+1}\log(bx+a)}{b(n+1)}-\frac{(bx+a)^{n+1}}{b(n+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="maxima")
[Out] (b*x + a)^(n + 1)*log(b*x + a)/(b*(n + 1)) - (b*x + a)^(n + 1)/(b*(n + 1)^2
)
```

Fricas [A]
time = 0.39, size = 47, normalized size = 1.07

$$-\frac{(bx-(an+(bn+b)x+a)\log(bx+a)+a)(bx+a)^n}{bn^2+2\,bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x - (a*n + (b*n + b)*x + a)*log(b*x + a) + a)*(b*x + a)^n/(b*n^2 + 2*b*
n + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(34) = 68$.
time = 0.77, size = 185, normalized size = 4.20

$$\begin{cases} \frac{x\log(a)}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n x \log(a) & \text{for } b = 0 \\ \frac{\log(a+bx)^2}{2b} & \text{for } n = -1 \\ \frac{an(a+bx)^n\log(a+bx)}{bn^2+2bn+b} + \frac{a(a+bx)^n\log(a+bx)}{bn^2+2bn+b} - \frac{a(a+bx)^n}{bn^2+2bn+b} + \frac{bnx(a+bx)^n\log(a+bx)}{bn^2+2bn+b} + \frac{bx(a+bx)^n\log(a+bx)}{bn^2+2bn+b} - \frac{bx(a+bx)^n}{bn^2+2bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*ln(b*x+a),x)
```

```
[Out] Piecewise((x*log(a)/a, Eq(b, 0) & Eq(n, -1)), (a**n*x*log(a), Eq(b, 0)), (l
og(a + b*x)**2/(2*b), Eq(n, -1)), (a*n*(a + b*x)**n*log(a + b*x)/(b*n**2 +
2*b*n + b) + a*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - a*(a + b*x)
**n/(b*n**2 + 2*b*n + b) + b*n*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n
+ b) + b*x*(a + b*x)**n*log(a + b*x)/(b*n**2 + 2*b*n + b) - b*x*(a + b*x)**
n/(b*n**2 + 2*b*n + b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*log(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*log(b*x + a), x)
```

Mupad [B]
time = 0.46, size = 52, normalized size = 1.18

$$\begin{cases} \frac{\ln(a+b\,x)^2}{2\,b} & \text{if } n = -1 \\ \frac{\left(\ln(a+b\,x)-\frac{1}{n+1}\right)\,(a+b\,x)^{n+1}}{b\,(n+1)} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(a + b*x)*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, log(a + b*x)^2/(2*b), n ~= -1, ((log(a + b*x) - 1/(n + 1
))*(a + b*x)^(n + 1))/(b*(n + 1)))
```

3.250 $\int \frac{1}{ax+bx\log(cx^n)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

[Out] ln(a+b*ln(c*x^n))/b/n

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x*Log[c*x^n])^(-1),x]

[Out] Log[a + b*Log[c*x^n]]/(b*n)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log\left(cx^n\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{n}\\ &= \frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x*Log[c*x^n])^(-1),x]

[Out] Log[a + b*Log[c*x^n]]/(b*n)

Maple [A]
time = 0.05, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(a+b\ln(c\,x^{n}))}{bn}$	19
norman	$\frac{\ln\left(b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+a\right)}{bn}$	21
risch	$\frac{\ln\left(\ln(x^{n})-\frac{i\pi b\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})-i\pi b\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^{n})^{2}-i\pi b\,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})^{2}+i\pi b\mathrm{csgn}(ic\,x^{n})^{3}-2b\ln(c)-2a}{2b}\right)}{bn}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+b*ln(c*x^n))/b/n
```

Maxima [A]
time = 0.29, size = 24, normalized size = 1.33

$$\frac{\log\left(\frac{b\log(c)+b\log(x^{n})+a}{b}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log((b*log(c) + b*log(x^n) + a)/b)/(b*n)
```

Fricas [A]
time = 0.40, size = 19, normalized size = 1.06

$$\frac{\log\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] log(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 0.39, size = 31, normalized size = 1.72

$$\begin{cases} \frac{\log\left(x\right)}{a} & \text{for}\: b = 0 \wedge \left(b = 0 \vee n = 0\right) \\ \frac{\log\left(x\right)}{a+b\log\left(c\right)} & \text{for}\: n = 0 \\ \frac{\log\left(\frac{a}{b}+\log\left(cx^{n}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*log(
c)), Eq(n, 0)), (log(a/b + log(c*x**n))/(b*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.
time = 3.76, size = 45, normalized size = 2.50

$$\frac{\log\left(\frac{1}{4}\left(\pi bn(\operatorname{sgn}(x)-1)+\pi b(\operatorname{sgn}(c)-1)\right)^{2}+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)+a\right)^{2}\right)}{2\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
+ b*log(abs(c)) + a)^2)/(b*n)
```

Mupad [B]
time = 0.38, size = 18, normalized size = 1.00

$$\frac{\ln\left(a+b\,\ln\left(c\,x^{n}\right)\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)),x)
```

```
[Out] log(a + b*log(c*x^n))/(b*n)
```

3.251 $\int \frac{1}{ax+bx\log^2(cx^n)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

```
[Out] arctan(ln(c*x^n)*b^(1/2)/a^(1/2))/n/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx\log^2(cx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\log(cx^n)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*Log[c*x^n])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*n)
```

Maple [A]
time = 0.14, size = 24, normalized size = 0.75

method	result
default	$\frac{\arctan\left(\frac{b\ln(c\,x^n)}{\sqrt{ab}}\right)}{n\sqrt{ab}}$
risch	$-\frac{\ln\left(\ln(x^n)-\frac{i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)\sqrt{-ab}-i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2\sqrt{-ab}-i\pi\,\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2\sqrt{-ab}+i\pi\mathrm{csgn}(ic\,x^n}{2\sqrt{-ab}}\right.}{2\sqrt{-ab}\,n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^2),x,method=_RETURNVERBOSE)
[Out] 1/n/(a*b)^(1/2)*arctan(b*ln(c*x^n)/(a*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="maxima")
[Out] integrate(1/(b*x*log(c*x^n)^2 + a*x), x)
```

Fricas [A]
time = 0.40, size = 121, normalized size = 3.78

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2-2\,\sqrt{-ab}\,(n\log(x)+\log(c))-a}{bn^2\log(x)^2+2\,bn\log(c)\log(x)+b\log(c)^2+a}\right)}{2\,abn},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(n\log(x)+\log(c))}{a}\right)}{abn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*n^2*log(x)^2 + 2*b*n*log(c)*log(x) + b*log(c)^2 - 2
*sqrt(-a*b)*(n*log(x) + log(c)) - a)/(b*n^2*log(x)^2 + 2*b*n*log(c)*log(x)
+ b*log(c)^2 + a))/(a*b*n), sqrt(a*b)*arctan(sqrt(a*b)*(n*log(x) + log(c))/
a)/(a*b*n)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(29) = 58$.
time = 3.63, size = 99, normalized size = 3.09

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b\log(c)^2} & \text{for } n = 0 \\ -\frac{1}{bn\log(cx^n)} & \text{for } a = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{-\frac{a}{b}}+\log(cx^n)\right)}{2bn\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a
, Eq(b, 0)), (log(x)/(a + b*log(c)**2), Eq(n, 0)), (-1/(b*n*log(c*x**n)), E
q(a, 0)), (log(-sqrt(-a/b) + log(c*x**n))/(2*b*n*sqrt(-a/b)) - log(sqrt(-a/
b) + log(c*x**n))/(2*b*n*sqrt(-a/b)), True))
```

Giac [A]
time = 3.72, size = 26, normalized size = 0.81

$$\frac{\arctan\left(\frac{bn\log(x)+b\log(c)}{\sqrt{ab}}\right)}{\sqrt{ab}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] arctan((b*n*log(x) + b*log(c))/sqrt(a*b))/(sqrt(a*b)*n)
```

Mupad [B]
time = 0.47, size = 71, normalized size = 2.22

$$-\frac{\ln\left(\frac{1}{b\,x}+\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)-\ln\left(\frac{1}{b\,x}-\frac{\ln(c\,x^n)}{\sqrt{-a}\,\sqrt{b}\,x}\right)}{2\sqrt{-a}\,\sqrt{b}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)^2),x)
```

```
[Out] -(log(1/(b*x) + log(c*x^n)/((-a)^(1/2)*b^(1/2)*x)) - log(1/(b*x) - log(c*x^
n)/((-a)^(1/2)*b^(1/2)*x)))/(2*(-a)^(1/2)*b^(1/2)*n)
```

3.252 $\int \frac{1}{ax+bx\log^3(cx^n)}\,dx$

Optimal. Leaf size=144

$$-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}\,n}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}\,n}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*ln(c*x^n))/a^(2/3)/b^(1/3)/n-1/6*ln(a^(2/3)-a^(1/3)*
b^(1/3)*ln(c*x^n)+b^(2/3)*ln(c*x^n)^2)/a^(2/3)/b^(1/3)/n-1/3*arctan(1/3*(a^
(1/3)-2*b^(1/3)*ln(c*x^n))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/n*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\log(cx^n)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,n}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}\,n}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\log\left(cx^n\right)\right)}{3a^{2/3}\sqrt[3]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Log[c*x^n])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/
3)*b^(1/3)*n)) + Log[a^(1/3) + b^(1/3)*Log[c*x^n]]/(3*a^(2/3)*b^(1/3)*n) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + b^(2/3)*Log[c*x^n]^2]/(6*a^(2/3)
*b^(1/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ax+bx\log^3(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \log(cx^n)\right)}{3a^{2/3}n} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \log(\right.}{3a^{2/3}n}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}\,n} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \log(cx^n)\right)}{2\sqrt[3]{a}\,n} - \frac{\text{S}}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}\,n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\log(cx^n)+b^{2/3}\log^2(cx^n)\right)}{6a^{2/3}\sqrt[3]{b}\,n} + \frac{\text{S}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,n} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\log(cx^n)\right)}{3a^{2/3}\sqrt[3]{b}\,n} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\text{lo}\right.}{6a^{2/}}$$

Mathematica [A]

time = 0.04, size = 112, normalized size = 0.78

$$-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\log(cx^n)}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\log\left(cx^n\right)\right)+\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\log\left(cx^n\right)+b^{2/3}\log^2\left(cx^n\right)\right)}{6a^{2/3}\sqrt[3]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Log[c*x^n])/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*Log[c*x^n]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n]
+ b^(2/3)*Log[c*x^n]^2])/(a^(2/3)*b^(1/3)*n)
```

Maple [A]
time = 0.18, size = 115, normalized size = 0.80

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(27a^2bn^3\_Z^3-1\right)} \_R\ln\left(\ln\left(x^n\right)+3an\_R-\frac{i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(icx^n)}{2}+\frac{i\pi\,\mathrm{csgn}(ic)\mathrm{csgn}(icx^n)^2}{2}\right.$
default	$\frac{\frac{\ln\left(\ln(cx^n)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\ln(cx^n)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\ln(cx^n)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(cx^n)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/3/b/(a/b)^(2/3)*ln(ln(c*x^n)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(ln(c*
x^n)^2-(a/b)^(1/3)*ln(c*x^n)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*ln(c*x^n)-1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^3 + a*x), x)
```

Fricas [A]
time = 0.42, size = 480, normalized size = 3.33

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abn^3\log(x)^3+6abn^2\log(c)\log(x)^2+6abn\log(c)^2\log(x)+2ab\log(c)^3-a^2+3\sqrt{\frac{1}{3}}\left(2abn^2\log(x)^2+4abn\log(c)\log(x)+2ab\log(c)^2+\left(a^2b\right)^{\frac{2}{3}}(n\log(x)+\log(c))-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}(an\log(x)+a\log(c))}{bn^3\log(x)^3+3bn^2\log(c)\log(x)^2+3bn\log(c)^2\log(x)+b\log(c)^3+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abn^2\log(x)^2+2abn\log(c)\log(x)+ab\log(c)^2-\left(a^2b\right)^{\frac{2}{3}}(n\log(x)+\log(c))+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abn\log(x)+ab\log(c)+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2bn}, \frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}(n\log(x)+\log(c))-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abn^2\log(x)^2+2abn\log(c)\log(x)+ab\log(c)^2-\left(a^2b\right)^{\frac{2}{3}}(n\log(x)+\log(c))+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abn\log(x)+ab\log(c)+\left(a^2b\right)^{\frac{2}{3}}\right)}{6a^2bn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*n^3*log(x)^3 + 6*a*
b*n^2*log(c)*log(x)^2 + 6*a*b*n*log(c)^2*log(x) + 2*a*b*log(c)^3 - a^2 + 3*
sqrt(1/3)*(2*a*b*n^2*log(x)^2 + 4*a*b*n*log(c)*log(x) + 2*a*b*log(c)^2 + (a
^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) -
 3*(a^2*b)^(1/3)*(a*n*log(x) + a*log(c)))/(b*n^3*log(x)^3 + 3*b*n^2*log(c)*
log(x)^2 + 3*b*n*log(c)^2*log(x) + b*log(c)^3 + a)) - (a^2*b)^(2/3)*log(a*b
*n^2*log(x)^2 + 2*a*b*n*log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log
(x) + log(c)) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*l
og(c) + (a^2*b)^(2/3)))/(a^2*b*n), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/
b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(n*log(x) + log(c)) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*n^2*log(x)^2 + 2*a*b*n*
log(c)*log(x) + a*b*log(c)^2 - (a^2*b)^(2/3)*(n*log(x) + log(c)) + (a^2*b)^
(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*n*log(x) + a*b*log(c) + (a^2*b)^(2/3)))/
(a^2*b*n)]
```

Sympy [A]
time = 40.69, size = 175, normalized size = 1.22

$$\begin{cases} \frac{\tilde{\infty}\log(x)}{\log(c)^3} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ -\frac{1}{2bn\log(cx^n)^2} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(x)}{a+b\log(c)^3} & \text{for } n = 0 \\ -\frac{\sqrt[3]{-\frac{a}{b}}\log\left(-\sqrt[3]{-\frac{a}{b}}+\log(cx^n)\right)}{3an} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4\left(-\frac{a}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{a}{b}}\log(cx^n)+4\log(cx^n)^2\right)}{6an} + \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(cx^n)}{3\sqrt[3]{-\frac{a}{b}}}\right)}{3an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(2*b*
n*log(c*x**n)**2), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)**
3), Eq(n, 0)), (-(-a/b)**(1/3)*log(-(-a/b)**(1/3) + log(c*x**n))/(3*a*n) +
(-a/b)**(1/3)*log(4*(-a/b)**(2/3) + 4*(-a/b)**(1/3)*log(c*x**n) + 4*log(c*x
**n)**2)/(6*a*n) + sqrt(3)*(-a/b)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*log(c*x
**n)/(3*(-a/b)**(1/3)))/(3*a*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(109) = 218.

time = 4.09, size = 239, normalized size = 1.66

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\pi b(\mathrm{sgn}(c)-1)-2\,bn\log(x)-2\,b\log(|c|)-2\,(ab^2)^{\frac{1}{3}}}{2\,\sqrt{3}\,bn\log(x)+\pi b(\mathrm{sgn}\,(c)-1)+2\,\sqrt{3}\,b\log(|c|)-2\,\sqrt{3}\,(ab^2)^{\frac{1}{3}}}\right)+\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}\,(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1))^2+\left(bn\log(|x|)+b\log(|c|)+(ab^2)^{\frac{1}{3}}\right)^2\right)-\frac{1}{6}\left(\frac{1}{a^2bn^3}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}\,\pi b(\mathrm{sgn}(c)-1)-2\,bn\log(x)-2\,b\log(|c|)-2\,(ab^2)^{\frac{1}{3}}\right)^2+\left(2\,\sqrt{3}\,bn\log(x)+\pi b(\mathrm{sgn}(c)-1)+2\,\sqrt{3}\,b\log(|c|)-2\,\sqrt{3}\,(ab^2)^{\frac{1}{3}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*n^3))^(1/3)*arctan((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n
*log(x) - 2*b*log(abs(c)) - 2*(a*b^2)^(1/3))/(2*sqrt(3)*b*n*log(x) + pi*b*(
sgn(c) - 1) + 2*sqrt(3)*b*log(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))) + 1/6*(1/
(a^2*b*n^3))^(1/3)*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b
*n*log(abs(x)) + b*log(abs(c)) + (a*b^2)^(1/3))^2) - 1/6*(1/(a^2*b*n^3))^(1
/3)*log((sqrt(3)*pi*b*(sgn(c) - 1) - 2*b*n*log(x) - 2*b*log(abs(c)) - 2*(a*
b^2)^(1/3))^2 + (2*sqrt(3)*b*n*log(x) + pi*b*(sgn(c) - 1) + 2*sqrt(3)*b*log
(abs(c)) - 2*sqrt(3)*(a*b^2)^(1/3))^2)
```

Mupad [B]
time = 2.28, size = 153, normalized size = 1.06

$$\frac{\ln\left(\frac{3\,a^{1/3}\,n}{b^{4/3}\,x^2}+\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}\right)}{3\,a^{2/3}\,b^{1/3}\,n}+\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}+\frac{3\,a^{1/3}\,n\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}-\frac{\ln\left(\frac{3\,n\,\ln(c\,x^n)}{b\,x^2}-\frac{3\,a^{1/3}\,n\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{4/3}\,x^2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)^3),x)
```

```
[Out] log((3*a^(1/3)*n)/(b^(4/3)*x^2) + (3*n*log(c*x^n))/(b*x^2))/(3*a^(2/3)*b^(1
/3)*n) + (log((3*n*log(c*x^n))/(b*x^2) + (3*a^(1/3)*n*(3^(1/2)*1i - 1))/(2*
b^(4/3)*x^2))*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)*n) - (log((3*n*log(c*x^n
))/(b*x^2) - (3*a^(1/3)*n*(3^(1/2)*1i + 1))/(2*b^(4/3)*x^2))*(3^(1/2)*1i +
1))/(6*a^(2/3)*b^(1/3)*n)
```

3.253 $\int \frac{1}{ax+bx\log^4(cx^n)}\,dx$

Optimal. Leaf size=227

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}$$

```
[Out] -1/4*arctan(1-b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)+
1/4*arctan(1+b^(1/4)*ln(c*x^n)*2^(1/2)/a^(1/4))/a^(3/4)/b^(1/4)/n*2^(1/2)-1
/8*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*x^n)^2*b^(1/2))/a^(3/
4)/b^(1/4)/n*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+a^(1/2)+ln(c*
x^n)^2*b^(1/2))/a^(3/4)/b^(1/4)/n*2^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(Sqrt[2]*a^(3/4)*b^(1
/4)*n) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)]/(2*Sqrt[2]*a^(3/4
)*b^(1/4)*n) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[b]*L
og[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Log[c*x^n] + Sqrt[b]*Log[c*x^n]^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{ax+bx\log^4(cx^n)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \log(cx^n)\right)}{2\sqrt{a}\,n} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, \log(cx^n)\right)}{2\sqrt{a}\,n}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{a}\sqrt{b}\,n} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\right.}{4\sqrt{a}\sqrt{b}\,n}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n} - \frac{\log\left(\sqrt{a}-\sqrt{2}\right.}{}$$

Mathematica [A]
time = 0.05, size = 167, normalized size = 0.74

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\log(cx^n)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{b}\log^2(cx^n)\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x*Log[c*x^n]^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Log[c*x^n])/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*Log[c*x^n])/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[
c*x^n] + Sqrt[b]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*n)
```

Maple [A]
time = 0.42, size = 136, normalized size = 0.60

method	result
risch	$\sum_{\_R=\text{RootOf}\left(256a^3bn^4\_Z^4+1\right)} \_R\ln\left(\ln(x^n)+4an\_R-\frac{i\pi\,\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(icx^n)}{2}+\frac{i\pi\,\text{csgn}(ic)\text{csgn}(icx^n)^2}{2}+\right.$

default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{\ln(c\,x^{n})^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} \ln(c\,x^{n}) \sqrt{2}+\sqrt{\frac{a}{b}}}{\ln(c\,x^{n})^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} \ln(c\,x^{n}) \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan\left(\frac{\sqrt{2}\, \ln(c\,x^{n})}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-2 \arctan\left(-\frac{\sqrt{2}\, \ln(c\,x^{n})}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right)}{8 n a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x*ln(c*x^n)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/n*(a/b)^(1/4)/a*2^(1/2)*(ln((ln(c*x^n)^2+(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+
(a/b)^(1/2))/(ln(c*x^n)^2-(a/b)^(1/4)*ln(c*x^n)*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*ln(c*x^n)+1)-2*arctan(-2^(1/2)/(a/b)^(1/4)*ln(c*x^n
)+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*x*log(c*x^n)^4 + a*x), x)
```

Fricas [A]
time = 0.44, size = 195, normalized size = 0.86

$$\left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{1}{4}} \arctan\left(\sqrt{a^{2} n^{2} \sqrt{-\frac{1}{a^{3} b n^{4}}}+n^{2} \log\left(x\right)^{2}+2\, n \log\left(c\right) \log\left(x\right)+\log\left(c\right)^{2}}\, a^{2} b n^{3} \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{3}{4}}-\left(a^{2} b n^{4} \log\left(x\right)+a^{2} b n^{3} \log\left(c\right)\right) \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{3}{4}}\right)+\frac{1}{4} \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{1}{4}} \log\left(a n \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{1}{4}}+n \log\left(x\right)+\log\left(c\right)\right)-\frac{1}{4} \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{1}{4}} \log\left(-a n \left(-\frac{1}{a^{3} b n^{4}}\right)^{\frac{1}{4}}+n \log\left(x\right)+\log\left(c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*n^4))^(1/4)*arctan(sqrt(a^2*n^2*sqrt(-1/(a^3*b*n^4)) + n^2*log(x
)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^2*b*n^3*(-1/(a^3*b*n^4))^(3/4) - (a^2
*b*n^4*log(x) + a^2*b*n^3*log(c))*(-1/(a^3*b*n^4))^(3/4)) + 1/4*(-1/(a^3*b*
n^4))^(1/4)*log(a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c)) - 1/4*(-1/(
a^3*b*n^4))^(1/4)*log(-a*n*(-1/(a^3*b*n^4))^(1/4) + n*log(x) + log(c))
```

Sympy [A]
time = 23.38, size = 133, normalized size = 0.59

$$\begin{cases} \frac{\tilde{\infty} \log\left(x\right)}{\log\left(c\right)^{4}} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 0 \\ - \frac{1}{3 b n \log\left(c x^{n}\right)^{3}} & \text{for}\: a = 0 \\ \frac{\log\left(x\right)}{a} & \text{for}\: b = 0 \\ \frac{\log\left(x\right)}{a + b \log\left(c\right)^{4}} & \text{for}\: n = 0 \\ - \frac{\sqrt[4]{- \frac{a}{b}} \log\left(- \sqrt[4]{- \frac{a}{b}} + \log\left(c x^{n}\right)\right)}{4 a n} + \frac{\sqrt[4]{- \frac{a}{b}} \log\left(\sqrt[4]{- \frac{a}{b}} + \log\left(c x^{n}\right)\right)}{4 a n} + \frac{\sqrt[4]{- \frac{a}{b}} \operatorname{atan}\left(\frac{\log\left(c x^{n}\right)}{\sqrt[4]{- \frac{a}{b}}}\right)}{2 a n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(x)/log(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-1/(3*b*
n*log(c*x**n)**3), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b*log(c)**
4), Eq(n, 0)), (-(-a/b)**(1/4)*log(-(-a/b)**(1/4) + log(c*x**n))/(4*a*n) +
(-a/b)**(1/4)*log((-a/b)**(1/4) + log(c*x**n))/(4*a*n) + (-a/b)**(1/4)*atan
(log(c*x**n)/(-a/b)**(1/4))/(2*a*n), True))
```

Giac [A]
time = 5.16, size = 170, normalized size = 0.75

$$-\frac{1}{2}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi b(\mathrm{sgn}(c)-1)+2\left(-ab^3\right)^{\frac{1}{4}}}{2\left(bn\log\left(x\right)+b\log\left(|c|\right)\right)}\right)+\frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1)\right)^2+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)+\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right)-\frac{1}{8}\left(-\frac{1}{a^3bn^4}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x)-1)+\pi b(\mathrm{sgn}(c)-1)\right)^2+\left(bn\log\left(|x|\right)+b\log\left(|c|\right)-\left(-ab^3\right)^{\frac{1}{4}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x*log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*n^4))^(1/4)*arctan(1/2*(pi*b*(sgn(c) - 1) + 2*(-a*b^3)^(1/4
))/(b*n*log(x) + b*log(abs(c)))) + 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b
*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) +
 (-a*b^3)^(1/4))^2) - 1/8*(-1/(a^3*b*n^4))^(1/4)*log(1/4*(pi*b*n*(sgn(x) -
1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) - (-a*b^3)^(1/
4))^2)
```

Mupad [B]
time = 2.24, size = 95, normalized size = 0.42

$$-\frac{\ln\left({\left(-a\right)}^{1/4}+b^{1/4}\,\ln\left(c\,x^n\right)\right)-\ln\left({\left(-a\right)}^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\right)+\ln\left({\left(-a\right)}^{1/4}-b^{1/4}\,\ln\left(c\,x^n\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}-\ln\left({\left(-a\right)}^{1/4}+b^{1/4}\,\ln\left(c\,x^n\right)\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{3/4}\,b^{1/4}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x*log(c*x^n)^4),x)
```

```
[Out] -(log((-a)^(1/4) + b^(1/4)*log(c*x^n)) - log((-a)^(1/4) - b^(1/4)*log(c*x^n
)) + log((-a)^(1/4) - b^(1/4)*log(c*x^n)*1i)*1i - log((-a)^(1/4) + b^(1/4)*
log(c*x^n)*1i)*1i)/(4*(-a)^(3/4)*b^(1/4)*n)
```

3.254 $\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{a}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}$$

[Out] ln(x)/a-b*ln(b+a*ln(c*x^n))/a^2/n

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{\log(x)}{a}-\frac{b\log\left(a\log\left(cx^n\right)+b\right)}{a^2n}$$

Antiderivative was successfully verified.

[In] Int[(a*x + (b*x)/Log[c*x^n])^(-1),x]

[Out] Log[x]/a - (b*Log[b + a*Log[c*x^n]])/(a^2*n)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+\frac{bx}{\log(cx^n)}}\,dx&=\frac{\text{Subst}\left(\int \frac{x}{b+ax}\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)\,dx,x,\log\left(cx^n\right)\right)}{n}\\&=\frac{\log(x)}{a}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.26

$$\frac{\log\left(cx^n\right)}{an}-\frac{b\log\left(b+a\log\left(cx^n\right)\right)}{a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n])^(-1),x]
```

```
[Out] Log[c*x^n]/(a*n) - (b*Log[b + a*Log[c*x^n]])/(a^2*n)
```

Maple [A]
time = 0.05, size = 33, normalized size = 1.22

method	result
norman	$\frac{\ln(x)}{a}-\frac{b\ln\left(a\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+b\right)}{a^2 n}$
default	$\frac{\frac{\ln(c\,x^n)}{a}-\frac{b\ln(b+a\ln(c\,x^n))}{a^2}}{n}$
risch	$\frac{\ln(x)}{a}-\frac{b\ln\left(\ln(x^n)-\frac{i\pi a\,\mathrm{csgn}(ic)\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)-i\pi a\,\mathrm{csgn}(ic)\mathrm{csgn}(ic\,x^n)^2-i\pi a\,\mathrm{csgn}(ix^n)\mathrm{csgn}(ic\,x^n)^2+i\pi a\mathrm{csgn}(ic\,x^n)^3-2\ln(c)a-2b}{2a}\right)}{a^2 n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/a*ln(c*x^n)-b/a^2*ln(b+a*ln(c*x^n)))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 1.22

$$\frac{\log(x)}{a}-\frac{b\log\left(\frac{a\log(c)+a\log(x^n)+b}{a}\right)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - b*log((a*log(c) + a*log(x^n) + b)/a)/(a^2*n)
```

Fricas [A]
time = 0.40, size = 28, normalized size = 1.04

$$\frac{an\log(x)-b\log(an\log(x)+a\log(c)+b)}{a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="fricas")
```

```
[Out] (a*n*log(x) - b*log(a*n*log(x) + a*log(c) + b))/(a^2*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(22) = 44$.

time = 2.05, size = 116, normalized size = 4.30

$$\begin{cases} \frac{\log(c)\log(x)}{b} & \text{for } a=0 \wedge n=0 \\ \frac{\begin{cases} 0 & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^2}{2n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^2}{2n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{G_{3,3}^{3,0}\left(\begin{matrix} & 1,1,1 \\ 0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{G_{3,3}^{0,3}\left(\begin{matrix} 1,1,1 & \\ & 0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a=0 \\ \frac{\log(c)\log(x)}{a\log(c)+b} & \text{for } n=0 \\ \frac{\log(cx^n)}{an} - \frac{b\log\left(\log(cx^n)+\frac{b}{a}\right)}{a^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)),x)
```

```
[Out] Piecewise((log(c)*log(x)/b, Eq(a, 0) & Eq(n, 0)), (Piecewise((0, (Abs(c*x**
n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**2/(2*n), Abs(c*x**n) < 1), (l
og(1/(c*x**n))**2/(2*n), 1/Abs(c*x**n) < 1), (meijerg(((), (1, 1, 1)), ((0,
 0, 0), ()), c*x**n)/n + meijerg(((1, 1, 1), ()), ((), (0, 0, 0)), c*x**n)/
n, True))/b, Eq(a, 0)), (log(c)*log(x)/(a*log(c) + b), Eq(n, 0)), (log(c*x*
*n)/(a*n) - b*log(log(c*x**n) + b/a)/(a**2*n), True))
```

Giac [A]

time = 4.03, size = 53, normalized size = 1.96

$$\frac{\log(x)}{a} - \frac{b\log\left(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1)+\pi a(\operatorname{sgn}(c)-1)\right)^2+\left(an\log(|x|)+a\log(|c|)+b\right)^2\right)}{2a^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/2*b*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n
*log(abs(x)) + a*log(abs(c)) + b)^2)/(a^2*n)
```

Mupad [B]

time = 0.36, size = 27, normalized size = 1.00

$$\frac{\ln(x)}{a} - \frac{b\ln\left(b+a\ln\left(c\,x^n\right)\right)}{a^2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)),x)
```

```
[Out] log(x)/a - (b*log(b + a*log(c*x^n)))/(a^2*n)
```

3.255 $\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}+\frac{\log(x)}{a}$$

[Out] ln(x)/a-arctan(ln(c*x^n)*a^(1/2)/b^(1/2))*b^(1/2)/a^(3/2)/n

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {327, 211}

$$\frac{\log(x)}{a}-\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

[In] Int[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]

[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[x]/a

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^2(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{b+ax^2}\,dx,x,\log\left(cx^n\right)\right)}{n}$$
$$= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx,x,\log\left(cx^n\right)\right)}{an}$$
$$= -\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n} + \frac{\log(x)}{a}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.18

$$-\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\,\log(cx^n)}{\sqrt{b}}\right)}{a^{3/2}n} + \frac{\log\left(cx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^2)^(-1),x]
[Out] -((Sqrt[b]*ArcTan[(Sqrt[a]*Log[c*x^n])/Sqrt[b]])/(a^(3/2)*n)) + Log[c*x^n]/
(a*n)
```

Maple [A]
time = 0.08, size = 41, normalized size = 1.02

method	result
default	$\frac{\frac{\ln(c\,x^n)}{a}-\frac{b\arctan\left(\frac{a\ln(c\,x^n)}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{n}$
risch	$\frac{\ln(x)}{a}+\frac{\sqrt{-ab}\,\ln\left(\ln(x^n)-\frac{i\pi a\,\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(ic\,x^n)-i\pi a\,\text{csgn}(ic)\text{csgn}(ic\,x^n)^2-i\pi a\,\text{csgn}(ix^n)\text{csgn}(ic\,x^n)^2+i\pi a\text{csgn}(ic\,x^n)^3-2\ln}{2a}\right.}{2a^2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^2),x,method=_RETURNVERBOSE)
[Out] 1/n*(1/a*ln(c*x^n)-b/a/(a*b)^(1/2)*arctan(a*ln(c*x^n)/(a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(2*a^2*x*log(c)*log(x^n) + a^2*x*log(x^n)^2 + (a^2*log(c)^2
+ a*b)*x), x) + log(x)/a
```

Fricas [A]
time = 0.42, size = 143, normalized size = 3.58

$$\left[\frac{2\,n\log\left(x\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{an^{2}\log(x)^{2}+2\,an\log(c)\log(x)+a\log(c)^{2}-2\,(an\log(x)+a\log(c))\sqrt{-\frac{b}{a}}-b}{an^{2}\log(x)^{2}+2\,an\log(c)\log(x)+a\log(c)^{2}+b}\right)}{2\,an},\ \frac{n\log\left(x\right)-\sqrt{\frac{b}{a}}\arctan\left(\frac{(an\log(x)+a\log(c))\sqrt{\frac{b}{a}}}{b}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*n*log(x) + sqrt(-b/a)*log((a*n^2*log(x)^2 + 2*a*n*log(c)*log(x) + a
*log(c)^2 - 2*(a*n*log(x) + a*log(c))*sqrt(-b/a) - b)/(a*n^2*log(x)^2 + 2*a
*n*log(c)*log(x) + a*log(c)^2 + b)))/(a*n), (n*log(x) - sqrt(b/a)*arctan((a
*n*log(x) + a*log(c))*sqrt(b/a)/b))/(a*n)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(34) = 68.
time = 6.14, size = 204, normalized size = 5.10

$$\begin{cases} \tilde{\infty}\log\left(c\right)^{2}\log\left(x\right) & \text{for}\ a=0\wedge b=0\wedge n=0 \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^{3}}{3n}+\frac{\log\left(cx^{n}\right)^{3}}{3n} & \text{for}\ \frac{1}{|cx^{n}|}<1\wedge|cx^{n}|<1 \\ \frac{\log\left(cx^{n}\right)^{3}}{3n} & \text{for}\ |cx^{n}|<1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^{3}}{3n} & \text{for}\ \frac{1}{|cx^{n}|}<1 \\ -\frac{2G_{4,4}^{4,0}\left(\begin{matrix} & 1,1,1,1 \\ 0,0,0,0 & \end{matrix}\middle|cx^{n}\right)}{n}+\frac{2G_{4,4}^{0,4}\left(\begin{matrix}1,1,1,1 & \\ & 0,0,0,0\end{matrix}\middle|cx^{n}\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for}\ a=0 \\ \frac{\log\left(c\right)^{2}\log\left(x\right)}{a\log\left(c\right)^{2}+b} & \text{for}\ n=0 \\ \frac{\log\left(x\right)}{a} & \text{for}\ b=0 \\ \frac{\log\left(cx^{n}\right)}{an}-\frac{b\log\left(-\sqrt{-\frac{b}{a}}+\log\left(cx^{n}\right)\right)}{2a^{2}n\sqrt{-\frac{b}{a}}}+\frac{b\log\left(\sqrt{-\frac{b}{a}}+\log\left(cx^{n}\right)\right)}{2a^{2}n\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**2),x)
```

```
[Out] Piecewise((zoo*log(c)**2*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((-log(1/(c*x**n))**3/(3*n) + log(c*x**n)**3/(3*n), (Abs(c*x**n) < 1) & (1
/Abs(c*x**n) < 1)), (log(c*x**n)**3/(3*n), Abs(c*x**n) < 1), (-log(1/(c*x**
n))**3/(3*n), 1/Abs(c*x**n) < 1), (-2*meijerg(((), (1, 1, 1, 1)), ((0, 0, 0
, 0), ()), c*x**n)/n + 2*meijerg(((1, 1, 1, 1), ()), ((), (0, 0, 0, 0)), c*
x**n)/n, True))/b, Eq(a, 0)), (log(c)**2*log(x)/(a*log(c)**2 + b), Eq(n, 0)
), (log(x)/a, Eq(b, 0)), (log(c*x**n)/(a*n) - b*log(-sqrt(-b/a) + log(c*x**
n))/(2*a**2*n*sqrt(-b/a)) + b*log(sqrt(-b/a) + log(c*x**n))/(2*a**2*n*sqrt(
-b/a)), True))
```

Giac [A]
time = 3.80, size = 38, normalized size = 0.95

$$\frac{\log\left(x\right)}{a} - \frac{b\arctan\left(\frac{an\log\left(x\right)+a\log\left(c\right)}{\sqrt{ab}}\right)}{\sqrt{ab}\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^2),x, algorithm="giac")
```

```
[Out] log(x)/a - b*arctan((a*n*log(x) + a*log(c))/sqrt(a*b))/(sqrt(a*b)*a*n)
```

Mupad [B]
time = 0.38, size = 45, normalized size = 1.12

$$\frac{\ln\left(x\right)}{a} - \frac{\sqrt{b}\,\operatorname{atan}\left(\frac{a^2\,n\,\ln\left(c\,x^n\right)}{\sqrt{b}\,\sqrt{a^3\,n^2}}\right)}{\sqrt{a^3\,n^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^2),x)
```

```
[Out] log(x)/a - (b^(1/2)*atan((a^2*n*log(c*x^n))/(b^(1/2)*(a^3*n^2)^(1/2))))/(a^
3*n^2)^(1/2)
```

3.256 $\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx$

Optimal. Leaf size=149

$$\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{4/3}n}+\frac{\log(x)}{a}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}+\sqrt[3]{a}\log\left(cx^n\right)\right)}{3a^{4/3}n}+\frac{\sqrt[3]{b}\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)\right.}{6a^{4/3}n}$$

```
[Out] ln(x)/a-1/3*b^(1/3)*ln(b^(1/3)+a^(1/3)*ln(c*x^n))/a^(4/3)/n+1/6*b^(1/3)*ln(
b^(2/3)-a^(1/3)*b^(1/3)*ln(c*x^n)+a^(2/3)*ln(c*x^n)^2)/a^(4/3)/n+1/3*b^(1/3
)*arctan(1/3*(b^(1/3)-2*a^(1/3)*ln(c*x^n))/b^(1/3)*3^(1/2))/a^(4/3)/n*3^(1/
2)
```

Rubi [A]

time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {327, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}\log(cx^n)}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,a^{4/3}n}+\frac{\sqrt[3]{b}\log\left(a^{2/3}\log^2\left(cx^n\right)-\sqrt[3]{a}\sqrt[3]{b}\log\left(cx^n\right)+b^{2/3}\right)}{6a^{4/3}n}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}\log\left(cx^n\right)+\sqrt[3]{b}\right)}{3a^{4/3}n}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (b^(1/3)*ArcTan[(b^(1/3) - 2*a^(1/3)*Log[c*x^n])/(Sqrt[3]*b^(1/3))])/(Sqrt[
3]*a^(4/3)*n) + Log[x]/a - (b^(1/3)*Log[b^(1/3) + a^(1/3)*Log[c*x^n]])/(3*a
^(4/3)*n) + (b^(1/3)*Log[b^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log
[c*x^n]^2])/(6*a^(4/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^3(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b+ax^3}\,dx,x,\log(cx^n)\right)}{n}$$
$$= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^3}\,dx,x,\log(cx^n)\right)}{an}$$
$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}\,x}\,dx,x,\log(cx^n)\right)}{3an} - \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}\,x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\right.}{3an}$$
$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+a^{2/3}x^2}\,dx,x,\log\right.}{6a^{4/3}n}$$
$$= \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\,\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\log(cx^n)+a^{2/3}\log\right.}{6a^{4/3}n}$$
$$= \frac{\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}n} + \frac{\log(x)}{a} - \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\log(cx^n)\right)}{3a^{4/3}n} + \frac{\sqrt[3]{b}\,\log\left(b\right.}{}$$

Mathematica [A]

time = 0.04, size = 132, normalized size = 0.89

$$\frac{2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}\,\log(cx^n)}{\sqrt[3]{b}}}{\sqrt{3}}\right)+6\sqrt[3]{a}\,\log(cx^n)+\sqrt[3]{b}\left(-2\log\left(\sqrt[3]{b}+\sqrt[3]{a}\,\log(cx^n)\right)+\log\left(b^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\log(cx^n)+a^{2/3}\log^2(cx^n)\right)\right)}{6a^{4/3}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*a^(1/3)*Log[c*x^n])/b^(1/3))/Sqrt[3]] + 6
*a^(1/3)*Log[c*x^n] + b^(1/3)*(-2*Log[b^(1/3) + a^(1/3)*Log[c*x^n]] + Log[b
^(2/3) - a^(1/3)*b^(1/3)*Log[c*x^n] + a^(2/3)*Log[c*x^n]^2]))/(6*a^(4/3)*n)
```

Maple [A]

time = 0.18, size = 132, normalized size = 0.89

method	result
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\text{RootOf}(27n^3a^4\_Z^3+b)}\_R\ln\left(\ln(x^n)-3an\_R-\frac{i\pi\,\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(icx^n)}{2}+\frac{i\pi\,\text{csgn}(ic)\text{csgn}(\right.\right.$

default	$\frac{\frac{\ln(c\,x^{n})}{a}-\frac{\left(\frac{\ln\left(\ln(c\,x^{n})+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}-\frac{\ln\left(\ln(c\,x^{n})^{2}-\left(\frac{b}{a}\right)^{\frac{1}{3}}\ln(c\,x^{n})+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{b}{a}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\ln(c\,x^{n})}{\left(\frac{b}{a}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{b}{a}\right)^{\frac{2}{3}}}\right)b}{a}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/a*ln(c*x^n)-(1/3/a/(b/a)^(2/3)*ln(ln(c*x^n)+(b/a)^(1/3))-1/6/a/(b/a)
^(2/3)*ln(ln(c*x^n)^2-(b/a)^(1/3)*ln(c*x^n)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*ln(c*x^n)-1)))*b/a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(3*a^2*x*log(c)^2*log(x^n) + 3*a^2*x*log(c)*log(x^n)^2 + a^2
*x*log(x^n)^3 + (a^2*log(c)^3 + a*b)*x), x) + log(x)/a
```

Fricas [A]
time = 0.39, size = 149, normalized size = 1.00

$$\frac{6\,n\log\left(x\right)+2\,\sqrt{3}\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2\left(\sqrt{3}\,an\log(x)+\sqrt{3}\,a\log(c)\right)\left(-\frac{b}{a}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(n^{2}\log\left(x\right)^{2}+2\,n\log\left(c\right)\log\left(x\right)+\log\left(c\right)^{2}+\left(n\log\left(x\right)+\log\left(c\right)\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}\right)+2\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(n\log\left(x\right)-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\log\left(c\right)\right)}{6\,an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="fricas")
```

```
[Out] 1/6*(6*n*log(x) + 2*sqrt(3)*(-b/a)^(1/3)*arctan(1/3*(2*(sqrt(3)*a*n*log(x)
+ sqrt(3)*a*log(c))*(-b/a)^(2/3) - sqrt(3)*b)/b) - (-b/a)^(1/3)*log(n^2*log
(x)^2 + 2*n*log(c)*log(x) + log(c)^2 + (n*log(x) + log(c))*(-b/a)^(1/3) + (
-b/a)^(2/3)) + 2*(-b/a)^(1/3)*log(n*log(x) - (-b/a)^(1/3) + log(c)))/(a*n)
```

Sympy [A]

time = 44.49, size = 245, normalized size = 1.64

$$\begin{cases} \tilde{\infty}\log(c)^3\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ \frac{\begin{cases} 0 & \text{for } \frac{1}{|cx^n|}<1 \wedge |cx^n|<1 \\ \frac{\log(cx^n)^4}{4n} & \text{for } |cx^n|<1 \\ \frac{\log\left(\frac{x^{-n}}{c}\right)^4}{4n} & \text{for } \frac{1}{|cx^n|}<1 \\ \frac{6G_{5,5}^{5,0}\left(\begin{matrix} & 1,1,1,1,1 \\ 0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{6G_{5,5}^{0,5}\left(\begin{matrix} 1,1,1,1,1 & \\ & 0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a=0 \\ \frac{\log(c)^3\log(x)}{a\log(c)^3+b} & \text{for } n=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\sqrt[3]{-\frac{b}{a}}\log\left(-\sqrt[3]{-\frac{b}{a}}+\log(cx^n)\right)}{3an} - \frac{\sqrt[3]{-\frac{b}{a}}\log\left(4\left(-\frac{b}{a}\right)^{\frac{2}{3}}+4\sqrt[3]{-\frac{b}{a}}\log(cx^n)+4\log(cx^n)^2\right)}{6an} - \frac{\sqrt{3}\sqrt[3]{-\frac{b}{a}}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\log(cx^n)}{3\sqrt[3]{-\frac{b}{a}}}\right)}{3an} + \frac{\log(cx^n)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**3),x)
```

```
[Out] Piecewise((zoo*log(c)**3*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((0, (Abs(c*x**n) < 1) & (1/Abs(c*x**n) < 1)), (log(c*x**n)**4/(4*n), Abs(
c*x**n) < 1), (log(1/(c*x**n))**4/(4*n), 1/Abs(c*x**n) < 1), (6*meijerg(((()
, (1, 1, 1, 1, 1)), ((0, 0, 0, 0, 0), ()), c*x**n)/n + 6*meijerg(((1, 1, 1,
 1, 1), ()), ((), (0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(c)
**3*log(x)/(a*log(c)**3 + b), Eq(n, 0)), (log(x)/a, Eq(b, 0)), ((-b/a)**(1/
3)*log(-(-b/a)**(1/3) + log(c*x**n))/(3*a*n) - (-b/a)**(1/3)*log(4*(-b/a)**
(2/3) + 4*(-b/a)**(1/3)*log(c*x**n) + 4*log(c*x**n)**2)/(6*a*n) - sqrt(3)*(
-b/a)**(1/3)*atan(sqrt(3)/3 + 2*sqrt(3)*log(c*x**n)/(3*(-b/a)**(1/3)))/(3*a
*n) + log(c*x**n)/(a*n), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(115) = 230.
time = 4.89, size = 257, normalized size = 1.72

$$\frac{\log(x)}{a} + \frac{2\sqrt{3}\left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\pi a(\mathrm{sgn}(c)-1)-2an\log(x)-2a\log(|c|)+2(-a^2b)^{\frac{1}{3}}}{2\sqrt{3}an\log(x)+\pi a(\mathrm{sgn}(c)-1)+2\sqrt{3}a\log(|c|)+2\sqrt{3}(-a^2b)^{\frac{1}{3}}}\right) + \left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\log\left(\frac{1}{4}(\pi an(\mathrm{sgn}(x)-1)+\pi a(\mathrm{sgn}(c)-1))^2+\left(an\log(|x|)+a\log(|c|)-(-a^2b)^{\frac{1}{3}}\right)^2\right) - \left(-\frac{bn^6}{a}\right)^{\frac{1}{3}}\log\left(\left(\sqrt{3}\pi a(\mathrm{sgn}(c)-1)-2an\log(x)-2a\log(|c|)+2(-a^2b)^{\frac{1}{3}}\right)^2+\left(2\sqrt{3}an\log(x)+\pi a(\mathrm{sgn}(c)-1)+2\sqrt{3}a\log(|c|)+2\sqrt{3}(-a^2b)^{\frac{1}{3}}\right)^2\right)}{6an^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^3),x, algorithm="giac")
```

```
[Out] log(x)/a + 1/6*(2*sqrt(3)*(-b*n^6/a)^(1/3)*arctan((sqrt(3)*pi*a*(sgn(c) - 1
) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^(1/3))/(2*sqrt(3)*a*n*log(x
) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(c)) + 2*sqrt(3)*(-a^2*b)^(1/3))
) + (-b*n^6/a)^(1/3)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 +
(a*n*log(abs(x)) + a*log(abs(c)) - (-a^2*b)^(1/3))^2) - (-b*n^6/a)^(1/3)*lo
g((sqrt(3)*pi*a*(sgn(c) - 1) - 2*a*n*log(x) - 2*a*log(abs(c)) + 2*(-a^2*b)^
(1/3))^2 + (2*sqrt(3)*a*n*log(x) + pi*a*(sgn(c) - 1) + 2*sqrt(3)*a*log(abs(
c)) + 2*sqrt(3)*(-a^2*b)^(1/3))^2))/(a*n^3)
```

Mupad [B]
time = 2.40, size = 174, normalized size = 1.17

$$\frac{\ln(x)}{a}+\frac{(-b)^{1/3}\,\ln\left(\frac{3\,(-b)^{4/3}\,n}{a^{7/3}\,x^2}+\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}\right)}{3\,a^{4/3}\,n}+\frac{(-b)^{1/3}\,\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}+\frac{3\,(-b)^{4/3}\,n\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n}-\frac{(-b)^{1/3}\,\ln\left(\frac{3\,b\,n\,\ln(c\,x^n)}{a^2\,x^2}-\frac{3\,(-b)^{4/3}\,n\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{7/3}\,x^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^3),x)
```

```
[Out] log(x)/a + ((-b)^(1/3)*log((3*(-b)^(4/3)*n)/(a^(7/3)*x^2) + (3*b*n*log(c*x^
n))/(a^2*x^2)))/(3*a^(4/3)*n) + ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2
) + (3*(-b)^(4/3)*n*((3^(1/2)*1i)/2 - 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2
- 1/2))/(3*a^(4/3)*n) - ((-b)^(1/3)*log((3*b*n*log(c*x^n))/(a^2*x^2) - (3*(
-b)^(4/3)*n*((3^(1/2)*1i)/2 + 1/2))/(a^(7/3)*x^2))*((3^(1/2)*1i)/2 + 1/2))/
(3*a^(4/3)*n)
```

3.257 $\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx$

Optimal. Leaf size=233

$$\frac{\sqrt[4]{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}n}-\frac{\sqrt[4]{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a}\,\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}n}+\frac{\log(x)}{a}+\frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\ \cdots\right.}{4\sqrt{2}\,a\cdots}$$

```
[Out] ln(x)/a-1/4*b^(1/4)*arctan(-1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*
2^(1/2)-1/4*b^(1/4)*arctan(1+a^(1/4)*ln(c*x^n)*2^(1/2)/b^(1/4))/a^(5/4)/n*2
^(1/2)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/2)+ln(c*x^n)^2*a^(1/2
)+b^(1/2))/a^(5/4)/n*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*ln(c*x^n)*2^(1/
2)+ln(c*x^n)^2*a^(1/2)+b^(1/2))/a^(5/4)/n*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {327, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a}\,\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}n}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{a}\,\log(cx^n)}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\,a^{5/4}n}+\frac{\sqrt[4]{b}\,\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\log(cx^n)+\sqrt{a}\,\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}\,a^{5/4}n}-\frac{\sqrt[4]{b}\,\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,\log(cx^n)+\sqrt{a}\,\log^2(cx^n)+\sqrt{b}\right)}{4\sqrt{2}\,a^{5/4}n}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[2]*a^(5/
4)*n) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)])/(2*Sqrt[
2]*a^(5/4)*n) + Log[x]/a + (b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*L
og[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[2]*a^(5/4)*n) - (b^(1/4)*Log[Sqr
t[b] + Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(4*Sqrt[
2]*a^(5/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{ax+\frac{bx}{\log^4(cx^n)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{b+ax^4}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\log(x)}{a} - \frac{b\text{Subst}\left(\int \frac{1}{b+ax^4}\,dx, x, \log(cx^n)\right)}{an}$$

$$= \frac{\log(x)}{a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{a}\,x^2}{b+ax^4}\,dx, x, \log(cx^n)\right)}{2an} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{a}\,x^2}{b+ax^4}\,dx, x,\right.}{2an}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}+2x}{-\frac{\sqrt{b}}{\sqrt{a}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}-x^2}\,dx, x, \log(cx^n)\right)}{4\sqrt{2}\,a^{5/4}n} + \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{\sqrt{2}}{-\frac{\sqrt{b}}{\sqrt{a}}+}\right.}{4}$$

$$= \frac{\log(x)}{a} + \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{4\sqrt{2}\,a^{5/4}n} - \frac{\sqrt[4]{b}\,\log\left(\sqrt{b}+\right.}{}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}n} - \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)}{2\sqrt{2}\,a^{5/4}n} + \frac{\log(x)}{a} + \frac{\sqrt[4]{b}}{}$$

Mathematica [A]

time = 0.05, size = 211, normalized size = 0.91

$$\frac{2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a}\log(cx^n)}{\sqrt[4]{b}}\right)+8\sqrt[4]{a}\log(cx^n)+\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\log(cx^n)+\sqrt{a}\log^2(cx^n)\right)}{8a^{5/4}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + (b*x)/Log[c*x^n]^4)^(-1),x]
```

```
[Out] (2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] - 2*Sqr
t[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*a^(1/4)*Log[c*x^n])/b^(1/4)] + 8*a^(1/4)*L
og[c*x^n] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*a^(1/4)*b^(1/4)*Log[c*x^n
] + Sqrt[a]*Log[c*x^n]^2] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*a^(1/4)*b
^(1/4)*Log[c*x^n] + Sqrt[a]*Log[c*x^n]^2])/(8*a^(5/4)*n)
```

Maple [A]

time = 0.42, size = 148, normalized size = 0.64

method	result
risch	$\frac{\ln(x)}{a} + \left(\sum_{\_R=\text{RootOf}\left(256n^4a^5\_Z^4+b\right)} \_R\ln\left(\ln(x^n) - 4an\_R - \frac{i\pi\,\text{csgn}(ic)\text{csgn}(ix^n)\text{csgn}(icx^n)}{2} + \frac{i\pi\,\text{csgn}(ic)\text{csgn}}{2}\right.\right.$

default	$\frac{\frac{\ln(c\,x^n)}{a}-\frac{\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\ln(c\,x^n)^2+\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{b}{a}}}{\ln(c\,x^n)^2-\left(\frac{b}{a}\right)^{\frac{1}{4}}\ln(c\,x^n)\sqrt{2}+\sqrt{\frac{b}{a}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\ln(c\,x^n)}{\left(\frac{b}{a}\right)^{\frac{1}{4}}}+1\right)\right)}{8a}}{n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x/ln(c*x^n)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(1/a*ln(c*x^n)-1/8/a*(b/a)^(1/4)*2^(1/2)*(ln((ln(c*x^n)^2+(b/a)^(1/4)*l
n(c*x^n)*2^(1/2)+(b/a)^(1/2))/(ln(c*x^n)^2-(b/a)^(1/4)*ln(c*x^n)*2^(1/2)+(b
/a)^(1/2)))+2*arctan(2^(1/2)/(b/a)^(1/4)*ln(c*x^n)+1)-2*arctan(-2^(1/2)/(b/
a)^(1/4)*ln(c*x^n)+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="maxima")
```

```
[Out] -b*integrate(1/(4*a^2*x*log(c)^3*log(x^n) + 6*a^2*x*log(c)^2*log(x^n)^2 + 4
*a^2*x*log(c)*log(x^n)^3 + a^2*x*log(x^n)^4 + (a^2*log(c)^4 + a*b)*x), x) +
 log(x)/a
```

Fricas [A]
time = 0.43, size = 192, normalized size = 0.82

$$-\frac{4\,a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{a^2n^2\sqrt{-\frac{b}{a^5n^4}}+n^2\log\left(x\right)^2+2\,n\log\left(c\right)\log\left(x\right)+\log\left(c\right)^2}\,a^4n^3\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}-\left(a^4n^4\log(x)+a^4n^3\log(c)\right)\left(-\frac{b}{a^5n^4}\right)^{\frac{3}{4}}}{b}\right)+a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log\left(x\right)+\log\left(c\right)\right)-a\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}\log\left(-an\left(-\frac{b}{a^5n^4}\right)^{\frac{1}{4}}+n\log\left(x\right)+\log\left(c\right)\right)-4\,\log\left(x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*(-b/(a^5*n^4))^(1/4)*arctan((sqrt(a^2*n^2*sqrt(-b/(a^5*n^4)) + n^
2*log(x)^2 + 2*n*log(c)*log(x) + log(c)^2)*a^4*n^3*(-b/(a^5*n^4))^(3/4) - (
a^4*n^4*log(x) + a^4*n^3*log(c))*(-b/(a^5*n^4))^(3/4))/b) + a*(-b/(a^5*n^4)
)^(1/4)*log(a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - a*(-b/(a^5*n^4)
)^(1/4)*log(-a*n*(-b/(a^5*n^4))^(1/4) + n*log(x) + log(c)) - 4*log(x))/a
```

Sympy [A]

time = 25.52, size = 228, normalized size = 0.98

$$\begin{cases} \tilde{\infty}\log(c)^4\log(x) & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\begin{cases} -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n} + \frac{\log(cx^n)^5}{5n} & \text{for } \frac{1}{|cx^n|} < 1 \wedge |cx^n| < 1 \\ \frac{\log(cx^n)^5}{5n} & \text{for } |cx^n| < 1 \\ -\frac{\log\left(\frac{x^{-n}}{c}\right)^5}{5n} & \text{for } \frac{1}{|cx^n|} < 1 \\ -\frac{24G_{6,6}^{6,0}\left(\begin{matrix} & 1,1,1,1,1,1 \\ 0,0,0,0,0,0 & \end{matrix}\middle| cx^n\right)}{n} + \frac{24G_{6,6}^{0,6}\left(\begin{matrix} 1,1,1,1,1,1 & \\ & 0,0,0,0,0,0 \end{matrix}\middle| cx^n\right)}{n} & \text{otherwise} \end{cases}}{b} & \text{for } a = 0 \\ \frac{\log(x)}{a} & \text{for } b = 0 \\ \frac{\log(c)^4\log(x)}{a\log(c)^4+b} & \text{for } n = 0 \\ \frac{\sqrt[4]{-\frac{b}{a}}\log\left(-\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an} - \frac{\sqrt[4]{-\frac{b}{a}}\log\left(\sqrt[4]{-\frac{b}{a}}+\log(cx^n)\right)}{4an} - \frac{\sqrt[4]{-\frac{b}{a}}\operatorname{atan}\left(\frac{\log(cx^n)}{\sqrt[4]{-\frac{b}{a}}}\right)}{2an} + \frac{\log(cx^n)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/ln(c*x**n)**4),x)
```

```
[Out] Piecewise((zoo*log(c)**4*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (Piecewis
e((-log(1/(c*x**n))**5/(5*n) + log(c*x**n)**5/(5*n), (Abs(c*x**n) < 1) & (1
/Abs(c*x**n) < 1)), (log(c*x**n)**5/(5*n), Abs(c*x**n) < 1), (-log(1/(c*x**
n))**5/(5*n), 1/Abs(c*x**n) < 1), (-24*meijerg(((), (1, 1, 1, 1, 1, 1)), ((
0, 0, 0, 0, 0, 0), ()), c*x**n)/n + 24*meijerg(((1, 1, 1, 1, 1, 1), ()), ((
), (0, 0, 0, 0, 0, 0)), c*x**n)/n, True))/b, Eq(a, 0)), (log(x)/a, Eq(b, 0)
), (log(c)**4*log(x)/(a*log(c)**4 + b), Eq(n, 0)), ((-b/a)**(1/4)*log(-(-b/
a)**(1/4) + log(c*x**n))/(4*a*n) - (-b/a)**(1/4)*log((-b/a)**(1/4) + log(c*
x**n))/(4*a*n) - (-b/a)**(1/4)*atan(log(c*x**n)/(-b/a)**(1/4))/(2*a*n) + lo
g(c*x**n)/(a*n), True))
```

Giac [A]
time = 3.98, size = 178, normalized size = 0.76

$$\frac{\log(x)}{a} - \frac{4\left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\arctan\left(\frac{\pi a(\operatorname{sgn}(c)-1)-2\left(-a^3b\right)^{\frac{1}{4}}}{2\left(an\log(x)+a\log(|c|)\right)}\right) + \left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1)+\pi a(\operatorname{sgn}(c)-1)\right)^2 + \left(an\log(|x|)+a\log(|c|)+\left(-a^3b\right)^{\frac{1}{4}}\right)^2\right) - \left(-\frac{bn^{12}}{a}\right)^{\frac{1}{4}}\log\left(\frac{1}{4}\left(\pi an(\operatorname{sgn}(x)-1)+\pi a(\operatorname{sgn}(c)-1)\right)^2 + \left(an\log(|x|)+a\log(|c|)-\left(-a^3b\right)^{\frac{1}{4}}\right)^2\right)}{8\,an^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x/log(c*x^n)^4),x, algorithm="giac")
```

```
[Out] log(x)/a - 1/8*(4*(-b*n^12/a)^(1/4)*arctan(1/2*(pi*a*(sgn(c) - 1) - 2*(-a^3
*b)^(1/4))/(a*n*log(x) + a*log(abs(c)))) + (-b*n^12/a)^(1/4)*log(1/4*(pi*a*
n*(sgn(x) - 1) + pi*a*(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) +
(-a^3*b)^(1/4))^2) - (-b*n^12/a)^(1/4)*log(1/4*(pi*a*n*(sgn(x) - 1) + pi*a*
(sgn(c) - 1))^2 + (a*n*log(abs(x)) + a*log(abs(c)) - (-a^3*b)^(1/4))^2))/(a
*n^4)
```

Mupad [B]
time = 2.21, size = 176, normalized size = 0.76

$$\frac{\ln(x)}{a}+\frac{(-b)^{1/4}\left(\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}-\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}-\ln\left(-\frac{(-b)^{5/2}}{a^{11/2}\,x^3}+\frac{(-b)^{9/4}\,\ln(c\,x^n)\,1\mathrm{i}}{a^{21/4}\,x^3}\right)\,1\mathrm{i}\right)}{4\,a^{5/4}\,n}-\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}+a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}+\frac{(-b)^{1/4}\,\ln\left(\frac{(-b)^{5/2}-a^{1/4}\,(-b)^{9/4}\,\ln(c\,x^n)}{x^3}\right)}{4\,a^{5/4}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + (b*x)/log(c*x^n)^4),x)
```

```
[Out] log(x)/a + ((-b)^(1/4)*(log(- (-b)^(5/2)/(a^(11/2)*x^3) - ((-b)^(9/4)*log(c
*x^n)*1i)/(a^(21/4)*x^3))*1i - log(((-b)^(9/4)*log(c*x^n)*1i)/(a^(21/4)*x^3
) - (-b)^(5/2)/(a^(11/2)*x^3))*1i))/(4*a^(5/4)*n) - ((-b)^(1/4)*log(((-b)^(
5/2) + a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n) + ((-b)^(1/4)*log
(((-b)^(5/2) - a^(1/4)*(-b)^(9/4)*log(c*x^n))/x^3))/(4*a^(5/4)*n)
```

3.258 $\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx$

Optimal. Leaf size=22

$$\frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2\log(7x)+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
```

```
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+x\log(7x)+x\log^2(7x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \log(7x)\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\log(7x)\right)\right)\\
&= \frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{1+2\log(7x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x*Log[7*x] + x*Log[7*x]^2)^(-1),x]
[Out] (2*ArcTan[(1 + 2*Log[7*x])/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.91

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
default	$\frac{2\arctan\left(\frac{(1+2\ln(7x))\sqrt{3}}{3}\right)\sqrt{3}}{3}$	20
risch	$\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{3}-\frac{i\sqrt{3}\,\ln\left(\ln(7x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x*ln(7*x)+x*ln(7*x)^2),x,method=_RETURNVERBOSE)
[Out] 2/3*arctan(1/3*(1+2*ln(7*x))*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="maxima")
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Fricas [A]
time = 0.43, size = 21, normalized size = 0.95

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(7x\right)+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*arctan(2/3*sqrt(3)*log(7*x) + 1/3*sqrt(3))
```

Sympy [A]
time = 0.07, size = 22, normalized size = 1.00

$$\operatorname{RootSum}\left(3z^2+1,\left(i\mapsto i\log\left(\frac{3i}{2}+\log\left(7x\right)+\frac{1}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*ln(7*x)+x*ln(7*x)**2),x)
[Out] RootSum(3*_z**2 + 1, Lambda(_i, _i*log(3*_i/2 + log(7*x) + 1/2)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x*log(7*x)+x*log(7*x)^2),x, algorithm="giac")
[Out] integrate(1/(x*log(7*x)^2 + x*log(7*x) + x), x)
```

Mupad [B]
time = 0.43, size = 19, normalized size = 0.86

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln\left(7\,x\right)+1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x*log(7*x) + x*log(7*x)^2),x)
[Out] (2*3^(1/2)*atan((3^(1/2)*(2*log(7*x) + 1))/3))/3
```

3.259 $\int \frac{-1+\log(3x)}{x(1-\log(3x)+\log^2(3x))}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\log(3x)}{x\left(1-\log(3x)+\log^2(3x)\right)}\,dx &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx\right.\\ &= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x\right.\\ &= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.02

$$-\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x])/(x*(1 - Log[3*x] + Log[3*x]^2)),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.93

method	result
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{}}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Maxima [A]
time = 0.56, size = 37, normalized size = 0.90

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\log\left(3x\right)-1\right)\right)+\frac{1}{2}\log\left(\log\left(3x\right)^{2}-\log\left(3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*log(3*x) - 1)) + 1/2*log(log(3*x)^2 - lo
g(3*x) + 1)
```

Fricas [A]
time = 0.36, size = 39, normalized size = 0.95

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3x\right)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log\left(3x\right)^{2}-\log\left(3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A]
time = 0.18, size = 22, normalized size = 0.54

$$\operatorname{RootSum}\left(3z^{2}-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x))/x/(1-ln(3*x)+ln(3*x)**2),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x))/x/(1-log(3*x)+log(3*x)^2),x, algorithm="giac")
[Out] integrate((log(3*x) - 1)/((log(3*x)^2 - log(3*x) + 1)*x), x)
```

Mupad [B]
time = 0.57, size = 37, normalized size = 0.90

$$\frac{\ln\left(\ln\left(3\,x\right)^2-\ln\left(3\,x\right)+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln\left(3\,x\right)-1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x) - 1)/(x*(log(3*x)^2 - log(3*x) + 1)),x)
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

3.260 $\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)+1/3*arctan(1/3*(1-2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(\log^2(3x)-\log(3x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
```

```
[Out] ArcTan[(1 - 2*Log[3*x])/Sqrt[3]]/Sqrt[3] + Log[1 - Log[3*x] + Log[3*x]^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\log^2(3x)}{x+x\log^3(3x)}\,dx &= \text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx, x, \log(3x)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \log(3x)\right)\right)+\frac{1}{2}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \log(3x)\right)\\ &= \frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\log(3x)\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.02

$$-\frac{\tan^{-1}\left(\frac{-1+2\log(3x)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-\log(3x)+\log^2(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x]^3),x]
[Out] -(ArcTan[(-1 + 2*Log[3*x])/Sqrt[3]]/Sqrt[3]) + Log[1 - Log[3*x] + Log[3*x]^
2]/2
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.93

method	result
derivativedivides	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$
default	$\frac{\ln\left(1-\ln(3x)+\ln(3x)^2\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(-1+2\ln(3x))\sqrt{3}}{3}\right)}{3}$
risch	$\frac{\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{}}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(1-ln(3*x)+ln(3*x)^2)-1/3*3^(1/2)*arctan(1/3*(-1+2*ln(3*x))*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="maxima")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.95

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log\left(3x\right)-\frac{1}{3}\sqrt{3}\right)+\frac{1}{2}\log\left(\log\left(3x\right)^{2}-\log\left(3x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) - 1/3*sqrt(3)) + 1/2*log(log(3*x)^
2 - log(3*x) + 1)
```

Sympy [A]
time = 0.20, size = 22, normalized size = 0.54

$$\operatorname{RootSum}\left(3z^{2}-3z+1,\left(i\mapsto i\log\left(-3i+\log\left(3x\right)+1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)**3),x)
```

```
[Out] RootSum(3*_z**2 - 3*_z + 1, Lambda(_i, _i*log(-3*_i + log(3*x) + 1)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)^3),x, algorithm="giac")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^3 + x), x)
```

Mupad [B]
time = 0.35, size = 37, normalized size = 0.90

$$\frac{\ln\left(\ln\left(3\,x\right)^2-\ln\left(3\,x\right)+1\right)}{2}-\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}\ \left(2\,\ln\left(3\,x\right)-1\right)}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x)^3),x)
[Out] log(log(3*x)^2 - log(3*x) + 1)/2 - (3^(1/2)*atan((3^(1/2)*(2*log(3*x) - 1))
/3))/3
```

3.261 $\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx$

Optimal. Leaf size=42

$$-\sqrt{3}\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

```
[Out] ln(x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1671, 648, 632, 210, 642}

$$-\sqrt{3}\,\text{ArcTan}\left(\frac{2\log(3x)+1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\log^2(3x)+\log(3x)+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[x] - Log[1 + Log[3*x] + L
og[3*x]^2]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\log^2(3x)}{x+x\log(3x)+x\log^2(3x)}\,dx &= \text{Subst}\left(\int \frac{-1+x^2}{1+x+x^2}\,dx, x, \log(3x)\right)\\ &= \text{Subst}\left(\int \left(1-\frac{2+x}{1+x+x^2}\right)\,dx, x, \log(3x)\right)\\ &= \log(x)-\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, \log(3x)\right)\\ &= \log(x)-\frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \log(3x)\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\right.\\ &= \log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)+3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.\\ &= -\sqrt{3}\,\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.05

$$-\sqrt{3}\,\tan^{-1}\left(\frac{1+2\log(3x)}{\sqrt{3}}\right)+\log(3x)-\frac{1}{2}\log\left(1+\log(3x)+\log^2(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Log[3*x]^2)/(x + x*Log[3*x] + x*Log[3*x]^2),x]
[Out] -(Sqrt[3]*ArcTan[(1 + 2*Log[3*x])/Sqrt[3]]) + Log[3*x] - Log[1 + Log[3*x] +
 Log[3*x]^2]/2
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.95

method	result
derivativedivides	$\ln(3x)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^2\right)}{2}-\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)\sqrt{3}$

default	$\ln(3x)-\frac{\ln\left(1+\ln(3x)+\ln(3x)^2\right)}{2}-\arctan\left(\frac{(1+2\ln(3x))\sqrt{3}}{3}\right)\sqrt{3}$
risch	$\ln(x)-\frac{\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{2}+\frac{i\ln\left(\ln(3x)+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{2}-\frac{\ln\left(\ln(3x)+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{2}-\frac{i\ln\left(\ln(3x)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+ln(3*x)^2)/(x+x*ln(3*x)+x*ln(3*x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(3*x)-1/2*ln(1+ln(3*x)+ln(3*x)^2)-arctan(1/3*(1+2*ln(3*x))*3^(1/2))*3^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="maxima"
)
```

```
[Out] -integrate((log(3) + log(x) + 2)/(x*(2*log(3) + 1)*log(x) + x*log(x)^2 + (l
og(3)^2 + log(3) + 1)*x), x) + log(x)
```

Fricas [A]
time = 0.39, size = 41, normalized size = 0.98

$$-\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\log(3x)+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\log\left(\log(3x)^2+\log(3x)+1\right)+\log(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="fricas"
)
```

```
[Out] -sqrt(3)*arctan(2/3*sqrt(3)*log(3*x) + 1/3*sqrt(3)) - 1/2*log(log(3*x)^2 +
log(3*x) + 1) + log(3*x)
```

Sympy [A]
time = 0.17, size = 19, normalized size = 0.45

$$\log(x)+\operatorname{RootSum}\left(z^2+z+1,\left(i\mapsto i\log(-i+\log(3x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+ln(3*x)**2)/(x+x*ln(3*x)+x*ln(3*x)**2),x)
```

```
[Out] log(x) + RootSum(_z**2 + _z + 1, Lambda(_i, _i*log(-_i + log(3*x))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+log(3*x)^2)/(x+x*log(3*x)+x*log(3*x)^2),x, algorithm="giac")
```

```
[Out] integrate((log(3*x)^2 - 1)/(x*log(3*x)^2 + x*log(3*x) + x), x)
```

Mupad [B]
time = 0.60, size = 37, normalized size = 0.88

$$\ln(x) - \frac{\ln\left(\ln(3\,x)^2 + \ln(3\,x) + 1\right)}{2} - \sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,\left(2\,\ln(3\,x) + 1\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(3*x)^2 - 1)/(x + x*log(3*x) + x*log(3*x)^2),x)
```

```
[Out] log(x) - log(log(3*x) + log(3*x)^2 + 1)/2 - 3^(1/2)*atan((3^(1/2)*(2*log(3*
x) + 1))/3)
```

3.262 $\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{32x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$-\frac{1}{32x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^(-1)]^2/x^5,x]
[Out] -1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2\left(\frac{1}{x}\right)}{x^5}\,dx &= -\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}-\frac{1}{2}\int \frac{\log\left(\frac{1}{x}\right)}{x^5}\,dx\\ &= -\frac{1}{32x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$-\frac{1}{32x^4}+\frac{\log\left(\frac{1}{x}\right)}{8x^4}-\frac{\log^2\left(\frac{1}{x}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-1)]^2/x^5,x]
```

```
[Out] -1/32*1/x^4 + Log[x^(-1)]/(8*x^4) - Log[x^(-1)]^2/(4*x^4)
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4}+\frac{\ln\left(\frac{1}{x}\right)}{8}}{x^{4}}$	21
derivativedivides	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
default	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27
risch	$-\frac{1}{32x^{4}}+\frac{\ln\left(\frac{1}{x}\right)}{8x^{4}}-\frac{\ln\left(\frac{1}{x}\right)^{2}}{4x^{4}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/x^4+1/8*ln(1/x)/x^4-1/4*ln(1/x)^2/x^4
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.53

$$-\frac{8\,\log\left(x\right)^{2}+4\,\log\left(x\right)+1}{32\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*(8*log(x)^2 + 4*log(x) + 1)/x^4
```

Fricas [A]
time = 0.42, size = 21, normalized size = 0.66

$$-\frac{8\,\log\left(\frac{1}{x}\right)^{2}-4\,\log\left(\frac{1}{x}\right)+1}{32\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/32*(8*log(1/x)^2 - 4*log(1/x) + 1)/x^4
```

Sympy [A]
time = 0.12, size = 27, normalized size = 0.84

$$-\frac{\log\left(\frac{1}{x}\right)^{2}}{4x^{4}}+\frac{\log\left(\frac{1}{x}\right)}{8x^{4}}-\frac{1}{32x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x)**2/x**5,x)
```

```
[Out] -log(1/x)**2/(4*x**4) + log(1/x)/(8*x**4) - 1/(32*x**4)
```

Giac [A]
time = 3.48, size = 22, normalized size = 0.69

$$-\frac{\log\left(x\right)^{2}}{4\,x^{4}}-\frac{\log\left(x\right)}{8\,x^{4}}-\frac{1}{32\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(x)^2/x^4 - 1/8*log(x)/x^4 - 1/32/x^4
```

Mupad [B]
time = 0.41, size = 21, normalized size = 0.66

$$-\frac{\frac{\ln\left(\frac{1}{x}\right)^{2}}{4}-\frac{\ln\left(\frac{1}{x}\right)}{8}+\frac{1}{32}}{x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(1/x)^2/x^5,x)
```

```
[Out] -(log(1/x)^2/4 - log(1/x)/8 + 1/32)/x^4
```

3.263 $\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{\frac{\pi}{2}}\,x\,\mathrm{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

```
[Out] -1/2*x*erf(1/2*(-ln(a*x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)/(a*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2337, 2211, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}}\,x\,\mathrm{Erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^2]],x]
```

```
[Out] -((Sqrt[Pi/2]*x*Erf[Sqrt[-Log[a*x^2]]/Sqrt[2]])/Sqrt[a*x^2])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx = \frac{x\text{Subst}\left(\int \frac{e^{x/2}}{\sqrt{-x}}\,dx, x, \log(ax^2)\right)}{2\sqrt{ax^2}}$$
$$= -\frac{x\text{Subst}\left(\int e^{-\frac{x^2}{2}}\,dx, x, \sqrt{-\log(ax^2)}\right)}{\sqrt{ax^2}}$$
$$= -\frac{\sqrt{\frac{\pi}{2}}\,x\text{erf}\left(\frac{\sqrt{-\log(ax^2)}}{\sqrt{2}}\right)}{\sqrt{ax^2}}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.48

$$\frac{\sqrt{\frac{\pi}{2}}\,x\text{erfi}\left(\frac{\sqrt{\log(ax^2)}}{\sqrt{2}}\right)\sqrt{\log(ax^2)}}{\sqrt{ax^2}\sqrt{-\log(ax^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^2]],x]
```

```
[Out] (Sqrt[Pi/2]*x*Erfi[Sqrt[Log[a*x^2]]/Sqrt[2]]*Sqrt[Log[a*x^2]])/(Sqrt[a*x^2]
*Sqrt[-Log[a*x^2]])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln(a\,x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^2))^(1/2),x)
```

```
[Out] int(1/(-ln(a*x^2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log(ax^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a*x^2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\ln(a\,x^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a*x^2))^(1/2),x)
```

```
[Out] int(1/(-log(a*x^2))^(1/2), x)
```

3.264 $\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$

Optimal. Leaf size=39

$$\sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}\,x\mathrm{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

```
[Out] 1/2*x*erfi(1/2*(-ln(a/x^2))^(1/2)*2^(1/2))*2^(1/2)*Pi^(1/2)*(a/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2337, 2211, 2235}

$$\sqrt{\frac{\pi}{2}}\,x\sqrt{\frac{a}{x^2}}\,\mathrm{Erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a/x^2]],x]
```

```
[Out] Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erfi[Sqrt[-Log[a/x^2]]/Sqrt[2]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx = -\left(\frac{1}{2}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{e^{-x/2}}{\sqrt{-x}}\,dx, x, \log\left(\frac{a}{x^2}\right)\right)\right)$$
$$= \left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int e^{\frac{x^2}{2}}\,dx, x, \sqrt{-\log\left(\frac{a}{x^2}\right)}\right)$$
$$= \sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}\,x\text{erfi}\left(\frac{\sqrt{-\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)$$

Mathematica [**A**]
time = 0.01, size = 60, normalized size = 1.54

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{\frac{a}{x^2}}\,x\text{erf}\left(\frac{\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{2}}\right)\sqrt{\log\left(\frac{a}{x^2}\right)}}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a/x^2]],x]
[Out] -((Sqrt[Pi/2]*Sqrt[a/x^2]*x*Erf[Sqrt[Log[a/x^2]]/Sqrt[2]]*Sqrt[Log[a/x^2]])
/Sqrt[-Log[a/x^2]])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a/x^2))^(1/2),x)
[Out] int(1/(-ln(a/x^2))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a/x**2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a/x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-log(a/x^2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-\ln\left(\frac{a}{x^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a/x^2))^(1/2),x)
```

```
[Out] int(1/(-log(a/x^2))^(1/2), x)
```

3.265 $\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\sqrt{\pi}\,x(ax^n)^{-1/n}\operatorname{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] -x*erf((-ln(a*x^n))^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^(1/n))/n^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2337, 2211, 2236}

$$-\frac{\sqrt{\pi}\,x(ax^n)^{-1/n}\operatorname{Erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] -((Sqrt[Pi]*x*Erf[Sqrt[-Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1)))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\log(ax^n)}}\,dx = \frac{\left(x(ax^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{-x}}\,dx, x, \log(ax^n)\right)}{n}$$
$$= -\frac{\left(2x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{-\log(ax^n)}\right)}{n}$$
$$= -\frac{\sqrt{\pi}\,x(ax^n)^{-1/n}\,\text{erf}\left(\frac{\sqrt{-\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Mathematica [**A**]
time = 0.01, size = 62, normalized size = 1.44

$$\frac{\sqrt{\pi}\,x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)\sqrt{\log(ax^n)}}{\sqrt{n}\,\sqrt{-\log(ax^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]]*Sqrt[Log[a*x^n]])/(Sqrt[n]*(a*x^
n)^n^(-1)*Sqrt[-Log[a*x^n]])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\ln(a\,x^n)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-ln(a*x^n))^(1/2),x)
```

```
[Out] int(1/(-ln(a*x^n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-log(a*x^n)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\log \left(ax^{n}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-ln(a*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-log(a*x**n)), x)
```

Giac [A]
time = 3.15, size = 32, normalized size = 0.74

$$\frac{\sqrt{\pi}\, \operatorname{erf}\left(-\frac{\sqrt{-n \log \left(x\right) - \log \left(a\right)}}{\sqrt{n}}\right)}{a^{\left(\frac{1}{n}\right)} \sqrt{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-log(a*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*erf(-sqrt(-n*log(x) - log(a))/sqrt(n))/(a^(1/n)*sqrt(n))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{-\ln \left(a\, x^n\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-log(a*x^n))^(1/2),x)
```

```
[Out] int(1/(-log(a*x^n))^(1/2), x)
```

3.266 $\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx$

Optimal. Leaf size=122

$$-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right)+2\log\left(1+\sqrt{x}-x\right)\log\left(\right.$$

```
[Out] -2*ln(1/2+1/2*5^(1/2))*ln(1+5^(1/2)-2*x^(1/2))+ln(x)*ln(1-x+x^(1/2))-ln(x)*
ln(1-2*x^(1/2)/(-5^(1/2)+1))-2*polylog(2,2*x^(1/2)/(-5^(1/2)+1))+2*polylog(
2,1-2*x^(1/2)/(5^(1/2)+1))
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2610, 2604, 2404, 2354, 2438, 2353, 2352}

$$2\text{PolyLog}\left(2,1-\frac{2\sqrt{x}}{1+\sqrt{5}}\right)-2\text{PolyLog}\left(2,\frac{2\sqrt{x}}{1-\sqrt{5}}\right)-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(-2\sqrt{x}+\sqrt{5}+1\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right)+2\log\left(-x+\sqrt{x}+1\right)\log\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] - 2*Log[1 - (2*Sqrt[x]
)/(1 - Sqrt[5])]*Log[Sqrt[x]] + 2*Log[1 + Sqrt[x] - x]*Log[Sqrt[x]] + 2*Pol
yLog[2, 1 - (2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (2*Sqrt[x])/(1 - Sqrt
[5])]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2353

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
(a + b*Log[(-c)*(d/e)])*(Log[d + e*x]/e), x] + Dist[b, Int[Log[(-e)*(x/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[(-c)*(d/e), 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2604

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[Log[d + e*x]*((a + b*Log[c*RFx^p])^n/e), x] - Dist[b*n*(p/e)
, Int[Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2610

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(1+\sqrt{x}-x\right)}{x}\,dx &= 2\text{Subst}\left(\int \frac{\log\left(1+x-x^2\right)}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right)-2\text{Subst}\left(\int \frac{(1-2x)\log(x)}{1+x-x^2}\,dx, x, \sqrt{x}\right)\\
&= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right)-2\text{Subst}\left(\int \left(-\frac{2\log(x)}{1-\sqrt{5}-2x}-\frac{2\log(x)}{1+\sqrt{5}-2x}\right)\right.\\
&= 2\log\left(1+\sqrt{x}-x\right)\log\left(\sqrt{x}\right)+4\text{Subst}\left(\int \frac{\log(x)}{1-\sqrt{5}-2x}\,dx, x, \sqrt{x}\right)+4\text{Subst}\\
&= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right)\\
&= -2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right)-2\log\left(1-\frac{2\sqrt{x}}{1-\sqrt{5}}\right)\log\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 121, normalized size = 0.99

$$-2\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\log\left(1+\sqrt{5}-2\sqrt{x}\right)+\left(\log\left(-1+\sqrt{5}\right)-\log\left(-1+\sqrt{5}+2\sqrt{x}\right)\right)\log(x)+\log\left(1+\sqrt{x}-x\right)\log(x)+2\mathrm{Li}_2\left(\frac{1+\sqrt{5}-2\sqrt{x}}{1+\sqrt{5}}\right)-2\mathrm{Li}_2\left(-\frac{2\sqrt{x}}{-1+\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[x] - x]/x,x]
```

```
[Out] -2*Log[(1 + Sqrt[5])/2]*Log[1 + Sqrt[5] - 2*Sqrt[x]] + (Log[-1 + Sqrt[5]] -
 Log[-1 + Sqrt[5] + 2*Sqrt[x]])*Log[x] + Log[1 + Sqrt[x] - x]*Log[x] + 2*Po
lyLog[2, (1 + Sqrt[5] - 2*Sqrt[x])/(1 + Sqrt[5])] - 2*PolyLog[2, (-2*Sqrt[x
])/(-1 + Sqrt[5])]
```

Maple [A]
time = 0.01, size = 102, normalized size = 0.84

method	result
derivativedivides	$\ln(x)\ln\left(1-x+\sqrt{x}\right)-\ln(x)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln(x)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\,\mathrm{d}$
default	$\ln(x)\ln\left(1-x+\sqrt{x}\right)-\ln(x)\ln\left(\frac{1+\sqrt{5}-2\sqrt{x}}{\sqrt{5}+1}\right)-\ln(x)\ln\left(\frac{-1+\sqrt{5}+2\sqrt{x}}{\sqrt{5}-1}\right)-2\,\mathrm{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-x+x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln(1-x+x^(1/2))-ln(x)*ln((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+1))-ln(x)*ln(
(-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))-2*dilog((1+5^(1/2)-2*x^(1/2))/(5^(1/2)+
1))-2*dilog((-1+5^(1/2)+2*x^(1/2))/(5^(1/2)-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(log(-x + sqrt(x) + 1)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(\sqrt{x} - x + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-x+x**(1/2))/x,x)
```

```
[Out] Integral(log(sqrt(x) - x + 1)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-x+x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(-x + sqrt(x) + 1)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln \left(\sqrt{x} - x + 1\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^(1/2) - x + 1)/x,x)
```

```
[Out] int(log(x^(1/2) - x + 1)/x, x)
```

3.267 $\int \frac{x\log(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=81

$$-\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2}-\frac{a\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}$$

```
[Out] -x/b+(d*x+c)*ln(d*x+c)/b/d-a*ln(-d*(b*x+a)/(-a*d+b*c))*ln(d*x+c)/b^2-a*poly
log(2,b*(d*x+c)/(-a*d+b*c))/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {45, 2463, 2436, 2332, 2441, 2440, 2438}

$$-\frac{a\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{b^2}-\frac{a\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{b^2}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] -(x/b) + ((c + d*x)*Log[c + d*x])/(b*d) - (a*Log[-((d*(a + b*x))/(b*c - a*d
))]*Log[c + d*x])/b^2 - (a*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\log(c+dx)}{a+bx}\,dx &= \int \left(\frac{\log(c+dx)}{b}-\frac{a\log(c+dx)}{b(a+bx)}\right)dx \\
&= \frac{\int \log(c+dx)\,dx}{b}-\frac{a\int \frac{\log(c+dx)}{a+bx}\,dx}{b} \\
&= -\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2}+\frac{\text{Subst}(\int \log(x)\,dx,x,c+dx)}{bd}+\frac{(ad)\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx}{b^2} \\
&= -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2}+\frac{a\text{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\right.}{b^2} \\
&= -\frac{x}{b}+\frac{(c+dx)\log(c+dx)}{bd}-\frac{a\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{b^2}-\frac{a\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.90

$$
\frac{-bdx+\left(bc+bdx-ad\log\left(\frac{d(a+bx)}{-bc+ad}\right)\right)\log(c+dx)-ad\text{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[c + d*x])/(a + b*x),x]
```

```
[Out] (-(b*d*x) + (b*c + b*d*x - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)])*Log[c + d
*x] - a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d)
```

Maple [A]
time = 0.12, size = 109, normalized size = 1.35

method	result	size
derivativedivides	$\frac{\frac{((dx+c)\ln(dx+c)-dx-c)d}{b}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b}\right)a\,d^{2}}{b}}{d^{2}}$	109
default	$\frac{\frac{((dx+c)\ln(dx+c)-dx-c)d}{b}-\frac{\left(\frac{\operatorname{dilog}\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b}\right)a\,d^{2}}{b}}{d^{2}}$	109
risch	$\frac{\ln(dx+c)x}{b}+\frac{\ln(dx+c)c}{db}-\frac{x}{b}-\frac{c}{db}-\frac{a\operatorname{dilog}\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b^{2}}-\frac{a\ln(dx+c)\ln\left(\frac{ad-bc+b(dx+c)}{ad-bc}\right)}{b^{2}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(((d*x+c)*ln(d*x+c)-d*x-c)*d/b-(dilog((a*d-b*c+b*(d*x+c))/(a*d-b*c))/
b+ln(d*x+c)*ln((a*d-b*c+b*(d*x+c))/(a*d-b*c))/b)*a*d^2/b)
```

Maxima [A]
time = 0.29, size = 111, normalized size = 1.37

$$d\left(\frac{\left(\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\operatorname{Li}_{2}\left(-\frac{bdx+ad}{bc-ad}\right)\right)a}{b^{2}d}-\frac{x}{bd}+\frac{c\log\left(dx+c\right)}{bd^{2}}\right)+\left(\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^{2}}\right)\log\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] d*((log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/
(b*c - a*d)))*a/(b^2*d) - x/(b*d) + c*log(d*x + c)/(b*d^2)) + (x/b - a*log(
b*x + a)/b^2)*log(d*x + c)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(d*x + c)/(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \log(c + dx)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*log(c + d*x)/(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(d*x + c)/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\ln(c + d\,x)}{a + b\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(c + d*x))/(a + b*x),x)
```

```
[Out] int((x*log(c + d*x))/(a + b*x), x)
```

3.268 $\int \frac{\log(x)}{-1+x} \, dx$

Optimal. Leaf size=9

$$-\text{Li}_2(1-x)$$

```
[Out] -polylog(2,1-x)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2352}

$$-\text{PolyLog}(2, 1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(-1 + x),x]
```

```
[Out] -PolyLog[2, 1 - x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{\log(x)}{-1+x} \, dx = -\text{Li}_2(1-x)$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\text{Li}_2(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(-1 + x),x]
```

```
[Out] -PolyLog[2, 1 - x]
```

Maple [A]
time = 0.07, size = 5, normalized size = 0.56

method	result	size

default	$-\operatorname{dilog}(x)$	5
risch	$-\operatorname{dilog}(x)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(-1+x),x,method=_RETURNVERBOSE)
```

```
[Out] -dilog(x)
```

Maxima [A]
time = 0.30, size = 12, normalized size = 1.33

$$\log(x)\log(-x+1) + \operatorname{Li}_2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="maxima")
```

```
[Out] log(x)*log(-x + 1) + dilog(x)
```

Fricas [A]
time = 0.40, size = 8, normalized size = 0.89

$$-\operatorname{Li}_2(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="fricas")
```

```
[Out] -dilog(-x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.92, size = 10, normalized size = 1.11

$$-\operatorname{Li}_2\left((x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(-1+x),x)
```

```
[Out] -polylog(2, (x - 1)*exp_polar(I*pi))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(-1+x),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(x - 1), x)
```

Mupad [B]
time = 0.02, size = 4, normalized size = 0.44

$$-\mathrm{Li}_2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x - 1),x)
```

```
[Out] -dilog(x)
```

3.269 $\int \frac{x\log(1-a-bx)}{a+bx}\,dx$

Optimal. Leaf size=43

$$-\frac{x}{b}-\frac{(1-a-bx)\log(1-a-bx)}{b^2}+\frac{a\mathrm{Li}_2(a+bx)}{b^2}$$

[Out] -x/b-(-b*x-a+1)*ln(-b*x-a+1)/b^2+a*polylog(2,b*x+a)/b^2

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {45, 2463, 2436, 2332, 2440, 2438}

$$\frac{a\mathrm{PolyLog}(2,a+bx)}{b^2}-\frac{(-a-bx+1)\log(-a-bx+1)}{b^2}-\frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[(x*Log[1 - a - b*x])/(a + b*x),x]

[Out] -(x/b) - ((1 - a - b*x)*Log[1 - a - b*x])/b^2 + (a*PolyLog[2, a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}\int \frac{x\log(1-a-bx)}{a+bx}\,dx &= \int \left(\frac{\log(1-a-bx)}{b}-\frac{a\log(1-a-bx)}{b(a+bx)}\right)dx\\ &= \frac{\int \log(1-a-bx)\,dx}{b}-\frac{a\int \frac{\log(1-a-bx)}{a+bx}\,dx}{b}\\ &= -\frac{\text{Subst}\left(\int \log(x)\,dx,x,1-a-bx\right)}{b^2}-\frac{a\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,a+bx\right)}{b^2}\\ &= -\frac{x}{b}-\frac{(1-a-bx)\log(1-a-bx)}{b^2}+\frac{a\text{Li}_2(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.81

$$\frac{-bx+(-1+a+bx)\log(1-a-bx)+a\text{Li}_2(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[1 - a - b*x])/(a + b*x),x]
[Out] (-(b*x) + (-1 + a + b*x)*Log[1 - a - b*x] + a*PolyLog[2, a + b*x])/b^2
```

Maple [A]
time = 0.06, size = 47, normalized size = 1.09

method	result	size
derivativedivides	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+\text{dilog}(-bx-a+1)a}{b^2}$	47
default	$\frac{-(-bx-a+1)\ln(-bx-a+1)-bx-a+1+\text{dilog}(-bx-a+1)a}{b^2}$	47

risch	$\frac{x\ln(-bx-a+1)}{b}+\frac{\operatorname{dilog}(-bx-a+1)a}{b^2}+\frac{\ln(-bx-a+1)a}{b^2}-\frac{x}{b}-\frac{\ln(-bx-a+1)}{b^2}-\frac{a}{b^2}+\frac{1}{b^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(-b*x-a+1)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(-(-b*x-a+1)*ln(-b*x-a+1)-b*x-a+1+dilog(-b*x-a+1)*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.
time = 0.31, size = 82, normalized size = 1.91

$$b\left(\frac{(\log(bx+a)\log(-bx-a+1)+\mathrm{Li}_2(bx+a))a}{b^3}-\frac{x}{b^2}+\frac{(a-1)\log(bx+a-1)}{b^3}\right)+\left(\frac{x}{b}-\frac{a\log(bx+a)}{b^2}\right)\log(-bx-a+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*((log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))*a/b^3 - x/b^2 + (a - 1
)*log(b*x + a - 1)/b^3) + (x/b - a*log(b*x + a)/b^2)*log(-b*x - a + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x*log(-b*x - a + 1)/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\log(-a-bx+1)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(-b*x-a+1)/(b*x+a),x)
```

```
[Out] Integral(x*log(-a - b*x + 1)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(-b*x-a+1)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*log(-b*x - a + 1)/(b*x + a), x)
```

Mupad [B]
time = 0.32, size = 59, normalized size = 1.37

$$-\frac{\ln\left(1-b\,x-a\right)+b\,\left(x-x\,\ln\left(1-b\,x-a\right)\right)-a\,\mathrm{Li}_2\left(1-b\,x-a\right)-a\,\ln\left(1-b\,x-a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*log(1 - b*x - a))/(a + b*x),x)
```

```
[Out] -(log(1 - b*x - a) + b*(x - x*log(1 - b*x - a)) - a*dilog(1 - b*x - a) - a*
log(1 - b*x - a))/b^2
```

3.270 $\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx$

Optimal. Leaf size=30

$$\frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)$$

```
[Out] 1/2*ln(x)^2+ln(x)*ln(1+c*x/b)+polylog(2,-c*x/b)
```

Rubi [A]
time = 0.07, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2404, 2338, 2354, 2438}

$$\text{PolyLog}\left(2,-\frac{cx}{b}\right)+\log(x)\log\left(\frac{cx}{b}+1\right)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
```

```
[Out] Log[x]^2/2 + Log[x]*Log[1 + (c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2404

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(b+2cx)\log(x)}{x(b+cx)}\,dx &= \int \left(\frac{\log(x)}{x}+\frac{c\log(x)}{b+cx}\right)dx\\ &= c\int \frac{\log(x)}{b+cx}\,dx+\int \frac{\log(x)}{x}\,dx\\ &= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)-\int \frac{\log\left(1+\frac{cx}{b}\right)}{x}\,dx\\ &= \frac{\log^2(x)}{2}+\log(x)\log\left(1+\frac{cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.03

$$\frac{\log^2(x)}{2}+\log(x)\log\left(\frac{b+cx}{b}\right)+\text{Li}_2\left(-\frac{cx}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((b + 2*c*x)*Log[x])/(x*(b + c*x)),x]
[Out] Log[x]^2/2 + Log[x]*Log[(b + c*x)/b] + PolyLog[2, -((c*x)/b)]
```

Maple [A]
time = 0.07, size = 41, normalized size = 1.37

method	result	size
risch	$\frac{\ln(x)^2}{2}+\ln(x)\ln\left(\frac{cx+b}{b}\right)+\text{dilog}\left(\frac{cx+b}{b}\right)$	31
default	$\frac{\ln(x)^2}{2}+\left(\frac{\text{dilog}\left(\frac{cx+b}{b}\right)}{c}+\frac{\ln(x)\ln\left(\frac{cx+b}{b}\right)}{c}\right)c$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*ln(x)/x/(c*x+b),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)^2+(dilog((c*x+b)/b)/c+ln(x)*ln((c*x+b)/b)/c)*c
```

Maxima [A]
time = 0.30, size = 49, normalized size = 1.63

$$(\log(cx+b)+\log(x))\log(x)-\log(cx+b)\log(x)+\log\left(\frac{cx}{b}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\text{Li}_2\left(-\frac{cx}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="maxima")
```

```
[Out] (log(c*x + b) + log(x))*log(x) - log(c*x + b)*log(x) + log(c*x/b + 1)*log(x
) - 1/2*log(x)^2 + dilog(-c*x/b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="fricas")
```

```
[Out] integral((2*c*x + b)*log(x)/(c*x^2 + b*x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 90.98, size = 228, normalized size = 7.60

$$b\left(\begin{cases} -\frac{1}{cx} & \text{for}\: b = 0 \\ \frac{\begin{cases} \operatorname{Li}_{2}\left(\frac{b e^{i\pi}}{cx}\right) & \text{for}\: \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log{\left(c \right)} \log{\left(x \right)} + \operatorname{Li}_{2}\left(\frac{b e^{i\pi}}{cx}\right) & \text{for}\: |x| < 1 \\ - \log{\left(c \right)} \log{\left(\frac{1}{x} \right)} + \operatorname{Li}_{2}\left(\frac{b e^{i\pi}}{cx}\right) & \text{for}\: \frac{1}{|x|} < 1 \\ - G_{2, 2}^{2, 0}\left(\begin{matrix} & 1, 1 \\0, 0 & \end{matrix} \middle| {x} \right) \log{\left(c \right)} + G_{2, 2}^{0, 2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| {x} \right) \log{\left(c \right)} + \operatorname{Li}_{2}\left(\frac{b e^{i\pi}}{cx}\right) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{1}{cx} & \text{for}\: b = 0 \\ \frac{\log{\left(\frac{b}{x} + c \right)}}{b} & \text{otherwise} \end{cases}\right) \log{\left(x \right)} - 2c\left(\begin{cases} \frac{x}{b} & \text{for}\: c = 0 \\ \frac{\begin{cases} - \operatorname{Li}_{2}\left(\frac{cx e^{i\pi}}{b}\right) & \text{for}\: \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log{\left(b \right)} \log{\left(x \right)} - \operatorname{Li}_{2}\left(\frac{cx e^{i\pi}}{b}\right) & \text{for}\: |x| < 1 \\ - \log{\left(b \right)} \log{\left(\frac{1}{x} \right)} - \operatorname{Li}_{2}\left(\frac{cx e^{i\pi}}{b}\right) & \text{for}\: \frac{1}{|x|} < 1 \\ - G_{2, 2}^{2, 0}\left(\begin{matrix} & 1, 1 \\0, 0 & \end{matrix} \middle| {x} \right) \log{\left(b \right)} + G_{2, 2}^{0, 2}\left(\begin{matrix} 1, 1 & \\ & 0, 0 \end{matrix} \middle| {x} \right) \log{\left(b \right)} - \operatorname{Li}_{2}\left(\frac{cx e^{i\pi}}{b}\right) & \text{otherwise} \end{cases}}{c} & \text{otherwise} \end{cases}\right) + 2c\left(\begin{cases} \frac{x}{b} & \text{for}\: c = 0 \\ \frac{\log{\left(b + cx \right)}}{c} & \text{otherwise} \end{cases}\right) \log{\left(x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*ln(x)/x/(c*x+b),x)
```

```
[Out] b*Piecewise((-1/(c*x), Eq(b, 0)), (Piecewise((polylog(2, b*exp_polar(I*pi)/
(c*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(c)*log(x) + polylog(2, b*exp_p
olar(I*pi)/(c*x)), Abs(x) < 1), (-log(c)*log(1/x) + polylog(2, b*exp_polar(
I*pi)/(c*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(c
) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(c) + polylog(2, b*exp_polar(
I*pi)/(c*x)), True))/b, True)) - b*Piecewise((1/(c*x), Eq(b, 0)), (log(b/x
+ c)/b, True))*log(x) - 2*c*Piecewise((x/b, Eq(c, 0)), (Piecewise((-polylog
(2, c*x*exp_polar(I*pi)/b), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(b)*log(x)
- polylog(2, c*x*exp_polar(I*pi)/b), Abs(x) < 1), (-log(b)*log(1/x) - polyl
og(2, c*x*exp_polar(I*pi)/b), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0
), ()), x)*log(b) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(b) - polylog
(2, c*x*exp_polar(I*pi)/b), True))/c, True)) + 2*c*Piecewise((x/b, Eq(c, 0)
), (log(b + c*x)/c, True))*log(x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*log(x)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] integrate((2*c*x + b)*log(x)/((c*x + b)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln(x)\ (b+2\,c\,x)}{x\ (b+c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)*(b + 2*c*x))/(x*(b + c*x)),x)
[Out] int((log(x)*(b + 2*c*x))/(x*(b + c*x)), x)
```

3.271 $\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx$

Optimal. Leaf size=7

$$-\cos(x\log(x))$$

```
[Out] -cos(x*ln(x))
```

Rubi [A]
time = 0.02, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {4607}

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

```
[Out] -Cos[x*Log[x]]
```

Rule 4607

```
Int[Log[(b_.)*(x_)]*Sin[Log[(b_.)*(x_)]*(a_.)*(x_)], x_Symbol] :> Simp[-Cos
[a*x*Log[b*x]]/a, x] - Int[Sin[a*x*Log[b*x]], x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (\sin(x\log(x)) + \log(x)\sin(x\log(x)))\,dx &= \int \sin(x\log(x))\,dx + \int \log(x)\sin(x\log(x))\,dx\\ &= -\cos(x\log(x))\end{aligned}$$

Mathematica [A]
time = 0.09, size = 7, normalized size = 1.00

$$-\cos(x\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x*Log[x]] + Log[x]*Sin[x*Log[x]],x]
```

```
[Out] -Cos[x*Log[x]]
```

Maple [A]
time = 0.07, size = 8, normalized size = 1.14

method	result	size
derivativedivides	$-\cos(x\ln(x))$	8
default	$-\cos(x\ln(x))$	8
norman	$-\frac{2}{1+\tan^2\left(\frac{x\ln(x)}{2}\right)}$	15
risch	$-\frac{x^{ix}}{2}-\frac{x^{-ix}}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x*ln(x))
```

Maxima [A]
time = 0.37, size = 7, normalized size = 1.00

$$-\cos(x\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="maxima")
```

```
[Out] -cos(x*log(x))
```

Fricas [A]
time = 0.41, size = 7, normalized size = 1.00

$$-\cos(x\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="fricas")
```

```
[Out] -cos(x*log(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(\log(x)+1)\sin(x\log(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*ln(x))+ln(x)*sin(x*ln(x)),x)
```

```
[Out] Integral((log(x) + 1)*sin(x*log(x)), x)
```

Giac [**A**]
time = 5.44, size = 7, normalized size = 1.00

$$-\cos(x\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
[Out] -cos(x*log(x))
```

Mupad [**B**]
time = 0.42, size = 7, normalized size = 1.00

$$-\cos(x\ \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x*log(x)) + sin(x*log(x))*log(x),x)
[Out] -cos(x*log(x))
```

3.272 $\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{x}+\tan^{-1}(1-x)-\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}-\frac{1}{2}\log\left(2-2x+x^2\right)$$

```
[Out] -1/x-arctan(-1+x)-ln((1-(1-x)^2)/(1+(-1+x)^2))/x+1/2*ln(2-x)+1/2*ln(x)-1/2*
ln(x^2-2*x+2)
```

Rubi [A]
time = 0.17, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2605, 12, 6860, 648, 631, 210, 642}

$$\text{ArcTan}(1-x)-\frac{1}{2}\log\left(x^2-2x+2\right)-\frac{1}{x}-\frac{\log\left(\frac{1-(1-x)^2}{(x-1)^2+1}\right)}{x}+\frac{1}{2}\log(2-x)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] - Log[(1 - (1 - x)^2)/(1 + (-1 + x)^2)]/x + Log[2 -
 x]/2 + Log[x]/2 - Log[2 - 2*x + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x^2}\,dx = -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \int \frac{4(1-x)}{(2-x)x^2\left(2-2x+x^2\right)}\,dx$$

$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \frac{1-x}{(2-x)x^2\left(2-2x+x^2\right)}\,dx$$

$$= -\frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + 4\int \left(\frac{1}{8(-2+x)} + \frac{1}{4x^2} + \frac{1}{8x} - \frac{x}{4\left(2-2x+x^2\right)}\right)dx$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \int \frac{x}{2-2x+x^2}\,dx$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx - \int \frac{}{2-2}$$

$$= -\frac{1}{x} - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log\left(2-2x+x^2\right) - \text{Subst}\Bigg(\int$$

$$= -\frac{1}{x} + \tan^{-1}(1-x) - \frac{\log\left(\frac{1-(1-x)^2}{1+(-1+x)^2}\right)}{x} + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{1}{2}\log(2-2x+$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 0.93

$$-\frac{1}{x} + \tan^{-1}(1-x) + \frac{1}{2}\log(2-x) + \frac{\log(x)}{2} - \frac{\log\left(-\frac{(-2+x)x}{2-2x+x^2}\right)}{x} - \frac{1}{2}\log\left(2-2x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - (-1 + x)^2)/(1 + (-1 + x)^2)]/x^2,x]
```

```
[Out] -x^(-1) + ArcTan[1 - x] + Log[2 - x]/2 + Log[x]/2 - Log[-(((-2 + x)*x)/(2 -
 2*x + x^2))]/x - Log[2 - 2*x + x^2]/2
```

Maple [A]
time = 0.04, size = 57, normalized size = 0.84

method	result	size
default	$-\frac{\ln\left(\frac{x(2-x)}{x^2-2x+2}\right)}{x} - \frac{1}{x} + \frac{\ln(x)}{2} - \frac{\ln(x^2-2x+2)}{2} - \arctan(-1+x) + \frac{\ln(x-2)}{2}$	57
risch	$-\frac{\ln\left(\frac{1-(-1+x)^2}{1+(-1+x)^2}\right)}{x} + \frac{i\ln(x-1-i)x - i\ln(x-1+i)x - \ln(x-1-i)x - \ln(x-1+i)x + \ln(x^2-2x)x - 2}{2x}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln(x*(2-x)/(x^2-2*x+2))-1/x+1/2*ln(x)-1/2*ln(x^2-2*x+2)-arctan(-1+x)+1
/2*ln(x-2)
```

Maxima [A]
time = 0.93, size = 57, normalized size = 0.84

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2\,x+2\right)+\frac{1}{2}\log\left(x-2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="maxima")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(x - 2) + 1/2*log(x)
```

Fricas [A]
time = 0.39, size = 58, normalized size = 0.85

$$-\frac{2\,x\arctan\left(x-1\right)+x\log\left(x^2-2\,x+2\right)-x\log\left(x^2-2\,x\right)+2\,\log\left(-\frac{x^2-2\,x}{x^2-2\,x+2}\right)+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*arctan(x - 1) + x*log(x^2 - 2*x + 2) - x*log(x^2 - 2*x) + 2*log(-
(x^2 - 2*x)/(x^2 - 2*x + 2)) + 2)/x
```

Sympy [A]
time = 0.09, size = 46, normalized size = 0.68

$$\frac{\log\left(x^2-2x\right)}{2}-\frac{\log\left(x^2-2x+2\right)}{2}-\operatorname{atan}\left(x-1\right)-\frac{\log\left(\frac{1-(x-1)^2}{(x-1)^2+1}\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((1-(-1+x)**2)/(1+(-1+x)**2))/x**2,x)
```

```
[Out] log(x**2 - 2*x)/2 - log(x**2 - 2*x + 2)/2 - atan(x - 1) - log((1 - (x - 1)*
*2)/((x - 1)**2 + 1))/x - 1/x
```

Giac [A]
time = 3.48, size = 59, normalized size = 0.87

$$-\frac{\log\left(-\frac{(x-1)^2-1}{(x-1)^2+1}\right)}{x}-\frac{1}{x}-\arctan\left(x-1\right)-\frac{1}{2}\log\left(x^2-2x+2\right)+\frac{1}{2}\log\left(|x-2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1-(-1+x)^2)/(1+(-1+x)^2))/x^2,x, algorithm="giac")
```

```
[Out] -log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x - 1/x - arctan(x - 1) - 1/2*log(x^
2 - 2*x + 2) + 1/2*log(abs(x - 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.46, size = 59, normalized size = 0.87

$$\frac{\ln\left(x\left(x-2\right)\right)}{2}-\mathrm{atan}(x-1)-\frac{\ln\left(x^2-2x+2\right)}{2}-\frac{\ln\left(2x-x^2\right)}{x}+\frac{\ln\left(x^2-2x+2\right)}{x}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-((x - 1)^2 - 1)/((x - 1)^2 + 1))/x^2,x)
```

```
[Out] log(x*(x - 2))/2 - atan(x - 1) - log(x^2 - 2*x + 2)/2 - log(2*x - x^2)/x +
log(x^2 - 2*x + 2)/x - 1/x
```

3.273 $\int \log\left(\sqrt{x}+x\right)\,dx$

Optimal. Leaf size=29

$$\sqrt{x}-x-\log\left(1+\sqrt{x}\right)+x\log\left(\sqrt{x}+x\right)$$

```
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2628, 383, 78}

$$-x+\sqrt{x}+x\log\left(x+\sqrt{x}\right)-\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x] + x],x]
```

```
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log \left(\sqrt{x}+x\right) dx &= x\log \left(\sqrt{x}+x\right)-\int \frac{1+2\sqrt{x}}{2+2\sqrt{x}}\,dx\\ &= x\log \left(\sqrt{x}+x\right)-2\text{Subst}\left(\int \frac{x(1+2x)}{2+2x}\,dx,x,\sqrt{x}\right)\\ &= x\log \left(\sqrt{x}+x\right)-2\text{Subst}\left(\int \left(-\frac{1}{2}+x+\frac{1}{2(1+x)}\right)dx,x,\sqrt{x}\right)\\ &= \sqrt{x}-x-\log \left(1+\sqrt{x}\right)+x\log \left(\sqrt{x}+x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$\sqrt{x}-x-\log \left(1+\sqrt{x}\right)+x\log \left(\sqrt{x}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x] + x],x]
[Out] Sqrt[x] - x - Log[1 + Sqrt[x]] + x*Log[Sqrt[x] + x]
```

Maple [A]
time = 0.01, size = 24, normalized size = 0.83

method	result	size
derivativedivides	$-x-\ln \left(1+\sqrt{x}\right)+x\ln \left(x+\sqrt{x}\right)+\sqrt{x}$	24
default	$-x-\ln \left(1+\sqrt{x}\right)+x\ln \left(x+\sqrt{x}\right)+\sqrt{x}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x-ln(1+x^(1/2))+x*ln(x+x^(1/2))+x^(1/2)
```

Maxima [A]
time = 0.30, size = 23, normalized size = 0.79

$$x\log \left(x+\sqrt{x}\right)-x+\sqrt{x}-\log \left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="maxima")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Fricas [A]
time = 0.41, size = 31, normalized size = 1.07

$$(x+1)\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-2\,\log\left(\sqrt{x}+1\right)-\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*log(x + sqrt(x)) - x + sqrt(x) - 2*log(sqrt(x) + 1) - log(sqrt(x))
```

Sympy [A]
time = 6.36, size = 24, normalized size = 0.83

$$\sqrt{x}+x\log\left(\sqrt{x}+x\right)-x-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+x**(1/2)),x)
[Out] sqrt(x) + x*log(sqrt(x) + x) - x - log(sqrt(x) + 1)
```

Giac [A]
time = 3.72, size = 23, normalized size = 0.79

$$x\log\left(x+\sqrt{x}\right)-x+\sqrt{x}-\log\left(\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+x^(1/2)),x, algorithm="giac")
[Out] x*log(x + sqrt(x)) - x + sqrt(x) - log(sqrt(x) + 1)
```

Mupad [B]
time = 0.08, size = 23, normalized size = 0.79

$$\sqrt{x}-\ln\left(\sqrt{x}+1\right)-x+x\,\ln\left(x+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x + x^(1/2)),x)
[Out] x^(1/2) - log(x^(1/2) + 1) - x + x*log(x + x^(1/2))
```

3.274 $\int \log\left(-\frac{x}{1+x}\right) dx$

Optimal. Leaf size=18

$$x\log\left(-\frac{x}{1+x}\right)-\log(1+x)$$

```
[Out] x*ln(-x/(1+x))-ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2536, 31}

$$x\log\left(-\frac{x}{x+1}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-(x/(1 + x))],x]
```

```
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2536

```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)^n/(c
+ d*x)^n)])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b
*x)^n/(c + d*x)^n)])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(-\frac{x}{1+x}\right) dx &= x\log\left(-\frac{x}{1+x}\right)-\int \frac{1}{1+x}\,dx\\ &= x\log\left(-\frac{x}{1+x}\right)-\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$x\log\left(-\frac{x}{1+x}\right)-\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-(x/(1 + x))],x]
```

```
[Out] x*Log[-(x/(1 + x))] - Log[1 + x]
```

Maple [A]
time = 0.04, size = 28, normalized size = 1.56

method	result	size
risch	$x \ln \left(-\frac{x}{1+x}\right)-\ln \left(1+x\right)$	19
derivativedivides	$\ln \left(\frac{1}{1+x}\right)-\ln \left(-1+\frac{1}{1+x}\right) \left(-1+\frac{1}{1+x}\right) \left(1+x\right)$	28
default	$\ln \left(\frac{1}{1+x}\right)-\ln \left(-1+\frac{1}{1+x}\right) \left(-1+\frac{1}{1+x}\right) \left(1+x\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-x/(1+x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1/(1+x))-ln(-1+1/(1+x))*(-1+1/(1+x))*(1+x)
```

Maxima [A]
time = 0.31, size = 18, normalized size = 1.00

$$x \log \left(-\frac{x}{x+1}\right) - \log \left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="maxima")
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Fricas [A]
time = 0.37, size = 18, normalized size = 1.00

$$x \log \left(-\frac{x}{x+1}\right) - \log \left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="fricas")
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.78

$$x \log \left(-\frac{x}{x+1}\right) - \log \left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-x/(1+x)),x)
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(18) = 36.
time = 3.73, size = 80, normalized size = 4.44

$$-\frac{\log\left(-\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)\left(\frac{x}{(x+1)\left(\frac{x}{x+1}-1\right)}-1\right)}\right)}{\frac{x}{x+1}-1}-\log\left(\frac{|x|}{|x+1|}\right)+\log\left(\left|-\frac{x}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-x/(1+x)),x, algorithm="giac")
```

```
[Out] -log(-x/((x + 1)*(x/(x + 1) - 1)*(x/((x + 1)*(x/(x + 1) - 1)) - 1)))/(x/(x
+ 1) - 1) - log(abs(x)/abs(x + 1)) + log(abs(-x/(x + 1) + 1))
```

Mupad [B]
time = 0.36, size = 18, normalized size = 1.00

$$x\,\ln\left(-\frac{x}{x+1}\right)-\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-x/(x + 1)),x)
```

```
[Out] x*log(-x/(x + 1)) - log(x + 1)
```

3.275 $\int \log \left(\frac{-1+x}{1+x}\right) \, dx$

Optimal. Leaf size=27

$$-\left((1-x)\log\left(-\frac{1-x}{1+x}\right)\right)-2\log(1+x)$$

```
[Out] -(1-x)*ln((-1+x)/(1+x))-2*ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2535, 31}

$$-\left((1-x)\log\left(-\frac{1-x}{x+1}\right)\right)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[(-1 + x)/(1 + x)],x]
```

```
[Out] -((1 - x)*Log[-((1 - x)/(1 + x))]) - 2*Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2535

```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.), x_Symbol] :> Simp[(a + b*x)*((A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p/b), x] - Dist[B*n*p*((b*c - a*d)/b), Int[(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, A, B, n}, x
] && NeQ[b*c - a*d, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{-1+x}{1+x}\right) dx &= -(1-x)\log\left(-\frac{1-x}{1+x}\right)-2\int\frac{1}{1+x}\,dx\\ &= -(1-x)\log\left(-\frac{1-x}{1+x}\right)-2\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.78

$$(-1+x)\log\left(\frac{-1+x}{1+x}\right)-2\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(-1 + x)/(1 + x)],x]
[Out] (-1 + x)*Log[(-1 + x)/(1 + x)] - 2*Log[1 + x]
```

Maple [A]
time = 0.03, size = 35, normalized size = 1.30

method	result	size
risch	$x\ln\left(\frac{-1+x}{1+x}\right)-\ln\left(x^2-1\right)$	22
derivativedivides	$2\ln\left(-\frac{2}{1+x}\right)+\ln\left(1-\frac{2}{1+x}\right)\left(1-\frac{2}{1+x}\right)(1+x)$	35
default	$2\ln\left(-\frac{2}{1+x}\right)+\ln\left(1-\frac{2}{1+x}\right)\left(1-\frac{2}{1+x}\right)(1+x)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-1+x)/(1+x)),x,method=_RETURNVERBOSE)
[Out] 2*ln(-2/(1+x))+ln(1-2/(1+x))*(1-2/(1+x))*(1+x)
```

Maxima [A]
time = 0.30, size = 25, normalized size = 0.93

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x+1\right)-\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="maxima")
[Out] x*log((x - 1)/(x + 1)) - log(x + 1) - log(x - 1)
```

Fricas [A]
time = 0.38, size = 21, normalized size = 0.78

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="fricas")
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

Sympy [A]
time = 0.10, size = 15, normalized size = 0.56

$$x\log\left(\frac{x-1}{x+1}\right)-\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-1+x)/(1+x)),x)
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x**2 - 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(21) = 42.
time = 5.91, size = 103, normalized size = 3.81

$$-\frac{2\log\left(\frac{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}+1}{\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1}-1}\right)}{\frac{x-1}{x+1}-1}-2\log\left(\frac{|x-1|}{|x+1|}\right)+2\log\left(\left|\frac{x-1}{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-1+x)/(1+x)),x, algorithm="giac")
```

```
[Out] -2*log((((x - 1)/(x + 1) + 1)/((x - 1)/(x + 1) - 1) + 1)/((((x - 1)/(x + 1)
+ 1)/((x - 1)/(x + 1) - 1) - 1))/((x - 1)/(x + 1) - 1) - 2*log(abs(x - 1)/a
bs(x + 1)) + 2*log(abs((x - 1)/(x + 1) - 1))
```

Mupad [B]
time = 0.08, size = 21, normalized size = 0.78

$$x\,\ln\left(\frac{x-1}{x+1}\right)-\ln\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x - 1)/(x + 1)),x)
```

```
[Out] x*log((x - 1)/(x + 1)) - log(x^2 - 1)
```

3.276 $\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{1+x}-\tan^{-1}(x)+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -1/(1+x)-arctan(x)+1/2*ln(-x^2+1)-ln((-x^2+1)/(x^2+1))/(1+x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2605, 12, 2099, 266, 649, 209}

$$-\text{ArcTan}(x)+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) - ArcTan[x] + Log[1 - x^2]/2 - Log[(1 - x^2)/(1 + x^2)]/(1 + 
x) - Log[1 + x^2]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 2605

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*Log[c*RFx^p])^n/(e*(m + 1)))
, x] - Dist[b*n*(p/(e*(m + 1))), Int[SimplifyIntegrand[(d + e*x)^(m + 1)*(a
 + b*Log[c*RFx^p])^(n - 1)*(D[RFx, x]/RFx), x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{(1+x)^2}\,dx &= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+\int \frac{4x}{-1-x+x^4+x^5}\,dx\\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+4\int \frac{x}{-1-x+x^4+x^5}\,dx\\
&= -\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+4\int \left(\frac{1}{4(1+x)^2}+\frac{x}{4\left(-1+x^2\right)}+\frac{-1-x}{4\left(1+x^2\right)}\right)dx\\
&= -\frac{1}{1+x}-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}+\int \frac{x}{-1+x^2}\,dx+\int \frac{-1-x}{1+x^2}\,dx\\
&= -\frac{1}{1+x}+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{1+x}-\tan^{-1}(x)+\frac{1}{2}\log\left(1-x^2\right)-\frac{\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 1.00

$$
\frac{1}{2}\left(-\frac{2}{1+x}-2\tan^{-1}(x)+\log(1-x)+\log(1+x)-\frac{2\log\left(\frac{1-x^2}{1+x^2}\right)}{1+x}-\log\left(1+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(1 - x^2)/(1 + x^2)]/(1 + x)^2,x]
```

```
[Out] (-2/(1 + x) - 2*ArcTan[x] + Log[1 - x] + Log[1 + x] - (2*Log[(1 - x^2)/(1 + x^2)])/(1 + x) - Log[1 + x^2])/2
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 112, normalized size = 1.96

method	result
risch	$-\frac{\ln\left(\frac{-x^2+1}{x^2+1}\right)}{1+x}+\frac{i\ln(x-i)x-i\ln(x+i)x+i\ln(x-i)-i\ln(x+i)-\ln(x-i)x-\ln(x+i)x+\ln(x^2-1)x-\ln(x-i)-\ln(x+i)+\ln(x^2-1)-2}{2+2x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln((-x^2+1)/(x^2+1))/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln((-x^2+1)/(x^2+1))/(1+x)+1/2*(I*ln(x-I)*x-I*ln(x+I)*x+I*ln(x-I)-I*ln(x+I)-ln(x-I)*x-ln(x+I)*x+ln(x^2-1)*x-ln(x-I)-ln(x+I)+ln(x^2-1)-2)/(1+x)
```

Maxima [A]
time = 0.50, size = 54, normalized size = 0.95

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 + 1) + 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.95

$$-\frac{2\left(x+1\right)\arctan\left(x\right)+\left(x+1\right)\log\left(x^2+1\right)-\left(x+1\right)\log\left(x^2-1\right)+2\log\left(-\frac{x^2-1}{x^2+1}\right)+2}{2\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(x + 1)*arctan(x) + (x + 1)*log(x^2 + 1) - (x + 1)*log(x^2 - 1) + 2*log(-(x^2 - 1)/(x^2 + 1)) + 2)/(x + 1)
```

Sympy [A]
time = 0.21, size = 41, normalized size = 0.72

$$\frac{\log\left(x^2-1\right)}{2}-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}\left(x\right)-\frac{4}{4x+4}-\frac{\log\left(\frac{1-x^2}{x^2+1}\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((-x**2+1)/(x**2+1))/(1+x)**2,x)
```

```
[Out] log(x**2 - 1)/2 - log(x**2 + 1)/2 - atan(x) - 4/(4*x + 4) - log((1 - x**2)/
(x**2 + 1))/(x + 1)
```

Giac [A]
time = 6.30, size = 56, normalized size = 0.98

$$-\frac{\log\left(-\frac{x^2-1}{x^2+1}\right)}{x+1}-\frac{1}{x+1}-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((-x^2+1)/(x^2+1))/(1+x)^2,x, algorithm="giac")
```

```
[Out] -log(-(x^2 - 1)/(x^2 + 1))/(x + 1) - 1/(x + 1) - arctan(x) - 1/2*log(x^2 +
1) + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

Mupad [B]
time = 0.42, size = 55, normalized size = 0.96

$$\frac{\ln\left(x^2-1\right)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\mathrm{atan}(x)-\frac{1}{x+1}+\frac{\ln\left(x^2+1\right)}{x+1}-\frac{\ln\left(1-x^2\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(-(x^2 - 1)/(x^2 + 1))/(x + 1)^2,x)
```

```
[Out] log(x^2 - 1)/2 - log(x^2 + 1)/2 - atan(x) - 1/(x + 1) + log(x^2 + 1)/(x + 1
) - log(1 - x^2)/(x + 1)
```

3.277 $\int \frac{\log\left(c(1+x^2)^n\right)}{1+x^2} dx$

Optimal. Leaf size=60

$$in\tan^{-1}(x)^2 + 2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) + \tan^{-1}(x)\log\left(c(1+x^2)^n\right) + in\mathrm{Li}_2\left(1-\frac{2}{1+ix}\right)$$

```
[Out] I*n*arctan(x)^2+2*n*arctan(x)*ln(2/(1+I*x))+arctan(x)*ln(c*(x^2+1)^n)+I*n*p
olylog(2,1-2/(1+I*x))
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {209, 2520, 5040, 4964, 2449, 2352}

$$in\text{PolyLog}\left(2, 1-\frac{2}{1+ix}\right) + \text{ArcTan}(x)\log\left(c(x^2+1)^n\right) + in\text{ArcTan}(x)^2 + 2n\text{ArcTan}(x)\log\left(\frac{2}{1+ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[2/(1 + I*x)] + ArcTan[x]*Log[c*(1 + x^2
)^n] + I*n*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(c\left(1+x^2\right)^n\right)}{1+x^2}\,dx &= \tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{x\tan^{-1}(x)}{1+x^2}\,dx \\
&= in\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2n)\int \frac{\tan^{-1}(x)}{i-x}\,dx \\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)-(2n)\int \frac{\log}{1} \\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+(2in)\text{Subst} \\
&= in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2}{1+ix}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+in\text{Li}_2\left(1-\right.
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 1.03

$$in\tan^{-1}(x)^2+2n\tan^{-1}(x)\log\left(\frac{2i}{i-x}\right)+\tan^{-1}(x)\log\left(c\left(1+x^2\right)^n\right)+in\text{Li}_2\left(\frac{i+x}{-i+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[c*(1 + x^2)^n]/(1 + x^2),x]
```

```
[Out] I*n*ArcTan[x]^2 + 2*n*ArcTan[x]*Log[(2*I)/(I - x)] + ArcTan[x]*Log[c*(1 + x
^2)^n] + I*n*PolyLog[2, (I + x)/(-I + x)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 249, normalized size = 4.15

method	result
risch	$\arctan(x)\ln\left(\left(x^2+1\right)^n\right)-n\ln\left(x^2+1\right)\arctan(x)-\frac{in\ln(x-i)\ln(x^2+1)}{2}+\frac{in\ln(x-i)^2}{4}+\frac{in\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*(x^2+1)^n)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)*ln((x^2+1)^n)-n*ln(x^2+1)*arctan(x)-1/2*I*n*ln(x-I)*ln(x^2+1)+1/4
*I*n*ln(x-I)^2+1/2*I*n*dilog(-1/2*I*(x+I))+1/2*I*n*ln(x-I)*ln(-1/2*I*(x+I))
+1/2*I*n*ln(x+I)*ln(x^2+1)-1/4*I*n*ln(x+I)^2-1/2*I*n*dilog(1/2*I*(x-I))-1/2
*I*n*ln(x+I)*ln(1/2*I*(x-I))+1/2*I*arctan(x)*Pi*csgn(I*(x^2+1)^n)*csgn(I*c*
(x^2+1)^n)^2-1/2*I*arctan(x)*Pi*csgn(I*(x^2+1)^n)*csgn(I*c*(x^2+1)^n)*csgn(
I*c)-1/2*I*arctan(x)*Pi*csgn(I*c*(x^2+1)^n)^3+1/2*I*arctan(x)*Pi*csgn(I*c*(
x^2+1)^n)^2*csgn(I*c)+ln(c)*arctan(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(c\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*(x**2+1)**n)/(x**2+1),x)
```

```
[Out] Integral(log(c*(x**2 + 1)**n)/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*(x^2+1)^n)/(x^2+1),x, algorithm="giac")
[Out] integrate(log((x^2 + 1)^n*c)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(c\left(x^2+1\right)^n\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(c*(x^2 + 1)^n)/(x^2 + 1),x)
[Out] int(log(c*(x^2 + 1)^n)/(x^2 + 1), x)
```

3.278 $\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx$

Optimal. Leaf size=61

$$i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+i\text{Li}_2\left(-1+\frac{2}{1-ix}\right)$$

```
[Out] I*arctan(x)^2-2*arctan(x)*ln(2-2/(1-I*x))+arctan(x)*ln(x^2/(x^2+1))+I*polyl
og(2,-1+2/(1-I*x))
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {209, 2606, 12, 5044, 4988, 2497}

$$i\text{PolyLog}\left(2,-1+\frac{2}{1-ix}\right)+\text{ArcTan}(x)\log\left(\frac{x^2}{x^2+1}\right)+i\text{ArcTan}(x)^2-2\text{ArcTan}(x)\log\left(2-\frac{2}{1-ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] I*ArcTan[x]^2 - 2*ArcTan[x]*Log[2 - 2/(1 - I*x)] + ArcTan[x]*Log[x^2/(1 + x
^2)] + I*PolyLog[2, -1 + 2/(1 - I*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2606

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
```

```
tegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log\left(\frac{x^2}{1+x^2}\right)}{1+x^2}\,dx &= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-\int \frac{2\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\
&= \tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2\int \frac{\tan^{-1}(x)}{x\left(1+x^2\right)}\,dx\\
&= i\tan^{-1}(x)^2+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)-2i\int \frac{\tan^{-1}(x)}{x(i+x)}\,dx\\
&= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+2\int \frac{\log\left(2-\frac{2}{1-ix}\right.}{1+x^2}\\
&= i\tan^{-1}(x)^2-2\tan^{-1}(x)\log\left(2-\frac{2}{1-ix}\right)+\tan^{-1}(x)\log\left(\frac{x^2}{1+x^2}\right)+i\mathrm{Li}_2\left(-1+\frac{2}{1-i\right.
\end{aligned}
$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 239 vs. $2(61) = 122$.
time = 0.04, size = 239, normalized size = 3.92

$$-\frac{1}{4}i\log^2(i-x)+i\log(i-x)\log(-ix)-\frac{1}{2}i\log(i-x)\log\left(-\frac{1}{2}i(i+x)\right)+\frac{1}{2}i\log\left(-\frac{1}{2}i(i-x)\right)\log(i+x)-i\log(ix)\log(i+x)+\frac{1}{4}i\log^2(i+x)-\frac{1}{2}i\log(i-x)\log\left(\frac{x^2}{1+x^2}\right)+\frac{1}{2}i\log(i+x)\log\left(\frac{x^2}{1+x^2}\right)-\frac{1}{2}i\mathrm{Li}_2\left(-\frac{1}{2}i(i-x)\right)+i\mathrm{Li}_2(-i(i-x))+\frac{1}{2}i\mathrm{Li}_2\left(-\frac{1}{2}i(i+x)\right)-i\mathrm{Li}_2(-i(i+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] (-1/4*I)*Log[I - x]^2 + I*Log[I - x]*Log[(-I)*x] - (I/2)*Log[I - x]*Log[(-1
/2*I)*(I + x)] + (I/2)*Log[(-1/2*I)*(I - x)]*Log[I + x] - I*Log[I*x]*Log[I
+ x] + (I/4)*Log[I + x]^2 - (I/2)*Log[I - x]*Log[x^2/(1 + x^2)] + (I/2)*Log
[I + x]*Log[x^2/(1 + x^2)] - (I/2)*PolyLog[2, (-1/2*I)*(I - x)] + I*PolyLog
[2, (-I)*(I - x)] + (I/2)*PolyLog[2, (-1/2*I)*(I + x)] - I*PolyLog[2, (-I)*
(I + x)]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(57) = 114$.
time = 0.06, size = 146, normalized size = 2.39

method	result
default	$-\frac{i\left(\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)-2\,\mathrm{dilog}(-ix)-2\ln(x-i)\ln(-ix)+\frac{\ln(x-i)^2}{2}+\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)+\ln(x-i)\ln\left(-\frac{i(x+i)}{2}\right)\right)}{2}+\frac{i\left(\ln(x+i)\ln\left(\frac{x}{x^2}\right.\right.}{}$
risch	$-\frac{i\ln(x-i)\ln\left(\frac{x^2}{x^2+1}\right)}{2}+i\,\mathrm{dilog}\left(-ix\right)+i\ln\left(x-i\right)\ln\left(-ix\right)-\frac{i\ln(x-i)^2}{4}-\frac{i\,\mathrm{dilog}\left(-\frac{i(x+i)}{2}\right)}{2}-\frac{i\ln(x-i)\ln\left(-\frac{i(}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2/(x^2+1))/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(ln(x-I)*ln(x^2/(x^2+1))-2*dilog(-I*x)-2*ln(x-I)*ln(-I*x)+1/2*ln(x-I
)^2+dilog(-1/2*I*(x+I))+ln(x-I)*ln(-1/2*I*(x+I)))+1/2*I*(ln(x+I)*ln(x^2/(x^
2+1))-2*dilog(I*x)-2*ln(x+I)*ln(I*x)+1/2*ln(x+I)^2+dilog(1/2*I*(x-I))+ln(x+
I)*ln(1/2*I*(x-I)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2/(x**2+1))/(x**2+1),x)
```

```
[Out] Integral(log(x**2/(x**2 + 1))/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2/(x^2+1))/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln\left(\frac{x^2}{x^2+1}\right)}{x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x^2/(x^2 + 1))/(x^2 + 1),x)
```

```
[Out] int(log(x^2/(x^2 + 1))/(x^2 + 1), x)
```

3.279 $\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx$

Optimal. Leaf size=165

$$\frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{i\text{Li}_2\left(-1+\frac{2\sqrt{\cdots}}{\sqrt{a}-\cdots}\right.}{\sqrt{a}\,\sqrt{b}}$$

[Out] I*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*ln(c*x^2/(b*x^2+a))/a^(1/2)/b^(1/2)-2*arctan(x*b^(1/2)/a^(1/2))*ln(2-2*a^(1/2)/(a^(1/2)-I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*polylog(2,-1+2*a^(1/2)/(a^(1/2)-I*x*b^(1/2)))/a^(1/2)/b^(1/2)

Rubi [A]

time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {211, 2606, 12, 5044, 4988, 2497}

$$\frac{i\text{PolyLog}\left(2,-1+\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{i\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\,x}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (I*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(c*x^2)/(a + b*x^2)])/(Sqrt[a]*Sqrt[b]) - (2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[2 - (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) + (I*PolyLog[2, -1 + (2*Sqrt[a])/(Sqrt[a] - I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 2606

```
Int[Log[(c_.)*(RFx_)^(n_.)]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{u = I
ntHide[1/(d + e*x^2), x]}, Simp[u*Log[c*RFx^n], x] - Dist[n, Int[SimplifyIn
tegrand[u*(D[RFx, x]/RFx), x], x], x]] /; FreeQ[{c, d, e, n}, x] && Rationa
lFunctionQ[RFx, x] &&  !PolynomialQ[RFx, x]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\log\left(\frac{cx^2}{a+bx^2}\right)}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \int \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\,x\,(a+bx^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{(2\sqrt{a}\,)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{x(a+bx^2)}\,dx}{\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{(2i)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{x\left(i+\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}\,dx}{\sqrt{a}\,\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\right.}{\sqrt{a}\,\sqrt{b}}$$

$$= \frac{i\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)^2}{\sqrt{a}\,\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(\frac{cx^2}{a+bx^2}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\log\left(2-\frac{2\sqrt{a}}{\sqrt{a}-i\sqrt{b}\right.}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 373 vs. $2(165) = 330$.
time = 0.14, size = 373, normalized size = 2.26

$$\frac{-4\log\left(\frac{\sqrt{b}\,x}{\sqrt{-a}}\right)\log\left(\sqrt{-a}-\sqrt{b}\,x\right)+\log^2\left(\sqrt{-a}-\sqrt{b}\,x\right)+4\log\left(\frac{a\sqrt{b}\,x}{(-a)^{3/2}}\right)\log\left(\sqrt{-a}+\sqrt{b}\,x\right)-\log^2\left(\sqrt{-a}+\sqrt{b}\,x\right)+2\log\left(\sqrt{-a}-\sqrt{b}\,x\right)\log\left(\frac{a-\sqrt{-a}\,\sqrt{b}\,x}{2a}\right)-2\log\left(\sqrt{-a}+\sqrt{b}\,x\right)\log\left(\frac{a+\sqrt{-a}\,\sqrt{b}\,x}{2a}\right)+2\log\left(\sqrt{-a}-\sqrt{b}\,x\right)\log\left(\frac{cx^2}{a+bx^2}\right)-2\log\left(\sqrt{-a}+\sqrt{b}\,x\right)\log\left(\frac{cx^2}{a+bx^2}\right)+4\mathrm{Li}_2\left(1+\frac{\sqrt{b}\,x}{\sqrt{-a}}\right)-2\mathrm{Li}_2\left(\frac{a-\sqrt{-a}\,\sqrt{b}\,x}{2a}\right)+2\mathrm{Li}_2\left(\frac{a+\sqrt{-a}\,\sqrt{b}\,x}{2a}\right)-4\mathrm{Li}_2\left(1+\frac{a\sqrt{b}\,x}{(-a)^{3/2}}\right)}{4\sqrt{-a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[(c*x^2)/(a + b*x^2)]/(a + b*x^2),x]
```

```
[Out] (-4*Log[(Sqrt[b]*x)/Sqrt[-a]]*Log[Sqrt[-a] - Sqrt[b]*x] + Log[Sqrt[-a] - Sq
rt[b]*x]^2 + 4*Log[(a*Sqrt[b]*x)/(-a)^(3/2)]*Log[Sqrt[-a] + Sqrt[b]*x] - Lo
g[Sqrt[-a] + Sqrt[b]*x]^2 + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(a - Sqrt[-a]*S
qrt[b]*x)/(2*a)] - 2*Log[Sqrt[-a] + Sqrt[b]*x]*Log[(a + Sqrt[-a]*Sqrt[b]*x)
/(2*a)] + 2*Log[Sqrt[-a] - Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] - 2*Log[Sqrt
[-a] + Sqrt[b]*x]*Log[(c*x^2)/(a + b*x^2)] + 4*PolyLog[2, 1 + (Sqrt[b]*x)/S
qrt[-a]] - 2*PolyLog[2, (a - Sqrt[-a]*Sqrt[b]*x)/(2*a)] + 2*PolyLog[2, (a +
 Sqrt[-a]*Sqrt[b]*x)/(2*a)] - 4*PolyLog[2, 1 + (a*Sqrt[b]*x)/(-a)^(3/2)])/(
4*Sqrt[-a]*Sqrt[b])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(\frac{c\,x^{2}}{b\,x^{2}+a}\right)}{b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)
[Out] int(ln(c*x^2/(b*x^2+a))/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="fricas")
[Out] integral(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\frac{cx^{2}}{a+bx^{2}}\right)}{a+bx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(c*x**2/(b*x**2+a))/(b*x**2+a),x)
[Out] Integral(log(c*x**2/(a + b*x**2))/(a + b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x^2/(b*x^2+a))/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(log(c*x^2/(b*x^2 + a))/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\ln\left(\frac{c\,x^2}{b\,x^2+a}\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2),x)
```

```
[Out] int(log((c*x^2)/(a + b*x^2))/(a + b*x^2), x)
```

3.280 $\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\operatorname{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] polylog(2,-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2598}

$$\frac{\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\operatorname{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.
time = 0.48, size = 134, normalized size = 4.62

$$\frac{4\tanh^{-1}(ax)\log\left(1+\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\operatorname{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-2\left(\tanh^{-1}(ax)\left(\log\left(1+e^{-2\tanh^{-1}(ax)}\right)-\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)-\operatorname{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)+\operatorname{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 + (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] - Log[1 -
I/E^ArcTanh[a*x]] + Log[1 + I/E^ArcTanh[a*x]]) - PolyLog[2, (-I)/E^ArcTanh[
a*x]] + PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1+\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^{2}x^{2}+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) + I*sqrt(-a*x + 1)))/a - integrate(-1/2*sqrt(a*x + 1)*
(log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) - (-I*a^2*x^2 +
 I)*sqrt(-a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 37, normalized size = 1.28

$$\frac{\operatorname{Li}_2\left(-\frac{ax-\sqrt{ax+1}\,\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x - sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\ln\left(1+\frac{\sqrt{1-a\,x}\,1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1),x)
```

```
[Out] int(-log(((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2) + 1)/(a^2*x^2 - 1), x)
```

3.281 $\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] polylog(2,I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = $\{2598\}$

$$\frac{\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rubi steps

$$\int \frac{\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. $2(29) = 58$.
time = 0.39, size = 134, normalized size = 4.62

$$\frac{4\tanh^{-1}(ax)\log\left(1-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{Li}_2\left(-e^{-2\tanh^{-1}(ax)}\right)-2\left(\tanh^{-1}(ax)\left(\log\left(1+e^{-2\tanh^{-1}(ax)}\right)+\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)+\text{Li}_2\left(-ie^{-\tanh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\tanh^{-1}(ax)}\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (4*ArcTanh[a*x]*Log[1 - (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + PolyLog[2, -E^(-
2*ArcTanh[a*x])] - 2*(ArcTanh[a*x]*(Log[1 + E^(-2*ArcTanh[a*x])] + Log[1 -
I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/(4*a)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\ln\left(1-\frac{i\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

```
[Out] int(ln(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
maxima")
```

```
[Out] -1/8*(2*(log(a*x + 1) - log(-a*x + 1))*log(a*x + 1) - log(a*x + 1)^2 + 2*lo
g(a*x + 1)*log(-a*x + 1) - log(-a*x + 1)^2 - 4*(log(a*x + 1) - log(-a*x + 1
))*log(sqrt(a*x + 1) - I*sqrt(-a*x + 1)))/a + integrate(1/2*sqrt(a*x + 1)*(
log(a*x + 1) - log(-a*x + 1))/((a^2*x^2 - 1)*sqrt(a*x + 1) + (-I*a^2*x^2 +
I)*sqrt(-a*x + 1)), x)
```

Fricas [A]
time = 0.38, size = 36, normalized size = 1.24

$$\frac{\mathrm{Li}_2\left(-\frac{ax+\sqrt{ax+1}\,\sqrt{ax-1}+1}{ax+1}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
fricas")
```

```
[Out] dilog(-(a*x + sqrt(a*x + 1)*sqrt(a*x - 1) + 1)/(a*x + 1) + 1)/a
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-I*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-I*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="
giac")
```

```
[Out] integrate(-log(-I*sqrt(-a*x + 1)/sqrt(a*x + 1) + 1)/(a^2*x^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int -\frac{\ln\left(1-\frac{\sqrt{1-a\,x}\;1\mathrm{i}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

```
[Out] int(-log(1 - ((1 - a*x)^(1/2)*1i)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.282 $\int \log \left(e^{a+bx}\right) \, dx$

Optimal. Leaf size=17

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

```
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2188, 30}

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2188

```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Dist[1/c, Subst[
Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log \left(e^{a+bx}\right) \, dx &= \frac{\text{Subst}\left(\int x \, dx, x, \log \left(e^{a+bx}\right)\right)}{b}\\ &= \frac{\log^2\left(e^{a+bx}\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log^2\left(e^{a+bx}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x)],x]
```

```
[Out] Log[E^(a + b*x)]^2/(2*b)
```

Maple [A]
time = 0.03, size = 15, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
default	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
norman	$\frac{\ln\left(\mathrm{e}^{bx+a}\right)^{2}}{2b}$	15
risch	$x\ln\left(\mathrm{e}^{bx+a}\right)-\frac{b\,x^{2}}{2}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(exp(b*x+a))^2/b
```

Maxima [A]
time = 0.31, size = 10, normalized size = 0.59

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.41, size = 10, normalized size = 0.59

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.47

$$ax+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(b*x+a)),x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A]
time = 3.15, size = 10, normalized size = 0.59

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(b*x+a)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.07, size = 17, normalized size = 1.00

$$x\,\ln\left(\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\right)-\frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(a + b*x)),x)
```

```
[Out] x*log(exp(b*x)*exp(a)) - (b*x^2)/2
```

3.283 $\int \log \left(e^{a+bx^n}\right) \, dx$

Optimal. Leaf size=27

$$-\frac{bnx^{1+n}}{1+n}+x\log \left(e^{a+bx^n}\right)$$

```
[Out] -b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2628, 12, 30}

$$x\log \left(e^{a+bx^n}\right)-\frac{bnx^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^(a + b*x^n)],x]
```

```
[Out] -((b*n*x^(1 + n))/(1 + n)) + x*Log[E^(a + b*x^n)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \log \left(e^{a+bx^n}\right) \, dx &= x\log \left(e^{a+bx^n}\right)-\int bnx^n \, dx\\
&= x\log \left(e^{a+bx^n}\right)-(bn)\int x^n \, dx\\
&= -\frac{bnx^{1+n}}{1+n}+x\log \left(e^{a+bx^n}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.93

$$x\left(-\frac{bnx^n}{1+n}+\log\left(e^{a+bx^n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^(a + b*x^n)],x]
```

```
[Out] x*(-((b*n*x^n)/(1 + n)) + Log[E^(a + b*x^n)])
```

Maple [A]
time = 0.02, size = 27, normalized size = 1.00

method	result	size
risch	$x\ln\left(\mathrm{e}^{a+b\,x^n}\right)-\frac{bnx\,x^n}{1+n}$	26
default	$-\frac{bn\,x^{1+n}}{1+n}+x\ln\left(\mathrm{e}^{a+b\,x^n}\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(a+b*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] -b*n*x^(1+n)/(1+n)+x*ln(exp(a+b*x^n))
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.59

$$ax+\frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="maxima")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.41, size = 20, normalized size = 0.74

$$\frac{bxx^n+(an+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

time = 0.47, size = 65, normalized size = 2.41

$$\begin{cases} -\frac{bnxx^n}{n+1} + \frac{nx\log\left(e^a e^{bx^n}\right)}{n+1} + \frac{x\log\left(e^a e^{bx^n}\right)}{n+1} & \text{for } n \neq -1 \\ b\log(x) + x\log\left(e^a e^{\frac{b}{x}}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(a+b*x**n)),x)
```

```
[Out] Piecewise((-b*n*x*x**n/(n + 1) + n*x*log(exp(a)*exp(b*x**n))/(n + 1) + x*lo
g(exp(a)*exp(b*x**n))/(n + 1), Ne(n, -1)), (b*log(x) + x*log(exp(a)*exp(b/x
)), True))
```

Giac [A]

time = 2.96, size = 16, normalized size = 0.59

$$ax + \frac{bx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(a+b*x^n)),x, algorithm="giac")
```

```
[Out] a*x + b*x^(n + 1)/(n + 1)
```

Mupad [B]

time = 0.60, size = 49, normalized size = 1.81

$$\begin{cases} x\ln\left(\mathrm{e}^{a+\frac{b}{x}}\right) + b\ln(x) & \text{if } n = -1 \\ x\ln\left(\mathrm{e}^{a+b\,x^n}\right) - \frac{b\,n\,x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(a + b*x^n)),x)
```

```
[Out] piecewise(n == -1, x*log(exp(a + b/x)) + b*log(x), n ~= -1, x*log(exp(a + b
*x^n)) - (b*n*x^(n + 1))/(n + 1))
```

3.284 $\int e^x \log(a+be^x)\,dx$

Optimal. Leaf size=25

$$-e^x+\frac{(a+be^x)\log(a+be^x)}{b}$$

```
[Out] -exp(x)+(a+b*exp(x))*ln(a+b*exp(x))/b
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {2225, 2634, 12, 2280, 45}

$$e^x\log(a+be^x)+\frac{a\log(a+be^x)}{b}-e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + (a*Log[a + b*E^x])/b + E^x*Log[a + b*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2280

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Dist[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F])), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/Den
ominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int e^x \log(a+be^x)\,dx &= e^x \log(a+be^x) - \int \frac{be^{2x}}{a+be^x}\,dx \\ &= e^x \log(a+be^x) - b\int \frac{e^{2x}}{a+be^x}\,dx \\ &= e^x \log(a+be^x) - b\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, e^x\right) \\ &= e^x \log(a+be^x) - b\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)\,dx, x, e^x\right) \\ &= -e^x + \frac{a\log(a+be^x)}{b} + e^x \log(a+be^x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-e^x + \frac{(a+be^x)\log(a+be^x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[a + b*E^x],x]
```

```
[Out] -E^x + ((a + b*E^x)*Log[a + b*E^x])/b
```

Maple [A]
time = 0.02, size = 28, normalized size = 1.12

method	result	size
derivativedivides	$\frac{(a+b\,e^x)\ln(a+b\,e^x)-b\,e^x-a}{b}$	28
default	$\frac{(a+b\,e^x)\ln(a+b\,e^x)-b\,e^x-a}{b}$	28
norman	$-e^x+\frac{a\ln(a+b\,e^x)}{b}+e^x\ln(a+b\,e^x)$	28
risch	$e^x\ln(a+b\,e^x)+\frac{a\ln\left(e^x+\frac{a}{b}\right)}{b}-e^x$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((a+b*exp(x))*ln(a+b*exp(x))-b*exp(x)-a)
```

Maxima [A]
time = 0.30, size = 26, normalized size = 1.04

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="maxima")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Fricas [A]
time = 0.37, size = 25, normalized size = 1.00

$$-\frac{be^x - (be^x + a)\log(be^x + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="fricas")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(a+b*exp(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.38, size = 26, normalized size = 1.04

$$-\frac{be^x - (be^x + a)\log(be^x + a) + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(a+b*exp(x)),x, algorithm="giac")
```

```
[Out] -(b*e^x - (b*e^x + a)*log(b*e^x + a) + a)/b
```

Mupad [B]
time = 0.66, size = 27, normalized size = 1.08

$$e^x \ln(a + b e^x) - e^x + \frac{a \ln(a + b e^x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*log(a + b*exp(x)),x)
[Out] exp(x)*log(a + b*exp(x)) - exp(x) + (a*log(a + b*exp(x)))/b
```

3.285 $\int e^{a+bx} \log(x) \, dx$

Optimal. Leaf size=26

$$-\frac{e^a \text{Ei}(bx)}{b} + \frac{e^{a+bx} \log(x)}{b}$$

```
[Out] -exp(a)*Ei(b*x)/b+exp(b*x+a)*ln(x)/b
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2225, 2634, 12, 2209}

$$\frac{\log(x) e^{a+bx}}{b} - \frac{e^a \text{Ei}(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a + b*x)*Log[x],x]
```

```
[Out] -((E^a*ExpIntegralEi[b*x])/b) + (E^(a + b*x)*Log[x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2634

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*(D[u, x]/u), x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int e^{a+bx}\log(x)\,dx = \frac{e^{a+bx}\log(x)}{b} - \int \frac{e^{a+bx}}{bx}\,dx$$
$$= \frac{e^{a+bx}\log(x)}{b} - \frac{\int \frac{e^{a+bx}}{x}\,dx}{b}$$
$$= -\frac{e^a\text{Ei}(bx)}{b} + \frac{e^{a+bx}\log(x)}{b}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.85

$$\frac{e^a\left(-\text{Ei}(bx) + e^{bx}\log(x)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a + b*x)*Log[x],x]
[Out] (E^a*(-ExpIntegralEi[b*x] + E^(b*x)*Log[x]))/b
```

Maple [A]
time = 0.02, size = 26, normalized size = 1.00

method	result	size
risch	$\frac{\mathrm{e}^{bx+a}\ln(x)}{b} + \frac{\mathrm{e}^{a}\,\text{expIntegral}(1,-bx)}{b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(b*x+a)*ln(x),x,method=_RETURNVERBOSE)
[Out] exp(b*x+a)*ln(x)/b+1/b*exp(a)*Ei(1,-b*x)
```

Maxima [A]
time = 0.34, size = 24, normalized size = 0.92

$$-\frac{\text{Ei}(bx)\,e^a}{b} + \frac{e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="maxima")
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Fricas [A]
time = 0.38, size = 23, normalized size = 0.88

$$-\frac{\text{Ei}(bx)\,e^a - e^{(bx+a)}\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -(Ei(b*x)*e^a - e^(b*x + a)*log(x))/b
```

Sympy [A]
time = 3.46, size = 26, normalized size = 1.00

$$\left(\begin{cases} x & \text{for}\: b = 0 \\ \frac{e^{bx}}{b} & \text{otherwise} \end{cases}\right) e^{a} \log{\left(x \right)} - \left(\begin{cases} x & \text{for}\: b = 0 \\ \frac{\operatorname{Ei}{\left(bx \right)}}{b} & \text{otherwise} \end{cases}\right) e^{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*ln(x),x)
```

```
[Out] Piecewise((x, Eq(b, 0)), (exp(b*x)/b, True))*exp(a)*log(x) - Piecewise((x,
Eq(b, 0)), (Ei(b*x)/b, True))*exp(a)
```

Giac [A]
time = 4.31, size = 24, normalized size = 0.92

$$-\frac{\text{Ei}(bx)\, e^{a}}{b} + \frac{e^{(bx+a)} \log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] -Ei(b*x)*e^a/b + e^(b*x + a)*log(x)/b
```

Mupad [B]
time = 0.37, size = 20, normalized size = 0.77

$$-\frac{\mathrm{e}^a\,\left(\mathrm{ei}(b\,x)-\mathrm{e}^{b\,x}\,\ln\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a + b*x)*log(x),x)
```

```
[Out] -(exp(a)*(ei(b*x) - exp(b*x)*log(x)))/b
```

3.286 $\int \frac{x^2}{x+\log(x)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x^2}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x^2/(x+ln(x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2/(x + Log[x]),x]
[Out] Defer[Int][x^2/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x^2}{x+\log(x)}\,dx = \int \frac{x^2}{x+\log(x)}\,dx$$

Mathematica [A]
time = 14.94, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/(x + Log[x]),x]
[Out] Integrate[x^2/(x + Log[x]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x+ln(x)),x)
[Out] int(x^2/(x+ln(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="maxima")
[Out] integrate(x^2/(x + log(x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(x^2/(x + log(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x + \log(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x+ln(x)),x)
[Out] Integral(x**2/(x + log(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(x^2/(x + log(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x^2}{x + \ln(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x + log(x)),x)
[Out] int(x^2/(x + log(x)), x)
```

3.287 $\int \frac{x}{x+\log(x)}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{x}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(x/(x+ln(x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(x + Log[x]),x]
```

```
[Out] Defer[Int][x/(x + Log[x]), x]
```

Rubi steps

$$\int \frac{x}{x+\log(x)}\,dx = \int \frac{x}{x+\log(x)}\,dx$$

Mathematica [A]
time = 9.03, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(x + Log[x]),x]
```

```
[Out] Integrate[x/(x + Log[x]), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+ln(x)),x)
```

```
[Out] int(x/(x+ln(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x + log(x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(x/(x + log(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x+\log(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+ln(x)),x)
```

```
[Out] Integral(x/(x + log(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(x/(x + log(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{x+\ln(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + log(x)),x)
[Out] int(x/(x + log(x)), x)
```

3.288 $\int \frac{1}{x+\log(x)}\,dx$

Optimal. Leaf size=9

$$\mathrm{Int}\left(\frac{1}{x+\log(x)},x\right)$$

```
[Out] CannotIntegrate(1/(x+ln(x)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x + Log[x])^(-1),x]
[Out] Defer[Int][(x + Log[x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{x+\log(x)}\,dx=\int \frac{1}{x+\log(x)}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\log(x)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x + Log[x])^(-1),x]
[Out] Integrate[(x + Log[x])^(-1), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\ln(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+ln(x)),x)
```

```
[Out] int(1/(x+ln(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + log(x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(x + log(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \log{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+ln(x)),x)
```

```
[Out] Integral(1/(x + log(x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+log(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(x + log(x)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{x+\ln\left(x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + log(x)),x)
[Out] int(1/(x + log(x)), x)
```

3.289 $\int \frac{1}{x(x+\log(x))} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate(1/x/(x+ln(x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(x+\log(x))} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(x + Log[x])),x]
[Out] Defer[Int][1/(x*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x(x+\log(x))} dx = \int \frac{1}{x(x+\log(x))} dx$$

Mathematica [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x+\log(x))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(x + Log[x])),x]
[Out] Integrate[1/(x*(x + Log[x])), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(x+\ln(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x+ln(x)),x)
[Out] int(1/x/(x+ln(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="maxima")
[Out] integrate(1/((x + log(x))*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^2 + x*log(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x + \log\left(x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+ln(x)),x)
[Out] Integral(1/(x*(x + log(x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x\,\left(x+\ln\left(x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x + log(x))),x)
[Out] int(1/(x*(x + log(x))), x)
```

3.290 $\int \frac{1}{x^2(x+\log(x))} dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x^2(x+\log(x))},x\right)$$

```
[Out] CannotIntegrate(1/x^2/(x+ln(x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(x+\log(x))} dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(x + Log[x])),x]
```

```
[Out] Defer[Int][1/(x^2*(x + Log[x])), x]
```

Rubi steps

$$\int \frac{1}{x^2(x+\log(x))} dx = \int \frac{1}{x^2(x+\log(x))} dx$$

Mathematica [A]
time = 17.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(x+\log(x))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(x + Log[x])),x]
```

```
[Out] Integrate[1/(x^2*(x + Log[x])), x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(x+\ln(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x+ln(x)),x)
[Out] int(1/x^2/(x+ln(x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="maxima")
[Out] integrate(1/((x + log(x))*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="fricas")
[Out] integral(1/(x^3 + x^2*log(x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (x + \log(x))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x+ln(x)),x)
[Out] Integral(1/(x**2*(x + log(x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x+log(x)),x, algorithm="giac")
[Out] integrate(1/((x + log(x))*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x^2\,\left(x + \ln(x)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x + log(x))),x)
[Out] int(1/(x^2*(x + log(x))), x)
```

3.291 $\int \frac{\log(x)}{x+4x\log^2(x)}\,dx$

Optimal. Leaf size=13

$$\frac{1}{8}\log\left(1+4\log^2(x)\right)$$

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {209, 266}

$$\frac{1}{8}\log\left(4\log^2(x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x + 4*x*Log[x]^2),x]
```

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{x+4x\log^2(x)}\,dx &= \text{Subst}\left(\int \frac{x}{1+4x^2}\,dx, x, \log(x)\right)\\ &= \frac{1}{8}\log\left(1+4\log^2(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{8}\log\left(1+4\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x + 4*x*Log[x]^2),x]
```

```
[Out] Log[1 + 4*Log[x]^2]/8
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.92

method	result	size
risch	$\frac{\ln\left(\ln(x)^2+\frac{1}{4}\right)}{8}$	10
default	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12
norman	$\frac{\ln\left(1+4\ln(x)^2\right)}{8}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(x+4*x*ln(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(1+4*ln(x)^2)
```

Maxima [A]
time = 0.32, size = 9, normalized size = 0.69

$$\frac{1}{8}\log\left(\log\left(x\right)^2+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="maxima")
```

```
[Out] 1/8*log(log(x)^2 + 1/4)
```

Fricas [A]
time = 0.36, size = 11, normalized size = 0.85

$$\frac{1}{8}\log\left(4\log\left(x\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="fricas")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

Sympy [A]
time = 0.09, size = 10, normalized size = 0.77

$$\frac{\log\left(\log\left(x\right)^2+\frac{1}{4}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(x+4*x*ln(x)**2),x)
```

```
[Out] log(log(x)**2 + 1/4)/8
```

Giac [A]
time = 3.65, size = 11, normalized size = 0.85

$$\frac{1}{8}\,\log\left(4\,\log\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(x+4*x*log(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*log(4*log(x)^2 + 1)
```

Mupad [B]
time = 0.46, size = 11, normalized size = 0.85

$$\frac{\ln\left(4\ln\left(x\right)^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(x + 4*x*log(x)^2),x)
```

```
[Out] log(4*log(x)^2 + 1)/8
```

3.292 $\int \frac{1-\log(x)}{x(x+\log(x))}\,dx$

Optimal. Leaf size=9

$$\log\left(1+\frac{\log(x)}{x}\right)$$

```
[Out] ln(1+ln(x)/x)
```

Rubi [A]
time = 0.05, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6844, 31}

$$\log\left(\frac{\log(x)}{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] Log[1 + Log[x]/x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6844

```
Int[(u_)*(v_)^(r_.)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol]
:> With[{c = Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[(-c)*q, Subst[I
nt[(a + b*x^q)^m, x], x, v^(m*p + r + 1)*w], x] /; FreeQ[c, x]] /; FreeQ[{a
, b, m, p, q, r}, x] && EqQ[p + q*(m*p + r + 1), 0] && IntegerQ[q] && Integ
erQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\log(x)}{x(x+\log(x))}\,dx &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\log(x)}{x}\right)\\ &= \log\left(1+\frac{\log(x)}{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.11

$$-\log(x)+\log(x+\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Log[x])/(x*(x + Log[x])),x]
```

```
[Out] -Log[x] + Log[x + Log[x]]
```

Maple [A]
time = 0.08, size = 11, normalized size = 1.22

method	result	size
default	$-\ln(x)+\ln(x+\ln(x))$	11
norman	$-\ln(x)+\ln(x+\ln(x))$	11
risch	$-\ln(x)+\ln(x+\ln(x))$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-ln(x))/x/(x+ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(x+ln(x))
```

Maxima [A]
time = 0.30, size = 10, normalized size = 1.11

$$\log(x+\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="maxima")
```

```
[Out] log(x + log(x)) - log(x)
```

Fricas [A]
time = 0.37, size = 10, normalized size = 1.11

$$\log(x+\log(x))-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="fricas")
```

```
[Out] log(x + log(x)) - log(x)
```

Sympy [A]
time = 0.10, size = 8, normalized size = 0.89

$$-\log(x)+\log(x+\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-ln(x))/x/(x+ln(x)),x)
```

```
[Out] -log(x) + log(x + log(x))
```

Giac [A]
time = 4.59, size = 14, normalized size = 1.56

$$-\log(x) + \log(-x - \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-log(x))/x/(x+log(x)),x, algorithm="giac")
```

```
[Out] -log(x) + log(-x - log(x))
```

Mupad [B]
time = 0.37, size = 10, normalized size = 1.11

$$\ln(x + \ln(x)) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(log(x) - 1)/(x*(x + log(x))),x)
```

```
[Out] log(x + log(x)) - log(x)
```

3.293 $\int \frac{1+x}{\log(x)(x+\log(x))} \, dx$

Optimal. Leaf size=13

$$\log(\log(x)) - \log(x + \log(x)) + \text{li}(x)$$

[Out] Li(x)+ln(ln(x))-ln(x+ln(x))

Rubi [A]
time = 0.09, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6874, 2395, 2335, 2339, 29, 6816}

$$\text{li}(x) + \log(\log(x)) - \log(x + \log(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(Log[x]*(x + Log[x])),x]

[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{\log(x)(x+\log(x))}\,dx &= \int \left(\frac{1+x}{x\log(x)}+\frac{-1-x}{x(x+\log(x))}\right)dx\\
&= \int \frac{1+x}{x\log(x)}\,dx+\int \frac{-1-x}{x(x+\log(x))}\,dx\\
&= -\log(x+\log(x))+\int \left(\frac{1}{\log(x)}+\frac{1}{x\log(x)}\right)dx\\
&= -\log(x+\log(x))+\int \frac{1}{\log(x)}\,dx+\int \frac{1}{x\log(x)}\,dx\\
&= -\log(x+\log(x))+\mathrm{li}(x)+\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,\log(x)\right)\\
&= \log(\log(x))-\log(x+\log(x))+\mathrm{li}(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$\log(\log(x))-\log(x+\log(x))+\mathrm{li}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(Log[x]*(x + Log[x])),x]
[Out] Log[Log[x]] - Log[x + Log[x]] + LogIntegral[x]
```

Maple [A]
time = 0.09, size = 20, normalized size = 1.54

method	result	size
default	$-\mathrm{expIntegral}\left(1,-\ln\left(x\right)\right)+\ln\left(\ln\left(x\right)\right)-\ln\left(x+\ln\left(x\right)\right)$	20
risch	$-\mathrm{expIntegral}\left(1,-\ln\left(x\right)\right)+\ln\left(\ln\left(x\right)\right)-\ln\left(x+\ln\left(x\right)\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/ln(x)/(x+ln(x)),x,method=_RETURNVERBOSE)
[Out] -Ei(1,-ln(x))+ln(ln(x))-ln(x+ln(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="maxima")
[Out] integrate((x + 1)/(x*log(x)), x) - log(x + log(x))
```

Fricas [A]
time = 0.40, size = 13, normalized size = 1.00

$$-\log\left(x+\log\left(x\right)\right)+\log\left(\log\left(x\right)\right)+\mathrm{log\_integral}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="fricas")
[Out] -log(x + log(x)) + log(log(x)) + log_integral(x)
```

Sympy [A]
time = 2.49, size = 15, normalized size = 1.15

$$-\log\left(x+\log\left(x\right)\right)+\log\left(\log\left(x\right)\right)+\mathrm{Ei}\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/ln(x)/(x+ln(x)),x)
[Out] -log(x + log(x)) + log(log(x)) + Ei(log(x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/log(x)/(x+log(x)),x, algorithm="giac")
[Out] integrate((x + 1)/((x + log(x))*log(x)), x)
```

Mupad [B]
time = 0.39, size = 13, normalized size = 1.00

$$\ln\left(\ln\left(x\right)\right)-\ln\left(x+\ln\left(x\right)\right)+\mathrm{logint}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(log(x)*(x + log(x))),x)
[Out] log(log(x)) - log(x + log(x)) + logint(x)
```

3.294 $\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=67

$$-\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right)+x\log\left(2+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] -1/6*ln(1-(1+1/x)^(1/2))+1/2*ln(1+(1+1/x)^(1/2))-1/3*ln(2+(1+1/x)^(1/2))+x*
ln(2+((1+x)/x)^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2628, 12, 2083}

$$-\frac{1}{6}\log\left(1-\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2}\log\left(\sqrt{\frac{1}{x}+1}+1\right)-\frac{1}{3}\log\left(\sqrt{\frac{1}{x}+1}+2\right)+x\log\left(\sqrt{\frac{x+1}{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/6*Log[1 - Sqrt[1 + x^(-1)]] + Log[1 + Sqrt[1 + x^(-1)]]/2 - Log[2 + Sqrt
[1 + x^(-1)]]/3 + x*Log[2 + Sqrt[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(2+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(2+\sqrt{\frac{1+x}{x}}\right) - \int \frac{1}{2\left(-1-x-2x\sqrt{\frac{1+x}{x}}\right)}\,dx$$

$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) - \frac{1}{2}\int \frac{1}{-1-x-2x\sqrt{\frac{1+x}{x}}}\,dx$$

$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) + \text{Subst}\left(\int \frac{1}{2+x-2x^2-x^3}\,dx, x, \sqrt{\frac{1+x}{x}}\right)$$

$$= x\log\left(2+\sqrt{\frac{1+x}{x}}\right) + \text{Subst}\left(\int \left(-\frac{1}{6(-1+x)} + \frac{1}{2(1+x)} - \frac{1}{3(2+x)}\right)\,dx$$

$$= -\frac{1}{6}\log\left(1-\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2}\log\left(1+\sqrt{1+\frac{1}{x}}\right) - \frac{1}{3}\log\left(2+\sqrt{1+\frac{1}{x}}\right) +$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.79

$$\frac{1}{3}\tanh^{-1}\left(\frac{1}{3}\left(1+2\sqrt{1+\frac{1}{x}}\right)\right) - \tanh^{-1}\left(3+2\sqrt{1+\frac{1}{x}}\right) + x\log\left(2+\sqrt{1+\frac{1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] ArcTanh[(1 + 2*Sqrt[1 + x^(-1)])/3]/3 - ArcTanh[3 + 2*Sqrt[1 + x^(-1)]] + x
*Log[2 + Sqrt[1 + x^(-1)]]
```

Maple [A]
time = 0.03, size = 107, normalized size = 1.60

method	result
default	$x\ln\left(2+\sqrt{\frac{1+x}{x}}\right) - \frac{3\sqrt{\frac{1+x}{x}}\,x\ln(-3x+1)+\sqrt{9}\,\ln\left(\frac{4\sqrt{9}\,\sqrt{x^2+x}+15x+3}{9x-3}\right)\sqrt{x(1+x)}-6\ln\left(\frac{1}{2}+x+\sqrt{x^2+\right.}}{18\sqrt{\frac{1+x}{x}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(2+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(2+((1+x)/x)^(1/2))-1/18/((1+x)/x)^(1/2)/x*(3*((1+x)/x)^(1/2)*x*ln(-3*x
+1)+9^(1/2)*ln(1/3*(4*9^(1/2)*(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1/2
)-6*ln(1/2+x+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Maxima [A]
time = 0.31, size = 67, normalized size = 1.00

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+2\right)}{\frac{x+1}{x}-1}-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 2)/((x + 1)/x - 1) - 1/3*log(sqrt((x + 1)/x) + 2) + 1
/2*log(sqrt((x + 1)/x) + 1) - 1/6*log(sqrt((x + 1)/x) - 1)
```

Fricas [A]
time = 0.39, size = 48, normalized size = 0.72

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}+2\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) + 2) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/6
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A]
time = 68.53, size = 53, normalized size = 0.79

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{6}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - log(sqrt(1 + 1/x) - 1)/6 + log(sqrt(1 + 1/x) +
 1)/2 - log(sqrt(1 + 1/x) + 2)/3
```

Giac [A]
time = 5.15, size = 88, normalized size = 1.31

$$x\log\left(\sqrt{\frac{x+1}{x}}+2\right)-\frac{\log\left(\left|-x+\sqrt{x^2+x}+1\right|\right)}{6\,\mathrm{sgn}(x)}-\frac{\log\left(\left|-2x+2\sqrt{x^2+x}-1\right|\right)}{3\,\mathrm{sgn}(x)}+\frac{\log\left(\left|-3x+3\sqrt{x^2+x}-1\right|\right)}{6\,\mathrm{sgn}(x)}-\frac{1}{6}\log\left(\left|3x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 2) - 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) -
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) + 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B]
time = 0.64, size = 63, normalized size = 0.94

$$\frac{\ln\left(-5\sqrt{\frac{x+1}{x}}-5\right)}{2}-\frac{\ln\left(\frac{\sqrt{\frac{x+1}{x}}}{9}-\frac{1}{9}\right)}{6}-\frac{\ln\left(-\frac{5\sqrt{\frac{x+1}{x}}}{9}-\frac{10}{9}\right)}{3}+x\ln\left(\sqrt{\frac{x+1}{x}}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) + 2),x)
```

```
[Out] log(- 5*((x + 1)/x)^(1/2) - 5)/2 - log(((x + 1)/x)^(1/2)/9 - 1/9)/6 - log(-
 (5*((x + 1)/x)^(1/2))/9 - 10/9)/3 + x*log(((x + 1)/x)^(1/2) + 2)
```

3.295 $\int \log\left(1+\sqrt{\frac{1+x}{x}}\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)}+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1+x}{x}}\right)+x\log\left(1+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] 1/2*arctanh(((1+x)/x)^(1/2))+x*ln(1+((1+x)/x)^(1/2))-1/2/(1+(1+1/x)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2628, 12, 46, 213}

$$-\frac{1}{2\left(\sqrt{\frac{1}{x}+1}+1\right)}+x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/2*1/(1 + Sqrt[1 + x^(-1)]) + ArcTanh[Sqrt[(1 + x)/x]]/2 + x*Log[1 + Sqrt
[(1 + x)/x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(1+\sqrt{\frac{1+x}{x}}\right) dx = x\log\left(1+\sqrt{\frac{1+x}{x}}\right) - \int \frac{1}{2\left(-1-x-x\sqrt{\frac{1+x}{x}}\right)} dx$$

$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right) - \frac{1}{2}\int \frac{1}{-1-x-x\sqrt{\frac{1+x}{x}}} dx$$

$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)(1+x)^2} dx, x, \sqrt{\frac{1+x}{x}}\right)$$

$$= x\log\left(1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \left(-\frac{1}{2(1+x)^2} + \frac{1}{2(-1+x^2)}\right) dx, x, \sqrt{\frac{1+}{x}}\right.$$

$$= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)} + x\log\left(1+\sqrt{\frac{1+x}{x}}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{\right.$$

$$= -\frac{1}{2\left(1+\sqrt{1+\frac{1}{x}}\right)} + \frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1+x}{x}}\right) + x\log\left(1+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.06

$$\frac{1}{4}\left(2x - 2\sqrt{1+\frac{1}{x}}\,x + 4x\log\left(1+\sqrt{1+\frac{1}{x}}\right) + \log\left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + Sqrt[(1 + x)/x]],x]
[Out] (2*x - 2*Sqrt[1 + x^(-1)]*x + 4*x*Log[1 + Sqrt[1 + x^(-1)]] + Log[1 + (2 +
2*Sqrt[1 + x^(-1)])*x])/4
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \ln\left(1+\sqrt{\frac{1+x}{x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+((1+x)/x)^(1/2)),x)
```

```
[Out] int(ln(1+((1+x)/x)^(1/2)),x)
```

Maxima [A]
time = 0.33, size = 68, normalized size = 1.36

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}+1\right)}{\frac{x+1}{x}-1}-\frac{1}{2\left(\sqrt{\frac{x+1}{x}}+1\right)}+\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) + 1)/((x + 1)/x - 1) - 1/2/(sqrt((x + 1)/x) + 1) + 1/4*
log(sqrt((x + 1)/x) + 1) - 1/4*log(sqrt((x + 1)/x) - 1)
```

Fricas [A]
time = 0.43, size = 49, normalized size = 0.98

$$\frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\,x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\,x-\frac{1}{4}\,\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) + 1) - 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) - 1)
```

Sympy [A]
time = 66.92, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}+\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}-\frac{1}{2\left(\sqrt{1+\frac{1}{x}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) + 1) - log(sqrt(1 + 1/x) - 1)/4 + log(sqrt(1 + 1/x) +
 1)/4 - 1/(2*(sqrt(1 + 1/x) + 1))
```

Giac [A]
time = 5.10, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\,x-\frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{4\,\mathrm{sgn}\left(x\right)}-\frac{\sqrt{x^2+x}}{2\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) + 1) + 1/2*x - 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) - 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B]
time = 0.45, size = 38, normalized size = 0.76

$$\frac{x}{2}+\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\,\ln\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{x\,\sqrt{\frac{1}{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) + 1),x)
```

```
[Out] x/2 + atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) + 1) - (x*(1/x + 1
)^(1/2))/2
```

3.296 $\int \log \left(\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=21

$$x \log \left(\sqrt{1+\frac{1}{x}}\right)+\frac{1}{2} \log(1+x)$$

```
[Out] 1/2*ln(1+x)+1/2*x*ln(1+1/x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2503, 2498, 269, 31}

$$x \log \left(\sqrt{\frac{1}{x}+1}\right)+\frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[(1 + x)/x]],x]
```

```
[Out] x*Log[Sqrt[1 + x^(-1)]] + Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2503

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.), x_Symbol] :> Int[(a + b*Lo
g[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b, c, p, q}, x] && BinomialQ[v
, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\int \log\left(\sqrt{\frac{1+x}{x}}\right) dx = \int \log\left(\sqrt{1+\frac{1}{x}}\right) dx$$
$$= x\log\left(\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)x} dx$$
$$= x\log\left(\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2}\int \frac{1}{1+x} dx$$
$$= x\log\left(\sqrt{1+\frac{1}{x}}\right) + \frac{1}{2}\log(1+x)$$

Mathematica [**A**]
time = 0.00, size = 19, normalized size = 0.90

$$\frac{1}{2}\left(\log(x) + (1+x)\log\left(\frac{1+x}{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[(1 + x)/x]],x]
[Out] (Log[x] + (1 + x)*Log[(1 + x)/x])/2
```

Maple [**A**]
time = 0.04, size = 22, normalized size = 1.05

method	result	size
risch	$\frac{x\ln\left(\frac{1+x}{x}\right)}{2}+\frac{\ln(1+x)}{2}$	19
derivativedivides	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22
default	$-\frac{\ln\left(\frac{1}{x}\right)}{2}+\frac{\ln\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x}\right)x}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln((1+x)/x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(1/x)+1/2*ln(1+1/x)*(1+1/x)*x
```

Maxima [**A**]
time = 0.30, size = 18, normalized size = 0.86

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right) + \frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="maxima")
```

```
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Fricas [A]
time = 0.41, size = 18, normalized size = 0.86

$$\frac{1}{2}\,x\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log((x + 1)/x) + 1/2*log(x + 1)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{x\log\left(\frac{x+1}{x}\right)}{2}+\frac{\log\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln((1+x)/x),x)
```

```
[Out] x*log((x + 1)/x)/2 + log(2*x + 2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.
time = 4.81, size = 47, normalized size = 2.24

$$\frac{\log\left(\frac{x+1}{x}\right)}{2\left(\frac{x+1}{x}-1\right)}+\frac{1}{2}\,\log\left(\frac{|x+1|}{|x|}\right)-\frac{1}{2}\,\log\left(\left|\frac{x+1}{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log((1+x)/x),x, algorithm="giac")
```

```
[Out] 1/2*log((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(abs(x + 1)/abs(x)) - 1/2*log(a
bs((x + 1)/x - 1))
```

Mupad [B]
time = 0.06, size = 18, normalized size = 0.86

$$\frac{\ln\left(x+1\right)}{2}+\frac{x\,\ln\left(\frac{x+1}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log((x + 1)/x)/2,x)
```

```
[Out] log(x + 1)/2 + (x*log((x + 1)/x))/2
```

3.297 $\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right) dx$

Optimal. Leaf size=50

$$-\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)}-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)+x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] -1/2*arctanh((1+1/x)^(1/2))+x*ln(-1+((1+x)/x)^(1/2))-1/2/(1-(1+1/x)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2628, 46, 213}

$$-\frac{1}{2\left(1-\sqrt{\frac{1}{x}+1}\right)}+x\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + Sqrt[(1 + x)/x]],x]
```

```
[Out] -1/2*1/(1 - Sqrt[1 + x^(-1)]) - ArcTanh[Sqrt[1 + x^(-1)]]/2 + x*Log[-1 + Sq
rt[(1 + x)/x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(-1+\sqrt{\frac{1+x}{x}}\right)dx = x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \int \frac{1}{-2+\left(-2+2\sqrt{1+\frac{1}{x}}\right)x}\,dx$$

$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)^2(1+x)}\,dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) - \text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2} - \frac{1}{2(-1+x^2)}\right)dx, x,$$

$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)} + x\log\left(-1+\sqrt{\frac{1+x}{x}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,\right.$$

$$= -\frac{1}{2\left(1-\sqrt{1+\frac{1}{x}}\right)} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right) + x\log\left(-1+\sqrt{\frac{1+x}{x}}\right)$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.06

$$\frac{1}{2}\left(1+\sqrt{1+\frac{1}{x}}\right)x + x\log\left(-1+\sqrt{1+\frac{1}{x}}\right) - \frac{1}{4}\log\left(1+\left(2+2\sqrt{1+\frac{1}{x}}\right)x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + Sqrt[(1 + x)/x]],x]
[Out] ((1 + Sqrt[1 + x^(-1)])*x)/2 + x*Log[-1 + Sqrt[1 + x^(-1)]] - Log[1 + (2 + 2*Sqrt[1 + x^(-1)])*x]/4
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \ln\left(-1+\sqrt{\frac{1+x}{x}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+((1+x)/x)^(1/2)),x)
[Out] int(ln(-1+((1+x)/x)^(1/2)),x)
```

Maxima [A]
time = 0.30, size = 68, normalized size = 1.36

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-1\right)}{\frac{x+1}{x}-1}+\frac{1}{2\left(\sqrt{\frac{x+1}{x}}-1\right)}-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 1)/((x + 1)/x - 1) + 1/2/(sqrt((x + 1)/x) - 1) - 1/4*
log(sqrt((x + 1)/x) + 1) + 1/4*log(sqrt((x + 1)/x) - 1)
```

Fricas [A]
time = 0.38, size = 49, normalized size = 0.98

$$\frac{1}{4}\,(4\,x+1)\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{1}{2}\,x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\,x-\frac{1}{4}\log\left(\sqrt{\frac{x+1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*x + 1)*log(sqrt((x + 1)/x) - 1) + 1/2*x*sqrt((x + 1)/x) + 1/2*x - 1/
4*log(sqrt((x + 1)/x) + 1)
```

Sympy [A]
time = 68.15, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{4}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{4}+\frac{1}{2\left(\sqrt{1+\frac{1}{x}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 1) + log(sqrt(1 + 1/x) - 1)/4 - log(sqrt(1 + 1/x) +
 1)/4 + 1/(2*(sqrt(1 + 1/x) - 1))
```

Giac [A]
time = 3.96, size = 53, normalized size = 1.06

$$x\log\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{1}{2}\,x+\frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)}{4\,\mathrm{sgn}\left(x\right)}+\frac{\sqrt{x^2+x}}{2\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 1) + 1/2*x + 1/4*log(abs(-2*x + 2*sqrt(x^2 + x) - 1
))/sgn(x) + 1/2*sqrt(x^2 + x)/sgn(x)
```

Mupad [B]
time = 0.43, size = 38, normalized size = 0.76

$$\frac{x}{2}-\frac{\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)}{2}+x\,\ln\left(\sqrt{\frac{x+1}{x}}-1\right)+\frac{x\,\sqrt{\frac{1}{x}+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) - 1),x)
```

```
[Out] x/2 - atanh((1/x + 1)^(1/2))/2 + x*log(((x + 1)/x)^(1/2) - 1) + (x*(1/x + 1
)^(1/2))/2
```

3.298 $\int \log \left(-2+\sqrt{\frac{1+x}{x}}\right) \, dx$

Optimal. Leaf size=69

$$\frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)$$

```
[Out] 1/2*ln(1-(1+1/x)^(1/2))-1/3*ln(2-(1+1/x)^(1/2))-1/6*ln(1+(1+1/x)^(1/2))+x*l
n(-2+((1+x)/x)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2628, 720, 31, 647}

$$\frac{1}{2}\log\left(1-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{3}\log\left(2-\sqrt{\frac{1}{x}+1}\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x}+1}+1\right)+x\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] Log[1 - Sqrt[1 + x^(-1)]]/2 - Log[2 - Sqrt[1 + x^(-1)]]/3 - Log[1 + Sqrt[1
+ x^(-1)]]/6 + x*Log[-2 + Sqrt[(1 + x)/x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2628

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*(D[u,
 x]/u), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \log\left(-2+\sqrt{\frac{1+x}{x}}\right)\,dx = x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\int \frac{1}{-2+\left(-2+4\sqrt{1+\frac{1}{x}}\right)x}\,dx$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\text{Subst}\left(\int \frac{1}{(-2+x)\,(-1+x^2)}\,dx,x,\sqrt{1+\frac{1}{x}}\right)$$

$$= x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx,x,\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\text{Subst}$$

$$= -\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)+x\log\left(-2+\sqrt{\frac{1+x}{x}}\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx\right.$$

$$= \frac{1}{2}\log\left(1-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{3}\log\left(2-\sqrt{1+\frac{1}{x}}\right)-\frac{1}{6}\log\left(1+\sqrt{1+\frac{1}{x}}\right)-$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 0.93

$$\frac{1}{6}\left(-6\tanh^{-1}\left(3-2\sqrt{1+\frac{1}{x}}\right)+\log\left(2-\sqrt{1+\frac{1}{x}}\right)+6x\log\left(-2+\sqrt{1+\frac{1}{x}}\right)-\log\left(1+\sqrt{1+\frac{1}{x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-2 + Sqrt[(1 + x)/x]],x]
```

```
[Out] (-6*ArcTanh[3 - 2*Sqrt[1 + x^(-1)]] + Log[2 - Sqrt[1 + x^(-1)]] + 6*x*Log[-
2 + Sqrt[1 + x^(-1)]] - Log[1 + Sqrt[1 + x^(-1)]])/6
```

Maple [A]
time = 0.02, size = 107, normalized size = 1.55

method	result
default	$x\ln\left(-2+\sqrt{\frac{1+x}{x}}\right)+\frac{\sqrt{9}\,\ln\left(\frac{4\sqrt{9}\,\sqrt{x^2+x}+15x+3}{9x-3}\right)\sqrt{x\,(1+x)}-3\sqrt{\frac{1+x}{x}}\,x\ln(-3x+1)-6\ln\left(\frac{1}{2}+x+\sqrt{x}\right.}{18\sqrt{\frac{1+x}{x}}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-2+((1+x)/x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(-2+((1+x)/x)^(1/2))+1/18/((1+x)/x)^(1/2)/x*(9^(1/2)*ln(1/3*(4*9^(1/2)*
(x^2+x)^(1/2)+15*x+3)/(3*x-1))*(x*(1+x))^(1/2)-3*((1+x)/x)^(1/2)*x*ln(-3*x+
1)-6*ln(1/2+x+(x^2+x)^(1/2))*(x*(1+x))^(1/2))
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.97

$$\frac{\log\left(\sqrt{\frac{x+1}{x}}-2\right)}{\frac{x+1}{x}-1}-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)-\frac{1}{3}\log\left(\sqrt{\frac{x+1}{x}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt((x + 1)/x) - 2)/((x + 1)/x - 1) - 1/6*log(sqrt((x + 1)/x) + 1) + 1
/2*log(sqrt((x + 1)/x) - 1) - 1/3*log(sqrt((x + 1)/x) - 2)
```

Fricas [A]
time = 0.42, size = 48, normalized size = 0.70

$$\frac{1}{3}(3x-1)\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{1}{6}\log\left(\sqrt{\frac{x+1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x - 1)*log(sqrt((x + 1)/x) - 2) - 1/6*log(sqrt((x + 1)/x) + 1) + 1/2
*log(sqrt((x + 1)/x) - 1)
```

Sympy [A]
time = 68.06, size = 53, normalized size = 0.77

$$x\log\left(\sqrt{\frac{x+1}{x}}-2\right)-\frac{\log\left(\sqrt{1+\frac{1}{x}}-2\right)}{3}+\frac{\log\left(\sqrt{1+\frac{1}{x}}-1\right)}{2}-\frac{\log\left(\sqrt{1+\frac{1}{x}}+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-2+((1+x)/x)**(1/2)),x)
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) - log(sqrt(1 + 1/x) - 2)/3 + log(sqrt(1 + 1/x) -
 1)/2 - log(sqrt(1 + 1/x) + 1)/6
```

Giac [A]
time = 2.74, size = 88, normalized size = 1.28

$$x \log \left(\sqrt{\frac{x+1}{x}}-2\right)+\frac{\log \left(\left|-x+\sqrt{x^{2}+x}+1\right|\right)}{6 \operatorname{sgn}(x)}+\frac{\log \left(\left|-2 x+2 \sqrt{x^{2}+x}-1\right|\right)}{3 \operatorname{sgn}(x)}-\frac{\log \left(\left|-3 x+3 \sqrt{x^{2}+x}-1\right|\right)}{6 \operatorname{sgn}(x)}-\frac{1}{6} \log (|3 x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-2+((1+x)/x)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(sqrt((x + 1)/x) - 2) + 1/6*log(abs(-x + sqrt(x^2 + x) + 1))/sgn(x) +
1/3*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(-3*x + 3*sqrt
(x^2 + x) - 1))/sgn(x) - 1/6*log(abs(3*x - 1))
```

Mupad [B]
time = 0.32, size = 63, normalized size = 0.91

$$\frac{\ln \left(5-5 \sqrt{\frac{x+1}{x}}\right)}{2}-\frac{\ln \left(\frac{\sqrt{\frac{x+1}{x}}}{9}+\frac{1}{9}\right)}{6}-\frac{\ln \left(\frac{10}{9}-\frac{5 \sqrt{\frac{x+1}{x}}}{9}\right)}{3}+x \ln \left(\sqrt{\frac{x+1}{x}}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(((x + 1)/x)^(1/2) - 2),x)
```

```
[Out] log(5 - 5*((x + 1)/x)^(1/2))/2 - log(((x + 1)/x)^(1/2)/9 + 1/9)/6 - log(10/
9 - (5*((x + 1)/x)^(1/2))/9)/3 + x*log(((x + 1)/x)^(1/2) - 2)
```

3.299 $\int (x^{ax} + x^{ax} \log(x))\, dx$

Optimal. Leaf size=9

$$\frac{x^{ax}}{a}$$

```
[Out] x^(a*x)/a
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2633}

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(a*x) + x^(a*x)*Log[x],x]
```

```
[Out] x^(a*x)/a
```

Rule 2633

```
Int[Log[u_]*(u_)^((a_.)*(x_)), x_Symbol] :> Simp[u^(a*x)/a, x] - Int[Simpli
fyIntegrand[x*u^(a*x - 1)*D[u, x], x], x] /; FreeQ[a, x] && InverseFunction
FreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (x^{ax} + x^{ax} \log(x))\, dx &= \int x^{ax}\, dx + \int x^{ax} \log(x)\, dx\\ &= \frac{x^{ax}}{a}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(a*x) + x^(a*x)*Log[x],x]
```

```
[Out] x^(a*x)/a
```

Maple [A]
time = 0.02, size = 10, normalized size = 1.11

method	result	size
risch	$\frac{x^{ax}}{a}$	10
norman	$\frac{e^{ax\ln(x)}}{a}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(a*x)+x^(a*x)*ln(x),x,method=_RETURNVERBOSE)
```

```
[Out] x^(a*x)/a
```

Maxima [A]
time = 0.33, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="maxima")
```

```
[Out] x^(a*x)/a
```

Fricas [A]
time = 0.40, size = 9, normalized size = 1.00

$$\frac{x^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="fricas")
```

```
[Out] x^(a*x)/a
```

Sympy [A]
time = 0.05, size = 10, normalized size = 1.11

$$\begin{cases} \frac{x^{ax}}{a} & \text{for}\: a \neq 0 \\ x \log{\left(x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(a*x)+x**(a*x)*ln(x),x)
```

```
[Out] Piecewise((x**(a*x)/a, Ne(a, 0)), (x*log(x), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(a*x)+x^(a*x)*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^(a*x)*log(x) + x^(a*x), x)
```

Mupad [**B**]
time = 0.37, size = 9, normalized size = 1.00

$$\frac{x^{a\,x}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(a*x) + x^(a*x)*log(x),x)
```

```
[Out] x^(a*x)/a
```

3.300 $\int \log^m(x)^p \, dx$

Optimal. Leaf size=26

$$\Gamma(1+mp,-\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

```
[Out] GAMMA(m*p+1,-ln(x))*(ln(x)^m)^p/((-ln(x))^(m*p))
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6852, 2336, 2212}

$$(-\log(x))^{-mp}\log^m(x)^p\text{Gamma}(mp+1,-\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Log[x]^m)^p,x]
```

```
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\int \log^m(x)^p\,dx = \left(\log^{-mp}(x)\log^m(x)^p\right)\int \log^{mp}(x)\,dx$$
$$= \left(\log^{-mp}(x)\log^m(x)^p\right)\text{Subst}\left(\int e^x x^{mp}\,dx, x, \log(x)\right)$$
$$= \Gamma(1+mp, -\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

Mathematica [**A**]
time = 0.02, size = 26, normalized size = 1.00

$$\Gamma(1+mp, -\log(x))(-\log(x))^{-mp}\log^m(x)^p$$

Antiderivative was successfully verified.

```
[In] Integrate[(Log[x]^m)^p,x]
[Out] (Gamma[1 + m*p, -Log[x]]*(Log[x]^m)^p)/(-Log[x])^(m*p)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(\ln\left(x\right)^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((ln(x)^m)^p,x)
[Out] int((ln(x)^m)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="maxima")
[Out] integrate((log(x)^m)^p, x)
```

Fricas [**A**]
time = 0.10, size = 16, normalized size = 0.62

$$\cos\left(\pi mp\right)\Gamma(mp+1, -\log\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="fricas")
[Out] cos(pi*m*p)*gamma(m*p + 1, -log(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\log\left(x\right)^{m}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((ln(x)**m)**p,x)
[Out] Integral((log(x)**m)**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((log(x)^m)^p,x, algorithm="giac")
[Out] integrate((log(x)^m)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left({\ln\left(x\right)}^m\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((log(x)^m)^p,x)
[Out] int((log(x)^m)^p, x)
```

3.301 $\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=60

$$-\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{x\sqrt{a+b\log(x)}}{b}$$

```
[Out] -1/2*(2*a+b)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a/b)+x*(a
+b*ln(x))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2399, 2336, 2211, 2235}

$$\frac{x\sqrt{a+b\log(x)}}{b}-\frac{\sqrt{\pi}\,(2a+b)e^{-\frac{a}{b}}\,\mathrm{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] -1/2*((2*a + b)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(b^(3/2)*E^(a/b)
) + (x*Sqrt[a + b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{\sqrt{a+b\log(x)}}\,dx &= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\ &= \frac{x\sqrt{a+b\log(x)}}{b}+\frac{(-2a-b)\text{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\ &= \frac{x\sqrt{a+b\log(x)}}{b}-\frac{(2a+b)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\ &= -\frac{(2a+b)e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{x\sqrt{a+b\log(x)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 72, normalized size = 1.20

$$\frac{2x(a+b\log(x))-(2a+b)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a + b*Log[x]],x]
```

```
[Out] (2*x*(a + b*Log[x]) - ((2*a + b)*Gamma[1/2, -((a + b*Log[x])/b)]*Sqrt[-((a
+ b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{\sqrt{a+b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a+b*ln(x))^(1/2),x)
```

[Out] int(ln(x)/(a+b*ln(x))^(1/2),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(47) = 94$.
time = 0.28, size = 108, normalized size = 1.80

$$-\frac{\frac{2\sqrt{\pi}a\operatorname{erf}\left(\sqrt{b\log\left(x\right)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}+\frac{\sqrt{\pi}b\operatorname{erf}\left(\sqrt{b\log\left(x\right)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log\left(x\right)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) +
 sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqrt
(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))/b^2
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x\right)}{\sqrt{a+b\log\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a + b*log(x)), x)
```

Giac [A]
time = 8.10, size = 89, normalized size = 1.48

$$\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{\sqrt{b\log\left(x\right)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}}+\frac{\sqrt{\pi}a\operatorname{erf}\left(-\frac{\sqrt{b\log\left(x\right)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}\,b}+\frac{\sqrt{b\log\left(x\right)+a}\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(pi)*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(p
i)*a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log
(x) + a)*x/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{\sqrt{a+b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a + b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a + b*log(x))^(1/2), x)
```

3.302 $\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=64

$$-\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*a-b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)-x*(a-
b*ln(x))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2399, 2336, 2211, 2236}

$$-\frac{\sqrt{\pi}\,(2a-b)e^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] -1/2*((2*a - b)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]])/b^(3/2) -
 (x*Sqrt[a - b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{\sqrt{a-b\log(x)}}\,dx &= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(-2a+b)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\
&= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(-2a+b)\text{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx,x,\log(x)\right)}{2b}\\
&= -\frac{x\sqrt{a-b\log(x)}}{b}-\frac{(2a-b)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a-b\log(x)}\right)}{b^2}\\
&= -\frac{(2a-b)e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{x\sqrt{a-b\log(x)}}{b}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 71, normalized size = 1.11

$$\frac{-\left((-2a+b)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}\right)-2x(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[a - b*Log[x]],x]
```

```
[Out] (-((-2*a + b)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]) - 2*x*(a
 - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\ln(x)}{\sqrt{a-b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

```
[Out] int(ln(x)/(a-b*ln(x))^(1/2),x)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 1.47

$$-\frac{2\sqrt{\pi}\,a\sqrt{b}\,\operatorname{erf}\left(\frac{\sqrt{-b\log\left(x\right)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}-\sqrt{\pi}\,b^{\frac{3}{2}}\operatorname{erf}\left(\frac{\sqrt{-b\log\left(x\right)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}+2\sqrt{-b\log\left(x\right)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*a*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - sqrt(
pi)*b^(3/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) + 2*sqrt(-b*log(x) + a
)*b*e^((b*log(x) - a)/b + a/b))/b^2
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\log\left(x\right)}{\sqrt{a-b\log\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral(log(x)/sqrt(a - b*log(x)), x)
```

Giac [A]
time = 4.80, size = 74, normalized size = 1.16

$$\frac{\sqrt{\pi}\,a\operatorname{erf}\left(-\frac{\sqrt{-b\log\left(x\right)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\frac{\sqrt{-b\log\left(x\right)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}}-\frac{\sqrt{-b\log\left(x\right)+a}\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) - 1/2*sqrt(pi)
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*x/
b
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\ln(x)}{\sqrt{a-b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(x)/(a - b*log(x))^(1/2),x)
```

```
[Out] int(log(x)/(a - b*log(x))^(1/2), x)
```

3.303 $\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx$

Optimal. Leaf size=69

$$\frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

```
[Out] 1/2*(2*A*b-(2*a+b)*B)*erfi((a+b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/exp(
a/b)+B*x*(a+b*ln(x))^(1/2)/b
```

Rubi [A]

time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2399, 2336, 2211, 2235}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}(2Ab-B(2a+b))\mathrm{Erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
```

```
[Out] ((2*A*b - (2*a + b)*B)*Sqrt[Pi]*Erfi[Sqrt[a + b*Log[x]]/Sqrt[b]])/(2*b^(3/2
)*E^(a/b)) + (B*x*Sqrt[a + b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\log(x)}{\sqrt{a+b\log(x)}}\,dx &= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\int \frac{1}{\sqrt{a+b\log(x)}}\,dx}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\text{Subst}\left(\int \frac{e^x}{\sqrt{a+bx}}\,dx,x,\log(x)\right)}{2b}\\
&= \frac{Bx\sqrt{a+b\log(x)}}{b}+\frac{(2Ab-(2a+b)B)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\log(x)}\right)}{b^2}\\
&= \frac{(2Ab-(2a+b)B)e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+b\log(x)}}{b}
\end{aligned}$$

Mathematica [**A**]
time = 0.19, size = 80, normalized size = 1.16

$$\frac{2Bx(a+b\log(x))+(2Ab-(2a+b)B)e^{-\frac{a}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\log(x)}{b}\right)\sqrt{-\frac{a+b\log(x)}{b}}}{2b\sqrt{a+b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a + b*Log[x]],x]
[Out] (2*B*x*(a + b*Log[x]) + ((2*A*b - (2*a + b)*B)*Gamma[1/2, -((a + b*Log[x])/
b)]*Sqrt[-((a + b*Log[x])/b)])/E^(a/b))/(2*b*Sqrt[a + b*Log[x]])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln(x)}{\sqrt{a+b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)
```

[Out] int((A+B*ln(x))/(a+b*ln(x))^(1/2),x)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(55) = 110$.
time = 0.31, size = 156, normalized size = 2.26

$$\frac{\frac{2\sqrt{\pi}A\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{b\sqrt{-\frac{1}{b}}}-\frac{\left(\frac{\sqrt{\pi}b\operatorname{erf}\left(\sqrt{b\log(x)+a}\sqrt{-\frac{1}{b}}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-\frac{1}{b}}}-2\sqrt{b\log(x)+a}\,be^{\left(\frac{b\log(x)+a}{b}-\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(pi)*A*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) -
2*sqrt(pi)*B*a*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/(b*sqrt(-1/b)) -
 (sqrt(pi)*b*erf(sqrt(b*log(x) + a)*sqrt(-1/b))*e^(-a/b)/sqrt(-1/b) - 2*sqr
t(b*log(x) + a)*b*e^((b*log(x) + a)/b - a/b))*B/b)/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\log(x)}{\sqrt{a + b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a+b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a + b*log(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(55) = 110$.
time = 4.50, size = 129, normalized size = 1.87

$$-\frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}}+\frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{2\sqrt{-b}}+\frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{b\log(x)+a}\sqrt{-b}}{b}\right)e^{\left(-\frac{a}{b}\right)}}{\sqrt{-b}\,b}+\frac{\sqrt{b\log(x)+a}\,Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a+b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(pi)*A*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + 1/2*sqr
t(pi)*B*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/sqrt(-b) + sqrt(pi)*B*
a*erf(-sqrt(b*log(x) + a)*sqrt(-b)/b)*e^(-a/b)/(sqrt(-b)*b) + sqrt(b*log(x)
 + a)*B*x/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln\left(x\right)}{\sqrt{a+b\,\ln\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(x))/(a + b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a + b*log(x))^(1/2), x)
```

3.304 $\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx$

Optimal. Leaf size=71

$$-\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

```
[Out] -1/2*(2*A*b+2*B*a-B*b)*exp(a/b)*erf((a-b*ln(x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(
3/2)-B*x*(a-b*ln(x))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2399, 2336, 2211, 2236}

$$-\frac{\sqrt{\pi}\,e^{a/b}(2aB+2Ab-bB)\mathrm{Erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
```

```
[Out] -1/2*((2*A*b + 2*a*B - b*B)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a - b*Log[x]]/Sqrt[b]
])/b^(3/2) - (B*x*Sqrt[a - b*Log[x]])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2399

```
Int[((A_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(B_.))/Sqrt[Log[(c_.)*((
d_.) + (e_.)*(x_))^(n_.)]*(b_.) + (a_)], x_Symbol] :> Simp[B*(d + e*x)*(Sqr
t[a + b*Log[c*(d + e*x)^n]]/(b*e)), x] + Dist[(2*A*b - B*(2*a + b*n))/(2*b)
, Int[1/Sqrt[a + b*Log[c*(d + e*x)^n]], x], x] /; FreeQ[{a, b, c, d, e, A,
B, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\log(x)}{\sqrt{a-b\log(x)}}\,dx &= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\int \frac{1}{\sqrt{a-b\log(x)}}\,dx}{2b}\\
&= -\frac{Bx\sqrt{a-b\log(x)}}{b}+\frac{(2Ab+2aB-bB)\mathrm{Subst}\left(\int \frac{e^x}{\sqrt{a-bx}}\,dx,x,\log(x)\right)}{2b}\\
&= -\frac{Bx\sqrt{a-b\log(x)}}{b}-\frac{(2Ab+2aB-bB)\mathrm{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a-b\log(x)}\right)}{b^2}\\
&= -\frac{(2Ab+2aB-bB)e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a-b\log(x)}}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{Bx\sqrt{a-b\log(x)}}{b}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 79, normalized size = 1.11

$$\frac{(2Ab+2aB-bB)e^{a/b}\Gamma\left(\frac{1}{2},\frac{a}{b}-\log(x)\right)\sqrt{\frac{a}{b}-\log(x)}-2Bx(a-b\log(x))}{2b\sqrt{a-b\log(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[x])/Sqrt[a - b*Log[x]],x]
[Out] ((2*A*b + 2*a*B - b*B)*E^(a/b)*Gamma[1/2, a/b - Log[x]]*Sqrt[a/b - Log[x]]
- 2*B*x*(a - b*Log[x]))/(2*b*Sqrt[a - b*Log[x]])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\ln(x)}{\sqrt{a-b\ln(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

```
[Out] int((A+B*ln(x))/(a-b*ln(x))^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.
time = 0.35, size = 130, normalized size = 1.83

$$-\frac{\frac{2\sqrt{\pi}Ba\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}} + 2\sqrt{\pi}A\sqrt{b}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - \frac{\left(\sqrt{\pi}b^{\frac{3}{2}}\operatorname{erf}\left(\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}} - 2\sqrt{-b\log(x)+a}\,be^{\left(\frac{b\log(x)-a}{b}+\frac{a}{b}\right)}\right)B}{b}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(pi)*B*a*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) + 2*s
qrt(pi)*A*sqrt(b)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - (sqrt(pi)*b^(3
/2)*erf(sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b) - 2*sqrt(-b*log(x) + a)*b*e^((
b*log(x) - a)/b + a/b))*B/b)/b
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\log(x)}{\sqrt{a - b\log(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(x))/(a-b*ln(x))**(1/2),x)
```

```
[Out] Integral((A + B*log(x))/sqrt(a - b*log(x)), x)
```

Giac [A]
time = 4.57, size = 106, normalized size = 1.49

$$\frac{\sqrt{\pi}Ba\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{b^{\frac{3}{2}}} + \frac{\sqrt{\pi}A\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{\sqrt{b}} - \frac{\sqrt{\pi}B\operatorname{erf}\left(-\frac{\sqrt{-b\log(x)+a}}{\sqrt{b}}\right)e^{\frac{a}{b}}}{2\sqrt{b}} - \frac{\sqrt{-b\log(x)+a}\,Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(x))/(a-b*log(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(pi)*B*a*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/b^(3/2) + sqrt(pi)*A
*erf(-sqrt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - 1/2*sqrt(pi)*B*erf(-sq
rt(-b*log(x) + a)/sqrt(b))*e^(a/b)/sqrt(b) - sqrt(-b*log(x) + a)*B*x/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\ln \left (x\right)}{\sqrt {a-b\,\ln \left (x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*log(x))/(a - b*log(x))^(1/2),x)
```

```
[Out] int((A + B*log(x))/(a - b*log(x))^(1/2), x)
```

3.305 $\int x^2 \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=98

$$\frac{ix^4}{12}-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log(\log(x)\sin(x))+\frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right)-\frac{1}{2}x\text{Li}_3\left(e^{2ix}\right)-\frac{1}{4}i\text{Li}_4\left(e^{2ix}\right)$$

```
[Out] 1/12*I*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(ln(x)*sin(x)
)+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x))-1/4*I*polylog
(4,exp(2*I*x))
```

Rubi [A]
time = 0.19, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {30, 2635, 12, 6820, 14, 3798, 2221, 2611, 6744, 2320, 6724, 2346, 2209}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\frac{ix^4}{12}-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])/3 + (x^
3*Log[Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (x*PolyLog[3,
 E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int x^2\log(\log(x)\sin(x))\,dx &= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \int \frac{x^2(1+x\cot(x)\log(x))}{3\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \frac{x^2(1+x\cot(x)\log(x))}{\log(x)}\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^2\left(x\cot(x)+\frac{1}{\log(x)}\right)\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int \left(x^3\cot(x)+\frac{x^2}{\log(x)}\right)\,dx \\
&= \frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{3}\int x^3\cot(x)\,dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\,dx \\
&= \frac{ix^4}{12} + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{2}{3}i\int \frac{e^{2ix}x^3}{1-e^{2ix}}\,dx - \frac{1}{3}\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(x)\right. \\
&= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \int x^2\log\left(1\right. \\
&= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2}\right. \\
&= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2}\right. \\
&= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2}\right. \\
&= \frac{ix^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log(\log(x)\sin(x)) + \frac{1}{2}ix^2\text{Li}_2\left(e^{2}\right.
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 95, normalized size = 0.97

$$\frac{1}{192}i\left(\pi^4-16x^4+64i\text{Ei}(3\log(x))+64ix^3\log\left(1-e^{-2ix}\right)-64ix^3\log(\log(x)\sin(x))-96x^2\text{Li}_2\left(e^{-2ix}\right)+96ix\text{Li}_3\left(e^{-2ix}\right)+48\text{Li}_4\left(e^{-2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - 16*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x^3*Log[1
- E^((-2*I)*x)] - (64*I)*x^3*Log[Log[x]*Sin[x]] - 96*x^2*PolyLog[2, E^((-2*
I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^((-2*I)*x)])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 462, normalized size = 4.71

method	result
risch	$ix^2\,\text{polylog}\left(2,-e^{ix}\right)+ix^2\,\text{polylog}\left(2,e^{ix}\right)-\frac{i\pi\,x^3}{6}+\frac{ix^4}{12}-2x\,\text{polylog}\left(3,-e^{ix}\right)-2x\,\text{polylog}\left(3,e^{ix}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^4-1/3*ln(2)*x^3+1/6*I*Pi*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x)
)^2*x^3+1/6*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))^2*x^3+I*x^
2*polylog(2,-exp(I*x))+I*x^2*polylog(2,exp(I*x))-1/6*I*Pi*x^3+1/3*Ei(1,-3*l
n(x))+1/6*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2*x^3+1/
6*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x^3+1/6*I*Pi*csgn(ln(x)*sin(x
))^3*x^3-2*x*polylog(3,-exp(I*x))-2*x*polylog(3,exp(I*x))-1/3*x^3*ln(exp(I*
x))-1/3*x^3*ln(exp(I*x)+1)-1/3*x^3*ln(1-exp(I*x))-1/6*I*Pi*csgn(I*ln(x)*sin
(x))^3*x^3-1/6*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))^3*x^3+1/3*x^3*ln(ln(x))+1/
6*I*Pi*csgn(I*ln(x)*sin(x))^2*x^3+1/6*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*
sin(x))*x^3-2*I*polylog(4,-exp(I*x))-2*I*polylog(4,exp(I*x))+1/6*I*Pi*csgn(
I*exp(-I*x))*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))*x^3+1/3*x^3*ln
(exp(2*I*x)-1)-1/6*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))^2*x^3-1/6*I
*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))*x^3
```

Maxima [A]
time = 0.58, size = 94, normalized size = 0.96

$$-\frac{1}{6}\left(-i\,\pi+2\,\log\left(2\right)\right)x^3-\frac{1}{4}i\,x^4+\frac{1}{3}x^3\log\left(\log\left(x\right)\right)+i\,x^2\text{Li}_2(-e^{(i\,x)})+i\,x^2\text{Li}_2(e^{(i\,x)})-2\,x\text{Li}_3(-e^{(i\,x)})-2\,x\text{Li}_3(e^{(i\,x)})-\frac{1}{3}\,\text{Ei}(3\,\log\left(x\right))-2i\,\text{Li}_4(-e^{(i\,x)})-2i\,\text{Li}_4(e^{(i\,x)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/6*(-I*pi + 2*log(2))*x^3 - 1/4*I*x^4 + 1/3*x^3*log(log(x)) + I*x^2*dilog
(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*polylog(
```

```
3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*polylog(4,
e^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(65) = 130$.
time = 0.46, size = 234, normalized size = 2.39

$$\frac{1}{3}x^3\log(\log(x)\sin(x)) - \frac{1}{6}x^3\log(\cos(x) + i\sin(x) + 1) - \frac{1}{6}x^3\log(\cos(x) - i\sin(x) + 1) - \frac{1}{6}x^3\log(-\cos(x) + i\sin(x) + 1) - \frac{1}{6}x^3\log(-\cos(x) - i\sin(x) + 1) + \frac{1}{2}ix^2\mathrm{Li}_2(\cos(x) + i\sin(x)) - \frac{1}{2}ix^2\mathrm{Li}_2(\cos(x) - i\sin(x)) - \frac{1}{2}ix^2\mathrm{Li}_2(-\cos(x) + i\sin(x)) + \frac{1}{2}ix^2\mathrm{Li}_2(-\cos(x) - i\sin(x)) - x\mathrm{polylog}(3, \cos(x) + i\sin(x)) - x\mathrm{polylog}(3, \cos(x) - i\sin(x)) - x\mathrm{polylog}(3, -\cos(x) + i\sin(x)) - x\mathrm{polylog}(3, -\cos(x) - i\sin(x)) - \frac{1}{3}\mathrm{log\_integral}(x^3) - i\,\mathrm{polylog}(4, \cos(x) + i\sin(x)) + i\,\mathrm{polylog}(4, \cos(x) - i\sin(x)) + i\,\mathrm{polylog}(4, -\cos(x) + i\sin(x)) - i\,\mathrm{polylog}(4, -\cos(x) - i\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x) + 1) - 1/6*x^3*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin(x)) - 1/2*I*x^2
*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin(x)) + 1/2*I*x^2
*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)) - x*polylog(3,
 cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*polylog(3, -cos(
x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x) + I*sin(x)) +
I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*sin(x)) - I*poly
log(4, -cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log(\log(x)\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(x**2*log(log(x)*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(log(x)*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln(\ln(x)\,\sin(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(log(x)*sin(x)),x)
[Out] int(x^2*log(log(x)*sin(x)), x)
```

3.306 $\int x \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=80

$$\frac{ix^3}{6}-\frac{1}{2}\mathrm{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))+\frac{1}{2}ix\mathrm{Li}_2\left(e^{2ix}\right)-\frac{1}{4}\mathrm{Li}_3\left(e^{2ix}\right)$$

```
[Out] 1/6*I*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(ln(x)*sin(x))
+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A]
time = 0.12, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {30, 2635, 12, 6820, 14, 3798, 2221, 2611, 2320, 6724, 2346, 2209}

$$\frac{1}{2}ix\mathrm{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\mathrm{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\mathrm{Ei}(2\log(x))+\frac{ix^3}{6}-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2 + (x^2
*Log[Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^((2
*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int x\log(\log(x)\sin(x))\,dx &= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \int \frac{x(1+x\cot(x)\log(x))}{2\log(x)}\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \frac{x(1+x\cot(x)\log(x))}{\log(x)}\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x\left(x\cot(x)+\frac{1}{\log(x)}\right)\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int \left(x^2\cot(x)+\frac{x}{\log(x)}\right)\,dx \\
&= \frac{1}{2}x^2\log(\log(x)\sin(x)) - \frac{1}{2}\int x^2\cot(x)\,dx - \frac{1}{2}\int \frac{x}{\log(x)}\,dx \\
&= \frac{ix^3}{6} + \frac{1}{2}x^2\log(\log(x)\sin(x)) + i\int \frac{e^{2ix}x^2}{1-e^{2ix}}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(x)\right) \\
&= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \int x\log\left(1-\right. \\
&= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right. \\
&= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right. \\
&= \frac{ix^3}{6} - \frac{1}{2}\text{Ei}(2\log(x)) - \frac{1}{2}x^2\log\left(1-e^{2ix}\right) + \frac{1}{2}x^2\log(\log(x)\sin(x)) + \frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right.
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.99

$$\frac{1}{48}\left(i\pi^3-8ix^3-24\text{Ei}(2\log(x))-24x^2\log\left(1-e^{-2ix}\right)+24x^2\log(\log(x)\sin(x))-24ix\text{Li}_2\left(e^{-2ix}\right)-12\text{Li}_3\left(e^{-2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-2*I)
*x)] + 24*x^2*Log[Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)] - 12*P
olyLog[3, E^((-2*I)*x)])/48
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 434, normalized size = 5.42

method	result
risch	$\frac{\text{expIntegral}(1,-2\ln(x))}{2}-\frac{i\pi\,\text{csgn}(i(e^{2ix}-1)\ln(x))^3x^2}{4}-\frac{i\pi\,\text{csgn}(i\ln(x)\sin(x))^3x^2}{4}+\frac{i\pi\,\text{csgn}(i\ln(x)\sin(x))^2x^2}{4}+\frac{i\pi\,\text{csgn}(\ln(x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*x^3-1/2*ln(2)*x^2-1/2*x^2*ln(exp(I*x))-1/2*x^2*ln(exp(I*x)+1)-1/2*x^2
*ln(1-exp(I*x))+1/4*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))^2*
x^2+1/4*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x^2+1/4*I*Pi*csgn(I*(ex
p(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2*x^2+1/4*I*Pi*csgn(I*exp(-I*x))*
csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))*x^2-1/4*I*Pi*csgn(I*(exp(2*
I*x)-1)*ln(x))^3*x^2-1/4*I*Pi*csgn(I*ln(x)*sin(x))^3*x^2+1/4*I*Pi*csgn(I*ln
(x))*csgn(I*(exp(2*I*x)-1)*ln(x))^2*x^2+1/2*ln(ln(x))*x^2+1/2*ln(exp(2*I*x)
-1)*x^2+1/4*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*sin(x))*x^2-1/4*I*Pi*csgn(
I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))*x^2+1/2*Ei(1,-
2*ln(x))-polylog(3,-exp(I*x))-polylog(3,exp(I*x))-1/4*I*Pi*x^2+I*x*polylog(
2,-exp(I*x))+I*x*polylog(2,exp(I*x))-1/4*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(
x)*sin(x))^2*x^2+1/4*I*Pi*csgn(I*ln(x)*sin(x))^2*x^2+1/4*I*Pi*csgn(ln(x)*si
n(x))^3*x^2
```

Maxima [A]
time = 0.59, size = 70, normalized size = 0.88

$$-\frac{1}{4}\left(-i\pi+2\log(2)\right)x^2-\frac{1}{3}i\,x^3+\frac{1}{2}\,x^2\log\left(\log(x)\right)+i\,x\mathrm{Li}_2(-e^{(i\,x)})+i\,x\mathrm{Li}_2(e^{(i\,x)})-\frac{1}{2}\,\mathrm{Ei}(2\,\log(x))-\mathrm{Li}_3(-e^{(i\,x)})-\mathrm{Li}_3(e^{(i\,x)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(-I*pi + 2*log(2))*x^2 - 1/3*I*x^3 + 1/2*x^2*log(log(x)) + I*x*dilog(-
e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I*x)) - p
olylog(3, e^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(54) = 108.
time = 0.43, size = 174, normalized size = 2.18

$$\frac{1}{2}\,x^2\log\left(\log(x)\sin(x)\right)-\frac{1}{4}\,x^2\log\left(\cos(x)+i\,\sin(x)+1\right)-\frac{1}{4}\,x^2\log\left(\cos(x)-i\,\sin(x)+1\right)-\frac{1}{4}\,x^2\log\left(-\cos(x)+i\,\sin(x)+1\right)-\frac{1}{4}\,x^2\log\left(-\cos(x)-i\,\sin(x)+1\right)+\frac{1}{2}i\,x\mathrm{Li}_2(\cos(x)+i\,\sin(x))-\frac{1}{2}i\,x\mathrm{Li}_2(\cos(x)-i\,\sin(x))-\frac{1}{2}i\,x\mathrm{Li}_2(-\cos(x)+i\,\sin(x))+\frac{1}{2}i\,x\mathrm{Li}_2(-\cos(x)-i\,\sin(x))-\frac{1}{2}\,\mathrm{log\_integral}\left(x^2\right)-\frac{1}{2}\,\mathrm{polylog}(3,\cos(x)+i\,\sin(x))-\frac{1}{2}\,\mathrm{polylog}(3,\cos(x)-i\,\sin(x))-\frac{1}{2}\,\mathrm{polylog}(3,-\cos(x)+i\,\sin(x))-\frac{1}{2}\,\mathrm{polylog}(3,-\cos(x)-i\,\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) + 1) - 1/4*x^2*l
og(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)) - 1/2*I*x*dil
og(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) + 1/2*I*x*dilog(-
cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3, cos(x) + I*sin(
x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -cos(x) + I*sin(x)
) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(\log\left(x\right)\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(x*log(log(x)*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(log(x)*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(log(x)*sin(x)),x)
```

```
[Out] int(x*log(log(x)*sin(x)), x)
```

3.307 $\int \log(\log(x)\sin(x))\,dx$

Optimal. Leaf size=52

$$\frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathrm{li}(x) + \frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)$$

```
[Out] 1/2*I*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(ln(x)*sin(x))+1/2*I*polylog(2,exp(2
*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2629, 3798, 2221, 2317, 2438, 2335}

$$\frac{1}{2}i\mathrm{PolyLog}\left(2,e^{2ix}\right) - \mathrm{li}(x) + \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]*Sin[x]],x]
```

```
[Out] (I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[Log[x]*Sin[x]] - LogIntegral[x]
+ (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2629

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \log(\log(x)\sin(x))\,dx &= x\log(\log(x)\sin(x)) - \int \left(x\cot(x) + \frac{1}{\log(x)}\right)\,dx \\
&= x\log(\log(x)\sin(x)) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx \\
&= \frac{ix^2}{2} + x\log(\log(x)\sin(x)) - \mathrm{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathrm{li}(x) + \int \log\left(1-e^{2ix}\right)\,dx \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathrm{li}(x) - \frac{1}{2}i\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, \right. \\
&= \frac{ix^2}{2} - x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathrm{li}(x) + \frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.90

$$-x\log\left(1-e^{2ix}\right) + x\log(\log(x)\sin(x)) - \mathrm{li}(x) + \frac{1}{2}i\left(x^2 + \mathrm{Li}_2\left(e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]*Sin[x]],x]
```

```
[Out] -(x*Log[1 - E^((2*I)*x)]) + x*Log[Log[x]*Sin[x]] - LogIntegral[x] + (I/2)*(
x^2 + PolyLog[2, E^((2*I)*x)])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 368, normalized size = 7.08

method	result
risch	$-x\ln\left(\mathrm{e}^{ix}\right)+i\,\mathrm{dilog}\left(\mathrm{e}^{ix}+1\right)+\frac{ix\pi\,\mathrm{csgn}(i\ln(x))\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\ln(x)\right)^2}{2}+\frac{ix\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2ix}-1\right)\ln(x)\right)\mathrm{csgn}(\ln(x)\sin(x))}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -x*ln(exp(I*x))+1/2*I*x*Pi*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))^2+1/2
*I*x*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))^2-1/2*I*x*Pi*csgn(l
n(x)*sin(x))*csgn(I*ln(x)*sin(x))^2-1/2*I*x*Pi-1/2*I*x*Pi*csgn(I*(exp(2*I*x
)-1)*ln(x))^3+1/2*I*x*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2-x*ln(2)-1/2
*I*x*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))+1
/2*I*x*Pi*csgn(ln(x)*sin(x))^3+1/2*I*x*Pi*csgn(I*ln(x)*sin(x))^2-I*ln(exp(I
*x))*ln(exp(2*I*x)-1)-1/2*I*x*Pi*csgn(I*ln(x)*sin(x))^3+I*dilog(exp(I*x)+1)
+1/2*I*x^2+1/2*I*x*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2
+1/2*I*x*Pi*csgn(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x
))+I*ln(exp(I*x))*ln(exp(I*x)+1)+1/2*I*x*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)
*sin(x))-I*dilog(exp(I*x))+ln(ln(x))*x+Ei(1,-ln(x))
```

Maxima [**A**]
time = 0.61, size = 43, normalized size = 0.83

$$\frac{1}{2}\,(i\,\pi-2\,\log\left(2\right))x-\frac{1}{2}i\,x^2+x\log\left(\log\left(x\right)\right)-\mathrm{Ei}(\log\left(x\right))+i\,\mathrm{Li}_2\left(-e^{(i\,x)}\right)+i\,\mathrm{Li}_2\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - 1/2*I*x^2 + x*log(log(x)) - Ei(log(x)) + I*dilog(
-e^(I*x)) + I*dilog(e^(I*x))
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(37) = 74.
time = 0.44, size = 109, normalized size = 2.10

$$x\log\left(\log\left(x\right)\sin\left(x\right)\right)-\frac{1}{2}x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\frac{1}{2}x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(\cos\left(x\right)-i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(-\cos\left(x\right)-i\,\sin\left(x\right))-\mathrm{log\_integral}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] x*log(log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/2*x*log(cos(x)
- I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*log(-cos(x) - I
*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(cos(x) - I*sin(
x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x) - I*sin(x)) - l
og_integral(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log\left(\log\left(x\right)\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x)),x)
```

```
[Out] Integral(log(log(x)*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(log(x)*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x)),x)
```

```
[Out] int(log(log(x)*sin(x)), x)
```

3.308 $\int \frac{\log(\log(x)\sin(x))}{x} \, dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{\log(\log(x)\sin(x))}{x},x\right)$$

```
[Out] CannotIntegrate(ln(ln(x)*sin(x))/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x,x]
```

```
[Out] Defer[Int][Log[Log[x]*Sin[x]]/x, x]
```

Rubi steps

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx = \int \frac{\log(\log(x)\sin(x))}{x} \, dx$$

Mathematica [A]
time = 1.63, size = 0, normalized size = 0.00

$$\int \frac{\log(\log(x)\sin(x))}{x} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x,x]
```

```
[Out] Integrate[Log[Log[x]*Sin[x]]/x, x]
```

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln(\ln(x)\sin(x))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(ln(x)*sin(x))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - integrat
e(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(log(x)*sin(x))/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(\log\left(x\right)\sin\left(x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x,x)
```

```
[Out] Integral(log(log(x)*sin(x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(log(x)*sin(x))/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{\ln (\ln (x) \sin (x))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x))/x,x)
```

```
[Out] int(log(log(x)*sin(x))/x, x)
```

3.309 $\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$

Optimal. Leaf size=26

$$\mathrm{Ei}(-\log(x))-\frac{\log(\log(x)\sin(x))}{x}+\mathrm{Int}\left(\frac{\cot(x)}{x},x\right)$$

```
[Out] Ei(-ln(x))-ln(ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [A]
time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Log[Log[x]*Sin[x]]/x^2,x]
[Out] ExpIntegralEi[-Log[x]] - Log[Log[x]*Sin[x]]/x + Defer[Int][Cot[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx &= -\frac{\log(\log(x)\sin(x))}{x}-\int \frac{-1-x\cot(x)\log(x)}{x^2\log(x)}\,dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}-\int \left(-\frac{\cot(x)}{x}-\frac{1}{x^2\log(x)}\right)dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx+\int \frac{1}{x^2\log(x)}\,dx\\
&= -\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx+\mathrm{Subst}\left(\int \frac{e^{-x}}{x}\,dx,x,\log(x)\right)\\
&= \mathrm{Ei}(-\log(x))-\frac{\log(\log(x)\sin(x))}{x}+\int \frac{\cot(x)}{x}\,dx
\end{aligned}$$

Mathematica [A]
time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{\log(\log(x)\sin(x))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[Log[x]*Sin[x]]/x^2,x]
```

```
[Out] Integrate[Log[Log[x]*Sin[x]]/x^2, x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(\ln \left(x \right) \sin \left(x \right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x)*sin(x))/x^2,x)
```

```
[Out] int(ln(ln(x)*sin(x))/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*log(2) - log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*log(log(x)))/x
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(log(log(x)*sin(x))/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log{\left(\log{\left(x \right)} \sin{\left(x \right)} \right)}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x)*sin(x))/x**2,x)
```

```
[Out] Integral(log(log(x)*sin(x))/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x)*sin(x))/x^2,x, algorithm="giac")
[Out] integrate(log(log(x)*sin(x))/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\ln\left(\ln\left(x\right)\,\sin\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(log(x)*sin(x))/x^2,x)
[Out] int(log(log(x)*sin(x))/x^2, x)
```

3.310 $\int x^2 \log (e^x \log(x) \sin(x))\, dx$

Optimal. Leaf size=103

$$\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}\text{Ei}(3\log(x))-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)+\frac{1}{2}ix^2\text{Li}_2\left(e^{2ix}\right)-\frac{1}{2}x\text{Li}_3\left(e^{2ix}\right)$$

```
[Out] (-1/12+1/12*I)*x^4-1/3*Ei(3*ln(x))-1/3*x^3*ln(1-exp(2*I*x))+1/3*x^3*ln(exp(
x)*ln(x)*sin(x))+1/2*I*x^2*polylog(2,exp(2*I*x))-1/2*x*polylog(3,exp(2*I*x)
)-1/4*I*polylog(4,exp(2*I*x))
```

Rubi [A]
time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {30, 2635, 12, 14, 3798, 2221, 2611, 6744, 2320, 6724, 2346, 2209}

$$\frac{1}{2}ix^2\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{4}i\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{3}\text{Ei}(3\log(x))+\left(-\frac{1}{12}+\frac{i}{12}\right)x^4-\frac{1}{3}x^3\log\left(1-e^{2ix}\right)+\frac{1}{3}x^3\log\left(e^x\log(x)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/12 + I/12)*x^4 - ExpIntegralEi[3*Log[x]]/3 - (x^3*Log[1 - E^((2*I)*x)])
/3 + (x^3*Log[E^x*Log[x]*Sin[x]])/3 + (I/2)*x^2*PolyLog[2, E^((2*I)*x)] - (
x*PolyLog[3, E^((2*I)*x)])/2 - (I/4)*PolyLog[4, E^((2*I)*x)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \log\left(e^x \log(x)\sin(x)\right) dx &= \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \int \frac{1}{3}x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right) dx \\
&= \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \frac{1}{3}\int x^3\left(1+\cot(x)+\frac{1}{x\log(x)}\right) dx \\
&= \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \frac{1}{3}\int \left(x^3(1+\cot(x))+\frac{x^2}{\log(x)}\right) dx \\
&= \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \frac{1}{3}\int x^3(1+\cot(x))\,dx - \frac{1}{3}\int \frac{x^2}{\log(x)}\,dx \\
&= \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \frac{1}{3}\int \left(x^3+x^3\cot(x)\right) dx - \frac{1}{3}\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx,\right. \\
&= -\frac{x^4}{12} - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) - \frac{1}{3}\int x^3\cot(x)\,dx \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) + \frac{1}{3}x^3 \log\left(e^x \log(x)\sin(x)\right) + \frac{2}{3}i\int \frac{e^{2ix}}{1-e} \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\right. \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\right. \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\right. \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\right. \\
&= \left(-\frac{1}{12}+\frac{i}{12}\right)x^4 - \frac{1}{3}\text{Ei}(3\log(x)) - \frac{1}{3}x^3\log\left(1-e^{2ix}\right) + \frac{1}{3}x^3\log\left(e^x\log(x)\right.
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 100, normalized size = 0.97

$$\frac{1}{192}i\left(\pi^4-(16-16i)x^4+64i\text{Ei}(3\log(x))+64ix^3\log\left(1-e^{-2ix}\right)-64ix^3\log\left(e^x\log(x)\sin(x)\right)-96x^2\text{Li}_2\left(e^{-2ix}\right)+96ix\text{Li}_3\left(e^{-2ix}\right)+48\text{Li}_4\left(e^{-2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I/192)*(Pi^4 - (16 - 16*I)*x^4 + (64*I)*ExpIntegralEi[3*Log[x]] + (64*I)*x
^3*Log[1 - E^((-2*I)*x)] - (64*I)*x^3*Log[E^x*Log[x]*Sin[x]] - 96*x^2*PolyL
og[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E^(
(-2*I)*x)])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.25, size = 691, normalized size = 6.71

method	result	size
risch	Expression too large to display	691

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*x^4-1/12*x^4+1/6*I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csgn(I*ln(x)
*(exp((1+I)*x)-exp((1-I)*x)))*x^3-1/3*ln(2)*x^3-1/6*I*Pi*csgn(I*ln(x)*(exp(
(1+I)*x)-exp((1-I)*x)))^3*x^3+1/6*I*Pi*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*
ln(x))^2*x^3+1/6*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))^2*x^3
-1/6*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x^
3+I*x^2*polylog(2,-exp(I*x))+I*x^2*polylog(2,exp(I*x))-1/6*I*Pi*x^3+1/3*Ei(
1,-3*ln(x))+1/6*I*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2*
x^3+1/6*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x^3+1/6*I*Pi*csgn(ln(x)
*sin(x))^3*x^3-2*x*polylog(3,-exp(I*x))-2*x*polylog(3,exp(I*x))-1/3*x^3*ln(
exp(I*x))-1/3*x^3*ln(exp(I*x)+1)-1/3*x^3*ln(1-exp(I*x))-1/6*I*Pi*csgn((exp(
(1+I)*x)-exp((1-I)*x))*ln(x))^3*x^3+1/3*x^3*ln(exp(x))+1/6*I*Pi*csgn((exp((
1+I)*x)-exp((1-I)*x))*ln(x))^2*x^3-1/6*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))^3*
x^3+1/3*x^3*ln(ln(x))-1/6*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*cs
gn((exp((1+I)*x)-exp((1-I)*x))*ln(x))*x^3-2*I*polylog(4,-exp(I*x))-2*I*poly
log(4,exp(I*x))+1/6*I*Pi*csgn(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1)*ln(x))*csg
n(ln(x)*sin(x))*x^3+1/6*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn
((exp((1+I)*x)-exp((1-I)*x))*ln(x))^2*x^3+1/6*I*Pi*csgn(I*exp(x))*csgn(I*ln
(x)*(exp((1+I)*x)-exp((1-I)*x)))^2*x^3+1/3*x^3*ln(exp(2*I*x)-1)-1/6*I*Pi*cs
gn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))*x^3
```

Maxima [A]
time = 0.59, size = 94, normalized size = 0.91

$$-\frac{1}{6}\left(-i\pi + 2\log\left(2\right)\right)x^{3} - \left(\frac{1}{4}i - \frac{1}{4}\right)x^{4} + \frac{1}{3}x^{3}\log\left(\log\left(x\right)\right) + i\,x^{2}\mathrm{Li}_{2}(-e^{(ix)}) + i\,x^{2}\mathrm{Li}_{2}(e^{(ix)}) - 2\,x\mathrm{Li}_{3}(-e^{(ix)}) - 2\,x\mathrm{Li}_{3}(e^{(ix)}) - \frac{1}{3}\,\mathrm{Ei}(3\log\left(x\right)) - 2i\,\mathrm{Li}_{4}(-e^{(ix)}) - 2i\,\mathrm{Li}_{4}(e^{(ix)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/6*(-I*pi + 2*log(2))*x^3 - (1/4*I - 1/4)*x^4 + 1/3*x^3*log(log(x)) + I*x
^2*dilog(-e^(I*x)) + I*x^2*dilog(e^(I*x)) - 2*x*polylog(3, -e^(I*x)) - 2*x*
polylog(3, e^(I*x)) - 1/3*Ei(3*log(x)) - 2*I*polylog(4, -e^(I*x)) - 2*I*pol
ylog(4, e^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(67) = 134$.
time = 0.44, size = 241, normalized size = 2.34

$$-\frac{1}{12}x^4+\frac{1}{3}x^3\log\left(e^x\log\left(x\right)\sin\left(x\right)\right)-\frac{1}{6}x^3\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)-\frac{1}{6}x^3\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)-\frac{1}{6}x^3\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)-\frac{1}{6}x^3\log\left(-\cos\left(x\right)-i\sin\left(x\right)+1\right)+\frac{1}{2}ix^2\mathrm{Li}_2(\cos\left(x\right)+i\sin\left(x\right))-\frac{1}{2}ix^2\mathrm{Li}_2(\cos\left(x\right)-i\sin\left(x\right))-\frac{1}{2}ix^2\mathrm{Li}_2(-\cos\left(x\right)+i\sin\left(x\right))+\frac{1}{2}ix^2\mathrm{Li}_2(-\cos\left(x\right)-i\sin\left(x\right))-x\mathrm{polylog}(3,\cos\left(x\right)+i\sin\left(x\right))-x\mathrm{polylog}(3,\cos\left(x\right)-i\sin\left(x\right))-x\mathrm{polylog}(3,-\cos\left(x\right)+i\sin\left(x\right))-x\mathrm{polylog}(3,-\cos\left(x\right)-i\sin\left(x\right))-\frac{1}{3}\mathrm{log\_integral}\left(x^3\right)-i\,\mathrm{polylog}(4,\cos\left(x\right)+i\sin\left(x\right))+i\,\mathrm{polylog}(4,\cos\left(x\right)-i\sin\left(x\right))+i\,\mathrm{polylog}(4,-\cos\left(x\right)+i\sin\left(x\right))-i\,\mathrm{polylog}(4,-\cos\left(x\right)-i\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/12*x^4 + 1/3*x^3*log(e^x*log(x)*sin(x)) - 1/6*x^3*log(cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(cos(x) - I*sin(x) + 1) - 1/6*x^3*log(-cos(x) + I*sin(x)
+ 1) - 1/6*x^3*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x^2*dilog(cos(x) + I*sin
(x)) - 1/2*I*x^2*dilog(cos(x) - I*sin(x)) - 1/2*I*x^2*dilog(-cos(x) + I*sin
(x)) + 1/2*I*x^2*dilog(-cos(x) - I*sin(x)) - x*polylog(3, cos(x) + I*sin(x)
) - x*polylog(3, cos(x) - I*sin(x)) - x*polylog(3, -cos(x) + I*sin(x)) - x*
polylog(3, -cos(x) - I*sin(x)) - 1/3*log_integral(x^3) - I*polylog(4, cos(x
) + I*sin(x)) + I*polylog(4, cos(x) - I*sin(x)) + I*polylog(4, -cos(x) + I*
sin(x)) - I*polylog(4, -cos(x) - I*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \log\left(e^x \log\left(x\right)\sin\left(x\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(x**2*log(exp(x)*log(x)*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(e^x*log(x)*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\ln\left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*log(exp(x)*log(x)*sin(x)),x)
[Out] int(x^2*log(exp(x)*log(x)*sin(x)), x)
```

3.311 $\int x \log\left(e^x \log(x) \sin(x)\right) \, dx$

Optimal. Leaf size=85

$$\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+\frac{1}{2}ix\text{Li}_2\left(e^{2ix}\right)-\frac{1}{4}\text{Li}_3\left(e^{2ix}\right)$$

```
[Out] (-1/6+1/6*I)*x^3-1/2*Ei(2*ln(x))-1/2*x^2*ln(1-exp(2*I*x))+1/2*x^2*ln(exp(x)
*ln(x)*sin(x))+1/2*I*x*polylog(2,exp(2*I*x))-1/4*polylog(3,exp(2*I*x))
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {30, 2635, 12, 14, 3798, 2221, 2611, 2320, 6724, 2346, 2209}

$$\frac{1}{2}ix\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{1}{2}\text{Ei}(2\log(x))+\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/6 + I/6)*x^3 - ExpIntegralEi[2*Log[x]]/2 - (x^2*Log[1 - E^((2*I)*x)])/2
 + (x^2*Log[E^x*Log[x]*Sin[x]])/2 + (I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyL
og[3, E^((2*I)*x)]/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[w*Simplify[D[u, x]/u], x], x] /; InverseFunctionFr
eeQ[w, x]] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x\log\left(e^x\log(x)\sin(x)\right)\,dx = \frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\int\frac{1}{2}x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx$$
$$=\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\left(1+\cot(x)+\frac{1}{x\log(x)}\right)\,dx$$
$$=\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2(1+\cot(x))+\frac{x}{\log(x)}\right)\,dx$$
$$=\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2(1+\cot(x))\,dx-\frac{1}{2}\int\frac{x}{\log(x)}\,dx$$
$$=\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int\left(x^2+x^2\cot(x)\right)\,dx-\frac{1}{2}\text{Subst}\left(\int\frac{e^{2x}}{x}\,dx,x$$
$$=-\frac{x^3}{6}-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)-\frac{1}{2}\int x^2\cot(x)\,dx$$
$$=\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))+\frac{1}{2}x^2\log\left(e^x\log(x)\sin(x)\right)+i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx$$
$$=\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$
$$=\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$
$$=\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$
$$=\left(-\frac{1}{6}+\frac{i}{6}\right)x^3-\frac{1}{2}\text{Ei}(2\log(x))-\frac{1}{2}x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\log\left(e^x\log(x)\sin\right.$$

Mathematica [A]
time = 0.06, size = 82, normalized size = 0.96

$$\frac{1}{48}\left(i\pi^3-(8+8i)x^3-24\text{Ei}(2\log(x))-24x^2\log\left(1-e^{-2ix}\right)+24x^2\log\left(e^x\log(x)\sin(x)\right)-24ix\text{Li}_2\left(e^{-2ix}\right)-12\text{Li}_3\left(e^{-2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (I*Pi^3 - (8 + 8*I)*x^3 - 24*ExpIntegralEi[2*Log[x]] - 24*x^2*Log[1 - E^((-
2*I)*x)] + 24*x^2*Log[E^x*Log[x]*Sin[x]] - (24*I)*x*PolyLog[2, E^((-2*I)*x)
] - 12*PolyLog[3, E^((-2*I)*x)])/48
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 663, normalized size = 7.80

method	result	size
risch	Expression too large to display	663

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*I*x^3-1/6*x^3-1/2*ln(2)*x^2-1/2*x^2*ln(exp(I*x))-1/2*x^2*ln(exp(I*x)+1)
-1/2*x^2*ln(1-exp(I*x))+1/4*I*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*si
n(x))^2*x^2+1/4*I*Pi*csgn(I*exp(-I*x))*csgn(ln(x)*sin(x))^2*x^2+1/4*I*Pi*cs
gn(I*(exp(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2*x^2+1/4*I*Pi*csgn(I*exp
(-I*x))*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))*x^2-1/4*I*Pi*csgn(I
*(exp(2*I*x)-1)*ln(x))^3*x^2+1/4*I*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x))*csg
n(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*x^2+1/4*I*Pi*csgn(I*ln(x))*csgn(I*(e
xp(2*I*x)-1)*ln(x))^2*x^2+1/2*ln(ln(x))*x^2+1/2*ln(exp(2*I*x)-1)*x^2-1/4*I*
Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))*x^2+1/
2*Ei(1,-2*ln(x))-polylog(3,-exp(I*x))-polylog(3,exp(I*x))-1/4*I*Pi*x^2+I*x*
polylog(2,-exp(I*x))+I*x*polylog(2,exp(I*x))+1/4*I*Pi*csgn((exp((1+I)*x)-ex
p((1-I)*x))*ln(x))^2*x^2-1/4*I*Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^3
*x^2-1/4*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^3*x^2+1/2*ln(exp(x)
)*x^2-1/4*I*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))
^2*x^2-1/4*I*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x
)-exp((1-I)*x))*ln(x))*x^2+1/4*I*Pi*csgn(ln(x)*sin(x))^3*x^2+1/4*I*Pi*csgn(
I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x)
)^2*x^2+1/4*I*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2
*x^2
```

<u>Maxima</u> [A]
time = 0.60, size = 70, normalized size = 0.82

$$-\frac{1}{4}\left(-i\pi+2\log\left(2\right)\right)x^{2}-\left(\frac{1}{3}i-\frac{1}{3}\right)x^{3}+\frac{1}{2}x^{2}\log\left(\log\left(x\right)\right)+ix\mathrm{Li}_{2}(-e^{(ix)})+ix\mathrm{Li}_{2}(e^{(ix)})-\frac{1}{2}\,\mathrm{Ei}(2\log\left(x\right))-\mathrm{Li}_{3}(-e^{(ix)})-\mathrm{Li}_{3}(e^{(ix)})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*(-I*pi + 2*log(2))*x^2 - (1/3*I - 1/3)*x^3 + 1/2*x^2*log(log(x)) + I*x
*dilog(-e^(I*x)) + I*x*dilog(e^(I*x)) - 1/2*Ei(2*log(x)) - polylog(3, -e^(I
*x)) - polylog(3, e^(I*x))
```

<u>Fricas</u> [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(56) = 112.
time = 0.42, size = 181, normalized size = 2.13

$$-\frac{1}{6}x^{3}+\frac{1}{2}x^{2}\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)-\frac{1}{4}x^{2}\log\left(\cos\left(x\right)+i\sin\left(x\right)+1\right)-\frac{1}{4}x^{2}\log\left(\cos\left(x\right)-i\sin\left(x\right)+1\right)-\frac{1}{4}x^{2}\log\left(-\cos\left(x\right)+i\sin\left(x\right)+1\right)-\frac{1}{4}x^{2}\log\left(-\cos\left(x\right)-i\sin\left(x\right)+1\right)+\frac{1}{2}ix\mathrm{Li}_{2}(\cos\left(x\right)+i\sin\left(x\right))-\frac{1}{2}ix\mathrm{Li}_{2}(\cos\left(x\right)-i\sin\left(x\right))-\frac{1}{2}ix\mathrm{Li}_{2}(-\cos\left(x\right)+i\sin\left(x\right))+\frac{1}{2}ix\mathrm{Li}_{2}(-\cos\left(x\right)-i\sin\left(x\right))-\frac{1}{2}\,\mathrm{log\_integral}\left(x^{2}\right)-\frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)+i\sin\left(x\right))-\frac{1}{2}\,\mathrm{polylog}(3,\cos\left(x\right)-i\sin\left(x\right))-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)+i\sin\left(x\right))-\frac{1}{2}\,\mathrm{polylog}(3,-\cos\left(x\right)-i\sin\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/6*x^3 + 1/2*x^2*log(e^x*log(x)*sin(x)) - 1/4*x^2*log(cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(cos(x) - I*sin(x) + 1) - 1/4*x^2*log(-cos(x) + I*sin(x) +
 1) - 1/4*x^2*log(-cos(x) - I*sin(x) + 1) + 1/2*I*x*dilog(cos(x) + I*sin(x)
) - 1/2*I*x*dilog(cos(x) - I*sin(x)) - 1/2*I*x*dilog(-cos(x) + I*sin(x)) +
1/2*I*x*dilog(-cos(x) - I*sin(x)) - 1/2*log_integral(x^2) - 1/2*polylog(3,
cos(x) + I*sin(x)) - 1/2*polylog(3, cos(x) - I*sin(x)) - 1/2*polylog(3, -co
s(x) + I*sin(x)) - 1/2*polylog(3, -cos(x) - I*sin(x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(x*log(exp(x)*log(x)*sin(x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(x*log(e^x*log(x)*sin(x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\ln\left(\mathrm{e}^{x}\,\ln\left(x\right)\,\sin\left(x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*log(exp(x)*log(x)*sin(x)),x)
```

```
[Out] int(x*log(exp(x)*log(x)*sin(x)), x)
```

3.312 $\int \log(e^x \log(x)\sin(x))\,dx$

Optimal. Leaf size=57

$$\left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)-\text{li}(x)+\frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)$$

```
[Out] (-1/2+1/2*I)*x^2-Li(x)-x*ln(1-exp(2*I*x))+x*ln(exp(x)*ln(x)*sin(x))+1/2*I*p
olylog(2,exp(2*I*x))
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2629, 3798, 2221, 2317, 2438, 2335}

$$\frac{1}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\text{li}(x)+\left(-\frac{1}{2}+\frac{i}{2}\right)x^2-x\log\left(1-e^{2ix}\right)+x\log\left(e^x\log(x)\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] (-1/2 + I/2)*x^2 - x*Log[1 - E^((2*I)*x)] + x*Log[E^x*Log[x]*Sin[x]] - LogI
ntegral[x] + (I/2)*PolyLog[2, E^((2*I)*x)]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2629

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[x*Simpl
ify[D[u, x]/u], x], x] /; ProductQ[u]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^x \log(x)\sin(x)\right)\,dx &= x\log\left(e^x\log(x)\sin(x)\right) - \int \left(x + x\cot(x) + \frac{1}{\log(x)}\right)dx\\ &= -\frac{x^2}{2} + x\log\left(e^x\log(x)\sin(x)\right) - \int x\cot(x)\,dx - \int \frac{1}{\log(x)}\,dx\\ &= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 + x\log\left(e^x\log(x)\sin(x)\right) - \mathrm{li}(x) + 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\\ &= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \mathrm{li}(x) + \int \log\left(1 \right.\\ &= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \mathrm{li}(x) - \frac{1}{2}i\mathrm{Subst}\left(\right.\\ &= \left(-\frac{1}{2}+\frac{i}{2}\right)x^2 - x\log\left(1-e^{2ix}\right) + x\log\left(e^x\log(x)\sin(x)\right) - \mathrm{li}(x) + \frac{1}{2}i\mathrm{Li}_2\left(e^{2i\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.98

$$\frac{1}{2}\left((-1+i)x^2 - 2x\log\left(1-e^{2ix}\right) + 2x\log\left(e^x\log(x)\sin(x)\right) - 2\mathrm{li}(x) + i\mathrm{Li}_2\left(e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]],x]
```

```
[Out] ((-1 + I)*x^2 - 2*x*Log[1 - E^((2*I)*x)] + 2*x*Log[E^x*Log[x]*Sin[x]] - 2*L
ogIntegral[x] + I*PolyLog[2, E^((2*I)*x)])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 583, normalized size = 10.23

method	result
risch	$-\frac{ix\pi \operatorname{csgn}\left(\left(\mathrm{e}^{(1+i)x}-\mathrm{e}^{(1-i)x}\right)\ln(x)\right)^{3}}{2}+\frac{ix\pi \operatorname{csgn}\left(\left(\mathrm{e}^{(1+i)x}-\mathrm{e}^{(1-i)x}\right)\ln(x)\right)^{2}}{2}-\frac{ix\pi \operatorname{csgn}\left(i\ln(x)\left(\mathrm{e}^{(1+i)x}-\mathrm{e}^{(1-i)x}\right)\right)^{3}}{2}+\mathrm{expInte}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x*Pi*csgn(ln(x)*sin(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2+
1/2*I*x*Pi*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln(x))^2-1/2*I*x*Pi*csgn(I*(
exp(2*I*x)-1)*ln(x))^3+1/2*I*x^2+1/2*I*x*Pi*csgn(I*exp(x))*csgn(ln(x)*sin(x
))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))-I*ln(exp(I*x))*ln(exp(2*I*x)-1
)+1/2*I*x*Pi*csgn(I*exp(x))*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))^2+1/2
*ln(exp(x))^2+I*dilog(exp(I*x)+1)-1/2*I*x*Pi+I*ln(exp(I*x))*ln(exp(I*x)+1)+
1/2*I*x*Pi*csgn(I*(exp(2*I*x)-1)*ln(x))*csgn(ln(x)*sin(x))^2+1/2*I*x*Pi*csg
n(I*(exp(2*I*x)-1))*csgn(I*(exp(2*I*x)-1)*ln(x))^2+1/2*I*x*Pi*csgn(I*exp(-I
*x))*csgn(ln(x)*sin(x))^2-x*ln(2)-x*ln(exp(I*x))-I*dilog(exp(I*x))-1/2*I*x*
Pi*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))^3+1/2*I*x*Pi*csgn((exp((1+I)*x)-
exp((1-I)*x))*ln(x))^2-1/2*I*x*Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))
^3-1/2*I*x*Pi*csgn(I*(exp(2*I*x)-1))*csgn(I*ln(x))*csgn(I*(exp(2*I*x)-1)*ln
(x))+ln(ln(x))*x+1/2*I*x*Pi*csgn(ln(x)*sin(x))^3-1/2*I*x*Pi*csgn(I*ln(x)*(e
xp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x))*ln(x))+1/2*I*x*
Pi*csgn(I*ln(x)*(exp((1+I)*x)-exp((1-I)*x)))*csgn((exp((1+I)*x)-exp((1-I)*x
))*ln(x))^2+Ei(1,-ln(x))+1/2*I*x*Pi*csgn(I*exp(-I*x))*csgn(I*(exp(2*I*x)-1)
*ln(x))*csgn(ln(x)*sin(x))
```

Maxima [A]
time = 0.59, size = 43, normalized size = 0.75

$$\frac{1}{2}\,(i\,\pi-2\,\log\left(2\right))x-\left(\frac{1}{2}i-\frac{1}{2}\right)\,x^{2}+x\log\left(\log\left(x\right)\right)-\mathrm{Ei}(\log\left(x\right))+i\,\mathrm{Li}_{2}\left(-e^{(i\,x)}\right)+i\,\mathrm{Li}_{2}\left(e^{(i\,x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/2*(I*pi - 2*log(2))*x - (1/2*I - 1/2)*x^2 + x*log(log(x)) - Ei(log(x)) +
I*dilog(-e^(I*x)) + I*dilog(e^(I*x))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(39) = 78.
time = 0.43, size = 116, normalized size = 2.04

$$-\frac{1}{2}\,x^{2}+x\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)-\frac{1}{2}\,x\log\left(\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(\cos\left(x\right)-i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(-\cos\left(x\right)+i\,\sin\left(x\right)+1\right)-\frac{1}{2}\,x\log\left(-\cos\left(x\right)-i\,\sin\left(x\right)+1\right)+\frac{1}{2}i\,\mathrm{Li}_{2}(\cos\left(x\right)+i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_{2}(\cos\left(x\right)-i\,\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_{2}(-\cos\left(x\right)+i\,\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_{2}(-\cos\left(x\right)-i\,\sin\left(x\right))-\mathrm{log\_integral}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*log(e^x*log(x)*sin(x)) - 1/2*x*log(cos(x) + I*sin(x) + 1) - 1/
2*x*log(cos(x) - I*sin(x) + 1) - 1/2*x*log(-cos(x) + I*sin(x) + 1) - 1/2*x*
log(-cos(x) - I*sin(x) + 1) + 1/2*I*dilog(cos(x) + I*sin(x)) - 1/2*I*dilog(
cos(x) - I*sin(x)) - 1/2*I*dilog(-cos(x) + I*sin(x)) + 1/2*I*dilog(-cos(x)
- I*sin(x)) - log_integral(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x)),x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x)),x, algorithm="giac")
```

```
[Out] integrate(log(e^x*log(x)*sin(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \ln\left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x)),x)
```

```
[Out] int(log(exp(x)*log(x)*sin(x)), x)
```

3.313 $\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$

Optimal. Leaf size=16

$$\text{Int}\left(\frac{\log(e^x \log(x) \sin(x))}{x}, x\right)$$

`[Out] CannotIntegrate(ln(exp(x)*ln(x)*sin(x))/x,x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$$

Verification is not applicable to the result.

`[In] Int[Log[E^x*Log[x]*Sin[x]]/x,x]`

`[Out] Defer[Int][Log[E^x*Log[x]*Sin[x]]/x, x]`

Rubi steps

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx = \int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$$

Mathematica [A]
time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\log(e^x \log(x) \sin(x))}{x} \, dx$$

Verification is not applicable to the result.

`[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x,x]`

`[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x, x]`

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\ln(e^x \ln(x) \sin(x))}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="maxima")
```

```
[Out] -(log(2) + 1)*log(x) + 1/2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*log(x) +
 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*log(x) + log(x)*log(log(x)) +
x + integrate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1), x) - inte
grate(log(x)*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="fricas")
```

```
[Out] integral(log(e^x*log(x)*sin(x))/x, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log\left(e^{x}\log\left(x\right)\sin\left(x\right)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x,x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x))/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x,x, algorithm="giac")
```

```
[Out] integrate(log(e^x*log(x)*sin(x))/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\ln\left(\mathrm{e}^{x}\ \ln\left(x\right)\ \sin\left(x\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x))/x,x)
[Out] int(log(exp(x)*log(x)*sin(x))/x, x)
```

3.314 $\int \frac{\log(e^x \log(x)\sin(x))}{x^2} \, dx$

Optimal. Leaf size=31

$$\mathrm{Ei}(-\log(x)) + \log(x) - \frac{\log(e^x \log(x)\sin(x))}{x} + \mathrm{Int}\left(\frac{\cot(x)}{x}, x\right)$$

```
[Out] Ei(-ln(x))+ln(x)-ln(exp(x)*ln(x)*sin(x))/x+Unintegrable(cot(x)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log(e^x \log(x)\sin(x))}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] ExpIntegralEi[-Log[x]] + Log[x] - Log[E^x*Log[x]*Sin[x]]/x + Defer[Int][Cot
[x]/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(e^x \log(x)\sin(x))}{x^2} \, dx &= -\frac{\log(e^x \log(x)\sin(x))}{x} + \int \frac{1+\cot(x)+\frac{1}{x\log(x)}}{x} \, dx \\
&= -\frac{\log(e^x \log(x)\sin(x))}{x} + \int \left(\frac{1+\cot(x)}{x} + \frac{1}{x^2\log(x)}\right) dx \\
&= -\frac{\log(e^x \log(x)\sin(x))}{x} + \int \frac{1+\cot(x)}{x} \, dx + \int \frac{1}{x^2\log(x)} \, dx \\
&= -\frac{\log(e^x \log(x)\sin(x))}{x} + \int \left(\frac{1}{x} + \frac{\cot(x)}{x}\right) dx + \mathrm{Subst}\left(\int \frac{e^{-x}}{x} \, dx, x, \log(x)\right) \\
&= \mathrm{Ei}(-\log(x)) + \log(x) - \frac{\log(e^x \log(x)\sin(x))}{x} + \int \frac{\cot(x)}{x} \, dx
\end{aligned}$$

Mathematica [A]
time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{\log(e^x \log(x)\sin(x))}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2,x]
```

```
[Out] Integrate[Log[E^x*Log[x]*Sin[x]]/x^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\ln \left(\mathrm{e}^{x} \ln \left(x\right) \sin \left(x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

```
[Out] int(ln(exp(x)*ln(x)*sin(x))/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*(Ei(-log(x)) + conjugate(Ei(-log(x)))) - 2*x*integrate(sin(x)/(x*cos
(x)^2 + x*sin(x)^2 + 2*x*cos(x) + x), x) + 2*x*integrate(sin(x)/(x*cos(x)^2
 + x*sin(x)^2 - 2*x*cos(x) + x), x) + 2*x*log(x) + 2*log(2) - log(cos(x)^2
+ sin(x)^2 + 2*cos(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*lo
g(log(x)))/x
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="fricas")
```

```
[Out] integral(log(e^x*log(x)*sin(x))/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\log \left(e^{x} \log \left(x\right) \sin \left(x\right)\right)}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(x)*ln(x)*sin(x))/x**2,x)
```

```
[Out] Integral(log(exp(x)*log(x)*sin(x))/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(x)*log(x)*sin(x))/x^2,x, algorithm="giac")
[Out] integrate(log(e^x*log(x)*sin(x))/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\ln\left(\mathrm{e}^x\,\ln\left(x\right)\,\sin\left(x\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(log(exp(x)*log(x)*sin(x))/x^2,x)
[Out] int(log(exp(x)*log(x)*sin(x))/x^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```